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TECHNICAL NOTE R-28

A FORTRAN PROGRAM TO CALCULATE
A BALLISTIC MISSILE TRAJECTORY
FROM BURN OUT TO IMPACT

Prepared By

Charles F. Ostner

November, 1962

BROWN

ENGINEERING COMPANY INC.
HUNTSVILLE, ALABAMA

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TECHNICAL NOTE R-28

A FORTRAN PROGRAM TO CALCULATE
A BALLISTIC MISSILE TRAJECTORY
FROM BURN OUT TO IMPACT

November, 1962

Prepared For
DIRECTORATE OF MISSILE INTELLIGENCE
ARMY MISSILE COMMAND

By
SCIENTIFIC RESEARCH STAFF
BROWN ENGINEERING COMPANY, INC.

Contract No. DA-01-009-ORD-1068

Prepared By:
Charles F. Ostner
Senior Project Engineer

ABSTRACT

This computer program computes a ballistic trajectory from burn out or any point thereafter to impact. It assumes point mass and a spherical earth. The program takes into account air resistance, the variation of the gravitational and centrifugal field with altitude, and earth's rotation. It uses a system of co-ordinates (X, Y, and Z) rigidly connected with the rotating earth.

The computer program is written in FORTRAN II and an autocoder deck has been compiled. The program is run on the IBM 1410 computer. It is being adapted for use on the IBM 1620 computer for use by the Army. This program may be obtained from the Scientific Programming Section; Program No. SP9. It was programmed by W. B. Warren of the Scientific Research Staff.

Approved By:


Harry C. Crews, Jr.
Director, Special Projects Office

SYMBOLS

<u>Symbol</u>	<u>Computer Symbol</u>	
β	BETA	Bearing angle - Clockwise horizontal angle from north to direction of flight
δ	DELTA	Flight path angle - Angle between missile axis and local horizontal. Angle is positive when missile is climbing.
ϕ	PHI	Latitude - Positive in northern hemisphere, negative in southern
θ	THETA	Longitude - Increases to the east. When firing east, $\theta_0 = 0$, to the west, $\theta_0 = \pi/2$
X	X	See Figure 1 - Co-ordinate system with center of earth as origin. Z is the polar axis. X and Y are in the plane of the Equator and are 90° apart. The X axis corresponds to the point where $\theta = 0$. All are positive as shown.
Y	Y	
Z	Z	
γ	GAMMA	
		Range angle - Vertex at center of earth measured from the point where data were put into the program to the position attained along the trajectory.
A_x	AX	Acceleration in the X direction
A_y	AY	Acceleration in the Y direction
A_z	AZ	Acceleration in the Z direction
A_t	AT	Acceleration along the trajectory
V_x	VX	Velocity in the X direction
V_y	VY	Velocity in the Y direction
V_z	VZ	Velocity in the Z direction
V_t	VT	Velocity along the trajectory
R_a	RA	Distance from center of earth to re-entry body

Symbols (Cont.)

<u>Symbol</u>	<u>Computer Symbol</u>	
H	ALT	Distance from surface of earth to re-entry body
R	R	Radius of earth - Constant (20, 902, 890 ft.)
$\bar{r}$	-	Geocentric location vector
g_o	GO	Acceleration of gravity at sea level - constant (32.174 ft/sec ²)
g	GRAV	Acceleration of gravity at altitude
U	U	Earth's rotation in radians per second - constant ($2\pi/24 \times 3600$)
σ	SIGMA	Angle of a spherical triangle - see Figure 3
u	AMU	$(g_o R^2)^{\frac{1}{2}}$
$\left(\frac{w}{C_D A}\right)$	BALCO	Ballistic coefficient - Estimated from shape and dimensions
ρ	DEN	Air density from tape for IBM 1410 program - computed internally in IBM 1620 program

PHYSICAL CONSTANTS

$$R = 20,902,890 \text{ feet}$$

$$g_o = 32.174 \text{ ft/sec}^2$$

$$U = 2\pi/24 \times 3600 \text{ radians/sec.}$$

$$u^2 = g_o R^2$$

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INTRODUCTION

This computer program may be used to study the free flight and re-entry portions of a ballistic trajectory. The information calculated and printed by the computer includes time of flight, location in both X, Y, and Z from earth's center and latitude, longitude and altitude from earth's surface, acceleration, velocity, flight path angle, bearing angle and range in nautical miles.

The model chosen for use in this computer program was taken from References 1 and 2. Point mass and a spherical earth are assumed. The basic acceleration equations are developed in Reference 1. Reference 2 discusses a similar computer program which uses the same acceleration equations and many of the other relationships used in this computer program.

A portion of the computer program discussed in Reference 3 was used to compute density in the IBM 1620 Computer program. This portion is an integral part of that program.

Library Functions Used:

SQRT
SIN
COS
EXPF
ATAN

ANALYSIS

Initial Conditions

The computer program will accept initial conditions at burn out or at any time thereafter until impact. The items of information needed are punched on two IBM cards which are placed at the end of the deck.

<u>Items</u>	<u>Comments</u>
δ	Positive when missile is climbing.
β	Clockwise horizontal angle from north
θ	Initial $\theta_0 = 0$ when flying eastward and $\theta_0 = \pi/2$ when flying toward the west
ϕ	Positive in northern hemisphere
H	Altitude above earth's surface
R	20,902,890 ft. (earth's radius)
V_t	Velocity along the trajectory
g_0	32.174 ft/sec ²
U	$2\pi/24 \times 3600$ radians per second
$\left(\frac{w}{C_D A}\right)$	Ballistic coefficient

All angles are put into the computer in radians. The computer program is designed to accept and compute English units. All angles are printed in degrees. (Program SP-9-E)

The computer program may be modified to accept, compute and print metric units by using metric units for H, R, V_t , g_0 and by using conversion

factors for the altitude and density in the density subroutine in the IBM 1620 computer program. A deck in metric units has been prepared. (SP-9-M)

Development of Equations

In order to provide the proper initial conditions, the following set of co-ordinate transformations were used as an integral part of the computer program. (See Figure 1):

$$X = R_a \cos \phi \cos \theta \quad (1)$$

$$Y = R_a \cos \phi \sin \theta \quad (2)$$

$$Z = R_a \sin \phi \quad (3)$$

By definition:

$$H = R_a - R \quad (4)$$

It follows that:

$$R_a = (X^2 + Y^2 + Z^2)^{\frac{1}{2}} \quad (5)$$

In order to convert the input velocity along the trajectory to components in the X, Y, and Z directions, the angles δ , β , ϕ , and θ are used. See Figure 1. δ is the angle the vehicle makes with the local horizontal.

$$V_t \sin \delta = \text{a velocity component along } R_a \quad (6)$$

$$V_t \sin \delta \sin \phi = \text{a velocity component along the Z axis} \quad (7)$$

$$V_t \sin \delta \cos \phi = \text{a velocity component along } R_a \text{ projected} \quad (8) \\ \text{into the plane of the Equator}$$

$$V_t \sin \delta \cos \phi \sin \theta = \text{a velocity component along the Y axis} \quad (9)$$

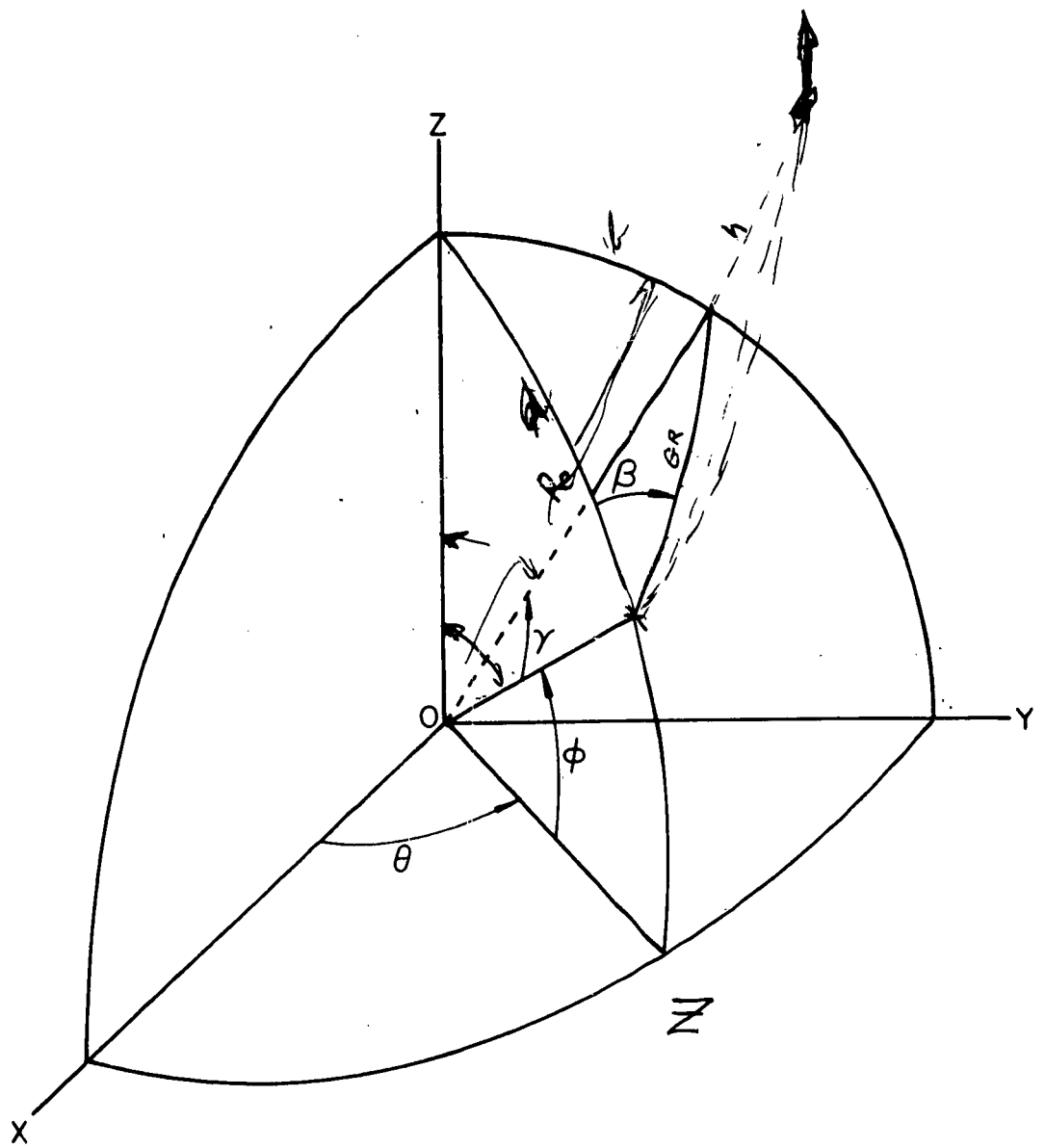


FIGURE 1

$$V_t \sin \delta \cos \phi \cos \theta = \text{a velocity component along the X axis} \quad (10)$$

$$V_t \cos \delta = \text{a velocity component parallel to the earth's surface} \quad (11)$$

$$V_t \cos \delta \sin \beta = \text{a velocity component along a parallel of latitude} \quad (12)$$

$$V_t \cos \delta \sin \beta \sin \theta = \text{a velocity component along the X axis} \quad (13)$$

$$V_t \cos \delta \sin \beta \cos \theta = \text{a velocity component along the Y axis} \quad (14)$$

$$V_t \cos \delta \cos \beta = \text{a velocity component along a meridian of longitude} \quad (15)$$

$$V_t \cos \delta \cos \beta \sin \phi = \text{a velocity component along } R_a \text{ projected into the plane of the Equator} \quad (16)$$

$$V_t \cos \delta \cos \beta \sin \phi \sin \theta = \text{a velocity component along the Y axis} \quad (17)$$

$$V_t \cos \delta \cos \beta \sin \phi \cos \theta = \text{a velocity component along the X axis} \quad (18)$$

$$V_t \cos \delta \cos \beta \cos \phi = \text{a velocity component along the Z axis} \quad (19)$$

The following three equations are integral parts of the computer program. The signs (+ or -) of the sin and cos functions of δ , β , ϕ , and θ were analyzed as the angles changed. The proper sign was given to the components used.

Combining (10), (13), and (18):

$$V_x = V_t (\sin \delta \cos \phi \cos \theta - \cos \delta \sin \beta \sin \theta - \cos \delta \cos \beta \sin \phi \cos \theta) \quad (20)$$

Combining (9), (14), and (17):

$$V_y = V_t (\sin \delta \cos \phi \sin \theta + \cos \delta \sin \beta \cos \theta - \cos \delta \cos \beta \sin \phi \sin \theta) \quad (21)$$

Combining (7) and (19):

$$V_z = V_t (\sin \delta \sin \phi + \cos \delta \cos \beta \cos \phi) \quad (22)$$

The component velocities can be used to find the velocity along the trajectory.

$$V_t = (V_x^2 + V_y^2 + V_z^2)^{\frac{1}{2}} \quad (23)$$

If one takes into account the air resistance, the variation of the gravitational and centrifugal field and earth's rotation, the equation of motion for the ballistic vehicle becomes:

$$m\ddot{\mathbf{r}} = m\bar{\mathbf{g}} + \bar{\mathbf{W}} + \bar{\mathbf{k}}_c + \mathbf{k}_n \quad (24)$$

where

m = mass of vehicle

$\bar{\mathbf{r}}$ = radius vector from center of earth to vehicle

$\bar{\mathbf{g}}$ = acceleration due to gravity

$\bar{\mathbf{W}}$ = air resistance

$\bar{\mathbf{k}}_c$ = Coriolis force

$\bar{\mathbf{k}}_n$ = centrifugal force

Examining equation (24) term by term, one obtains:

$$\bar{\mathbf{g}} = -\frac{mu^2}{R_a^2} \frac{\bar{\mathbf{r}}}{R_a} \quad (25)$$

where

$$u^2 = g_o R^2$$

R = radius of earth

R_a = distance from earth's center to vehicle

$$\overline{W} = -\overline{v} c v^{n-1} \quad (26)$$

where c and n are factors which depend on air density, the shape of the vehicle and the velocity;

$$\overline{k}_c = 2m (\overline{v} \times \overline{U}) \quad (27)$$

where

$\overline{U}$ = angular velocity of the rotation of earth in radians per second

and

$$\overline{k}_n = m U^2 r \cos \phi = m U^2 (X^2 + Y^2)^{\frac{1}{2}} \quad (28)$$

In this analysis, the air resistance was assumed to be proportional to the square of velocity so that equation (26) becomes

$$\overline{W} = \frac{\overline{v} \rho g V m}{2 \left(\frac{w}{C_D A} \right)} \quad (29)$$

Writing equations (25) through (29) in component form, we obtain:

$$A_x = -\frac{u^2 X}{R_a^3} - \frac{\rho V_t g V_x}{2 \left(\frac{w}{C_D A} \right)} + 2 V_y U + U^2 X \quad (30)$$

$$A_y = -\frac{u^2 Y}{R_a^3} - \frac{\rho V_t g V_y}{2 \left(\frac{w}{C_D A} \right)} - 2 V_x U + U^2 Y \quad (31)$$

$$A_z = - \frac{u^2 Z}{R_a^3} - \frac{\rho V_t g V_z}{2 \left(\frac{w}{C_D A} \right)} \quad (32)$$

Integrating these equations in computer form, we obtain

$$V_{x_n} = V_{x(n-1)} + A_x \Delta t \quad (33)$$

$$V_{y_n} = V_{y(n-1)} + A_y \Delta t \quad (34)$$

$$V_{z_n} = V_{z(n-1)} + A_z \Delta t \quad (35)$$

$$X_n = X_{n-1} + V_x \Delta t \quad (36)$$

$$Y_n = Y_{n-1} + V_y \Delta t \quad (37)$$

$$Z_n = Z_{n-1} + V_z \Delta t \quad (38)$$

From Figure 1, the relationships pertaining to the angles are:

$$\phi = \arcsin \frac{Z}{R_a} \quad (39)$$

$$\theta = \arctan \frac{Y}{X} \quad (40)$$

$$\delta = \arcsin \frac{\Delta R_a / \Delta t}{V_t} \quad \text{where } \Delta R_a / \Delta t = \text{average rate of change in altitude} \quad (41)$$

In spherical trigonometry, the law of cosines states:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \quad (42)$$

The law of sines states:

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \quad (43)$$

where angles A, B, and C represent the angles of the spherical triangle on the earth's surface and a, b, and c represent the sides of the triangle and the angles subtended by them at earth's center. See Figure 2.

Now refer to Figure 3.

$$\cos \gamma = \cos \text{Co}\phi_0 \cos \text{Co}\phi + \sin \text{Co}\phi_0 \sin \text{Co}\phi \cos \Delta\theta \quad (44)$$

where

$$\Delta\theta = \theta - \theta_0 \text{ and}$$

$$\text{Co}\phi_0 = 90^\circ - \phi_0, \text{ then } \cos \text{Co}\phi_0 = \sin \phi_0 \text{ and } \sin \text{Co}\phi_0 = \cos \phi_0$$

$$\text{Co}\phi = 90^\circ - \phi, \text{ then } \cos \text{Co}\phi = \sin \phi \text{ and } \sin \text{Co}\phi = \cos \phi$$

Equation (44) then becomes

$$\cos \gamma = \sin \phi_0 \sin \phi + \cos \phi_0 \cos \phi \cos \Delta\theta \quad (45)$$

$$\gamma = \arccos(\sin \phi_0 \sin \phi + \cos \phi_0 \cos \phi \cos \Delta\theta) \quad (46)$$

Then range = γR , γ is in radians. Earth's radius, $R = 3440.239$ nautical miles.

$$\text{Range} = \gamma (3440.239) \quad (47)$$

Refer again to Figure 3.

$$\frac{\sin \sigma}{\sin \text{Co}\phi_0} = \frac{\sin \Delta\theta}{\sin \gamma} \quad \text{but } \sin \text{Co}\phi_0 = \cos \phi_0 \quad (48)$$

$$\sin \sigma = \frac{\cos \phi_0 \sin \Delta\theta}{\sin \gamma} \quad (49)$$

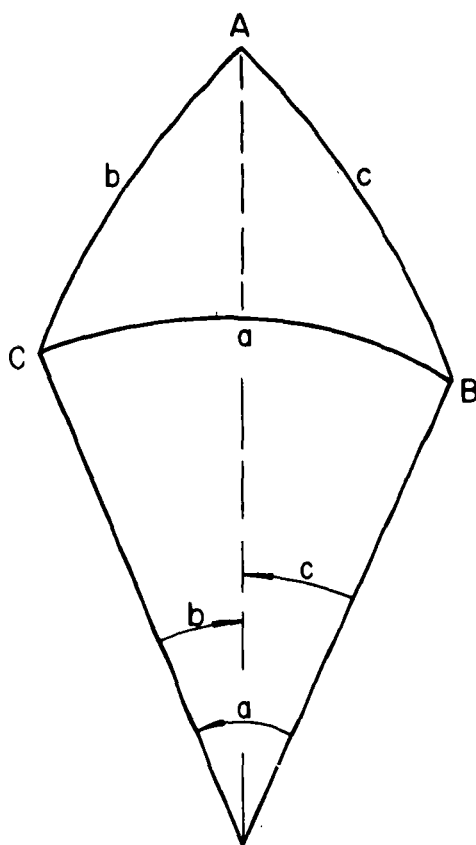


FIGURE 2

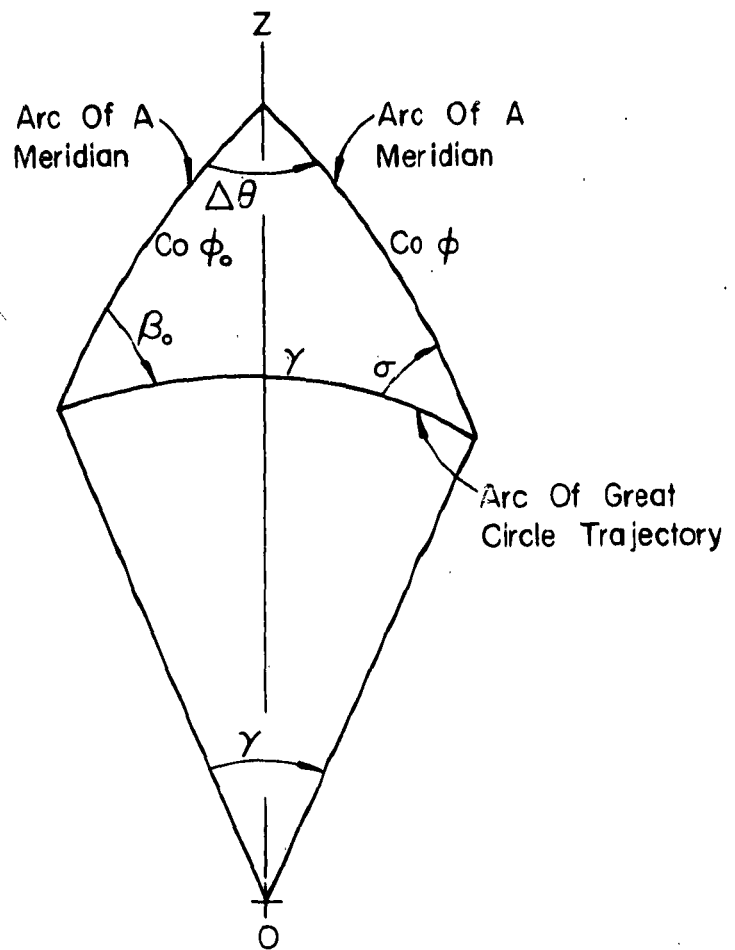


FIGURE 3

$$\sigma = \arcsin\left(\frac{\cos \phi_0 \sin \Delta\theta}{\sin \gamma}\right) \quad (50)$$

Sigma (σ) is used to compute bearing (β). In order to minimize the error caused by assuming that the trajectory follows an exact great circle (it would follow an exact great circle if the earth were not rotating), a smaller spherical triangle is used in the computer program.

Let symbols with subscript 1 stand for the next to last values computed. The symbols without subscripts stand for the newest values computed. Let γ_3 equal the range increment angle.

Equation (46) now becomes:

$$\gamma_3 = \arccos\left[\sin \phi_1 \sin \phi + \cos \phi_1 \cos \phi \cos (\theta - \theta_1)\right] \quad (51)$$

Equation (50) becomes

$$\sigma = \arcsin\left(\frac{\cos \phi_1 \sin (\theta - \theta_1)}{\sin \gamma_3}\right) \quad (52)$$

The computer uses the tangent function to determine the value of sigma (σ). The value determined will lie between -90° and $+90^\circ$. A graph showing sigma plotted against beta is shown in Figure 4.

From Figure 4 we can see that if beta is between 0 and 90°

$$\beta = \sigma \quad (53)$$

If beta is between 90° and 270° ,

$$\beta = \pi - \sigma \quad (54)$$

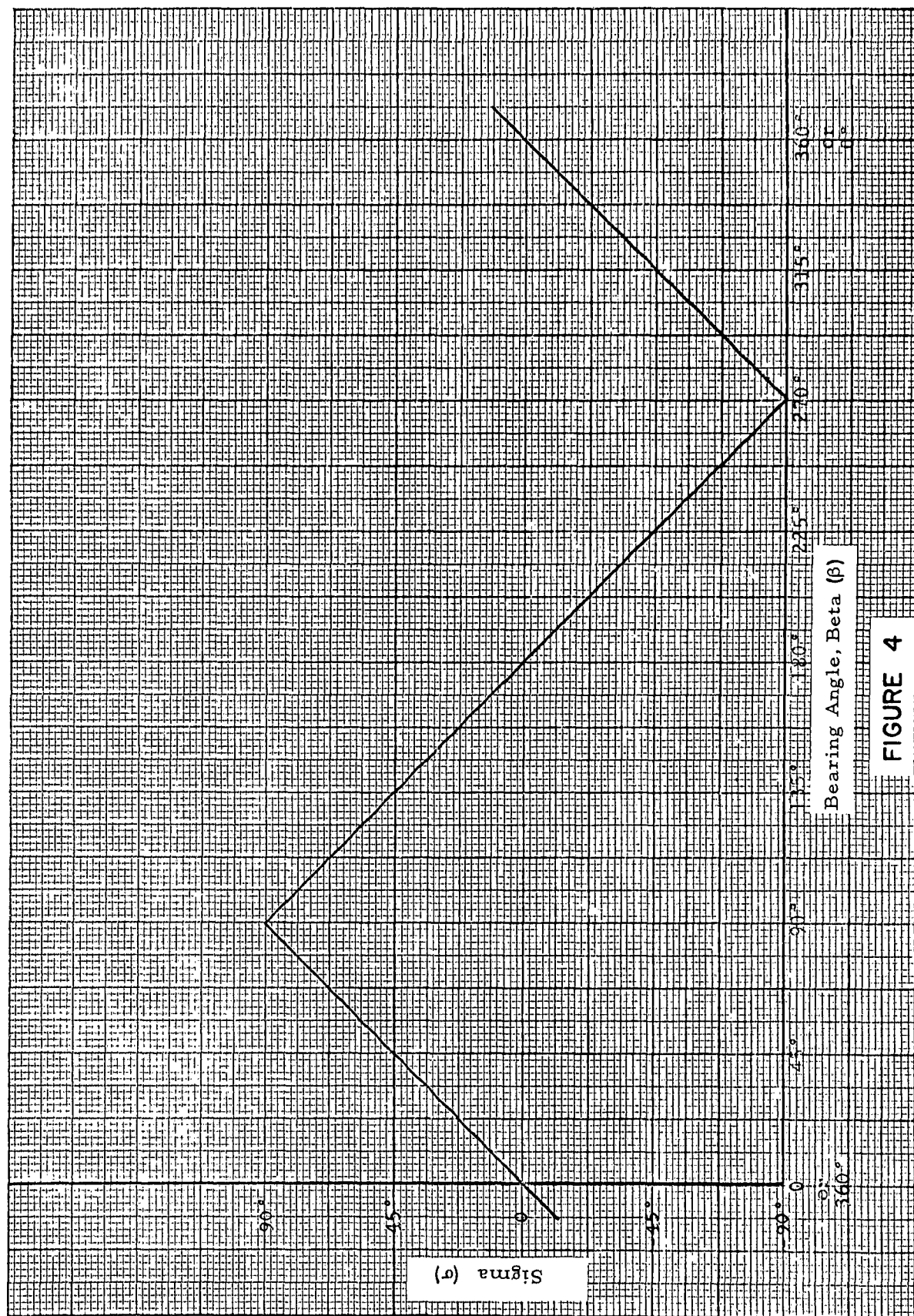


FIGURE 4

If beta is greater than 270° ,

$$\beta = 2\pi + \sigma \quad (55)$$

The five equations (51) through (55) are integral parts of the computer program.

In order to compute A_t , which is the acceleration along the trajectory, let V_t = the last velocity computed along the trajectory,

V_{t_1} = the next to last velocity computed along the trajectory

Δt = the time increment being used the computer

then

$$A_t = \frac{V_t - V_{t_1}}{\Delta t} \quad (56)$$

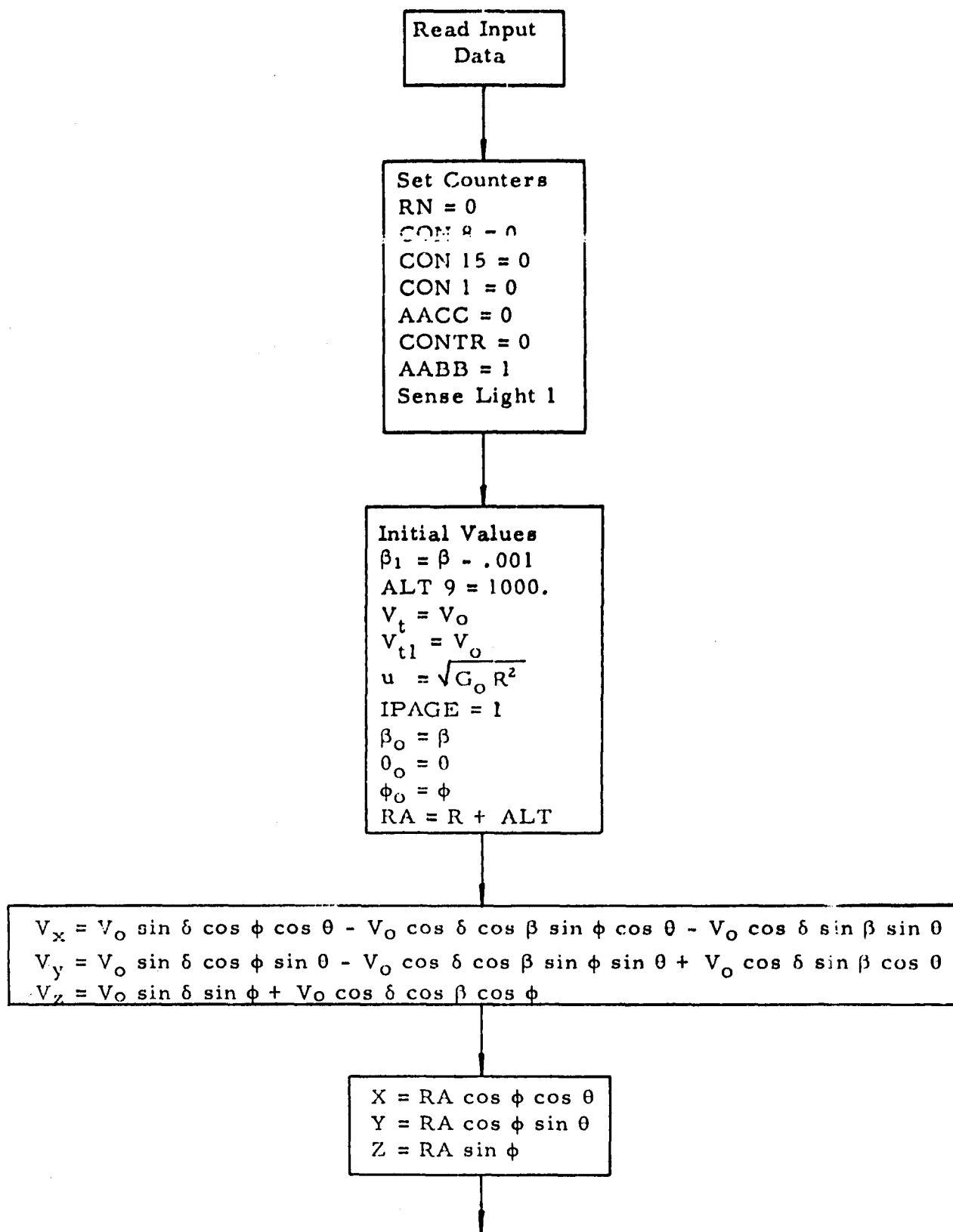
DISCUSSION

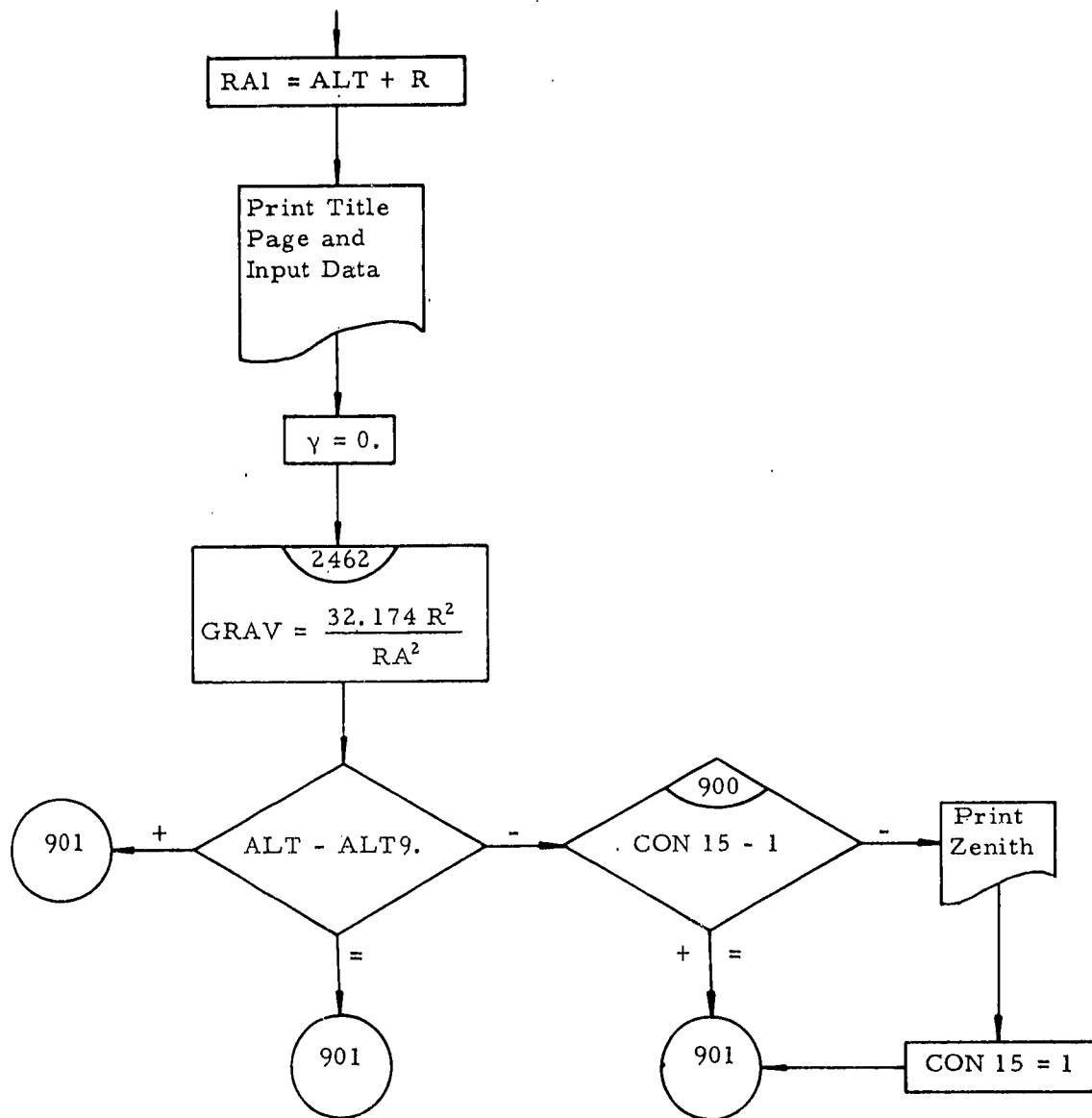
The computer program assumes a spherical earth and a point-mass. Other conditions are those which actually exist at earth's surface and at altitude from the surface.

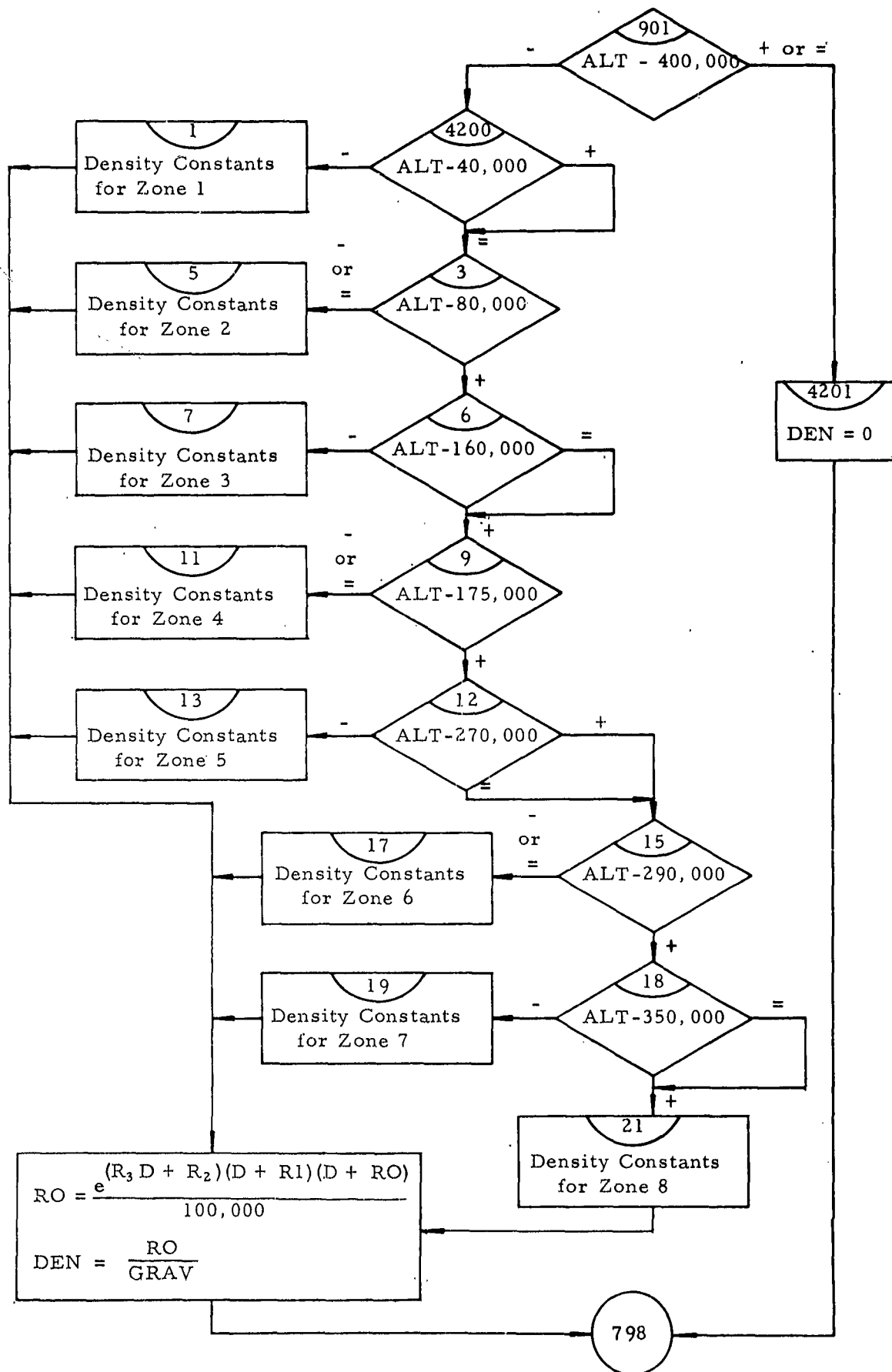
The time increment used by the computer is one second during the portion of the trajectory where conditions change slowly. During the re-entry portion of the trajectory when the altitude of the re-entry body is below 150,000 feet, the time increment is decreased to 0.1 second to increase accuracy. In order to give desired accuracy, this program was written in double precision. The program uses about 39,950 digits of memory. It is believed that this program produces highly accurate results.

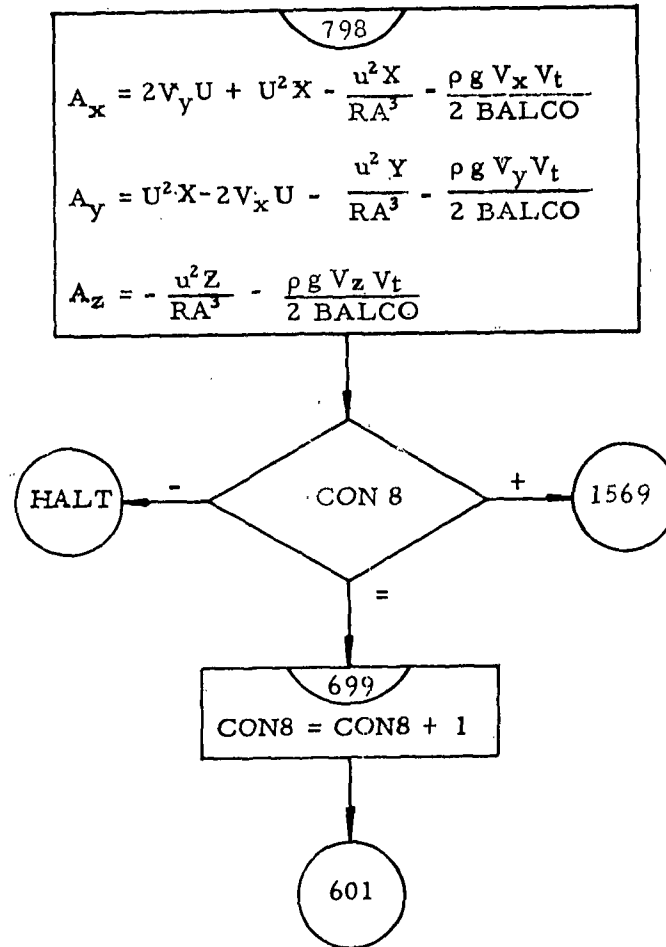
For some of the angle relationships, it is assumed that the trajectory follows an exact great circle. The errors caused by this assumption are very small because of the short time increments used and the methods chosen to compute these relationships. These relationships are computer outputs and are not used to drive the computer.

FLOW CHART FOR FREE FLIGHT AND RE-ENTRY TRAJECTORY

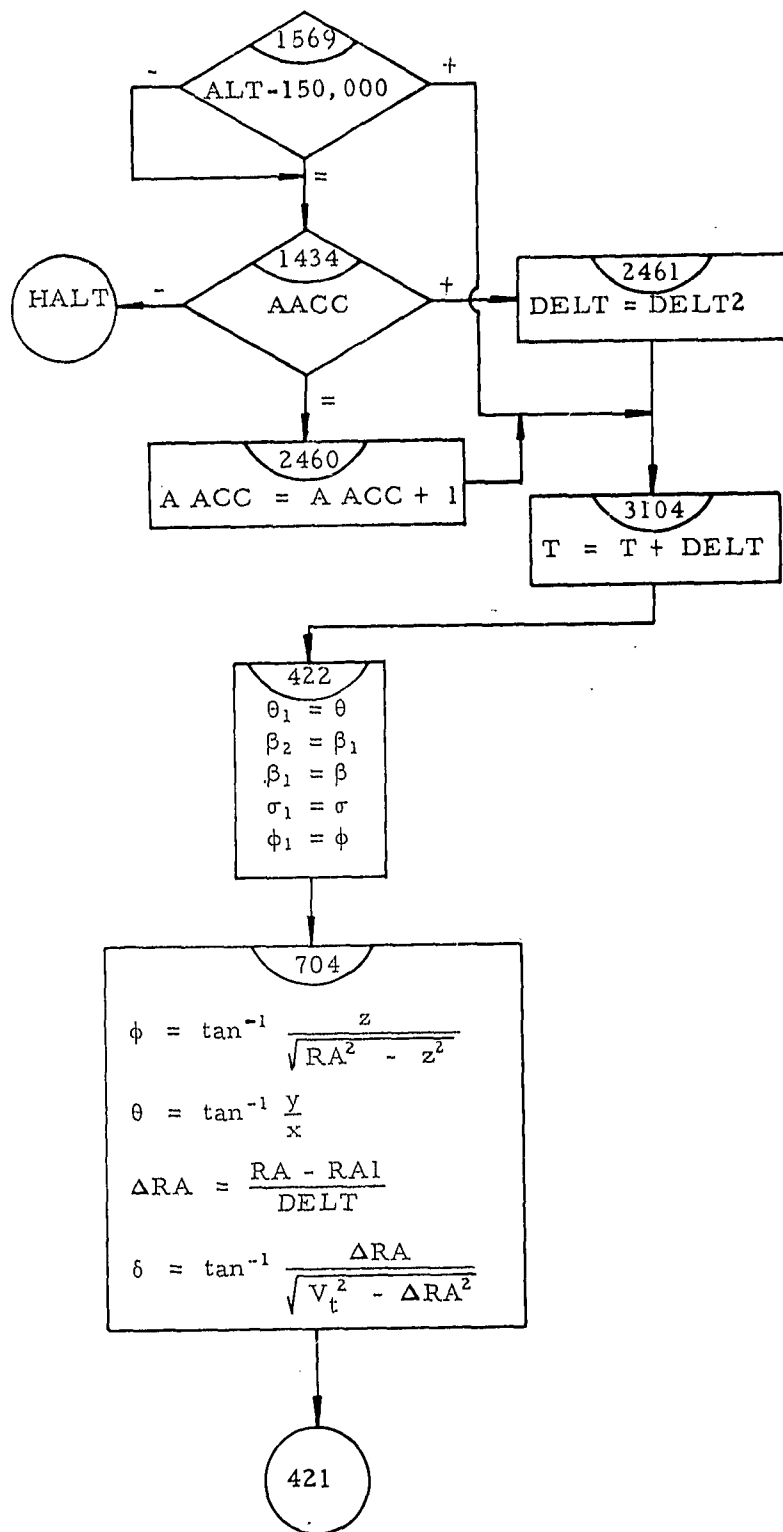


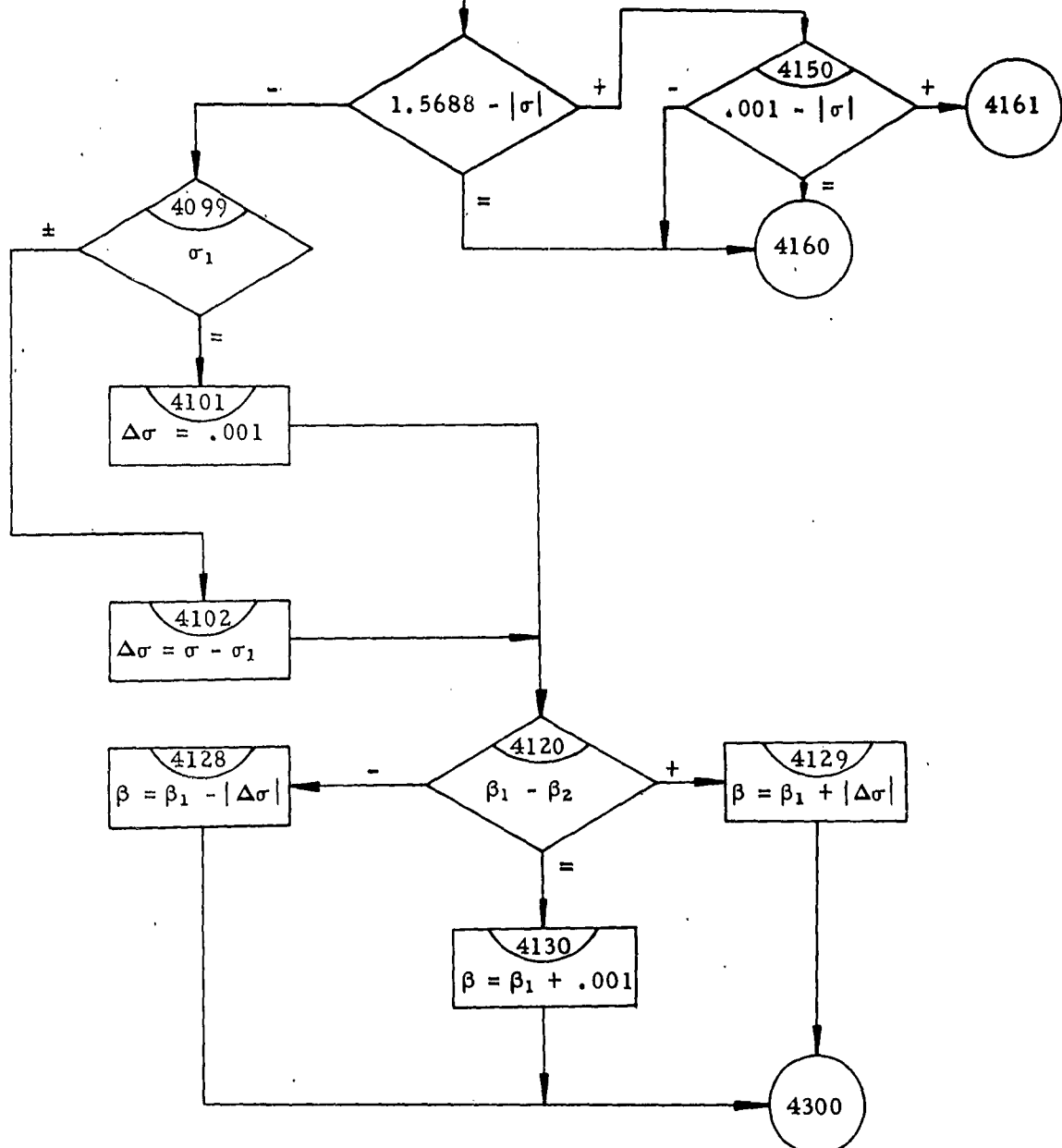
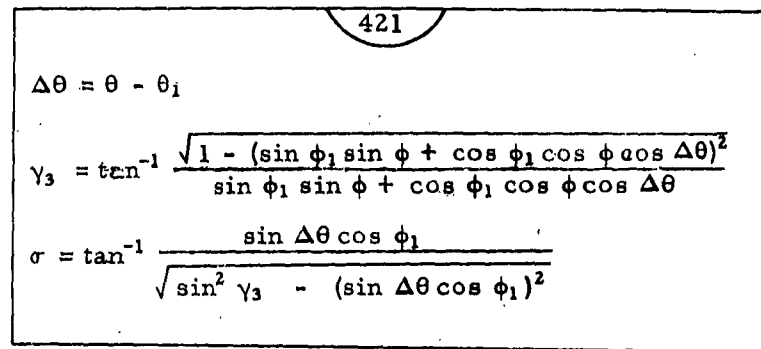


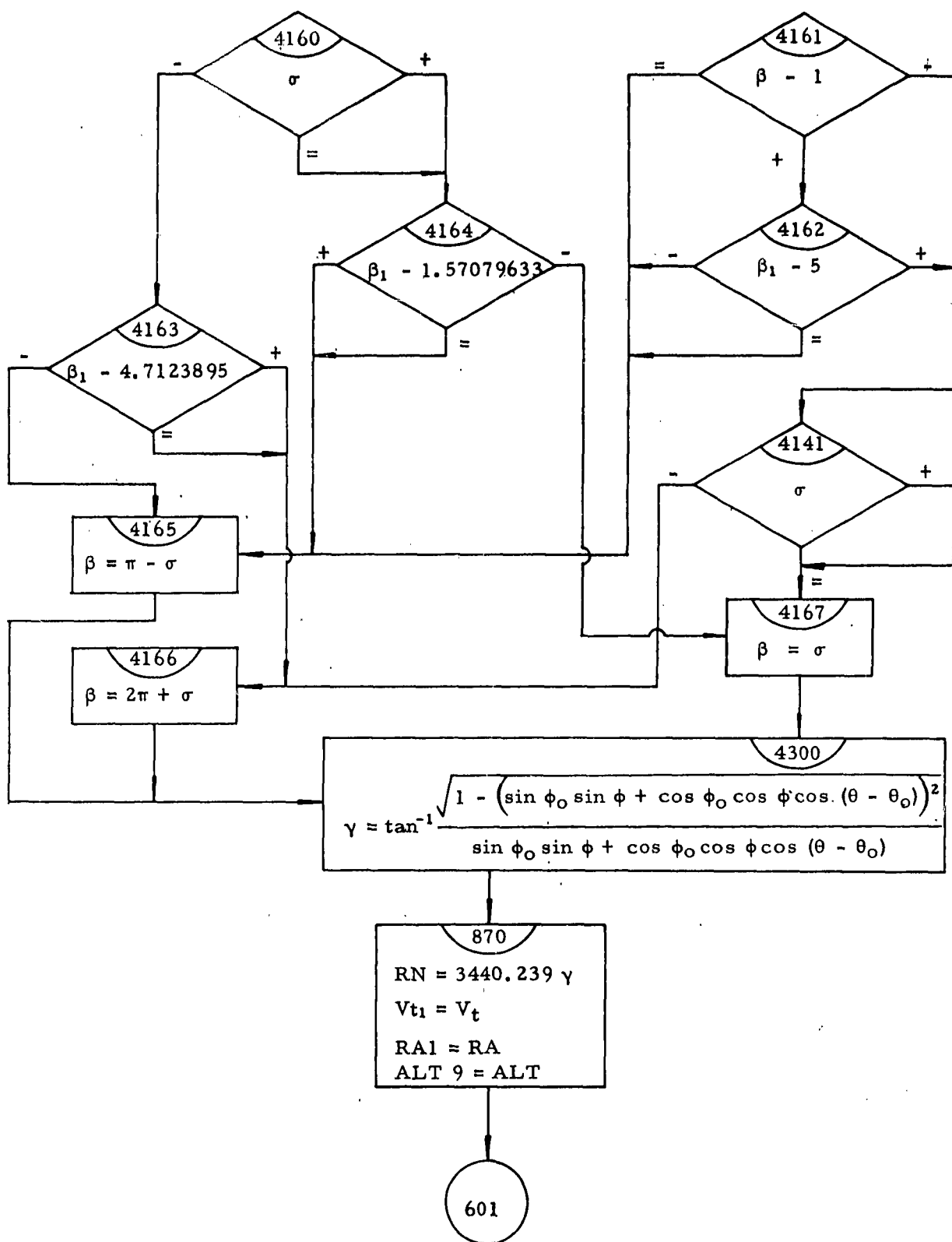


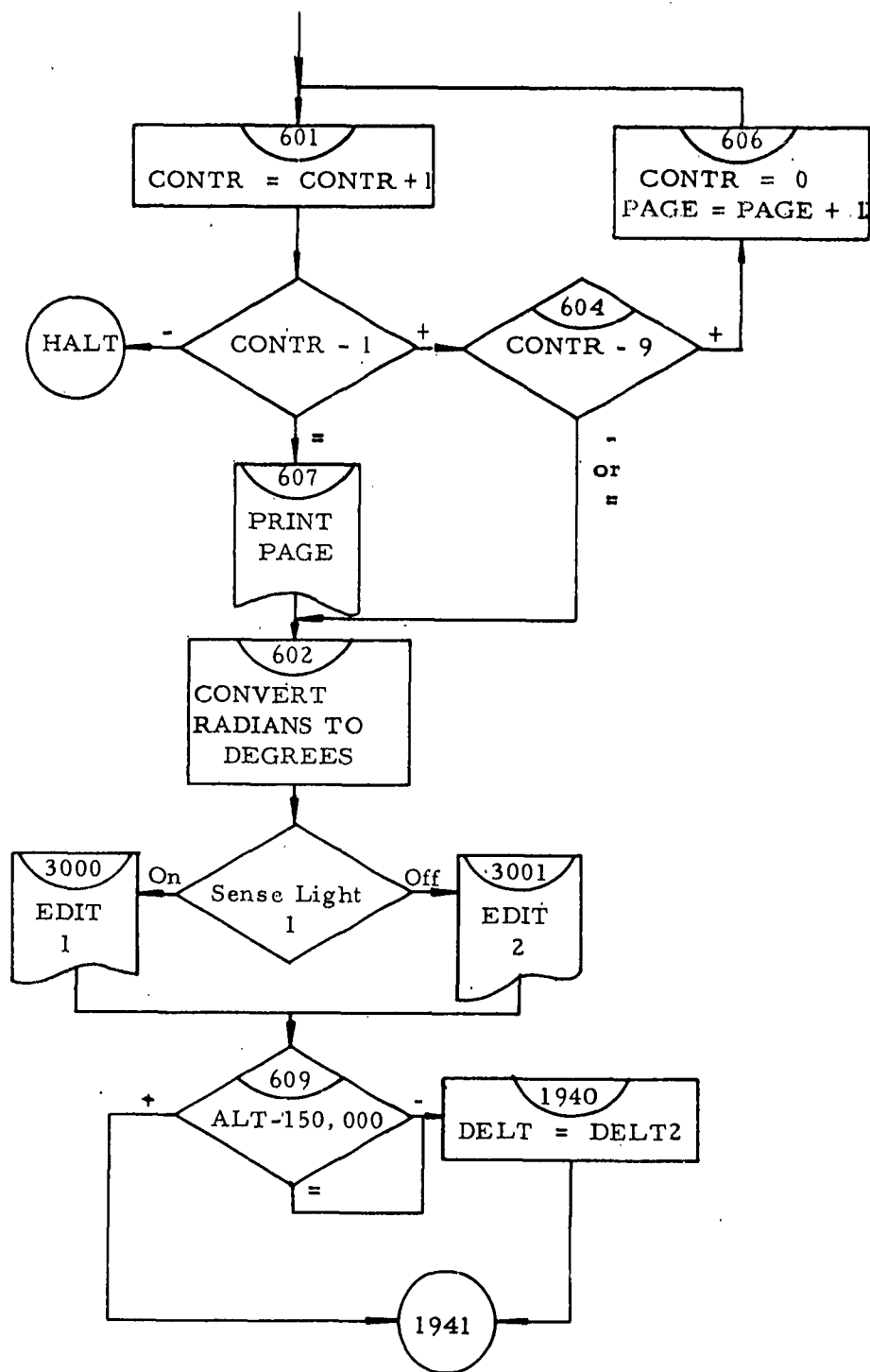


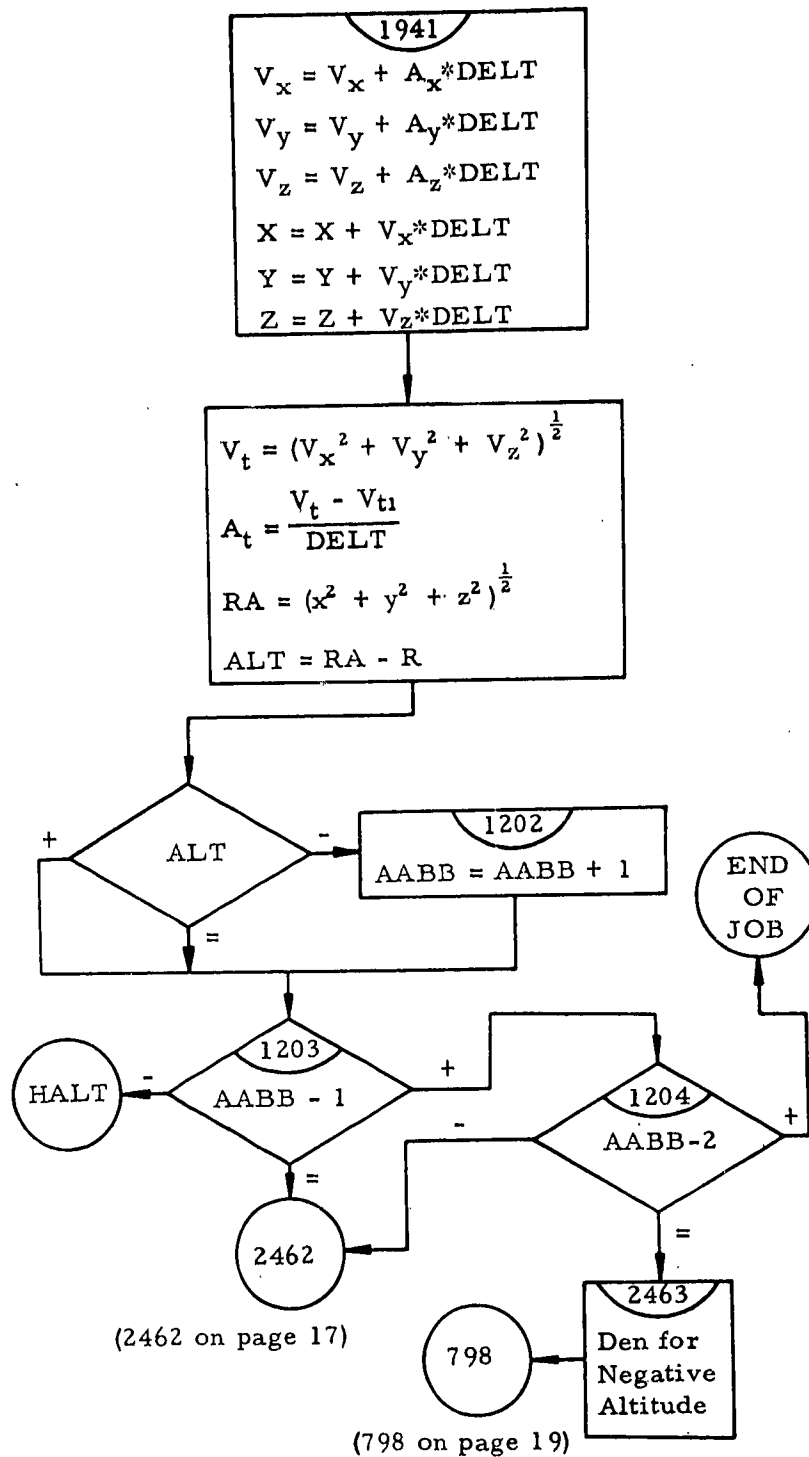
(601 on page 23)











FORTTRAN STATEMENTS

English Units

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C   FREE FLIGHT AND RE-ENTRY
C   PROGRAMMED BY W.B.WARREN
C   FOR RESEARCH STAFF
C   10 NOVEMBER 1962
C   PROGRAM NO. SP 9 ENGLISH UNITS.

C
C   PHYSICAL CONSTANTS
C   GO # GRAVITATIONAL ACCELERATION AT SEA LEVEL - 32.174 FT/SEC**2
C   U # EARTH ROTATION - %2*PI/%24*3600 RADIANS/SEC
C   R # RADIUS OF EARTH - 20,902,890 FEET
C   BALCO # BALLISTIC COEFFICIENT - CONSTANT
C   MU**2 # GO*R**2 # 32.174*20,902,890**2

C   OTHER INPUT DATA
C   DEL #ANGLE BETWEEN HORIZONTAL AND MISSILE AXIS--POSITIVE UP
C   BETA #AZIMUTH ANGLE CLOCKWISE FROM NORTH TO MISSILE AXIS
C   THETA #LONGITUDE OF MISSILE
C   PHI #LATITUDE OF MISSILE
C   H #HEIGHT OF MISSILE ABOVE EARTH SURFACE
C   RHO #ATMOSPHERIC DENSITY --TABLE
C   RA # MISSILE ALTITUDE FROM CENTER OF EARTH
C   VO # MISSILE BURNOUT VELOCITY #VT AT T#0
C   REAC 100,DEL,BETA,THETA,PHI,ALT
C   REAC100,R,VO,GO,U,BALCO
C   REAC 100,T,DELT,DELT2
C   CON8 # 0.
C   SENSE LIGHT 1
C   CON15 # 0.
C   CON1 # 0.
C   AACC # 0.
C   CONTR#0.
C   AABBB # 1.
C   BETA1# BETA - .001
C   ALT9 #1000.
C   RN#C.
C   VT # VO
C   VT1 #VO
C   AMU # SQRTF%GO*R**2
C   IPAGE # 1
C   BETAO # BETA
C   THETO # THETA
C   PHIC # PHI
C   RA # ALT&R
C   OVX #VO*SINF%DEL*%COSF%PHI*%COSF%THETA-VO*%COSF%DEL*%COSF%BETA*%SIN
C   1F%PHI*%COSF%THETA-VO*%COSF%DEL*%SINF%BETA*%SINF%THETA
C   OVY #VO*SINF%DEL*%COSF%PHI*%SINF%THETA-VO*%COSF%DEL*%COSF%BETA*%SIN

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IF%PHI□*SINF%THETA□&VO*COSF%DEL□*SINF%BETA□*COSF%THETA□
OVZ #VO*SINF%DEL□*SINF%PHI□&VO*COSF%DEL□*COSF%BETA□*COSF%PHI□
X #RA*COSF%PHI□*COSF%THETA□
Y #RA*COSF%PHI□*SINF%THETA□
Z # RA*SINF%PHI□
RA1 # ALT & R
PRINT 4999,IPAGE
PRINT 5000
PRINT 5001
PRINT 5027
PRINT 5002
PRINT 5003
PRINT 5004
PRINT 5010
PRINT 5011
PRINT 5012
PRINT 5013
PRINT 5014
PRINT 5015,DEL,BETA,THETA,PHI
PRINT 5016,ALT,BALCO,RA,VO
GAMMA # 0.
2462 GRAV # %32.174*R**2□/RA**2
IF %ALT - ALT9□900,901,901
900 IF %CON15 - 1.□3840,901,901
3840 PRINT 5050
5050 FORMAT %1HK,35X,6HZENITH□
CON15 # 1.
901 IF %ALT-400000.□4200,4201,4201
4200 IF %ALT - 40000.□1,2,3
1 D # ALT/10000.
R3# -0.37728875E-02
R2# 0.52352523E-02
R1# -0.31047929E&00
R0# 0.89444960E&01
GO TO 50
2 CONTINUE
3 IF %ALT-80000.□4,5,6
4 CONTINUE
5 D # %ALT-40000.□/10000.
R3# 0.23298604E-05
R2# 0.21620268E-03
R1#-0.47974207E&00
R0# 0.75402419E&01
GO TO 50
6 IF %ALT - 160000.□7,8,9
7 D # %ALT-80000.□/10000.
R3# 0.26359811E-04
R2# 0.80943560E-02
R1# -0.51509227E&00
R0# 0.56272847E&01
GO TO 50
8 CONTINUE
9 IF %ALT-175000.□10,11,12

```

```

10 CONTINUE
11 D # %ALT-160000.□/10000.
   R3# -0.57066660E-04
   R2#  0.30080CC0E-03
   R1# -0.36384173E&00
   R0#  0.20423758E&01
   GO TO 50
12 IF %ALT-270000.□13,14,15
13 D # %ALT-175000.□/10000.
   R3# -0.84014648E-03
   R2# -0.16797257E-02
   R1# -0.32931657E&00
   R0#  0.15021266E&01
   GO TO 50
14 CONTINUE
15 IF %ALT-290000.□16,17,18
16 CONTINUE
17 D # %ALT-270000.□/10000.
   R3#  0.
   R2#  0.36480C09E-03
   R1# -0.6137624CE&00
   R0# -0.25094864E&01
   GO TO 50
18 IF %ALT-350000.□19,20,21
19 D # %ALT-290000.□/10000.
   R3#  0.17070042E-02
   R2# -0.16579218E-02
   R1# -0.63898718E&00
   R0# -0.37292327E&01
   GO TO 50
20 CONTINUE
21 D # %ALT-350000.□/10000.
   R3# -0.38550824E-02
   R2#  0.65901945E-01
   R1# -0.68705559E&00
   R0# -0.72589563E&01
50 RO# %EXPFF%%R3*DER2□*DER1□*DER0□□/100000.
   DEN # RO/GRAV
7980 AX#2.*VY*U&U**2*X-%AMU**2*X□/RA**3□-%DEN*GRAV  *VX*VT□/%2.*BALCO
1□□
   OAY#U**2*Y-2.*VX*U-%AMU**2*Y□/RA**3□-%DEN*GRAV  *VY*VT□/%2.*BALCO
1□□
   AZ#%-1.□*%%AMU**2*Z□/RA**3□&%%DEN*GRAV  *VZ*VT□/%2.*BALCO□□□
   IF %CON8□4000,699,1569
699 CON8 # CON8 & 1.
   GO TO 601
4201 DEN # 0.
   GO TO 798
1569 IF %ALT -150000.□1434,1434,3104
1434 IF %AACC□4000,2460,2461
2460 AACC # AACC & 1.
   GO TO 3104
2461 DELT #DELT2

```

```

3104 T # T & DELT
422 THET1 # THETA
    BETA2 #BETA1
    BETA1 # BETA
    SIGMA1#SIGMA
    PHI1 # PHI
704 PHI # ATANF%Z/%SQRTF%RA**2-Z**2
    THETA #ATANF%Y/X
    DELRA # %RA - RA1/DELT
    DEL # ATANF%DELRA/%SQRTF%VT**2-DELRA**2
421 DTHET#THETA-THET1
    GAMMA3# ATANF%SQRTF%1.-%SINF%PHI1*SINF%PHI1&%COSF%PHI1*COSF%P
    1HI1*COSF%DTHET**2/%SINF%PHI1*SINF%PHI1&%COSF%PHI1*COSF%PHI1*
    2COSF%DTHET
    SIGMA # ATANF%SINF%DTHET*COSF%PHI1/%SQRTF%SINF%GAMMA3**2-%S
    1INF%DTHET*COSF%PHI1**2
    IF %1.5688 - %ABSF%SIGMA4099,4160,4150
4099 IF %SIGMA14102,4101,4102
4101 DELSIG # .001
    GO TO 4120
4102 DELSIG # SIGMA -SIGMA1
4120 IF%BETA1-BETA24128,4130,4129
4128 BETA# BETA1-%ABSF%DELSIG
    GO TO 4300
4130 BETA# BETA1&.001
    GO TO 4300
4129 BETA# BETA1 &%ABSF%DELSIG
    GO TO 4300
4150 IF %.C01-%ABSF%SIGMA4160,4160,4161
4161 IF %BETA - 1.4141,4165,4162
4141 IF %SIGMA4166,4167,4167
4162 IF %BETA1 - 5.4165,4165,4141
4160 IF %SIGMA 4163,4164,4164
4163 IF %BETA1-4.71238954165,4166,4166
4165 BETA #3.14159265-SIGMA
    GO TO 4300
4166 BETA #6.28318530 & SIGMA
    GO TO 4300
4164 IF %BETA1 - 1.570796326754167,4165,4165
4167 BETA # SIGMA
4300 GAMMA # ATANF%SQRTF%1.-%SINF%PHI0*SINF%PHI0&%COSF%PHI0*COSF%P
    1HI0*COSF%THETA-THET0**2/%SINF%PHI0*SINF%PHI0&%COSF%PHI0*COSF
    2%PHI0*COSF%THETA-THET0
870 RN #GAMMA * 3440.239
    VT1#VT
    RA1# RA
    ALT9 #ALT
601 CONTR #CONTR & 1.
    IF %CONTR -1.4000,607,604
604 IF %CONTR-9.602,602,606
606 CONTR #0.
    IPAGE #IPAGE & 1
    GO TO 601

```

```

607 PRINT 5017,IPAGE
602 AA1 #DEL* 57.2958
    AA2 #BETA*57.2958
    AA3 #THETA*57.2958
    AA4 #PHI *57.2958
    PRINT 5005,T,X,Y,Z
    AA6 # SIGMA * 57.2958
    AA7 # GAMMA3 * 57.2958
    PRINT 5006,ALT,VX,VY,VZ
    PRINT 5007,VT,AX,AY,AZ
    PRINT 5008,AA2,RN,DEN,GRAV
    IF %SENSE LIGHT 1= 3000,3001
3000 PRINT 5028,AA4,AA3,AA1
    GO TO 609
3001 PRINT 5025,AA4,AA3,AA1,AT
    609 IF %ALT - 150000.=1940,1940,1941
1940 DELT #DELT2
1941 VX # VX & AX*DELT
    VY # VY & AY * DELT
    VZ # VZ & AZ * DELT
    X # X & VX * DELT
    Y # Y & VY * DELT
    Z # Z & VZ * DELT
    VT # SQRTF%VX**2 & VY**2 & VZ**2=
    AT #%VT-VT1=DELT
    RA # SQRTF%X**2 & Y**2 & Z**2=
    ALT #RA - R
    IF %ALT=1202,1203,1203
1202 AAB8 # AAB8 & 1.
1203 IF %AAB8-1.=4000,2462,1204
1204 IF %AAB8-2.=2462,2463,8000
2463 DEN #.C02376E%%ALT/%-5000.=.0027448-.0023769=
    GO TO 798
5028 FORMAT %1H ,2X,5HPHI ,E15.8,2X,5HTHETA,E15.8,2X,5HDELTA,E15.8=
    100 FORMAT %5E15.8=
5010 FORMAT %1HK,25X,42HPHYSICAL CONSTANTS -SAME FOR ALL MISSILES-=
5011 FORMAT %1H ,25X,57HGRAVITATIONAL ACCELERATION AT SEA LEVEL 32.174
    1 FT/SEC**2=
5012 FORMAT %1H ,30X,44HEARTH ROTATION 2PI/24 X 3600 RADIANS/SECOND=
5013 FORMAT %1H ,30X,32HRADIUS OF EARTH 20,902,890 FEET=
5014 FORMAT %1HK,5X,12HOTHER INPUTS=
5015 FORMAT %1H ,5HDEL ,E15.8,2X,5HBETA ,E15.8,2X,5HTHETA,E15.8,2X
    1,5HPHI ,E15.8=
5016 FORMAT %1H ,5HALT ,E15.8,2X,5HBALCO,E15.8,2X,5HRA ,E15.8,2X
    1,5HVO ,E15.8=
5017 FORMAT %1H1,79X,5HPAGE ,14=
8000 PRINT 8001
8001 FORMAT %1H1,/////////20X,32H*****END OF JOB*****//1H1=
4999 FORMAT %1H1,32X,31HBROWN ENGINEERING COMPANY, INC.,4X,5HPAGE ,14=
5000 FORMAT %1H ,30X,35HFREE FLIGHT AND RE-ENTRY TRAJECTORY=
5001 FORMAT %1H ,35X,24HPROGRAMMED BY W.B.WARREN=
5027 FORMAI %1H ,30X,35HTHEORETICAL DERIVATION - C.F.OSTNER=
5002 FORMAT %1H ,38X,18H RESEARCH STAFF =

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CARD TO PRINT 80/80

5003 FORMAT %1H ,39X,16H10 NOVEMBER 1962□
 5004 FORMAT %1HK,39X,31HPROGRAM NO. SP 9 ENGLISH UNITS.□
 50050FORMAT %1HK,2X,5HTIME ,E15.8,2X,5HX ,E15.8,2X,5HY ,E15.8,2X,
 15HZ ,E15.8□
 5006 FORMAT %1H ,2X,5HALT ,E15.8,2X,5HVX ,E15.8,2X,5HVV ,E15.8,2X,
 15HVZ ,E15.8□
 5007 FORMAT %1H ,2X,5HVT ,E15.8,2X,5HAX ,E15.8,2X,5HAY ,E15.8,2X,
 15HAZ ,E15.8□
 5008 FORMAT %1H ,2X,5HBETA ,E15.8,2X,5HRANGE,E15.8,2X,5HDEN ,E15.8,2X,
 15HGRAY ,E15.8□
 5025 FORMAT %1H ,2X,5HPhi ,E15.8,2X,5HTheta,E15.8,2X,5HDelta,E15.8,2X,
 15HACCEL,E15.8□
 4000 STOP
 END

FORTTRAN STATEMENTS

Metric Units

FREE FLIGHT AND RE-ENTRY
 PROGRAMMED BY W.B. WARREN
 FOR RESEARCH STAFF
 10 NOVEMBER 1962

PROGRAM NO. SP 9 METRIC UNITS.

PHYSICAL CONSTANTS

GO#GRAVITATIONAL ACCELERATION AT SEA LEVEL - 9.80665 METERS/SEC**2
 U # EARTH ROTATION - $2\pi/24 \times 3600$ RADIANS/SEC
 R # RADIUS OF EARTH - 6,371,230 METERS
 BALCO # BALLISTIC COEFFICIENT - CONSTANT
 MU**2 # $GO \times R**2$ # $9.80665 \times 6,371,230**2$

OTHER INPUT DATA

DEL #ANGLE BETWEEN HORIZONTAL AND MISSILE AXIS--POSITIVE UP
 BETA #AZIMUTH ANGLE CLOCKWISE FROM NORTH TO MISSILE AXIS
 THETA #LONGITUDE OF MISSILE
 PHI #LATITUDE OF MISSILE
 ALT #HEIGHT OF MISSILE ABOVE EARTH SURFACE
 RA # MISSILE ALTITUDE FROM CENTER OF EARTH
 VO # MISSILE BURNOUT VELOCITY #VT AT T#0
 READ 100,DEL,BETA,THETA,PHI,ALT
 READ 100,R,VO,GO,U,BALCO
 READ 100,T,DELT,DELT2
 CON8 # 0.
 SENSE LIGHT 1
 CON15 # 0.
 CON1 # 0.
 AACC # 0.
 CONTR#0.
 AAB8 # 1.
 BETA1# BETA - .001
 ALT9 #1000.
 RN#0.
 VT # VO
 VT1 #VO
 AMU # $\sqrt{GO \times R**2}$
 IPAGE # 1
 BETAO # BETA
 THETO # THETA
 PHIC # PHI
 RA # ALT&R
 AK # 16.0184
 OVX # $VO \times \sin\%DEL \times \cos\%PHI \times \cos\%THETA - VO \times \cos\%DEL \times \cos\%BETA \times \sin$
 1F%PHI $\times \cos\%THETA - VO \times \cos\%DEL \times \sin\%BETA \times \sin\%THETA$
 OVY # $VO \times \sin\%DEL \times \cos\%PHI \times \sin\%THETA - VO \times \cos\%DEL \times \cos\%BETA \times \sin$
 1F%PHI $\times \sin\%THETA + VO \times \cos\%DEL \times \sin\%BETA \times \cos\%THETA$
 OVZ # $VO \times \sin\%DEL \times \sin\%PHI + VO \times \cos\%DEL \times \cos\%BETA \times \cos\%PHI$
 X # $RA \times \cos\%PHI \times \cos\%THETA$
 Y # $RA \times \cos\%PHI \times \sin\%THETA$

```

Z # RA*SINF%PHI
RA1 # ALT & R
PRINT 4999,IPAGE
PRINT 5000
PRINT 5001
PRINT 5027
PRINT 5002
PRINT 5003
PRINT 5004
PRINT 5010
PRINT 5011
PRINT 5012
PRINT 5013
PRINT 5014
PRINT 5015,DEL,BETA,THETA,PHI
PRINT 5016,ALT,BALCO,RA,VO
GAMMA # 0.
2462 GRAV # %GO*R**2/RA**2
IF %ALT - ALT900,901,901
900 IF %CON15 - 1.03840,901,901
3840 PRINT 5050
5050 FORMAT %1HK,35X,6HZENITH
CON15 # 1.
901 ALT # ALT * 3.280833
IF %ALT-400000.04200,4201,4201
4200 IF %ALT - 40000.01,2,3
1 D # ALT/10000.
R3# -0.37728875E-02
R2# 0.52352523E-02
R1# -0.31047929E00
R0# 0.89444960E01
GO TO 50
2 CONTINUE
3 IF %ALT-80000.04,5,6
4 CONTINUE
5 D # %ALT-40000.0/10000.
R3# 0.23298604E-05
R2# 0.21620268E-03
R1#-0.47974207E00
R0# 0.75402419E01
GO TO 50
6 IF %ALT - 160000.07,8,9
7 D # %ALT-80000.0/10000.
R3# 0.26359811E-04
R2# 0.80943560E-02
R1# -0.51509227E00
R0# 0.56272847E01
GO TO 50
8 CONTINUE
9 IF %ALT-175000.010,11,12
10 CONTINUE
11 D #%ALT-160000.0/10000.
R3#-0.57066660E-04

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```

R2# 0.30080000E-03
R1#-0.36384173E00
R0# 0.20423758E01
GO TO 50
12 IF %ALT-270000.□13,14,15
13 D # %ALT-175000.□/10000.
R3# -0.84014648E-03
R2# -0.16797257E-02
R1# -0.32931657E00
R0# 0.15021266E01
GO TO 50
14 CONTINUE
15 IF %ALT-290000.□16,17,18
16 CONTINUE
17 D # %ALT-270000.□/10000.
R3# 0.
R2# 0.36480009E-03
R1#-0.61376240E00
R0#-0.25094864E01
GO TO 50
18 IF %ALT-350000.□19,20,21
19 D # %ALT-290000.□/10000.
R3# 0.17070042E-02
R2# -0.16579218E-02
R1# -0.63898718E00
R0# -0.37292327E01
GO TO 50
20 CONTINUE
21 D # %ALT-350000.□/10000.
R3# -0.38550824E-02
R2# 0.65901945E-01
R1# -0.68705559E00
R0# -0.72589563E01
50 RO#%EXP%R3*DER2□*DER1□*DER0□/100000.
DEN #RO*AK*RA**2□/R**2
ALT # ALT / 3.280833
7980AX#2.*VY*U&U**2*X-%AMU**2*X□/RA**3□-%DEN*GRAV *VX*VT□/%2.*BALCO
1□□
0AY#U**2*Y-2.*VX*U-%AMU**2*Y□/RA**3□-%DEN*GRAV *VY*VT□/%2.*BALCO
1□□
AZ#%-1.□**%AMU**2*Z□/RA**3□&%DEN*GRAV *VZ*VT□/%2.*BALCO□□□
IF %CON8□4000,699,1569
699 CON8 # CON8 & 1.
GO TO 601
4201 DEN # 0.
ALT # ALT / 3.280833
GO TO 798
1569 IF %ALT - 45720.096□1434,1434,3104
1434 IF %AACC□4000,2460,2461
2460 AACC # AACC & 1.
GO TO 3104
2461 DELT #DELT2
3104 T # T & DELT

```

```

422 THET1 # THETA
    BETA2 #BETA1
    BETA1 # BETA
    SIGMA1#SIGMA
    PHI1 # PHI
704 PHI # ATANF%Z/%SQRTF%RA**2-Z**2
    THETA #ATANF%Y/X
    DELRA # %RA - RA1/DELT
    DEL # ATANF%DELRA/%SQRTF%VT**2-DELRA**2
421 DTHET#THETA -THET1
    GAMMA3# ATANF%SQRTF%1.-%SINF%PHI1*%SINF%PHI1*%COSF%PHI1*%COSF%P
1HI*%COSF%DTHET**2/%SINF%PHI1*%SINF%PHI1*%COSF%PHI1*%COSF%PHI1*
2COSF%DTHET
    SIGMA # ATANF%SINF%DTHET*%COSF%PHI1/%SQRTF%SINF%GAMMA3**2-%S
1INF%DTHET*%COSF%PHI1**2
    IF %1.5688 - %ABSF%SIGMA4099,4160,4150
4099 IF %SIGMA4102,4101,4102
4101 DELSIG # .001
    GO TO 4120
4102 DELSIG # SIGMA -SIGMA1
4120 IF%BETA1-BETA24128,4130,4129
4128 BETA# BETA1-%ABSF%DELSIG
    GO TO 4300
4130 BETA# BETA1*.001
    GO TO 4300
4129 BETA# BETA1 &%ABSF%DELSIG
    GO TO 4300
4150 IF %.001-%ABSF%SIGMA4160,4160,4161
4161 IF %BETA - 1.4141,4165,4162
4141 IF %SIGMA4166,4167,4167
4162 IF %BETA1 - 5.4165,4165,4141
4160 IF %SIGMA 4163,4164,4164
4163 IF %BETA1-4.71238954165,4166,4166
4165 BETA #3.14159265-SIGMA
    GO TO 4300
4166 BETA #6.28318530 & SIGMA
    GO TO 4300
4164 IF %BETA1 - 1.570796326754167,4165,4165
4167 BETA # SIGMA
4300 GAMMA # ATANF%SQRTF%1.-%SINF%PHI0*%SINF%PHI0*%COSF%PHI0*%COSF%P
1HI*%COSF%THETA-THET0**2/%SINF%PHI0*%SINF%PHI0*%COSF%PHI0*%COSF
2%PHI*%COSF%THETA-THET0
870 RN #GAMMA * 3440.239
    VT1#VT
    RA1# RA
    ALT9 #ALT
601 CONTR #CONTR & 1.
    IF %CONTR -1.4000,607,604
604 IF %CONTR-9.602,602,606
606 CONTR #0.
    IPAGE #IPAGE & 1
    GO TO 601
607 PRINT 5017,IPAGE

```

```

602 AA1 #DEL * 57.2958
    AA2 #BETA*57.2958
    AA3 #THETA*57.2958
    AA4 #PHI *57.2958
    PRINT 5005,T,X,Y,Z
    AA6 # SIGMA * 57.2958
    AA7 # GAMMA3 * 57.2958
    PRINT 5006,ALT,VX,VY,VZ
    PRINT 5007,VT,AX,AY,AZ
    PRINT 5008,AA2,RN,DEN,GRAV
    IF %SENSE LIGHT 1= 3000,3001
3000 PRINT 5028,AA4,AA3,AA1
    GO TO 609
3001 PRINT 5025,AA4,AA3,AA1,AT
    609 IF %ALT - 45720.096=1940,1940,1941
1940 DELT #DELT2
1941 VX # VX & AX*DELT
    VY # VY & AY * DELT
    VZ # VZ & AZ * DELT
    X # X & VX * DELT
    Y # Y & VY * DELT
    Z # Z & VZ * DELT
    VT # SQRTF%VX**2 & VY**2 & VZ**2=
    AT #%VT-VT1= /DELT
    RA # SQRTF%X**2 & Y**2 & Z**2=
    ALT #RA - R
    IF %ALT=1202,1203,1203
1202 AABBB # AABBB & 1.
1203 IF %AABBB-1.=4000,2462,1204
1204 IF %AABBB-2.=2462,2463,8000
2463 DEN #.002376&%ALT/%-5000.=*%.0027448-.0023769=
    GO TO 798
5028 FORMAT %1H ,2X,5HPhi ,E15.8,2X,5HTheta,E15.8,2X,5HDelta,E15.8=
    100 FORMAT %5E15.8=
5010 FORMAT %1HK,25X,42HPhysical constants -SAME FOR ALL MISSILES-=
5011 FORMAT %1H ,25X,62HGravitational acceleration at sea level 9.8066
    15 METERS/SEC**2=
5012 FORMAT %1H ,30X,44HEarth rotation 2PI/24 X 3600 RADIANS/SECOND=
5013 FORMAT %1H ,30X,33HRadius of earth 6,371,230 METERS=
5014 FORMAT %1HK,5X,12HOther inputs=
5015 FORMAT %1H ,5HDEL ,E15.8,2X,5HBETA ,E15.8,2X,5HTheta,E15.8,2X
    1,5HPhi ,E15.8=
5016 FORMAT %1H ,5HALT ,E15.8,2X,5HBALCO,E15.8,2X,5HRA ,E15.8,2X
    1,5HVO ,E15.8=
5017 FORMAT %1H1,79X,5HPAGE ,14=
8000 PRINT 8001
8001 FORMAT %1H1,/////////20X,32H*****END OF JOB*****//1H1=
4999 FORMAT %1H1,32X,31HBrown Engineering Company, INC.,4X,5HPAGE ,14=
5000 FORMAT %1H ,30X,35HFree flight and re-entry trajectory=
5001 FORMAT %1H ,35X,24HProgrammed by W.B.Warren=
5027 FORMAT %1H ,30X,35HTheoretical derivation - C.F.Ostner=
5002 FORMAt %1H ,38X,18H RESEARCH STAFF =
5003 FORMAT %1H ,39X,16H10 NOVEMBER 1962=

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CARD TO PRINT 80/80

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5004 FORMAT %1HK,39X,31HPROGRAM NO. SP 9 METRIC UNITS.
50050FORMAT %1HK,2X,5HTIME ,E15.8,2X,5HX ,E15.8,2X,5HY ,E15.8,2X,
      15HZ ,E15.8
5006 FORMAT %1H ,2X,5HALT ,E15.8,2X,5HVX ,E15.8,2X,5HVV ,E15.8,2X,
      15HVZ ,E15.8
5007 FORMAT %1H ,2X,5HVT ,E15.8,2X,5HAX ,E15.8,2X,5HAY ,E15.8,2X,
      15HAZ ,E15.8
5008 FORMAT %1H ,2X,5HBETA ,E15.8,2X,5HRANGE,E15.8,2X,5HDEN ,E15.8,2X,
      15HGRAV ,E15.8
5025 FORMAT %1H ,2X,5HPHI ,E15.8,2X,5HTHETA,E15.8,2X,5HDELTA,E15.8,2X,
      15HACCEL,E15.8
4000 STOP
      END

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REFERENCES

1. Kooy, Uytenbogaart, Ballistics of the Future, N. V. Detechnische Uitgeverij H. Stam.
2. Pope, Bennie E., "Body Dynamics During Re-Entry", Brown Engineering Company, Inc. Technical Note R-5A, (Unclassified Version), August 28, 1962.
3. Minshew, H. M., "A FORTRAN Program to Calculate Atmospheric Properties", Brown Engineering Company, Inc. Technical Note R-27, October, 1962.