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AN ANALYTICAL INVESTIGATION OF CAVITY OSCILLATIONS

by

Eugene E. Covert

MIT DSR 8951
Contract Nonr 1841(79)

October 1963

Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Aerospace Research Division
Aerophysics Laboratory

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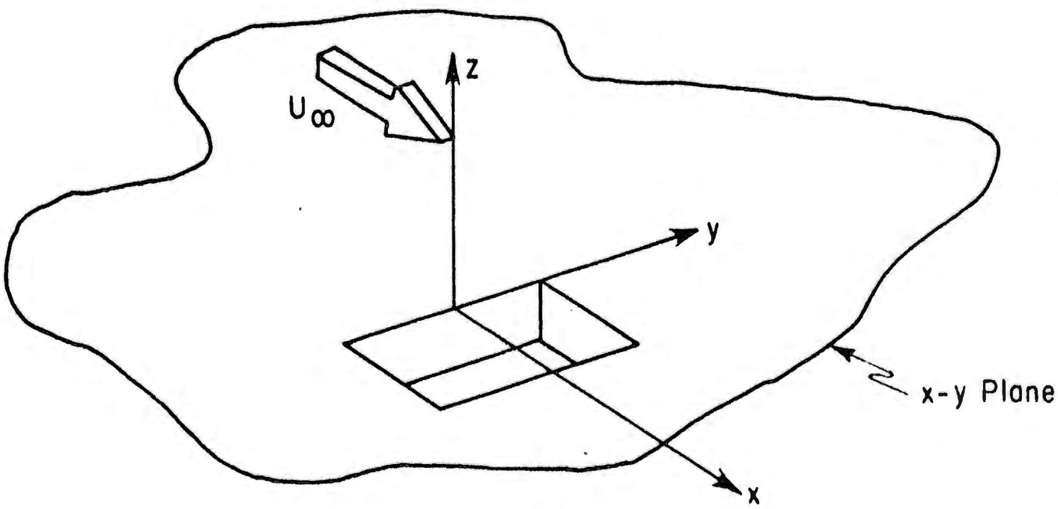
Under contract Nonr 1841(79) the Aerophysics Laboratory, Massachusetts Institute of Technology, carried out analytical studies of the stability of the flow past cavities filled with fluid. The basic geometry is shown in the sketch below



Sketch 1
Basic geometry

The problem was solved in the following way. Some particular fluid property (pressure, density, etc.) was found in the upper half space $(z > 0)$ in terms of the known distribution of that property over the boundaries and the unknown distribution over the opening. The same property was found in the cavity in terms of the distribution over the cavity boundaries and the opening. The two distributions were then equated at the opening. The resulting equation provided a means of computing the entire flow field, including its stability. Two cases were studied. One case was an accoustic cavity with energy storage in the fluid and the other case corresponded to a hydro-elastic cavity with energy storage in the cavity walls. The calculated results were in good general agreement with the experimental results. The details are contained in Ref. 1.

It was recommended in Ref. 1 that the study be continued with the goal of calculating the amplitude of the oscillations through the use of nonlinear mechanics. Subsequently it was proposed to carry out additional studies in the following way:

Each variable would be assumed to be represented by the product of an unknown time part and a spatial distribution, corresponding to that found by extending the stability analysis. Since the stability analysis provided an estimate of the eigenvalues, a preliminary step of calculating the eigenfunctions for the perturbed flow would be taken. After the equations of motion were put into their integral form, these assumed functions for the variables would be substituted and the integrations over space would then be carried out. This would result in a set of nonlinear ordinary differential equations that should exhibit the following properties:

a. Stable equilibrium points corresponding to the stable points from the linear analysis
b. Unstable equilibrium point corresponding to the unstable point from the linear analysis
c. The rate of growth of the instability
d. The existence of limit cycles corresponding to the observations of continuing finite amplitude oscillations

The period and amplitude of the oscillation could be calculated from item (d). The numerical results would then be compared with data available in the literature.

The detailed procedure for this analysis is outlined below. The integral form of the equations of motion is the basic form, i.e., the equation of continuity of mass is the difference between the density change in the volume and the mass flux across the boundaries of the volume; thus

$$\int_{\text{vol}} \frac{\partial \rho}{\partial t}\, d\,\text{vol} + \int_{\text{area}} \rho \vec{v} \cdot d\vec{A} = 0$$

Similarly if the stress tensor is $\overleftrightarrow{\pi}$, the law of conservation of momentum takes the form

$$\int_{\text{vol}} \frac{\partial}{\partial t} (\rho \vec{v})\, d\text{vol} + \int_{\text{area}} (\rho \vec{v}\vec{v} + \overleftrightarrow{\pi}) \cdot d\vec{A} = 0$$

To the approximation that we intended to use, the law of conservation of energy is included in the statement that the flow is isentropic and the entropy is uniform at infinity. Thus it can be included in the state equation, that is

$$p = R_e \frac{S}{cp} \rho^\gamma = \text{constant } \rho^\gamma$$

Now let

$$\rho = R(\vec{r})\, \theta(t) \; ; \; \vec{v} = B_i(\vec{r})\, T_i(t)\, \hat{a}_i \; ; \; \overleftrightarrow{\pi} = p \overleftrightarrow{\mathcal{I}}$$

where $\hat{a}_i$ is a unit vector in the i^{th} direction

Substituting

$$\frac{\partial \theta}{\partial t} \int_{vol} R\, d\,vol + \theta(t) \sum_{i=1}^{3} T_i(t) \int_{area} R\, B_i\, \hat{a}_i \cdot d\vec{A} = 0$$

$$\frac{\partial(\theta T_i)}{\partial t} \int_{vol} R\, B_i\, d\,vol + \sum_n \theta\, T_i\, T_n \int_{area} R\, B_i\, B_n\, dS_n + \text{const } \theta^\gamma \hat{a}_i \int R^\gamma dS = 0$$

which is a set of simultaneous equations for θ, T_i. Let integrals be denoted by capital A, thus

$$A_0 \frac{\partial \theta}{\partial T} + \theta(t) \sum_{n=1} A_n\, T_n(t) = 0$$

$$a_i \left\{ A_{4i} \frac{\partial}{\partial t} (\theta t_i) + \sum_n A_{5i}\, \theta\, T_i\, T_n + A_{6i}\, \theta^\gamma(t) \right\} = 0$$

is the set of simultaneous equation to be studied. The values of the coefficients, A_{ji}, are dependent upon the particular vibration and can include flexible as well as rigid wall.

It was suggested that the subject contract be extended (at no additional cost) until 1 October 1963 and that the additional time be used to study the literature of nonlinear mechanics as it applied to the problem of calculating the limiting amplitude of the oscillation. In this way some preliminary studies required for the proposed work would be completed.

In particular, some of the properties of a set of simultaneous equations similar to those displayed above were studied. By application of Bendixon's theorem[2,3] (the so-called negative criteria) to a simplified model it was found that it was not impossible for the system to have limit cycles. (A conclusion that the experimental data had already made evident). The application of Bendixson's theorem to a more realistic model was postponed until the further extension since this application is more difficult and requires more sophisticated approach. Unfortunately the additional support for this work is not forthcoming and the studies have been stopped at this point.

A shortened version of Ref. 1 has been submitted to the AIAA Journal for possible publication.

REFERENCES

1. Covert, Eugene E., An Analytical Investigation of Cavity Oscillations - Cavities with Unobstructed Openings and Discontinuous Velocity Profile, Massachusetts Institute of Technology, Aerophysics Laboratory, TR 38, October 1962.

2. Minorsky, N., Introduction to Nonlinear Mechanics, Edwards Brothers, Ann Arbor, Michigan (1947).

3. Cunningham, W. J., Introduction to Nonlinear Analysis, McGraw Hill Book Company, 1958.

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