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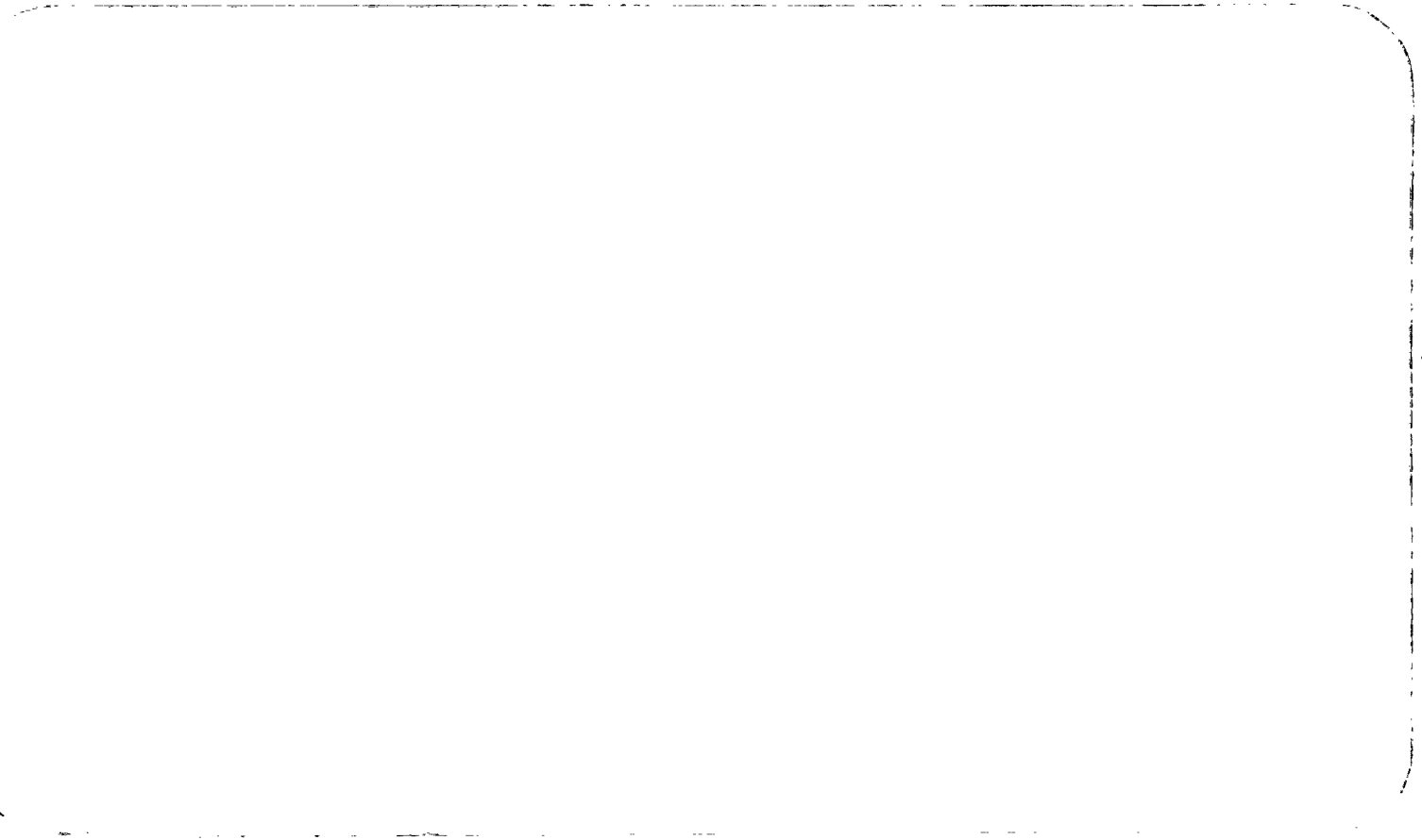
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⑥ Analytical Approximations
Volume 6.

⑩ by Cecil Hastings, Jr. and
James P. Wong, Jr. ✓

P-358

31 December 1952

The RAND Corporation

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0035 over (0, 3),

$$q(3, 3-y) \doteq \frac{.568}{[1 + .157y + .107y^2 + .017y^3]^4}$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .00011 over (0,1),

$$q(1, x) \doteq .6066 + .1500x^2 - .0238x^4.$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0013 over $(0, \infty)$,

$$\lim_{R \rightarrow \infty} q(R, R-y) = \int_{-\infty}^{-y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt$$

$$\doteq \frac{.5}{[1 + .209y + .061y^2 + .062y^3]^4}$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0028 over $(-\infty, \infty)$,

$$\lim_{R \rightarrow 0} \frac{1 - q(R, x)}{1 - q(R, 0)} = e^{-\frac{1}{2}x^2} \doteq \frac{1}{[1 + .123x^2 + .010x^4]^4}$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(r^2 + x^2)} I_0(rx) r dr$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0019 over (0,3),

$$q(3, x) \doteq \left[.105 + .930 \left(\frac{x}{3} \right)^2 - .282 \left(\frac{x}{3} \right)^4 \right]^2 .$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0011 over (0, 3),

$$q(3, x) \doteq \left[.105 + .954 \left(\frac{x}{3} \right)^2 - .349 \left(\frac{x}{3} \right)^4 + .043 \left(\frac{x}{3} \right)^6 \right]^2 .$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .002 over (0, 4),

$$q(4, x) \doteq \left[.018 + .581 \left(\frac{x}{4} \right)^2 + .515 \left(\frac{x}{4} \right)^4 - .372 \left(\frac{x}{4} \right)^6 \right]^2 .$$

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