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A BRIEF SURVEY OF TRAJECTORY, GUIDANCE, AND PROPULSION ASPECTS OF ORBITAL RENDEZVOUS
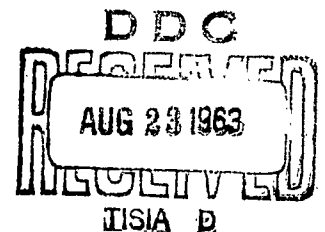
H. B. Schechter

The RAND Corporation

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H. B. Schechter

The RAND *Corporation*

PREFACE

This Memorandum summarizes the result of a literature survey deal-
ing with the problem of orbital rendezvous, which was made as part of
Project RAND continuing studies of orbital and flight mechanics. Papers
which appeared in the open literature prior to mid-1962 were scanned.
Of these, a small but fairly representative fraction were singled out
for inclusion here. The material presented here in condensed form
should be of interest to Air Force personnel concerned with space mis-
sions involving orbital transfer and rendezvous.

SUMMARY

This Memorandum summarizes some aspects of the problem of orbital rendezvous that have emerged from a survey of the open literature. The papers studied are discussed briefly, and some of the interesting results and data are compared.

Most of the papers can be grouped into either of two classes: The first class tackles the problem from the point of view of impulsive Keplerian orbital transfers; the second class analyzes the terminal portion of the rendezvous maneuver, or more specifically, the selection of thrusting and guidance laws required to insure a soft contact between the maneuverable interceptor and the target satellite.

The survey pointed out the need for a more general parametric study of the terminal phase of orbital rendezvous; in particular, optimal guidance laws.

ACKNOWLEDGMENT

CONTENTS

I. INTRODUCTION

In the last few years a profusion of papers dealing with specific aspects of the problem of satellite orbital rendezvous have appeared in the literature. Most of the papers can be grouped quite generally into two separate though overlapping classes, depending on the portion of the intercept path with which the paper is primarily concerned.

One class of papers tackles the problem essentially from the point of view of impulsive Keplerian orbital transfers; this class is concerned with the influence of launch-point location, destination-orbit geometry, satellite position in orbit, etc., on the impulsive-velocity requirements of the ascent trajectory.

The papers in the second class are somewhat more analytical in nature and are devoted to an analysis of the terminal portion of the rendezvous maneuver, or more specifically, the selection of thrusting and guidance laws required to insure a soft contact between the maneuverable interceptor and the target satellite. Within each class of papers a great number of different basic assumptions for the computation of arbitrarily selected numerical cases have been made. A host of new techniques have been generated, or modifications of a detailed nature have been introduced into the existing methods of solution. As a result, it becomes extremely difficult to assess the relative merits of the various approaches and to compare the data generated on a common basis.

This study will attempt to indicate briefly some of the more important avenues of approach followed by previous investigators and to present some of the results and conclusions which have emerged from a survey of a fairly representative cross section of papers published in the open literature. On the basis of the foregoing it should then become possible to decide what, if any, additional work might be required in this area in order to round out the picture and increase our understanding of the problem.

II. BREAKDOWN OF RENDEZVOUS PROBLEM

For convenience the rendezvous problem is usually broken down into a number of separate but interrelated components which can be treated either simultaneously or separately and pieced together at the end, depending on the generality or accuracy of the results desired.

Starting out with the terminal conditions, we have a satellite moving in an orbit of known geometry and orientation in space; the actual position of that satellite in its orbit is predictable at any instant of time within some degree of accuracy. Soft contact with that satellite of an interceptor launched from a predetermined site is desired. This contact is possibly subject to some additional constraints, such as time or place of rendezvous, or maximum thrust available, which might have to be imposed.

The ascent trajectory of the interceptor is assumed to contain some or all of the following components:

1. Impulsive launch into a ballistic intercept trajectory, or programmed powered ascent through the atmosphere and beyond to a properly selected thrust-cutoff point.

2. Coast up to an intermediate parking orbit and injection into it, or direct ascent to some suitable point in the neighborhood of the destination orbit.

3. Initiation of a radar search pattern leading to the acquisition of the target satellite, followed by a powered-flight phase as commanded by the particular guidance law selected to control the rendezvous mission.

The impulsive transfer presupposes perfect position-matching at destination and an impulsive cancellation of the relative closing velocity. The finite-magnitude thrust program can consist of either intermittent periods of thrust separated by intervals of coasting or continuous thrusting of constant or variable magnitude and/or orientation.

Phases 1 and 2 are generally investigated in a geocentric coordinate system, while Phase 3 is usually analyzed in a target-centered coordinate system that can be either rotating or inertially stabilized.

III. SATELLITE ORBITS

The target orbits employed most frequently in the analysis of rendezvous maneuvers were circular orbits at an altitude of 300 mi.[*] Whenever elliptical orbits were treated, no uniformity in the choice of orbital parameters prevailed, different numerical values being chosen by the different investigators. Both coplanar and out-of-plane intercepts have received wide attention.

One class of satellite orbits of particular interest (if rendezvous with friendly satellites is contemplated) are the circular rendezvous compatible orbits (RCO) first investigated by Swanson and Petersen.[1,2] The altitudes and inclinations of RCO's have been chosen so that the ratio of the number of satellite revolutions N to the synchronous number m of effective rotational periods of the earth (i.e., rotational periods measured relative to the satellite orbital plane, which regresses because of the earth's oblateness) is an integer. For m = 1, N will thus be the integral number of satellite revolutions elapsed between two consecutive passes in the same direction over the same fixed point on the earth. RCO's can be selected which allow two rendezvous daily from a given launch base. Of special interest is the AMR-launched RCO which has N/m = 15 and which crosses the launch base again in the opposite direction slightly more than two orbital periods later. This orbit has an inclination i = 31.03 deg to the equator and is at an altitude h = 262.08 n mi. The above values have to be modified slightly if account is taken of the fact that an interceptor, also launched from AMR, does not make rendezvous exactly overhead but slightly downrange because of the ground range covered during ascent to orbital altitude. Perturbations, principally due to drag, would tend to destroy this periodicity unless some corrective measures were taken. The investigators have found that a thrusting program to counteract the drag dissipation and maintain orbital stability is quite inexpensive and can be accomplished by means of an intermittent limit-cycle type of program. For a satellite with a weight-to-area ratio W/A =

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[*]Statute miles unless otherwise specified.

100 lb/ft^2, a representative value for the characteristic velocity expenditure V_{CH} is around 32 ft/sec each year. Maximum displacements from the nominal position in orbit encountered during the limit-cycle motion and characteristic velocity requirements for other values of W/A were investigated in a Northrop study[3] and are shown in Figs. 1, 2, and 3. The regression rate of the line of nodes for various satellite orbital altitudes and inclinations is presented in Fig. 4.
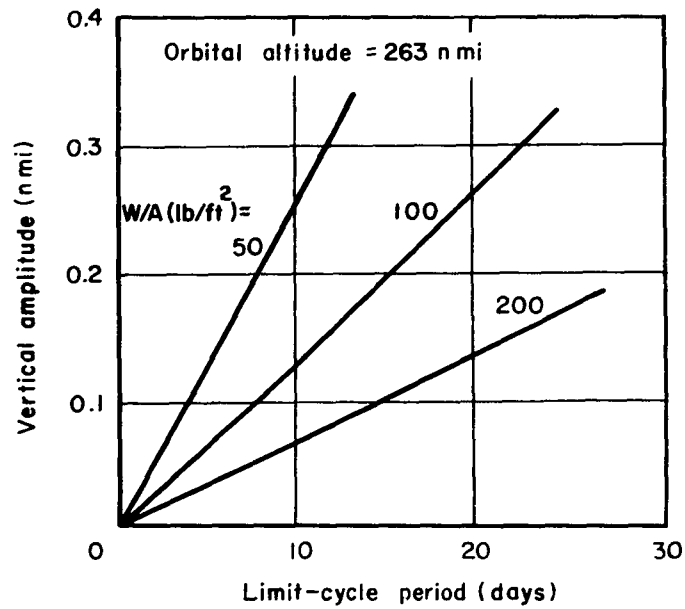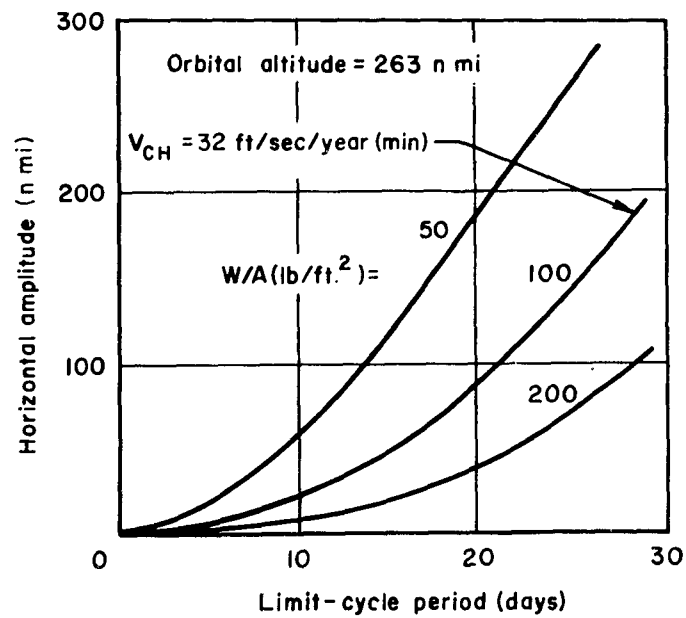
Fig. 1 — Vertical limit-cycle amplitude

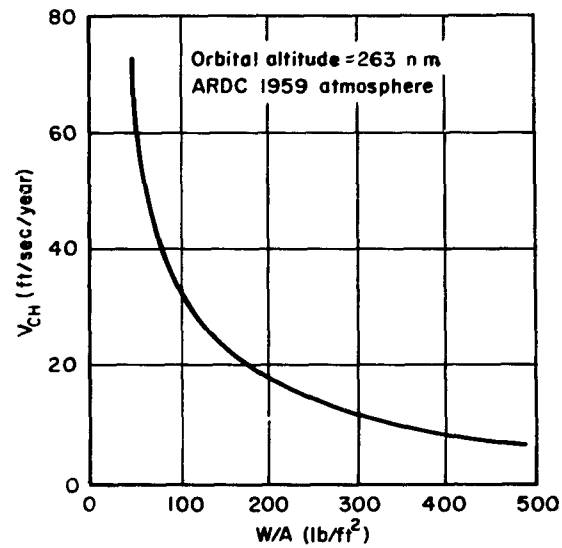
Fig. 2 — Horizontal limit-cycle amplitude

Fig. 3—Characteristic velocity per year against
area loading for N/m=15 RCO
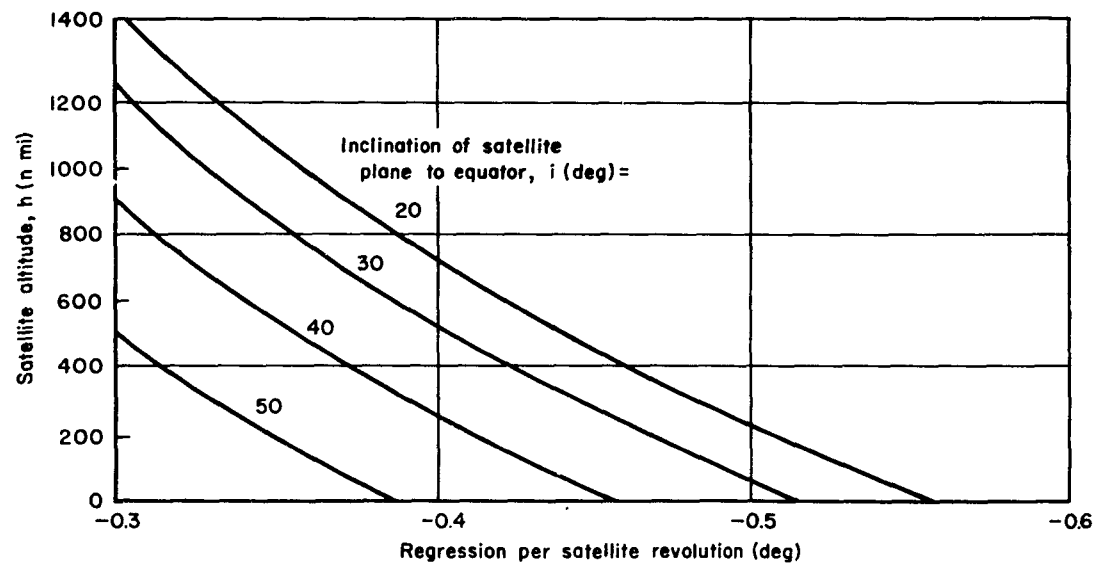

Fig. 4—Regression rate of an earth satellite in a circular orbit

IV. BOOST PHASE

The impulsive-transfer trajectories and minimal velocity require-
ments for launching an interceptor into a noncoplanar circular satel-
lite orbit were investigated in a study by Carstens and Edelbaum.[4]
The geometry of the transfer is indicated in Fig. 5. The velocity re-
quirements (nondimensionalized with respect to circular velocity at
the launch radius r_1) for optimal transfer trajectories from an ini-
tial radial distance r_1 to a final distance r_2 are shown in Fig. 6.
The angle β is used as a parameter (β = angle included between the
launch radius r_1 and the plane of the destination orbit).

When a vehicle is launched into a circular orbit from a launch
point not contained in the orbital plane, the vehicle will arrive at
the orbital altitude with a velocity vector inclined to the destina-
tion plane. Since velocity-orientation corrections are rather expen-
sive, it has been frequently suggested that a transfer trajectory con-
tained in a plane having the least inclination i to the plane of the
destination orbit will tend to minimize the velocity penalties. A
transfer trajectory of the above kind will subtend a range (or trans-
fer) angle of ϕ = 90 deg with the earth's center, as can be easily de-
termined by referring to Fig. 5. Once a specific orbital plane and
launch site have been selected and oblateness effects neglected, then
ϕ_{min} (i.e., β) remains a fixed quantity. From spherical trigonometry

$$\frac{\sin \phi}{\sin \frac{\pi}{2}} = \frac{\sin \beta}{\sin i}$$

so that

$$\sin i = \frac{\sin \beta}{\sin \phi}$$

and i_{min} is approached as $\phi \to \frac{\pi}{2}$.

While the above reasoning is valid as far as the terminal velo-
city increment is concerned, it turns out that the trajectories so gen-
erated are not optimal from an over-all point of view.[4] Figure 7

8

Note: Dashed lines indicate
projections of
trajectories

—Launch point

—Circular target orbit

Transfer ellipse —

—Point of interception

—Intersection of
transfer plane
with sphere of
radius r_2

r_2

r_1

ϕ

β

90°

90°

90°

90°

Direction of satellite in target orbit
moving away from launch point after
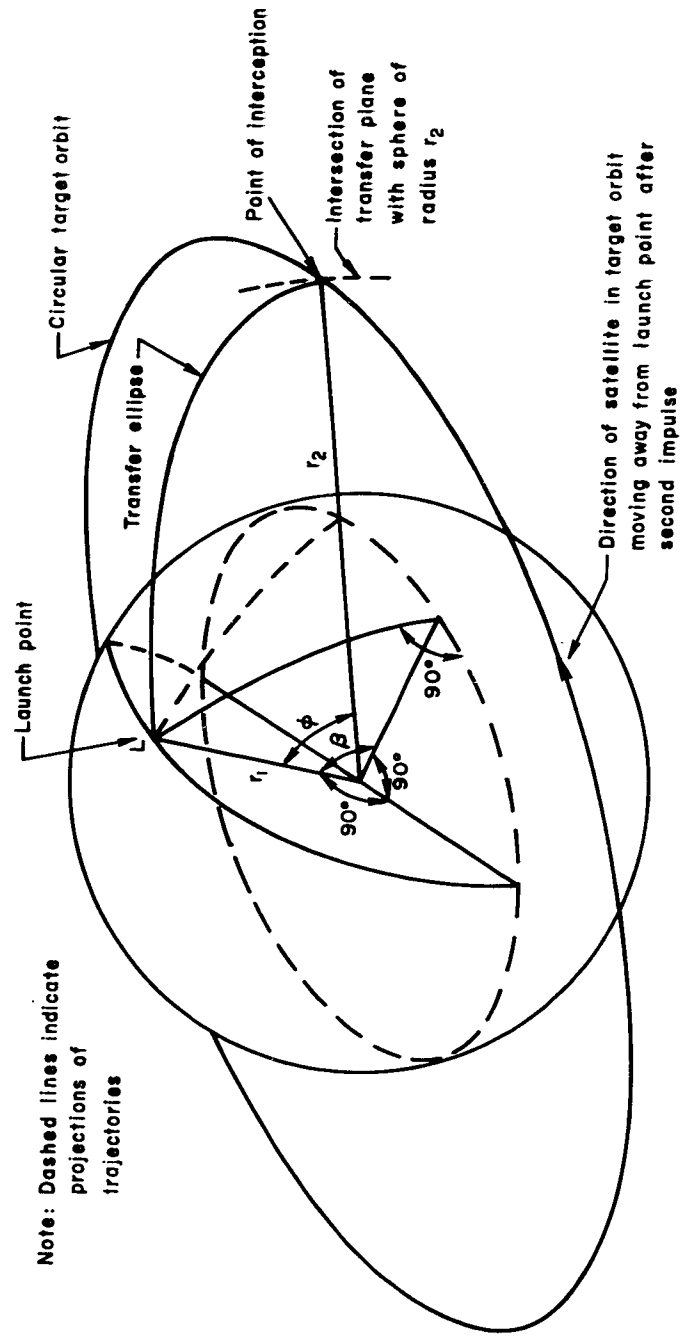second impulse
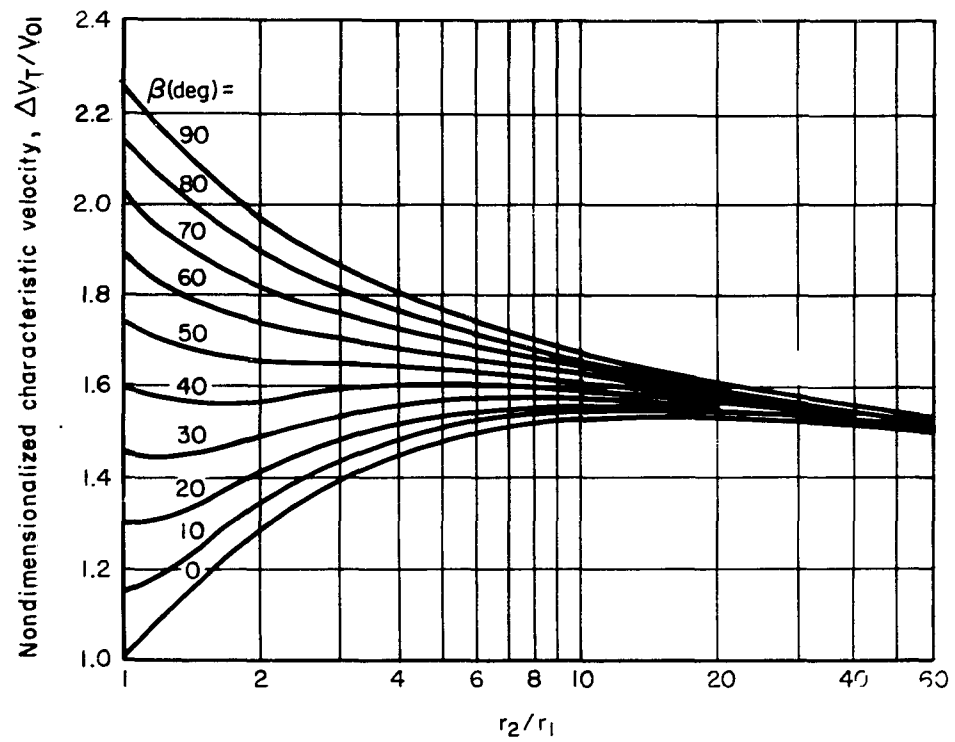
Fig. 5 — Two-impulse—trajectory geometry

9

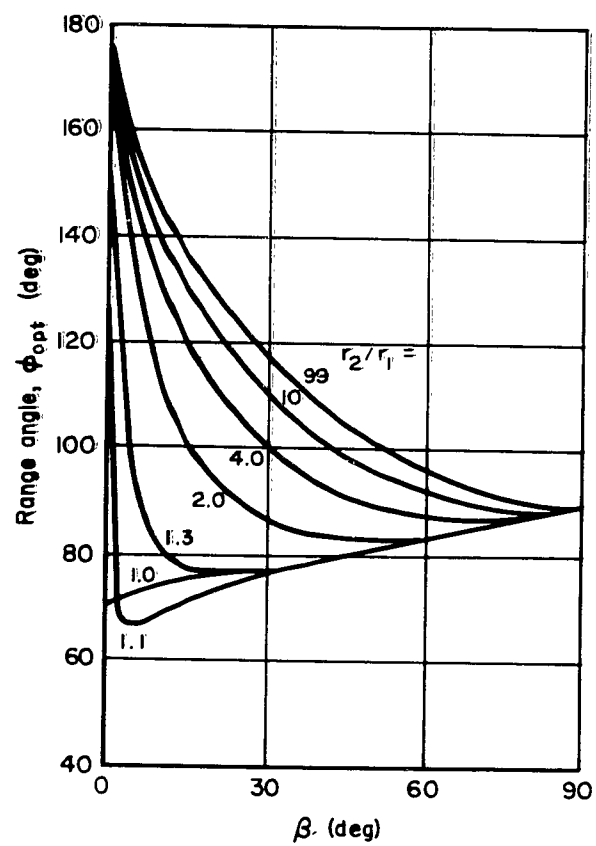Fig. 6— Minimum characteristic velocity requirements of two-impulse trajectory

Fig. 7— Range angles for optimum
two-impulse trajectories

shows the actual variation of ϕ_{opt} with β for various values of r_2/r_1. The angle ϕ_{opt} denotes the range angle of the optimum transfer curves (the least total V_{CH} at both ends of the trajectory). It is apparent that the optimum transfer trajectories, particularly those which occur for low values of r_2/r_1, are not contained in transfer planes for which $i = i_{min}$, except when the angle β approaches 90 deg. In most other cases, range angles smaller than 90 deg are called for.

For impulsive Hohmann-type transfers between inclined circular orbits, small velocity savings can be achieved if the transfer plane is inclined with respect to the plane of the departure orbit by some small angle i_T. The results of some studies by Horner and Silber[5] are shown in Figs. 8 and 9. It appears that the maximum velocity savings which can be achieved by utilizing the intermediate transfer plane are on the order of 3 per cent.

For ascent into an orbit of $h > 150$ mi, the booster burnout conditions denote the beginning of the coast phase.

These conditions vary over a relatively wide range of values, depending on the type of mission, position of satellite in orbit, time available to rendezvous, etc. For ascent into a 300-mi circular orbit, for instance, the following set of values for the burnout conditions seems to have been widely used and can be considered to be fairly representative:

$t_{bo} \cong 275$ sec (time of powered ascent)

$V_{bo} \cong 26,000$ ft/sec (geocentric velocity of vehicle)

$\gamma_{bo} \cong 2.4$ deg (inclination of velocity vector to local horizontal)

$h_{bo} \cong 60$ mi (altitude)

$X_{bo} \cong 530$ mi (ground range covered during powered portion of the ascent)

If the design burnout conditions have been exactly attained, the coasting interceptor will approach the satellite in accordance with
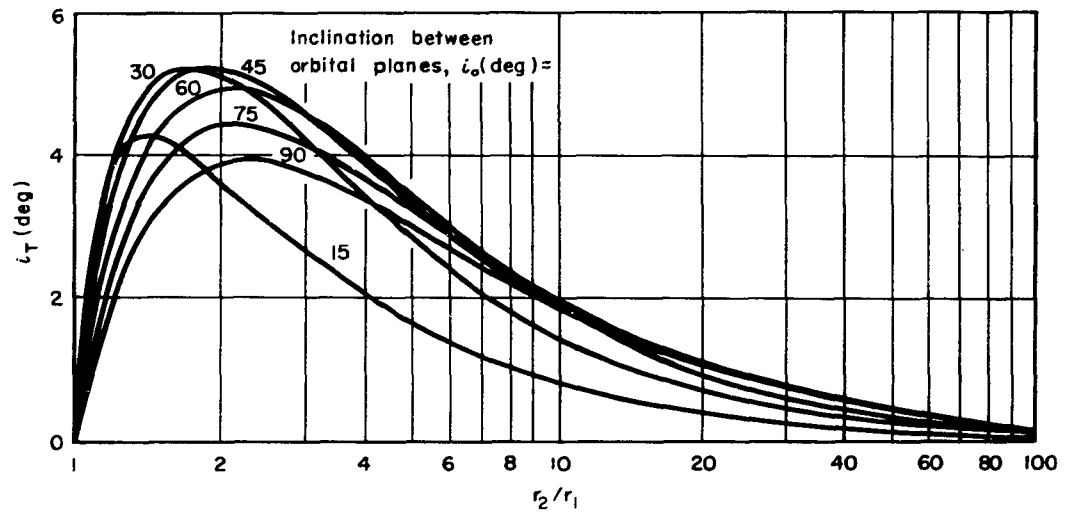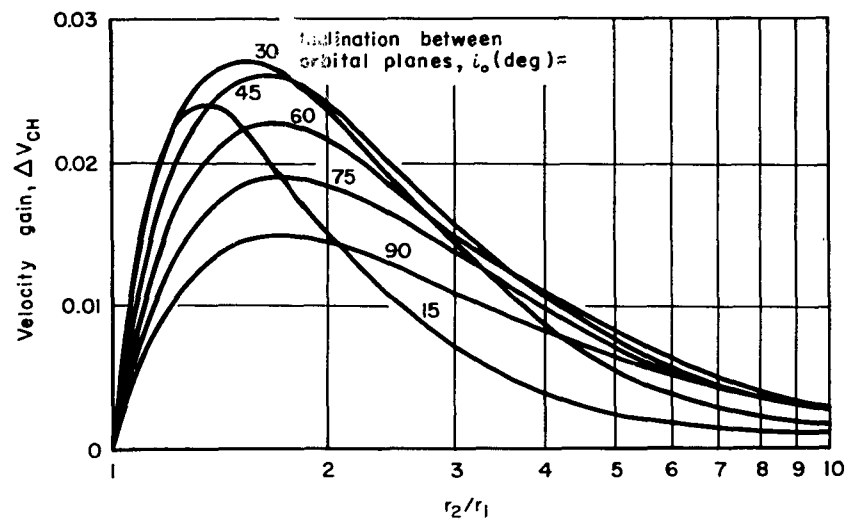
Fig. 8 — Optimum transit-plane inclination

Fig.9— Velocity gain due to inclined transfer
nondimensionalized with respect to initial
circular velocity

known laws of variation of range rate and close in just before intercept with a nearly constant value of the final impact velocity. The velocity and elevation-angle requirements at a burnout altitude h = 60 mi needed to intercept a satellite located in a 300-mi orbit with various preselected closing speeds were computed by Eggleston and Beck.[6] The results are shown in Figs. 10 and 11. While the figures themselves are self-explanatory, a few interesting results concerning permissible launch-delay times can be extracted from them. For a chosen value of the relative closing velocity ΔV_{REL} at impact, a value of $\Delta\theta_s$ ($\Delta\theta_s$ is the central angle between satellite and interceptor at burnout and is positive when satellite is ahead of interceptor), or the equivalent ground range $r_e\Delta\theta_s$, can be read off for each available burnout velocity V_{bo}. Increasing V_{bo} increases $\Delta\theta_s$ up to a maximum lead angle of around 8 deg. Approximate delay times can be computed by dividing the projected ground range by an average satellite ground speed of approximately 5 mi/sec. Thus, if a ΔV_{REL} of 1000 ft/sec is specified, a maximum delay time of around 2 min can be tolerated. If negative values of γ_{bo} have to be excluded because of aerodynamic considerations, it is seen that no intercept trajectories to a 300-mi station leading the burnout point by more than 8 deg are possible, even if no limitations exist on the magnitude of the burnout velocity. It is interesting to note that in the majority of cases, the position of the station at burnout is ahead of the interceptor.

The investigators have also examined rendezvous requirements with a satellite in an elliptical orbit having its apogee at 500 mi and perigee at 100 mi.[6] These two altitudes were selected because they yield an elliptical orbit with a semimajor axis equal to that of the previously discussed 300-mi circular satellite orbit. The results are presented in Figs. 12 and 13. The following conclusions can be drawn from these curves:

1. Station position at burnout is now equally divided between lead and lag.

2. Intercepts which occur when the station is at a true anomaly angle of 90 deg provide less delay times than intercepts which occur at larger distances from perigee.

Fig. 10 — Launch velocity and flight-path-angle variation
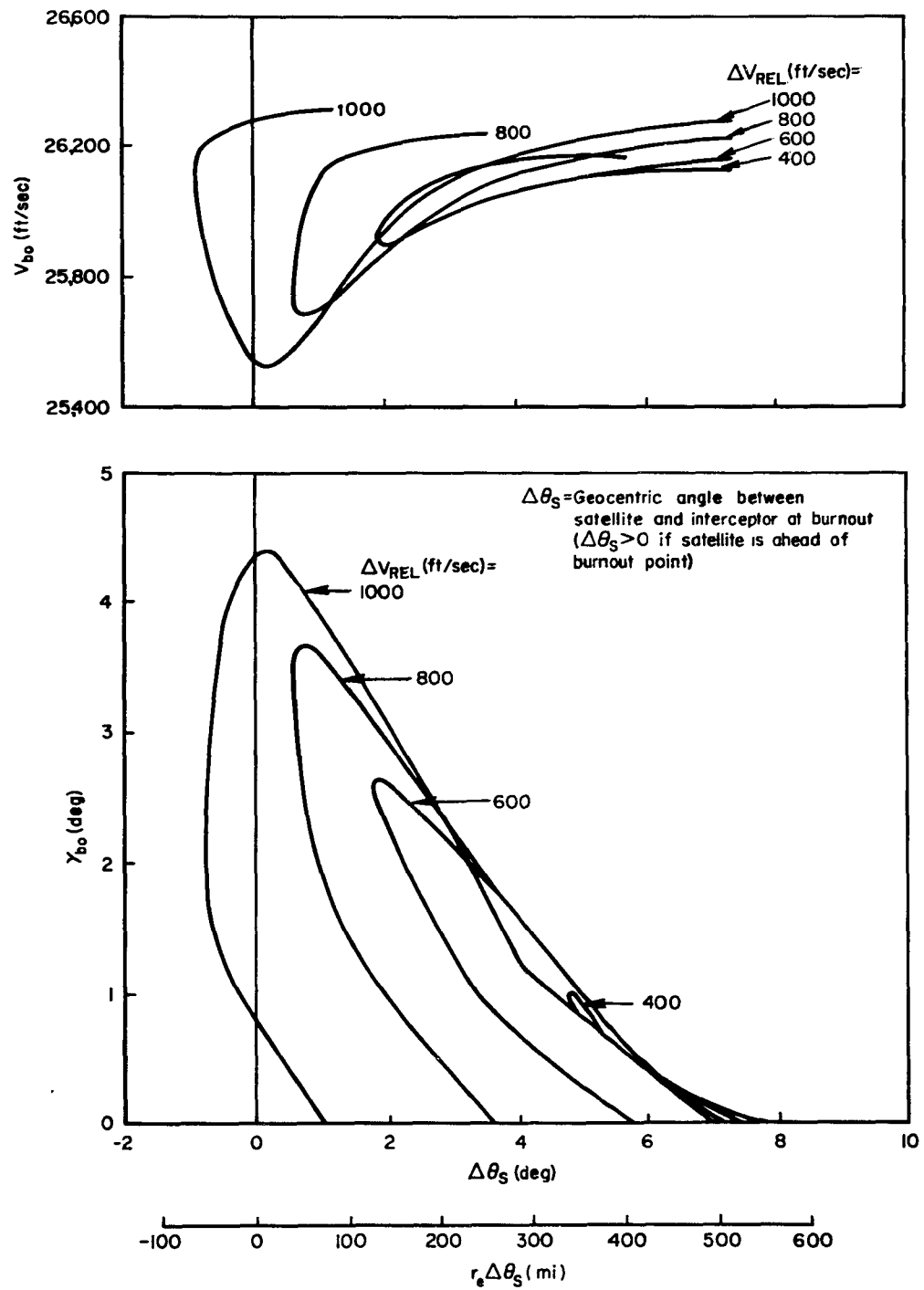with initial satellite position (circular orbit)

Fig. 11 — Launch velocity and flight-path-angle variation with initial satellite position (circular orbit)

16
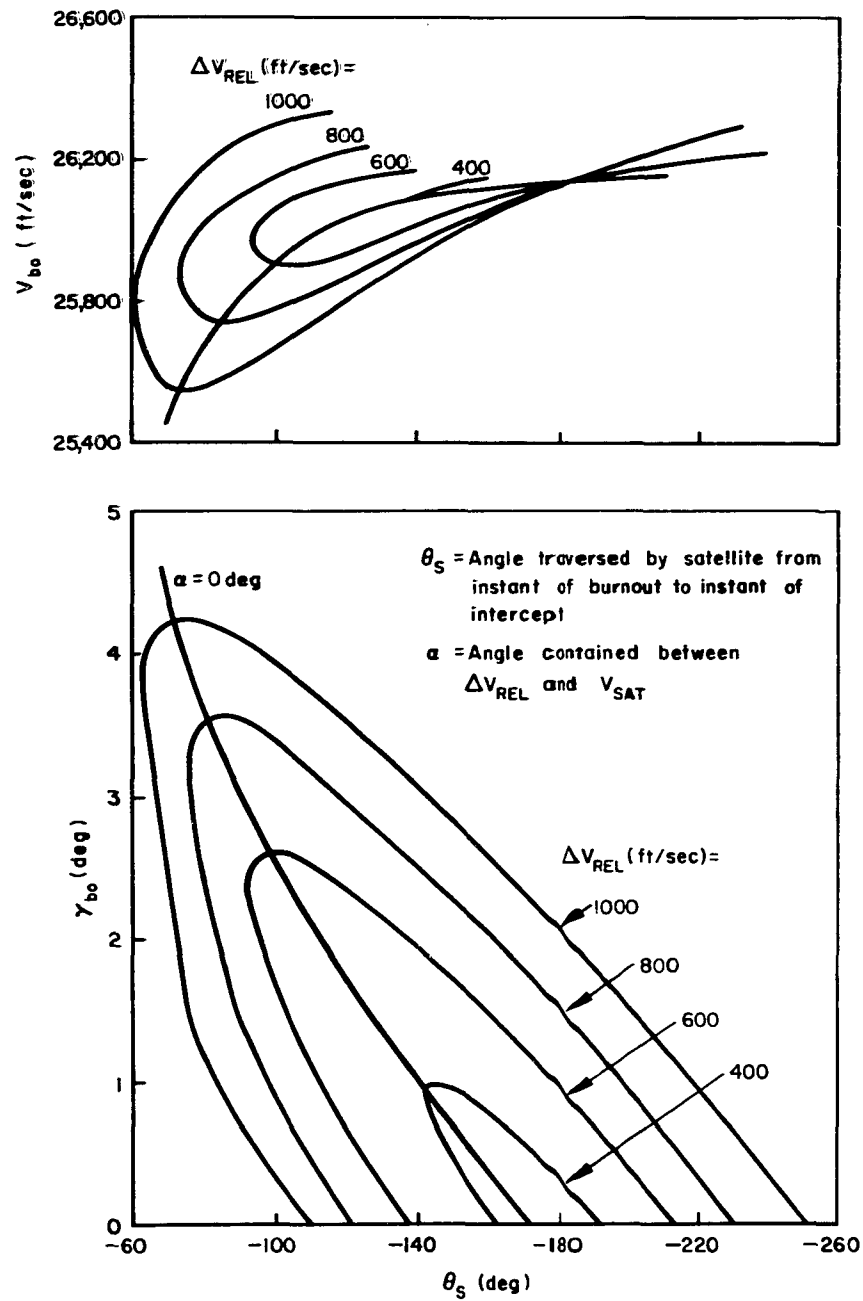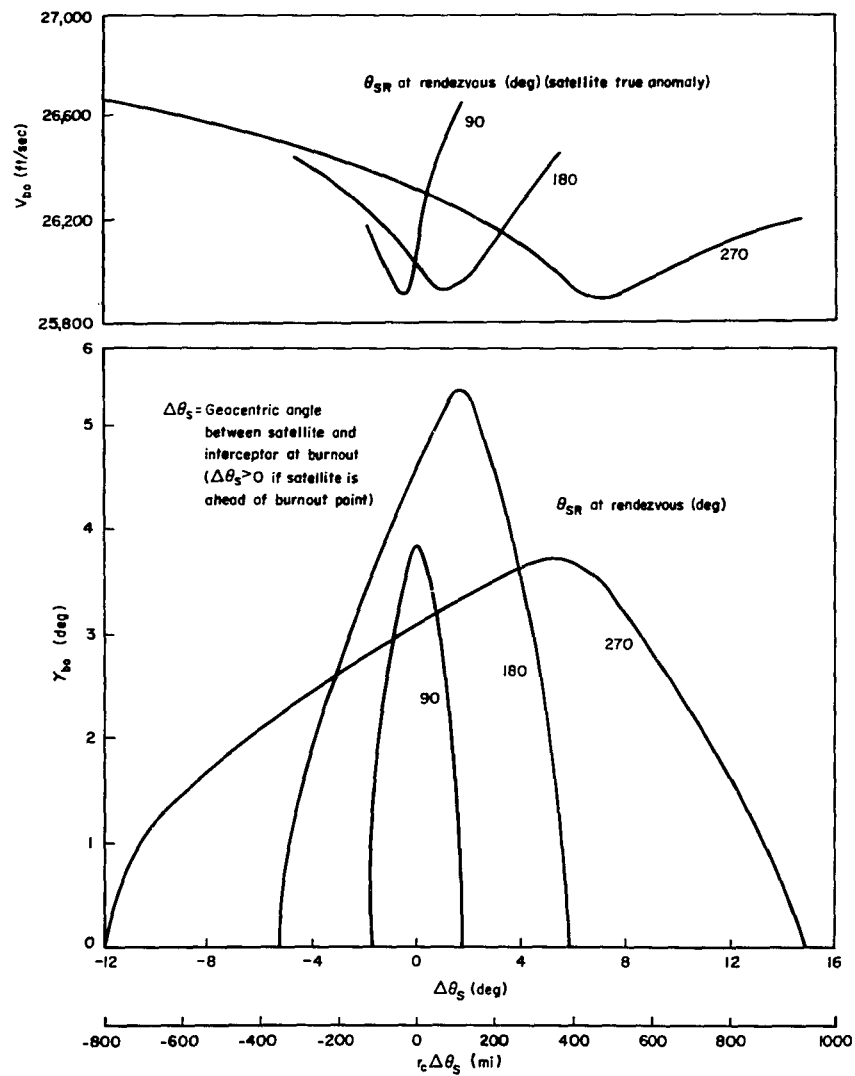
Fig. I2 — Launch-velocity and flight-path-angle variation with
initial satellite position (elliptical orbit)
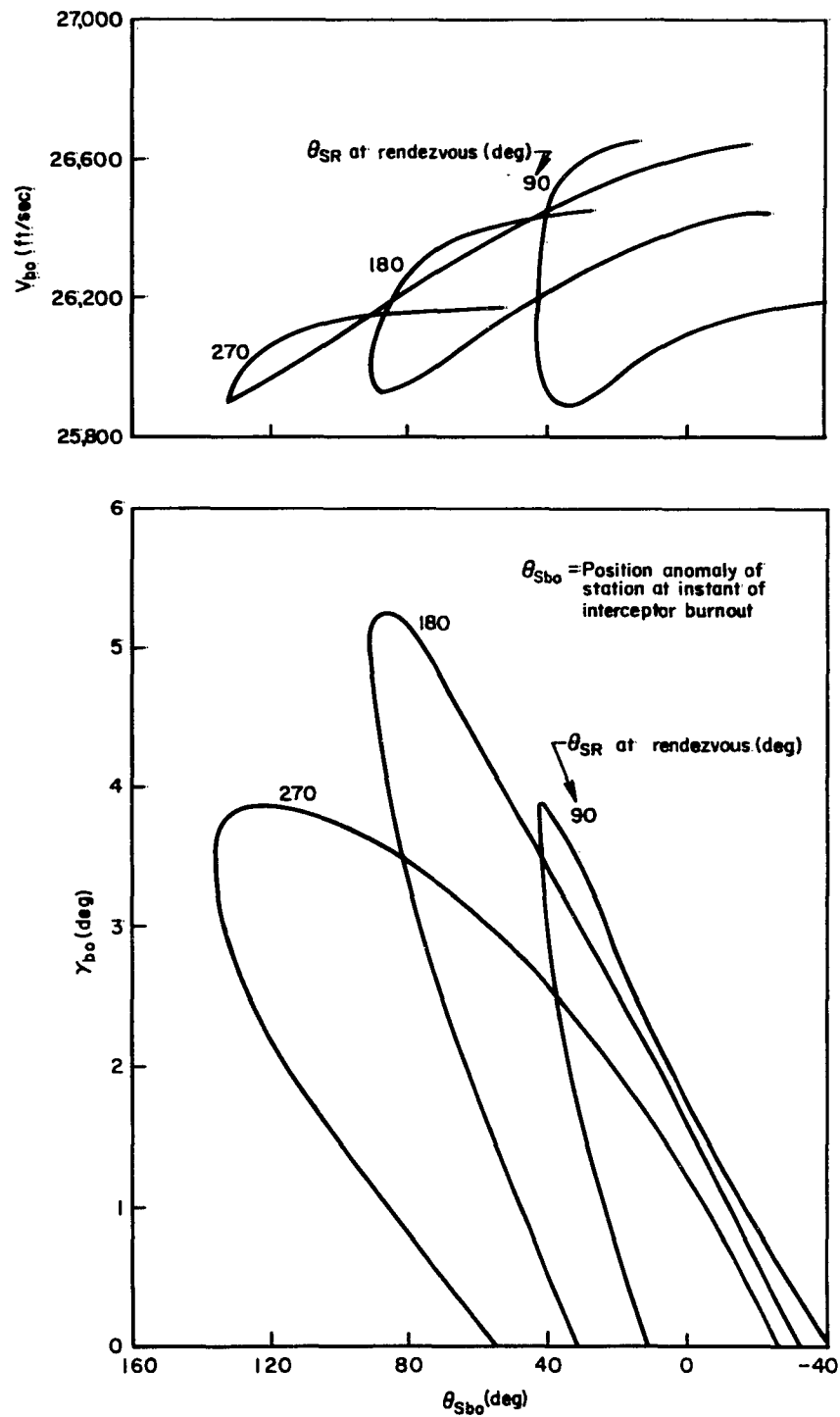
Fig.13 — Launch-velocity and flight-path-angle variation
with initial satellite position (elliptical orbit)

3. Higher values of V_{bo} are now generally needed, and the velocity requirements rise more sharply as one deviates from optimum conditions.

In addition to the above specific conclusions, it is necessary to keep in mind that launch delays tend to alter the propulsive burdens imposed on the power plants at either one or the other terminal of the coast phase. As a rule of thumb, later launches require higher burnout velocities; but they also delivery the interceptor to the orbital altitude with a higher geocentric velocity, thereby decreasing the relative velocity of intercept and easing the task of the terminal-propulsion power plant. Conversely, earlier launches tend to place a heavier burden on the terminal-propulsion device. All these factors must be carefully considered in a decision on delay requirements.

V. COAST PHASE

When the orbital altitude of the satellite is low, no coast phase
is required; the interceptor proceeds along a powered trajectory all
the way into orbit. But in purely impulsive rendezvous maneuvers to
orbits of higher altitudes, the interceptor follows the coasting arc
until satellite position coordinates are matched. In actuality, the
finite time interval needed for thrusting in order to bring the vehi-
cle up to orbital speed makes it necessary to locate the terminal point
of the coasting arc--the so-called aim point--slightly above and ahead
of the satellite position. The exact location of the aim point varies
from one case to the other, depending on the specific rendezvous guid-
ance law employed for each mission and on the characteristics of the
engine.

Many of the rendezvous guidance laws proposed in the past have
been based on variations and adaptations of the proportional naviga-
tion law that was first introduced as a steering program for intercep-
tor missiles. This law calls for a thrusting program aimed at cancel-
ing the angular velocity of the line of sight (LOS) or relative range
vector from the satellite to the interceptor, thus insuring that the
approach path will lead to a collision. This condition is fulfilled
when the velocity components of the satellite and vehicle normal to
the LOS are equal in magnitude and pointing in the same direction.
The relative velocity between the two bodies will then be oriented
along the LOS, and this will result in an eventual collision. An ap-
propriate braking program which controls the range rate would have to
be employed so as to reduce gradually the closing velocity to zero be-
tween the two bodies. As soon as the terminal guidance command takes
over and controls the approach, a nonrotating observer located on board
the satellite would see the path of the interceptor as a straight line.
This is shown in the first sketch on page 20.

When proportional navigation is used, the coast path selected
must be such as to avoid large values of ω_{LOS} (angular velocity of
the LOS) at the aim point, the cancellation of which would necessitate
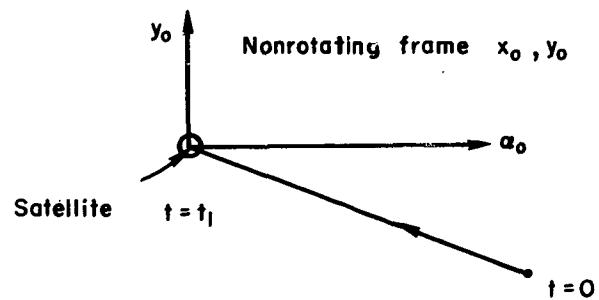
the use of power plants with large initial thrust-to-mass ratios. One possible aim point which assures a $\omega_{LOS} = 0$ at the start of rendezvous is shown below in geocentric coordinates.
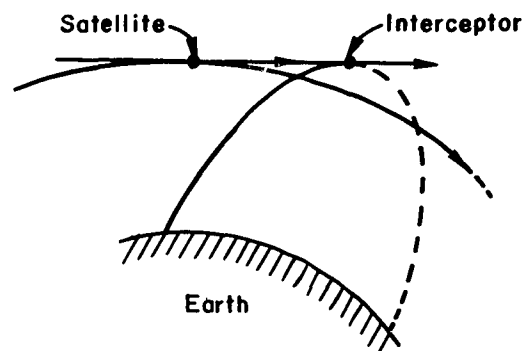
For the above starting condition, neither vehicle possesses a velocity component normal to the LOS.

VI. PARKING ORBITS

The use of intermediate parking orbits is sometimes necessary if delay times anticipated during launching are longer than the ones permissible for ground takeoffs, or if available velocity capabilities are inadequate to cope with an unfavorable launch configuration. Some of the delays incurred might also give rise to acceleration load factors in a direct ascent path which may exceed those considered safe for manned vehicles. If the launching cannot easily be postponed to another day, the use of intermediate parking orbits becomes necessary.

While a general comparison of the use of parking orbits as opposed to direct ascents for the complete spectrum of all the variables involved is not available at the present time, specific comparisons based on various simplifying assumptions have been made by some authors. The region of applicability of these results and the conclusions which can be drawn are of necessity rather limited. Carstens and Edelbaum[4] have taken a quick look at the problem by comparing their optimum two-impulse transfer trajectories mentioned earlier with three-impulse indirect transfers. The indirect transfers involved entry into a circular orbit at essentially the original launch radius, followed by the inclined Hohmann transfers of Ref. 5 to the destination orbit. The results are shown in Fig. 14. From this figure one gathers that except for low-inclination angles β and high orbital-radii ratios, the direct-ascent velocity requirements are lower than the corresponding three-impulse requirements. It should be noted that in the present three-impulse transfer a circular parking orbit located near the earth's surface was arbitrarily chosen.
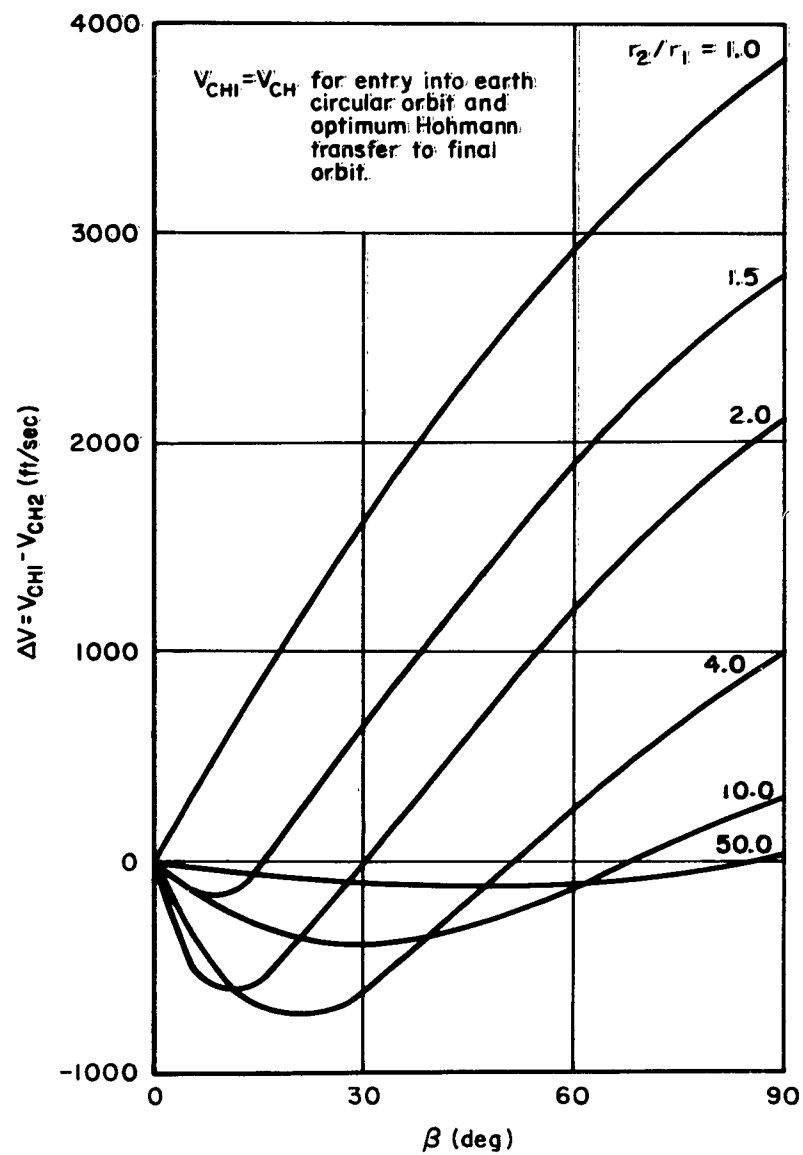
Fig. 14 — Difference in required characteristic velocities between indirect (V_{CHI}) and direct (V_{CH2}) trajectories

VII. TERMINAL RENDEZVOUS PHASE

AIM POINT

The terminal portion of the rendezvous maneuver is generally de-
fined as that phase in which the vehicle is close to the orbital alti-
tude of the satellite at the start but separated anywhere from 50 to
100 mi from it. The relative velocity between interceptor and satel-
lite may be anywhere from 200 to 1000 ft/sec.

The starting point (or aim point) best suited to a given system
is preselected at launch. This point is rarely reached in actuality
because of the various errors and perturbations which accumulate in
the previous phases; it becomes the task of the guidance system to
compensate and correct for those deviations. The factor common to
most of the papers dealing with this portion of the flight path is the
attempt to select a suitable analytical expression for the thrust con-
trol that will insure an eventual soft contact with the satellite.
Ground-controlled transfer from the aim-point error envelope (the n-
dimensional surface surrounding the nominal aim point) to the target
satellite, be it impulsive or otherwise, is not very realistic at the
ranges contemplated, because of the instrument inaccuracies in the meas-
urement of the differential corrections needed. Furthermore, such guid-
ance schemes require an accurate knowledge of the orbital elements of
the satellite, which might not always be available on short notice at
a given instant of time. Most guidance schemes therefore use relative-
range and range-rate information measured on board the interceptor.
This has the advantage that the exact orbit of the satellite does not
have to be known in order to perform the mission.

PROPORTIONAL-NAVIGATION GUIDANCE

As mentioned earlier, the scheme which has received the widest
attention is based on the proportional-navigation law, which can be
written quite generally in the form

$$\bar{\omega}_V = S \bar{\omega}_{LOS} \qquad S > 1 \tag{1}$$

This equation states that the thrust program must impart to the relative-velocity vector $\overline{V}$ an angular velocity which is a preselected multiple of ω_{LOS}, the angular velocity of the line of sight (LOS). This will cause the vector $\overline{V}$ to rotate towards the range vector $\overline{R}$, thereby forcing the vehicle onto a direct collision course. The lead angle L will be continuously reduced in the process.
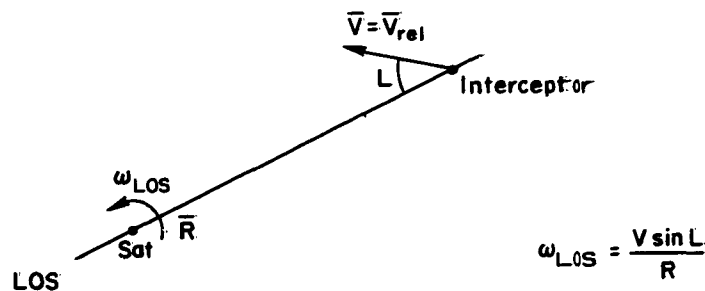
In addition to Eq. (1) or some equivalent expression for the proportional-navigation law, provision for a braking program must be made in the guidance scheme to decrease the range rate, $V \cos L$, to some very small value, or zero, at impact. One major disadvantage of the proportional-navigational scheme is the high starting-thrust requirement and large throttling ratios needed if the initial value of ω_{LOS} is not vanishingly small. Some of the examples show that throttling requirements in excess of 100:1 are not uncommon.

Sears and Felleman[7] define a variable command acceleration $\overline{a}_{com}$ by means of a proportional-navigation law of the form

$$\overline{a}_{com} = S_1 \overline{i}_R \left[\dot{R} + k \sqrt{R} \right] + S_2 \left(\overline{\omega}_{LOS} \times \overline{R} \right) \qquad (2)$$

The second term of Eq. (2) serves to nullify the angular rate of the LOS, while the first term ensures that closing velocity $\dot{R}$ goes to zero with decrease in range R; S_1 and S_2 are suitably chosen constant system sensitivities.

For the initial condition shown in Fig. 15, flight path and time variation of thrust acceleration $\bar{a}_{com}$ are presented in Figs. 16 and 17. In Fig. 17 it is observed that the thrust acceleration remains at a constant level after the first 40 sec have elapsed. This constant-acceleration program is a consequence of the $\dot{R}$-versus-R relationship chosen for the first term of Eq. (2). Kidd and Soule[8] have shown that once an intercept trajectory has been established, the requirement that R be proportional to $\dot{R}^2$ during the braking phase implies a constant-acceleration thrust program. Figure 17, which is typical of proportional-navigation thrusting programs, conveys an idea of the severity of the throttling requirements encountered.

Very large thrust variations are required in order to satisfy the constraints imposed by the proportional-navigation scheme. Because throttling ratios of this magnitude are not feasible with a single engine, multiple engines with the attendant system complications might be required to produce such throttling ratios.

The thrust reversals commanded by the present thrust program would also prove to be rather wasteful in terms of fuel consumption. The initial radial acceleration and subsequent deceleration indicated by the α-curve is not an efficient process for accomplishing rendezvous.

In the analysis of noncoplanar transfers to elliptical satellite orbits, investigators have found that the thrust variations called for by the program were even larger than those shown above for the circular cases.

Harrison,[9] starting out with the initial conditions shown in Fig. 18 and using a navigation law of the general form

$$a_R(R,\dot{R})$$
$$a_n(\dot{\delta}_{LOS})$$
(3)

where

$$a_R = \text{radial thrust acceleration}$$

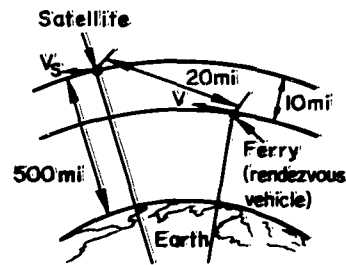$$a_n = \text{thrust acceleration normal to LOS}$$
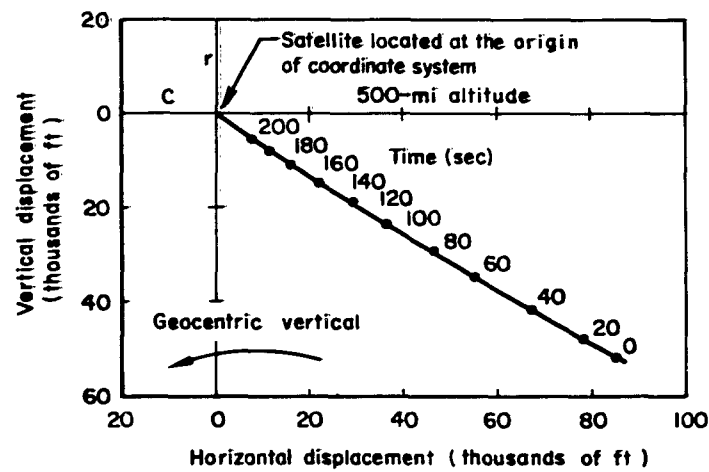
Fig. 15— Initial conditions for terminal phase

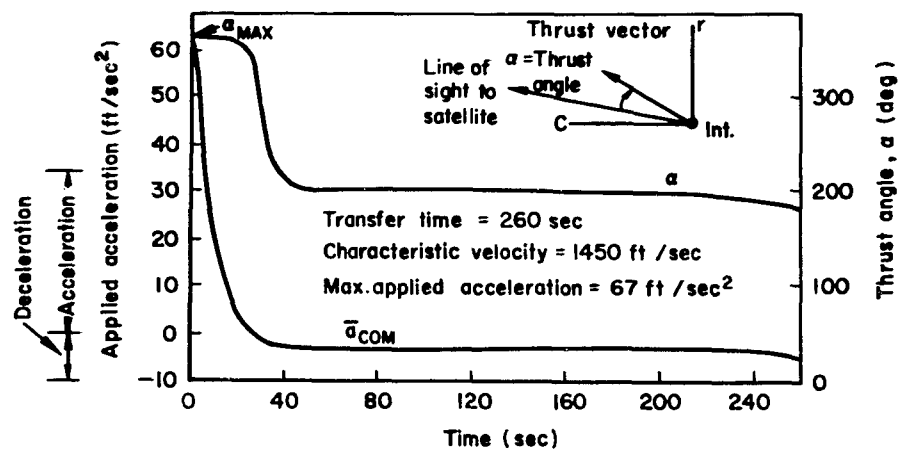Fig. 16— Standard trajectory in satellite coordinate system

Fig. 17— Thrust and thrust-angle profile
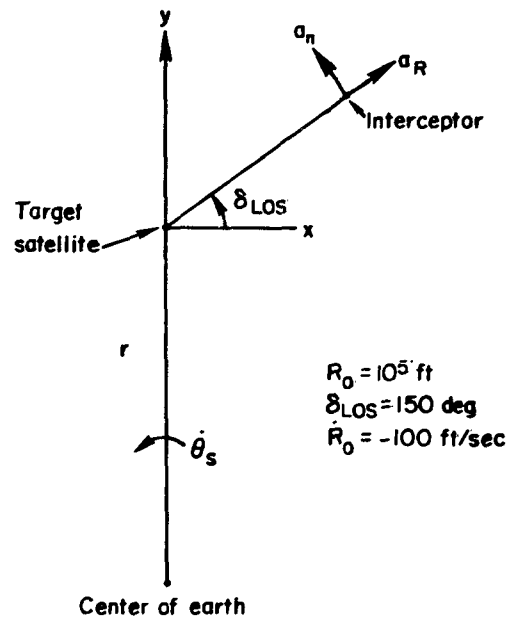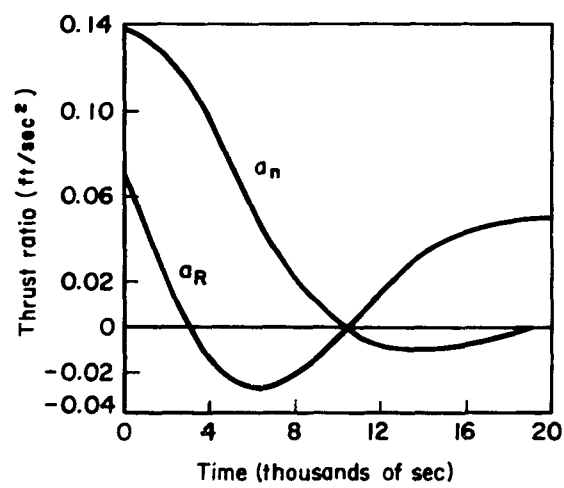
Fig. 18 — Initial conditions

Fig. 19 — Thrust versus time

$$\dot{\delta}_{LOS} + \dot{\theta}_{s} = 0 \text{ (zero angular rate of LOS)}$$

comes up with the acceleration profiles shown in Fig. 19.

The thrust-reversal program imposed by this guidance law is particularly noticeable in the a_R curve of the closing acceleration. The low values of starting-thrust acceleration found in this study, as compared with those obtained by Sears and Felleman, are due mainly to the absence of any initial ω_{LOS}.

Many other guidance schemes have been proposed by other investigators. Although they are based essentially on the principle of proportional navigation, these schemes introduce certain modifications that permit the use of constant-magnitude-thrust power plants, burning either continuously or intermittently. Lineberry and Foudriat,[10] for example, looked at a scheme whereby the normal velocity components are first cancelled by thrusting with a fixed magnitude in a direction normal to the LOS and then reorienting the engine along the LOS to decrease the range rate. The braking mode was performed at a constant continuous-thrust acceleration and required a slight amount of thrust modulation (throttling) to compensate for mass variations and errors in thrust alignment. An on-off constant-thrust radial-braking program was also proposed in Ref. 10. The switching points selected were the potential breakout points of the trajectory in the $R,\dot{R}$ phase plane from the region bounded by two constant radial-acceleration curves.

A constant radial acceleration (actually deceleration) implies that

$$\ddot{R} = a = \text{constant} \tag{4}$$

Two integrations give

$$\dot{R} = at + \dot{R}_o \tag{5}$$

$$R = \frac{1}{2} at^2 + \dot{R}_o t + R_o \tag{6}$$

Eliminating time between Eqs. (5) and (6) leads to the acceleration expression

$$a = \frac{\dot{R}_o^2}{2R_o} \tag{7}$$

"Switching" curves are obtained by plotting $\dot{R}$ versus R for two values of a, i.e., $a = a_1$ and $a = a_2$ $(a_1 > a_2)$. Thrust is switched on whenever the phase plane trajectory reaches the a_1 curve and is shut off when the a_2 curve is touched. The number of switchings is inversely related to the spread between a_1 and a_2. At the upper end, a_1 is limited by the value of T/m_o, the initial acceleration available. Too narrow a range between a_1 and a_2 will result in larger velocity expenditures because of the excessive number of times the power plants will have to be switched off and on, whereas too wide a gap could jeopardize the attainment of the desired final conditions, because the control available would be too coarse.

Two cases of range control employing this method are shown in Fig. 20.

The coupling that exists between the radial and normal components of the vehicle's relative equations of motion and any residual normal velocities left over from the normal thrusting mode would lead to gradual increases in ω_{LOS} that would also have to be cancelled during the braking mode. It has been suggested that this be accomplished by offsetting slightly the thrust orientation from the radial direction during the final braking phase.

Another paper, by Steffan,[11] employs two constant-thrust engines, one pointing in the normal direction, the other, in the radial direction. An on-off switching logic is programmed separately for each engine, which handles only the velocity component along its own channel. The existing cross-coupling effects between the two directions are also considered insofar as they affect the parameters of the radial thrusting mode.

The thrust program for the present case is relatively straightforward. The angular velocity ω_{LOS} is kept within a deadband region
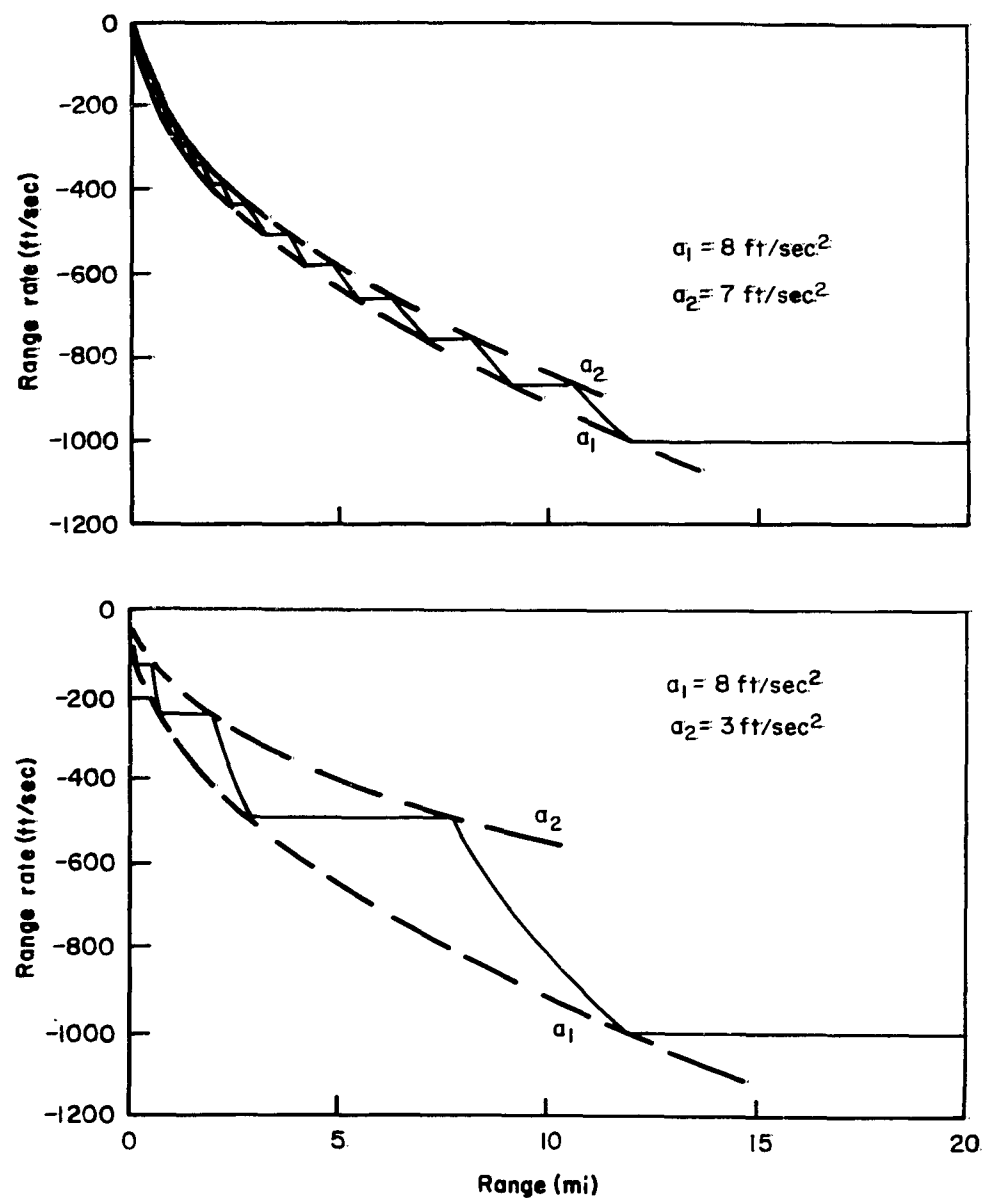
Fig. 20——Variation of range rate with range for on-off
thrust control during braking maneuver

ω_{LOS1} to ω_{LOS2} by the normal thrusting program, as shown in Fig. 21. Coast time t_c between successive normal correction periods must be kept above a certain minimum value to permit the instruments to complete position and rate measurements in the interval between the thrusting cycles. This instrumentation limitation in turn causes the initial time to closest approach τ (defined by the ratio $\tau = r/\dot{r}$) to be bounded from below. An upper bound on τ is chosen from such considerations as total rendezvous time and time required for instrumentation to settle after an acceleration input. The range of 30 sec $< \tau <$ 50 sec chosen by Steffan led to the radial phase plane trajectory shown in Fig. 22.

The proportional navigation schemes discussed so far all started by first defining a thrusting program and then computing the resultant vehicle trajectory by integration of the differential equations of the powered motion. An inverse approach was proposed by Cicolani.[12] Starting out with the kinematical relations for velocity and acceleration, Cicolani selected a suitable, monotonically decreasing time function for the lead angle L which decreased to zero at the same time that R and $\ddot{R}$ approached that value. The powered-flight trajectory thus artificially generated led to a soft contact between satellite and vehicle. The merit of such an approach lies primarily in the fact that no integration of complicated nonlinear differential equations of motion is necessary because the flight path is known from the very beginning through the chosen time-dependence of the path coordinates. The differential equations are used only as algebraic relations which provide us with a time expression for the thrust acceleration needed to fly the selected trajectory.

While this approach differed conceptually from the previous ones, it did not lead to significantly different or more convenient thrust requirements. This approach also required the use of thrust-reversal and throttling programs similar to those found by the approaches described earlier.
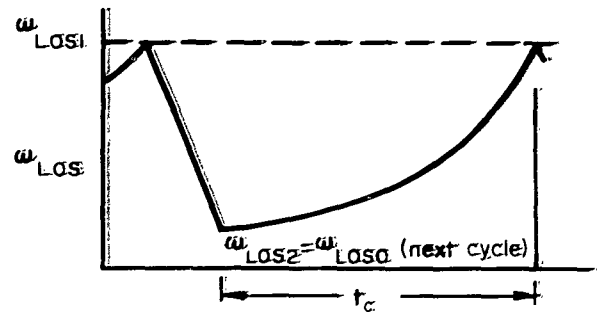
Fig. 21 — Approximate behavior
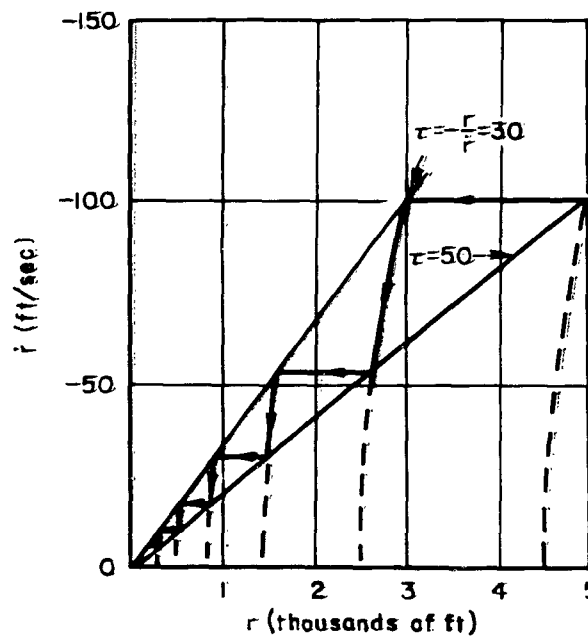of line-of-sight angular rate

Fig. 22 — Radial-phase plane

VIII. IMPULSIVE MANEUVERS

It was stated earlier that ground-controlled Keplerian transfers were not feasible because of instrument limitations in discerning the differential corrections needed at the ranges considered.

The many advantages inherent in the impulsive schemes of orbital transfer and the extensive familiarity and experience gained in their application prompted some investigators to search for possible ways to adapt them to the problem of orbital rendezvous. In an early paper, Clohessy and Wiltshire[13] demonstrated that impulsive-thrusting schemes can also be applied to rendezvous maneuvers, provided the equations of motion of the interceptor set up in a satellite-centered coordinate system are suitably linearized so that a closed-form solution to them can be obtained. Their scheme is most useful if the satellite orbit is circular, although extensions to elliptical satellite orbits have also been made available since then.

Viewed in the satellite-attached coordinate system shown in the sketch below, the exact equations of motion of the interceptor are given by

$$\frac{T_x}{m} - \frac{Gx}{r^3} = \ddot{x} - 2\omega\dot{y} - \omega^2 x$$

$$T_y/m - \frac{G(y + r_o)}{r^3} = \ddot{y} + 2\omega\dot{x} - \omega^2(y + r_o) \tag{8}$$

$$T_z/m - \frac{Gz}{r^3} = \ddot{z}$$

where

T_x, T_y, T_z = components of thrust vector

G = gravity constant

r_o = radius of circular satellite orbit

For unpowered flight in the vicinity of the satellite (i.e., $\sqrt{x^2 + y^2 + z^2} \leq 200$ mi), Eqs. (8) can be linearized, resulting in the relations

$$\ddot{x} - 2\omega\dot{y} = 0$$
$$\ddot{y} + 2\omega\dot{x} - 3\omega^2 y = 0$$
$$\ddot{z} + \omega^2 z = 0$$

(9)
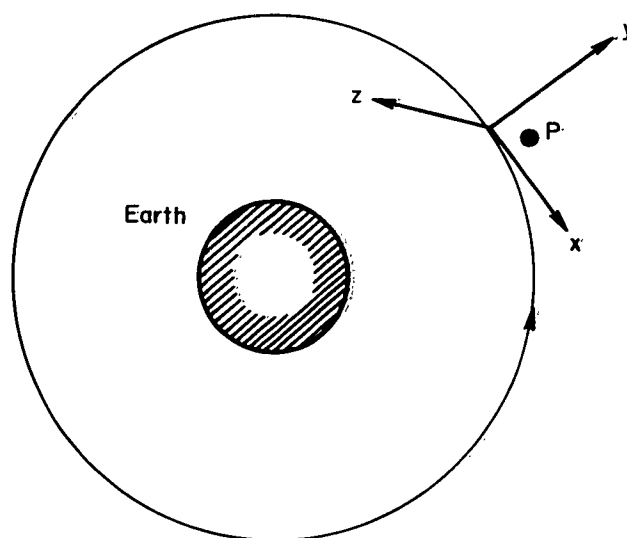
System (9) has a closed-form solution given by

$$x = 2\left(\frac{2\dot{x}_o}{\omega} - 3y_o\right)\sin \omega t - \frac{2\dot{y}_o}{\omega}\cos \omega t + (6\omega y_o - 3\dot{x}_o)t + const$$

$$y = \left(\frac{2\dot{x}_o}{\omega} - 3y_o\right)\cos \omega t + \frac{\dot{y}_o}{\omega}\sin \omega t + \left(4y_o - \frac{2\dot{x}_o}{\omega}\right)$$

(10)

$$z = z_o \cos \omega t + \frac{\dot{z}_o}{\omega}\sin \omega t$$

Equations (10) indicate that the motion in the z direction is periodic. If $6\omega y_0 - 3\dot{x}_0 = 0$, then the interceptor will move in an elliptical orbit contained in the xy plane around the center of coordinates, while if $6\omega y_0 - 3\dot{x}_0 \neq 0$, the elliptical motion now occurs around a center translating with uniform velocity, $6\omega y_0 - 3\dot{x}_0$, in the x direction. The path thus generated would be a cycloid. If intercept is desired to occur at time $t = \tau$, say, the velocity components needed at $t = 0$ can be found from Eqs. (10) by solving for the quantities $\dot{x}_0$, $\dot{y}_0$, $\dot{z}_0$.

The required starting velocities are

$$\dot{x}_0 = \frac{x_0 \sin \omega_T + y_0 \left[6\omega_T \sin \omega_T - 14(1 - \cos \omega_T) \right]}{\Delta}$$

$$\dot{y}_0 = \frac{2x_0 (1 - \cos \omega_T) + y_0 \left[4 \sin \omega_T - 3\omega_T \cos \omega_T \right]}{\Delta} \qquad (11)$$

$$\Delta = 3\omega_T \sin \omega_T - 8(1 - \cos \omega_T)$$

The rendezvous velocity components obtained above are only approximate, because they were obtained from the solution of a linearized system of equations and will consequently cause the vehicle to miss the target by some small distance. The accuracy of the approximation improves if one decreases the required time to go until intercept occurs. The actual relative velocity components of the vehicle, as determined from measurements carried out on board, have to be compared at frequent intervals with those demanded by a guidance scheme based on Eqs. (11); the necessary small impulsive velocity corrections are applied until impact is achieved. The terminal approach velocity also has to be cancelled impulsively.

Some numerical values for a specific rendezvous mission based on the above guidance scheme are presented in a paper by Soule.[14] For the numerical example, he chose to consider a rendezvous with a satellite in an elliptical orbit of eccentricity e = 0.01, and semimajor axis a = 3592 n mi. Rendezvous was required to occur after 270 deg of satellite travel, and velocity corrections were spaced every 10 deg

of satellite travel. No relative motion was assumed to exist between satellite and interceptor at the start of the maneuver. The interceptor path is shown in Fig. 23 in a satellite-centered rotating coordinate system. The coordinate system was chosen such that at $t = 0$, the x-axis passed through the interceptor.

The necessary velocity impulses are marked off at the appropriate points. A velocity expenditure of 452 ft/sec was required to perform the rendezvous.

In assessing the efficiency and domain of applicability of the present guidance scheme, it should be remembered that in essence the simplifications obtained from the linearization of the equations of motion were bought at the expense of introducing distortions into the gravitational force field through which the two vehicles move. The assumed gravitational field is unidirectional and decreases linearly with increase in altitude. The extent of the position and velocity errors introduced by this hypothetical gravity field will depend on the length of time during which the two bodies are exposed to it. The linearized approximation becomes progressively worse as the time of flight τ increases, and this leads to increasing errors in the guidance velocities computed.

The domain of applicability of this approximation is the subject of a recent study by Eggleston and Dunning,[15] who employed the same guidance law but spaced their impulsive-velocity corrections in accordance with the geometric range progression $Ro/2^n$ ($n = 0,1,2...$). Two of the conclusions of this study follow.

1. If the central range angle $\omega\tau$ traversed by the satellite until rendezvous is smaller than 90 deg, the errors introduced by the linearization of the equations are relatively small. The approximate guidance equations lead to increasing errors in the in-plane velocity-component requirements when the angle $\omega\tau$ increases beyond 90 deg. This is demonstrated in Fig. 24 for the case in which the satellite and interceptor are moving initially in coplanar orbits. The solid lines show the time variation of the actual velocity components when the vehicle follows an exact intercept trajectory. The dashed curves show the velocity components required by the approximate guidance scheme. It is

Scale for
velocity—change vectors

Distance (n mi)

Initial position of
rendezvous vehicle

187 ft/sec

Starting relative
velocity

Target

Velocity change
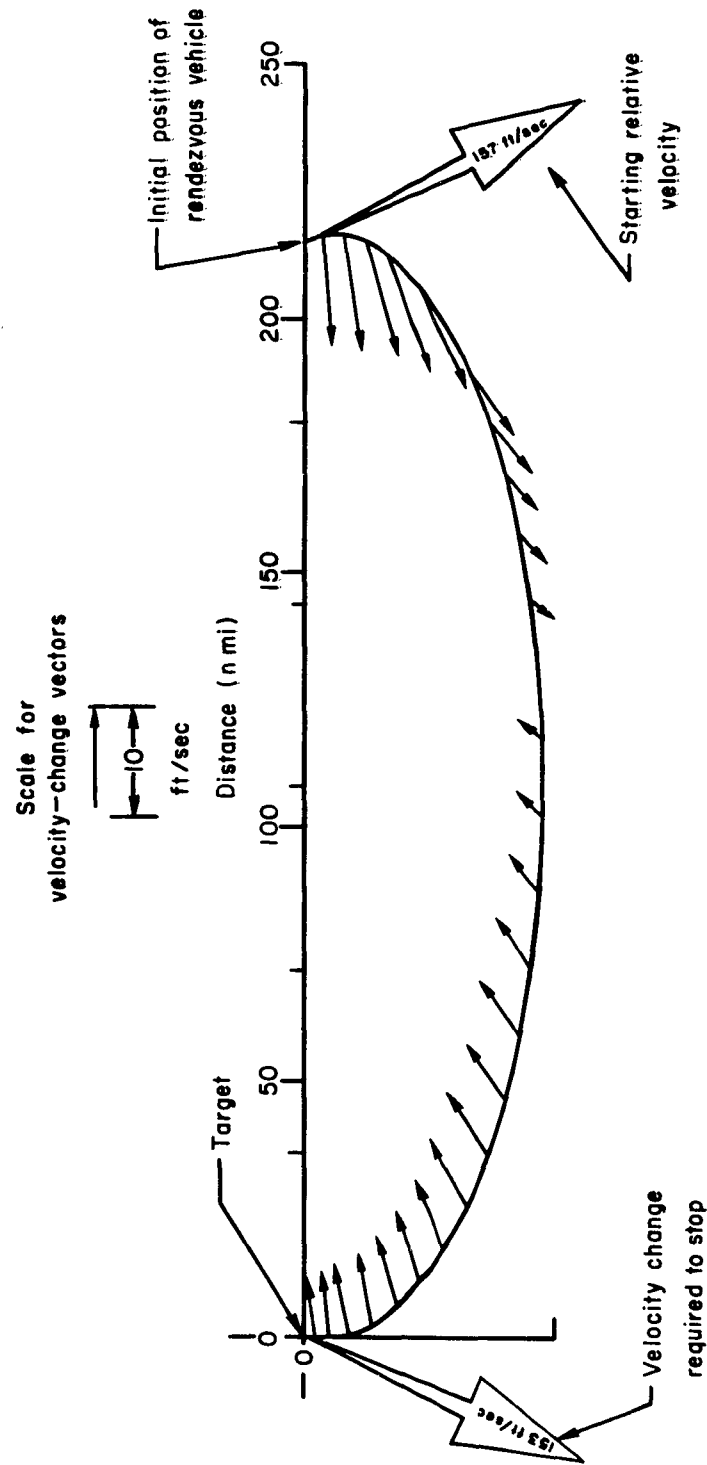required to stop

193 ft/sec

Fig. 23 — Relative motion of rendezvous vehicle with
respect to the target
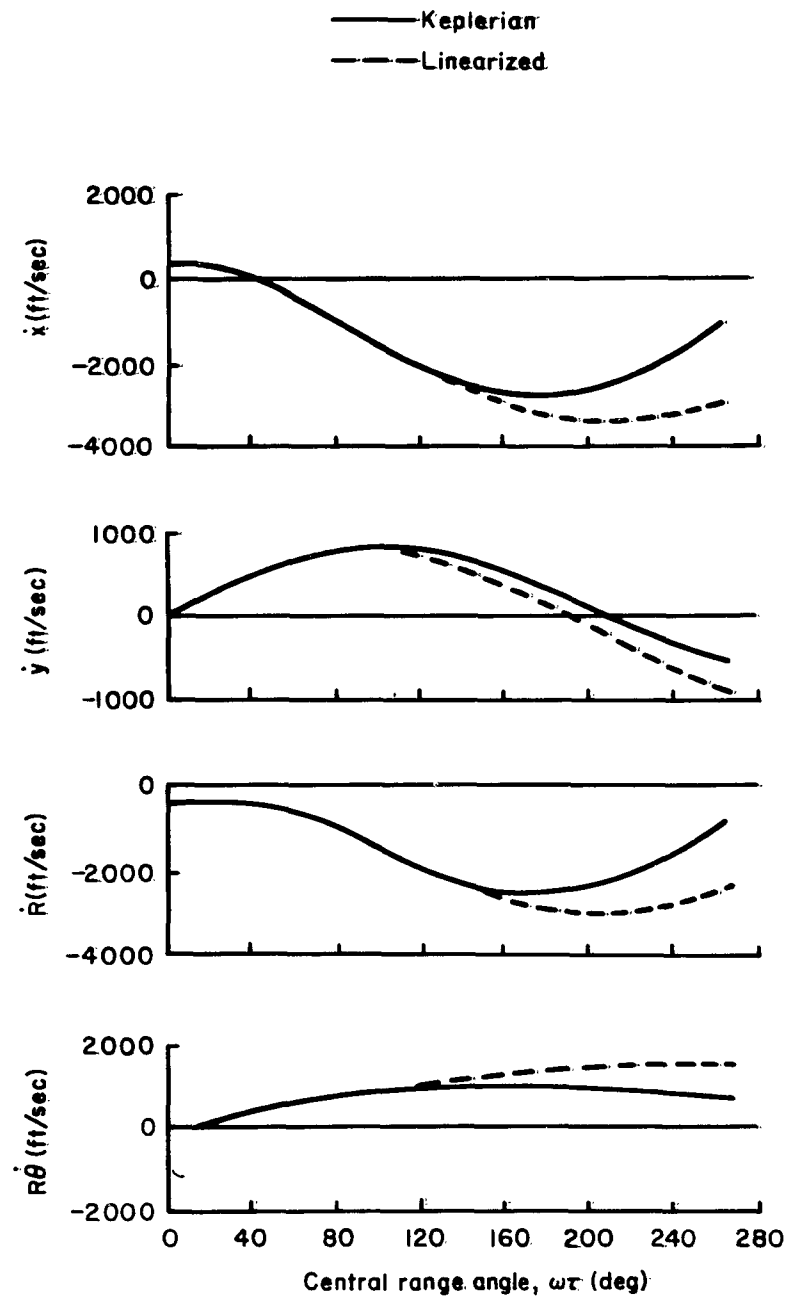
Fig. 24—Magnitude of the velocity requirements

seen that even if the vehicle would start out initially on the correct intercept trajectory, the guidance scheme would still demand a series of unnecessary velocity corrections before intercept could occur.

2. Errors in the time of booster burnout up to 5 sec do not materially affect the required velocity corrections. This result is due to the present method of guidance, which does not depend as much on the specified time to rendezvous as it does on the instantaneous relative-position and velocity conditions at the instant a velocity correction is called for.

IX. LINE-OF-SIGHT ORIENTATION DURING INTERCEPT MANEUVERS

A factor common to all the nonimpulsive rendezvous guidance laws that have been discussed so far is the requirement of cancelling the angular rate of the LOS, thereby forcing the vehicle onto a collision course with the target.

The impulsive guidance schemes discussed earlier are not concerned explicitly with the rotational behavior of the LOS during the terminal phase, since they accomplish intercept by the application of discrete velocity corrections. Whether the above velocity increments are indeed of such a magnitude as to bring about a stabilization of the LOS is not readily apparent.

It is clear that if the requirement that ω_{LOS} vanish is a sufficient but not a necessary condition for intercept, imposing it would result in an increase in the characteristic velocity expended by the interceptor over that which would have to be expended by using the same power plant but a more efficient guidance scheme.

The question of the necessity of the condition $\omega_{LOS} \rightarrow 0$ is quickly settled with the aid of Figs. 25 and 26, which present three arbitrary trajectories that result in an intercept, and the corresponding behavior of ω_{LOS}. It is seen that except for the instant of colinear closure in the case of the Hohmann-transfer case, no restriction on the size of ω_{LOS} or uniformity in its time variation is evident from the Figs. 25 and 26.

It would thus appear that the use of a guidance scheme based on proportional navigation tends to place excessive and unnecessary demands on the flexibility required of the rendezvous power plant and in all likelihood will increase the velocity expended. This conclusion has been borne out by the work of some other investigators.
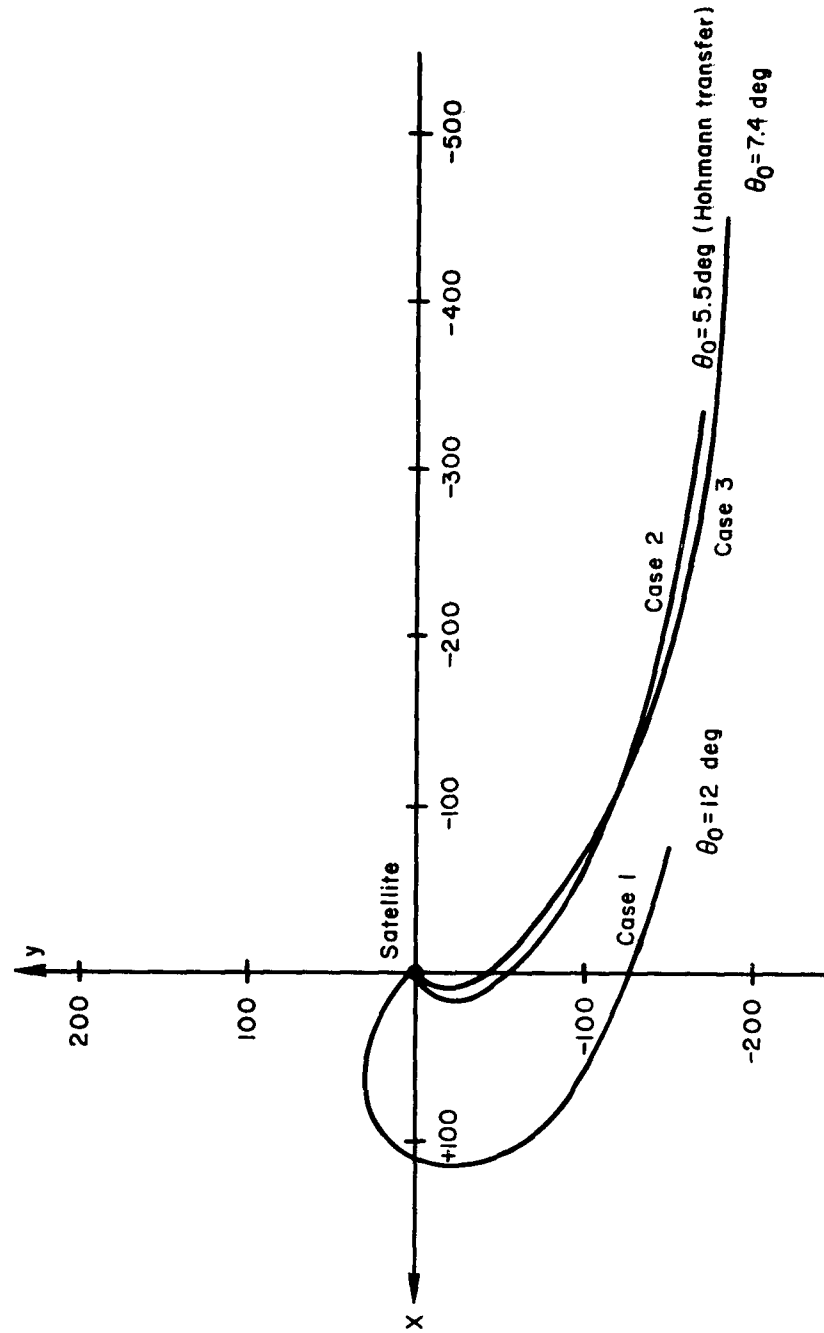
Fig.—25 Intercept trajectories from h=150 n mi to satellite in 300−n mi orbit (transfer angle in all cases is 180 deg; initial lead angle, θ_O, specified)
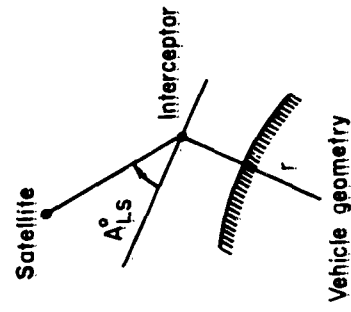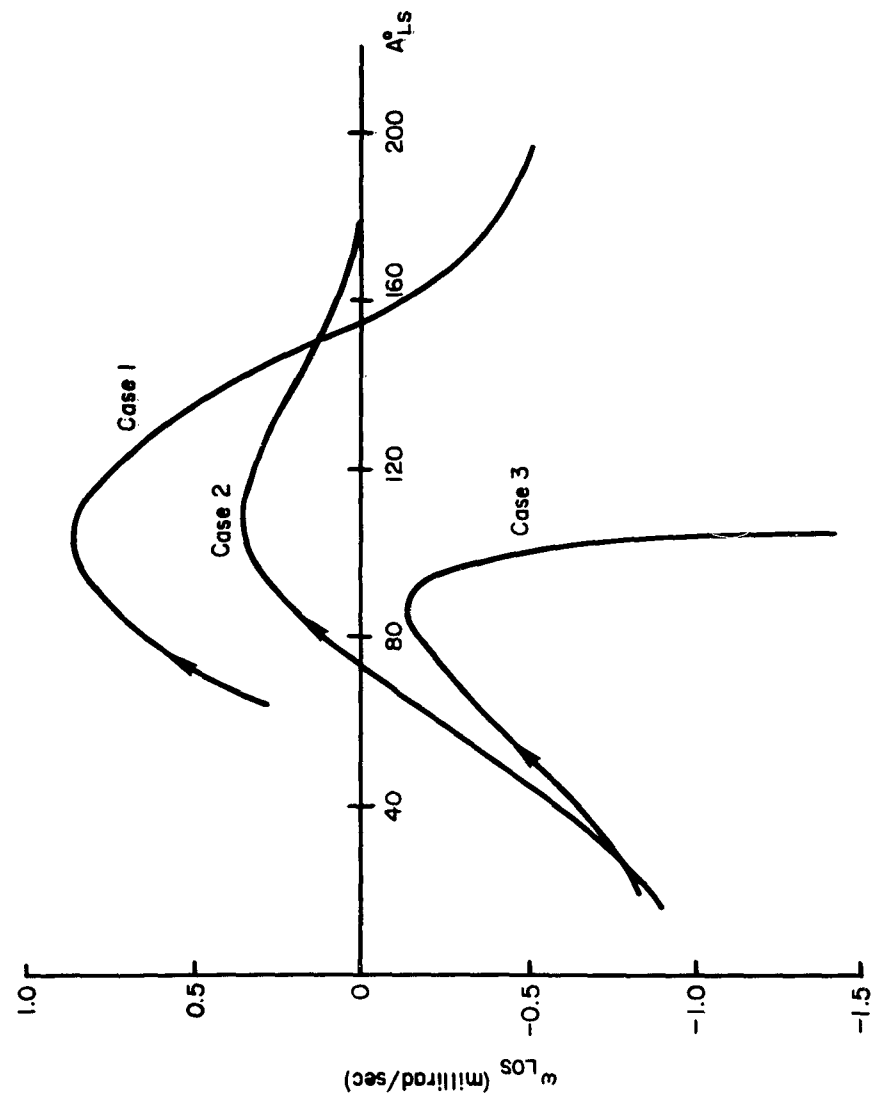
Fig. 26 — Behavior of ω_{LOS} during intercept

X. ADAPTATION OF IMPULSIVE INTERCEPT TO FINITE THRUSTING

The large thrust variations necessary for the cancellation of ω_{LOS}, as well as the large power plants needed to implement the impulsive guidance schemes, have prompted some investigators to search further for ways to accomplish rendezvous more efficiently and with lower and more realistic thrust requirements. An interesting scheme, based essentially on the impulsive guidance scheme of Clohessy and Wiltshire, was proposed in a recent paper by Shapiro.[16] The proposed modifications result in a rendezvous that avoids the velocity penalties associated with previous finite-magnitude-thrust guidance schemes based on the unnecessary stabilization of the LOS.

Briefly, the attenuated intercept guidance scheme proposed by Shapiro works as follows:

Thrust is activated and suitably vectored until the actual measured velocity components of the interceptor, $\dot{x}$ and $\dot{y}$, are made to fall within a certain deadband region around the desired intercept velocities $\dot{x}_D$ and $\dot{y}_D$ obtained from Eqs. (11) for an intercept assumed to occur τ sec later (the velocity components $\dot{x}_D$ and $\dot{y}_D$ used by Shapiro correspond to $\dot{x}_o$ and $\dot{y}_o$ in the previous notation). Thrust is activated and shut down any time the velocity error V_δ, given by

$$V_\delta = \left[(\dot{x} - \dot{x}_D)^2 + (\dot{y} - \dot{y}_D)^2 \right]^{1/2} \tag{12}$$

is about to pass out of the deadband region $V_a - V_d$

$$V_d < V_\delta < V_a \tag{13}$$

where

V_a = velocity for thrust activation

V_d = velocity for thrust deactivation

The instantaneous direction of thrusting chosen by Shapiro, as seen in an $\dot{x}$ versus $\dot{y}$ coordinate plane was from point $\dot{x},\dot{y}$ to point $\dot{x}_D,\dot{y}_D$. Introducing no other corrections, and neglecting the errors due to linearization, intercept (i.e., a hard rendezvous) would occur at the end of the prescribed time period τ. A suitable stretching constant $k \leq 1/2$ is now introduced into Eqs. (11) by substituting the expression $\tau - kt$ for previous time τ. The guidance system continuously compares the measured vehicle velocities with the variable desired velocities of Eqs. (11), initiating and terminating thrust in accordance with the constraint condition of Eq. (13) while flight time t increases monotonically from 0.

When a value of $t = \tau/k$ is reached, the above thrusting scheme brings about a soft intercept (zero closing speed). The parameters τ and k are amenable to optimization although this was not attempted in Ref. 16, where a τ of one-half the satellite orbital period and a $k = 1/2$ were arbitrarily chosen for the majority of examples computed. Shapiro also investigated the performance of 50 per cent and 100 per cent variable-thrust power plants, concluding that the gains possible were not sufficiently large to offset the additional complexities introduced into the system.

The velocity expenditures ΔV to rendezvous, plotted against initial target true anomaly θ_{T_0}, are shown in Fig. 27 for an elliptical target orbit of e = 0.1 and a = 5000 mi. Relative conditions at departure are indicated in the figure. The curves for variable throttling are also presented. In all cases a maximum value of 0.01g was chosen for the thrust acceleration.

The thrusting program that results from the use of a constant-magnitude stretching factor places a lower bound on the least permissible acceleration level which allows rendezvous to be accomplished for a given initial condition. To circumvent this restriction, Shapiro also suggests a modification to his program, based on a k-switching technique which could be used to accomplish a rendezvous irrespective of the thrust magnitude available. He suggests starting the thrusting program with a $k = 0$ (i.e., infinite time to rendezvous) and then switching to a $k = 1/2$ as soon as the velocity error V_δ has been brought
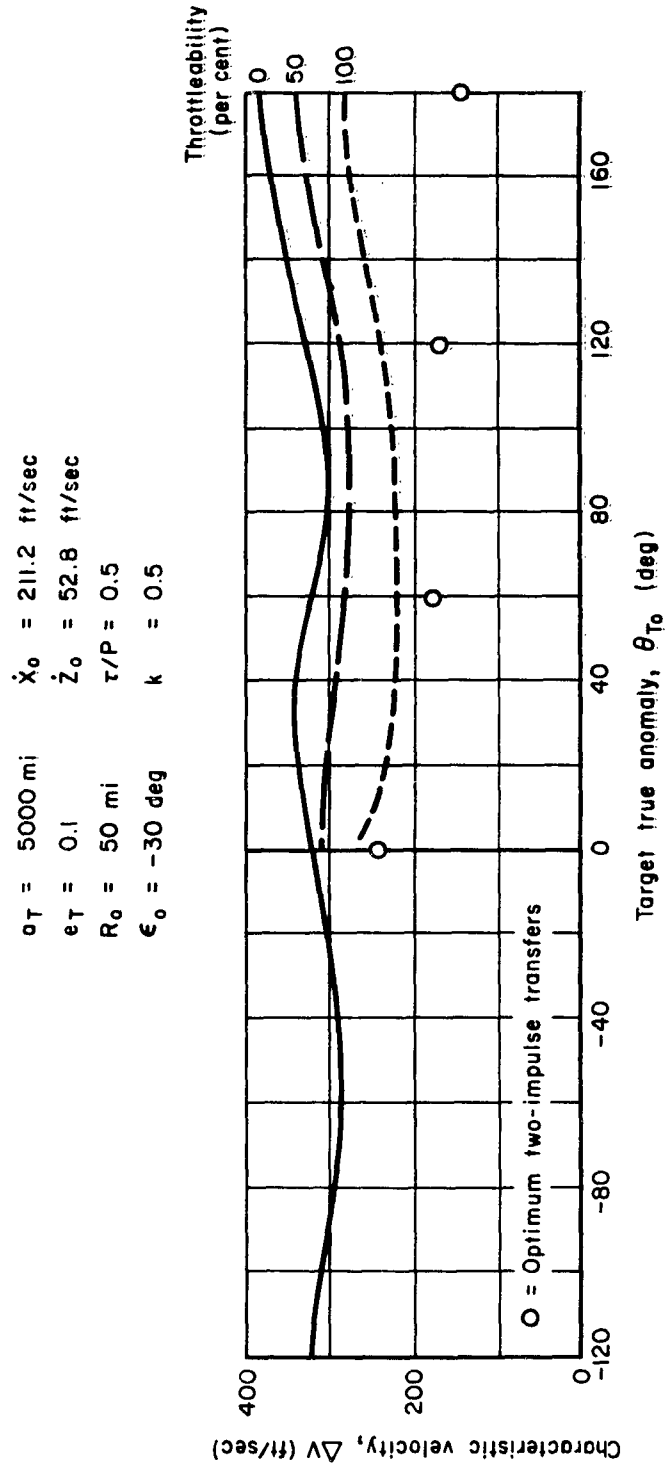
Fig. 27 — Propellant assessment based on degree of engine throttleability

into the deadband region $V_a - V_d$. He compared his results with the equivalent results obtained from an application of the proportional navigation scheme and found that the latter scheme consumed characteristic velocities that were an order of magnitude larger than the values obtained by the above scheme. The data for any actual comparison, however, were not presented in the paper.

XI. CONCLUDING REMARKS

In the preceding pages some aspects of the problem of orbital rendezvous have been examined.

A review of the available unclassified literature disclosed that the research of the various investigators fell, by and large, into two more-or-less distinct categories: In one, the gross impulsive motion of the interceptor between the launch (or burnout) point and the moving-target point is studied; in the other, attention is focused on a more thorough and detailed analysis of the powered-flight maneuver during the terminal portion of the mission. In each category, a number of the more representative and informative papers published were singled out for specific mention and discussion. Those graphs and figures which best helped tell the study were reproduced here.

Based on the material presented, a few concluding remarks can be made.

The two-impulse rendezvous maneuvers have received thorough coverage and are well-enough understood and documented that any numerical information concerning trajectory shape, velocity expenditures, launch delays, frequency of intercepts, etc., can be either read off existing curves or easily generated.

No general, conclusive comparisons of the relative merits of the various terminal-guidance schemes seem to have been undertaken so far, although some favorable comments concerning the relative advantages of their own schemes are occasionally made by some of the investigators. Some of the schemes discussed appear to make excessively large demands on the terminal power-plant unit in terms of throttling ratios required.

What seems to be lacking also is information concerning the effect of maximum thrust acceleration available on the various parameters of the rendezvous mission. In particular, it would be useful to get a clearer understanding of the relationships and tradeoffs that exist between such quantities as thrust acceleration, time to rendezvous, velocity expenditure, and their combined effect on the initial kinematic conditions which can be satisfied.

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