TECHNICAL MEMORANDUM TM-1263

TEA - 2

TRAJECTORY ERROR ANALYSIS COMPUTER PROGRAM FOR TWO-DEGREE OF FREEDOM TRAJECTORIES

BY

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PICATINNY ARSENAL
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ABSTRACT

Estimates of the cumulative error associated with the third-order Runge-Kutta solution of the two-degree of freedom equations of motion are presented. These estimates constitute a simple, but not rigorous, approach to automatic increment selection. Details of the computer program TEA-2 which utilize these equations are described. Sample calculations are included. TEA-2 is between 2.0 and 1.7 times faster than the previous trajectory program in addition to providing greater consistency and reliability.

INTRODUCTION

Numerical solutions of two-degree of freedom trajectory problems are approximations to the exact solutions. Intelligent and efficient employment of high-speed computers for solving these trajectory problems requires that the errors resulting from numerical methods be controlled. This report presents the equations and describes the computer techniques used in TEA-2, a computer program which calculates integration step length automatically. The integration step length is computed such that the estimated error in the range and/or percentage error in altitude at impact compared to maximum altitude lies within prescribed limits. This program, therefore, provides ammunition designers with the ability to calculate trajectories such that the cumulative numerical error at the impact point is tolerable and of pre-specified magnitude.

The TEA-2 trajectory program requires the usual input of drag, thrust, and mass tables as well as initial conditions for velocity, angle, and the coordinate position. In addition, one must furnish allowable percentage errors for the range and/or maximum altitude. Details concerning input and output are included in Section C of the Procedure.

Equations for error estimation and automatic step selection are presented in Section A. Program limitations and computer flow are described in Section B. Appendix I contains a FORTRAN listing of the program. Lastly, sample calculations are presented in Appendix II.

PROCEDURE

A previous report, Technical Memorandum 1262, established a rigorous mathematical foundation for this work. Results from that report are utilized in conjunction with additional approximations to provide a set of equations for TEA-2. Section A contains this aspect of the analysis. The general features of TEA-2 and detailed instructions for its application are presented in Sections B and C, respectively.

A. Mathematical Basis

Trajectory equations of motion for a point-mass, two-degree of freedom system are:

$$
\begin{aligned}
\dot{X} &= V \cos \theta & \quad X\,(t=0) &= 0 \\
\dot{Y} &= V \sin \theta & \quad Y\,(t=0) &= Y_o \\
\dot{V} &= \frac{T}{m} - g \sin \theta - \rho K_D d^2 V^2/m & \quad V\,(t=0) &= V_o \\
\dot{\theta} &= -g \cos \theta / V & \quad \theta\,(t=0) &= \theta_o
\end{aligned}
\qquad (1)
$$

where $T(t)$, $m(t)$, $\rho(y)$, $K_D(V, \rho)$, $g(y)$, and d are given. They symbolize thrust, mass, air density, the drag coefficient, gravitational acceleration, and particle diameter, respectively. The quantities V and θ are the magnitude and orientation of the particle velocity, respectively. Particle position coordinates are specified by x and y. Figure 1 illustrates these quantities.

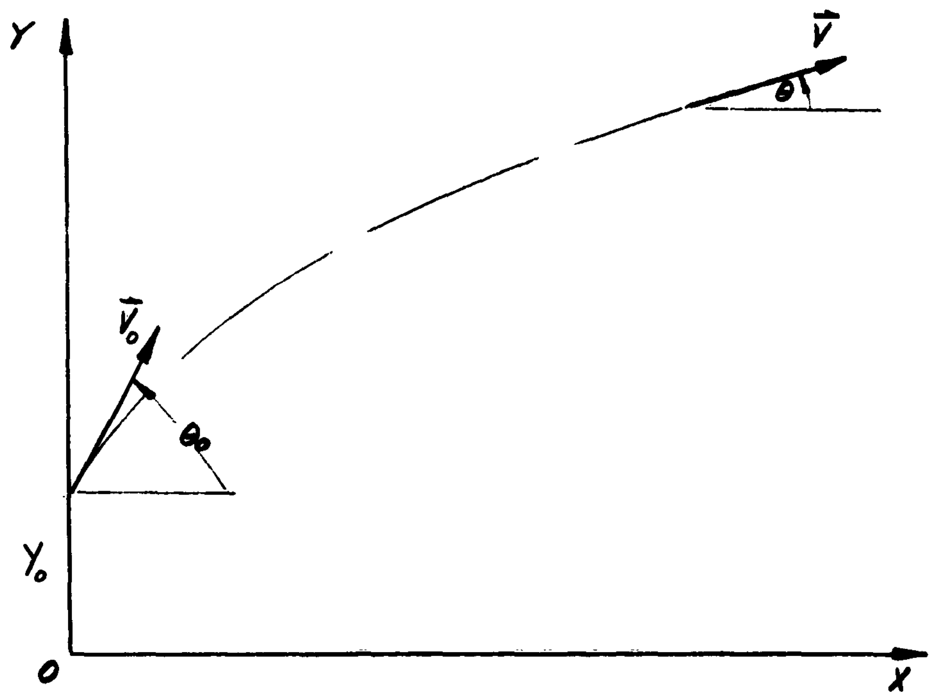


Figure 1
The Coordinate System

An approximate solution to the set of equations (1) is obtained by using a third-order Runge-Kutta method. The approximate solution is given by:

$$U_{i+1} = U_i + c_1 K_{1U} + c_5 K_{2U} + c_4 K_{3U} \qquad U = x, y, V, \theta \qquad (2)$$

where

$$K_{1U} = h\dot{U}(x_i, y_i, V_i, \theta_i, t_i) \qquad (3)$$

$$K_{2U} = h\dot{U}(x_i + c_1 K_{1x}, y_i + c_1 K_{1y}, V_i + c_1 K_{1V}, \theta_i + c_1 K_{1\theta}, t_i + c_1 h)$$

$$K_{3U} = h\dot{U}(x_i + c_1 K_{1x} + c_2 K_{2x}, \ldots, t_i + c_1 h + c_2 h)$$

Coefficients corresponding to the Gill modification are used. They are:

$$C_1 = .62653829$$

$$C_2 = -.55111241$$

$$C_4 = -.48268182$$

$$C_5 = .85614329$$

Application of the Runge-Kutta solution method to equations (1) introduces truncation error which is dependent on the degree of approximation and rounding error which follows from the limitations of the digital computer. Errors at the i-th time step propagate forward in time and introduce "propagation error."

The aforementioned errors were treated at length in TM-1262. It was shown that a bound for the cumulative error can be obtained from consideration of propagation, truncation, and rounding errors. Results from TM-1262 are combined with assumptions and approximations in this section. The set of equations so derived provide estimates of the cumulative error (not rigorous error bounds), and is employed in the TEA-2 program. The assumptions, limitations, and equations used in TEA-2 follow.

Assumptions:

1. The speed of the point-mass does not change by more than 5% per time increment.

2. The orientation of the velocity vector does not change by more than 2 degrees per time increment.

3. The difference between the values generated by a single step of length (h) and two steps of length $\frac{h}{2}$ is only due to truncation and rounding errors.

4. Quantities undergoing arithmetic operations and those in storage possess a relative error less than 10^{-8}.

5. Sine and cosine routines used on the Picatinny Arsenal IBM 709 have a relative error less than 10^{-8}.

Derivations contained in TM-1262 established rigorous error bounds. This report is not concerned with rigorous error bounds, but with error estimates. Consequently, bounding values for the variables used in TM-1262 will be replaced by local values. Moreover, estimates for truncation error based upon the difference in the variables for a single step of length (h) and two steps of length (h/2) will be employed. Appropriately modified equations from TM-1262 follow. Estimates of cumulative errors at the i-th point which result from errors at the n-th point are:

$$e_{vi} = V\left[-c_1 + (c_1 + c_2)\,\lambda_1^{\,i-n}\right] \tag{4}$$

$$e_{\theta i} = \frac{|K|}{V}\, e_{vi} \tag{5}$$

$$e_{xi} = e_{xn} + Vh\left[(i-n)(\alpha_{xT}h^3 + \alpha_{xR}) - c_1' c_3' + (c_1' + c_2')\, c_3' \,\frac{\lambda_1^{i-n} - 1}{\lambda_1 - 1}\right] \tag{6}$$

$$e_{yi} = e_{yn} + Vh\left[(i-n)(\alpha_{yT}h^3 + \alpha_{yR}) - c_1' c_4' + (c_1' + c_2')\, c_4' \,\frac{\lambda_1^{i-n} - 1}{\lambda_1 - 1}\right] \tag{7}$$

where

$$c_2' = \max\left\langle \frac{e_{vn}}{V}\,,\ \frac{e_{\theta n}}{|K|}\right\rangle \tag{8}$$

$$c_3' = \alpha_{13} + K\alpha_{14} \tag{9}$$

$$c_4' = \alpha_{23} + K\alpha_{24} \tag{10}$$

$$c_1' = \frac{1}{|\Delta_1|}\max\left\langle \left|\bar{\Phi}h^3 + \bar{\bar{\Phi}}\right|,\ \left|\bar{\Psi}h^3 + \bar{\bar{\Psi}}\right|\right\rangle \tag{11}$$

and $$\bar{\phi} = \alpha_{44}\alpha_{VT} - \alpha_{34}\alpha_{\theta T}$$

$$\bar{\bar{\phi}} = \alpha_{44}\alpha_{VR} - \alpha_{34}\alpha_{\theta R} \qquad (12)$$

$$\bar{\psi} = -\alpha_{43}\alpha_{VT} + \alpha_{32}\alpha_{\theta T}$$

$$\bar{\bar{\psi}} = -\alpha_{43}\alpha_{VR} + \alpha_{33}\alpha_{\theta R}$$

also, $$K = \frac{\epsilon - \alpha_{33}}{\alpha_{34}} \qquad (13)$$

$$\epsilon = \frac{1}{2}\left[\alpha_{33} + \alpha_{44} + \sqrt{(\alpha_{33} - \alpha_{44})^2 + 4\alpha_{34}\alpha_{43}}\,\right] \qquad (14)$$

$$\lambda_1 = 1 + \sigma\epsilon h \qquad (15)$$

$$\Delta_1 = \alpha_{33}\alpha_{44} - \alpha_{34}\alpha_{43} \quad , \quad \sigma = \frac{g}{V} \qquad (16, 17)$$

The propagation error coefficients are given by:

$$\rho_c = \cos\theta \ , \quad \rho_s = |\sin\theta| \qquad (18)$$

$$\alpha_{13} = \rho_c(1.03 - .014c + .0003c^2) \qquad (19.1)$$

$$\alpha_{14} = 1.20\rho_s + 0.28 - .0001c \qquad (19.2)$$

$$\alpha_{23} = (\rho_s + .08)(1.0 - .02c + .0004c^2) + .10 \qquad (19.3)$$

$$\alpha_{24} = (\rho_c + .07)\alpha_{44} \qquad (19.4)$$

$$\alpha_{33} = .019 - 1.10c + .04c^2 - .0002c^3 \qquad (19.5)$$

$$\alpha_{34} = (\rho_c + .10)(1.02 + .018c) \qquad (19.6)$$

$$\alpha_{43} = (\rho_c + .07)(1.40 - .02c + .0003c^2) \qquad (19.7)$$

$$\alpha_{44} = 1.20\rho_s + .21 + .003c \qquad (19.8)$$

where $$c = \frac{2\rho K_D d^2 V^2}{mg} \qquad (20)$$

Next, one invokes the assumption that the difference between the values resulting from a single step of length (h) and two steps of length (h/2) is only due to truncation and rounding errors. Subtraction of the rounding error yields:

$$\Delta x = \max \left\langle \left| x(h) - x\left(2\cdot\frac{h}{2}\right)\right| - \alpha_{XR} h V, 0 \right\rangle \tag{21.1}$$

$$\Delta y = \max \left\langle \left| y(h) - y\left(2\cdot\frac{h}{2}\right)\right| - \alpha_{YR} h V, 0 \right\rangle \tag{21.2}$$

$$\Delta V = \max \left\langle \left| V(h) - V\left(2\cdot\frac{h}{2}\right)\right| - \alpha_{VR}\, g h, 0 \right\rangle \tag{21.3}$$

$$\Delta \theta = \max \left\langle \left| \theta(h) - \theta\left(2\cdot\frac{h}{2}\right)\right| - \alpha_{\theta R} \frac{hg}{V}, 0 \right\rangle \tag{21.4}$$

where the rounding error coefficients may be approximated by:

$$\begin{aligned}
\alpha_{XR} &= 1.9 \times 10^{-7} \\
\alpha_{YR} &= 1.9 \times 10^{-7} \\
\alpha_{VR} &= \left[2.9\frac{T}{mg} + 2.0 + 2.3c\right] \times 10^{-7} \\
\alpha_{\theta R} &= 2.6 \times 10^{-7}
\end{aligned} \tag{22}$$

It follows from equations (15), (23), (92), (93), (94), and (95) of FRL-TM-1262 that

$$\Delta V \approx \frac{7}{4}\left|\frac{\bar{\phi}}{\Delta_1}\right| h^3 \tag{23}$$

$$\Delta \theta \approx \frac{7}{4}\left|\frac{\bar{\chi}}{\Delta_1}\right| h^3 \tag{24}$$

Thus,

$$\alpha_{VT} \approx \left(|\alpha_{33}|\,\Delta V + \alpha_{34} V \Delta\theta\right)\frac{4|\Delta_1|}{7Vh^3} \tag{25}$$

$$\alpha_{\theta T} \approx \left(|\alpha_{43}|\,\Delta V + \alpha_{44} V \Delta\theta\right)\frac{4|\Delta_1|}{7Vh^3} \tag{26}$$

Also,

$$\alpha_{xT} \approx \frac{8\Delta x}{7Vh^4} \tag{27}$$

$$\alpha_{yT} \approx \frac{8\Delta y}{7Vh^4} \tag{27.1}$$

Time increments are selected such that the estimated cumulative errors based upon local (that is, n-th step) propagation, truncation, and rounding errors satisfy specified allowable error criteria for the x and/or y variables at the impact point. That is,

$$\Phi_1(h) = e_{xf}(h) - \bar{e}_x = 0 \tag{28}$$

and/or $$\Phi_2(h) = e_{yf}(h) - \bar{e}_y = 0 \tag{29}$$

where the subscript f denotes the impact point and the quantities $\bar{e}_x$ and $\bar{e}_y$ are the allowable errors in the x and y variables at impact, respectively. Equations (7) and (8) require the number of steps between the n-th and f-th points. Moreover, estimates of the range and/or maximum altitude will be required since the allowable error is specified as a certain percentage of these quantities.

Equations (28) and/or (29) are solved by the method of double false position with a convergence criterion of $\frac{h_1 - h_2}{h_1} < .05$ where $\Phi_i(h_1) \geq 0$ and $\Phi_i(h_2) < 0$, $i = 1, 2$. The starting value of the iteration $h_0 = .01V/g$ followed by $3.5\, h_0 = h_2$ if $\Phi_i(h_0) < 0$ or by $.1\, h_0 = h_1$ if $\Phi(h_0) > 0$. The maximum step size is .035 V/g which is consistent with assumption 2. Time increments less than .000035V/g are not permitted.

The following formulas are employed to obtain the required estimates:

$$(f-n) = \Delta t / h \tag{30}$$

where $$\Delta t = \frac{1}{g}\left[\dot{y}_m + \sqrt{\dot{y}_m^2 + 2gy_m}\right] + t_m - t_n \tag{31}$$

or $$\Delta t = t^* - t_n \quad \text{such that} \quad y(t^*) = 0 \tag{31.1}$$

where t_m is the starting point for the terminal ballistic phase of the trajectory. The time t_m will be specified in the thrust table.

The terminal ballistic phase (Phase IV) impact range and maximum altitude are estimated from a linear drag model. They are given by:

$$x_{max} = x_n - \frac{1}{g} \sum_{i=n}^{f} \frac{\Delta U_i}{\sqrt{1-U_i^2}\; A(U_i)} \tag{32}$$

$$y = y_n - \frac{1}{g} \sum_{i=n}^{f} \frac{U_i \Delta U_i}{(1-U_i^2)\, A(U_i)} \tag{32.1}$$

$$y_{max} = \sup. \left\{ y \,\|\, y_i \,,\ i = n, \ldots, f \right\} \tag{33}$$

$$U_{i+1} = U_i + \Delta U_i \tag{33.1}$$

$$U_n = \sin \theta_n \tag{33.2}$$

where

$$y_f \leq 0 \qquad \text{defines "} f \text{"}$$

$$A(U) = \left\{ \sqrt{1-U^2} \left[\frac{1}{V_n \cos\theta_n} + \frac{\beta}{g} \tan\theta_n \right] - \frac{\beta}{g} U \right\}^2 \tag{33.3}$$

$$\Delta U_i = \begin{cases} (U_n + .9995)/20 & U > U_{f-1} \\ (U_n + .9995)\, y_{f-1} / \left[20 (y_{f-1} - y_f) \right] & \end{cases} \tag{33.4}$$

$$\beta = \rho d^2 V_n K_D \tag{33.5}$$

The following approximation is made for the model trajectory:

$$\rho = \left[.00234 \text{ lb/ft}^3\right] \times 10^{-m}$$

where

m=0	0 ft $\leq$ y $<$ 58,000 ft
m=1	58,000 ft $\leq$ y $<$ 100,000 ft
m=2	100,000 ft $\leq$ y $<$ 160,000 ft
m=3	160,000 ft $\leq$ y $<$ 230,000 ft
m=4	230,000 ft $\leq$ y $<$ 300,000 ft
m=5	300,000 ft $\leq$ y $<$ 400,000 ft
m=6	400,000 ft $\leq$ y $<$ 600,000 ft
m=7	600,000 ft $\leq$ y

Estimates of the dependent variables for an intermediate ballistic phase (Phase II) from t_j to t_g follow:

$$V_g = V_j - (g \sin\theta_j + \rho K_D d^2 V_j^2/m)(t_j - t_g) \quad (34)$$

$$\bar{V} = 0.5\,(V_j + V_g) \quad (34.1)$$

$$\theta_g = \theta_j - g\cos\theta_j\,(t_g - t_j)/\bar{V} \quad (35)$$

$$\theta_1 = 0.5\,(\theta_g + \theta_j) \quad (35.1)$$

$$x_g = x_j + 0.8\,V_1 \cos\theta_1 (t_g - t_j) \quad (36)$$

$$y_g = y_j + 0.8\,V_1 \sin\theta_1 (t_g - t_j) \quad (37)$$

The velocity change for a thrust phase from t_h to t_k has been calculated from linear drag approximation. Estimates for thrust phases are:

$$\alpha = g \sin\theta_h \quad (38)$$

$$\beta = \rho K_D d^2 V_h \quad (39)$$

$$V_K = \frac{T(t_K)}{\beta} - \frac{m(t_K)}{\beta - \dot{m}}\left[\frac{\dot{T}}{\beta} + \alpha\right] + \left[V_h + \frac{m(t_K)}{\beta - \dot{m}}\,\frac{\dot{T}}{\beta} + \alpha - \frac{T(t_h)}{\beta}\right]\left[\frac{m(t_K)}{m(t_h)}\right]^{\frac{\beta}{\dot{m}}} \quad (40)$$

where

$$T = T_h + \dot{T}(t - t_h)$$

$$m = m_h - \dot{m}(t - t_h) \qquad t_h \leq t \leq t_K \tag{41}$$

$$V_2 = 0.5\,(V_h + V_K) \tag{42}$$

$$\theta_K = \theta_h - g\cos\theta_h\,(t_K - t_h)/V_2 \tag{43}$$

$$\theta_2 = 0.5\,(\theta_h + \theta_K) \tag{43.1}$$

$$X_K = X_h + 0.8\,V_2\cos\theta_2\,(t_K - t_h) \tag{44}$$

$$y_K = y_h + 0.8\,V_2\sin\theta_2\,(t_K - t_h) \tag{45}$$

B. Computer Technique

A number of refinements have been incorporated in TEA-2 in order to generate reasonable error estimates. These refinements include:

1. Conservative range estimates.

2. Insuring that the truncation error coefficients do not decrease by a factor of more than .8 from the last time they were computed unless a phase change occurs.

3. Averaging the estimated error at the (n+N)-th step such that it is one-half the sum of estimates based upon local values at the n-th and (n+N)-th steps provided that N steps have been taken since the last step selection and that a phase change does not occur at n+N.

4. The acceptable percentage error is doubled if the criteria cannot be satisfied.

The TEA-2 trajectory program may be utilized to compute two-dimensional, point mass trajectories for one and two stage rockets and/or ballistic projectiles and consists of the following phases:

Phase I	Acceleration of Booster and Main Stage
Phase II	Coasting of Main Stage
Phase III	Acceleration of Main Stage
Phase IV	Free-flight of Main Stage

Any of the above phases may be excluded in the computations; for ballistic trajectory computations, only Phase IV is required.

The following are program conditions:

1. Control systems are not considered
2. A flat earth model is assumed
3. Winds are not considered
4. ARDC Standard Atmosphere, 1959
5. Constant thrust or thrust variable with time
6. Thrust modification for atmospheric pressure changes
7. Constant linear weight change or weight variable with time for rocket boosted phases
8. k_D or C_D drag coefficients may be used
9. Form factor to alter drag coefficient

Thrust values for the acceleration phases (I & III) are obtained in the computations by linear interpolation of a thrust (lb) versus time (seconds) table when table values are presented; however, a constant thrust force may be presented instead of tables.

A constant weight change (burning rate, lb/sec) or a table of weight (lb) versus time (seconds) may be introduced into the computations for rocket boosted missiles. Using the table, weight is obtained in the program by linear interpolation. When tables are not presented, the weight is decreased during thrust phases by a factor of integration increment (seconds) multiplied by the burning rate for each step of integration.

Tables of drag coefficients (C_D or k_D) versus mach numbers may be presented for each acceleration phase to be computed and for the free-flight phase. If only Phase IV (i.e., ballistic trajectory) is to be computed, then only the drag coefficient table for free-flight is required.

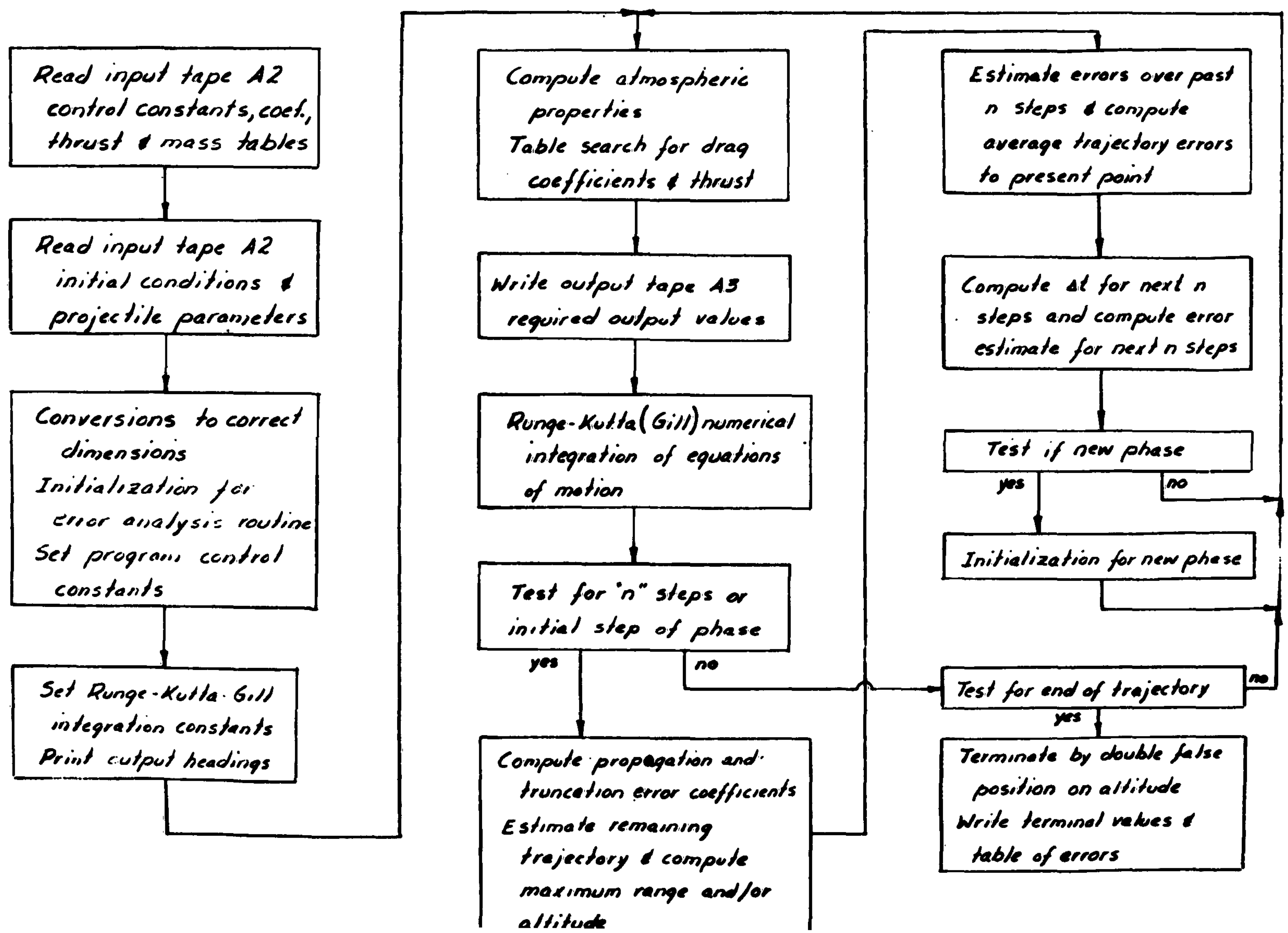
A constant drag coefficient may be presented for the entire trajectory. (See description for Card #1.)

A generalized and a detailed flow chart are presented in order to depict the program logic. The corresponding FORTRAN listing can be found in Appendix I. Reference should be made to the flow charts prior to any modification due to the complexity of the branching operations in the body of the program. This program requires 7024 words of core storage.

TRAJECTORY ERROR ANALYSIS for TWO-DEGREE-OF-FREEDOM TRAJECTORIES

Computer Program TEA-2

Generalized Flow Chart



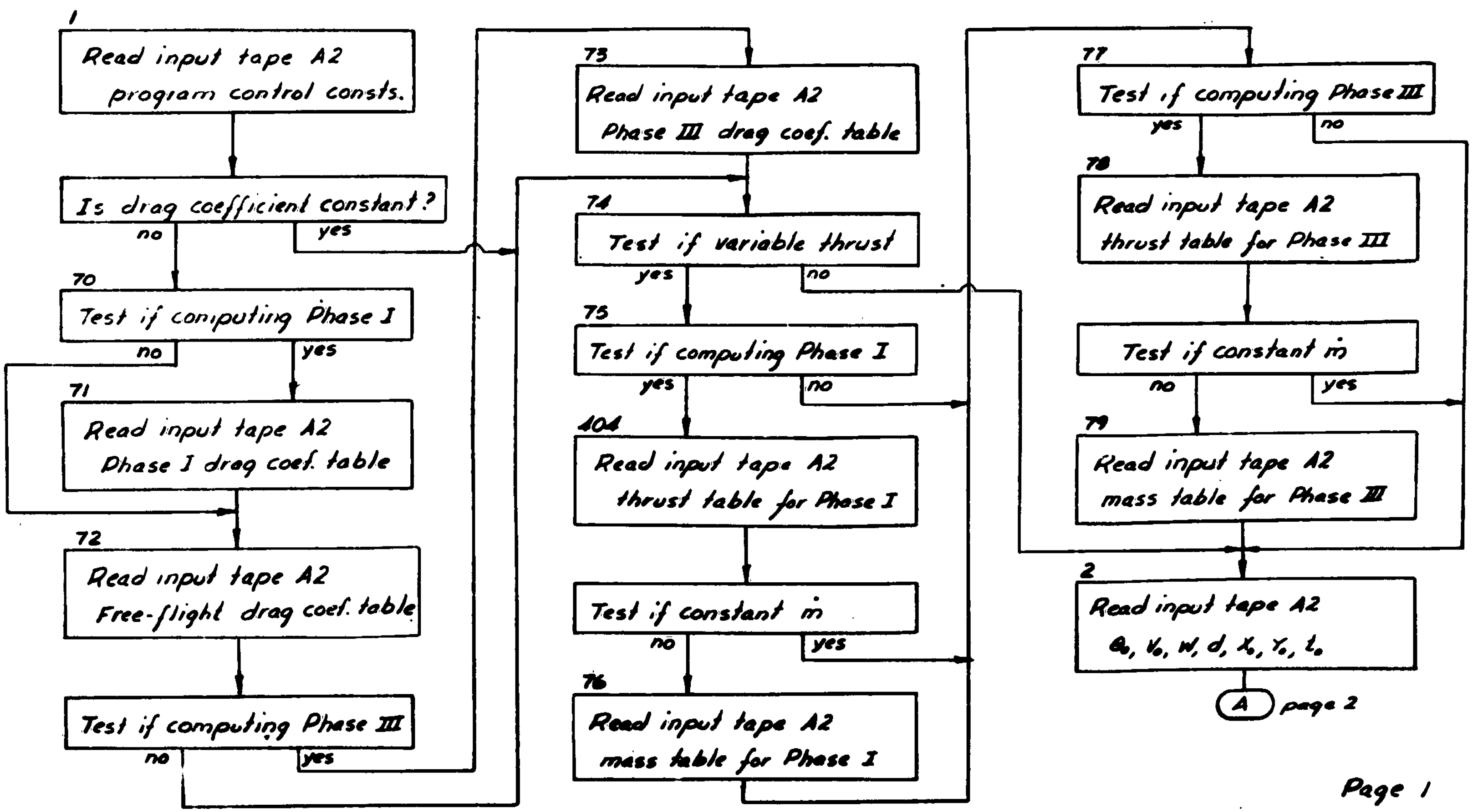
TRAJECTORY ERROR ANALYSIS for TWO-DEGREE-OF-FREEDOM TRAJECTORIES

Computer Program TEA-2

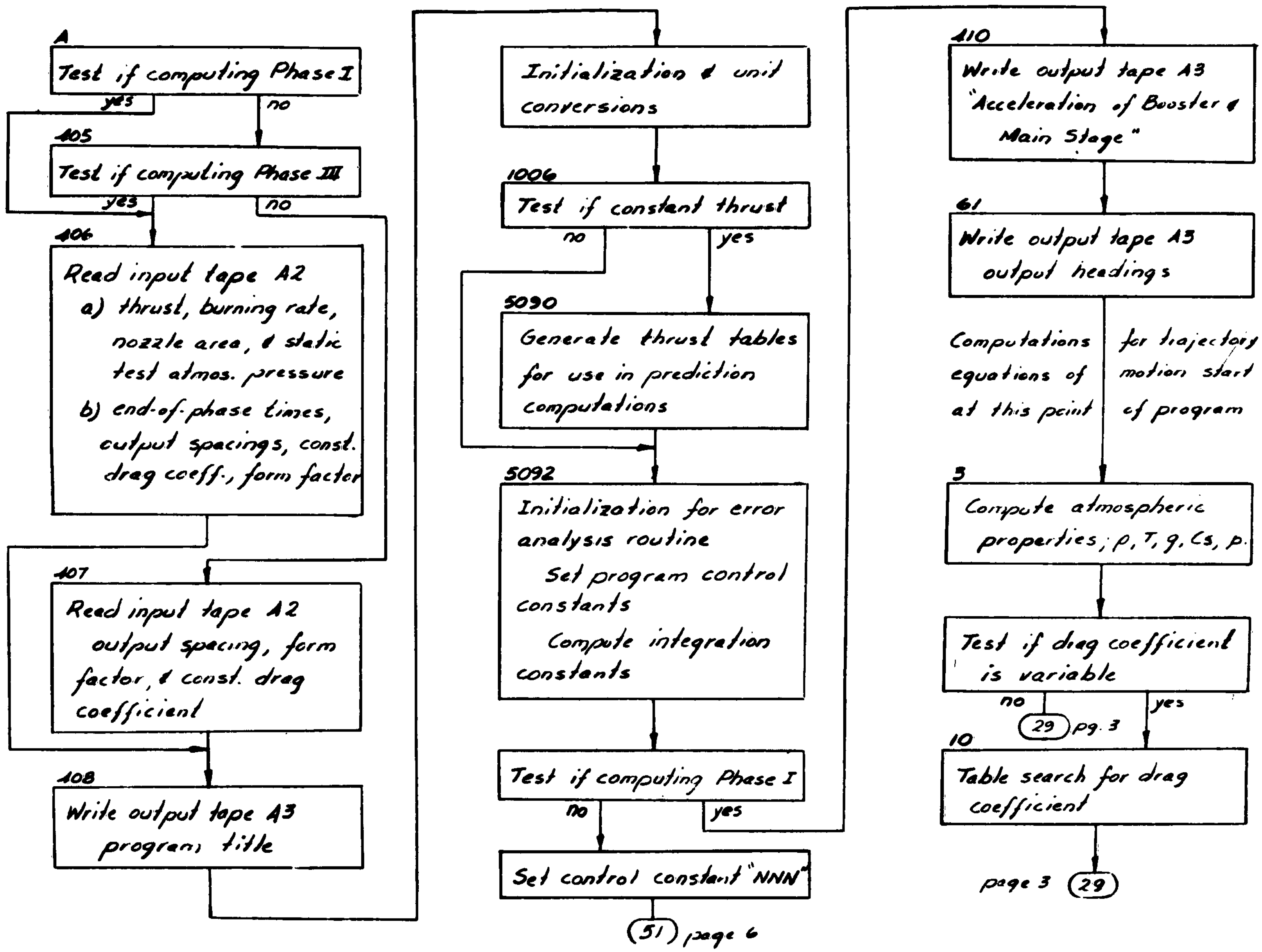
ANALYSIS : Dr. H.J. Kopp
PROGRAMMING : Mr. J.N. Nielsen

ORGANIZATION: Computing & Analysis Section
Engineering Sciences Lab.
Picatinny Arsenal

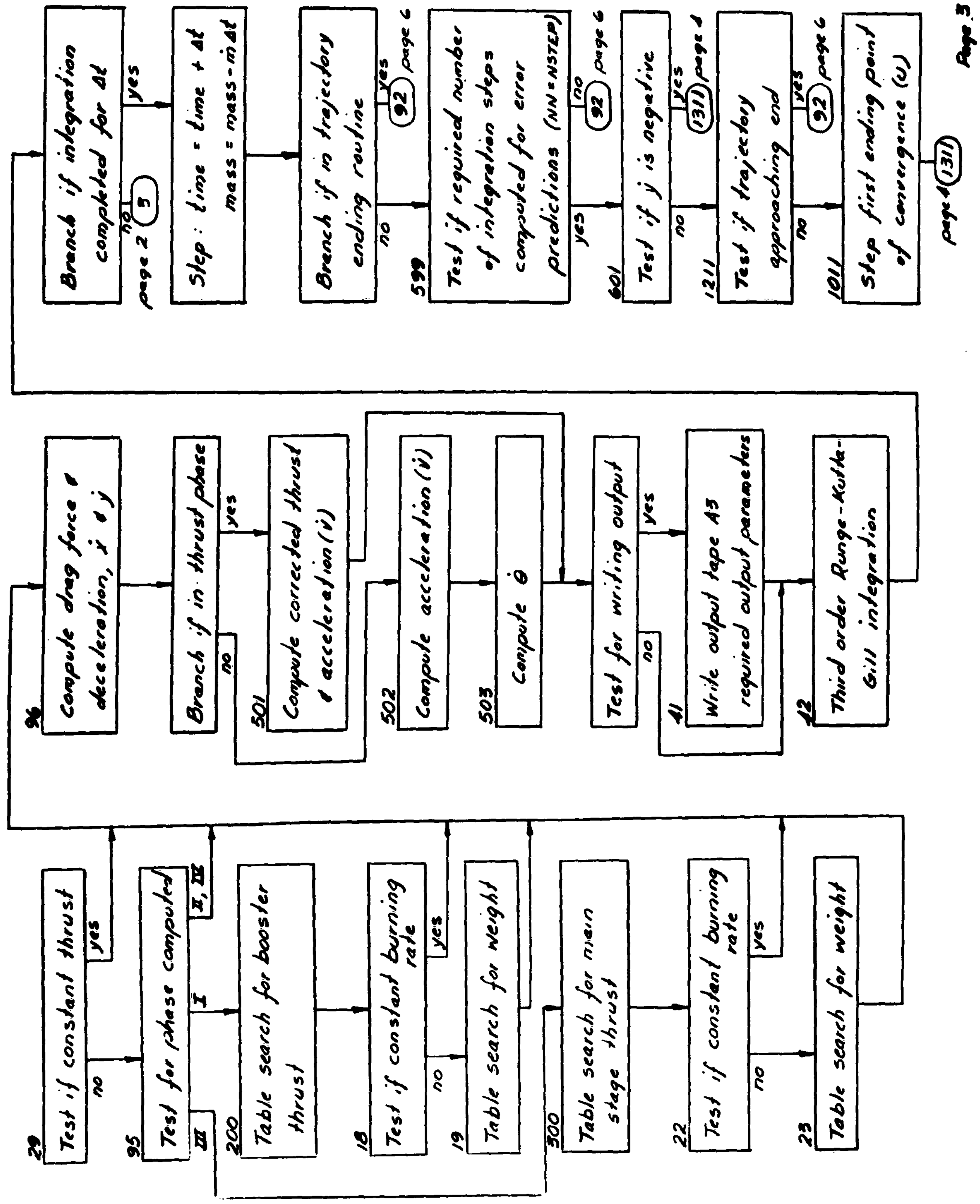
Detailed Flow Chart



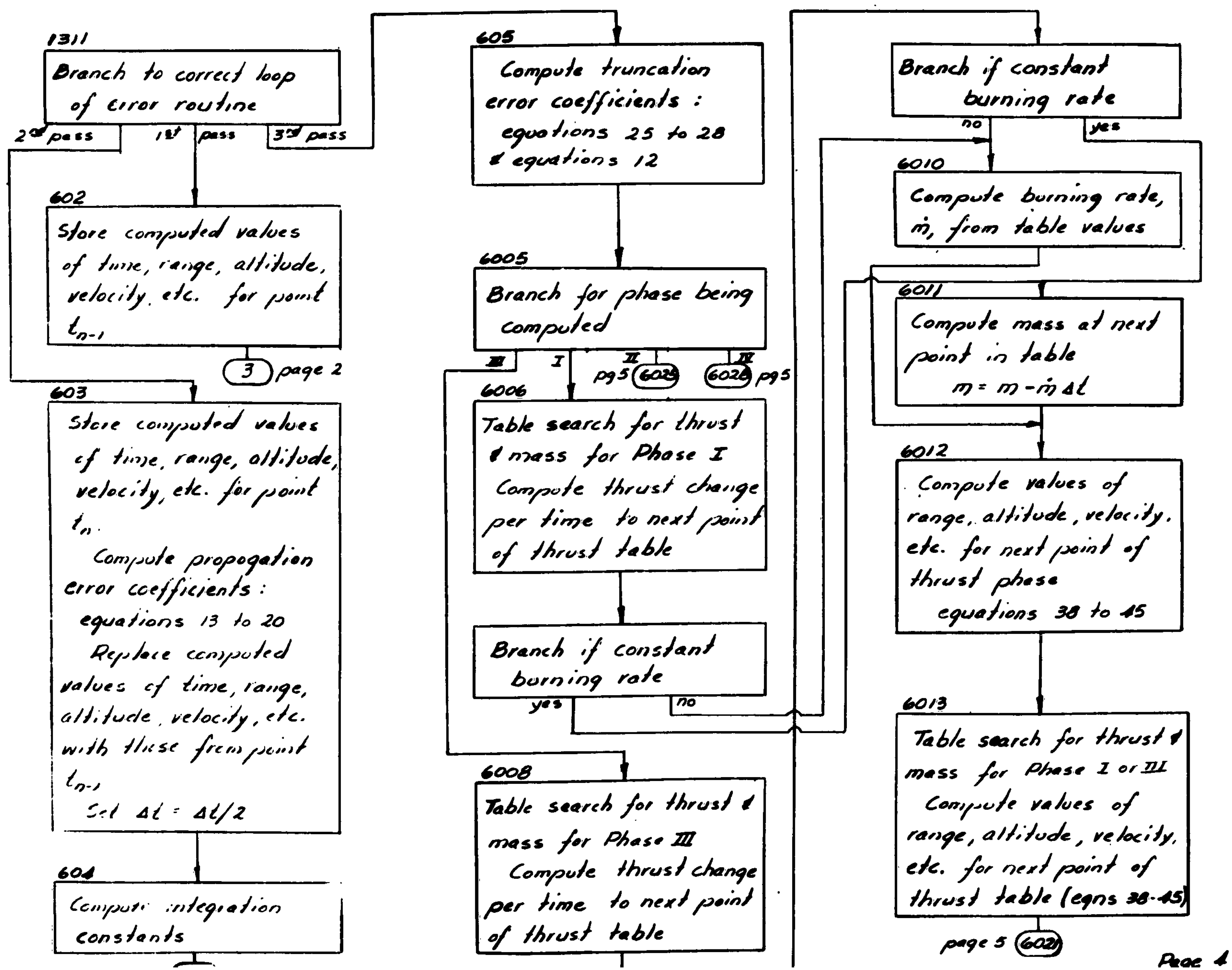
Page 1

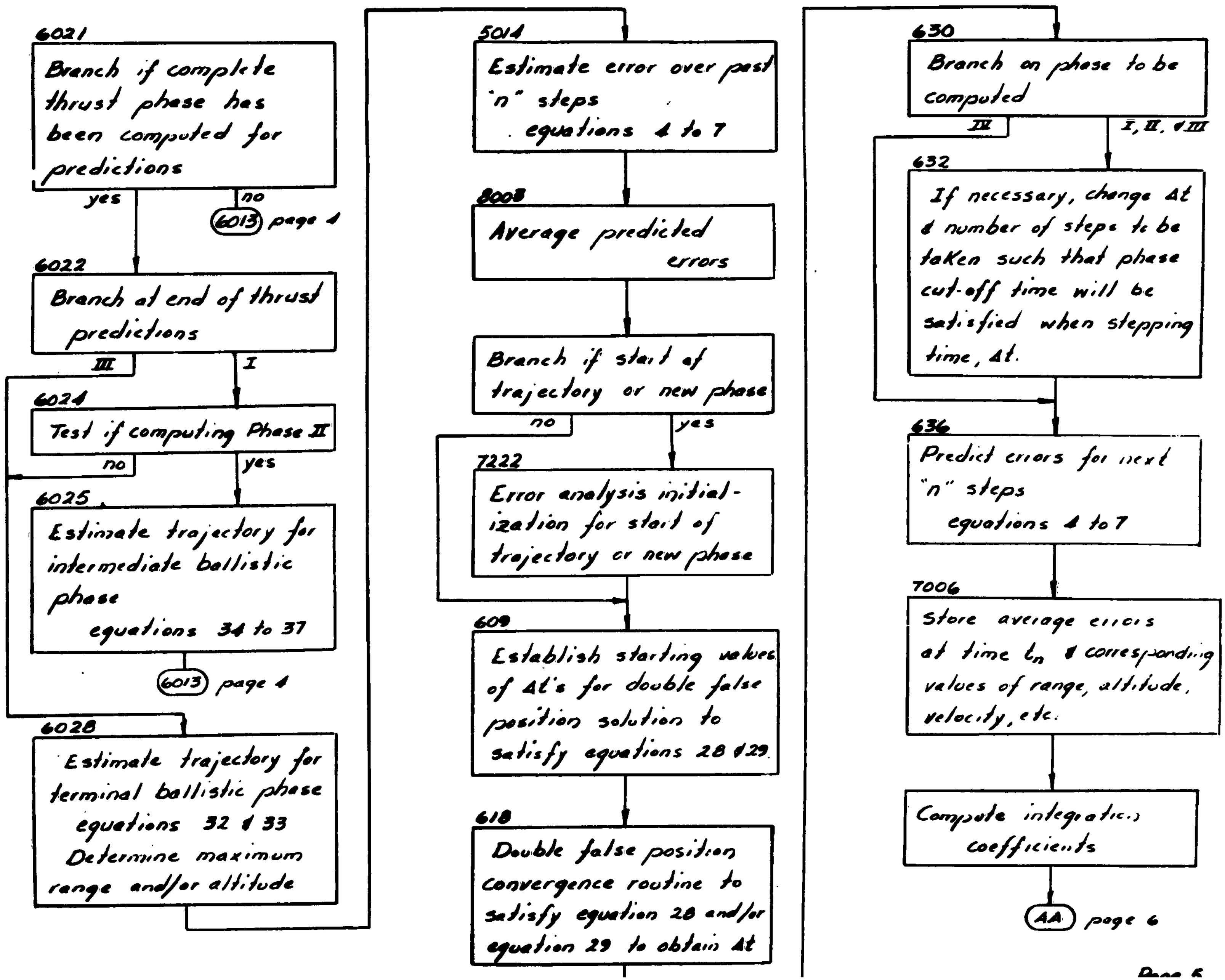

A
Test if computing Phase I
yes
no
405
Test if computing Phase III
yes
no
406
Read input tape A2
a) thrust, burning rate, nozzle area, & static test atmos. pressure
b) end-of-phase times, output spacings, const. drag coeff., form factor
407
Read input tape A2 output spacing, form factor, & const. drag coefficient
408
Write output tape A3 program title
Initialization & unit conversions
1006
Test if constant thrust
no
yes
5090
Generate thrust tables for use in prediction computations
5092
Initialization for error analysis routine
Set program control constants
Compute integration constants
Test if computing Phase I
no
yes
Set control constant "NNN"
51
page 6
410
Write output tape A3 "Acceleration of Booster & Main Stage"
61
Write output tape A3 output headings
Computations for trajectory equations of motion start at this point of program
3
Compute atmospheric properties; ρ, T, g, Cs, p.
Test if drag coefficient is variable
no
yes
29
pg. 3
10
Table search for drag coefficient
page 3
29

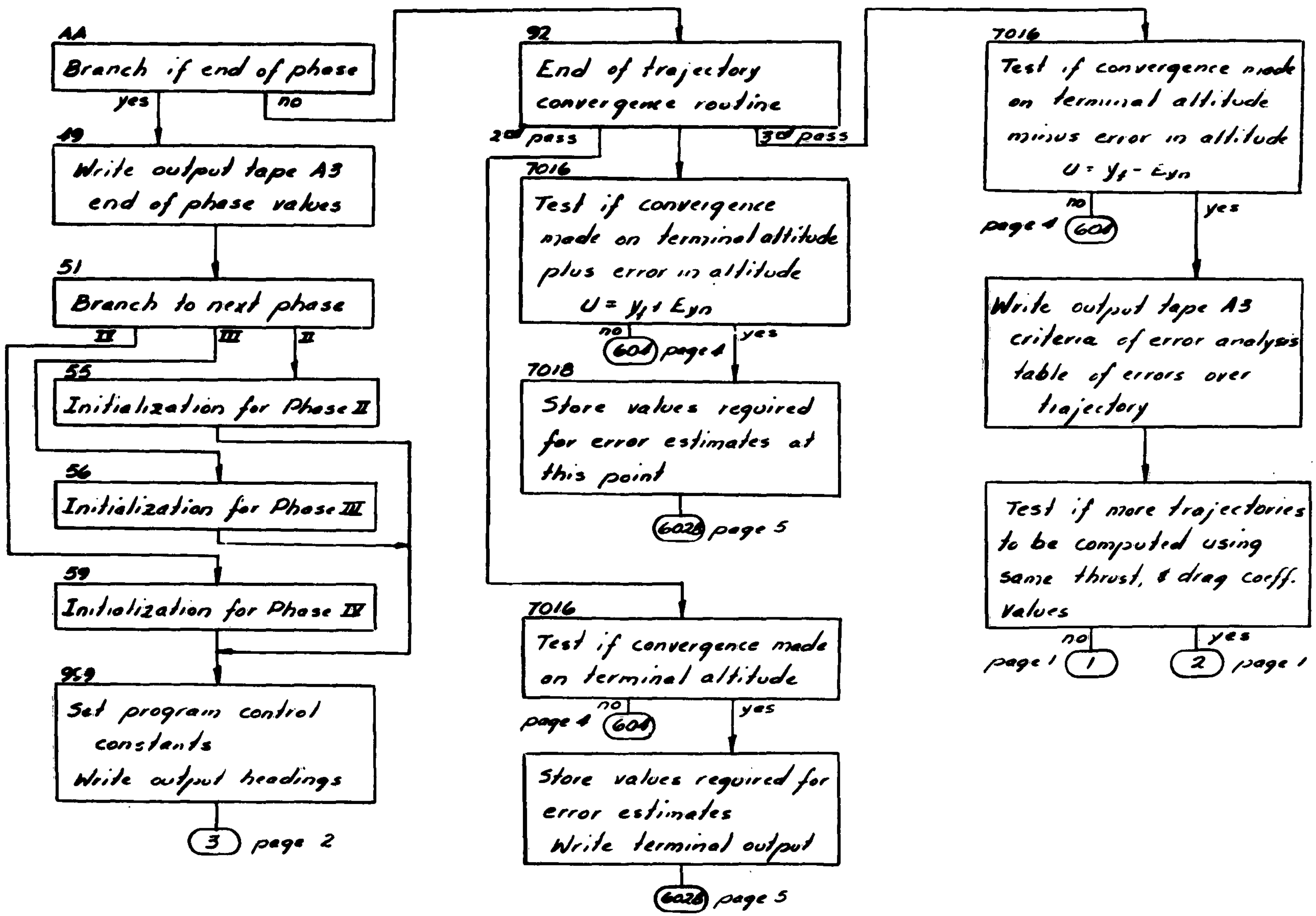
Page 2



29 Test if constant thrust
yes
no
95 Test for phase computed
I
II, IV
III
200 Table search for booster thrust
18 Test if constant burning rate
yes
no
19 Table search for weight
300 Table search for main stage thrust
22 Test if constant burning rate
yes
no
23 Table search for weight
96 Compute drag force & deceleration, ẋ & ẏ
Branch if in thrust phase
yes
no
501 Compute corrected thrust & acceleration (v̇)
502 Compute acceleration (v̇)
503 Compute θ̇
Test for writing output
yes
no
41 Write output tape A3 required output parameters
42 Third order Runge-Kutta-Gill integration
Branch if integration completed for Δt
yes
no
page 2 (5)
Step: time = time + Δt mass = mass - ṁΔt
Branch if in trajectory ending routine
yes
(92) page 6
no
599 Test if required number of integration steps computed for error predictions (NN=NSTEP)
no
(92) page 6
yes
601 Test if y is negative
yes
(131) page 4
no
1211 Test if trajectory approaching end
yes
(92) page 6
no
1011 Step first ending point of convergence (u)
page 4 (131)
Page 3


1311
Branch to correct loop of error routine
2nd pass
1st pass
3rd pass
602
Store computed values of time, range, altitude, velocity, etc. for point t_{n-1}
3 page 2
603
Store computed values of time, range, altitude, velocity, etc. for point t_n.
Compute propogation error coefficients: equations 13 to 20
Replace computed values of time, range, altitude, velocity, etc. with these from point t_{n-1}
Set $\Delta t = \Delta t/2$
604
Compute integration constants
605
Compute truncation error coefficients: equations 25 to 28 & equations 12
6005
Branch for phase being computed
III
I
II
IV
pg 5
pg 5
6006
Table search for thrust & mass for Phase I
Compute thrust change per time to next point of thrust table
Branch if constant burning rate
yes
no
6008
Table search for thrust & mass for Phase III
Compute thrust change per time to next point of thrust table
Branch if constant burning rate
no
yes
6010
Compute burning rate, $\dot{m}$, from table values
6011
Compute mass at next point in table
$m = m - \dot{m}\Delta t$
6012
Compute values of range, altitude, velocity, etc. for next point of thrust phase
equations 38 to 45
6013
Table search for thrust & mass for Phase I or III
Compute values of range, altitude, velocity, etc. for next point of thrust table (eqns 38-45)
page 5
Page 4


6021
Branch if complete thrust phase has been computed for predictions
yes
no
6013 page 4
6022
Branch at end of thrust predictions
III
I
6024
Test if computing Phase II
no
yes
6025
Estimate trajectory for intermediate ballistic phase
equations 34 to 37
6013 page 4
6028
Estimate trajectory for terminal ballistic phase
equations 32 & 33
Determine maximum range and/or altitude
5014
Estimate error over past "n" steps
equations 4 to 7
8008
Average predicted errors
Branch if start of trajectory or new phase
no
yes
7222
Error analysis initial-ization for start of trajectory or new phase
609
Establish starting values of Δt's for double false position solution to satisfy equations 28 & 29
618
Double false position convergence routine to satisfy equation 28 and/or equation 29 to obtain Δt
630
Branch on phase to be computed
IV
I, II, & III
632
If necessary, change Δt & number of steps to be taken such that phase cut-off time will be satisfied when stepping time, Δt.
636
Predict errors for next "n" steps
equations 4 to 7
7006
Store average errors at time t_n & corresponding values of range, altitude, velocity, etc.
Compute integration coefficients
AA page 6


AA
Branch if end of phase
yes
no
49
Write output tape A3 end of phase values
51
Branch to next phase
IV
III
II
55
Initialization for Phase II
56
Initialization for Phase III
59
Initialization for Phase IV
959
Set program control constants
Write output headings
3
page 2
92
End of trajectory convergence routine
2d pass
3d pass
7016
Test if convergence made on terminal altitude plus error in altitude
U = y_f + Eyn
no
604
page 4
yes
7018
Store values required for error estimates at this point
602B
page 5
7016
Test if convergence made on terminal altitude
no
page 4
604
yes
Store values required for error estimates
Write terminal output
602B
page 5
7016
Test if convergence made on terminal altitude minus error in altitude
U = y_f - Eyn
no
page 4
604
yes
Write output tape A3
criteria of error analysis
table of errors over trajectory
Test if more trajectories to be computed using same thrust, & drag coeff. Values
no
page 1
1
yes
2
page 1

Page 6

C. Computer Usage

The procedures for preparing input data to trajectory program TEA-2 are discussed in this section, and examples of typical cases are presented and described.

Input data is punched on cards which are loaded with the program deck onto tape. Data sheets which may be used for presenting required data for key-punching are illustrated in the descriptions of sample trajectory cases to be computed. The computer program reads input data from Tape A2 which is the standard for IBM FORTRAN Monitor Systems.

Descriptions of input requirements will consist of two parts: (1) Rocket trajectories and, (2) Ballistic trajectories.

1. Rocket Trajectory Input

Card #1 contains nine one-digit and three two-digit control parameters from columns 1 through 15. Columns 16 to 20 are for a decimal input number. These parameters are to be presented across card #1 as follows:

Card #1 .... NEXT(1), NEXT(2), NEXT(3), NEXT(4), KDCON, NTHRST, KD, M, NXY, JN, NJR, NS, GAMMA
Format (9I1, 3I2, F5.0)

where:

NEXT(1) .... equal to 0 for not computing Phase I
equal to 1 for computing Phase I

NEXT(2) .... equal to 0 for not computing Phase II
equal to 1 for computing Phase II

NEXT(3) .... equal to 0 for not computing Phase III
equal to 1 for computing Phase III

NEXT(4) .... equal to 0 for not computing Phase IV
equal to 1 for computing Phase IV

KDCON .... equal to 1 for variable drag coefficients (use tables)
equal to 2 for constant drag coefficient

NTHRST .... equal to 1 for using thrust tables
equal to 2 for not using thrust tables

KD .... equal to 1 for using k_D coefficients
equal to 2 for using C_D coefficients

M .... equal to 1 for using weight table
equal to 2 for not using weight table

NXY...... equal to 1 for step selection based on range error criterion
equal to 2 for step selection based on maximum altitude
equal to 3 for step selection based on total range and maximum altitude

JN........ number of trajectories to be computed for the same conditions of error limit, thrust and drag coefficients (a two digit integer)

NJR...... trajectory identification number for first trajectory to be computed (a two digit integer)

NS....... number of integration steps to be computed for error estimations (a two digit integer; if left blank on card, set equal to ten in program)

GAMMA... percent error limit for trajectory computations (if left blank on input card, GAMMA is set equal to 5.0% in program)

Cards #2 to #7 contain the drag coefficient table for Phase I. Cards #2 to #4 consist of the table mach numbers, and cards #5 to #7 contain the corresponding drag coefficients.

Cards #8 to #13 contain drag coefficient tables for free-flight phases (Phases II and IV). Cards #8 to #10 are for the table mach numbers, and cards #11 to #13 are for the corresponding drag coefficients.

Cards #14 to #19 contain the drag coefficient for Phase III. Cards #14 to #16 are for the table mach numbers, and cards #17 to #19 are for the corresponding drag coefficients.

Either k_D or C_D coefficients may be used in the tables; however, values that are to be used must be specified by the control parameter KD in card #1. All of the coefficient tables may be eliminated as input when in card #1 KDCON = 2.

Cards #20 to #25 contain the thrust table for Phase I. This table is not required when a constant thrust force is to be used; and for this case, the control parameter NTHRST in card #1 must equal 2, and a thrust value must be presented in card #46. Cards #20 to #22 are for the table time values, and cards #23 to #25 are for the corresponding table thrust values.

Cards #26 to #31 contain the weight table for Phase I and must be presented only if in card #1 M=1. Cards #26 to #28 are for table time values, and cards #29 to #31 are the corresponding table values of weight.

Cards #32 to #37 contain the thrust table for Phase III. These cards are not required when a constant thrust value is to be used; and for this case, NTHRST must equal 2 in card #1 and a constant thrust value must be presented in card #46. Cards #32 to #34 are for table time values, and cards #35 to #37 are for the corresponding table thrust values.

Cards #38 to #43 contain the weight table for Phase III and must be presented when constant burning rates are not used (in card #1 M=1). Cards #38 to #40 are for table time values, and cards #41 to #43 are the corresponding table values of weight.

Data field widths for all input values are nine columns, and eight values may be introduced on one card. Tables are required only for phases that are to be computed. Each table that is required by the program will consist of six input cards; therefore, if data is not available for completing the maximum table requirements, blank cards must be used to supply the proper number of cards.

The last three cards required for input (cards #44, 45, and 46) will be listed along with input symbols, and these cards are required for each rocket trajectory to be computed. Using the same error criteria, thrust, weight, and drag coefficient tables, "JN" trajectories may be computed by varying initial conditions, configuration, etc., in cards #44, 45 and 46.

Card #44.....THDEGO, VO, WGT, DIA, XO, YO, TO, YF

where:

THDEGO ... initial quadrant elevation, degrees
VO .... initial velocity, feet per second
WGT ...... total weight, pounds
DIA .... missile diameter, inches
XO .... initial range, feet
YO .... initial altitude, feet
TO .... time at start of trajectory, seconds
YF .... terminal altitude, feet

Card #45..... THRST, DOTM, AREA, BWGT, THR2, DOTM2, AREA2, PRESO

WHERE:

THRST ..... booster thrust, pounds
DOTM ..... booster burning rate, lb per second
AREA ..... nozzle throat area of booster, sq. in.
BWGT ..... booster weight (w.o. propellant), pounds
THR2 ...... main stage thrust, pounds
DOTM2 .... main stage burning rate, lb per second
AREA2 .... nozzle throat area of main stage, sq. in.
PRESO .... static test atmospheric pressure

AREA, AREA2 and PRESO are to be presented for input only when the thrust is to be modified in the computations for atmospheric pressure changes. For this correction not to be performed, AREA and AREA2 must be made equal to zero or left blank on the input card.

Card #46 .... TIM(1), TIM(2), TIM(3), DWRT(1), DWRT(2), DWRT(4), CKD, FACT

where:

TIM(1)	end of Phase I, seconds
TIM(2)	end of Phase II, seconds
TIM(3)	end of Phase III, seconds
DWRT(1)	output spacing for Phases I and III, seconds
DWRT(2)	output spacing for Phase II, seconds
DWRT(4)	output spacing for Phase IV, seconds
CKD	constant drag coefficient
FACT	form factor (if left blank on input card, form factor is set equal to one)

ALL DATA PRESENTED ON CARDS #2 TO #46 MUST HAVE DECIMAL POINTS AND HAVE FIELD WIDTHS OF NINE COLUMNS.

Rocket Trajectory Sample Input Case

The rocket sample case is for a one stage rocket which would require computing Phases I and IV of the program.

The following conditions represent the trajectory input requirements:

1. C_D drag coefficient tables
2. constant burning rate (5 lbs/sec)
3. constant thrust (100 lbs)
4. step selection based on range error criteria
5. 10 integration steps per estimate
6. quadrant elevation = 45.0°
7. initial velocity = 1000.0 fps
8. weight = 100.0 lbs
9. diameter = 11.0 inches
10. burning time = 4.0 seconds
11. output spacing every Δt
12. form factor = 1.0
13. 5.0% error limit

For the above conditions, the following data must be presented in card #1:

NEXT(1)	= 1, compute Phase I
NEXT(2)	= 0, do not compute Phase II
NEXT(3)	= 0, do not compute Phase III
NEXT(4)	= 1, compute Phase IV
KDCON	= 1, variable drag coefficients presented
NTHRST	= 2, constant thrust
KD	= 2, using C_D drag coefficients
M	= 2, not presenting weight tables (constant burning rate)
NXY	= 1, step selection based on range error criteria
JN	= 01, compute one trajectory
NJR	= 01, trajectory numbered 1
NS (blank)	, ten integration steps per estimate
GAMMA (blank)	, 5.0% acceptable error

An illustration of the presentation of data on card #1 may be seen on page 33.

The drag coefficient tables for Phase I and Phase IV as presented for key-punching are shown on the table data sheet on page 32. These tables are punched onto cards #2 to #13, and the punched cards are illustrated on page 33.

Cards #14 to #19 are not required for input as Phase III is not to be computed.

Cards #20 to #43 are not required for input as thrust and weight tables are not to be presented.

The following data must be punched on card #44:

THDEGO (45.0), quadrant elevation, degrees
VO (1000.0), initial velocity, fps
WGT (100.0), missile weight, lbs
DIA (11.0), missile diameter, inches

Values for XO, YO, TO, and YF do not have to be punched on the card for this case as they equal zero.

The following data must be punched on card #45:

THRST (100.0), constant thrust force, lbs
DOTM (5.0), burning rate, lbs/sec

Values for AREA, BWGT, THR2, DOTM2, AREA2, and PRESO do not have to be punched on the card as this data is not required for the given rocket.

In card #46, only the following need be punched:

TIM(1) (4.0), end of Phase I, seconds

TIM(2) and TIM(3) are not required as Phases II and III are not being computed.

DWRT(1), DWRT(2), and DWRT(4) are left blank as output is to be printed for every integration increment, Δt.

CKD is left blank as table values of drag coefficients are presented.

FACT is left blank as a form factor equal to 1.0 is required; however, a value of 1.0 may be punched for FACT.

Data presented for key-punching onto cards #44, #45 and #46 may be seen on the input sheet on page 31. An illustration of this data on cards may be seen on page 33.

INPUT FOR TRAJECTORY PROGRAM TEA-2

For Ballistic and Rocket Trajectory

QUADRANT ELEVATION, degrees	45.
INITIAL VELOCITY, ft/sec	1000.
TOTAL WEIGHT, lbs	100.
DIAMETER, inches	11.0
INITIAL RANGE, ft	
INITIAL ALTITUDE, ft	
INITIAL TIME, sec	
TERMINAL ALTITUDE, ft	

For Rocket Trajectory

BOOSTER THRUST, lbs	100.
BOOSTER BURNING RATE, lb/sec	5.0
BOOSTER NOZZLE AREA, sq. inches	
BOOSTER WEIGHT (EMPTY), lbs	
MAIN STAGE THRUST, lbs	
MAIN STAGE BURNING RATE, lbs/sec	
MAIN STAGE NOZZLE AREA, sq. inches	
STATIC TEST ATMOS. PRESSURE, lb $sec^2 ft^{-4}$	

For Rocket Trajectory

TIME AT END OF PHASE I, sec	4.0
TIME AT END OF PHASE II, sec	
TIME AT END OF PHASE III, sec	
OUTPUT SPACING FOR PHASES I & III, sec	
OUTPUT SPACING FOR PHASE II, sec	
OUTPUT SPACING FOR PHASE IV, sec	
CONSTANT DRAG COEFFICIENT	
FORM FACTOR	

For Ballistic Trajectory

OUTPUT SPACING, sec	
FORM FACTOR	
CONSTANT DRAG COEFFICIENT	

Table Data Sheet for TEA-2 Trajectory Program

Name ____________________ Date ____________________

Argument mach no.	0.0	0.6	0.8	0.9	1.0.	1.1	1.2	1.3
	1.4	1.5	1.6	1.8	2.0	2.5	3.0	4.0
	5.0							
Function C_D (Phase I)	.13	.13	.14	.175	.255	.277	.284	.275
	.26	.241	.228	.204	.186	.161	.153	.141
	.132							

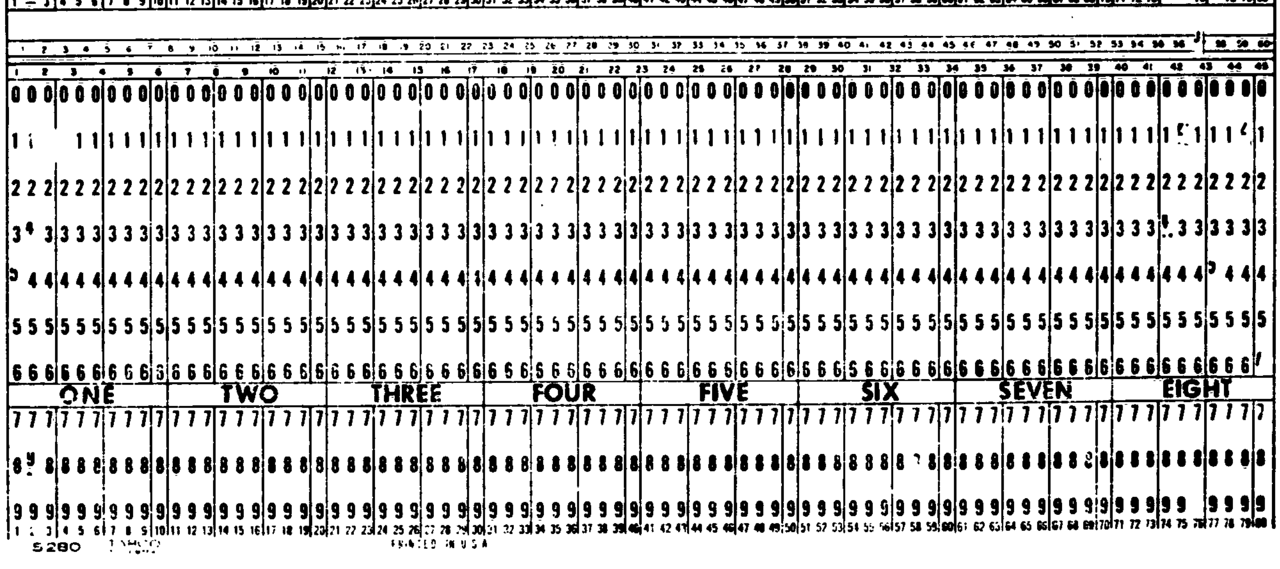
Argument mach no.	0.0	0.6	0.8	0.9	1.0	1.1	1.2	1.3
	1.4	1.5	1.6	1.8	2.0	2.5	3.0	4.0
	5.0							
Function C_D (Phase IV)	.13	.13	.14	.175	.255	.277	.284	.275
	.26	.241	.228	.204	.186	.161	.153	.141
	.132							

Argument								
Function								

Argument								
Function								

Argument								
Function								

CARD 1	1001122210101							
CARD 2	0.0	.6	.8	.9	1.0	1.1	1.2	1.3
CARD 3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	4.0
CARD 4	5.0	7.0	9.0	11.0	15.0	30.0		
CARD 5	.13	.13	.14	.175	.255	.277	.284	.275
CARD 6	.26	.241	.228	.204	.186	.161	.153	.141
CARD 7	.132	.122	.112	.102	.10	.10		
CARD 8	0.0	.6	.8	.9	1.0	1.1	1.2	1.3
CARD 9	1.4	1.5	1.6	1.8	2.0	2.5	3.0	4.0
CARD 10	5.0	7.0	9.0	11.0	15.0	30.0		
CARD 11	.13	.13	.14	.175	.255	.277	.284	.275
CARD 12	.26	.241	.228	.204	.186	.161	.153	.141
CARD 13	.132	.122	.112	.102	.10	.10		
CARD 44	45.	1000.0	100.0	11.0				
CARD 45	100.	5.0						
CARD 46	4.							



2. Ballistic Trajectory Input

Card #1 contains the same values as in card #1 described for rocket trajectories on page 24.

Card #2 to #7 contain the drag coefficient table. Card #2 to #4 are for the table mach numbers, and cards #5 to #7 are for the corresponding drag coefficients. This table is not required when KDCON = 2 in card #1. For this case, the constant drag coefficient must be presented in card #9.

Either k_D or C_D drag coefficients may be used, and values employed must be specified by the control parameter KD in card #1.

Card #8 ...... THDEGO, VO, WGT, DIA, XO, YO, TO, YF

where:

THDEGO ..... initial quadrant elevation, degrees
VO .... initial velocity, feet per second
WGT .... total weight, pounds
DIA .... missile diameter, inches
XO .... initial range, feet
YO .... initial altitude, feet
TO .... time at start of trajectory, seconds
YF .... terminal altitude, feet

Card #9 ..... DWRT(4), FACT, CKD

where:

DWRT(4) ..... output spacing, seconds
FACT .... form factor
CKD .... constant drag coefficient

ALL DATA PRESENTED ON CARDS #2 TO #9 MUST HAVE DECIMAL POINTS AND HAVE FIELD WIDTHS OF NINE COLUMNS.

Ballistic Trajectory Sample Input Case

The following conditions are for the sample ballistic trajectory:

1. C_D drag coefficient table
2. initial velocity = 1000. fps
3. quadrant elevation = 45.0°
4. weight = 100.0 lbs
5. diameter = 11.0 inches
6. output spacing every Δt
7. form factor = 1.0
8. ten integration steps per estimate
9. 0.5% error limit

For the above conditions, the following data must be presented in card #1:

NEXT(1)	= 0,	do not compute Phase I
NEXT(2)	= 0,	do not compute Phase II
NEXT(3)	= 0,	do not compute Phase III
NEXT(4)	= 1,	compute Phase IV
KDCON	= 1,	variable drag coefficients presented
NTHRST	= 2,	no thrust
KD	= 2,	using C_D drag coefficients
M	= 2,	not presenting weight tables
NXY	= 2,	step selection based on altitude error criteria
JN	= 01,	compute one trajectory
NJR	= 01,	trajectory numbered 1
NS	= 10,	integration steps per estimate
GAMMA	= 0.5,	percent acceptable error

An illustration of the presentation of data on card #1 may be seen on page 37.

The drag coefficient table for the ballistic trajectory as presented for key-punching is shown on the table data sheet on page 38. This table is punched onto cards #2 to #7, and the punched cards are illustrated on page 39.

The following data must be punched on card #8:

THDEGO (45.0), quadrant elevation, degrees
VO (1000.0), initial velocity, fps
WGT (100.0), missile weight, lbs
DIA (11.0), missile diameter, inches

Values of XO, YO, TO, and YF do not have to be punched on the card for this case as they equal zero.

Card #9 may be left blank. No value is required for CKD as a table of drag coefficients is to be presented. No value is required for DWRT(4) as output is required every integration step, Δt. No value is required for FACT as the form factor must equal one.

Data presented for key-punching onto cards #8 and #9 may be seen on the input sheet on page 37 . An illustration of this data on cards may be seen on page 38 .

INPUT FOR TRAJECTORY PROGRAM TEA-2

For Ballistic and Rocket Trajectory

QUADRANT ELEVATION, degrees	45.0
INITIAL VELOCITY, ft/sec	1000.
TOTAL WEIGHT, lbs	100.
DIAMETER, inches	11.
INITIAL RANGE, ft	
INITIAL ALTITUDE, ft	
INITIAL TIME, sec	
TERMINAL ALTITUDE, ft	

For Rocket Trajectory

BOOSTER THRUST, lbs	
BOOSTER BURNING RATE, lb/sec	
BOOSTER NOZZLE AREA, sq. inches	
BOOSTER WEIGHT (EMPTY), lbs	
MAIN STAGE THRUST, lbs	
MAIN STAGE BURNING RATE, lbs/sec	
MAIN STAGE NOZZLE AREA, sq. inches	
STATIC TEST ATMOS. PRESSURE, lb $sec^2 ft^{-4}$	

For Rocket Trajectory

TIME AT END OF PHASE I, sec	
TIME AT END OF PHASE II, sec	
TIME AT END OF PHASE III, sec	
OUTPUT SPACING FOR PHASES I & III, sec	
OUTPUT SPACING FOR PHASE II, sec	
OUTPUT SPACING FOR PHASE IV, sec	
CONSTANT DRAG COEFFICIENT	
FORM FACTOR	

For Ballistic Trajectory

OUTPUT SPACING, sec	
FORM FACTOR	
CONSTANT DRAG COEFFICIENT	

Table Data Sheet for TEA-2 Trajectory Program

Name ________________ Date ______________

Argument	0.0	0.6	0.8	0.9	1.0	1.1	1.2	1.3
mach no.	1.4	1.5	1.6	1.8	2.0	2.5	3.0	4.0
	5.0	7.0	9.0	11.0	15.0	30.		
Function	.13	.13	.14	.175	.255	.277	.284	.275
C_D	.26	.241	.228	.204	.186	.161	.153	.141
	.132	.122	.112	.102	.10	.10		

Argument								
Function								

Argument								
Function								

Argument								
Function								

Argument								
Function								

BALLISTIC TRAJECTORY INPUT

CARD 1: 000112222010110 0.5

CARD 2: 0.0 .6 .8 .9 1.0 1.1 1.2 1.3

CARD 3: 1.4 1.5 1.6 1.8 2.0 2.5 3.0 4.0

CARD 4: 3.0 7.0 9.0 11.0 15.0 30.0

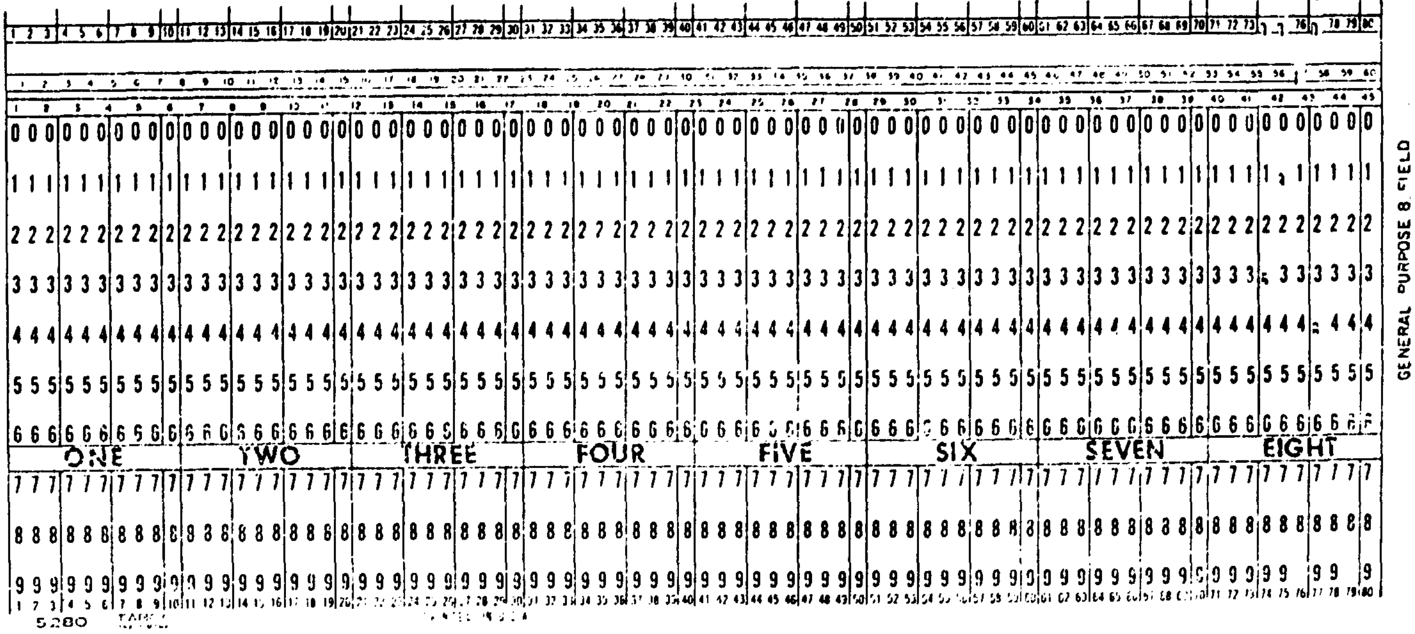
CARD 5: .13 .13 .14 .175 .255 .277 .284 .275

CARD 6: .26 .241 .228 .204 .186 .161 .153 .141

CARD 7: .132 .122 .112 .102 .10 .10

CARD 8: 45.0 1000.0 100.0 11.0

CARD 9



RESULTS AND DISCUSSION

The TEA-2 digital computer program for the IBM 709 has been tested on a substantial number of trajectories. This program has always selected reasonable time steps. Time steps for TEA-2 are dependent upon the accuracy specified by the user as well as the problem characteristics. That is, problem initial conditions, body drag characteristics, and vehicle thrust characteristics are important parameters and exert a significant influence on the time increments used by TEA-2.

Analysis of output for typical ballistic flights has shown that the estimated range error is somewhat more than 10 times too conservative. That is, an estimated range error of 1% by TEA-2 would correspond to an actual range error less than 0. 1%.

Four typical trajectories were computed in order to establish timing estimates. The program limitation to a maximum angular change per time step of two degrees places an upper limit on computational speed. This limitation explains the small difference in computation time between the 5% and the 0. 5% ballistic trajectories presented in Table 1.

The first ballistic trajectory corresponds to the sample input, and sample output presented in Section C and Appendix II, respectively. A similar trajectory and the time steps required for less than 0. 5% range error are presented in Figure 2. Actual range error is probably much less than 0. 5% in this case. Figure 2 illustrates that maximum time steps were taken from 10. 483 seconds to 33. 837 seconds.

Figure 3 illustrates a typical rocket trajectory which is similar to the ballistic case. Initial step size is much smaller in this case. The average step size is 25% less than for the ballistic case.

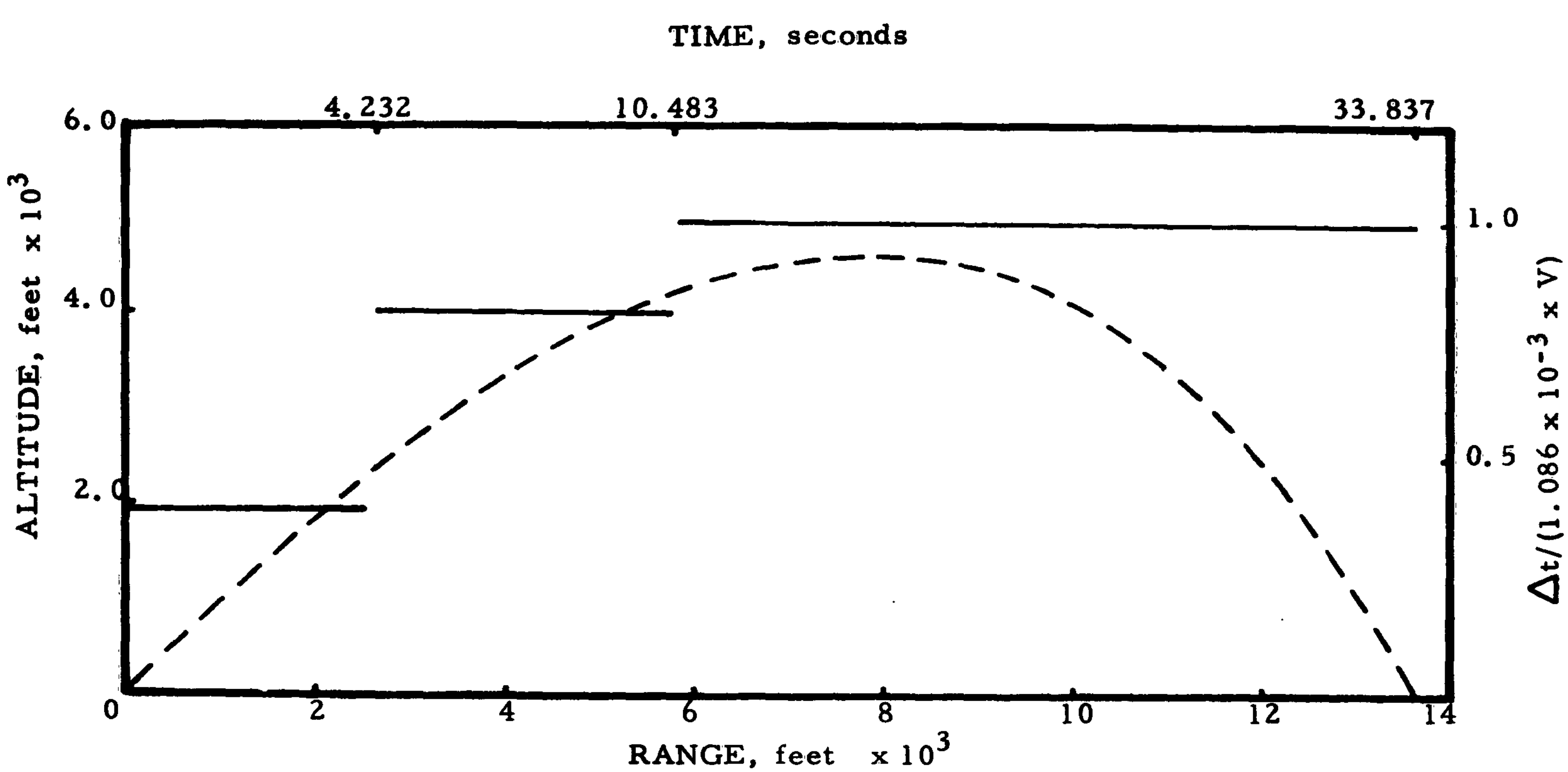
Table 1 lists two typical ballistic trajectories and two typical rocket trajectories. Both 5% and 0.5% range accuracies were requested for these trajectories. These problems were run both on TEA-2 and on the "standard" two-degree of freedom program which utilizes a fixed time increment which is an input quantity. This input time increment is based on the user's experience. The values reported in the table are typical of those presently used. Under the assumption that the four cases presented with flight history constitute the "average" problems, the use of TEA-2 constitutes a time savings of 2.1 and 1.7 for the 5% and 0.5% accuracy specifications, respectively. This will result in a cost reduction of 35,900 dollars per year or 28,500 dollars per year for the respective required accuracies. Cost savings are based on current usage of 19 hours per month at a cost of 300 dollars per hour. The 5% requested accuracy should be the most widely used case.

Computation time is approximately 2.5 to 3.5 times faster than the "standard" program for the usual ballistic and rocket trajectories. However, TEA-2 speed is restricted by output time which results in the computation times reflected in Table 1. Computation times including terminal output and error table output only are also included in Table 1. Clearly, detailed flight history should be printed out only when absolutely necessary.

Very high drag bodies such as parachutes cannot be treated by this program.

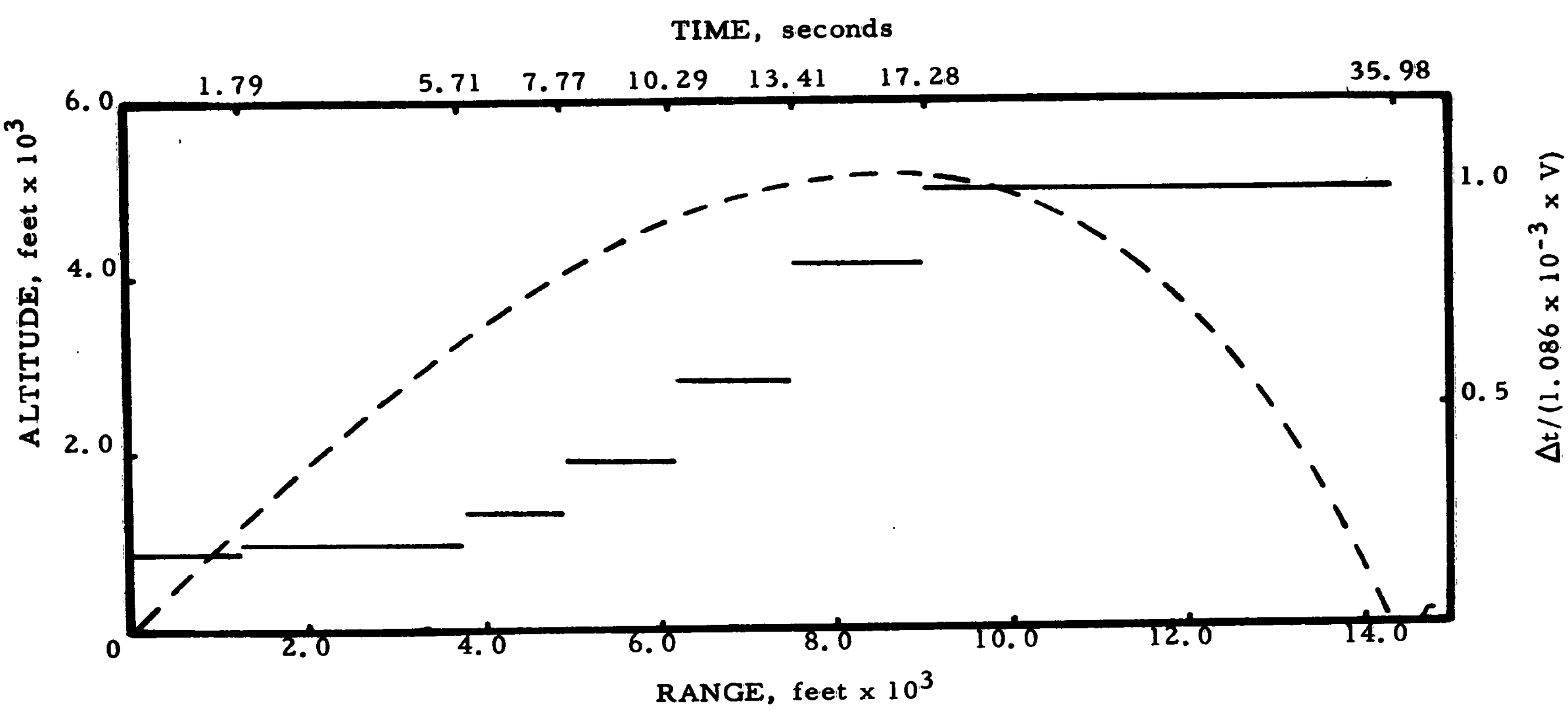
Type of Flight	COMPUTATION TIME (Seconds)				
	Flight History Output		Terminal Output Only		Standard Program (RKTCN)
	TEA-2 5%	TEA-2 0.5%	TEA-2 5%	TEA-2 0.5%	
Ballistic Trajectory 1	12.0	13.8	9.6	10.2	29.4
Ballistic Trajectory 2	16.8	19.2	12.6	15.0	36.0
Rocket Trajectory 1	15.6	22.2	10.8	16.2	33.0
Rocket Trajectory 2	21.0	28.8	15.6	21.0	39.6

TABLE 1
Comparison of Computation Time



BALLISTIC TRAJECTORY
LESS THAN 0.5% RANGE ERROR REQUESTED
AVERAGE TIME INCREMENT = 0.44 SECONDS

Figure 2



ROCKET TRAJECTORY
LESS THAN 0.5% RANGE ERROR REQUESTED
AVERAGE TIME INCREMENT = 0.33 SECONDS

Figure 3

APPENDIX I

FØRTRAN LISTING

THE TEA-2 SØURCE PRØGRAM IS WRITTEN IN FØRTRAN LANGUAGE. IT REQUIRES SINE, CØSINE, SQUARE RØØT, AND EXPØNENTIAL LIBRARY SUBRØUTINES. ALSØ REQUIRED ARE THE ATMØSPHERE (ATMØS) AND TABLE SEARCH (TABLE) SUB-PRØGRAMS WHICH ARE LISTED AFTER THE MAIN PRØGRAM. A FØRTRAN LISTING ØF THE MAIN PRØGRAM FØLLØWS.

```
CTEA-2     TWØ-DIMENSIØNAL, TWØ-STAGE RØCKET ØR BALLISTIC TRAJECTØRY
C                    (WITH 1959 ARDC STANDARD ATMØSPHERE)
C                       CØMPUTING AND ANALYSIS SECTIØN
C
C          TRAJECTØRY ERRØR ANALYSIS ** TWØ DEGREES ØF FREEDØM
C     ANALYSIS ** LT. H.J. KØPP,PH.D * EXTENSIØN 72264
C     PRØGRAMMING ** MR. J.N. NIELSEN * EXTENSIØN 73230
C
C        THIS PRØGRAM SELECTS THE INTEGRATIØN STEP LENGTH SUCH THAT THE
C     ERRØR IN THE TØTAL RANGE IS LESS THAN A PRESCRIBED AMØUNT AND/ØR
C     THE ERRØR IN ALTITUDE IS LESS THAN A PRESCRIBED AMØUNT.
C
      DIMENSIØN  TABL1(24,2),TABL2(24,2),TABL3(24,2),TRST1(24,2),
     1           TRST2(24,2),TMAS(24,2),TMAS2(24,2),NEXT(5),TIM(3),
     2           DWRT(4),XDØT(3),YDØT(3),VDØT(3),THDØT(3)
      DIMENSIØN  TN(200),XN(200),YN(200),THN(200),VN(200),EX(200),
     1           EY(200),ET(200),EV(200)
      CØMMØN RHØ,PRSB,GRAV,CMACH,KDCØN,NATMØS
C
C                              FØRMAT STATEMENTS
C
  100 FØRMAT (9I1,3I2,F5.0)
  101 FØRMAT (8F9.0)
  104 FØRMAT (7F10.0)
  105 FØRMAT (8F5.0)
  106 FØRMAT (1H1,43X,31H CØMPUTING AND ANALYSIS SECTIØN/27X,66H TWØ DIM
     1ENSIØNAL, TWØ-STAGE RØCKET ØR BALLISTIC TRAJECTØRY PRØGRAM/41X,37H
     2 *WITH 1959 ARDC STANDARD ATMØSPHERE*//)
  108 FØRMAT (8H1    TIME,7X,6H RANGE,6X,9H ALTITUDE,4X,9H VELØCITY,4X,
     1          6H THETA,5X,5H MACH,5X,3H KD,7X,7H THRUST,6X,7H WEIGHT,5X,
     2          8H DENSITY)
  109 FØRMAT (8H     TIME,7X,6H RANGE,6X,9H ALTITUDE,4X,9H VELØCITY,4X,
     1          6H THETA,5X,5H MACH,5X,3H KD,7X,7H THRUST,6X,7H WEIGHT,5X,
     2          8H DENSITY)
  110 FØRMAT (14X,8H X-DERIV,5X,8H Y-DERIV,5X,8H V-DERIV,3X,9H TH-DERIV,
     1          38X,5H DRAG,6X,9H PRESSURE//)
  111 FØRMAT (F9.3,2F14.3,F12.3,F11.3,F10.4,F9.4,F14.3,F11.3,E15.5)
  112 FØRMAT (9X,2F14.3,F12.3,F11.3,33X,F11.3,E15.5)
  113 FØRMAT (46X,20H END  ØF  TRAJECTØRY,I3)
  115 FØRMAT (1H1,46X,26H CØASTING  ØF  MAIN  STAGE)
  114 FØRMAT (38X,44H ACCELERATIØN  ØF  BØØSTER  AND  MAIN  STAGE)
  116 FØRMAT (1H1,44X,30H ACCELERATIØN  ØF  MAIN  STAGE)
  117 FØRMAT (1H1,45X,29H FREE-FLIGHT  ØF  MAIN  STAGE)
  118 FØRMAT (1H1,36X,59H ACCELERATIØN  ØF  BØØSTER  AND  MAIN  STAGE Ø
     1FF  LAUNCHER)
  121 FØRMAT (47X,26H CØASTING  ØF  MAIN  STAGE)
  122 FØRMAT (45X,30H ACCELERATIØN  ØF  MAIN  STAGE)
  123 FØRMAT (46X,29H FREE-FLIGHT  ØF  MAIN  STAGE)
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  126 FØRMAT (1H1,56X,6H TEA-2/34X,52H TRAJECTØRY ERRØR ANALYSIS ** TWØ
     1DEGREES ØF FREEDØM//)
  127 FØRMAT (5X,71H ANALYSIS ** DR. H.J. KØPP * EXTENSIØN 72264  *  REP
     1ØRT TØ BE PUBLISHED/5X,50H PRØGRAMMING ** MR. J.N. NIELSEN * EXTEN
     2SIØN 73230//)
  128 FØRMAT (8X,103H THIS PRØGRAM SELECTS THE INTEGRATIØN STEP LENGTH S
     1UCH THAT THE ERRØR IN THE TØTAL RANGE IS LESS THAN A/5X,111H PRESC
     2RIBED AMØUNT AND/ØR THE ERRØR IN ALTITUDE IS LESS THAN A PRESCRIBE
     3D AMØUNT.  THE ERRØR CRITERIØN USED FØR/5X,23H THIS TRAJECTØRY WAS
     4 **)
  129 FØRMAT (8X,85H THE ABØVE CØNDITIØN WAS SATISFIED, AND THE CØMPUTED
     1 ERRØR IN TØTAL RANGE IS EQUAL TØ,E15.8,9H PERCENT.//)
  130 FØRMAT (8X,93H THE ABØVE CØNDITIØN CØULD NØT BE SATISFIED, AS THE
     1CØMPUTED ERRØR IN TØTAL RANGE IS EQUAL TØ/6X,E15.8,9H PERCENT.//)
  131 FØRMAT (28X,39H THE ERRØR IN TØTAL RANGE IS LESS THAN ,F6.2,
     1         9H PERCENT.//)
  132 FØRMAT (28X,45H THE ERRØR IN TERMINAL ALTITUDE IS LESS THAN ,F6.2,
     1        29H PERCENT ØF MAXIMUM ALTITUDE.//)
  133 FØRMAT (50X,22H FINAL ERRØR ESTIMATES/57X,6H *****)
  134 FØRMAT (5X,7H RANGE ,E15.8,14H  RANGE ERRØR ,E15.8,11H  ALTITUDE ,
     1         E15.8,17H  ALTITUDE ERRØR ,E15.8/5X,7H THETA ,E15.8,
     2        14H  THETA ERRØR ,E15.8,11H  VELØCITY ,E15.8,
     3        17H  VELØCITY ERRØR ,E15.8//)
  135 FØRMAT (51X,19H INITIAL CØNDITIØNS/57X,6H *****)
  136 FØRMAT (8X,7H RANGE ,E15.8,3X,10H ALTITUDE ,E15.8,3X,7H THETA ,
     1         E15.8,3X,10H VELØCITY ,E15.8//)
  137 FØRMAT (47X,25H TABLE ØF ERRØR ESTIMATES/56X,6H *****)
  138 FØRMAT (1H1,40X,37H TABLE ØF ERRØR ESTIMATES (CØNTINUED)/56X,
     1         6H *****)
  139 FØRMAT (11X,5H TIME,17X,6H RANGE,16X,9H ALTITUDE,15X,6H THETA,16X,
     1         9H VELØCITY/30X,12H RANGE ERRØR,10X,15H ALTITUDE ERRØR,9X,
     2         12H THETA ERRØR,10X,15H VELØCITY ERRØR//)
  140 FØRMAT (6X,E15.8,4(8X,E15.8)/21X,4(8X,E15.8)//)
  141 FØRMAT (5X,24H IMPACT BETWEEN RANGE ØF,E15.8,9H FEET AND,E15.8,
     1         5H FEET//5X,36H IMPACT CENTER PREDICTED AT RANGE ØF,E15.8,
     2         21H FEET AND ALTITUDE ØF,E15.8,5H FEET)
  142 FØRMAT (8X,90H THE ABØVE CØNDITIØN CØULD NØT BE SATISFIED, AS THE
     1CØMPUTED ERRØR IN ALTITUDE IS EQUAL TØ/6X,E15.8,29H PERCENT ØF MAX
     2IMUM ALTITUDE.//)
  143 FØRMAT (8X,91H THE ABØVE CØNDITIØNS CØULD NØT BE SATISFIED. THE CØ
     1MPUTED ERRØR IN TØTAL RANGE IS EQUAL TØ/3X,E15.8,56H PERCENT, AND
     2THE CØMPUTED ERRØR IN ALTITUDE IS EQUAL TØ,E15.8,29H PERCENT ØF MA
     3XIMUM ALTITUDE.//)
  144 FØRMAT (8X,82H THE ABØVE CØNDITIØN WAS SATISFIED, AND THE CØMPUTED
     1 ERRØR IN ALTITUDE IS EQUAL TØ,E15.8,11H PERCENT ØF/5X,17H MAXIMUM
     2 ALTITUDE//)
  145 FØRMAT (8X,83H THE ABØVE CØNDITIØNS WERE SATISFIED. THE CØMPUTED E
     1RRØR IN TØTAL RANGE IS EQUAL TØ,E15.8,9H PERCENT,/4X,48H  AND THE
     2CØMPUTED ERRØR IN ALTITUDE IS EQUAL TØ,E15.8,29H PERCENT ØF MAXIMU
     3M ALTITUDE.//)
    1 READ INPUT TAPE 2,100, NEXT(1),NEXT(2),NEXT(3),NEXT(4),KDCØN,
     1                        NTHRST,KC,M,NXY,JN,NJR,NS,GAMMA
C
      GØ TØ (70,74),KDCØN
C
C                             READ STATEMENTS
C
   70 IF (NEXT(1)) 72,72,71
   71 READ INPUT TAPE 2,101, TABL1
   72 READ INPUT TAPE 2,101, TABL2
```

```
      IF (NEXT(3)) 74,74,73
   73 READ INPUT TAPE 2,101, TABL3
   74 GØ TØ (75,2),NTHRST
   75 IF (NEXT(1)) 77,77,404
  404 READ INPUT TAPE 2,101, TRST1
      GØ TØ (76,77),M
   76 READ INPUT TAPE 2,101, TMAS
   77 IF (NEXT(3)) 2,2,78
   78 READ INPUT TAPE 2,101, TRST2
      GØ TØ (79,2),M
   79 READ INPUT TAPE 2,101, TMAS2
    2 READ INPUT TAPE 2,101, THDEGØ,VØ,WGT,DIA,XØ,YØ,TØ,YF
      IF (NEXT(1)) 405,405,406
  405 IF (NEXT(3)) 407,407,406
  406 READ INPUT TAPE 2,101, THRST,DØTM,AREA,BWGT,THR2,DØTM2,AREA2,PRESØ
      READ INPUT TAPE 2,101, TIM,DWRT(1),DWRT(2),DWRT(4),CKD,FACT
      GØ TØ 408
  407 READ INPUT TAPE 2,101, DWRT(4),FACT,CKD
  408 WRITE ØUTPUT TAPE 3,106
C
C                CØNVERSIØN ØF INITIAL VALUES TØ CØRRECT UNITS
C
       T = TØ
       X = XØ
       Y = YØ
       V = VØ
       THET = THDEGØ/57.29577
      THETØ = THET
      RWGT = WGT
      RMASS = WGT/32.17405
      RMASSØ = RMASS
      DØTM = DØTM/32.17405
      DØTM2 = DØTM2/32.17405
      BMASS = BWGT/32.17405
      CMACH = 0.0
      DIA = DIA/12.0
      DIASQ = DIA*DIA
      AREA = AREA/144.
      PRESØ = 144.0*PRESØ
      DWRT(3) = DWRT(1)
      IF (NS) 1001,1001,1002
 1001 NS = 10
 1002 IF (GAMMA) 1003,1003,1004
 1003 GAMMA = 5.0
 1004 IF (FACT) 1005,1005,1006
 1005 FACT = 1.0
 1006 GØ TØ (5092,5090),NTHRST
 5090 TRST1(1,1) = TØ
      TRST1(1,2) = THRST
      TRST2(1,1) = TIM(2)
      TRST2(1,2) = THR2
      DELTHR = 0.1*TIM(1)
      DELTRS = 0.1*(TIM(3)-TIM(2))
      DØ 5091 I = 2,24
      TRST1(I,1) = TRST1(I-1,1) + DELTHR
      TRST1(I,2) = THRST
      TRST2(I,1) = TRST2(I-1,1) + DELTRS
      TRST2(I,2) = THR2
 5091 CØNTINUE
C
```

```
C          INITIAL VALUES FØR ERRØR ANALYSIS RØUTINE
C
 5092 AXR = 1.9E-7
      AYR = 1.9E-7
      ATR = 2.6E-7
      AVTL = 0.0
      ATTL = 0.0
      AXTL = 0.0
      AYTL = 0.0
      EXLN = 0.0
      EYLN = 0.0
      EVLN = 0.0
      ETLN = 0.0
      EXNN = 0.0
      EYNN = 0.0
      EVNN = 0.0
      ETNN = 0.0
      EVN = 0.0
      ETN = 0.0
      EXN = 0.0
      EYN = 0.0
      YØU = 0.0
      DLEY = 0.0
      DTØNE = 0.0
      TIME = TIM(1)
      NEG = 0
      NPØS = 0
      NA = 0
      NB = 0
      NTERM = 1
      NEST = 1
      NN = 2
      CNN = NN
      NØ = 1
      NSTEP = 1
      YMAX = Y
      IF (THET) 5991,5991,5992
 5991 JMP = 2
      GØ TØ 5993
 5992 JMP = 1
 5993 JUMP = 1
      GAM2 = .01*GAMMA
       KPHASE = 1
      DELT = V/32174.05
      DTH = DELT
      EPS = 1.0
      KU = 1
      KUK = 1
      ASSIGN 7007 TØ KAP
      ASSIGN 42 TØ NWRT
      GØ TØ (6001,6002,6001),NXY
 6001 NNXY = 1
      GØ TØ 6003
 6002 NNXY = 2
 6003 ASSIGN 602 TØ NERR
C
      GØ TØ (80,81),KD
   80 CF = FACT
      GØ TØ 82
   81 CF = 0.39269908*FACT
```

```
   82 J = 1
      NNN = 1
      NATMØS = 1
   88 N = 1
      LINE = 2
      ASSIGN 64 TØ NCNT
C
C                         INTEGRATIØN CØNSTANTS
C
      C1 = 0.62653829 * DELT
      C2 = -0.55111241 * DELT
      C3 = 1.4072559 * DELT
      C4 = -0.48268182 * DELT
      EPSLØN = 0.001 * DELT
      WRIT = T-EPSLØN
C
      IF (NEXT(N))409,409,410
  409 NNN = 2
      GØ TØ 50
  410 WRITE ØUTPUT TAPE 3,114
   61 WRITE ØUTPUT TAPE 3,109
      WRITE ØUTPUT TAPE 3,110
C
    3 DØ 45 K = 1,3
      GØ TØ NCNT,(62,64)
   62 WRITE ØUTPUT TAPE 3,108
      WRITE ØUTPUT TAPE 3,110
      ASSIGN 64 TØ NCNT
      CØNTINUE
C
   64 CALL ATMØS(Y,V)
      GØ TØ (10,29),KDCØN
   10 GØ TØ (17,25,21,25),N
C
C                      TABLE LØØK-UPS FØR DRAG CØEFFICIENT
C
C                        DURING FIRST BURNING STAGE
C
   17 CALL TABLE(TABL1,CMACH,CKD,24)
      GØ TØ 29
C
C                       DURING SECØND BURNING STAGE
C
   21 CALL TABLE(TABL3,CMACH,CKD,24)
      GØ TØ 29
C
C                              DURING FREE-FLIGHT
C
   25 CALL TABLE(TABL2,CMACH,CKD,24)
      CØNTINUE
   29 GØ TØ (95,96),NTHRST
   95 GØ TØ (200,96,300,96),N
C
C                   THRUST TABLE FØR FIRST BURNING STAGE
C
  200 CALL TABLE(TRST1,T,THRST,24)
      GØ TØ (18,96,96),K
   18 GØ TØ (19,96),M
   19 CALL TABLE(TMAS,T,RMASS,24)
      RMASS = RMASS/32.17405
```

```
      GØ TØ 96
C
C                 THRUST TABLE FØR SECØND BURNING STAGE
C
  300 CALL TABLE(TRST2,T,THRST,24)
      GØ TØ (22,96,96),K
   22 GØ TØ (23,96),M
   23 CALL TABLE(TMAS2,T,RMASS,24)
      RMASS = RMASS/32.17405
      CØNTINUE
C
C                              DERIVATIVES
C
   96 CCD = CF*CKD
      DRAG = CCD*RHØ*V*V*DIASQ
      DØM = DRAG/RMASS
      CCØS = CØSF(THET)
      CSIN = SINF(THET)
      XDØT(K) = V*CCØS
      YDØT(K) = V*CSIN
      GØ TØ (501,502,501,502),N
  501 PRESS = PRSB*ARGU
      THRUST = THRST + AREA*(PRESØ-PRESS)
      VDØT(K) = (THRUST/RMASS) - GRAV*CSIN - DØM
      RWGT = 32.17405*RMASS
      GØ TØ 503
  502 VDØT(K) = -GRAV*CSIN - DØM
  503 THDØT(K) = -GRAV*CCØS/V
C
   34 GØ TØ (35,43,44),K
   35 GØ TØ NWRT,(36,39,41,42)
   36 IF(DWRT(N)) 37,37,38
   37 ASSIGN 41 TØ NWRT
      GØ TØ 41
   38 ASSIGN 39 TØ NWRT
      CØNTINUE
   39 IF(WRIT-T) 40,40,42
   40 WRIT = T+DWRT(N)-EPSLØN
   41 THDEG = 57.29577*THET
      THDDT = 57.29577*THDØT(1)
      WRITE ØUTPUT TAPE 3,111, T,X,Y,V,THDEG,CMACH,CCD,THRUST,RWGT,RHØ
      WRITE ØUTPUT TAPE 3,112, XDØT(1),YDØT(1),VDØT(1),THDDT,DRAG,PRESS
      LINE = LINE + 1
      IF (28-LINE) 63,63, 42
   63 ASSIGN 62 TØ NCNT
      LINE = 1
      CØNTINUE
C
C                 MØDIFIED THIRD ØRDER RUNGE-KUTTA INTEGRATIØN
C
   42 X = X + C1*XDØT(K)
      Y = Y+C1*YDØT(K)
      V = V+C1*VDØT(K)
      THET = THET+C1*THDØT(K)
      GØ TØ 45
C
   43 X = X + C2*XDØT(K)
      Y = Y+C2*YDØT(K)
      V = V+C2*VDØT(K)
      THET = THET+C2*THDØT(K)
```

```
      GØ TØ 45
C
   44 X = X + C3*XDØT(K-1) + C4*XDØT(K)
      Y = Y+ C3*YDØT(K-1)+C4*YDØT(K)
      V = V+ C3*VDØT(K-1)+C4*VDØT(K)
      THET = THET+C3*THDØT(K-1)+C4*THDØT(K)
      T = T+DELT
      RMASS = RMASS-DØTM*DELT
   45 CØNTINUE
C
C           H.KØPP ERRØR ANALYSIS RØUTINE (SIMPLIFIED)
C
      NSTEP = NSTEP + 1
      GØ TØ (599,599,92,92),KU
  599 IF (NN - NSTEP) 601,601,92
  601 GØ TØ (1311,92),NTERM
 1311 IF (Y-YØU) 92,1312,1312
 1312 GØ TØ NERR,(602,603,605)
C
  602 ASSIGN 603 TØ NERR
      NSTEP = NN - 1
      VV = V
      THTH = THET
      XX = X
      YY = Y
      RMT = RMASS
      TT = T
      DDTT = DELT
      GØ TØ 3
  603 ASSIGN 605 TØ NERR
      ASSIGN 42 TØ NWRT
      NSTEP = NN - 2
      DH = DELT
      VH = V
      THH = THET
      XH = X
      YH = Y
C
C         CØMPUTATIØN ØF PRØPAGATIØN ERRØR CØEFFICIENTS
C
      C = (2.0*RHØ*CCD*DIASQ*V*V)/(RMASS*GRAV)
      CSQ = C*C
      PC = CØSF(THET)
      PS = SINF(THET)
      PS = ABSF(PS)
      A13 = PC*(1.03 - .014*C + .0003*CSQ)
      A14 = 1.2*PS + .28 - .0001*C
      A23 = (PS+.08)*(1.0-.02*C+.0004*CSQ)
      A44 = 1.2*PS + .21 + .0003*C
      PCP7 = PC + .07
      A24 = A44*PCP7
      A33 = .019 +C*(-1.1 + C*(.04 - .0002*C))
      A34 = (PC+.1)*(1.02+.018*C)
      A43 = PCP7*(1.4-.02*C+.0003*CSQ)
C
      DELTA = A33*A44 - A34*A43
      AVR = (THRUST/RWGT)*2.9E-7 + C*3.3E-7 + 2.0E-7
      AFT = A33-A44
      ARGM = AFT*AFT + 4.0*A34*A43
      EPS = 0.5*(A33 + A44 + SQRTF(ARGM))
```

```
      GKK = (EPS-A33)/A34
      EPS = GRAV*EPS/V
       V = VV
       THET = THTH
       X = XX
       Y = YY
       T = TT
       RMASS = RMT
      DELT = 0.5*DELT
C
  604 C1 = 0.62653829*DELT
      C2 = -0.55111241*DELT
      C3 = 1.4072559*DELT
      C4 = -0.48268182*DELT
       GØ TØ 3
  605 EH = EPS*DH
      ASSIGN 602 TØ NERR
      CLAM = 1.0 + EH
      EHØ2 = 0.5*EH*EH
C
C       CØMPUTATIØN ØF TRUNCATIØN ERRØR CØEFFICIENTS
C
      A = ABSF(VH-V) - DH*((THRUST/RMASS)*11.0E-8 + GRAV*7.0E-8 + DØM*
     1    19.0E-8)
      IF (A) 5001,5002,5002
 5001 A = 0.0
 5002 B = ABSF(THH-THET) - (GRAV*DH*1.0E-8)/V
      IF (B) 5003,5004,5004
 5003 B = 0.0
 5004 DH3 = DH**3
      CC = 1.75*V*DH3
      AVT = (A*ABSF(A33) + VH*A34*B)/CC
      IF (AVT-AVTL) 4002,4001,4001
 4001 AVTL = 0.8*AVT
       GØ TØ 4003
 4002 AVT = AVTL
      AVTL = 0.8*AVT
 4003 CØNTINUE
      ATT = (A43*A + VH*A44*B)/CC
      IF (ATT-ATTL) 4005,4004,4004
 4004 ATTL = 0.8*ATT
       GØ TØ 4006
 4005 ATT = ATTL
      ATTL = 0.8*ATT
 4006 CØNTINUE
      PHB = A44*AVT-A34*ATT
      PHBB = A44*AVR - A34*ATR
      PSB = -A43*AVT + A33*ATT
      PSBB = -A43*AVR + A33*ATR
      EVØV = EVN/VH
      VH4 = VH*DH*DH3
      TH3 = 2.0*DH3
      QQ = 1.1428571/VH4
      DHV7 = DH*V*7.0E-8
      DX2 = ABSF(XH-X) - DHV7
      IF (DX2) 5005,5006,5006
 5005 DX2 = 0.0
 5006 AXT = QQ*DX2
      IF (AXT-AXTL) 4008,4007,4007
 4007 AXTL = 0.8*AXT
```

```
      GØ TØ 4009
 4008 AXT = AXTL
      AXTL = 0.8*AXT
 4009 CØNTINUE
      DY2 = ABSF(YH-Y) - DHV7
      IF (DY2) 5007,5008,5008
 5007 DY2 = 0.0
 5008 AYT = QQ*DY2
      IF (AYT-AYTL) 4011,4010,4010
 4010 AYTL = 0.8*AYT
      GØ TØ 6005
 4011 AYT = AYTL
      AYTL = 0.8*AYT
C
C          TRAJECTØRY ESTIMATES FØR THRUST PHASES
C
 6005 QYK = Y
      QYH = Y
      QXK = X
      QXH = X
      QVK = V
      QVH = V
      QTHK = THET
      QMH = RMASS
      DMDØT = DØTM
      GØ TØ (6006,6026,6008,6028),NEST
 6006 DØ 6366 II = 1,30
      IF (TRST1(II,1) - T) 6366,6166,6266
 6166 QTK = TRST1(II,2)
      QTH = TRST1(II+1,2)
      QMH = TMAS(II+1,2)/32.17405
      CØNT = TRST1(II+1,1) - TRST1(II,1)
      GØ TØ 6466
 6266 QTK = TRST1(II-1,2) + ((T-TRST1(II-1,1))/(TRST1(II,1)-TRST1(II-1,1
     1))) * (TRST1(II,2)-TRST1(II-1,2))
      QTH = TRST1(II,2)
      QMH = TMAS(II,2)/32.17405
      CØNT = TRST1(II,1) - T
      GØ TØ 6466
 6366 CØNTINUE
 6466 KLER = 1
      GØ TØ (6010,6011),M
 6008 DØ 6388 II = 1,30
      IF (TRST2(II,1) - T) 6388,6188,6288
 6188 QTK = TRST2(II,2)
      QTH = TRST2(II+1,2)
      QMH = TMAS2(II+1,2)/32.17405
      CØNT = TRST2(II+1,1) - TRST2(II,1)
      GØ TØ 6488
 6288 QTK = TRST2(II-1,2) + ((T-TRST2(II-1,1))/(TRST2(II,1)-TRST2(II-1,1
     1)))* (TRST2(II,2) - TRST2(II-1,2))
      QTH = TRST2(II,2)
      QMH = TMAS2(II,2)/32.17405
      CØNT = TRST2(II,1) - T
      GØ TØ 6488
 6388 CØNTINUE
 6488 KLER = 2
      DMDØT = DØTM2
      GØ TØ (6010,6011),M
 6010 QM = (QMH-RMASS)/CØNT
```

```
      GØ TØ 6012
 6011 QM = -DMDØT
      QMH = RMASS - DMDØT*CØNT
 6012 QT = (QTH - THRST)/CØNT
      ALPHA = GRAV*SINF(THET)
      BETA = RHØ*DIASQ*QVK*CKD*CF
      AQTØB = ALPHA + QT/BETA
      QEX = BETA/QM
      UN = QM + BETA
      QVH = QTH/BETA - (QMH*AQTØB/UN) + (QVK + AQTØB*RMASS/UN - THRST/
     1        BETA)*((RMASS/QMH)**QEX)
      PT3V = 0.4*(QVH+QVK)*CØNT
      CSKAY = CØSF(QTHK)
      QXK = QXK + PT3V*CSKAY
      QYK = QYK + PT3V*SINF(QTHK)
      QTHK = QTHK - 2.0*GRAV*CSKAY*CØNT/(QVK+QVH)
      QVK = QVH
      II = II + 1
 6013 DØ 6021 I = II,30
      CALL ATMØS(QYK,QVK)
      GØ TØ (6014,6016),KLER
 6014 QTK = QTH
      QTH = TRST1(I,2)
      CØNT = TRST1(I,1) - TRST1(I-1,1)
      TMØUT = TRST1(I,1)
      GØ TØ (6614,6615),KDCØN
 6614 CALL TABLE(TABL1,CMACH,CKD,24)
 6615 GØ TØ (6015,6019),M
 6015 QMK = QMH
      QMH = TMAS(I,2)/32.17405
      GØ TØ 6018
C
 6016 QTK = QTH
      QTH = TRST2(I,2)
      CØNT = TRST2(I,1) - TRST2(I-1,1)
      TMØUT = TRST2(I,1)
      GØ TØ (6616,6617),KDCØN
 6616 CALL TABLE(TABL3,CMACH,CKD,24)
 6617 GØ TØ (6017,6019),M
 6017 QMK = QMH
      QMH = TMAS2(I,2)/32.17405
 6018 QM = (QMH-QMK)/CØNT
      GØ TØ 6020
 6019 QM = -DMDØT
      QMK = QMH
      QMH = QMK - DMDØT*CØNT
 6020 QT = (QTH-QTK)/CØNT
      SNKAY = SINF(QTHK)
      CSKAY = CØSF(QTHK)
      ALPHA = GRAV*SNKAY
      BETA = RHØ*DIASQ*QVK*CKD*CF
      AQTØB = ALPHA + QT/BETA
      UN = QM + BETA
      QEX = BETA/QM
      QVH = (QTH/BETA) - (QMH*AQTØB/UN) + (QVK + (AQTØB*QMK/UN) - (QTK/
     1        BETA))*((QMK/QMH)**QEX)
      PT3V = 0.4*(QVH+QVK)*CØNT
      QXK = QXK + PT3V*CSKAY
      QYK = QYK + PT3V*SNKAY
      QTHK = QTHK - 2.0*GRAV*CSKAY*CØNT/(QVK+QVH)
```

```
      QVK = QVH
      IF (TIME-TMØUT-CØNT*1.0E-4) 6022,6022,6021
 6021 CØNTINUE
 6022 GØ TØ (6024,6028),KLER
 6024 IF (NEXT(2))6028,6028,6025
C
C          TRAJECTØRY ESTIMATE FØR INTERMEDIATE BALLISTIC PHASE
C
 6025 QMK = QMH - BMASS
      CØNT = TIM(2) - TIM(1)
       GØ TØ 6027
 6026 QMK = RMASS
      CØNT = TIM(2) - T
 6027 PT6V = 0.6*QVK*CØNT
      CSKAY = CØSF(QTHK)
      SNKAY = SINF(QTHK)
      QXK = QXK + PT6V*CSKAY
      QYK = QYK + PT6V*SNKAY
      QTHK = QTHK - (GRAV*CSKAY/QVK)*CØNT
      GØ TØ (6627,6628),KDCØN
 6627 CALL TABLE(TABL2,CMACH,CKD,24)
 6628 QVK = QVK + (-GRAV*CSKAY - RHØ*CF*CKD*QVK*QVK*DIASQ/QMK)*CØNT
      KLER = 2
      II = 2
      DMDØT = DØTM2
      QTH = TRST2(II-1,2)
      GØ TØ 6013
C
C          TRAJECTØRY ESTIMATE FØR TERMINAL BALLISTIC PHASE
C
 6028 U = SINF(QTHK)
      QDØTY = QVK*U
      TMTN = (QDØTY + SQRTF(QDØTY*QDØTY + 2.0*GRAV*QYK))/GRAV
      GØ TØ (6129,6128,6129),NXY
 6128 GØ TØ (6129,5014),JMP
 6129 CSK = CØSF(QTHK)
      QDØTX = 0.6*QVK*CSK
      PLUS = 1.0/(QVK*CSK)
      GØ TØ (6227,6228),KDCØN
 6227 CALL ATMØS(QYK,QVK)
      CALL TABLE(TABL2,CMACH,CKD,24)
 6228 BET = 0.0023769*DIASQ*QVK*CKD*CF/32.17405
      SNØCS = U/CSK
      DU = 0.05*(U+0.9995)
      DUØG = DU/32.17405
C
 6229 IF (QYK-5.8E4) 6029,6030,6031
 6029 BØG = BET
       GØ TØ 6043
 6030 BØG = 0.1*BET
       GØ TØ 6043
 6031 IF (QYK-1.0E5) 6030,6032,6033
 6032 BØG = 0.01*BET
       GØ TØ 6043
 6033 IF (QYK-1.6E5) 6032,6034,6035
 6034 BØG = 0.001*BET
       GØ TØ 6043
 6035 IF (QYK-2.3E5) 6034,6036,6037
 6036 BØG = BET*1.0E-4
       GØ TØ 6043
```

```
 6037 IF (QYK-3.0E5) 6036,6038,6039
 6038 BØG = BET*1.0E-6
       GØ TØ 6043
 6039 IF (QYK-4.0E5) 6038,6040,6041
 6040 BØG = BET*1.0E-8
       GØ TØ 6043
 6041 IF (QYK-6.0E5) 6040,6042,6042
 6042 BØG = BET*1.0E-13
C
 6043 AA = BØG*SNØCS + PLUS
      ØMU = (1.0+U)*(1.0-U)
      RØØT = SQRTF(ØMU)
      AU = (RØØT*AA - BØG*U)**2
C
      QDØTY =  U/(ØMU*AU)
      QYH = QYK + 0.8*QDØTY*DUØG
      QDØTX = 1.0/(AU*RØØT)
      QXH = QXK + QDØTX*DUØG
      GØ TØ (6444,6046),JMP
 6444 GØ TØ (6044,6046),JUMP
 6044 IF (QYH - QYK) 6045,6045,6046
 6045 YMAX = QYK
      JUMP = 2
 6046 IF (QYH-YF) 6049,6048,6047
 6047 QYK = QYH
      QXK = QXH
      U = U - DU
      IF (1.0+U) 6048,6048,6229
C
 6048 XMAX = QXH
       GØ TØ 5014
 6049 XMAX = QXK + (QXH-QXK)*(QYK-YF)/(QYK-QYH)
C
C          ESTIMATES ØF ERRØRS ØVER N STEPS
C
 5014 JUMP = 1
      IF (NN-NSTEP) 8002,8002,8001
 8001 CNN = NSTEP
 8002 CLAMN = CLAM**CNN
      GL = (CLAMN-1.0)/(CLAM-1.0)
      VHH = VH*DH
      PHH = ABSF(PHB*DH3 + PHBB)
      PSH = ABSF(PSB*DH3 + PSBB)
      IF (PHH-PSH) 4013,4014,4014
 4013 CØNE = PSH/ABSF(DELTA)
      GØ TØ 4015
 4014 CØNE = PHH/ABSF(DELTA)
 4015 ETØK = ETN/ABSF(GKK)
      IF (EVØV-ETØK) 4016,4017,4C17
 4016 CTWØ = ETØK
      GØ TØ 4018
 4017 CTWØ = EVØV
 4018 CTHR = A13 + GKK*A14
      CFØR = A23 + GKK*A24
      C12 = CØNE + CTWØ
      EVLN = V*(-CØNE + CLAMN*C12)
      ETLN = GKK*EVLN/V
      EXLN = EXN + VHH*(CNN*(AXT*DH3 + AXR) - CØNE*CTHR + CTHR*C12*GL)
      EYLN = EYN + VHH*(CNN*(AYT*DH3 + AYR) - CØNE*CFØR + CFØR*C12*GL)
C
```

```
      IF (NN-NSTEP) 8003,8003,8004
 8003 EVN = 0.5*(EVLN + EVNN)
      ETN = 0.5*(ETLN + ETNN)
      EXN = 0.5*(EXLN + EXNN)
      EYN = 0.5*(EYLN + EYNN)
      GØ TØ 8005
C
 8004 EVN = EVLN
      ETN = ETLN
      EXN = EXLN
      EYN = EYLN
 8005 GØ TØ (7111,7222,7333),KPHASE
C
C         ERRØR ANALYSIS INITIALIZATIØN FØR START ØF PHASE
C
 7222 KPHASE = 3
      NN = 4
      NSTEP = 2
      CNN = 2.0
      AVTL = 0.0
      ATTL = 0.0
      AXTL = 0.0
      AYTL = 0.0
      EVNN = 0.0
      ETNN = 0.0
      EXNN = 0.0
      EYNN = 0.0
      EXAVE = EXN
      EYAVE = EYN
      EVAVE = EVN
      ETAVE = ETN
      TØØ = T
      XØØ = X
      YØØ = Y
      VØØ = V
      THØØ = THET
      RMØØ = RMASS
      DTH = V/32174.05
      ASSIGN 42 TØ NWRT
      GØ TØ 637
C
 7333 KPHASE = 1
      T = TØØ
      X = XØØ
      Y = YØØ
      V = VØØ
      THET = THØØ
      RMASS = RMØØ
      EXN = EXAVE
      EYN = EYAVE
      EVN = EVAVE
      ETN = ETAVE
      ASSIGN 36 TØ NWRT
      GØ TØ 609
C
 7111 IF (NN-2) 7001,7001,7002
 7001 T = TØ
      X = XØ
      Y = YØ
      V = VØ
```

```
      THET = THETØ
      RMASS = RMASSØ
      EXN = 0.0
      EYN = 0.0
      EVN = 0.0
      ETN = 0.0
 7002 CNN = NS
      NN = NS
       NSTEP = 1
      ASSIGN 36 TØ NWRT
      IF (SENSE SWITCH6) 3004,3006
 3004 CALL PDUMP(ZM,A13,1)
 3006 CØNTINUE
C
C         ESTABLISHING STARTING VALUES FØR DØUBLE FALSE PØSITIØN
C
  609 KLK = 1
      DT = 0.035*VH/GRAV
  610 CLAM = 1.0 + EPS*DT
      GNN = TMTN/DT
      CLAMN = CLAM**GNN
      GL = (CLAMN-1.0)/(CLAM-1.0)
      DT3 = DT**3
      PHT = ABSF(PHB*DT3 + PHBB)
      PST = ABSF(PSB*DT3 + PSBB)
      IF (PHT-PST) 4020,4021,4021
 4020 CØNE = PST/ABSF(DELTA)
      GØ TØ 4022
 4021 CØNE = PHT/ABSF(DELTA)
 4022 ETØK = ETN/ABSF(GKK)
      EVØV = EVN/VH
      IF (EVØV-ETØK) 4023,4024,4024
 4023 CTWØ = ETØK
      GØ TØ 4025
 4024 CTWØ = EVØV
 4025 C12 = CØNE + CTWØ
      VHH = VH*DT
      GØ TØ (4026,4027),NNXY
 4026 EXNN = EXN + VHH*(GNN*(AXT*DT3 + AXR) - CØNE*CTHR + CTHR*C12*GL)
      XIDT = EXNN - GAM2*XMAX
      GØ TØ 4028
 4027 EYNN = EYN + VHH*(GNN*(AYT*DT3 + AYR) - CØNE*CFØR + CFØR*C12*GL)
      XIDT = EYNN - GAM2*YMAX
C
 4028 GØ TØ (611,615,615,615,617),KLK
  611 IF (XIDT) 612,613,613
  612 DTH = DT
       GØ TØ 630
  613 XID2 = XIDT
      DT2 = DT
  614 DT = 0.1*DT
      KLK = KLK + 1
       GØ TØ 610
  615 IF (XIDT) 616,614,614
  616 XID1 = XIDT
      DT1 = DT
       GØ TØ 618
  617 GAM2 = 2.0*GAM2
       GØ TØ 609
C
```

```
C
C               DØUBLE FALSE PØSITIØN RØUTINE FØR F(DELT)
C
  618 DTØ = (DT1*XID2 - DT2*XID1)/(XID2 - XID1)
      GNN = TMTN/DTØ
       CLAM = 1.0 + EPS*DTØ
      CLAMN = CLAM**GNN
       DT3 = DTØ**3
      VHH = VH*DTØ
      GL = (CLAMN-1.0)/(CLAM-1.0)
      PHT = ABSF(PHB*DT3 + PHBB)
      PST = ABSF(PSB*DT3 + PSBB)
      IF (PHT-PST) 4030,4031,4031
 4030 CØNE = PST/ABSF(DELTA)
      GØ TØ 4032
 4031 CØNE = PHT/ABSF(DELTA)
 4032 ETØK = ETN/ABSF(GKK)
      IF (EVØV-ETØK) 4033,4034,4034
 4033 CTWØ = ETØK
      GØ TØ 4035
 4034 CTWØ = EVØV
 4035 C12 = CØNE + CTWØ
      GØ TØ (5020,5021),NNXY
 5020 EXNN = EXN + VHH*(GNN*(AXT*DT3 + AXR) - CØNE*CTHR + CTHR*C12*GL)
      XIDØ = EXNN - GAM2*XMAX
       GØ TØ 5022
 5021 EYNN = EYN + VHH*(GNN*(AYT*DT3 + AYR) - CØNE*CFØR + CFØR*C12*GL)
      XIDØ = EYNN - GAM2*YMAX
 5022 DIFH = (DT1-DT2)/DT2
      IF (0.05-ABSF(DIFH)) 661,660,660
  660 GØ TØ (5025,5026),NNXY
 5025 GØ TØ (5029,5029,5027),NXY
 5026 GØ TØ (5029,5029,5028),NXY
 5027 KLK = 1
      DT = DTØ
      NNXY = 2
      GØ TØ 610
 5028 NNXY = 1
 5029 DTH = DTØ
      NEG = 0
      NPØS = 0
       GØ TØ 630
C
  661 IF (XIDØ) 619,620,620
  619 NEG = NEG + 1
      NPØS = 0
      GØ TØ (621,626),NEG
  620 NPØS = NPØS + 1
      NEG = 0
      GØ TØ (621,629),NPØS
  621 IF (XIDØ) 622,623,623
  622 DT1 = DTØ
      XID1 = XIDØ
      GØ TØ 618
  623 DT2 = DTØ
      XID2 = XIDØ
       GØ TØ 618
C
  626 IF (XID1) 627,628,628
  627 XID2 = XID2/2.0
```

```
      NEG = 0
      NPØS = 0
      GØ TØ 618
  628 XIDL = XIDL/2.0
      NEG = 0
      NPØS = 0
      GØ TØ 618
  629 IF (XIDL) 628,627,627
C
  630 GØ TØ (632,632,632,631),N
  631 NN = NS
      CNN = NS
      KPHASE = 1
      NSTEP = 1
       GØ TØ 636
  632 CNN = NS
      IF (CNN*DTH+T - TIM(N)) 636,636,633
  633 CNN = (TIM(N)-T)/DTH
      NN = CNN
      CCN = NN
      IF (CNN-CCN) 634,634,635
  634 CNN = CCN
       GØ TØ 6355
  635 CNN = CCN + 1.0
 6355 NSTEP = 1
      NN = CNN
 6356 NEST = NEST + 1
      IF (NEXT(NEST)) 6356,6356,6357
 6357 KPHASE = 2
      DTH = (TIM(N)-T)/CNN
  636 CLAMN = CLAM**CNN
      GL = (CLAMN-1.0)/(CLAM-1.0)
      EVNN =VH*(-CØNE + CLAMN*C12)
      ETNN = GKK*EVNN/VH
      EXNN = EXN + VHH*(CNN*(AXT*DT3 + AXR) - CØNE*CTHR + CTHR*C12*GL)
      EYNN = EYN + VHH*(CNN*(AXT*DT3 + AYR) - CØNE*CFØR + CFØR*C12*GL)
C
      GØ TØ (7006,7003,7006,637),KU
 7003 YØU = YF + EYN
      DLEY = (EYNN - EYN)/CNN
      ERRA = -DLEY
 7006 EV(NØ) = EVN
      ET(NØ) = ETN*57.29577
      EX(NØ) = EXN
      EY(NØ) = EYN
      TN(NØ) = TT + DH
      XN(NØ) = XH
      YN(NØ) = YH
      VN(NØ) = VH
      THN(NØ) = THH * 57.29577
      NØ = NØ + 1
  637 DELT = DTH
      C1 = 0.62653829*DELT
      C2 = -0.55111241*DELT
      C3 = 1.4072559*DELT
      C4 = -0.48268182*DELT
      GØ TØ (92,92,49),KPHASE
C
   49 THDEG = 57.29577*THET
      THDOT = 57.29577*THDØT(3)
```

```
      RWGT = 32.17405 * RMASS
      WRITE ØUTPUT TAPE 3,111, T,X,Y,V,THDEG,CMACH,CCD,THRUST,RWGT,RHØ
      WRITE ØUTPUT TAPE 3,112, XDØT(1),YDØT(1),VDØT(1),THDDT,DRAG,PRESS
   50 N = N+1
      IF(NEXT(N)) 50,50,51
   51 GØ TØ (3,55,56,59,7020),N
C
   55 THRUST = 0.0
      DØTM = 0.0
      AREA = 0.0
      RMASS = RMASS-BMASS
      RMØØ = RMASS
      RWGT = 32.17405 * RMASS
      GØ TØ (801,802),NNN
  801 WRITE ØUTPUT TAPE 3,115
      GØ TØ 999
  802 WRITE ØUTPUT TAPE 3,121
      NNN = 1
      GØ TØ 999
C
   56 THRST = THR2
      DØTM = DØTM2
      AREA = AREA2/144.0
      KASE = 2
      GØ TØ (803,804),NNN
  803 WRITE ØUTPUT TAPE 3,116
      GØ TØ 999
  804 WRITE ØUTPUT TAPE 3,122
      NNN = 1
      GØ TØ 999
C
   59 THRUST = 0.0
      DØTM = 0.0
      AREA = 0.0
      GØ TØ (805,806),NNN
  805 WRITE ØUTPUT TAPE 3,117
      GØ TØ 999
  806 WRITE ØUTPUT TAPE 3,123
      NNN = 1
C
  999 EPSLØN = 0.001*DELT
      NEST = N
      TIME = TIM(N)
      ASSIGN 42 TØ NWRT
      ASSIGN 64 TØ NCNT
      LINE = 2
      WRITE ØUTPUT TAPE 3,109
      WRITE ØUTPUT TAPE 3,110
      GØ TØ 3
C
C                       TEST FØR END ØF TRAJECTØRY
C
   92 GØ TØ KAP,(7007,7010,7013)
 7007 IF (YDØT(1)) 7008,3,3
 7008 KU = 2
      ASSIGN 7010 TØ KAP
      YMAX = Y
      JMP = 2
 7010 IF (Y-YØU) 7012,7011,7011
 7011 YØU = YØU + DLEY
```

```
      YLST = Y
      GØ TØ 3
7012  ASSIGN 42 TØ NWRT
      NTERM = 2
      YØNE = Y
      YTWØ = YLST
      DTTWØ = -DELT
      DELT3 = -DTTWØ*(YØNE-YØU)/(YTWØ-YØNE)
      DELT = DELT3
      ASSIGN 7013 TØ KAP
      GØ TØ 604
7013  IF(Y-YØU) 7014,7015,7015
7014  YØNE = Y
      NB = 0
      NA = NA + 1
      IF (NA-2) 6081,6080,6080
6080  YTWØ = YØU + 0.5*(YTWØ-YØU)
6081  DTØNE = DELT3
      DELT3 = ((YTWØ-YØU)*DTØNE - (YØNE-YØU)*DTTWØ)/(YTWØ-YØNE)
      DELT = DELT3 - DTØNE
      GØ TØ 7016
7015  YTWØ = Y
      NA = 0
      NB = NB + 1
      IF (NB-2)6083,6082,6082
6082  YØNE = YØU + 0.5*(YØNE-YØU)
6083  DTTWØ = DELT3
      DELT3 = ((YTWØ-YØU)*DTØNE - (YØNE-YØU)*DTTWØ)/(YTWØ-YØNE)
      DELT = DELT3 - DTTWØ
7016  IF (YTWØ-YØNE-.01*EYN) 7017,7017,604
7017  GØ TØ (7018,7018,7019,7020),KU
7018  XUNØ = X
      KU = 3
      NN = NS
      CNN = NSTEP
      NSTEP = 1
      TT = T
      XH = X
      THH = THET
      YH = YØU
      YØU = YF
      DLEY = 0.0
      VH = V
      YLST = Y
      DTØNE = 0.0
      DH = 0.0
      ASSIGN 7010 TØ KAP
      GØ TØ 6005
7019  EV(NØ) = EVN
      ET(NØ) = ETN*57.29577
      EX(NØ) = EXN
      EY(NØ) = EYN
      TN(NØ) = T
      XN(NØ) = X
      YN(NØ) = Y
      VN(NØ) = V
      THN(NØ) = THET * 57.29577
      KU = 4
      NN = NS
      CNN = NSTEP
```

```
      NSTEP = 1
      TT = T
      XH = X
      THH = THET
      YH = YØU
      VH = V
      YLST = Y
      DH = 0.0
      YØU = YF-EYN
      DLEY = ERRA
      DTØNE = 0.0
      ASSIGN 7010 TØ KAP
      THDEG = THET*57.29577
      WRITE ØUTPUT TAPE 3,111, T,X,Y,V,THDEG,CMACH,CCD,THRUST,RWGT,RHØ
  401 WRITE ØUTPUT TAPE 3,113, NJR
      WRIT = T + 1000.
      ASSIGN 42 TØ NWRT
      GØ TØ 6005
C
 7020 LINE = 27
      XDØS = X
      WRITE ØUTPUT TAPE 3,126
      WRITE ØUTPUT TAPE 3,127
      WRITE ØUTPUT TAPE 3,128
      GØ TØ (5051,5053,5052),NXY
 5051 WRITE ØUTPUT TAPE 3,131, GAMMA
      PERX = 100.*EX(NØ)/XN(NØ)
      GØ TØ 5054
 5052 WRITE ØUTPUT TAPE 3,131, GAMMA
 5053 WRITE ØUTPUT TAPE 3,132, GAMMA
      PERY = 100.*EY(NØ)/YMAX
 5054 IF (GAMMA-100.*GAM2) 5055,5056,5056
 5055 GØ TØ (5551,5552,5553),NXY
 5551 WRITE ØUTPUT TAPE 3,130, PERX
      GØ TØ 5057
 5552 WRITE ØUTPUT TAPE 3,142, PERY
      GØ TØ 5057
 5553 WRITE ØUTPUT TAPE 3,143, PERX,PERY
      GØ TØ 5057
 5056 GØ TØ (5554,5555,5556),NXY
 5554 WRITE ØUTPUT TAPE 3,129, PERX
      GØ TØ 5057
 5555 WRITE ØUTPUT TAPE 3,144, PERY
      GØ TØ 5057
 5556 WRITE ØUTPUT TAPE 3,145, PERX,PERY
 5057 WRITE ØUTPUT TAPE 3,133
      WRITE ØUTPUT TAPE 3,134, XN(NØ),EX(NØ),YN(NØ),EY(NØ),THN(NØ),
     1                         ET(NØ),VN(NØ),EV(NØ)
      WRITE ØUTPUT TAPE 3,135
      WRITE ØUTPUT TAPE 3,136, XØ,YØ,THDEGØ,VØ
      WRITE ØUTPUT TAPE 3,137
      WRITE ØUTPUT TAPE 3,139
      NØØ = NØ - 1
      DØ 5059 KE =2,NØØ
      WRITE ØUTPUT TAPE 3,140, TN(KE),XN(KE),YN(KE),THN(KE),VN(KE),
     1                         EX(KE),EY(KE),ET(KE),EV(KE)
      LINE = LINE + 3
      IF (56-LINE) 5058,5058,5059
 5058 WRITE ØUTPUT TAPE 3,138
      WRITE ØUTPUT TAPE 3,139
```

```
      LINE = 5
 5059 CØNTINUE
      GØ TØ (5060,5060,5060,5060,5061),N
 5060 WRITE ØUTPUT TAPE 3,141, XUNØ,XDØS,XN(NØ),YF
 5061 NJR = NJR + 1
      JN = JN-1
      IF (JN) 1,1,2
C
      END
```

APPENDIX II

This appendix contains output for the sample calculation depicted by Figure 2. The input for this computation is illustrated in Section C. The first portion of the output consists of a table of range, altitude, speed and angle as a function of time. Also included are the time derivatives of these quantities, the mach number, the drag coefficient, the thrust, vehicle weight and drag, the air density, and the air pressure when required.

The second section of the output is devoted to the error analysis routine. This begins with a description of the error specification and provides the estimated error for comparison. A table of range error (feet), altitude error (feet), speed error (feet/second), and angular error (degrees) as a function of time follow. This page is terminated by bounds for the impact range which are based upon a consideration of the numerical error envelope surroundin g the computed trajectory.

CØMPUTING AND ANALYSIS SECTIØN
TWØ DIMENSIØNAL, TWØ-STAGE RØCKET ØR BALLISTIC TRAJECTØRY PRØGRAM
WITH 1959 ARDC STANDARD ATMØSPHERE

FREE-FLIGHT ØF MAIN STAGE

TIME	RANGE X-DERIV	ALTITUDE Y-DERIV	VELØCITY V-DERIV	THETA TH-DERIV	MACH	KD	THRUST	WEIGHT DRAG	DENSITY PRESSURE
-0.	-0.	-0.	1000.000	45.000	0.8957	0.0950	0.	100.000	0.23769E-02
	707.107	707.107	-83.772	-1.304				189.662	C.
0.423	295.453	292.596	965.299	44.436	0.8655	0.0972	0.	100.000	0.23566E-02
	689.258	675.814	-80.238	-1.364				179.378	0.
0.846	583.524	572.190	932.065	43.845	0.8365	0.0994	0.	100.000	0.23374E-02
	672.215	645.656	-76.842	-1.426				169.564	C.
1.270	864.546	839.251	900.238	43.228	0.8087	0.1015	0.	100.000	0.23191E-02
	655.944	616.576	-73.584	-1.492				160.222	0.
1.693	1138.846	1094.229	869.827	42.582	0.7821	0.1026	0.	100.000	0.23017E-02
	640.460	588.567	-70.088	-1.560				150.182	0.
2.116	1406.773	1337.588	840.901	41.907	0.7567	0.1034	0.	100.000	0.22853E-02
	625.824	561.657	-66.650	-1.631				140.370	0.
2.539	1668.676	1569.778	813.379	41.201	0.7325	0.1041	0.	100.000	0.22696E-02
	611.989	535.776	-63.445	-1.705				131.332	0.
2.963	1924.884	1791.219	787.169	40.463	0.7094	0.1048	0.	100.000	0.22548E-02
	598.894	510.843	-60.450	-1.781				122.998	0.
3.386	2175.696	2002.295	762.186	39.693	0.6874	0.1054	0.	100.000	0.22407E-02
	586.487	486.786	-57.643	-1.861				115.304	0.
3.809	2421.394	2203.363	738.354	38.888	0.6664	0.1060	0.	100.000	0.22274E-02
	574.716	463.539	-55.005	-1.943				108.194	0.
4.232	2662.237	2394.754	715.606	38.048	0.6463	0.1066	0.	100.000	0.22148E-02
	563.538	441.042	-52.520	-2.028				101.618	0.
4.857	3009.582	2660.369	683.862	36.739	0.6182	0.1075	0.	100.000	0.21973E-02
	548.024	409.069	-49.094	-2.160				92.786	0.
5.482	3347.574	2906.432	654.178	35.346	0.5919	0.1080	0.	100.000	0.21813E-02
	533.592	378.455	-45.873	-2.298				84.743	0.
6.107	3676.870	3133.756	626.478	33.865	0.5673	0.1082	0.	100.000	0.21665E-02
	520.197	349.100	-42.793	-2.443				77.296	0.
6.732	3998.093	3343.092	600.635	32.291	0.5443	0.1083	0.	100.000	0.21530E-02
	507.741	320.875	-39.928	-2.594				70.695	0.
7.357	4311.798	3535.107	576.524	30.621	0.5228	0.1084	0.	100.000	0.21406E-02
	496.128	293.661	-37.243	-2.751				64.836	0.
7.983	4618.485	3710.398	554.044	28.852	0.5027	0.1086	0.	100.000	0.21294E-02
	485.272	267.349	-34.705	-2.913				59.629	0.
8.608	4918.602	3869.501	533.112	26.978	0.4840	0.1087	0.	100.000	0.21192E-02
	475.097	241.850	-32.287	-3.080				55.001	0.
9.233	5212.554	4012.897	513.661	25.000	0.4666	0.1088	0.	100.000	0.21101E-02
	465.536	217.080	-29.965	-3.251				50.889	0.
9.858	5500.706	4141.020	495.635	22.913	0.4504	0.1089	0.	100.000	0.21020E-02
	456.527	192.969	-27.720	-3.425				47.239	0.
10.483	5783.382	4254.261	478.994	20.718	0.4354	0.1090	0.	100.000	0.20948E-02
	448.017	169.456	-25.536	-3.598				44.005	0.
11.004	6015.151	4337.576	466.149	18.805	0.4239	0.1090	0.	100.000	0.20896E-02
	441.266	150.264	-23.751	-3.742				41.598	0.
11.525	6243.476	4410.980	454.227	16.818	0.4132	0.1091	0.	100.000	0.20850E-02
	434.800	131.420	-21.992	-3.883				39.433	0.
12.047	6468.500	4474.646	443.217	14.757	0.4032	0.1092	0.	100.000	0.20810E-02
	428.596	112.899	-20.255	-4.020				37.493	0.
12.568	6690.353	4528.737	433.107	12.627	0.3941	0.1092	0.	100.000	0.20776E-02
	422.632	94.680	-18.536	-4.152				35.761	0.
13.089	6909.155	4573.405	423.889	10.430	0.3858	0.1093	0.	100.000	0.20748E-02
	416.884	76.742	-16.834	-4.275				34.224	0.

TIME	RANGE X-DERIV	ALTITUDE Y-DERIV	VELOCITY V-DERIV	THETA TH-DERIV	MACH	KD	THRUST	WEIGHT DRAG	DENSITY PRESSURE
13.610	7125.013	4608.791	415.554	8.172	0.3783	0.1093	0.	100.000	0.20726E-02
	411.335	59.067	-15.147	-4.389				32.870	0.
14.132	7338.026	4635.030	408.095	5.857	0.3715	0.1093	0.	100.000	0.20709E-02
	405.965	41.641	-13.476	-4.492				31.687	0.
14.653	7548.282	4652.247	401.501	3.491	0.3655	0.1094	0.	100.000	0.20698E-02
	400.756	24.450	-11.825	-4.581				30.665	0.
15.174	7755.861	4660.562	395.763	1.083	0.3603	0.1094	0.	100.000	0.20693E-02
	395.692	7.483	-10.195	-4.655				29.796	0.
15.696	7960.834	4660.087	390.868	-1.359	0.3558	0.1094	0.	100.000	0.20693E-02
	390.758	-9.270	-8.590	-4.713				29.071	0.
16.121	8126.219	4653.264	387.488	-3.371	0.3528	0.1094	0.	100.000	0.20698E-02
	386.818	-22.788	-7.304	-4.747				28.581	0.
16.546	8289.941	4640.719	384.651	-5.396	0.3502	0.1095	0.	100.000	0.20706E-02
	382.946	-36.172	-6.042	-4.769				28.178	0.
16.972	8452.031	4622.509	382.344	-7.427	0.3480	0.1095	0.	100.000	0.20717E-02
	379.136	-49.424	-4.806	-4.779				27.860	0.
17.397	8612.511	4598.689	380.557	-9.460	0.3464	0.1095	0.	100.000	0.20732E-02
	375.382	-62.548	-3.601	-4.776				27.622	0.
17.823	8771.405	4569.313	379.275	-11.489	0.3452	0.1095	0.	100.000	0.20750E-02
	371.676	-75.543	-2.430	-4.761				27.463	0.
18.248	8928.732	4534.437	378.484	-13.509	0.3444	0.1095	0.	100.000	0.20772E-02
	368.013	-88.411	-1.296	-4.734				27.378	0.
18.673	9084.508	4494.113	378.167	-15.515	0.3441	0.1095	0.	100.000	0.20797E-02
	364.387	-101.153	-0.202	-4.695				27.366	0.
19.099	9238.750	4448.396	378.306	-17.502	0.3442	0.1095	0.	100.000	0.20826E-02
	360.793	-113.769	0.848	-4.645				27.424	0.
19.524	9391.468	4397.340	378.882	-19.465	0.3446	0.1095	0.	100.000	0.20858E-02
	357.227	-126.257	1.853	-4.585				27.549	0.
19.949	9542.674	4340.998	379.876	-21.401	0.3454	0.1095	0.	100.000	0.20894E-02
	353.682	-138.617	2.810	-4.516				27.740	0.
20.363	9688.182	4281.230	381.222	-23.253	0.3466	0.1095	0.	100.000	0.20931E-02
	350.255	-150.505	3.693	-4.441				27.985	0.
20.776	9832.276	4216.574	382.922	-25.072	0.3481	0.1095	0.	100.000	0.20972E-02
	346.841	-162.268	4.527	-4.359				28.288	0.
21.190	9974.962	4147.080	384.958	-26.856	0.3498	0.1095	0.	100.000	0.21016E-02
	343.437	-173.906	5.312	-4.270				28.647	0.
21.603	10116.241	4072.800	387.308	-28.603	0.3519	0.1095	0.	100.000	0.21063E-02
	340.041	-185.417	6.047	-4.177				29.060	0.
22.017	10256.118	3993.790	389.951	-30.309	0.3542	0.1094	0.	100.000	0.21113E-02
	336.649	-196.797	6.732	-4.080				29.524	0.
22.430	10394.593	3910.102	392.867	-31.975	0.3567	0.1094	0.	100.000	0.21166E-02
	333.260	-208.044	7.367	-3.979				30.039	0.
22.843	10531.667	3821.792	396.035	-33.599	0.3595	0.1094	0.	100.000	0.21223E-02
	329.870	-219.156	7.952	-3.876				30.602	0.
23.257	10667.339	3728.916	399.435	-35.180	0.3625	0.1094	0.	100.000	0.21282E-02
	326.478	-230.131	8.488	-3.771				31.211	0.
23.670	10801.608	3631.532	403.047	-36.717	0.3656	0.1094	0.	100.000	0.21344E-02
	323.083	-240.965	8.976	-3.665				31.866	0.
24.084	10934.472	3529.699	406.850	-38.210	0.3689	0.1094	0.	100.000	0.21410E-02
	319.683	-251.655	9.417	-3.559				32.564	0.
24.526	11075.200	3415.777	411.115	-39.761	0.3726	0.1093	0.	100.000	0.21483E-02
	316.034	-262.941	9.839	-3.446				33.357	0.
24.969	11214.311	3296.897	415.555	-41.261	0.3765	0.1093	0.	100.000	0.21559E-02
	312.377	-274.055	10.210	-3.334				34.196	0.

TIME	RANGE X-DERIV	ALTITUDE Y-DERIV	VELØCITY V-DERIV	THETA TH-DERIV	MACH	KD	THRUST	WEIGHT DRAG	CENSITY PRESSURE
25.412	11351.801	3173.134	420.148	-42.713	0.3805	0.1093	0.	100.000	0.21639E-02
	308.711	-284.995	10.532	-3.223				35.078	0.
25.855	11487.665	3044.567	424.873	-44.115	0.3846	0.1093	0.	100.000	0.21723E-02
	305.034	-295.756	10.807	-3.114				36.002	0.
26.297	11621.899	2911.276	429.711	-45.470	0.3888	0.1092	0.	100.000	0.21809E-02
	301.347	-306.335	11.037	-3.008				36.965	0.
26.740	11754.498	2773.342	434.640	-46.779	0.3931	0.1092	0.	100.000	0.21899E-02
	297.649	-316.729	11.225	-2.904				37.965	0.
27.183	11885.457	2630.848	439.644	-48.042	0.3974	0.1092	0.	100.000	0.21993E-02
	293.940	-326.934	11.372	-2.803				39.000	0.
27.626	12014.772	2483.878	444.704	-49.261	0.4018	0.1092	0.	100.000	0.22089E-02
	290.220	-336.947	11.480	-2.705				40.069	0.
28.068	12142.438	2332.518	449.804	-50.437	0.4062	0.1091	0.	100.000	0.22189E-02
	286.490	-346.766	11.553	-2.610				41.168	0.
28.511	12268.450	2176.854	454.928	-51.572	0.4106	0.1091	0.	100.000	0.22292E-02
	282.750	-356.388	11.591	-2.518				42.297	0.
29.006	12407.372	1997.831	460.669	-52.794	0.4155	0.1091	0.	100.000	0.22410E-02
	278.558	-366.907	11.596	-2.419				43.590	0.
29.501	12544.216	1813.663	466.402	-53.968	0.4204	0.1091	0.	100.000	0.22533E-02
	274.355	-377.174	11.563	-2.325				44.915	0.
29.996	12678.978	1624.476	472.110	-55.096	0.4253	0.1090	0.	100.000	0.22660E-02
	270.143	-387.183	11.496	-2.234				46.267	0.
30.491	12811.652	1430.399	477.778	-56.180	0.4301	0.1090	0.	100.000	0.22790E-02
	265.923	-396.934	11.397	-2.147				47.645	0.
30.986	12942.236	1231.561	483.388	-57.222	0.4348	0.1090	0.	100.000	0.22924E-02
	261.697	-406.422	11.268	-2.064				49.045	0.
31.481	13070.727	1028.090	488.928	-58.224	0.4395	0.1089	0.	100.000	0.23062E-02
	257.466	-415.647	11.112	-1.985				50.465	0.
31.976	13197.123	820.120	494.385	-59.188	0.4441	0.1089	0.	100.000	0.23204E-02
	253.233	-424.605	10.932	-1.910				51.902	0.
32.471	13321.423	607.781	499.747	-60.116	0.4486	0.1089	0.	100.000	0.23349E-02
	248.999	-433.297	10.728	-1.838				53.353	0.
32.966	13443.628	391.206	505.003	-61.008	0.4529	0.1089	0.	100.000	0.23498E-02
	244.766	-441.721	10.505	-1.769				54.816	0.
33.461	13563.738	170.528	510.143	-61.868	0.4572	0.1088	0.	100.000	0.23651E-02
	240.537	-449.875	10.263	-1.704				56.287	0.
33.790	13642.371	21.726	513.490	-62.421	0.4600	0.1088	0.	100.000	0.23754E-02
	237.731	-455.144	10.092	-1.662				57.269	0.
33.837	13653.700	-0.002	513.971	-62.500	0.4604	0.1088	0.	100.000	0.23769E-02
			END	ØF TRAJECTØRY 1					
33.837	13653.700	-0.002	513.971	-62.500	0.4604	0.1088	0.	100.000	0.23769E-02
	237.324	-455.898	10.067	-1.656				57.411	0.

TEA-2
TRAJECTØRY ERRØR ANALYSIS ** TWØ DEGREES ØF FREEDØM

ANALYSIS ** DR. H.J. KØPP * EXTENSIØN 72264 * REPØRT TØ BE PUBLISHED
PRØGRAMMING ** MR. J.N. NIELSEN * EXTENSIØN 73230

THIS PRØGRAM SELECTS THE INTEGRATIØN STEP LENGTH SUCH THAT THE ERRØR IN THE TØTAL RANGE IS LESS THAN A PRESCRIBED AMØUNT AND/ØR THE ERRØR IN ALTITUDE IS LESS THAN A PRESCRIBED AMØUNT. THE ERRØR CRITERIØN USEC FØR THIS TRAJECTØRY WAS **

THE ERRØR IN TØTAL RANGE IS LESS THAN 0.50 PERCENT.

THE ABØVE CØNDITIØN WAS SATISFIED, AND THE CØMPUTED ERRØR IN TØTAL RANGE IS EQUAL TØ 0.20411938E-00 PERCENT.

FINAL ERRØR ESTIMATES
*****

RANGE 0.13653699E 05 RANGE ERRØR 0.27869847E 02 ALTITUDE -0.16103294E-02 ALTITUDE ERRØR 0.19756567E C2
THETA -0.62500186E 02 THETA ERRØR 0.36391161E-00 VELØCITY 0.51397064E 03 VELØCITY ERRØR 0.74526225E 00

INITIAL CØNDITIØNS
*****

RANGE -0. ALTITUDE -0. THETA 0.45000000E 02 VELØCITY 0.09999999E C4

TABLE ØF ERRØR ESTIMATES
*****

TIME	RANGE RANGE ERRØR	ALTITUDE ALTITUDE ERRØR	THETA THETA ERRØR	VELØCITY VELØCITY ERRØR
0.42321443E 01	0.26622376E 04 0.79198267E 00	0.23947539E 04 0.61440352E 00	0.38047796E 02 0.21010649E-02	0.71560609E 03 0.70057453E-02
0.10482828E 02	0.57833844E 04 0.27059503E 01	0.42542616E 04 0.21146028E 01	0.20718350E 02 0.12748724E-01	0.47899339E 03 0.48066485E-01
0.15695586E 02	0.79608352E 04 0.37017895E 01	0.46600877E 04 0.26971755E 01	-0.13590061E 01 0.24096826E-01	0.39086834E 03 0.10064537E-00
0.19949489E 02	0.95426740E 04 0.48154643E 01	0.43409983E 04 0.33297788E 01	-0.21401435E 02 0.42529281E-01	0.37987571E 03 0.16588639E-00
0.24083638E 02	0.10934472E 05 0.72549541E 01	0.35296989E 04 0.52193458E 01	-0.38209875E 02 0.82345665E-01	0.40685038E 03 0.27014931E-00
0.28511017E 02	0.12268449E 05 0.13093567E 02	0.21768546E 04 0.97570217E 01	-0.51572245E 02 0.15795733E-00	0.45492835E 03 0.43775219E-00
0.33460953E 02	0.13563738E 05 0.27869847E 02	0.17052772E 03 0.19756567E 02	-0.61867693E 02 0.31086409E-00	0.51014319E 03 0.70454574E 00
0.33789781E 02	0.13642371E 05 0.27869847E 02	0.21728190E 02 0.19756567E 02	-0.62421049E 02 0.36391161E-00	0.51348998E 03 0.74526225E 00

IMPACT BETWEEN RANGE ØF 0.13642371E 05 FEET AND 0.13664990E 05 FEET

IMPACT CENTER PREDICTED AT RANGE ØF 0.13653699E 05 FEET AND ALTITUDE ØF-0. FEET

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