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Application of the Similarity Method to Investigate the Movements of Powerful Beams of Charged Particles in High Voltage Electrostatic Fields.

by

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To solve a series of physical and technical problems we by necessity encounter the problem of motion of a powerful beam of charged particles in a high voltage electrostatic field. In this case, when it is neccessary to take into consideration the effect of the space charge, as well as the statistical discrepancy of initial velocities of particles, analytical calculation of the form of the beam and dimensions of the spot, obtained when the beam strikes the target, is connected with greater difficulties. Consequently an experimental solution of this problem by means of experiments on similar low voltage models is of great interest.

To derive similarity criteria of the beams we will discuss at first a similar partial case, when the beam of particles satisfies the following condition:

1) the beam consists of identical particles;

2) the velocity of the particles appears to be a synonymous function of the coordinate i.e. through one point of space cannot pass particles, the velocities or which differ in magnitude or direction;

3) the velocities of particles v connected with the field potential φ by the ratio

$$\frac{mv^2}{2} + e\varphi = 0, \qquad (1)$$

i.e. $v = 0$ at $\varphi = 0$.

In this case the movement of the beam is described by three differential equations: Poisson equation, continuum equation and equation of motion of particles, binding the values, describing the system (field potential φ, current density j and ort of the velocity vector of the particle v^0) and space coordinate (x,y,z)

$$\Delta\varphi = -\frac{4\pi}{\sqrt{-2\varphi}}\cdot\frac{j}{\sqrt{e/m}}; \qquad (2a)$$

$$\mathrm{div}\,(j\bar{v}^0) = 0; \qquad (2b)$$

$$(\sqrt{\varphi}\bar{v}^0\nabla)(\sqrt{\varphi}\bar{v}^0) = -\frac{1}{2}\nabla\varphi. \qquad (2c)$$

Time is excluded from the equation of motion, because under the assumption made the time derivative can be expressed through derivatives in accordance with space coordinates

$$\frac{d\bar{v}}{dt} = (\bar{v}\nabla)\bar{v} = -\frac{2e}{m}(\sqrt{\varphi}\bar{v}^0\nabla)(\sqrt{\varphi}\bar{v}^0). \qquad (3)$$

The system of these three equations together with boundary conditions (potentials of electrodes, limiting the discussed space, $\varphi = \varphi\,(_c)$ and current density j = j (c) of the beam, included in this space), determines synonymously the movement of the discussed beam and the distribution of the field potential.

The system of equations (2) can be easily generalized for this important case, when the beam consists of particles with different specific charges $\frac{e_k}{m_k}$, i.e. from various components, which almost always take place for ion beams.

In this case equations of continuum and motion should be written for each component individually. Consequently, the system will consist of n + n + 1 = 2n + 1 equation (n - number of components) with 2n + 1 unknown φ; $j_1, \ldots, j_n; \bar{v}_n^0$

$$\Delta\varphi = -\frac{4\pi}{\sqrt{-2\varphi}}\left|\frac{j_1}{\sqrt{e_1/m_1}} + \cdots + \frac{j_n}{\sqrt{e_n/m_n}}\right|; \qquad (4a)$$

$$\mathrm{div}\,(j_k\bar{v}_k^0) = 0; \qquad (4b)$$

$$(\sqrt{\varphi}\bar{v}_k^0\nabla)(\sqrt{\varphi}\bar{v}_k^0) = -\frac{1}{2}\nabla\varphi. \qquad (4c)$$

$$(k = 1, 2, \ldots, n).$$

Members with index k pertain to the k-compoent of the beam. Term is square parenthesis of equation (4a) is the general space charge of the beam.

In cases, when the ratio of current densities of various components is identical in all points of output from the source (this assumption if strictly valid in case, when the source has no magnetic field, and is approximately avlid in the case of presence of magnetic fields in the source) system (4) can be simplified considerably. The fact is, the trajectories of particles in a stationary electric field, which is established in the system, as is evident from equation (4c), does not depend upon $\frac{e_k}{m_k}$. Consequently the current tubes of all components, coming out from section of the source, coincide. Thus, the current density ratio of the components $i_{kv} = \frac{j_e}{j_k}$ in all sections of one current tube is constant and equal to the value of this ratis at the beginning of the tube, i.e. at the outlet from the source, and since i_{ke} is identical in all points of output from the source, then it is evident, that i_{ke} is identical in all points of space.

And so

$$\vec{v}_k^{\,0}(x, y, z) = \vec{v}^{\,0}(x, y, z),$$
$$j_k(x, y, z) = i_k I(x, y, z), \qquad (5)$$

where I -total current density of the beam, and i_k-constant, not depending upon the coordinate. As result of this system (3) contains only three independent variables φ $\bar{v}_k^o$ j_k (k-arbitrary) and all the remaining ones are expressed by same.

We shall introduce a new variable j, proportional to the current density of the complex beam

$$\frac{j}{\sqrt{\frac{e}{m}}} = \left[\frac{j_1}{\sqrt{\frac{e_1}{m_1}}} + \dots + \frac{j_n}{\sqrt{\frac{e_n}{m_n}}}\right] = I\left[\frac{i_1}{\sqrt{\frac{e_1}{m_1}}} + \dots + \frac{i_n}{\sqrt{\frac{e_n}{m_n}}}\right], \qquad (6)$$

where $\frac{e}{m}$ is a certain (sofar arbitrary) constant, having the dimension of a specific charge. The substitution of (5) and (6) in system (4) reduces the solution of system 2n + 1 of the equations to a solution of a system of three equations

$$\Delta\varphi = -\frac{4\pi}{\sqrt{-2\varphi}}\frac{j}{\sqrt{e/m}}; \qquad (7a)$$
$$\operatorname{div}(j\vec{v}^{\,0}) = 0; \qquad (7b)$$
$$(\sqrt{\varphi}\,\vec{v}^{\,0}\,\nabla)(\sqrt{\varphi}\,\vec{v}^{\,0}) = -\frac{1}{2}\nabla\varphi. \qquad (7c)$$

i.e. a system of equations, describing the complex beam, equivalent to the system, describing a simple beam, consisting of particles with specific charge $\frac{e}{m}$.

We shall now examine the case, when into the investigated space enter a beam,consisting of identical particles, but for which the conditions (2) and (3) are not fulfilled, i.e. through one point can pass particles, the velocities of which differ in magnitude and direction as well. Let us break up this beam into a series of component beams (components) so that for each one of them condition (2) is fulfilled individually. It means that the velocities of particles, belonging to one component, are determined by equations

$$\frac{mv_k^2}{2} + e(\varphi + c_k) = 0, \qquad c_k = \text{const}, \tag{1a}$$

whereby $\bar{v}_k$ appears to be already a synonymous function of the coordinate. (single-values function of coordinate).

The movement of such a beam can be described by a system of equations analogous to system (4)

$$\Delta\varphi = -\frac{4\pi}{\sqrt{e/m}}\left[\frac{j_1}{\sqrt{-2(\varphi + c_1)}} + \dots + \frac{j_n}{\sqrt{-2(\varphi + c_n)}}\right] \tag{8a}$$

$$\operatorname{div}(j_k \bar{v}_k^0) = 0 \tag{8b}$$

$$\left(\sqrt{(\varphi + c_k)}\,\bar{v}_k^0\,\nabla\right)\left(\sqrt{(\varphi + c_k)}\,\bar{v}_k^0\right) = -\frac{1}{2}\nabla\varphi \tag{8c}$$

$k = 1, 2, \dots, n$, where n – number of components. In case of a complex beam the system of equations (8) can be generalized in the same manner as the generalization of system (2). It will have the very same form as (8) but j_k will no longer be current density of components, but values, determinable by formulas, analogous to (6)

$$\frac{j_k}{\sqrt{\frac{e}{m}}} = \frac{j_{k1}}{\sqrt{e_1/m_1}} + \dots + \frac{j_{kp}}{\sqrt{e_p/m_p}}. \tag{6a}$$

where j_{kp} – current density of particles with specific charge e_p/m_p and energy $e(\varphi + c_k)$.

To solve system (8) is very difficult, because even the method of breaking up the beam into individual components may be found only after the movement of particles beam is known. However for our purposes – to explain similarity conditions – this is of no value. Important is only the fact that the beam in general case can be described by system (8), even though it is unknown how it should be broken down into components.

We will imagine, that there a[illegible]lar systems accelerating electrodes, in which similar beams of particles are moving (in first case particles with specific charge $\frac{e_1}{m_1}$, in the second – $\frac{e_2}{m_2}$).

In compatible points of similar systems

$$\bar{r}_2 = \lambda \bar{r}_1, \qquad (l_2 = \lambda l_1) \tag{9a}$$

the values of physical magnitudes are connected by ratios

$$\frac{e_2}{m_2} = \frac{1}{\mu}\frac{e_1}{m_1}; \quad \varphi_2 = \varepsilon\varphi_1, \quad j_2 = ij_1. \tag{9b, 9c, 9d}$$

whereby

$$\bar{v}_2^0 = \bar{v}_1^0. \tag{9e}$$

because the trajectories of particles are similar.

The values pertaining to the first ($\frac{e_1}{m_1}$, $\bar{r}_1$, φ_1, j_1 $\bar{v}^0_1$) as well as to the second ($\frac{e_2}{m_2}$, $\bar{r}_2$, φ_2, $\bar{v}^0_1$) system should satisfy the system of equation (2). Consequently, substituting in (2) the values ($\frac{e_2}{m_2}$, $\bar{r}_2$, φ_2, j_2, $\bar{v}^0_2$), expressed through ($\frac{e_1}{m_2}$, $\bar{r}_1$, φ_1, j_1, $\bar{v}^0_1$) is possible to obtain formulas, binding the similarity constants. The substitution offers

$$\frac{\varepsilon}{\lambda^2} = \frac{i}{\mu^{-1/2}} \cdot \frac{1}{\varepsilon^{1/2}}, \tag{10a}$$

$$\frac{i}{\lambda} \times 0 = 0, \tag{10b}$$

$$\left(\varepsilon^{1/2}\frac{1}{\lambda}\right)\varepsilon^{1/2} = \frac{\varepsilon}{\lambda}. \tag{10c}$$

(10b) and (10c) appear to be identical and do not give limiting conditions. But equality (10a) offers a condition binding similarity constants

$$i = \frac{\varepsilon^{3/2}}{\mu^{1/2}\lambda^2}. \tag{11}$$

Condition (11) can be represented in form of a similarity criterion

$$\frac{jr^2}{\sqrt{\frac{e}{m}}\,\varphi^{3/2}} = \text{idem}. \tag{12}$$

From the condition, binding the density of currents, is possible to obtain a condition binding the currents of similar beams (J_2 and J_1). Since $J = \int \bar{j}\, d\bar{s}$, then

$$\frac{J}{\sqrt{\frac{e}{m}}\,\varphi^{3/2}} = \text{idem}. \tag{13}$$

Equations, de[illegible] it was shown, are equivalent to equations, describing the simple beam with specific charge $\frac{e}{m}$ and current density

$$j = I\left[i_1\sqrt{\frac{m_1}{e_1}}\sqrt{\frac{e}{m}} + \dots + i_n\sqrt{\frac{m_n}{e_n}}\sqrt{\frac{e}{m}}\right]. \quad (6a)$$

Values I, i_k, $\frac{e_K}{m_K}$ are the very same as in (6).

Such a simple beam, apparently, is similar to a complex one, because in similar points the potentials and velocity orts (crosscuts) of the particles are identical and the current densities of the beams are directly proportional. In general, the case is not only when the compositions of similar beams are different, but also the dimensions and potentials of the electrode system, the similarity condition appears to be the sole generalization of condition (11)

$$i = \frac{\varepsilon^{3/2}}{\lambda^2}\left[i_1\sqrt{\frac{m_1}{e_1}}\sqrt{\frac{e}{m}} + \dots + i_n\sqrt{\frac{m_n}{e_n}}\sqrt{\frac{e}{m}}\right]. \quad (11a)$$

Finally, when the beam is determined by the system of equations (8) the system of equations will be invariant with respect to a similar transform (9) in this and only in this case, when to each individual component of the beam will correspond a similar individual beam, the current density of which is determined by equation (11) (or 11a for the complex beam).

Since the movement of the beam is determined uniquely by the boundary conditions then for the realization of beam similarity it is necessary and it is sufficient, that the boundary conditions should be sufficient, i.e., that the electrode accelerating systems, their potentials (coefficients of similarity λ and ε) and the functions of distribution of initial particle velocities (in magnitude and direction) should be similar, whereby the initial energies of similar particles should pertain like ε, and the current densities of similar beams in analogous initial points as $i = \frac{\varepsilon^{3/2}}{\mu^{1/2}\lambda^2}$ or (11a). Especially, if the distribution of initial velocities of particles, included in the system, is described by the Maxwell distribution, corresponding to a temperature εT.

If the emissivity of the sources can be considered infinite and the current intensity of beams is limited only by the space charge, then for the similarity of beams is sufficient a similarity of the electrode system. Hence, among other things, it follows, that all three Child-Langmuir formulas can be obtained with an accuracy to constant multiple from (12).

Equation (12) is valid not only for nonrelativistic particles, which we are considering here, but also for relativistic. But in the latter case in addition to similarity criteria (12) should also be fulfilled the criterion

$$\frac{v}{c} = \text{idem} \qquad (14)$$

(c - speed of light), indicating, that the relativistic effect in similar beams is identical. Since v is connected with φ and $\frac{e}{m}$, then the change in particle mass at a similar transform in relativistic case is equally connected with the change in potential, which limits the possibilities of utilizing models for relativistic systems.

In this way, on the basis of the similarity criterion with the aid of model experiments can be investigated focusing properties of high voltage systems, whereby it is fully considered as a statistical discrepancy in initial velocities of particles and as the effect of a space charge of the beam.

With the aid of such experiments we have investigated the focusing properties of lengthy accelerating systems. Not stopping on the obtained results, we will try one technical problem, essential when conducting such experiments.

The experimental results may be distorted by the ionization of gases remaining in the tube by particles of the beam and formation of a secondary particles cloud, changing the general space charge. That is why it is necessary to produce in the tube a sufficiently good vacuum (of the order of 10^{-6} mm Hg). In addition, in cases (practically most important), when there is even the slightest field in the tube (greater than 10 v/cm), the effect of ionization of gas residues on the performance of the model can be evaluated experimentally. For this in the tube is first accelerated the electron beam, and then in exactly the very same conditions the ion bea (sodium or lithium thermoions). Due to the presence of an accelerating field secondary electrons are rapidly resorbed and their charge can be disregarded in comparison with the charge of the positive ion cloud. Therefore, if in first case ionization of gas residues reduces the space charge of the beam, then in the second it increases same. Consequently, if the results of both experiments are insufficiently good agreement (with consideration of similarity formula) then it can be said, that gas ionization does not distort the obtained result s.

Corresponding experiments showed, that at pressures of the order of 10^{-5} mm Hg the coincidence of both experiments appears to be sufficiently good and consequently, during the interpretation of results of the experiments, secondary phenomena may be left out of consideration.

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