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LECTURE NOTES  
on  
SHIP MOTIONS IN IRREGULAR SEAS  
by  
Edward V. Lewis  
and  
Rutger Bennet



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## LECTURE NOTES ON SHIP MOTION

### IN IRREGULAR SEAS.

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ABSTRACT

*(Two papers)*  
↓ Two papers are presented. The first is  
• The Superposition Principle Applied to the Prediction of Ship Behavior ,

An outline is given of the procedure usually followed in predicting the response of a ship to an irregular storm sea on the basis of the superposition of its responses to all the components which make up that sea. A non-dimensional form of presentation is then developed which makes it possible to visualize and interpret the results of comparative calculations for different ships.

*The second paper is*  
• Acceleration due to Vertical Motions of a Ship ,

The vertical acceleration of a point in a ship subject to pitching and heaving motions is considered by combining vectorially the pitching and heaving accelerations. A method of estimating the expected extreme values in irregular seas is then presented and compared with full-scale ship data.



THE SUPERPOSITION PRINCIPLE APPLIED TO THE

PREDICTION OF SHIP BEHAVIOR \*

By

Edward V. Lewis

Introduction

It has been a number of years since St. Denis and Pierson demonstrated the application of the superposition principle to the study of ship behavior in irregular seas. The original paper (1)\*\* was somewhat difficult, but subsequent work has clarified the application of the technique (2), verified its validity when applied to model and full-scale data (3) (4), and developed new aspects (5). It has been shown that the difficult case of the short-crested irregular sea can be conveniently treated in a simpler manner than originally proposed (6).

A typical plot is given in Fig. 1, showing how an oceanographer's sea spectrum plotted on the basis of frequency,  $\omega$ , can be converted to frequency of encounter,  $\omega_e$ , and then multiplied by a "response amplitude operator" (for pitch, bending moment, etc.) to obtain a response spectrum, assuming a long-crested sea. The response spectrum, of course, provides the basis for determining various statistical properties of the ship response. The reason for working on the basis of frequency of encounter has been that model tests in regular waves show the peaks of the response curves (pitch or heave) occurring in the vicinity of the ship's natural frequency of oscillation.

If a short-crested sea is considered, a family of wave spectra is needed to indicate the energy in different angular bands of wave components; there will be a separate response amplitude operator curve for

\* Presented to the Seakeeping Session, 10th International Towing Tank Conference, London, September 1963.

\*\* See references listed at the end of the paper.

each incremental angle; and there will be a family of response spectra. The latter can be integrated over direction to obtain a single response spectrum whose area and shape again define the statistical characteristics of the response (6).

As the results of systematic model tests have become available (7), it is now for the first time possible to predict trends of ship behavior in realistic sea conditions (8). However, it has also become clear that the oceanographers' way of plotting spectra as a function of frequency makes it difficult to understand and interpret the results of the calculations. In comparing the behavior of ships of different hull form, proportions, sizes, and speeds in irregular seas, the present system submerges all of these factors in such a way that their individual effects cannot be separated and significant comparisons cannot be made.

For example, Fig. 1 shows the comparison of a 250 ft. and a 500 ft. ship, geometrically similar and operating at the same Froude Number in the same seaway. This form of plotting involves first of all a difference in the sea spectra for each ship when plotted on the basis of frequency of encounter,  $\omega_e$ . This is because the two ship speeds are not the same at constant Froude Number. (If, on the other hand, the ships were compared at the same absolute speed, then completely unrelated response operators would have to be used).

The figure also shows that the response operators have a very different appearance, even when the ships are geometrically similar and the Froude Numbers are the same. Hence, although the final comparison of response spectra is significant, there is no way to judge the results or to account for the differences displayed.

Furthermore, a simple change of ship speed for one ship requires a transformation of the sea spectrum and makes it difficult to predict the result. An obvious modification is to plot on the basis of  $\omega$  (9). In effect this means the elimination of the influence of ship speed on frequency, and the use of a frequency scale then loses its significance. A completely new form of presentation is clearly needed, oriented to the needs of the designer.

### Non-dimensional Representation

In order to offer a solution to the problem of presenting data in a form suitable for the use of ship designers, a non-dimensional scheme will be presented step by step and some interesting conclusions drawn therefrom. Strictly speaking the proposed scheme is not completely non-dimensional. The term is used here only in the sense that geometrically similar ships will have similar responses to an ideal sea-way having the same component slopes at all frequencies.

First of all, we may recognize that the parameters describing ship performance should be non-dimensional. Pitch is a satisfactory measure of angular motion, for it will be the same for ships of different size in comparable situations. But heave or vertical bow motion,  $s$ , should be non-dimensionalized. This can best be done by relating these quantities to a ship dimension such as length,  $L$ , giving a ratio such as  $s/L$ . This means that we consider two ships to have equivalent heaving behavior in comparable conditions if the ratios of heave amplitude to ship length are the same. (This is in contrast to the conventional procedure of comparing heave amplitudes directly).

Once we adopt a non-dimensional response parameter (an angle or a ratio) the most significant wave characteristic to use is the wave slope, which is also non-dimensional. This means that for the present purpose a wave slope spectrum would be more significant than an amplitude spectrum. Such a slope spectrum would show the relative amplitude/length ratios of component waves, and it will be shown that it can assist greatly in visualizing and comparing ship behavior in an irregular sea. Strictly speaking maximum wave slope is  $2\pi\zeta_0/\lambda$ , where  $\zeta_0$  is wave amplitude,  $2\zeta_0$  is wave height, and  $\lambda$  is wave length. Here the term wave slope is intended to mean the ratio  $\zeta_0/\lambda$ .

The question then arises as to what basis for plotting sea spectra and ship response would be more suitable than wave frequency or frequency of encounter? Consideration of the behavior of ship models in regular waves leads to the realization that the wave length,  $\lambda$ , in relation to ship length,  $L$ , is of equal importance to frequency of encounter in determining a model's response to waves. Furthermore, one could

make a non-dimensional plot of response on the basis of the  $\lambda/L$  ratio. Such a diagram would apply to any size of ship or model, provided the Froude Number was constant. However, for predicting ship behavior in actual storm seas, which are absolute in size, we must have a scale which directly represents wave frequency or length, not a ratio.

A little experimentation shows that a suitable scale is  $\log_e \lambda$ . If at the same time we use a non-dimensional parameter for ship response operator, such as pitch angle/wave slope, we find that the response operator curves for geometrically similar ships of different size (at the same Froude Number) will be identical in shape, but separated by an amount equal to  $\log_e L_1/L_2$  (Fig. 3 (b)). Furthermore, points at corresponding values of  $\lambda/L$  will have the same ordinates.\*

The reason that a log scale makes the response operators identical for different sizes of ships is that it makes the increments  $\delta(\log_e \lambda)$  the same in terms of  $\lambda/L$  for different sizes. For example, the increment of  $\log_e \lambda$  from 1.0L to 1.1L is the same distance on the log scale for a 250 ft. ship as for a 500 ft. ship. (The distance is  $\log_e 1.1L - \log_e 1.0L = \log_e 1.1$ ). It will be shown that this is a very desirable characteristic in dealing with wave spectra.

The relationship between this proposed system and the old can now be derived. Since  $\lambda = c/\omega^2$ , where  $c = 2\pi g = 202$  (ft., sec. system), we have:

$$\log_e \lambda = \log_e c - 2(\log_e \omega) \quad \text{AND}$$

$$\delta(\log_e \lambda) = \frac{d \log_e \lambda}{d \omega} \delta \omega = -\frac{2}{\omega} \delta \omega, \quad \text{OR } \delta \omega = -\frac{\omega}{2} \delta(\log_e \lambda).$$

Hence, when we make  $\delta(\log_e \lambda)$  constant in the new system, the corresponding increments  $\delta \omega$  in the old system vary with frequency.

Typical response amplitude operators on the proposed basis are shown in

\*When this paper was presented at the 10th International Towing Tank Conference, London, September 1963, it was pointed out by Dr. Y. Yamanouchi of the Ship Research Institute, Tokyo, that the plot could also be based on  $\log_e \omega$  instead of  $\log_e \lambda$ . This has a precedent in other analogous work in the field of communication theory and might be preferred by some.

Fig. 3 (b) for pitching motion. Note that, although the curves for different sizes of ships are geometrically similar, they both fair into a value of 1.0 and extend to infinity. This means simply that in very long waves maximum pitch angle is equal to maximum wave slope.

The next question is what form will the wave spectrum take when it is plotted on the basis of  $\log_e \lambda$  instead of frequency. The answer is that the new spectrum  $[r(\log_e \lambda)]^2$  is related to the conventional energy spectrum  $[r(\omega)]^2$  (in Pierson's notation) thus:

$$[r(\log_e \lambda)]^2 = -\frac{\omega}{2} [r(\omega)]^2$$

See Fig. 2 (b). (The minus sign merely indicates a reversal of positive direction.) To prove the above relationship it is simply necessary to establish that differential areas (representing squares of component wave amplitudes) are the same in the two systems. Writing a differential area in the new system, we have,

$$[r(\log_e \lambda)]^2 \delta(\log_e \lambda).$$

Making substitutions from above leads to

$$\begin{aligned} [r(\log_e \lambda)]^2 \delta(\log_e \lambda) &= \\ -\frac{\omega}{2} [r(\omega)]^2 \left(-\frac{2}{\omega} \delta\omega\right) &= \\ [r(\omega)]^2 \delta\omega & \end{aligned}$$

The latter is a differential area in the original system, and thus the requirement is satisfied. (See Figure 2). Consequently the area under the two spectra must be the same; i.e.:

$$\int [r(\log_e \lambda)]^2 d(\log_e \lambda) = \int [r(\omega)]^2 d\omega$$

In order to permit the direct comparison of ships of different size, this form of wave spectrum is still not satisfactory. For this purpose, as previously noted, the important consideration is the slope, or amplitude/length ratio, of component waves. Hence, it is of interest to divide points on the spectrum by the appropriate wave length squared, in order to obtain a slope spectrum. See Fig. 2 (c). Thus a differential area,

$$\left[ \frac{r(\log_e \lambda)}{\lambda} \right]^2 \delta(\log_e \lambda) = \frac{[r(\log_e \lambda)]^2 \delta(\log_e \lambda)}{\lambda^2}$$

represents the component wave slope squared -- except for a factor  $(2\pi)^2$  -- i.e. (amplitude/length)<sup>2</sup>.

Expressing the above slope spectrum in terms of  $\omega$  we obtain,

$$\left[ \frac{r(\log_e \lambda)}{\lambda} \right]^2 = -\frac{\omega}{2} [r(\omega)]^2 \frac{1}{\lambda^2} = -\frac{\omega^5}{2c^2} [r(\omega)]^2$$

In other words, a spectrum which is proportional to  $1/\omega^5$  when plotted on the basis of  $\omega$ , would be a horizontal straight line on the new proposed basis and would represent an ideal sea in which all wave components in terms of  $\log_e \lambda$  had the same slope. Or strictly speaking the mean square slopes of all the components in any increment of  $\log_e \lambda$  would be the same. It is of interest to note that there is some evidence that the wave spectrum  $[r(\omega)]^2$  should be proportional to  $1/\omega^5$  under the influence of a strong steady wind -- particularly at the high frequency end (10). Actual sea spectra in strong winds are found generally to be less steep than this (11), as well as to fall off rapidly at low frequencies (long wave components). (See the example plotted in Fig. 2). But the spectrum proportional to  $1/\omega^5$  is of particular interest as an upper limit of sea spectra, in which we can determine how geometrically similar ships behave. Note that the sharp peak in Fig. 2a disappears in 2c (arrows).

Finally, we may multiply the wave slope spectrum (Fig. 3a) by the pitch response operators (Fig. 3b) to give the non-dimensional response spectra (Fig. 3c). These non-dimensional response spectra are of direct quantitative significance, since they represent (pitch amplitude)<sup>2</sup>, and the mean pitch amplitude will be a function of the area under the curves.

Comparing the two responses, we can see that in this case the shape of the sea spectrum has little to do with the result. For even if the sea spectrum were a horizontal line (constant slope) there would be little difference in response above  $\log_e \lambda = 6.5$ . Hence, the larger pitching response of the small ship results almost entirely from its smaller size -- as indicated by the horizontal separation of the response curves -- which adds the shaded area to the response spectrum.

A similar treatment of heaving motion leads to the rather startling results shown in Fig. 4. Again the response operators are geomet-

rically similar and displaced horizontally by a constant amount ( $\log_e L_1/L_2 = \log_e 2$ ). But in this case they increase without limit as  $\log_e \lambda$  increases. This puts an even more severe penalty on the smaller ship, no matter what the shape of the sea spectrum.

However, it must be recognized that heave amplitude is in itself of no great significance in ship behavior and that the associated vertical accelerations may be of much greater importance. Accordingly, a plot has been prepared of non-dimensional heave acceleration,  $a/g$ . (Fig. 5). In this case the heave acceleration operators,

$$\left(\frac{a/g}{\phi_0/\lambda}\right)^2$$

must fall into curves representing amplitudes of vertical wave acceleration at very long wave lengths. (Value at infinity is  $4\pi^2$ ).

The resulting response curves (Fig. 5) are surprisingly similar to those obtained for pitch (Fig. 3). And the same conclusion can be drawn: the larger response of the small ship results almost entirely from its smaller size. And, again, a smaller ship will always be worse off than a similar larger ship in most typical seas.

Usually the vertical acceleration due to the combined effect of heave and pitch at a particular location in the ship is of more importance than heave acceleration. But this can be plotted in the same way and can be expected to give a similar picture to Fig. 5.

Another aspect of ship motion that is usually of even greater significance than pitch angle is the motion of the bow relative to the water surface -- or simply "relative bow motion." (See Fig. 6). This can be handled in the same manner as other motions, but there is a significant difference. As indicated in Fig. 7 the response amplitude operator for relative bow motion tends to zero at moderately large values of  $\log_e \lambda$ . Hence, it can be seen that the comparative performance of ships of different size depends mainly on the shape of the sea spectrum and not on ship size, per se.

If one imagines the ideal limiting case of a horizontal slope spectrum, the behavior of the two ships would be identical. This is a case of truly non-dimensional behavior in irregular seas and contrasts sharply with the situations previously discussed of pitch angle and

heave acceleration. If the sea spectrum corresponds to a decreasing slope, the large ship will have a definite advantage over the small ship. In the general case, the comparative performance of two similar ships will depend mainly on the values of the spectrum ordinate at the peaks of the two response operators (approximately at  $\log_e \lambda = \log_e 1.2 L$ ).

Finally, consideration should be given to the response of a ship in terms of wave induced bending moment. It is convenient to use a non-dimensional coefficient,

$$\frac{h_e}{L} = \frac{\text{Bending Moment}}{c \rho g L^3 B C_{wp}}$$

where  $h_e$  is "effective" wave height, i.e. the height of a wave of ship length which would give the desired bending moment by conventional static calculation.  $B$  is the beam,  $C_{wp}$  is the waterplane coefficient,  $\rho$  is density,  $g$  is the acceleration of gravity, and the coefficient  $c$  is the static bending moment coefficient in the relation,

$$\text{Static bending mt.} = c \rho g L^2 B h C_{wp}$$

Then the bending moment response operator may be expressed in non-dimensional form as

$$\left( \frac{h_e/L}{z_0/\lambda} \right)^2$$

From the sample plot in Fig. 8 it may be seen that the wave spectrum shape has a large effect on the response spectrum as in the case of relative bow motion.

The same type of plotting can also be applied for the case of directional sea spectra. Instead of a single wave spectrum for each case, there will be a family of them representing different component wave directions. Similarly there will be a family of response operator curves and a family of response spectra. The mean square amplitude of response will be a function of the double integration of this family of curves.

### Conclusions

Reviewing the proposed system of representation for sea spectra and the response of different ships thereto, the following features may be noted:

1. The sea spectrum is presented in a manner which shows clearly the relative importance of different components in relation to ship behavior. It shows directly the relative slopes (squared) of the component waves, which are more significant for ship motion than wave energy or amplitudes, per se.
2. The  $\log_e \lambda$  scale provides a system in which the response amplitude operators for geometrically similar ships are identical in shape but shifted along the axis in accordance with absolute size, whereas a change in hull form or proportions will show as a change in shape of the response operator. A direct comparison of response operators for different ships plotted in this manner will permit a judgment to be made of the relative merits of the ships.
3. The plots of the final response spectra give a direct visual indication of the performance of different ships or of the same ship at different speeds. Furthermore, one can see by a glance at the other curves whether the superior performance of one ship is the result of better motion characteristics (response operator curves) or of its larger size.

It has been shown that for relative bow motions the ideal limiting case of a constant slope spectrum results in identical responses for geometrically similar ships of any size, i.e. a truly non-dimensional irregular wave situation. Actual sea spectra do not have such constant slopes, except perhaps at the short wave length (high frequency) end. Nevertheless, plotting of actual sea spectra on this basis assists in showing the relative effect on ship behavior to be expected from different wave length components.

In the case of pitching and heaving it has been shown that size in itself has a tremendous effect on the performance of geometrically similar ships, regardless of the shape of the sea spectrum.

The proposed method of presentation does not change the calculated prediction of ship behavior in irregular seas in any way. But it is hoped that it will aid in the understanding and interpretation of the superposition technique and thereby will encourage its use among Naval Architects and research people.

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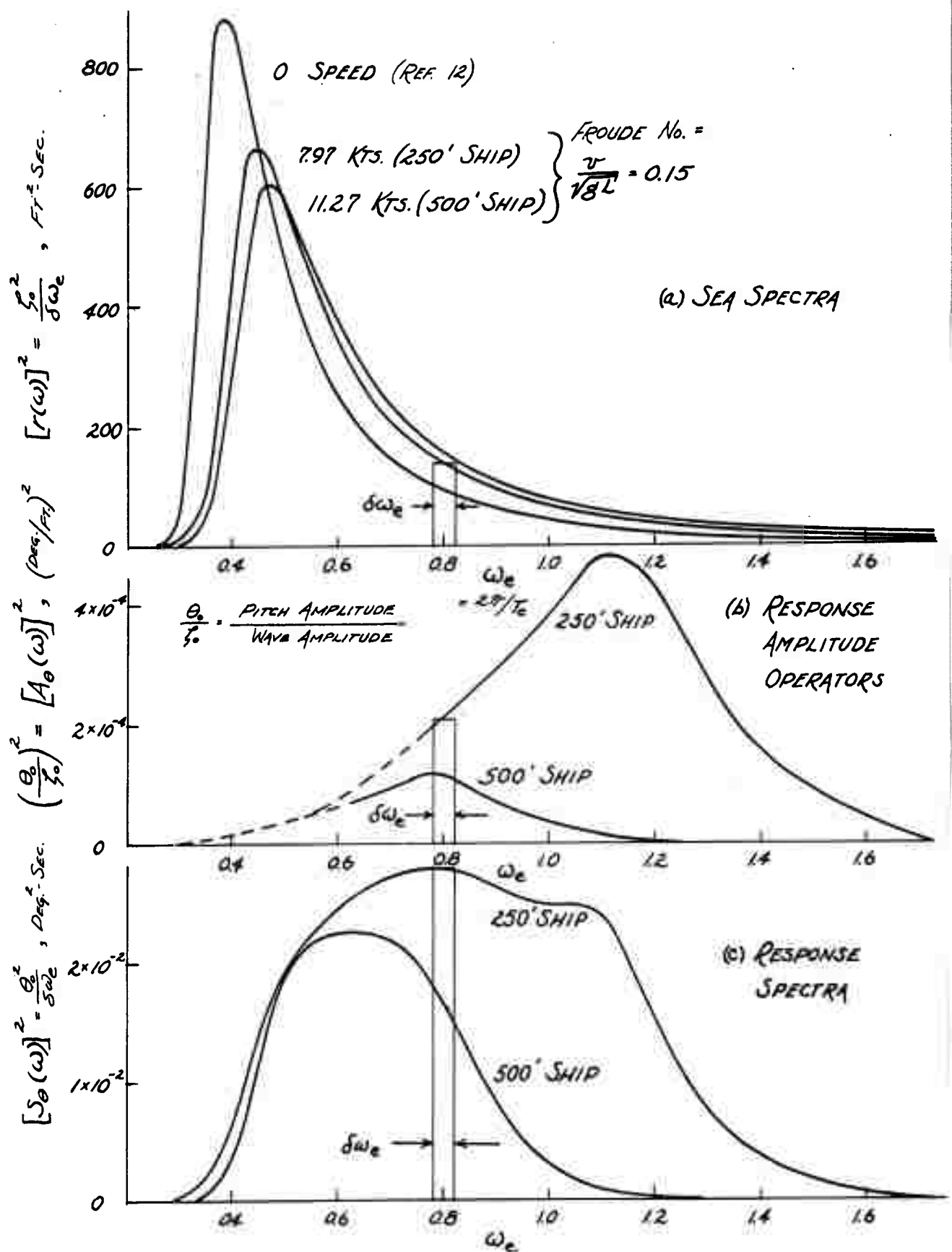


FIGURE 1: TYPICAL PITCHING RESPONSE TO IRREGULAR SEA

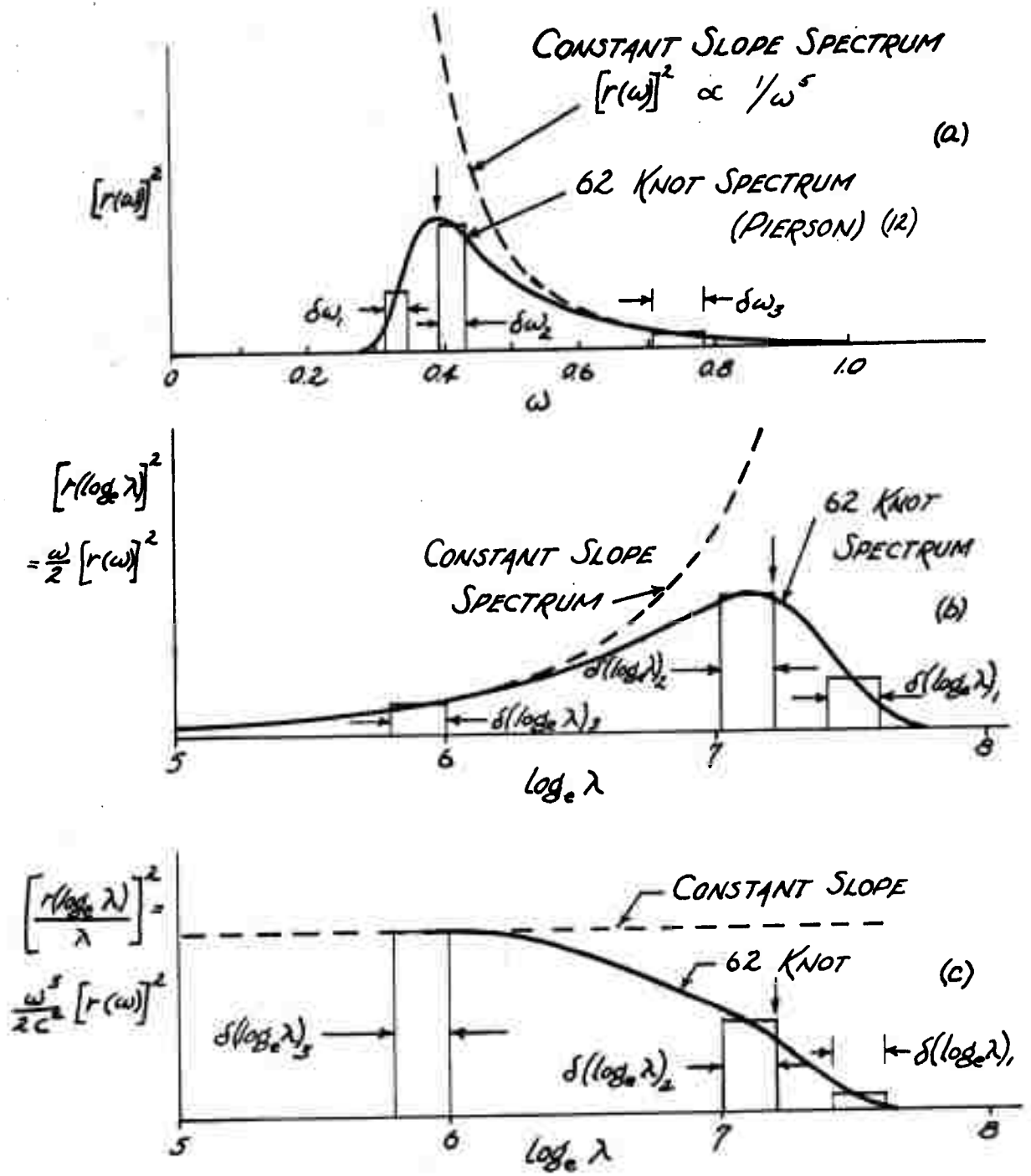


FIGURE 2: TRANSFORMATIONS OF SEA SPECTRA

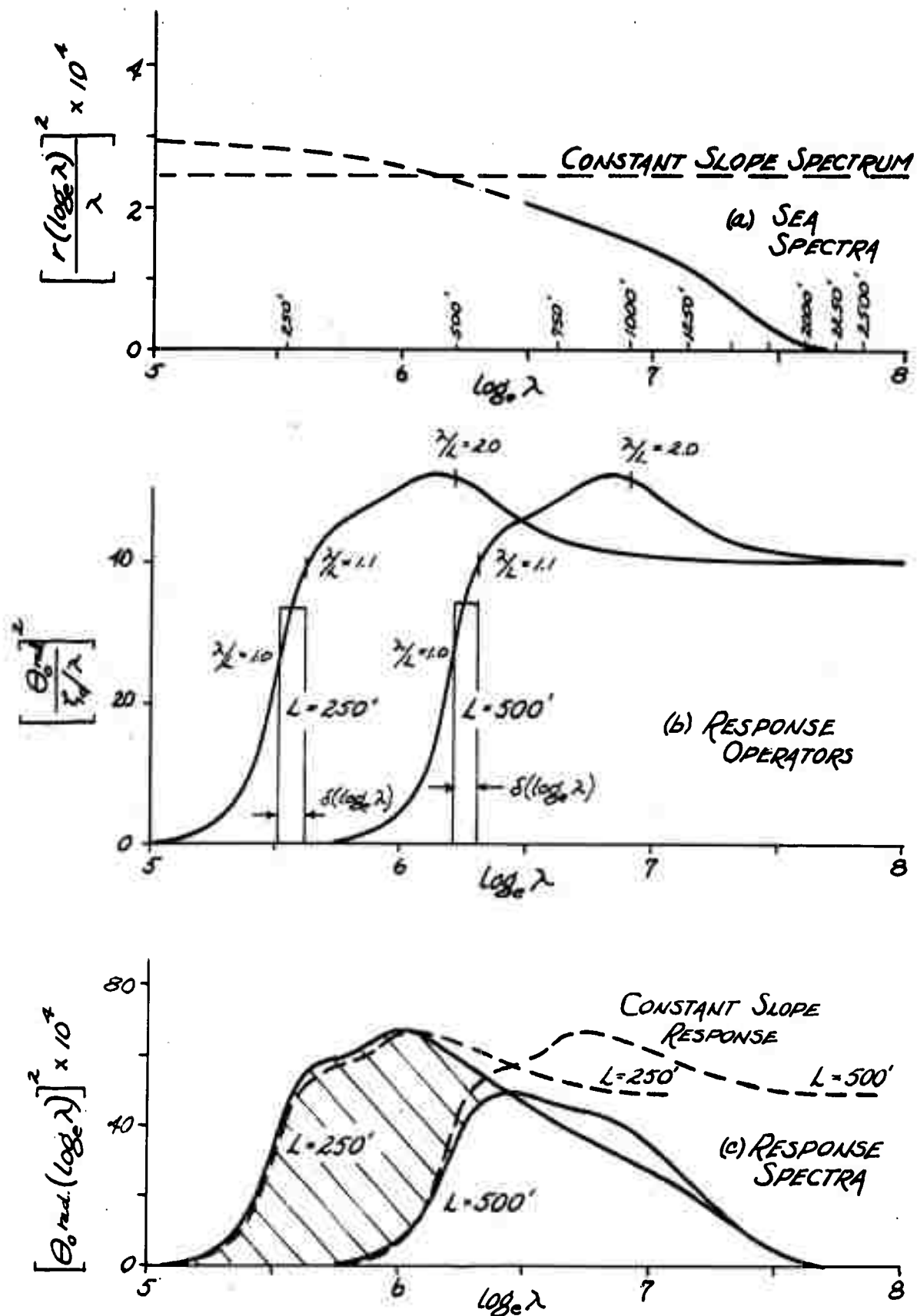


FIGURE 3: NON-DIMENSIONAL REPRESENTATION OF PITCHING RESPONSE TO IRREGULAR SEA

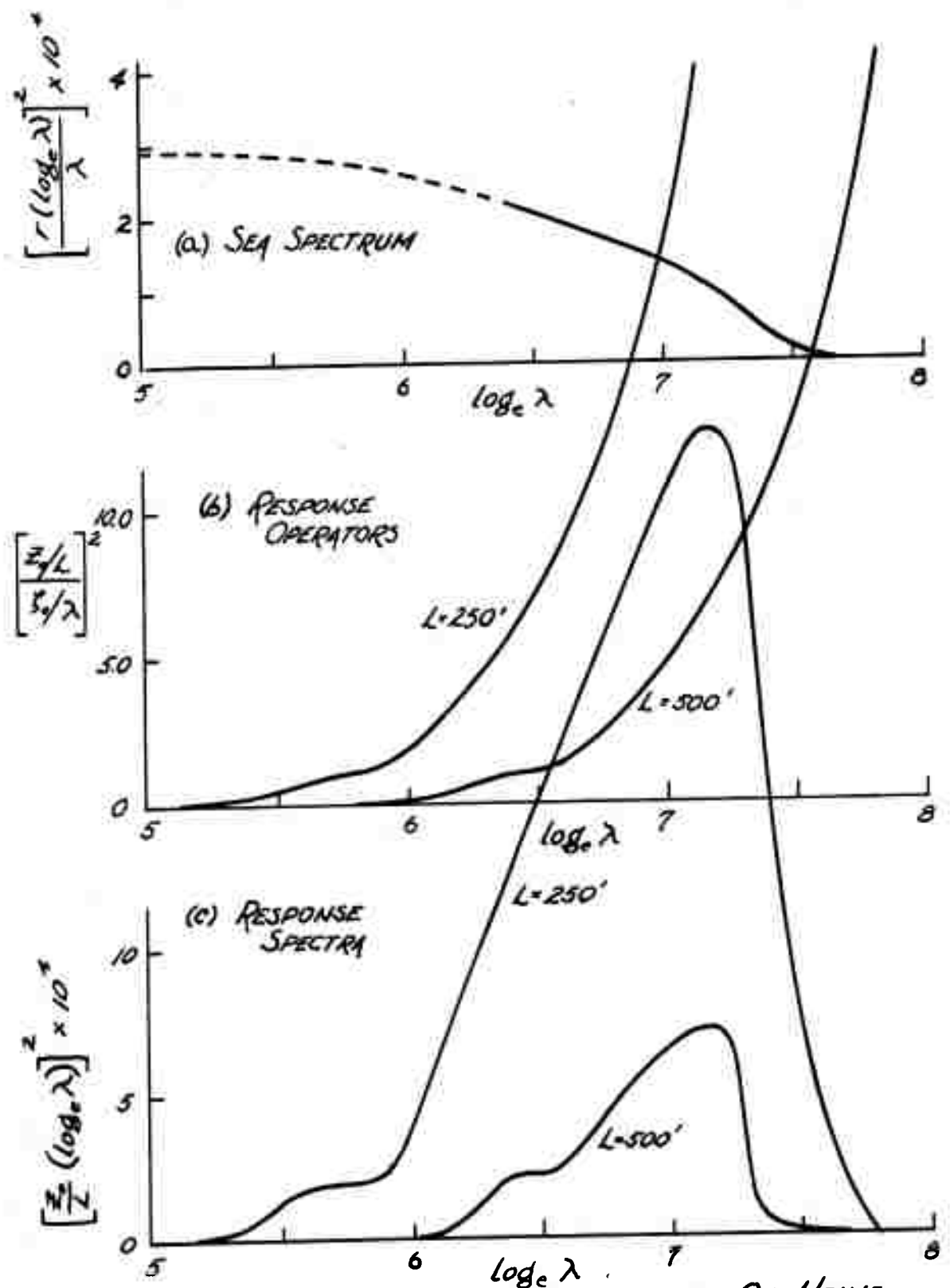


FIGURE 4: NON-DIMENSIONAL REPRESENTATION OF HEAVE RESPONSE TO IRREGULAR SEA

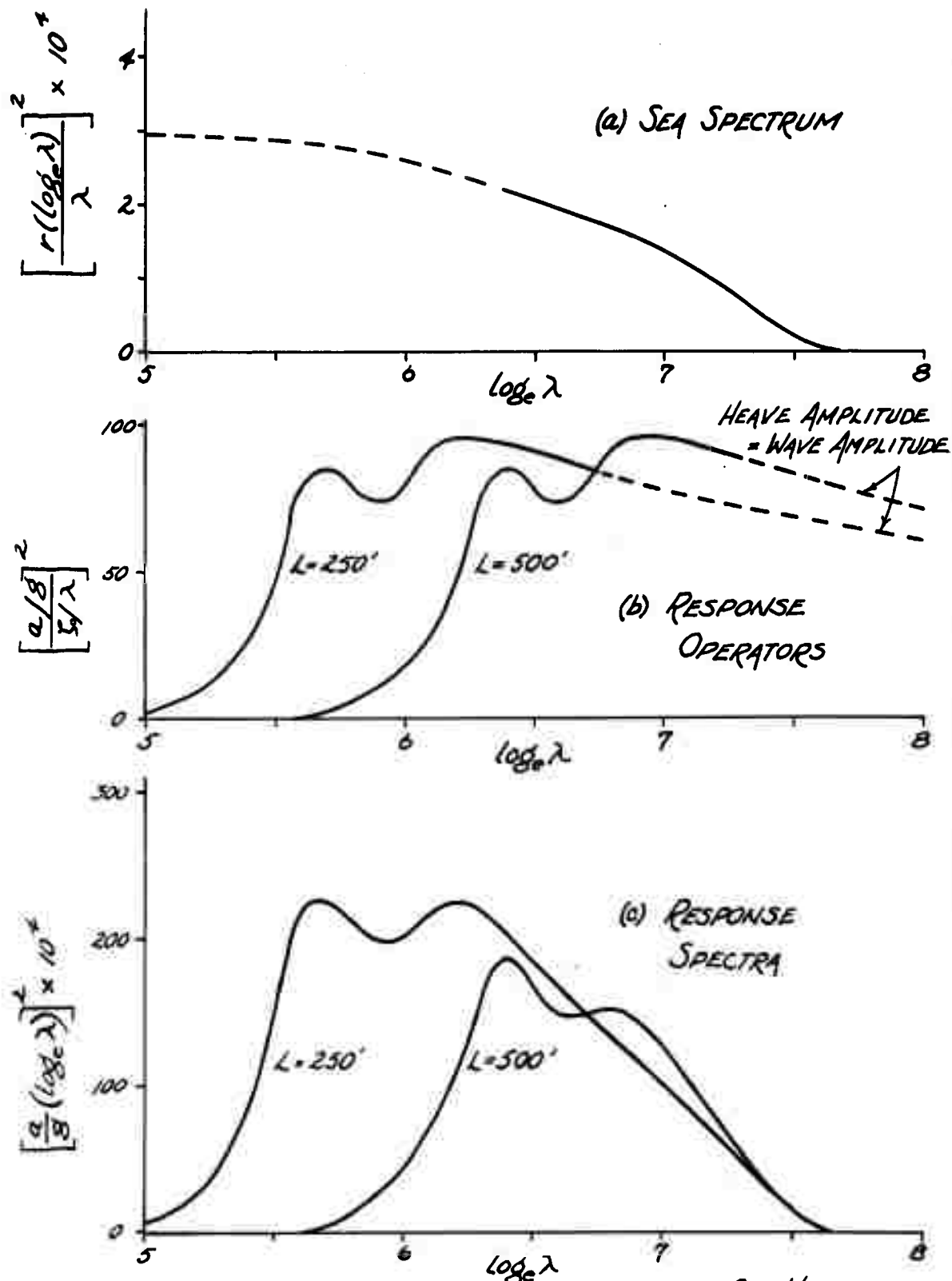


FIGURE 5: NON-DIMENSIONAL REPRESENTATION OF HEAVE ACCELERATION RESPONSE TO IRREGULAR SEA

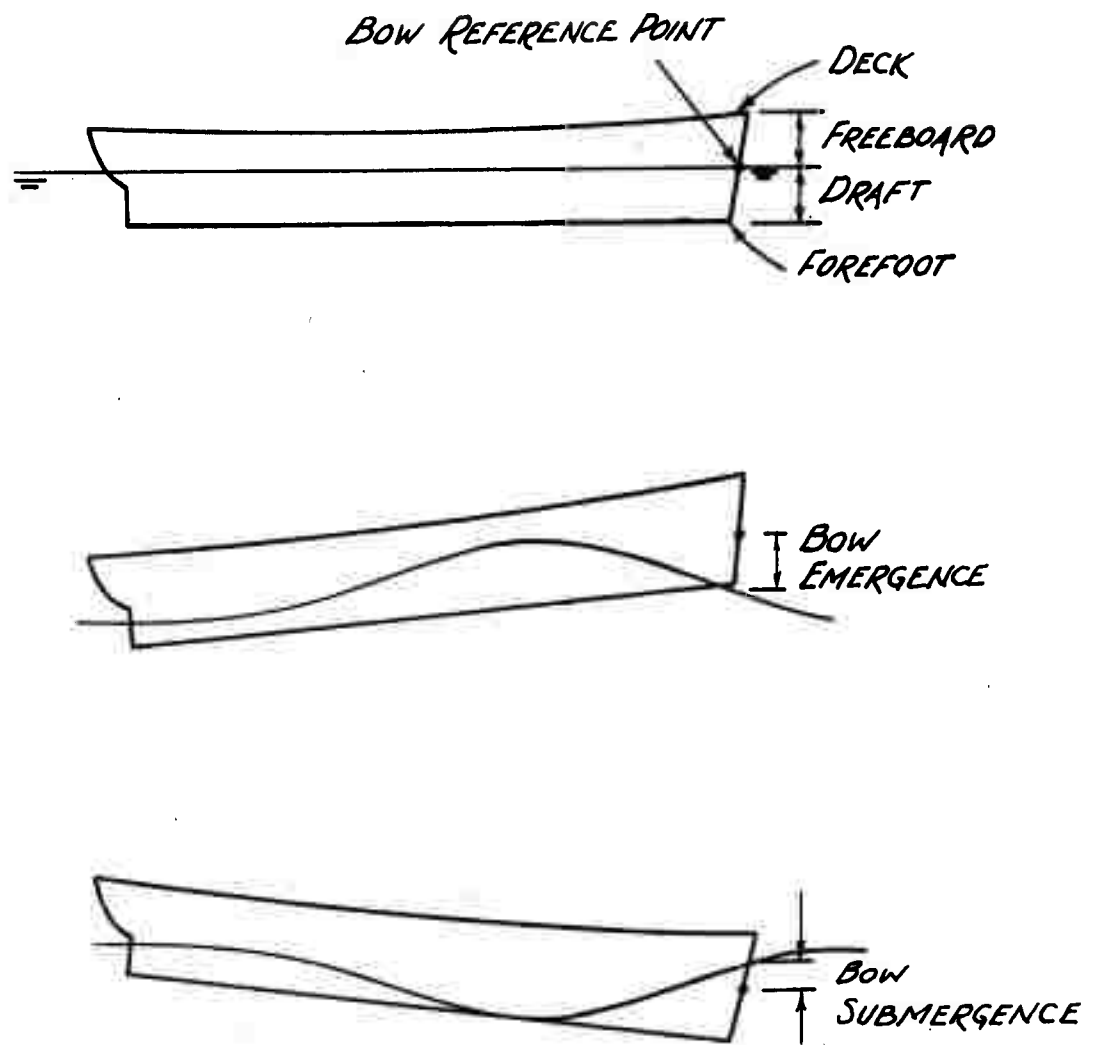


FIGURE 6 : SKETCH DEFINING RELATIVE BOW MOTION

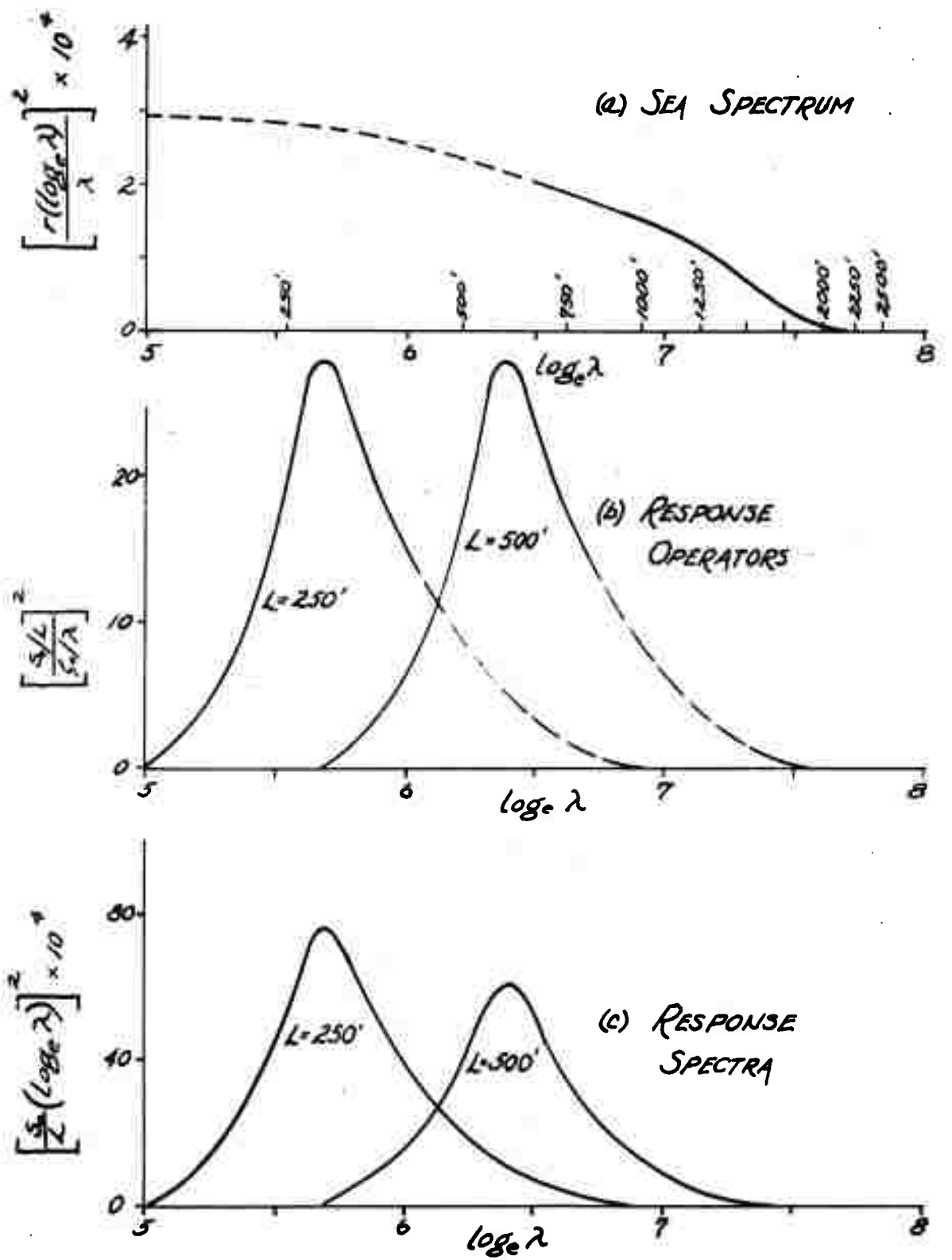


FIGURE 7: NON-DIMENSIONAL REPRESENTATION OF RELATIVE BOW MOTION RESPONSE TO IRREGULAR SEAS

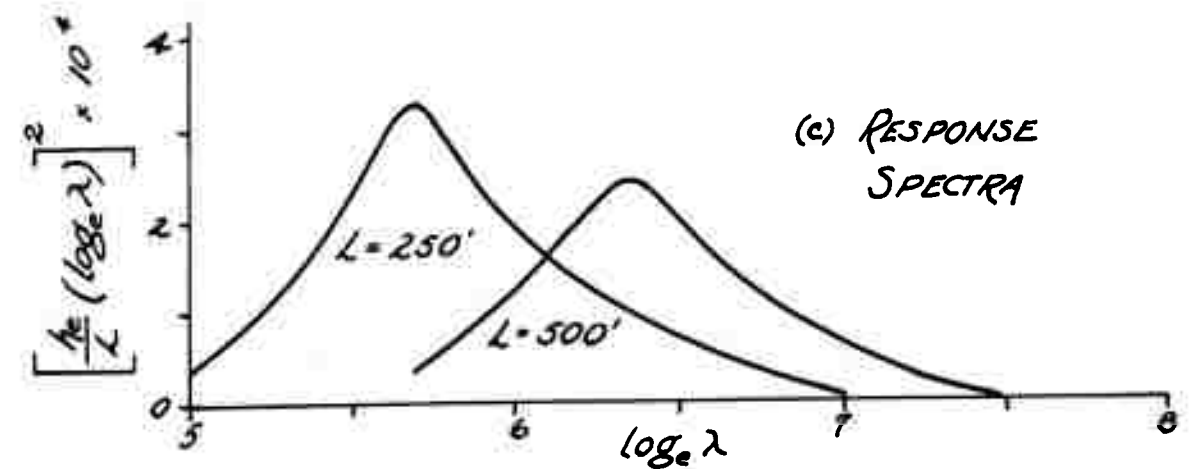
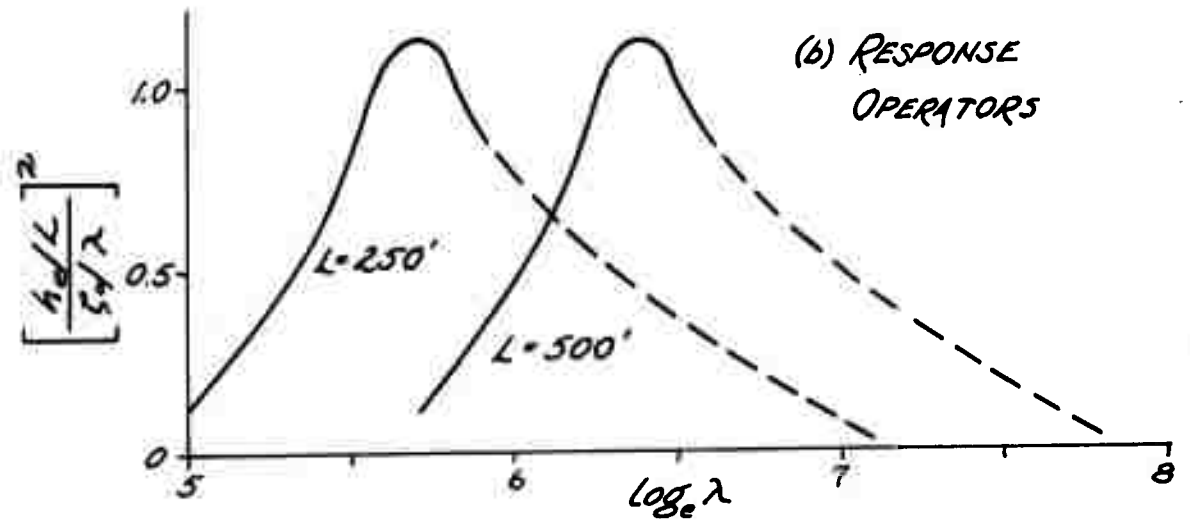
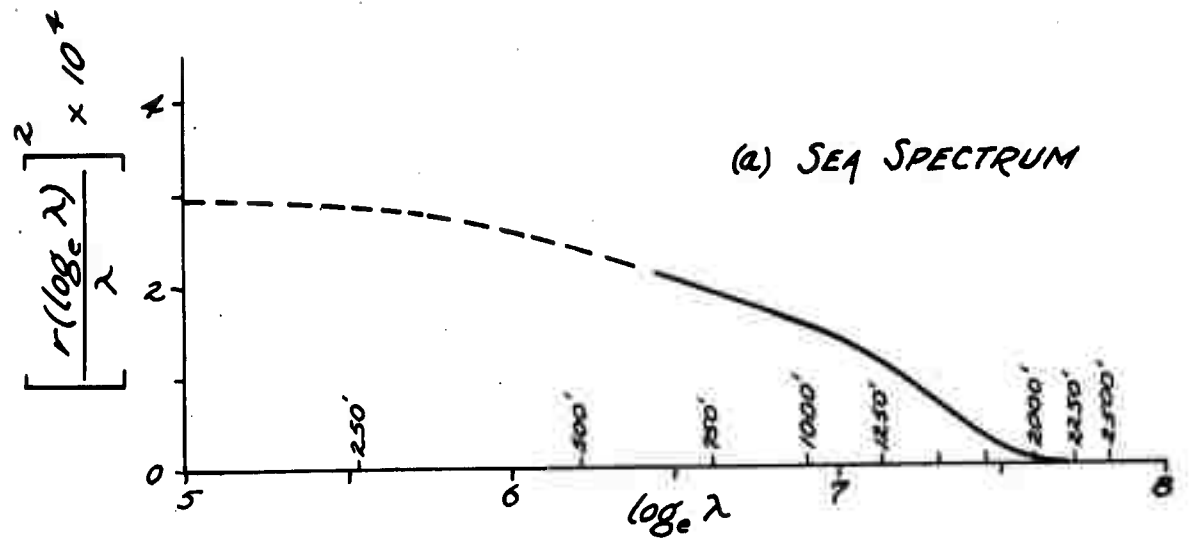


FIGURE 8: NON-DIMENSIONAL REPRESENTATION OF BENDING  
MOMENT COEFFICIENT,  $h_e/L$ , RESPONSE TO IRREGULAR



## ACCELERATION DUE TO VERTICAL MOTIONS OF A SHIP \*

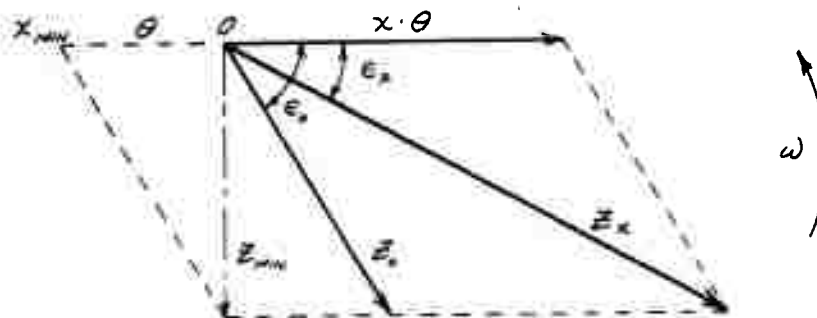
By  
Rutger Bennet

The vertical acceleration at a certain point in a ship is composed of components from three modes of motion: heaving, pitching and rolling. Rolling in general gives the least contribution, and the greatest part is caused by heaving if the point is near amidships, and by pitching if it is near the ends. The acceleration amplitude at any point is very much dependent on the phase angle between heaving and pitching.

### "Pitching Axis"

Considering only heave and pitch, the motion of a ship is considered to be made up of the vertical translation (heave) of the center of gravity,  $Z_0$ , and a rotation (pitch,  $\theta$ ). In general there is no point which during any finite length of time has zero velocity or acceleration; hence there is usually no real pitching axis.

The motion may be represented by a vector diagram as follows:



The vector  $x \cdot \theta$  is the vertical displacement at distance  $x$  from amidships at an instant when the angle is  $\theta$ . The various amplitudes

\* Based on Report No. 22 of the Swedish Shipbuilding Research Foundation, "Accelerationer pa Grund av Fartygs Rorelser i Sjo," by Rutger Bennet, 1961 (in Swedish).

are related by: \*\*

$$Z_x^2 = Z_0^2 + x^2 \theta^2 + 2 Z_0 x \theta \cos \epsilon_0 \quad (1)$$

where  $Z_x$  = amplitude of resulting motion at distance  $x$  from C.G.

$Z_0$  = heave amplitude at C.G. ( $x = 0$ )

$\theta$  = amplitude of angular motion

$\epsilon_0$  = phase lag, heave  $Z_0$  after pitch  $\theta$

The minimum amplitude of vertical motion,

$$\left. \begin{aligned} Z_{MIN} &= Z_0 \sin \epsilon_0 \\ \text{For } x_{MIN} &= -\frac{Z_0 \cos \epsilon_0}{\theta} \end{aligned} \right\} (2)$$

Szebehely (Ref. 1) calls this point of least vertical motion the "apparent pitching axis." For  $-\pi/2 < \epsilon_0 < \pi/2$ ,  $x_{MIN}$  is negative, meaning that the apparent pitching axis is aft of CG, which is usually the case.

If both motions are harmonic, the variables in equations (1) and (2) may be directly replaced by their first or second derivatives. Therefore, the point with the least vertical motion will also have the least vertical velocity and the least vertical acceleration.

#### Irregular Motions

In irregular waves, when motions are no longer harmonic, conditions become more complicated. The point with least acceleration will not in general have the least displacement, and none of the minimum points will be fixed but will move between plus and minus infinity. The distance  $x_{MIN}$  will vary according to a statistical distribution with mean value according to equation (2) (see Ref. 2). It is now generally

\*\* See summary of Notation at the end of this report.

accepted that all ship motions may be represented by stochastic processes which are Gaussian or normal. This means that the distribution of equally spaced values of  $\ddot{z}$  and  $\ddot{\theta}$  should be normal; thus:

$$p(\ddot{z}) = \frac{1}{\sigma_{\ddot{z}} \sqrt{2\pi}} \cdot e^{-\frac{\ddot{z}^2}{2\sigma_{\ddot{z}}^2}} \quad (3)$$

with a similar expression for  $p(\ddot{\theta})$ .  
The amplitudes should be Rayleigh distributed:

$$p(a) = \frac{2a}{R} \cdot e^{-\frac{a^2}{R}} \quad (4)$$

where  $a$  is the amplitude of any motion and  $R$  is the mean square amplitude.

At any point in the length of the ship, the sum of accelerations due to pitch and heave will also be normal, with Rayleigh distributed total amplitude. The parameters of the distributions will be functions of the correlation between pitch and heave (Ref. 2).

#### Energy Spectrum

Any irregular motion, e.g. pitching, may be written as a sum of a large number of regular components:

$$\theta(t) = \sum_{n \rightarrow \infty} a_n \cos(\omega_n t + \epsilon_n) \quad (5)$$

The energy spectrum is defined as the spectral density in a frequency interval  $\delta\omega$ :

$$E_{\theta} = \frac{\sum_{\delta\omega} a_n^2}{\delta\omega} \quad (6)$$

For each component the acceleration is the second derivative:

$$\ddot{\theta}_n(t) = -\omega_n^2 a_n \cos(\omega_n t + \epsilon_n) \quad (7)$$

The acceleration spectrum is

$$E_{\ddot{\theta}} = \omega^4 E_{\theta} \quad (8)$$

The variance of the statistical distribution is the area of the spectrum; and this is also equal to half the parameter R in eq. (4):

$$\frac{R_{\theta}}{2} = \sigma_{\theta}^2 = \int_0^{\infty} E_{\theta} d\omega \quad (9a)$$

$$\frac{R_{\ddot{\theta}}}{2} = \sigma_{\ddot{\theta}}^2 = \int_0^{\infty} \omega^4 E_{\theta} d\omega \quad (9b)$$

If the combined effect of pitch and heave on the motion of a point is known, then spectra of motion and of acceleration of the point can be obtained in the same manner. However, in many cases the pitch and heave motions are only known separately. It is of value, therefore, to develop approximate methods of estimating accelerations at a point from pitch and heave motion data, preferably without actually computing the spectra of either.

#### Angular Acceleration Deduced from the Angle

If every period of the motion in irregular waves were harmonic, the acceleration amplitude would be

$$a_{\ddot{\theta}} = \frac{4\pi^2}{T^2} a_{\theta} \quad (10)$$

If the period T could be considered as constant also in an irregular sea, there would be a simple relationship between the statistical parameter R (eq. 4) for pitch amplitude and acceleration:

$$R_{\ddot{\theta}} = \left(\frac{4\pi^2}{T^2}\right)^2 R_{\theta} \quad (11)$$

This is equivalent to  $\omega = \text{const.}$  in equation (9b).

But as the period T is varying, equation (11) will not hold in an irregular sea. It is necessary to apply a correction factor K, so that

$$R_{\ddot{\theta}} = K^2 \left(\frac{4\pi^2}{T^2}\right)^2 R_{\theta} \quad (12)$$

or, with  $r = \sqrt{R}$

$$\ddot{r}_0 = K \cdot \frac{4\pi^2}{T^2} \cdot r_0 \quad (13)$$

An investigation of measurements of pitching angle and angular accelerations in Ref. (4) has given values of K between 1.3 and 1.6. In ref. (5) K = 1.5 is given as a recommended value. If the spectrum has an equation of the same type as the Newmann wave spectrum, K = 1.5 is obtained exactly.

In Ref. (3) spectra of pitching angle for a cargo ship are presented, and for two of these the acceleration spectra have been derived according to equation (8). From the relation between the areas of the angle and acceleration spectra, K was found to be 1.2 and 1.35, respectively.

It may be concluded that the acceleration caused by an irregular pitching motion can be up to 50% greater than in a regular motion with the same amplitude.

#### Vertical Acceleration from Heave and Pitch

At a distance  $\chi$  from C.G. the amplitude of vertical acceleration according to eq. (1) is:

$$a_\chi = \sqrt{a_0^2 + \chi^2 a_{\ddot{\theta}}^2 + 2a_0 \chi a_{\ddot{\theta}} \cos \epsilon_0} \quad (14)$$

In terms of a known acceleration  $a_\chi$  at a point  $\chi = A$  this equation becomes

$$a_\chi = \sqrt{a_A^2 + (\chi - A)^2 a_{\ddot{\theta}}^2 + 2a_A(\chi - A) a_{\ddot{\theta}} \cos \epsilon_A} \quad (15)$$

where  $\epsilon_A$  is phase lag between heave and pitch at  $\chi = A$ .

The time phase angle is

$$\epsilon_0 = \text{ARCTAN} \frac{a_A \sin \epsilon_A}{a_A \cos \epsilon_A - A a_{\ddot{\theta}}} \quad (16)$$

In analyzing measurements it is an advantage to use mean values

instead of individual amplitudes. The mean square of the resulting amplitude is

$$\overline{a_x^2} = \overline{a_1^2} + (x-A)^2 \overline{a_\theta^2} + 2(x-A) \overline{a_1 a_\theta} \cos \epsilon_A \quad (17)$$

The last term is the mean of a three-dimensional distribution. If the three variables are independent, the mean of the product is the product of the means:

$$\overline{a_1 a_\theta \cos \epsilon_A} = \overline{a_1} \cdot \overline{a_\theta} \cdot \overline{\cos \epsilon_A} \quad (18)$$

Calculations and measurements have shown that the coupling between pitch and heave has only a small influence (Ref. 2); so eq. (18) may be assumed to be valid.

The Rayleigh distribution has a constant ratio between mean value  $\bar{y}$  and the mean square:

$$\bar{y} = \frac{\sqrt{\pi}}{2} \sqrt{\overline{y^2}} \quad (19)$$

where  $y$  is any Rayleighian variable.

Eq. (17) can then be written in the more useful form:

$$R_x = R_1 + (x-A)^2 R_\theta + 2 \cdot \frac{\pi}{4} (x-A) (R_1 R_\theta)^{1/2} \cos \epsilon_A \quad (20)$$

If the angular acceleration is not known, but only the angle,  $R_\theta$  is calculated from eq. (12) with  $K = 1.5$ .

#### Expected Extreme Values

The value that is expected to be exceeded with the probability  $Q$  is determined from the properties of the Rayleigh distribution:

$$a_x(Q) = V_Q r_x \quad (21)$$

where  $V_Q$  is a function only of  $Q$ . In ref. (6) it is shown that

$V_Q = 3.5$  is a reasonable extreme value for practical consideration. Theoretically it is the value which with 99% probability will only be exceeded once during 2000 cycles, corresponding to about four hours sailing time. This value will be used below to predict expected extreme accelerations.

#### Results of Measurements

Since actual data on observed accelerations are scarce, a summary may be given of available published information.

##### DTMB Full Scale Tests

Results of motion and acceleration measurements, carried out by David Taylor Model Basin on the SILVER MARINER, are published in Ref. (4). The observations were made at various speeds and headings in a fairly moderate sea with a significant wave height of 9.4 feet and a wind speed of 23-28 knots. The highest recorded heave acceleration at C.G. was 0.26 g, and during the same cycle the angular acceleration was 0.11 rad/sec.<sup>2</sup> with a phase angle of 40°. This was also the highest recorded value of  $\ddot{\theta}$ . The maximum root mean square amplitudes were:

$$\text{Max } v_{\theta} = 0.11 g$$

$$\text{Max } v_{\ddot{\theta}} = 0.049 \text{ rad./sec}^2$$

##### Japanese Model Tests

Model tests are published in Ref. (6) for three different ship types:

Tanker, Displacement 87,500 tons,  $C_B = .80$ ,

Cargo liner, Displacement 17,400 tons,  $C_B = .646$ ,

"Experimental Ship", Displacement 3,460 tons,  $C_B = .52$ .

The tests were made in regular waves, and vertical accelerations were measured at two points.

The superposition method was used to calculate the response to

a Neumann wave spectrum. Root-mean-square acceleration amplitude was determined from equation (9), and expected extreme values from eq. (21). Fig. 1 from Ref. (6) gives values which with 90% probability will only be exceeded once in 5000 cycles. This means  $\sqrt{Q} = 3.3$  in (21), which is close to the value 3.5 suggested above. The Japanese investigation was made in order to determine the best position in a ship for a nuclear reactor.

#### Swedish Tests

The Swedish Shipbuilding Research Foundation has measured stresses and motions in a cargo liner and an ore carrier, Ref. (3).

M/S CANADA is a 465', 19.5 knot cargo liner. Pitch angle and vertical acceleration at a point 42 feet aft of C.G. were measured. The highest recorded acceleration was 0.25g, and during that cycle the pitch angle was  $4^\circ$  with  $90^\circ$  phase lag. The highest recorded pitch angle was  $7.5^\circ$ , with acceleration 0.21 g, also with  $90^\circ$  phase difference. The period T was 6.5 sec. assuming  $K = 1.5$ , this means an angular acceleration of  $0.184 \text{ rad/sec}^2$ .

From eq. (15) and (16) is found:

Heave acceleration at C.G.	$a_0 = 0.32 \text{ g}$
Phase angle at C.G.	$\epsilon_0 = 41^\circ$
Angular acceleration	$a_{\ddot{\theta}} = 0.184 \text{ rad/sec}^2$

The distribution of this acceleration along the length of the ship is shown in Fig. 2, labeled "Max. recorded."

The same record gave the following root mean square values:

$$\begin{aligned} r_{\theta} &= 3^\circ \\ r_{\ddot{\theta}} &= 0.073 \text{ rad/sec}^2 \\ r_A &= 0.114 \text{ g} \end{aligned}$$

At C.G.	$r_{\theta} = 0.14 \text{ g}$	
At F P	$r_x = 0.58 \text{ g}$	Total vertical acceleration
At A P	$r_x = 0.37 \text{ g}$	)

The number of cycles in the record was 300, and the most probable highest value is then  $2.4r$  ( $V_q = 2.4$ ). Table I shows good agreement between theoretical and measured maxima. The table also shows the highest value expected in 2000 cycles according to eq. (21) ( $V_q = 3.5$ ). The longitudinal distribution is shown in Fig. 2.

A limited number of measurements was also made on a 560' ore carrier, M/S VASSIJAURE (Ref. 3). The results are given in Table II and Fig. 3.

These examples show how limited information on pitching and heaving motions will permit good approximate estimates to be made of maximum expected accelerations at a particular point.

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5. Buchmann and McConnell, "Environmental Conditions of Ship Motions and Vibrations for Design of Radar Systems on Destroyers and Air Craft Carriers," David Taylor Model Basin, Report 1298.
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TABLE I  
M/S "CANADA"

L = 465', B = 64', d = 26.7',  
C<sub>B</sub> = 0.65, Displ. = 14,500 ts.

	r	Theor. max ampl. during measurement	Maximum measured amplitude	Max. expected amplitude in 2000 cycles
Heave acceleration at C.G., g's	0.14	0.33	0.32	0.5
Pitch angular accelera- tion, rad/sec <sup>2</sup>	0.07	0.175	0.184	0.25
Vertical acc. 42' abaft C.G., g's	0.11	0.27	0.25	
Vertical acc. at FP, g's	0.58	1.40	1.60	2.0
Vertical acc. at AP, g's	0.37	0.90	1.07	1.3

TABLE II  
M/S "VASSIJAURE"

L = 560', B = 74.5', d = 31',  
C<sub>B</sub> = 0.78, Displ. = 29,000 ts.

	r	Maximum measured amplitude	Theoretical max. amplitude during measurement
Heave acceleration at C.G., g's	0.035	0.12	0.07
Pitch angular acceleration, rad/sec <sup>2</sup>	0.046	0.16	0.13
Vertical acc. at FP, g's	0.42	1.45	1.15
Vertical acc. at AP, g's	0.38	1.35	1.09

NOTATION

- $\theta$  Pitching angle  
 $Z_0$  Heave at C.G.  
 $Z_x = Z_0 + x \theta$  = Vertical displacement at a distance  
x from C.G.  
 $A$  x-coordinate of accelerometer  
 $\theta_0$  Amplitude of pitch angle  
 $Z_0$  Amplitude of heave at C.G.  
 $Z_x$  Amplitude of heave at x from C.G.  
 $a$  Amplitude of motion or acceleration  
 $a_\theta$  Amplitude of angular acceleration  
 $a_0$  Amplitude of heave acceleration at C.G.  
 $a_x$  Amplitude of heave acceleration at x from C.G.  
 $\omega$  Angular velocity  
 $\epsilon_0$  Phase lag, heave after pitch at C.G.  
 $\epsilon_x$  Phase lag, at x from C.G.  
 $\sigma^2$  Variance of a statistical variable  
 $R$  Mean square amplitude  
 $r = \sqrt{R}$   
 $x$  Coordinate in longitudinal direction

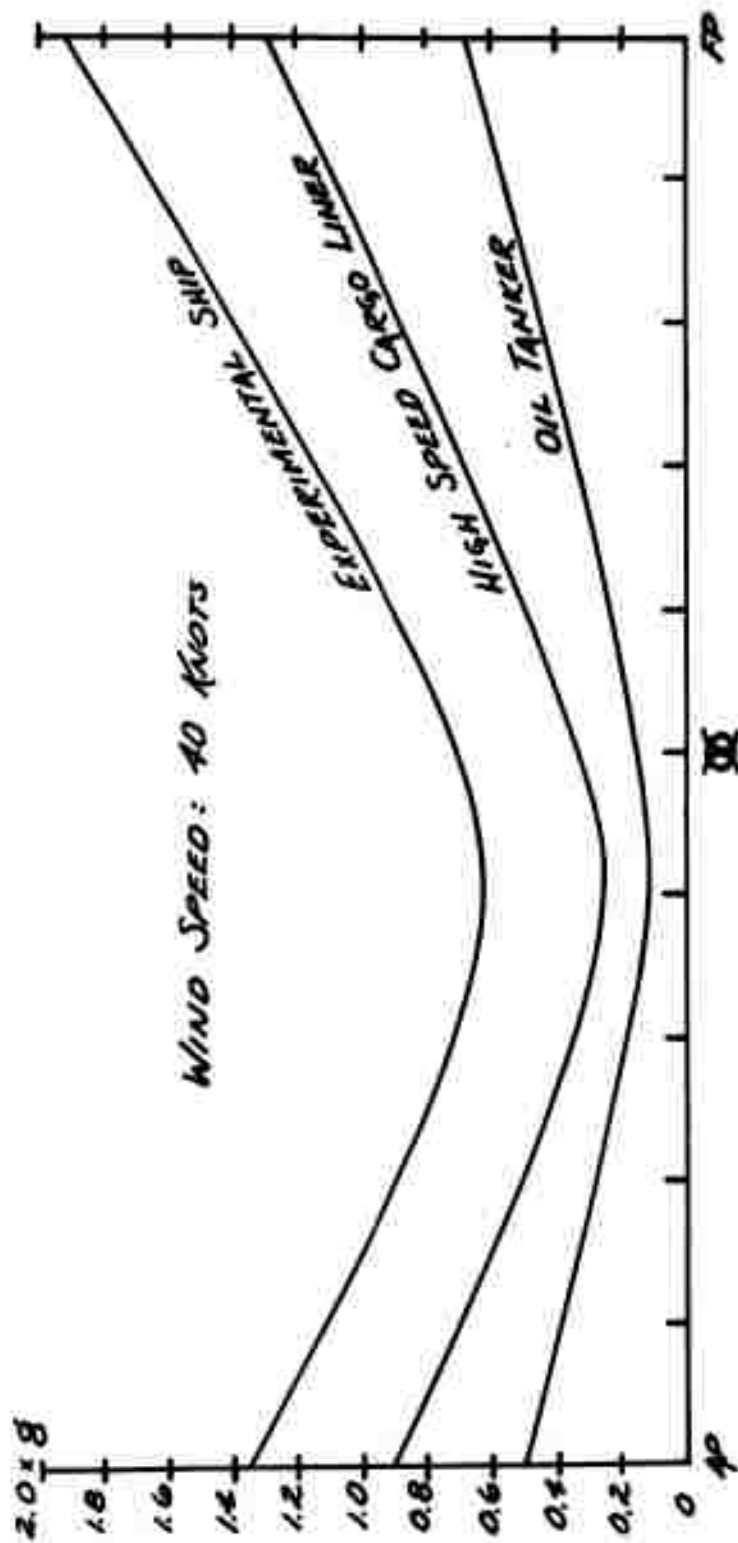


FIGURE 1: MAXIMUM EXPECTED VERTICAL ACCELERATION CALCULATED FROM MODEL TESTS IN REGULAR WAVES (REF. 6)

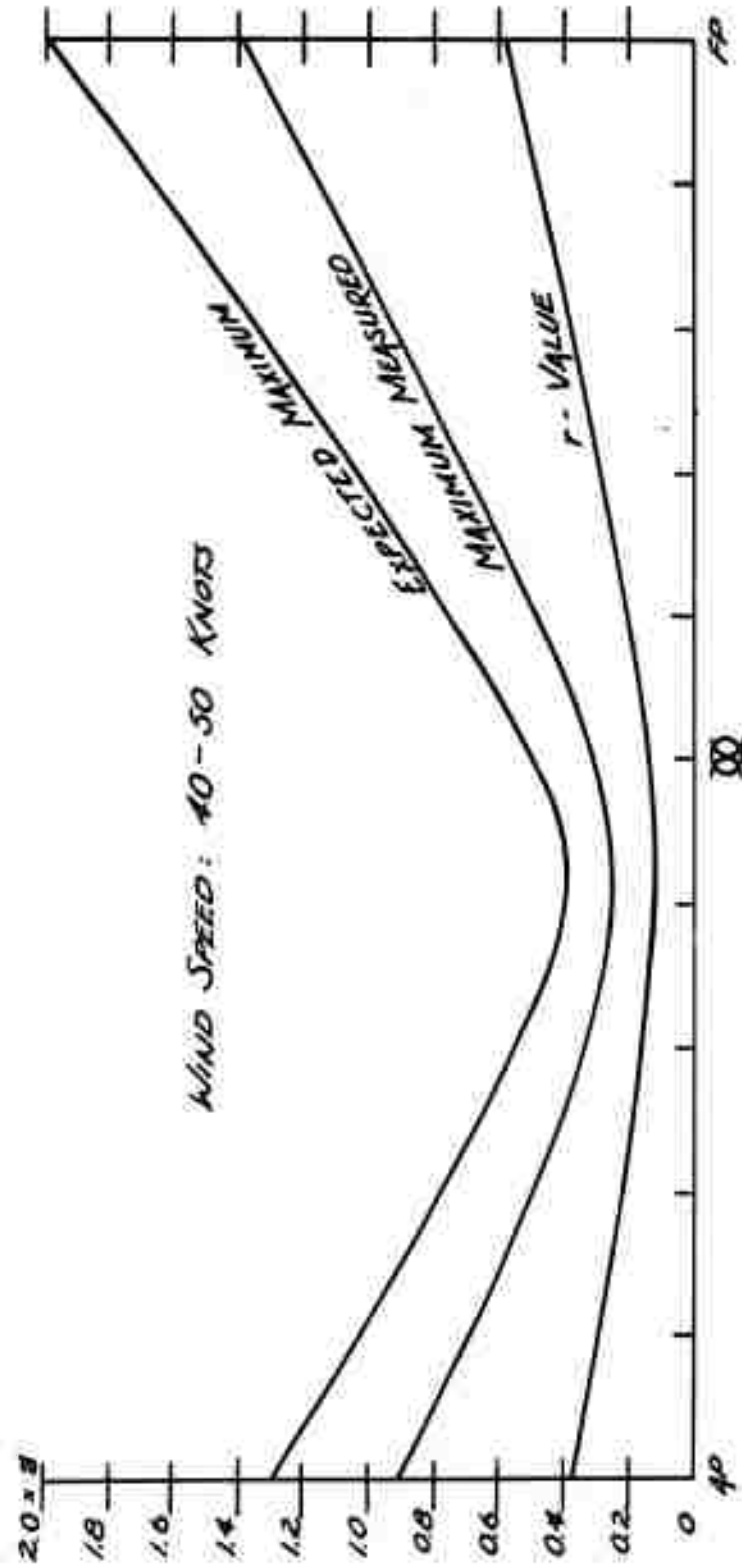


FIGURE 2: VERTICAL ACCELERATION CAUSED BY PITCHING AND HEAVING

M/S CANADA

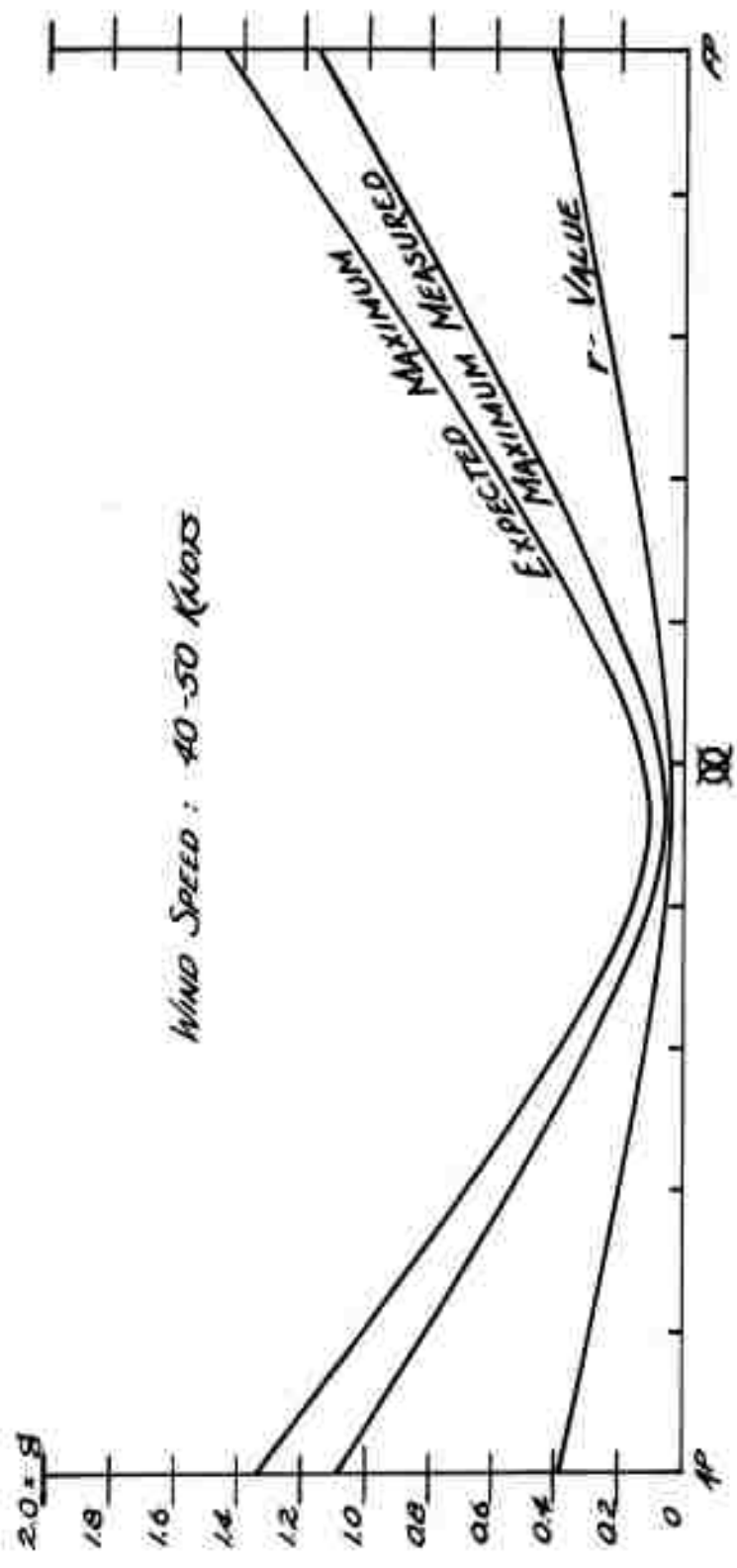


FIGURE 3: VERTICAL ACCELERATION CAUSED BY PITCHING AND HEAVING

MT VASSINAURE

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