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TRANSLATION

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By

B. M. Bublik

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

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VIBRATIONS AND STABILITY OF A RIBBED CYLINDRICAL SHELL IN A FLOW OF COMPRESSIBLE LIQUID

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An analogous problem for a cylindrical shell without rigidity ribs has been discussed in report [1]. In this report were investigated certain questions concerning vibration and stability of a closed cylindrical elastic shell, reinforced by elastic ribs (rigidity ribs), which is in a non-viscous flow of a compressible fluid, equations are formulated for the natural frequencies of this elastic system and critical rate of liquid flow are determined. For the sake of simplicity only normal shell vibrations are discussed here. The rigidity ribs are considered as distributed over shell discretely.

1. Assuming that along the closed cylindrical shell, reinforced by rigidity ribs, flows a potential flow with unperturbed velocity U. Utilizing the basic conditions of reports [1] and [2] the equations of motion of the shell with bulkheads will be written in form of:

$$\frac{\partial^2 u}{\partial \alpha^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial \beta^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial \alpha \partial \beta} + \nu \frac{\partial w}{\partial \alpha} = 0,$$

$$\frac{\partial^2 v}{\partial \beta^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial \alpha^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial \alpha \partial \beta} + \frac{\partial w}{\partial \beta} + \frac{1}{B}\sum_{i=1}^{N}\delta(\alpha-\alpha_i)E_iF_i\left(\frac{\partial^2 v}{\partial \beta^2} + \frac{\partial w}{\partial \beta}\right) = 0,$$

$$\nu\frac{\partial u}{\partial \alpha} + \frac{\partial v}{\partial \beta} + c^2(\nabla^2+1)^2 w + w - 2c^2(1-\nu)\frac{\partial^2 w}{\partial \alpha^2} + \frac{1}{B}\sum_{i=1}^{N}\delta(\alpha-\alpha_i)\times$$

$$\times\left[\frac{E_iF_i}{R^2}\left(\frac{\partial^2}{\partial \beta^2}+1\right)^2 w + E_iF_i\left(\frac{\partial v}{\partial \beta} + w\right)\right] + \frac{R^2}{B}\left[m_0 + \sum_{i=1}^{N} m_i\delta(\alpha-\alpha_i)\right]\times$$

$$\times\frac{\partial^2 w}{\partial t^2} = \frac{1}{B}q_r. \qquad (1)$$

where $\delta(\alpha-\alpha_i)$ - delta-function with peculiarity on line $\alpha=\alpha_i$; E_iF_i and E_iI_i - of rigidity ribs, which are distributed over lines $\alpha = \alpha_i (i=1,2,\ldots,N)$; q_r-normal load component referred to a unit of area of the center surface of the shell.

If p designates the pressure against the shell from the side of the flow, then

$$q_r = -m_0R^2\frac{\partial^2 w}{\partial t^2} - R^2\sum_{i=1}^{N} m_i\delta(\alpha-\alpha_i)\frac{\partial^2 w}{\partial t^2} - R^2 p. \qquad (2)$$

Here m_0 - mass per unit of shell area, and m_i-linear mass of i-rigidity rib.

As is shown in report [2] the tangential effects of rigidity ribs can be averaged with a slight error, the evaluation of which is shown there analytically. Accepting these assumptions, the system of equations (I) can be reduced to one equation:

$$c^2(\nabla^2\nabla^2 + 2\nabla^2 + 1)\tilde{\nabla}\Phi + \left(1 - \nu^2 + \frac{h_1}{h}\right)\frac{\partial^4\Phi}{\partial\alpha^4} - 2(1-\nu)c^2\frac{\partial^2}{\partial\alpha^2}\tilde{\nabla}\Phi +$$

$$+ \sum_{i=1}^{N}\delta(\alpha-\alpha_i)\frac{E_iI_i}{BR^2}\left(\frac{\partial^2}{\partial\beta^2} + 1\right)^2\tilde{\nabla}\Phi = \frac{q_r}{B}. \qquad (3)$$

In it:

$$\text{В нему: } q_r = -\left[R^2m_0 + R^2\sum_{i=1}^{N}\delta(\alpha-\alpha_i)\,m_i\right]\frac{\partial^2\tilde{\Delta}\Phi}{\partial t^2} - R^2p,$$

$$\tilde{\nabla} = \nabla^2\nabla^2 + \frac{h_1}{h}\cdot\frac{\partial^4}{\partial\beta^4} + \frac{2}{1-\nu}\cdot\frac{h_1}{h}\cdot\frac{\partial^4}{\partial\alpha^2\partial\beta^2}. \qquad (4)$$

$$h_1 = \frac{1}{l}\sum_{i=1}^{N}F_i, \quad c^2 = \frac{h^2}{12R^2}. \qquad (5)$$

In this case displacements of the shell u, v, w will be expressed through the function Φ with the aid of such formulas:

$$u = \nu\frac{\partial^3\Phi}{\partial\alpha^3} + \left(1 + \frac{h_1}{h}\right)\frac{\partial^3\Phi}{\partial\alpha\partial\beta^2}, \quad w = \tilde{\nabla}\Phi,$$

$$v = \left(1 + \frac{h_1}{h}\right)\frac{\partial^3\Phi}{\partial\beta^3} + \left[2 + \nu + \frac{2}{1-\nu}\frac{h_1}{h}\right]\frac{\partial^3\Phi}{\partial\alpha^2\partial\beta}. \qquad (6)$$

2. We shall now discuss the determination of liquid pressure. In report [1] is shown, that in linear approximation for an arbitrary barotropic liquid the liquid pressure against the shell can be determined by formula:

$$p = p_0 - \varrho\left(\frac{U}{R}\frac{\partial\varphi}{\partial\alpha} + \frac{\partial\varphi}{\partial t}\right)\Big|_{r=R}. \qquad (7)$$

in which p_0 and q - pressure and density of liquid in nonturbulent state, φ- potential of rate of motion of the liquid in turbulent state .

To determine φ in case of small shell deformations it is possible to obtain such linearized equations:

$$(1-M^2)\frac{1}{R^2}\frac{\partial^2\varphi}{\partial\alpha^2}+\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\beta^2}-\frac{2M}{a}\cdot\frac{1}{R}\cdot\frac{\partial^2\varphi}{\partial\alpha\partial t}-\frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}=0. \tag{8}$$

where a –speed of sound in nonturbulent flow of liquid, and $M=\frac{u}{a}$.

Equations (8) are accompanied by conditions of cylinder wall impenetrability, which after modifications are reduced to the fulfillment of such a relation:

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r=R}=\frac{U}{R}\frac{\partial w}{\partial\alpha}+\frac{\partial w}{\partial t} \tag{9}$$

and the condition of damping and (or) radiation:

$$\varphi\to 0,\ \frac{\partial\varphi}{\partial r}\to 0,\ \text{when}\ r\to\infty, \tag{10}$$

$$\varphi=0(r^{-\frac{1}{2}}),\ \frac{\partial\varphi}{\partial r}+i\omega\varphi=0(r^{-\frac{1}{2}}). \tag{11}$$

The conditions of radiation (11) should be satisfied when reference is made to oscillatory motion and the velocity potential has a factor $e^{i\omega t}$.

Solution of equation (8) is sought in form of:

$$\varphi=f(r)e^{i(\omega t-kR\alpha)}\cos n\beta. \tag{12}$$

After substituting (12) in (8) to determine f(r) we will obtain

$$\frac{d^2f}{dr^2}+\frac{1}{r}\frac{\partial f}{dr}-\left[\left(\frac{\omega}{a}-MkR\right)^2-(kR)^2-\frac{n^2}{r^2}\right]f(r)=0. \tag{13}$$

Solution of equation (13) offers the possibility of determining φ and then also p. It is shown in report [1] that between pressure and buckling of shell exists such a relationship:

$$p=p_0+M_{kn}^{**}\left(\frac{\partial}{\partial t}+\frac{U}{R}\frac{\partial}{\partial\alpha}\right)^2 w(\alpha,\beta,t), \tag{14}$$

where

$$M_{kn}^{**}=\frac{p_0Z_n(\mu R)}{\mu Z_n'(\mu R)},\quad \mu r=x.$$

$$Z_n(x)=\begin{cases}J_n(x),\ \text{when}\ \left|\frac{\omega}{a}-MkR\right|<kR,\\ H_n(x),\ \text{when}\ \left|\frac{\omega}{a}-MkR\right|>kR,\end{cases} \tag{14a}$$

$$\mu^2=\left|\left(\frac{\omega}{a}-MkR\right)^2-(kR)^2\right|.$$

Here $I_n(x)$ - Bessel function of the real argument, and $H_n(x)$ - Henkel function of different type.

Figuring (4) and (6) for the pressure in our case, we will obtain

$$q_r = -R^2\left[m_0 + \sum_{i=1}^{N} \delta(\alpha - \alpha_i) m_i\right]\frac{\partial^2}{\partial t^2}\tilde{\nabla}\Phi - M^{**}_{km}R^2\left(\frac{\partial}{\partial t} + \frac{U}{R}\frac{\partial}{\partial \alpha}\right)^2\tilde{\nabla}\Phi - p_0 \quad (15)$$

3. We will find a solution for equation (3). Replacing in it q_r we will have

$$c^2(\nabla^2 + 1)^2\tilde{\nabla}\Phi + \left(1 - \nu^2 + \frac{h_1}{h}\right)\frac{\partial^4\Phi}{\partial\alpha^4} - 2(1-\nu)c^2\frac{\partial^2}{\partial\alpha^2}\tilde{\nabla}\Phi + \sum_{i=1}^{N}\delta(\alpha - \alpha_i) \times$$

$$\times \frac{E_i I_i}{BR^2}\left(\frac{\partial^2}{\partial\beta^2} + 1\right)^2\tilde{\nabla}\Phi = R^2\left[m_0 - \sum_{i=1}^{N}\delta(\alpha - \alpha_i) m_i\right]\frac{\partial^2}{\partial t^2}\tilde{\nabla}\Phi -$$

$$- M^{**}_{km}R^2\left(\frac{\partial}{\partial t} + \frac{U}{R}\right)^2\tilde{\nabla}\Phi. \quad (16)$$

Comparing these equations with analogous equations of free oscillations of a ribbed shell, it is necessary first of all to determine, that the value M^{**}_{nk} is in a certain sense an "added" mass of the liquid to the mass of the ribbed shell. The member $M^{**}_{km}R^2\frac{\partial}{\partial\alpha}\tilde{\nabla}\frac{\partial\varphi}{\partial t}$ expresses dampings of the motion of the system as result of irradiated energy.

Solution of equation (16) can be realized by the Galorkin method. For this we seek the unknown in form of:

$$\Phi(\alpha, \beta, t) = \sum_{k}^{\infty}\sum_{n}^{\infty} C_{kn}e^{-i\omega t}\sin\lambda_k\alpha\cos n\beta. \quad (17)$$

Selection of systems of functions $\sin\lambda_k a \left|\lambda_k = \frac{k\pi R}{l}\right|$ due to hinged conditions of fastening the edges of the shell $a = 0$ and $a = \frac{l}{R}$ which we are assuming.

Substituting (17) in (16) and carrying out the ordinary Galorkin procedure, we obtain a system of linear algebraic equations for the determination of C_{kn} constants. in the presence of rigidity ribs is interminably connected. But with an error of the order of ε^2 during the realization of inequalities

$$\sum_{i=1}^{N}\frac{2E_iF_i}{BR^2 l}\sin\lambda_k\alpha_i\sin\lambda_q\alpha_i < \varepsilon,$$

$$\frac{2}{lm_0}\sum_{i=1}^{N} m_i\sin\lambda_k\alpha_i\sin\lambda_q\alpha_i < \varepsilon \quad (18)$$

the system under question is simplified considerably (see [2]). Having no possibility of describing same here, we shall mention only the conditions of its non-zero solutions:

$$c^2(\lambda_k^2 + n^2 - 1)^2 a_{kn} + \left(1 - \nu^2 + \frac{h_1}{h}\right)\lambda_k^4 + 2(1-\nu)c^2\lambda_k^2 a_{kn} + I_k(n^2-1)^2 a_{kn} =$$

$$= R^2(m_0 + M_k^*)a_{kn}\omega_{kn}^2 + R^2 M_{kn}^{**}\left(\omega_{kn} + \frac{U}{R}\lambda_k\right)^2 a_{kn}. \quad (19)$$

$$a_{kn} = (\lambda_k^2 + n^2)^2 + \frac{h_1}{h}h^4 + \frac{2}{1-\nu}\frac{h_1}{h}\lambda_k^2 n^2,$$

here

$$I_k = \sum_{i=1}^{N}\frac{2E_i I_i}{BR^2 l}\sin^2\lambda_k\alpha_i, \quad M_k = \sum_{i=1}^{N}\frac{2m_i}{l}\sin^2\lambda_k\alpha_i. \quad (20)$$

If in equation (19) is written $M_{kn}^{**} = 0$, then the obtained equation will enable to find the frequency of the shell's natural oscillations a shell with ribs in vacuo. Designating same by Ω_{kn} we will obtain

$$\Omega_{kn}^2 = \frac{c^2(\lambda_k^2 + n^2 - 1)^2 a_{kn} + \left(1 - \nu^2 + \frac{h_1}{h}\right)\lambda_k^4 + 2(1-\nu)c^2\lambda_k^2 a_{kn} + I_k(n^2-1)^2 a_{kn}}{R^2(m_0 + M_k^*)a_{kn}}. \quad (21)$$

Now equation (19) can be written as:

$$\Omega_{kn}^2 - \omega_{kn}^2 = \frac{M_{kn}^{**}}{m_0 + M_k^*}\left(\omega_{kn} + \frac{U}{R}\lambda_k\right)^2. \quad (22)$$

If in it is known the rate of nonturbulent flow U, then we have the possibility of finding the frequency of the discussed shell, which moves in a liquid. From equation (22) can also be obtained the critical rate of motion of the liquid, which corresponds to loss in stability of the nonperturbed state of the shell with rigidity ribs. For this we write in equation (22) $\omega_{kn} = 0$. Then we will obtain

$$U_{kp} = \frac{\Omega_{kn}}{\lambda_k}\sqrt{\frac{m_0 + M_k^*}{M_{kn}^{**}}}. \quad (23)$$

It should be mentioned, that during determination of frequency and during the determination of critical velocities it is necessary to solve transcendent equations, since ω and U are included in M_{kn}^{**}.

Literature

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