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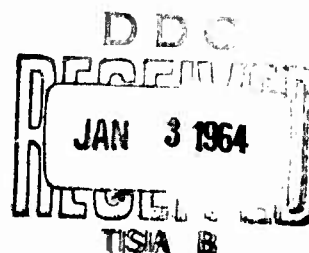
VISCOUS EFFECTS ON BALANCED JETS
IN GROUND PROXIMITY

Task 1D021701A04804
(Formerly Task 9R99-01-005-04)
Contract DA 44-177-TC-845

October 1963

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HEADQUARTERS
U S ARMY TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA

This is an interim report on a theoretical investigation of the fluid flow of jets in ground proximity. Previous investigators have obtained analytical models of the flow of annular jets which serve to predict accurately the average pressure rise between the jets. This is sufficient for performance purposes; however, to obtain an improved understanding of the flow and to correlate the theory with observed experimental phenomena, this investigation was undertaken.

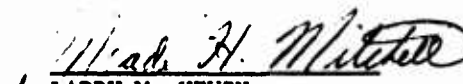
This report presents the results of the investigation to date, which indicate that, by use of simple approximations for the effects of jet mixing and vorticity in real fluids, excellent correlation of experiment and theory can be obtained. The investigation is continuing and will attempt to extend the theory to the unsymmetric cases of rotation and translation.


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Task 1D021701A04804
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VISCOUS EFFECTS ON BALANCED JETS
IN GROUND PROXIMITY

Technical Report 241-1

One of a Series of Reports Pertaining to the Theoretical
Investigation of Air Jet Flow Fields in Ground Proximity

Prepared by
HYDRONAUTICS, Incorporated
Laurel, Maryland

for
U. S. ARMY TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA.

PREFACE

This report has been prepared by Mr. C. C. Hsu, who carried out the work described herein. Advisory supervision has been provided by Mr. M. P. Tulin.

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SYMBOLS

A	augmentation factor
A_b	base thrust augmentation factor
$A_{inv.}$	augmentation factor based on inviscid theory
a	ratio of mean jet velocity $\bar{u}$ and outward velocity $\bar{u}_d$
b	ratio of mean velocities of inward and outward mass flow
C	perimeter of the nozzle base
D	diameter of the nozzle base
f	entrainment function
g	a function defined by Equation [12]
H	nozzle base height above the ground
J	jet momentum per unit length of nozzle slot
m_c	total mass flow
m_{ec}	entrained mass flow
m_d	outward mass flow
m_u	inward mass flow
p_{vortex}	pressure loss due to vortex flow
Δp_b	average base pressure distribution
R	radius of curvature
S	nozzle base area
s_c	distance along the jet path from the nozzle exit

t nozzle thickness
 U_e jet exit velocity
 u tangential component velocity.
 $\bar{u}$ the average velocity of the jet before impingement
 $\bar{u}_u, \bar{u}_d$ the average velocities of the inward and outward mass flow respectively
 $\bar{u}_{ave}$ the average induced velocity inside the cavity
 v normal component velocity
 W half width of the nozzle base
 x, y axes of the Rectangular Coordinate System
 γ a constant, depending on the local shearing stress
 θ momentum thickness
 κ strength of vorticity
 ρ density of the air
 ϕ_o angle of divergence of the jet
 ϕ_c angle of impingement of the jet
 ψ Lagrange's Stream Function

SUMMARY

A theoretical investigation to determine the viscous effects on balanced jets in ground proximity has been carried out. Pertinent available literature concerning inviscid jet impingement, vortex generation and viscous mixing is, briefly, reviewed. The mean flow pattern of an annular jet in ground proximity is seen to be that of diffusing jets which are deflected laterally by their interaction with the central pressure zone. With simple and plausible approximations, the effects of a standing vortex and jet mixing can be theoretically determined, resulting in good agreement between predicted and measured augmentation factors. It is found that in ground proximity the effect of jet mixing is always adverse and dominant. Curves which allow the rapid estimation of augmentation factors are presented.

CONCLUSIONS

The assumptions and approximations involved in the analysis presented are quite broad. However, the analytic representation of the major effects and the important trends of the results are believed to be both qualitatively and quantitatively correct. This is borne out by the good general agreement between predicted and measured augmentation factors. Several tentative conclusions are suggested by the results for air jet flow fields in ground proximity:

- 1) The mathematical representation of the jet flow field, assuming complete potential flow, is inadequate.
- 2) The effect of jet mixing is always adverse and is dominant with respect to the effect of the standing vortex.
- 3) The viscous effect is significantly less for an inwardly inclined annular jet than for a vertically oriented jet.
- 4) For a fixed ratio of operating height to machine width, the effect of jet mixing generally increases with increasing nozzle aspect ratio, $2W/t$ or D/t .

INTRODUCTION

Since 1956, considerable theoretical and experimental efforts have been applied to the understanding of the basic principles involved in what has become known as the GEM, i.e., a machine supported on a cushion of compressed air sustained and contained by a peripheral jet. Most available theories are essentially based on inviscid, incompressible jet theory. Chaplin (1) presents a very simple approximation based on the assumption that the jet is very thin, and of circular arc cross section. Some refinements have been added by Pinnes (2), Chaplin and Stephenson (3), Stanton Jones (4), and Lin (5) to consider the effect of finite thickness of the jet. Strand (6) and Ehrich (7), who used a free streamline model, develop an exact perfect incompressible fluid theory for the two-dimensional model. However, the details of the jet and of base pressure distribution are, of course, very much influenced by turbulent mixing and vortex generation. These effects have only to a limited extent been studied analytically or through experiments. Since the jet delivered by the nozzle, except at very low Reynolds numbers, is turbulent in nature, it is felt, that a correct and adequate representation of turbulent mixing and vortex effects, rather than precise mathematical formulation of the jet flow field assuming completely potential flow, is needed. The object of this report is to partially fulfill this requirement.

It is very difficult to solve for the details of the jet flow in ground proximity; fortunately, simple and plausible approximations for the quantities in the present problem are afforded by assuming that the jet velocity distribution and the entrainment process therein are the same as for a single jet discharging into an infinite fluid. For high Reynolds number, the problem of determining the effect of the standing vortex is reduced to that of calculating the inviscid rotational flow pattern in a closed region with uniform but undetermined vorticity; the associated indeterminacy of the inviscid motion may be resolved by a simple analysis of the closed boundary layer. As to the effect of jet mixing, the calculations are based on the entrainment process and momentum balance consideration in the impinge-

ment region. The overall viscous effects on augmentation factor of a ground effect machine for various nozzle geometries are computed; the results are in good agreement with experimental findings.

METHOD OF ANALYSIS

MEAN FLOW PATTERN OF AN ANNULAR JET IN GROUND PROXIMITY

We shall, first, discuss the mean flow pattern of a two-dimensional, incompressible annular jet in ground proximity. As air discharges from the nozzle, high velocity gradients prevail along both boundaries of the jet and high intensities of shear result. Except at very low Reynolds numbers, turbulence is generated, and the mixing process further reduces the velocity near its boundaries and brings the neighboring fluid into motion. Under the nozzle base this leads to the formation of a vortex standing alongside the main jet; this vortex may in turn generate a second weak vortex of opposite sense and so on. Along the open side of the jet, entrainment of fluid into the jet occurs as a result of turbulent mixing. The vertical momentum of the jet is reduced by the deflection action of the ground. In the horizontal direction, the increased pressure, between the base plate and ground, acts to curve the jet outward. The mean flow pattern of an annular jet in ground proximity is, in general, that of a diffusing jet curving outward with respect to the centerline of the GEM as sketched in Figure 1. The significant variables of the problem are the nozzle base width $2W$, the nozzle thickness t , the nozzle divergence angle ϕ_0 , the nozzle base height above the ground H , the jet velocity at the nozzle exit U_e and the fluid properties, ρ (density) and μ (viscosity). If these quantities are specified, the physical features of the flow are determined. By dimensional analysis, the pressure difference across the jet, Δp_b , and the augmentation factor, A , defined as the ratio of total lift to the total jet momentum flux, may be expressed as follows:

$$\frac{\Delta p_b}{J/W} = F_1 \left(\frac{H}{W}, \frac{H}{t}, \phi_0; \frac{\rho U_e t}{\mu} \right)$$

where J is the jet momentum per unit length of slot, and

$$A = F_2 \left(\frac{H}{W}, \frac{H}{t}, \phi_0; \frac{\rho U_e t}{\mu} \right)$$

If the Reynolds number ($= \rho U_e t / \mu$) is sufficiently high, it is expected that $\Delta p_b / J/W$ and A should be independent of $\rho U_e t / \mu$. The functions F_1 , F_2 are, of course, very much influenced by the turbulent mixing and vortex generation; it is then necessary to study, beforehand, the approximate characteristics of the turbulent jet.

APPROXIMATE CHARACTERISTICS OF A TURBULENT JET

As the direct result of turbulence generated at the boundaries of a jet, the fluid within the jet undergoes both lateral diffusion and deceleration, and at the same time fluid from the surrounding region is brought into motion. The actual treatment of the flow in these regions varies with distance from the jet origin; with reference to Figure 2, it will be seen that an initial zone of establishment must exist beyond the efflux section. Since the fluid discharged from the boundary opening may be assumed to be of relatively constant velocity, at the efflux section there will necessarily be a very sharp velocity gradient between the jet and surrounding fluid. The eddies generated in this region of high shear will immediately result in a lateral mixing process which diffuses the shear region both inwards and outwards with distance from the efflux section. Such lateral mixing produces a necessarily balanced action and reaction. On the one hand, the fluid within the jet is gradually decelerated, while on the other hand, the fluid from the surrounding region is gradually entrained; as a result, the constant velocity core of the jet will steadily decrease in lateral extent, while both the rate of flow and overall breadth of the jet will steadily increase in magnitude, with distance from the efflux section. The limit of this initial zone of flow establishment is reached when the mixing region has penetrated to the centerline of the jet. Once the entire central portion of the jet becomes turbulent, the flow may be considered as fully established, for the diffusion process continues thereafter without essential change in character.

Conditions within both the zone of flow establishment and

the zone of established flow for a free jet were first investigated theoretically by Tollmien (8), on the assumption that (i) the sole effective force was the tangential shear (no pressure gradients), (ii) the mixing length varied with the first power of the longitudinal distance from the efflux section but was constant across the jet, and (iii) the intensity of the turbulence was proportional to the product of the mixing length and the mean velocity gradient. Later experimental studies by Corrsin (9) and by Liepmann and Laufer (10) indicated that there exists considerable discrepancy between assumption (ii) and the facts. However, for approximate analysis of the mean velocity distribution within either zone, the theory of Tollmien is adequate.

Albertson, et al (11), developed approximate analytical expressions for the zone of establishment and the zone of established flow based upon the assumptions: (i) the distribution of the mean longitudinal velocity follows the error law in the region of diffusion, (ii) the region of diffusion expands linearly with distance from the efflux section, and (iii) the pressure distribution is hydrostatic.

Although, in the present problem, the prevailing centrifugal force field in the core of a curved jet may have some effect on the turbulence, for the present calculation, simple and plausible approximations for the quantities required are afforded, by assuming that the jet velocity distribution and entrainment process therein are the same as for a single jet discharging into an infinite fluid. For this case, according to Reference 11, the velocity distributions are found empirically to be:

$$\left. \begin{aligned} \log_{10} \frac{\bar{u}}{U_e} &= -18.4 \left(0.096 + \frac{y-t/2}{x} \right)^2 && \text{(zone of establishment)} \\ \log_{10} \frac{\bar{u}}{U_e} &= 0.36 - 1.84 \frac{y^2}{x^2} && \text{(zone of established flow)} \end{aligned} \right\} [1]$$

where $\bar{u}$ is the mean tangential velocity, and the entrainment function, f , may be approximated as:

$$f\left(\frac{s_c}{t}\right) = \begin{cases} \frac{0.080 s_c/t}{1+0.080 s_c/t} & s_c/t < 5.2 \\ 1 - 1.613 \sqrt{\frac{t}{s_c}} & s_c/t \geq 5.2 \end{cases} \quad [2]$$

where s_c is the distance along the jet path.

In the following we shall discuss approximately the effects of the standing vortex and of jet mixing by utilizing the above-mentioned results.

EFFECT OF THE STANDING VORTEX

It had been found by Nixon and Sweeny (12) and Poisson-Quinton (13) in their experimental studies on ground effect phenomena that "vortices" exist near the nozzle exits. It was indicated in their reports that the "vortices" are responsible for the non-uniformity of the base pressure distributions; however, no effort was made to correlate the strength of the vortices with the jet parameter or to study in detail the flow within the vortex. The first theoretical account of the subject was given by Shen (14), who drew attention to the theory of flows with closed streamlines developed by Batchelor (15) but used a simple single concentrated vortex model to evaluate approximately the pressure distribution on the base plate due to vorticity. A similar engineering procedure for estimating the pressure distribution on the bottom of a stationary ground effect machine of the annular-jet type is suggested by Magnus (16), who develops empirical relations for the placement and strength of the "vortices", distributed halfway between the vehicle and the ground based upon available model test data. We feel that models such as those of Shen and Magnus, which involve point vortices or simple distributions of point vortices, are too restrictive and that in order to study actual air jet flow fields in ground proximity, a more realistic flow model is needed.

The two-dimensional air jet flow field under the base of a GEM has been discussed in previous sections and the mean

flow pattern is shown in Figure 1. Since the strength of the second induced vortex is much smaller than that of the first, for a first-order analysis, one can practically assume that beyond the first induced vortex adjacent to the main jet the flow field in the closed region is stagnant. The simplified rotational flow model is shown schematically in Figure 3. The geometrical designation of the model is: Width W , which is half the width of a GEM, height H of base plate above the ground, H_1 approximately the longitudinal size of the induced vortex. The reduced problem is somewhat similar to the problem of steady flow with closed streamline studied by Batchelor (15). For sufficiently high Reynolds number, the rotational flow inside the closed region with the exception of the wall boundary layer may be considered as inviscid, incompressible and confined by solid walls; the vorticity is approximately constant in this region. Taking the center of the rectangular ABCD as the origin of cartesian coordinates, the equation governing steady two-dimensional rotational flow of an inviscid, incompressible fluid may be expressed as:

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \kappa \quad [3]$$

where ψ is Lagrange's stream function, and $-\kappa$ represents the uniform vorticity. At the boundary, the velocity of the circulating fluid is in the direction of the tangent to the boundary. The problem is identical to that of the torsional problem with rectangular cross section in elasticity and may be solved by an energy method; the approximate solution is readily given in the book "Theory of Elasticity", by Timoshenko (17):

$$\psi = \left[x^2 - \left(\frac{H_1}{2} \right)^2 \right] \left[y^2 - \left(\frac{H}{2} \right)^2 \right] \sum_m \sum_n a_{mn} x^m y^n \quad [4]$$

in which, from symmetry; m and n must be even numbers. It is observed in experimental studies, References 12 and 13, that the longitudinal size of the vortex, H_1 , is approximately equal to H , and is insensitive to other parameters; hence, by assuming $H_1 = H$ and taking the first three terms in the series, [4] becomes

$$\psi = \left[x^2 - \left(\frac{H}{2} \right)^2 \right] \left[y^2 - \left(\frac{H}{2} \right)^2 \right] \left[a_0 + a_1 (x^2 + y^2) \right] \quad [5]$$

where a_0 , a_1 are found to be, by using the variational principle,

$$a_0 = -\frac{1}{2} \cdot \frac{5}{8} \cdot \frac{259}{277} \cdot \frac{\kappa}{(H/2)^2},$$

$$a_1 = -\frac{1}{2} \cdot \frac{5}{8} \cdot \frac{3}{2} \cdot \frac{35}{277} \cdot \frac{\kappa}{(H/2)^4}.$$

The induced velocity may be written as:

$$\left. \begin{aligned} u &= \frac{\partial \psi}{\partial y} = -\frac{5}{8} \frac{\kappa}{(H/2)^2} y \left[x^2 - \left(\frac{H}{2} \right)^2 \right] \left\{ .935 + \frac{0.19}{(H/2)^2} x^2 + 2y^2 - \left(\frac{H}{2} \right)^2 \right\} \\ v &= -\frac{\partial \psi}{\partial x} = \frac{5}{8} \frac{\kappa}{(H/2)^2} x \left[y^2 - \left(\frac{H}{2} \right)^2 \right] \left\{ .935 + \frac{0.19}{(H/2)^2} 2x^2 + y^2 - \left(\frac{H}{2} \right)^2 \right\} \end{aligned} \right\} \quad [6]$$

The pressure, p , acting on the walls may be obtained by integrating the momentum equation:

$$\left. \begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} \\ v \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} \end{aligned} \right\} \quad [7]$$

with the condition of uniform vorticity

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -\kappa \quad [8]$$

i.e.,

$$\frac{p}{\rho} + \frac{1}{2} (u^2 + v^2) = \kappa \psi + \text{const} = \text{Constant}, \quad [9]$$

since ψ is taken to be zero on the walls. The motion of the fluid in the "inviscid" region is solved up to an arbitrary vorticity, $-\kappa$, which remains to be determined from the viscous aspect of the problem.

The associated indeterminacy of the inviscid motion may be resolved by an analysis of thin turbulent boundary layer along the boundary ABCD. The velocity distribution along

the jet boundary AB is assumed to be tangential and given by

$$\left. \begin{aligned} \frac{\bar{u}_{AB}}{U_e} &\approx 0.66 & x/t < 5.2 \\ \frac{\bar{u}_{AB}}{U_e} &\approx 0.66 \sqrt{\frac{5.2t}{x}} & x/t \geq 5.2 \end{aligned} \right\} [10]$$

and the velocities along BC, CD, DA are generally regarded as negligible. The calculation of the turbulent boundary layer is, in general, very difficult; for a rough estimation, we will use the semi-empirical method of Truckenbrodt (for details, refer to pp. 470-472, Reference 18). The momentum thickness, θ , is found to be

$$\frac{\theta(x)}{H} = \left(\frac{\bar{u}}{U_e} \right)^3 \left\{ \left(\frac{C_f}{2} \right)^{n+1/n} \int_0^{x/H} \left(\frac{\bar{u}}{U_e} \right)^{3+2/n} d \left(\frac{x}{H} \right) \right\}^{n/n+1} [11]$$

where

C_f is the skin friction coefficient = $\frac{\text{friction drag}}{\frac{1}{2} \rho \bar{u}^2 H}$

n is a positive real number = $\begin{cases} 4 & \text{for small Reynolds number} \\ 6 & \text{for large Reynolds number.} \end{cases}$

If we assume that the boundary layer is closed and the general magnitude of momentum thickness is presumed to be the same in the entire closed region, for a first-order approximation, it can be shown

$$\begin{aligned}
\frac{\bar{u}_{ave}}{U_e} &= g\left(\frac{t}{H}, \gamma, n\right) \\
&= .66 \left\{ 3 \gamma \right\}^{n+1/2n+3} & x/t < 5.2 \\
&= .66 \left\{ 1 + \left(\frac{2n}{n+2}\right)^{n/n+1} \left(5.2 \frac{t}{H}\right)^{-1/2(n+1)} \times \right. \\
&\quad \left. \left[\left(5.2 \frac{t}{H}\right)^{-(n+2/2n)} - .1 \right]^{n/n+1} \right\}^{-(n+1/2n+3)} \times \\
&\quad \left\{ (3\gamma) \left(5.2 \frac{t}{H}\right) \right\}^{n+1/2n+3} & x/t \geq 5.2 \quad [12]
\end{aligned}$$

where

$\bar{u}_{ave}$ is the average induced velocity inside the cavity and approximately equal to $0.24 H \kappa$ (from Equation [6])

γ is a constant, depending on the local shearing stress.

The strength of vortex, $-\kappa$, becomes

$$-\kappa \approx 4.16 \frac{U_e}{H} g\left(\frac{t}{H}, \gamma, n\right) \quad [13]$$

and the mean pressure loss due to vortex flow inside the cavity may be approximated as

$$\frac{p_{vortex}}{\frac{1}{2}\rho U_e^2} \propto \left(\frac{\bar{u}_{ave}}{U_e}\right)^2 \left(= \left[g\left(\frac{t}{H}, \gamma, n\right) \right]^2 \right) \quad [14]$$

Comparisons of velocity and pressure distributions are made, for $\gamma = 0.05$, $n = 6$, between present calculations and experimental studies by Roshko (19) of the flow within a square cavity in a wind tunnel wall; the results are in excellent agreement as shown in Figures 4 and 5. Note that the velocity increases continuously from the center of the "vortex" toward the walls in contrast with the field that obtains for a point vortex.

The effect of a standing vortex on the average base pressure distribution, Δp_b , may be expressed as

$$\Delta p_b \Big|_{\text{vortex}} - \Delta p_b \propto \frac{H}{W} \left(\frac{\bar{u}_{\text{ave}}}{U_e} \right)^2 \quad [15]$$

Since $\left(\frac{\bar{u}_{\text{ave}}}{U_e} \right)^2 \ll 1$, it can be seen, in ground proximity, that the effects of the standing vortex are relatively small. It is conjectured here that the effect of jet mixing may have a dominant role.

EFFECT OF JET MIXING

The first publication dealing with the effect of jet mixing on the annular jet is that of Chaplin (20). The theory makes use of the analysis of Tollmein (8) and measurements of Föhlmann (21) for the fully-developed free turbulent jet; his results are, therefore, limited to altitudes above the ground sufficiently large that mixing has become fully established before the jet reaches the ground. Mack and Yen (22) extended Chaplin's result to the case in which impingement occurs while the jet is still in the zone of flow establishment. In our treatment, a more realistic approximation is made to account for the effect of velocity distribution of the jet. The basic assumptions, similar to that given by Chaplin (20), are made as follows:

- 1) The jet curves at a constant rate, until it impinges on the ground at an angle ϕ_c from the vertical.
- 2) The velocity distributions in the jet just before

and after the impingement are similar except very near the ground.

3) The ratio of entrained mass flow, m_{ec} , to total mass flow, m_c , in the jet just before impingement can be represented by

$$\frac{m_{ec}}{m_c} = f\left(\frac{s_c}{t}\right) \quad [16]$$

where s_c is the distance along the jet path from the nozzle exit to the impingement point.

Within ground effect, the flow pattern is assumed to be as sketched in Figure 6a. The jet curves until it impinges on the ground at an angle ϕ_c from the vertical. From the impingement, a part, m_u , of the mass flow of the jet flows inward into the base cavity, and the rest, m_d , flows outward along the ground. In steady flow, the inward mass flow, m_u , must be equal the mass entrainment from the base cavity.

The entrainment from the base cavity into the jet upstream of the impingement is assumed to be

$$\frac{m_u}{m_u + m_d} = \lambda \frac{m_{ec}}{m_c} = \lambda f\left(\frac{s_c}{t}\right) \quad [17]$$

where λ is a constant ($= 1/2$ for jet discharging into an infinite fluid). After impingement, the jet flow behaves more or less like a wall jet as studied by Glauert (23); if we assume that, except very near to the ground, the velocity distribution, as shown in Figure 6b, is similar to that of a free half jet, a second expression for the mass flow, from the momentum balance, can be obtained:

$$m_d \bar{u}_d - m_u \bar{u}_u = (m_d + m_u) \bar{u} \sin \phi_c \quad [18]$$

or

$$\frac{m_u}{m_u + m_d} = (1 - a \sin \phi_c) / (1 + b) \quad [19]$$

where

$$a = \frac{\bar{u}}{\bar{u}_d}$$

$$b = \frac{\bar{u}_u}{\bar{u}_d}$$

$\bar{u}$ is the average velocity of the jet before impingement

$\bar{u}_u$, $\bar{u}_d$ are the average velocities of the inward and outward mass flows respectively.

It may be noted here that Chaplin (20) and Mack and Yen (22) assumed that $a = b = 1$.

Combining [17] and [19] gives

$$\sin \phi_c = \frac{1}{a} \left[1 - (1 + b) \lambda f \left(\frac{s_c}{t} \right) \right] \quad [20]$$

From the geometry, assuming radius of curvature, R , to be constant,

$$\left. \begin{aligned} s_c &= R (\phi_c - \phi_o) \\ H &= R (\sin \phi_c - \sin \phi_o) \end{aligned} \right\} \quad [21]$$

where ϕ_o is the angle of divergence of the jet from the vertical at the jet exit.

The effect of jet mixing on the base pressure distribution can be shown as

$$\frac{\Delta p_b}{2J/W} = \frac{\sin \phi_c - \sin \phi_o}{2H/W} + \lambda f \left(\phi_o, \frac{H}{t} \right) \quad [22]$$

If the velocity distributions of the jet and entrainment function are assumed to be given by Equations [1] and [2], the effect of jet mixing can easily be evaluated.

RESULTS OF ANALYSIS

In the following, the overall viscous effect, based on the previous analysis, on the augmentation factor for a two-dimensional ground effect machine, in ground proximity, is computed and can be shown as

$$A = \cos \phi_o + \frac{\sin \phi_c - \sin \phi_o}{2 H/2W} + \lambda f \left(\phi_o, \frac{H}{t} \right) - \frac{1}{4} \frac{H}{t} \left[g \left(\frac{t}{H}, \gamma, n \right) \right]^2 \quad [23]$$

In ground proximity, the two-dimensional solution affords a good approximation to the corresponding axially symmetric solution if $2W$ is replaced by $D/2$ as was shown by Chaplin in References 1 and 20. The augmentation factor in this case may be expressed as

$$A = \cos \phi_o + \frac{\sin \phi_c - \sin \phi_o}{4 H/D} + \lambda f \left(\phi_o, \frac{H}{t} \right) - \frac{1}{2} \left(1 - \frac{H}{D} \right) \frac{H}{t} \left[g \left(\frac{t}{H}, \gamma, n \right) \right]^2 \quad [24]$$

where D is the diameter of the nozzle base plate.

It is well known, for the inviscid, incompressible thin annular jet, that the augmentation factor is given by

$$\left. \begin{aligned} A_{inv} &= \cos \phi_o + \frac{1 - \sin \phi_o}{2 H/2W} \quad \text{(2-dimensional)} \\ &= \cos \phi_o + \frac{1 - \sin \phi_o}{4 H/D} \quad \text{(axi-symmetrical)} \end{aligned} \right\} \quad [25]$$

The viscous effects, i.e., the effects of standing vortex and jet mixing, can thus be conveniently represented by the ratio

$$\begin{aligned}
\frac{A - \cos \phi_o}{A_{\text{inv}} - \cos \phi_o} &= \frac{\sin \phi_c - \sin \phi_o}{1 - \sin \phi_o} + \frac{\lambda f \left(\phi_o, \frac{H}{t} \right) \left(\frac{H}{W} \right)}{1 - \sin \phi_o} - \\
&\quad \cdot \frac{1}{4} \frac{H}{W} \frac{H}{t} \left[g \left(\frac{t}{H}, \gamma, n \right) \right]^2 \quad (2\text{-dimen.}) \\
&= \frac{\sin \phi_c - \sin \phi_o}{1 - \sin \phi_o} + \frac{\lambda f \left(\phi_o, \frac{H}{t} \right) \left(\frac{4H}{D} \right)}{1 - \sin \phi_o} - \\
&\quad \cdot \frac{1}{2} \frac{4H}{D} \left[1 - \frac{H}{D} \frac{H}{t} \left[g \left(\frac{t}{H}, \gamma, n \right) \right]^2 \right] \quad (\text{axi-symm.})
\end{aligned} \quad [26]$$

Numerical calculations are made for the jet in balanced operation. In this case,

$$a = \frac{\bar{u}}{\bar{u}_d} \approx \begin{cases} 1.40 & \text{in the zone of establishment} \\ 1.38 & \text{in the zone of established flow} \end{cases}$$

$$b = \frac{\bar{u}_u}{\bar{u}_d} \approx \begin{cases} 0.30 & \text{in the zone of establishment} \\ 0.39 & \text{in the zone of established flow} \end{cases}$$

$$\lambda = \frac{1}{2} b.$$

For the better fitting of the experimental data of Yen (24), the function $g \left(\frac{t}{H}, \gamma, n \right) \left(= \frac{\bar{u}_{\text{ave}}}{\bar{u}_e} \right)$, in [26], is calculated by assuming $\gamma = 0.05$, $n = 6$. The results of Equation [26], for various nozzle geometries, are shown in Figures 7 and 8. It can be seen that the augmentation factor predicted by thin jet theory can be over-estimated by more than 50 percent. It may be noted here that in our calculations the last term in Equations [23], [24], and [26] is small in comparison with the first two terms.

Simple experimental studies for a small GEM model with

nozzle base cross-section $16" \times 24"$, $t = 0.15"$, $\phi_o = -45^\circ$

in balanced operation have been conducted by HYDRONAUTICS, Incorporated, for various height-thickness ratios, H/t . The experimental data are shown in Figure 9 with theoretical calculations, including planform correction, given by

$$A = \cos \phi_o + \frac{\sin \phi_c - \sin \phi_o}{H \cdot C/S} + \lambda f\left(\phi_o, \frac{H}{t}\right) - \frac{1}{2} \frac{H}{t} \left(1 - \frac{1}{4} \frac{HC}{S}\right) \left[g\left(\frac{t}{H}, \gamma, n\right)\right]^2 \quad [27]$$

where C , S are the perimeter and area of nozzle base respectively. The experimental findings seem to support our theoretical investigations.

Comparisons are also made, for the axi-symmetrical case, between present calculations and experimental studies by Kuhn and Carter (25), Smith (26), and the Societe Bertin from Reference 13, for various nozzle geometries and machine heights. The results, as shown in Figures 10, 11, 12, are in good agreement.

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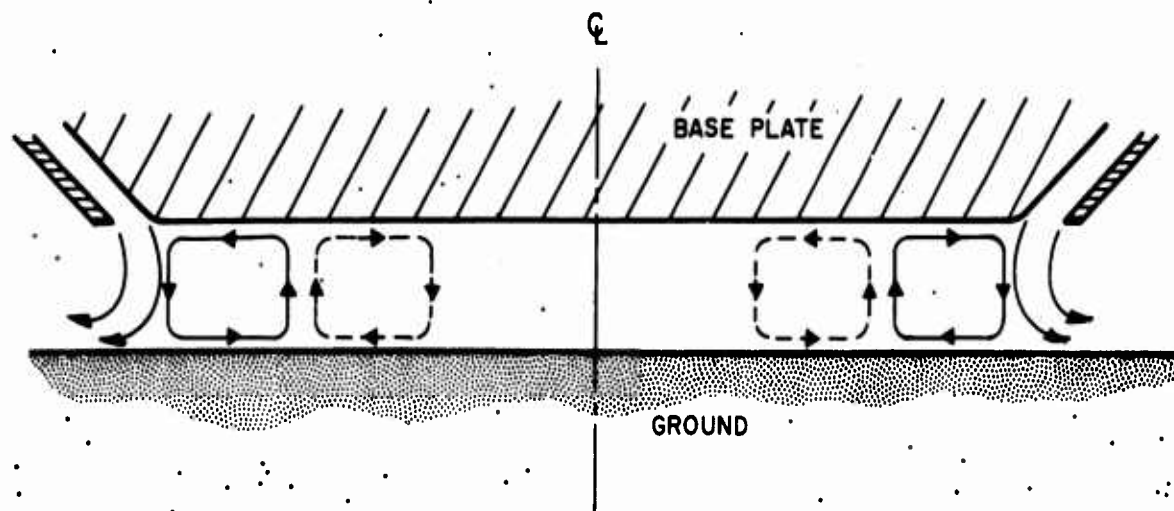


FIGURE 1—TWO-DIMENSIONAL REPRESENTATION OF ANNULAR JET.

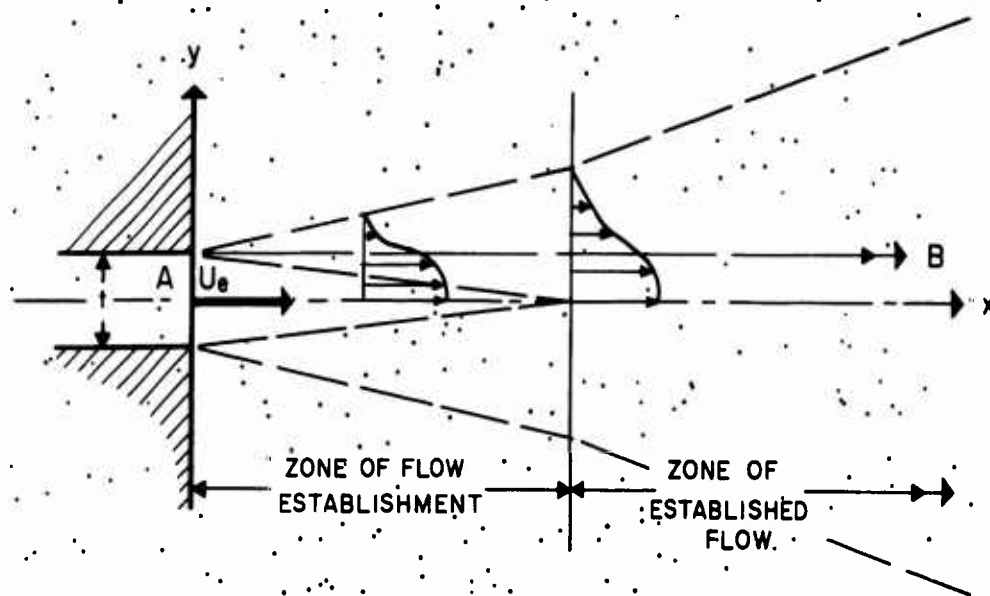


FIGURE 2—SCHEMATIC REPRESENTATION OF JET DIFFUSION.

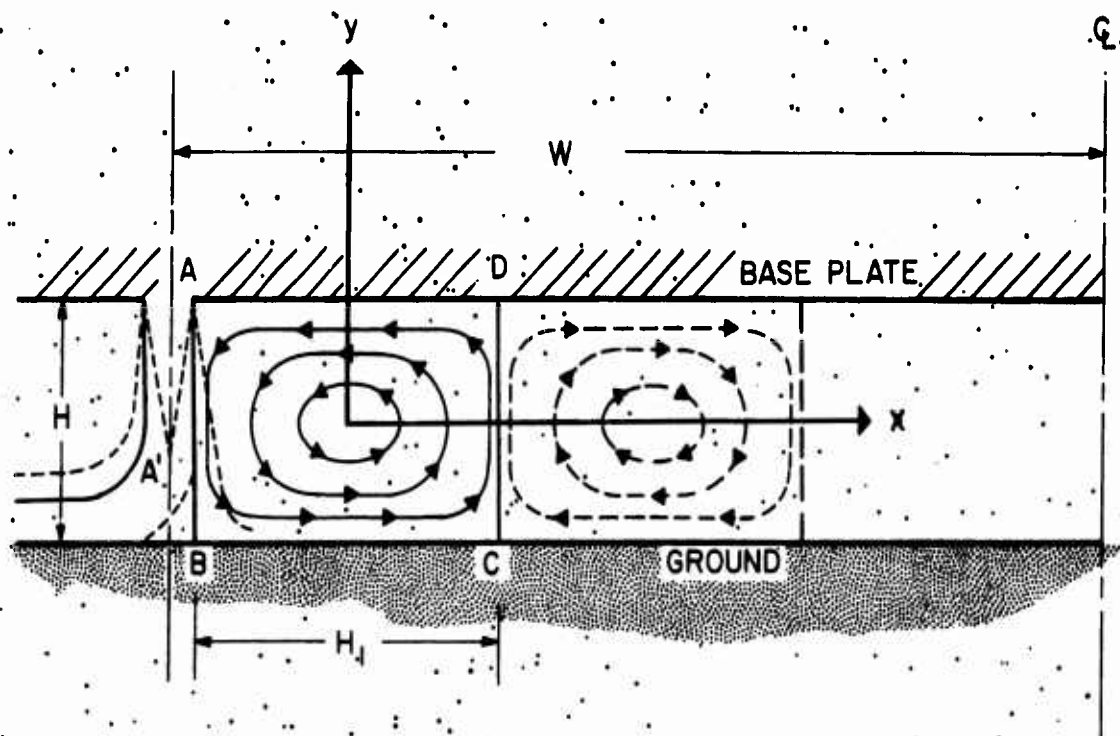


FIGURE 3- SCHEMATIC OF ROTATIONAL FLOW UNDER
THE BASE OF A GEM.

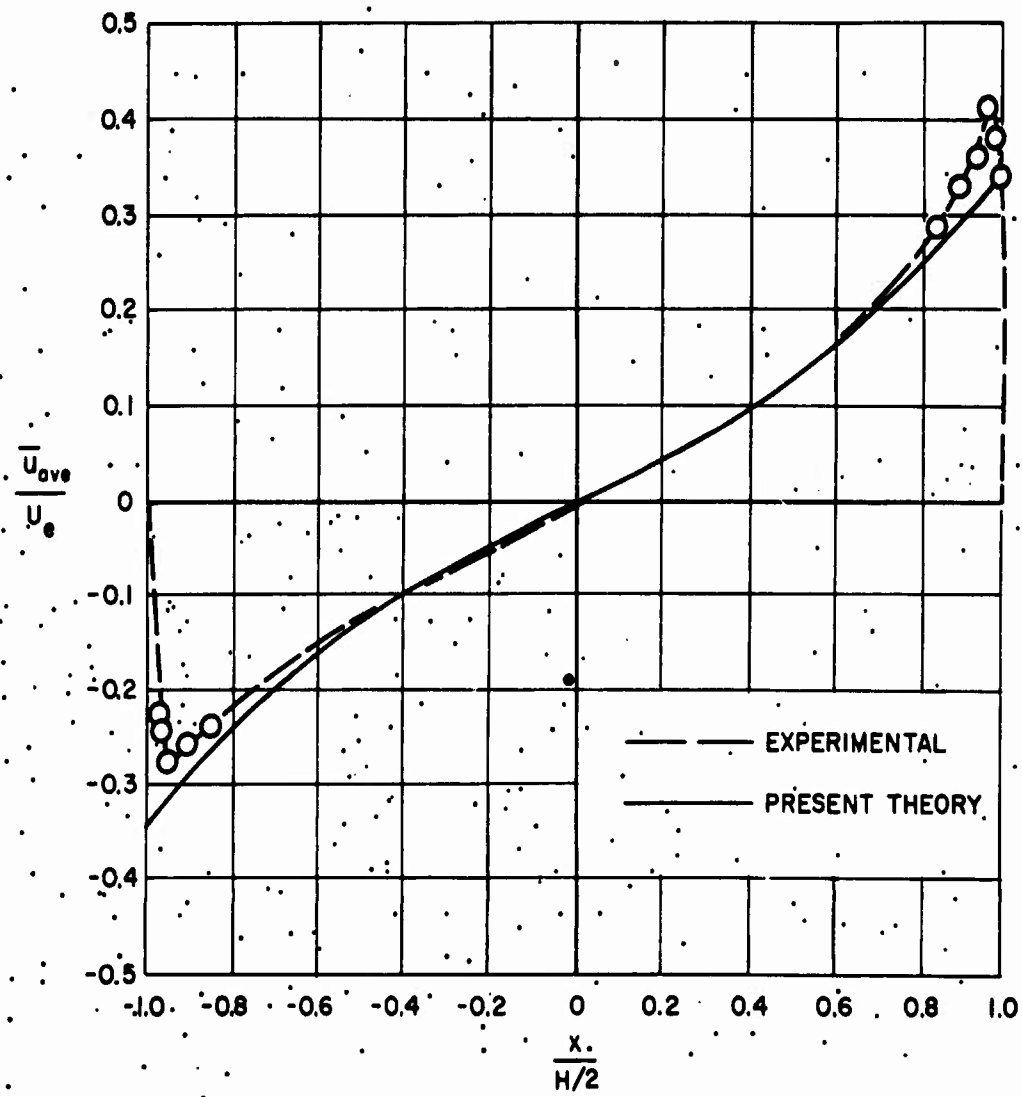


FIGURE 4 - VELOCITY INDUCED BY VORTICITY IN A SQUARE CAVITY

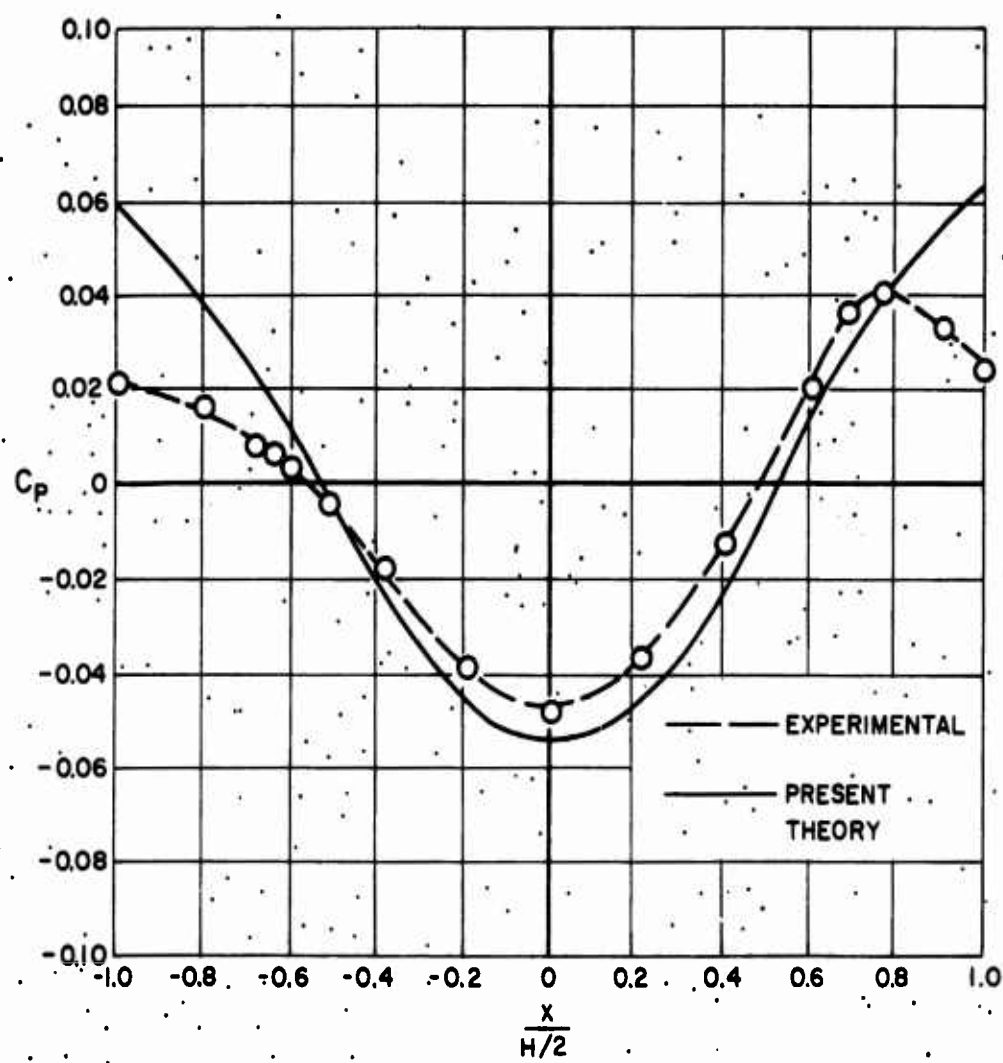


FIGURE 5- PRESSURE DISTRIBUTION ON CAVITY WALL

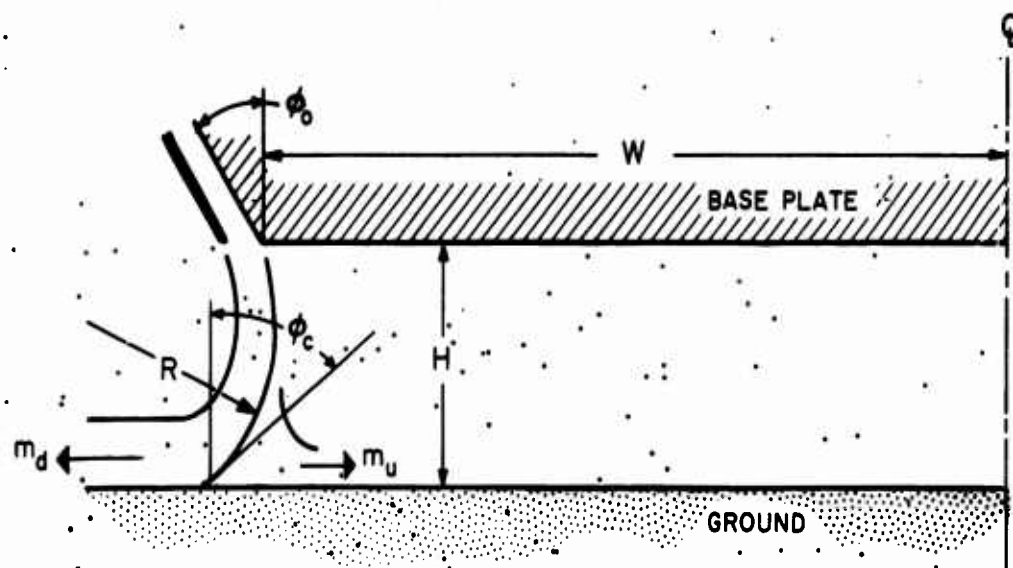


FIGURE 6a - DEFINITION SKETCH

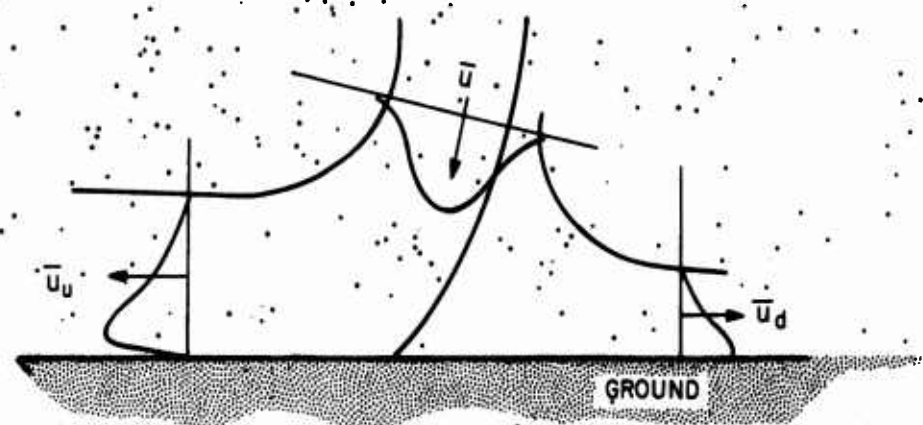


FIGURE 6b - VELOCITY DISTRIBUTION BEFORE AND AFTER IMPINGEMENT

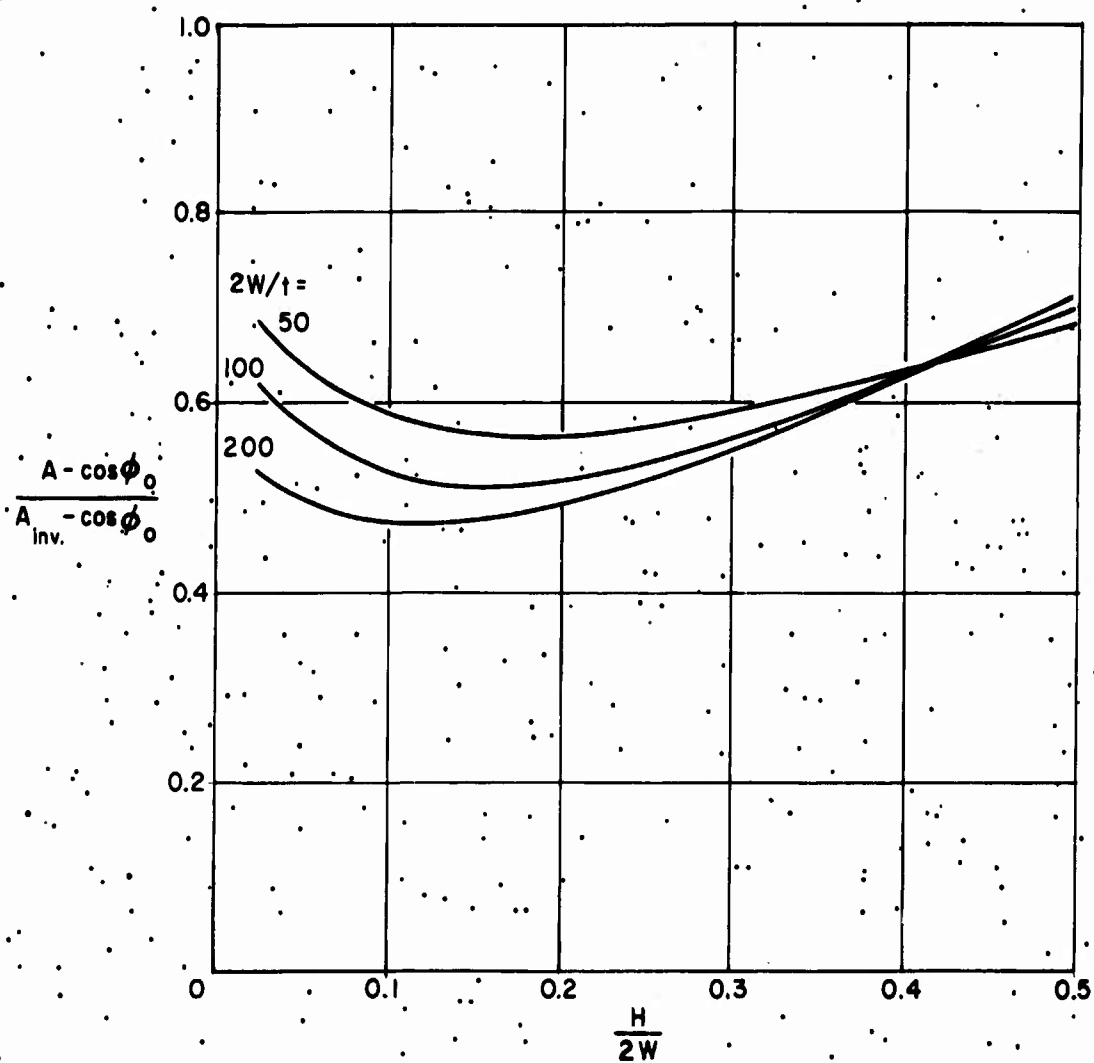


FIGURE 7a- VISCOUS EFFECTS ON AUGMENTATION, TWO-DIMENSIONAL
CASE, $\phi_0 = 0^\circ$

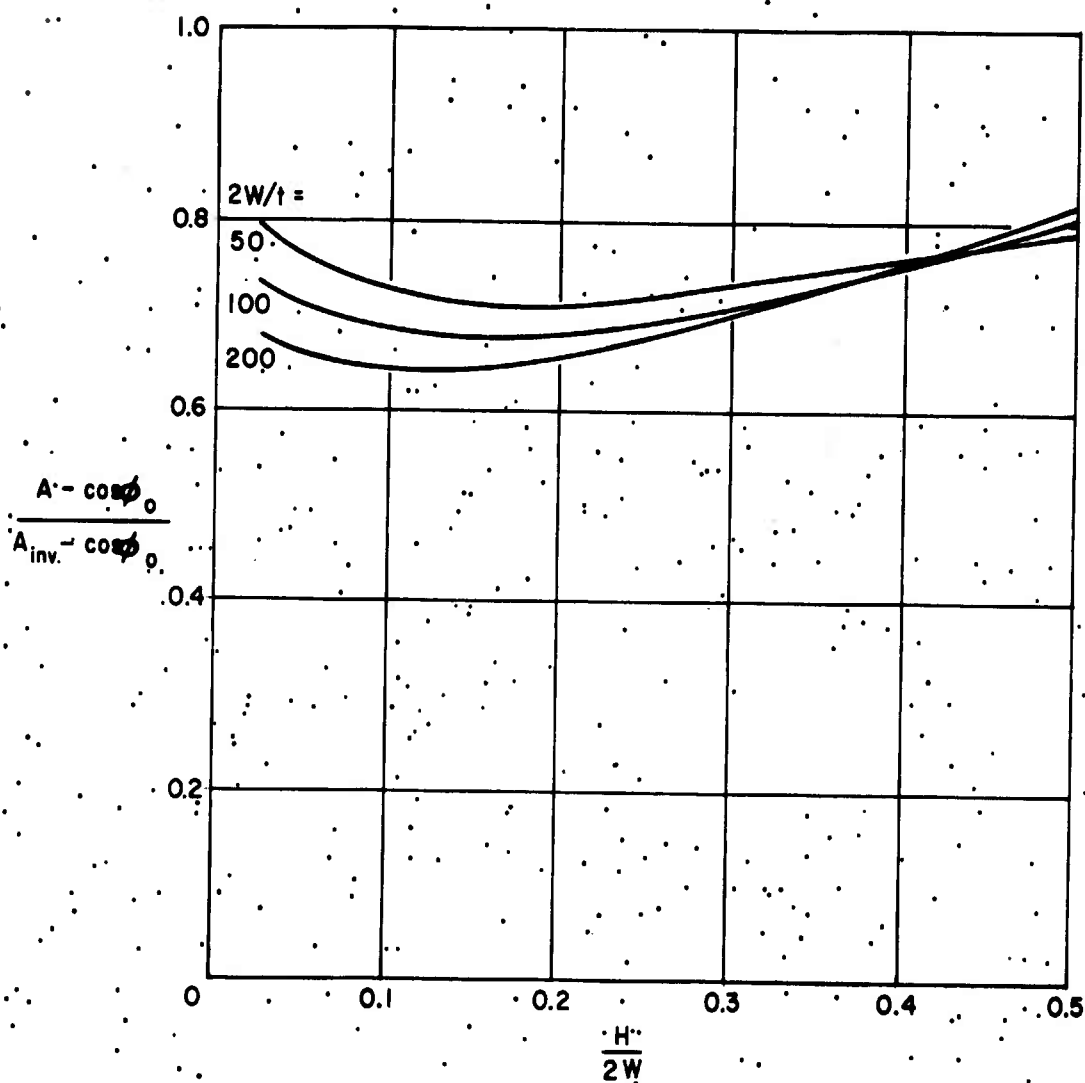


FIGURE 7b- VISCOUS EFFECTS ON AUGMENTATION, TWO-DIMENSIONAL

CASE, $\phi_0 = -30^\circ$

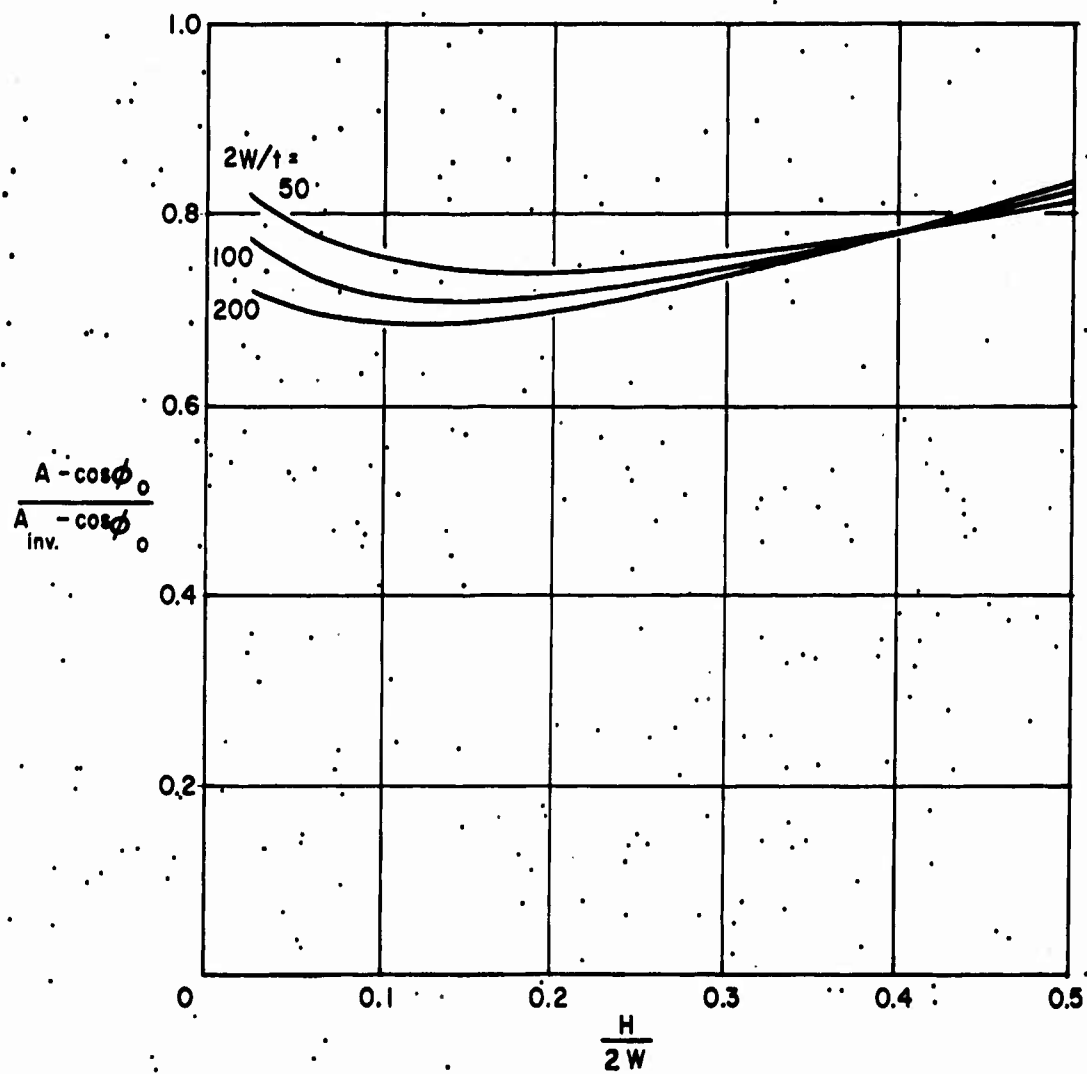


FIGURE 7c - VISCOUS EFFECTS ON AUGMENTATION, TWO-DIMENSIONAL
CASE, $\phi_0 = -45^\circ$

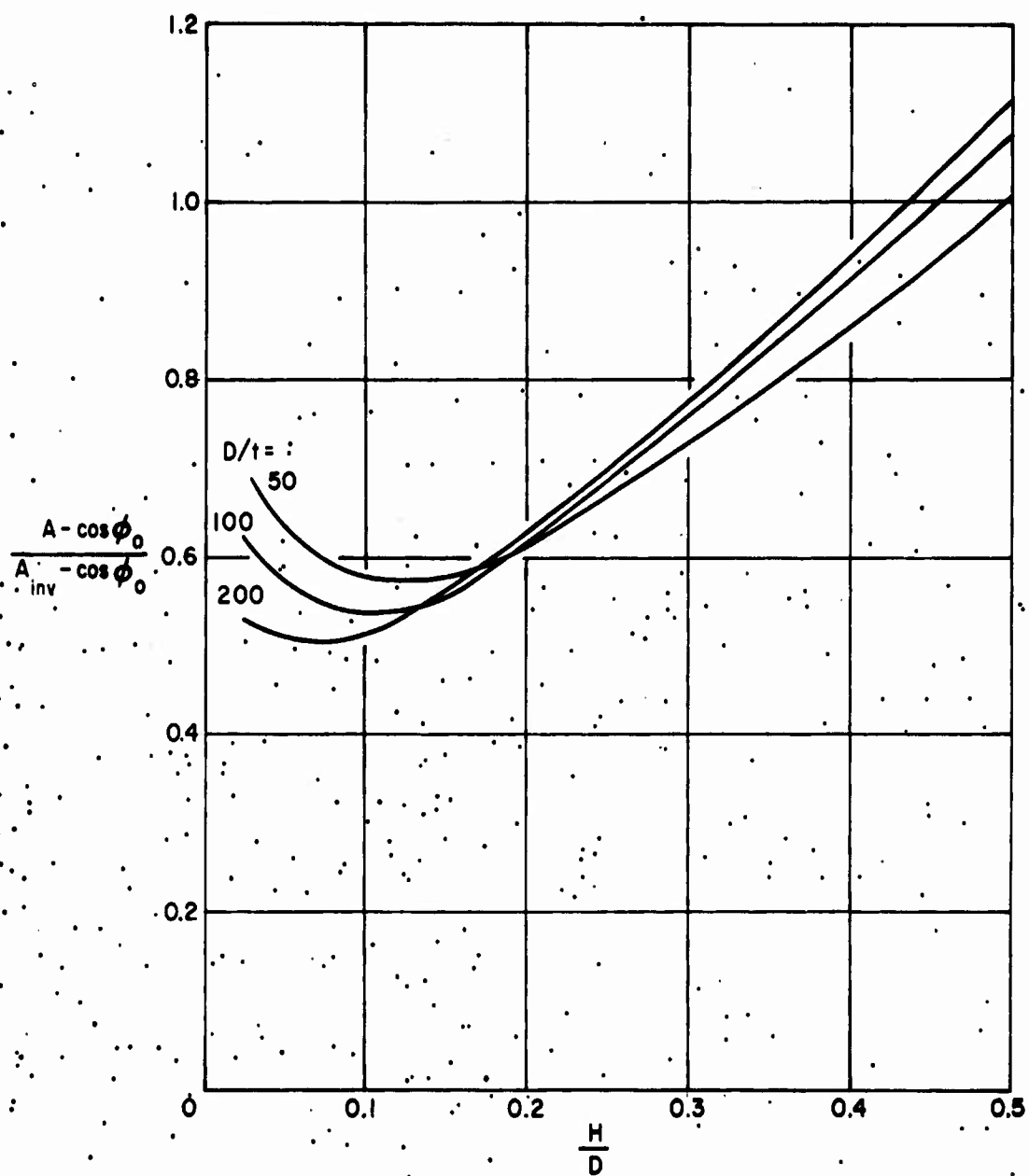


FIGURE 8a - VISCOUS EFFECTS ON AUGMENTATION, AXI-SYMMETRICAL
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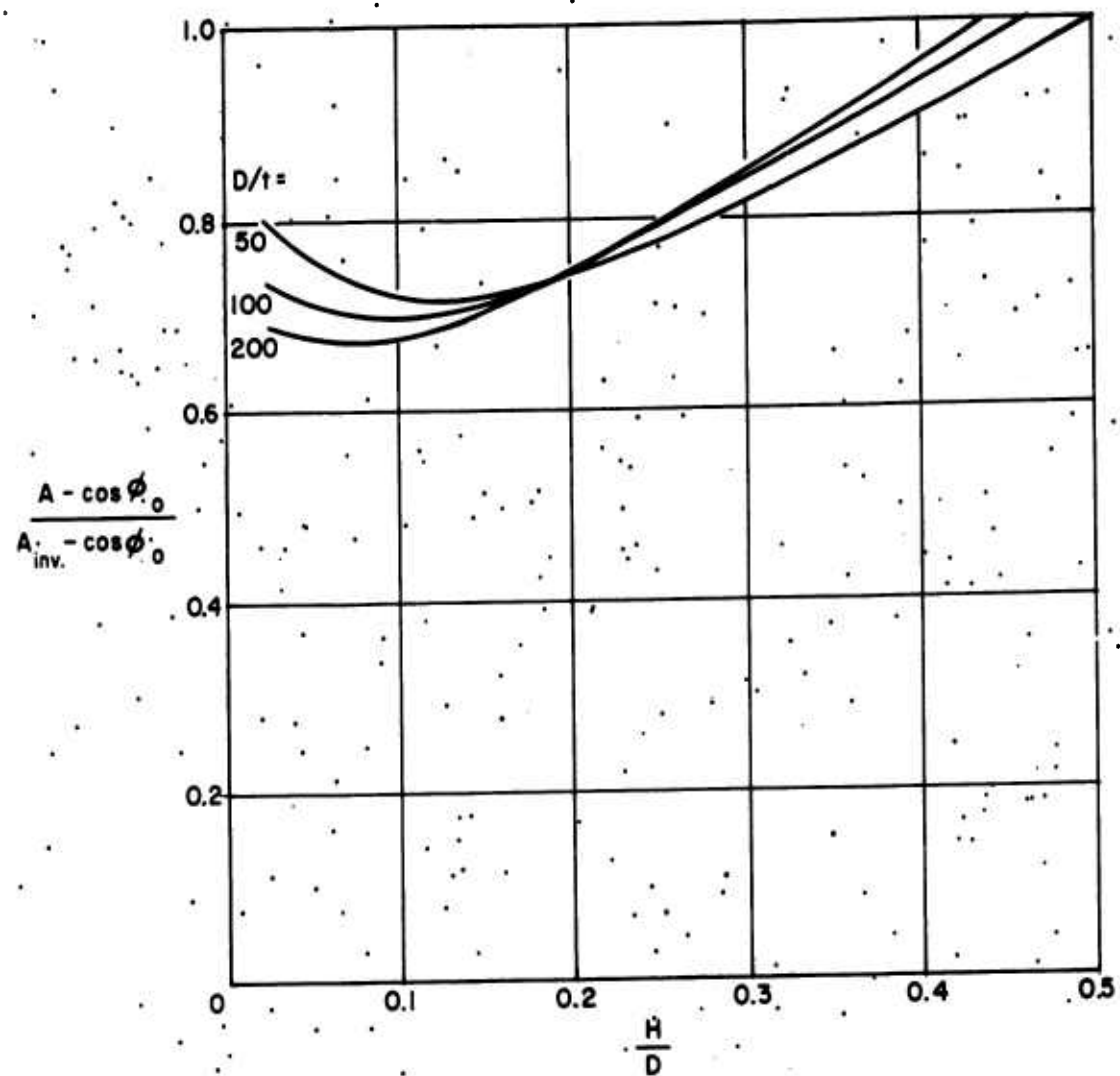


FIGURE 8b- VISCIOUS EFFECTS ON AUGMENTATION, AXI-SYMMETRICAL
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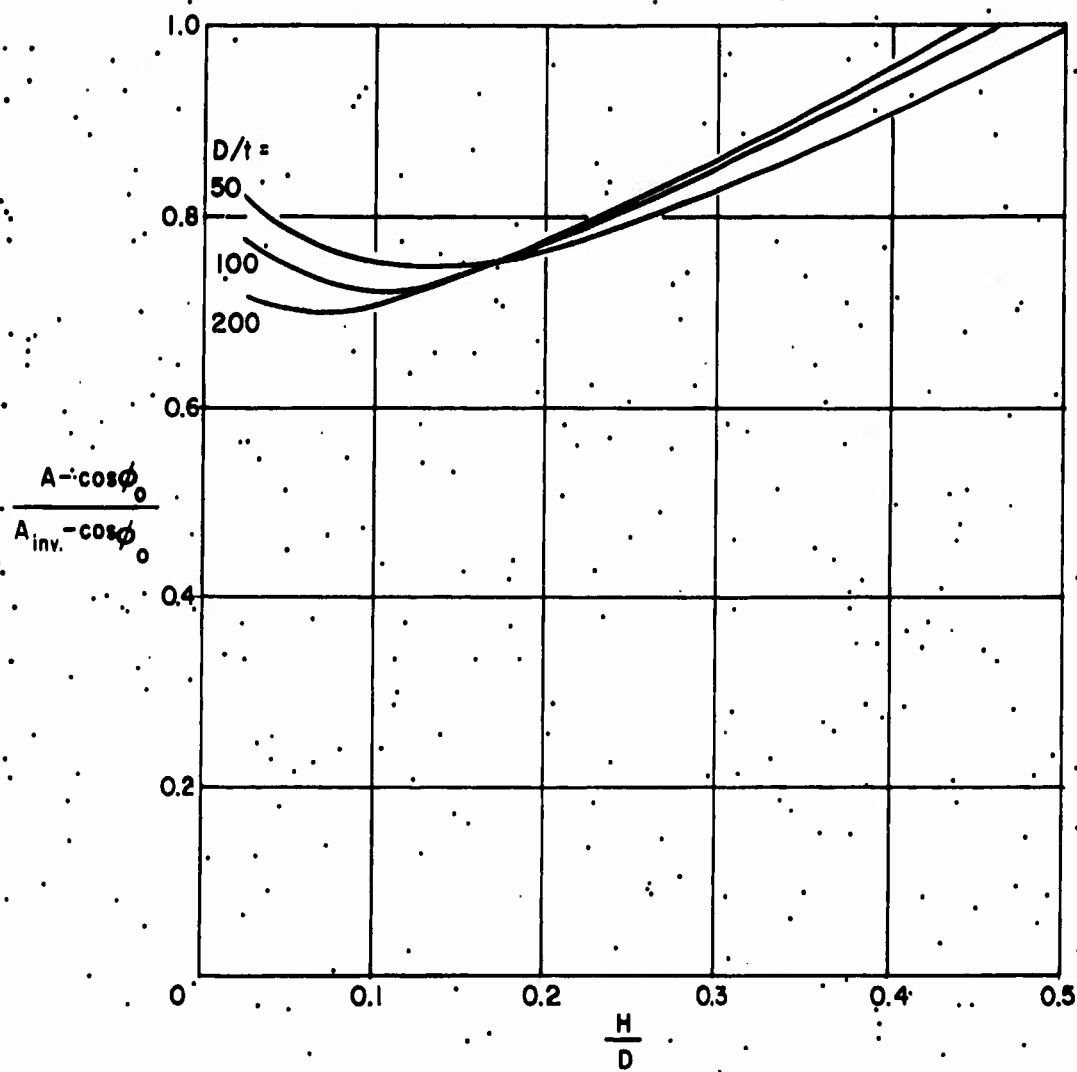


FIGURE 8c - VISCOUS EFFECTS ON AUGMENTATION, AXI-SYMMETRICAL
CASE, $\phi_0 = -45^\circ$

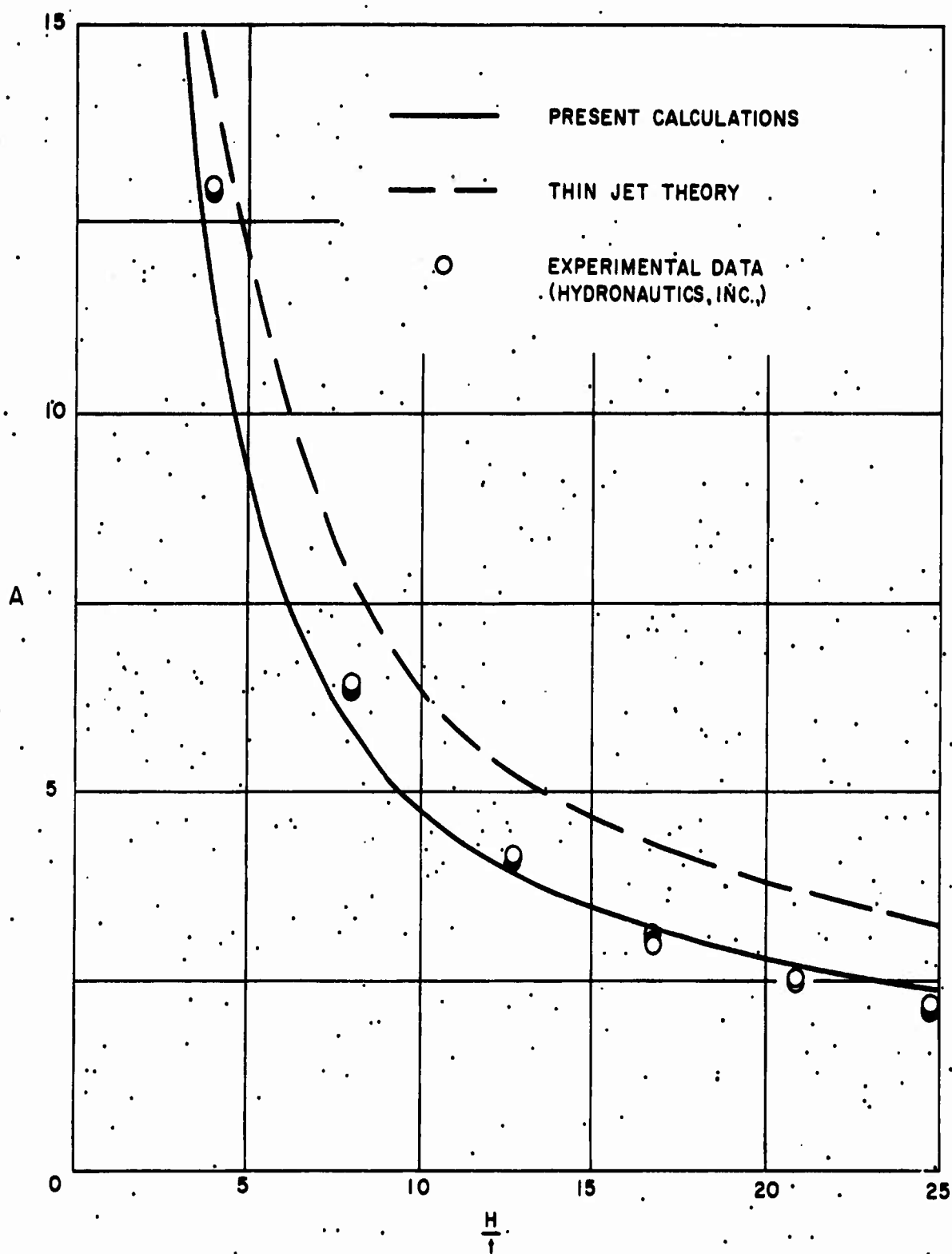


FIGURE 9- EFFECT OF GROUND PROXIMITY ON AUGMENTATION, $\phi_0 \approx -45^\circ$,
ASPECT RATIO = 160

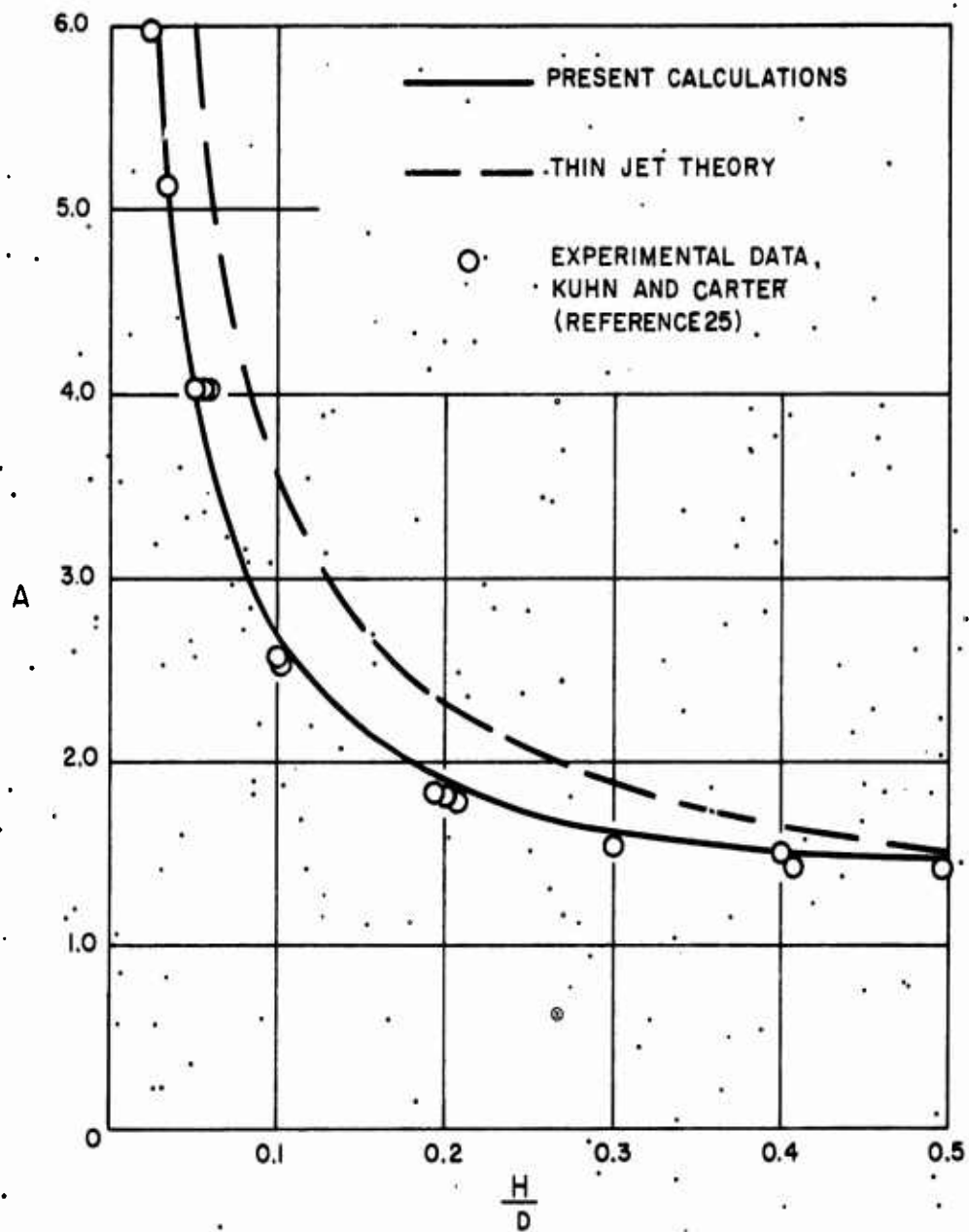


FIGURE 10- EFFECT OF GROUND PROXIMITY ON AUGMENTATION, $\phi_0 = 0^\circ$, $D/t = 27$.

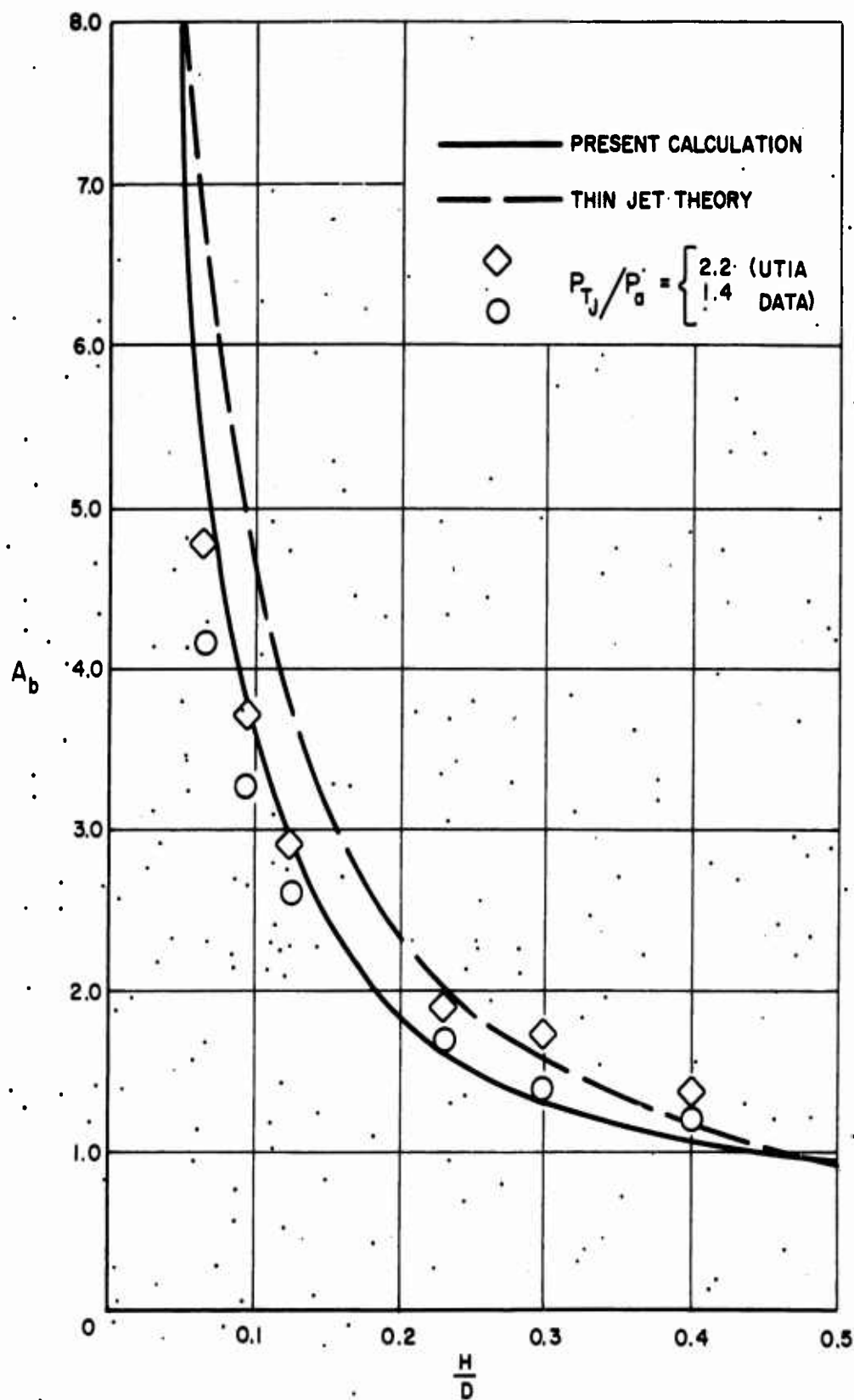


FIGURE 11a - EFFECT OF GROUND PROXIMITY ON BASE THRUST AUGMENTATION, $\phi_0 = -60^\circ$, $AR = 132$

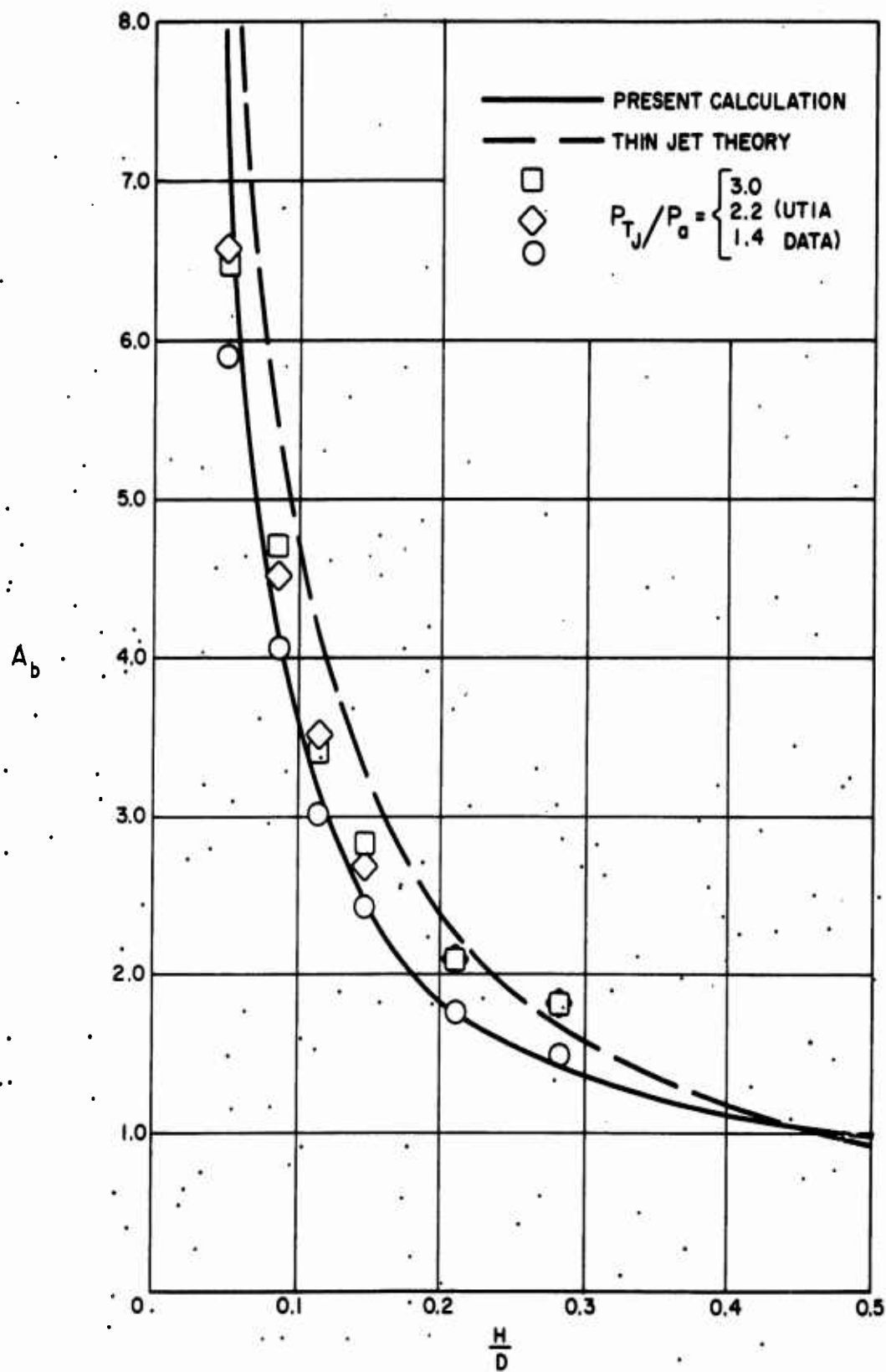


FIGURE 11b- EFFECT OF GROUND PROXIMITY ON BASE THRUST
 AUGMENTATION, $\phi_o = -60^\circ$, $AR = 263$

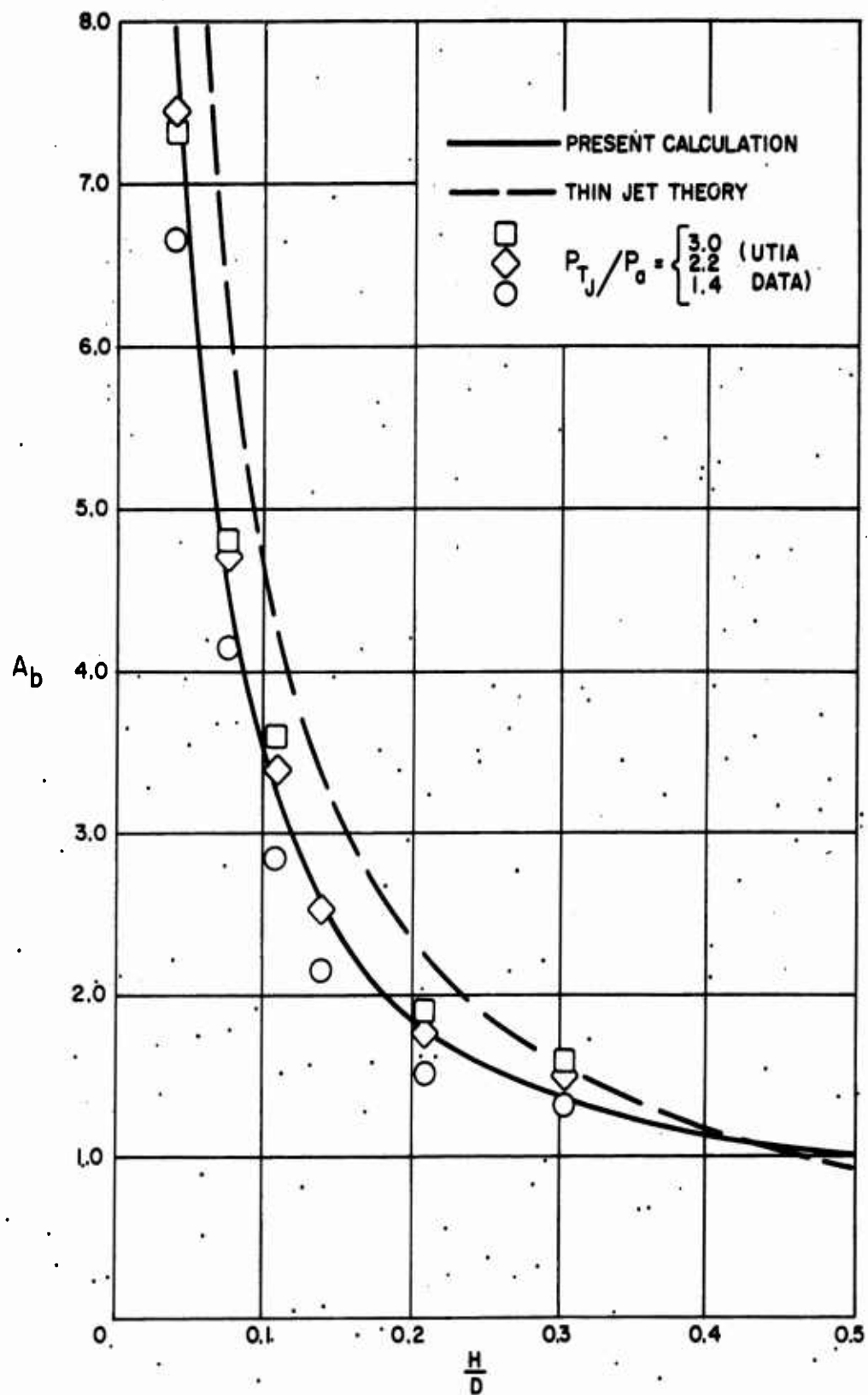


FIGURE 11c- EFFECT OF GROUND PROXIMITY ON BASE THRUST AUGMENTATION, $\phi_o = -60^\circ$, $AR = 524$

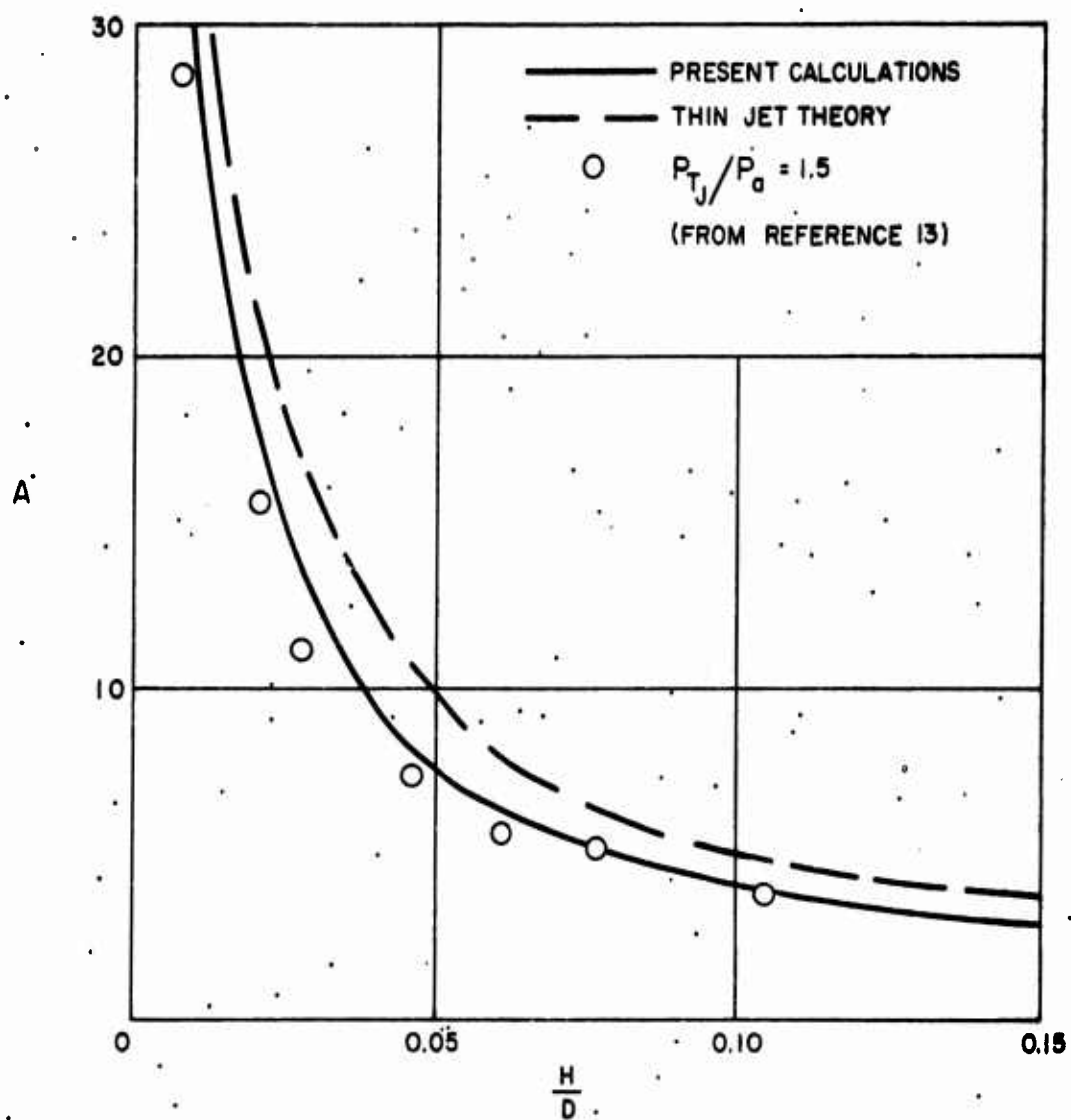


FIGURE 12 - EFFECT OF GROUND PROXIMITY ON AUGMENTATION, $\phi_0 = -45^\circ$
AR = 250.

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