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6 Analytical Approximations

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho\, d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0035 over (0,3),

$$q(3,3-y) \doteq \frac{.568}{\left[1 + .157y + .107y^2 + .017y^3\right]^4} .$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho \, d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .00011 over (0,1),

$$q(1,x) \doteq .6066 + .1500x^2 - .0238x^4.$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\,\rho\,d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0013 over (o,∞),

$$\lim_{R\to\infty} q(R,R-y) = \int_{-\infty}^{-y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}dt$$

$$\doteq \frac{.5}{\left[1 + .209y + .061y^2 + .062y^3\right]^4}$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_{R}^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_o(\rho x)\,\rho\, d\rho$$

in which $I_o(z)$ is the usual Bessel function.

To better than .0028 over $(-\infty,\infty)$,

$$\lim_{R\to 0} \frac{1 - q(R,x)}{1 - q(R,0)} = e^{-\frac{1}{2}x^2} \doteq \frac{1}{\left[1 + .123x^2 + .010x^4\right]^4}$$

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12-29-52

Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\, \rho\, d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0019 over (0,3),

$$q(3,x) \doteq \left[.105 + .930\left(\frac{x}{3}\right)^2 - .282\left(\frac{x}{3}\right)^4\right]^2 .$$

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12-30-52

Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_o(\rho x)\rho d\rho$$

in which $I_o(z)$ is the usual Bessel function.

To better than .0011 over (0,3),

$$q(3,x) \doteq \left[.105 + .954\left(\frac{x}{3}\right)^2 - .349\left(\frac{x}{3}\right)^4 + .043\left(\frac{x}{3}\right)^6\right]^2 .$$

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_o(\rho x)\,\rho\,d\rho$$

in which $I_o(z)$ is the usual Bessel function.

To better than .002 over (0,4),

$$q(4,x) \doteq \left[.018 + .581\left(\frac{x}{4}\right)^2 + .515\left(\frac{x}{4}\right)^4 - .372\left(\frac{x}{4}\right)^6\right]^2 .$$

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