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**DETERMINATION OF THE
ORIENTATION OF THE AXIS
OF A ROCKET OR SATELLITE
IN ITS TRAJECTORY OR ORBIT**

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ABSTRACT

A method is developed for determining the aspect of the axis of a rocket or satellite with respect to an earth based system of coordinates for the case where these bodies undergo a constant precessional motion about some fixed direction. The analysis is based on data obtained from a magnetometer mounted on the body so as to give the axial component of the Earth's magnetic field and a sun sensor which measures the angle between the Sun Vector and the axis of the body. The only information needed for the determination of the aspect of the axis is the maximum and minimum axial components of the Earth's magnetic field, the maximum and minimum angles between the axis of the body and the Sun Vector, the time sequence in which these maximums and minimums occur, and the angular velocity of precession.

1. ANGLE OF THE CONE OF PRECESSION; ANGLE BETWEEN AXIS OF PRECESSION, EARTH'S MAGNETIC FIELD β_H , AND SUN β_S .

We will assume in what follows that when the rocket or satellite has reached a certain altitude in the atmosphere, the atmospheric friction becomes negligible and the precessional motion remains such that the axis of precession makes a constant angle with the axis of the rocket.

To describe the motion of the rocket or satellite after its motion of precession becomes constant, let X, Y, Z be a right handed system of rectangular coordinate axes with the X-Y plane parallel to the equatorial plane of the Earth, the origin at any point of the trajectory or orbit of the moving rocket or satellite, and with the X axis pointing toward the vernal equinox.

Let i, j, k be a system of unit vectors parallel to the X, Y, Z axes respectively. Then the position vector from the origin (0, 0, 0) to x, y, z is given by

$$R = i x + j y + k z$$

if θ and ϕ are the latitude and longitude respectively and r is the distance

$$R = r (i \cos \theta \cos \phi + j \cos \theta \sin \phi + k \sin \theta).$$

We may introduce a new system of base unit vectors e_r , e_θ , and e_ϕ by

$$\begin{aligned} e_r &= \frac{\partial R}{\partial r} = i \cos \theta \cos \phi + j \cos \theta \sin \phi + k \sin \theta \\ e_\theta &= \frac{1}{r} \frac{\partial R}{\partial \theta} = -i \sin \theta \cos \phi - j \sin \theta \sin \phi + k \cos \theta \\ e_\phi &= \frac{1}{r \cos \theta} \frac{\partial R}{\partial \phi} = -i \sin \phi + j \cos \phi \end{aligned} \quad (1-1),$$

Let e_r be parallel to the axis of precession of the rocket or satellite. Let e_1' , e_2' , and e_r' be a third system of base unit vectors with e_1' in the plane of e_θ and e_ϕ , e_2' making an angle α with the $e_\theta e_\phi$ plane and e_r' parallel to the axis of the rocket or satellite. if w_0 is the angular velocity of rotation of the axis of the rocket or satellite about its axis of precession then,

$$\begin{aligned} e_1' &= e_\theta \cos w_0 t + e_\phi \sin w_0 t \\ e_2' &= e_r \sin \alpha - e_\theta \cos \alpha \sin w_0 t + e_\phi \cos \alpha \cos w_0 t \\ e_r' &= e_r \cos \alpha + (e_\theta \sin w_0 t - e_\phi \cos w_0 t) \sin \alpha. \end{aligned}$$

To take into account the rotation of the rocket or satellite about its axis, let w be the angular velocity of rotation of the rocket or satellite about its axis, e_r'' , e_1'' , e_2'' a fourth system of base unit vectors with e_r'' parallel to e_r' and with e_1'' and e_2'' in the plane of e_1' and e_2' , then

$$\begin{aligned} e_r'' &= e_r \cos \alpha + (e_\theta \sin w_0 t - e_\phi \cos w_0 t) \sin \alpha \\ (2-1) \quad e_1'' &= e_r \sin \alpha \sin w t + e_\theta (\cos w_0 t \cos w t - \cos \alpha \sin w_0 t \sin w t) \\ &\quad + e_\phi (\sin w_0 t \cos w t + \cos \alpha \cos w_0 t \sin w t) \\ e_2'' &= e_r \sin \alpha \cos w t - e_\theta (\cos w_0 t \sin w t + \cos \alpha \sin w_0 t \cos w t) \\ &\quad - e_\phi (\sin w_0 t \sin w t - \cos \alpha \cos w_0 t \cos w t). \end{aligned}$$

Let H_0 be the magnitude of the Earth's magnetic field and M a unit vector along the Earth's field then

$$e_r'' H_r'' + e_1'' H_1'' + e_2'' H_2'' = M H_0.$$

Thus the scalar product of e_r'' with the vector $M H_0$ gives

$$H_r'' = H_0 e_r'' \cdot M$$

$$\frac{\ddot{H}_r}{H_0} = e_r'' \cdot M'' \cos(\angle e_r'', M) = \cos \gamma_H(t)$$

$$\cos \gamma_H(t) = M \cdot e_r \cos \alpha + M \cdot e_\theta \sin \alpha \sin w_0 t - M \cdot e_\phi \sin \alpha \cos w_0 t$$

$$\text{If we set } M \cdot e_\theta = m, \quad M \cdot e_\phi = n, \quad M \cdot e_r = L = \cos \beta_H$$

$$H_e'' = H_0 (\cos \beta_H \cos \alpha + \sin \alpha (m \sin w_0 t - n \cos w_0 t)), \text{ the critical}$$

$$\text{values are given by } \frac{dH_r''}{dt} = 0, \quad \tan w_0 t = -\frac{m}{n}$$

$$m^2 + n^2 = 1 - \cos^2 \beta_H = \sin^2 \beta_H.$$

$$(3-1) \quad \text{The minimum is given by } w_0 t_0 = \arcsin \frac{-m}{\sin \beta_H} = \arccos \frac{+n}{\sin \beta_H}$$

$$\text{and the maximum by } w_0 t_1 = \arcsin \frac{m}{\sin \beta_H} = \arccos \frac{-n}{\sin \beta_H}$$

Thus for the minimum

$$(3-2) \quad H_r'' = H_0 (\cos \beta_H \cos \alpha - \sin \beta_H \sin \alpha) = H_0 \cos (\beta_H + \alpha)$$

and for the maximum

$$H_r'' = H_0 (\cos \beta_H \cos \alpha + \sin \beta_H \sin \alpha) = H_0 \cos (\beta_H - \alpha).$$

Since

$$\frac{H_r''}{H_0} = \cos (\angle e_r'', M) = \cos \gamma_H(t),$$

From (3-1) and (3-2)

$$\cos \gamma_H(t_0) = \cos (\beta_H + \alpha), \quad \cos \gamma_H(t_1) = \cos (\beta_H - \alpha).$$

Writing

$$\gamma_0 = \gamma_H(t_0), \quad \gamma_1 = \gamma_H(t_1),$$

we find

$$\gamma_0 = \beta_H + \alpha, \quad \gamma_1 = \pm (\beta_H - \alpha)$$

$$\beta_H = \frac{\gamma_0 \pm \gamma_1}{2}, \quad \alpha = \frac{\gamma_0 \mp \gamma_1}{2},$$

where the upper signs are to be taken if $\beta_H > \alpha$ and the lower signs if

$$\beta_H < \alpha.$$

Similarly if S is a unit vector parallel but in the opposite sense to the Sun's rays,

$$S = S_r'' e_r'' + S_1'' e_1'' + S_2'' e_2''$$

$$S \cdot e_r'' = \cos \gamma_S(t) = S_r''.$$

A calculation identical to that in the preceding section where S replaces H leads immediately to

$$(4-1) \quad \beta_S = \frac{\gamma_{0S} \pm \gamma_{1S}}{2} \quad \alpha = \frac{\gamma_{0S} \pm \gamma_{1S}}{2}$$

where the upper signs are to be taken if $\beta_S > \alpha$, and the lower signs are to be taken if $\beta_S < \alpha$,*

where

$$\gamma_{0S} = \gamma_S(t_3) = \arccos(S_r''(t_3))$$

$$\gamma_{1S} = \gamma_S(t_4) = \arccos(S_r''(t_4)).$$

However, here $\gamma_S(t_3)$ and $\gamma_S(t_4)$ are the actual measured angles between e_r'' and the Sun vector. Thus α and β_S are uniquely determined. β_H is thus uniquely determined by the formula

$$(4-2) \quad \sin \beta_H = \frac{\cos \gamma_H(t_1) - \cos \gamma_H(t_0)}{2 \sin \alpha}$$

If the rocket or satellite has no precessional motion then

$$\alpha = 0, \quad \omega_0 = 0, \quad e_r'' = e_r$$

and

$$\cos \beta_H = \cos \gamma_H(t) = \text{constant}$$

$$\cos \beta_S = \cos \gamma_S(t) = \text{constant}$$

* If the period of rotation of the rocket about its axis, as determined by the magnetometers, is greater than that determined by the Sun sensors, then $\beta_H < \alpha$, $\beta_S > \alpha$, if the opposite is true then $\beta_S < \alpha$, $\beta_H > \alpha$.

2. CALCULATIONS OF θ , AND ϕ , THE LATITUDE AND LONGITUDE OF THE AXIS OF PRECESSION

Let θ_H be the angle between the unit vector M and the equatorial plane, and ϕ_H be the angle between the vertical plane containing M and the X-Z plane, then:

$$M = i \cos \theta_H \cos \phi_H + j \cos \theta_H \sin \phi_H + k \sin \theta_H.$$

Making use of the first of (1-1) we find

$$\cos \theta \cos \theta_H \cos (\phi - \phi_H) + \sin \theta_H \sin \theta = \cos \beta_H.$$

If θ_S and ϕ_S are the corresponding angles for S, then:

$$S = i \cos \theta_S \cos \phi_S + j \cos \theta_S \sin \phi_S + k \sin \theta_S$$

$$\cos \theta \cos \theta_S \cos (\phi - \phi_S) + \sin \theta_S \sin \theta = \cos \beta_S.$$

Thus for the equations:

$$\cos \theta \cos \theta_H \cos (\phi - \phi_H) + \sin \theta_H \sin \theta = \cos \beta_H$$

$$\cos \theta \cos \theta_S \cos (\phi - \phi_S) + \sin \theta_S \sin \theta = \cos \beta_S.$$

Eliminating $\sin \theta$

$$\cos \theta = \frac{\cos \beta_H \sin \theta_S - \cos \beta_S \sin \theta_H}{\cos \theta_H \sin \theta_S \cos (\phi - \phi_H) - \cos \theta_S \sin \theta_H \cos (\phi - \phi_S)}$$

and eliminating $\cos \theta$

$$\sin \theta = \frac{\cos \beta_S \cos \theta_H \cos (\phi - \phi_H) - \cos \beta_H \cos \theta_S \cos (\phi - \phi_S)}{\cos \theta_H \sin \theta_H \cos (\phi - \phi_S) - \cos \theta_S \sin \theta_H \cos (\phi - \phi_H)}$$

If we let a, b_1, b_2, c_1, c_2 , denote the following expressions:

$$a = \cos \beta_H \sin \theta_S - \cos \beta_S \sin \theta_H$$

$$b_1 = \cos \theta_H \sin \theta_S \cos \phi_H - \cos \theta_S \sin \theta_H \cos \phi_S$$

$$\begin{aligned}b_2 &= \cos \theta_H \sin \theta_S \sin \phi_H - \cos \theta_S \sin \theta_H \sin \phi_S \\c_1 &= \cos \theta_H \cos \beta_S \cos \phi_H - \cos \theta_S \cos \beta_H \cos \phi_S \\c_2 &= \cos \theta_H \cos \beta_S \sin \phi_H - \cos \theta_S \cos \beta_H \sin \phi_S\end{aligned}$$

$$\begin{aligned}\cos \theta &= \frac{a}{b_1 \cos \phi + b_2 \sin \phi} \\(6-1) \quad \sin \theta &= \frac{c_1 \cos \phi + c_2 \sin \phi}{b_1 \cos \phi + b_2 \sin \phi}\end{aligned}$$

The equations (6-1) may be written

$$\begin{aligned}b_1 \cos \theta \cos \phi + b_2 \cos \theta \sin \phi &= a \\(b_1 \sin \theta - c_1) \cos \phi + (b_2 \sin \theta - c_2) \sin \phi &= 0\end{aligned}$$

The solution of these equations is

$$\begin{aligned}(6-2) \quad \sin \phi &= \frac{a}{\cos \theta} \frac{b_1 \sin \theta - c_1}{b_1 c_2 - b_2 c_1} \\ \cos \phi &= - \frac{a}{\cos \theta} \frac{b_2 \sin \theta - c_2}{b_1 c_2 - b_2 c_1} \\ \tan \phi &= - \frac{b_1 \sin \theta - c_1}{b_2 \sin \theta - c_2}\end{aligned}$$

Eliminating ϕ from the equation (6-2)

$$(6-3) \quad \sin \theta = \frac{a^2(b_1 c_1 + b_2 c_2) + (b_1 c_2 - b_2 c_1) \sqrt{(b_1 c_2 - b_2 c_1)^2 + a^2(b_1^2 + b_2^2 - c_1^2 - c_2^2)}}{(b_1^2 + b_2^2) a^2 + (b_1 c_2 - b_2 c_1)^2}$$

Substituting for a, b_1, b_2, c_1, c_2 we find

$$\begin{aligned}(b_1 c_2 - b_2 c_1)^2 + a^2(b_1^2 + b_2^2 - c_1^2 - c_2^2 - a^2) &= a^2 \{ \sin^2 \beta_S \sin^2 \beta_H - (M \cdot S - \cos \beta \\ b_1 c_1 + b_2 c_2 &= \sin \theta_S \cos \beta_S + \sin \theta_H \cos \beta_H - M \cdot S (\cos \beta_H \sin \theta_S + \cos \beta_S \sin \theta \\ b_1 c_2 - b_2 c_1 &= a \cos \theta_H \cos \theta_S \sin (\phi_H - \phi_S) \\ (b_1 c_2 - b_2 c_1)^2 + (b_1^2 + b_2^2) a^2 &= a^2 \{ 1 - (M \cdot S)^2 \}\end{aligned}$$

Thus (6-3) may be written

$$(7-1) \quad \sin \theta = \frac{\sin \theta_S \cos \beta_S + \sin \theta_H \cos \beta_H - M \cdot S (\cos \beta_H \sin \theta_S + \cos \beta_S \sin \theta_H)}{1 - (M \cdot S)^2} \\ \pm \frac{\cos \theta_H \cos \theta_S \sin (\phi_H - \phi_S)}{1 - (M \cdot S)^2} \sqrt{\sin^2 \beta_S \sin^2 \beta_H - (M \cdot S - \cos \beta_S \cos \beta_H)^2}$$

With this determination of ϕ and θ the base vectors e_θ , e_ϕ , and e_r in (1-1)

$$(7-2) \quad \begin{aligned} e_\theta &= -i \sin \theta \cos \phi - j \sin \theta \sin \phi + k \cos \theta \\ e_\phi &= -i \sin \phi + j \cos \phi \\ e_r &= i \cos \theta \cos \phi + j \cos \theta \sin \phi + k \sin \theta \end{aligned}$$

are fully determined, and so also

$M \cdot e_r$, $M \cdot e_\theta$, and $M \cdot e_\phi$, i.e. $\cos \beta_H$, m , n , $S \cdot e_r$, $S \cdot e_\theta$, $S \cdot e_\phi$, $\cos \beta_S$, m_S , n_S ,

Now from (2-1), when $\alpha \neq 0$

$$(7-3) \quad e_r'' = e_r \cos \alpha + (e_\theta \sin w_0 t - e_\phi \cos w_0 t) \sin \alpha$$

and e_r'' is determined for any t , however this last expression does not specify when t is to be counted zero. It is convenient to start counting the time at a time when S_r'' is a minimum, i.e. according to (3-1).

$$(7-4) \quad w_0 t_3 = \arcsin \frac{-m_S}{\sin \beta_S} = \arccos \frac{+n_S}{\sin \beta_S}$$

Let t be defined

$$\begin{aligned} t &= t_3 + T \\ w_0 t &= w_0 t_3 + w_0 T \end{aligned}$$

* see Appendix A for the sign before the radical. pp13-14

$$\begin{aligned}\sin w_0 t &= \sin (w_0 t_3 + w_0 T) + \sin w_0 t_3 \cos w_0 T + \cos w_0 t_3 \sin w_0 T \\ \cos w_0 t + \cos (w_0 T_3 + w_0 T) &= \cos w_0 t_3 \cos w_0 T - \sin w_0 t_3 \sin w_0 T.\end{aligned}$$

Substituting (7-4) in these two equations we find

$$\begin{aligned}(8-1) \quad \sin w_0 t &= \frac{n_S \sin w_0 T - m_S \cos w_0 T}{\sin \beta_S} \\ \cos w_0 t &= \frac{n_S \cos w_0 T + m_S \sin w_0 T}{\sin \beta_S}\end{aligned}$$

Substituting (8-1) in (7-3)

$$(8-2) \quad e_r'' = e_r \cos \alpha + \left[e_\theta \frac{n_S \sin w_0 T - m_S \cos w_0 T}{\sin \beta_S} - e_\phi \frac{n_S \cos w_0 T + m_S \sin w_0 T}{\sin \beta_S} \right] \sin \alpha.$$

3.

ASPECT WITH RESPECT TO THE ROTATING EARTH

In order to find the aspect of the rocket or satellite with respect to the Earth based system of coordinates axes, let w_e be the angular velocity of rotation of the Earth about its axis, t the time in seconds measured from midnight December 31 - January 1. Then if I_1 , I_2 and I_3 are a system of orthonormal base vectors parallel to the X' , Y' , Z' axes respectively, with X' and Y' in the equatorial plane of the Earth, and the X' axis in the Greenwich Meridian plane, and $w_e t$ measured from the X axis or from the base vector i , we have

$$\begin{aligned}I_1 &= i \cos (w_e t - \delta) + j \sin (w_e t - \delta) \\ I_2 &= -i \sin (w_e t - \delta) + j \cos (w_e t - \delta) \\ I_3 &= k\end{aligned}$$

and

$$i = I_1 \cos (w_e t - \delta) - I_2 \sin (w_e t - \delta)$$

$$j = I_1 \sin (w_e t - \delta) + I_2 \cos (w_e t - \delta)$$

$$k = I_3$$

where δ is the angle between the vectors i and I_1 measured clockwise from i to I_1 at midnight December 31 - January 1. Substituting the relations above in (7-2) i.e. in

$$e_r = i \cos \theta \cos \phi + j \cos \theta \sin \phi + k \sin \theta$$

$$e_\theta = -i \sin \theta \cos \phi - j \sin \theta \sin \phi + k \cos \theta$$

$$e_\phi = -i \sin \phi + j \cos \phi,$$

we find

$$\begin{aligned} (9-1) \quad e_r &= I_1 \cos \theta \cos (w_e t - \phi - \delta) - I_2 \cos \theta \sin (w_e t - \phi - \delta) + I_3 \sin \theta \\ e_\theta &= -I_1 \sin \theta \cos (w_e t - \phi - \delta) + I_2 \sin \theta \sin (w_e t - \phi - \delta) + I_3 \cos \theta \\ e_\phi &= I_1 \sin (w_e t - \phi - \delta) + I_2 \cos (w_e t - \phi - \delta). \end{aligned}$$

Now (7-3)

$$(9-2) \quad e_r'' = e_r \cos \alpha + (e_\theta \sin w_o t - e_\phi \cos w_o t) \sin \alpha.$$

Again let t_3 be the time corresponding to the minimum angle for the first Sun fix, then

$$t = t_3 + T.$$

and according to (8-1) we find

$$\begin{aligned} \sin w_o t &= \frac{n_S \sin w_o T - m_S \cos w_o T}{\sin \beta_S} \\ \cos w_o t &= \frac{n_S \cos w_o T + m_S \sin w_o T}{\sin \beta_S} \end{aligned}$$

To simplify these expressions let γ be defined by

$$\tan \gamma = \frac{m_S}{n_S}$$

$$m_S^2 + n_S^2 = \sin^2 \beta_S$$

$$m_S = \sin \beta_S \sin \gamma,$$

$$n_S = \sin \beta_S \cos \gamma$$

then,

$$\sin w_0 t = \sin (w_0 T - \gamma),$$

$$\cos w_0 t = \cos (w_0 T - \gamma).$$

In order to simplify the expression (9-1) and (9-2), let

$$\phi = w_e t - \phi - \delta$$

then the equations (9-1) become,

$$\begin{aligned} (10-1) \quad e_r &= I_1 \cos \phi \cos \theta - I_2 \sin \phi \cos \theta + I_3 \sin \theta \\ e_\theta &= -I_1 \cos \phi \sin \theta + I_2 \sin \phi \sin \theta + I_3 \cos \theta \\ e_\phi &= I_1 \sin \phi + I_2 \cos \phi \end{aligned}$$

and (9-2) becomes

$$e_r'' = e_r \cos \alpha + \{e_\theta \sin (w_0 T - \gamma) - e_\phi \cos (w_0 T - \gamma)\} \sin \alpha$$

The expression for e_r'' in terms of I_1 , I_2 , and I_3 becomes

$$\begin{aligned} (10-2) \quad e_r'' &= I_1 \left[\cos \phi \{ \cos \theta \cos \alpha - \sin \theta \sin \alpha \sin (w_0 T - \gamma) \} - \sin \phi \sin \alpha \cos (w_0 T - \gamma) \right. \\ &\quad - I_2 \left[\sin \phi \{ \cos \theta \cos \alpha - \sin \theta \sin \alpha \sin (w_0 T - \gamma) \} + \cos \phi \sin \alpha \cos (w_0 T - \gamma) \right. \\ &\quad \left. \left. + I_3 \left[\sin \theta \cos \alpha + \cos \theta \sin \alpha \sin (w_0 T - \gamma) \right] \right] \right] \end{aligned}$$

If the rocket or satellite has no precessional motion $\alpha = 0$ and,

$$e_r'' = I_1 \cos \phi \cos \theta - I_2 \sin \phi \cos \theta + I_3 \sin \theta.$$

The expression (10-2) is the representation of the unit vector parallel to the rocket or satellite axis. Thus if V is any vector with components (V_1, V_2, V_3) in the base I_1, I_2 , and I_3 i.e. if

$$V = I_1 V_1 + I_2 V_2 + I_3 V_3,$$

the component of V along e_r'' is given by $e_r'' \cdot V = V_r''$, i.e.

$$\begin{aligned} V_r'' = & V_1 \left[\cos \phi \left\{ \cos \theta \cos \alpha - \sin \theta \sin \alpha \sin (w_0 T - \gamma) \right\} - \sin \phi \sin \alpha \cos (w_0 T - \gamma) \right] \\ & - V_2 \left[\sin \phi \left\{ \cos \theta \cos \alpha - \sin \theta \sin \alpha \sin (w_0 T - \gamma) \right\} + \cos \phi \sin \alpha \cos (w_0 T - \gamma) \right] \\ & + V_3 \left[\sin \theta \cos \alpha + \cos \theta \sin \alpha \sin (w_0 T - \gamma) \right]. \end{aligned}$$

If the rocket or satellite has a Sun sensor mounted on it such that the axis of the sensor makes an angle of 116° with the rocket or satellite axis and if π is the angle which the Sun vector S makes with the axis of the sensor, the relation between the angle $\gamma_S(t) = (\angle S, e_r'')$ and π is given by

$$\begin{aligned} \gamma_S(t) &= 116^\circ - \pi(t) \\ (11-1) \quad \frac{d\gamma_S(t)}{dt} &= - \frac{d\pi}{dt}. \end{aligned}$$

The critical values of $\gamma_S(t)$ are given by

$$\frac{d\gamma_S(t)}{dt} = 0$$

and therefore also by

$$\frac{d\pi}{dt} = 0,$$

and

$$\frac{d^2 \gamma_S(t)}{dt^2} = - \frac{d^2 \pi}{dt^2}.$$

Thus for the maximum values of $\gamma_S(t)$, $\frac{d^2 \gamma_S(t)}{dt} < 0$ and $\frac{d^2 \pi(t)}{dt} > 0$ therefore, $\pi(t)$ is maximum when $\gamma_S(t)$ is minimum and $\pi(t)$ is minimum when $\gamma_S(t)$ is maximum. Now we have seen on page 4 that maximum $\gamma_S(t_3) = \beta_S - \alpha$ and minimum $\gamma_S(t_4) = \beta_S + \alpha$. Thus from (12-1)

$$\begin{aligned} \pi(t) &= 116^\circ - \gamma_S(t) \\ (12-1) \quad \pi(t_3) &= 116^\circ - \beta_S - \alpha \\ \pi(t_4) &= 116^\circ + (\beta_S - \alpha). \end{aligned}$$

Thus if $\beta_S = 0$

$$\begin{aligned} 116^\circ - \beta_S &= \frac{\pi(t_4) + \pi(t_3)}{2} \\ \alpha &= \frac{\pi(t_4) - \pi(t_3)}{2} \end{aligned}$$

and

$$\begin{aligned} \beta_S &= 116^\circ - \frac{\pi(t_4) + \pi(t_3)}{2} \\ (12-2) \quad \alpha &= \frac{\pi(t_4) - \pi(t_3)}{2} \end{aligned}$$

If $\beta_S = \alpha$

$$\begin{aligned} \beta_S &= \frac{\pi(t_4) - \pi(t_3)}{2} \\ \alpha &= 116^\circ - \frac{\pi(t_4) + \pi(t_3)}{2} \end{aligned}$$

The Adcole Sun Sensor, manufactured by the Adcole Research Corporation, is designed so that $\pi < 0$ for $116^\circ < \gamma_S(t) < 180^\circ$, $\pi > 0$ $52^\circ < \gamma_S(t) < 116^\circ$.

APPENDIX A

DETERMINATION OF THE SIGN IN THE EXPRESSION FOR SIN θ

If e_{r_1} and e_{r_2} lie in a plane perpendicular to the M-S plane and make equal angles with that plane, with e_{r_1} on one side and e_{r_2} on the other side of the M-S plane, then

$$e_{r_1} \times (M \times S) = e_{r_2} \times (M \times S).$$

Thus

$$(e_{r_1} \cdot S)M - (e_{r_1} \cdot M)S = (e_{r_2} \cdot S)M - (e_{r_2} \cdot M)S$$

and

$$\cos \beta_H = e_{r_1} \cdot M = e_{r_2} \cdot M, \quad \cos \beta_S = e_{r_1} \cdot S = e_{r_2} \cdot S.$$

One root of $\sin \theta$ corresponds to e_r on one side of the M-S plane and the other root corresponds to e_r on the other side of the M-S plane. Now it is easy to show that the triple scalar product of M, S, e_r

$$(MSe_r) = \pm \sqrt{\sin^2 \beta_S \sin^2 \beta_H - (M \cdot S - \cos \beta_S \cos \beta_H)^2}.$$

Obviously if the angle between $M \times S$ and e_r is less than 90° the + sign has to be taken for (M, S, e_r) and the - sign if e_r makes an angle greater than 90° .

Let γ_{H_0} and γ_{H_1} correspond to the maximum and minimum angles between the axis of the rocket or satellite and the magnetic field respectively, and γ_{S_2} and γ_{S_3} the corresponding angles between the Sun vector and the rocket or satellite axis respectively. If the rocket is precessing in a clockwise direction with respect to an observer on the ground, then in the records of γ_H and γ_S transmitted to the ground the following sequences will be observed for γ_{H_0} , γ_{H_1} , γ_{S_2} , γ_{S_3} :

If e_r makes an angle of less than 90° with MxS the sequence of maximum and minimum angles will be as follows:

$$\gamma_{S_3}, \gamma_{H_0}, \gamma_{S_2}, \gamma_{H_1},$$

that is, if this sequence occurs in a given flight the sign + must be taken. If e_r makes an angle greater than 90° with MxS , the sequence of maximum and minimum will be as follows:

$$\gamma_{S_3}, \gamma_{H_1}, \gamma_{S_2}, \gamma_{H_0},$$

in this case then, the negative sign before the radical must be taken. The system breaks down only if $(MSe_r) = 0$, but in this case there is only one root for $\sin \theta$, namely,

$$\sin \theta = \frac{\sin \theta_S \cos \beta_S + \sin \theta_H \cos \beta_H - M \cdot S (\cos \beta_H \sin \theta_S + \cos \beta_S \sin \theta_H)}{1 - (M \cdot S)^2}.$$

AF Cambridge Research Laboratories, Bedford Mass. Geophysics Research Directorate. ON DETERMINATION OF THE ORIENTATION OF THE AXIS OF A ROCKET OR SATELLITE IN ITS TRAJECTORY OR ORBIT. by R.J. Marcou, October 1963. 14 pp. AFCRL -63-871 Unclassified report. A method is developed for determining the aspect of the axis of a rocket or satellite with respect to an earth based system of coordinates for the case where these bodies undergo a constant precessional motion about some fixed direction. The analysis is based on data obtained from a magnetometer mounted on the body so as to give the axial component of the Earth's magnetic field and a sun sensor which measures the angle between the Sun vector and the axis of the body.	UNCLASSIFIED 1. Equations 2. Kinematics 3. Aspect of rocket or satellite	AF Cambridge Research Laboratories, Bedford Mass. Geophysics Research Directorate. ON DETERMINATION OF THE ORIENTATION OF THE AXIS OF A ROCKET OR SATELLITE IN ITS TRAJECTORY OR ORBIT. by R.J. Marcou, October 1963. 14 pp. AFCRL -63-871 Unclassified report. A method is developed for determining the aspect for the axis of a rocket or satellite with respect to an earth based system of coordinates for the case where these bodies undergo a constant precessional motion about some fixed direction. The analysis is based on data obtained from a magnetometer mounted on the body so as to give the axial component of the Earth's magnetic field and a sun sensor which measures the angle between the Sun vector and the axis of the body.	UNCLASSIFIED 1. Equations 2. Kinematics 3. Aspect of rocket or satellite
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