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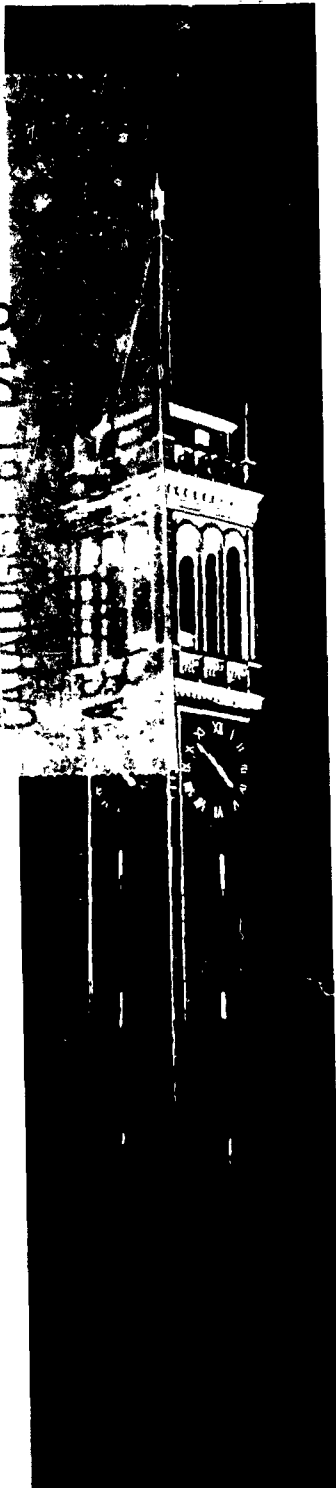


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Anode Structures For Cold-Cathode High-Power Magnetron

by
Y. Ikeda

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Electronics Research Laboratory
University of California
Berkeley, California

ANODE STRUCTURES FOR COLD-CATHODE
HIGH-POWER MAGNETRON

by

Y. Ikeda

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SUMMARY

Rf interaction properties of several structures suitable for cold-cathode high-power magnetrons have been investigated analytically and experimentally, with special emphasis on increasing the understanding of the interaction and maximizing the area of coherent interaction with the electron beam at a given frequency. The structures analyzed were designed for large mode separation, maximum interaction impedance, and easy coupling to the output circuit.

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I. INTRODUCTION

This report deals with the rf interaction properties of several anode structures for high-power, cold-cathode magnetrons. Emphasis is on understanding the interaction and on determining how the region of coherent interaction with the electron beam can be made as large as possible at a given frequency so as to maximize the output power.

In a conventional magnetron, the number of resonators is limited by the requirement of reasonable separation in frequency of the modes. As the number of resonators increases, the mode separation of even the "strapped" and "rising-sun" configurations begin to be insufficient.

The structures investigated in this report were chosen as (1) suitable for construction in the cold-cathode magnetron experiments in this Laboratory, (2) possessing large mode separation, and (3) having a circuit that is easily coupled to the output circuit.

One circuit that was tried was a corrugated cylindrical wall with interaction bars and coupling channels. This circuit has a 60-percent mode separation but a rather low interaction impedance for the TM_0 mode, since the minimum electric field occurs at the wall.

The new structures analyzed in this report are free from this difficulty; however, they do suffer from the disadvantage of being susceptible to possible multipactor action along the magnetic field resulting from the rf electric field. This problem probably can be avoided by a careful design of the circuit.

The main assumption made in the analysis is that the axial length of the structure is much shorter than a free-space wavelength, so that field variations along the structure can be neglected. Further, it is assumed that since many bars oriented parallel to the axis are to be used in structure, the capacity between them and the cylindrical wall is taken to be uniformly distributed along the cavity circumference. In many cases the capacity between anode and cathode is neglected.

MKS units are used throughout and only the symbols given other than common meaning are explained in the text.

II. INVERTED MAGNETRON STRUCTURE

A. PROPOSED ANODE STRUCTURE, SYMBOLS AND DEFINITIONS

In the first anode structure to be discussed (Fig. 1), the points at

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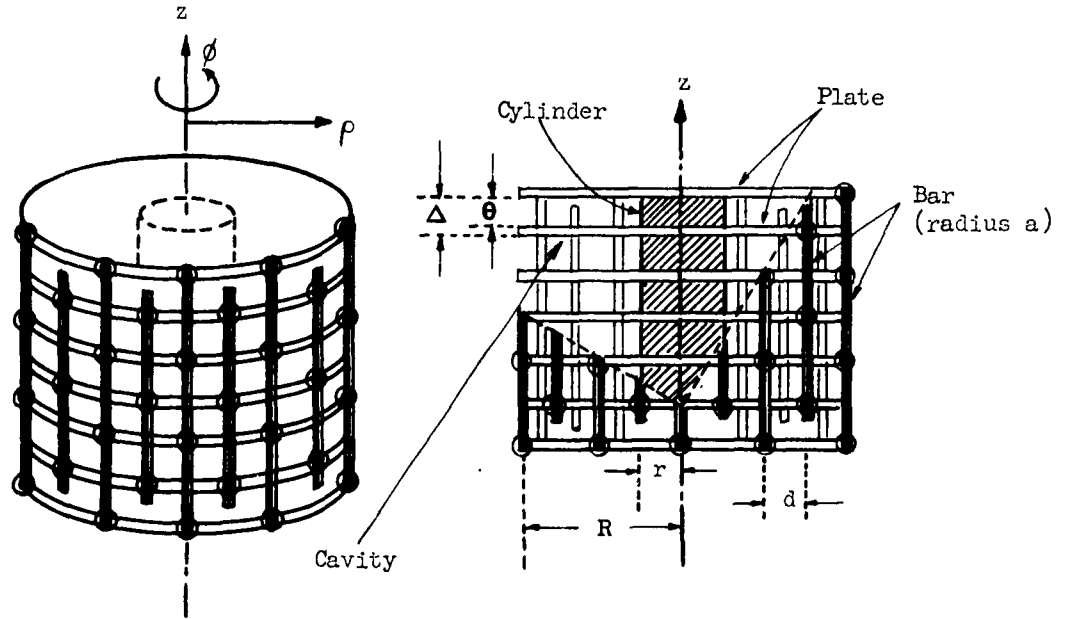


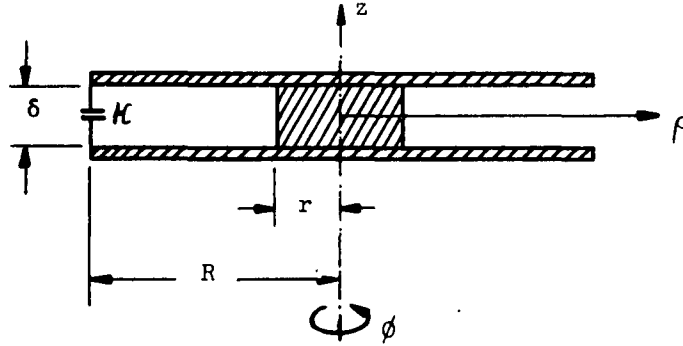
FIG. 1.--Inverted magnetron structure.

which the plate electrodes and bars are connected have been indicated by dots. The slow-wave structure is on the inside and is surrounded by a concentric cold cathode that depends on secondary emission for its electron current. The TM_0 mode that is excited in each cavity makes the bars alternately plus and minus and consequently provides the circumferential rf fields necessary for magnetron operation. There is a dc magnetic field in the direction of the z axis.

The gap between the plates of the anode structure in Fig. 1 is denoted by δ , a parameter chosen so as to minimize the multipactor problem and to optimize the interaction impedance. In general δ is small and as a consequence the capacitance for a unit cavity is large, so that the frequency of the cavity is insensitive to the electron beam interaction as well as to the proximity of the cathode cylinder. For this reason no conventional "straps" are required for this structure.

B. RESONANCE FREQUENCY FOR THE INDIVIDUAL CAVITY

The anode structure shown in Fig. 1 is made with a large number of bars. Their effect is therefore approximated by an equivalent uniform capacitance $\mathcal{K}$ per unit length in the circumferential direction (Fig. 2). The edge effect

FIG. 2.--Equivalent capacitance $\mathcal{K}$.

of the plate may also be included in this equivalent capacity. It is assumed that only the z component of the electric field exists inside the cavity because the gap dimension δ is small.

In cylindrical coordinates the solution of Maxwell's equations is given by the following:

$$E_z = E_0 [J_n(k\rho) + AN_n(k\rho)] \cos n\phi$$

$$H_\phi = -j \frac{kE_0}{\omega\mu_0} [J_n'(k\rho) + AN_n'(k\rho)] \cos n\phi \quad (1)$$

$$H = -j \frac{nE_0}{\omega\mu_0 \rho} [J_n(k\rho) + AN_n(k\rho)] \sin n\phi$$

where $k^2 = \omega^2 \epsilon_0 \mu_0$, $n = 0, \pm 1, \pm 2, \dots$ and $A = -J_n(kr)/N_n(kr)$. Also, since at $\rho = R$, $H_\phi(R)/V(R) = I_\rho(R)/V(R) = j\omega\mathcal{K}$, where $\mathcal{K}$ is the capacitance per unit length, we have

$$B_n(kR) = \frac{J_n'(kR) + AN_n'(kR)}{J_n(kR) + AN_n(kR)} = \mathcal{K} \delta \sqrt{\frac{\mu_0}{\epsilon_0}} \quad (2)$$

The resonant frequency for the various modes is determined by the above equations. It is convenient to rewrite these equations in terms of some new

(II. INVERTED MAGNETRON STRUCTURE)

variables. Letting $kR = 2\pi R/\lambda_0 = x$, where λ_0 is the free-space wavelength and $r/R = \xi$, we have

$$B_n(x, \xi) = \frac{J_n'(x)N_n(\xi x) - N_n'(x)J_n(\xi x)}{J_n(x)N_n(\xi x) - N_n(x)J_n(\xi x)} = \frac{\kappa \delta}{\epsilon_0 R} x \quad (2a)$$

The capacitance per unit length between parallel bars of radius a separated a distance d is

$$C_0 = \pi \epsilon_0 / \log [(d + \sqrt{(d^2 - 4a^2)})/2a] \quad (3)$$

From Fig. 1

$$\kappa = C_0 \Delta / d$$

As an example, we take $R = 66$ mm, $\xi = 1/8$, $\delta = 3.5$ mm, $\Delta = 5$ mm, $d = 7.2$ mm, and $a = 2.5$ mm. Then $\kappa \delta / \epsilon_0 R \approx 0.15$. The resonant frequency for the $n = 0$ and $n = 1$ modes can be determined as shown graphically in Fig. 3. The results of these calculations are given in Fig. 4. The rate of mode separation is also given.

If the central cylinder is removed, kR is the first root of $J_0(kR) = 0$ for the TM_{001} mode; the rate of mode separation is about 60 per cent. The radial dimension of the cavity resulting from Fig. 4 is bigger than the cavity for $J_0(kR) = 0$ if $\xi > 0.45$ and the mode separation is good if $\xi < 0.14$.

This structure has a large mode separation for small values of ξ ; the rate of mode separation is not significantly affected by the capacitance of the bars.

C. MEASUREMENT OF THE RESONANT FREQUENCY

A schematic diagram of the cavity used to check the calculated frequencies is shown in Fig. 5. The cavity was excited and the fields in the cavity were probed by small antennas inserted into holes as shown in the figure. The various modes of oscillation were detected by perturbing the fields with small wires that could be inserted into the drill holes at various places in the cavity.

The calculated and measured frequencies are compared in Table 1. The end capacitance per unit circumferential length is taken to be $1/4$ of the

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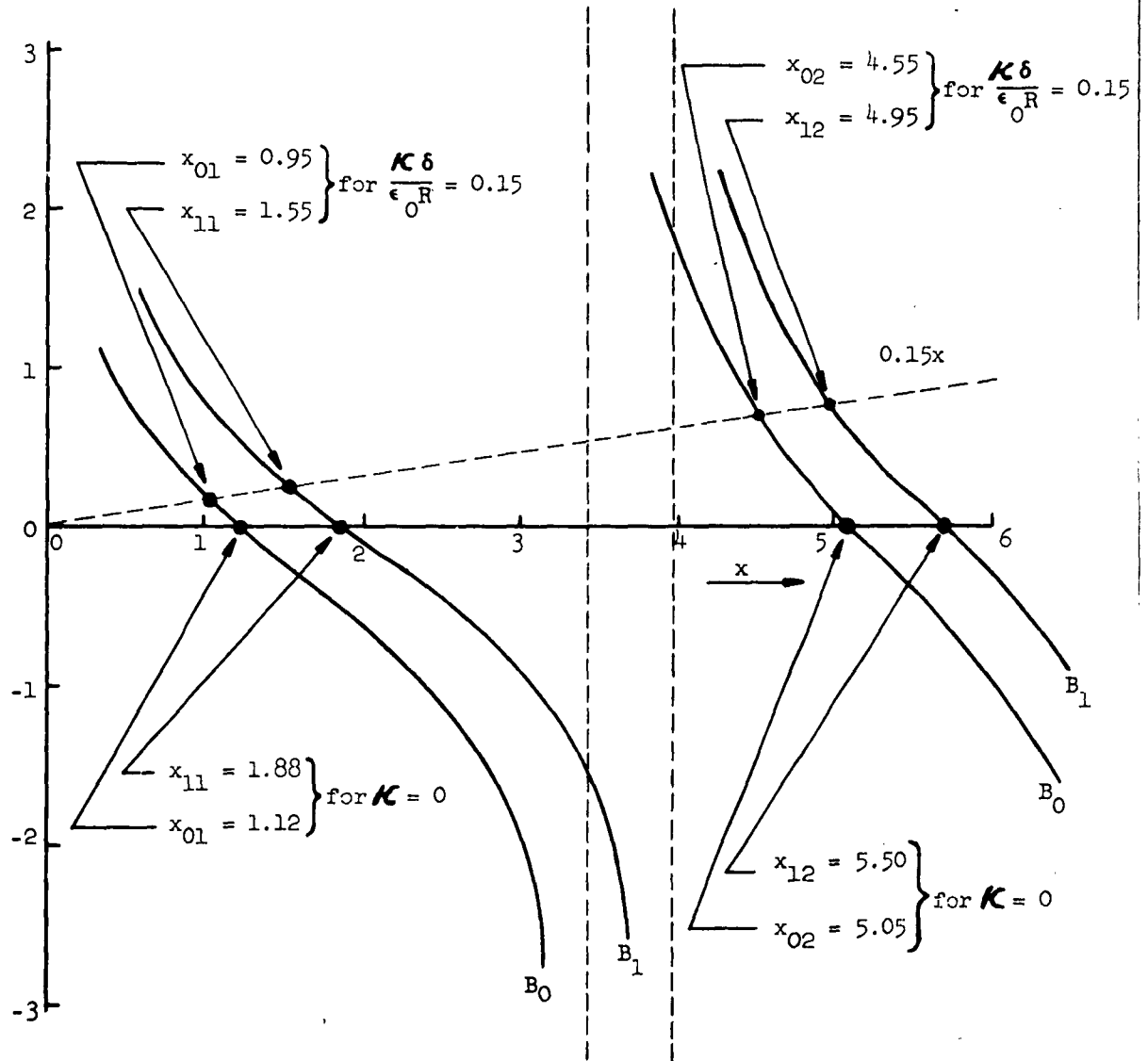


FIG. 3.--Graphical determination of resonant frequency.

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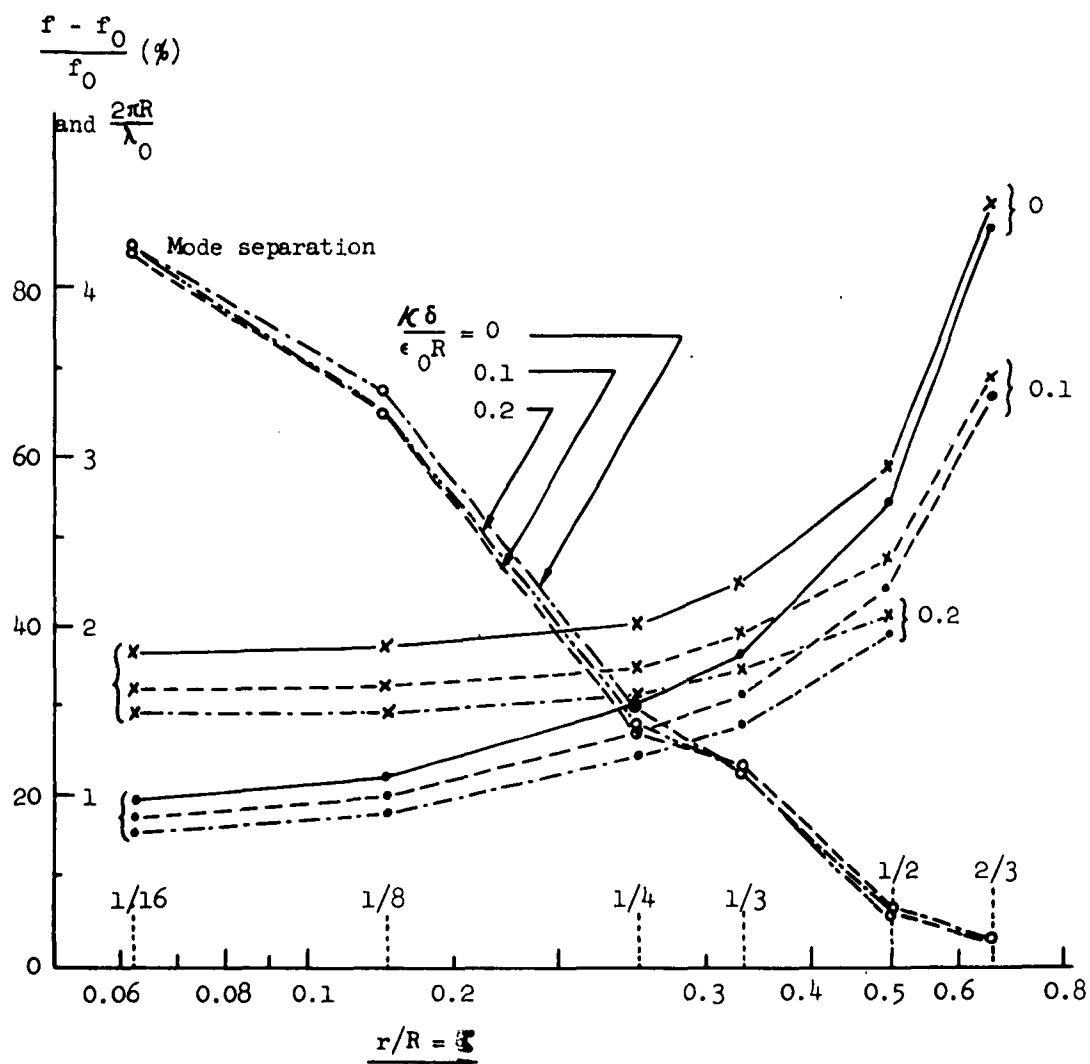


FIG. 4.--Results of the calculation of resonant frequencies.

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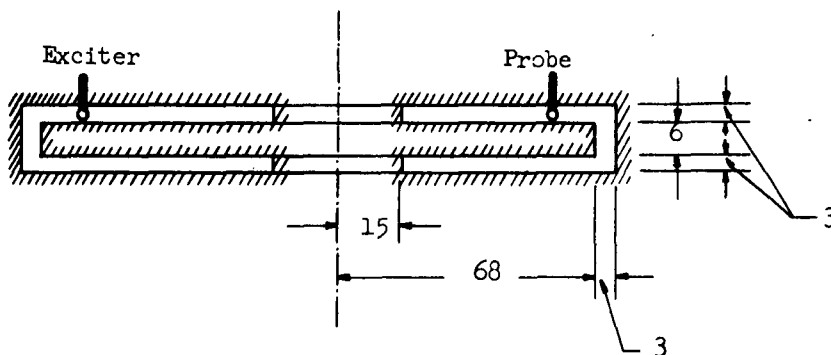


FIG. 5.--Cavity used in experimental confirmation of calculated frequencies. $R = 68$ mm, $\epsilon = 0.22$. (Dimensions in millimeters.)

Table 1.--Resonant frequencies for the structure of Fig. 5
($K\delta/\epsilon_0 R \approx 1$).

Mode	Frequency (Mc)	
	Measured	Calculated
0	957	950
1	1326	1287
2	2010	-

capacitance of a coaxial line that has an inner radius of 3 mm and an outer radius of 6 mm. The calculated and measured values agree to within 3 per cent. In the calculation of the resonant frequency for this case it was possible to assume the effective radius R to be equal to $1/2$ the distance along the gap in the cross section of Fig. 5; it is also assumed that K is equal to 0. In addition to the tabulated results a 1308-Mc mode was found. This is a weakly excited $n = 1$ mode that exists in the coupled structure.

These measurements were repeated with the structure shown in Fig. 6. The results of these measurements are given in Fig. 7. This structure has many resonances because of the complicated coupling of the structure. These frequencies are shown in Fig. 7 in relation to the resonant frequency for the single cavity, and will be given more detailed consideration below. Results of another experiment with a similar structure but of different dimensions are shown in Table 2.

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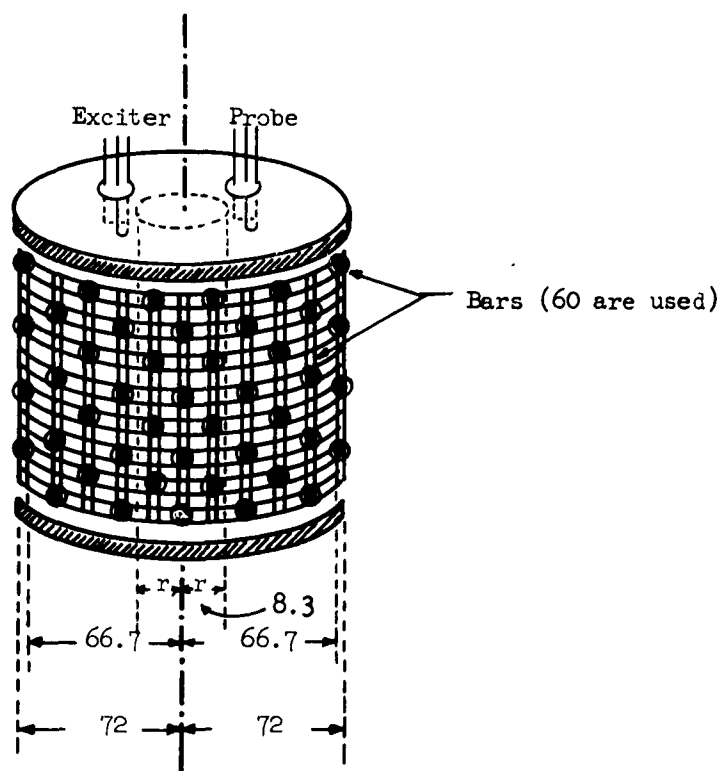


FIG. 6.--Alternate inverted structure. (All dimensions in millimeters.) $\delta = 3.5$, $\Delta = 5.0$, $d = 7.2$, a , 2.5 , $\delta/\epsilon_0 R = 0.15$.

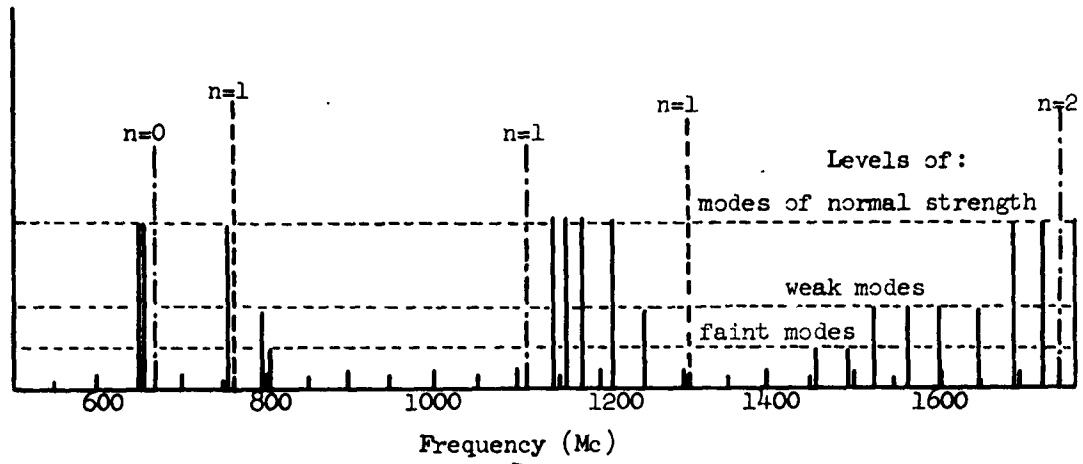


FIG. 7.--Measured spectra (solid lines) and calculated frequencies for unit cavity with $R = 66.7$ mm (-----) and for cavities at both ends with $R = 72$ m (-----).

Table 2.--Resonant frequencies for another structure: $R = 60$, $\xi = 0.125$, $\delta = 3$, $\Delta = 12$, $d = 10$, $a = 3$, $K\delta/\epsilon_0 R \approx 0.2$; 40 bars. Measured frequencies marked by an asterisk are caused by end cavities (cf. Fig. 6).

Mode (measured circumferentially)	Frequency (Mc)	
	Measured	Calculated
0	660	716
(1)	750*	(890)
0	975*	(890)
1	1125	1175
0	1230	-
0	1270	-

(II. INVERTED MAGNETRON STRUCTURE)

In Fig. 7 the detected frequencies are classified into three levels according to the amplitude of the oscillation. The amplitude depends on the method of excitation and thus the classification is not particularly valuable, although the probes were situated at the correct positions to measure TM modes. The agreement of theory and experiment is perhaps better than would be expected in view of the approximations used in determining the constants.

The end-cavities that were installed for the purpose of terminating the structure have a resonant frequency different from that of the main cavities. Therefore they are detected as a different group and are shown both in Fig. 7 and Table 2. These additional modes are characterized by a certain number of voltage nodes p of the voltage between adjacent bars in the z direction. In Fig. 7 the mode for 758 Mc has one node for the value of p and the configuration given in Table 2 has two nodes for the value of p . This is a result of the axial length of the latter structure being twice as long as that of the former.

These additional modes can be avoided by selection of an even number of main cavities. Then the end plates of the main cavities are connected with the same bars. It is also important for the number of main cavities to be even from the viewpoint of eliminating the additional end cavities, which generally have different resonant frequencies. In general an unsymmetrical structure at both ends introduces extraneous modes. Care must be taken to introduce the detecting probe and exciting antenna so as not to excite extraneous modes. For instance if the probe and antenna are inserted in the radial direction separated by an angle of 120° , the usual $n = 1$ mode-region has several $n = 3$ modes apparently excited. Similarly when the angle is 90° some of the $n = 2$ modes are excited.

D. AN EQUIVALENT CIRCUIT REPRESENTATION

We can synthesize an equivalent circuit for the structure of Fig. 1 if we assume that the two bars constitute a parallel-wire transmission line periodically loaded by the reactance of the cavities between the parallel plates. This equivalent circuit is shown in Fig. 8. The symbol L_{eq} denotes the reactance that changes from inductive to capacitive as the frequency is changed. The expression for L_{eq} is

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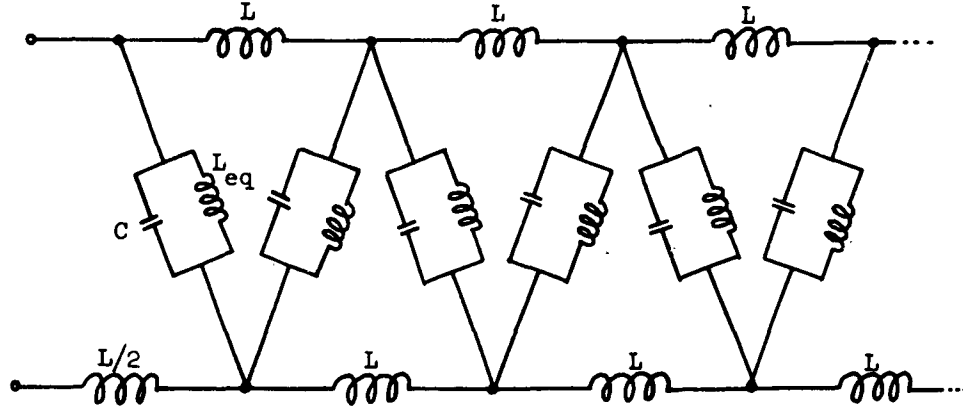


FIG. 8.--Equivalent circuit for the structure of Fig. 1.

$$L_{eq} = \frac{-1}{\omega^2 C_{eq}} = -\frac{\epsilon_0 \mu_0 R^2}{x^2 C_{eq}} = \frac{\delta \mu_0 R}{x d} \frac{J_n(x) N_n(\xi x) - N_n(x) J_n(\xi x)}{J_n'(x) N_n(\xi x) - N_n'(x) J_n(\xi x)} = \frac{\delta \mu_0 R}{x d} B_n^{-1}(x, \xi) \quad (4)$$

where $x = 2\pi R / \lambda_0$ and $\xi = r/R$.

The quantity C in Fig. 8 denotes the capacitance between the bars in a length Δ and should also contain the effect of the plate edges. In this calculation we shall include only the capacitance between the bars because the effect of the plate edges is small compared to the other terms in L_{eq} . Thus

$$C = \pi \epsilon_0 \Delta / \log \left[\frac{d + \sqrt{d^2 - 4a^2}}{2a} \right] = C_0 \Delta \quad (5)$$

The quantity L in Fig. 8 is given approximately by

$$L = \frac{\epsilon_0 \mu_0}{C} \quad (6)$$

The circuit of Fig. 8 is a lumped continuous transmission line made up of elementary four-terminal networks such as shown in Fig. 9. The reactance

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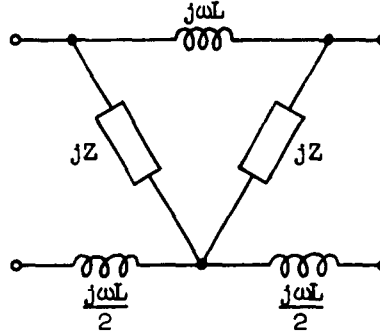


FIG. 9.--Four-terminal network elements of the line of Fig. 8.

Z is given by

$$Z = \omega L_{eq} / (1 - \omega^2 C L_{eq}) = \omega L_{eq} / [1 - (C/C_{eq})] \quad (7)$$

The characteristic impedance Z_c and the phase delay β of the wave propagating along the bars can be expressed in terms of the open-circuit and short-circuit impedance of the transmission line:

$$Z_c = \pm \sqrt{Z_0 Z_s}$$

$$\cos \theta = \cos \beta \Delta = \pm [1 - (Z_s/Z_0)]^{-1/2}$$

where

$$Z_0 = j \frac{2A^2 + 4Z\omega L + \omega^2 L^2}{2(2Z + \omega L)}$$

$$Z_s = j \frac{\omega L(2Z + \omega L)(4Z + \omega L)}{2(2Z^2 + Z\omega L + \omega^2 L^2)} \quad (8)$$

Equation (8) is a good approximation for the $n = 0$ mode; however, it is not a good approximation for higher-order modes because of the voltage variation along the circumference of the plates.

From Eqs. (4) through (8) the phase characteristic diagram and the characteristic impedance as a function of frequency have been calculated approximately for the structure shown in Fig. 6. The results of this calculation are shown in Fig. 10, where the $n = 0$ (TM_{001} and TM_{002}) and the $n = 1$ (TM_{011}) modes are shown. The frequency f is the ordinate and the abscissa

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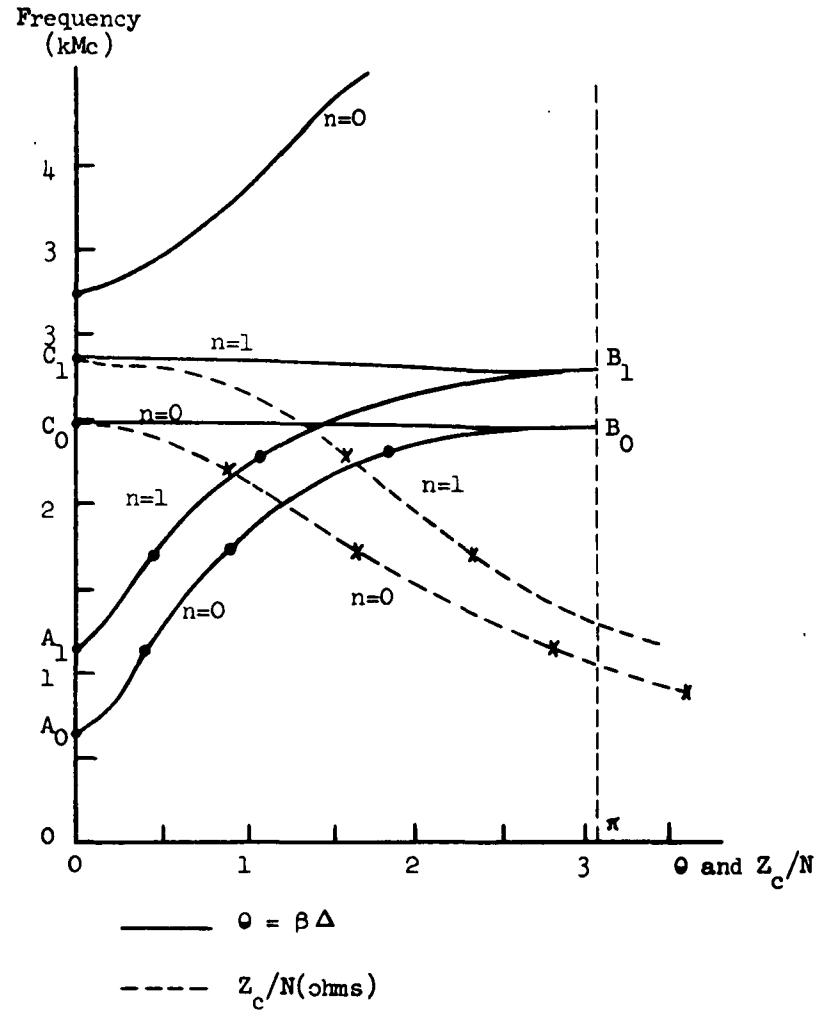


FIG. 10.--Phase characteristics and characteristic impedance for the structure of Fig. 6.

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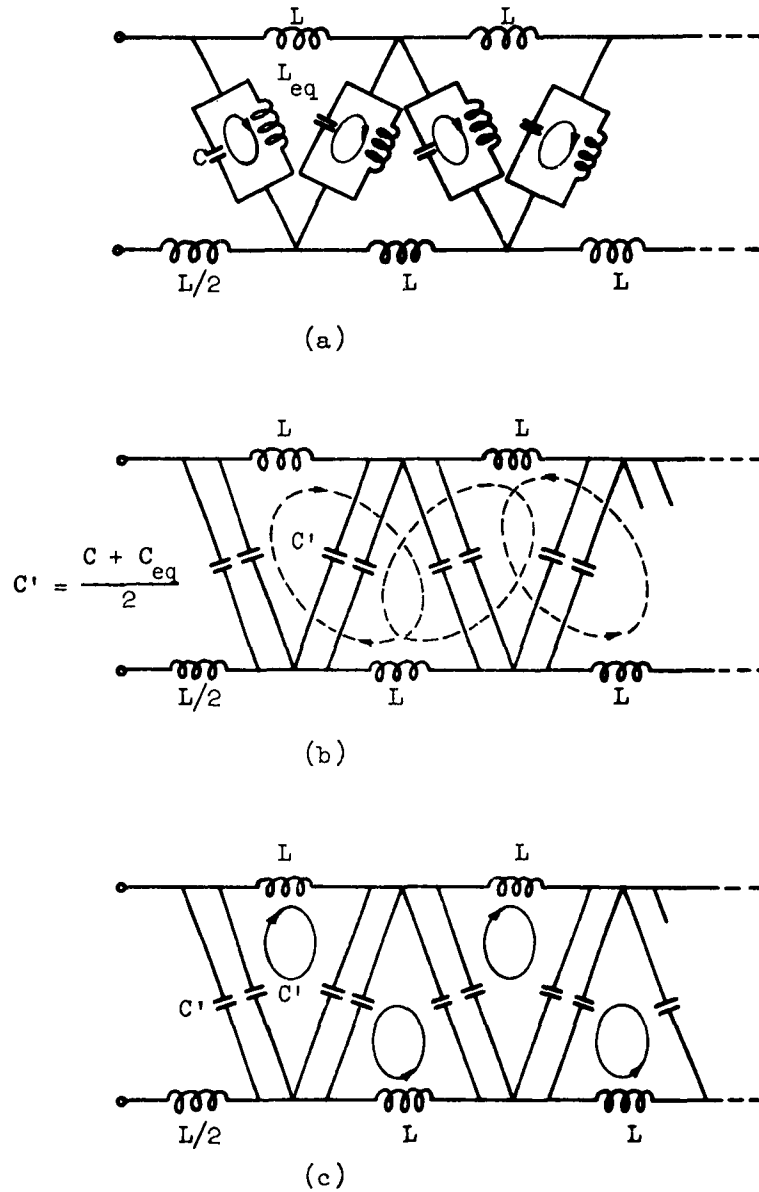


FIG. 11.--Equivalent-circuit representation for various resonant conditions: (a) parallel resonance, (b) series resonance, (c) combined resonance.

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is $\theta = \beta\Delta$. Also on the abscissa is the characteristic impedance Z_c divided by the number of bars N (here, 60).

The points A_0 and A_1 denote the conditions where the wave velocity along the bars is zero and C and L_{eq} have the value corresponding to parallel resonance for each mode (Fig. 11a). The points B_0 and B_1 are at the place where the conditions for series resonance exist (Fig. 11b). Two groups of circuit elements are possible for such series circuit, as shown in Fig. 11b by the solid and dotted lines. The points C_0 and C_1 are another resonant condition that can occur for each mode (Fig. 11c). The phase difference per section is π in Fig. 11b and 2π or zero in Fig. 11c.

The resonances of the structure of Fig. 6 are easily determined from the result shown in Fig. 10. Since seven main cavities are used, the resonant conditions are given by $\beta\Delta = q\pi/7$, where q is an integer. Thus, from Fig. 10, $f = 1250$ Mc for $q = 1$ and $f = 1755$ Mc for $q = 2$ for the $n = 0$ mode. These frequencies are a little higher than the lowest cutoff frequency for the $n = 1$ mode, which explains the spectrum of Fig. 7.

E. CAVITY LOSSES AND Q

In this section we shall obtain an estimate of the value of Q for the unit cavity and the distribution of losses along the cavity surface, assuming the bars to be absent. The power dissipated per unit area is given by

$$\sigma_p = \frac{R_s}{2} \left(\frac{k_0 E_0}{\omega_0 \mu_0} \right)^2 \left[J_1(k_0 \rho) - \frac{J_0(k_0 r)}{N_0(k_0 r)} N_1(k_0 \rho) \right]^2 \quad (9a)$$

$$\sigma_a = \frac{R_s}{2} \left(\frac{k_0 E_0}{\omega_0 \mu_0} \right)^2 \left[J_1(k_0 r) - \frac{J_0(k_0 r)}{N_0(k_0 r)} N_1(k_0 r) \right]^2$$

The maximum value of the time-average stored energy density is given by

$$\sigma_m = \frac{\mu_0}{2} \frac{k_0 E_0}{\omega_0 \mu_0} \left[J_1(k_0 \rho) - \frac{J_0(k_0 r)}{N_0(k_0 r)} N_1(k_0 \rho) \right]^2 \quad (9b)$$

The surface resistance is

$$R_s = \frac{\omega \mu}{2\sigma}$$

where σ is the conductivity of the cavity material.

(II. INVERTED MAGNETRON STRUCTURE)

Total power dissipated in the structure is given by

$$P_L = 2 \int_r^R 2\pi \rho \sigma_p d\rho + 2\pi r \delta \sigma_a \quad (10)$$

The total stored energy is

$$\bar{W}_s = \int_r^R 2\pi \rho \sigma_m d\rho$$

Using the relation between the constant E_0 and the voltage at the gap V_0 (Fig. 12)

$$E_0 = \frac{V_0}{\delta} \left[J_0(k_0 R) - \frac{J_0(k_0 r)}{N_0(k_0 r)} N_0(k_0 r) \right]^{-1}$$

we can determine the cavity losses from (9a) and determine the cavity Q from

$$Q = \frac{\omega_0 W}{P_L} \quad (11)$$

Taking the conductivity of copper as 10^8 mhos/m, we have for the cavity Q

$$\frac{1}{Q} = 8.26 \times 10^{-6} \frac{\sqrt{R}}{\delta} + 18.8 \times 10^{-6} \frac{1}{\sqrt{R}} \quad \text{for } \xi = \frac{r}{R} = \frac{1}{4}$$

$$\frac{1}{Q} = 9.18 \times 10^{-6} \frac{\sqrt{R}}{\delta} + 20.7 \times 10^{-6} \frac{1}{\sqrt{R}} \quad \text{for } \xi = \frac{1}{6}$$

If we use the values $R = 60$ mm and $\delta = 1.5$ mm

$$Q = 697 \quad (\xi = 1/4)$$

$$Q = 636 \quad (\xi = 1/6)$$

The distributions of losses on the plate surfaces are shown in Fig. 13 where the small contribution from σ_a has been neglected.

The expressions for the total losses can be simplified as follows

$$P_L = \left(3.06 \times 10^{-8} \frac{R^{3/2}}{\delta^2} + 6.92 \times 10^{-8} \frac{\sqrt{R}}{\delta} \right) V_0^2 \quad (\xi = 1/4)$$

and

$$P_L = 3.69 \times 10^{-8} \frac{R^{3/2}}{\delta^2} + 8.33 \times 10^{-8} \frac{\sqrt{R}}{\delta} V_0^2 \quad (\xi = 1/6)$$

For a value $V_0 = 5000$ v and the same dimensions ($R = 60$ mm and $\delta = 1.5$ mm),

(II. INVERTED MAGNETRON STRUCTURE)

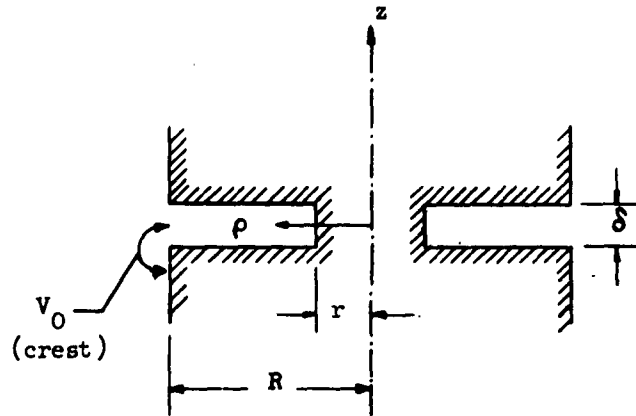


FIG. 12.--Schematic representation of gap.

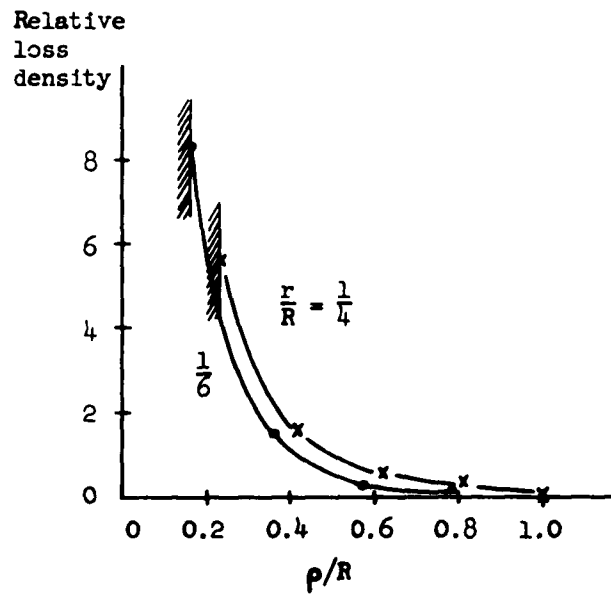


FIG. 13.--Distribution of loss density.

(II. INVERTED MAGNETRON STRUCTURE)

$$P_L = 4990 + 283 w \quad (\xi = 1/4)$$

$$P_L = 6010 + 340 w \quad (\xi = 1/6)$$

The first loss term is the plate loss and the second the losses on the cylindrical portion ($\rho = r$).

From Fig. 13 it is obvious that the losses are largest near the central cylinder. This may present a difficult problem for cooling a high-power magnetron structure. The problem will be worst for small values of ξ , which give good mode separation.

F. MODE-SEPARATION IMPROVEMENT

Mode separation can be improved by changing the gap length as a function of radius, as shown in Fig. 14, since the frequency for each mode changes differently from the uniform-gap case.

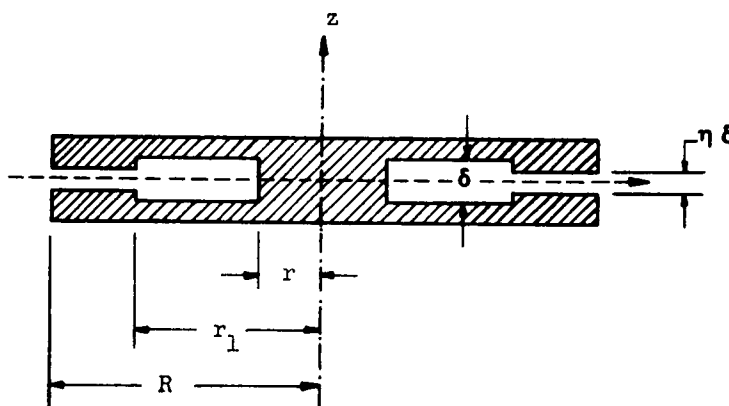


FIG. 14.--Gap modified for improved mode separation. $r/R = \xi$, $r_1/R = \xi_1$, $\delta \ll r$.

We assume as approximate boundary conditions for the TM_0 mode

$$\left. \begin{aligned} \delta E_z &= \eta \delta E_{z1} \\ I_\rho &= I_{\rho 1} \\ I_{\rho 1} &= 0 \end{aligned} \right\} \quad \begin{aligned} &\text{at } \rho = r_1 \\ &\text{at } \rho = R \end{aligned} \quad (13)$$

where the subscript 1 denotes the region $r_1 < \rho < R$.

The expressions for the electric and magnetic field are similar to Eqs. (1) and (2). The characteristic equations for the cavity resonance are

(II. INVERTED MAGNETRON STRUCTURE)

$$\frac{J_n'(x)}{N_n'(x)} = \frac{\eta J_n(\xi_1 x) \alpha - J_n'(\xi_1 x) \beta}{\eta N_n(\xi_1 x) \alpha - N_n'(\xi_1 x) \beta} \quad (14)$$

where

$$\alpha = J_n'(\xi_1 x) N_n(\xi x) - J_n(\xi x) N_n'(\xi_1 x)$$

$$\beta = J_n(\xi_1 x) N_n(\xi x) - J_n(\xi x) N_n(\xi_1 x)$$

and $x = kR$. It is easily seen that Eq. (14) reduces to (2b) for $\eta = 1$ and $k = 0$.

For the case $\xi = 1/8$ and $\xi_1 = 6/8$, the resonant frequencies for the $n = 0$ and $n = 1$ modes were calculated from Eq. (14) as a function n . These results, which are given in Fig. 15, show that a better mode separation is obtained for small η .

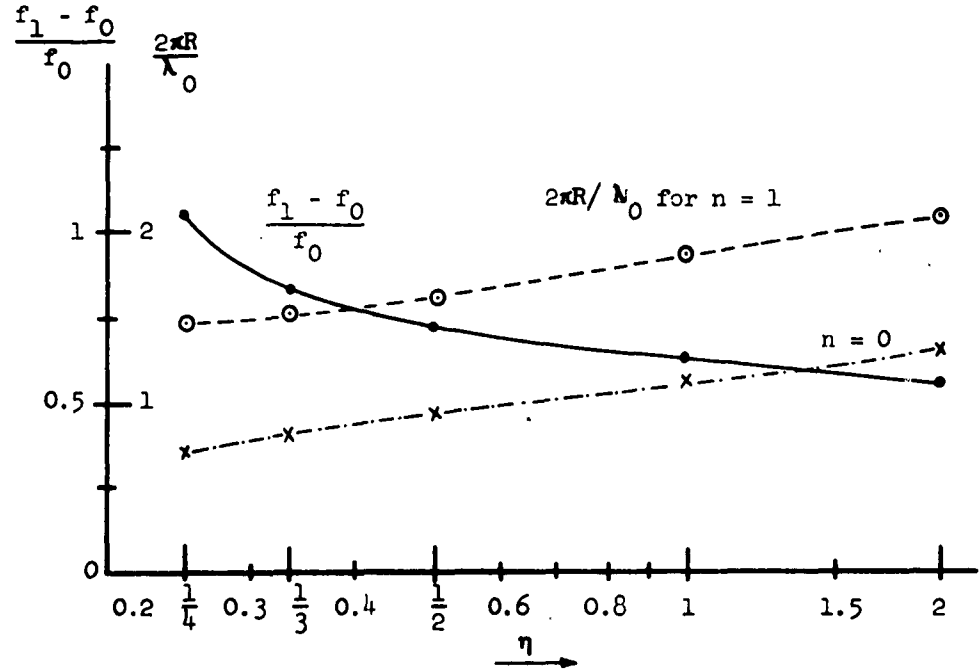


FIG. 15.--Calculation of resonant frequency from Eq. (14) with $\xi = 1/8$ and $\xi_1 = 6/8$.

III. CONVENTIONAL-MAGNETRON ANODE STRUCTURE

A. ANODE STRUCTURE, SYMBOLS, AND DEFINITIONS

In this section we shall analyze the rf properties of structures such as shown in Figs. 16 and 17. These configurations are of a conventional magnetron design where the anode is the outer structure. The configuration shown in Fig. 16 is a cylindrical one, similar to that analyzed in Sec. II. The configuration shown in Fig. 17 is suitable for a linear magnetron amplifier. A direct current flowing along the central structure can produce a dc magnetic field, so that electrons move in the axial direction owing to both dc electric and magnetic fields.

B. RESONANT FREQUENCY OF AN INDIVIDUAL CAVITY

Following the analysis given in Sec. II the resonant frequency for a unit cavity is calculated with the assumption of uniform capacitive loading to replace the effects of the bars and the edges at the inner opening. The resonant frequency is determined from the following formula:

$$B_n(x, \xi) = \frac{J_n'(x)N_n(\xi x) - N_n'(x)J_n(\xi x)}{N_n(x)J_n(\xi x) - J_n(x)N_n(\xi x)} = \frac{\kappa \delta}{\epsilon_0 r} x \quad (15)$$

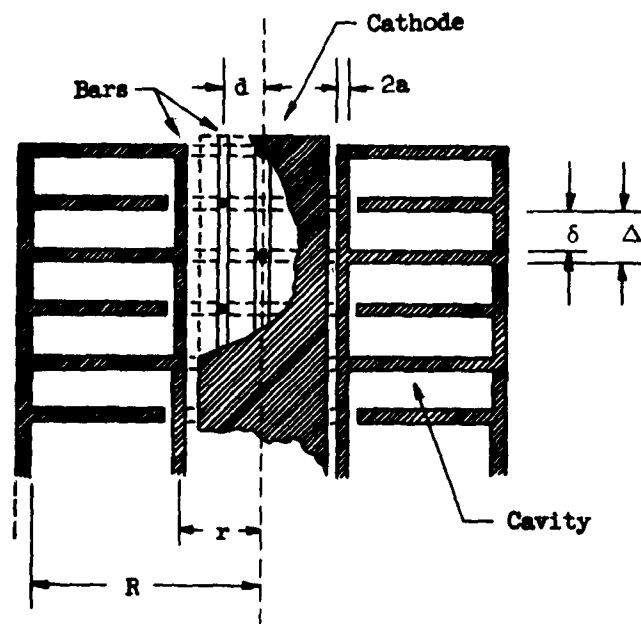
where $x = kr = 2\pi r/\lambda_0$, $\xi = R/r$, and κ is given by the same expression as in Eq. (3). For $k = 0$, the resonance condition is given simply by

$$J_n'(x)N_n(\xi x) = N_n'(x)J_n(\xi x) \quad (16)$$

The solutions of Eq. (16) for $n = 0$ and $n = 1$ modes are shown in Fig. 18. The mode separation is better for high values of ξ . However, this means a smaller value of r for a particular R value, and therefore the interaction space for the electron beam is limited.

Figure 19 gives the relation between $r\xi$ and the rate of mode separation for a fixed resonant frequency. For comparison with Fig. 4, Fig. 18 is also replotted in terms of kR instead of kr and $\xi = (r/R)$ instead of $\xi = (R/r)$. The resonant frequency of a cavity for a given value of outer radius and inner to outer radius ratio are much higher and the interaction space is smaller.

21.
(III. OUTER-ANODE STRUCTURE)



CI FIG. 16.--Structure for cylindrical magnetron.

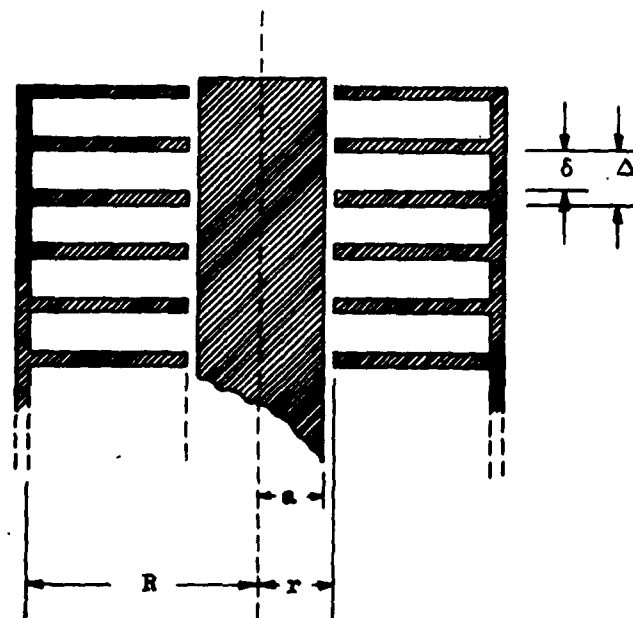


FIG. 17.--Structure for linear magnetron.

(III. OUTER-ANODE STRUCTURE)

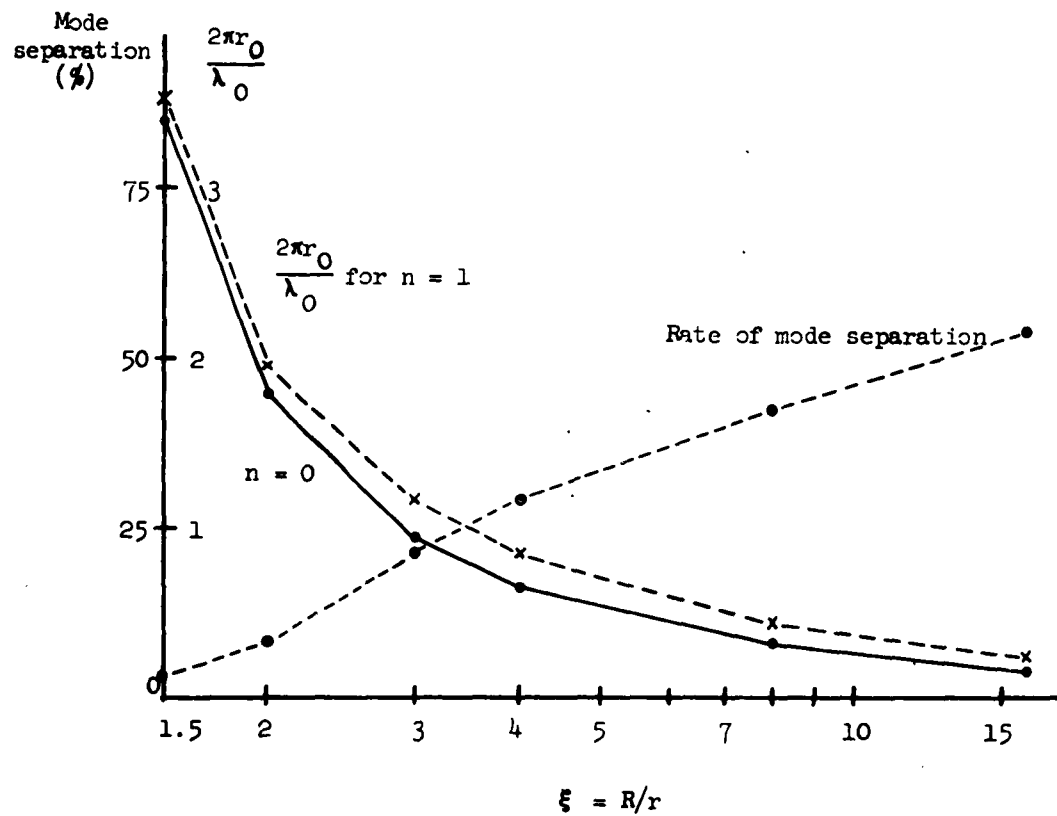


FIG. 18.--Solutions of Eq. (18) for $\kappa = 0$.

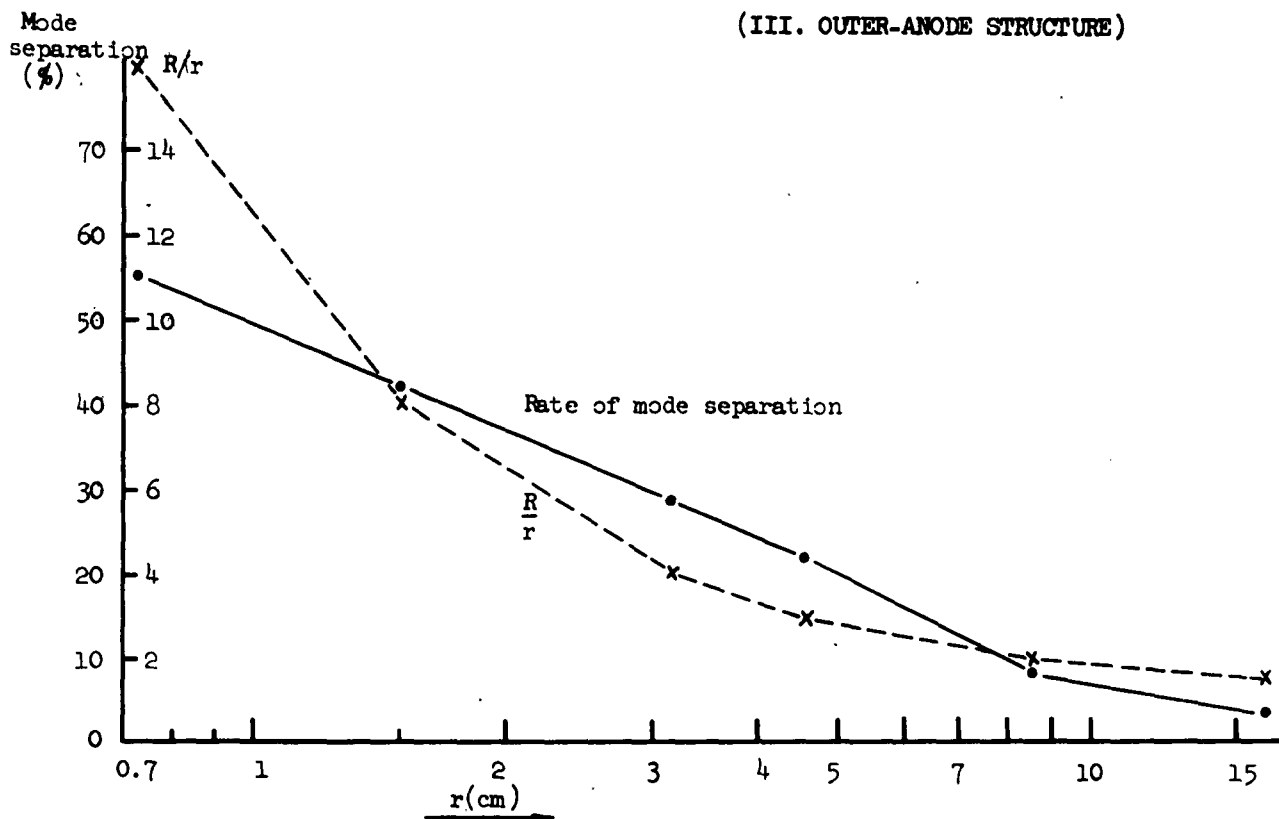
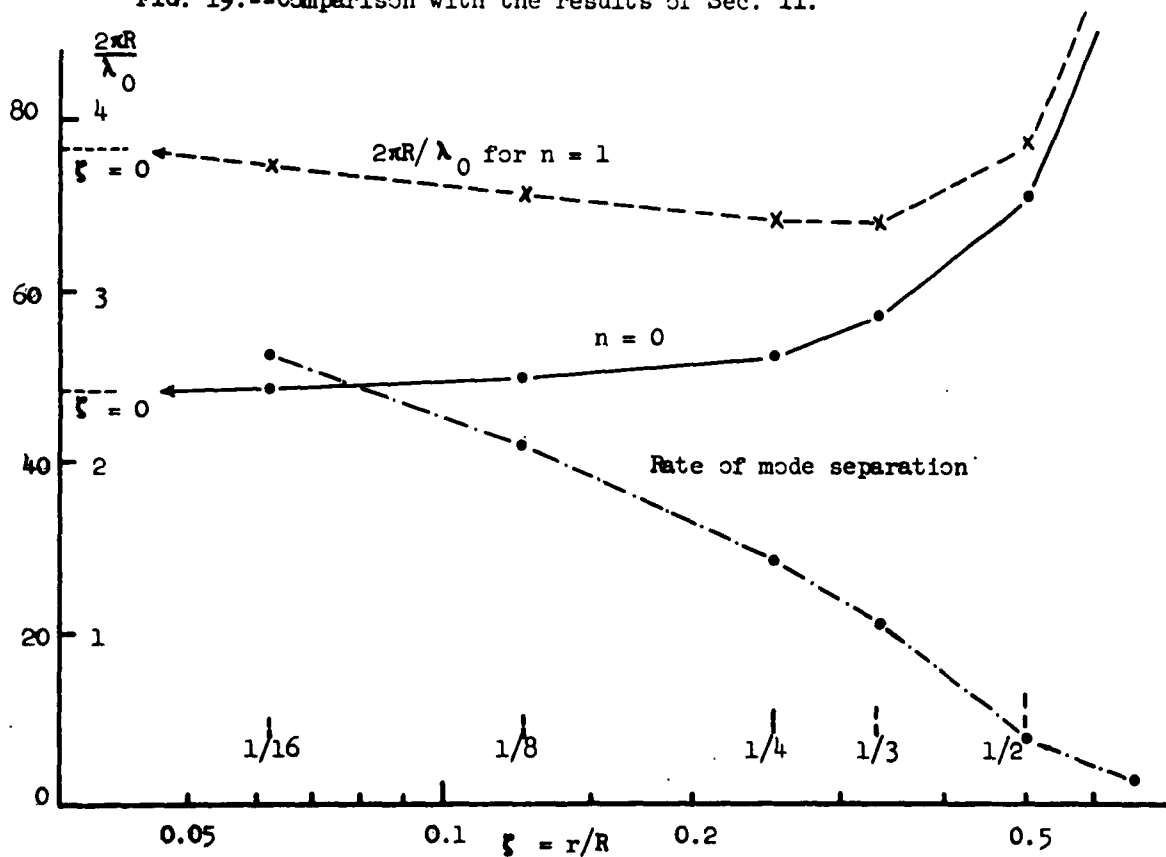


FIG. 19.--Comparison with the results of Sec. II.



(III. OUTER-ANODE STRUCTURE)

C. MEASUREMENT OF RESONANT FREQUENCIES

Figure 20 is a schematic diagram of the experimental setup used to test the theory presented in this analysis. Table 3 is a tabulation of the measured results compared with the theoretically calculated values of resonant frequency.

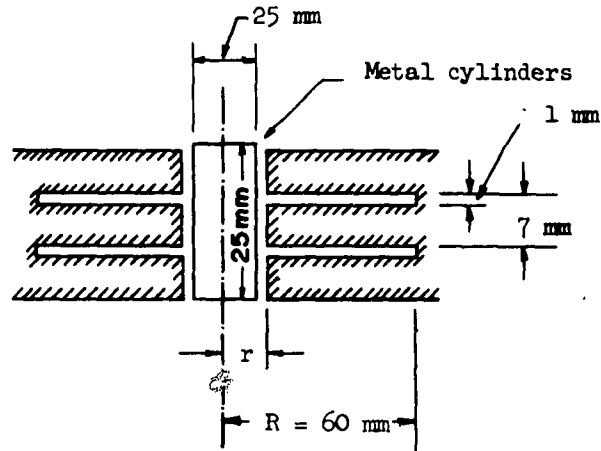


FIG. 20.--Test configuration.

Table 3.--Resonant frequencies (in megacycles) for the structure of Fig. 20.

r (mm)	n = 0 mode		n = 1 mode		Note	Mode separation(%)
	Measured	Calculated	Measured	Calculated		
6	1935	1980	2980	2900	No metal cylinder	54
10	1975,1968	2055	2915,2900	2830	"	> 46.8
12.5	2017,2027	2080	2820,2845	2810	"	> 39.8
15.7	2082,2099	2105	2765,2793	2790	"	> 31.7
15.7	1920,2082	-	2768,2800	-	With metal cylinder	> 37.6
15.7	1865,2070	-	2785,2855	-	"(somewhat eccentric)	-

(III. OUTER-ANODE STRUCTURE)

Measured values for the structure are in good agreement with the critically calculated values. The metal cylinder has little effect on the resonant frequency because of the capacity it introduces. The next step in the measurements was to investigate the effect of a small number of connecting bars. The results of these measurements are tabulated in Table 4. It was expected that the addition of the bars would reduce the $n = 0$ mode frequency and improve the mode separation; however, the addition of the bars did not produce this result as may be seen from a comparison of Parts A and B of Table 4.

If a metal shaft is inserted along the axis, it is expected to produce the same effect as the bars insofar as their influence on the mode can be included in the equivalent capacitance. According to Table 4 the mode separation is worse with the shaft in place whenever the value of R/r is less than 4 or 5, and the mode separation is better if this ratio is larger. The resonant frequency calculated for a Type C structure in Table 4 assumes no capacitive loading and gives $f_0 \approx 2500$ Mc and $f_1 \approx 3380$ Mc. The capacitance of the bars is thus seen to cause a considerable reduction in the resonant frequency. The Q of some of the cavities was measured approximately from the resonant curves. The Q for smaller gap lengths was found to be the order of hundreds and for larger gap lengths, of the order of 1000.

D. EQUIVALENT CIRCUITS FOR LINEAR MAGNETRON STRUCTURE

The equivalent circuit for the structure shown in Fig. 16 is the same as that given in Fig. 8. The equivalent loading reactance in this case is

$$L_{eq} = \frac{-1}{\omega^2 C_{eq}} = \frac{\delta \mu_0 r}{\omega^2 x d} B_n^{-1}(x, \xi) \quad (17)$$

Since the circuit is the same, similar phase characteristics for the two cases are expected.

The analysis will be carried out for the linear magnetron structure shown in Fig. 17.

1. ANALYSIS FOR $n = 0$ MODE. Figure 21 gives the equivalent circuit with the gap length assumed to be much smaller than a wavelength. The equivalent inductance for a unit cell is

$$L_{x0} = \frac{\mu_0 \delta}{2\pi x} B_0^{-1}(x, \xi) \quad (18)$$

Table 4.--Resonant frequencies measured with structures with bars

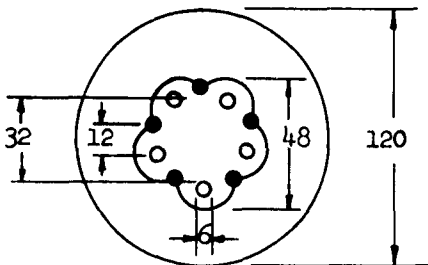
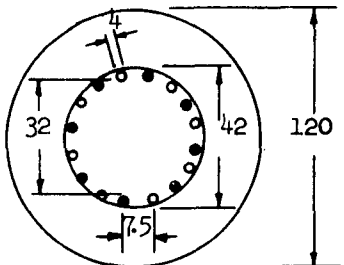
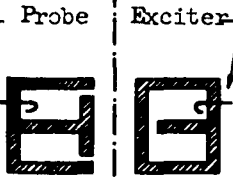
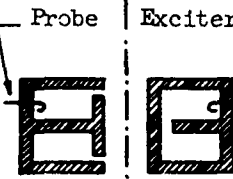
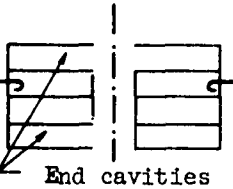
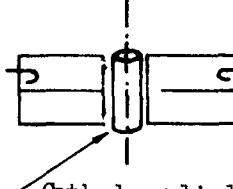
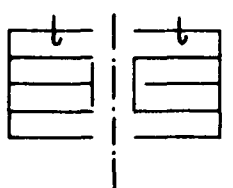
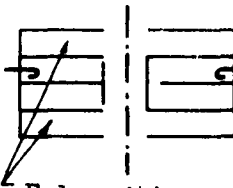
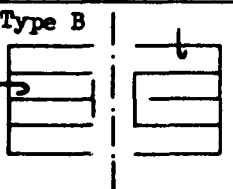
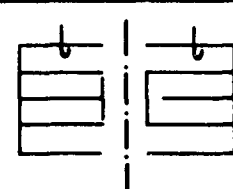
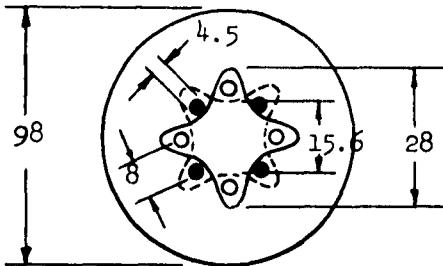
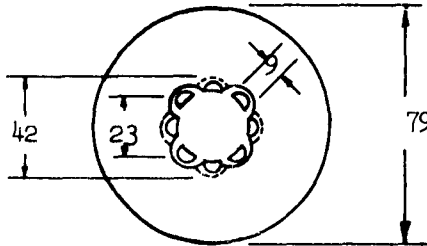
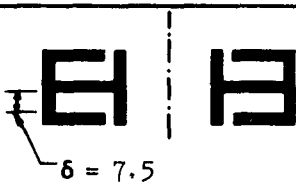
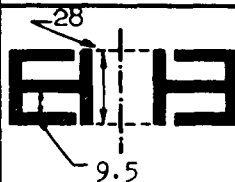
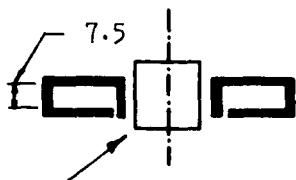
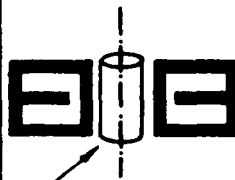
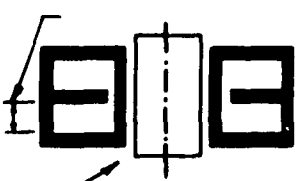
	Type A		Type B	
Schematic view of the structure (all dimensions in millimeters)	 $R/r \approx 3, \delta = 1.5, \Delta = 7.5$		 $R/r \approx 3, \delta = 1.5, \Delta = 7.5$	
	Schematic cross-section view	Frequencies measured	Schematic cross-section view	Frequencies measured
Basic measurements		$n = 0$ mode 2205 Mc $n = 1$ mode 2580 Mc $\frac{f_1 - f_0}{f_0} = 0.170$		$n = 0$ mode 1934 Mc $n = 1$ mode 2430 Mc $\frac{f_1 - f_0}{f_0} = 0.257$
Miscellaneous measurements	 End cavities	$n = 0$ 2055 Mc 2276 Mc $n = 1$ 2468 Mc 2692 Mc	 Cathode cylinder	$n = 0$ mode 1920 Mc $n = 1$ mode 2400 Mc $\frac{f_1 - f_0}{f_0} = 0.250$
	 Both probes in end cavity	$n = 0$ 2055 Mc (weak mode) 2155 Mc 2276 Mc $n = 1$ 2685 Mc 2700 Mc	 End cavities	$n = 0$ 1918 Mc 2140 Mc (faint mode) $n = 1$ 2410 Mc
	 One probe in end cavity	$n = 0$ 1918 Mc 2140 Mc (very strong) $n = 1$ 2405 Mc 2710 Mc	 Both probes in end cavity	$n = 0$ 1918 Mc (weak) 2112 Mc 2142 Mc $n = 1$ 2402 Mc (weak) 2646 Mc (weak) 2710 Mc 2726 Mc

Table 4 (Continued)

Table 4 (Continued)				
	Type C		Type D	
Schematic view of the structure (all dimensions in millimeters)				
	$R/r \approx 5, \quad \delta = 7.5 \text{ and } 12 \text{ mm}$		$R/r = 2.5, \quad \delta = 9.5 \text{ mm}$	
	Schematic cross-section view	Frequencies measured	Schematic cross-section view	Frequencies measured
Basic measurements	 $\delta = 7.5$	$f_0 = 2020 \text{ Mc}$ $f_1 = 2920 \text{ Mc}$ $\frac{f_1 - f_0}{f_0} = 0.446$	 28 9.5	$f_0 = 1490 \text{ Mc}$ $f_1 = 1896 \text{ Mc}$ $\frac{f_1 - f_0}{f_0} = 0.272$
Miscellaneous Measurements	 7.5 Cathode cylinder 13(diam.) x 13	without cylinder $f_0 = 1854 \text{ Mc}$ $\frac{f_1 - f_0}{f_0} = 0.545$ with cylinder $f_0 = 1821 \text{ Mc}$ $\frac{f_1 - f_0}{f_0} = 0.566$	 28 9.5 Cathode cylinder 21.5(diam.) x 30	$f_0 = 1386$ $f_1 = 1670$ $\frac{f_1 - f_0}{f_0} = 0.205$
	 $\delta = 12$	$f_0 = 1348 \text{ Mc}$ $f_1 = 2162 \text{ Mc}$ $\frac{f_1 - f_0}{f_0} = 0.568$	Cathode cylinder 21.5(diam.) x 50 Cathode cylinder 19(diam.) x 30	$f_0 = 1380$ $\frac{f_1 - f_0}{f_0} = 0.21$ $f_0 = 1460$ $\frac{f_1 - f_0}{f_0} = 0.243$
	 12 Cathode cylinder 13(diam.) x 28	$f_0 = 1348$ $f_1 = 2098$ $\frac{f_1 - f_0}{f_0} = 0.556$	Cathode cylinder 12.5 (diam.) x 30 Cathode cylinder 12.5 (diam.) x 30 Eccentric position	$f_0 = 1486$ $\frac{f_1 - f_0}{f_0} = 0.268$ $f_0 = 1456$ $\frac{f_1 - f_0}{f_0} = 0.237$

(III. OUTER-ANODE STRUCTURE)

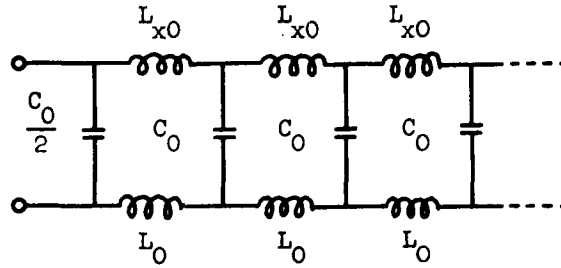


FIG. 21.--Equivalent circuit for $n = 0$ mode.

The capacitance between cathode and anode per section is

$$C_0 = 2\pi \epsilon_0 \Delta / \log (r/a)$$

and the corresponding inductance is

$$L_0 = \frac{\mu_0 \Delta}{2\pi} \log (r/a) \quad (19)$$

The phase characteristics for waves propagating in the axial direction are given by

$$\cos \theta = 1 - \frac{\chi}{2} \quad (20)$$

where $\beta \Delta = \theta$ and $\chi = \omega^2 V_0 (L_0 + L_{x0})$. The characteristic admittance is

$$Y_{ch} = \omega C_0 \sqrt{\frac{4}{\chi} - 1} \quad (21)$$

2. ANALYSIS FOR $n = 1$ MODE. A good equivalent circuit representation is more difficult to obtain in this because of the field distribution. An equivalent circuit that is a good approximation to the structure is shown in Fig. 22. The quantity ω_c is the resonant frequency of the TE coaxial line mode and is given by the first root of

$$\frac{J_1'(ka)}{J_1'(kr)} = \frac{N_1'(ka)}{N_1'(kr)} \quad (22)$$

where

$$k = 2\pi/\lambda_0 = \omega_c \sqrt{\mu_0 \epsilon_0}$$

The effective capacitance C_1 is determined from the electric stored energy:

(III. OUTER-ANODE STRUCTURE)

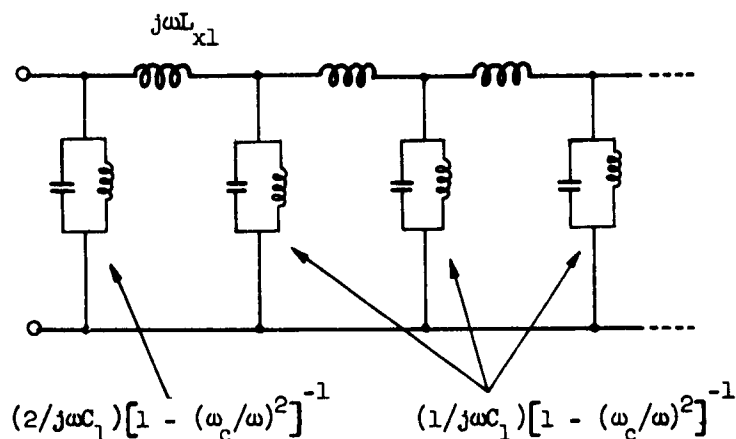


FIG. 22.--Equivalent circuit for the $n = 1$ mode.

$$\frac{1}{2} C_1 V_1^2 = \int \frac{\epsilon_0 E_1^2}{2} dv \quad (23)$$

so that

$$C_1 \approx \pi \epsilon_0 (\Delta - \delta)/2 \log(r/a)$$

where V_1 and E_1 are respectively the voltage and field between the anode and cathode for the $n = 1$ mode. The circuit of Fig. 22 has its cutoff at ω_c^3

The effective inductance can also be determined from energy considerations:

$$L_{x1} = \frac{\pi \epsilon_0 \delta}{8x} B_1^{-1}(x, \xi) \quad (24)$$

The propagation characteristics of the circuit are given by

$$\cos \theta = 1 - \frac{\chi}{2} \quad (25)$$

where $\theta = \beta \Delta$ and $\chi = \omega^2 L_{x1} C_1 [1 - (\omega_c/\omega)^2]$

The characteristic admittance is

$$Y_{ch} = \omega C_1 [1 - (\omega_c/\omega)^2] \sqrt{\frac{4}{\chi} - 1} \quad (26)$$

3. NUMERICAL RESULTS AND EXPERIMENT. The phase characteristics and the characteristic admittances for the $n = 0$ and $n = 1$ modes for the

(III. OUTER-ANODE STRUCTURE)

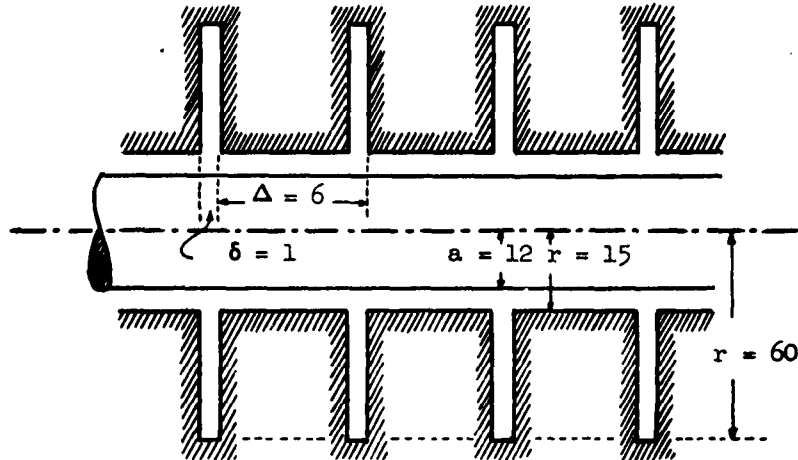


FIG. 23.--Experimental configuration.

structure shown schematically in Fig. 23 are shown in Fig. 24 for the case $\xi = R/r = 4$. These results were obtained by using Eqs. (17) through (26) and the impedance for the mode from these equations is the order of a few tens of ohms.

Selecting an axial length of $q \Delta$, where q denotes the total number of sections, we can predict all the resonances of this structure from the phase characteristic diagram of Fig. 24. Table 5 gives the results between calculated and measured resonant frequencies for a structure having six sections. As can be seen there is good agreement between the calculated and measured values of the resonant frequency. Not every frequency was measured because of superposition of some modes or difficulty of excitation of some modes.

E. CAVITY LOSSES AND Q

Following the analysis of Sec. II, we find the approximate Q of the structure shown in Fig. 25 from

$$\frac{1}{Q} = 12.6 \times 10^{-6} \sqrt{\frac{r}{\delta}} + 3.21 \times 10^{-6} \frac{1}{\sqrt{r}} (\xi = 4)$$

and

$$\frac{1}{Q} = 10.6 \times 10^{-6} \sqrt{\frac{r}{\delta}} + 3.99 \times 10^{-6} \frac{1}{\sqrt{r}} (\xi = 3)$$

(III. OUTER-ANODE STRUCTURE)

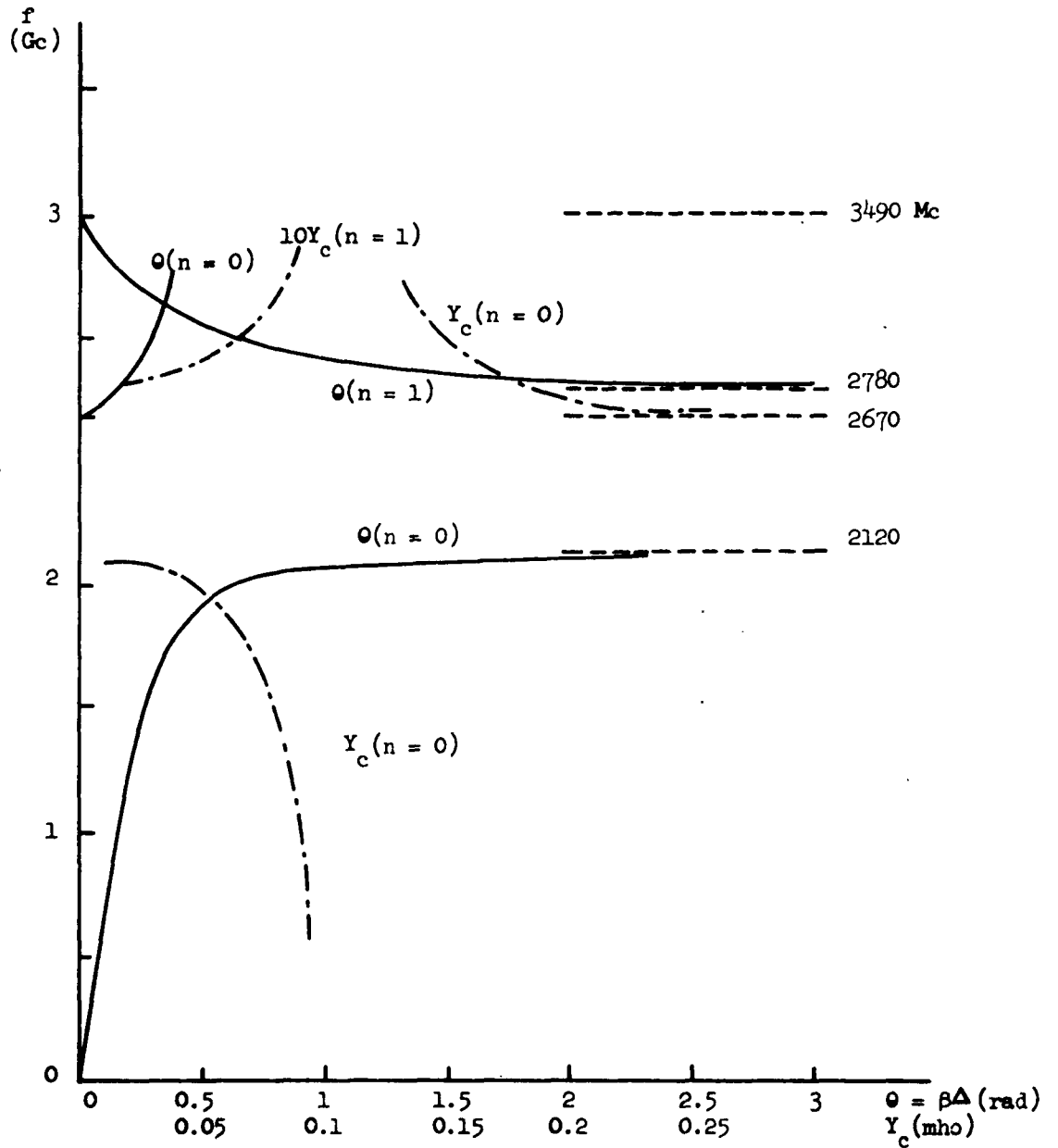


FIG. 24.--Summary of experimental results. $R/r = 4$, $r = 15$, $a = 12$, $\Delta = 6$, $\delta = 1$ mm.

(III. OUTER-ANODE STRUCTURE)

Table 5.--Calculated and measured frequencies (in megacycles).

$r = 15 \text{ mm}$, $R/r = 4$, $a = 12 \text{ mm}$, $\Delta = 6 \text{ mm}$, $\delta = 1 \text{ mm}$, $q = 6$

$n = 0$		$n = 1$	
Calculated	Measured	Calculated	Measured
2120	2091	2780	2763
2120	2078	2780	2769
2110	2041	2800	2785
2100	1922	2830	2813
2075		2890	
1930		3030	

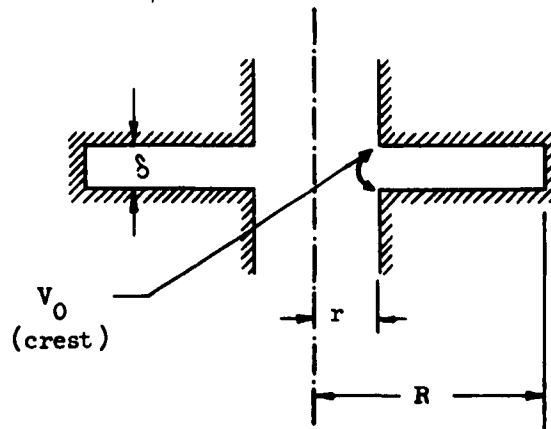


FIG. 25.--Schematic representation of gap.

Taking $R = 60 \text{ mm}$ and $\delta = 1.5 \text{ mm}$ we have

$$Q = 945 \quad (\xi = 4)$$

$$Q = 976 \quad (\xi = 3)$$

These Q values are about 50 per cent higher than those of the structures analyzed in Sec. II. However, for the same resonant frequency and same r/R , the Q 's in the two cases are about the same.

The total losses for this structure are

(III. OUTER-ANODE STRUCTURE)

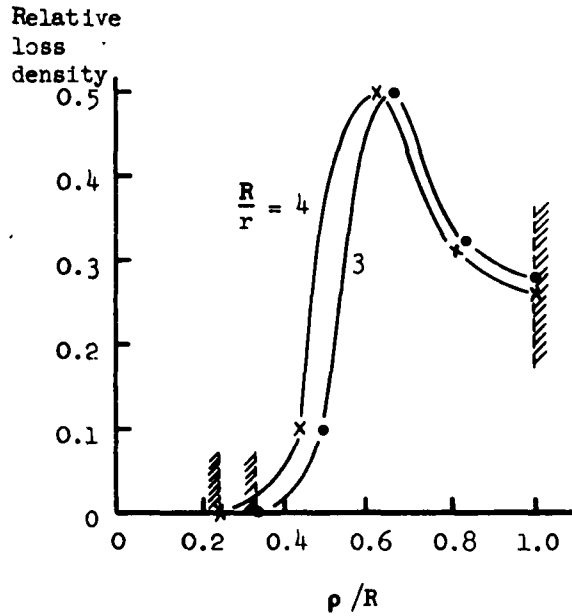


FIG. 26.--Loss-density distribution over plate surface.

$$P_L = \left(21.0 \times 10^{-8} \frac{r^{3/2}}{\delta^2} + 5.32 \times 10^{-8} \frac{\sqrt{r}}{\delta} \right) V_0^2 \quad (\xi = 4)$$

$$P_L = \left(14.3 \times 10^{-8} \frac{r^{3/2}}{\delta^2} + 5.36 \times 10^{-8} \frac{\sqrt{r}}{\delta} \right) V_0^2 \quad (\xi = 3)$$

Again we take $V_0 = 5000$ v, $R = 60$ mm, and $\delta = 1.5$ mm:

$$P_L = 4270 + 109 w \quad (\xi = 4)$$

$$P_L = 4430 + 126 w \quad (\xi = 3)$$

Although these losses are smaller than the values of Sec. II, it must be noted that they would be larger if the frequency was the same as for structure of Sec. II.

Figure 26 gives the distribution of losses over the surfaces of the plates.

(III. OUTER-ANODE STRUCTURE)

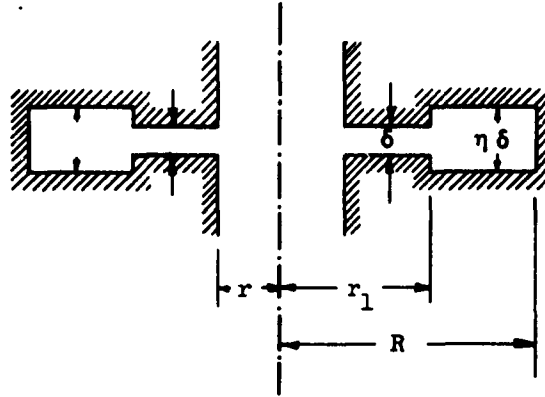


FIG. 27.--Structure used for mode-separation improvement.
($\xi = R/r$, $\xi_1 = r_1/r$, $\delta < r$.)

F. IMPROVEMENT OF MODE SEPARATION

The method of Sec. II has been applied to the structure depicted in Fig. 27. The equations for determining the resonant frequency for various values of ξ and η are

$$\frac{J_n'(x)}{N_n'(x)} = \frac{J_n'(\xi_1 x)\alpha - J_n(\xi_1 x)\beta}{N_n'(\xi_1 x)\alpha - N_n(\xi_1 x)\beta} \quad (27)$$

$$\alpha = J_n(\xi_1 x)N_n(\xi x) - N_n(\xi_1 x)J_n(\xi x)$$

$$\beta = J_n'(\xi_1 x)N_n(\xi x) - N_n'(\xi_1 x)J_n(\xi x)$$

where $x = kr$.

Table 6 gives the value of x and the mode separation for several conditions. It is seen that the mode separation is improved by a large η . This has the disadvantage of a small r , which may make fabrication of the structure difficult.

G. COMPARISON OF STRUCTURES OF SECS. II AND III

At a fixed frequency of 1000 Mc, the mode separation and R/r ratio is given as a function of the interaction space radius R_0 in Fig. 28. We see that the mode separation is always better for the inverted magnetron structure

Figure 29 gives the mode separation and R/r ratio as a function of the

(III. OUTER-ANODE STRUCTURE)

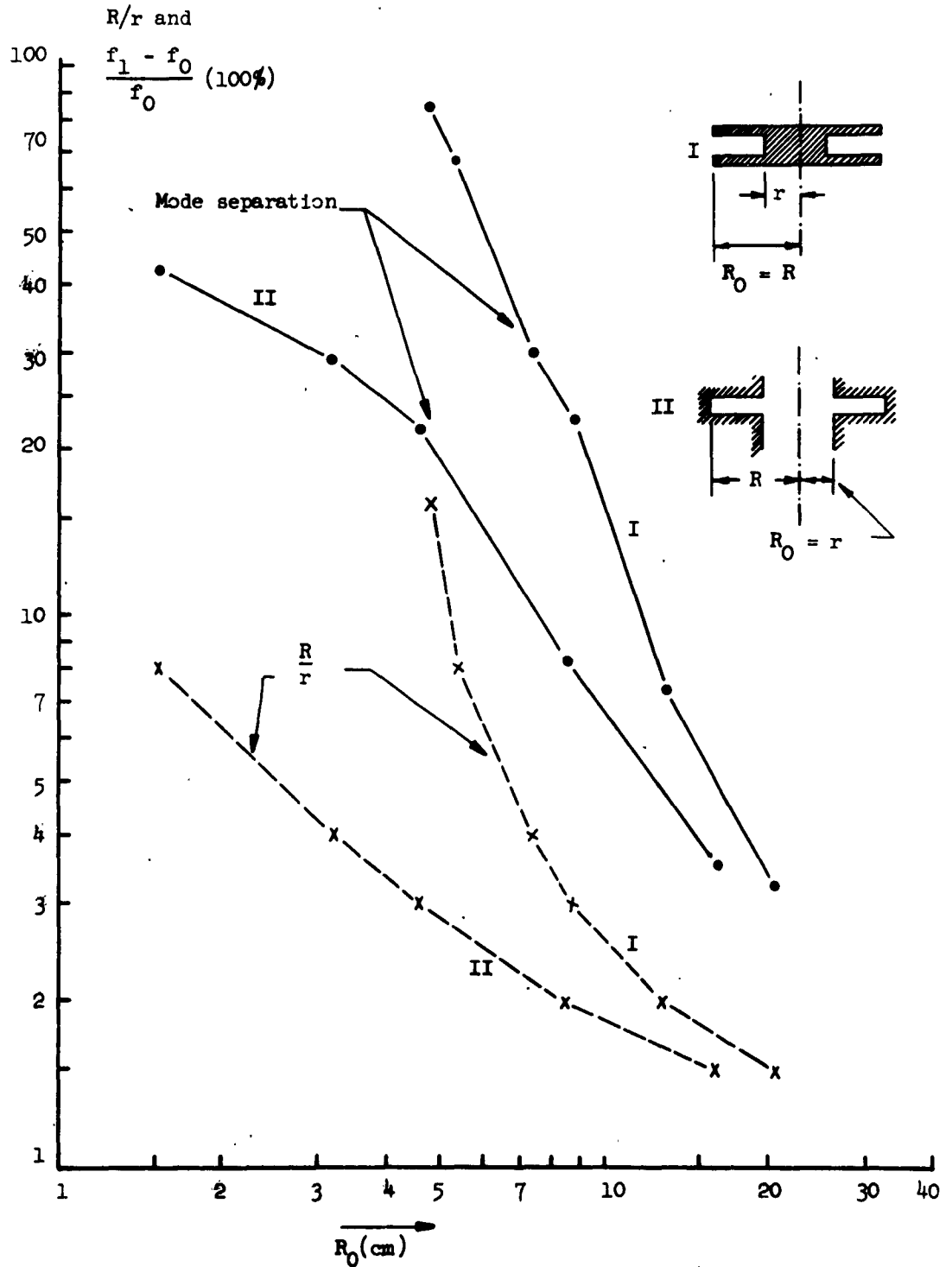


FIG. 28.--Mode separation and R/r ratio at 1000 Mc as a function of R_0 .

(III. OUTER-ANODE STRUCTURE)

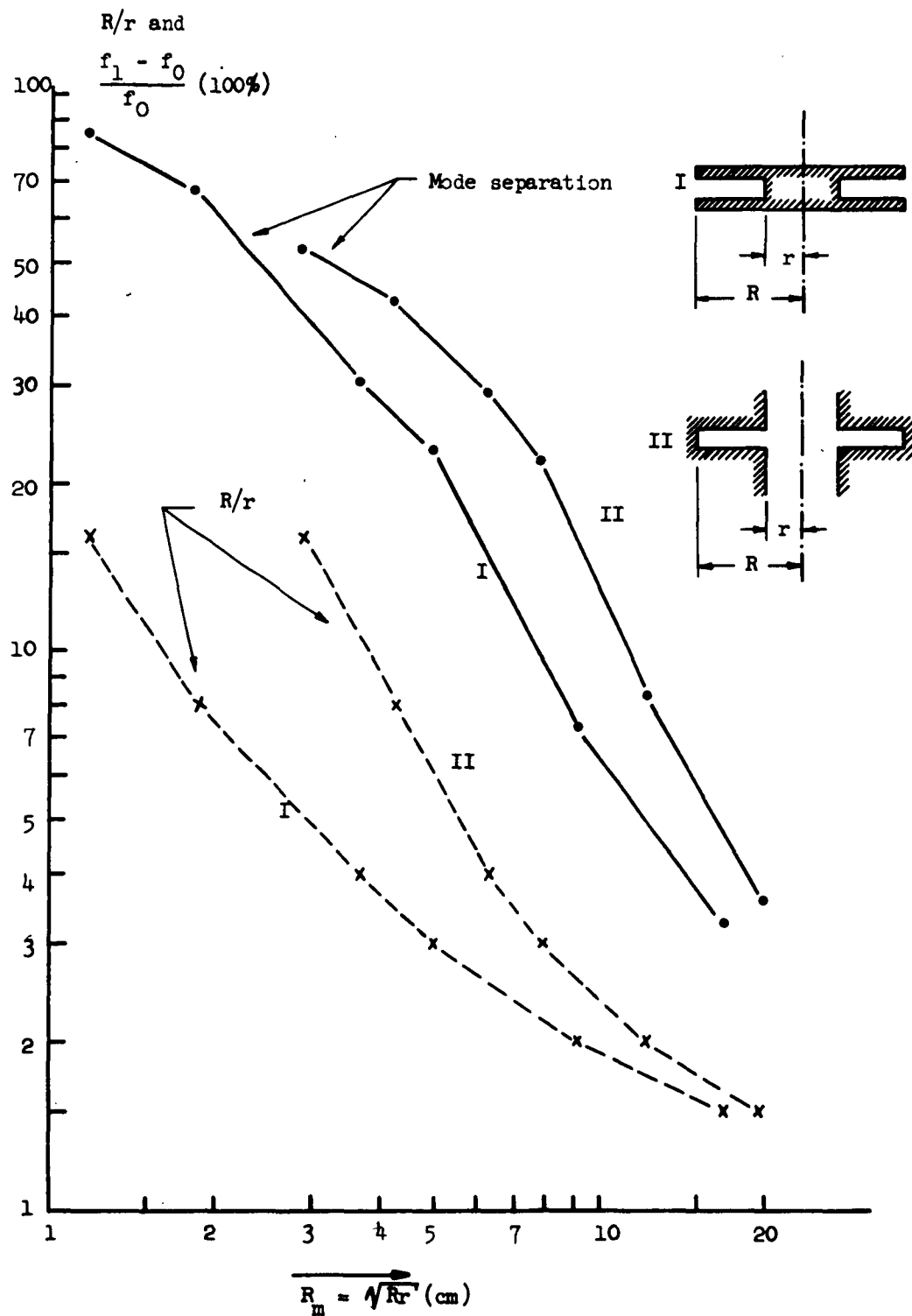


FIG. 29.--Mode separation and R/r ratio as a function of mean radius $(Rr)^{1/2}$.

Table 6.-- $x = 2\pi r/\lambda_0$ for various ξ and η .

η	Mode (n)	$\xi = 4$ $\xi_1 = 2$	$\xi = 4$ $\xi_1 = 3$	$\xi = 2$ $\xi_1 = 1.5$
1	0	0.666	0.666	1.80
	1	0.860	0.860	1.95
4	0	0.433	0.418	1.08
	1	0.763	0.673	1.35
8	0	0.325	0.313	0.775
	1	0.755	0.608	1.14
1	$\frac{f_1 - f_0}{f_0}$	29.1%	29.1%	8.33%
4		76.0%	61.0%	25.6%
8		132.0%	94.3%	46.5%

geometric mean radius $R_m = Rr$. We see that for a fixed effective size the conventional magnetron has a better mode separation.

IV. ANODE-STRUCTURE CONSIDERATIONS

A. REDUCTION OF OUTER RADIUS

The structure of Fig. 30 is proposed as a means of reducing the size of the anode.

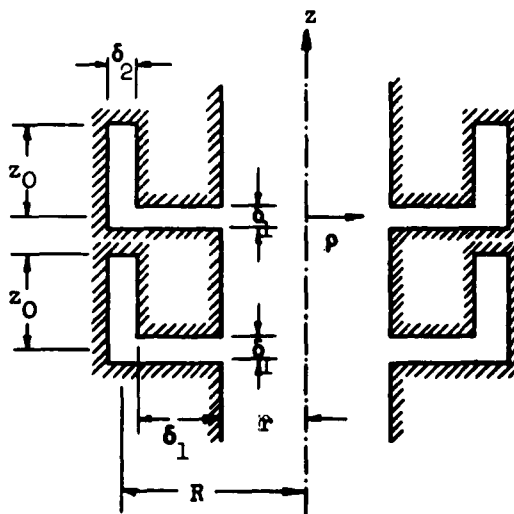


FIG. 30.--Proposed reduced-size anode.

(IV. ANODE CONSIDERATIONS)

This structure is analyzed for the $n = 0$ mode with the assumption of $R \gg \delta$ and of the boundary conditions of continuous voltage and current at $z = 0, \rho = R$. The resonant frequency is determined by

$$\frac{\delta_2}{\delta_1} \tan \beta_0 z_0 = \frac{J_0'(\beta_0 r) N_0(\beta_0 R) - N_0'(\beta_0 r) J_0(\beta_0 R)}{N_0'(\beta_0 r) J_0'(\beta_0 R) - J_0'(\beta_0 r) N_0'(\beta_0 R)} \quad (28)$$

where $\beta_0 = \omega \sqrt{\mu_0 \epsilon_0}$. Table 7 gives the results for both the $n = 0$ and $n = 1$ modes.

Table 7.-- $2\pi r/\lambda_0$ ($= \beta r$) for $R/r = 2$				
n	$\frac{z_0/r}{\delta_2/\delta_1}$	$\frac{1}{2}$	1	2
0	1	1.20	0.875	0.575
	2	0.965	0.696	0.470
	3	0.835	0.605	0.405
1	1	~ 1.4	~ 1.1	~ 0.76
	2	~ 1.2	~ 0.9	~ 0.67
	3	~ 1.06	~ 0.78	~ 0.55

An experimental structure with $r = 26$ mm, $R/r = 2$, $z_0/r = 1$, $\delta_2 = 3$ mm, and $\delta_1 = 1.5$ mm was used to test the calculated results. The measured frequencies were 1305 Mc ($n = 0$) and 1752 Mc ($n = 1$). The calculated values were 1286 Mc ($n = 0$) and 1645 Mc ($n = 1$).

B. MAGNETRON STRUCTURE WITH LARGE MODE SEPARATION

The conventional-magnetron structure shown in Fig. 3la has good mode separation. The structure shown in Fig. 3lb is one half of a simplified version of the structure, suitable for calculation of frequencies.

The lower resonances are given by

$$\tan \frac{\pi \beta x}{2} = \frac{2b}{\delta} \cot x_0 \beta_x \quad (29)$$

and

(IV. ANODE CONSIDERATIONS)

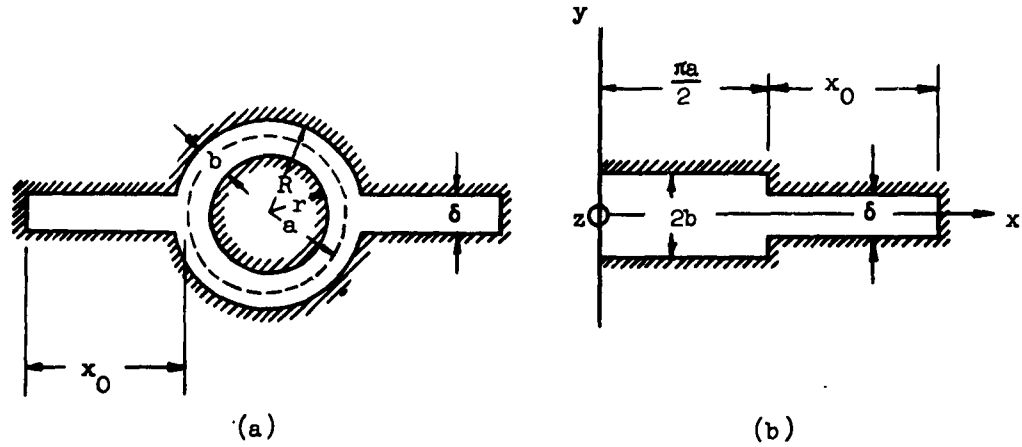


FIG. 31.--Structure with improved mode separation: (a) proposed model (length z_0 along axis); (b) simplified model (open-ended at $z = \pm z_0/2$).

$$\frac{\pi^2}{4z_0^2} + \beta_x^2 = \omega^2 \mu_0 \epsilon_0 = \beta^2$$

The lowest frequency is given by $\beta_x = \beta$. The results from these equations are given in Table 8 and as can be seen the mode separation is the better the shorter z_0 . Values of 100 per cent or more are easily obtained. However, this structure is not satisfactory for high-power operation.

Table 8.--Resonant frequencies in terms of βa .							
$\frac{x_0}{a}$	$\frac{b}{\delta}$	$\beta = \beta_x$		$\beta^2 = \beta^2 + \pi^2/4z_0^2$		Mode separation	
		First mode	Next mode	$z_0/a = 1$	$z_0/a = 2$	$z_0/a = 1$	$z_0/a = 2$
1	1	0.725	1.75	1.73	1.07	139%	47.6%
	2	0.826	1.70	1.78	1.13	106%	36.8%
2	1	0.528	1.24	1.66	0.946	134%	79.2%
	2	0.611	1.16	1.68	0.994	90.1%	62.6%

(IV. ANODE CONSIDERATIONS)

C. OUTER-CATHODE ANODE STRUCTURE WITH SHORT-CIRCUITING BARS ALONG OUTER EDGE

Inside the cavity of Fig. 32, the fields are given by the following expressions:

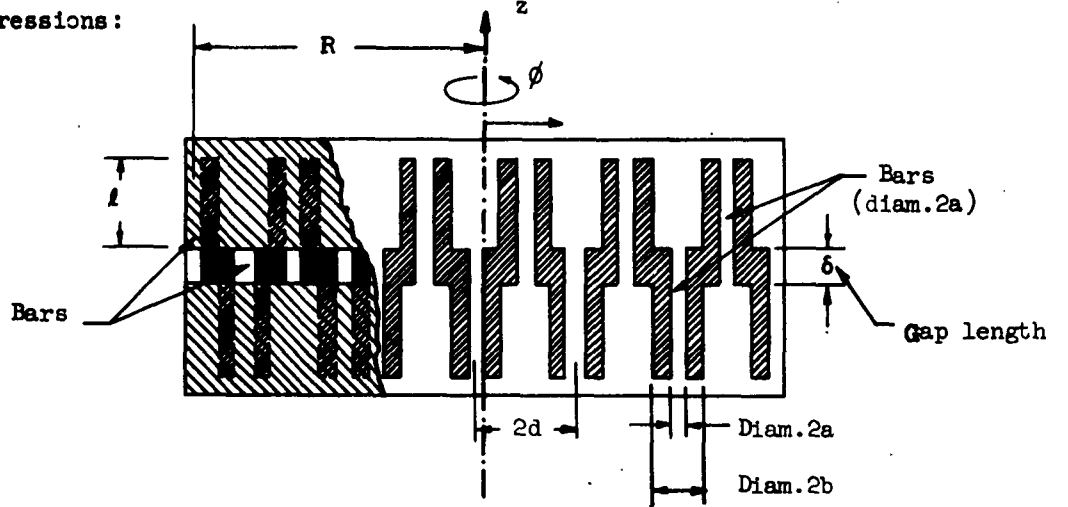


FIG. 32.--Anode structure with outer short-circuiting bars.

$$E_z = E_0 J_0(k\rho) \cos n\phi \quad (30)$$

where $k = 2\pi/\lambda_n$ and λ_n is the wavelength in free space for the n th mode. Then

$$H_\rho = -j \frac{nE}{\omega \mu_0} J_n(k\rho) \sin n\phi = -j \frac{nE}{k\rho \mu_0} J_n(k\rho) \sin n\phi$$

$$H_\phi = -j \frac{kE}{\omega \mu_0} J_n'(k\rho) \cos n\phi = -j \frac{\epsilon_0}{\mu_0} E J_n'(k\rho) \cos n\phi$$

The capacitance between two adjacent bars and the inductance for each bar are assumed to be given by

$$\omega C = \frac{\pi \delta}{R} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{kR}{\log \left(\frac{d + d^2 - 4a^2}{2a} \right)} \quad (31)$$

$$\omega L = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \log \left(\frac{b}{a} \right) \tan kR \left(\frac{l}{R} \right)$$

Since the boundary condition is taken as

$$-\frac{H_\phi}{\delta E_z} \bigg|_{\rho=R} = j\omega C + \frac{1}{j\omega L} \quad (32)$$

(IV. ANODE CONSIDERATIONS)

we obtain, from Eqs. (28) through (30),

$$\frac{J'_n(kR)}{J_n(kR)} = -\frac{2\pi\delta}{d} \left[\log \frac{b}{a} \tan \left(kR \frac{l}{R} \right) \right]^{-1} + \frac{\pi\delta^2}{Rd} \frac{kR}{\log[(d + \sqrt{d^2 - 4a^2})/2a]} \quad (33)$$

Some numerical results derived from Eqs. (31) are shown in Table 9.

Table 9.--Results for Fig. 32.							
$\frac{l}{R}$	$\frac{b}{a}$	$\frac{d}{a}$	$\frac{d}{\delta}$	$\frac{R}{\delta}$	$k_0 R$	$k_1 R$	$\frac{f_1 - f_0}{f_0}$
$\frac{1}{4}$	1.56	2	2.5	12	2.28	3.58	0.57
$\frac{1}{4}$	2	2	4	20	2.14	3.30	0.54
$\frac{1}{10}$	2	2	1	10	2.40	3.83	0.60

D. CYLINDRICAL CAVITY

Some experiments in this Laboratory have made use of the TE-mode structure depicted schematically in Fig. 33. The resonant frequencies of this structure are given by

$$J'_n(kR) = 0 \quad (34)$$

For the $n = 0$ mode the frequency separation is 20.3% assuming no variation of the fields along the axis. To realize this situation in practice, end cavities such as shown in Fig. 34 must be used. The dimensions of the end cavity are obtained by solving Maxwell's equations subject to the boundary condition of $E_\phi = j\sqrt{\mu_0/\epsilon_0} H_0 J'_0(k\rho)$ for $0 \leq \rho \leq R$, and $E_\phi = 0$ for $R \leq \rho \leq R_1$ at the plane $z = 0$. The expression for E_ϕ at $z = 0$ is

$$E_\phi = \sum_{s=1}^{\infty} A_s J_1(k_s \rho)$$

(IV. ANODE CONSIDERATIONS)

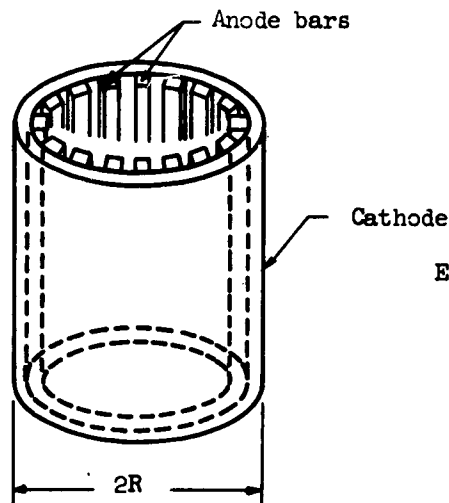


FIG. 33.--TE-mode structure.

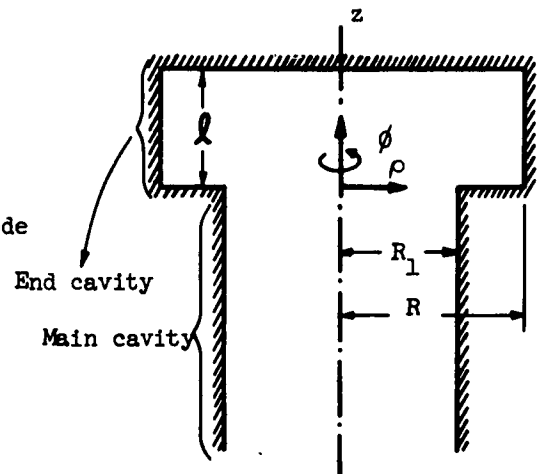


FIG. 34.--End cavities for TE-mode structure.

$$A_s = -j \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{H_0}{R_1^2} \frac{2}{J_2^2(k_s R_1)} \int_0^R J_1(k\rho) J_1(k_s \rho) \rho \, d\rho \quad (35)$$

where $k_s R_1$ are the roots of $J_1(x) = 0$.

Matching only the first term, the approximate cavity dimensions are

$$\left(\frac{l}{R}\right)^2 = \frac{0.168(R_1/R)^2}{(R_1/R)^2 - 1} \quad (36)$$

The matching condition for the s -th mode

$$\left(\frac{l}{R}\right)^2 = \frac{\pi^2 \left(\frac{1}{2} + m\right)^2}{(k_s R_1)^2} \frac{(R_1/R)^2}{(R_1/R)^2 - 1} \quad (37)$$

where m is an integer.

V. CONCLUSIONS

Resonant frequencies and mode separation have been determined for a conventional and for an inverted magnetron structure. These structures were chosen for their desirability as high-power magnetron circuits from the point of view of having the largest possible interaction volume.

It is shown that the circuits proposed here have a better mode separation than conventional structures such as the "rising sun."⁴

The structures proposed are easier to fabricate than conventional circuits.

REFERENCES

1. C. W. Hartman, "Production and interactions of electron beams in crossed fields," Series 60, Issue 325, University of California (Electronics Research Laboratory), 31 October 1960.
2. S. P. Yu and P. N. Hess, "Slow-wave structures for M-type devices," IRE Trans. ED-9: 51-57, 1962.
3. J. R. Pierce, "Traveling-Wave Tubes," Bell Syst. Tech. J. 29:390-460, 1950.
4. G. B. Collins, "Microwave Magnetrons" (vol. 6, M.I.T. Rad. Lab. Series), McGraw-Hill Book Co., New York, 1948; pp. 739-796.

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