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ESD-TDR-63-637 Publication No. U-2333

ANALYSIS OF A GENERAL PERTURBATIONS SATELLITE ORBIT COMPUTATION TECHNIQUE

Technical Documentary Report
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15 November 1963

J. R. Kuhlman

Aeronutronic
A Division of Ford Motor Company
Newport Beach, California

Prepared under Contract No. AF 19(628)-562 for
496L System Program Office
Electronic Systems Division
Air Force Systems Command
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L. G. Hanscom Field, Bedford, Mass.

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ABSTRACT

Results obtained from the analysis conducted with the Aeronutronic General Perturbations Differential Correction Program (AGPDC) described in ESD-TDR-63-632 (Aeronutronic Publication U-2201) are presented. The effects of the individual zonal harmonic bulge terms on prediction accuracy are itemized. Improvements in the solar radiation pressure perturbation formulation used in the AGPDC are discussed. AGPDC is compared with the Brouwer General Perturbations Differential Correction Program (BGPDC) in terms of the relative accuracy and treatment of low eccentricity orbits.

CONTENTS

SECTION 1

INTRODUCTION

The results of experimentation concerning the perturbations in the orbit of an artificial Earth Satellite caused by the asphericity (herein considered as the zonal harmonic bulge terms) of the Earth and by direct solar radiation pressure are presented in this report. The Aeronutronic General Perturbations Differential Correction Program (AGPDC),[1] used for the experimentation, is equipped to calculate the ephemeris of an Earth Satellite by a General Perturbations technique and to improve the orbital elements by a differential correction process using satellite observations.

The perturbation theory for the zonal harmonics of the Earth's gravitational field contains first-order short-period terms and secular and long-period terms to the second order of J_2. The expressions incorporated in the formulation are free of low-eccentricity singularities due to the choice of the parameters $e \sin \omega$, $e \cos \omega$ and the argument of latitude, u, instead of using the corresponding classical elements or the true anomaly, v.

The program was organized to facilitate the comparison of the complete first-order asphericity theory with simplified theories, in which selected perturbation terms are omitted. The comparison is

[1] See Arsenault, J. L., Kuhlman, J. R., and Stumpf, L. W., "A General Perturbations Differential Correction Program," ESD-TDR-63-432, Aeronutronic Publication U-2201, 1 August 1963. This document is cited throughout the following presentation to designate the general perturbation terms and to describe their individual effects on ephemeris computation.

made in the vector magnitude and in the radial, transverse and orthogonal components of the displacement vector from the position obtained with the complete theory. By this means, the effects on prediction accuracy of the individual terms can be determined.

Section 2 presents the results obtained from a prediction error analysis designed to determine which terms are to be retained from the complete theory for a specified prediction accuracy. These results show immediately that many of the terms are not required for most ephemeris predictions, particularly for orbits with moderately low or zero eccentricities. Application of the results of this analysis will effect a substantial improvement in computational efficiency without loss of accuracy.

The position prediction formulation also includes the effects of direct solar radiation on the orbit of the satellite. These effects are introduced through the perturbations in a set of elements[2] which define nearly-circular orbits without the difficulties caused by the singularities of the classical elements at zero eccentricity. The effects of the satellite being eclipsed by the Earth during part of its orbit are accounted for.

Section 3 describes experimentation performed with the radiation pressure section of the program. The results of mean element prediction in comparison with reduced mean elements for satellite Echo I, issued by the Smithsonian Astrophysical Observatory (SAO), are presented. Recent improvements in the radiation pressure formulation are discussed and the resultant effects are shown by comparison with earlier data. Also shown is a comparison of the differential correction of the orbital elements of satellite Echo I with and without the radiation pressure perturbations included. The results of this latter comparison emphasize that accurate orbit determination can be obtained for satellites with large area to mass ratios only if radiation pressure perturbations are accounted for in the formulation.

Section 4 discusses the experimentation performed with the differential correction feature of the program. Included in this section is a comparison of the AGPDC program with the Brouwer General Perturbations Differential Correction Program (BGPDC). These were compared for two reasons. First, to show that, under average conditions, the two programs produce essentially the same results. Second, to demonstrate that the AGPDC program is better able to handle a nearly circular orbit.

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[2] op. cit., pp. 23-30

In this second comparison, the AGPDC program was able to correct all of the elements to a good orbital fit in the first attempt, while the BGPDC program required considerable recycling and more than twice as much calculation to correct the elements at all.

SECTION 2

PREDICTION ERROR ANALYSIS

The prediction error analysis was conducted to determine the relative effects of the individual asphericity (bulge) perturbation terms on prediction accuracy and also to determine which terms to retain for a given prediction accuracy under given initial conditions. The results clearly show that a large amount of computer time can be saved by simply eliminating superfluous terms in the ephemeris calculation.

The analysis was performed by assuming as a nominal standard the complete first-order theory (all terms included) as a model. Simpler theories (selected terms omitted) were then compared to this by means of the following equations.[3] The subscript[4] n refers to nominal (all terms included) standard quantities.

$$\left.\begin{aligned} \Delta r &= r_n - r \\ \Delta u &= u_n - u \\ \Delta \Omega &= \Omega_n - \Omega \\ \Delta i &= i_n - i \end{aligned}\right\}$$

The second quantity on the right of each of these equations is obtained from the omitted term comparison case.

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[3] cf., p. 48 for a complete formulation of the error analysis.

[4] See Appendix A for glossary of terms.

$$r\Delta\theta_3 = r_n\Delta u + r_n \cos i_{oL_n}\Delta\Omega \qquad \text{circumferential displacement}$$

$$r\Delta\theta_1 = r_n \sin u_n \Delta i - r_n \sin i_{oL_n} \cos u_n \Delta\Omega \qquad \text{binormal displacement}$$

$$|\Delta \underline{r}| = [(\Delta r)^2 + (r\Delta\theta_3)^2 + (r\Delta\theta_1)^2]^{\frac{1}{2}} \qquad \text{total displacement}$$

where Δr, $r\Delta\theta_3$ and $r\Delta\theta_1$, are three mutually perpendicular components of the vector $\Delta \underline{r}$ which is given by $\underline{r}_n - \underline{r}$.

For the analysis, a number of sample cases were first obtained These included an ephemeris computation, including the printout of the values of the 89 bulge perturbation terms, at $t = 0.0$ and $t = 20{,}000$ minutes (about two weeks) past the epoch. The cases obtained included all combinations of the following initial orbital parameters:

perigee distance	$q_o = 1.022494217$ Earth Radii
eccentricity	$e_o = 0.001, 0.01, 0.1, 0.5$
inclination angle	$i_o = 1^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$
longitude of ascending node	$\Omega_o = 0^\circ$
argument of perigee	$\omega_o = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$
mean longitude	$L_o = 60^\circ$

In the asphericity theory adopted for this program, the terms are divided into three classes: secular, short-period and long-period. The secular terms are those which increase steadily with time. In this formulation. the secular variations in the longitude of the ascending node, the argument of perigee, the mean anomaly, and the mean longitude are each considered. Representative values of these secular terms are shown in Appendix B, Tables B4 through B7, where the relative size of the terms are shown as a function of the eccentricity and the inclination angle.

The short-period terms are those which oscillate in value with a period less than or equal to the satellite's orbital period The period of each term is determined by the trigonometric function of the argument of latitude, u, which appears as a factor in the term; the quantity, u, refers the satellite's position to the ascending nodal crossing point. Thus, the short-period term,

$$r_2 = \frac{1}{4} J_2 \frac{a_e^2}{p_L} \sin^2 i_{oL} \cos 2u$$

which represents a component of the short-period change in the length of the radius vector, will have a period equal to one-half the orbital period. Other terms appear with trigonometric arguments of u, 3u and 4u. The short-period variations in the argument of latitude, the radial distance, the longitude of the ascending node, the inclination angle, and the radial and circumferential components of the velocity are each represented in this formulation.

The long-period terms are those whose period of oscillation is greater than one orbital period and include a trigonometric function of the argument of perigee, ω. The period of these terms may be as short as a few days or as long as several months, depending on the rate of change of the argument of perigee and on whether ω enters the term as ω, 2ω, or 3ω. The long-period perturbations considered in this formulation include the effects on the longitude of the ascending node, the quantity $a_{xN} = e \cos \omega$, the quantity $a_{yN} = e \sin \omega$, the mean longitude, and the angle of inclination. Many of these long-period terms have an apparent singularity at an inclination angle of approximately $63^\circ.43$ caused by the appearance of the quantity $(4-5 \sin^2 i)$ in their denominator. There are several ways of treating this apparent singularity: one is to evaluate the term by a secular approximation in the neighborhood of this "critical" inclination; another way is to simply ignore the term at inclinations near this critical point, assuming that the perturbative contribution of the term will be small.

Analysis of the effects of the individual terms on prediction accuracy was divided into three categories: secular term effects, short-period term effects, and long-period term effects. The effects of the short-period and long-period terms on prediction accuracy are discussed in Sections 2.1 and 2.2, respectively. Section 2.3 contains a discussion of the secular term effects along with a description of techniques used in the overall prediction error analysis.

2.1 ANALYSIS OF SHORT-PERIOD TERMS

The short-period terms were analyzed in three ways. First, the values of the terms at three-minute intervals over a time-span of ninety minutes were obtained, in order to establish the relative sizes of the terms in each group (e.g., short-period terms in the inclination angle, i). The values of these are shown in Appendix B, Tables B8 through B11.

Second, each group of short-period terms was eliminated, one group at a time, from the ephemeris calculation and the prediction error was obtained at three minute intervals over a time-span of ninety minutes. Figures B13 through B16 in Appendix B show how the various groups of short-period terms affect the prediction accuracy around the orbit. In the third approach, the variation in position due to short-period terms was computed directly, as follows:

$$\Delta_r = \frac{1}{2} J_2 \frac{a_e^2}{p} \left(1 - \frac{3}{2} \sin^2 i\right) \left[\frac{r}{a\sqrt{1-e^2}} - \frac{1}{e}\left(1 - \sqrt{1-e^2}\right) \cos v - 1\right]$$

$$+ \frac{1}{4} J_2 \frac{a_e^2}{p} \sin^2 i \cos 2u$$

$$r\Delta\theta_3 = r\Delta\,\Omega \cos i + r\Delta u$$

where $r\Delta\,\Omega\cos i = \frac{3}{4} J_2 \frac{a_e^2}{p^2} r \cos^2 i \,[\sin 2u - 2(v - M + e \sin v)$

$$+ e \sin (v + 2\omega) + \frac{e}{3} \sin (3v + 2\omega)\,]$$

and $$r\,\Delta u = \frac{3}{4} J_2 \frac{a_e^2}{p^2} r \left[\left(1 - \frac{3}{2}\sin^2 i\right)\left\{\frac{4}{3e}\left(1 - \frac{e^2}{2} - \sqrt{1-e^2}\right)\sin v\right.\right.$$

$$\left.+ \frac{1}{3}\left(1 - \sqrt{1-e^2}\right) \sin 2v\right\} - \left(1 - \frac{7}{6}\sin^2 i\right) \sin 2u$$

$$+ (4 - 5\sin^2 i)(v - M + e \sin v)$$

$$\left. - \left(1 - \frac{5}{3}\sin^2 i\right) e \sin (v + 2\omega) - \frac{e}{3} \cos^2 i \sin (3v + 2\omega)\right]$$

where $v - M + e \sin v = 3e \sin v$ for $e = 0.01$

and $v - M + e \sin v = 3e \sin v - \frac{3}{4} e^2 \sin 2v$ for $e = 0.1$

for $e = 0.5$, M was computed directly.

The quantity $r\Delta\theta_1 = -\frac{3}{8} J_2 \frac{a_e^2}{p^2}\, r \sin 2i \left[\sin u + e\left\{\sin \omega + \frac{1}{3}\sin(2v+\omega)\right\}\right.$

$$\left. -2\,(v - M + e \sin v) \cos u\right]$$

Finally,

$$|\Delta \underline{r}| = [(\Delta r)^2 + (r\Delta\theta_3)^2 + (r\Delta\theta_1)^2]^{\frac{1}{2}}$$

In Appendix B, Figures B17 and B28, the three components of $\Delta \underline{r}$, along with $|\Delta \underline{r}|$, are shown to illustrate their relative effect on the overall position error. These charts indicate that for the higher eccentricities, the position-displacement vector magnitude $|\Delta \underline{r}|$ is determined mainly from the circumferential component $r\,\Delta\theta_3$. For the low eccentricity case (e=0.01), the three components contribute almost equally to the position-displacement.

2.2 ANALYSIS OF LONG-PERIOD TERMS

The long-period terms were analyzed in two ways. First, the variation in the value of the individual terms was calculated as a function of the argument of perigee, ω. Samples of these values are shown in Appendix B, Tables B12 through B16, where the relative size of the terms can be seen as a function of ω. Second, each group of long-period terms was eliminated, one group at a time, from the ephemeris calculation, and the position-error was obtained as a function of the argument of perigee. These results are illustrated in Appendix B, Figures B29 through B33.

2.3 POSITION PREDICTION ANALYSIS

The analyses discussed above were applied in an investigation of the terms required in the ephemeris formulation for a given prediction accuracy. The perturbative terms required for specified initial conditions (including accuracy desired) were determined a priori. The reference, "A General Perturbations Differential Correction Program" (Aeronutronic Publication U-2201), illustrates how the individual terms depend on secular, long- and/or short-period arguments. Prediction from this formulation of the prediction-error caused by any given secular term is straightforward. If the prediction-error obtained in omitting the group of secular perturbative terms in the longitude of the ascending node (terms 1 through 5 in Appendix A) from the ephemeris calculation is 4×10^4 km over a two-week interval; and further, if the largest value of a term in this group is 10^{-4}, while the next largest is 10^{-7}, then the contribution of the former term to the prediction-error is 10^3 times greater than that of the latter.

As an example, with an eccentricity of 0.01 and an inclination angle of 45°, the prediction-error obtained by omitting terms 1 through 5 in a prediction of 20,000 minutes was 6400 km. Within this group of terms, term 1 has a magnitude of 10^{-5}, terms 2 and 3 each have a magnitude of 10^{-8}, and terms 4 and 5 each have a magnitude of 10^{-12}. Thus, terms 4 and 5 can certainly be omitted from the ephemeris calculation, but term 1 must definitely be included. Terms 2 and 3 must be included for accurate prediction since they contribute of the order of 1 to 10 km to the total error.

Using the information obtained from the analysis described in Section 2.2, it was possible to determine the relative effects of the long-period terms on prediction accuracy.

The short-period terms were analyzed as described in Section 2.1. Drawing from information similar to that displayed in Appendix B (Figures B13 to B16) for the magnitude of the error attributed to these terms, and knowing the real value of each term, the relative effect of each term on prediction accuracy was deduced.

Knowing the relative effects of the individual secular, long- and short-period terms on position error, it was a simple matter to approximate the minimum number of terms necessary for a given prediction-accuracy.

Tables B1 through B3, in Appendix B, contain the results of this analysis and clearly show that many of the terms are not required for accurate position prediction. This is especially true for low-eccentricity orbits. One conclusion to be drawn from this analysis is that many general perturbations programs in operation contain superfluous terms which consume valuable computer time unnecessarily. Application of these results, by having the program select terms on the basis of accuracy required and initial orbital parameters, will result in a large decrease in the computer time needed for a given ephemeris prediction.

The results of this section, shown in Tables B1 through B3, were tested for prediction-accuracy at nine-minute intervals over a time-span of ninety minutes starting at the epoch time and over a similar time-span starting at 20,000 minutes past the epoch. Representative results of these tests are shown graphically in Appendix B, Figures B1 through B12. Six cases were selected to exhibit the prediction-accuracy at evenly spaced points around the orbit. The first two cases, Figures B1 through B4, show results obtained when using terms specified in Table B1 and indicate that the prediction-accuracy varies with the orbital position but does not exceed 10 km at any point.

Figures B5 through B12 illustrate analogous results obtained when using terms specified in Tables B2 and B3. These tables contain the terms necessary for a prediction-accuracy with a tolerance of 1 km and 0.1 km, respectively.

SECTION 3

SOLAR RADIATION PRESSURE EXPERIMENTATION

A general perturbations formulation, designed to account for the long-period variations in the orbital elements caused by direct solar radiation pressure, has recently been incorporated into the Aeronutronic General Perturbations Differential Correction Program. Several computer runs were made to ascertain the accuracy of this formulation with regard to mean element updating. The results of these runs are presented in Section 3.1.

The effect of the radiation pressure perturbations formulation on the differential correction process was also investigated. Here, the orbital elements of satellite 1960 Iota 1 were corrected with and without the radiation pressure perturbations. The results of this comparison are presented in Section 3.2.

Several other areas of this formulation were investigated during the experimentation. Section 3.3 presents and evaluates a new method of using the eclipse factor, ν. This method provides a substantial increase in prediction accuracy and a corresponding increase in the prediction interval allowable in one integration step. Section 3.4 discusses the calculation of the mean argument of perigee, ω. In this section it is shown that accurate prediction of ω is possible only if radiation pressure perturbations are included. In Section 3.5 a new method of treating apparent singularities in the radiation pressure formulation is presented. Incorporation of this method into the program will increase prediction accuracy and eliminate possible discontinuities in the formulation.

3.1 MEAN ELEMENT COMPARISON

The radiation pressure perturbation section of the program was evaluated by making several ephemeris runs starting with mean elements of satellite 1960 Iota 1 (Echo I) obtained from Smithsonian[5] tables. Because of its large area-to-mass ratio, this satellite is subject to relatively large solar radiation perturbations in its orbital elements.

Three runs were made. The first was a two-day prediction in which the satellite was in the Earth's shadow for part of each revolution. The second was a one-week prediction during which the satellite was exposed to continuous direct radiation pressure. The third run was a two-week prediction where the satellite's orbit entered the Earth's shadow during the second week.

The mean elements were obtained as output from the program for each of these prediction runs; no differential correction was used. These are shown, in comparison with mean elements from the Smithsonian tables for the same time-periods, in Table I. Examination of this table shows that the elements obtained from the program agree closely with those shown in the Smithsonian tables. This agreement is impossible to obtain when considering only bulge and drag perturbations. It emphasizes the need to account for solar radiation pressure perturbations whenever satellites with relatively large area-to-mass ratios are considered.

3.2 EFFECT OF RADIATION PRESSURE FORMULATION IN DIFFERENTIAL CORRECTION

For purpose of comparison, two differential correction computer runs were made using file data observations of Echo 1. The first run included the radiation pressure perturbations in the calculation. The second run bypassed these and included only the bulge and drag perturbations.

Several items of interest arise from this comparison. First, the final rms of the residuals was considerably higher in the second run, indicating that the program was unable to obtain as good a fit of the elements to the observations without the addition of the radiation pressure perturbations.

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[5] Smithsonian Institution Astrophysical Observatory, Research in Space Science, Special Reports Numbers 61 and 103, p. 12 and p. 20, respectively.

TABLE 1

MEAN ELEMENTS OF 1960 IOTA 1 (ECHO I)

Elements	Epoch Elements		Updated Elements		Epoch Elements		Updated Elements		Epoch Elements		Updated Elements	
	Input to Program	SAO	Output from Program	SAO	Input to Program	SAO	Output from Program	SAO	Input to Program	SAO	Output from Program	SAO
Epoch	1961 Jan 10.0	1961 Jan 10.0	1961 Jan 12.0	1961 Jan 12.0	1960 Aug 17.0	1960 Aug 17.0	1960 Aug 24.0	1960 Aug 24.0	1960 Aug 17.0	1960 Aug 17.0	1960 Aug 31.0	1960 Aug 31.0
e	0.07543	0.07543	0.07467	0.07445	0.01162	0.01162	0.01522	0.01522	0.01162	0.01162	0.01930	0.01910
i	47°.301	47°.301	47°.316	47°.292	47°.230	47°.230	47°.220	47°.238	47°.230	47°.230	47°.230	47°.222
Ω	145°.013	145°.013	138°.636	138°.638	242°.298	242°.298	220°.679	220°.678	242°.298	242°.298	199°.060	199°.062
ω	142°.62	142°.62	149°.62	149°.86	19°.90	19°.90	36°.74	36°.87	19°.90	19°.90	59°.60	58°.91

Second, the program did not succeed in correcting all seven elements in the second run (it corrected only a and U successfully) because the residuals were too large due to neglect of radiation pressure perturbations.

Third, the updated elements, especially the eccentricity, e, were more accurate when radiation pressure perturbations were included. The actual mean eccentricity, obtained from Smithsonian tables[6], near the final revolution time, was 0.01910. The updating, with the radiation pressure perturbations included, gave a value of 0.01879, while that obtained without these perturbations gave a value of 0.01598.

This comparison shows clearly that without radiation pressure perturbations included the program was unable to obtain a good set of mean orbital elements, thus further emphasizing the need to include these perturbations in the calculation of ephemerides for satellites with relatively large area-to-mass ratios.

3.3 EXPERIMENTATION WITH THE ECLIPSE FACTOR

The eclipse factor, ν, equivalent to the fractional part of a revolution spent in the sunlight by the satellite, is used as a factor in the calculation of the perturbations to allow a time-step of more than one revolution per integration. The time spent in the Earth's shadow per revolution changes continually due to the perturbations of the satellite orbit and the apparent motion of the sun[7]. This variation necessitates a recalculation of the eclipse factor for any time-interval greater than one day.

The experimentation was concerned with determining the best value of ν to use for long predictions. Originally, for integration steps greater than one day, the formulation used a value of ν calculated at the upper time limit. For a prediction of one week, this caused ν to vary from the average, between epoch ν and final ν, by as much as 0.22. This resulted in an error of more than 20% in the calculation of the perturbations of the elements.

To increase the prediction accuracy, the calculation of ν was changed so that the new ν, computed whenever the sum of the integration steps exceeds one day, is averaged with the previous ν. For example: if a four-day prediction were to be made using a two-day time increment, the program would first calculate a value of ν at the epoch time; it would then calculate a value of ν at two days past the epoch, then it would

[6] See footnote 5, p. 12

[7] op. cit., p. 11

recalculate ν at four days past the epoch, average this with the ν calculated at two days past the epoch, and use this average in the calculation of the ephemeris at four days past the epoch.

As shown in Table I, this formulation can be used for accurate predictions with a time-step as long as two weeks. This makes it easy to handle most orbit prediction requirements with only one integration step. When this is compared to the hundreds of steps necessary for an updating with comparable special perturbations techniques, the resulting improvement in program efficiency becomes obvious.

3.4 EXPERIMENTATION WITH THE CALCULATION OF THE MEAN ARGUMENT OF PERIGEE

When radiation pressure perturbations are not considered, the mean argument of perigee[8], ω_{so}, is calculated from

$$\omega_{so} = \omega_o + \frac{d\omega}{dt}(t - t_o)$$

where ω_o is the epoch mean argument of perigee and $\frac{d\omega}{dt}$ includes only secular bulge perturbations.

Originally, ω_{so} was calculated from this equation even when radiation pressure perturbations were considered. Experimentation showed that this formulation was inadequate for satellites such as Echo I. To improve the prediction accuracy the formulation was changed so that when radiation pressure perturbations are considered, the mean argument of perigee is calculated as:

$$\omega_{so} = \tan^{-1} \frac{a_{yN}}{a_{xN}}$$

where a_{xN} and a_{yN} include both secular bulge perturbations and radiation pressure perturbations. The resulting improvement in prediction accuracy is evident in Table II below.

[8] ibid., p. 23

TABLE II

ARGUMENT OF PERIGEE PREDICTION COMPARISON FOR SATELLITE 1960 IOTA 1

Epoch Time	ω_o at epoch (Note 3)	Prediction Time	ω_{so} Updated (Note 1)	ω_{so} Calculated (Note 2)	ω_{so} (Note 3)
1961 Jan 10.0	142°.62	1961 Jan 12.0	148°.72	149°.62	149°.86
1960 Aug 17.0	19°.90	1960 Aug 24.0	40°.66	36°.74	36°.87
1960 Aug 17.0	19°.90	1960 Aug 31.0	61°.42	59°.60	58°.91

Notes

1. Calculated from $\omega_{so} = \omega_o + \frac{d\omega}{dt}(t-t_o)$

2. Calculated from $\omega_{so} = \tan^{-1} \frac{a_{yN}}{a_{xN}}$

3. Obtained from SAO special reports, see footnote 4

3.5 TREATMENT OF APPARENT SINGULARITIES

Table A1, in Appendix A, contains a listing of the a_i^{-1} coefficients used in the radiation pressure perturbations formulation[9]. A knowledge of the behavior of these coefficients is important for two related reasons. First, they appear in the denominator of the expressions where they are used; and, second, as is evident from the graphs (Figures B34 through B77) shown in Appendix B, most of them pass through zero at various points dependent on the values of the orbital parameters.

At present, in the AGPDC program, the a_i^{-1} coefficients are individually compared to 10^{-4} as they are calculated. If they are less than 10^{-4}, then the term in which they appear is ignored.

[9] op. cit., pp. 6-10

The perturbations in the elements due to solar radiation pressure are each computed as a summation of terms of the form

$$k_1 \frac{\sin \gamma_t - \sin \gamma_o}{\dot{\gamma}}$$

or

$$k_2 \frac{\cos \gamma_t - \cos \gamma_o}{\dot{\gamma}}$$

where $\gamma_t = \gamma_o + \dot{\gamma} (t - t_o)$,

γ_i corresponds to b_i

and $\dot{\gamma}_i$ corresponds to a_i in the formulation.

Initial scrutiny of these expressions may indicate that as $\dot{\gamma}$ decreases and the period of the trigonometric function increases, the perturbations caused by these terms should decrease as shown in the sketch below, where ΔP is the perturbation and Δt is the prediction time, which is the same in both cases.

More careful scrutiny, however, reveals that this is not the case and that in actuality the perturbation increases to a limiting value as the period increases. This is because the denominator decreases as the period increases, and the perturbation is determined by the whole expression.

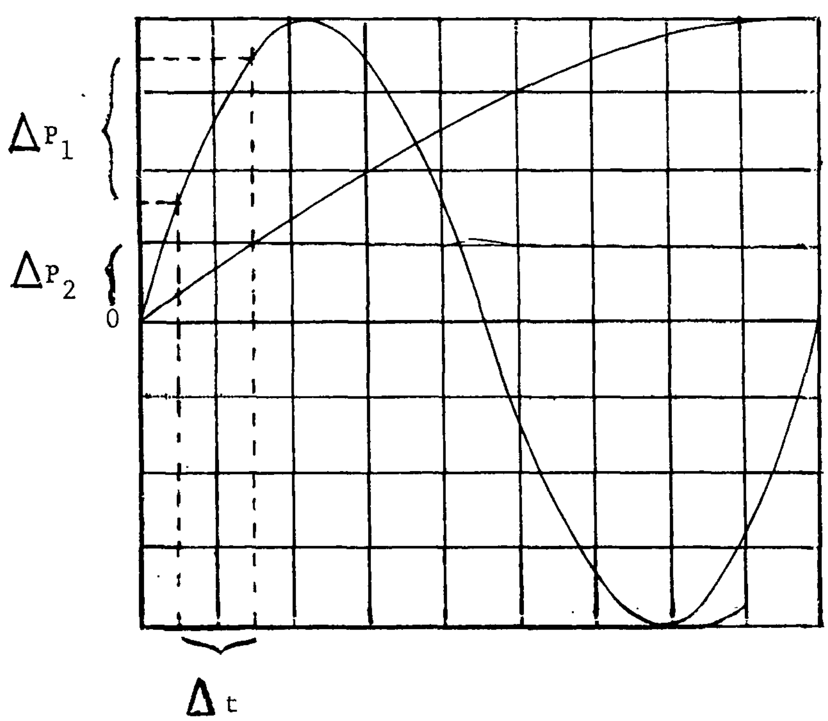
The limiting value of the first expression is given by

$$\lim_{\dot{\gamma} \to 0} \frac{\sin \gamma_t - \sin \gamma_o}{\dot{\gamma}} = \Delta t \cos \gamma_o .$$

Similarly, it follows from the definition of a derivative that

$$\lim_{\dot{\gamma} \to 0} \frac{\cos \gamma_t - \cos \gamma_o}{\dot{\gamma}} = \Delta t \sin \gamma_o .$$

FIGURE 1



If the function $\frac{\sin \gamma_t - \sin \gamma_o}{\dot{\gamma}}$ is considered, it can be seen that as $\dot{\gamma}$ approaches zero, there results a very large sine function whose amplitude must be $1/\dot{\gamma}$. If the prediction time, Δt, is much less than the period of this sine function, then the function may be represented in this region by a straight line with a slope of $\cos \gamma_o$. Thus $\Delta t \cos \gamma_o$ must represent the change in $\sin \gamma_t$ as t varies from t_o to $t_o + \Delta t$.

Therefore, when the denominator becomes less than 10^{-6}, these terms will be calculated by this "secular" approximation instead of being set equal to zero.

Graphs of the inverse functions of the a_i coefficients are shown in Appendix B (Figures B34 through B77). These graphs depict the magnitude and the critical points of the a_i coefficients as functions of the inclination angle, the eccentricity and the semi-major axis of the orbit.

These graphs may be of service in functional use of the equations in this section. One can readily obtain rough values of the a_i^{-1} coefficients for given conditions and subsequently ascertain whether these conditions produce a singularity in any of the coefficients. Knowing that the value of the a_i^{-1} is zero, in the light of the previous discussion, the program behavior is at once apparent.

SECTION 4

DIFFERENTIAL CORRECTION EXPERIMENTATION COMPARING THE AGPDC AND BGPDC PROGRAMS

The Aeronutronic General Perturbations Differential Correction Program (AGPDC) was tested in comparison with the Brouwer General Perturbations Differential correction (BGPDC); first, to show that the two programs produce essentially the same results under average conditions and second, to show that the AGPDC program is better able to handle a circular orbit (eccentricity,e, essentially zero) because its formulation avoids division by e. The following subsections present these comparisons in the order stated above.

4.1 COMPARISON USING BAKER-NUNN OBSERVATIONS

Two differential correction computer runs, using Baker-Nunn observations of satellite 1959 Eta, Vanguard III, were made. The first run was made with the AGPDC program, the second with the BGPDC program. The corrected epoch elements from both runs are shown, for comparison, in Table II below. These runs were designed to illustrate that the corrected elements obtained from the two programs are essentially the same. Examination of Table III reveals this to be the case.

TABLE III

COMPARISON OF DIFFERENTIALLY CORRECTED EPOCH ELEMENTS

Program	t_o (days)	L(deg)	a(E.R.)	e	i(deg)	Ω (deg)	ω (deg)
AGPDC	268.14984	216.10171	1.3338	0.18997	33.353	210.547	167.283
BGPDC	268.14984	216.11278	1.3345	0.18999	33.352	210.549	167.264

4.2 CIRCULAR ORBIT COMPARISON

In further comparison of the AGPDC and BGPDC programs, observations were simulated from an orbit with an orbit with an eccentricity,e, of zero. Using these observations and an input eccentricity of 10^{-2}, the two programs were matched.

In the formulation of the BGPDC program, several of the perturbative terms contain the eccentricity as a factor in their denominator. With a truly circular orbit this program should try to correct the eccentricity to zero. It would then use this eccentricity to compute the residuals in the next iterative pass. For a very samll e, the program should have trouble obtaining a good fit to the orbit because of the generation of large residuals.

Comparison of the output obtained from the two runs shows that the AGPDC run obtained a final rms of 0.3 km, while the BGPDC run was only able to reduce the size of the residuals to an rms of 8.7 km. In addition to this, the BGPDC program was unable to correct the elements on the first attempt, requiring more than twice as much calculation, and hence computer time, as the AGPDC run. This indicates that the AGPDC program is better suited for effective correction of the elements of a circular orbit.

SECTION 5

CONCLUSIONS

This report has been concerned with experimentation covering three aspects of the Aeronutronic General Perturbations Differential Correction Program: (1) analysis of the effects of the bulge perturbations terms on prediction accuracy, (2) experimentation with the radiation pressure formulation, and (3) comparison of the AGPDC program with the BGPDC program. Significant points which may be derived from this experimentation are as follows:

Selection of Terms

(1) The complete first order general perturbation (Earth's bulge) theory may be truncated into abbreviated packages depending on the eccentricity, the inclination and the desired prediction accuracy.

(b) These abbreviated packages would obviously circumvent superfluous computation, leading to a substantial saving in computer time.

This feature is highly desirable when the ephemeris routine is used in a differential correction process. In this process, where a large group of observations must be represented in several iterations, it is extremely advantageous to reduce the computational complexity of the routine.

(3) As a result of this analysis it is recommended that this term selection technique be considered for use in SPADATS operational differential correction programs. This approach would definitely aid in the current attempt to obtain a solution of the system saturation problem.

Radiation Pressure Formulation

(1) The ephemeris formulation in the AGPDC program can provide accurate prediction of the mean elements of satellites with large area-to-mass ratios when radiation pressure perturbation is a factor. This is substantiated in the differential correction of this type of satellite, where the updating of the eccentricity has been shown to be significantly more accurate than it would be neglecting radiation pressure.

(2) Averaging of the eclipse factor, ν, results in a considerable increase in position accuracy for long prediction periods.

(3) When radiation pressure perturbations are considered, they must be included in the calculation of the mean argument of perigee.

(4) The apparent singularities in the radiation pressure formulation, caused by the appearance of zero in the denominator of the perturbative terms, can be effectively treated with no loss in prediction accuracy.

(5) As an extension of this experimentation, a method similar to that developed for treatment of the radiation pressure singularities is currently being investigated for use with the singularities in the long-period bulge perturbations terms. When incorporated, this will improve the prediction accuracy by eliminating discontinuities in the formulation near the critical inclination angle of $63^\circ\!.43\cdots$.

Differential Correction Comparison of AGPDC (Aeronutronic) and BGPDC (Brouwer)

(1) Under most conditions the AGPDC and BGPDC programs produce essentially the same results.

(2) For circular orbits AGPDC is better able to correct the elements. The AGPDC formulation is based on elements which avoid zero eccentricity singularities. Thus, AGPDC provides a package suitable for SPADATS operational use devoid of low eccentricity problems. In addition, it operates with $\underline{M}$, $\underline{N}$ elements as do the routines currently in the operational system.

Thus, the results show that the Aeronutronic General Perturbations Differential Correction Program provides a versatile and accurate orbit determination scheme, well suited for operational use in a variety of satellite orbits.

APPENDIX A

DEFINITION OF SYMBOLS AND TERM CORRESPONDENCE

Symbol	Definition
a	semi-major axis of satellite orbit
a_e	equatorial radius of the Earth
a_{xN}	equivalent to $e \cos \omega$
a_{yN}	equivalent to $e \sin \omega$
e	eccentricity of satellite orbit
i	inclination of satellite orbit plane
i_{oL_n}	inclination obtained from nominal standard theory, including only long-period perturbations
i_n	osculating inclination obtained from nominal standard theory
J_2	second zonal harmonic, taken as 1.08245×10^{-3}
J_3	third zonal harmonic, taken as -2.5×10^{-6}
J_4	fourth zonal harmonic, taken as -1.85×10^{-6}
L	mean longitude of satellite, equivalent to $M + \Omega + \omega$
M	mean anomaly of satellite
p	semi-latus rectum of satellite orbit
q	geocentric perigee distance
r	magnitude of geocentric radius vector to satellite
$\underline{r}$	geocentric radius vector to satellite

r_n	magnitude of osculating radius vector from nominal standard theory
$\underline{r}_n$	radius vector to satellite from nominal standard theory
t	time past epoch, in minutes
t_o	time of epoch
u_n	argument of latitude obtained from nominal standard theory
v	true anomaly of satellite
$\underline{U}$	unit vector toward satellite, (radial)
$\underline{V}$	circumferential unit vector, completing right hand orthogonal system with $\underline{U}$ and $\underline{W}$
$\underline{W}$	unit vector perpendicular to orbit plane (binormal)
$\frac{d\omega}{dt}$	secular rate of change of ω
π	longitude of perigee, equivalent to $\Omega + \omega$
ω	argument of perigee
Ω_n	longitude of ascending node obtained from nominal standard theory

TABLE A1

The Coefficients a_i^{-1} (radians/k_e^{-1} min) used in the Radiation Pressure Formulation

$a_1^{-1} = (\Omega' + \omega' + n_\odot)$	$a_{12}^{-1} = (\Omega' - n_\odot)$
$a_2^{-1} = (\Omega' + \omega' - n_\odot)$	$a_{13}^{-1} = (2\,\Omega' + n_\odot)$
$a_3^{-1} = (\Omega' - \omega' + n_\odot)$	$a_{14}^{-1} = (2\,\Omega' - n_\odot)$
$a_4^{-1} = (\Omega' - \omega' - n_\odot)$	$a_{15}^{-1} = (n_\odot)$
$a_5^{-1} = (\omega' + n_\odot)$	$a_{16}^{-1} = (\Omega' + 2\,\omega' + n_\odot)$
$a_6^{-1} = (\omega' - n_\odot)$	$a_{17}^{-1} = (\Omega' + 2\,\omega' - n_\odot)$
$a_7^{-1} = (2\,\Omega' - \omega' - n_\odot)$	$a_{18}^{-1} = (2\,\omega' + n_\odot)$
$a_8^{-1} = (2\,\Omega' - \omega' + n_\odot)$	$a_{19}^{-1} = (2\,\omega' - n_\odot)$
$a_9^{-1} = (2\,\Omega' + \omega' - n_\odot)$	$a_{20}^{-1} = (2\,\Omega' + 2\,\omega' - n_\odot)$
$a_{10}^{-1} = (2\,\Omega' + \omega' + n_\odot)$	$a_{21}^{-1} = (2\Omega' + 2\omega' + n_\odot)$
$a_{11}^{-1} = (\Omega' + n_\odot)$	

Note: $n_\odot = 1.6064 \times 10^{-4} \dfrac{\text{radians}}{k_e^{-1}\,\text{min}}$

TABLE A2

BULGE PERTURBATIONS TERM CORRESPONDENCE*

Number	Name	Number	Name	Number	Name	Number	Name
1	Ω_1	18	a_{xN_3}	35	$a_{yN_{10}}$	52	L_6
2	Ω_2	19	a_{xN_4}	36	$a_{yN_{11}}$	53	i_1
3	Ω_3	20	a_{xN_5}	37	$a_{yN_{12}}$	54	i_2
4	Ω_4	21	a_{xN_6}	38	$a_{yN_{13}}$	55	i_3
5	Ω_5	22	a_{xN_7}	39	M_1	56	u_1
6	ω_1	23	a_{xN_8}	40	M_2	57	u_2
7	ω_2	24	a_{xN_9}	41	M_3	58	u_3
8	ω_3	25	$a_{xN_{10}}$	42	π_1	59	u_4
9	ω_4	26	$a_{xN_{11}}$	43	π_2	60	u_5
10	ω_5	27	a_{yN_1}	44	π_3	61	u_6
11	Ω_6	28	a_{yN_2}	45	π_4	62	r_1
12	Ω_7	29	a_{yN_3}	46	π_5	63	r_2
13	Ω_8	30	a_{yN_4}	47	L_1	64	r_3
14	Ω_9	31	a_{yN_5}	48	L_2	65	$\dot{r}_1$
15	Ω_{10}	32	a_{yN_7}	49	L_3	66	$\dot{r}_2$
16	a_{xN_1}	33	a_{yN_8}	50	L_4	67	$\dot{r}_3$
17	a_{xN_2}	34	a_{yN_9}	51	L_5	68	$\dot{r}_4$

*According to the notation adopted in "Aeronutronic General Perturbations Differential Correction Program" (Aeronutronic Publ. No. U-2201, pp 21-44).

Number	Name	Number	Name	Number	Name	Number	Name
69	$\dot{r}_5$	74	$r\dot{v}_1$	79	Ω_6	84	Ω_{14}
70	$\dot{r}_6$	75	$r\dot{v}_2$	80	Ω_7	85	i_4
71	$\dot{r}_7$	76	$r\dot{v}_3$	81	Ω_{11}	86	i_5
72	$\dot{r}_8$	77	$r\dot{v}_4$	82	Ω_{12}	87	i_6
73	$\dot{r}_9$	78	$r\dot{v}_5$	83	Ω_{13}	88	a_{yN_6}
						89	$r\dot{v}_8$

APPENDIX B

DATA AND FIGURES

This appendix contains data obtained from experimentation conducted with the Aeronutronic General Perturbations Differential Correction Program. Tables are presented showing the bulge perturbations terms necessary for a prediction accuracy within tolerances of 10 km, 1 km or 0.1 km. Following these are tables showing the values of the secular, short- and long-period bulge perturbation terms at various values of inclination and eccentricity. Also presented is a section of graphs, which include plots of the results in Section 2 on prediction accuracy, and graphs of the a^{-1} coefficients discussed in Section 3 on radiation pressure.

B.1 REQUIRED TERMS FOR A GIVEN ACCURACY

The following tables show the terms necessary for a given position accuracy, with the eccentricity e and the inclination i as parameters.

TABLE B1

TERMS NECESSARY FOR A POSITION PREDICTION ERROR OF LESS THAN 10 KILOMETERS

	e = 0.001
i = 1°	1,2,3,6,39,42,43,45
i = 15°	1,2,3,6,39,42,43,45
i = 30°	1,2,3,6,42,43,45,88
i = 45°	1,2,3,6,42,43,45,88
i = 60°	1,42,43,45,88
i = 75°	1,6,42,43,45,88
i = 90°	6,39,42,43,45,88

	e = 0.01
i = 1°	1,2,3,6,39,42,43,45,56,81
i = 15°	1,2,3,6,39,42,43,45
i = 30°	1,2,3,6,42,43,45,88
i = 45°	1,2,3,6,42,43,45,88
i = 60°	1,6,42,43,45,88
i = 75°	1,6,42,43,45,88
i = 90°	6,42,43,45,88

	e = 0.1
i = 1°	1,2,3,4,5,6,8,13,21,32,33,39,41,42,43,45,46
i = 15°	1,2,3,5,6,8,42,43,45
i = 30°	1,2,3,6,42,43,45,88
i = 45°	1,2,3,6,7,8,39,42,43,45,54,56,63,81,85,88
i = 60°	1,2,3,6,7,8,22,43,43,45,49,56,63,81,85,88
i = 75°	1,6,42,43,45,88
i = 90°	6,8,42,43,45,88

TABLE B1 (Continued)

	e = 0.5
i = 1°	1,2,3,5,6,7,8,9,10,11,13,14,21,25,32,33,39,40,41, 42,43,44,45,46,54,56,57,60,61,62,64,81,82,83,84
i = 15°	1,3,5,6,8,10,13,21,32,33,39,40,41,42,45,49,50,51, 54,56,57,62,81,82,83,88
i = 30°	1,3,6,8,13,18,21,22,24,25,26,32,42,43,45,49,54, 56,57,62,63,64,81,82,83,85,88
i = 45°	1,6,8,13,15,16,18,19,20,22,23,24,25,26,27,29,30, 31,32,34,35,36,37,38,42,43,45,46,47,48,49,50,51, 52,54,56,57,60,62,63,64,81,82,83,88
i = 60°	1,2,3,4,5,6,7,8,9,10,11,13,14,15,17,21,22,23,24, 25,26,28,30,32,33,34,35,36,37,38,39,42,43,45,46, 47,49,50,51,52,54,57,63,82,85,87,88
i = 75°	1,2,3,5,6,7,8,14,21,22,24,32,33,34,37,38,42,47, 49,50,51,52,57,88
i = 90°	6,7,8,10,22,32,33,34,36,37,38,42,43,45,46,47,49, 51,52,57,60,88

TABLE B2

TERMS NECESSARY FOR A POSITION PREDICTION ERROR OF LESS THAN 1 KILOMETER

	e = 0.001
i = 1°	1,2,3,6,39,42,43,45,56,81
i = 15°	1,2,3,6,39,42,43,45,56,81,85,88
i = 30°	1,2,3,6,39,42,43,45,56,81,85,88
i = 45°	1,2,3,6,8,39,42.43,45,56,63,81,85,88
i = 60°	1,2,3,6,42,43,45,56,63,81,85,88
i = 75°	1,2,3,6,39,42,43,45,56,63,81,85,88
i = 90°	6,39,42.43,45,63,88
	e = 0.01
i = 1°	1,2,3,6,8,39,42,43,45,56,57,81
i = 15°	1,2,3,6,8,39,42,43,45,54,56,57,81,82,85,88
i = 30°	1,2,3,6,39,42,43,45,56,81,85,88
i = 45°	1,2,3,6,7,8,39,42,43,45,56,57,63,81,82,85,88
i = 60°	1,2,3,6,42,43,45,56,57,60,63,81,85,88
i = 75°	1,2,3,6,39,42,43,45,56,57,63,81,85,88
i = 90°	6,8,39,42,43,45,56,63,88
	e = 0.1
i = 1°	1,2,3,4,5,6,8,13,21,32,33,39,40,41,42,43,44,45, 46,54,56,57,62,64,81,82,88
i = 15°	1,2,3,4,5,6,7,8,10,13,32,33,39,41,42,43,44,45, 46,49,50,56,57,60,61,62,63,64,81,82,83,84,85,88
i = 30°	1,2,3,5,6,7,8,13,22,39,42,43,45,49,50,54,56,57, 60,61,62,63,64,81,82,83,84,85,88
i = 45°	1,2,3,6,7,8,27,34,39,42,43,45,49,54,56,57,63,81, 82,83,84,85,86,88
i = 60°	1,2,3,6,7,8,15,16,22,27,34,39,42,43,45,47,48,49, 50,51,52,54,56,57,60,63,81,82,83,85,88
i = 75°	1,2,3,6,7,8,13,22,32,33,39,42,43,45,49,50,56,57, 60,61,62,63,81,82,85,86,88
i = 90°	6,7,8,22,27,32,33,34,39,42,43,45,49,56,57,62,63,88

TABLE B2 (Continued)

e = 0.5

i = 1°	1,2,3,5,6,7,8,9,10,11,13,14,17,18,19,21,23,25,29, 32,33,35,37,39,40,41,42,43,44,45,46,49,50,54,56, 57,60,61,62,64,81,82,83,84,85,86,88
i = 15°	1,2,3,5,6,7,8,9,10,11,13,14,21,25,29,32,33,34,35, 39,40,41,42,43,44,45,46,47,48,49,50,51,52,54,55, 56,57,60,61,62,63,64,81,82,83,84,88
i = 30°	1,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20, 21,22,23,24,25,26,27,29,32,33,34,35,36,37,38,39, 40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55, 56,57,60,61,62,63,64,81,82,83,84,85,86,87,88
i = 45°	1,2,3,5,6,7,8,10,12,13,14,15,16,17,18,19,20,22, 23,24,25,26,27,29,30,31,32,34,35,36,37,38,41,42, 43,44,45,46,47,48,49,50,51,52,54,56,57,60,62,63, 64,81,82,83,85,86,87,88
i = 60°	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19, 20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35, 36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51, 52,53,54,55,56,57,59,60,61,62,63,64,81,82,83,85, 86,87,88
i = 75°	1,2,3,4,5,6,7,8,9,10,11,13,14,15,17,19,21,22,23, 24,25,26,28,30,32,33,34,35,36,37,38,39,40,41,42, 43,44,45,46,47,48,49,50,51,52,54,55,56,57,60,62, 63,64,81,82,83,85,87,88
i = 90°	6,7,8,10,16,21,22,23,24,25,26,32,33,34,36,37,38, 39,41,42,43,45,46,47,49,51,52,56,57,58,59,60,62, 63,64,88

TABLE B3

TERMS NECESSARY FOR A POSITION PREDICTION ERROR OF LESS THAN 0.1 KILOMETER

	e = 0.001
i = 1°	1,2,3,6,8,39,42,43,45,49,50,56,81,85,88
i = 15°	1,2,3,6,8,39,42,43,45,54,56,57,63,81,85,88
i = 30°	1,2,3,6,39,42,43,45,56,63,81,85,88
i = 45°	1,2,3,6,8,39,42,43,45,56,63,81,85,88
i = 60°	1,2,3,6,39,42,43,45,56,63,81,85,88
i = 75°	1,2,3,6,39,42,43,45,56,63,81,85,88
i = 90°	6,39,42,43,45,56,57,63,88

	e = 0.01
i = 1°	1,2,3,6,8,39,42,43,45,54,56,57,81,82,85,88
i = 15°	1,2,3,6,7,8,13,39,41,42,43,45,46,54,56,57,60,61,62,63,64,81,82,83,84,85,86,88
i = 30°	1,2,3,5,6,7,8,13,22,39,42,43,45,49,50,54,56,57,60,61,62,63,64,81,82,83,84,85,86,88
i = 45°	1,2,3,6,7,8,39,42,43,45,49,50,56,57,63,81,82,85,88
i = 60°	1,2,3,6,7,8,13,15,16,22,27,34,39,42,43,45,47,48,49,50,51,52,54,56,57,60,63,64,81,82,83,85,86,88
i = 75°	1,2,3,6,39,42,43,45,49,56,57,63,81,85,88
i = 90°	6,7,8,39,42,43,45,49,56,57,63,88

	e = 0.1
i = 1°	1,2,3,4,5,6,7,8,10,13,21,32,33,39,40,41,42,43,44,45,46,49,54,56,57,60,62,64,81,82,83,85,88
i = 15°	1,2,3,4,5,6,7,8,9,10,13,16,21,22,32,33,34,39,40,41,42,43,44,45,56,49,50,51,54,56,57,60,61,62,63,64,81,82,83,84,85,88
i = 30°	1,2,3,4,5,6,7,8,10,13,16,21,22,27,32,33,34,39,40,42,43,44,45,46,47,49,50,51,52,54,55,56,57,60,61,62,63,64,81,82,83,84,85,86,88
i = 45°	1,2,3,4,5,6,7,8,9,10,13,16,22,25,26,27,32,34,39,40,41,42,43,45,46,47,49,50,51,52,53,54,55,56,57,60,61,62,63,81,82,83,84,85,86,87,88
i = 60°	1,2,3,4,5,6,7,8,10,11,12,13,14,15,16,18,21,22,23,24,25,26,27,29,31,32,34,35,36,37,38,39,41,42,43,44,45,46,47,48,49,50,51,52,54,56,57,60,61,63,81,82,83,84,85,86,87,88
i = 75°	1,2,3,4,5,6,7,8,13,21,22,24,32,33,34,39,40,41,42,43,44,45,46,49,50,51,52,54,55,56,57,60,61,62,63,64,81,82,83,84,85,86,88
i = 90°	6,7,8,9,10,16,21,22,27,32,33,34,36,37,38,39,40,41,42,43,44,45,46,47,49,51,56,57,60,62,63,64,88

TABLE B3 (Continued)

	e = 0.5
$i = 1^o$	1,2,3,4,5,6,7,8,9,10,11,13,14,17,18,19,21,23,25, 28,29,30,32,33,35,37,39,40,41,42,43,44,45,46,49, 50,54,55,56,57,60,61,62,64,81,82,83,84,85,86,87, 88
$i = 15^o$	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19, 20,21,22,23,24,25,26,27,28,29,30,32,33,34,35,36, 37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52, 53,54,55,56,57,58,59,60,61,62,63,64,81,82,83,84, 85,86,87,88
$i = 30^o$	1,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20, 21,22,23,24,25,26,27,29,32,33,34,35,36,37,38,39, 40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56, 57,58,59,60,61,62,63,64,81,82,83,84,85,86,87,88
$i = 45^o$	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19, 20,22,23,24,25,26,27,28,29,30,31,32,34,35,36,37, 38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,54,55,56, 57,59,60,61,62,63,64,81,82,83,84,85,86,87,88
$i = 60^o$	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19, 20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35, 36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51, 52,53,54,55,56,57,58,59,60,61,62,63,64,81,82,83, 84,85,86,87,88
$i = 75^o$	1,2,3,4,5,6,7,8,9,10,11,13,14,15,17,19,21,22,23, 24,25,26,28,30,32,33,34,35,36,37,38,39,40,41,42, 43,44,45,46,47,48,49,50,51,52,54,55,56,57,58,59, 60,61,62,63,64,81,82,83,84,85,86,87,88
$i = 90^o$	6,7,8,9,10,16,21,22,23,24,25,26,27,29,30,31,32, 33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48, 49,50,51,52,56,57,58,59,60,62,63,64,88

B.2 TABLES OF SECULAR TERM VALUES WITH ECCENTRICITY AND INCLINATION AS PARAMETERS

Tables B4, B5, B6, and B7 contain values of the secular terms used in the formulation. The initial parameters are those listed at the beginning of Section 2. The terms with a factor of cos i are not identically equal to zero at i = 90° because the computer uses a series approximation to obtain the cosine.

TABLE B4

VALUES OF THE SECULAR TERMS WITH THE ECCENTRICITY EQUAL TO 0.001

Term #	i = 1°	i = 30°	i = 60°	i = 90°
1	-1.112-04	-9.636-05	-5.566-05	-2.012-13
2	8.612-08	2.489-08	-4.314-08	-2.599-16
3	-4.901-07	-2.390-07	7.670-08	6.654-16
4	-2.874-14	-3.267-14	-2.786-14	-1.170-22
5	-7.532-13	-3.585-13	1.150-13	9.981-22
6	2.224-04	1.530-04	1.392-05	-5.568-05
7	1.272-10	7.163-08	1.955-08	-1.043-07
8	9.797-07	2.185-07	-1.802-07	1.841-07
9	1.006-13	7.938-14	6.628-15	-4.495-14
10	1.102-12	3.191-13	-1.639-13	1.381-13
39	-8.610-08	-2.133-08	2.022-09	-3.955-08
40	7.181-14	5.109-14	-7.302-15	-4.495-14
41	3.673-13	8.623-15	-1.064-13	1.381-13
42	1.112-04	5.663-05	-4.175-05	-5.568-05
43	8.625-08	9.652-08	-2.359-08	-1.043-07
44	7.181-14	4.671-14	-2.123-14	-4.495-14
45	4.896-07	-2.052-08	-1.035-07	1.841-07
46	3.671-13	-3.940-14	-4.889-14	1.381-13

TABLE B5

VALUES OF THE SECULAR TERMS WITH THE ECCENTRICITY EQUAL TO 0.01

Term #	i = 1°	i = 30°	i = 60°	i = 90°
1	-1.078-04	-9.337-05	-5.394-05	-1.950-13
2	8.196-08	2.368-08	-4.106-08	-2.474-16
3	-4.665-07	-2.275-07	7.300-08	6.334-16
4	-2.736-12	-3.110-12	-2.652-12	-1.113-20
5	-6.997-11	-3.412-11	1.095-11	9.500-20
6	2.155-04	1.482-04	1.348-05	-5.395-05
7	1.535-10	6.820-08	1.861-08	-9.925-08
8	9.325-07	2.079-07	-1.716-07	1.753-07
9	9.571-12	7.556-12	6.309-13	-4.278-12
10	1.049-10	3.037-11	-1.560-11	1.314-11
39	-8.194-08	-2.030-08	1.925-09	-3.764-08
40	6.835-12	4.862-12	-6.950-13	-4.278-12
41	3.496-11	8.207-13	-1.013-11	1.314-11
42	1.077-04	5.488-05	-4.045-05	-5.395-05
43	8.211-08	9.188-08	-2.245-08	-9.925-08
44	6.835-12	4.446-12	-2.012-12	-4.278-12
45	4.660-07	-1.953-08	-9.855-08	1.753-07
46	3.495-11	-3.750-12	-4.654-12	1.314-11

TABLE B6

VALUES OF THE SECULAR TERMS WITH THE ECCENTRICITY EQUAL TO 0.1

Term #	i = 1°	i = 30°	i = 60°	i = 90°
1	-7.876-05	-6.824-05	-3.924-05	-1.425-13
2	4.949-08	1.405-08	-2.523-08	-1.515-16
3	-2.874-07	-1.401-07	4.497-08	3.902-16
4	-1.686-10	-1.916-10	-1.634-10	-6.858-19
5	-4.311-09	-2.102-09	6.746-10	5.853-18
6	1.575-04	1.083-04	9.854-06	-3.942-05
7	2.100-09	4.288-08	1.145-08	-6.089-08
8	5.746-07	1.281-07	-1.057-07	1.080-07
9	5.897-10	4.655-10	3.887-11	-2.636-10
10	6.464-09	1.871-09	-9.613-10	8.097-10
39	-4.957-08	-1.281-08	1.190-09	-2.291-08
40	4.191-10	2.981-10	-4.260-11	-2.622-10
41	2.143-09	5.032-11	-6.209-10	8.056-10
42	7.873-05	4.010-05	-2.956-05	-3.942-05
43	5.159-08	5.693-08	-1.379-08	-6.089-08
44	4.211-10	2.739-10	-1.245-10	-2.636-10
45	2.871-07	-1.203-08	-6.071-08	1.080-07
46	2.153-09	-2.311-10	-2.867-10	8.097-10

TABLE B7

VALUES OF THE SECULAR TERMS WITH ECCENTRICITY EQUAL TO 0.5

Term #	i = 1°	i = 30°	i = 60°	i = 90°
1	-1.755-05	-1.520-05	-8.777-06	-3.172-14
2	2.807-09	-8.409-12	-2.826-09	-1.531-17
3	-3.443-08	-1.679-08	5.385-09	4.672-17
4	-5.048-10	-5.737-10	-4.891-10	-2.053-18
5	-1.291-08	-6.295-09	2.020-09	1.752-17
6	3.508-05	2.413-05	2.194-06	-8.778-06
7	6.495-09	7.821-09	1.322-09	-6.510-09
8	6.883-08	1.535-08	-1.266-08	1.293-08
9	1.766-09	1.394-09	1.164-10	-7.889-10
10	1.936-08	5.603-09	-2.878-09	2.424-09
39	-3.367-09	-5.657-10	1.522-10	-1.936-09
40	1.092-09	7.769-10	-1.110-10	-6.832-10
41	5.586-09	1.311-10	-1.618-09	2.099-09
42	1.754-05	8.932-06	-6.583-06	-8.778-06
43	9.301-09	7.813-09	-1.504-09	-6.510-09
44	1.261-09	8.202-10	-3.727-10	-7.889-10
45	3.440-08	-1.441-09	-7.270-09	1.293-08
46	6.449-09	-6.919-10	-8.583-10	2.424-09

TABLE B8

SHORT PERIOD TERMS IN u

Term #	Time (minutes past epoch)						
	15,000	15,015	15,030	15,045	15,060	15,075	15,090
	e = 0.5			i = 1°			
56	2.312-04	3.223-04	3.319-04	9.697-05	-3.449-04	2.757-04	1.818-04
57	-1.547-03	-1.867-03	-2.065-03	-1.942-03	-8.711-04	1.150-03	2.002-03
58	-4.869-06	-6.290-06	-7.655-06	-8.200-06	-4.215-06	5.467-06	8.257-06
59	1.468-05	1.521-05	1.072-05	-3.641-06	-1.353-05	1.529-05	5.337-07
60	-1.787-05	-5.755-05	-1.060-04	-1.586-04	-1.503-04	3.465-05	1.482-04
61	-5.630-05	-5.045-05	-6.160-06	5.673-05	-4.877-05	5.734-05	-4.835-05
	e = 0.5			i = 45°			
56	2.582-05	9.053-05	1.414-04	8.379-05	-1.431-04	9.735-05	1.175-04
57	-6.107-04	-7.240-04	-7.813-04	-6.845-04	-1.600-04	5.496-04	7.721-04
58	-1.301-06	-1.660-06	-1.985-06	-1.996-06	-5.257-07	1.697-06	2.062-06
59	3.779-06	3.714-06	2.147-06	-2.031-06	-1.896-06	3.641-06	-8.369-07
60	1.378-05	7.072-06	-2.951-06	-1.801-05	-2.859-05	-6.171-06	1.407-05
61	-2.180-05	-2.880-05	-1.347-05	2.713-05	-2.686-05	2.858-05	-1.912-05
	e = 0.5			i = 89°			
56	2.106-05	-8.972-06	-4.588-05	-4.546-05	5.647-05	-2.705-05	-5.653-05
57	4.248-04	4.944-04	5.182-04	4.103-04	-1.590-05	-4.234-04	-5.191-04
58	2.753-06	3.469-06	4.062-06	3.710-06	-1.574-07	-3.796-06	-4.039-06
59	-7.665-06	-7.055-06	-2.901-06	6.132-06	-5.867-07	-5.638-06	3.286-06
60	-1.028-04	-8.550-05	-5.040-05	2.270-05	1.123-04	7.157-05	4.778-06
61	-6.060-09	-1.603-08	-1.306-08	1.578-08	-1.737-08	1.750-08	-6.810-09

TABLE B9

SHORT PERIOD TERMS IN THE MAGNITUDE OF THE RADIUS VECTOR r

Term #	Time (minutes past epoch) 15.000	15.015	15.030	15.045	15.060	15.075	15.090
			e = 0.5	i = 1°			
62	1.597-04	9.918-05	2.326-05	-6.422-05	-1.391-04	-1.305-04	-4.989-05
63	-3.825-08	-1.838-08	1.410-08	4.944-08	1.178-09	-3.098-08	4.379-08
64	7.635-05	6.126-05	3.546-05	-1.125-05	-8.129-05	-7.084-05	-1.637-06
			e = 0.5	i = 45°			
62	3.703-05	2.098-05	1.131-06	-2.106-05	-3.682-05	-2.897-05	-7.453-06
63	-8.679-05	-6.862-05	-1.682-05	7.176-05	1.012-05	-6.502-05	5.099-05
64	1.847-05	1.423-05	6.874-06	-6.470-06	-2.293-05	-1.364-05	2.579-06
			e = 0.5	i = 89°			
62	-6.741-05	-3.365-05	7.477-06	5.137-05	7.456-05	4.903-05	4.948-06
63	-1.642-04	-1.743-04	-1.062-04	1.079-04	3.243-05	-1.557-04	3.132-05
64	-3.531-05	-2.579-05	-9.058-06	2.096-05	4.726-05	1.883-05	-1.032-05

TABLE B10

SHORT PERIOD TERMS IN THE LONGITUDE OF THE ASCENDING NODE Ω

Term #	Time (minutes past epoch)						
	15.000	15.015	15.030	15.045	15.060	15.075	15.090
			e = 0.5	i = 1°			
81	-2.312-04	-3.224-04	-3.319-04	-9.700-05	3.450-04	-2.758-04	-1.818-04
82	7.737-04	9.338-04	1.033-03	9.711-04	4.356-04	-5.749-04	-1.001-03
83	1.787-05	5.757-05	1.060-04	1.587-04	1.504-04	-3.466-05	-1.482-04
84	5.630-05	5.045-05	6.161-06	-5.673-05	4.878-05	-5.734-05	4.836-05
			e = 0.5	i = 45°			
81	-4.381-05	-1.536-04	-2.398-04	-1.421-04	2.427-04	-1.651-04	-1.994-04
82	5.754-04	6.822-04	7.362-04	6.450-04	1.508-04	-5.179-04	-7.275-04
83	-5.832-05	-2.994-05	1.250-05	7.625-05	1.210-04	2.613-05	-5.939-05
84	3.083-05	4.072-05	1.904-05	-3.836-05	3.798-05	-4.041-05	2.703-05
			e = 0.5	i = 89°			
81	2.210-06	-9.417-07	-4.815-06	-4.772-06	5.927-06	-2.840-06	-5.934-06
82	1.485-05	1.729-05	1.812-05	1.435-05	-5.561-07	-1.480-05	-1.815-05
83	-2.693-06	-2.241-06	-1.321-06	5.949-07	2.942-06	1.875-06	1.252-07
84	3.471-07	9.182-07	7.479-07	9.038-07	9.948-07	-1.002-06	3.901-07

TABLE B11

SHORT PERIOD TERMS IN THE INCLINATION i

Term #	Time (minutes past epoch) 15.000	15.015	15.030	15.045	15.060	15.075	15.090
			e = 0.5		i = 1°		
85	-4.378-06	-2.104-06	1.614-06	5.659-06	1.349-07	-3.545-06	5.012-06
86	2.933-06	2.780-06	2.326-06	1.160-06	-1.447-06	-2.889-06	-1.510-06
87	2.012-07	-4.720-07	-9.772-07	-1.620-07	5.209-07	7.703-08	5.322-07
			e = 0.5		i = 45°		
85	-1.699-04	-1.343-04	-3.294-05	1.405-04	1.981-05	-1.273-04	9.984-05
86	7.595-05	8.379-05	8.597-05	6.755-05	-1.217-05	-8.443-05	-7.554-05
87	1.884-05	-1.037-06	-2.547-05	-9.726-06	1.045-05	3.706-06	2.156-05
			e = 0.5		i = 89°		
85	-5.609-06	-5.955-06	-3.629-06	3.685-06	1.108-06	-5.318-06	1.070-06
86	1.360-06	2.020-06	2.712-06	2.957-06	6.668-07	-2.363-06	-3.014-06
87	9.436-07	4.101-07	-6.721-07	-4.409-07	1.470-07	8.105-08	9.267-07

TABLE B12

LONG-PERIOD TERMS IN THE LONGITUDE OF THE ASCENDING NODE Ω

Term #	$\omega = 0°$	$\omega = 15°$	$\omega = 30°$	$\omega = 45°$	$\omega = 60°$	$\omega = 75°$	$\omega = 90°$
		$e = 0.01$		$i = 30°$			
11	0.0	-6.493-09	-1.125-08	-1.299-08	-1.125-08	-6.493-09	-4.693-17
12	0.0	-4.654-09	-8.061-09	-9.308-09	-8.061-09	-4.654-09	-3.364-17
13	1.859-05	1.796-05	1.610-05	1.315-05	9.296-06	4.812-06	3.360-14
14	0.0	2.131-08	3.691-08	4.262-08	3.691-08	2.131-08	1.540-16
15	0.0	1.647-08	2.852-08	3.294-08	2.852-08	1.647-08	1.190-16
		$e = 0.1$		$i = 30°$			
11	0.0	-5.474-07	-9.481-07	-1.095-06	-9.481-07	-5.474-07	-3.956-15
12	0.0	-3.924-07	-6.796-07	-7.847-07	-6.796-07	-3.924-07	-2.836-15
13	1.707-04	1.649-04	1.478-04	1.207-04	8.536-05	4.418-05	3.085-13
14	0.0	1.797-06	3.112-06	3.593-06	3.111-06	1.797-06	1.299-14
15	0.0	1.388-06	2.405-06	2.777-06	2.405-06	1.388-06	1.004-14
		$e = 0.5$		$i = 30°$			
11	0.0	-7.359-06	-1.275-05	-1.472-05	-1.275-05	-7.359-06	-5.319-14
12	0.0	-5.275-06	-9.136-06	-1.055-05	-9.136-06	-5.275-06	-3.813-14
13	6.259-04	6.046-04	5.421-04	4.426-04	3.130-04	1.620-04	1.131-12
14	0.0	2.415-05	4.184-05	4.831-05	4.184-05	2.415-05	1.746-13
15	0.0	1.867-05	3.233-05	3.733-05	3.233-05	1.867-05	1.349-13

TABLE B13

LONG-PERIOD TERMS IN THE LONGITUDE OF THE ASCENDING NODE Ω

Term #	$\omega = 0^{\circ}$	$\omega = 15^{\circ}$	$\omega = 30^{\circ}$	$\omega = 45^{\circ}$	$\omega = 60^{\circ}$	$\omega = 75^{\circ}$	$\omega = 90^{\circ}$
		$e = 0.01$		$i = 60^{\circ}$			
11	0.0	5.392-08	9.340-08	1.078-07	9.340-08	5.392-08	3.898-16
12	0.0	-2.617-07	-4.532-07	-5.234-07	-4.532-07	-2.617-08	-1.891-15
13	6.197-06	5.986-06	5.368-06	4.382-06	3.099-06	1.604-06	1.120-14
14	0.0	-2.436-07	-4.220-07	-4.872-07	-4.220-07	-2.436-07	-1.761-15
15	0.0	6.090-07	1.055-06	1.218-06	1.055-06	6.090-07	4.402-15
		$e = 0.1$		$i = 60^{\circ}$			
11	0.0	4.546-06	7.874-06	9.092-06	7.874-06	4.546-06	3.286-14
12	0.0	-2.206-05	-3.821-05	-4.412-05	-3.821-05	-2.207-05	-1.595-13
13	5.690-05	5.497-05	4.928-05	4.024-05	2.845-05	1.473-05	1.028-13
14	0.0	-2.054-05	-3.557-05	-4.108-05	-3.557-05	-2.054-05	-1.485-13
15	0.0	5.135-05	8.893-05	1.027-04	8.893-05	5.135-05	3.711-13
		$e = 0.5$		$i = 60^{\circ}$			
11	0.0	6.112-05	1.059-04	1.222-04	1.059-04	6.112-05	4.418-13
12	0.0	-2.966-04	-5.137-04	-5.932-04	-5.137-04	-2.966-04	-2.144-12
13	2.087-04	2.015-04	1.807-04	1.475-04	1.043-04	5.400-05	3.770-13
14	0.0	-2.761-04	-4.783-04	-5.522-04	-4.783-04	-2.761-04	-1.996-12
15	0.0	6.903-04	1.196-03	1.381-03	1.196-03	6.903-04	4.990-12

TABLE B14

LONG PERIOD TERMS IN a_{xN}

Term #	ω = 0°	ω = 15°	ω = 30°	ω = 45°	ω = 60°	ω = 75°	ω = 90°
			e = 0.01		i = 30°		
16	5.911-07	5.710-07	5.119-07	4.180-07	2.956-07	1.530-07	1.068-15
17	-1.189-11	-1.149-11	-1.030-11	-8.411-12	-5.947-12	-3.079-12	-2.149-20
18	-3.977-10	-3.841-10	-3.444-10	-2.812-10	-1.988-10	-1.029-10	-7.186-19
19	-4.722-11	-3.339-11	4.644-19	3.339-11	4.722-11	3.339-11	2.505-19
20	9.942-11	7.030-11	-9.777-19	-7.030-11	-9.942-11	-7.030-11	-5.274-19
21	0.0	2.684-08	4.648-08	5.367-08	4.648-08	2.684-08	1.940-16
22	-2.092-06	-2.020-06	-1.811-06	-1.479-06	-1.046-06	-5.414-07	-3.780-15
23	2.768-11	2.674-11	2.397-11	1.958-11	1.384-11	7.165-12	5.002-20
24	3.518-10	3.398-10	3.046-10	2.487-10	1.795-10	9.105-11	6.357-19
25	1.815-10	1.283-10	-1.785-18	-1.283-10	-1.815-10	-1.283-10	-9.627-19
26	-3.518-10	-2.487-10	3.460-18	2.487-10	3.518-10	2.487-10	1.866-18
			e = 0.1		i = 30°		
16	4.984-06	4.814-06	4.316-06	3.524-06	2.492-06	1.290-06	9.005-15
17	-1.003-08	-9.686-09	-8.684-09	-7.091-09	-5.014-09	-2.595-09	-1.812-17
18	-3.353-07	-3.238-07	-2.903-07	-2.371-07	-1.676-07	-8.677-08	-6.058-16
19	-3.981-08	-2.815-08	3.915-16	2.815-08	3.981-08	2.815-08	2.112-16
20	8.381-08	5.927-08	-8.243-16	-5.927-08	-8.381-08	-5.927-08	-4.446-16
21	0.0	2.464-06	4.268-06	4.928-06	4.268-06	2.464-06	1.781-14
22	-1.763-05	-1.703-05	-1.527-05	-1.247-05	-8.817-06	-4.564-06	-3.186-14
23	2.334-08	2.254-08	2.021-08	1.650-08	1.167-08	6.041-09	4.217-17
24	2.966-07	2.865-07	2.568-07	2.097-07	1.483-07	7.676-08	5.359-16
25	1.530-07	1.082-07	-1.505-15	-1.082-07	-1.530-07	-1.082-07	-8.116-16
26	-2.966-07	-2.097-07	2.917-15	2.097-07	2.966-07	2.097-07	1.573-15
			e = 0.5		i = 30°		
16	1.340-05	1.294-05	1.161-05	9.475-06	6.700-06	3.468-06	2.421-14
17	-6.741-07	-6.511-07	-5.838-07	-4.767-07	-3.370-07	-1.745-07	-1.218-15
18	-2.254-05	-2.177-05	-1.952-05	-1.594-05	-1.127-05	-5.833-06	-4.072-14
19	-2.676-06	-1.892-06	2.663-14	1.892-06	2.676-06	1.892-06	1.420-14
20	5.634-06	3.984-06	-5.607-14	-3.984-06	-5.634-06	-3.984-06	-2.989-14
21	0.0	4.517-05	7.824-05	9.035-05	7.824-05	4.517-05	3.265-13
22	-4.742-05	-4.580-05	-4.106-05	-3.353-05	-2.371-05	-1.227-05	-8.568-14
23	1.569-06	1.515-06	1.359-06	1.109-06	7.844-07	4.061-07	2.835-15
24	1.994-05	1.926-05	1.727-05	1.410-05	9.968-06	5.160-06	3.602-14
25	1.029-05	7.273-06	-1.023-13	-7.273-06	-1.029-05	-7.273-06	-5.456-14
26	-1.994-05	-1.410-05	1.984-13	1.410-05	1.994-05	1.410-05	1.058-13

TABLE B15

LONG PERIOD TERMS IN a_{xN}

Term #	ω = 0°	ω = 15°	ω = 30°	ω = 45°	ω = 60°	ω = 75°	ω = 90°
			e = 0.01		i = 60°		
16	5.234-06	5.055-06	4.532-06	3.701-06	2.617-06	1.355-06	9.457-15
17	6.621-10	6.396-10	5.734-10	4.682-10	3.311-10	1.714-10	1.197-18
18	-2.442-09	-2.359-09	-2.115-09	-1.727-09	-1.221-09	-6.321-10	-4.413-18
19	-1.186-09	-8.383-10	1.166-17	8.383-10	1.186-09	8.383-10	6.289-18
20	1.832-09	1.295-09	-1.801-17	-1.295-09	-1.832-09	-1.295-09	-9.717-18
21	0.0	-1.549-08	-2.684-08	-3.099-08	-2.684-08	-1.549-08	-1.120-16
22	-1.218-05	-1.177-05	-1.055-05	-8.613-06	-6.090-06	-3.153-06	-2.201-14
23	-2.132-09	-2.059-09	-1.846-09	-1.507-09	-1.066-09	-5.517-10	-3.852-18
24	4.263-09	4.118-09	3.692-09	3.015-09	2.132-09	1.103-09	7.704-18
25	3.350-09	2.369-09	-3.294-17	-2.369-09	-3.350-09	-2.369-09	-1.777-17
26	-4.263-09	-3.015-09	4.193-17	3.015-09	4.263-09	3.015-09	2.262-17
			e = 0.1		i = 60°		
16	4.412-05	4.262-05	3.821-05	3.120-05	2.206-05	1.142-05	7.973-14
17	5.582-07	5.392-07	4.834-07	3.947-07	2.791-07	1.445-07	1.009-15
18	-2.059-06	-1.989-06	-1.783-06	-1.456-06	-1.030-06	-5.329-07	-3.721-15
19	-9.994-07	-7.067-07	9.830-15	7.067-07	9.994-07	7.067-07	5.302-15
20	1.544-06	1.092-06	-1.519-14	-1.092-06	-1.544-06	-1.092-06	-8.192-15
21	0.0	-1.423-06	-2.464-06	-2.845-06	-2.464-06	-1.423-06	-1.028-14
22	-1.027-04	-9.191-05	-8.893-05	-7.261-05	-5.135-05	-2.658-05	-1.856-13
23	-1.797-06	-1.736-06	-1.556-06	-1.271-06	-8.985-07	-4.651-07	-3.247-15
24	3.594-06	3.472-06	3.113-06	2.541-06	1.797-06	9.302-07	6.495-15
25	2.824-06	1.997-06	-2.777-14	-1.997-06	-2.824-06	-1.997-06	-1.498-14
26	-3.594-06	-2.541-06	3.535-14	2.541-06	3.594-06	2.541-06	1.907-14
			e = 0.5		i = 60°		
16	1.186-04	1.146-04	1.027-04	8.389-05	5.932-05	3.071-05	2.144-13
17	3.752-05	3.625-05	3.250-05	2.653-05	1.876-05	9.712-06	6.781-14
18	-1.384-04	-1.337-04	-1.199-04	-9.787-05	-6.921-05	-3.582-05	-2.501-13
19	-6.718-05	-4.751-05	6.685-13	4.751-05	6.718-05	4.751-05	3.564-13
20	1.038-04	7.340-05	-1.033-12	-7.340-05	-1.038-04	-7.340-05	-5.507-13
21	0.0	-2.608-05	-4.517-05	-5.216-05	-4.517-05	-2.608-05	-1.885-13
22	-2.761-04	-2.667-04	-2.391-04	-1.953-04	-1.381-04	-7.147-05	-4.990-13
23	-1.208-04	-1.167-04	-1.046-04	-8.542-05	-6.041-05	-3.127-05	-2.183-13
24	2.416-04	2.334-04	2.092-04	1.708-04	1.208-04	6.253-05	4.366-13
25	1.898-04	1.342-04	-1.889-12	-1.342-04	-1.898-04	-1.342-04	-1.007-12
26	-2.416-04	-1.708-04	2.404-12	1.708-04	2.416-04	1.708-04	1.282-12

TABLE B16

LONG PERIOD TERMS IN a_{yN}

Term #	ω = 0°	ω = 15°	ω = 30°	ω = 45°	ω = 60°	ω = 75°	ω = 90°
			e = 0.01		i = 30°		
27	0.0	-1.530-07	-2.956-07	-4.180-07	-5.119-07	-5.710-07	-5.911-07
28	0.0	3.079-12	5.947-12	8.411-12	1.030-11	1.149-11	1.189-11
29	0.0	1.029-10	1.988-10	2.812-10	3.444-10	3.841-10	3.977-10
30	0.0	-3.339-11	-4.722-11	-3.339-11	-7.093-19	3.339-11	4.722-11
31	0.0	7.030-11	9.942-11	7.030-11	1.494-18	-7.030-11	-9.942-11
88	5.367-04	5.367-04	5.367-04	5.367-04	5.367-04	5.367-04	5.367-04
32	-1.073-07	-1.073-07	-1.073-07	-1.073-07	-1.073-07	-1.073-07	-1.073-07
33	-5.367-08	-4.648-08	-2.684-08	-1.095-16	2.684-08	4.648-08	5.367-08
34	0.0	-5.414-07	1.046-06	1.479-06	1.811-06	2.020-06	2.092-06
35	0.0	-7.165-12	-1.384-11	-1.958-11	-2.397-11	-2.674-11	-2.768-11
36	0.0	9.105-11	-1.759-10	-2.487-10	-3.046-10	-3.398-10	-3.518-10
37	0.0	1.283-10	1.815-10	1.283-10	2.726-18	-1.283-10	-1.815-10
38	0.0	-2.487-10	-3.518-10	-2.487-10	-5.285-18	2.487-10	3.518-10
			e = 0.1		i = 30°		
27	0.0	-1.290-06	-2.492-06	-3.524-06	-4.316-06	-4.814-06	-4.984-06
28	0.0	2.595-09	5.014-09	7.091-09	8.684-09	9.686-09	1.003-08
29	0.0	8.677-08	1.676-07	2.371-07	2.903-07	3.238-07	3.353-07
30	0.0	-2.815-08	-3.981-08	-2.815-08	-5.980-16	2.815-08	3.981-08
31	0.0	5.927-08	8.381-08	5.927-08	1.259-15	-5.927-08	-8.381-08
88	4.928-04	4.928-04	4.928-04	4.928-04	4.928-04	4.928-04	4.928-04
32	-9.856-06	-9.856-06	-9.856-06	-9.856-06	-9.856-06	-9.856-06	-9.856-06
33	-4.928-06	-4.268-06	-2.464-06	-1.005-14	2.464-06	4.268-06	4.928-06
34	0.0	4.564-06	8.817-06	1.247-05	1.527-05	1.703-05	1.763-05
35	0.0	-6.041-09	-1.167-08	-1.650-08	-2.021-08	-2.254-08	-2.334-08
36	0.0	-7.676-08	-1.483-07	-2.097-07	-2.568-07	-2.865-07	-2.966-07
37	0.0	1.082-07	1.530-07	1.082-07	2.298-15	-1.082-07	-1.530-07
38	0.0	-2.097-07	-2.966-07	-2.097-07	-4.455-15	2.097-07	2.966-07
			e = 0.5		i - 30°		
27	0.0	-3.468-06	-6.700-06	-9.475-06	-1.161-05	-1.294-05	-1.340-05
28	0.0	1.745-07	3.370-07	4.767-07	5.838-07	6.511-07	6.741-07
29	0.0	5.833-06	1.127-05	1.594-05	1.952-05	2.177-05	2.254-05
30	0.0	-1.892-06	-2.676-06	-1.892-06	-4.020-14	1.892-06	2.675-06
31	0.0	3.984-06	5.634-06	3.984-06	8.464-14	-3.984-06	-5.634-06
88	3.614-04	3.614-04	3.614-04	3.614-04	3.614-04	3.614-04	3.614-04

TABLE B16 (Continued)

LONG-PERIOD TERMS IN a_{yN}

Term #	ω = 0°	ω = 15°	ω = 30°	ω = 45°	ω = 60°	ω = 75°	ω = 90°
		e = 0.5			i = 30°		
32	-1.807-04	-1.807-04	-1.807-04	-1.807-04	-1.807-04	-1.807-04	-1.807-04
33	-9.035-05	-7.824-05	-4.517-05	-1.843-13	4.517-05	7.824-05	9.035-05
34	0.0	1.227-05	2.371-05	3.353-05	4.106-05	4.580-05	4.742-05
35	0.0	-4.062-07	-7.844-07	-1.109-06	-1.359-06	-1.515-06	-1.569-06
36	0.0	-5.160-06	-9.968-06	-1.410-05	-1.727-05	-1.926-05	-1.994-05
37	0.0	7.273-06	1.029-05	7.273-06	1.545-13	-7.273-06	-1.029-05
38	0.0	-1.410-05	-1.994-05	-1.410-05	-2.995-13	1.410-05	1.994-05

TABLE B17

LONG PERIOD TERMS IN a_{yN}

$e = 0.01$ $i = 60^\circ$

Term #	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$
27	0.0	-1.355-06	-2.617-06	-3.701-06	-4.532-06	-5.055-06	-5.234-06
28	0.0	-1.714-10	-3.311-10	-4.682-10	-5.734-10	-6.396-10	-6.621-10
29	0.0	6.321-10	1.221-09	1.727-09	2.115-09	2.359-09	2.442-09
30	0.0	-8.383-10	-1.186-09	-8.383-10	-1.781-17	8.383-10	1.186-09
31	0.0	1.295-09	1.832-09	1.295-09	2.752-17	-1.295-09	-1.832-09
88	9.296-04	9.296-04	9.296-04	9.296-04	9.296-04	9.296-04	9.296-04
32	-6.197-08	-6.197-08	-6.197-08	-6.197-08	-6.197-08	-6.197-08	-6.197-08
33	3.099-08	2.684-08	1.549-08	6.321-17	1.549-08	-2.684-08	-3.099-08
34	0.0	3.153-06	6.090-06	8.613-06	1.055-05	1.177-05	1.218-05
35	0.0	5.517-10	1.066-09	1.507-09	1.846-09	2.059-09	2.132-09
36	0.0	-1.103-09	-2.132-09	-3.015-09	-3.692-09	-4.118-09	-4.263-09
37	0.0	2.369-09	3.350-09	2.369-09	5.032-17	-2.369-09	-3.350-09
38	0.0	-3.015-09	-4.263-09	-3.015-09	-6.405-17	3.015-09	4.263-09

$e = 0.1$ $i = 60^\circ$

Term #	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$
27	0.0	-1.142-05	-2.206-05	-3.120-05	-3.821-05	-4.262-05	-4.412-05
28	0.0	-1.445-07	-2.791-07	-3.947-07	-4.834-07	-5.392-07	-5.582-07
29	0.0	5.329-07	1.030-06	1.456-06	1.783-06	1.989-06	2.059-06
30	0.0	-7.067-07	-9.994-07	-7.067-07	-1.501-14	7.067-07	9.994-07
31	0.0	1.092-06	1.544-06	1.092-06	2.320-14	-1.092-06	-1.544-06
88	8.536-04	8.536-04	8.536-04	8.536-04	8.536-04	8.536-04	8.536-04
32	-5.690-06	-5.690-06	-5.690-06	-5.690-06	5.690-06	5.690-06	5.690-06
33	2.845-06	2.464-06	1.423-06	5.804-15	-1.423-06	-2.464-06	-2.845-06
34	0.0	2.658-05	5.135-05	7.261-05	8.893-05	9.919-05	1.027-04
35	0.0	4.651-07	8.985-07	1.271-06	1.556-06	1.736-06	1.797-06
36	0.0	-9.302-07	-1.797-06	-2.541-06	-3.113-06	-3.472-06	-3.594-06
37	0.0	1.997-06	2.824-06	1.997-06	4.242-14	-1.997-06	-2.824-06
38	0.0	-2.541-06	-3.594-06	-2.541-06	-5.399-14	2.541-06	3.594-06

TABLE B17 (Continued)

LONG PERIOD TERMS IN a_{yN}

e = 0.5 i = 60°

Term #	ω = 0°	ω = 15°	ω = 30°	ω = 45°	ω = 60°	ω = 75°	ω = 90°
27	0.0	-3.071-05	-5.932-05	-8.389-05	-1.027-04	-1.146-04	-1.186-04
28	0.0	-9.712-06	-1.876-05	-2.653-05	-3.250-05	-3.625-05	-3.752-05
29	0.0	3.582-05	6.921-05	9.787-05	1.199-04	1.337-04	1.384-04
30	0.0	-4.751-05	-6.718-05	-4.751-05	-1.009-12	4.751-05	6.718-05
31	0.0	7.340-05	1.038-04	7.340-05	1.560-12	-7.340-05	-1.038-04
88	6.259-04	6.259-04	6.259-04	6.259-04	6.259-04	6.259-04	6.259-04
32	-1.043-04	-1.043-04	-1.043-04	-1.043-04	-1.043-04	-1.043-04	-1.043-04
33	5.216-05	4.517-05	2.608-05	1.064-13	-2.608-05	-4.517-05	-5.216-05
34	0.0	7.147-05	1.381-04	1.953-04	2.391-04	2.667-04	2.761-04
35	0.0	3.127-05	6.040-05	8.542-05	1.046-04	1.167-04	1.208-04
36	0.0	-6.253-05	-1.208-04	-1.708-04	-2.092.04	-2.334-04	-2.416-04
37	0.0	1.342-04	1.898-04	1.342-04	2.852-12	-1.342-04	-1.898-04
38	0.0	-1.708-04	-2.416-04	-1.708-04	-3.630-12	1.708-04	2.416-04

TABLE B18

LONG PERIOD TERMS IN L

$e = 0.01$ $i = 30^{o}$

Term #	$\omega = 0^{o}$	$\omega = 15^{o}$	$\omega = 30^{o}$	$\omega = 45^{o}$	$\omega = 60^{o}$	$\omega = 75^{o}$	$\omega = 90^{o}$
47	0.0	-1.269-08	-2.198-08	-2.538-08	-2.198-08	-1.269-08	-9.174-17
48	0.0	5.288-09	9.159-09	1.058-08	9.159-09	5.288-09	3.822-17
49	8.051-06	7.776-06	6.972-06	5.693-06	4.025-06	2.084-06	1.455-14
50	2.491-06	2.406-06	2.157-06	1.761-06	1.245-06	6.447-07	4.501-15
51	0.0	4.469-08	7.740-08	8.937-08	7.740-08	4.469-08	3.230-16
52	0.0	-1.871-08	-3.241-08	-3.742-08	-3.241-08	-1.871-08	-1.352-16

$e = 0.1$ $i = 30^{o}$

Term #	$\omega = 0^{o}$	$\omega = 15^{o}$	$\omega = 30^{o}$	$\omega = 45^{o}$	$\omega = 60^{o}$	$\omega = 75^{o}$	$\omega = 90^{o}$
47	0.0	-1.069-06	-1.852-06	-2.138-06	-1.852-06	-1.069-06	-7.727-15
48	0.0	4.458-07	7.721-07	8.916-07	7.721-07	4.458-07	3.222-15
49	7.335-05	7.122-05	6.386-05	5.214-05	3.687-05	1.908-05	1.332-13
50	2.287-05	2.209-05	1.981-05	1.617-05	1.144-05	5.919-06	4.133-14
51	0.0	3.764-06	6.520-06	7.528-06	6.520-06	3.764-06	2.721-14
52	0.0	-1.577-06	-2.732-06	-3.155-06	-2.732-06	-1.577-06	-1.140-14

$e = 0.5$ $i = 30^{o}$

Term #	$\omega = 0^{o}$	$\omega = 15^{o}$	$\omega = 30^{o}$	$\omega = 45^{o}$	$\omega = 60^{o}$	$\omega = 75^{o}$	$\omega = 90^{o}$
47	0.0	-1.406-05	-2.435-05	-2.812-05	-2.435-05	-1.406-05	-1.016-13
48	0.0	5.993-06	1.038-05	1.199-05	1.038-05	5.993-06	4.332-14
49	2.533-04	2.447-04	2.194-04	1.791-04	1.267-04	6.556-05	4.578-13
50	8.386-05	8.100-05	7.263-05	5.930-05	4.193-05	2.171-05	1.515-13
51	0.0	4.949-08	8.572-05	9.898-05	8.572-05	4.949-05	3.578-13
52	0.0	-2.121-05	-3.673-05	-4.241-05	-3.673-05	-2.121-05	-1.533-13

TABLE B19

LONG PERIOD TERMS IN L

e = 0.01 $i = 60^{\circ}$

Term #	$\omega = 0^{\circ}$	$\omega = 15^{\circ}$	$\omega = 30^{\circ}$	$\omega = 45^{\circ}$	$\omega = 60^{\circ}$	$\omega = 75^{\circ}$	$\omega = 90^{\circ}$
47	0.0	-7.771-08	-1.346-07	-1.554-07	-1.346-07	-7.771-08	-5.617-16
48	0.0	-7.850-08	-1.360-07	-1.570-07	-1.360-07	-7.850-08	-5.674-16
49	1.344-05	1.347-05	1.208-05	9.860-06	6.972-06	3.609-06	2.520-14
50	3.099-06	2.993-06	2.684-06	2.191-06	1.549-06	8.020-07	5.599-15
51	0.0	1.218-07	2.110-07	2.436-07	2.110-07	1.218-07	8.804-16
52	0.0	1.827-07	3.165-07	3.654-07	3.165-07	1.827-07	1.321-15

e = 0.1 $i = 60^{\circ}$

Term #	$\omega = 0^{\circ}$	$\omega = 15^{\circ}$	$\omega = 30^{\circ}$	$\omega = 45^{\circ}$	$\omega = 60^{\circ}$	$\omega = 75^{\circ}$	$\omega = 90^{\circ}$
47	0.0	-6.543-06	-1.133-05	-1.309-05	-1.133-05	-6.543-06	-4.729-14
48	0.0	-6.618-06	-1.146-05	-1.324-05	-1.146-05	-6.618-06	-4.784-14
49	1.277-04	1.234-04	1.106-05	9.031-05	6.386-05	3.305-05	2.308-13
50	2.845-05	2.748-05	2.464-05	2.012-05	1.423-05	7.364-06	5.141-14
51	0.0	1.025-05	1.775-05	2.050-05	1.775-05	1.025-05	7.409-14
52	0.0	1.540-05	2.668-05	3.081-05	2.668-05	1.540-05	1.113-13

e = 0.5 $i = 60^{\circ}$

Term #	$\omega = 0^{\circ}$	$\omega = 15^{\circ}$	$\omega = 30^{\circ}$	$\omega = 45^{\circ}$	$\omega = 60^{\circ}$	$\omega = 75^{\circ}$	$\omega = 90^{\circ}$
47	0.0	-8.517-05	-1.475-04	-1.703-04	-1.476-04	-8.517-05	-6.156-13
48	0.0	-8.898-05	-1.542-04	-1.800-04	-1.541-04	-8.898-05	-6.431-13
49	4.388-04	4.238-04	3.800-04	3.103-04	2.194-04	1.136-04	7.928-13
50	1.043-04	1.008-04	9.035-05	7.377-05	5.216-05	2.700-05	1.885-13
51	0.0	1.313-04	2.274-04	2.626-04	2.274-04	1.313-04	9.490-13
52	0.0	2.071-04	3.587-04	4.142-04	3.587-04	2.071-04	1.497-12

TABLE B20

LONG-PERIOD TERMS IN THE INCLINATION i

$e = 0.01 \qquad i = 30^{\circ}$

Term #	$\omega = 0^{\circ}$	$\omega = 15^{\circ}$	$\omega = 30^{\circ}$	$\omega = 45^{\circ}$	$\omega = 60^{\circ}$	$\omega = 75^{\circ}$	$\omega = 90^{\circ}$
53	-1.024-08	-8.867-09	-5.119-09	-2.089-17	5.119-09	8.867-09	1.024-08
54	0.0	-2.406-06	-4.648-06	-6.573-06	-8.051-06	-8.979-06	-9.296-06
55	3.623-08	3.137-08	1.811-08	7.390-17	-1.811-08	-3.137-08	-3.623-08
			$e = 0.1$		$i = 30^{\circ}$		
53	-8.632-07	-7.475-07	-4.316-07	-1.761-15	4.316-07	7.475-07	8.632-07
54	0.0	-2.209-05	-4.268-05	-6.036-05	-7.392-05	-8.245-05	-8.536-05
55	3.054-06	2.645-06	1.527-06	6.230-15	-1.527-06	-2.645-06	-3.054-06
			$e = 0.5$		$i = 30^{\circ}$		
53	-1.161-05	-1.005-05	-5.802-06	-2.367-14	5.802-06	1.005-05	1.161-05
54	0.0	-8.100-05	-1.565-04	-2.213-04	-2.710-04	-3.023-04	-3.130-04
55	4.106-05	3.556-05	2.053-05	8.376-14	-2.053-05	-3.556-05	-4.106-05

TABLE B21

LONG-PERIOD TERMS IN THE INCLINATION i

Term #	$\omega = 0^o$	$\omega = 15^o$	$\omega = 30^o$	$\omega = 45^o$	$\omega = 60^o$	$\omega = 75^o$	$\omega = 90^o$
			e = 0.01		$i = 60^o$		
53	-3.022-08	-2.617-08	-1.511-08	-6.164-17	1.511-08	2.617-08	3.022-08
54	0.0	-1.389-06	-2.684-06	-3.795-06	-4.648-06	-5.184-06	-5.367-06
55	7.033-08	6.090-08	3.516-08	1.435-16	-3.516-08	-6.090-08	-7.033-08
			e = 0.1		$i = 60^o$		
53	-2.547-06	-2.206-06	-1.274-06	-5.196-15	1.274-06	2.206-06	2.547-06
54	0.0	-1.276-05	-2.464-03	-3.485-05	-4.268-05	-4.760-05	-4.928-05
55	5.929-06	5.135-06	2.964-06	1.209-14	-2.964-06	-5.135-06	-5.929-06
			e = 0.5		$i = 60^o$		
53	-3.425-05	-2.966-05	-1.712-05	-6.986-14	1.712-05	2.966-05	3.425-05
54	0.0	-4.677-05	-9.035-05	-1.278-04	-1.565-04	-1.745-04	-1.807-04
55	7.971-05	6.903-05	3.986-05	1.626-13	-3.986-05	-6.903-05	-7.971-05

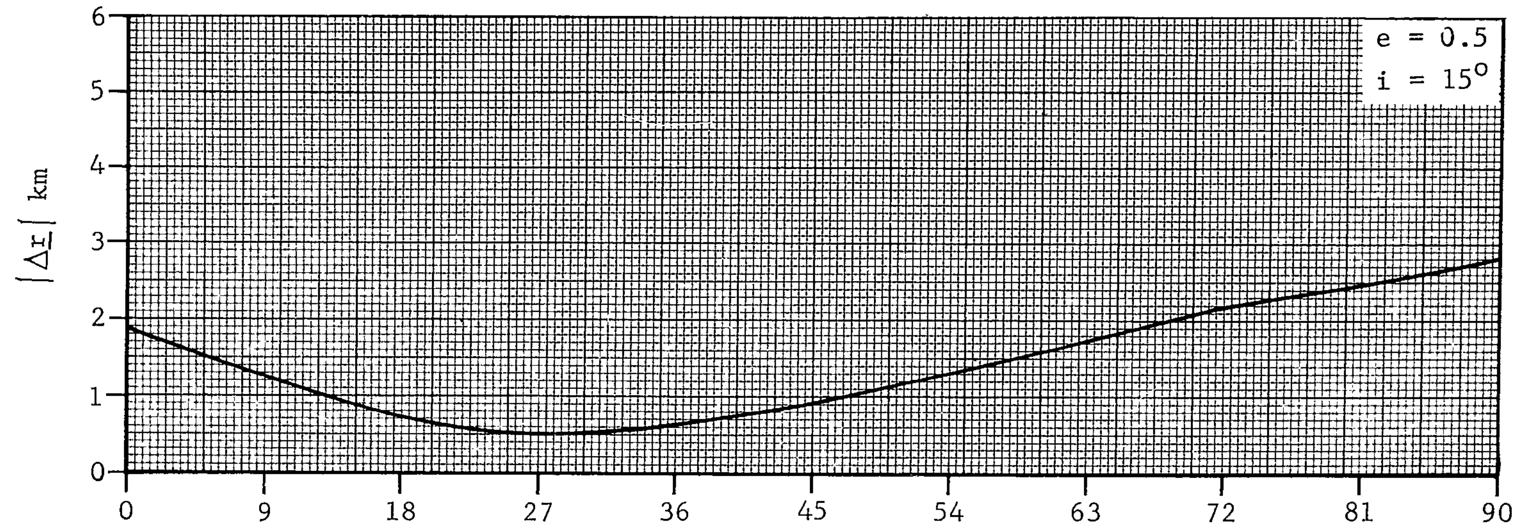


FIGURE B1 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B1

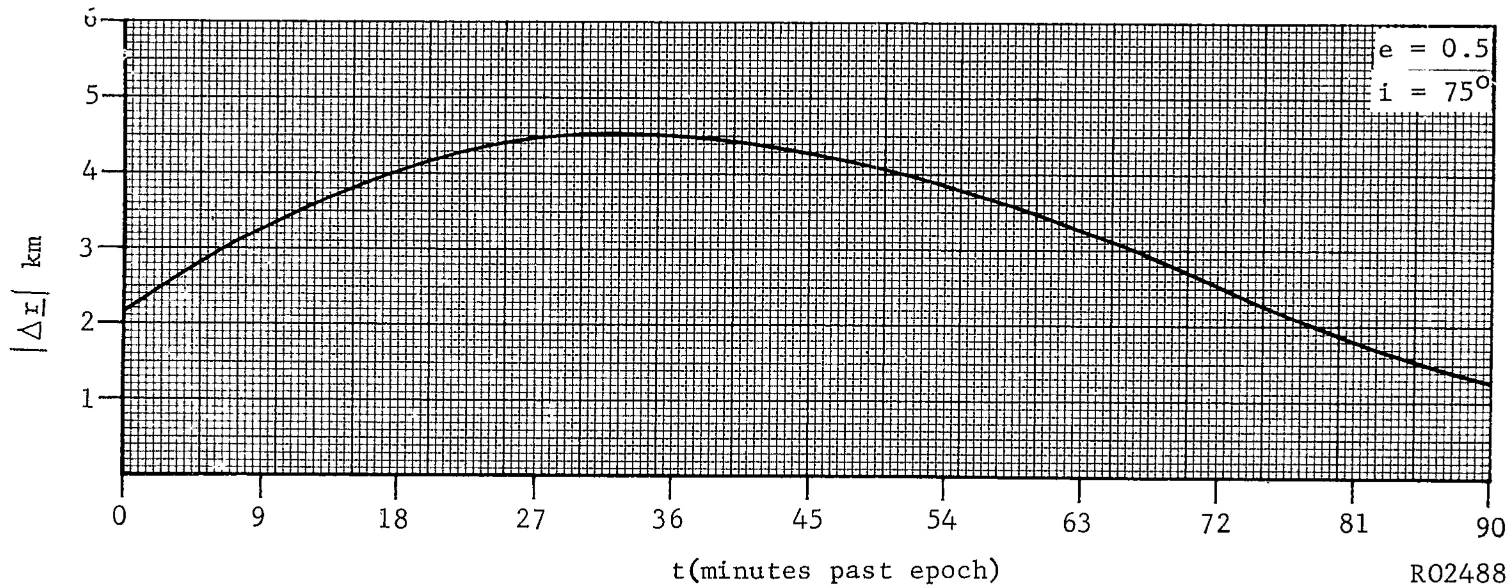


FIGURE B2 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B1

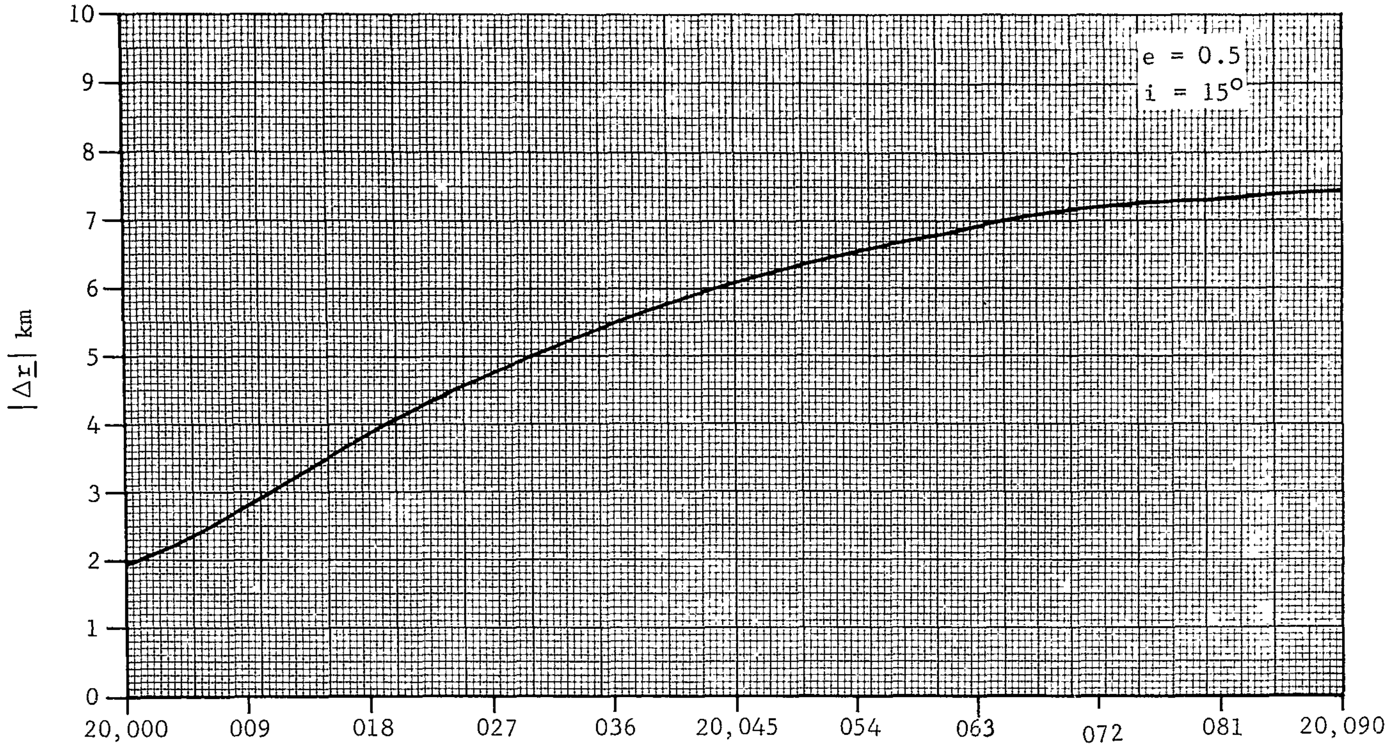


FIGURE B3 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B1

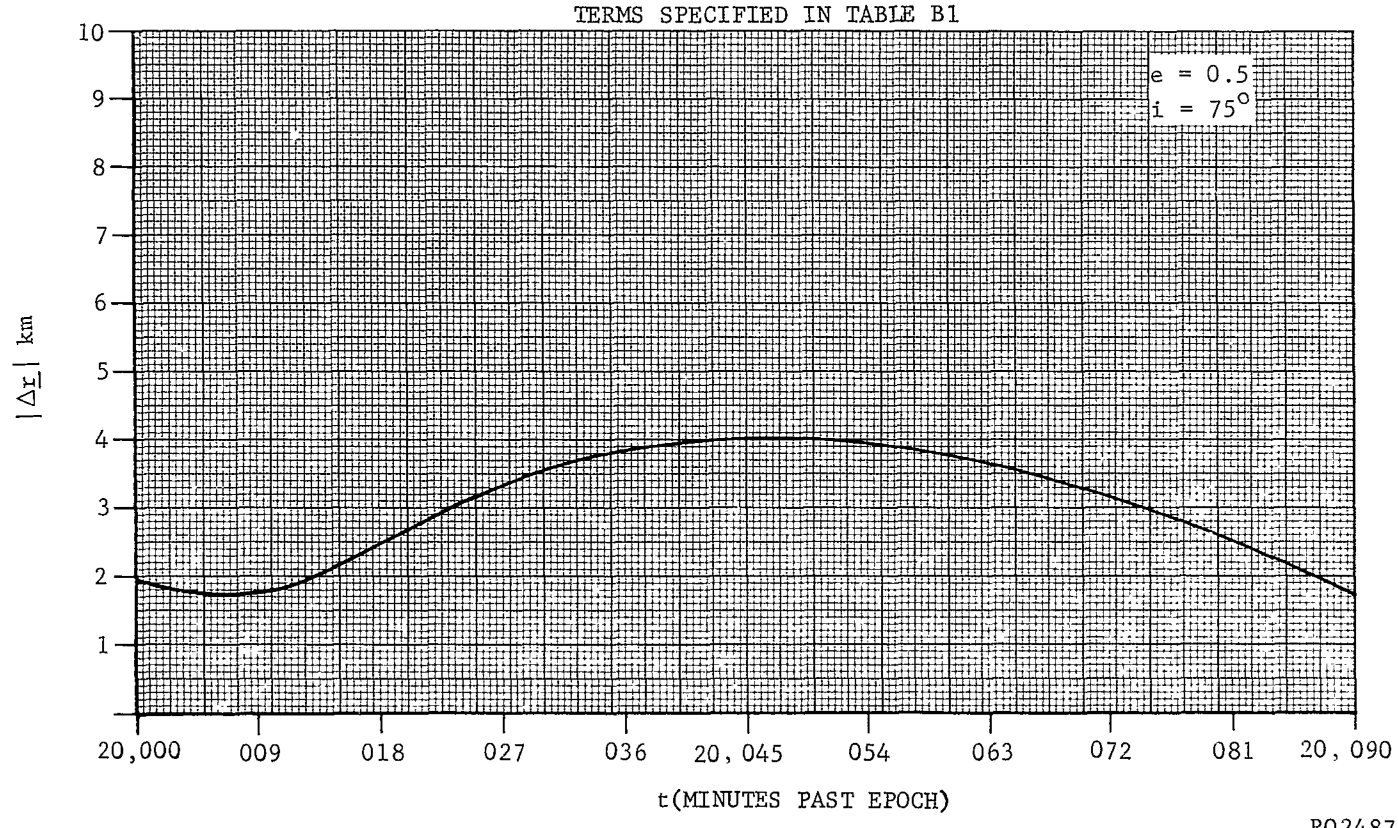


FIGURE B4 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B1

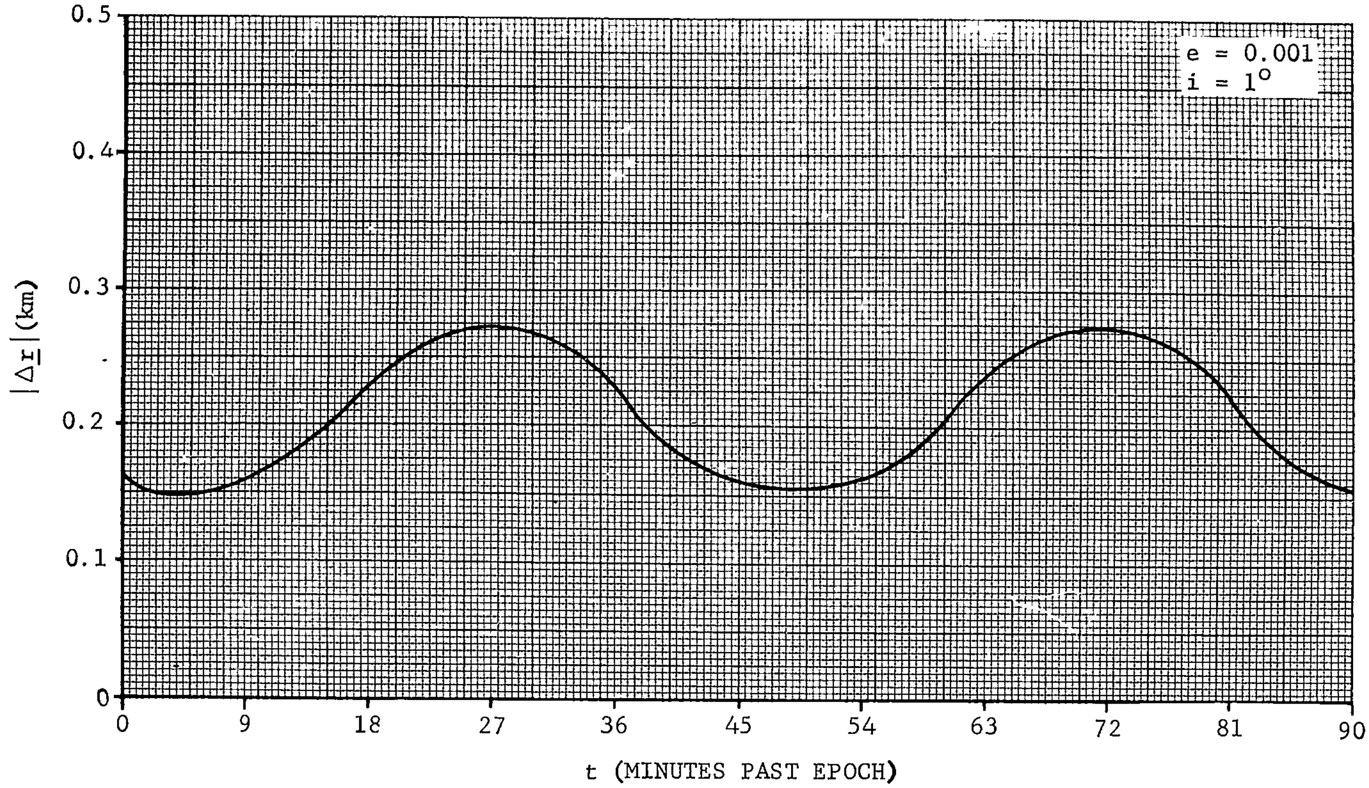


FIGURE B5 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B2

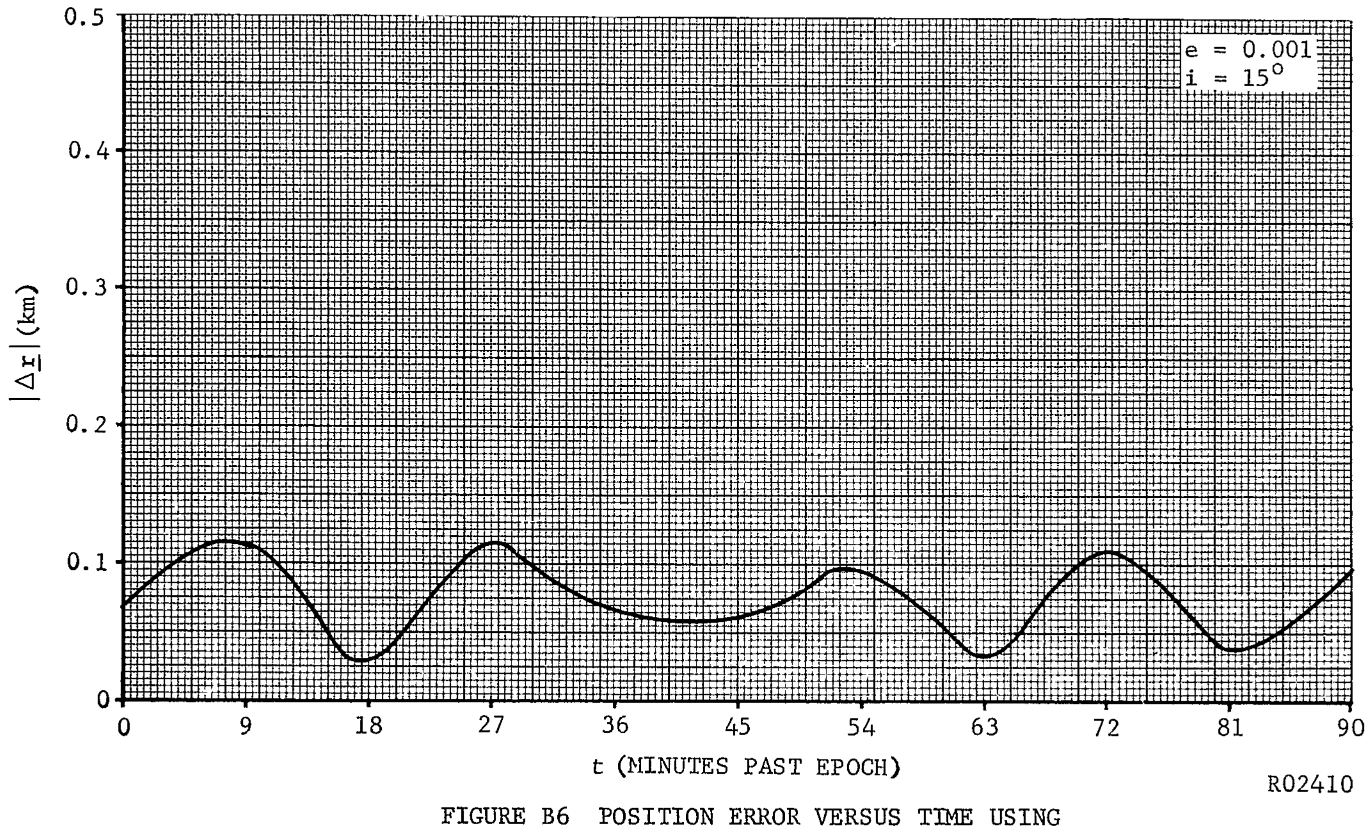


FIGURE B6 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B2

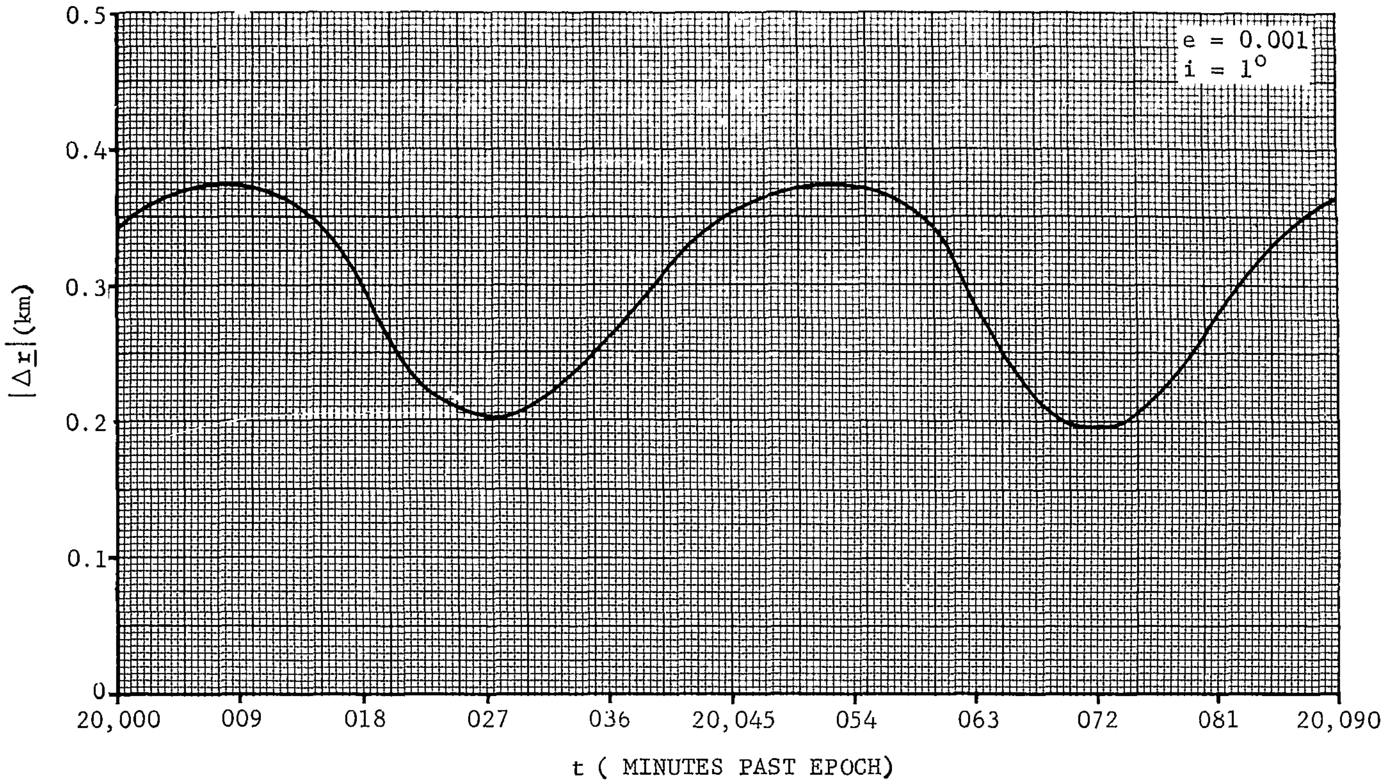


FIGURE B7 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B2

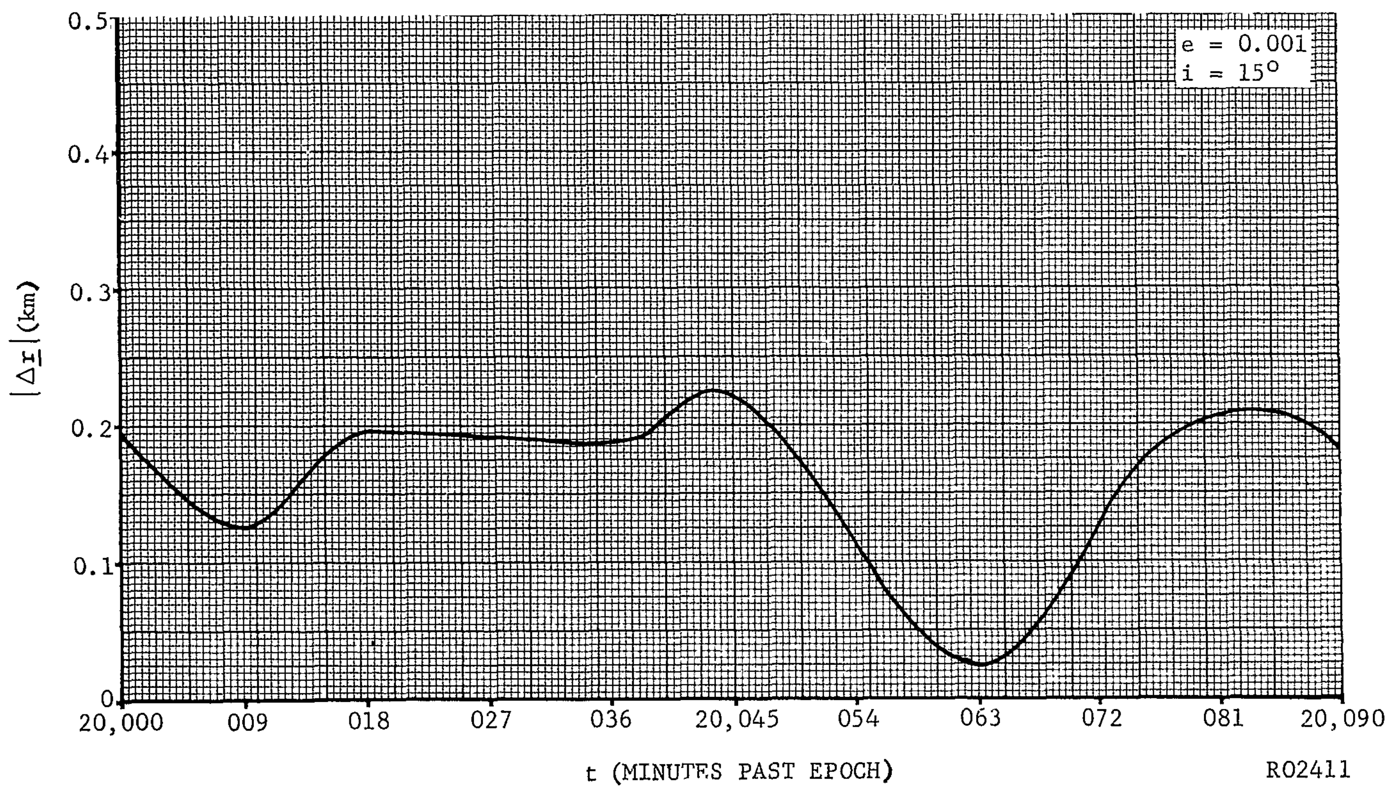


FIGURE B8 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B2

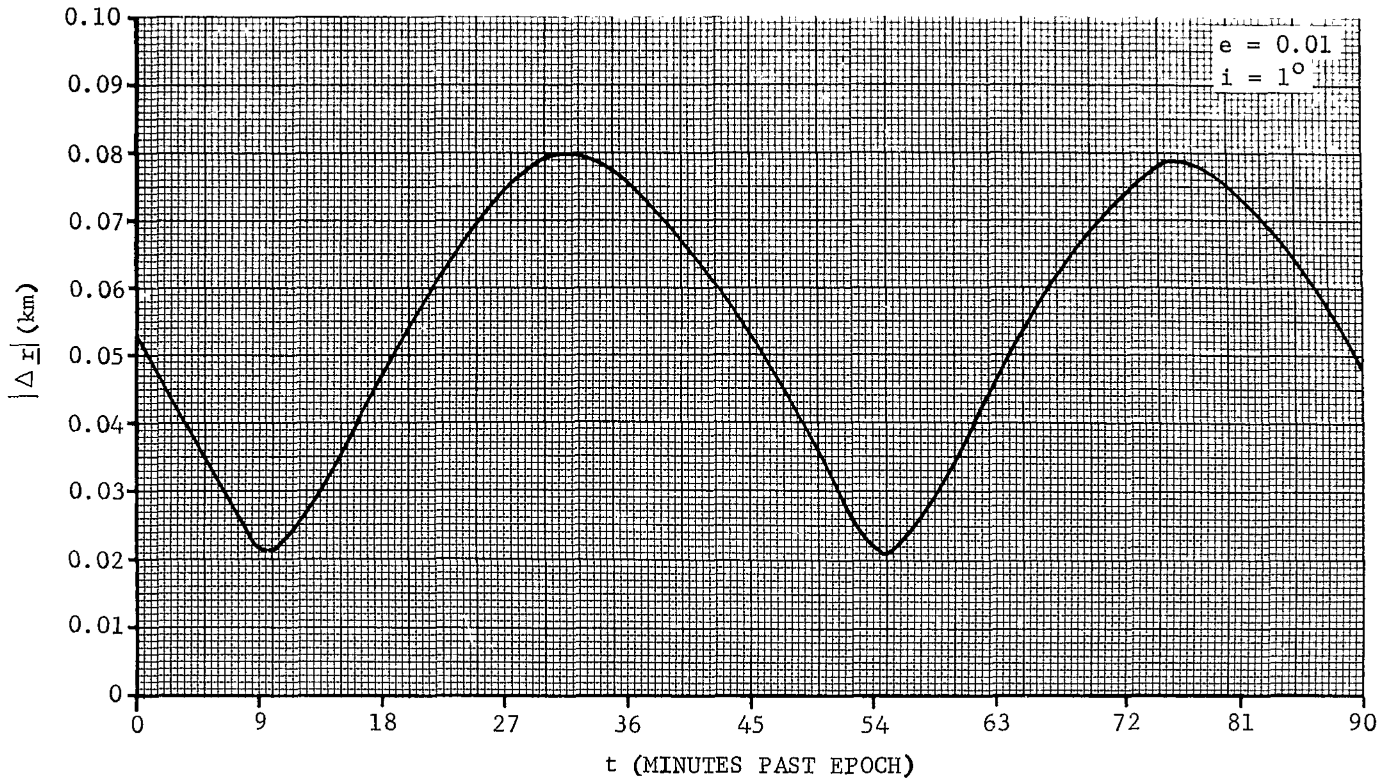
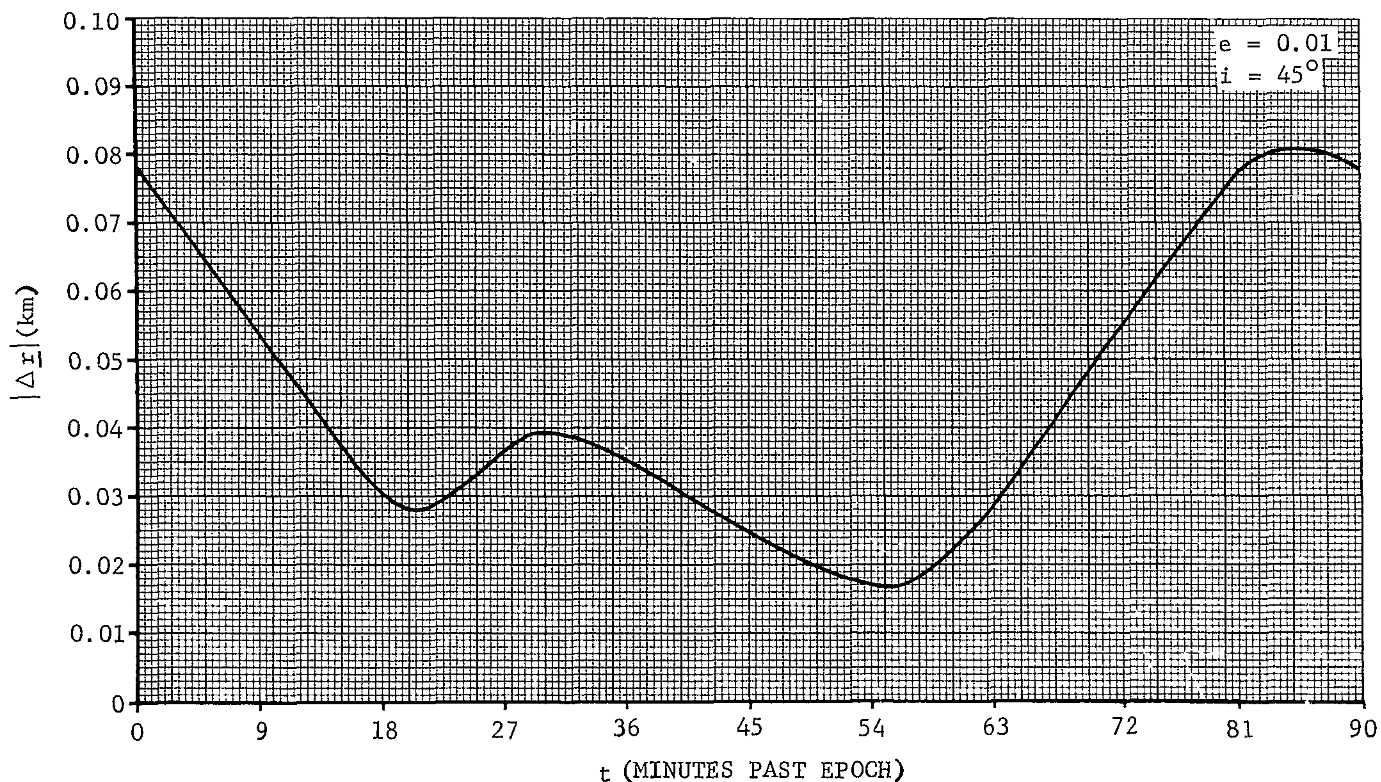


FIGURE B9 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B3



R02412

FIGURE B10 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B3

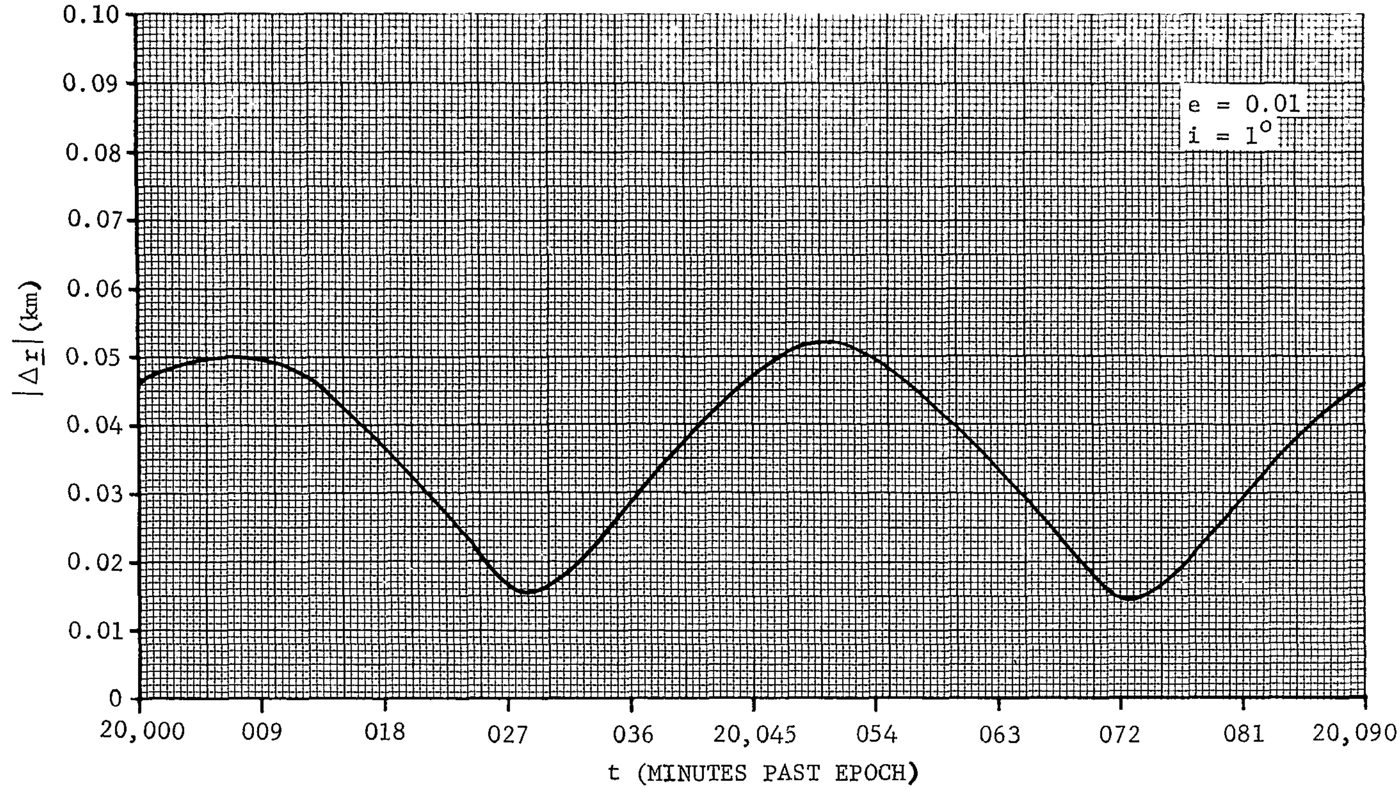
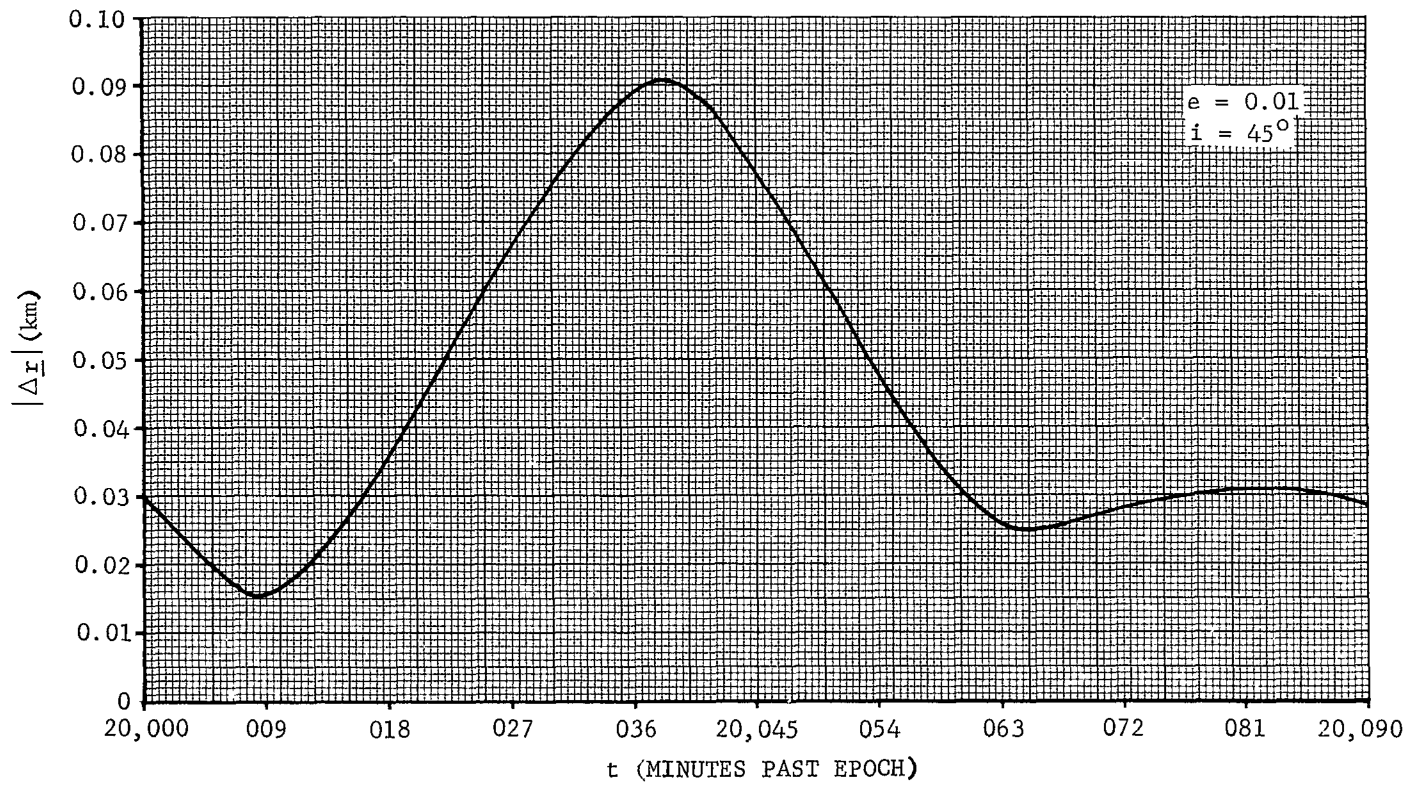


FIGURE B11 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B3



R02413

FIGURE B12 POSITION ERROR VERSUS TIME USING TERMS SPECIFIED IN TABLE B3

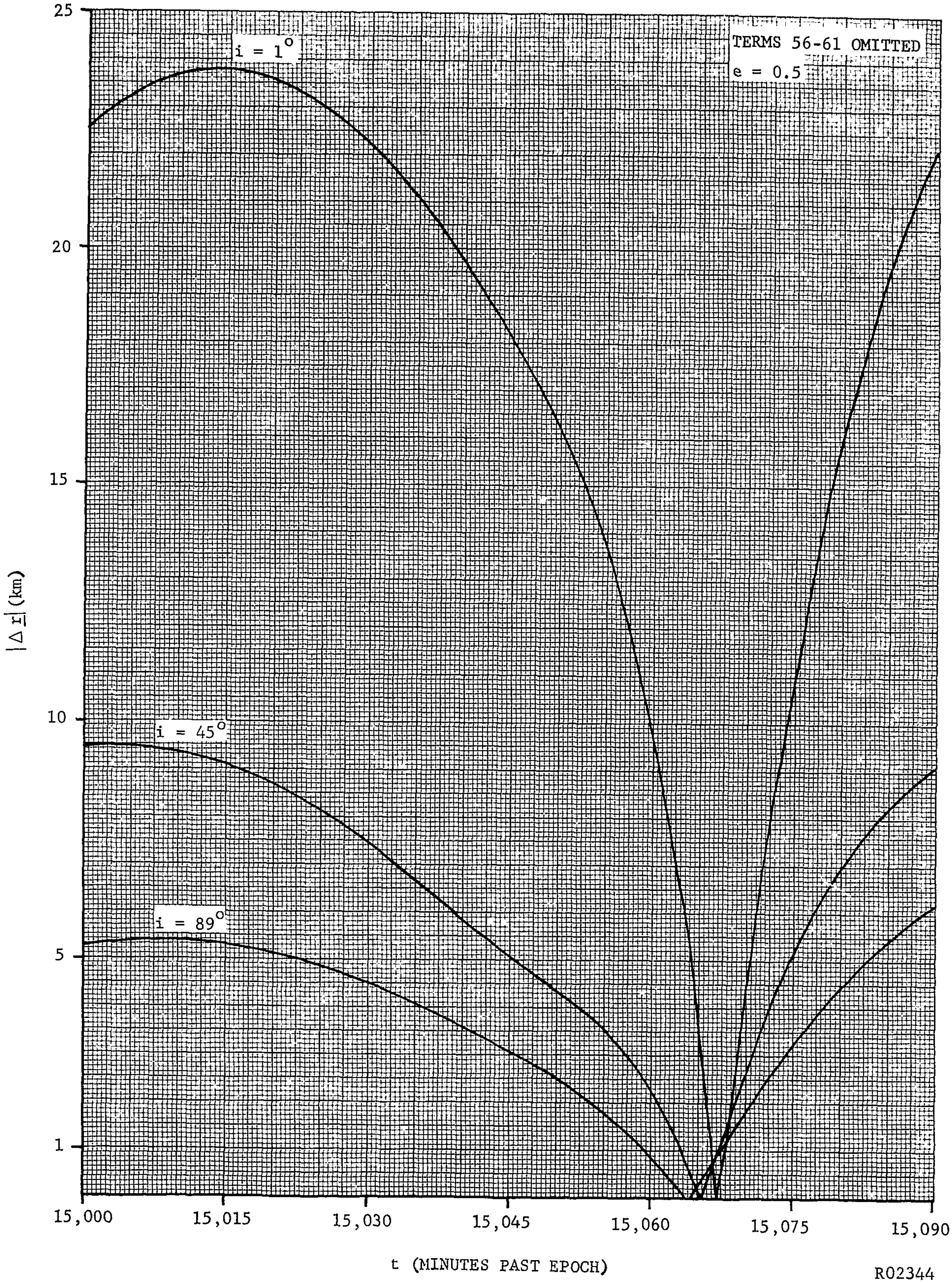


FIGURE B13 TOTAL DISPLACEMENT VERSUS TIME WITH TERMS 56-61 (SHORT-PERIOD TERMS IN u) OMITTED

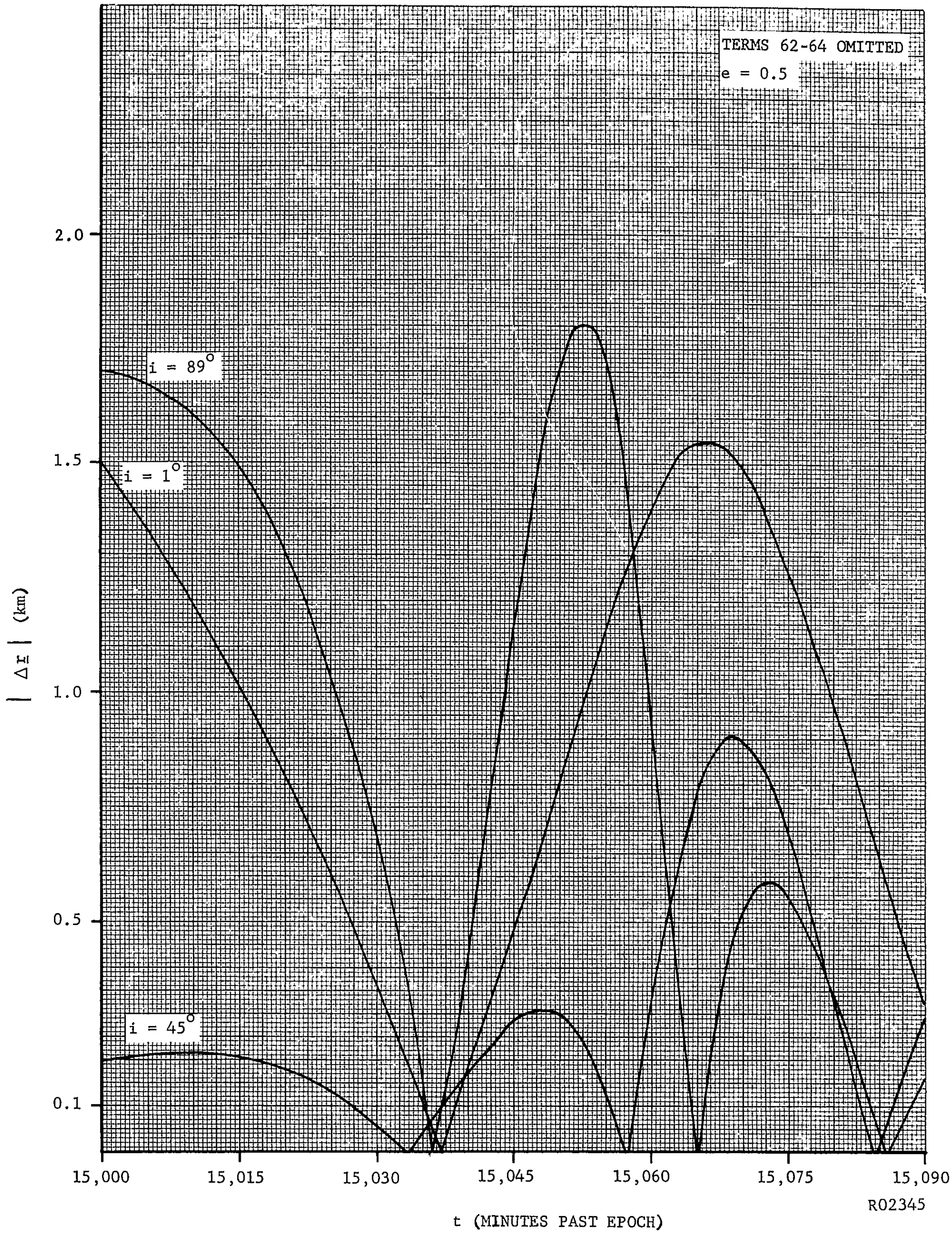


FIGURE B14 TOTAL DISPLACEMENT VERSUS TIME WITH TERMS 62-64 (SHORT-PERIOD TERMS IN r) OMITTED

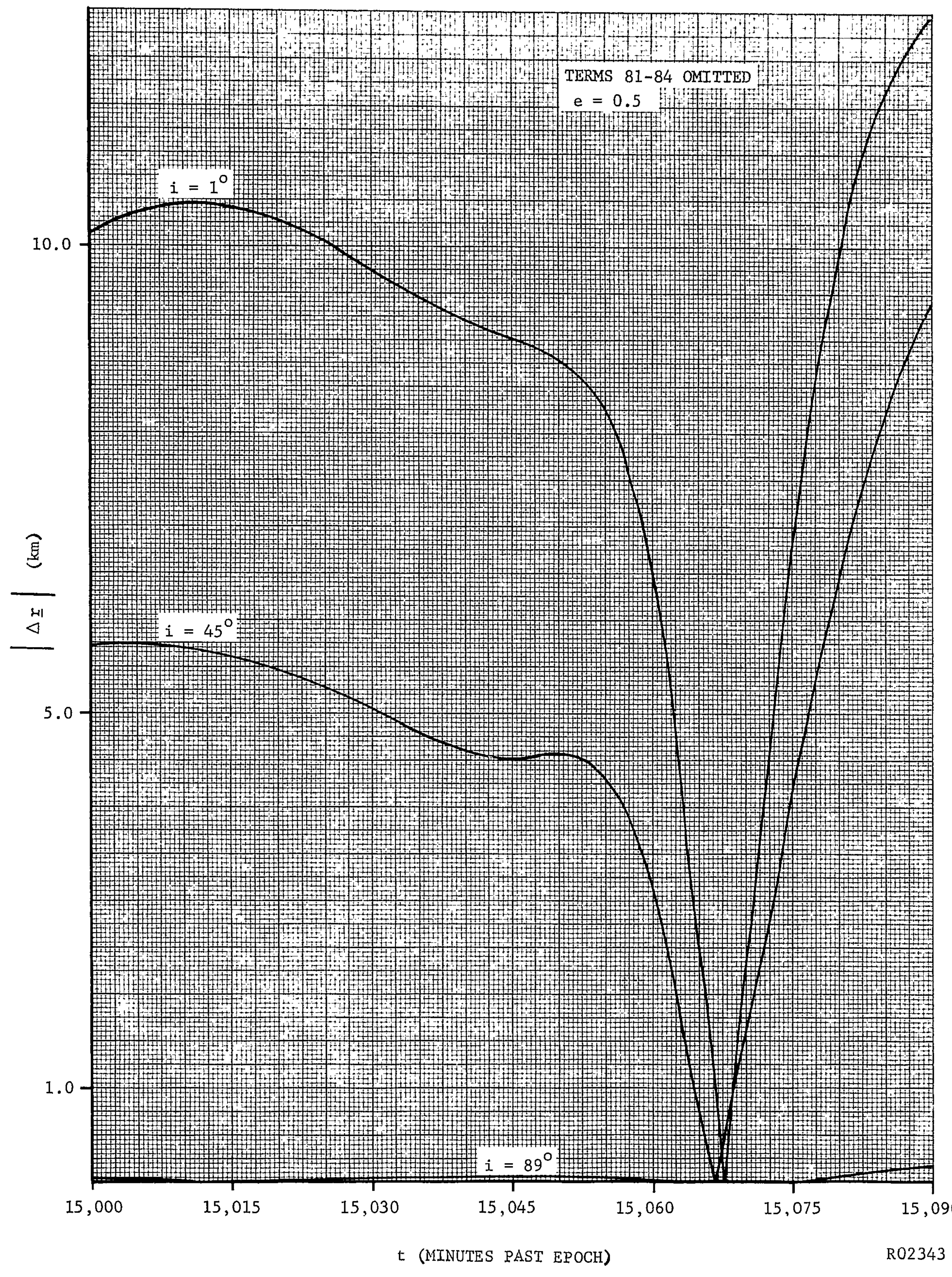


FIGURE B15 TOTAL DISPLACEMENT VERSUS TIME WITH TERMS 81-84 (SHORT-PERIOD TERMS IN $\mathcal{\partial\beta}$) OMITTED

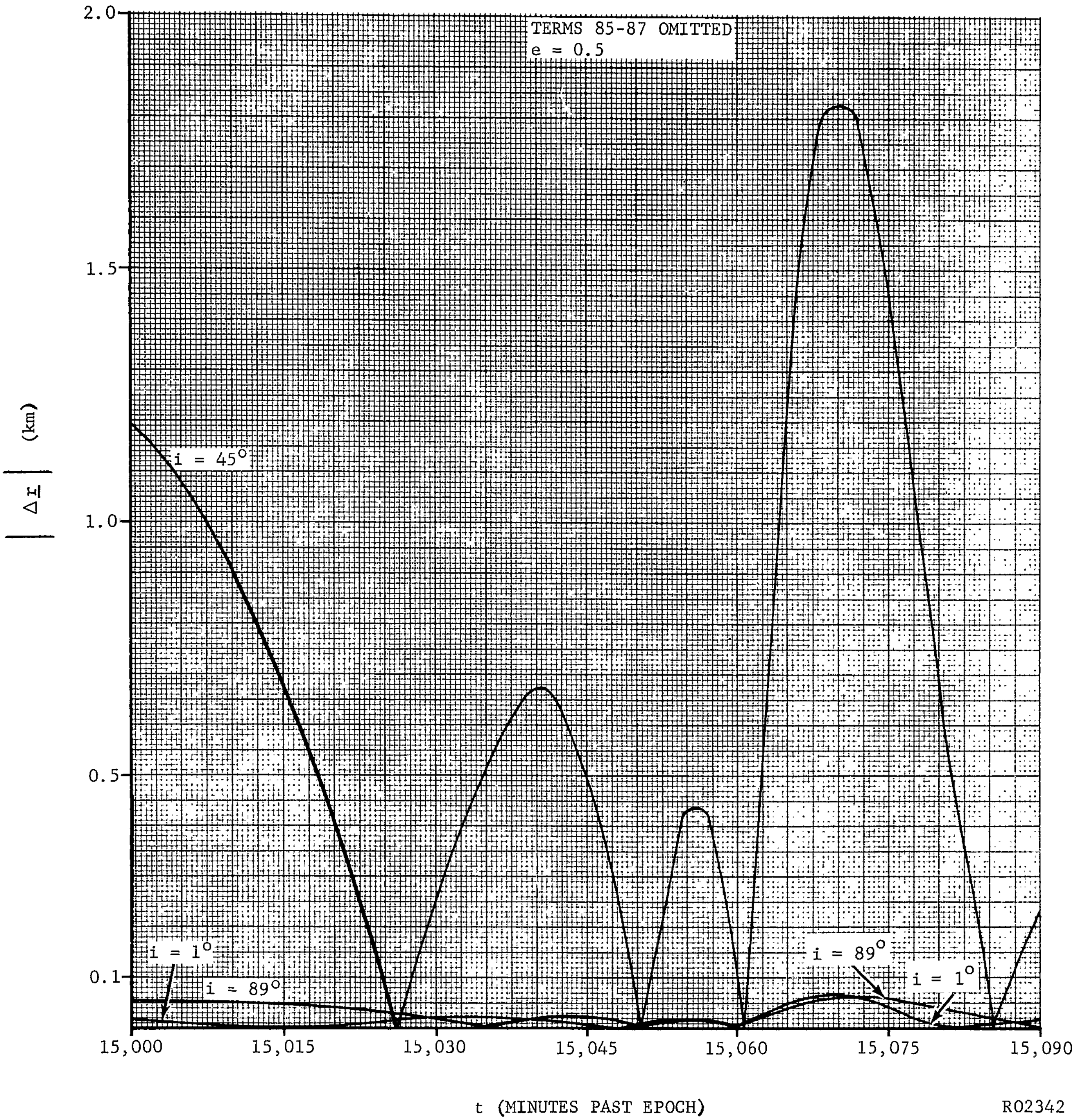


FIGURE B16 TOTAL DISPLACEMENT VERSUS TIME WITH TERMS 85-87 (SHORT-PERIOD TERMS IN i) OMITTED

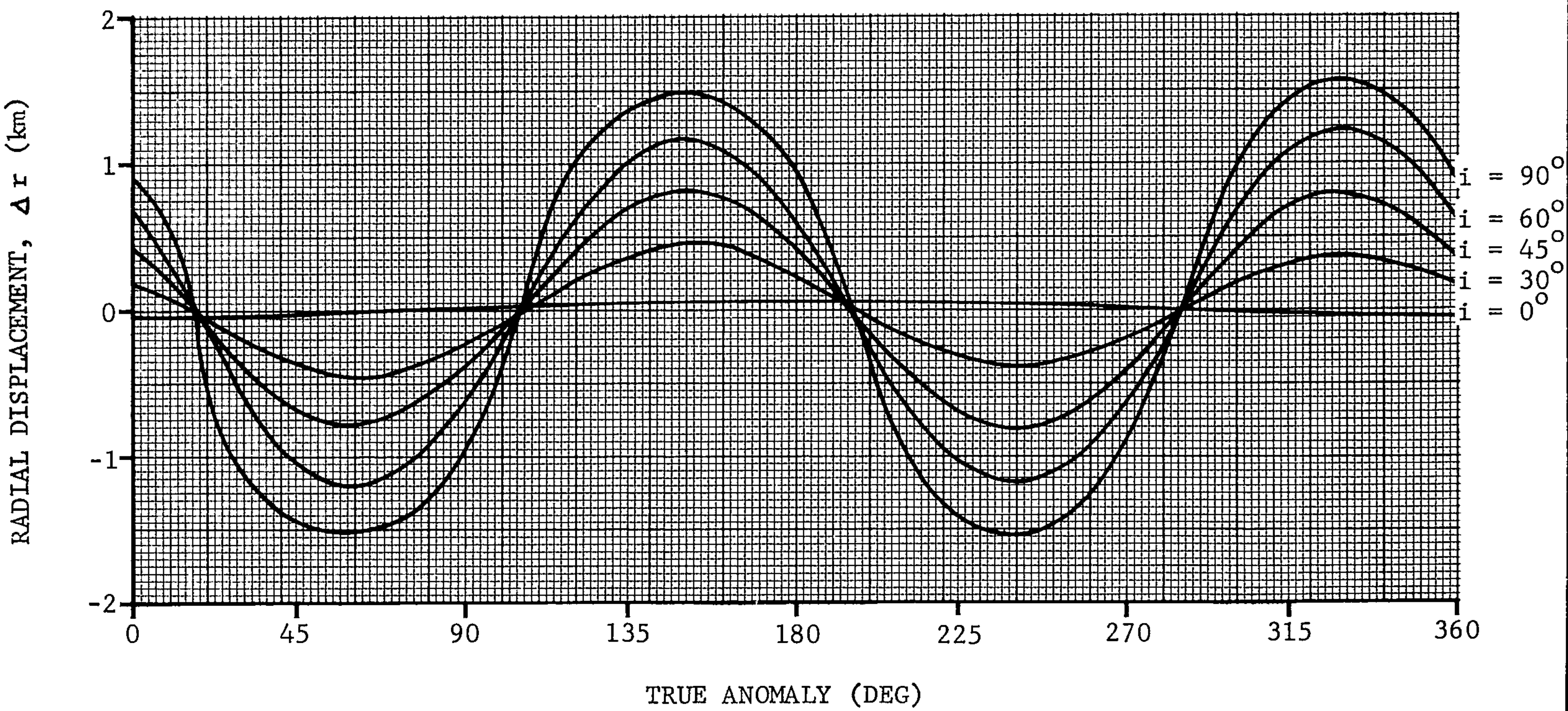


FIGURE B17 RADIAL DISPLACEMENT; e = 0.01, $\omega = 30^{\circ}$

R02431

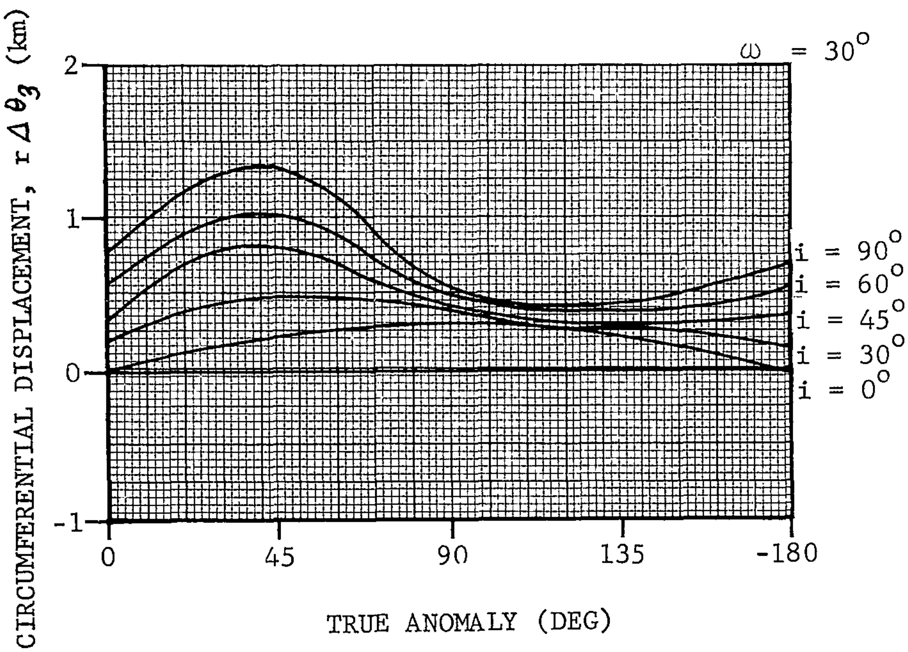


FIGURE B18 CIRCUMFERENTIAL DISPLACEMENT;
$e = 0.01, \quad \omega = 30^{\circ}$

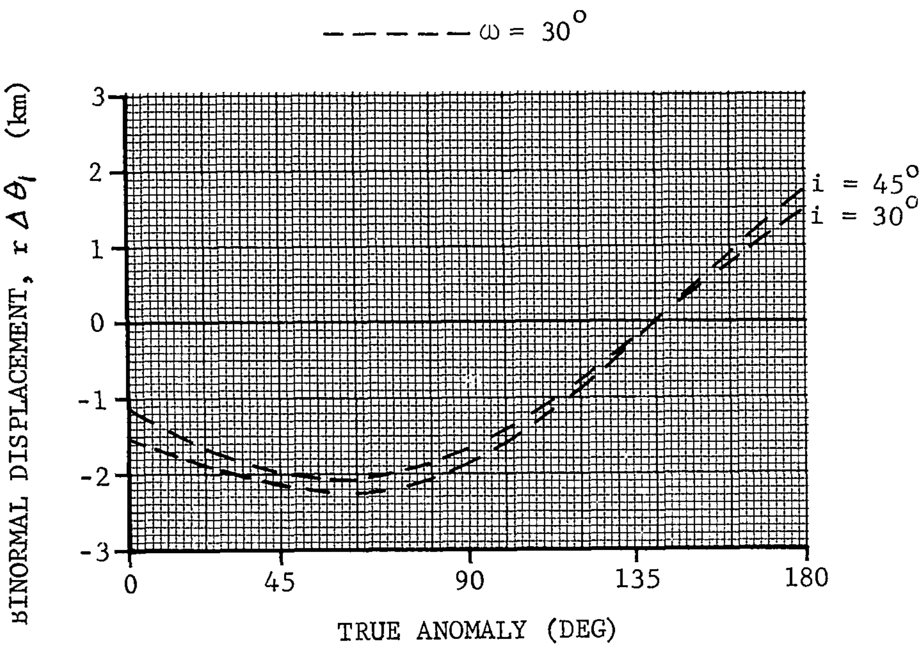


FIGURE B19 BINORMAL DISPLACEMENT;
$e = 0.01$, $\omega = 30^{\circ}$

R02433

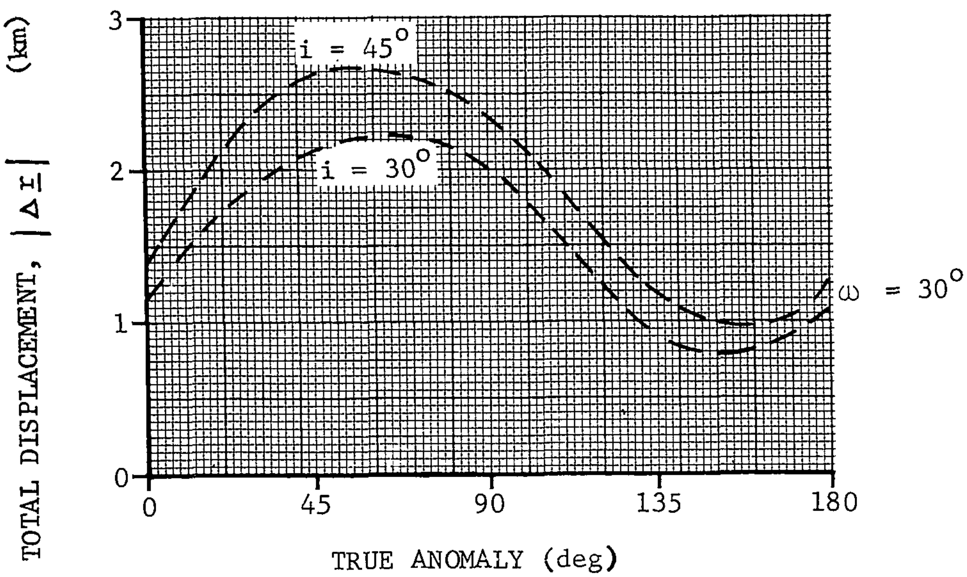


FIGURE B20 TOTAL DISPLACEMENT;
$e = 0.01$, $\omega = 30^\circ$

R02434

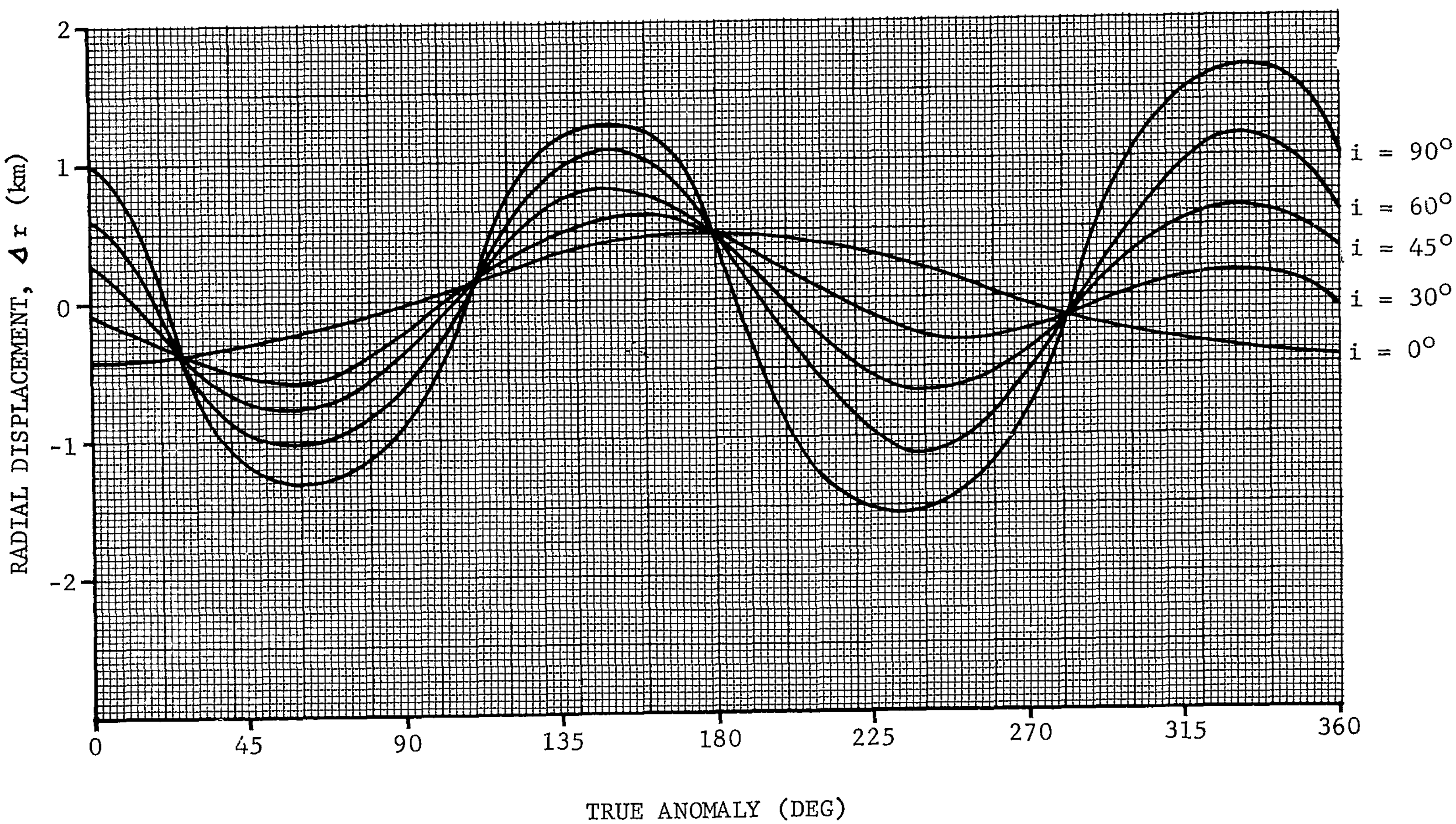


FIGURE B21 RADIAL DISPLACEMENT;
$e = 0.1, \quad \omega = 30^{\circ}$

R-02435

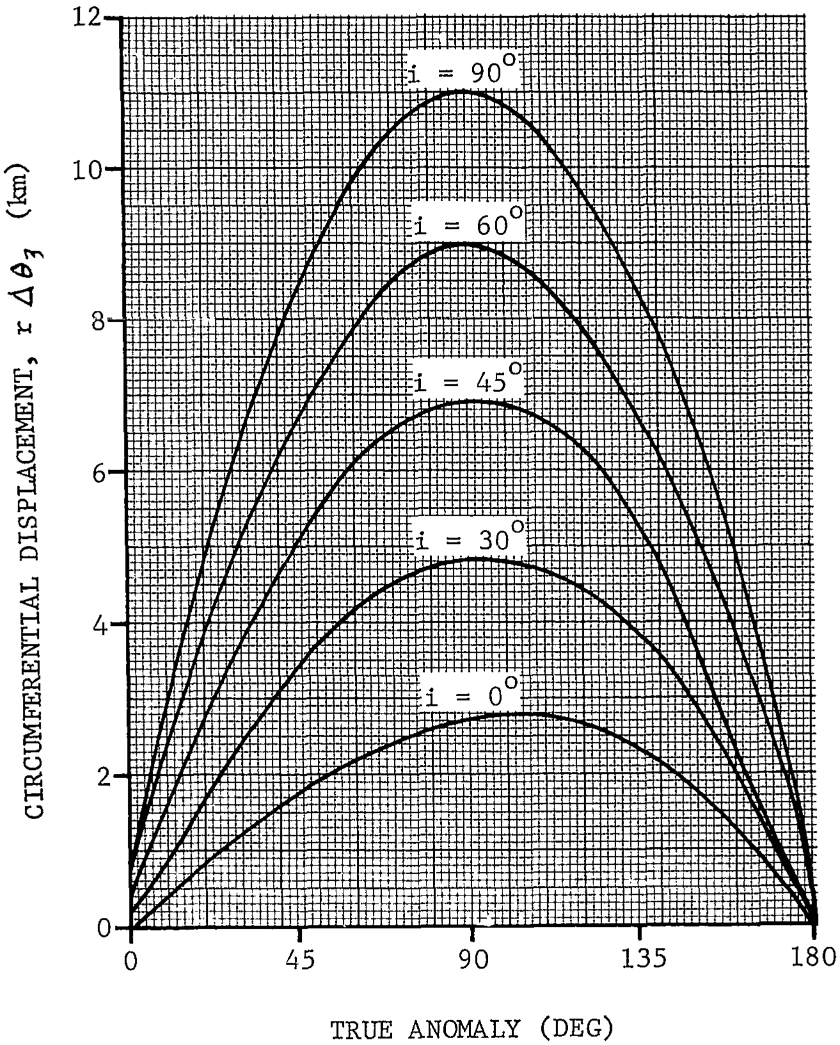


FIGURE B22 CIRCUMFERENTIAL DISPLACEMENT; $e = 0.1$, $\omega = 30^{\circ}$

R02436

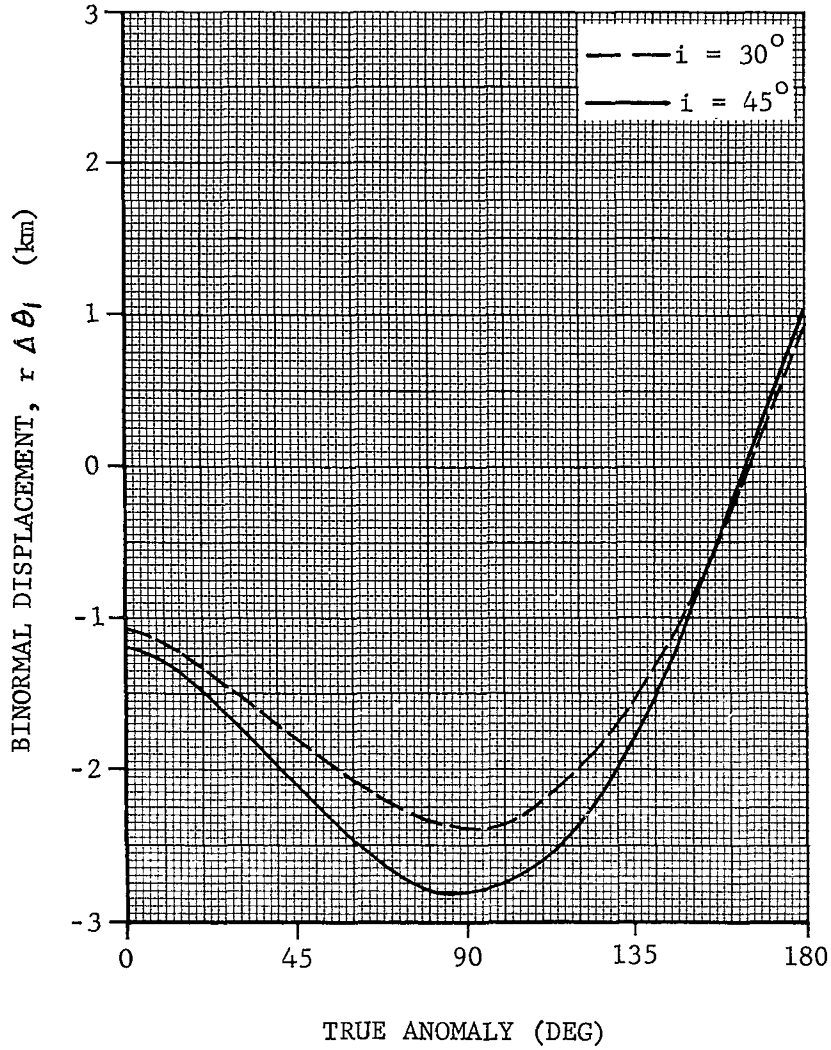


FIGURE B23 BINORMAL DISPLACEMENT;
e = 0.1, $\omega = 30^{\circ}$

R02437

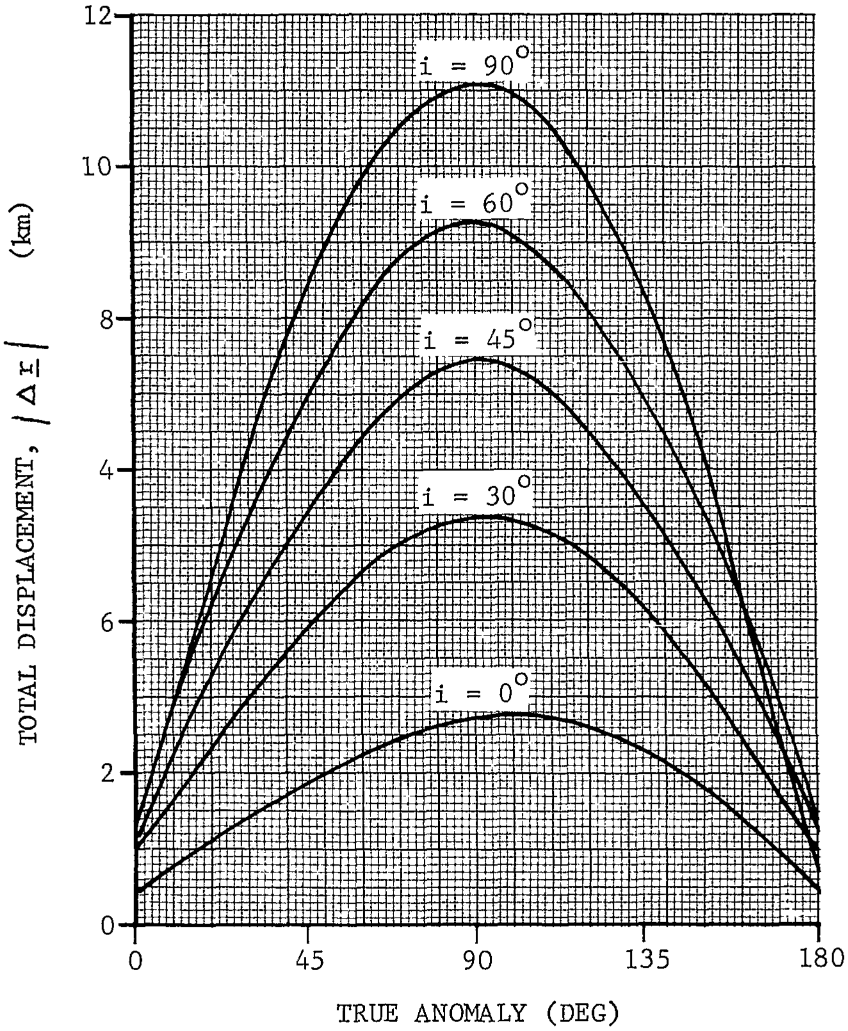


FIGURE B24 TOTAL DISPLACEMENT;
$e = 0.1, \quad \omega = 30^{\circ}$

R02438

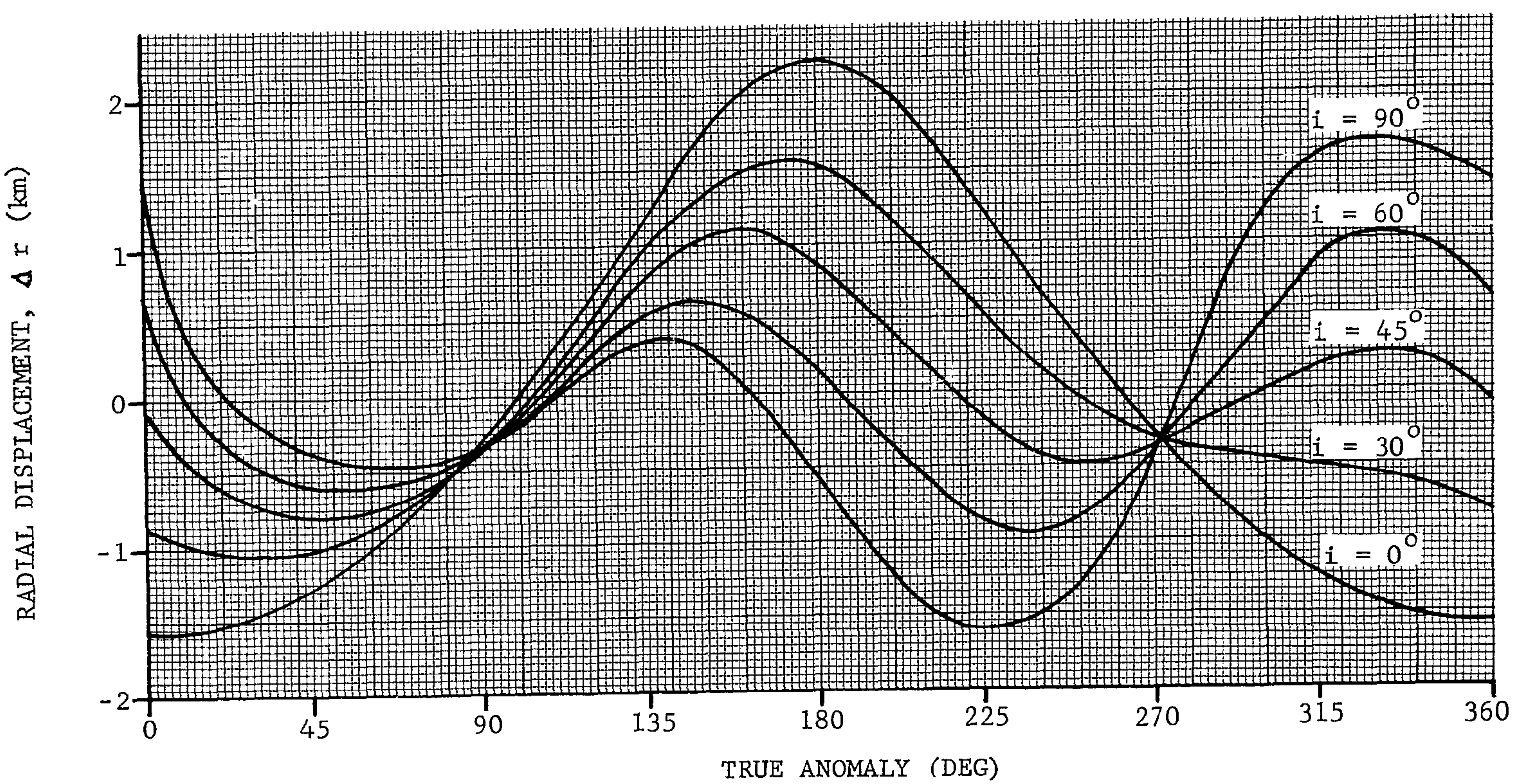


FIGURE B25 RADIAL DISPLACEMENT;
$e = 0.5, \quad \omega = 30^{\circ}$

R02439

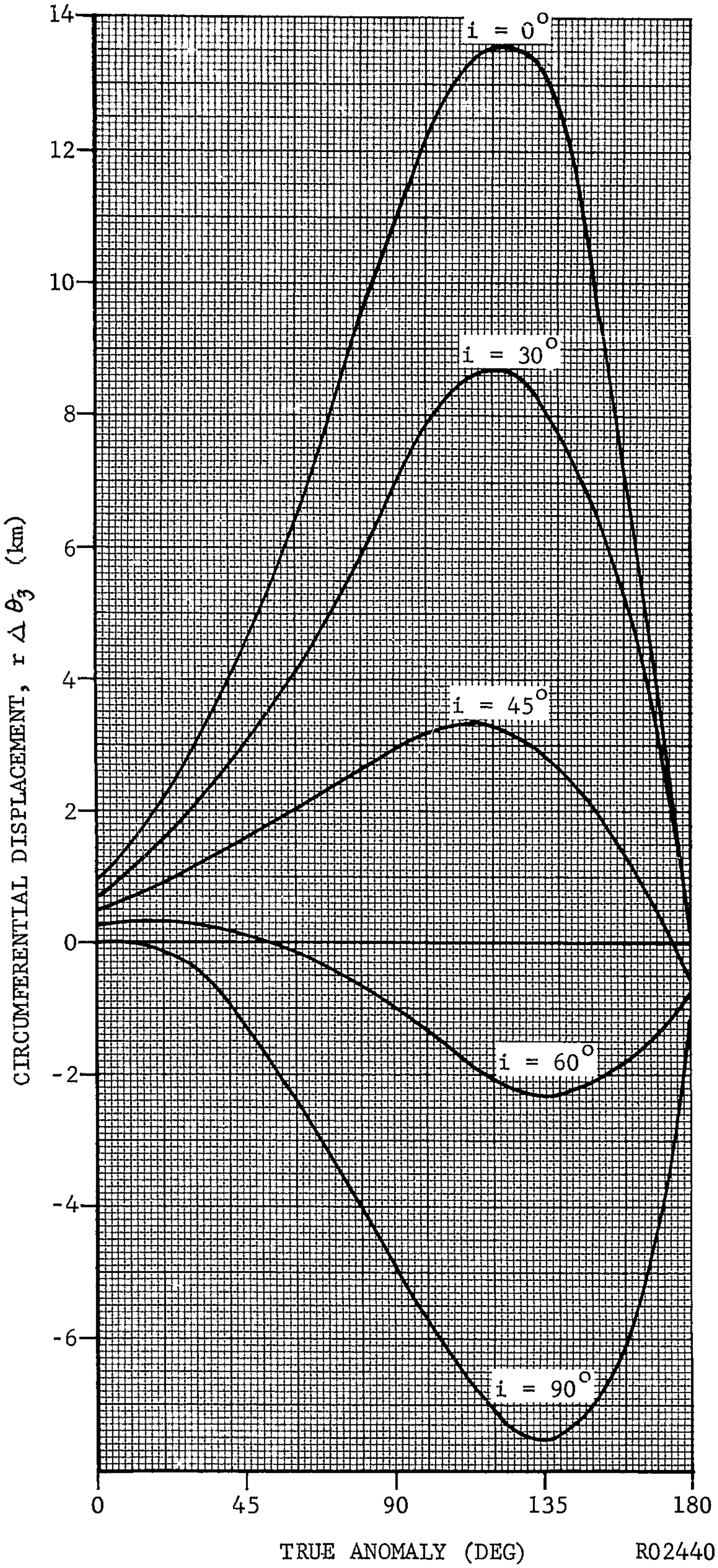


FIGURE B26 CIRCUMFERENTIAL DISPLACEMENT;
$e = 0.5$, $\omega = 30^{\circ}$

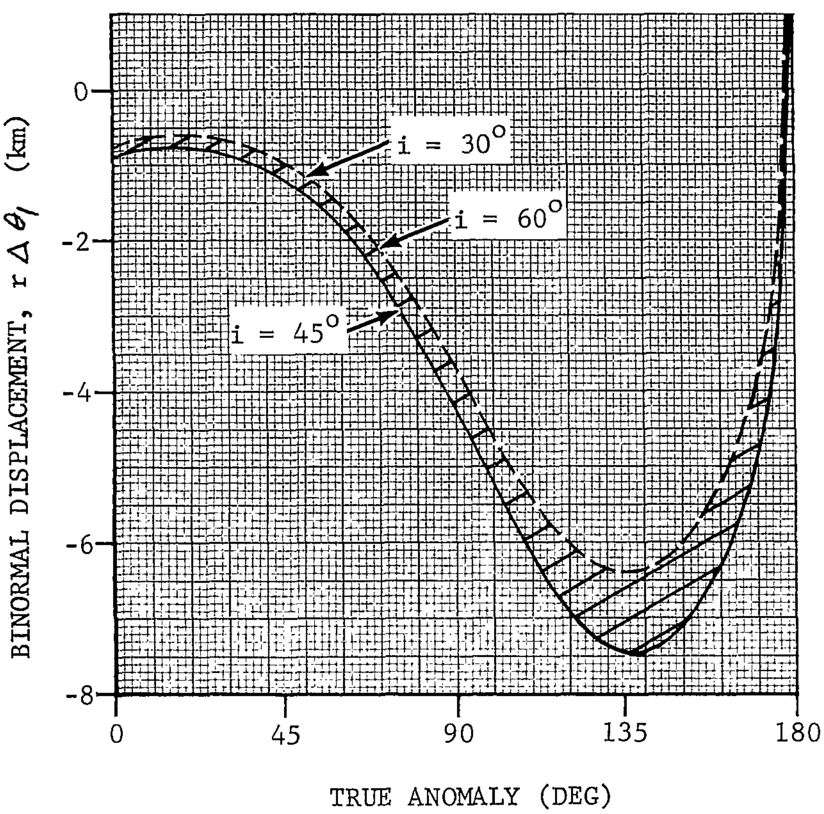


FIGURE B27 BINORMAL DISPLACEMENT;
$e = 0.5$, $\omega = 30^{\circ}$

R02441

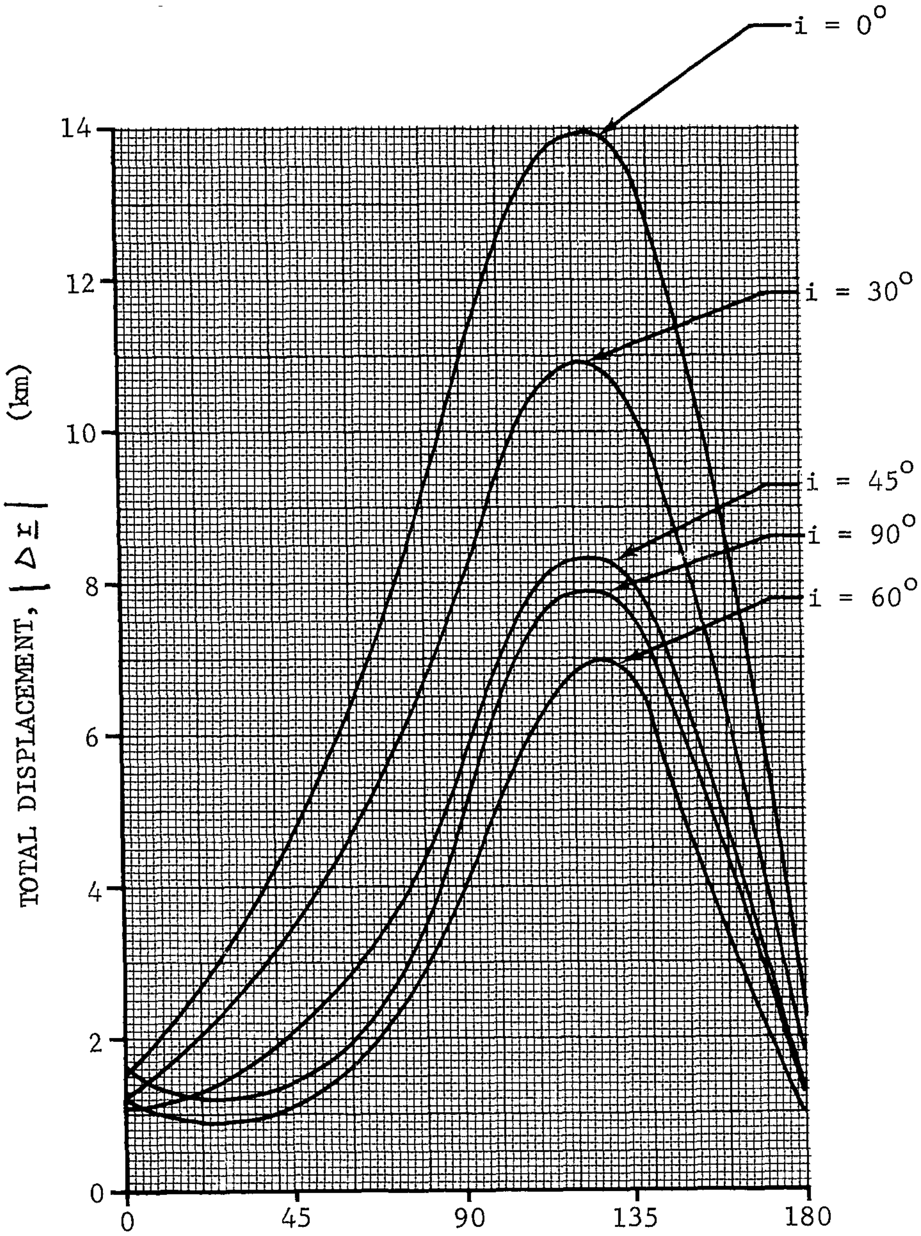


FIGURE B28 TOTAL DISPLACEMENT;
$e = 0.5, \quad \omega = 30^\circ$

R02442

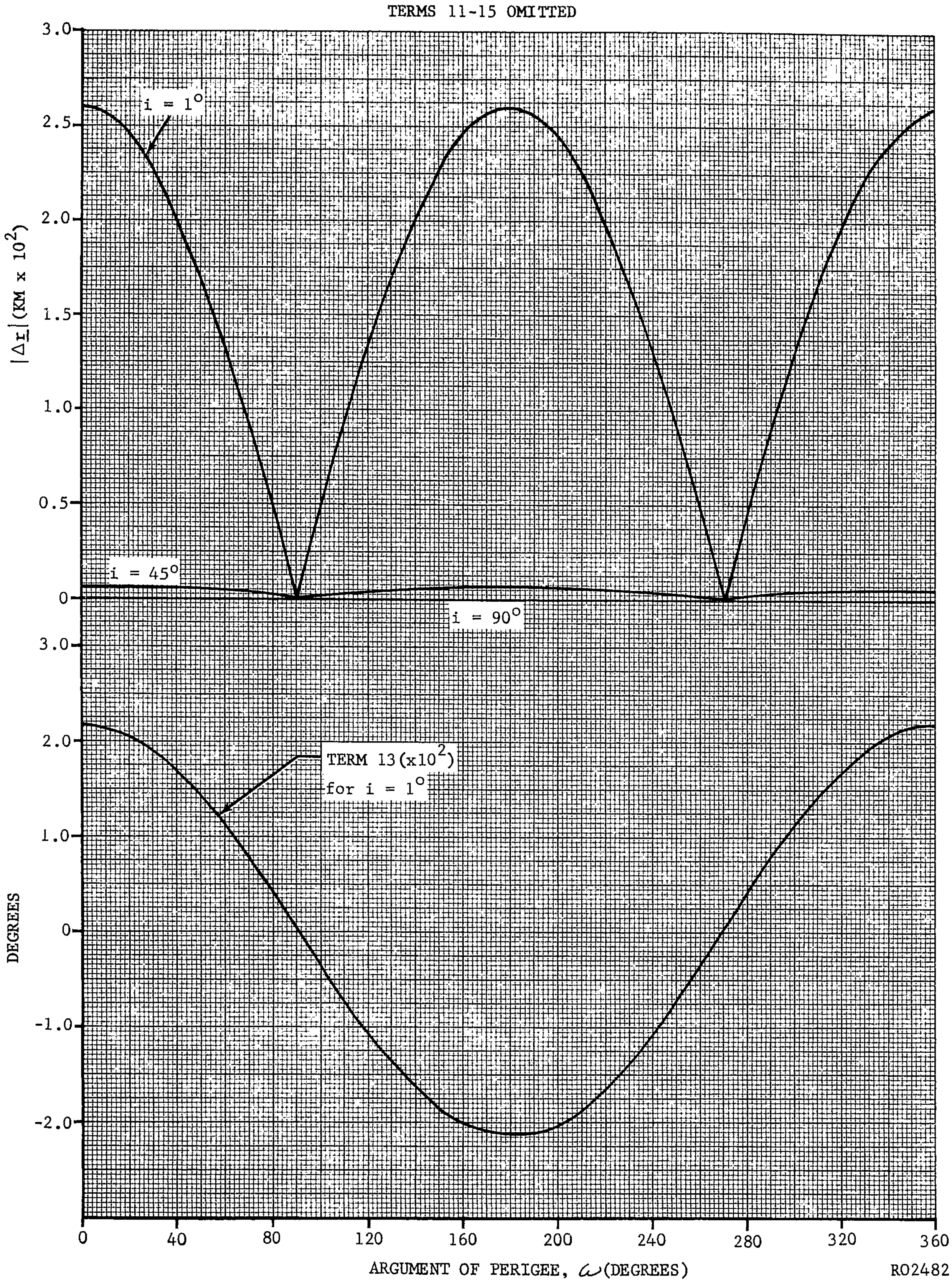


FIGURE B29 TOTAL DISPLACEMENT WITH TERMS 11-15 (LONG-PERIOD TERMS IN Ω) OMITTED

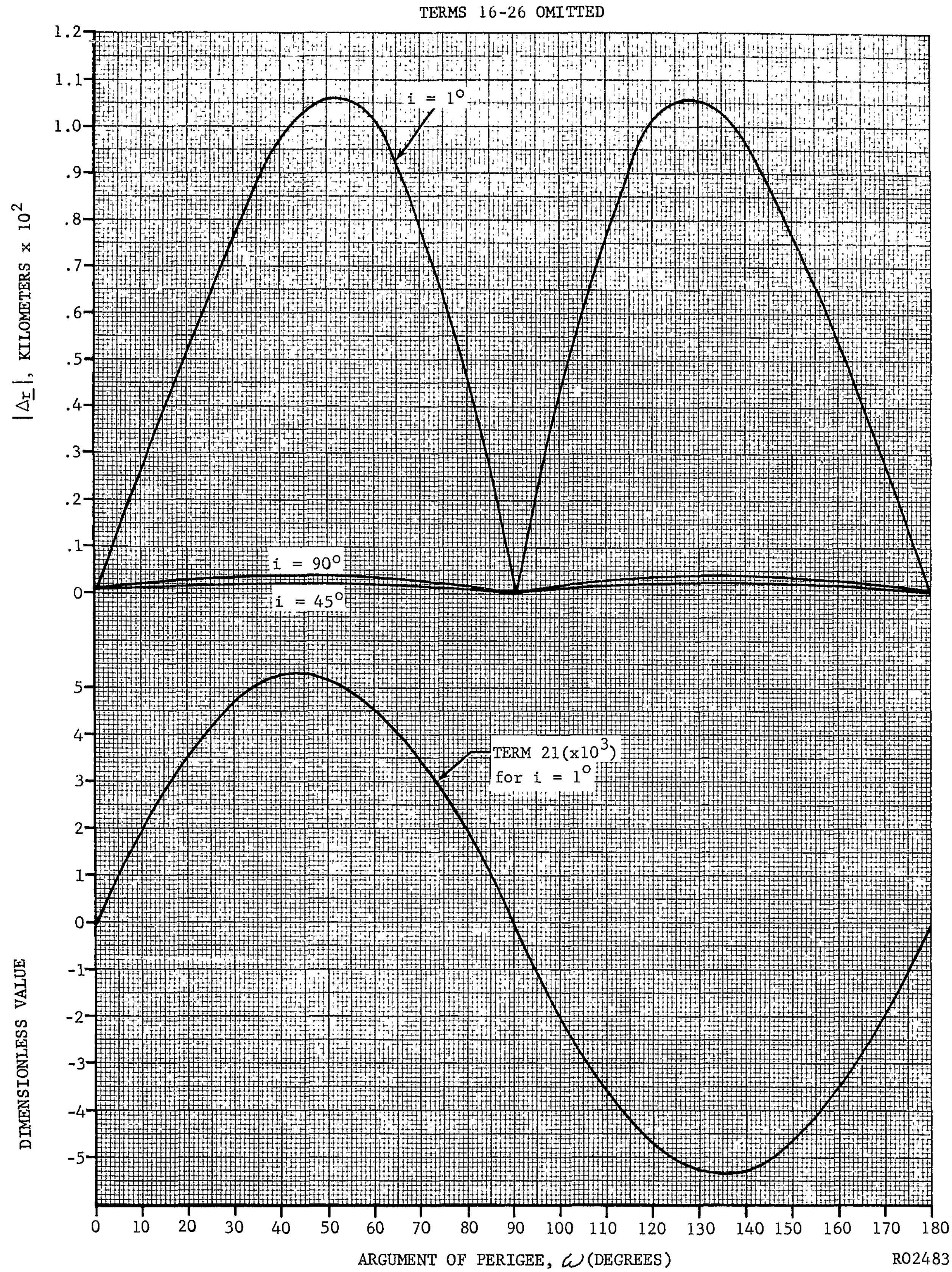


FIGURE B30 TOTAL DISPLACEMENT WITH TERMS 16-26 (LONG-PERIOD TERMS IN $a_{\chi N}$) OMITTED

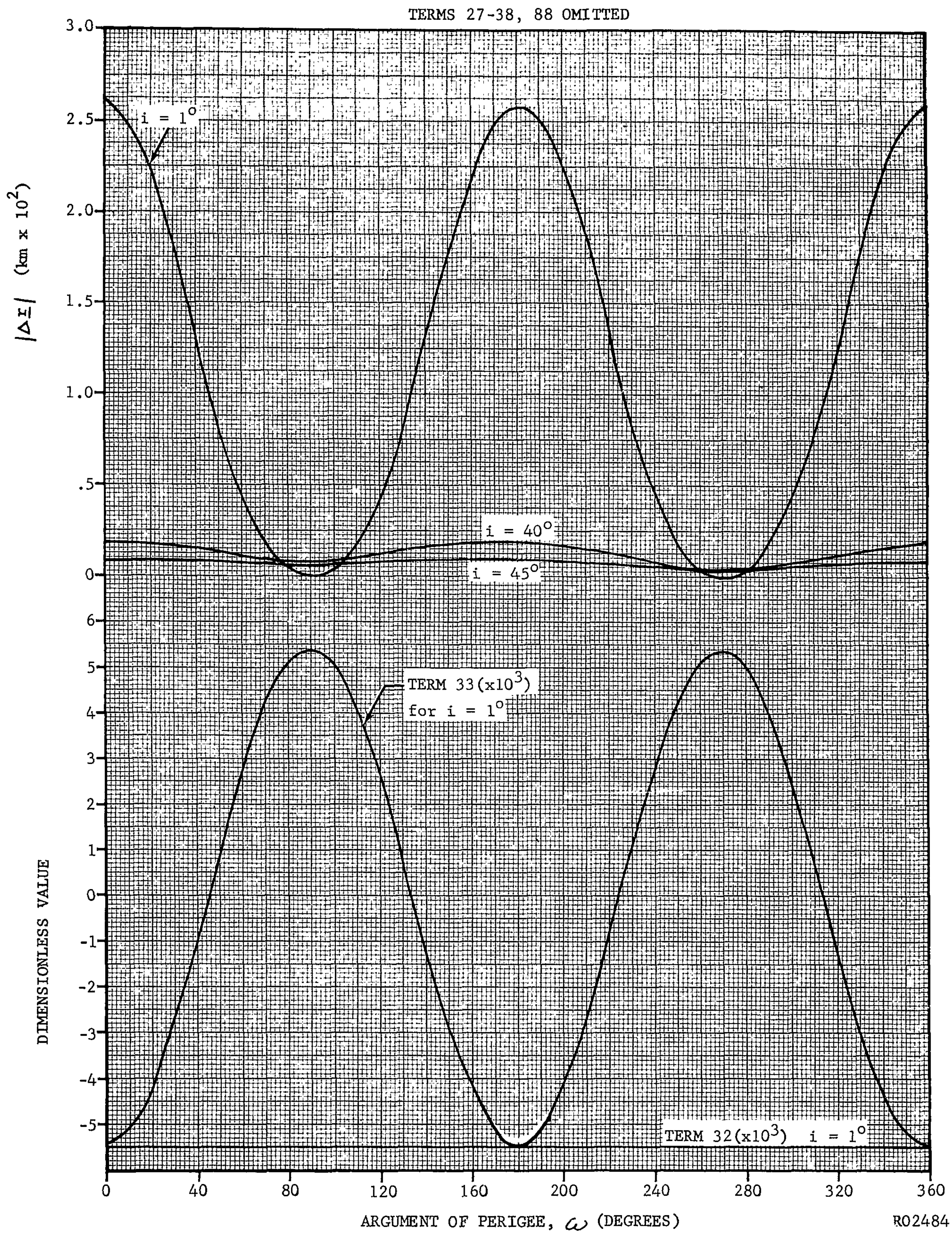


FIGURE B31 TOTAL DISPLACEMENT WITH TERMS 27-38, 88 (LONG-PERIOD TERMS IN a_{yN}) OMITTED

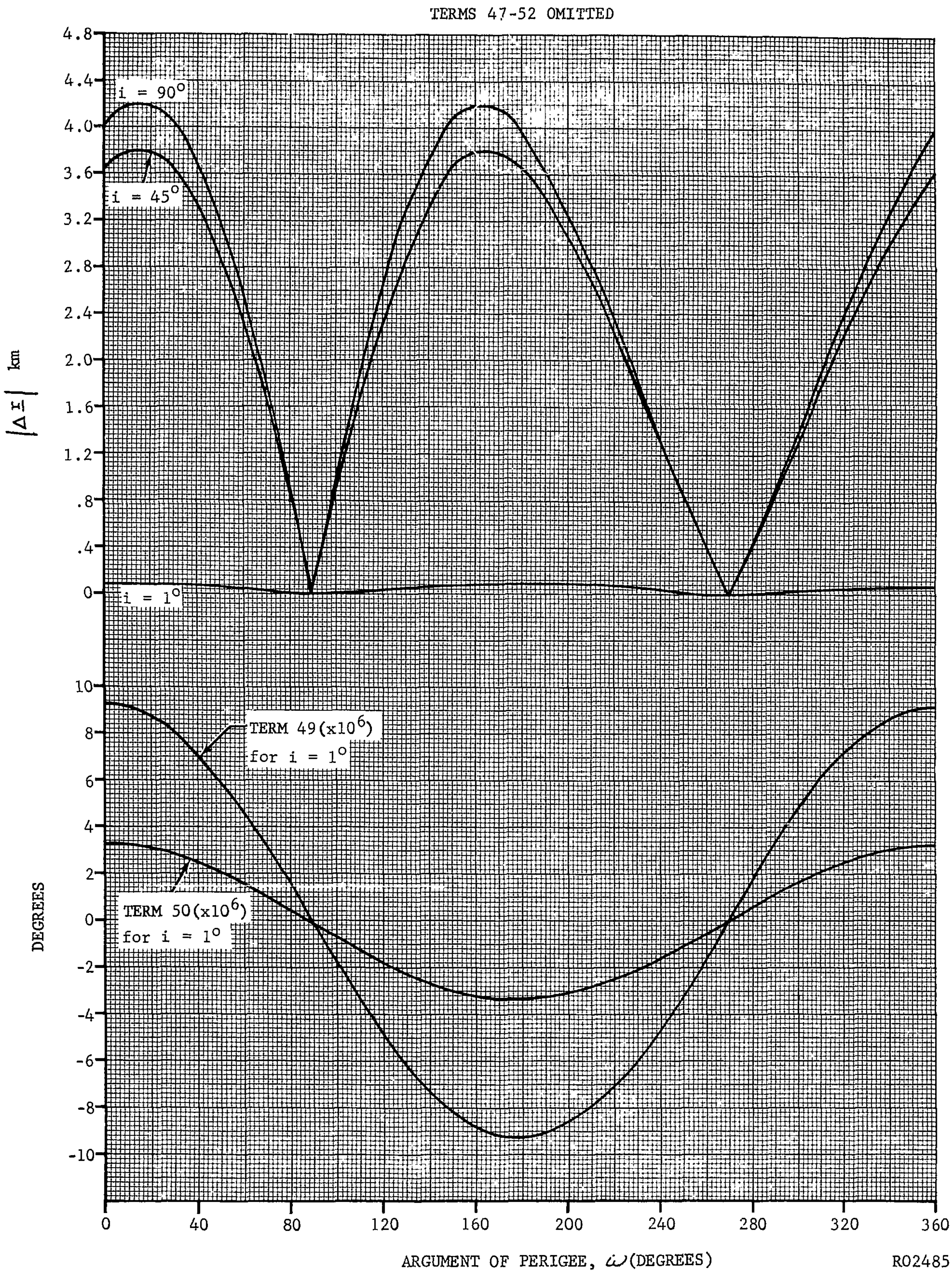


FIGURE B32 TOTAL DISPLACEMENT WITH TERMS 47-52 (LONG-PERIOD TERMS IN L) OMITTED

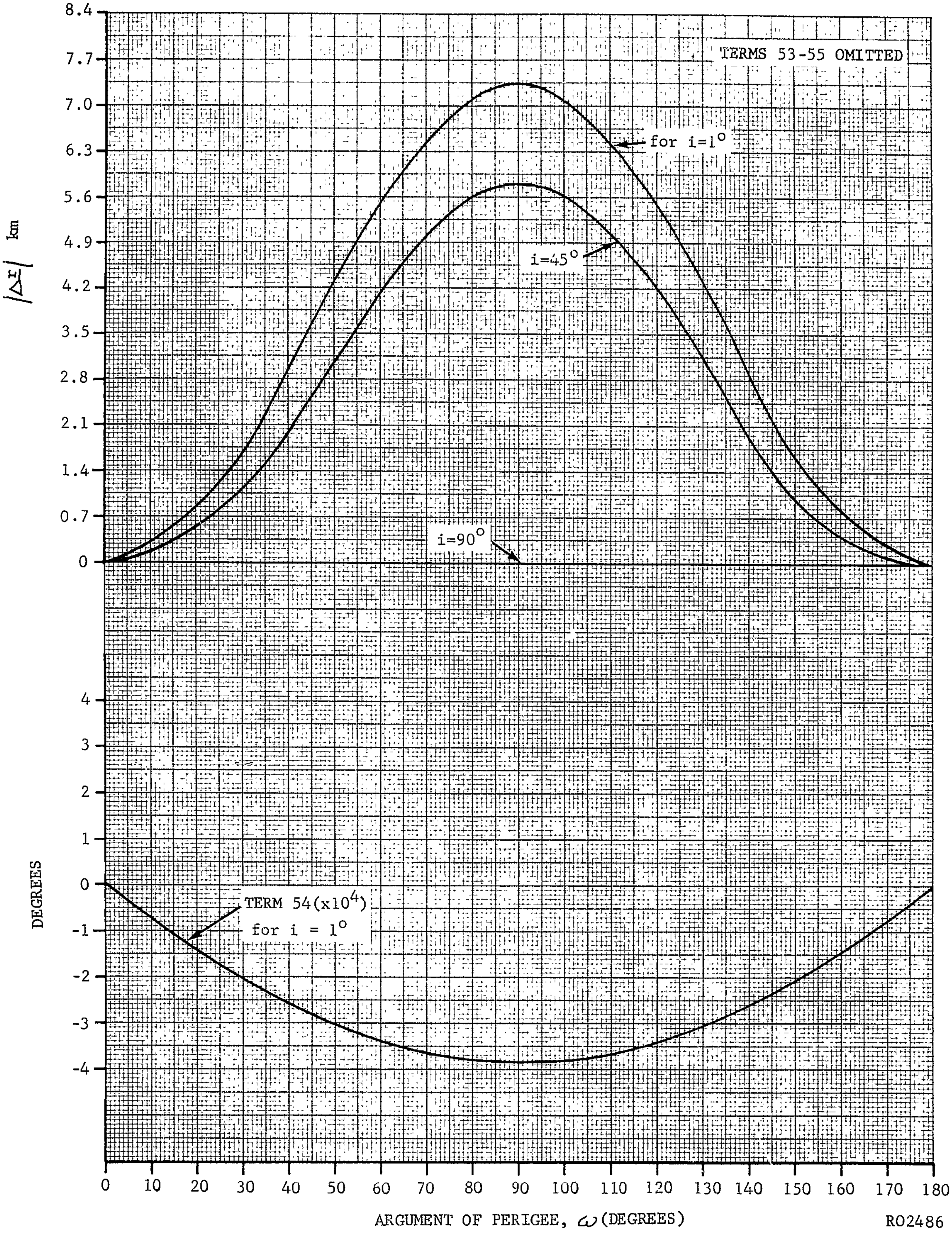


FIGURE B33 TOTAL DISPLACEMENT WITH TERMS 53-55 (LONG-PERIOD TERMS IN i) OMITTED

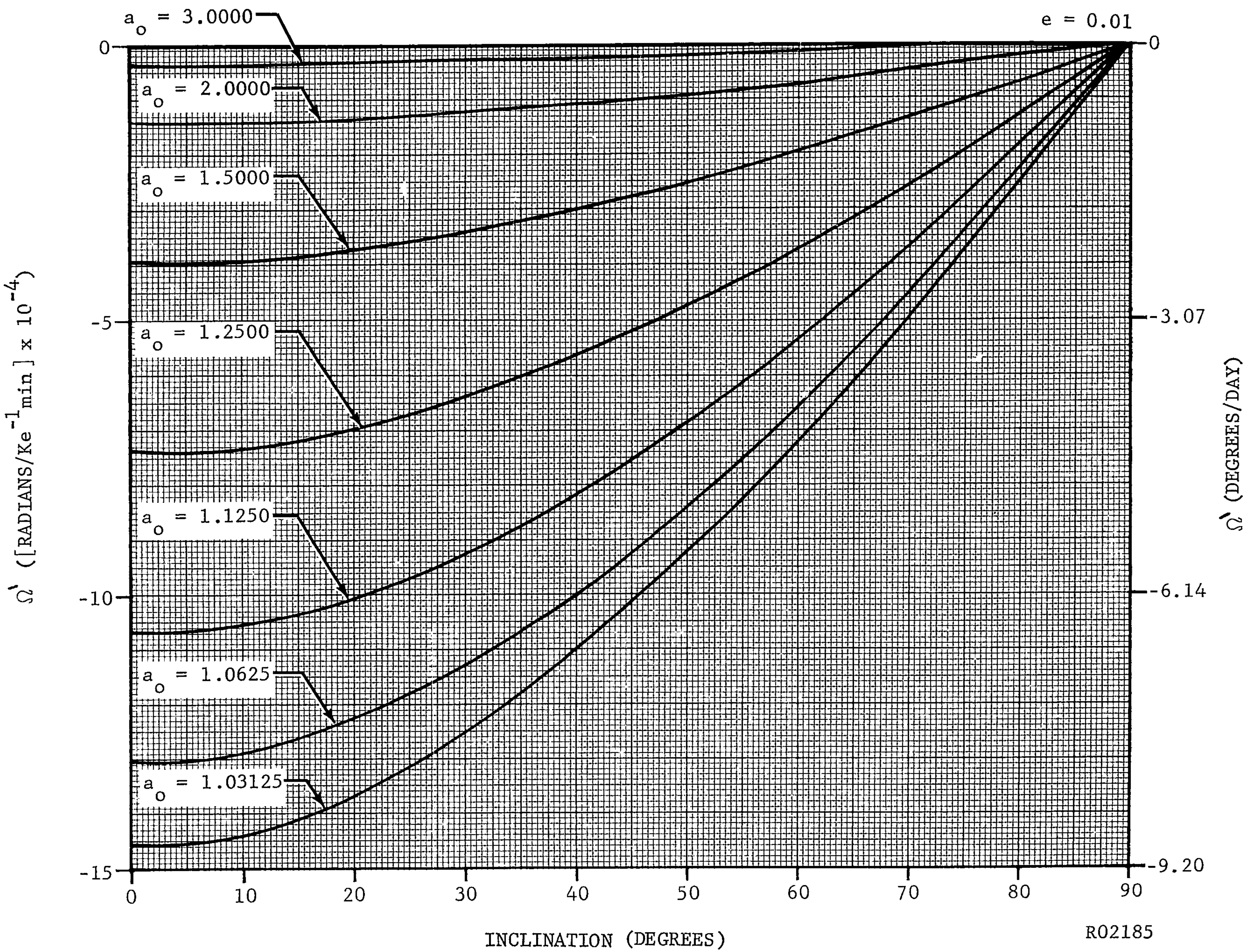


FIGURE B34 NODAL REGRESSION RATE VERSUS INCLINATION

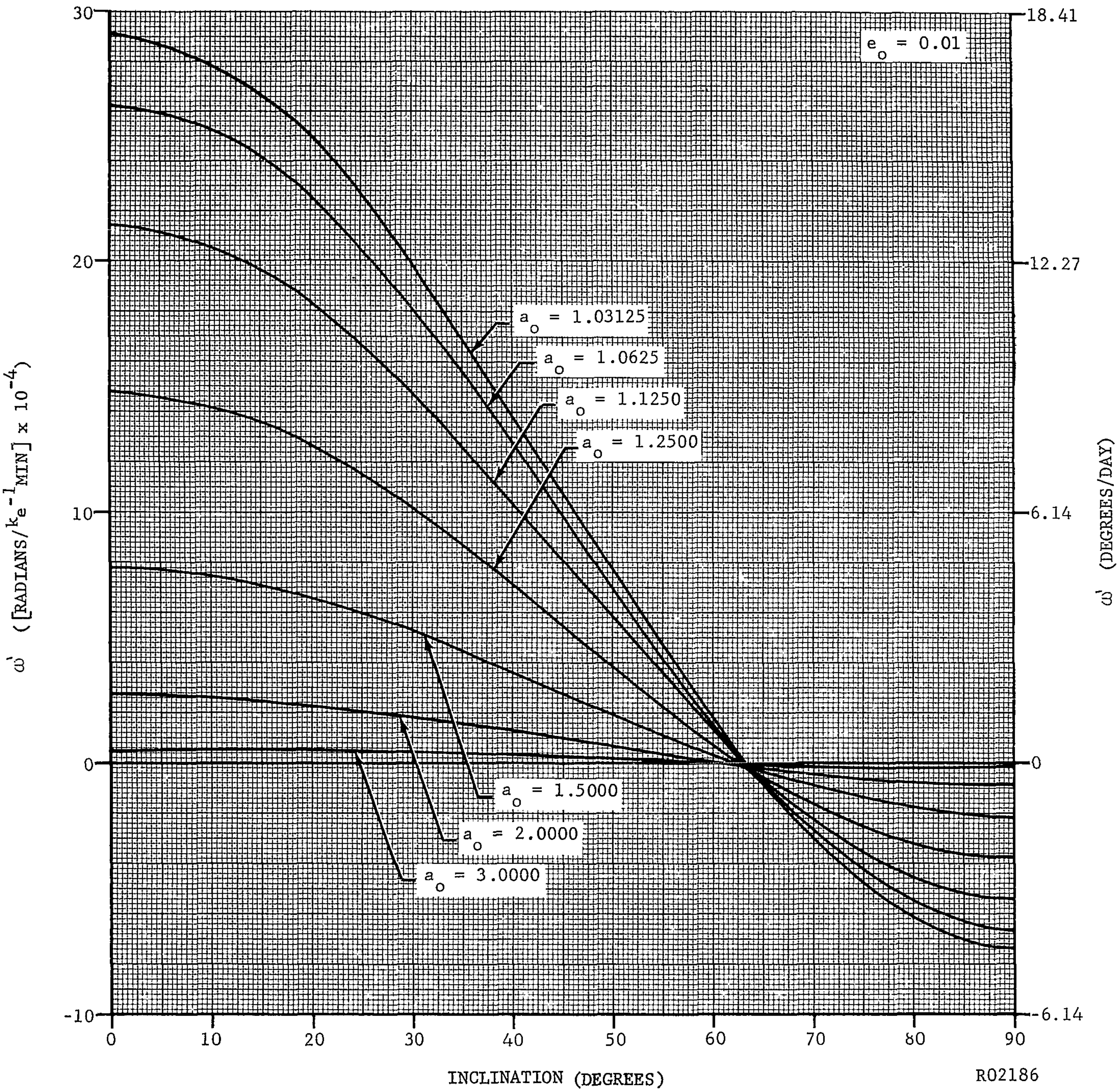


FIGURE B35 APSIDAL RATE VERSUS INCLINATION

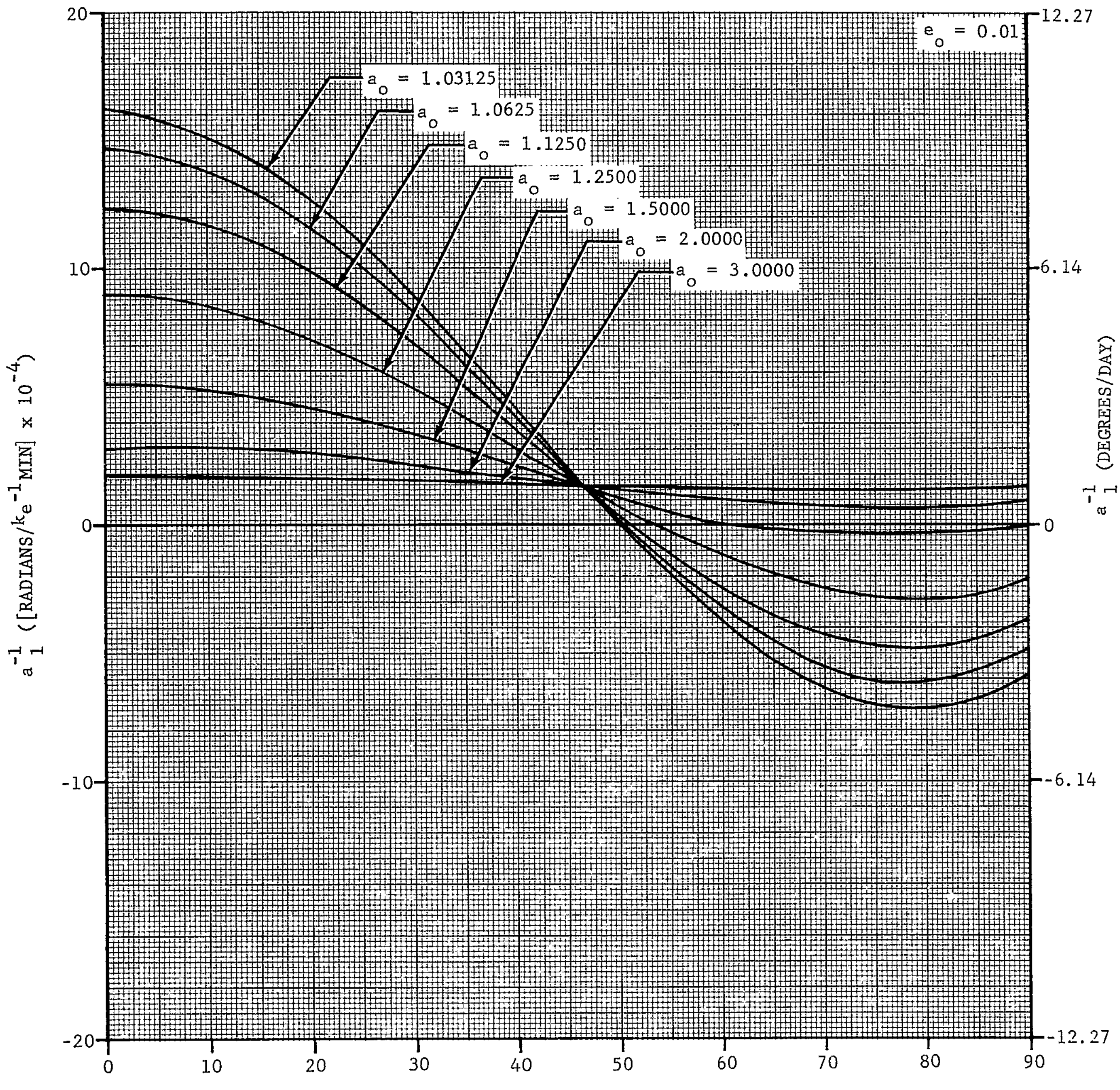


FIGURE B36 THE QUANTITY a_1^{-1} VERSUS INCLINATION

THE QUANTITY a_2^{-1} VERSUS INCLINATION OF ORBIT PLANE

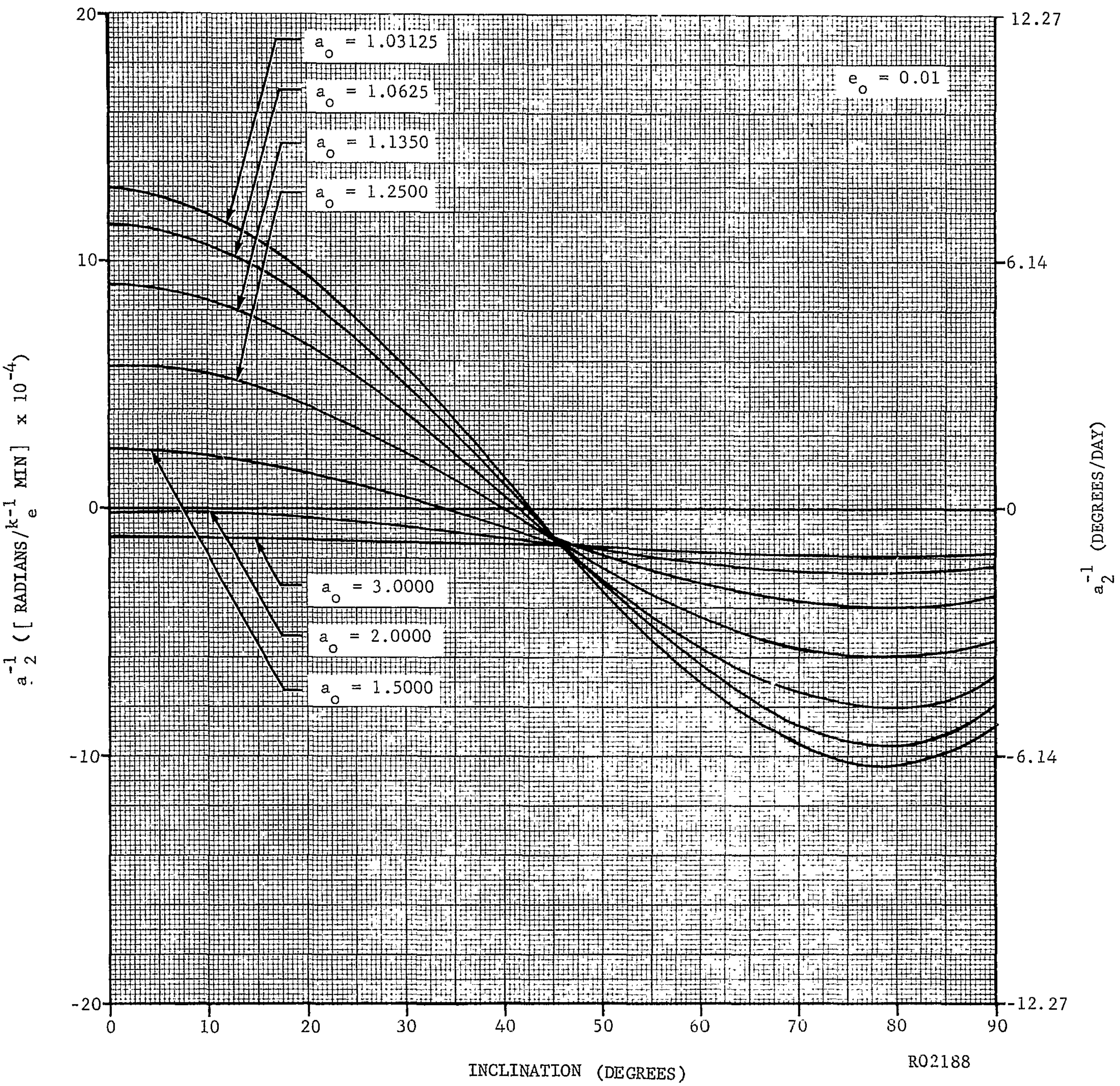


FIGURE B37 THE QUANTITY a_2^{-1} VERSUS INCLINATION

THE QUANTITY a_3^{-1} VERSUS INCLINATION OF ORBIT PLANE

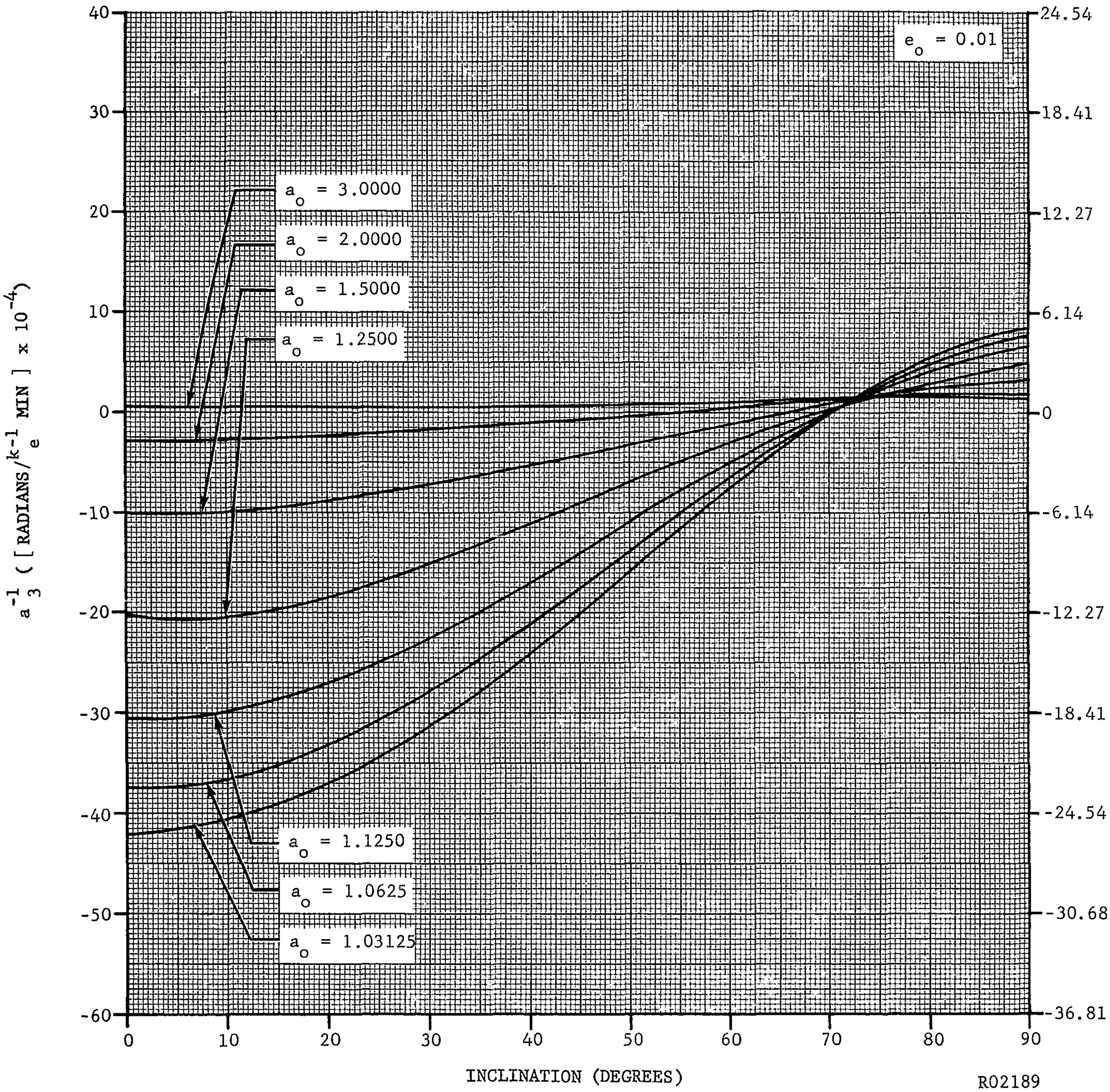


FIGURE B38 THE QUANTITY a_3^{-1} VERSUS INCLINATION

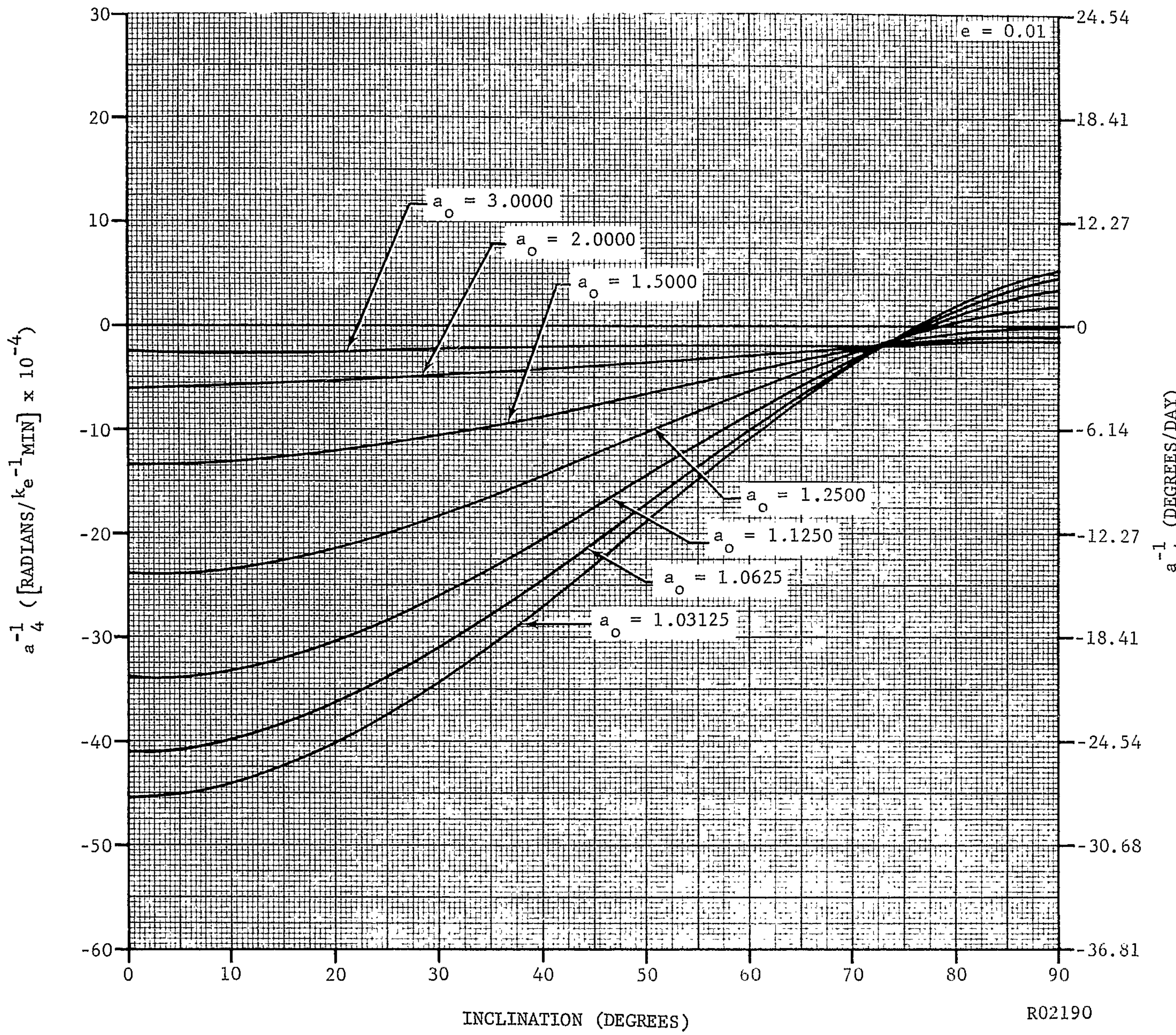


FIGURE B39 THE QUANTITY a_4^{-1} VERSUS INCLINATION

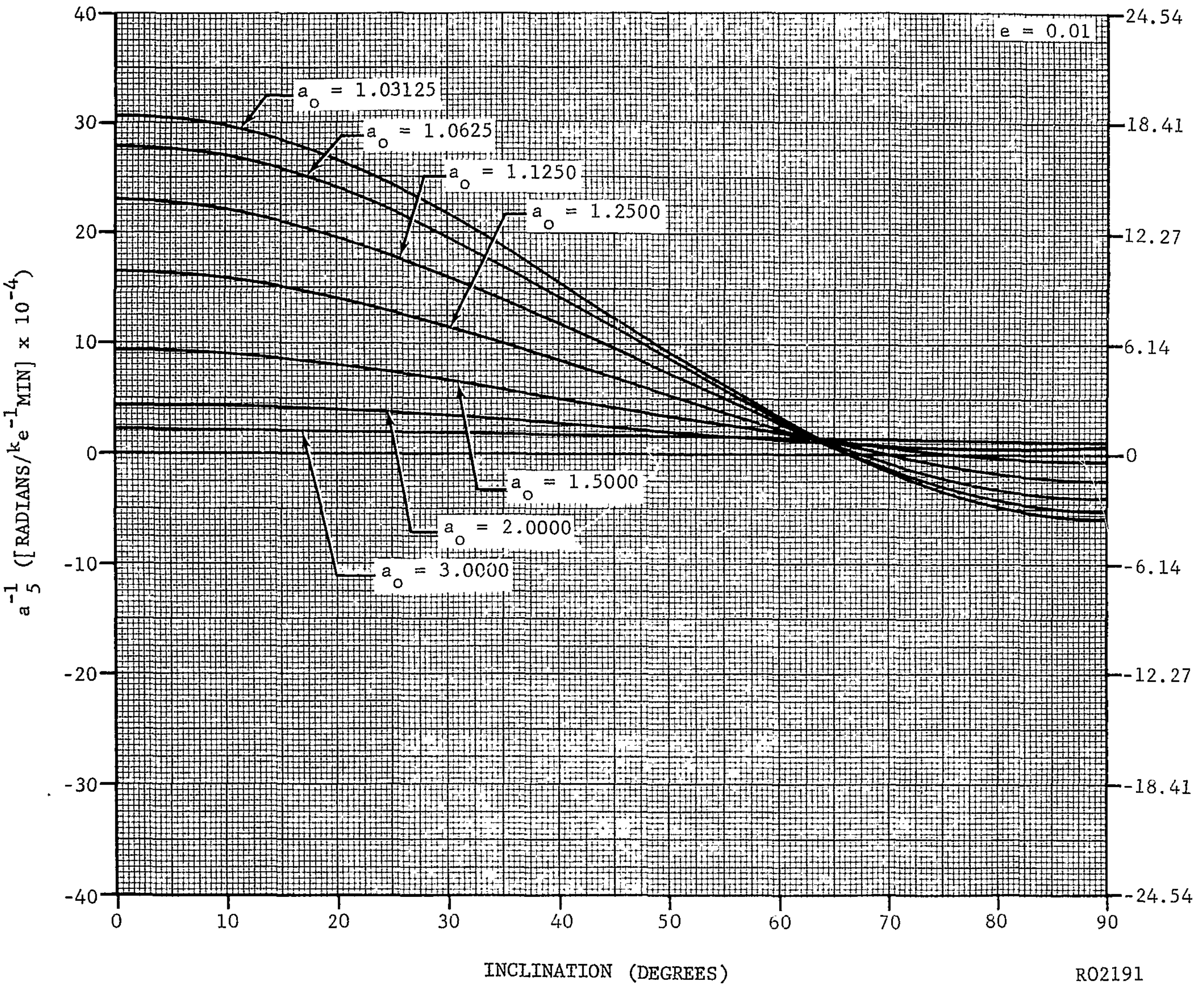


FIGURE B40 THE QUANTITY a_5^{-1} VERSUS INCLINATION

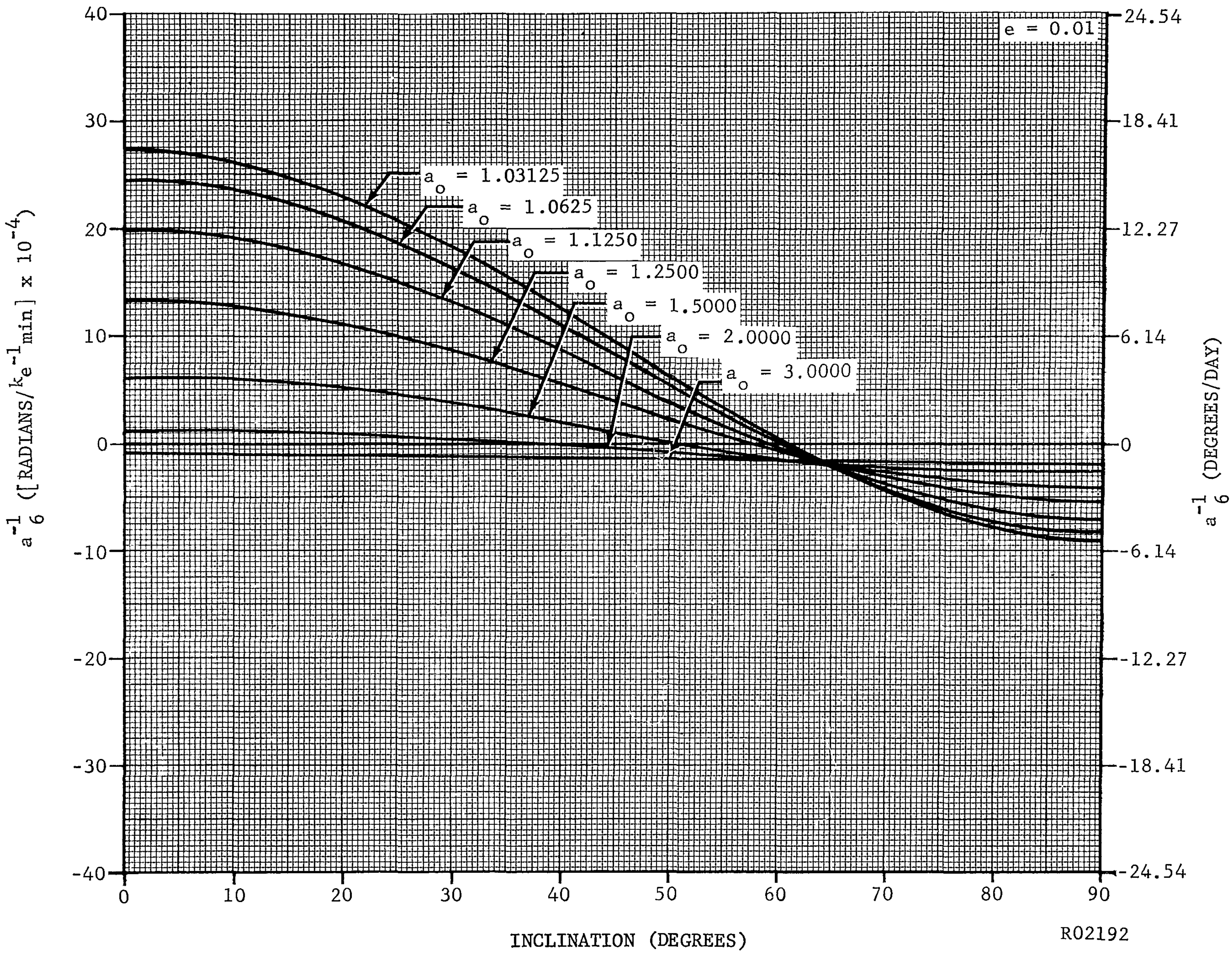


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FIGURE B41 THE QUANTITY a_6^{-1} VERSUS INCLINATION

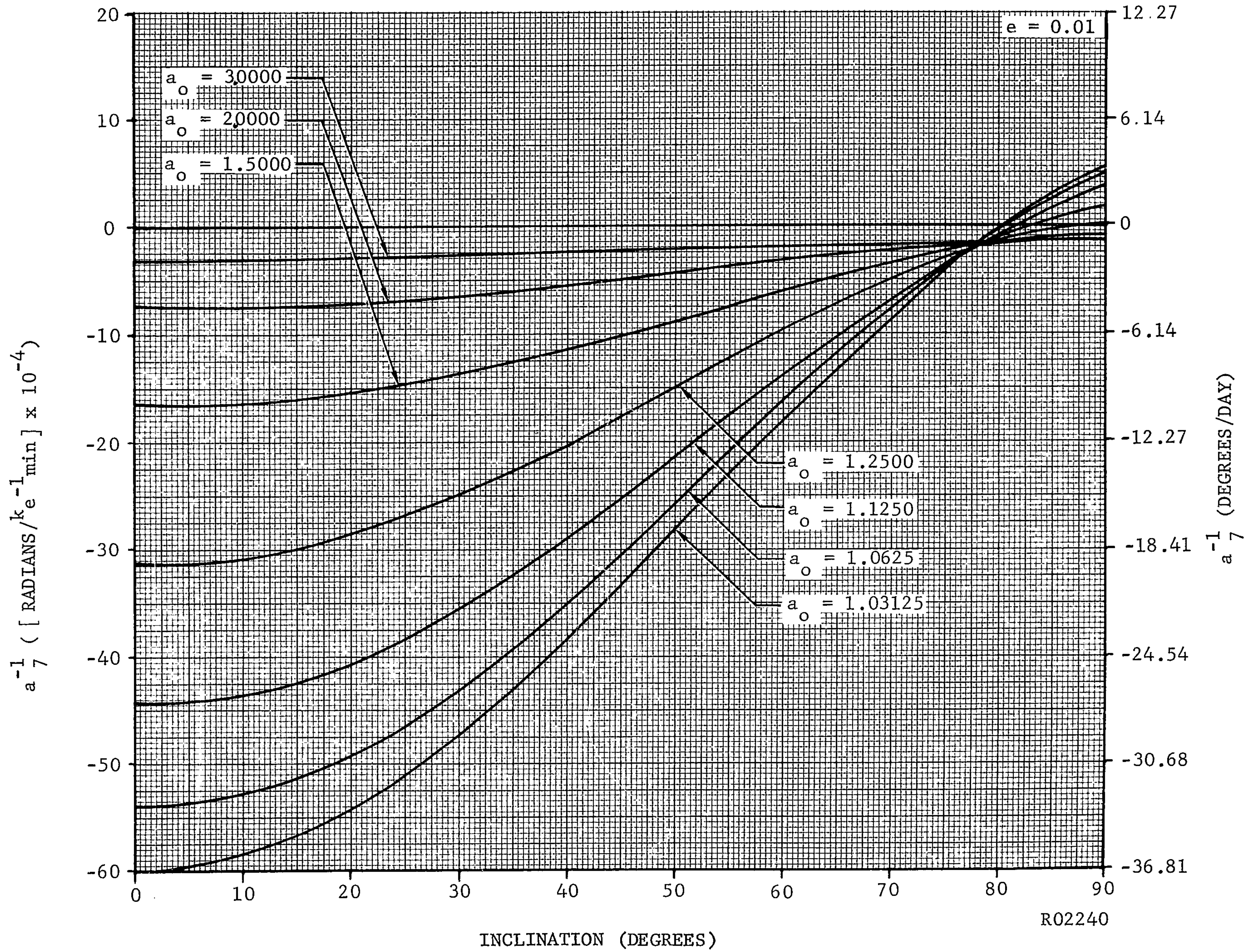


FIGURE B42 THE QUANTITY a_7^{-1} VERSUS INCLINATION

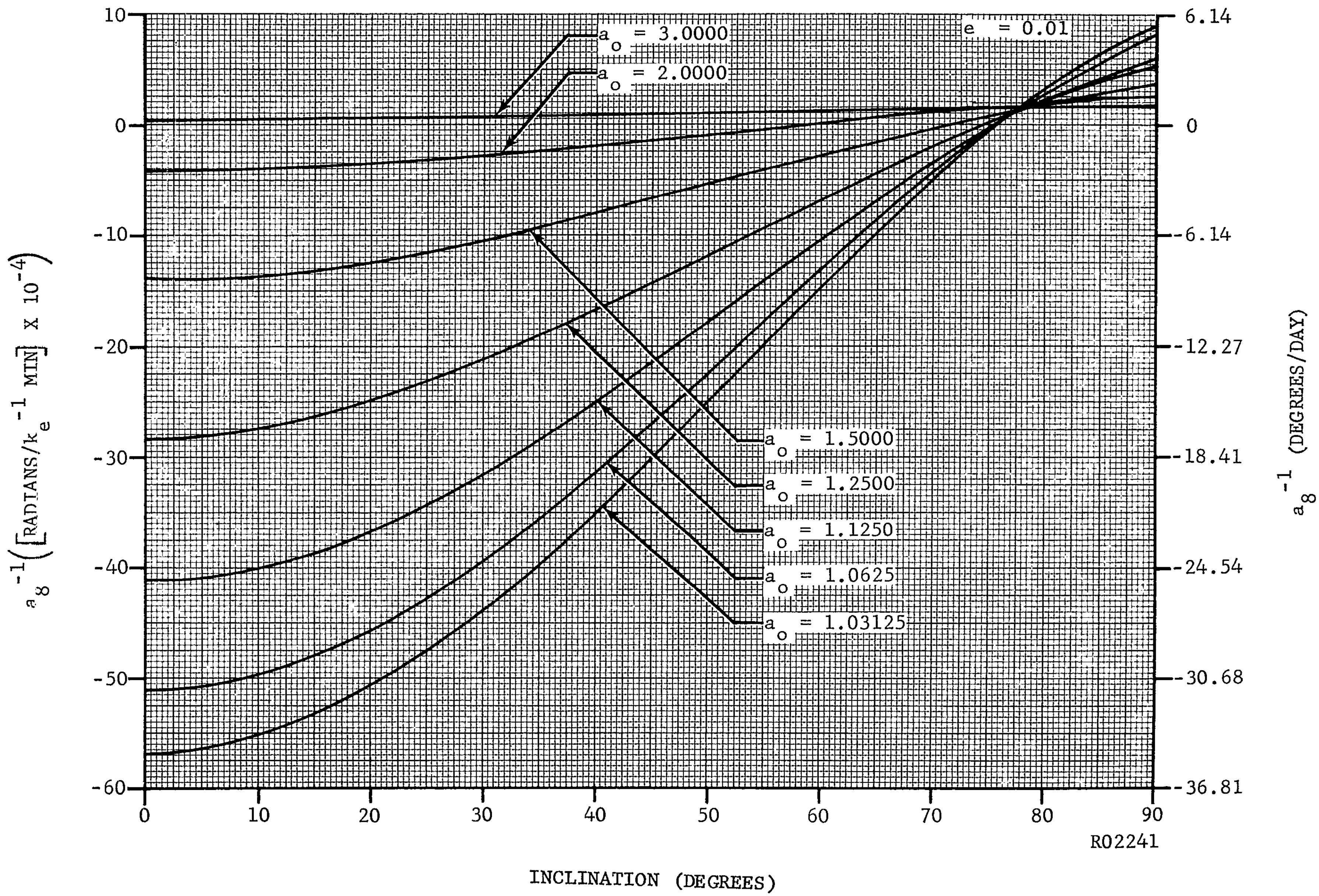
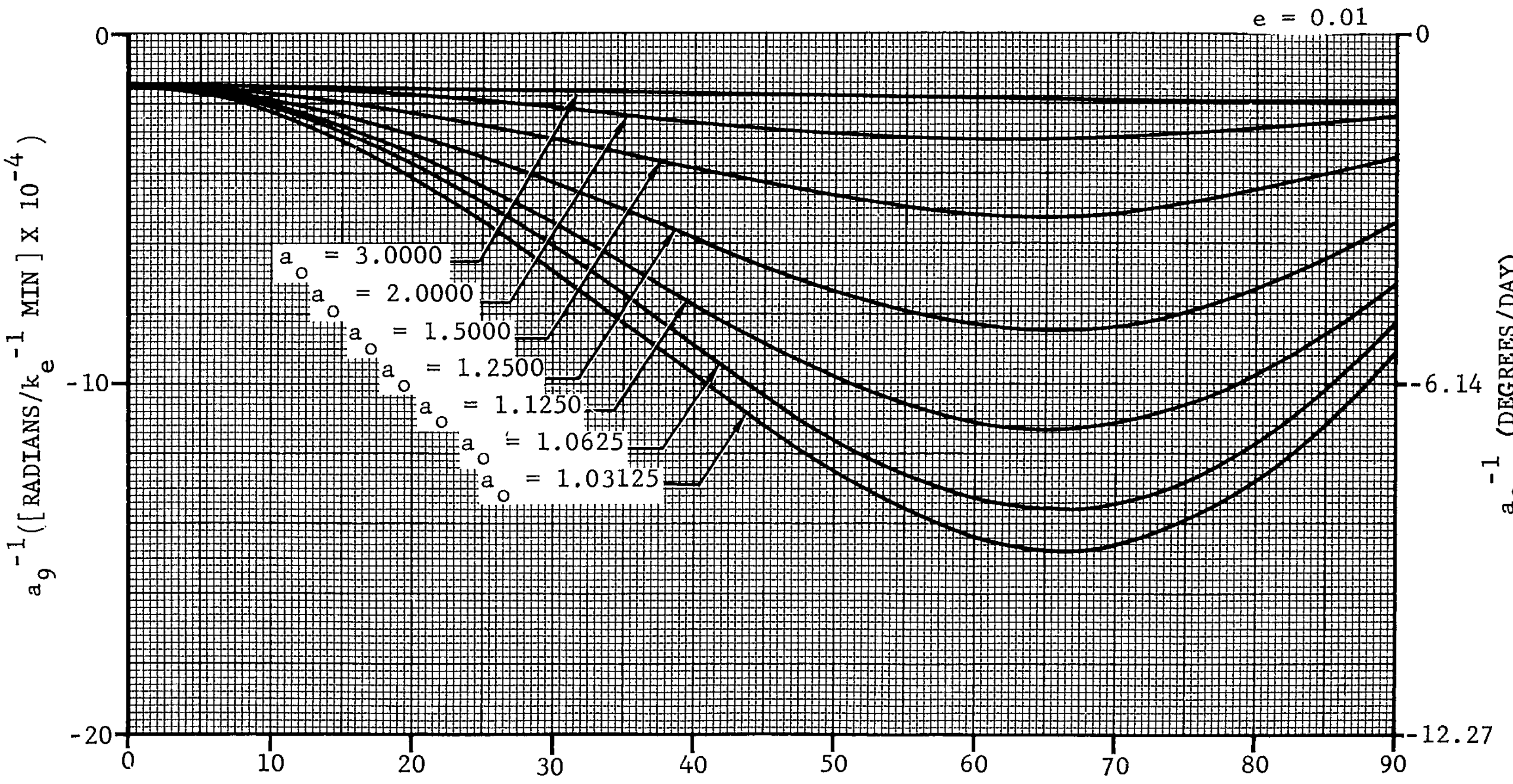


FIGURE B43 THE QUANTITY a_8^{-1} VERSUS INCLINATION



R02242

FIGURE B44 THE QUANTITY a_9^{-1} VERSUS INCLINATION

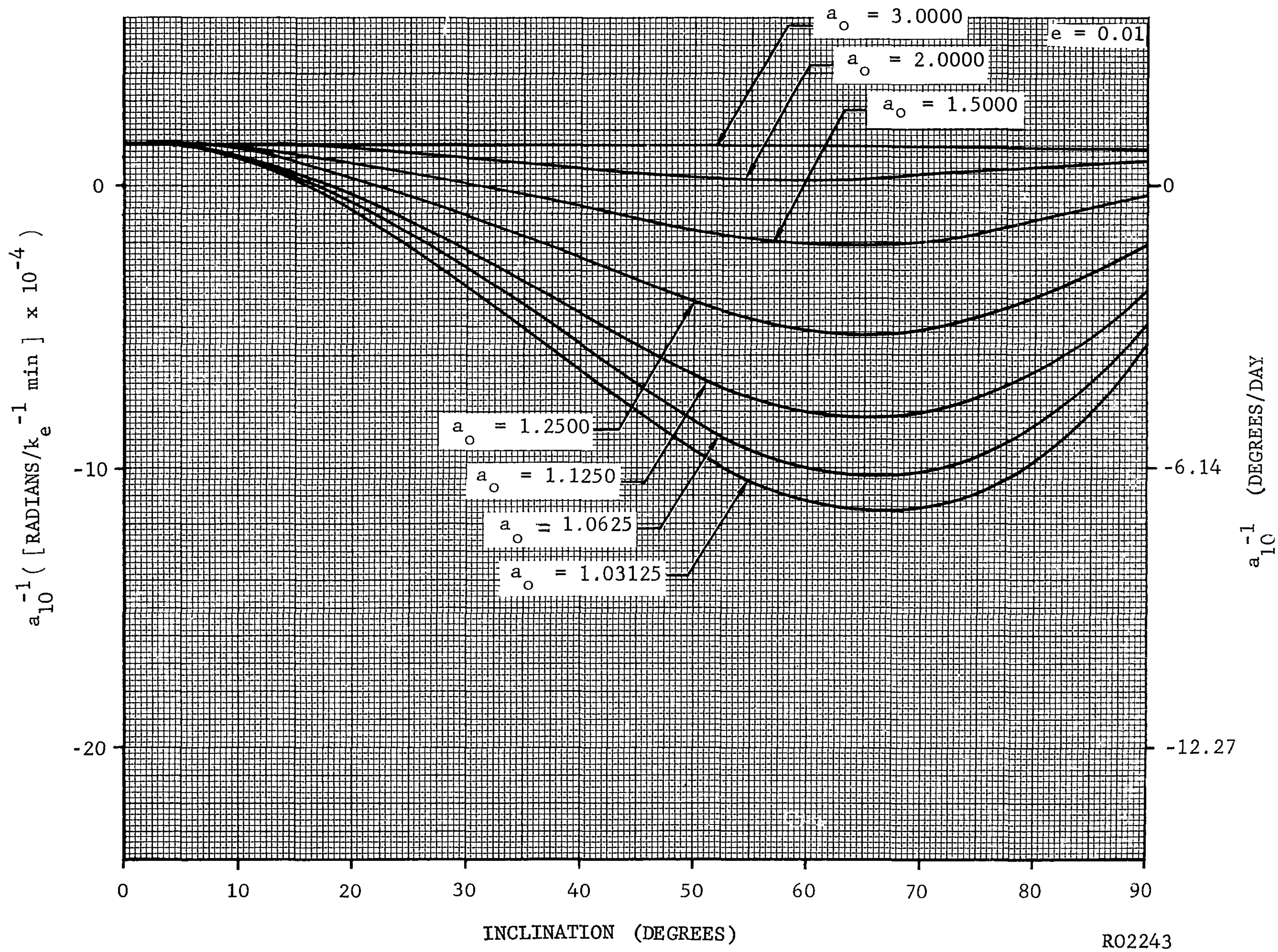


FIGURE B45 THE QUANTITY a_{10}^{-1} VERSUS INCLINATION

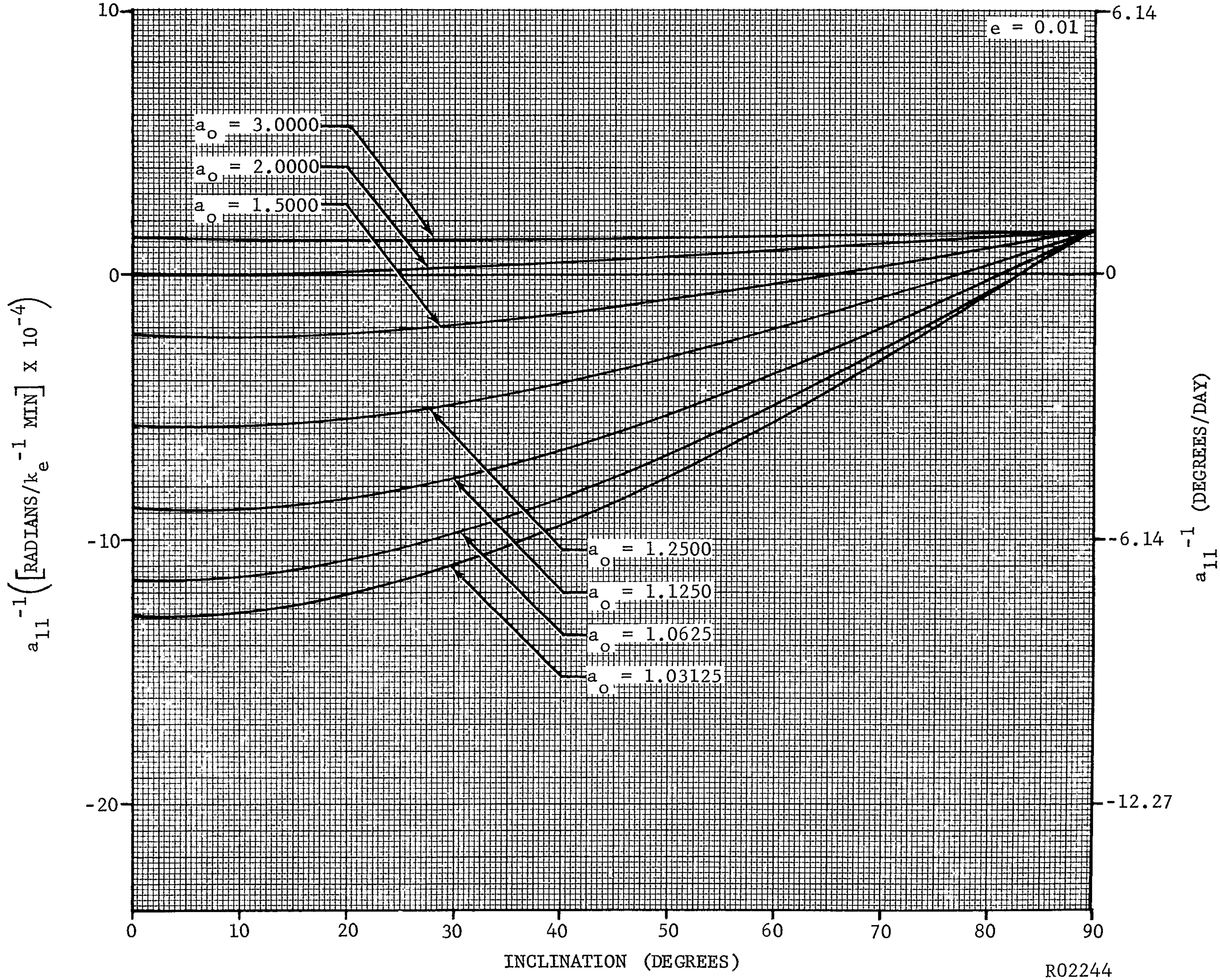


FIGURE B46 THE QUANTITY a_{11}^{-1} VERSUS INCLINATION

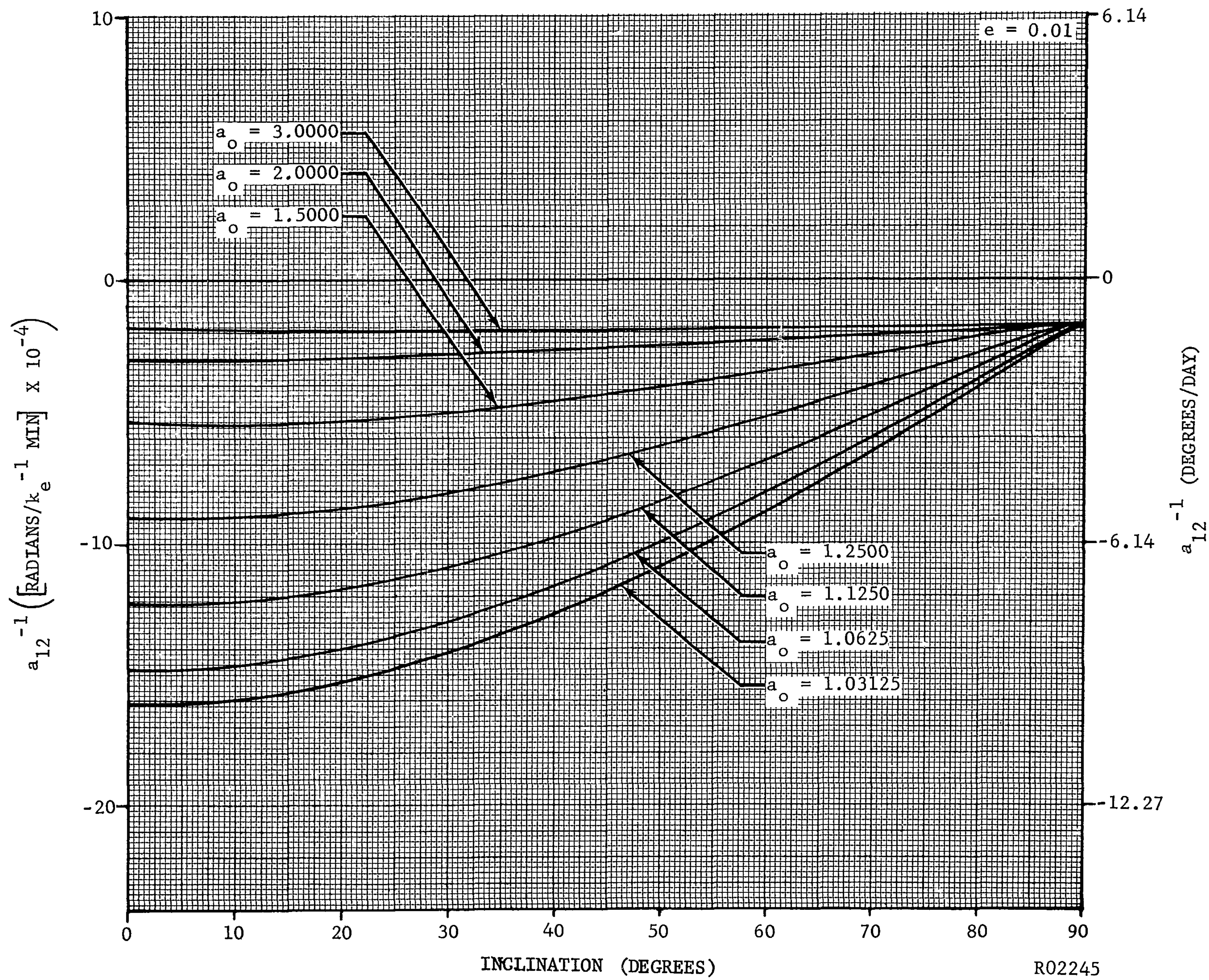


FIGURE B47 THE QUANTITY a_{12}^{-1} VERSUS INCLINATION

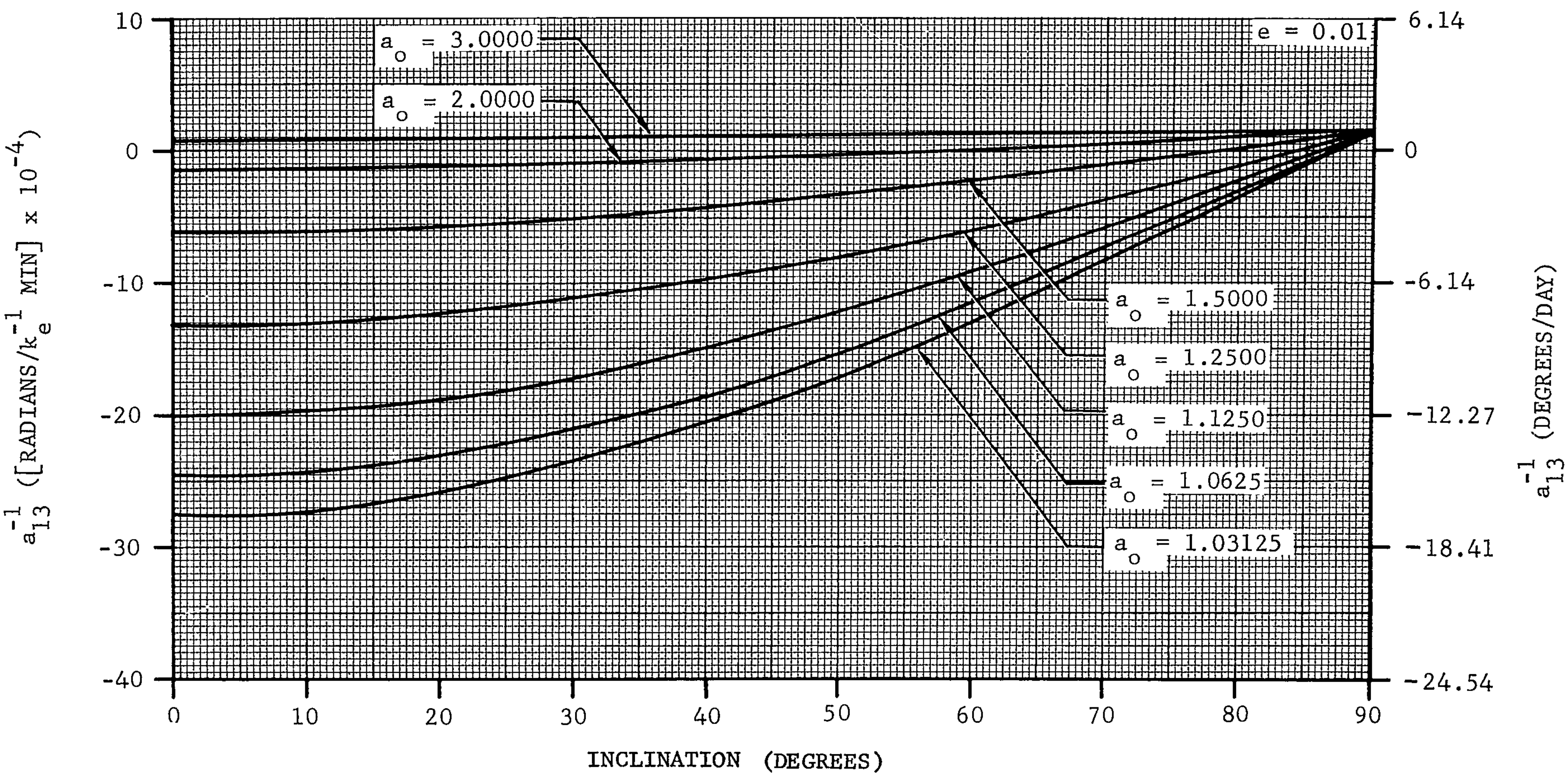


FIGURE B48 THE QUANTITY a_{13}^{-1} VERSUS INCLINATION

R02246

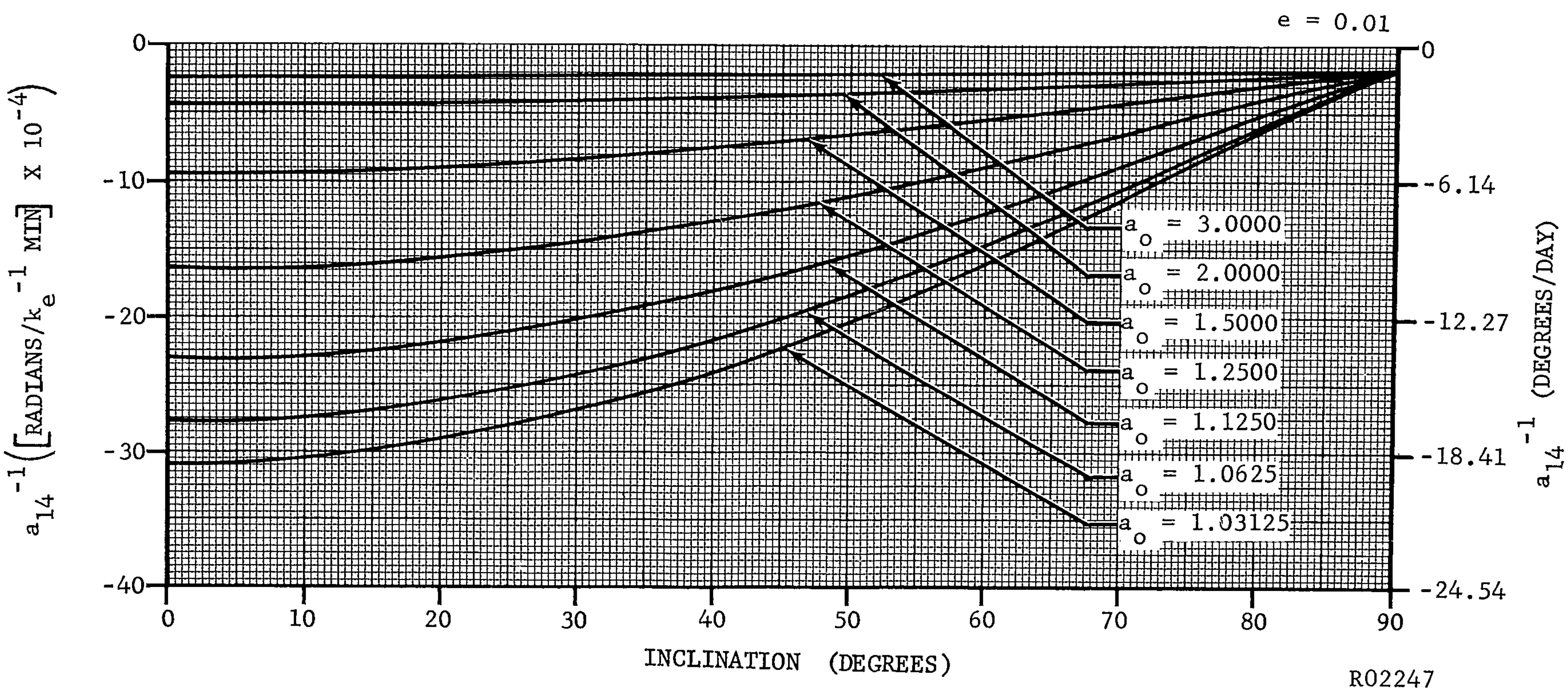


FIGURE B49 THE QUANTITY a_{14}^{-1} VERSUS INCLINATION

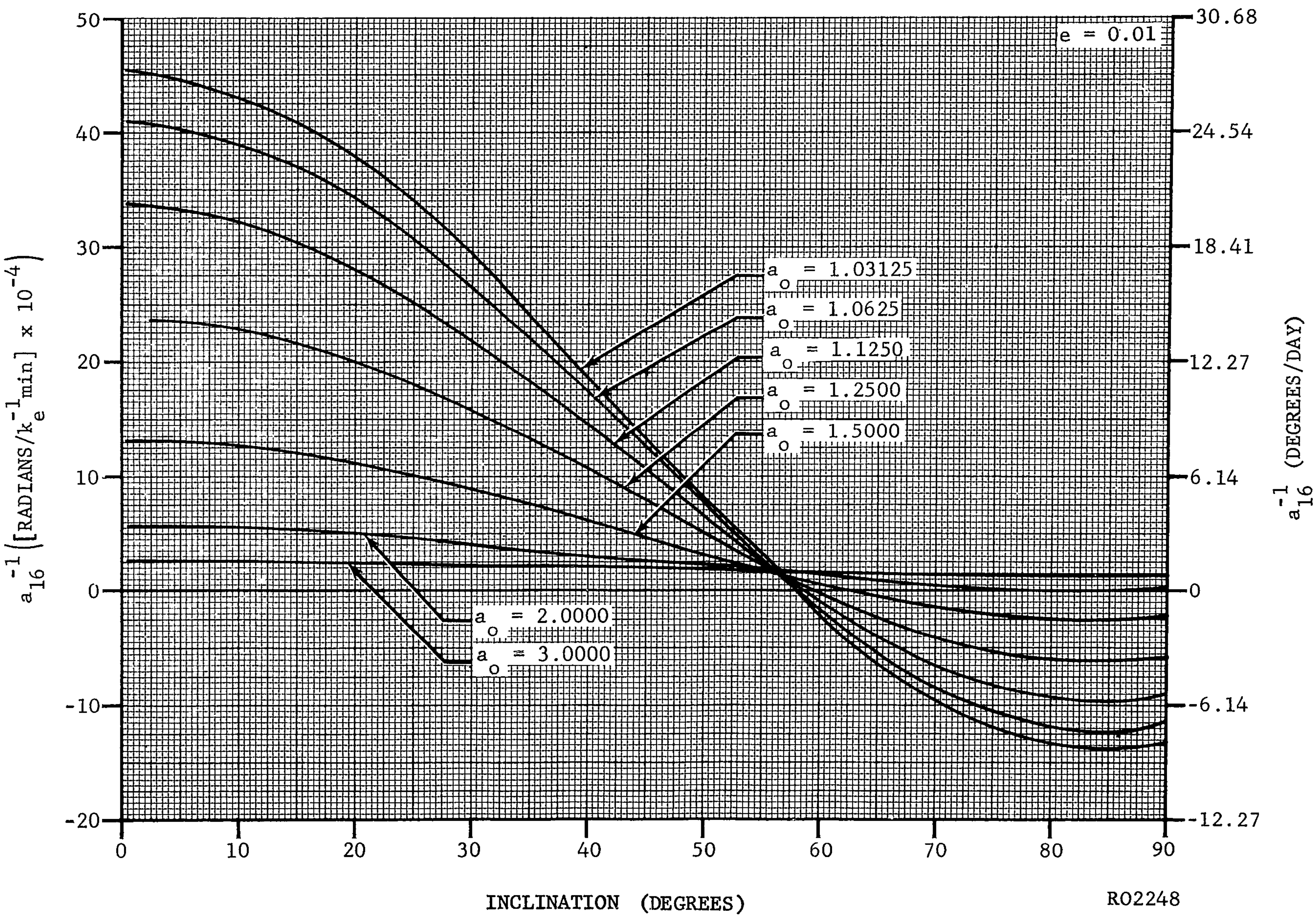


FIGURE B50 THE QUANTITY a_{16}^{-1} VERSUS INCLINATION

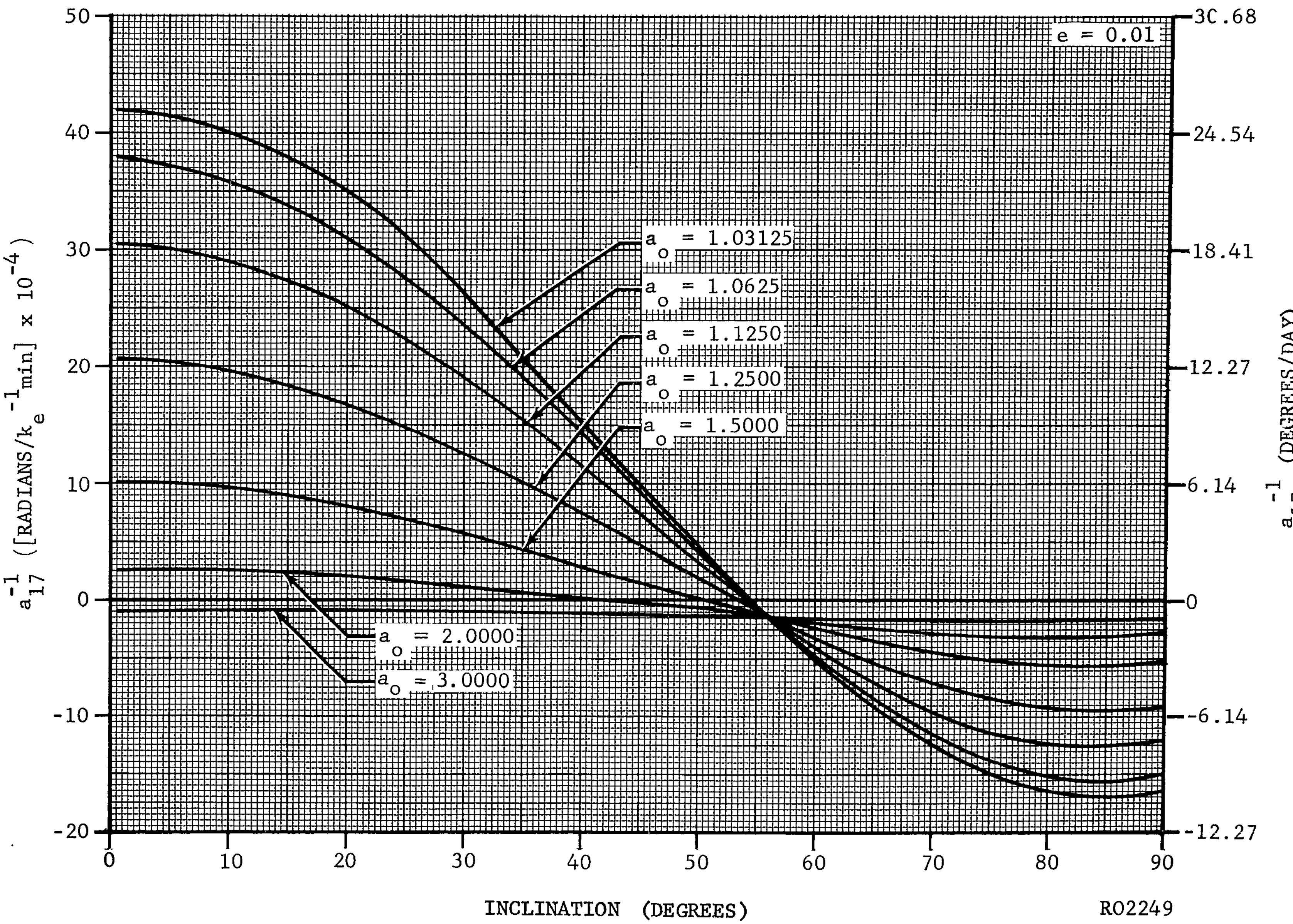


FIGURE B51 THE QUANTITY a_{17}^{-1} VERSUS INCLINATION

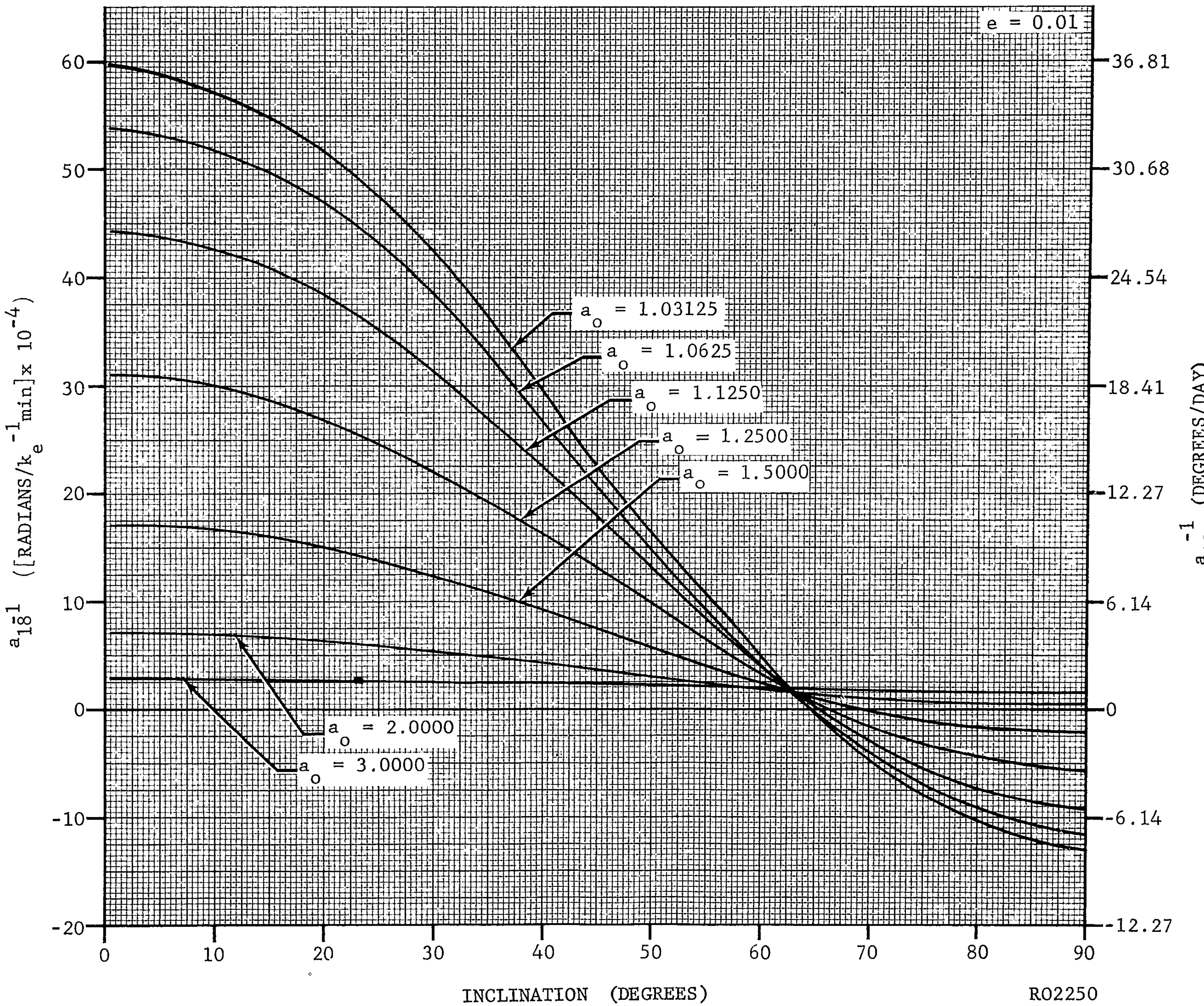


FIGURE B52 THE QUANTITY a_{18}^{-1} VERSUS INCLINATION

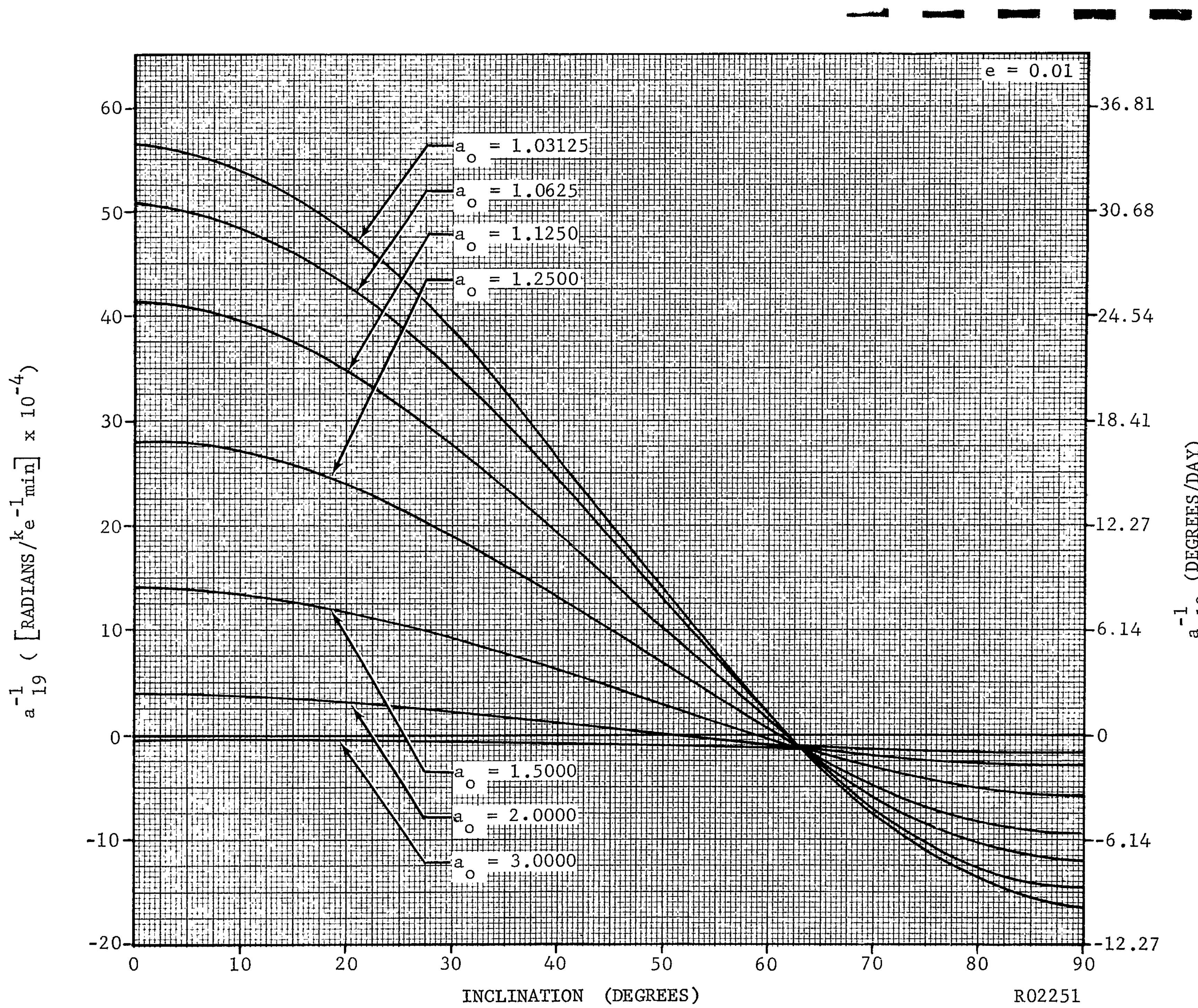


FIGURE B53 THE QUANTITY a_{19}^{-1} VERSUS INCLINATION

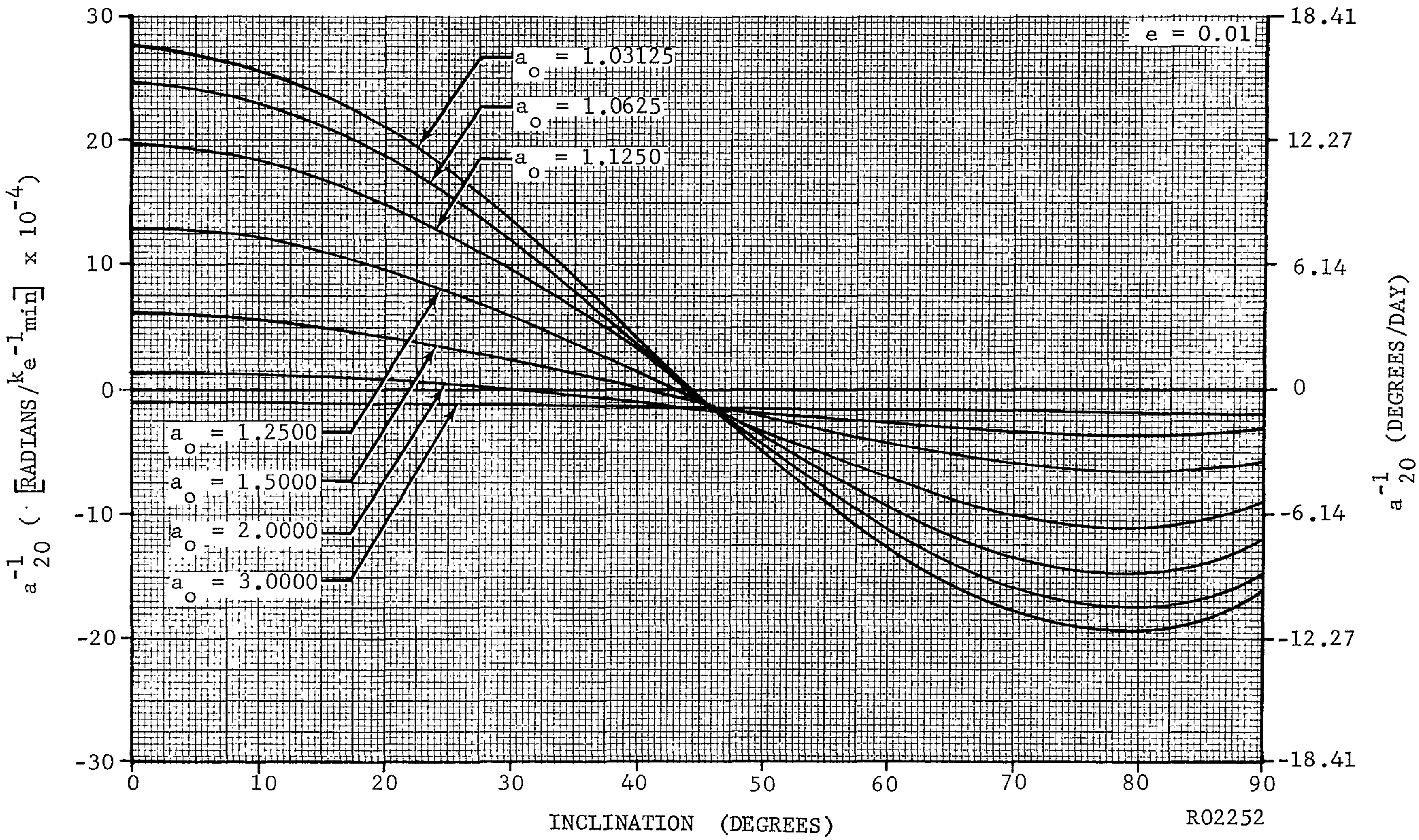


FIGURE B54 THE QUANTITY a_{20}^{-1} VERSUS INCLINATION

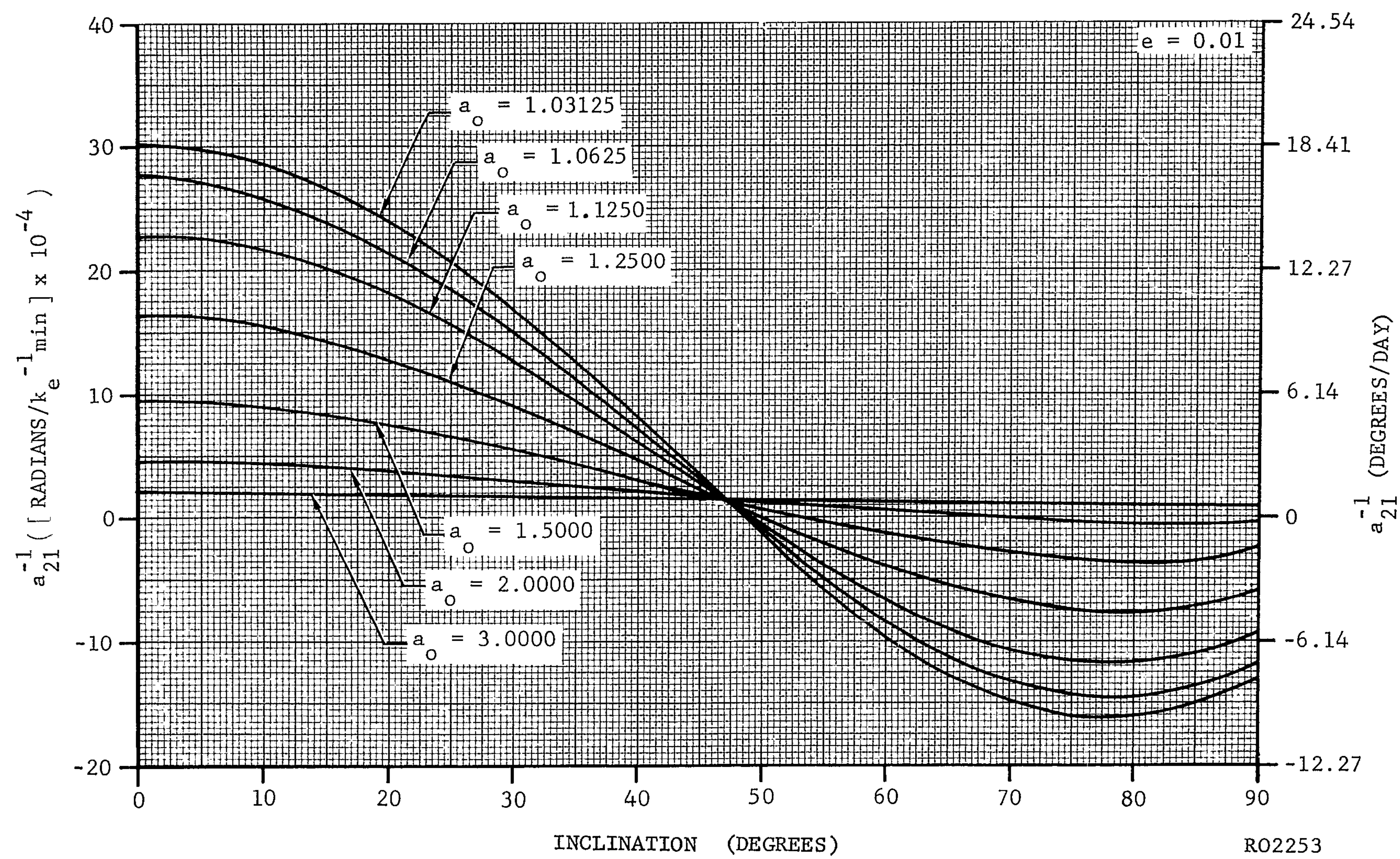


FIGURE B55 THE QUANTITY a_{21}^{-1} VERSUS INCLINATION

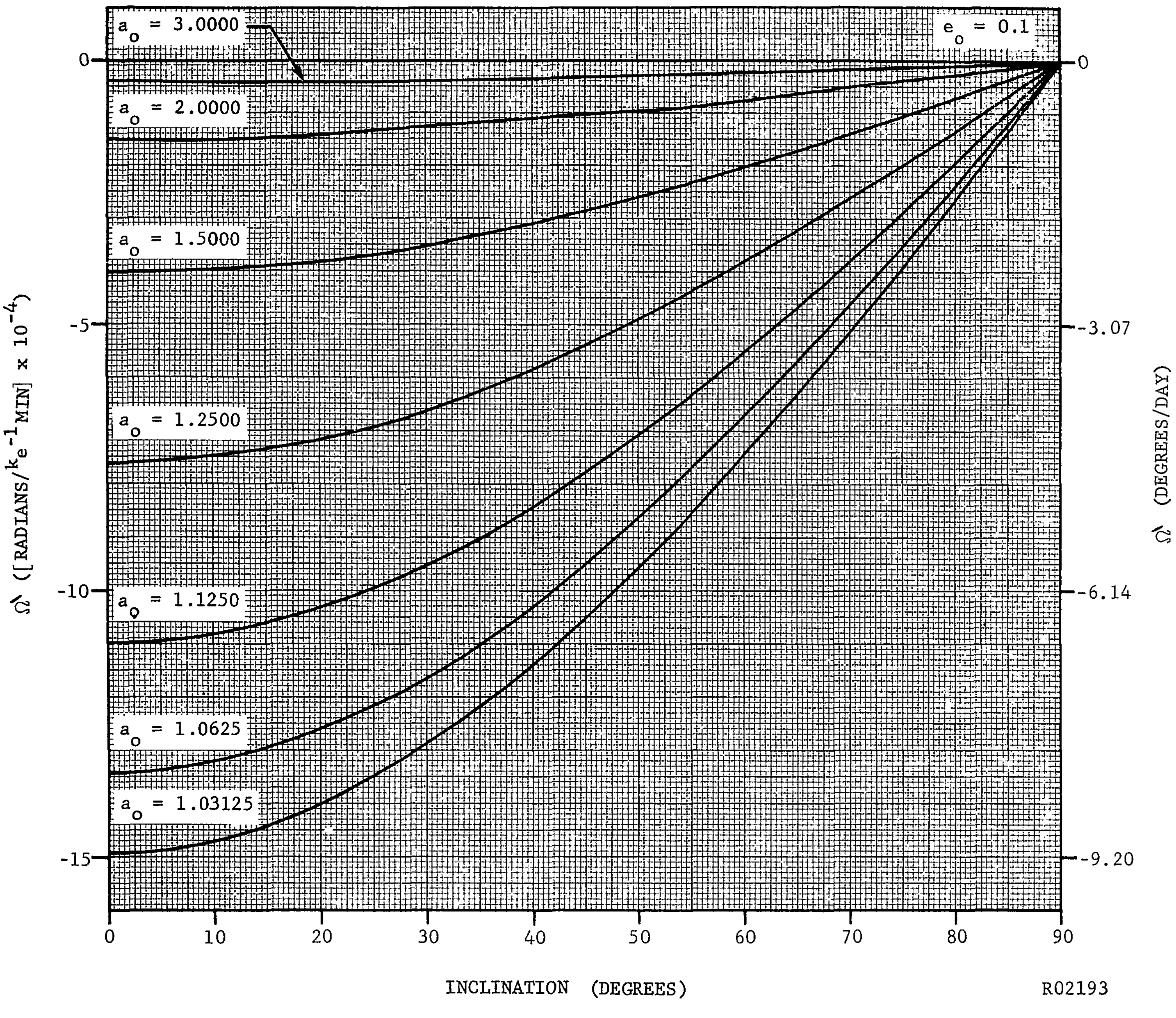


FIGURE B56 NODAL REGRESSION RATE VERSUS INCLINATION

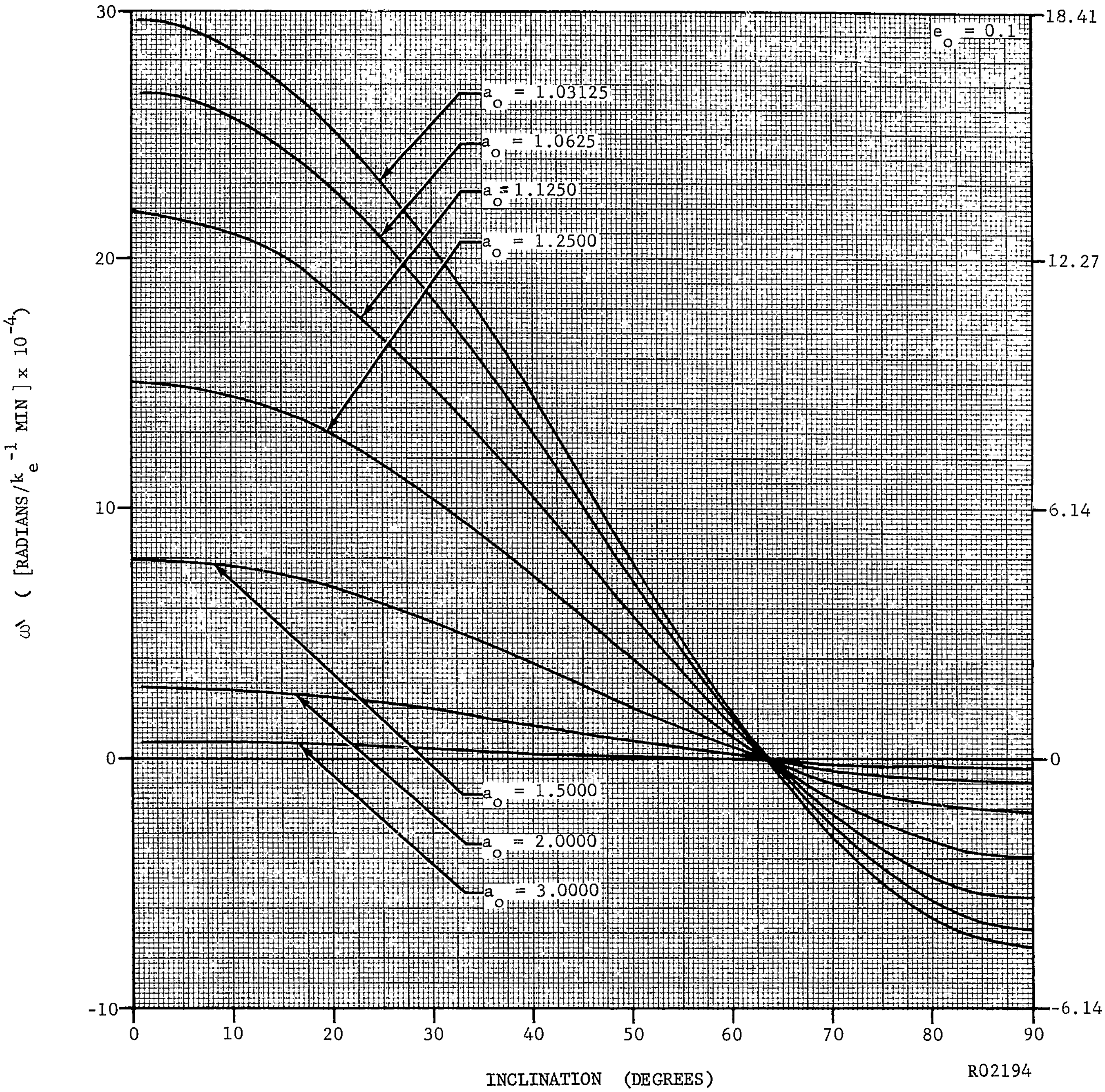


FIGURE B57 APSIDAL RATE VERSUS INCLINATION

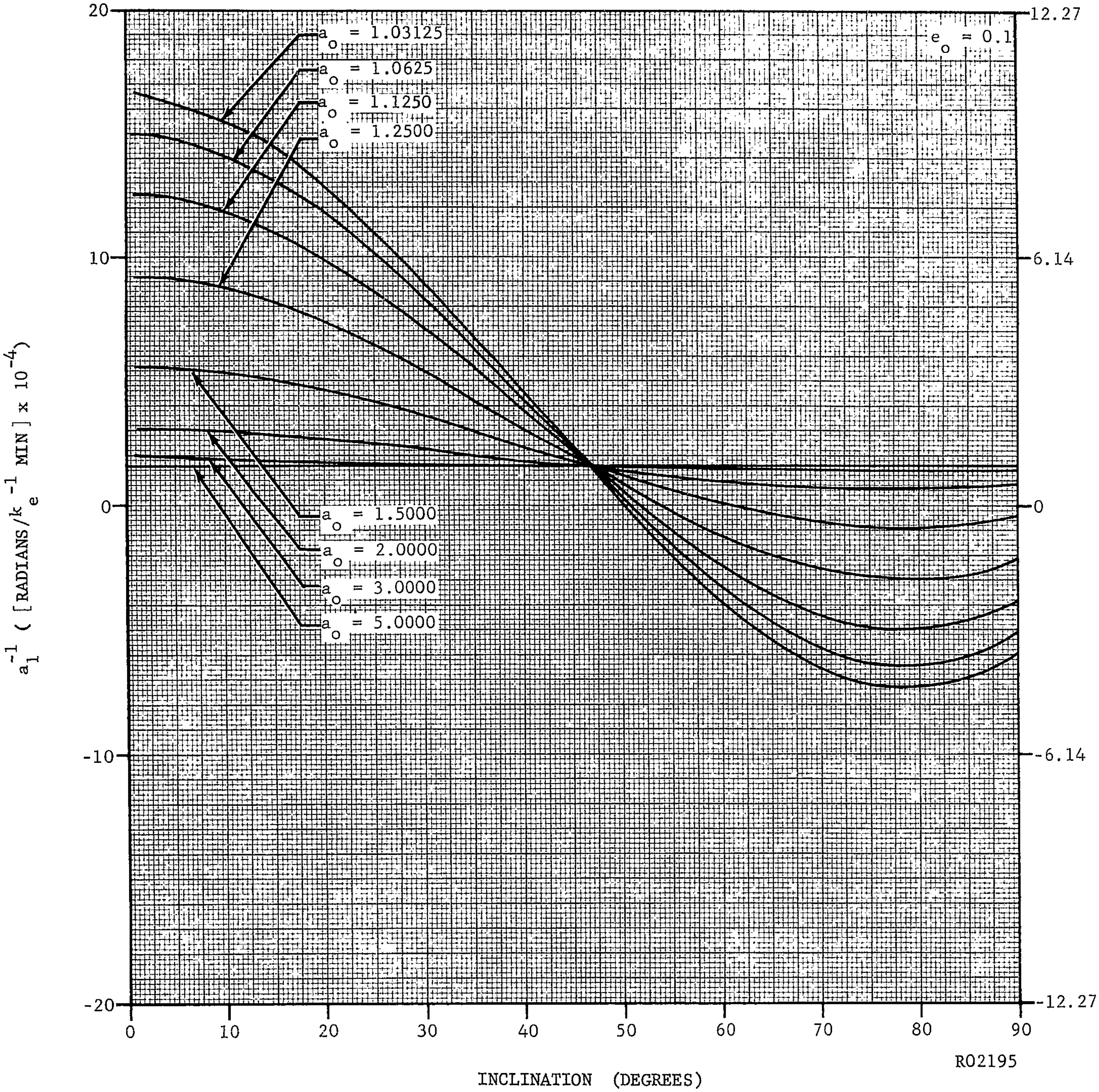


FIGURE B58 THE QUANTITY a_1^{-1} VERSUS INCLINATION

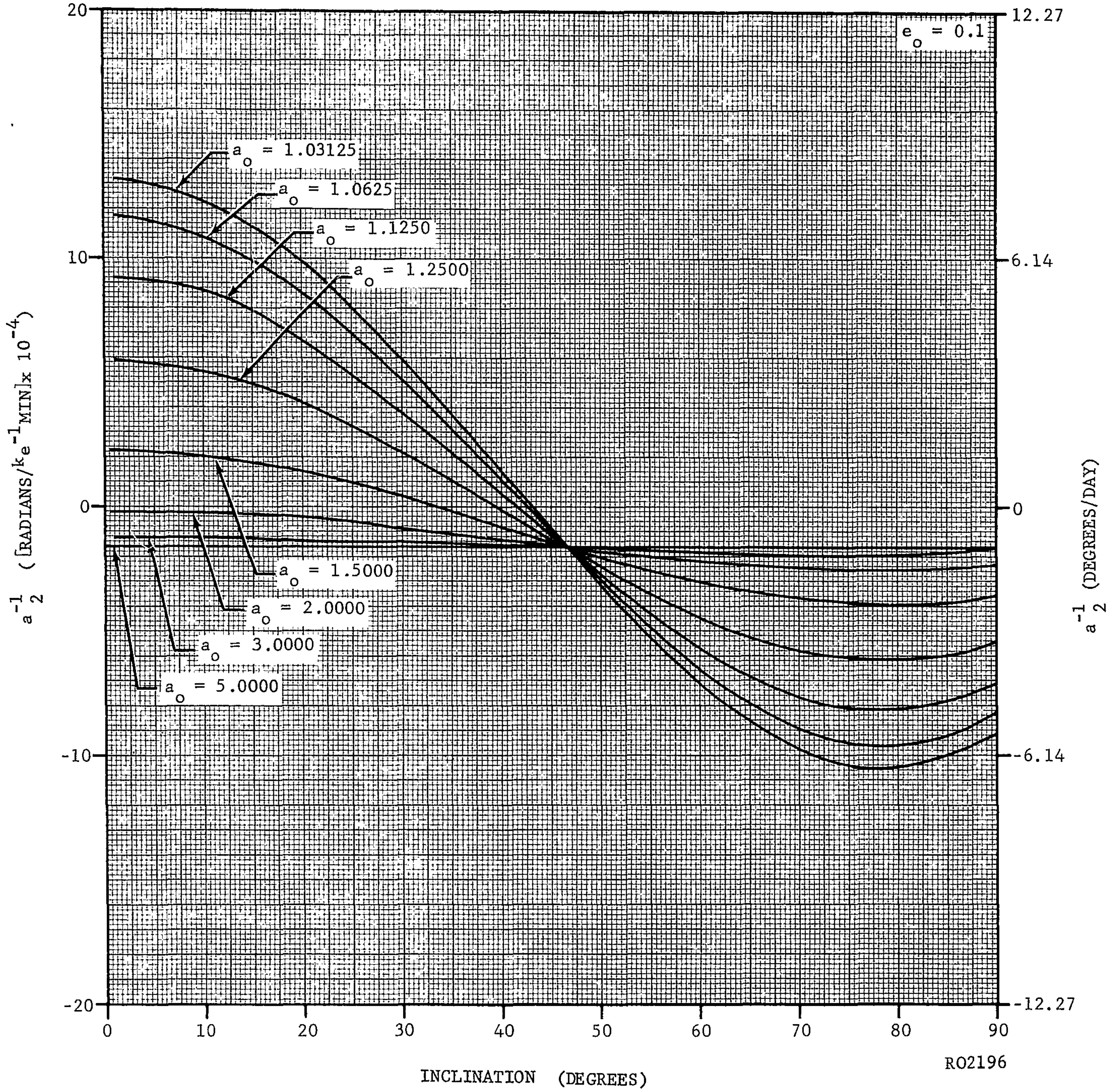


FIGURE B59 THE QUANTITY a_2^{-1} VERSUS INCLINATION

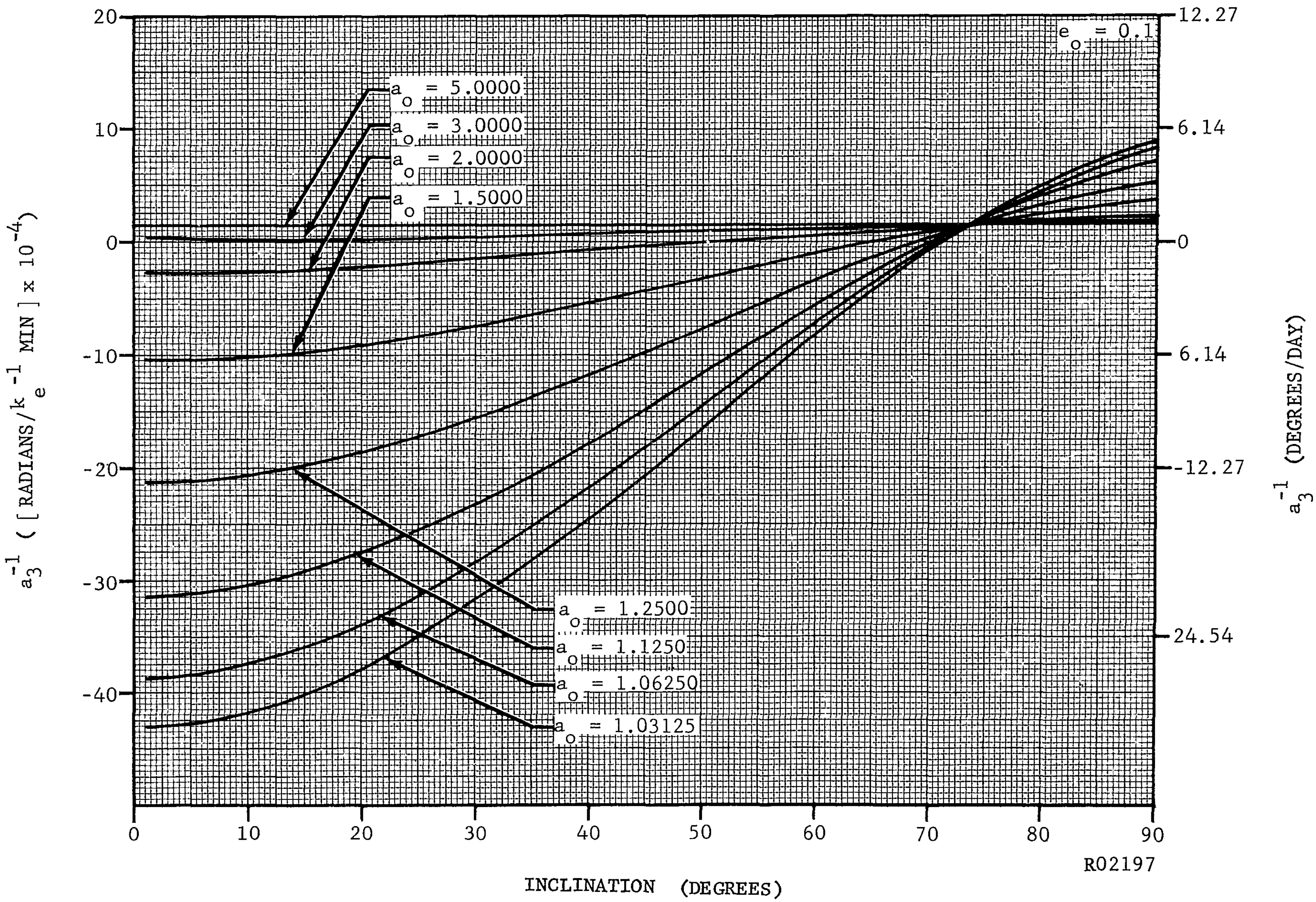


FIGURE B60 THE QUANTITY a_3^{-1} VERSUS INCLINATION

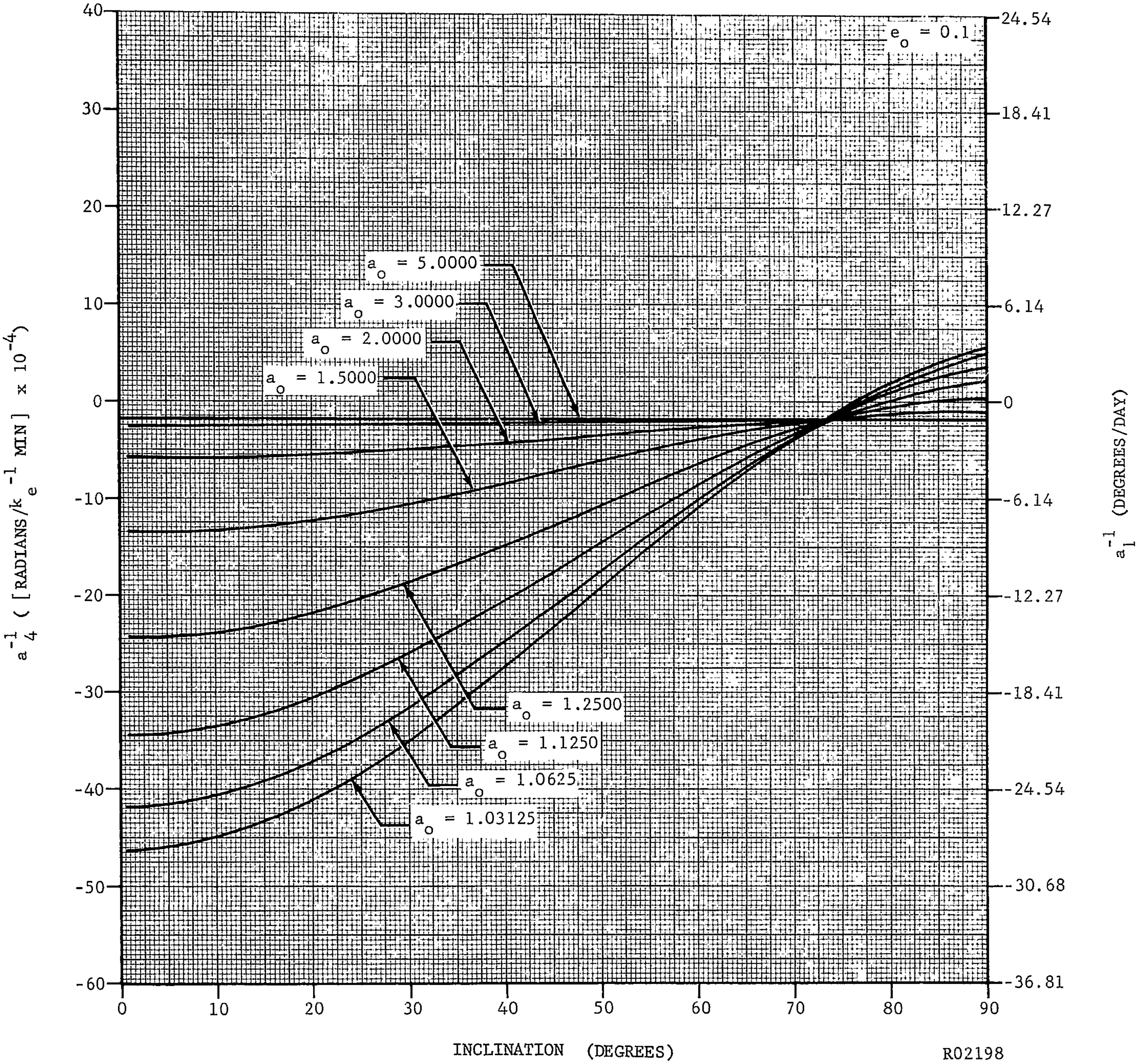


FIGURE B61 THE QUANTITY a_4^{-1} VERSUS INCLINATION

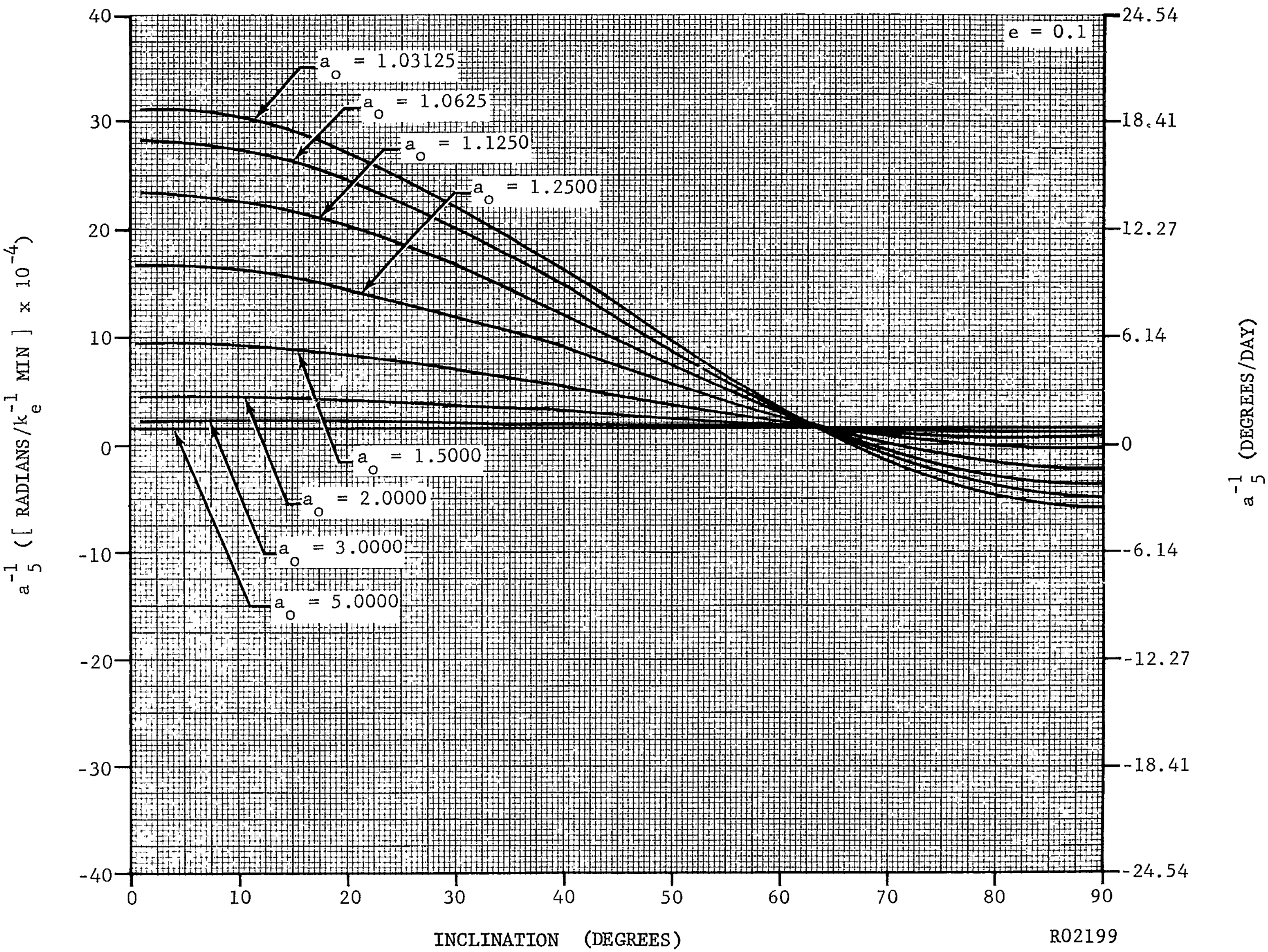


FIGURE B62 THE QUANTITY a_5^{-1} VERSUS INCLINATION

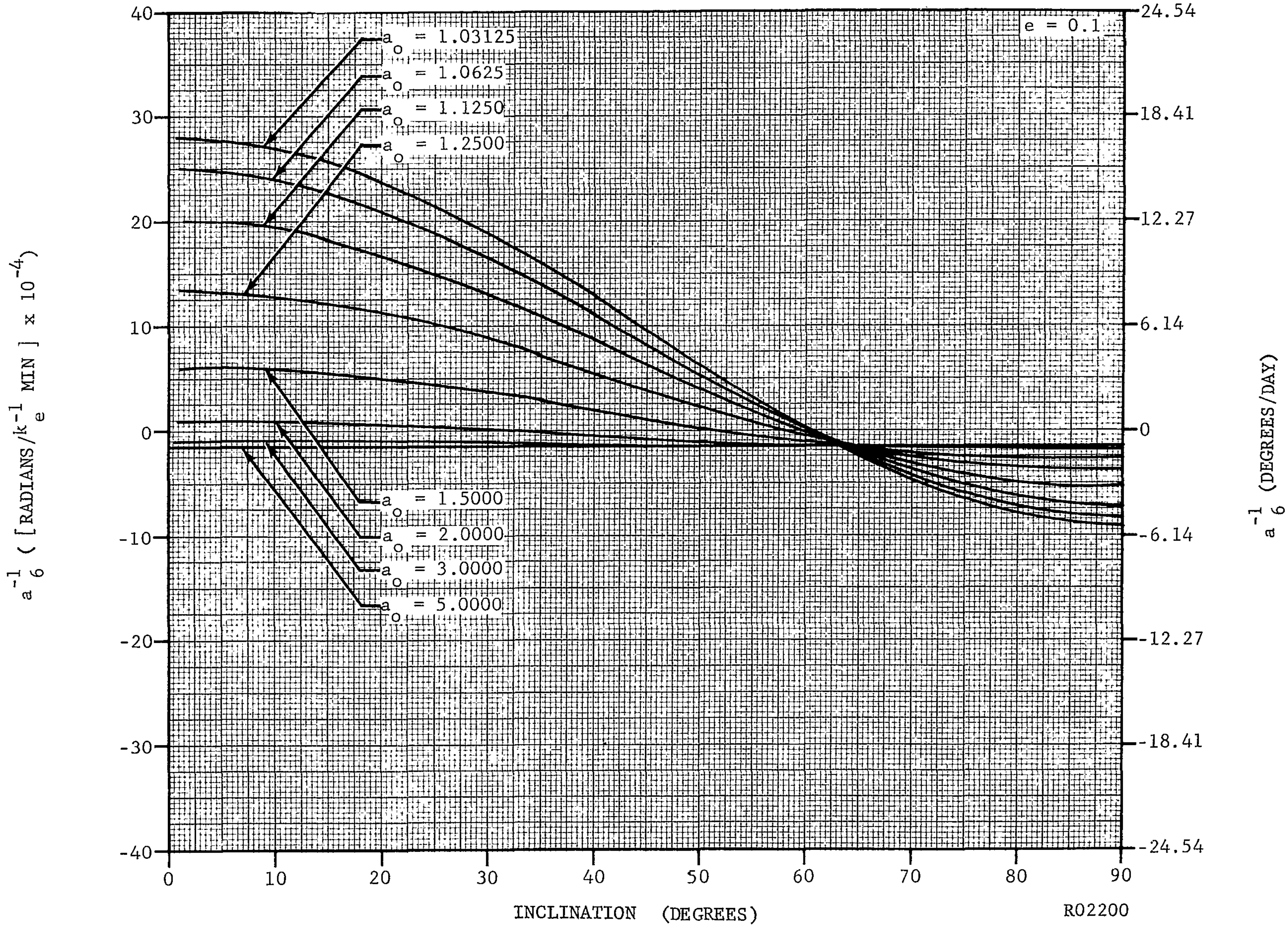


FIGURE B63 THE QUANTITY a_6^{-1} VERSUS INCLINATION

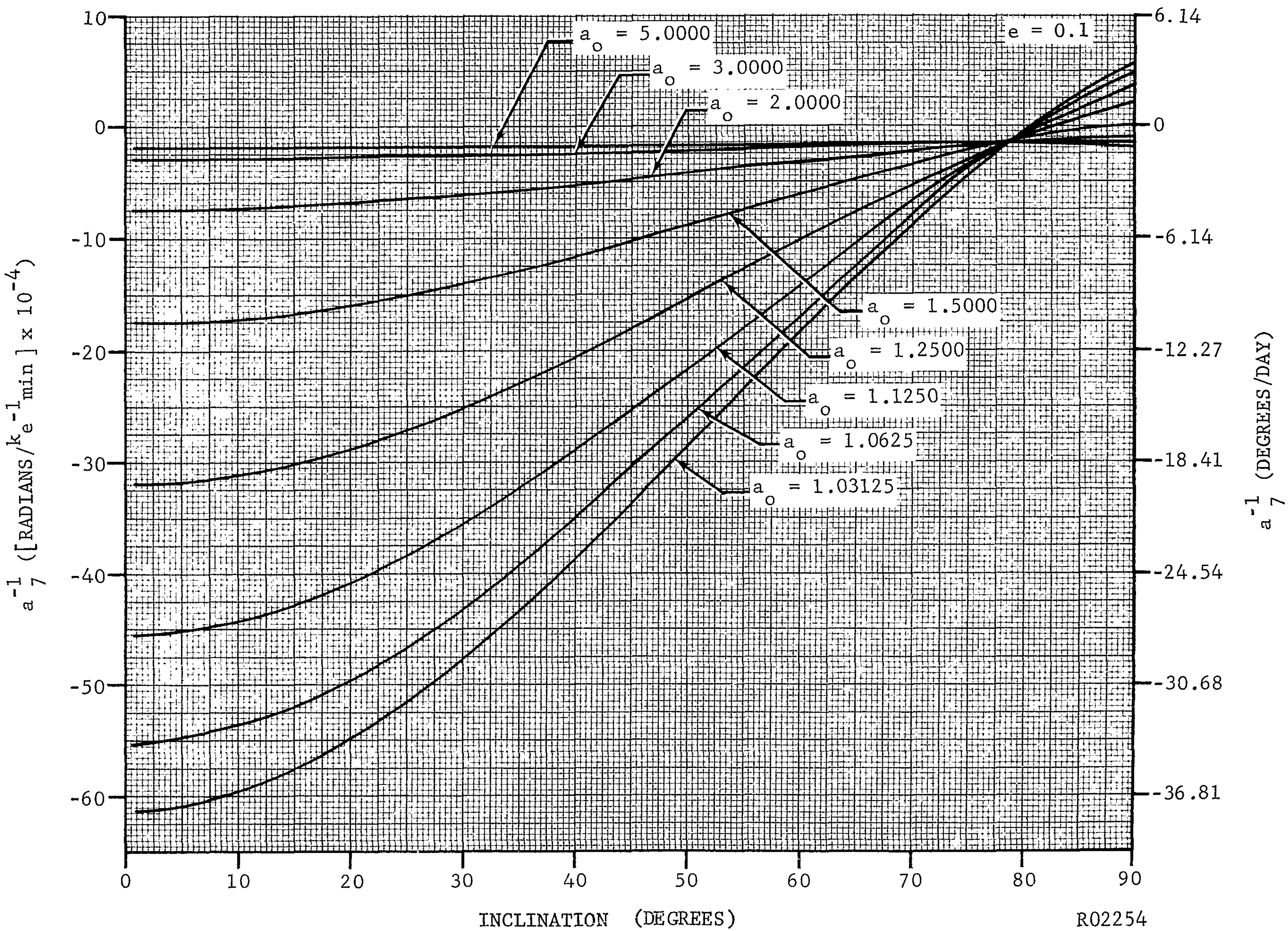


FIGURE B64 THE QUANTITY a_7^{-1} VERSUS INCLINATION

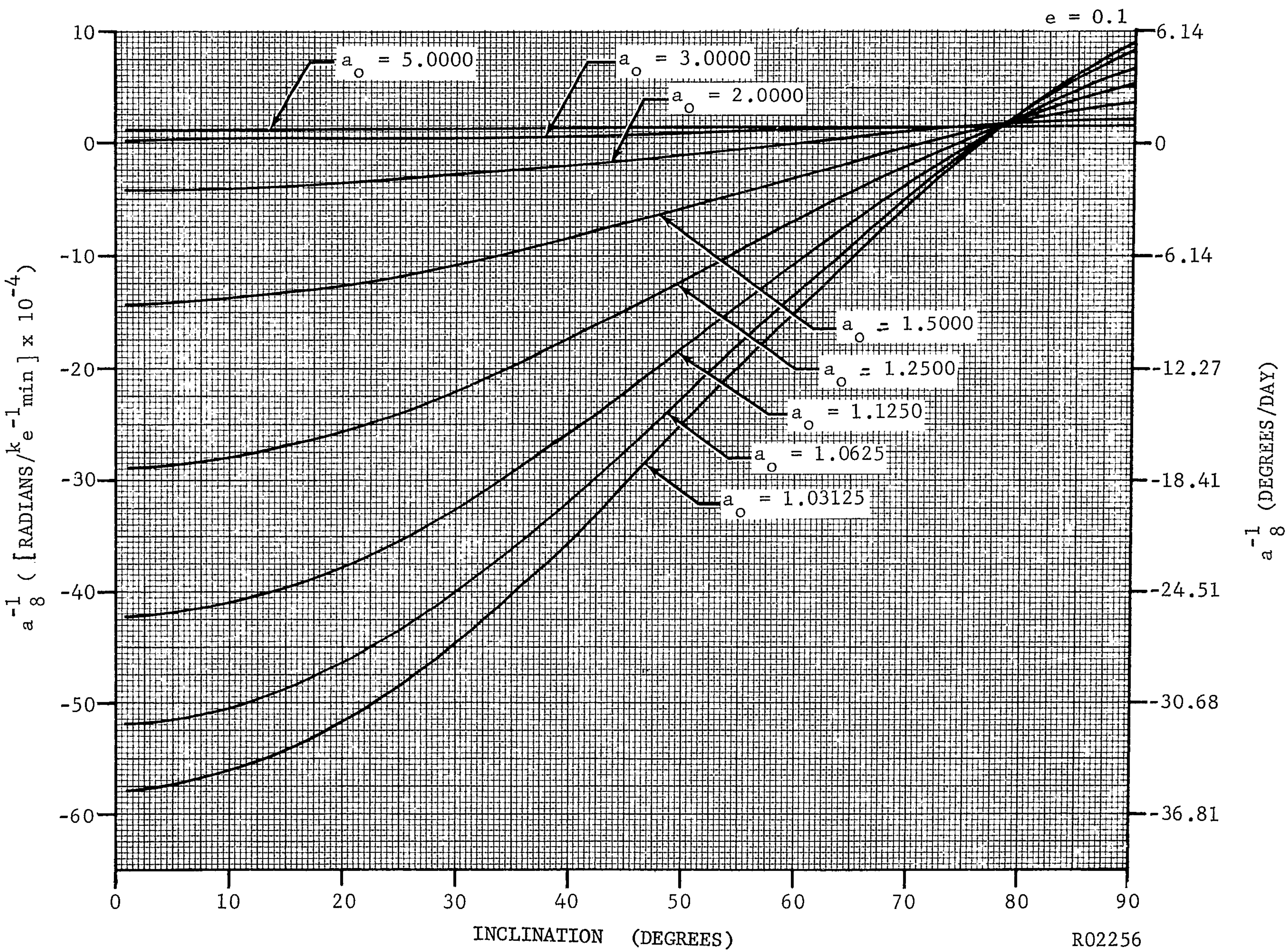


FIGURE B65 THE QUANTITY a_8^{-1} VERSUS INCLINATION

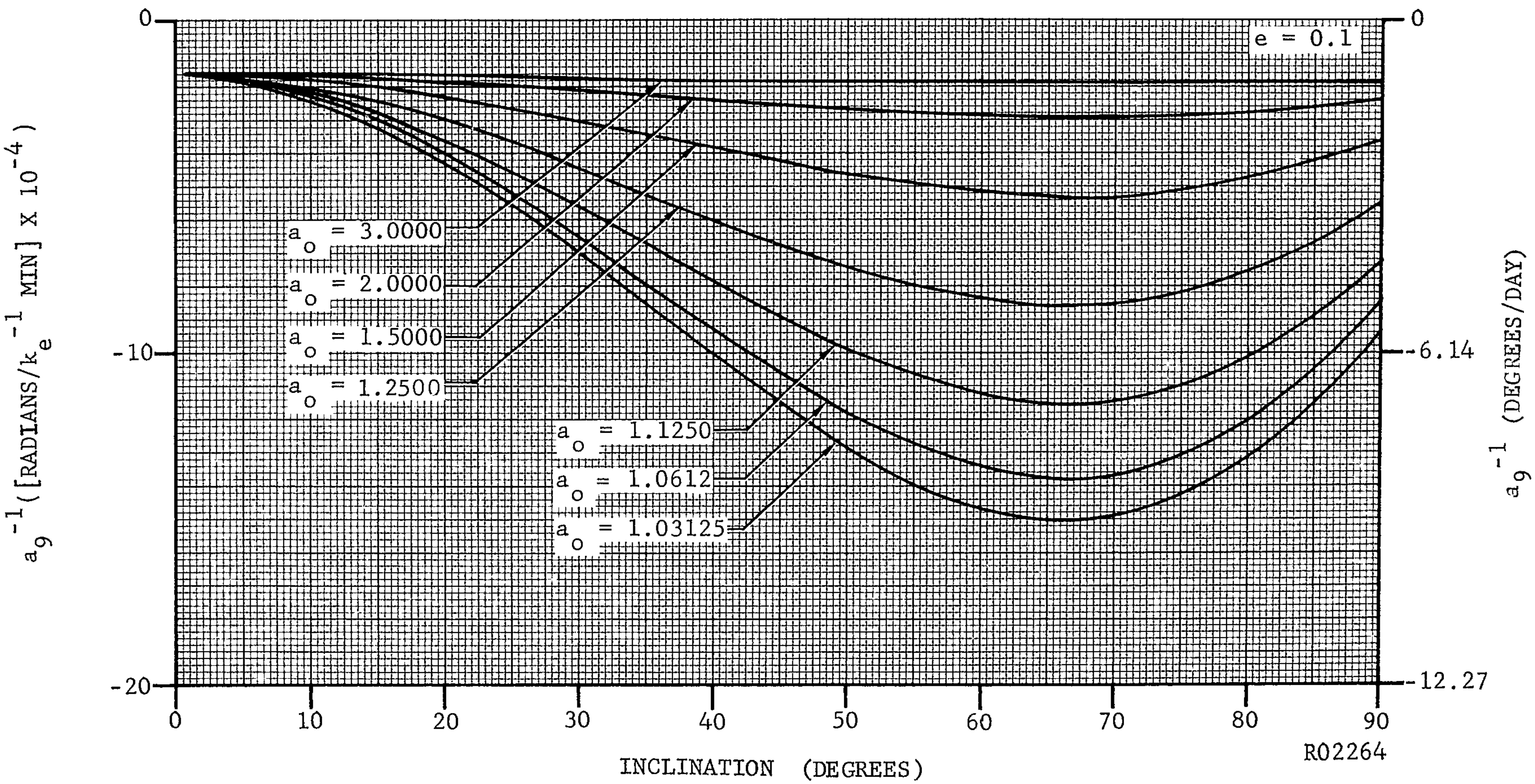


FIGURE B66 THE QUANTITY a_9^{-1} VERSUS INCLINATION

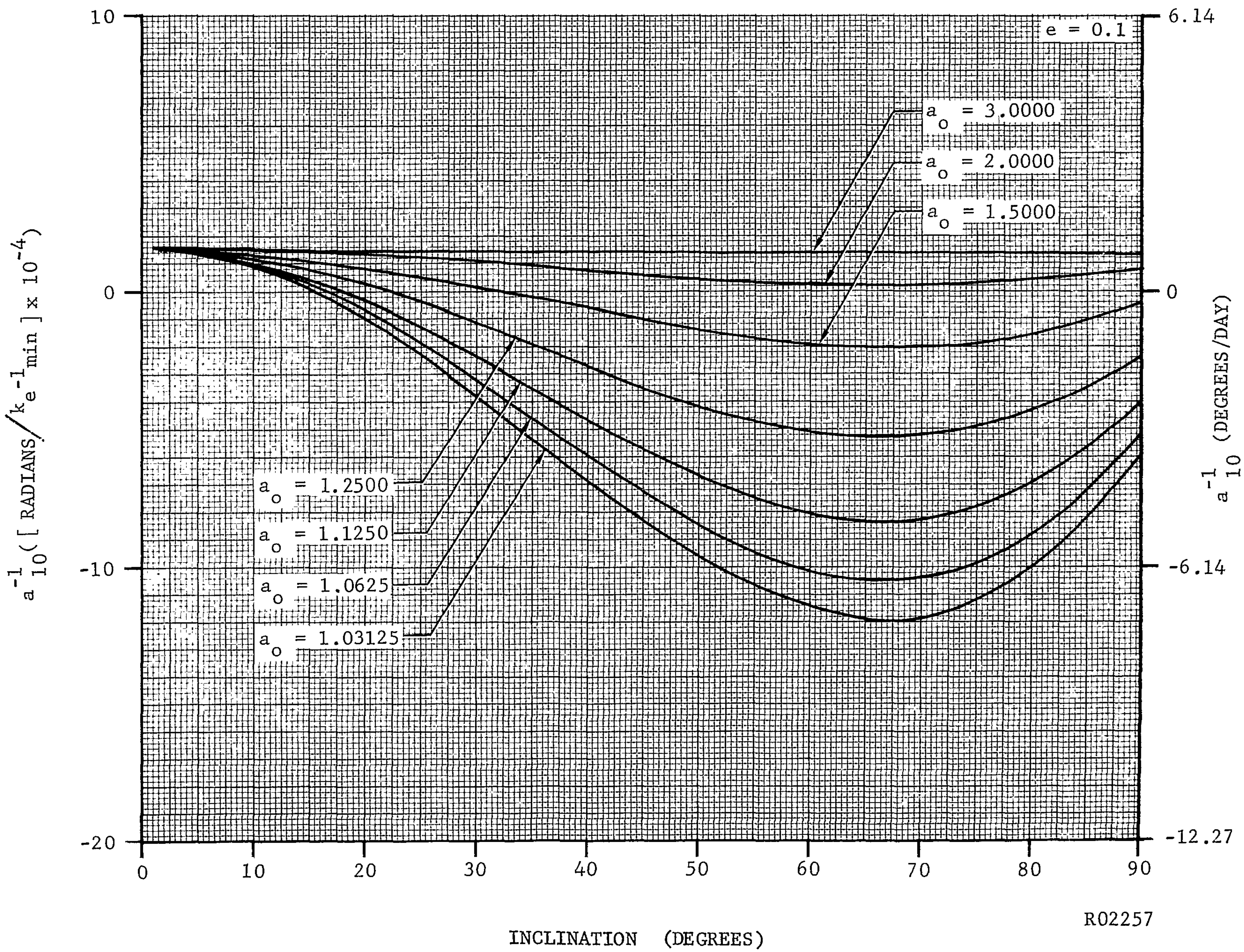


FIGURE B67 THE QUANTITY a_{10}^{-1} VERSUS INCLINATION

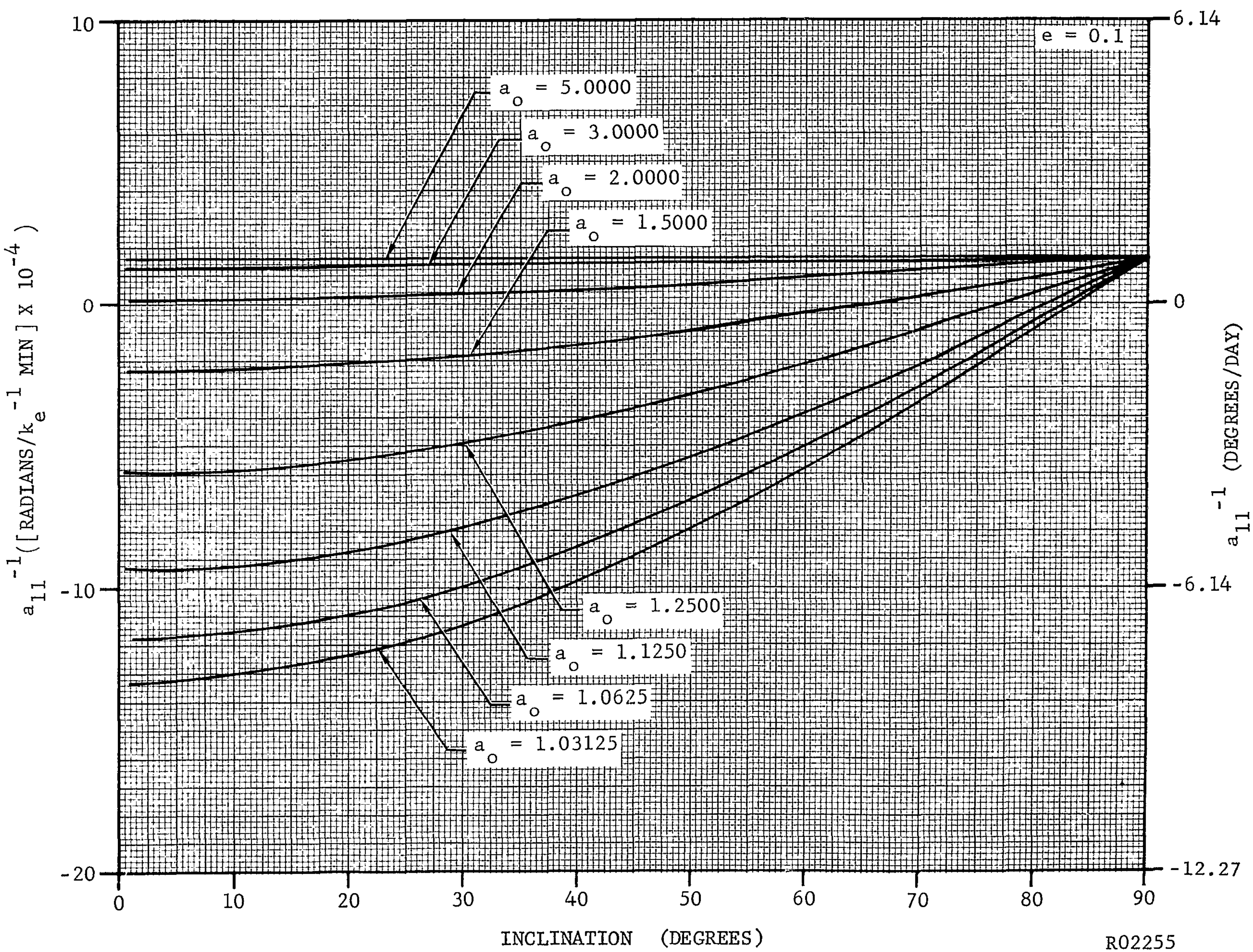


FIGURE B68 THE QUANTITY a_{11}^{-1} VERSUS INCLINATION

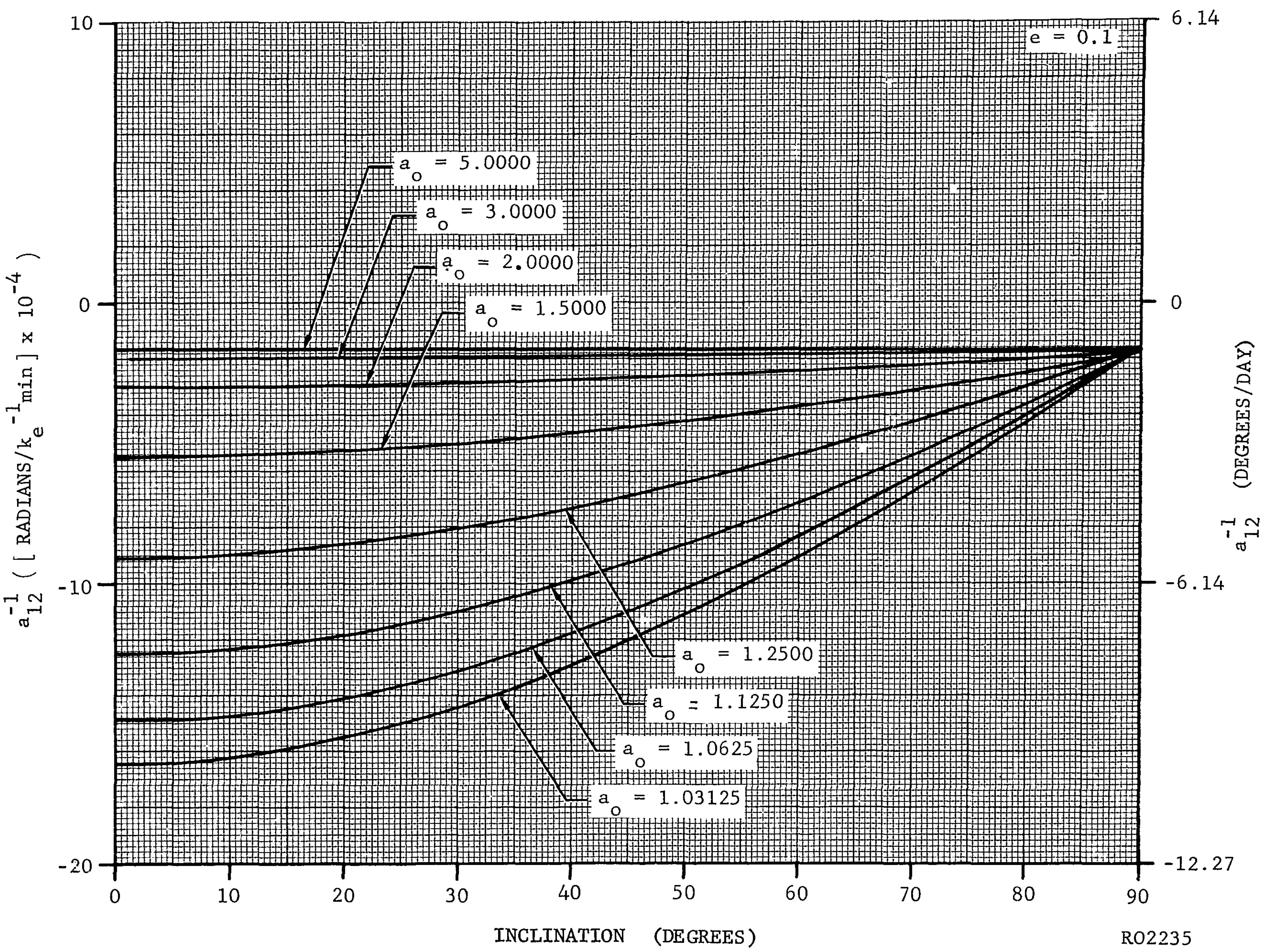
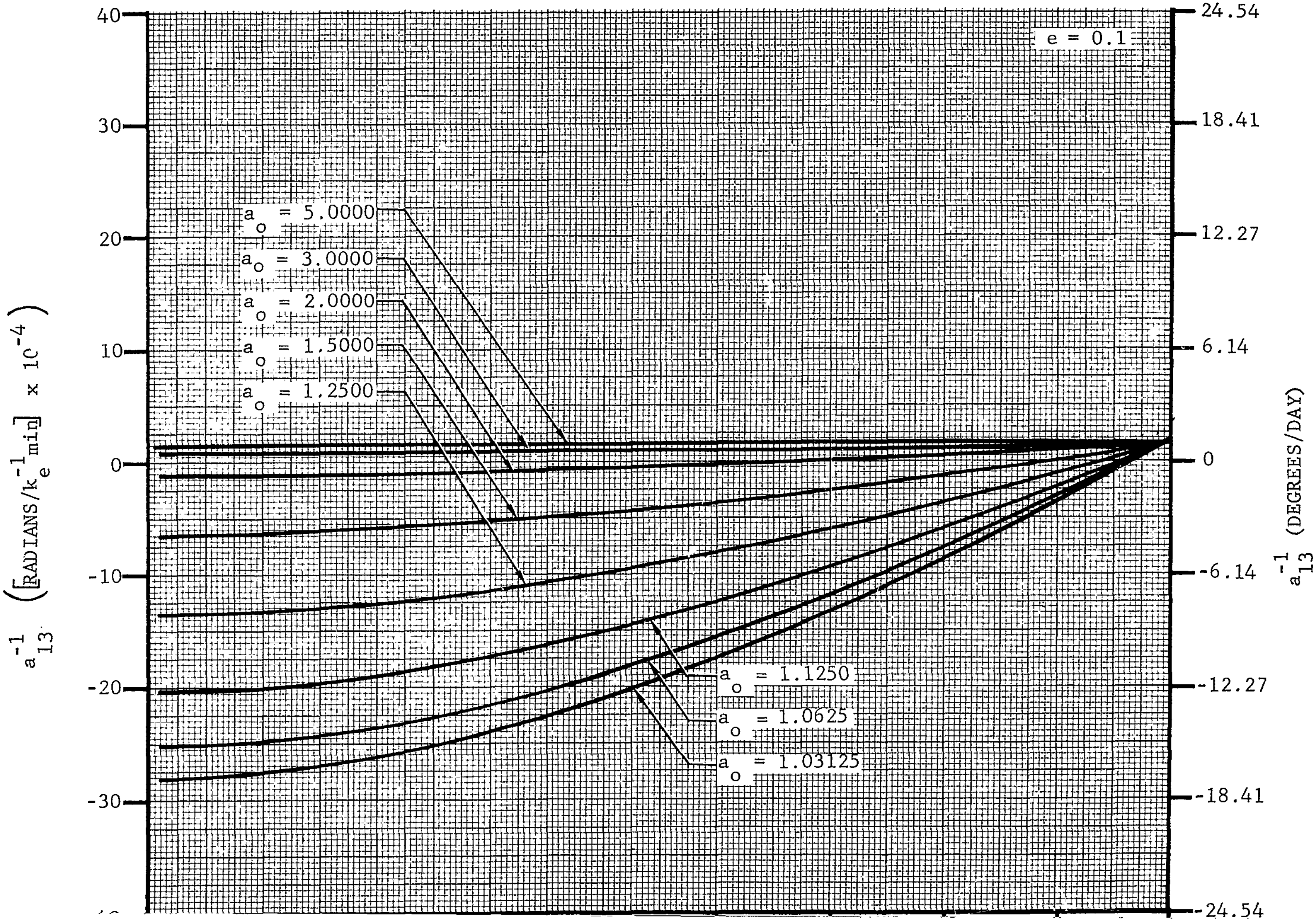


FIGURE B69 THE QUANTITY a_{12}^{-1} VERSUS INCLINATION


e = 0.1
a_o = 5.0000
a_o = 3.0000
a_o = 2.0000
a_o = 1.5000
a_o = 1.2500
a_o = 1.1250
a_o = 1.0625
a_o = 1.03125
40
30
20
10
0
-10
-20
-30
24.54
18.41
12.27
6.14
0
-6.14
-12.27
-18.41
-24.54
a_{13}^{-1} $\left([\text{RADIANS}/k_e^{-1}\text{min}] \times 10^{-4}\right)$
a_{13}^{-1} (DEGREES/DAY)

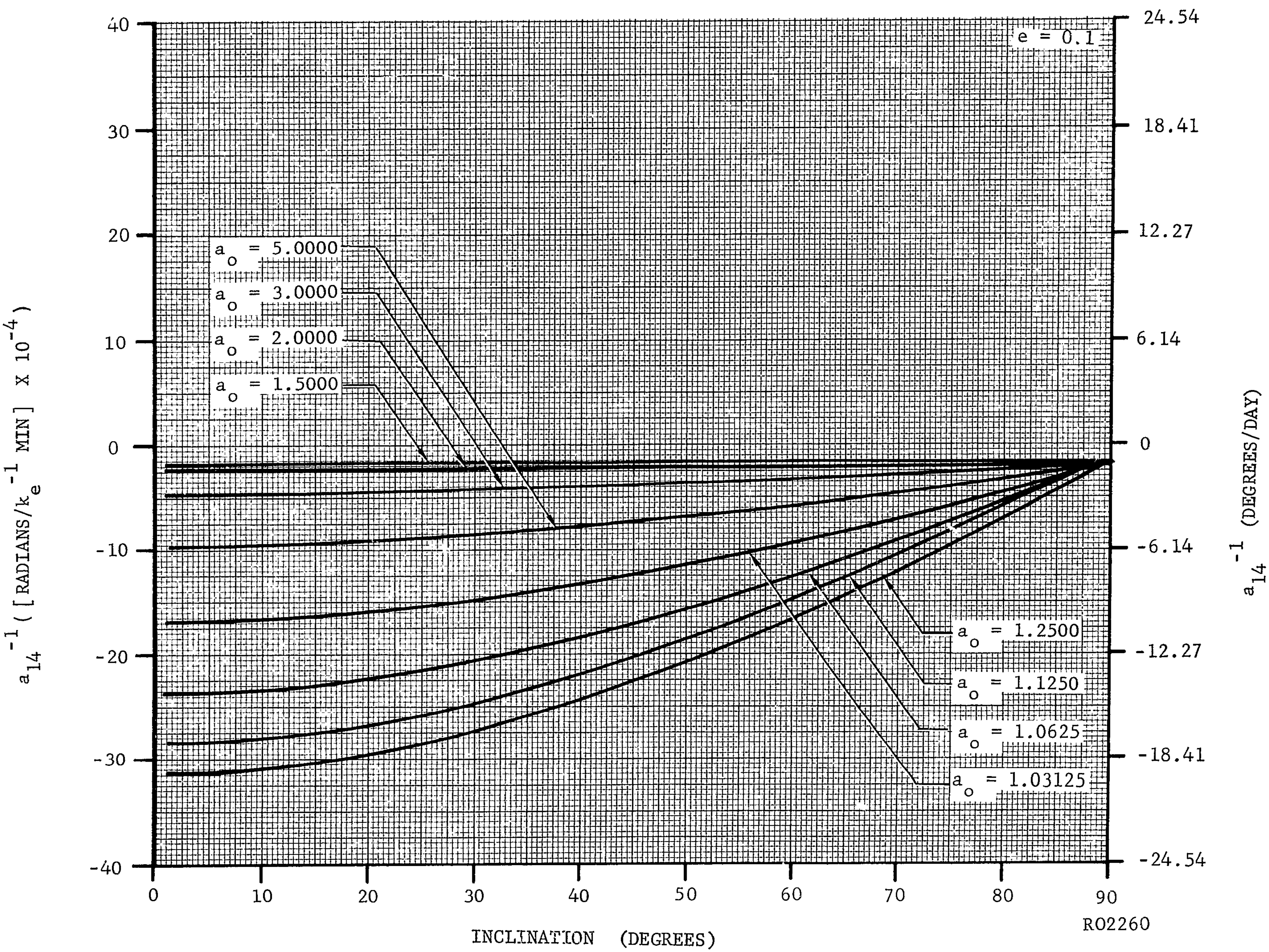


FIGURE B71 THE QUANTITY a_{14}^{-1} VERSUS INCLINATION

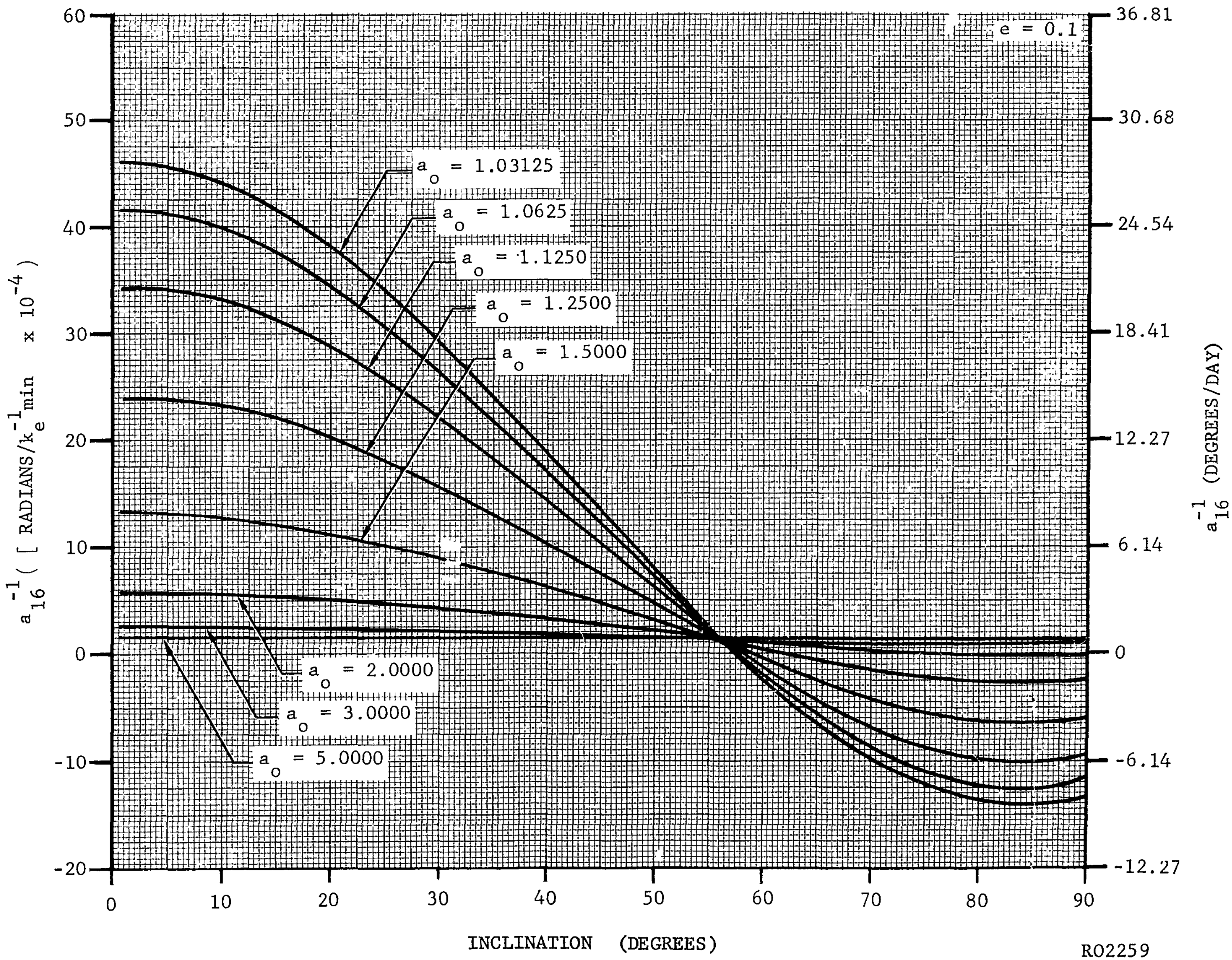


FIGURE B72 THE QUANTITY $\dot{a}_{16}^{-1}$ VERSUS INCLINATION

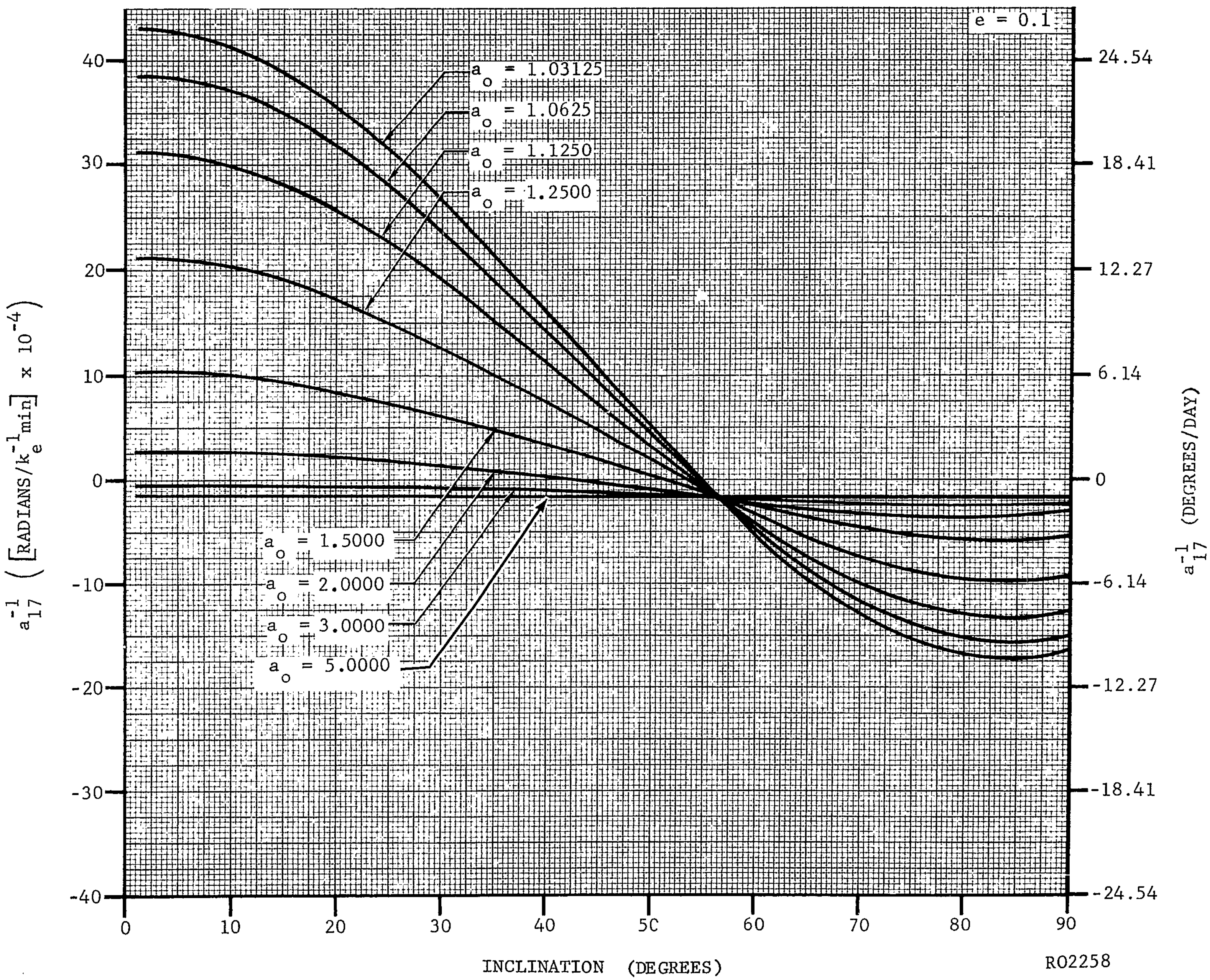


FIGURE B73 THE QUANTITY a_{17}^{-1} VERSUS INCLINATION

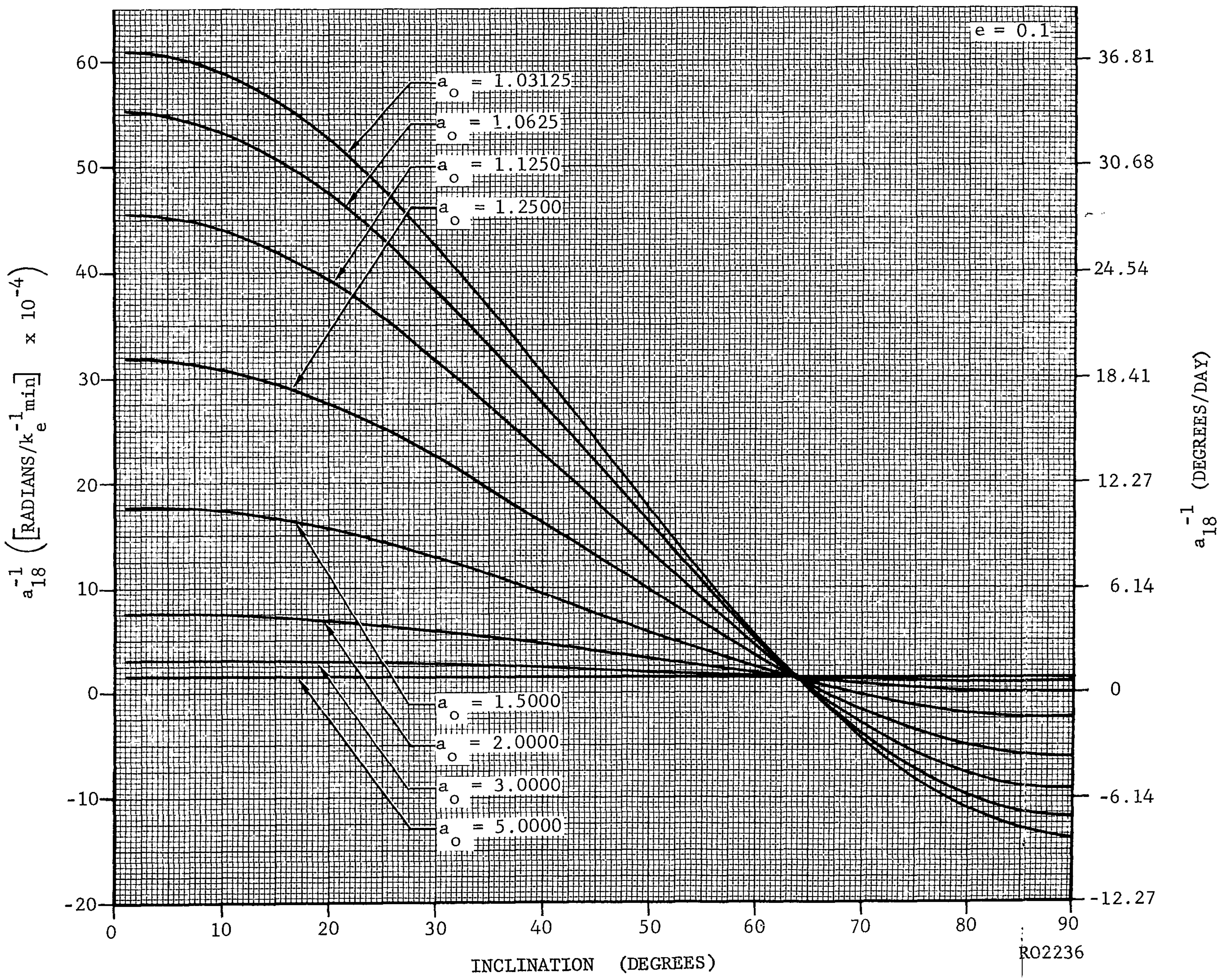


FIGURE B74 THE QUANTITY a_{18}^{-1} VERSUS INCLINATION

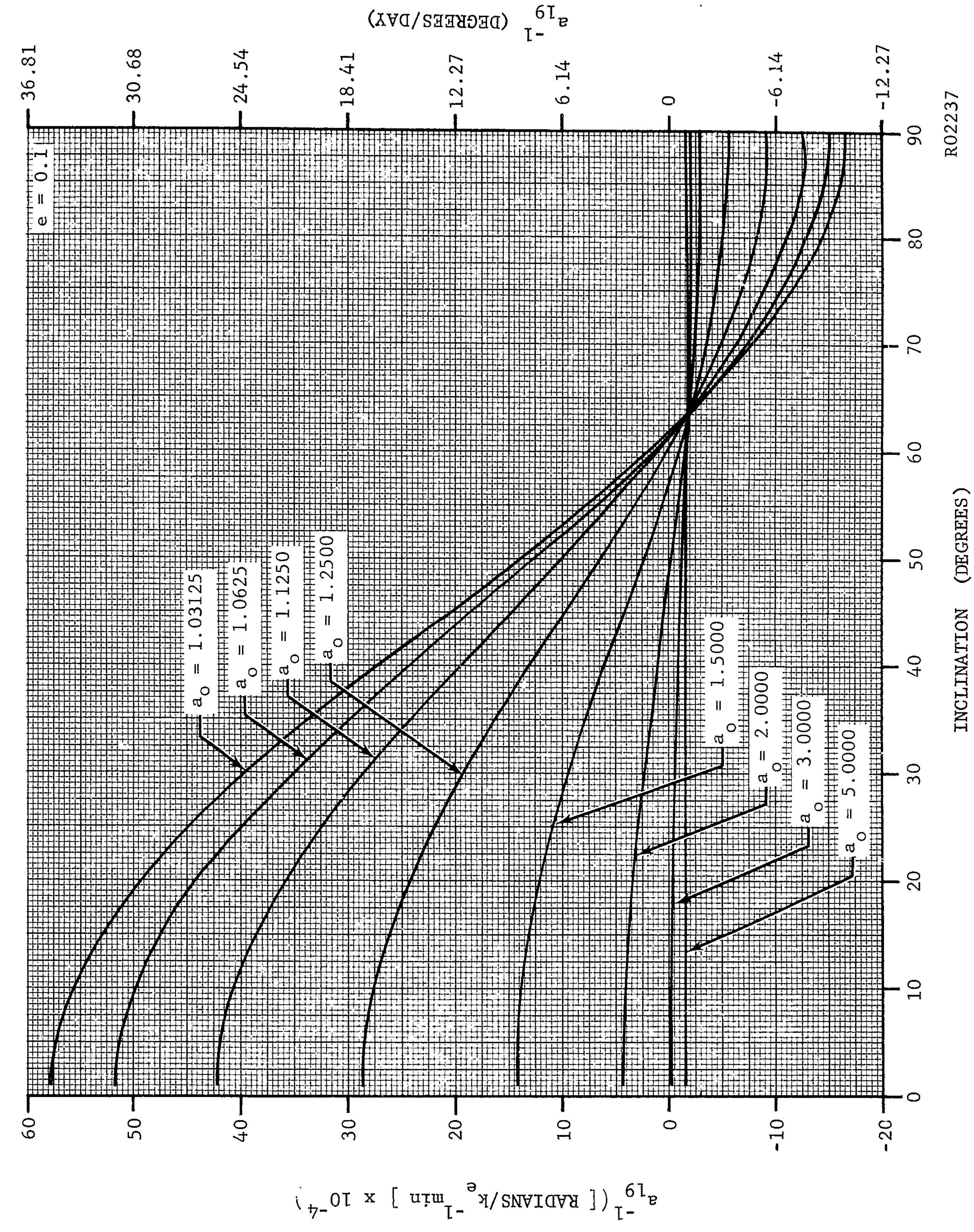


FIGURE B75 THE QUANTITY a_{19}^{-1} VERSUS INCLINATION

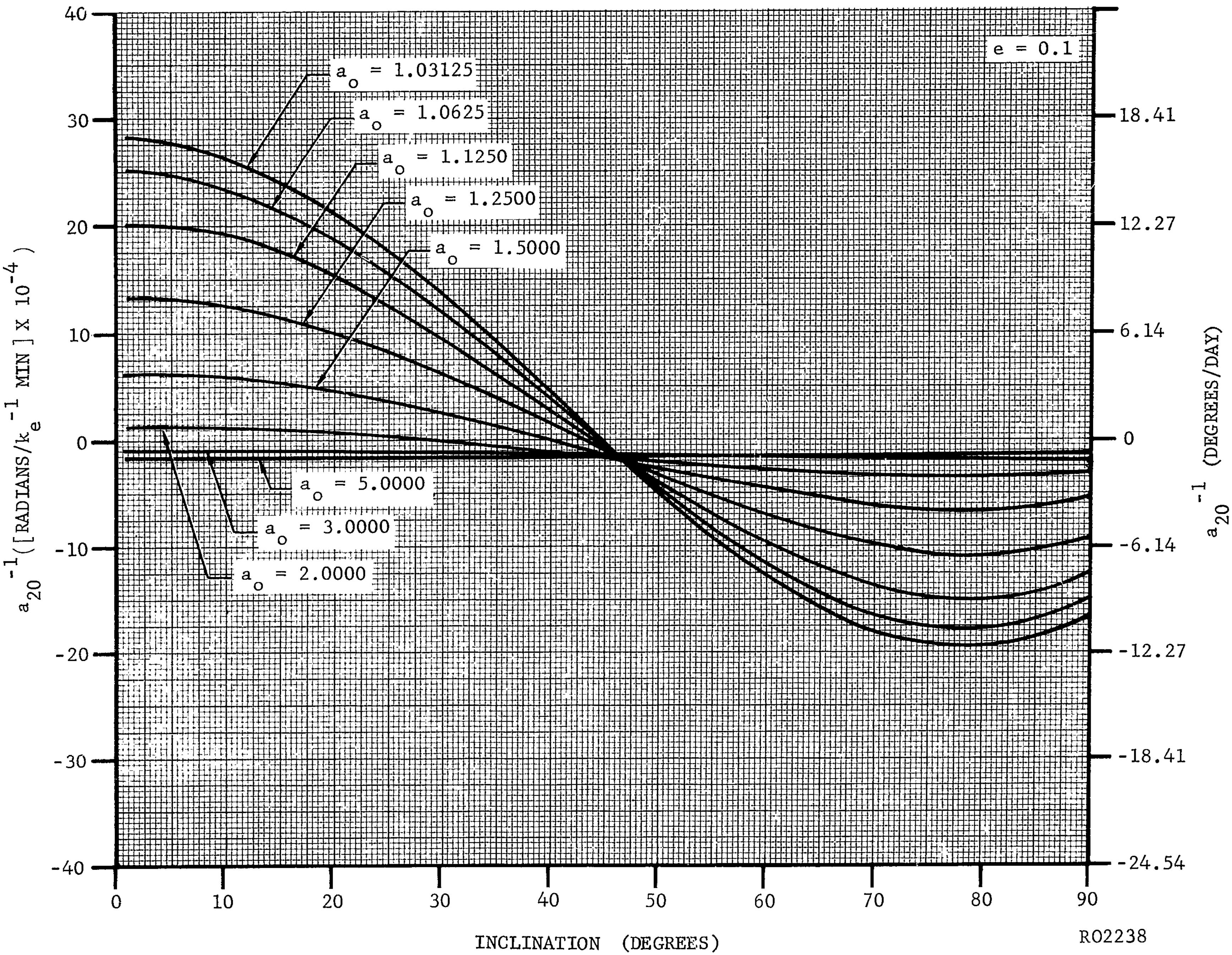


FIGURE B76 THE QUANTITY a_{20}^{-1} VERSUS INCLINATION

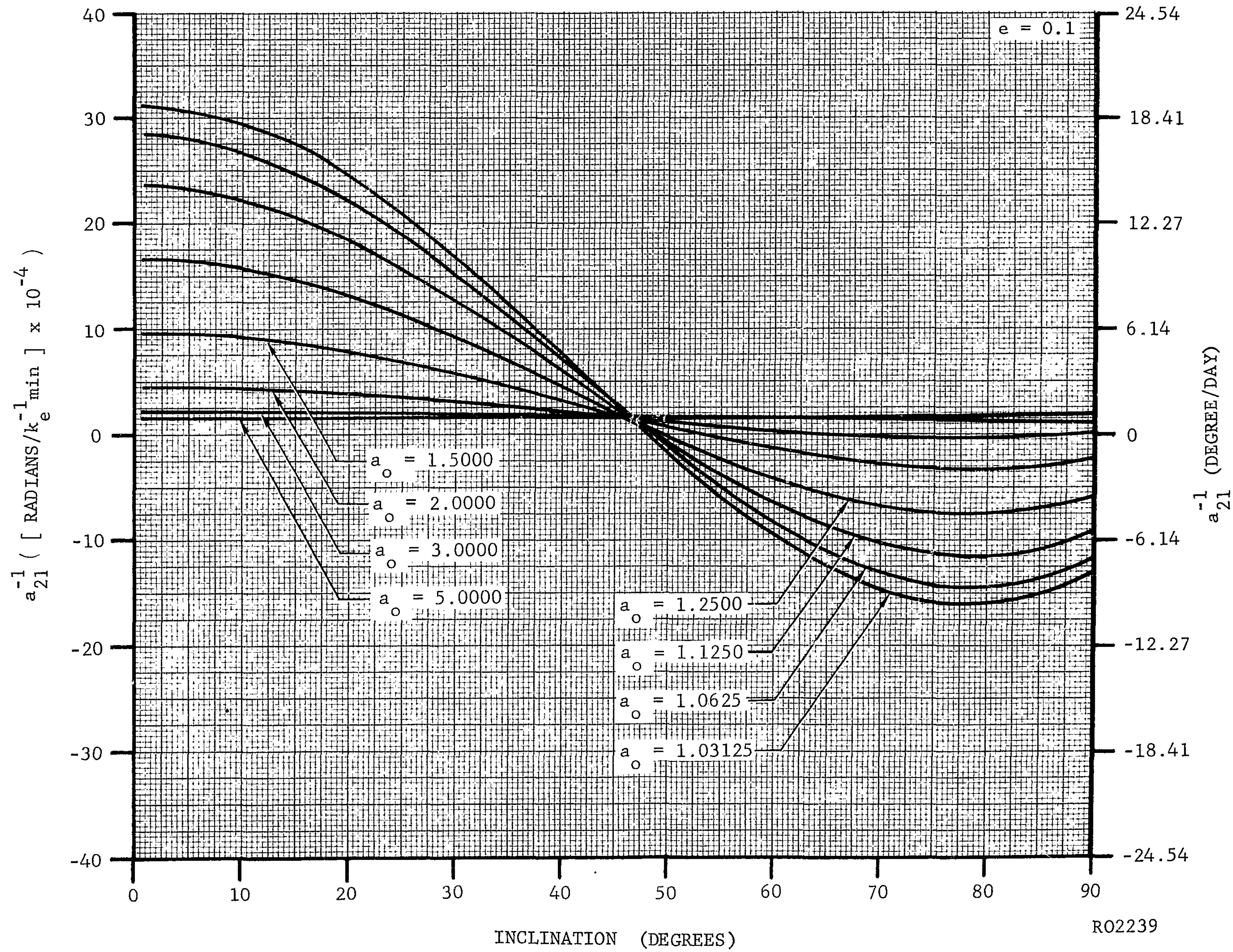


FIGURE B77 THE QUANTITY a_{21}^{-1} VERSUS INCLINATION

Air Force Systems Command, Electronic Systems Division, 496L System Program Office, L. G. Hanscom Field, Bedford, Mass.
Rpt. No. ESD-TDR-63-637 ,
ANALYSIS OF ARTIFICIAL EARTH SATELLITE ORBIT COMPUTATION (U). Tech. rpt., November 1963 133 p. incl. diagrams, and tables.

Unclassified Report

Results obtained from the analysis conducted with the Aeronutronic General Perturbations Differential

1. Satellite (artificial)
2. Orbital Trajectories
3. Celestial Mechanics
4. Computer Programming

I. 496L SPO
II. Contract AF19(628)-562
III. Aeronutronic, a Div. of Ford Motor Company, Newport Beach, California

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Air Force Systems Command, Electronic Systems Division, 496L System Program Office, L. G. Hanscom Field, Bedford Mass.
Rpt. No. ESD-TDR-63- 637,
ANALYSIS OF ARTIFICIAL EARTH SATELLITE ORBIT COMPUTATION (U). Tech. rpt., November 1963 133 p. incl. diagrams and tables.

Unclassified Report

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II. Contract AF19(628)-562
III. Aeronutronic a Div. of Ford Motor Company, Newport Beach, California

Correction Program (AGPDC) described in ESD-TDR-63-632 (Aeronutronic Publication U-2201) are presented. The effects of the individual zonal harmonic bulge terms on prediction accuracy are itemized. Improvements in the solar radiation pressure perturbation formulation used in the AGPDC are discussed. AGPDC is compared with the Brouwer General Perturbations Differential Correction Program (BGPDC) in terms of the relative accuracy and treatment of low eccentricity orbits.

IV. J. R. Kuhlman
V. Technical Report No. U-2333
VI. In DDC Collection

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