AFCRL-63-394
AD 425 301

ADMITTANCE OF A WAVEGUIDE RADIATING INTO STRATIFIED PLASMA

SCIENTIFIC REPORT NO. 2

CONTRACT NO. AF19(628)-2410

AUGUST 1963

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
LAURENCE G. HANSCOM FIELD
BEDFORD, MASSACHUSETTS

APPLIED RESEARCH LABORATORY

SYLVANIA ELECTRONIC SYSTEMS
Government Systems Management
for GENERAL TELEPHONE & ELECTRONICS

AFCRL-63-394

ADMITTANCE OF A WAVEGUIDE RADIATING INTO STRATIFIED PLASMA

By

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August 1963

Contract No. AF19(628)-2410
Project No. 4642
Task No. 464202

SCIENTIFIC REPORT NO. 2

Air Force Cambridge Research Laboratories
Office of Aerospace Research
Laurence G. Hanscom Field
Bedford, Massachusetts

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ABSTRACT

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY Applied Research Laboratory Sylvania Electronic Systems, A Division of Sylvania Electric Prod. Inc., 40 Sylvan Rd, Waltham 54, Massachusetts

2a. REPORT SECURITY CLASSIFICATION Unclassified

2b. GROUP (For ASTIA use only)

3. REPORT TITLE

Admittance of a Waveguide Radiating into Stratified Plasma

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Scientific Report No. 2

5. AUTHOR(S) (Last name, first name, initial)

Galejs, Janis

6. PUBLICATION DATE August 1963

7. TOTAL NO OF PAGES 59

8. ORIGINATOR'S REPORT NO(S)

S-3040-2

9a. CONTRACT OR GRANT NO

AF19(628)-2410

b. PROJECT NO 4642

c. TASK NO 464202

10. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

AFCRL-63-394

11. SUPPLEMENTARY NOTES (For ASTIA use only)

12. RELEASE STATEMENTS (For ASTIA use only)

13. AUTHORS' KEY TERMS - UNCLASSIFIED ONLY

1. Slot Antennas	7.	13.
2. Slot Admittance	8.	14.
3. Plasma Layers	9.	15.
4. Reentry Communication	10.	16.
5. Layers of Lossy Dielectric	11.	17.
6. Stratified Plasma	12.	18.

14. ASTIA DESCRIPTORS (For ASTIA use only)

15. IDENTIFIERS - UNCLASSIFIED ONLY (e.g., Model numbers; weapon system, project, chemical compound and trade names)

16. BODY OF ABSTRACT *(Including if applicable, Purpose; Method or approach; Results; Conclusions, applications and recommendations.)*

A slot covered by a stratified plasma is assumed to radiate into a wide waveguide instead of free space. The slot admittance approximates the free space admittance of the slot for waveguide diameters exceeding 6 to 10 λ. For thick plasma layers the computed slot admittance checks with earlier admittance calculations for a laterally unbounded plasma. When approximating a plasma profile of a typical hypersonic re-entry, a multi-layer plasma model in a wide waveguide appears to provide a more accurate slot admittance than a single layer approximation in a laterally unbounded geometry.

17. INDEXING ANNOTATION

The admittance is obtained for a rectangular slot antenna covered by a stratified plasma approximating a typical re-entry. radiating into a wide rectangular waveguide instead of free space.

ADMITTANCE OF A WAVEGUIDE RADIATING INTO STRATIFIED PLASMA

I. INTRODUCTION

The admittance of a waveguide or cavity backed slot which radiates into a homogeneous isotropic plasma or dielectric layer of a thickness that is larger than the wavelength has been recently computed by Galejs [1]. This paper also contains numerous references to earlier work on related problems. The formulation [1] is rather involved and it is not suited for determining the admittance for thin or stratified plasma layers.

In this latter problem it may be advantageous to invoke geometrical approximations and to consider the admittance of a plasma or dielectric covered slot which radiates into a wide waveguide instead of free space. Cohn and Flesher [2] have discussed radiation from a coaxial line into a waveguide the walls of which are removed to infinity, and their results are in agreement with other analyses not employing such an artificial boundary [Levine and Papas, 3]. The waveguide boundary greatly simplifies the admittance calculations for a plasma or dielectric covered slot, if it is permissible to use a waveguide of finite diameter. Heuristic reasoning suggests that the fields in the vicinity of the slot should not be affected by the presence of the waveguide walls if the distance between the walls is large enough. Hence, this method can be expected to yield realistic susceptances and also realistic estimates of losses in the plasma layer. However, the radiation fields of the slot will be modified by the waveguide walls. This model is therefore not suitable for determining radiation patterns of the slot and criteria must be developed for establishing the validity of conductance estimates. Also a large number of terms is required in the field representations for large waveguide cross-sections, which makes this approach useful only if a highspeed computer is available for the numerical work.

The specific problem to be considered in this paper is the admittance of a rectangular waveguide, excited in its principal mode which radiates into a wide waveguide containing layered isotropic plasma of arbitrary parameters. For the purposes of this paper the plasma is represented as a lossy dielectric of a relative dielectric constant less than unity. This simple representation is valid only in low power applications. The analytical formulation of the slot admittance can be considered as straight forward, and only the principal results of the development are summarized in Section II. Convergence considerations and the numerical results will be discussed in Section III, which contains also a detailed comparison with admittance calculations for a laterally unbounded plasma layer [1].

II. SLOT ADMITTANCE

2.1 The general formulation

The waveguide geometry under consideration is depicted in Figure 1. A small rectangular waveguide of cross-sectional area $x_i y_i$ is joined symmetrically to a larger waveguide of area $x_o y_o$, by a symmetrical aperture of width 2ϵ and length 2ℓ. The smaller guide is excited in the principal (TE_{10}) mode, but the change of the waveguide cross-section and the dielectric discontinuities generate a large number of TE and TM modes which must be considered in the formulation of the slot admittance. Such calculations have been carried out in Appendix A and the following stationary expression for the normalized slot admittance y_s is derived:

$$y_s = \frac{x_i y_i}{\gamma_{i1} D^2} \left\{ \sum_n \sum_m{}' \frac{\epsilon_m}{x_i y_i \gamma_i} \left[\left(\beta_{xi}{}^2 - k_i{}^2\right)\left(\iint E_{yi}\right)^2 + \left(k_i^2 - \beta_{yi}{}^2\right)\left(\iint E_{xi}\right)^2 \right] + \sum_n \sum_m \frac{\epsilon_m}{x_o y_o \gamma_p} \left[F(R) \left(\iint E_{yp}\right)^2 + G(R) \left(\iint E_{xp}\right)^2 \right] \right\} \qquad (1)$$

where

$$F(R) = \frac{1}{\beta_{xp}^2 + \beta_{yp}^2} \left[\gamma_p^2 \beta_{xp}^2 \frac{1 - R_a}{1 + R_a} - k_p^2 \beta_{yp}^2 \frac{1 + R_b}{1 - R_b} \right] \qquad (2)$$

$$G(R) = \frac{1}{\beta_{xp}^2 + \beta_{yp}^2} \left[- \gamma_p^2 \beta_{yp}^2 \frac{1 - R_a}{1 + R_a} + k_p^2 \beta_{xp}^2 \frac{1 - R_b}{1 - R_b} \right] \qquad (3)$$

$$D = \iint E_y(x', y') \sin \beta_{xi1} x' \, dx' \, dy' \qquad (4)$$

and

$$\iint E_{xj} = \iint E_x \cos \beta_{xj} x \sin \beta_{yj} y \, dx \, dy$$

$$\iint E_{yj} = \iint E_y \sin \beta_{xj} x \cos \beta_{yj} y \, dx \, dy$$

E_x and E_y refer to the aperture fields at $z = 0$ and $j = i$ or p. Also, the primed coordinates are used with subscript i, and the unprimed with p. The other symbols are defined as

$$\beta_{xi} = \frac{n\pi}{x_i} \qquad \beta_{yi} = \frac{m\pi}{y_i}$$

$$\beta_{xi1} = \frac{\pi}{x_i}$$

$$\beta_{xp} = \frac{n\pi}{x_o} \qquad \beta_{yp} = \frac{m\pi}{y_o}$$

$$k_j = \sqrt{\omega^2\mu_o\epsilon_j + i\omega\mu_o\sigma_j}$$

$$\gamma_j = i\sqrt{k_j^2 - \beta_{xj}^2 - \beta_{yj}^2}$$

$$\gamma_{il} = i\sqrt{k_i^2 - \beta_{xil}^2}$$

$$\epsilon_m = \begin{cases} 1 & \text{for } m = 0 \\ 2 & \text{for } m \neq 0 \end{cases}$$

The prime on the double summation (1) designates the omission of its $n = 1$, $m = 0$ term. The reflection coefficients of the TE modes R_a and of the TM modes R_b depend on the dielectric structure of the larger waveguide. For stratified dielectric layers shown in Figure 2 the reflection coefficient R_{dj} ($d = a$ or b) in the region of $z_{j+1} \leq z \leq z_j$ is related to the reflection coefficient $R_{d(j-1)}$ of the region $z_j \leq z \leq z_{j-1}$ by the expressions

$$R_{aj} = e^{2\gamma_j z_j} \frac{\left[e^{2\gamma_{j-1} z_j} + R_{a(j-1)}\right] - \frac{\gamma_{j-1}}{\gamma_j}\left[e^{2\gamma_{j-1} z_j} - R_{a(j-1)}\right]}{\left[e^{2\gamma_{j-1} z_j} + R_{a(j-1)}\right] + \frac{\gamma_{j-1}}{\gamma_j}\left[e^{2\gamma_{j-1} z_j} - R_{a(j-1)}\right]} \quad (5)$$

$$R_{bj} = e^{2\gamma_j z_j} \frac{\left(\frac{k_{j-1}}{k_j}\right)^2 \left[e^{2\gamma_{j-1} z_j} + R_{b(j-1)}\right] - \frac{\gamma_{j-1}}{\gamma_j}\left[e^{2\gamma_{j-1} z_j} - R_{b(j-1)}\right]}{\left(\frac{k_{j-1}}{k_j}\right)^2 \left[e^{2\gamma_{j-1} z_j} + R_{b(j-1)}\right] + \frac{\gamma_{j-1}}{\gamma_j}\left[e^{2\gamma_{j-1} z_j} - R_{b(j-1)}\right]} \quad (6)$$

R_{dj} is derived by considering the scalar functions (54) and (55) in two adjacent layers and by requiring the tangential field components (56), (57), (59) and (60) to be continuous across the interface. The computations should start with $j = 1$ where $R_{d(j-1)} = R_{do} = 0$ in (5) and (6). A series of computations gives then R_{d1}, R_{d2}, R_{d3} and finally R_{dp} as R_a or R_b in (2) and (3).

This completes the formal specification of the slot admittance y_s in terms of the electric field components E_y and E_x in the aperture at $z = 0$, and it remains to find a compatible set of E_y and E_x for the aperture.

In deriving the slot admittance the continuity of the tangential electric field components is assured by the definition of mode amplitudes following (65) and (66). The continuity of the magnetic field components H_x and H_y is specified by (68) and (69). The above equations still apply to the derivation of y_s as E_x is made very small, and the amplitude of the TE and TM modes of the waveguide may be selected in such a way that $\left(\iint E_{xj}\right)^2$ is negligible relative to $\left(\iint E_{yj}\right)^2$ in (1). The admittance is now determined by specifying only a single electric field component E_y in the aperture, which is in line with earlier work on radiation from waveguide backed slots in absence of dielectric discontinuities (Lewin [4], Stevenson [5], Watson [6]). $E_x = 0$ has been also used in computations for radiation across a dielectric discontinuity from an oil filled guide into free space, and the accuracy of such calculations have been checked experimentally (Figures 4 and 5 of Cohen, Crowley and Levis [7]). It may be noted that the continuity condition of H_y in (69) leads after multiplication with E_x and an integration to the terms proportional $\left(\iint E_{xj}\right)^2$ in (1). The continuity of H_y is not required for deriving y_s with $E_x = 0$, and it is possible that simple trial functions of the aperture fields which violate the condition of H_y continuity still lead to nearly correct results of y_s, if $\left(\iint E_{xj}\right)^2$ is equal to zero in this approximation.* The problem of non-physical solutions is avoided if a neglect of $\left(\int\int E_{xj}\right)^2$ is interpreted as E_x negligible, but non-zero in the cases where $E_x = 0$ is not a strict solution. The calculations of Section 2.2 are based on (1) with E_x negligible relative to E_y.

It is also possible to select reasonable trial functions with $E_x \neq 0$. When specifying the aperture field component H_x as that of the principal waveguide mode ($H_x = H_o \sin \beta_{xi1} x'$) in a completely open smaller guide, $E_x \neq 0$ if y variations are allowed for the E_y component. In this particular case the E_x integrals of the slot admittance in (1) may be expressed in terms of E_y integrals, and a separate trial function is not required for the E_x component. This development, which is carried out in Appendix B, is summarized in Section 2.3. It is shown that the presence of the E_x component gives only a minor correction to the real part of y_s, but the net change of y_s remains to be determined by numerical computations.

2.2 Slot Admittance with a Negligible E_x component.

With $E_x(x,y)$ negligible relative to $E_y(x,y)$ the electric fields in the slot are assumed to be of the form

$$E_y(x,y) = \left[Af_A(x) + Bf_B(x) \right] f(y) \tag{7}$$

where A and B are the undetermined amplitudes of the trial functions $f_A(x)$ and $f_B(x)$ and where $f(y)$ is yet arbitrary. Substitution of (7) in (1) results

* Such an approximate solution is obtained by assuming $E_x = 0$ in a plasma layer of infinite thickness, and an aperture of of $2\ell = x_i$, $2\epsilon = y_i$ when $E_y = f(y)$ violates the condition of continuous H_y (See Appendix D). However, numerical calculations show that y_s is nearly the same in this case as for $E_x = 0$, $E_y \neq f(y)$ and $E_x \neq 0$, $E_y = f(y)$ where H_y is continuous. (see Figure 15).

$$y_s = \frac{x_i y_i}{\gamma_{i1}} \frac{A^2\gamma_{AA} + 2AB\gamma_{AB} + B^2\gamma_{BB}}{\left[AF_A + BF_B\right]^2} \tag{8}$$

where

$$\gamma_{MN} = \gamma_{MN}^{\ i} + \gamma_{MN}^{\ p} \tag{9}$$

$$\gamma^j_{MN} = \left(\frac{1}{2k}\right)^2 \iint_{(slot)} dx\, dy\, f_M(x)\, f(y) \iint_{(slot)} d\xi\, d\eta\, f_N(\xi)\, f(\eta)\, G_j \tag{10}$$

$$F_M = \frac{1}{2k} \iint_{(slot)} f_M(x')\, f(y')\, f(y')\, \sin \beta_{xi1} x'\, dx'\, dy' \tag{11}$$

and where G_i and G_p represent the two double summations of (70). Considering the ratio of (A/B) as a variable in (12), the requirement that y_s be stationary leads to

$$\frac{dy_s}{d(A/B)} = 0 \tag{12}$$

Equation (12) is solved for A/B

$$\frac{A}{B} = \frac{\gamma_{BB} F_A - \gamma_{AB} F_B}{\gamma_{AA} F_B - \gamma_{AB} F_A} \tag{13}$$

Substituting (13) in (8) it follows that

$$y_s = \frac{x_i y_i}{\gamma_{i1}} \frac{\gamma_{AA}\gamma_{BB} - \gamma^2_{AB}}{\Delta} \tag{14}$$

with

$$\Delta = F^2_B \gamma_{AA} - 2F_A F_B \gamma_{AB} + F^2_A \gamma_{BB} \tag{15}$$

The slot admittance (14) has been computed for the field distribution $E_y(x,y)$ of (7) with "optimized" coefficients A and B. The trial functions $f_M(x)$ of (7) should satisfy the condition

$$f_M(\frac{x_o}{2} \pm \ell) = 0 \qquad (16)$$

Suitable trial functions are [1,8]

$$f_A(x) = \sin\left\{ k \left[\ell - \left| x - \frac{x_o}{2} \right| \right] \right\} \qquad (17)$$

and

$$f_B(x) = 1 - \cos\left\{ k \left[\ell - \left| x - \frac{x_o}{2} \right| \right] \right\} \qquad (18)$$

The parameter k of the trial functions in (17) and (18) may be made equal to k_i, k_p, k_o or kept arbitrary. The y variation of $E_y(x,y)$ is neglected and f(y) becomes

$$f(y) = -\frac{1}{2\epsilon} \qquad (19)$$

Evaluation of the integrals (10) and (11) results in

$$\gamma^i_{AA} = \sum_{n=1}^{\infty} F_n (g'_{ni})^2 \qquad (20)$$

$$\gamma^i_{AB} = \sum_{n=1}^{\infty} F_n g'_{ni} g''_{ni} \qquad (21)$$

$$\gamma^i_{BB} = \sum_{n=1}^{\infty} F_n (g''_{ni})^2 \qquad (22)$$

$$F_n = \sum_{m=o}^{\infty}{}' \frac{\epsilon_m}{x_i y_i \gamma_i} \frac{\beta^2_{xi} - k^2_i}{(k^2 - \beta^2_{xi})^2} \left(\frac{\sin \beta_{yi}\epsilon}{\beta_{yi}\epsilon}\right)^2 \qquad (23)$$

$$\gamma^p_{AA} = \sum_{n=1}^{\infty} H_n (g'_{np})^2 \qquad (24)$$

$$\gamma_{AB}^{p} = \sum_{n=1}^{\infty} H_n g'_{np} g''_{np} \tag{25}$$

$$\gamma_{BB}^{p} = \sum_{n=1}^{\infty} H_n (g''_{np})^2 \tag{26}$$

$$H_n = \sum_{m=o}^{\infty} \frac{\epsilon_m}{x_o y_o \gamma_p} \frac{F(R)}{(k^2 - \beta_{xp}^2)^2} \left(\frac{\sin \beta_{yp}\epsilon}{\beta_{yp}\epsilon}\right)^2 \tag{27}$$

$$F(R) = \frac{1}{\beta_{xp}^2 + \beta_{yp}^2} \left[\gamma_p^2 \beta_{xp}^2 \frac{1 - R_{ap}}{1 + R_{ap}} - k_p^2 \beta_{yp}^2 \frac{1 + R_{bp}}{1 - R_{bp}} \right] \tag{28}$$

$$g'_{nj} = \cos \beta_{xj}\ell - \cos k\ell \tag{29}$$

$$g''_{nj} = \frac{k}{\beta_{xj}} \sin \beta_{xj}\ell - \sin k\ell \tag{30}$$

$$F_A = g'_{1i}/(k^2 - \beta_{xi1}^2) \tag{31}$$

$$F_B = g''_{1i}/(k^2 - \beta_{xi1}^2) \tag{32}$$

In the n and m summations n is restricted to odd and m to even integers. The computations are particularly simple when the width of the slot 2ϵ is equal to the height of the small guide y_i; F_n of (23) contains only the m=o term in this case.

The slot admittance can be also computed for the sinusoidal field distribution $f_A(x)$ of (17) with $B = 0$ in (7). This gives

$$y_s = \frac{x_i y_i}{\gamma_{i1}} \frac{\gamma_{AA}}{F_A^2} \tag{33}$$

2.3 Slot Admittance with a Non-Negligible E_x Component

The discussion of this Section is restricted to small guides of $y_i = 2\epsilon$ and $x_i = 2\ell$ with an assumed magnetic field distribution $H_x = H_o \sin \beta_{xi1} x'$ in the aperture at z = 0. The absence of higher order modes of H_x in the aperture establishes a relation between the amplitudes a_{nm} and b_{nm} of TE and TM modes following (52), (53) and (59). Applying this result to (65) and (66) $\iint E_{xi}$ is related to $\iint E_{yi}$ by

$$\iint E_{xi} = \frac{\beta_{xi}^2 - k_i^2}{\beta_{xi}\beta_{yi}} \iint E_{yi} \tag{34}$$

for $m \neq 0$. Also the amplitudes of the TE_{no} modes of the smaller guide are equal to zero and E_y exhibits the same x-variation as H_x but it may have a different y-variation. With the relative amplitudes of TE and TM modes determined, E_x of (56) can be also expressed in terms of ($\iint E_{yi}$). Hence $\iint E_{xp}$ of (1) can be related to E_y. These calculations have been carried out in the Appendix C. The integral $\iint E_{xp}$ is computed as

$$\iint E_{xp} = \frac{2x_i^3 x_o V_o}{\pi^2 \beta_{yp}} \frac{n(\beta_{xi1}^2 - k_i^2)(-1)^{\frac{n-1}{2}}}{x_o^2 - n^2 x_i^2} \cos \beta_{xp}\ell \tag{35}$$

$$.(-1)^{\frac{m}{2}} \left[\frac{\sin \beta_{yp}\epsilon}{\beta_{yp}\epsilon} - Q(\beta_{yp}\epsilon) \right]$$

where V_o is the potential across the slot and Q(w) is defined as

$$Q(w) = Q(\beta\epsilon) = -\frac{2}{V_o} \int_o^\epsilon E_y(u) \cos \beta u \, du \tag{36}$$

with

$$u = y' - \epsilon \tag{37}$$

The integrals of E_{yp}, E_{yi} and D are expressed as

$$\iint E_{yp} = \frac{2x_i x_o^2 V_o (-1)^{\frac{n-1}{2}}}{\pi(x_o^2 - n^2 x_i^2)} (-1)^{\frac{m}{2}} \cos \beta_{xp} \ell \; Q(\beta_{yp} \epsilon) \tag{38}$$

$$\iint E_{yi} = \begin{cases} \frac{V_o x_i}{2} (-1)^{\frac{m}{2}+1} Q(\frac{m\pi}{2}) & \text{for } n = 1,\ m \neq o \\ 0 & \text{for } n \neq 1 \end{cases} \tag{39}$$

$$D = - V_o x_i / 2 \tag{40}$$

The above integrals depend on the function Q(w) or on the E_y variation in the y direction. The static solution for the waveguide profile of Figure 1 with $x_o \gg x_i$, carried out in Appendix B, shows that in the vicinity of the waveguide edge at $u = \epsilon - \delta (\delta \to o)$ the electric field component E_y is represented by

$$E_y = - V_o \frac{\sqrt{3}}{4} \left(\frac{2}{3\pi\epsilon^2 \delta} \right)^{1/3} \tag{41}$$

On the symmetry line of the guide at u = o it is

$$E_y = - \frac{V_o}{2\epsilon} \; 0.833 \tag{42}$$

This field variation will be approximated by

$$E_y(u) = \frac{A + B\, u^2/\epsilon^2}{(\epsilon^2 - u^2)^{1/3}} \; (-V_o/\epsilon^{1/3}) \tag{43}$$

Substituting (43) in (36), Q(w) can be evaluated using the integrals (1.3.8) and (1.1.5) of Erdélyi [9]. A calculation shows that

$$Q(w) = A\, Q_1(w) + B\, Q_2(w) \tag{44}$$

where

$$Q_1(w) = 2^{1/6} \; \Gamma(2/3) \sqrt{\pi} \; \frac{J_{1/6}(w)}{w^{1/6}} \tag{45}$$

$$Q_2(w) = 2^{1/6} \, \Gamma(2/3) \sqrt{\pi} \left[\frac{J_{1/6}(w)}{w^{1/6}} - \frac{4}{3} \frac{J_{7/6}(w)}{w^{7/6}} \right] \tag{46}$$

and $J_n(w)$ is the Bessel function of first kind of order n and $\Gamma(z)$ is the gamma function. The constants A and B are determined by requiring that E_y of (43) is equal to (41) near the edge and that the integral of $E_y(u)$ across the aperture is equal to $-V_o$ (or $Q(w) = 1$ for $\beta = o$). It follows that

$$A = \frac{7}{4} \frac{\Gamma(7/6)}{\sqrt{\pi}\,\Gamma(2/3)} - \frac{21}{56} \left(\frac{\sqrt{3}}{2\pi}\right)^{1/3} \tag{47}$$

$$B = - A + \frac{1}{2} \left(\frac{\sqrt{3}}{2\pi} \right)^{1/3} \tag{48}$$

With A and B determined as above, the $E_y(o)$ of (43) is

$$E_y(o) = - \frac{V}{2\epsilon} \; 0.852 \tag{49}$$

which is close to the correct static value of (42). The slot admittance y_s may be computed after substituting (34) to (40) into (1). The summations are extended over even integers m and odd integers n, except forthe primed summation where $n = 1$ because of (39).

The static solution for the slot in an infinitely thin screen gives $E_y(u) = - (V_o/\pi)(\epsilon^2 - u^2)^{-1/2}$ and $Q(w) = J_o(w)$. This will give higher values of integrals than the more accurate forms (43) and (44). When ignoring the y variation of $E_y(x,y)$ in the aperture $Q(w)$ becomes sin w/w and the integrals $\iint E_{xp}$, $\iint E_{yi}$ and $\iint E_{xi}$ are equal to zero according to (35), (39) and (34). The slot admittance computed from (1) using this particular form of $Q(w)$ in (38) is the same as (33) in the limit of $k \to \beta_{xi1}$.

The error of y_s which results from the neglect of E_x in the aperture (E_x may be non-negligible for $E_y(u)$ of (43)) can be estimated by computing the ratio between terms proportional to $\iint E_{xj}$ and $\iint E_{yj}$.

Using the small argument approximations of $\sin w/w$ and of $Q(w)$ of (44), where $w = \beta_{yp}\epsilon$, it follows that

$$\frac{\iint E_{xp}}{\iint E_{yp}} = \frac{n[\pi^2 - (k_i x_i)^2]}{12\ m} \quad 0.032 w^2 \tag{50}$$

$$\approx 0.008\pi^2 \quad nm \left(\frac{y_i}{y_o}\right)^2 \quad \text{for } w < 1$$

For $y_o/y_i > 30$, $\iint E_{xp}/\iint E_{yp} < 0.1$ for $nm < 1140$. This range of nm encompasses the more significant terms of (1) and the terms proportional to $(\iint E_{xp})^2$ will cause only a minor change in y_s, as long as $F(R)$ and $G(R)$ remain of approximately the same magnitude.

The ratio of the terms proportional to $(\iint E_{xi})^2$ and $(\iint E_{yi})^2$ in the primed summation of (1) is computed with the aid of (34) as

$$\frac{k_i^2 - \beta_{yi}^2}{\beta_{xi1}^2 - k_i^2} \; \frac{(\iint E_{xi})^2}{(\iint E_{yi})^2} \approx \frac{k_i^2 - \beta_{xi1}^2}{\beta_{xi1}^2} \tag{51}$$

For $x_i \geq 0.6\lambda$ the above ratio is more than 0.44, and the terms proportional to $(\iint E_{xi})^2$ are significant, particularly for larger values of x_i. This summation is real and therefore contributes only to the imaginary part of y_s in (1) (γ_{i1} is imaginary).

The presence of an E_x component will give only a minor correction to Re y_s ; the change in Im y_s will depend on the ratio of the primed summations to the other summations and will be examined by numerical calculations.

III DISCUSSION

The slot admittance y_s has been derived in Section 2.2 with a two term trial function for the aperture field component E_y, which was assumed to exhibit no y-variations. E_x was neglected in these calculations. The numerical results shown in Figures 3 to 11 and 13 to 14 are obtained using this formulation. The derivation of Section 2.3 assumes the principal mode H_x distribution in the slot and it allows y-variations of the E_y component. $E_x \neq 0$ in this approach. The results obtained by using the two different formulations are compared in Table 1 and in Figures 15 to 17.

The numerical calculations will illustrate first the variations of the slot admittance when a slot radiates into an open waveguide of increasing size. Calculations were made for x_o/x_i increasing in increments of 0.1 and $y_o = x_o$. The slot admittance of Figure 3 exhibits systematic variations for smaller values of x_o/x_i, but the admittance points appear to be scattered with smaller deviations for $x_o/x_i > 20$, although there are several widely scattered points. Also, there are deviations in both admittance components G and B, which is contrary to the elementary reasoning presented in Section I. Examination of (14) shows that y_s may become infinite if $H_n = \infty$ in (24) to (27) which occurs in absence of plasma ($R_a = R_b = o$) at the mode cut-off when $\gamma_p = \gamma_o \to 0$. (With a plasma layer R_a, $R_b \neq 0$. If $\gamma_o \neq 0$ and $\gamma_p \to 0$, R_a and $R_b \to -1$, but the ratios $\gamma_p/(1 + R_a)$ and $(1 + R_b)/\gamma_p$ and also H_n remain finite. With a single plasma layer and $\gamma_o \to 0$, H_n may become infinite only if $\exp(2\gamma_p z_o) = \pm 1$). Considering the free space admittance, $H_n = \infty$

occurs for $x_o = \frac{\lambda}{2}\sqrt{n^2 + m^2}$ where n = odd and m = even integers. These values of x_o are indicated by vertical lines in Figure 3 for the smaller values of x_o/x_i. The above values of x_o correspond to the apparent discontinuity points of the slot admittance. The widely scattered admittance points for larger values of x_o/x_i are also very close to these cut-off values of x_o. Although the separation between the cut-off points of the modes is decreased for increasing values of x_o in Figure 3, one has to approach the cut-off point more closely to obtain a sizeable deviation from the mean of the calculated admittance figures. This accounts for the smaller number of widely scattered admittance points for x_o/x_i large. The free space admittance of the present slot is y = 0.63 - i 0.35 following Lewin [4] or y = 0.65 - i 0.38 following Galejs [1]. The waveguide model is therefore effective for obtaining an approximation to the free space slot admittance provided the ratio of slot length to the outer waveguide dimension exceeds 10. In order to exclude the widely scattered admittance points from consideration, the slot admittance is required to be a continuous, gradually changing function of the guide dimensions in the vicinity of the selected outer guide dimension.

Examples of admittance calculations are shown for x_o/x_i = 10.5 and 19.5 in Figures 4 to 9 for a single layer plasma of relative dielectric constant ϵ_p/ϵ_o, less tangent $\tan\delta_p$ and thickness $h = z_p$. The slot admittance starts out with a free space admittance for h/λ much less than 1 and approximates the slot admittance of a semi-infinite plasma layer for h/λ much larger than 1. Although the admittance variation is relatively smooth for h/λ much less than 1, it becomes oscillatory for h/λ larger than 1. For relatively thin layers of plasma the admittance is less dependent of guide dimensions, if the plasma layer provides a sharp discontinuity with respect to free space (e.g., $\epsilon_p/\epsilon_o \ll 1$). In this case the guide admittance changes are determined principally by the effects of the nearby plasma and free space interface, with guide walls being of only a minor significance. However, for thick plasma layers the plasma and air interface is at a distance comparable to the distance from slot to the guide walls and the admittance depends more on guide dimensions. Also, there are oscillations of the admittance for h/λ larger than 1. The slot admittance for a semi-infinite homogeneous plasma of Galejs [1], which is indicated dashed in Figures 4 to 9, is approximately equal to the average of the oscillatory conductance curve for h and x_o/x_i large. The admittance curves of Figures 8 and 9 for $x_o = y_o = 19.5\ x_i$ appear to approximate the admittance of the semi-infinite plasma closer than Figures 6 and 7 for $x_o = y_o = 10.5\ x_i$. For $h > \lambda$ the slot admittance is about the same as the slot admittance for a semi-infinite plasma layer, provided the conductivity oscillation is smoothed out. The slot admittance tends toward the free slot admittance for $h < \lambda/2$. The above conclusions support the findings of plasma layers analysis of Galejs [1] which was strictly applicable only to layers of $h \gg \lambda$.

The variational approximation to the voltage distribution along the slots is computed from (7) for the slot and plasma parameters of Figures 4, 5, 8, 9 and are shown in Figure 10. The voltage distribution is seen to change only slightly with varying plasma parameters. These calculations are made with $k = k_o$ in the trial functions (17) and (18). For $k = k_p$ the voltage distribution is changed for the same plasma parameters to the form indicated in Figure 11. These distributions are about the same as for the laterally unbounded plasma in Figure 7 of Galejs [1]. The calculated admittance is nearly the same for both values of k of the trial functions, as may be seen from the examples of Table 1.

The accuracy of the voltage distribution shown in Figures 10 and 11 can be strictly determined only by comparing these variational approximations with a more accurate result, if this were available. However, the assumed form of trial function is general enough to allow for wide variations of the field distribution along the slot, such as observed in cavity backed slot antennas [Galejs 1, 7]. The close resemblance between the voltage distributions computed with $k = k_o$ and k_p suggests that the true distribution may be intermediate. These distributions may be approximated by the principal waveguide mode ($E_y \sim \sin \pi x'/x_i$) for $x_i = 2\ell$.

It was also pointed out that the present model is suited for computing the effects of a stratified dielectric or plasma layer. This will be shown for a re-entry plasma sheath of Figure 20 by Rotman and Meltz [8] which is reproduced in Figure 12. The relative dielectric constant and the loss tangent are computed from standard formulas for various frequencies below and above the gyrofrequency of plasma. The slot dimensions are kept constant in relation to the wavelength. The perturbation of the plasma profile by the antenna fields is ignored and the slot admittance is computed for a seven (7) layer approximation to the profile of Figure 12, which is depicted in Figure 13. The voltage distribution is shown in Figure 14. The calculations made for estimated average plasma parameters ($N_e = 2.5 \times 10^9$ cm^{-3} and $\nu = 3.8 \times 10^8$ sec^{-1}) are shown dashed in Figure 13. These average data agree with computations made for a laterally unbounded plasma layer [1] but they provide significant errors relative to the multilayer model particularly for frequencies below the gyrofrequency of the plasma.

The preceeding numerical calculations were based on electric aperture fields with neglected E_x and with a y-independent E_y. These calculations are compared in Table 1 with calculations made for the principal mode H_x field in the aperture, where E_y exhibits the static y-variation and $E_x \neq 0$. The differences between the two sets of calculations are small. Even in the case where $E_x \neq 0$, the neglect of it does not affect the slot conductance and changes the susceptance by less than 1%. The change from the static $E_y(y)$ to a y-independent E_y causes a negligible change in G and about 7% change in B.

The changes of the susceptance B are larger than the changes of G for the cases listed in Table 1 and further susceptance calculations are shown in Figure 15 for the same plasma and antenna parameters as in Figure 7. There is a change of B caused by a change of $E_y(y)$, but none by the neglect of E_x within the accuracy of the plot.

The slot admittance of an open waveguide ($x_i = 2\ell$, $y_i = 2\epsilon$) radiating into a laterally unbounded plasma has been recently calculated by Villeneuve [11]. He assumes the principal waveguide mode as the aperture field E_y with $E_x = 0$, and the slot admittance which is obtained after numerically evaluating a two-fold Fourier transform should correspond to the admittance computed from (33) in the limit of $k \to \beta_{xi1} = \pi/x_i$. A comparison of the results by Villeneuve [11] with computations following Section 2.2 ($E_y \neq f(y)$, E_x neglected) and of Section 2.3 (H_x of the principal waveguide mode in the aperture, static $E_y(y)$ and $E_x \neq 0$ or $= 0$) is shown in Figures 16 and 17. The free space slot admittance of Lewis [4] or Cohen et al. [7] is also indicated in Figures 16 and 17. The results of Villeneuve [11] agree with those for $E_y \neq f(y)$ of the smaller values of ϵ_p, but there are differences up to 5% for $\epsilon_p/\epsilon_o > 0.6$. The neglect of E_x in the solution with the static $E_y(y)$ dependence gives a detectable change in the susceptance of Figure 17 because of the increase in x_i/λ relative to Figure 15. (This is in agreement with the discussion following (51)).

The comparison of slot admittances in Table 1 or Figures 15 to 17 which are based on principal mode electric or magnetic fields as the aperture trial functions may be also of interest in connection with a recent discussion of these Transactions (Unz and Hodara, [9]).

The above work has shown that the plasma layers in a large waveguide may be considered as a substitute for laterally unbounded plasma layers. The waveguide model makes the problem mathematically simple and suitable for computer calculation. The dimensions of the large guide may be selected to be such that the slot has approximately the correct admittance in the absence of plasma layers. The slot will exhibit the correct admittance behavior of a slot in an infinite screen for thin plasma layers ($h < \lambda$). Furthermore, the dimensions of the large guide are less critical if the electrical characteristics of the thin plasma layer differ significantly from those of free space ($\epsilon_p/\epsilon_o \ll 1$ or $\tan \delta \gg 1$). For thick plasma layers ($h > \lambda$) the calculated conductance exhibits oscillations which are absent in laterally unbounded plasma layers. However, the average of this oscillatory plasma conductance gives the correct admittance figure in the cases discussed earlier. It appears that a multilayer plasma model in a wide waveguide provides a more accurate slot admittance than a single layer approximation of a plasma profile in a laterally unbounded geometry. Most of the calculations make use of a two term trial function for the slot fields, but the fields do not differ significantly from the principal waveguide mode and the trial function and hence the calculations may be further simplified.

The technique described in this paper is applicable for obtaining admittance estimates for slot antennas in presence of layered isotropic plasmas or dielectrics. At present, there are no such treatments for a laterally unbounded plasma. The present calculations will be correlated with antenna impedance measurements on hypersonic reentry vehicles and also with laboratory measurements on a glow discharge. The multi-layer formalism accounts also for glass wall effects of discharge vessels. The lateral bounds of the plasma layer assumed in the analysis appear of no consequence in such applications. The waveguide model has been also applied to plasma or dielectric covered cavity backed annular slots [Galejs, 13]. This model is useful for representing debris covered hardened slot antennas operating in the LF or HF bands.

ACKNOWLEDGMENT

Appreciation is expressed to Miss E. A. Marley and Mr. N. Ciccia of Sylvania's Applied Research Laboratory for computer programming. This research was sponsored by the Air Force Cambridge Research Laboratories Office of Aerospace Research, under Contract No. AF19(628)-2410.

APPENDIX A

SLOT ADMITTANCE

The waveguide geometry under consideration is depicted in Figure 1. The smaller guide ($z \leq 0$) is excited by waves in its principal TE_{10} mode which propagates in the positive z-direction. There are also reflected waves in this guide. Waves propagating in the positive and negative z directions are excited in the larger guide ($z \geq 0$). The fields in the guides consist of a superposition of TE and TM modes that can be derived from the scalar functions Ψ_j and Φ_j respectively. The subscripts j=i or p designate the smaller guide or the initial portion of the larger guide. For an assumed $\exp(-i\omega t)$ time dependence of the fields it follows that

$$\Psi_i = a_{10} \cos \beta_{xi1} x' (e^{\gamma_{i1} z} + R\, e^{-\gamma_{i1} z})$$

$$+ \sum_n \sum_m{}' a_{nm} \cos \beta_{xi} x' \cos \beta_{yi} y' \, e^{-\gamma_i z} \tag{52}$$

$$\Phi_i = \sum_n \sum_m b_{nm} \sin \beta_{xi} x' \sin \beta_{yi} y' \, e^{-\gamma_i z} \tag{53}$$

$$\Psi_p = \sum_n \sum_m A_{nm} \cos \beta_{xp} x \cos \beta_{yp} y \, (e^{\gamma_p z} + R_a \, e^{-\gamma_p z}) \tag{54}$$

$$\Phi_p = \sum_n \sum_m B_{nm} \sin \beta_{xp} x \sin \beta_{yp} y (e^{\gamma_p z} + R_b \, e^{-\gamma_p z}) \tag{55}$$

where

$$\beta_{xi} = \frac{n\pi}{x_i} \qquad \beta_{yi} = \frac{m\pi}{y_i}$$

$$\beta_{xi1} = \frac{\pi}{x_i}$$

$$\beta_{xp} = \frac{n\pi}{x_o} \qquad \beta_{yp} = \frac{m\pi}{y_o}$$

$$\gamma_j = i\sqrt{k_j^2 - \beta_{xj}^2 - \beta_{yj}^2}$$

$$k_j = \sqrt{\omega^2\mu_o\epsilon_j + i\omega\mu_o\sigma_j}$$

$$\gamma_{i1} = \sqrt{k_i^2 - \beta_{xi1}^2}$$

R, R_a and R_b are reflection coefficients. The prime on the double summation (52) designates the omission of its n=1, m=0 term. The field components are related to Ψ_j and Φ_j by

$$E_{xj} = \frac{\partial}{\partial y}\Psi_j + \frac{\partial^2}{\partial z\,\partial x}\Phi_j \tag{56}$$

$$E_{yj} = -\frac{\partial}{\partial x}\Psi_j + \frac{\partial^2}{\partial z\,\partial y}\Phi_j \tag{57}$$

$$E_{zj} = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi_j \tag{58}$$

$$H_{xj} = \frac{1}{i\omega\mu_o}\left[\frac{\partial^2}{\partial z\,\partial x}\Psi_j + k_j^2\frac{\partial}{\partial y}\Phi_j\right] \tag{59}$$

$$H_{yj} = \frac{1}{i\omega\mu_o} \left[\frac{\partial^2}{\partial z\, \partial y} \Psi_j - k_j^2 \frac{\partial}{\partial x} \Phi_j \right] \tag{60}$$

$$H_{zj} = - \frac{1}{i\omega\mu_o} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Psi_j \tag{61}$$

After multiplying (57) with sin $\beta_{xi1} x'$ and integrating over the aperture at $z=0$ it follows that

$$a_{10} = \frac{2}{\beta_{xi1} x_i y_i (1+R)} \iint E_y \sin \beta_{xi1} x' \, dx' \, dy' \tag{62}$$

Equation (56) is multiplied with cos $\beta_{xj} x$ sin $\beta_{yj} y$ and (57) with sin $\beta_{xj} x$ cos $\beta_{yj} y$. After integrating over the aperture this gives two pairs of equations ($j=i$ or p) with the unknowns a_{nm} and b_{nm} for $j=i$ or A_{nm} and B_{nm} for $j=p$. After introducing the abbreviations

$$\iint E_{xj} = \iint E_x \cos \beta_{xj} x \sin \beta_{yj} y \, dx \, dy \tag{63}$$

and

$$\iint E_{yj} = \iint E_y \sin \beta_{xj} x \cos \beta_{yj} y \, dx \, dy \tag{64}$$

where E_x and E_y refers to the aperture fields $z=0$, these coefficients may be computed as

$$a_{nm} = - \frac{2\epsilon_m}{x_i y_i (\beta_{xi}^2 + \beta_{yi}^2)} \left[\beta_{yi} \iint E_{xi} - \beta_{xi} \iint E_{yi} \right] \tag{65}$$

$$b_{nm} = - \frac{2\,\epsilon_m}{x_i y_i (\beta_{xi}^2 + \beta_{yi}^2)\gamma_i} \left[\beta_{xi} \iint E_{xi} + \beta_{yi} \iint E_{yi} \right] \tag{66}$$

where

$$\epsilon_m = \begin{cases} 1 & \text{for } m=0 \\ 2 & \text{for } m\neq 0 \end{cases}$$

The subscripts i should be replaced by p for obtaining A_{nm} and B_{nm} from (65) and (66) respectively. Also, the right hand sides of (65) and (66) should be multiplied with $(1 + R_a)^{-1}$ and $(R_b - 1)^{-1}$ respectively. After substituting a_{nm}, b_{nm}, A_{nm} and B_{nm} in (52) to (55), the magnetic field components H_{xj} and H_{yj} of (59) and (60) are related to the integrals of the electric field components (63) and (64). After equating the tangential magnetic field components in the aperture one obtains two coupled equations. Letting

$$D = \iint E_y \sin \beta_{xi1} x' \, dx' \, dy' \tag{67}$$

it follows that

$$\frac{\gamma_{i1}}{x_i y_i} \frac{1 - R}{1 + R} D \sin \beta_{xi1} \xi = \iint E_y(x,y) G_1 \, dx \, dy + \iint E_x(x,y) G_2 \, dx \, dy \tag{68}$$

$$\iint E_y(x,y) G_2 \, dx \, dy = \iint E_x(x,y) G_3 \, dx \, dy \tag{69}$$

where

$$G_1 = \sum_n \sum_m{}' \frac{\epsilon_m}{x_i y_i \gamma_i} (\beta_{xi}^2 - k_i^2) \sin \beta_{xi} x' \cos \beta_{yi} y' \sin \beta_{xi} \xi' \cos \beta_{yi} \eta'$$

$$+ \sum_n \sum_m \frac{\epsilon_m}{x_o y_c (\beta_{xp}^2 + \beta_{yp}^2} \Bigg[\gamma_p \beta_{xp}^2 \frac{1 - R_a}{1 + R_a}$$

$$- \beta_{yp}^2 \frac{k_p^2}{\gamma_p} \frac{1 + R_b}{1 - R_b} \Bigg] \sin \beta_{xp} x \cos \beta_{yp} y \sin \beta_{xp} \xi \cos \beta_{yp} \eta \tag{70}$$

$$G_2 = -\sum_n \sum_m{}' \frac{\epsilon_m \beta_{xi}\beta_{yi}}{x_i y_i \gamma_i} \sin \beta_{xi}x' \cos \beta_{yi}y' \cos \beta_{xi}\xi' \sin \beta_{yi}\eta'$$

$$- \sum_n \sum_m \frac{\epsilon_m \beta_{xp}\beta_{yp}}{x_o y_o (\beta_{xp}^2 + \beta_{yp}^2)} \left[\gamma_p \frac{1 - R_a}{1 + R_a} \right.$$

$$\left. + \frac{k_p^2}{\gamma_p} \frac{1 + R_b}{1 - R_b} \right] \sin \beta_{xp}x \cos \beta_{yp}y \cos \beta_{xp}\xi \sin \beta_{yp}\eta \qquad (71)$$

$$G_3 = \sum_n \sum_m{}' \frac{\epsilon_m (k_i^2 - \beta_{yi}^2)}{x_i y_i \gamma_i} \cos \beta_{xi}x' \sin \beta_{yi}y' \cos \beta_{xi}\xi' \sin \beta_{yi}\eta'$$

$$+ \sum_n \sum_m \frac{\epsilon_m}{x_o y_o (\beta_{xp}^2 + \beta_{yp}^2)} \left[-\gamma_p \beta_{yp}^2 \frac{1 - R_a}{1 + R_a} \right.$$

$$\left. + \frac{\beta_{xp}^2 k_p^2}{\gamma_p} \frac{1 + R_b}{1 - R_b} \right] \cos \beta_{xp}x \sin \beta_{yp}y \cos \beta_{xp}\xi \sin \beta_{yp}\eta \qquad (72)$$

The normalized slot admittance of the waveguide is

$$y_s = \frac{1 - R}{1 + R} \qquad (73)$$

A variational principle for computing y_s may be constructed by multiplying (68) with E_y and (69) with E_x and by integrating over the slot. The two terms of (68) and (69) proportional to G_2 are equal and

$$y_s = \frac{\iiiint E_y(x,y)\, E_y(\xi,\eta)\, G_1 \, dx\, dy\, d\xi\, d\eta + \iiiint E_x(x,y)\, E_x(\xi,\eta)\, G_3 \, dx\, dy\, d\xi\, d\eta}{\frac{\gamma_{i1}}{x_i y_i} D^2} \qquad (74)$$

The stationarity of (74) can be demonstrated by examining the first order change δy_s due to small changes δE_x and δE_y about their correct values E_x and E_y. As a result of these calculations δy_s is shown to be zero.

Because of the symmetry of G_1 and G_3 the variation of y_s due to small changes δE_y and δE_x can be written as

$$\delta y_s = \frac{2x_i y_i}{\gamma_{i1} D^2} \Big\{ \iiiint E_y \, \delta E_y \, G_1 + \iiiint E_x \, \delta E_x \, G_3$$

$$- \left[\iiiint E_y E_y G_1 + \iiiint E_x E_x G_3 \right] \left[\iint \delta E_y(\xi,\eta) \sin \beta_{xi1} \xi \, d\xi \, d\eta \right] / D \Big\} \tag{75}$$

where the differentials dx dy dξ dη have been omitted from the four fold integrals for the sake of brevity. Equation (68) is first multiplied with $\delta E_y(\xi,\eta)$ and integrated over the aperture. Then (68) is multiplied with $E_y(\xi,\eta) \iint \delta E_y(\xi,\eta) \sin \beta_{xi1} \xi \, d\xi \, d\eta$ and again integrated over the aperture. Combining the resulting two equations it follows that

$$\iiiint E_y \, \delta E_y \, G_1 + \iiiint E_x \, \delta E_y \, G_2 = \left[\iiiint E_y E_y G_1 + \iiiint E_x E_y G_2 \right]$$

$$\cdot \frac{\iint \delta E_y(\xi,\eta) \sin \beta_{xi1} \xi \, d\xi \, d\eta}{D} \tag{76}$$

Taking the variation of (69), multiplying the resulting equation with $E_x(\xi,\eta)$ and integrating over the aperture results in

$$\iiiint E_x \delta E_y G_2 = \iiiint E_x \delta E_x G_3 \tag{77}$$

Multiplying (69) with $E_x(\xi,\eta)$ and integrating over the aperture

$$\iiiint E_x E_y G_2 = \iiiint E_x E_x G_3 \tag{78}$$

After using (77) and (78) to eliminate G_2 from (76), the result shows that δy_s of (75) is equal to zero.

APPENDIX B

STATIC SOLUTION FOR THE WAVEGUIDE PROFILE

The waveguide profile of Figure 1 can be represented by the t-plane ($t = z + iy$) configuration of Figure 18 if a conducting plane is substituted for the symmetry plane $y = y_o/2$ of Figure 1 and if the effects of the outer guide wall are neglected ($x_o \gg x_i$). The space between the electrodes of the t-plane is mapped into the upper-half of the w-plane ($w = u + iv$) by the transformation

$$\frac{dt}{dw} = \frac{C_1}{w} \sqrt{w - 1} \tag{79}$$

where C_1 is a constant. Integrating (79) between the two B points of the t-plane and around the B point of the w-plane gives the constant as $C_1 = i\epsilon/\pi$. Integrating (79) and noting that the $t = i\epsilon$ point is mapped into $w = 1$ results in

$$t = i\epsilon \left[1 + \frac{2}{\pi} \left(\sqrt{w-1} - \tan^{-1} \sqrt{w-1} \right) \right] \tag{80}$$

The complex potential function for the electrodes of the w-plane is

$$W\,[w(t)] = - \frac{V_o}{2\pi} \ln w \tag{81}$$

The potential is the imaginary part of W and the electric field is given by

$$E_z + i\,E_y = -i \left(\frac{dW}{dt} \right)^* \tag{82}$$

$$= -i \left(\frac{dW}{dw} \Big/ \frac{dt}{dw} \right)^*$$

$$= \frac{V_o}{2\epsilon} \Big/ \sqrt{w^* - 1}$$

where the asterisk indicates a complex conjugate. The electric field component E_y can be determined as the imaginary part of (82) in the z = 0 plane for $0 \leq iy \leq i\epsilon$ only after inverting the mapping function (80). The general inversion problem will not be attempted here, and the consideration will be restricted to points in the vicinity of the corner $t = i\epsilon$ and on the ground plane $t = 0$.

The point $w = 1$ corresponds to $t = i\epsilon$ and $\sqrt{w-1}$ will be small for $t = i(\epsilon - \delta)$ with $\delta \to 0$. Expanding the inverse tangent of (80) results in

$$- \delta = \frac{2\epsilon}{3\pi} (w - 1)^{3/2} \tag{83}$$

or

$$\sqrt{w^* - 1} = \left(\frac{3\pi\delta}{2\epsilon} \right)^{1/3} \exp (i\pi/3) \tag{84}$$

Substituting (84) in (82) gives

$$E_z + i E_y = \frac{V_o}{2} \left(\frac{2}{3\pi\epsilon^2\delta} \right)^{1/3} \left(\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) \tag{85}$$

(The ratio of E_z and iE_y depends on the direction along which the corner is approached. When approaching along the symmetry line of the corner, $t = i\epsilon + \delta(1-i)$ and a similar calculation shows that $E_z + i E_y \sim (1 - i)$, which follows also from elementary considerations).

The $t = 0$ point is mapped on the negative real axis of the w-plane and the corresponding value of w can be determined from (80) as a solution of the transcendental equation

$$i\zeta - \tan^{-1} (i\zeta) = - \frac{\pi}{2} \tag{86}$$

or

$$\zeta = \coth \zeta \tag{87}$$

where

$$i\zeta = \sqrt{w - 1}$$

The solution of (87) can be found from tables as $\zeta = 1.1997 \approx 1.2$ or $w = -0.44$. Substituting this value of w in (82) the electric field is computed as

$$E_x + i\,E_y = \frac{V_o}{2\epsilon}\,\frac{1}{i\,1.2} = -\,i\,\frac{V_o}{2\epsilon}\,0.833 \qquad (88)$$

APPENDIX C

FIELD COMPONENTS ASSOCIATED WITH THE PRINCIPAL WAVEGUIDE MODE AS THE H_x COMPONENT IN THE APERTURE

The magnetic field component H_{xi} can be computed from (52), (53) and (59). The aperture field component H_{xi} is given by

$$H_{xi} = H_o \sin \frac{\pi x'}{x_i} \tag{89}$$

over the aperture of the small guide ($y_i = 2\epsilon$, $x_i = 2\ell$) only if

$$a_{nm}\beta_{xi}\gamma_i + b_{nm}k_i^2 \beta_{yi} = 0 \tag{90}$$

for $nm \neq 10$. The coefficients a_{nm} and b_{nm} are defined by (65) and (66) in terms of E_x and E_y integrals (63) and (64). Substituting (65) and (66) into (90) the integrals E_{yi} are related to integrals of E_{xi} as

$$E_{xi} = \frac{\beta_{xi}^2 - k_i^2}{\beta_{xi}\,\beta_{yi}} \iint E_{yi} \tag{91}$$

where $m \neq 0$. For $m = 0$, $\beta_{yi} = 0$ and $a_{no} = 0$ for $n \neq 1$ according to (90). But a_{no} of (65) is zero only if $\iint E_{yi} = 0$, which implies that

$$E_{yi}(x,y) = E(y) \sin \frac{\pi x'}{x_i} \tag{92}$$

After substituting (91) in (65) and (66) the coefficients a_{nm} and b_{nm} can be expressed in terms of $\iint E_{yi}$ only and E_{xi} may be computed from (52), (53) and (56) as

$$E_{xi} = \sum_{q=2}^{\infty} \frac{4\left(\beta_{xil}^2 - k_i^2\right)}{x_i y_i\, \beta_{xil}\, \beta_{yi}} \left[\iint E_{yi} \sin \beta_{xil}x' \cos \beta_{yi}y'\, dx'\, dy' \right] \cdot \cos \beta_{xil}x' \sin \beta_{yi}y' \tag{93}$$

where the definition of β_{yi} is changed to $q\pi/y_i$ in order to distinguish the index of the summation from the constant value of m in $\beta_{yp} = m\pi/y_o$. Because of the continuity of E_x in the aperture, the integrals $\iint E_{xp}$ of the y_s expression may be evaluated as

$$\iint E_{xp} = \iint E_{xi} \cos \beta_{xp} x \quad \sin \beta_{yp} y \, dx \, dy \tag{94}$$

Equation (93) is substituted in (94) and the x', x and y integrals are seen to be elementary. The q-summation is evaluated prior to the y' integral with the aid of the relation

$$\sum_{n=1}^{\infty} \frac{\cos 2n\theta}{(2n)^2 - a^2} = \frac{1}{2a} \left[\frac{1}{a} - \frac{\pi}{2} \frac{\cos a\theta + \cos a(\pi-\theta)}{\sin a\pi} \right] \tag{95}$$

which has been derived from (558) of Jolley [14] and Ex. 3 p. 371 of Bromwich [15]. After carrying out the above steps (94) becomes

$$\iint E_{xp} = \frac{8 \, x_i^3 x_o}{y_i \pi^2} \left(\beta_{xi1}^2 - k_i^2\right) \frac{(-1)^{\frac{n-1}{2}} n}{x_o^2 - n^2 x_i^2} \cos \left[\beta_{xp} \frac{x_i}{2}\right] (-1)^{\frac{m}{2}+1} S \tag{96}$$

where

$$S = \frac{\sin(\beta_{yp} y_i/2)}{2\beta_{yp}} \int_0^{y_i} dy' \, E_y(y') \left\{ \frac{1}{\beta_{yp}} - \frac{y_i}{2} \left[\cos \beta_{yp} y' \, \frac{1+\cos\beta_{yp} y_i}{\sin\beta_{yp} y_i} + \sin \beta_{yp} y' \right] \right\} \tag{97}$$

The numerical value of S and of $\iint E_{xp}$ depends on the estimate of $E_y(y')$. For $E_y(y')$ = const. the y' integral of (97) is equal to zero, which can be seen more readily from (93). This makes $\iint E_{xp} = 0$ and the solution is reduced to the form discussed in Section 2.2. The static solution of Appendix B for the wave-guide profile along the $y' = y_i/2$ plane provides another estimate of $E_y(y')$.

If $E_y(y')$ is an even function about the slot center $(y' = y_i/2)$, S can be expressed with the aid of Q(w) of (36) as

$$S = \frac{V_o y_i}{4\beta_{yp}} \left[- \frac{\sin(\beta_{yp}\epsilon)}{(\beta_{yp}\epsilon)} + Q(\beta_{yp}\epsilon) \right] \tag{98}$$

which completes the evaluation of $\iint E_{xp}$.

$\iint E_{yp}$, $\iint E_{yi}$ and D are evaluated after substituting (92). The results of these integrations are shown as (38), (39) and (40).

APPENDIX D

FIELD COMPONENTS FOR $E_x = 0$

The field components are examined in this Appendix with $E_x = 0$ when the larger guide is filled by a homogeneous dielectric ($R_a = R_b = 0$). Substituting (52) and (53) in (56) and requiring that $E_{xi} = 0$ relates b_{nm} to a_{nm} as

$$b_{nm} = - a_{nm} \frac{\beta_{yi}}{\gamma_i \beta_{xi}} \tag{99}$$

B_{nm} is related similarly to A_{nm} by

$$B_{nm} = A_{nm} \frac{\beta_{yp}}{\gamma_p \beta_{xp}} \tag{100}$$

After introducing the notation

$$d_{10} = a_{10} \frac{\beta_{xi1}}{i\omega\mu_o \gamma_{i1}} \tag{101}$$

$$d_{nm} = - a_{nm} \frac{\beta_{xi}^2 + \beta_{yi}^2}{i\omega\mu_o \gamma_i \beta_{xi}} \tag{102}$$

$$D_{nm} = A_{nm} \frac{\beta_{xp}^2 + \beta_{yp}^2}{i\omega\mu_o \gamma_p \beta_{xp}} \tag{103}$$

The field components E_{yj}, H_{xj} and H_{yj} may be computed from (52) to (55), (57), (58), (59), (99) and (100) as

$$E_{yj} = i\omega\mu_o \frac{\partial}{\partial z} \theta_j \tag{104}$$

$$H_{xj} = (k_j^2 + \frac{\partial^2}{\partial x^2}) \theta_j \tag{105}$$

$$H_{yj} = \frac{\partial^2}{\partial y\, \partial x}\, \theta_j \qquad (106)$$

where

$$\theta_i = d_{10}\left(e^{\gamma_{i1} z} - R e^{-\gamma_{i1} z}\right) \sin \beta_{xi1} x' \qquad (107)$$

$$+ \sum_n \sum_m{}' \, d_{nm} \sin \beta_{xi} x' \cos \beta_{yi} y' \, e^{-\gamma_i z}$$

$$\theta_p = \sum_n \sum_m D_{nm} \sin \beta_{xp} x \cos \beta_{yp} y \, e^{\gamma_p z} \qquad (108)$$

The scalar function θ_j can be identified as the x-component of a magnetic Hertz vector $\underline{\Pi}_j = i_x\, \theta_j$, which satisfies the wave equation

$$\nabla^2 \underline{\Pi}_j + k_j^2 \underline{\Pi}_j = 0 \qquad (109)$$

The electric and magnetic fields are related to $\underline{\Pi}_j$ as

$$\underline{E}_j = i\omega\mu_o \nabla \times \underline{\Pi}_j \qquad (110)$$

and

$$\underline{H}_j = k_j^2 \underline{\Pi}_j + \nabla (\nabla \cdot \underline{\Pi}_j) \qquad (111)$$

Continuity of the tangential electric and magnetic field components across the aperture requires that

$$\frac{\partial}{\partial z}\, \theta_i = \frac{\partial}{\partial z}\, \theta_p \qquad (112)$$

$$(k_i^2 + \frac{\partial^2}{\partial x^2}) \ \theta_i = (k_p^2 + \frac{\partial^2}{\partial x^2}) \ \theta_p \tag{113}$$

$$\frac{\partial^2}{\partial y \ \partial x} \ \theta_i = \frac{\partial^2}{\partial y \ \partial x} \ \theta_p \tag{114}$$

The last two conditions can be satisfied for a y-dependent θ_j only if $k_i = k_p$, which is the trivial case of no dielectric discontinuities. However, there are no contradictions if θ_i does not deperd on y within the aperture. In this case $H_{yj} = 0$, H_{xj} does not depend on y, and the continuity of the tangential field components requires only (112) and (113) to be satisfied.

In a completely open small guide ($y_i = 2\epsilon$, $x_i = 2\ell$) the function θ_i, which does not depend on y, can be represented by the m = 0 terms of the double summation (107). Hence E_{yi} (and also E_{yp}) will exhibit no y variations over the aperture. Conversely E_{yi}, which depends on y, leads to a y-dependent θ_i and to a y-dependent H_{xi}, which violates the continuity of H_y. A y-dependent E_{yi} and a y-independent H_{xi} lead to $E_{xi} \neq 0$. (This can be seen from (89), (92) and (93) of Appendix C in the case where H_{xi} is the same as in the principal wave guide mode).

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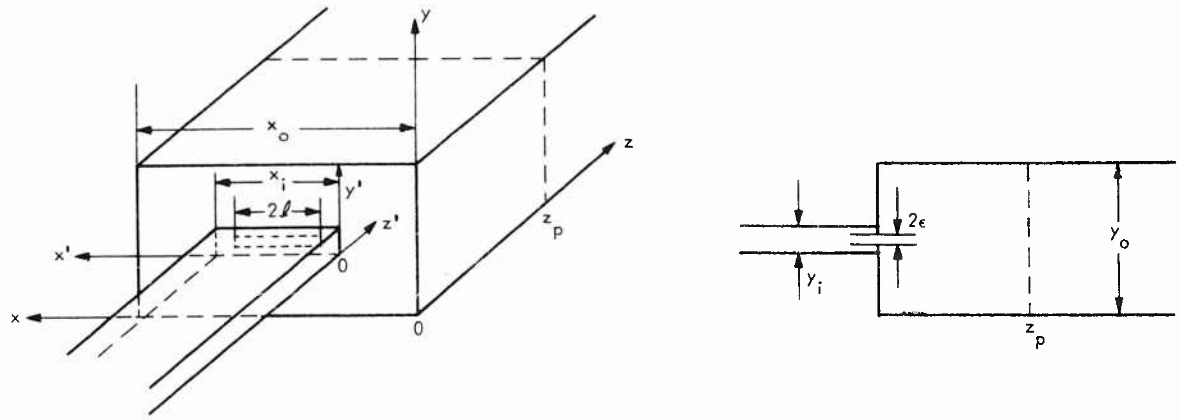
		$\epsilon_p/\epsilon_o = 0.1 \quad h/\lambda = 0.1$	$\epsilon_p/\epsilon_o = 0.5 \quad h/\lambda = 0.5$
Two term trial function $E_y \neq f(y)$	$k = k_o$	$0.1913 + i\ 0.4276$	$0.2160 + i\ 0.05626$
	$k = k_p$	$0.1934 + i\ 0.4153$	$0.2174 + i\ 0.04758$
Principal Mode E - Field, $E_y \neq f(y)$		$0.1881 + i\ 0.4120$	$0.2143 + i\ 0.05226$
Principal Mode H_x and $E_y(x)$ Static $E_y(y)$	$E_x \neq 0$	$0.1875 + i\ 0.3857$	$0.2128 + i\ 0.04928$
Principal Mode $E_y(x)$ Static $E_y(y)$	$E_x = 0$	$0.1875 + i\ 0.3863$	$0.2128 + i\ 0.04985$

TABLE 1 - Comparison of Slot Admittances.

$$x_o = y_o = 10.5 x_i \ , \quad x_i = 2\ell = 0.6\lambda \ , \quad y_i = 2\epsilon = 0.2\lambda$$



$$x = x' + \frac{x_o - x_i}{2}\ ; \quad y = y' + \frac{y_o - y_i}{2}\ ; \quad z = z'$$

Figure 1. Waveguide Geometry

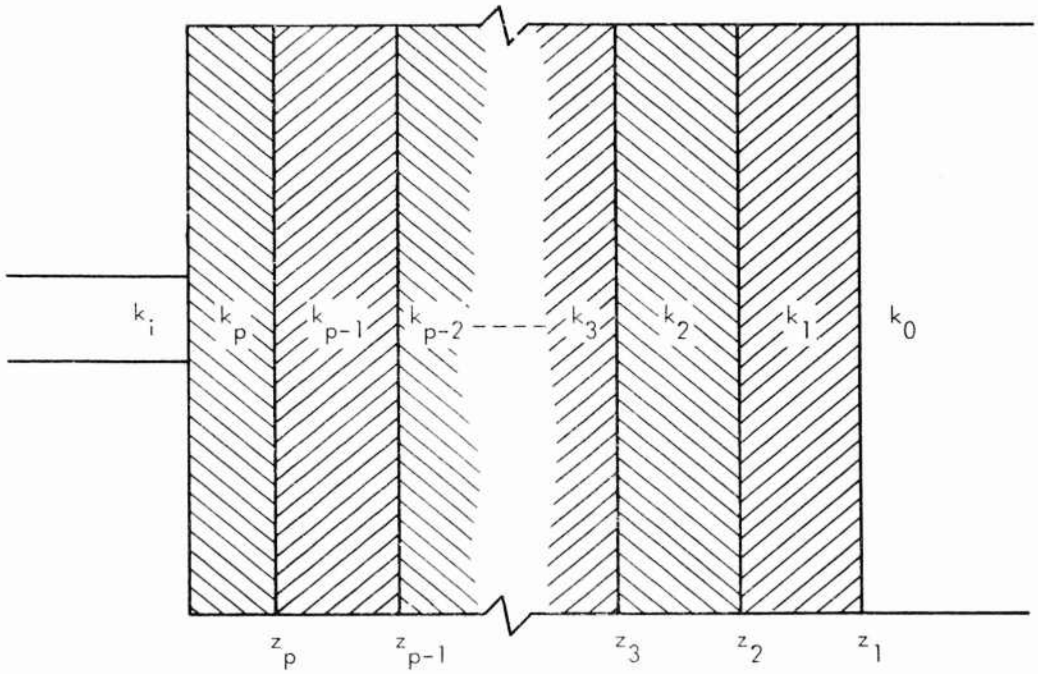


Figure 2. Model of the Stratified Dielectric

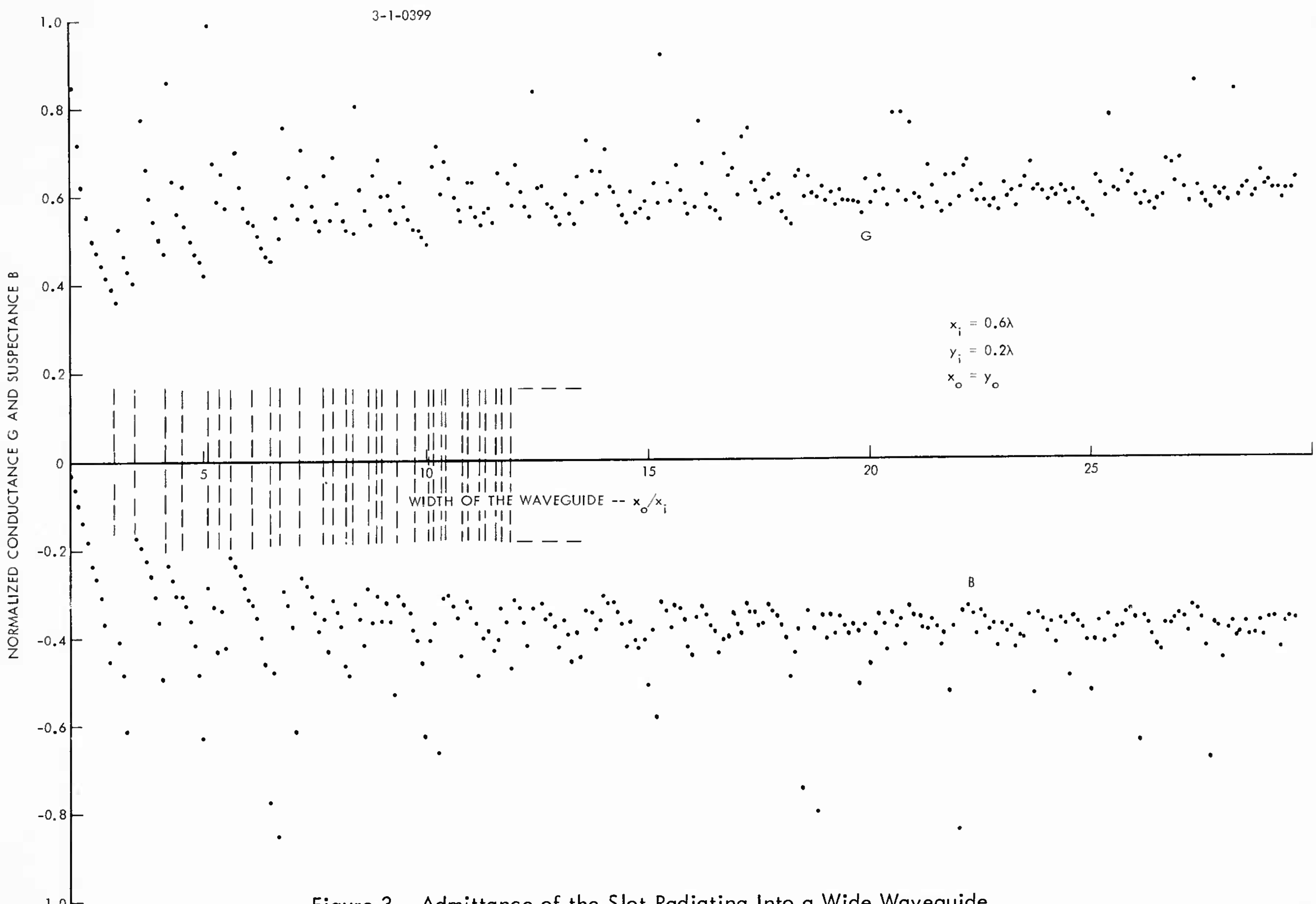


Figure 3. Admittance of the Slot Radiating Into a Wide Waveguide

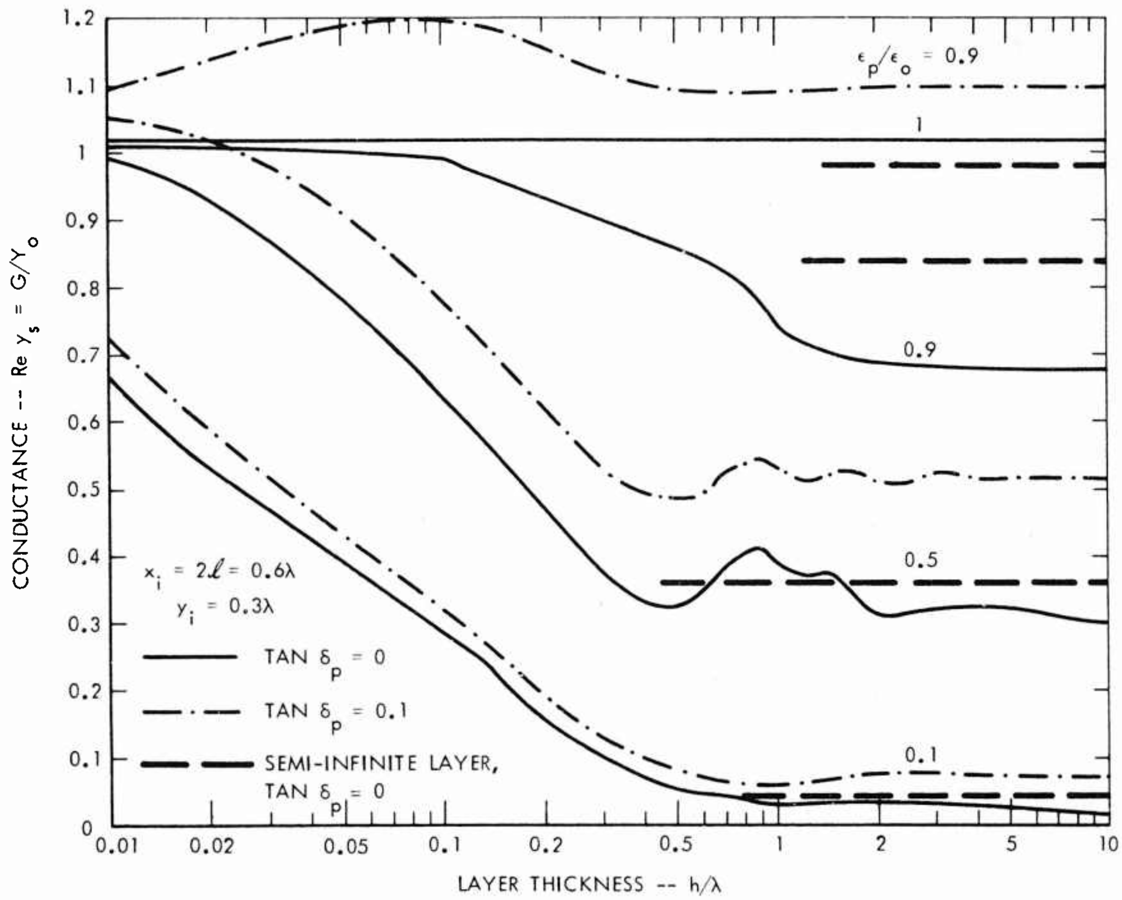


Figure 4. Waveguide Conductance $x_o = y_o = 10.5x_i$; $\epsilon = 0.05y_i$

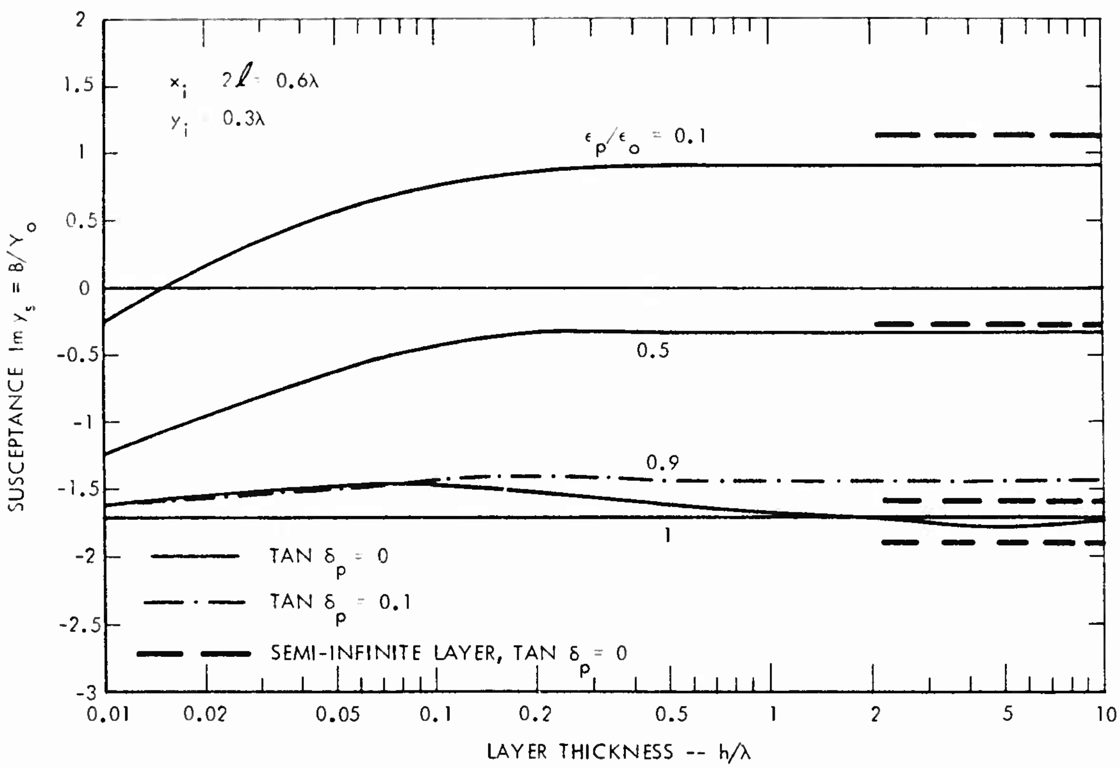


Figure 5. Waveguide Susceptance $x_o = y_o = 10.5x_i$; $\epsilon = 0.05y_i$

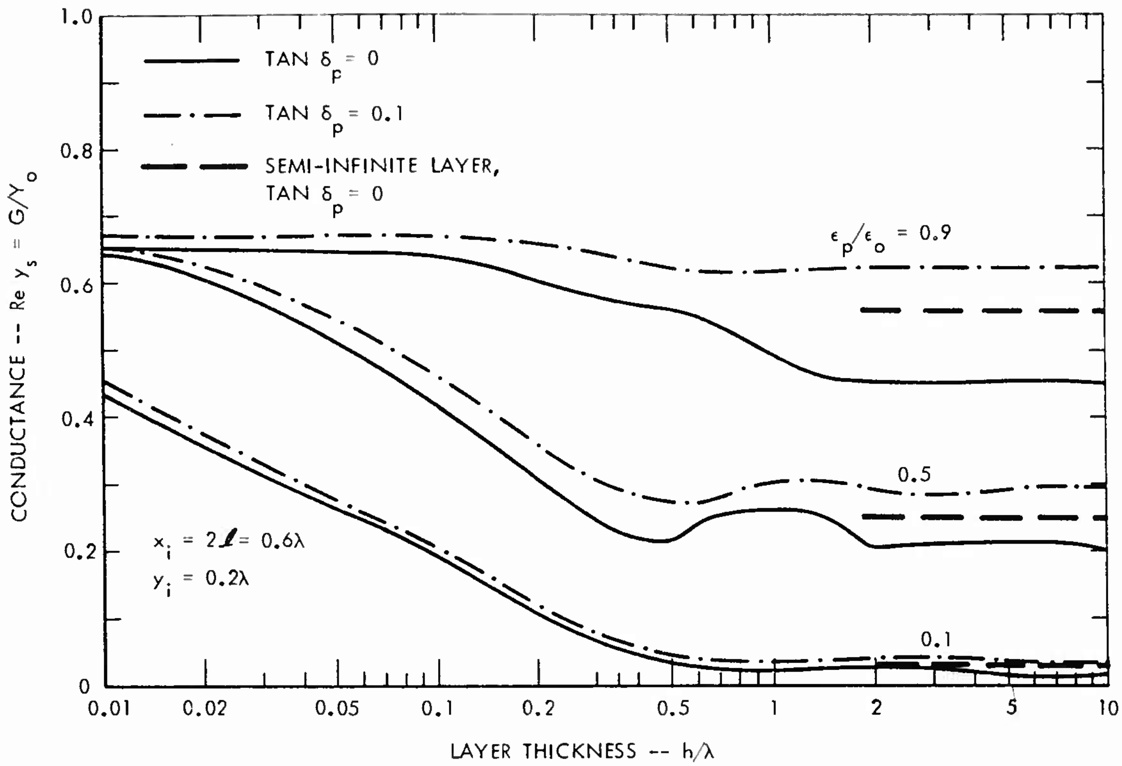


Figure 6. Waveguide Conductance $x_o = y_o = 10.5x_i$; $\epsilon = 0.5y_i$

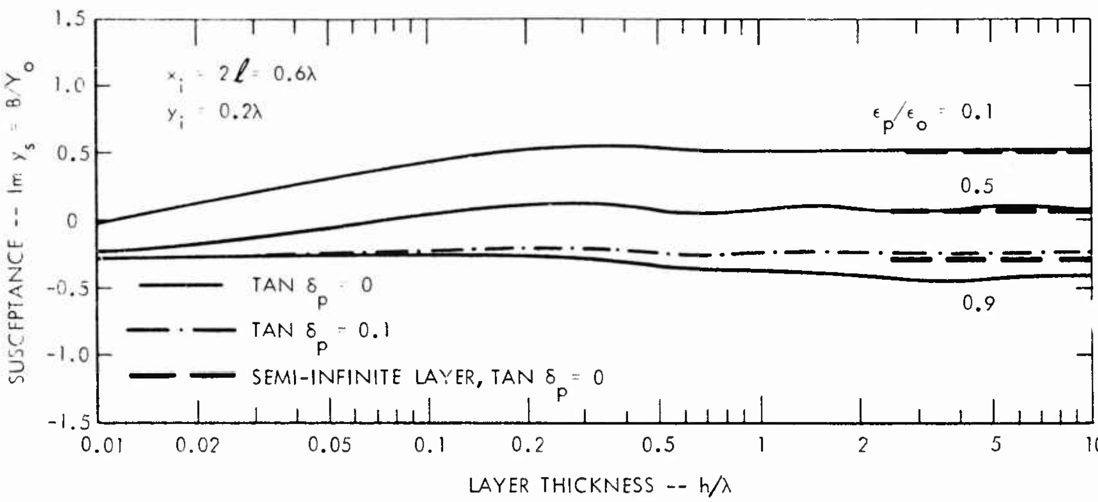


Figure 7. Waveguide Susceptance $x_o = y_o = 10.5x_i$; $\epsilon = 0.5y_i$

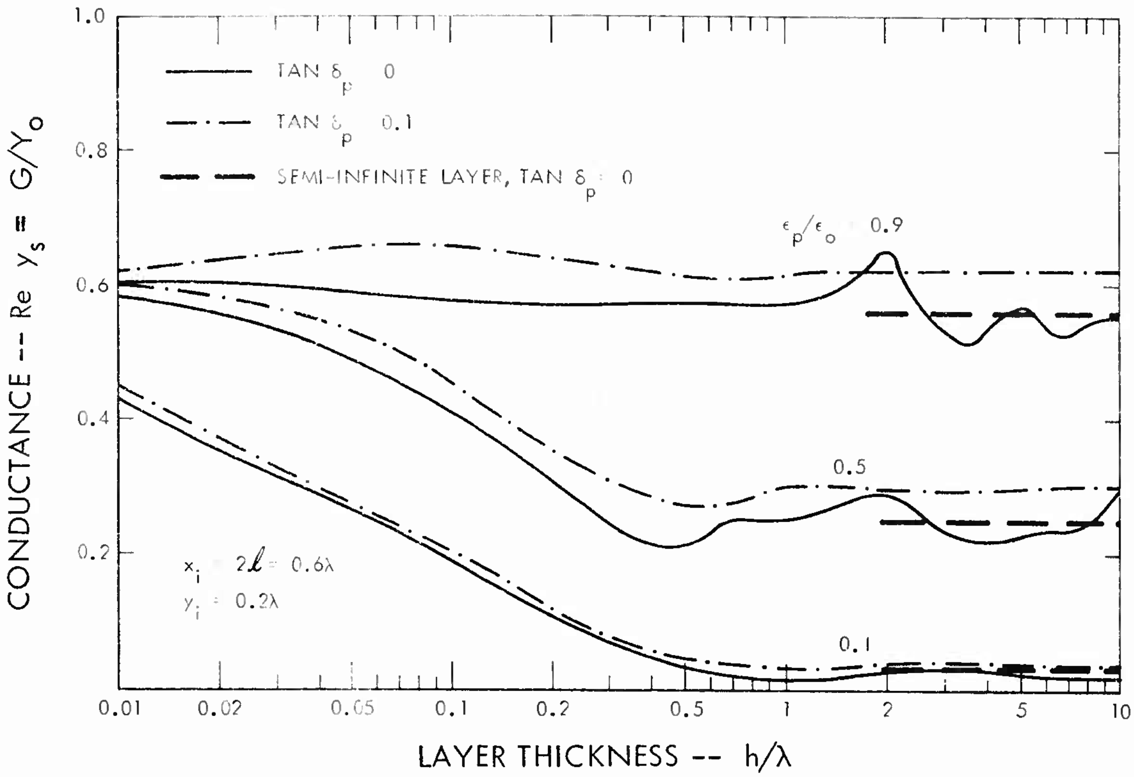


Figure 8. Slot Conductance $x_o = y_o = 19.5x_i$ $\epsilon = 0.5y_i$

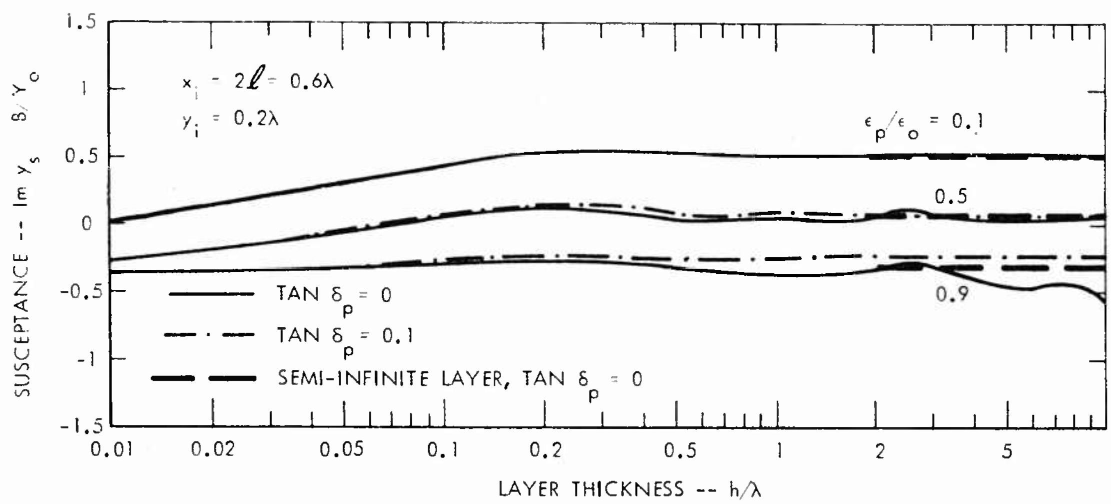


Figure 9. Slot Susceptance $x_o = y_o = 19.5x_i$, $\epsilon = 0.5y_i$

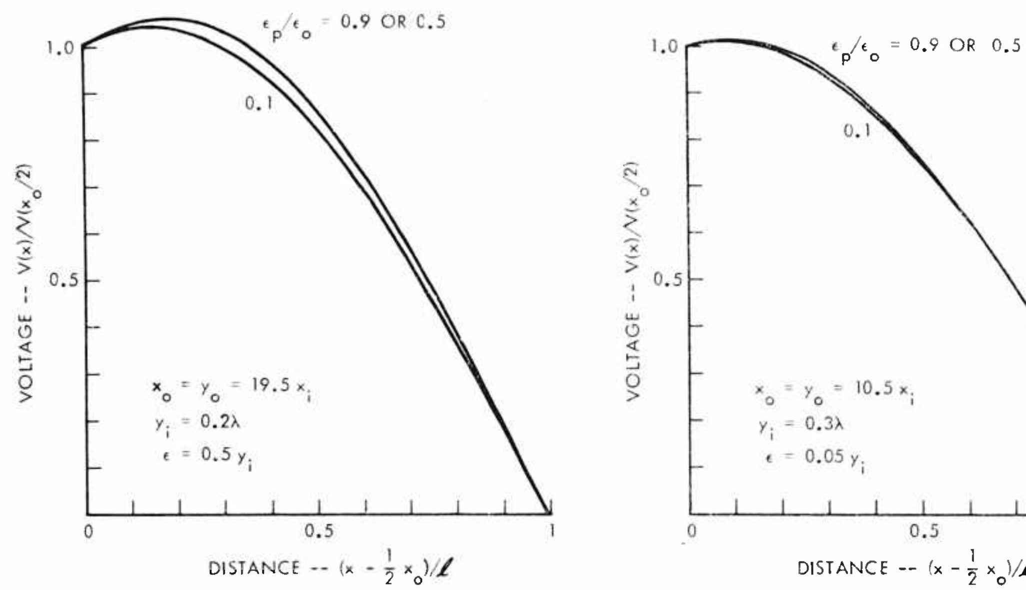


Figure 10. Variational Approximation to the Voltage Distribution Along the Waveguide Excited Slot $K = K_o$

$x_i = 2\ell = 0.6\lambda$ $\qquad$ $h = 0.5\lambda$ $\qquad$ $\tan \delta_p = 0$

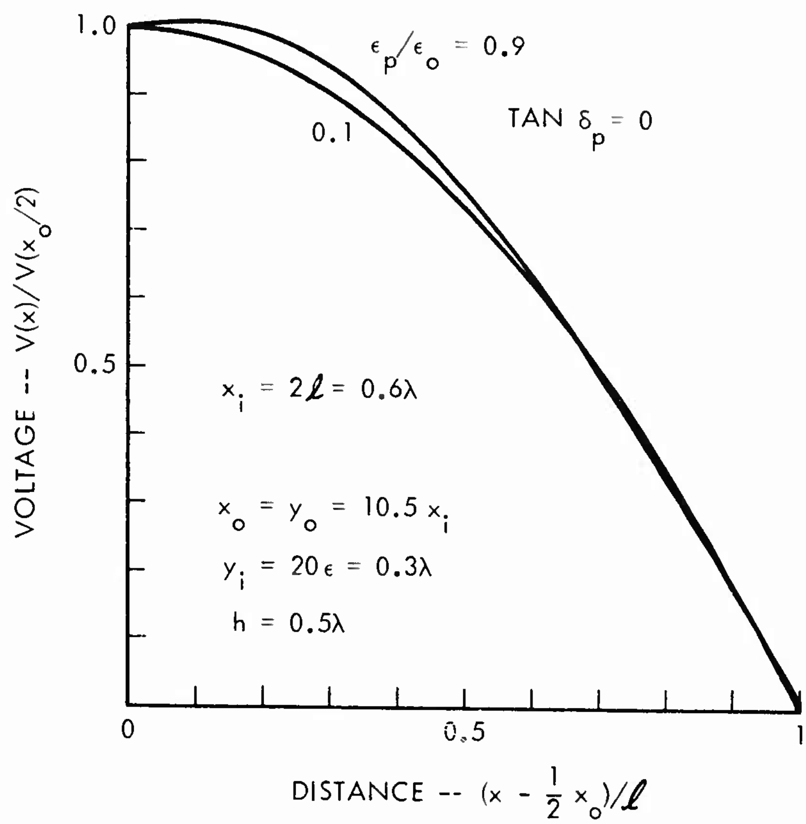


Figure 11. Variational Approximation to the Voltage Distribution Along the Waveguide Excited Slot. $K = K_p$

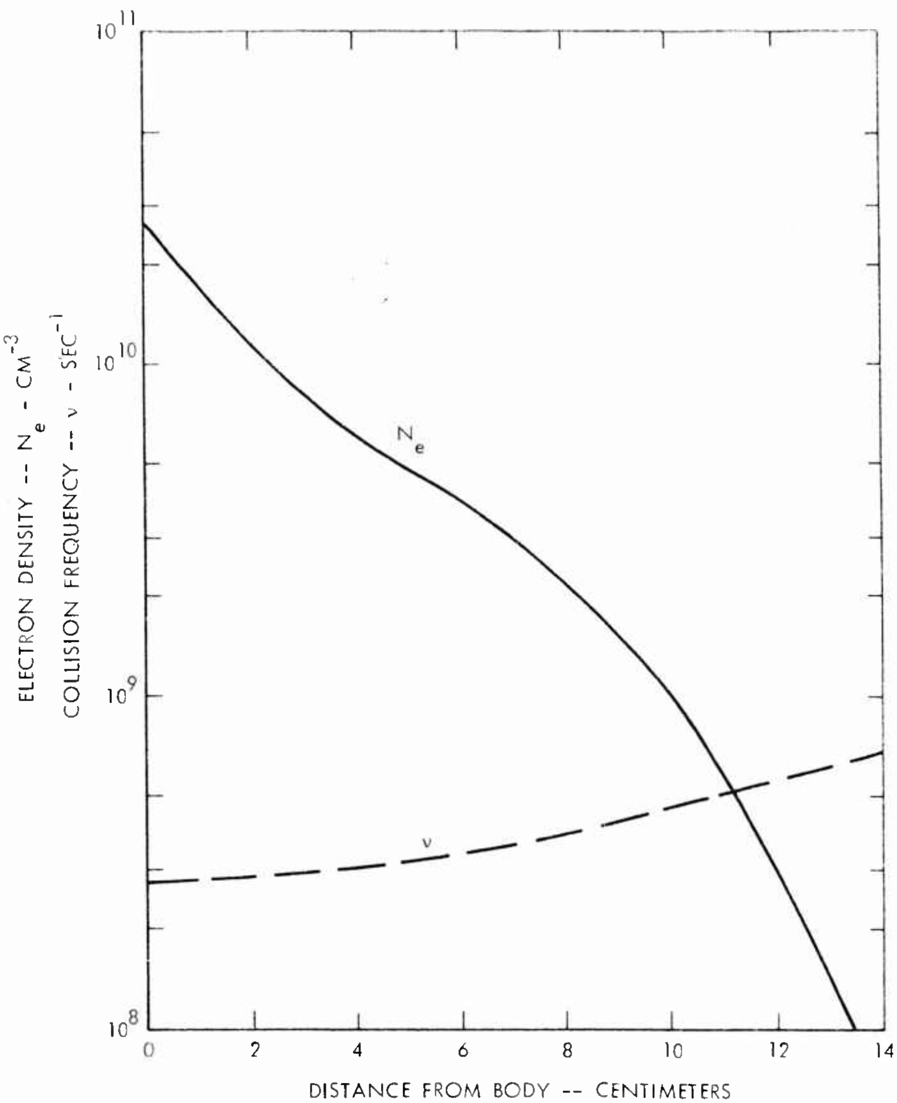


Figure 12. Profile of Electron Density and Collision Frequency for a Velocity of 17.5 K FPS and Altitude of 200 K FT

3-1-0409

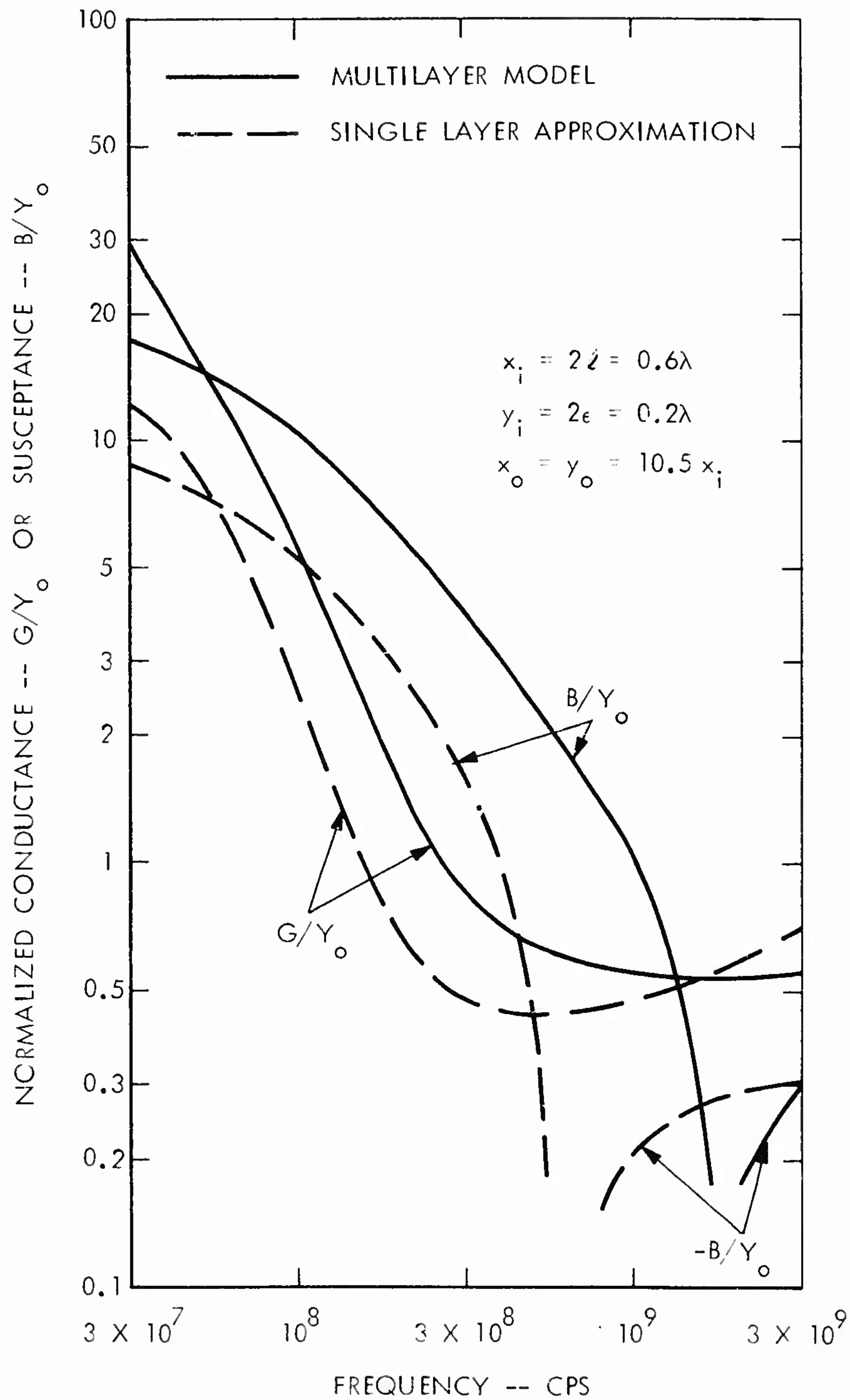


Figure 13. Admittance for a Multilayer Plasma and Its Single Layer Approximation

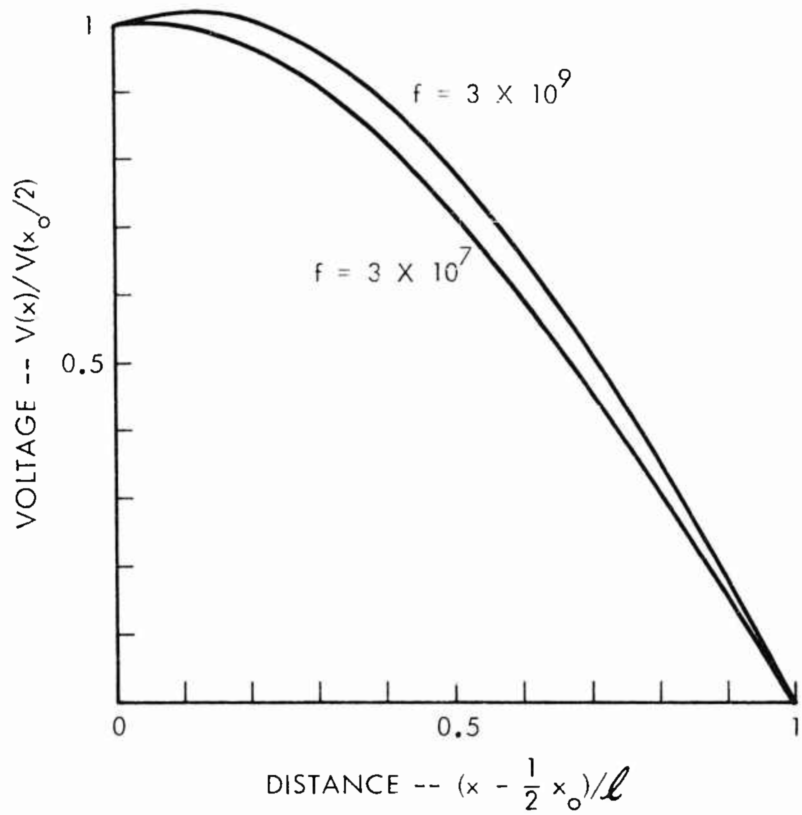


Figure 14. Variational Approximation to the Voltage Distribution Along the Slot of Figure 13

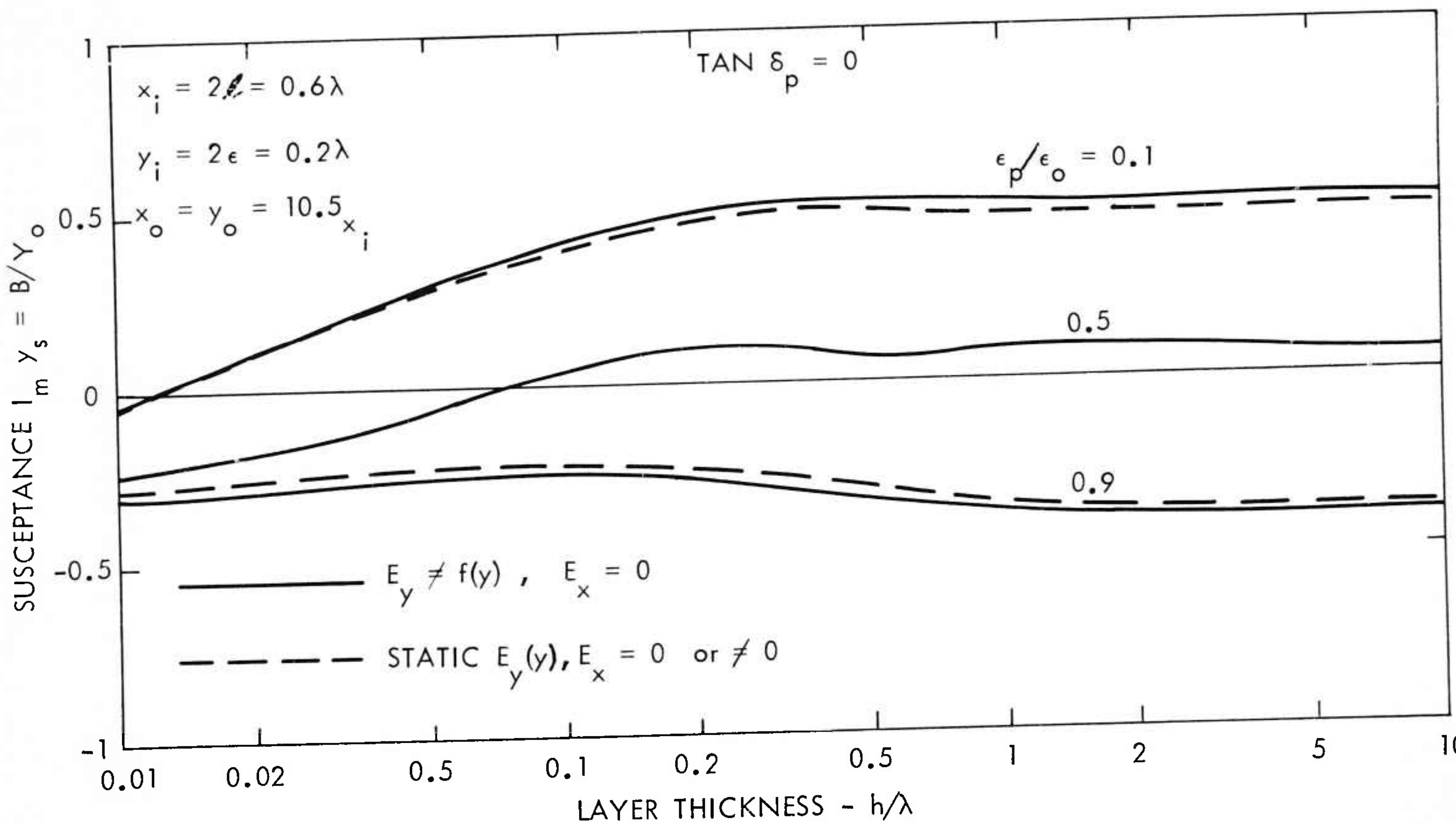


Figure 15. Waveguide Susceptance Comparison of Solutions for $E_x = 0$ and $E_x \neq 0$

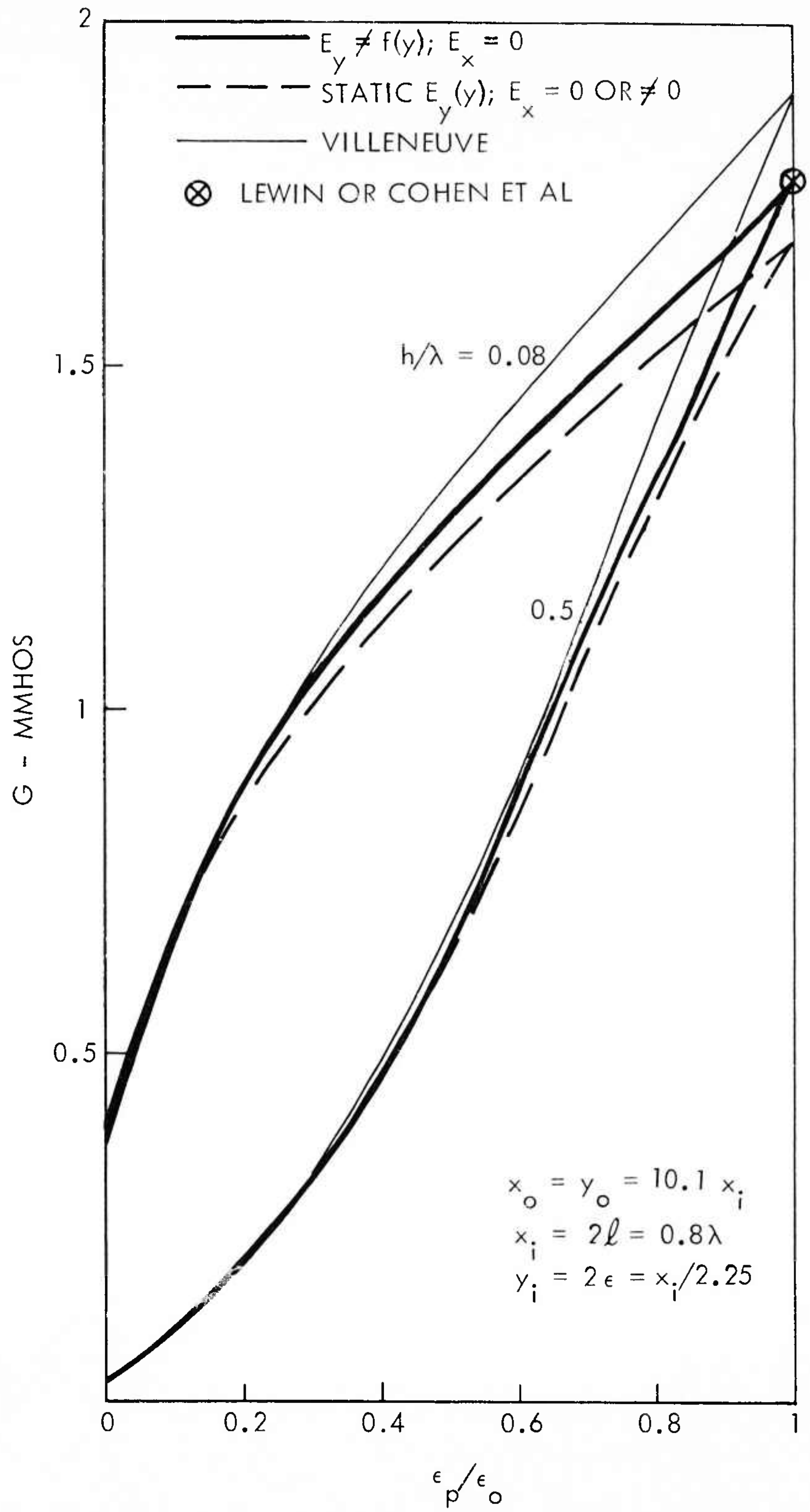


Figure 16. Conductance of the Waveguide Slot

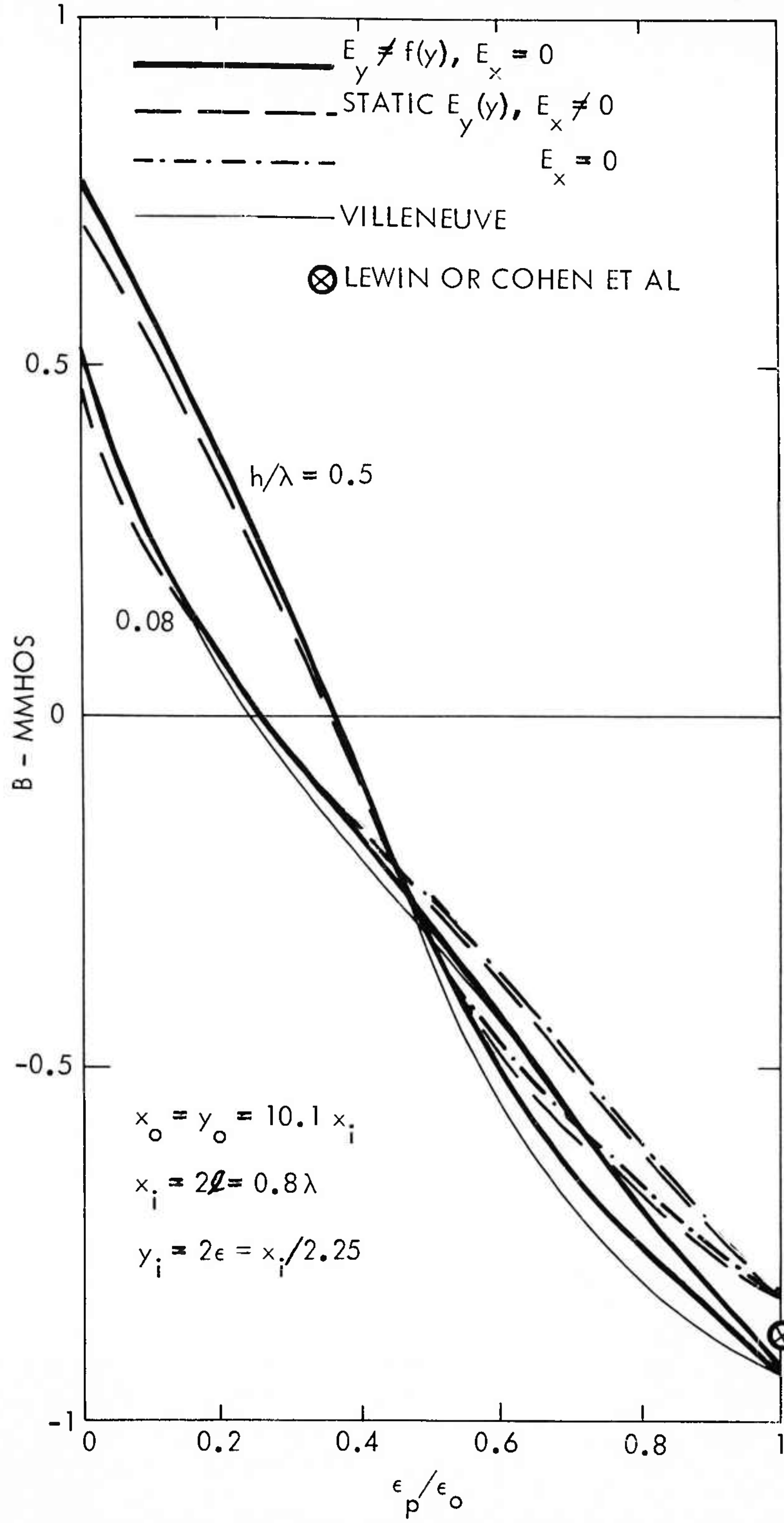


Figure 17 Susceptance of the Waveguide Slot

3-1-0556

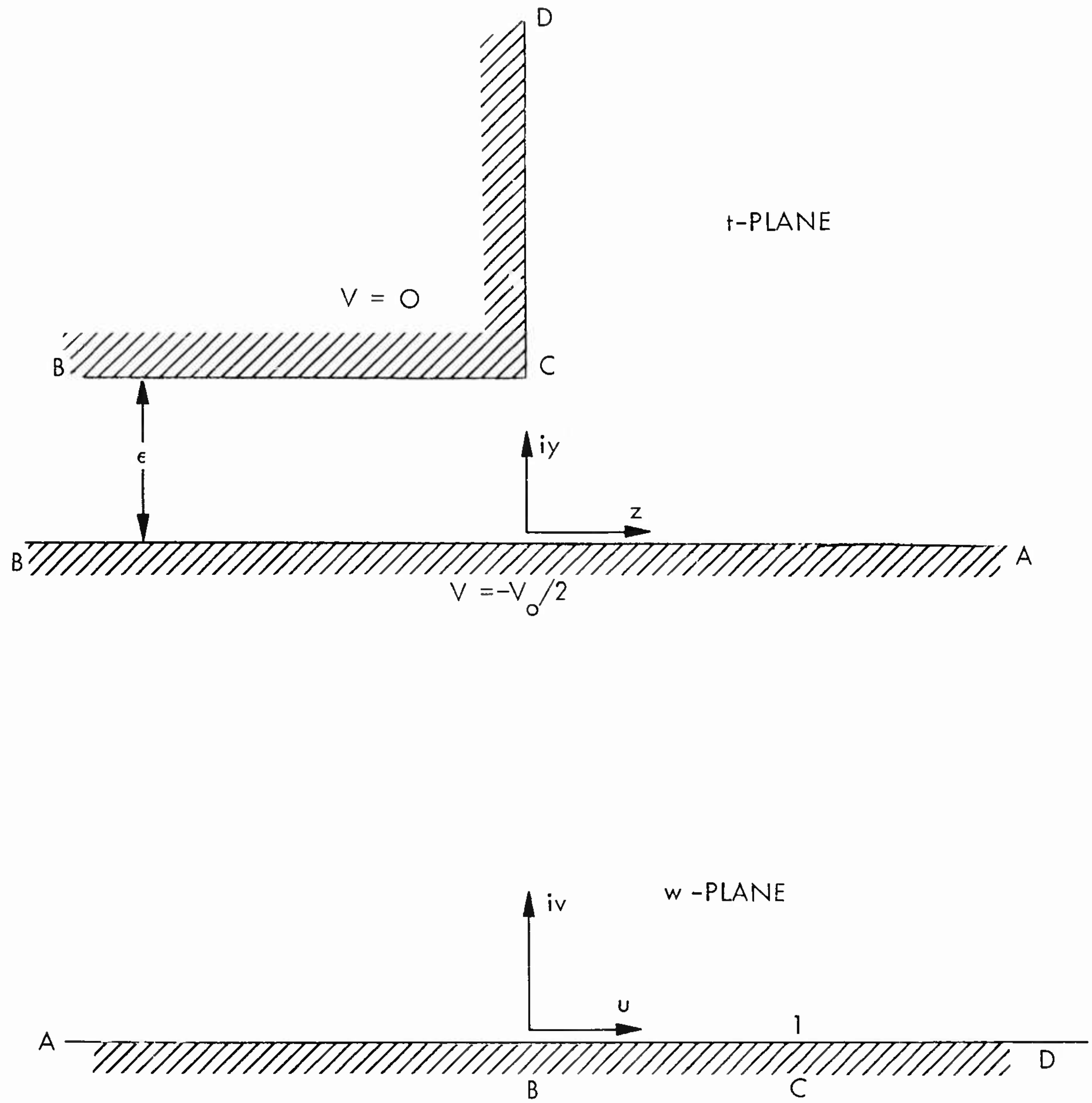


Figure 18. Conformal Mapping of the Waveguide Profile

Air Force Cambridge Research Laboratories, Office of Aerospace Research, Laurence G. Hanscom Field, Bedford, Massachusetts.
AFCRL-63-394 **ADMITTANCE OF A WAVEGUIDE RADIATING INTO STRATIFIED PLASMA**, Scientific Report, August 1963. 42p., incl. illus., 8 refs. Unclassified Report

A slot covered by a stratified plasma is assumed to radiate into a wide waveguide instead of free space. The slot admittance approximates the free space admittance of the slot for waveguide diameters exceeding 6 to 10 λ. For thick plasma layers the computed slot admittance checks with earlier admittance calculations for a laterally unbounded plasma. When approximating a plasma profile of a typical hypersonic re-entry, a multi-layer plasma model in a wide waveguide appears to provide a more accurate slot admittance than a single layer approximation in a laterally unbounded geometry.

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4. Reentry Communication
5. Layers of Lossy Dielectric
6. Stratified Plasma

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Project No. 4642
Task No. 464202
II. Applied Research Laboratory Sylvania Electronic Systems, 40 Sylvan Road, Waltham, Mass.
III. J. Galejs
IV. Secondary Report No. S-3040-2
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