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TECHNICAL NOTE R-48

A FORTRAN PROGRAM FOR THREE-DEGREE OF FREEDOM
TRAJECTORIES, REFERENCED TO GEOCENTRIC COORDINATES
AND TO AN ARBITRARY POINT ON THE EARTH'S SURFACE

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Prepared By

H. M. Minshew

May, 1963

BROWN

ENGINEERING COMPANY INC.
HUNTSVILLE, ALABAMA

**A FORTRAN PROGRAM FOR THREE-DEGREE OF FREEDOM
TRAJECTORIES, REFERENCED TO GEOCENTRIC COORDINATES
AND TO AN ARBITRARY POINT ON THE EARTH'S SURFACE**

May, 1963

Prepared Under the Direction Of

**PHYSICAL SCIENCES LABORATORY
ARMY MISSILE COMMAND
REDSTONE ARSENAL, ALABAMA
(AMM Systems Division Scope of Work SW-Z-48-61)**

By

**SCIENTIFIC RESEARCH LABORATORIES
BROWN ENGINEERING COMPANY, INC.**

Technical Note R-48

Prepared By

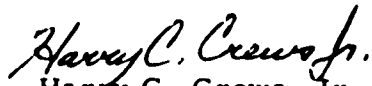
H. M. Minshew

ABSTRACT

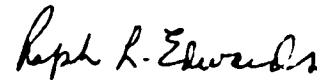
This report describes a FORTRAN II computer program for generating synthetic ballistic vehicle trajectories. The vehicle is considered to have a constant ballistic coefficient and to be under the influence of gravitational, aerodynamic, centrifugal and Coriolis forces. The program contains provisions for computing the trajectory in the reference frame of an arbitrarily located radar station.

A copy of the program may be obtained from the Scientific Programming Library, Program No. SP-66.

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LIST OF SYMBOLS

A	Vehicle reference area
A_z	Vehicle azimuth angle (measured in radar system)
C_D	Drag coefficient (a characteristic of the vehicle)
E	Vehicle elevation angle (measured in radar system)
g_0	Acceleration of gravity at sea-level
H	Height above the earth's surface
k	A force constant defined on page 4
$\vec{i}, \vec{j}, \vec{k}$	Unit vectors along the X, Y, Z axes respectively of the earth-fixed coordinate system
$\vec{R}$	Radius vector from earth's center to the vehicle
$\vec{R}_0$	A previous value of $\vec{R}$ used to compute ground range
R_1	Slant range to vehicle measured from radar site
$\vec{R}_1$	R_1 in vector form
$\dot{R}_1$	Time rate of change of the slant range
R_e	Mean radius of the earth
$R_e(\phi)$	Radius of the earth at latitude ϕ
R_{er}	Radius of the earth at the latitude of the radar site
S	Ground range defined on page 7
ΔS	An increment of ground range
t	Time
Δt	An increment of time

LIST OF SYMBOLS (cont.)

$\vec{V}$	Vector velocity of the vehicle in the earth-fixed X, Y, Z system
V	Magnitude of $\vec{V}$
$V_{x_m}, V_{y_m}, V_{z_m}$	Velocity components in the local reference system of the vehicle
W	Weight of the vehicle
X, Y, Z	Earth-fixed coordinate system (Figure 1)
$\dot{X}, \dot{Y}, \dot{Z}$	First time derivatives of X, Y, Z
$\ddot{X}, \ddot{Y}, \ddot{Z}$	Second time derivatives of X, Y, Z
X_m, Y_m, Z_m	Local reference system of the vehicle (Figure 2)
X_r, Y_r, Z_r	Coordinate of the radar in the earth-fixed X, Y, Z system
X_l, Y_l, Z_l	Radar coordinate system (Figure 1)
β	Ballistic coefficient
γ	Velocity aspect angle defined on page 11
δ	Vehicle re-entry angle
θ	Longitude of the vehicle
θ_r	Longitude of the radar
μ	A constant defined on page 4
ξ	Angle between $\vec{R}$ and $\vec{R}_0$ used in computing ground range
ρ	Atmospheric density

LIST OF SYMBOLS (cont.)

ϕ	Latitude of the vehicle
ϕ_r	Latitude of the radar
ψ	Bearing angle of the vehicle
ω	Earth's rotation rate

NOTE: With the exception of ground range all distances are in feet.
Ground range is in nautical miles. β has units of lb/ft^2
and ρ is in slugs/ft^2

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INTRODUCTION

This trajectory program is intended for use when the vehicle can be considered as a point mass under the influence of gravity, atmospheric drag force, Coriolis force and centrifugal force. The input parameters have been chosen to be as few in number as possible and, at the same time, the ones most often used. For instance, the initial position of the vehicle is determined by its longitude as measured from Greenwich, its latitude above or below the equator and its height above the earth's surface. The position of the vehicle in a radar system is calculated from a knowledge of the longitude and latitude of the radar site.

The equations of motion, coordinate transformations and auxiliary computations are contained in the main body of the report. Appendix A contains the numerical integration procedure used to solve the equations of motion. Appendix B contains a list of the Fortran symbols and corresponding mathematical symbols, a complete listing of the Fortran statements and a flow diagram of the program. The input-output quantities are also defined in Appendix B.

The author would like to thank Mr. Thomas J. Kroupa III for his assistance in programming the equations for an IBM 1410 computer.

EQUATIONS OF MOTION

The trajectory of the vehicle is referenced to a right-handed rectangular co-ordinate system, x, y, z , rigidly connected to the rotating earth and with the origin at the earth's center. The z -axis is along the earth's polar axis and the xy plane is in the plane of the equator with the x -axis located at the meridian of Greenwich. (See Figure 1).

The forces acting on the vehicle in this system are gravitational, air resistance, Coriolis, and centrifugal. With the assumption that the force due to air resistance varies as $-kV^2$, the equations of motion along each of the co-ordinate axes are: (Reference 1)

$$\ddot{x} = \frac{-\mu^2 x}{(x^2 + y^2 + z^2)^{3/2}} - k \dot{x}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} + 2\dot{y}\omega + \omega^2 x \quad (1)$$

$$\ddot{y} = \frac{-\mu^2 y}{(x^2 + y^2 + z^2)^{3/2}} - k \dot{y}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} - 2\dot{x}\omega + \omega^2 y \quad (2)$$

$$\ddot{z} = \frac{-\mu^2 z}{(x^2 + y^2 + z^2)^{3/2}} - k \dot{z}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \quad (3)$$

where $\ddot{x}, \ddot{y}, \ddot{z} = \frac{d^2 x}{dt^2}, \frac{d^2 y}{dt^2}, \frac{d^2 z}{dt^2}$ respectively, and

$\dot{x}, \dot{y}, \dot{z} = \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$ respectively.

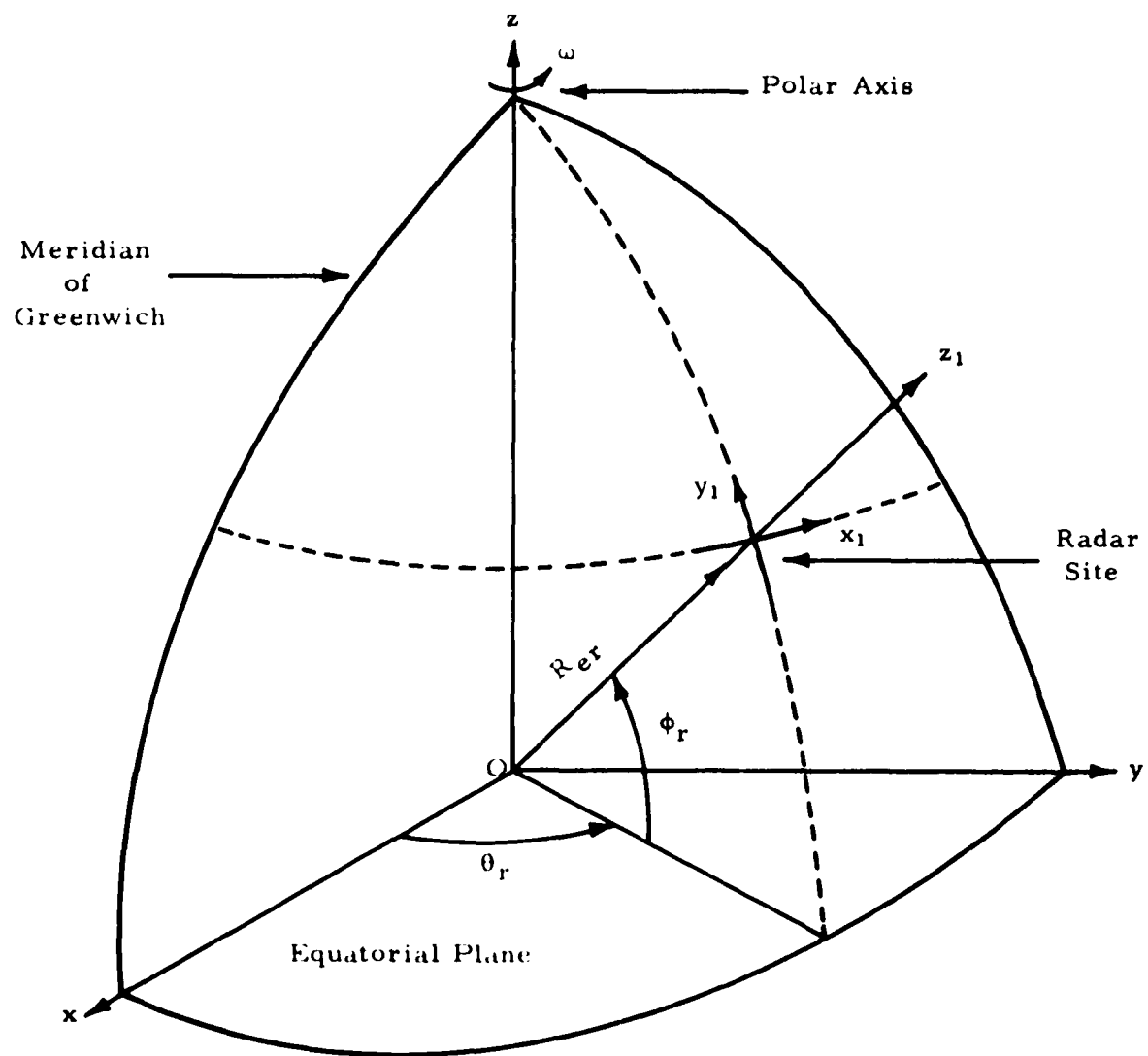


Figure 1

Relationship Between the Earth-Fixed x, y, z System
and the Radar x_1, y_1, z_1 System

$$\mu^2 = g_0 R_e^2$$

g_0 = acceleration of gravity at sea-level

R_e = mean radius of the earth

$$k = \frac{1}{2} g_0 \rho / \beta$$

ρ = atmospheric density
 ρ is computed as a function of altitude by a subroutine based on the ARDC Model Atmosphere, 1959 (Ref. 4)

$\beta = W/C_D A$, the ballistic coefficient

W = the weight of the vehicle

C_D = drag coefficient

A = reference area

ω = earth's rotation rate

Equations (1), (2), and (3) were numerically integrated by the fourth order method of Runge-Kutta as outlined in Appendix A. To start the integration procedure, a point in the 7-dimensional configuration space $t \ x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}$ must be known. This point is determined from the usual earth referenced trajectory parameters (speed, altitude, latitude, longitude, bearing angle and re-entry angle) by the following transformations. (See Figure 2)

$$x = [R_e(\phi) + H] \cos \phi \cos \theta$$

$$y = [R_e(\phi) + H] \cos \phi \sin \theta \tag{4}$$

$$z = [R_e(\phi) + H] \sin \theta$$

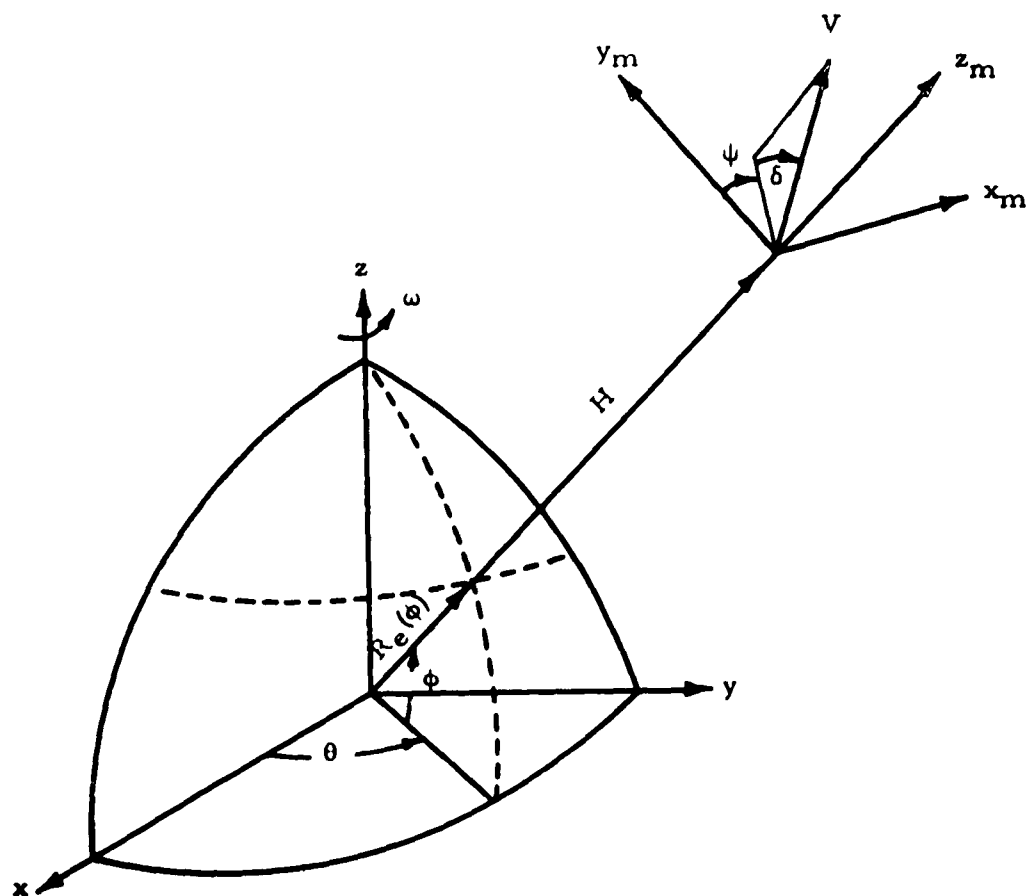


Figure 2

Relationship Between the Earth-Fixed x, y, z System
and the Vehicle Local Reference System x_m, y_m, z_m

$$\begin{vmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{vmatrix} = \begin{vmatrix} -\sin \theta & -\sin \phi \cos \theta & \cos \phi \cos \theta \\ \cos \theta & -\sin \phi \sin \theta & \cos \phi \sin \theta \\ 0 & \cos \phi & \sin \phi \end{vmatrix} \begin{vmatrix} V_{x_m} \\ V_{y_m} \\ V_{z_m} \end{vmatrix} \quad (5)$$

where $V_{x_m} = V \cos \delta \sin \psi$

$V_{y_m} = V \cos \delta \cos \psi$

$V_{z_m} = V \sin \psi$

$\theta =$ longitude of the vehicle

$\phi =$ latitude of the vehicle

$\delta =$ re-entry angle (positive upward - see Figure 2)

$\psi =$ bearing angle (positive clockwise from north - see Figure 2)

$V_{x_m}, V_{y_m},$

$V_{z_m} =$ the components of V in the vehicle's local reference system - see Figure 2

$H =$ altitude above the earth's surface

$R_e(\phi) =$ radius of the earth at latitude ϕ

$R_e(\phi) = 20855967(1 - .00672267 \cos^2 \phi)^{-\frac{1}{2}}$

Equations (4) and (5) are integral parts of the computer program.

The vehicle's position at any time during the integration is

given by:

$$H = (x^2 + y^2 + z^2)^{\frac{1}{2}} - R_e(\phi) \quad (6)$$

$$\phi = \tan^{-1} \frac{z}{(x^2 + y^2)^{\frac{1}{2}}} \quad (7)$$

where if z is positive, $0 < \phi \leq 90^\circ$ (north latitude), and if z is negative, $-90^\circ \leq \phi < 0$ (south latitude).

$$\theta = \tan^{-1} \frac{y}{x} \quad (8)$$

To remove the ambiguity from θ , east longitudes were chosen to be positive, and west longitudes negative. The value of θ is determined from Subroutine QUAD by the following scheme:

y	x	θ
+	+	$0^\circ \leq \theta \leq 90^\circ$
+	-	$90^\circ \leq \theta \leq 180^\circ$
-	+	$-90^\circ \leq \theta \leq 0^\circ$
-	-	$-180^\circ \leq \theta \leq -90^\circ$

Ground range (S) is defined to be the distance traveled from the initial point along the earth's surface and is computed in increments as follows:

$$\begin{aligned} \text{Let } \vec{R} &= \vec{i}x + \vec{j}y + \vec{k}z \text{ at time } t \\ \text{and } \vec{R}_0 &= \vec{i}x_0 + \vec{j}y_0 + \vec{k}z_0 \text{ at time } t - \Delta t. \end{aligned}$$

Then:

$$|\vec{R}_0 \times \vec{R}| = |\vec{R}_0| |\vec{R}| \sin \xi$$

where ξ is the angle between the two vectors. Since the computation

interval Δt is very small, $\vec{R}$ will differ very little from $\vec{R}_0$, and
and $\sin \xi \cong \xi$.

Thus,

$$\xi \cong |\vec{R}_0 \times \vec{R}| / |\vec{R}_0| |\vec{R}|, \text{ and}$$

$$\Delta S \cong R_e \xi \quad . \quad (9)$$

ΔS is summed at the end of each computation interval to give S .

The re-entry angle, δ , is given at any time by:

$$\begin{aligned} \vec{R} \cdot \vec{V} &= |\vec{R}| |\vec{V}| \cos (\pi/2 - \delta) \\ &= |\vec{R}| |\vec{V}| \sin \delta \end{aligned}$$

$$\delta = \sin^{-1} \frac{\vec{R} \cdot \vec{V}}{|\vec{R}| |\vec{V}|} \quad (10)$$

where:

$$\vec{R} = \vec{i}x + \vec{j}y + \vec{k}z$$

and,

$$\vec{V} = \vec{i}x + \vec{j}y + \vec{k}z \quad .$$

δ is defined to be positive when above the local horizontal and negative when below. (See Figure 2).

TRAJECTORY PARAMETERS IN A RADAR REFERENCE SYSTEM

The co-ordinate system x_1, y_1, z_1 with origin O_1 at the radar is defined as follows: (See Figure 1)

x_1, y_1, z_1 are co-ordinate axes with origin O_1 at the surface of the earth, the x_1, y_1 plane is perpendicular to a radius vector drawn from the center of the earth and the z_1 axis is along the radius vector, the positive direction for x_1 and y_1 are taken to be due east and due north respectively.

For a station at latitude ϕ_r and longitude θ_r , the co-ordinates of O_1 in the earth-fixed x, y, z system are:

$$\begin{aligned} x_r &= R_{er} \cos \phi_r \cos \theta_r \\ y_r &= R_{er} \cos \phi_r \sin \theta_r \\ z_r &= R_{er} \sin \phi_r \end{aligned} \tag{11}$$

where R_{er} is the value of $R_e(\phi)$ at ϕ_r .

Using the standard equation for translation and rotation of co-ordinate axes, the following relationship between the two systems is obtained. From x, y, z to x_1, y_1, z_1 :

$$\begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix} = \begin{vmatrix} -\sin \theta_r & \cos \theta_r & 0 \\ -\sin \phi_r \cos \theta_r & -\sin \phi_r \sin \theta_r & \cos \phi_r \\ \cos \phi_r \cos \theta_r & \cos \phi_r \sin \theta_r & \sin \phi_r \end{vmatrix} \begin{vmatrix} x - x_r \\ y - y_r \\ z - z_r \end{vmatrix} \tag{12}$$

from x_1, y_1, z_1 to x, y, z :

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} -\sin \theta_r & -\sin \phi_r \cos \theta_r & \cos \phi_r \cos \theta_r \\ \cos \theta_r & -\sin \phi_r \sin \theta_r & \cos \phi_r \sin \theta_r \\ 0 & \cos \phi_r & \sin \phi_r \end{vmatrix} \begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix} + \begin{vmatrix} x_r \\ y_r \\ z_r \end{vmatrix} \quad (13)$$

In the computer program, these transformations are executed by the subroutines COOD and COODI. Thus, if one wishes to define the radar system in some other manner, only the subroutines will have to be changed.

After the vehicle's position has been transformed from the x, y, z system to the x_1, y_1, z_1 system, the slant range, azimuth angle, and elevation angle are computed as follows:

$$R_1 = (x_1^2 + y_1^2 + z_1^2)^{\frac{1}{2}} \quad (14)$$

$$E_f = \tan^{-1} [z_1 / (x_1^2 + y_1^2)^{\frac{1}{2}}] \quad (15)$$

$$A_z = \tan^{-1} \left(\frac{x_1}{y_1} \right) \quad (16)$$

E_f ranges from 0° to 90° and is positive if the vehicle is above the horizon.

A_z ranges from 0° to 360° and is measured positive clockwise from north.

R_1 , E_f , and A_z are computed in subroutine RAE. The comments made about COOD and COODI also apply to RAE.

$\vec{R}_1$ can be expressed in the x, y, z system as:

$$\vec{R}_1 = \vec{i} (x - x_r) + \vec{j} (y - y_r) + \vec{k} (z - z_r) ,$$

and the velocity vector in the same system is:

$$\vec{V} = \vec{i} x + \vec{j} y + \vec{k} z .$$

Using these two equations, the velocity aspect angle and the range rate can be computed.

The velocity aspect angle (angle between the radar line of sight and the velocity vector) is given by:

$$\gamma = \cos^{-1} \frac{\vec{R}_1 \cdot \vec{V}}{|\vec{R}_1| |\vec{V}|} \quad (17)$$

The component of $|\vec{V}|$ along R_1 is the range-rate ($\dot{R}_1$). Thus,

$$\dot{R}_1 = |\vec{V}| \cos \gamma \quad (18)$$

Since $\dot{R}_1$ is negative when the vehicle is approaching the radar, γ is chosen to range from 0° to 180° . $\gamma = 0^\circ$ when the vehicle is going directly away from the radar, and $\gamma = 180^\circ$ when the vehicle is headed straight in.

CONCLUSIONS

The output of the computer program has been compared to actual radar data and found to be in good agreement. It is felt that the program will be useful for generating theoretical slowdown curves and for determining range, range rates, and look-angles from arbitrary radar locations.

APPENDIX A

NUMERICAL INTEGRATION OF THE EQUATIONS OF MOTION

The method described below in the classic fourth order procedure of Runge-Kutta (Reference 2). Only one point on the integral curves is needed to start the integration, and with the aid of high speed computing machines, any degree of accuracy can be achieved by choosing a sufficiently small increment of the independent variable.

Writing equations (1), (2), and (3) as:

$$\ddot{x} = f_1(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \quad (1A)$$

$$\ddot{y} = f_2(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \quad (2A)$$

$$\ddot{z} = f_3(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \quad (3A)$$

The integration proceeds as follows.

Let t take on an increment Δt ; then $x, y, z, \dot{x}, \dot{y},$ and $\dot{z}$ receive increments $K_1, K_2, K_3, K_4, K_5,$ and K_6 respectively.

$$K_1 = 1/6 (k_{11} + 2k_{21} + 2k_{31} + k_{41}) \quad (4A)$$

$$K_2 = 1/6 (k_{12} + 2k_{22} + 2k_{32} + k_{42}) \quad (5A)$$

$$K_3 = 1/6 (k_{13} + 2k_{23} + 2k_{33} + k_{43}) \quad (6A)$$

$$K_4 = 1/6 (k_{14} + 2 k_{24} + 2 k_{34} + k_{44}) \quad (7A)$$

$$K_5 = 1/6 (k_{15} + 2 k_{25} + 2 k_{35} + k_{45}) \quad (8A)$$

$$K_6 = 1/6 (k_{16} + 2 k_{26} + 2 k_{36} + k_{46}) \quad (9A)$$

$$k_{11} = \dot{x} \Delta t \quad (10A)$$

$$k_{12} = \dot{y} \Delta t \quad (11A)$$

$$k_{13} = \dot{z} \Delta t \quad (12A)$$

$$k_{14} = f_1(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \Delta t \quad (13A)$$

$$k_{15} = f_2(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \Delta t \quad (14A)$$

$$k_{16} = f_3(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \Delta t \quad (15A)$$

$$k_{21} = (\dot{x} + \frac{1}{2} k_{14}) \Delta t \quad (16A)$$

$$k_{22} = (\dot{y} + \frac{1}{2} k_{15}) \Delta t \quad (17A)$$

$$k_{23} = (\dot{z} + \frac{1}{2} k_{16}) \Delta t \quad (18A)$$

$$k_{24} = f_1(t + \frac{1}{2} \Delta t, x + \frac{1}{2} k_{11}, y + \frac{1}{2} k_{12}, z + \frac{1}{2} k_{13}, \dot{x} + \frac{1}{2} k_{14}, \dot{y} + \frac{1}{2} k_{15}, \dot{z} + \frac{1}{2} k_{16}) \Delta t \quad (19A)$$

$$k_{25} = f_2 \left(t + \frac{1}{2} \Delta t, x + \frac{1}{2} k_{11}, y + \frac{1}{2} k_{12}, z + \frac{1}{2} k_{13}, \dot{x} + \frac{1}{2} k_{14}, \right. \\ \left. \dot{y} + \frac{1}{2} k_{15}, \dot{z} + \frac{1}{2} k_{16}, \right) \Delta t \quad (20A)$$

$$k_{26} = f_3 \left(t + \frac{1}{2} \Delta t, x + \frac{1}{2} k_{11}, y + \frac{1}{2} k_{12}, z + \frac{1}{2} k_{13}, \dot{x} + \frac{1}{2} k_{14}, \right. \\ \left. \dot{y} + \frac{1}{2} k_{15}, \dot{z} + \frac{1}{2} k_{16}, \right) \Delta t \quad (21A)$$

$$k_{31} = (\dot{x} + \frac{1}{2} k_{24}) \Delta t \quad (22A)$$

$$k_{32} = (\dot{y} + \frac{1}{2} k_{25}) \Delta t \quad (23A)$$

$$k_{33} = (\dot{z} + \frac{1}{2} k_{26}) \Delta t \quad (24A)$$

$$k_{34} = f_1 \left(t + \frac{1}{2} \Delta t, x + \frac{1}{2} k_{21}, y + \frac{1}{2} k_{22}, z + \frac{1}{2} k_{23}, \dot{x} + \frac{1}{2} k_{24}, \right. \\ \left. \dot{y} + \frac{1}{2} k_{25}, \dot{z} + \frac{1}{2} k_{26} \right) \Delta t \quad (25A)$$

$$k_{35} = f_2 \left(t + \frac{1}{2} \Delta t, x + \frac{1}{2} k_{21}, y + \frac{1}{2} k_{22}, z + \frac{1}{2} k_{23}, \dot{x} + \frac{1}{2} k_{24}, \right. \\ \left. \dot{y} + \frac{1}{2} k_{25}, \dot{z} + \frac{1}{2} k_{26} \right) \Delta t \quad (26A)$$

$$k_{36} = f_3 \left(t + \frac{1}{2} \Delta t, x + \frac{1}{2} k_{21}, y + \frac{1}{2} k_{22}, z + \frac{1}{2} k_{23}, \dot{x} + \frac{1}{2} k_{24}, \right. \\ \left. \dot{y} + \frac{1}{2} k_{25}, \dot{z} + \frac{1}{2} k_{26} \right) \Delta t \quad (27A)$$

$$k_{41} = (\dot{x} + k_{34}) \Delta t \quad (28A)$$

$$k_{42} = (\dot{y} + k_{35}) \Delta t \quad (29A)$$

$$k_{43} = (\dot{z} + k_{36}) \Delta t \quad (30A)$$

$$k_{44} = f_1 \left(t + \Delta t, x + k_{31}, y + k_{32}, z + k_{33}, \dot{x} + k_{34}, \dot{y} + k_{35}, \right. \\ \left. \dot{z} + k_{36} \right) \Delta t \quad (31A)$$

$$k_{45} = f_2(t + \Delta t, x + k_{31}, y + k_{32}, z + k_{33}, \dot{x} + k_{34}, \dot{y} + k_{35}, \\ z + k_{36}) \Delta t \quad (32A)$$

$$k_{46} = f_3(t + \Delta t, x + k_{31}, y + k_{32}, z + k_{33}, \dot{x} + k_{34}, \dot{y} + k_{35}, \\ \dot{z} + k_{36}) \Delta t \quad (33A)$$

On the first pass through equations 10A to 33A, the variables t , x , y , z , $\dot{x}$, $\dot{y}$, $\dot{z}$ will have their initial values. After equation 33A has been executed, t is incremented by Δt , x by K_1 , y by K_2 , z by K_3 , $\dot{x}$ by K_4 , $\dot{y}$ by K_5 , and $\dot{z}$ by K_6 and the procedure beginning at 10A is repeated.

During portions of the trajectory where the acceleration is small Δt may be chosen fairly large (around $\frac{1}{4}$ sec), but when the acceleration is large, Δt must be small (around 1/100 sec).

Equations f_1 , f_2 , and f_3 are evaluated in the program by subroutine FU123.

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APPENDIX B

List of FORTRAN Symbols

Inputs to the Program

Output of the Program

Sample Print-Out

List of FORTRAN Statements

Flow Diagram

I
I
I
I
I
I

LIST OF FORTRAN SYMBOLS

<u>FORTRAN Symbols</u>	<u>Mathematical Symbols</u>
AK1, AK2, AK3	K_1, K_2, K_3
AK4, AK5, AK6	K_4, K_5, K_6
AK11, AK12, AK13	k_{11}, k_{12}, k_{13}
AK14, AK15, AK16	k_{14}, k_{15}, k_{16}
AK21, AK22, AK23	k_{21}, k_{22}, k_{23}
AK24, AK25, AK26	k_{24}, k_{25}, k_{26}
AK31, AK32, AK33	k_{31}, k_{32}, k_{33}
AK34, AK35, AK36	k_{34}, k_{35}, k_{36}
AK41, AK42, AK43	k_{41}, k_{42}, k_{43}
AK44, AK45, AK46	k_{44}, k_{45}, k_{46}
ALA	ϕ_r
ALO	θ_r
AT	$\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2$
AZ	A_z
BC	β
BETA	ψ
DEL	δ
EL	Ξ_l
DT	Δt
Gamma	γ

FORTTRAN SymbolsMathematical Symbols

H

H

PHI

 ϕ

RAN

S

RER

 $R_e(\phi) \text{ at } \phi = \phi_r$

RO

 ρ

RV

 $H + R_e(\phi)$

R1

 R_1

RR1

 $\dot{R}_1$

T

t

THETA

 θ

V

V

VT

 $(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}}$

VX, VY, VZ

 $V_{x_m}, V_{y_m}, V_{z_m}$

X, Y, Z

X, Y, Z

XD, YD, ZD

 $\dot{X}, \dot{Y}, \dot{Z}$

XDD, YDD, ZDD

 $\ddot{X}, \ddot{Y}, \ddot{Z}$

XR, YR, ZR

 X_r, Y_r, Z_r

X1, Y1, Z1

 X_1, Y_1, Z_1

INPUTS TO THE PROGRAM

The initial conditions are read into the computer from four input cards containing the following information:

Card #1

BC, DEL, BETA, V

BC = the ballistic coefficient ($W/C_D A$) in lbs/ft^2

DEL = the re-entry angle (negative when re-entering) in degrees

BETA = the velocity bearing angle (positive clockwise from north) in degrees

V = magnitude of the velocity in ft/sec

Card #2

THETA, PHI, H

THETA = longitude of the vehicle in degrees. If the longitude is given as θ degrees west of Greenwich, change to $360^\circ - \theta$.

PHI = latitude of vehicle in degrees - input as positive when above the Equator and negative when below

H = altitude above the earth's surface in feet

Card #3

ALO, ALA

ALO = longitude of radar site (input in the same manner as THETA)

ALA = latitude of the radar site (input in the same manner as PHI)

Card #4

DT, HEND, N

DT = increment of the independent variable time

HEND = altitude at which it is desired that the computation
be halted

N = print rate - controls the number of times through the
integration loop before printing. For instance, if DT
is 1/10 sec and trajectory data is desired at 1 sec
intervals, then N should be read in as 10.

OUTPUT OF THE PROGRAM

The quantities shown on the sample print-out are defined as follows:

TIME = Elapsed time in seconds from initial point

LATITUDE = Latitude of the vehicle in degrees (positive when north of the equator)

LONGITUDE = Longitude of the vehicle in degrees (positive when east of Greenwich)

ALTITUDE = Height above the earth's surface in feet

TOTAL ACCELERATION = Absolute value of the acceleration (ft/sec^2) in the earth-fixed reference system

TOTAL VELOCITY = Absolute value of the velocity (ft/sec) in the earth-fixed reference system

GROUND RANGE = Distance traveled over the earth's surface from the initial point (nautical miles)

RE-ENTRY ANGLE = Angle between the velocity vector and the local horizontal (degrees)

X RADAR

Y RADAR = Co-ordinates of the vehicle in the radar reference system (feet)

AZIMUTH ANGLE

ELEVATION ANGLE = Radar look-angles in degrees (azimuth is no. of degrees clockwise from north, and elevation is no. of degrees above the horizon).

GAMMA = Angle between the radar line-of-sight and the velocity vector (degrees)

SLANT RANGE = Distance from the radar to the vehicle (feet)

RANGE RATE = Rate of change of the slant range (ft/sec)

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

FORTRAN RUN

BCP

LINE # C

IPAGE # 1

READ 100,BC,DEL,BETA,V

READ 100,THETA,PHI,H

READ 100,ALO,ALA

READ 101,DT,HEND,N

PRINT 1C7

PRINT 1C6

PRINT 1C2,H,V ,DEL,BC,BETA,PHI,THETA

RAN#0.

TWO.

M#0

DEL#DEL*.01745

BETA#BETA*.01745

THETA#THETA*.01745

PHI#PHI*.01745

ALO#ALO*.01745

ALA#ALA*.01745

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
RV#H&20855967./SQRTF%1.-.00672267*COSF%PHI□**2□
KER#20855967./SQRTF%1.-.00672267*COSF%ALAO□**2□
XR#RER*COSF%ALAO□*COSF%ALAO□
YR#RER*COSF%ALAO□*SINF%ALAO□
ZR#RER*SINF%ALAO□
VX#V*COSF%DELO□*SINF%BETA□
VY#V*COSF%DELO□*COSF%BETA□
VZ#V*SINF%DELO□
CALL COCO%XD ,YD ,ZD ,VX,VY,VZ,THETA,PHI□
X#RV*COSF%PHI□*COSF%THETA□
Y#RV*COSF%PHI□*SINF%THETA□
Z#RV*SINF%PHI□
1 XD#X
YD#Y
ZD#Z
RV0#RV
CALL ALT%H,RO,A,B,C,D,E,F,G□
RC#RO/32.174
AK11#XD*DT
AK12#YD*DT
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
AK13#ZD*DT
CALL FU123%AK14,AK15,AK16,X,Y,Z,XD,YD,ZD,RO,BC,DT
AK21#%XD*.5*AK14*DT
AK22#%YD*.5*AK15*DT
AK23#%ZD*.5*AK16*DT
OCALL FU123%AK24,AK25,AK26,X*.5*AK11,Y*.5*AK12,Z*.5*AK13,XD*.5*AK14
1,YD*.5*AK15,ZD*.5*AK16,RO,BC,DT
AK31#%XD*.5*AK24*DT
AK32#%YD*.5*AK25*DT
AK33#%ZD*.5*AK26*DT
OCALL FU123%AK34,AK35,AK36,X*.5*AK21,Y*.5*AK22,Z*.5*AK23,XD*.5*AK24
1,YD*.5*AK25,ZD*.5*AK26,RO,BC,DT
AK41#%XD%AK34*DT
AK42#%YD%AK35*DT
AK43#%ZD%AK36*DT
OCALL FU123%AK44,AK45,AK46,X%AK31,Y%AK32,Z%AK33,XD%AK34,YD%AK35,ZD%
1AK36,RO,BC,DT
AK1#%AK11%2.*AK21%2.*AK31%AK41%/6.
AK2#%AK12%2.*AK22%2.*AK32%AK42%/6.
AK3#%AK13%2.*AK23%2.*AK33%AK43%/6.
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```

AK4#%AK14E2.*AK24E2.*AK34EAK44□/6.
AK5#%AK15E2.*AK25E2.*AK35EAK45□/6.
AK6#%AK16E2.*AK26E2.*AK36EAK46□/6.
X#XEAK1
Y#YEAK2
Z#ZEAK3
XD#XDCAK4
YD#YDEAK5
ZD#ZDEAK6
RV#SQRTF%X=XEY+YEZ+Z□
H#RV-20855967./SQRTF%1.-.00672267*%X=XEY+Y□/RV**2□
B1#%Z=YC-Y+Z0□**2E%X=Z0-Z*XC□**2E%Y=X0-X+Y0□**2
C#SQRTF%B1□/%RV=RV0□
RAN#C*2.078505EE07/6076.1E#RAN
T#T&C#
M#ME1
IF%N-M□2,2,1
2 CALL QUAD%THETA,PHI,X,Y,Z□
M#0
VT#SQRTF%X=XD+XD&YD+YD&ZD+ZD□

```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```

CALL FU123(XDD,YDD,ZDD,X,Y,Z,XD,YD,ZD,RO,BC,I.0
AT#SQRTF(XDD*XDD&YDD*YDD&ZDD*ZDD
D#X*X&Y&Y&Z&Z/D#RV=VT
D1#SQRTF(1.-D#D
DEL#ATANF(D/D1#/.0174533
CALL COCD(1,X1,Y1,Z1,X-XR,Y-YR,Z-ZR,ALO,ALAD
CALL RAE(R1,AZ,EL,X1,Y1,Z1
RR1#X-XR&Y-YR&Z-ZR/D#R1
CGAMA#RR1/VT
SGAMA#SQRTF(1.-CGAMA**2
IF(RR1#12,12,13
13 GAMA#ATANF(SGAMA/CGAMA#/.01745
GO TO 15
12 GAMA#180.-ABSF(ATANF(SGAMA/CGAMA#/.01745#
15 CONTINUE
PRINT 200,T,PHI,AZ
PRINT 201,H,THETA,EL
PRINT 202,AT,DEL,GAMA
PRINT 1111,X1,Y1,Z'
PRINT 203,VT,R1

```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
PRINT 204,RAN,RR1
LINE # LINE & 1
IF%LINE - 601234,1423,1234
1423 LINE # 0
IPAGE # IPAGE & 1
PRINT 1532,IPAGE
1532 FORMAT%1H1,75X,4HPAGE,150
1234 CONTINUE
IF%H-HEND04,4,1
4 PRINT 500
STOP
100 FORMAT%8E15.80
101 FORMAT%2E15.8,13
107 FORMAT%1H1,57X,18HINITIAL CCNDITIONSD
1060FORMAT%1HK,25X,3HALT,5X,8HVELOCITY,3X,10HRE-ENT ANG,7X,6HBAL CO,5X
1,8HBEAR ANG,9X,4HLATO,9X,4HLON00
102 FORMAT%1H ,17X,10E13.50
2000FORMAT%///,16X,4HTIME,18X,E12.5,6X,8HLATITUDE,9X,E12.5,5X,13HAZIMU
1TH ANGLE,5X,E12.50
2010FORMAT%1H ,15X,8HALTITUDE,14X,E12.5,6X,9HLONGITUDE,8X,E12.5,5X,15H
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
1 ELEVATION ANGLE,3X,E12.5□
2020FORMAT%1H ,15X,18HTOTAL ACCELERATION,4X,E12.5,6X,14HRE-ENTRY ANGLE
    1,3X,E12.5,5X,5HGAMMA,13X,E12.5□
2030FORMAT%1H ,15X,14HTOTAL VELCCITY,8X,E12.5,6X,11HSLANT RANGE,6X,E12
    1.5□
204 FORMAT%1H ,15X,12HGROUND RANGE,10X,E12.5,6X,10HRANGE RATE,7X,E12.5
    1□
1111 FORMAT%1H ,15X,8HX RADAR ,14X,E12.5,6X,8HY RADAR ,9X,E12.5,5X,7HZ
    1RADAR,11X,E12.5□
500 FORMAT%1H1,60X,10HEND OF JOB //////////////□
END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE COOD

BCP COOD

SUBROUTINE COOD(A,B,C,D,E,F,O,P)

A#-C*SINF%O-E*SINF%P*COSF%O&F*COSF%P*COSF%O

B#D*COSF%O-E*SINF%P*SINF%O&F*COSF%P*SINF%O

C#I*COSI%P&F*SINF%P

RETURN

END

LIST OF FORTRAN PROGRAM

SUBROUTINE QUAD

BCP QUAD

SUBROUTINE QUAD(A,B,X,Y,Z)

A=ATANF(Y/X)/.01745

IF(Y)1,2,2

1 IF(X)3,4,20

3 A=-180.6A

GO TO 20

4 A=90.

GO TO 20

2 IF(X)6,4,20

6 A=180.6A

GO TO 20

20 B=ATANF(Z/((SQRTF(X*X+Y*Y))))/.01745

RETURN

END

LIST OF FORTRAN PROGRAM

SUBROUTINE RAF

BCP RAE

SUBROUTINE RAE%*A*,*B*,*C*,*X1*,*Y1*,*Z1**□*

A = $\sqrt{\%X1 \cdot X1 \ \& \ Y1 \cdot Y1 \ \& \ Z1 \cdot Z1}$

C = $\text{ATANF}\%Z1 / \sqrt{\%X1 \cdot X1 \ \& \ Y1 \cdot Y1} \cdot .01745$

B = $\text{ATANF}\%X1 / Y1 \cdot .01745$

IF%*X1**□*1,2,2

1 IF%*Y1**□*3,4,5

3 *B* = 180. *EB*

GO TO 15

4 *B* = 90.

GO TO 15

5 *B* = 360. *EB*

GO TO 15

2 IF%*Y1**□*6,4,15

6 *B* = 180. *EB*

GO TO 15

15 RETURN

END

LIST OF FORTRAN PROGRAM

SUBROUTINE COODI

BCP COODI

SUBROUTINE COODI(A,B,C,D,E,F,G,P)

A=D*SINF%G+E*COSE%G

B=D*SINF%P+COSE%G-E*SINF%P+SINF%G*F*COSE%P

C=D*COSE%P+COSE%G+E*COSE%P+SINF%G*F*SINF%P

RETURN

END

LIST OF FORTRAN PROGRAM

SUBROUTINE FU123

BCP FU123

SUBROUTINE FU123(A,B,C,X,Y,Z,XD,YD,ZD,RO,BC,DT)

C2=1.38999091E6

U=.72918296E-04

C3=16.087*RC/BC

D1=X*X+Y*Y+Z*Z*.5

D2=SQRT(XD*XD+YD*YD+ZD*ZD)

A=C2*X/D1-C3*XD*D2*.U*Y+U*X*DT

B=C2*Y/D1-C3*YD*D2*.U*X+U*Y*DT

C=C2*Z/D1-C3*ZD*D2*.U*DT

RETURN

END

LIST OF FORTRAN PROGRAM

SUBROUTINE ALT

BCP ALT

SUBROUTINE ALT%H,RO,PR,CF,FP,TEP,GRA,WM,SOS□

IF%F-40000.□30,30,31

31 RC#0.

RETURN

30 IF%H-40000.□1,2,3

1 X#H/10000.

P3# -0.56721846E-03

P2# -0.95808049E-02

P1# -0.37347339E00

PO# 0.19173574E02

T3# 0.12881606E01

T2# -0.60534827E01

T1# -0.28813737E02

TO# 0.51772365E03

GO TO 50

2 CONTINUE

3 IF%F-80000.□4,5,6

4 CONTINUE

5 X#%F-40000.□/10000.

LIST OF FORTRAN PROGRAM

SUBROUTINE ALT

P3#-0.39193446E-05
P2# 0.25197324E-03
P1#-0.47883423E00
P0# 0.17487069E02
I3# 0.
I2# 0.
I1# 0.
IC# 0.38999000E03
GO TO 50
/ IEXH-160000.#7,8,9
/ X#XH-80000.#/10000.
P3# -0.19423133E-03
P2# 0.99676253E-02
P1# -0.48090213E00
P0# 0.15575633E02
I3# -0.10962868E00
I2# 0.12311886E01
I1# 0.12416913E02
I0# 0.38925805E03
GO TO 50

LIST OF FORTRAN PROGRAM

SUBROUTINE ALT

```
8 CONTINUE
9 IFZH-17500C.D10,11,12
10 CONTINUE
11 X#ZH-16000C.D/10000.
    P3# 0.0000
    P2# 0.16800000E-03
    P1#-0.36283040E&00
    P0# 0.12266563E&02
    T3# 0.
    T2# 0.
    T1# 0.
    TC# 0.50879000E&03
    GC TC 50
12 IFZH-27000C.D13,14,15
13 X#ZH-17500C.D/10000.
    P3# -0.50571711E-03
    P2# -0.71458221E-02
    P1# -0.36369539E&00
    P0# 0.11723187E&02
    T3# 0.12190679E&00
```

LIST OF FORTRAN PROGRAM

SUBROUTINE ALT

```
T2# -0.13745684E01
T1# -0.20357798E02
T0# 0.50755152E03
GO TO 50
14 CONTINUE
15 IF#H-290000.#16,17,18
16 CONTINUE
17 X#H-270000.#/10000.
P3# 0.
P2# 0.25095006E-03
P1#-0.61251035E00
P0# 0.71906764E01
T3# 0.
T2# 0.
T1# 0.
T0# 0.29820000E03
GO TO 50
18 IF#H-350000.#19,20,21
19 X#H-290000.#/10000.
P3# -0.12243014E-03
```

LIST OF FORTRAN PROGRAM

SUBROUTINE ALT

```
P2# 0.16998582E-01
P1# -0.63252519E00
P0# 0.59679151E01
T3# -0.55833219E00
T2# 0.61892744E01
T1# 0.66193506E00
T0# 0.29675713E03
GC TO 50
20 CONTINUE
21 X#H-350000.#/10000.
P3# -0.26309606E-02
P2# 0.43390852E-01
P1# -0.43767363E00
P0# 0.27568534E01
T3# -0.10924334E00
T2# 0.84074361E00
T1# 0.10253056E03
T0# 0.40381341E03
50 IF#H-295000.#22,22,23
22 WM#28.966
```

LIST OF FORTRAN PROGRAM

SUBROUTINE ALT

GO TO 51

23 IF%F-350000.024,25,25

24 WM#28.968858-%0.11714744E-010*X-%0.14285128E-020*X*X

GO TO 51

25 WM#28.848927-%0.26963111E-010*X-%0.89303508E-030*X*X

51 PR#XEXPFX%3*X&P20*X&P10*X&P000/100000.

TEM#%T3*X&T20*X&T10*X&T0

RO#3PR*WM0/%1545.*TEM0

RETURN

SCS#SQRTF%45.0436*PR/RO0

FP#1./1.7406976E&C9*WM/RO

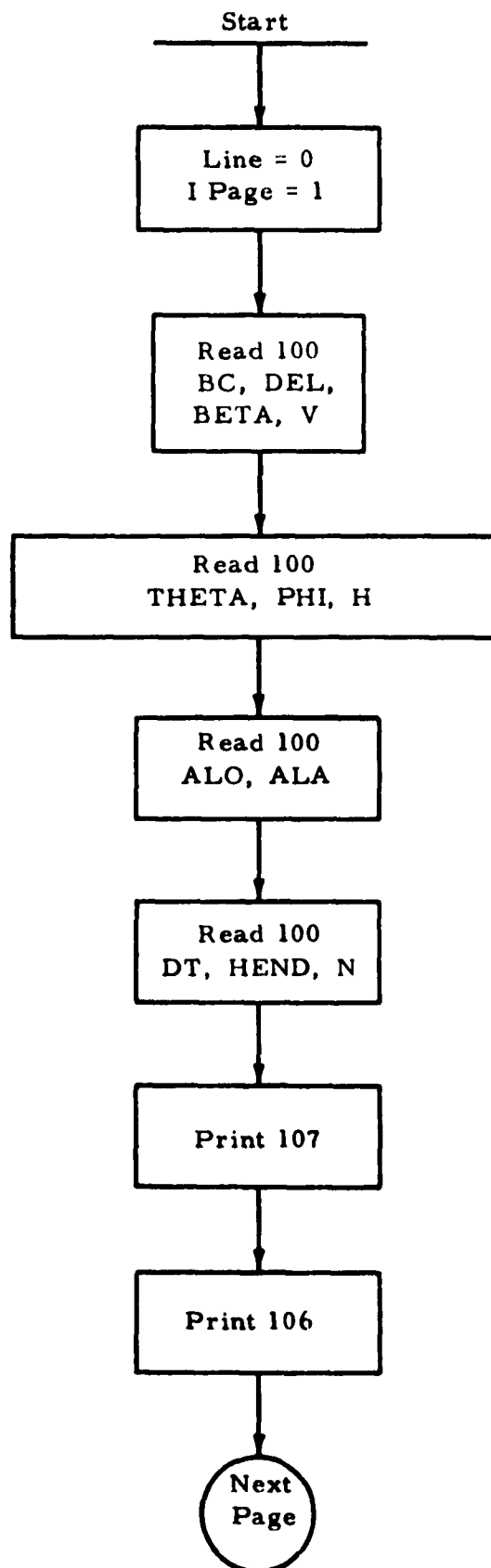
VB#SQRTF%3.6666166E&06*TEM0/WM

CF#VB/FP

GRAN1.3994182E&16/%H&2.085553E&C700.2

RETURN

END



ENTER FROM PREVIOUS PAGE

Print 102
H, V, DEL, BC, BETA
PHI, THETA

RAN = 0
T = 0
M = 0

DEL =
DEL (.01745)

BETA =
BETA (.01745)

THETA =
THETA (.01745)

PHI =
PHI (.01745)

ALO =
ALO (.01745)

Next
Page

ENTER FROM PREVIOUS PAGE

$$\begin{aligned} \text{ALA} &= \\ \text{ALA} &(.01745) \end{aligned}$$

$$\begin{aligned} \text{RV} &= \text{H} + 20855967 / \\ &\sqrt{1 - .00672267 [\text{COSF}(\text{PHI})]^2} \end{aligned}$$

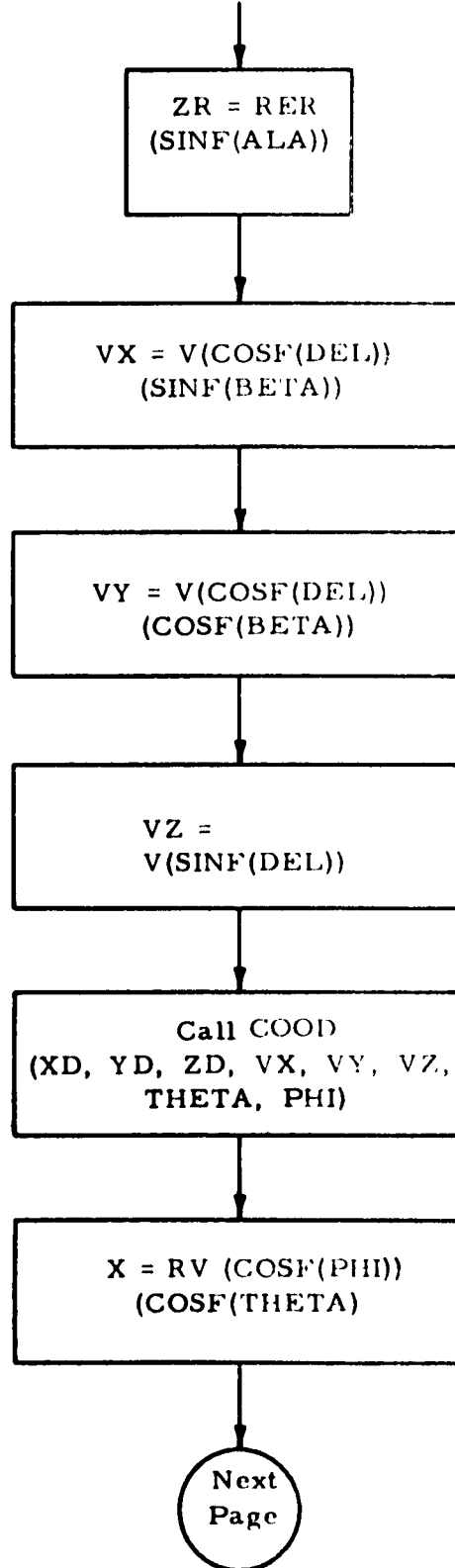
$$\begin{aligned} \text{RER} &= 20855967 / \\ &\sqrt{1 - .00672267 [\text{COSF}(\text{ALA})]^2} \end{aligned}$$

$$\begin{aligned} \text{XR} &= \text{RER} (\text{COSF}(\text{ALA})) \\ &(\text{COSF}(\text{ALO})) \end{aligned}$$

$$\begin{aligned} \text{YR} &= \text{RER} (\text{COSF}(\text{ALA})) \\ &(\text{SINF}(\text{ALO})) \end{aligned}$$

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Page

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ENTER FROM PREVIOUS PAGE

$Y = RV (\cos(\Phi))$
 $(\sin(\Theta))$

$Z = RV (\sin(\Phi))$

Enter No. 1
From Page 54
or 58

$XO = X$

$YO = Y$

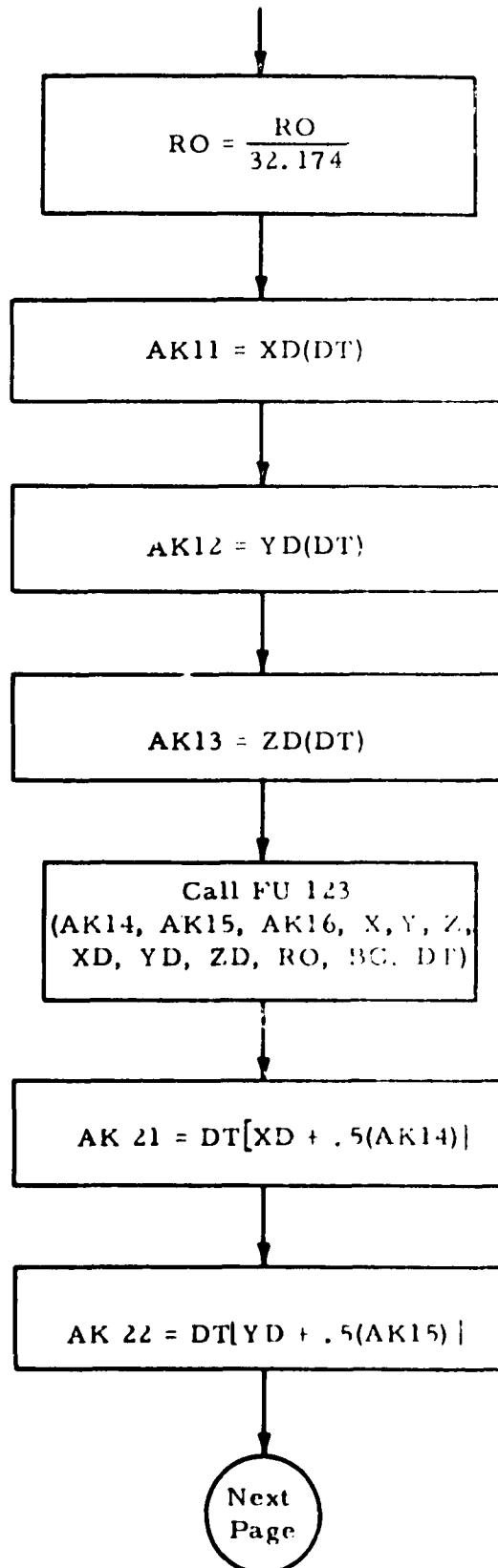
$ZO = Z$

$RVO = RV$

Call Alt
(H, RO, A, B, C,
D, E, F, G)

Next
Page

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$AK23 + DT [ZD + .5(AK16)]$

CALL FU 123
(AK24, AK25, AK26, X + .5 (AK11),
Y + .5 (AK12), Z + .5 (AK13),
XD + .5 (AK14), YD + .5 (AK15),
ZD + .5 (AK16), RO BC, DT)

$AK31 = DT$
 $[XD + .5 (AK24)]$

$AK32 = DT$
 $[YD + .5 (AK25)]$

$AK33 = DT$
 $[ZD + .5 (AK26)]$

CALL FU 123
(AK34, AK35, AK36, X + .5 (AK21),
Y + .5 (AK22), Z + .5 (AK23),
XD + .5 (AK24), YD + .5 (AK25),
ZD + .5 (AK26), RO, BC, DT)

Next
Page

ENTER FROM PREVIOUS PAGE

AK41 = DT
[XD + AK34]

AK43 = DT
[YD + AK35]

AK43 = DT
[ZD + AK36]

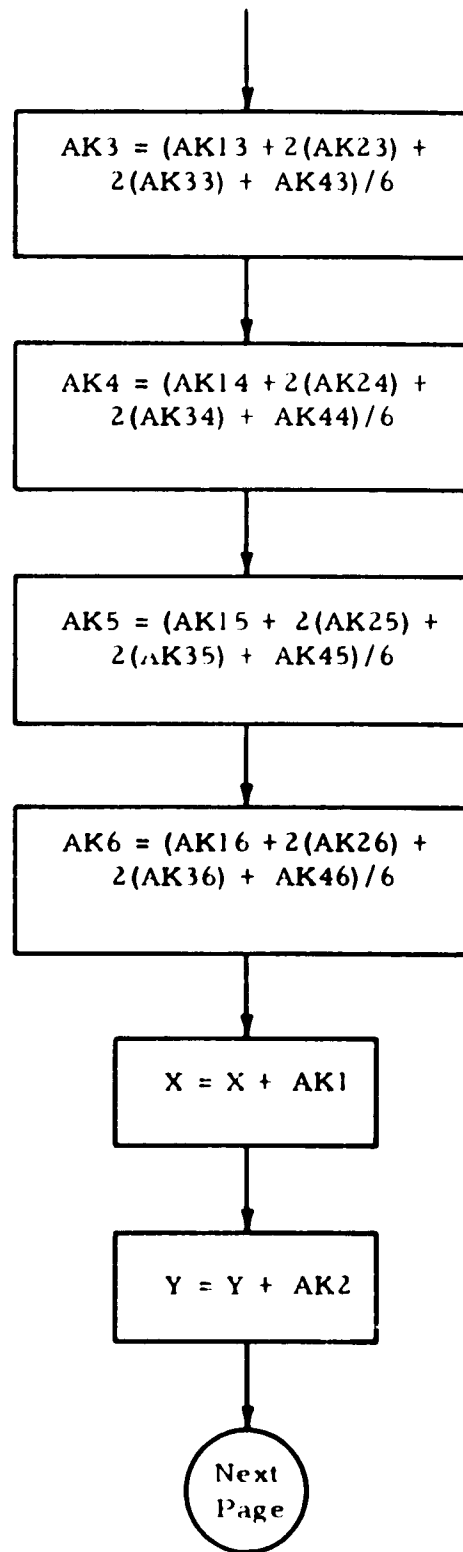
CALL FU 123
(AK44, AK45, AK46,
X+AK31, Y+AK32, Z+AK33,
XD + AK34, YD + AK35,
ZD + AK36, RO, BC, DT)

$AK1 = (AK11 + 2(AK21) +$
 $2(AK31) + AK41)/6$

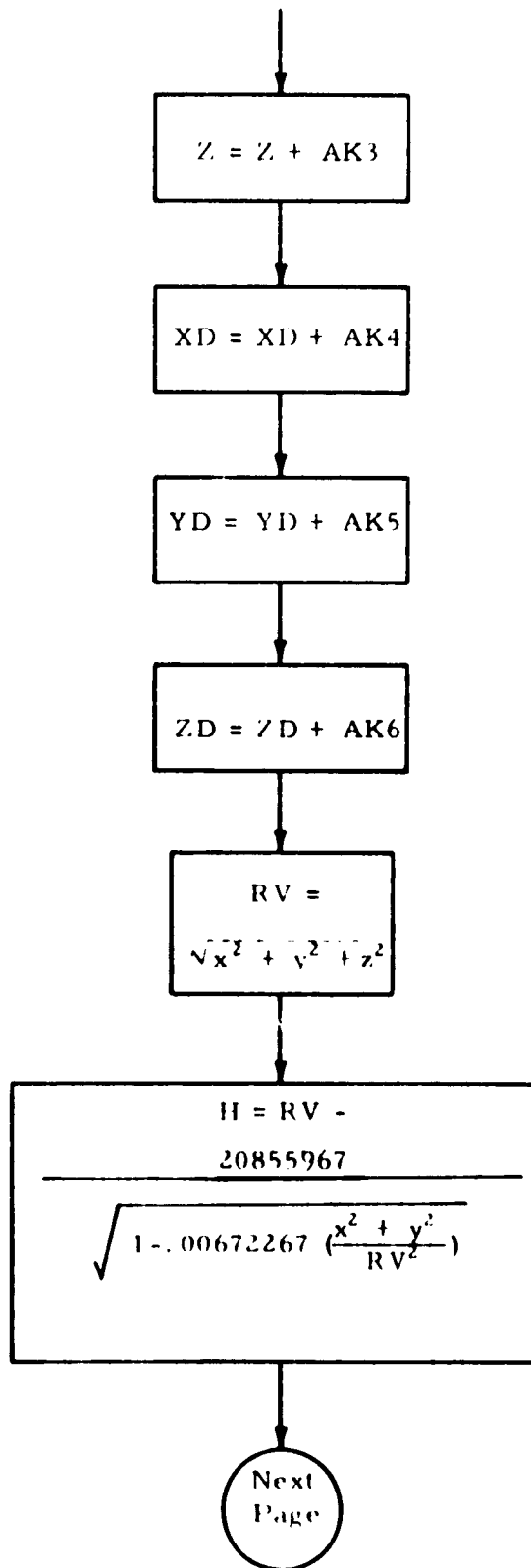
$AK2 = (AK12 + 2(AK22) +$
 $2(AK32) + AK42)/6$

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Page

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$$B1 = [Z(YO) - Y(ZO)]^2 \\ + [X(ZO) - Z(XO)]^2 \\ + [Y(XO) - X(YO)]^2$$

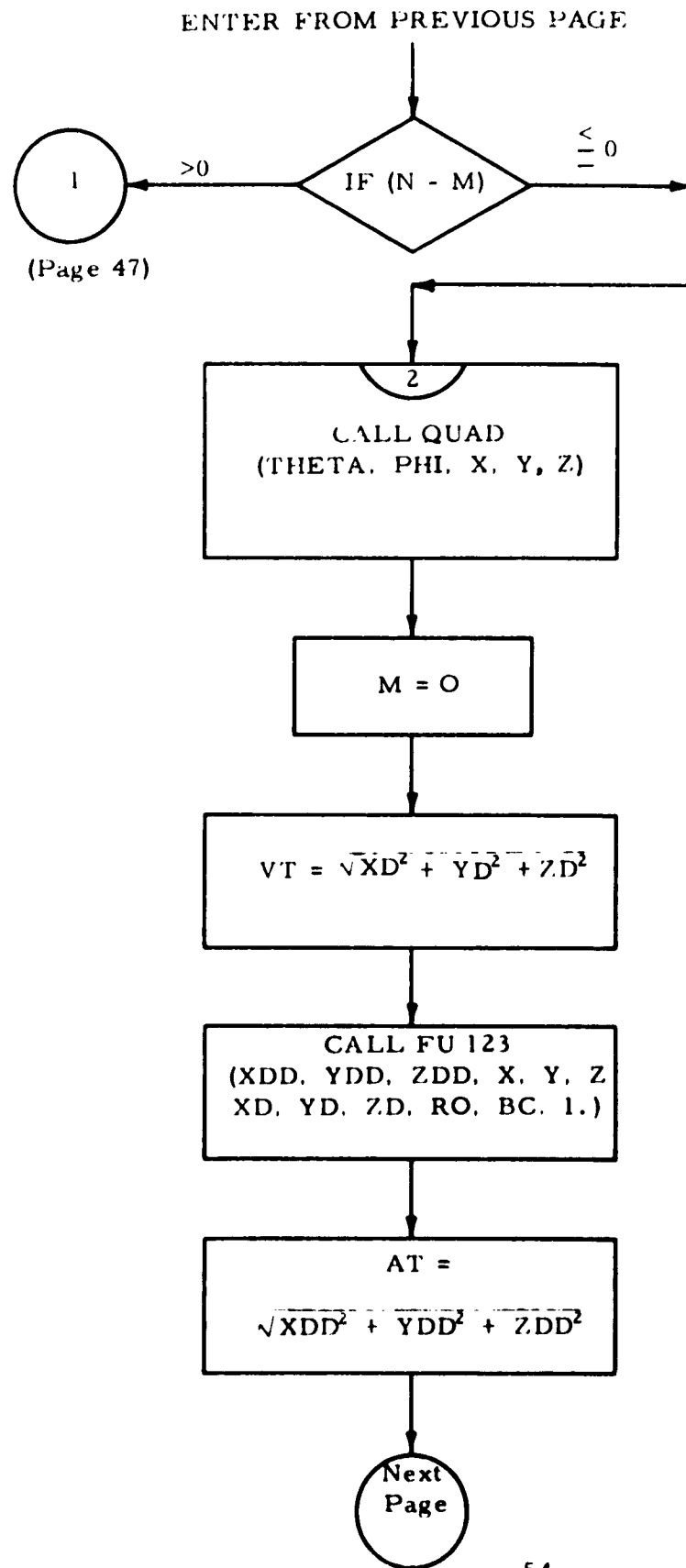
$$C = \frac{\sqrt{B1}}{RV(RVO)}$$

$$RAN = RAN + \\ \frac{C(2.078505E + 07)}{6076.1}$$

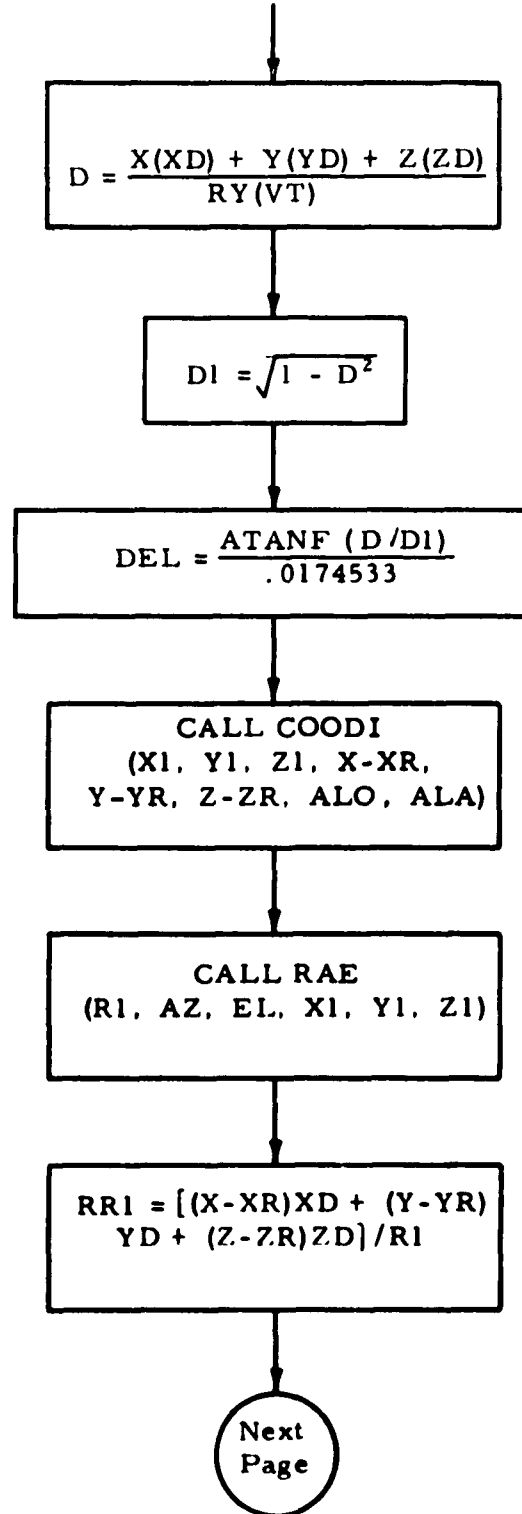
$$T = T + DT$$

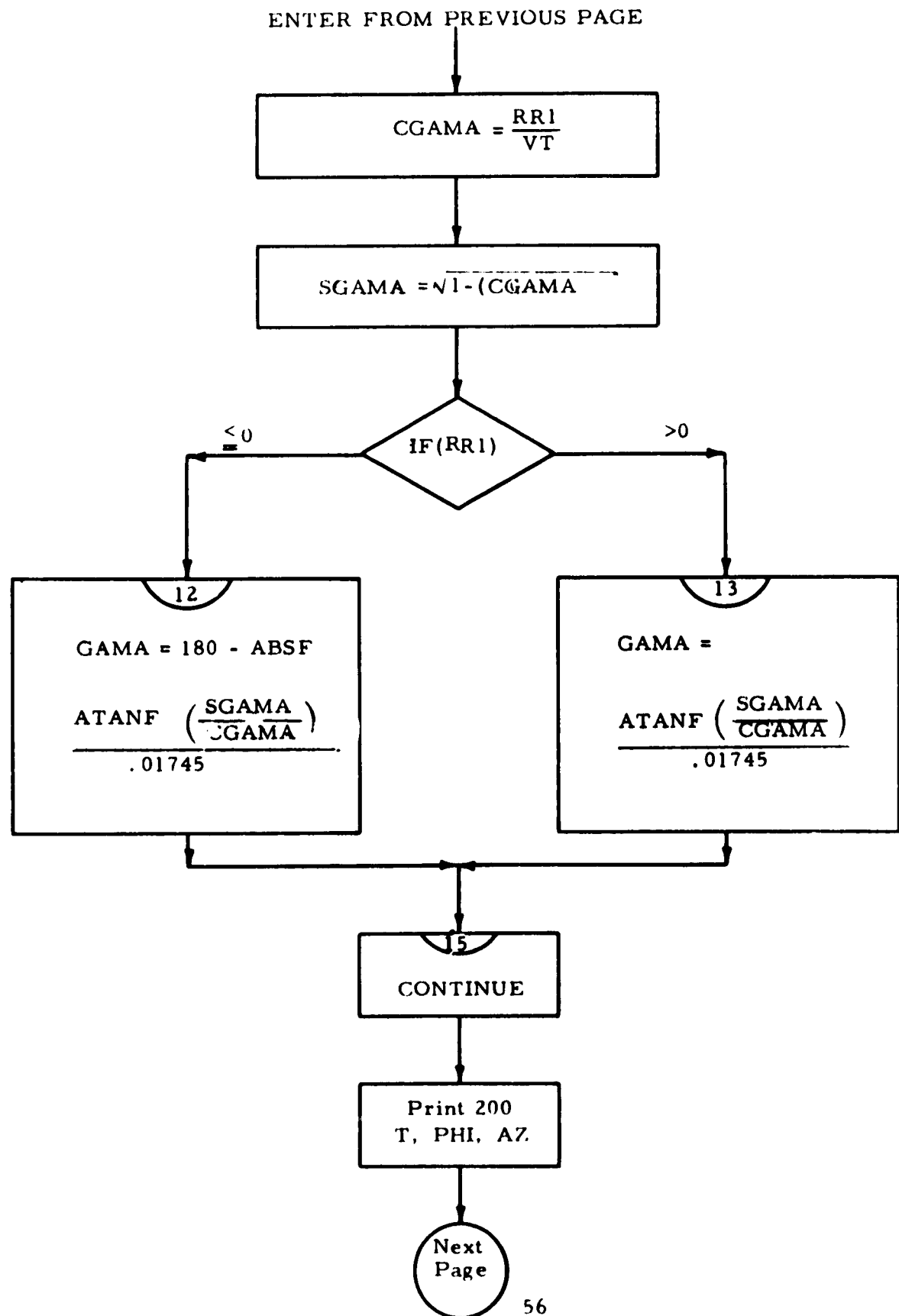
$$M = M + 1$$

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Page

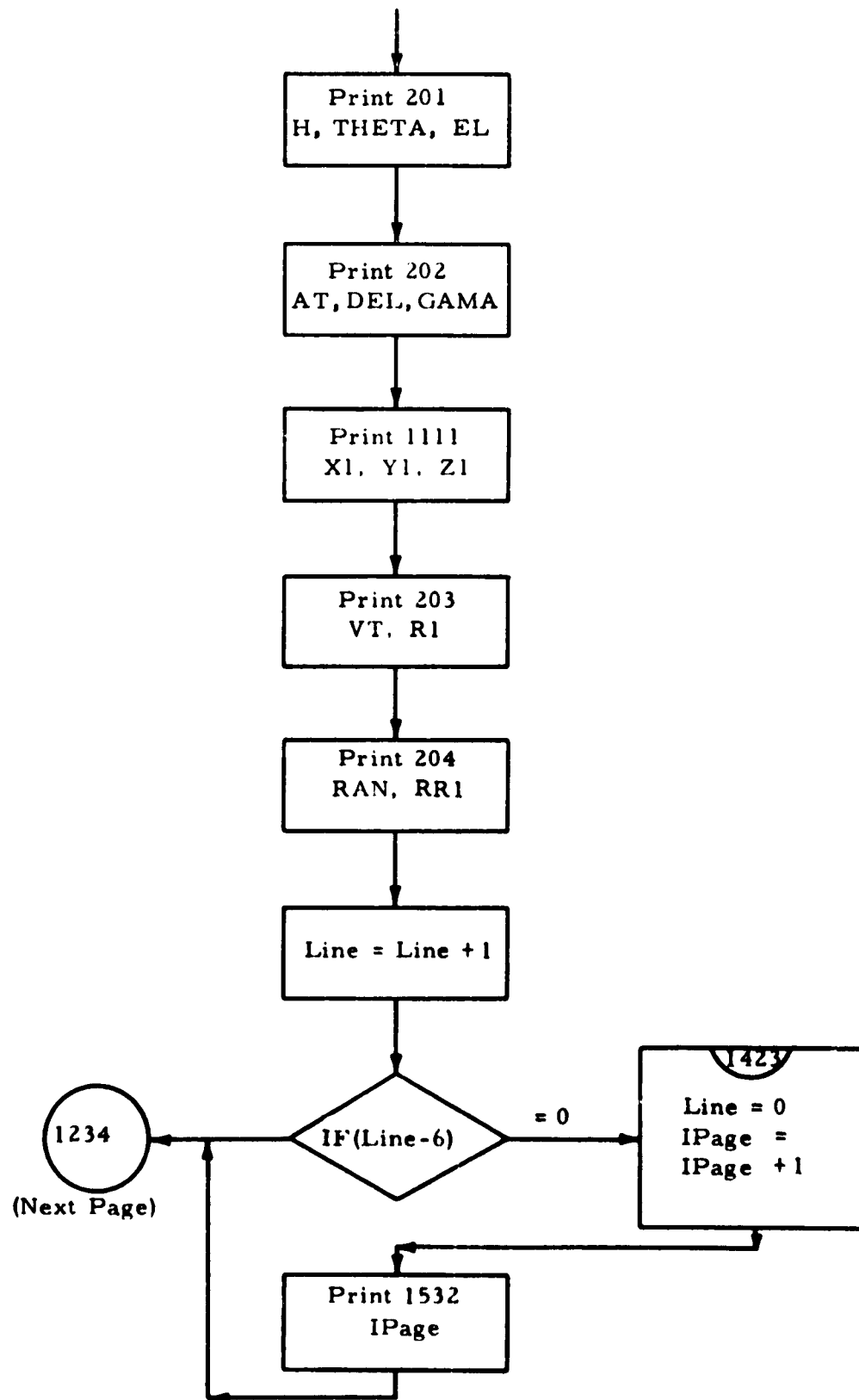


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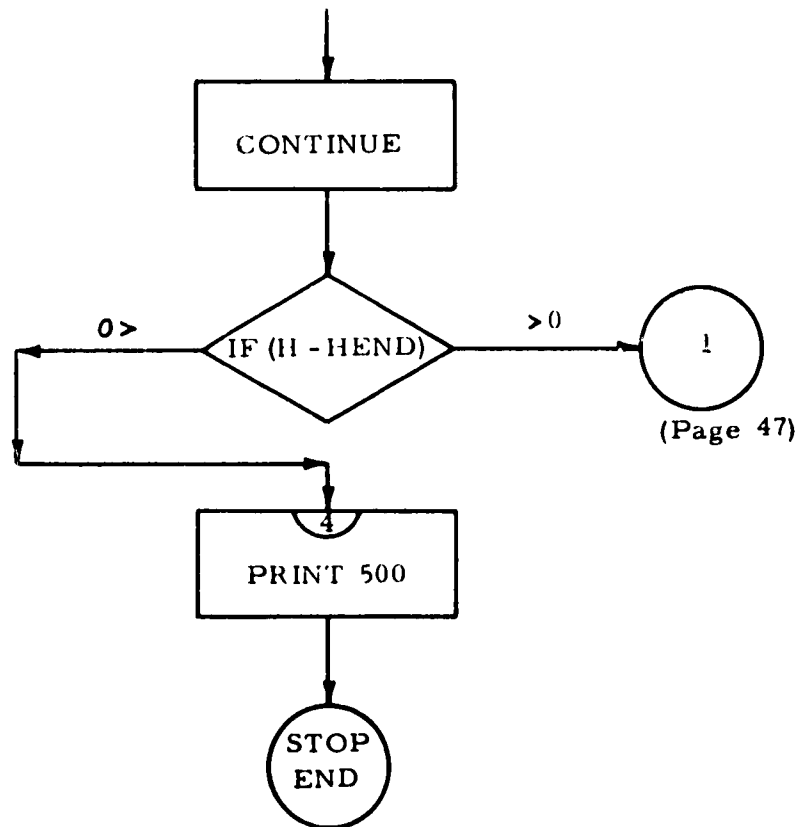




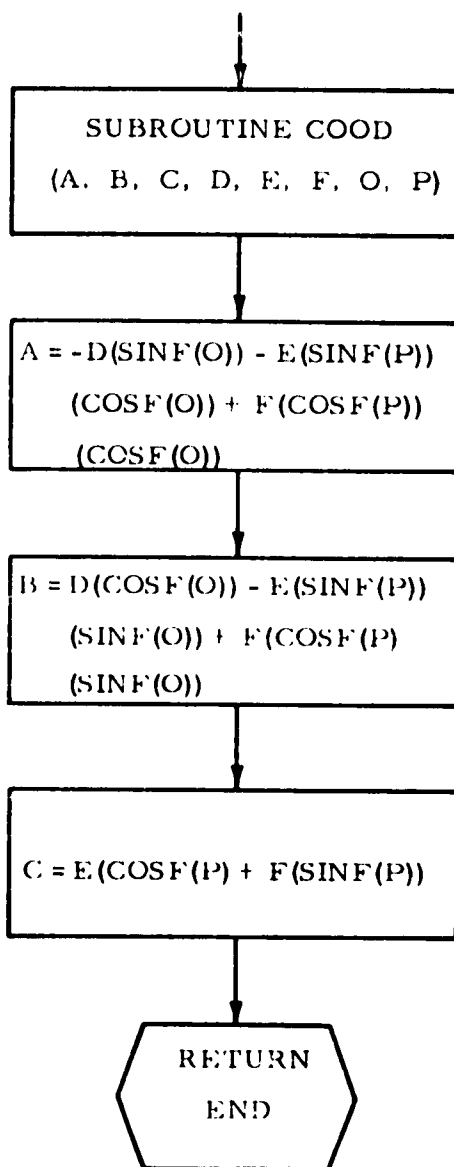
ENTER FROM PREVIOUS PAGE

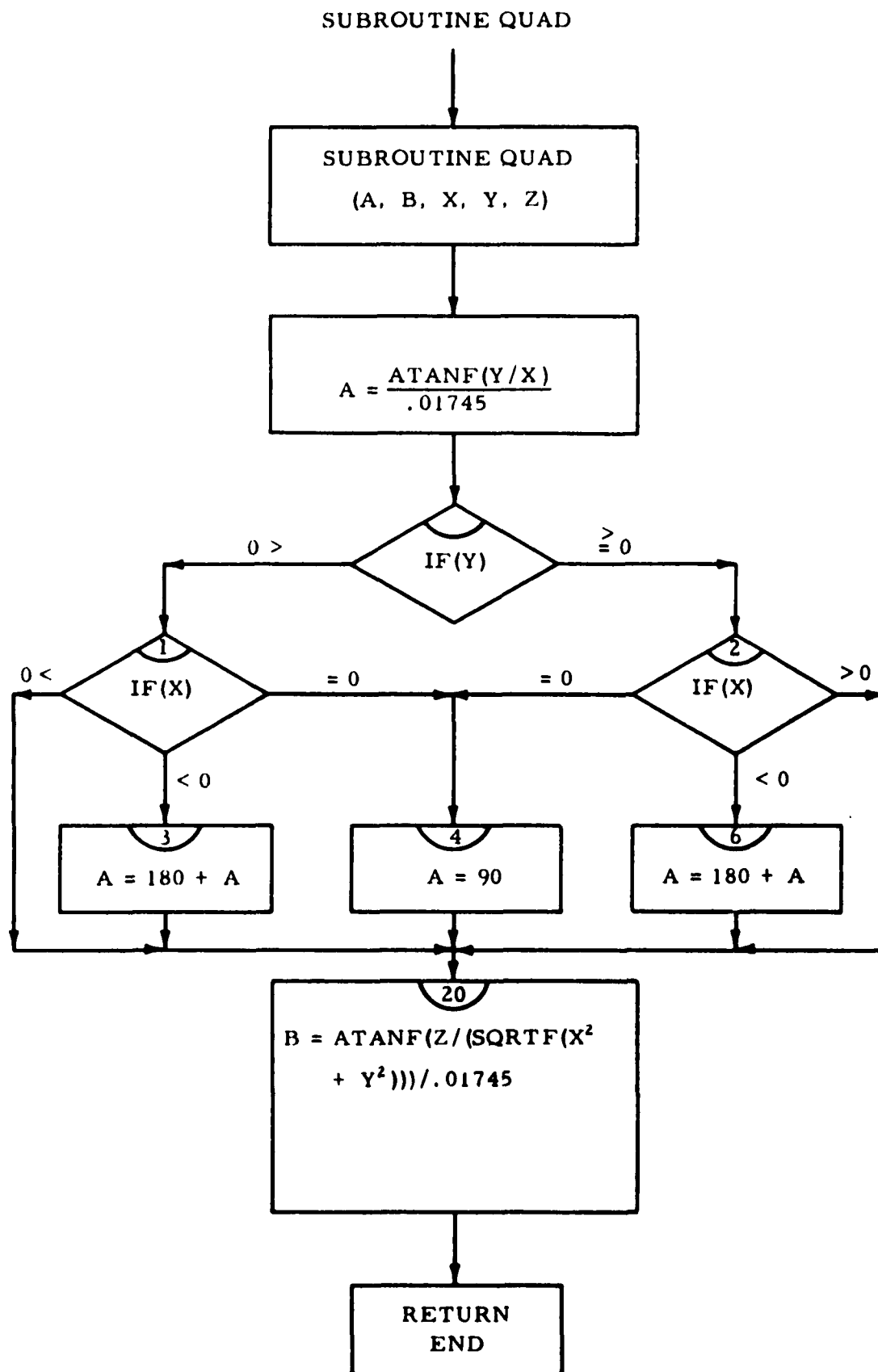


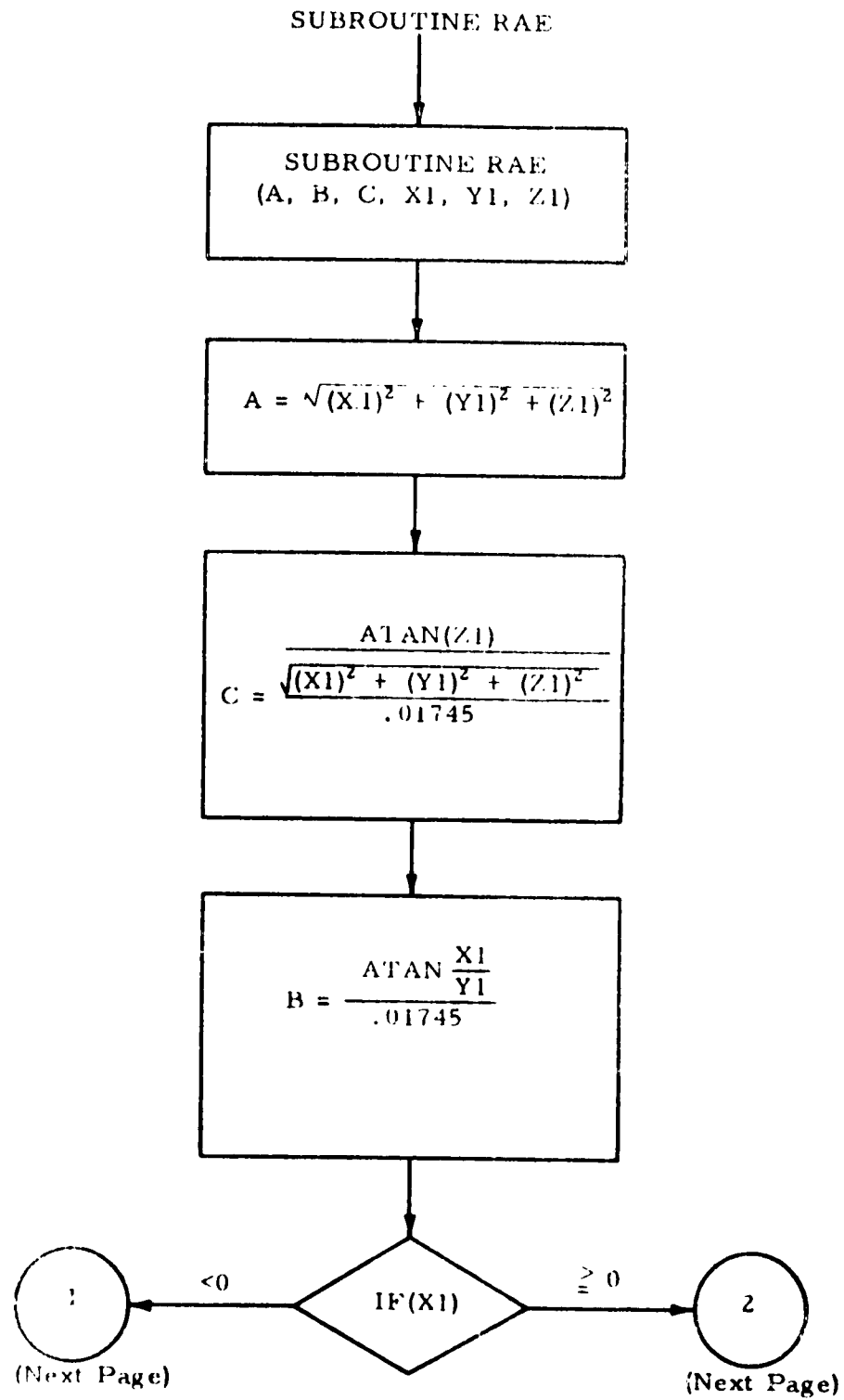
ENTER 1234 FROM PAGE 57



SUBROUTINE COOD

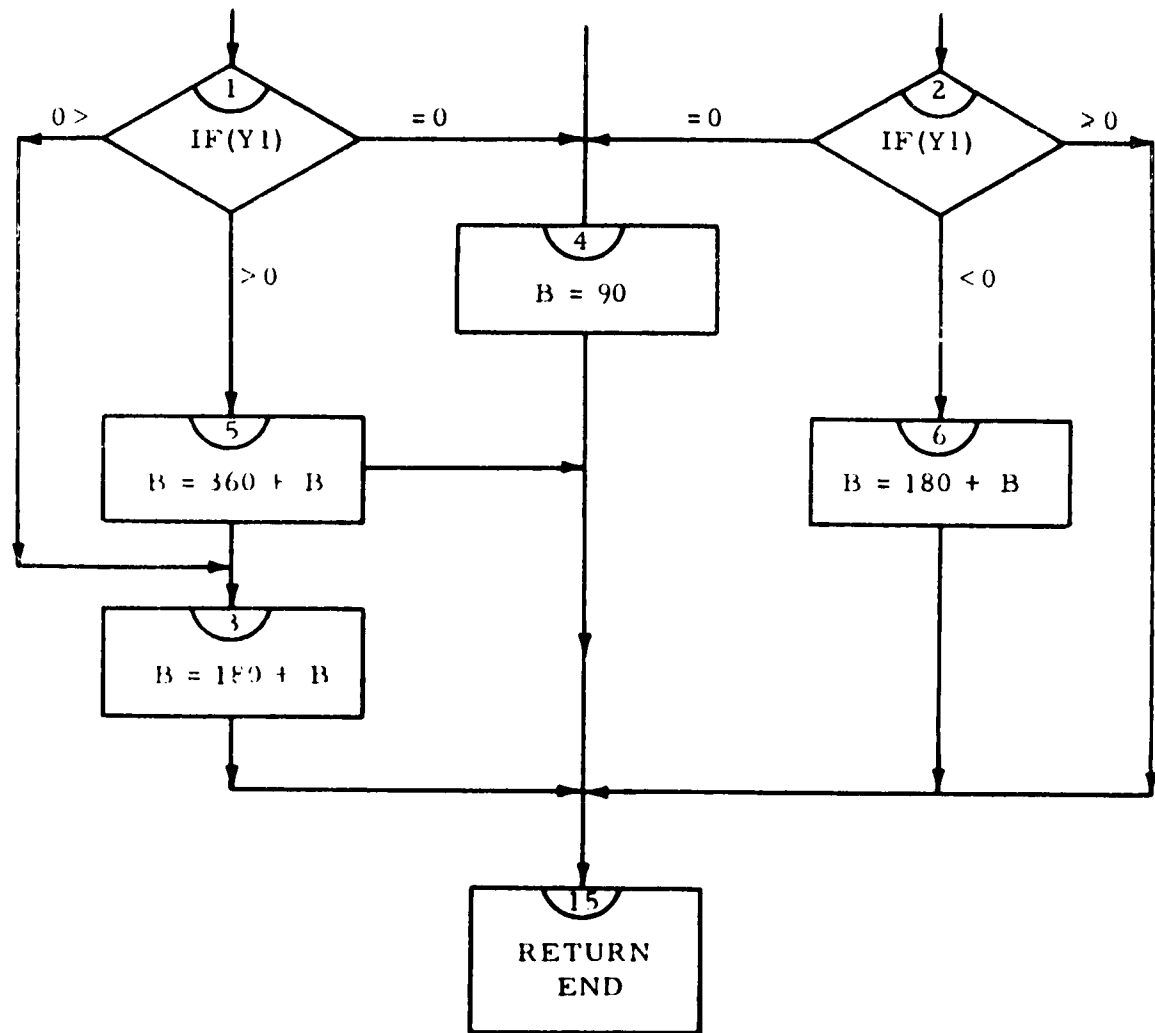






ENTER 1 FROM
PREVIOUS PAGE

ENTER 2 FROM
PREVIOUS PAGE



SUBROUTINE COODI

SUBROUTINE COODI
(A, B, C, D, E, F, O, P)

$A = -D(\text{SINF}(O))$
 $+ E(\text{COSF}(O))$

$B = -D(\text{SINF}(P))(\text{COSF}(O))$
 $- E(\text{SINF}(P)(\text{SINF}(O)))$
 $+ F(\text{COSF}(P))$

$C = D(\text{COSF}(P))(\text{COSF}(O))$
 $+ E(\text{COSF}(P))(\text{SINF}(O))$
 $+ F(\text{SINF}(P))$

RETURN
END

SUBROUTINE FU123

SUBROUTINE FU123

(A, B, C, X, Y, Z,
XD, YD, ZD, RO,
BC, DT)

$C = 1.38999091E + 16$

$U = .72918296E - 04$

$C3 = 16.087 \left(\frac{RO}{BC} \right)$

$D1 = (x^2 + y^2 + z^2)$

Next
Page

SUBROUTINE FU123

