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(6) Analytical Approximations

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6)

Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_o(\rho x)\, \rho d\rho$$

in which $I_o(z)$ is the usual Bessel function.

To better than .0017 over (0,2),

$$q(2,x) \doteq .135 + .566\left(\frac{x}{2}\right)^2 - .096\left(\frac{x}{2}\right)^4 .$$

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12-11-52

Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_o(\rho x)\,\rho\,d\rho$$

in which $I_o(z)$ is the usual Bessel function.

To better than .0008 over (0,3),

$$q(3,x) \doteq .011 + .231\left(\frac{x}{3}\right)^2 + .654\left(\frac{x}{3}\right)^4 - .329\left(\frac{x}{3}\right)^6.$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\,\rho\, d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0011 over $(1, \infty)$,

$$q(R,R) \doteq \frac{R + .183}{2R - .388} .$$

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12-18-52

Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_o(\rho x)\rho d\rho$$

in which $I_o(z)$ is the usual Bessel function.

To better than .0004 over (0,1),

$$q(R,R) = \tfrac{1}{2}\left\{1 + e^{-R^2} I_o(R^2)\right\}$$

$$\doteq 1 - .4921R^2 + .3212R^4 - .0966R^6 .$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_o(\rho x)\rho \, d\rho$$

in which $I_o(z)$ is the usual Bessel function.

To better than .0013 over (0,4),

$$q(4,4-y) \doteq \frac{.551}{\left[1 + .187y + .055y^2 + .051y^3\right]^4} \; .$$

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