

UNCLASSIFIED

AD NUMBER
AD427030
LIMITATION CHANGES
TO: Approved for public release; distribution is unlimited.
FROM: Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; 31 OCT 1963. Other requests shall be referred to Air Force Systems Command, Wright-Patterson AFB, OH 45433.
AUTHORITY
CFSTI per ltr, 12 Nov 1965

THIS PAGE IS UNCLASSIFIED

UNCLASSIFIED

AD **427 030**

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

427030

THE ANTENNA LABORATORY

RESEARCH ACTIVITIES in ---

<i>Automatic Controls</i>	<i>Antennas</i>	<i>Echo Area Studies</i>
<i>Microwave Circuits</i>	<i>Astronautics</i>	<i>EM Field Theory</i>
<i>Terrain Investigations</i>	<i>Radomes</i>	<i>Systems Analysis</i>
<i>Wave Propagation</i>		<i>Submillimeter Applications</i>

CATALOGED BY DDC

AS AD NO.

427030

NO OTS

Calculation of Transmission and Surface-
Wave Data for Plane Multilayers and
Inhomogeneous Plane Layers

Jack H. Richmond

Contract Number AF 33(615)-1081
Task Number 416103

1751-2

31 October 1963

JAN 21 1964

Department of ELECTRICAL ENGINEERING



THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION
Columbus, Ohio

NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

The Government has the right to reproduce, use, and distribute this report for governmental purposes in accordance with the contract under which the report was produced. To protect the proprietary interests of the contractor and to avoid jeopardy of its obligations to the Government, the report may not be released for non-governmental use such as might constitute general publication without the express prior consent of The Ohio State University Research Foundation.

Qualified requesters may obtain copies of this report from the Defense Documentation Center, Cameron Station, Alexandria, Virginia. Department of Defense contractors must be established for DDC services, or have their "need-to-know" certified by the cognizant military agency of their project or contract.

REPORT
by
THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION
COLUMBUS 12, OHIO

Sponsor	Research Technology Division Air Force Systems Command United States Air Force Wright-Patterson Air Force Base, Ohio
Contract Number	AF 33(615)-1081
Task Number	416103
Investigation of	Research on Integrated Radome-Scanner Design Techniques
Subject of Report	Calculation of Transmission and Surface- Wave Data for Plane Multilayers and Inhomogeneous Plane Layers
Submitted by	Jack H. Richmond Antenna Laboratory Department of Electrical Engineering
Date	31 October 1963

ABSTRACT

Digital computer programs are given for the transmission and reflection coefficients of low-loss, multilayer, radome sandwiches. Using matrix multiplication, these programs yield design data for parallel and perpendicular polarization in the most efficient manner. Typical data are included for a five-layer sandwich for normal and oblique incidence at frequencies from zero to 40 gigacycles.

Matrix multiplication is also employed to calculate the phase velocity of the surface waves which may propagate on a plane multilayer sandwich. Typical results are given for three-layer and five-layer sandwiches.

The transmission coefficient of an inhomogeneous plane layer is determined with step-by-step numerical integration of the wave equation. A computer program is included for this problem, along with typical computed data for an exponentially inhomogeneous layer.

TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. TRANSMISSION COEFFICIENTS OF LOW-LOSS PLANE MULTILAYERS	1
III. SURFACE WAVES ON LOSSLESS PLANE MULTILAYERS	18
IV. TRANSMISSION COEFFICIENT OF AN INHOMOGENEOUS LAYER	25
V. CONCLUSIONS	31
VI. REFERENCES	32

CALCULATION OF TRANSMISSION AND SURFACE-WAVE DATA FOR PLANE MULTILAYERS AND INHOMOGENEOUS PLANE LAYERS

I. INTRODUCTION

To facilitate the investigation of electromagnetic windows and radomes, digital computer programs have been developed for the transmission and reflection coefficients of low-loss, multilayer, radome sandwiches. Using matrix multiplication, these programs yield design data for parallel and perpendicular polarization in the most rapid and efficient manner. They have been employed to study the performance of a five-layer unsymmetrical sandwich for normal and oblique incidence over a broad band of frequencies.

An understanding of the electromagnetic effects of dielectric windows requires some knowledge of the surface waves that may be excited in the windows by a radar antenna. This knowledge can be used to minimize the excitation of such waves and to predict the effects of the modes that are excited. Therefore, a computer program was developed for identifying all the modes that can propagate on a given multilayer sandwich, and for calculating the phase velocity of each propagating mode. The application of the matrix formulation to surface-wave analysis is believed to be new.

Another computer program produces transmission-coefficient data for inhomogeneous radomes in which the dielectric constant varies continuously as a function of distance from the radome surface. Discontinuities are allowed for at both surfaces. Since the program is based on step-by-step numerical integration, accurate data can be obtained even for rapidly varying dielectric constants.

II. TRANSMISSION COEFFICIENTS OF LOW-LOSS PLANE MULTILAYERS

Although the matrix formulation for the analysis of plane multilayers is well known,^{1,2,3} an outline of the theory is included here to facilitate the definition of the quantities in the computer programs.

Consider a harmonic plane wave obliquely incident on the surface of a stack of plane, homogeneous, isotropic, dielectric slabs of finite thickness and infinite width as shown in Fig. 1. Assume the surrounding medium is free space.

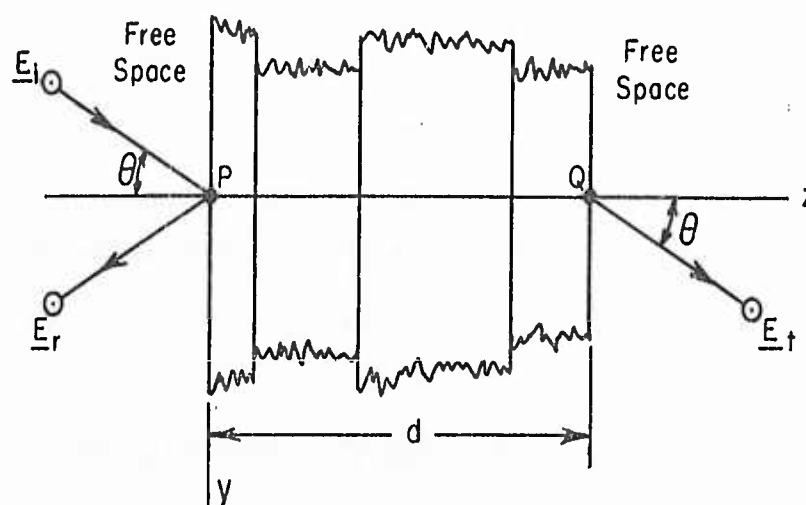


Fig. 1. Plane wave incident on plane multilayer.

In the case illustrated, the wave is said to have "perpendicular polarization" since the electric field intensity vector is perpendicular to the "plane of incidence" (defined by the propagation axis of the incident wave and the normal to the dielectric surface). Let the symbols $\underline{E}_i$ and $\underline{E}_r$ represent the electric field intensities of the incident and reflected plane waves evaluated at the "incident point," point P in Fig. 1; and let $\underline{E}_t$ represent the electric field intensity of the transmitted plane wave at the "normal exit point" Q. The reflection coefficient R and the "normal transmission coefficient" T_n of the plane multilayer are defined by

$$(1) \quad R = \frac{E_r(P)}{E_i(P)} \quad (\text{perpendicular polarization});$$

$$(2) \quad T_n = \frac{E_t(Q)}{E_i(P)} \quad (\text{perpendicular polarization}).$$

It is also useful to define the "insertion transmission coefficient" T as follows:

$$(3) \quad T = \frac{E_t(Q)}{E_i(Q)} \quad (\text{perpendicular polarization})$$

$$= T_n e^{jkd \cos \theta}$$

where d is the total thickness, θ is the angle of incidence measured from the normal, and k is the free-space phase constant $\omega\sqrt{\mu_0\epsilon_0}$.

The resultant field in each layer consists of an outgoing plane wave and a reflected plane wave. Let the complex constants A_n and C_n represent the electric field intensity E_x of the outgoing wave in layer n , evaluated at its two boundaries. Similarly, let B_n and D_n represent the reflected electric field intensity at the two boundaries in layer n as shown in Fig. 2.

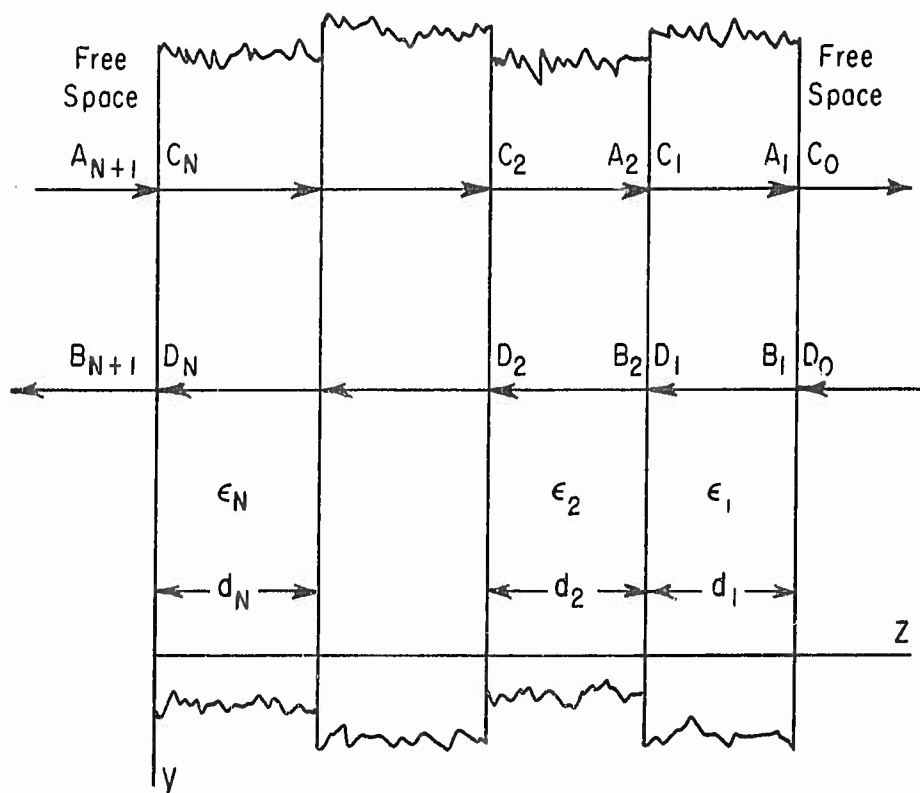


Fig. 2. Plane wave incident on plane multilayer.

The field in an arbitrary layer, say layer n , is given by

$$(4) \quad E_x = (a e^{-\gamma_n x} + b e^{\gamma_n x}) e^{-jky \sin \theta}.$$

The propagation constant γ_n is expressed in terms of the attenuation constant α_n and the phase constant β_n as follows:

$$(5) \quad \gamma_n = \alpha_n + j\beta_n.$$

Let us assume the permeability of each layer is real and the complex permittivity is expressed as follows:

$$(6) \quad \epsilon = \epsilon' (1 - j \tan \delta).$$

From the wave equations and Eqs. (4), (5), and (6) it is found that

$$(7) \quad \alpha = (k/\sqrt{2}) \sqrt{(\mu_r \epsilon'_r - \sin^2 \theta)^2 + (\mu_r \epsilon'_r \tan \delta)^2 - (\mu_r \epsilon'_r - \sin^2 \theta)}$$

and

$$(8) \quad \beta = (k/\sqrt{2}) \sqrt{(\mu_r \epsilon'_r - \sin^2 \theta)^2 + (\mu_r \epsilon'_r \tan \delta)^2 + (\mu_r \epsilon'_r - \sin^2 \theta)};$$

where μ_r and ϵ'_r are the relative permeability and permittivity:

$$(9) \quad \mu_r = \mu/\mu_0$$

and

$$(10) \quad \epsilon'_r = \epsilon'/\epsilon_0.$$

By evaluating E_x in Eq. (4) at the two points $(x, y, z) = (0, 0, z_1)$ and $(0, 0, z_2)$, where z_1 and z_2 are the coordinates of the left and right boundaries of layer n , it can be shown that

$$(11) \quad A_n = C_n e^{-\gamma_n d_n}$$

and

$$(12) \quad B_n = D_n e^{\gamma_n d_n}$$

where d_n is the thickness of layer n . Equations (11) and (12) can be expressed by the following matrix equation:

$$(13) \quad \begin{pmatrix} A_n \\ B_n \end{pmatrix} = \begin{pmatrix} e^{-\gamma_n d_n} & 0 \\ 0 & e^{\gamma_n d_n} \end{pmatrix} \begin{pmatrix} C_n \\ D_n \end{pmatrix}.$$

Let $t_{n+1,n}$ and $r_{n+1,n}$ denote the interface transmission and reflection coefficients for a wave in layer $n+1$ incident on the boundary of layer n . Further, let $t_{n,n+1}$ and $r_{n,n+1}$ represent the interface coefficients for a wave in layer n incident on the boundary of layer $n+1$. In terms of these coefficients, the electric field intensities of the outgoing and reflected waves, evaluated at both sides of this boundary (between layers n and $n+1$), are related linearly as follows:

$$(14) \quad C_n = t_{n+1,n} A_{n+1} + r_{n,n+1} D_n$$

and

$$(15) \quad B_{n+1} = t_{n,n+1} D_n + r_{n+1,n} A_{n+1}.$$

These relations follow from the superposition theorem and the definitions of the interface coefficients.

It can be shown that

$$(16) \quad r_{n,n+1} = -r_{n+1,n},$$

$$(17) \quad t_{n+1,n} = 1 + r_{n+1,n},$$

$$(18) \quad t_{n,n+1} = 1 + r_{n,n+1} = 1 - r_{n+1,n} ,$$

and

$$(19) \quad t_{n+1,n} t_{n,n+1} - r_{n+1,n} r_{n,n+1} = 1 .$$

By using Eqs. (16) through (19), Eqs. (14) and (15) can be rearranged as follows:

$$(20) \quad C_n = (A_{n+1} + r_{n,n+1} B_{n+1})/t_{n,n+1}$$

and

$$(21) \quad D_n = (B_{n+1} - r_{n+1,n} A_{n+1})/t_{n,n+1} .$$

These can be expressed in matrix form as follows:

$$(22) \quad \begin{pmatrix} C_n \\ D_n \end{pmatrix} = \frac{1}{t_{n,n+1}} \begin{pmatrix} 1 & -r_{n+1,n} \\ -r_{n+1,n} & 1 \end{pmatrix} \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} .$$

The matrix Eqs. (13) and (22) can be combined to obtain the following:

$$(23) \quad \begin{pmatrix} C_{n-1} \\ D_{n-1} \end{pmatrix} = \frac{1}{t_{n-1,n}} \begin{pmatrix} e^{-\gamma_n d_n} & -r_{n,n-1} e^{\gamma_n d_n} \\ -r_{n,n-1} e^{-\gamma_n d_n} & e^{\gamma_n d_n} \end{pmatrix} \begin{pmatrix} C_n \\ D_n \end{pmatrix} .$$

Let us denote the two-by-two matrix in Eq. (23) by the symbol M_n :

$$(24) \quad M_n = \begin{pmatrix} e^{-\gamma_n d_n} & -r_{n,n-1} e^{\gamma_n d_n} \\ -r_{n,n-1} e^{-\gamma_n d_n} & e^{\gamma_n d_n} \end{pmatrix} .$$

Repeated application of Eq. (23) yields the following matrix relation among the electric field intensities A_{N+1} and B_{N+1} at the "incidence surface" $z = 0$, and the intensities C_0 and D_0 at the "exit surface" $z=d$:*

*The dots in Eq. (25) denote matrix multiplication.

$$(25) \quad \begin{pmatrix} C_0 \\ D_0 \end{pmatrix} = (1/t) M_1 \cdot M_2 \cdot M_3 \cdot M_4 \cdots M_N \cdot S \cdot \begin{pmatrix} A_{N+1} \\ B_{N+1} \end{pmatrix}$$

where N represents the total number of layers, S denotes the matrix

$$(26) \quad S = \begin{pmatrix} 1 & -r_{N+1, N} \\ -r_{N+1, N} & 1 \end{pmatrix},$$

and

$$(27) \quad t = t_{0,1} t_{1,2} t_{2,3} \cdots t_{N,N+1}.$$

Equation (25) applies even in the general case where plane waves are incident on both outer surfaces of the multilayer, provided the two incident plane waves have the same frequency, angle of incidence, and polarization. In the situation used to define the transmission and reflection coefficients of the structure, a wave of unit amplitude is assumed to be incident on one outer surface only (say surface $z = 0$), so that

$$(28) \quad A_{N+1} = 1,$$

$$(29) \quad B_{n+1} = R,$$

$$(30) \quad C_0 = T_n,$$

and

$$(31) \quad D_0 = 0.$$

Thus Eq. (25) becomes

$$(32) \quad \begin{pmatrix} T_n \\ 0 \end{pmatrix} = (1/t) M_1 \cdot M_2 \cdot M_3 \cdot M_4 \cdots M_N \cdot S \cdot \begin{pmatrix} 1 \\ R \end{pmatrix}.$$

The solution for "parallel polarization" (where the electric field intensity vector is parallel with the plane of incidence) is obtained by applying the theorem of duality to the solution given above. Thus, the reflection and transmission coefficients are defined by

$$(33) \quad R = \frac{H_r(P)}{H_i(P)} \quad (\text{parallel polarization})$$

and

$$(34) \quad T_n = \frac{H_t(Q)}{H_i(P)} \quad (\text{parallel polarization}).$$

The matrix equations given above apply also for parallel polarization, in which case the complex constants A_n , B_n , C_n , and D_n represent the amplitudes of the magnetic field intensities H_x of the traveling waves in layer n . Equations (16) through (19) also apply for parallel polarization, in which case the interface reflection and transmission coefficients are defined by the ratios of the magnetic field intensities H_x . The interface reflection coefficients are given by³

$$(35) \quad r_{n+1,n} = \frac{\mu_n \gamma_{n+1} - \mu_{n+1} \gamma_n}{\mu_n \gamma_{n+1} + \mu_{n+1} \gamma_n} \quad (\text{perpendicular polarization})$$

and

$$(36) \quad r_{n+1,n} = \frac{\epsilon_n \gamma_{n+1} - \epsilon_{n+1} \gamma_n}{\epsilon_n \gamma_{n+1} + \epsilon_{n+1} \gamma_n} \quad (\text{parallel polarization})$$

where γ is given by Eqs. (5), (7), and (8) if the permeability of each layer is real.

After the indicated matrix multiplications in Eq. (32) are performed, and the division by t has been carried out, the equation has the form

$$(37) \quad \begin{pmatrix} T_n \\ 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ R \end{pmatrix} .$$

Thus,

$$(38) \quad T_n = a + bR$$

and

$$(39) \quad 0 = c + dR.$$

In view of the reciprocity theorem, the transmission coefficient of a given multilayer is unaffected by a reversal in the direction of propagation of the incident plane wave. If the incident wave direction is reversed (see Fig. 1 or Fig. 2), Eqs. (28) through (31) are replaced by the following:

$$(40) \quad C_o = R',$$

$$(41) \quad D_o = 1,$$

$$(42) \quad A_{N+1} = 0,$$

and

$$(43) \quad B_{N+1} = T_n$$

where R' denotes the reflection coefficient for a wave in free space incident on the right-hand surface of the multilayer in Fig. 2. (In general R' differs from R , where R is the reflection coefficient for a wave in free space incident on the left-hand surface of the multilayer.) From Eqs. (25) and (40) through (43),

$$(44) \quad \begin{pmatrix} R' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ T_n \end{pmatrix}$$

where a , b , c , and d are the same as in Eq. (37). Thus,

$$(45) \quad R' = bT_n,$$

$$(46) \quad 1 = dT_n,$$

$$(47) \quad ad - bc = 1,$$

$$(48) \quad T_n = 1/d,$$

$$(49) \quad R = -c/d,$$

and

$$(50) \quad R' = b/d.$$

In comparison with other methods, the repetitive matrix multiplication indicated in Eq. (32) is systematic and easy to program on a digital computer. It is efficient with respect to memory storage requirements and running time. Figures 3 and 4 show Fortran programs (OSU Version 2) for the transmission and reflection coefficients of low-loss plane multilayers for perpendicular and parallel polarization, using the matrix formulation. These computer programs apply to non-magnetic media for which $\mu = \mu_0$ and $\mu_r = 1$. To reduce the memory storage requirements and the running time, the programs utilize the following approximations for low-loss media such as those used in radome construction.

$$(51) \quad r_{n+1,n} \approx \frac{\sqrt{\epsilon'_{n+1} - \sin^2 \theta} - \sqrt{\epsilon'_n - \sin^2 \theta}}{\sqrt{\epsilon'_{n+1} - \sin^2 \theta} + \sqrt{\epsilon'_n - \sin^2 \theta}} \quad (\text{perpendicular polarization}),$$

$$(52) \quad r_{n+1,n} \approx \frac{\epsilon'_n \sqrt{\epsilon'_{n+1} - \sin^2 \theta} - \epsilon'_{n+1} \sqrt{\epsilon'_n - \sin^2 \theta}}{\epsilon'_n \sqrt{\epsilon'_{n+1} - \sin^2 \theta} + \epsilon'_{n+1} \sqrt{\epsilon'_n - \sin^2 \theta}} \quad (\text{parallel polarization}),$$

and

$$(53) \quad e^{Y_n d_n} = e^{\alpha_n d_n} e^{j\beta_n d_n} \approx (1 + \alpha_n d_n) e^{j\beta_n d_n}$$

where

$$(54) \quad \alpha_n \approx \frac{k \epsilon'_n \tan \delta_n}{2 \sqrt{\epsilon'_n - \sin^2 \theta}}$$

```

C J RICHMOND, TRANS AND REFL COEFF OF PLANE LOWLOSS MULTILAYERS
C PERPENDICULAR POLARIZATION (MAXIMUM NUMBER OF LAYERS = 8)
      DIMENSION D(8),E(9),R(8),G(8),SR(9),TD(8)
2     FORMAT (F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6)
3     FORMAT (115,F15.9,F15.9,F15.9,F15.9)
4     FORMAT (F15.7,F10.6,F10.6,F10.6,F10.6,F10.6,F15.7)
      READ 3,N,T,F,DF,FM
      READ 2,D(1),D(2),D(3),D(4),D(5),D(6),D(7),D(8)
      READ 2,E(1),E(2),E(3),E(4),E(5),E(6),E(7),E(8)
      READ 2,TD(1),TD(2),TD(3),TD(4),TD(5),TD(6),TD(7),TD(8)
      NN = N+1
      E(N+1) = 1.
      T = .01745329*T
      DD = 6.2831853*COS(T)*(D(1)+D(2)+D(3)+D(4)+D(5)+D(6)+D(7)+D(8))
      S = SIN(T)*SIN(T)
      SR(1) = SQRT(E(1)-S)
      RR = (SR(1)-COS(T))/(SR(1)+COS(T))
      DO 10 I = 1,N
      II = I+1
      SR(II) = SQRT(E(II)-S)
      G(I) = 6.2831853*D(I)*SR(I)
10    R(I) = (SR(II)-SR(I))/(SR(II)+SR(I))
      A = 1.-RR
      DO 15 I = 1,N
15    A = A*(1.-R(I))
      A = 1./A
      M = FM
      DO 49 J = 1,M
      W = 1.180314E+4/F
      GG = G(I)/W
      CG = COS(GG)
      SG = SIN(GG)
      AD = 3.1415926*E(I)*TD(I)*D(I)/(W*SR(I))
      X1 = CG*(1.-AD)
      Y1 = -SG*(1.-AD)
      X2 = -RR*CG*(1.+AD)
      Y2 = -RR*SG*(1.+AD)
      X3 = -RR*CG*(1.-AD)
      Y3 = RR*SG*(1.-AD)
      X4 = CG*(1.+AD)
      Y4 = SG*(1.+AD)
      DO 35 I = 2,NN
      IF(1-NN)25,20,50
20    U1 = 1.
      U2 = -R(N)
      U3 = -R(N)
      U4 = 1.
      V1 = 0.
      V2 = 0.
      V3 = 0.
      V4 = 0.
      GO TO 30
25    II = I-1
      AD = 3.1415926*E(I)*TD(I)*D(I)/(W*SR(I))
      GG = G(I)/W
      CG = COS(GG)
      SG = SIN(GG)
      U1 = CG*(1.-AD)
      V1 = -SG*(1.-AD)
      U2 = -R(II)*CG*(1.+AD)
      V2 = -R(II)*SG*(1.+AD)
      U3 = -R(II)*CG*(1.-AD)
      V3 = R(II)*SG*(1.-AD)
      U4 = CG*(1.+AD)

```

Fig. 3. Computer program for transmission and reflection coefficients of low-loww multilayer for perpendicular polarization.

```

V4 = SG*(1.+AD)
30 P1 = X1*U1-Y1*V1+X2*U3-Y2*V3
   Q1 = Y1*U1+X1*V1+Y2*U3+X2*V3
   P2 = X1*U2-Y1*V2+X2*U4-Y2*V4
   Q2 = Y1*U2+X1*V2+X2*V4+Y2*U4
   P3 = X3*U1-Y3*V1+X4*U3-Y4*V3
   Q3 = Y3*U1+X3*V1+X4*V3+Y4*U3
   P4 = X3*U2-Y3*V2+X4*U4-Y4*V4
   Q4 = Y3*U2+X3*V2+X4*V4+Y4*U4
   X1 = P1
   X2 = P2
   X3 = P3
   X4 = P4
   Y1 = Q1
   Y2 = Q2
   Y3 = Q3
35 Y4 = Q4
   RCR = (-X3*X4-Y3*Y4)/(X4*X4+Y4*Y4)
   RCI = (-Y3*X4+X3*Y4)/(X4*X4+Y4*Y4)
   RC2 = RCR*RCR+RCI*RCI
   RC = SQRT(RC2)
   TR = (X1+X2*RCR-Y2*RCI)*A
   TI = (Y1+Y2*RCR+X2*RCI)*A
   TC2 = TR*TR+TI*TI
   TC = SQRT(TC2)
   PR = ATAN(RCI/RCR)
   IF(RCR)40,40,45
40 PR = 3.1415926 + PR
45 PR = 57.295779*PR
   XX = ATAN(TI/TR)
   IF(TR)47,47,48
47 XX = 3.1415926 + XX
48 FIPD = -57.295779*(XX+DD/W)
   PUNCH 4,F,RCR,RCI,RC2,RC,PR
   PUNCH 4,F,TR,TI,TC2,TC,FIPD
49 F = F + DF
50 STOP
   END

```

Fig. 3.

```

C J RICHMOND, TRANS AND REFL COEFF OF PLANE LOWLOSS MULTILAYERS
C PARALLEL POLARIZATION (MAXIMUM NUMBER OF LAYERS = 5)
  DIMENSION D(5),E(6),R(5),G(5),SR(6),TD(5)
  2  FORMAT (F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6)
  3  FORMAT (I15,F15.9,F15.9,F15.9,F15.9)
  4  FORMAT (F15.7,F10.6,F10.6,F10.6,F10.6,F10.6,F15.7)
  READ 3,N,T,F,DF,FM
  READ 2,D(1),D(2),D(3),D(4),D(5)
  READ 2,E(1),E(2),E(3),E(4),E(5)
  READ 2,TD(1),TD(2),TD(3),TD(4),TD(5)
  NN = N+1
  E(N+1) = 1.
  T = .01745329*T
  DD = 6.2831853*COS(T)*(D(1)+D(2)+D(3)+D(4)+D(5))
  S = SIN(T)*SIN(T)
  SR(1) = SQRT(E(1)-S)
  RR = (SR(1)-E(1)*COS(T))/(SR(1)+E(1)*COS(T))
  DO 10 I = 1,N
  II = I+1
  SR(II) = SQRT(E(II)-S)
  G(I) = 6.2831853*D(I)*SR(I)
10  R(I) = (E(I)*SR(II)-E(II)*SR(I))/(E(I)*SR(II)+E(II)*SR(I))
  A = 1.-RR
  DO 15 I = 1,N
15  A = A*(1.-R(I))
  A = 1./A
  M = FM
  DO 49 J = 1,M
  W = 1.180314E+4/F
  GG = G(I)/W
  CG = COS(GG)
  SG = SIN(GG)
  AD = 3.1415926*E(1)*TD(1)*D(1)/(W*SR(1))
  X1 = CG*(1.-AD)
  Y1 = -SG*(1.-AD)
  X2 = -RR*CG*(1.+AD)
  Y2 = -RR*SG*(1.+AD)
  X3 = -RR*CG*(1.-AD)
  Y3 = RR*SG*(1.-AD)
  X4 = CG*(1.+AD)
  Y4 = SG*(1.+AD)
  DO 35 I = 2,NN
  IF(I-NN)25,20,50
20  U1 = 1.
  U2 = -R(N)
  U3 = -R(N)
  U4 = 1.
  V1 = 0.
  V2 = 0.
  V3 = 0.
  V4 = 0.
  GO TO 30
25  II = I-1
  AD = 3.1415926*E(I)*TD(I)*D(I)/(W*SR(I))
  GG = G(I)/W
  CG = COS(GG)
  SG = SIN(GG)
  U1 = CG*(1.-AD)
  V1 = -SG*(1.-AD)
  U2 = -R(II)*CG*(1.+AD)
  V2 = -R(II)*SG*(1.+AD)
  U3 = -R(II)*CG*(1.-AD)
  V3 = R(II)*SG*(1.-AD)
  U4 = CG*(1.+AD)

```

Fig. 4. Computer program for transmission and reflection coefficients of low-loss multilayer for parallel polarization.

```

V4 = SG*(1.+AD)
30 P1 = X1*U1-Y1*V1+X2*U3-Y2*V3
   Q1 = Y1*U1+X1*V1+Y2*U3+X2*V3
   P2 = X1*U2-Y1*V2+X2*U4-Y2*V4
   U2 = Y1*U2+X1*V2+X2*V4+Y2*U4
   P3 = X3*U1-Y3*V1+X4*U3-Y4*V3
   Q3 = Y3*U1+X3*V1+X4*V3+Y4*U3
   P4 = X3*U2-Y3*V2+X4*U4-Y4*V4
   Q4 = Y3*U2+X3*V2+X4*V4+Y4*U4
   X1 = P1
   X2 = P2
   X3 = P3
   X4 = P4
   Y1 = Q1
   Y2 = Q2
   Y3 = Q3
35 Y4 = Q4
   RCR = (-X3*X4-Y3*Y4)/(X4*X4+Y4*Y4)
   RCI = (-Y3*X4+X3*Y4)/(X4*X4+Y4*Y4)
   RC2 = RCR*RCR+RCI*RCI
   RC = SQRT(RC2)
   TR = (X1+X2*RCR-Y2*RCI)*A
   TI = (Y1+Y2*RCR+X2*RCI)*A
   TC2 = TR*TR+TI*TI
   TC = SQRT(TC2)
   PR = ATAN(RCI/RCR)
   IF(RCR)40,40,45
40 PR = 3.1415926 + PR
45 PR = 57.295779*PR
   XX = ATAN(TI/TR)
   IF(TR)47,47,48
47 XX = 3.1415926 + XX
48 FIPD = -57.295779*(XX+DD/W)
   PUNCH 4,F,RCR,RCI,RC2,RC,PR
   PUNCH 4,F,TR,TI,TC2,TC,FIPD
49 F = F + DF
50 STOP
   END

```

Fig. 4.

and

$$(55) \quad \beta_n \approx k \sqrt{\epsilon'_n - \sin^2 \theta}.$$

In Eqs. (51) through (55), ϵ'_n and ϵ'_{n+1} represent the real part of the relative complex permittivity of layers n and $n+1$. The errors in the transmission-coefficient data arising from these approximations are of second order, and the resulting data are accurate for low-loss multilayers such as radome sandwiches.

The symbols used in the computer programs in Figs. 3 and 4 are defined in Table I.

TABLE I

Symbols Used in the Computer Programs in Figs. 3 and 4

N	= number of layers
T	= θ , angle of incidence, degrees
F	= lowest frequency, megacycles
DF	= increment in frequency, megacycles
FM	= number of frequencies to be calculated
D(I)	= thickness of layer I, inches
E(I)	= real part of relative permittivity of layer I
TD(I)	= loss tangent of layer I
S	= $\sin^2 \theta$
SR(I)	= $\sqrt{\epsilon'_I - \sin^2 \theta}$
G(I)	= $2\pi d_I \sqrt{\epsilon'_I - \sin^2 \theta}$
R(I)	= interface reflection coefficient, $r_{I+1,I}$
W	= wavelength in free space, inches

The quantities calculated and punched as output data are given in Table II.

These computer programs were used with an IBM 1620 digital computer to study the broadband characteristics of a sandwich with five layers having dielectric constants of 5, 4, 3, 2, and 1.5 and thicknesses of 0.15, 0.15, 0.15, 0.26, and 0.26 inches. The results for frequencies from zero to 40 gigacycles are shown in Figs. 5, 6, and 7 for the lossless and the low-loss cases. Each of the six curves involved calculation of the transmission coefficient at 80 frequencies. The calculation time was

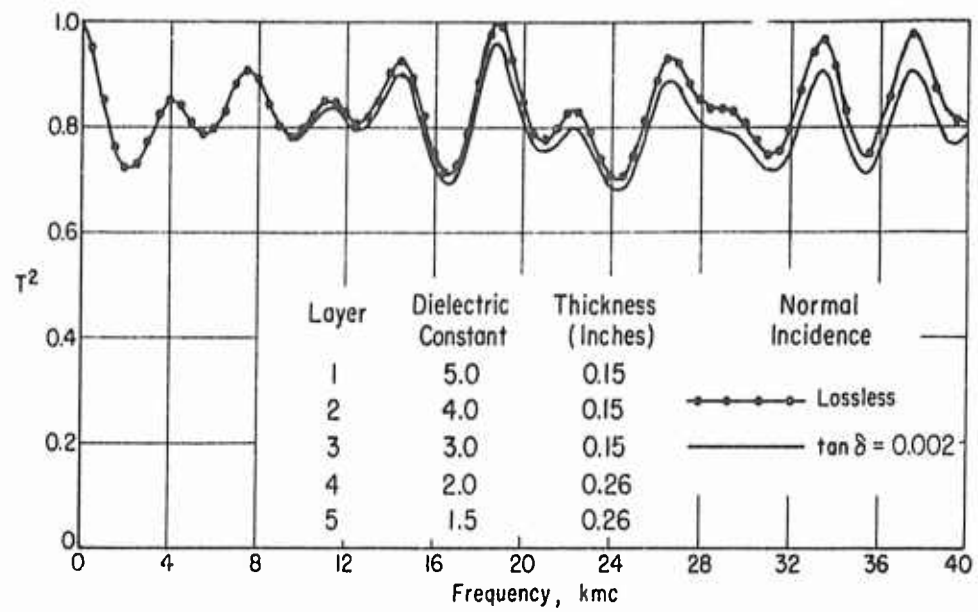


Fig. 5. Power transmission coefficient versus frequency for five-layer sandwich.

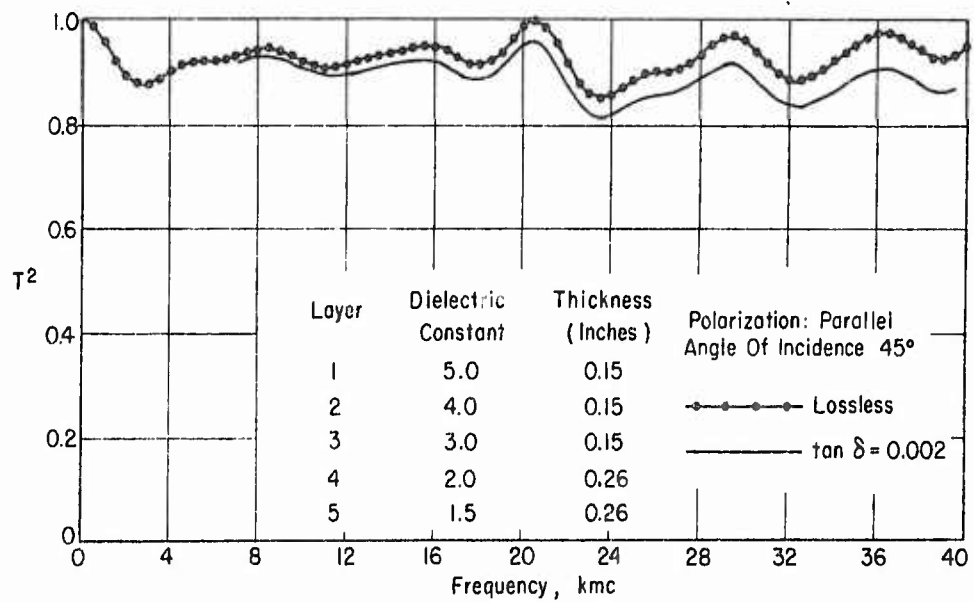


Fig. 6. Power transmission coefficient versus frequency for five-layer sandwich.

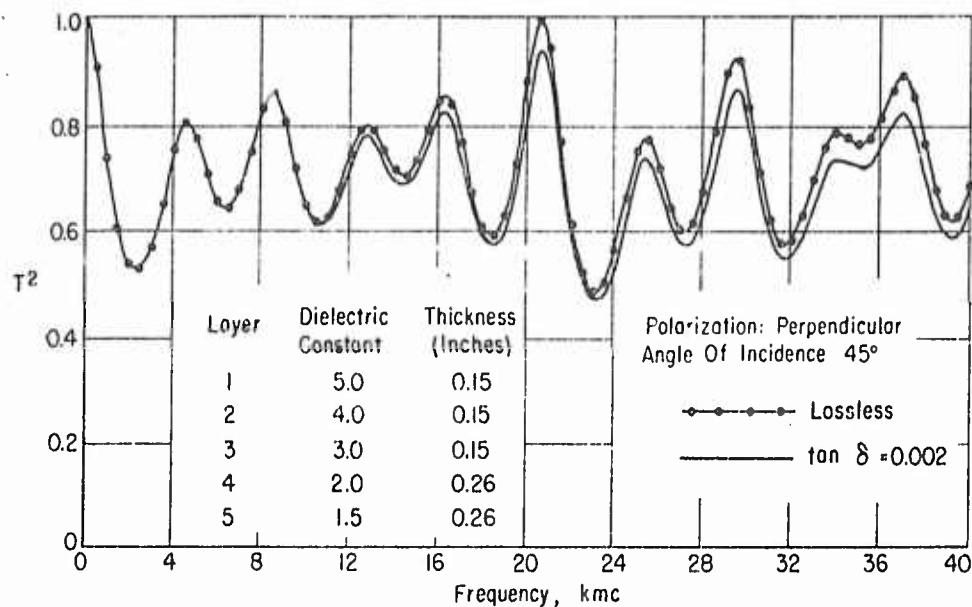


Fig. 7. Power transmission coefficient versus frequency for five-layer sandwich.

15 seconds for each frequency, which amounts to 20 minutes for each curve. It is apparent from Fig. 6 that this particular sandwich has excellent broadband properties for parallel polarization at oblique incidence.

TABLE II

Symbols for Output Data of Computer Programs in Figs. 3 and 4

$R = RCR + jRCI$ = reflection coefficient
 $RC2 = |R|^2$ = power reflection coefficient
 $RC = |R|$ = voltage reflection coefficient
 PR = reflection phase angle, degrees
 $T_n = TR + jTI$ = normal transmission coefficient
 $TC2 = |T_n|^2$ = power transmission coefficient
 $TC = |T_n|$ = voltage transmission coefficient
 $FIPD$ = insertion phase delay, degrees

When used with the IBM 1620 equipment at The Ohio State University, these programs handle a maximum of eight layers for perpendicular polarization and five layers for parallel polarization. With slight modifications in these programs, the maximum number of layers can be increased by using equipment with greater storage capacity. If the reflection phase, the transmission phase, and the insertion phase delay need not be calculated, the maximum number of layers can be increased, even while using the IBM 1620 equipment, by deleting the statements which call for the arc-tangent subroutine.

III. SURFACE WAVES ON LOSSLESS PLANE MULTILAYERS

Consider a surface wave propagating on a plane, lossless multilayer. Let h represent the phase constant of the surface wave, and let α represent the normal attenuation constant in the free-space regions on both sides of the multilayer. Both h and α are real quantities. For a TE surface wave, the field in the free-space regions on both sides of the multilayer has the form

$$(56) \quad E_x = e^{-jhy} e^{-\alpha|z|}.$$

The multilayer and the coordinate system are shown in Figs. 1 and 2.

The electric field intensity E_x in layer n is given by

$$(57) \quad E_n = (a e^{-\gamma_n z} + b e^{\gamma_n z}) e^{-jhy}$$

where

$$(58) \quad \gamma_n = k \sqrt{h^2/k^2 - \mu_n \epsilon_n}.$$

The quantity under the square-root sign in Eq. (58) may be positive in some layers and negative in others. Thus, the field in some layers is "evanescent" while in other layers it is the sum of two criss-crossing plane-wave fields.

It can be shown that

$$(59) \quad h/k = \lambda/\lambda_g = c/v$$

where λ is the wavelength in free space, c is the velocity of light in free space, λ_g is the "guide wavelength," and v is the phase velocity of the surface wave.

The equations for a surface wave on a plane multilayer differ from those in Section II only in that the quantity $k \sin \theta$ is replaced by h , and in the surface-wave case the amplitude of the incident plane wave is zero. Thus, the matrix Eq. (25) applies also for surface waves, for which case

$$(60) \quad D_0 = 0$$

and

$$(61) \quad A_{N+1} = 0.$$

Now Eq. (25) becomes

$$(62) \quad \begin{pmatrix} C_0 \\ 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ B_{N+1} \end{pmatrix}$$

from which

$$(63) \quad dB_{N+1} = 0.$$

But B_{N+1} represents the amplitude of the surface-wave field at one of the outer surfaces of the multilayer, and it may be assumed to be non-zero. Hence,

$$(64) \quad d = 0.$$

Equation (64) represents the "transcendental equation for surface waves," since d represents the lower right element in the matrix $(1/t)M_1 \cdot M_2 \cdots M_N \cdot S$ (see Eq. (25)) and the equation $d = 0$ cannot be

rearranged to obtain an explicit expression for h . It is, however, a useful implicit equation for h .

Since the zeros of the factor $1/t$ are not significant here, the transcendental equation for surface waves can be written as follows:

$$(65) \quad M_1 \cdot M_2 \cdot M_3 \cdots M_N \cdot S = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$$

in which d' is required to vanish; and

$$(66) \quad M_n = \begin{pmatrix} e^{-g_n} & -r_n e^{g_n} \\ -r_n e^{-g_n} & e^{g_n} \end{pmatrix},$$

$$(67) \quad S = \begin{pmatrix} 1 & -r_{N+1} \\ -r_{N+1} & 1 \end{pmatrix},$$

$$(68) \quad g_n = \gamma_n d_n,$$

and

$$(69) \quad r_n = r_{n,n-1} = \frac{\mu_{n-1} \gamma_n - \mu_n \gamma_{n-1}}{\mu_{n-1} \gamma_n + \mu_n \gamma_{n-1}} \quad (\text{for TE waves}).$$

It can be shown that the range of h/k is from unity to $\sqrt{\mu_m \epsilon_m}$ where μ_m and ϵ_m are the relative permeability and permittivity of the layer having the greatest value of the product $\mu\epsilon$. If $h/k = 1$, the surface wave is said to be at cutoff. The value $h/k = \sqrt{\mu_m \epsilon_m}$ is the high-frequency limit. To determine the phase constants of all the surface waves that may propagate on a given multilayer, one may calculate the lower-right matrix element d' in Eq. (65) for many values of h/k in the range from unity to $\sqrt{\mu_m \epsilon_m}$. The zeros of d' can then be found by making a graph of $|d'|$ versus h/k , or by finding the roots of a polynomial which fits the calculated values of d' .

The matrix multiplications indicated in Eq. (65) are programmed in Fortran as shown in Fig. 8. The symbols in this program are defined in Table III. The program applies to nonmagnetic multilayers ($\mu = \mu_0$ and $\mu_r = 1$).

TABLE III

Symbols Used in the Computer Program in Fig. 8

N	= number of layers
M	= number of values of h/k to be used
T	= initial value of h/k , on input data card
T	= current value of h/k , in output data
DT	= increment in h/k
F	= frequency, mc
D(I)	= thickness of layer I, inches
E(I)	= relative dielectric constant of layer I
W	= wavelength in free space, inches
T2	= $(h/k)^2$
d'	= $X_4 + j Y_4$
Z4	= $ d' $

Figure 9 is a graph of the data obtained with this program on an IBM 1620 digital computer for a sandwich having three layers with dielectric constants of 4, 1, and 4, and thicknesses of 0.25, 0.20, and 0.25 inches. From Fig. 9, this sandwich supports TE modes having $h/k = 1.62$ (TE_1 mode) and 1.70 (TE_0 mode) at 11,803 mc. This agrees accurately with independent calculations.

This computer program was also used to obtain surface-wave data for the five-layer sandwich described in Figs. 5, 6, and 7. At 10 gigacycles the only TE modes it supports are the TE_0 and TE_1 modes, for which h/k is 1.85 and 1.30, respectively.

Since

$$(70) \quad d' = t_{0,1} t_{1,2} t_{2,3} \cdots t_{N,N+1} d$$

and

$$(71) \quad t_{n,n+1} = 1 + r_{n,n+1} = 1 - r_{n+1,n}$$

$|d'|$ has some zeros which $|d|$ does not have. These extraneous zeros occur at values of h/k for which one of the interface reflection coefficients r_n takes on the value $+1$. From Eqs. (58) and (69), this occurs

```

C      J RICHMOND, SURFACE WAVES ON PLANE LOSSLESS MULTILAYER, TE
C      MAXIMUM NUMBER OF LAYERS = 8
      DIMENSION D(8),E(8)
2     FORMAT (F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6)
3     FORMAT (I15,I15,F15.9,F15.9,F15.9)
4     FORMAT (F15.9,F15.9,F15.9,F15.9,F15.9)
      READ 3,N,M,T,DT,F
      READ 2,D(1),D(2),D(3),D(4),D(5),D(6),D(7),D(8)
      READ 2,E(1),E(2),E(3),E(4),E(5),E(6),E(7),E(8)
      N1 = N+1
      PI = 3.1415926
      T = T-DT
      W = 1.180314E+4/F
      DO 70 L = 1,M
      T = T + DT
      T2 = T*T
      SS = SQRT(T2-1.)
      A = ABS(T2-E(1))
      S = SQRT(A)
      G = 2.*PI*D(1)*S/W
      IF (T2-E(1))10,10,11
10     GR = COS(G)
      GI = SIN(G)
      FR = GR
      FI = -GI
      GO TO 15
11     GR = EXP(G)
      GI = 0.
      FR = 1./GR
      FI = 0.
15     IF (T2-E(1))16,16,17
16     PH = PI-2.*ATAN(S/SS)
      RR = COS(PH)
      RI = SIN(PH)
      GO TO 20
17     RR = (S-SS)/(S+SS)
      RI = 0.
20     X1 = FR
      Y1 = FI
      X2 = -RR*GR+RI*GI
      Y2 = -RI*GR-RR*GI
      X3 = -RR*FR+RI*FI
      Y3 = -RI*FR-RR*FI
      X4 = GR
      Y4 = GI
      DO 65 I = 2,N1
      IF (I-N1)35,25,80
25     A = ABS(T2-E(N))
      S = SQRT(A)
      IF (T2-E(N))26,26,27
26     PH = 2.*ATAN(S/SS)
      RR = COS(PH)
      RI = -SIN(PH)
      GO TO 30
27     RR = (SS-S)/(SS+S)
      RI = 0.
30     U1 = 1.
      V1 = 0.
      U2 = -RR
      V2 = -RI
      U3 = -RR
      V3 = -RI
      U4 = 1.

```

Fig. 8. Computer program for phase constants of TE surface waves on plane multilayers.

```

V4 = 0.
GO TO 60
35 A = ABS(T2-E(1))
S = SQRT(A)
G = 2.*PI*D(1)*S/W
I1 = I-1
A1 = ABS(T2-E(I1))
S1 = SQRT(A1)
Z = (T2-E(I1))*(T2-E(I1))
IF(Z)40,36,36
36 RR = (S-S1)/(S+S1)
RI = 0.
GO TO 50
40 IF(T2-E(I1))41,41,45
41 PH = PI-2.*ATAN(S/S1)
RR = COS(PH)
RI = SIN(PH)
GO TO 50
45 PH = 2.*ATAN(S1/S)
RR = COS(PH)
RI = -SIN(PH)
50 IF(T2-E(I1))51,51,52
51 GR = COS(G)
GI = SIN(G)
FR = GR
FI = -GI
GO TO 55
52 GR = EXP(G)
GI = 0.
FR = 1./GR
FI = 0.
55 U1 = FR
V1 = FI
U2 = -RR*GR+RI*GI
V2 = -RI*GR-RR*GI
U3 = -RR*FR+RI*FI
V3 = -RI*FR-RR*FI
U4 = GR
V4 = GI
60 P1 = X1*U1-Y1*V1+X2*U3-Y2*V3
Q1 = Y1*U1+X1*V1+Y2*U3+X2*V3
P2 = X1*U2-Y1*V2+X2*U4-Y2*V4
Q2 = Y1*U2+X1*V2+Y2*U4+X2*V4
P3 = X3*U1-Y3*V1+X4*U3-Y4*V3
Q3 = Y3*U1+X3*V1+Y4*U3+X4*V3
P4 = X3*U2-Y3*V2+X4*U4-Y4*V4
Q4 = Y3*U2+X3*V2+Y4*U4+X4*V4
X1 = P1
X2 = P2
X3 = P3
X4 = P4
Y1 = Q1
Y2 = Q2
Y3 = Q3
65 Y4 = Q4
Z4 = SQRT(X4*X4+Y4*Y4)
70 PUNCH 4,T,X4,Y4,Z4
30 STOP
END

```

Fig. 8.

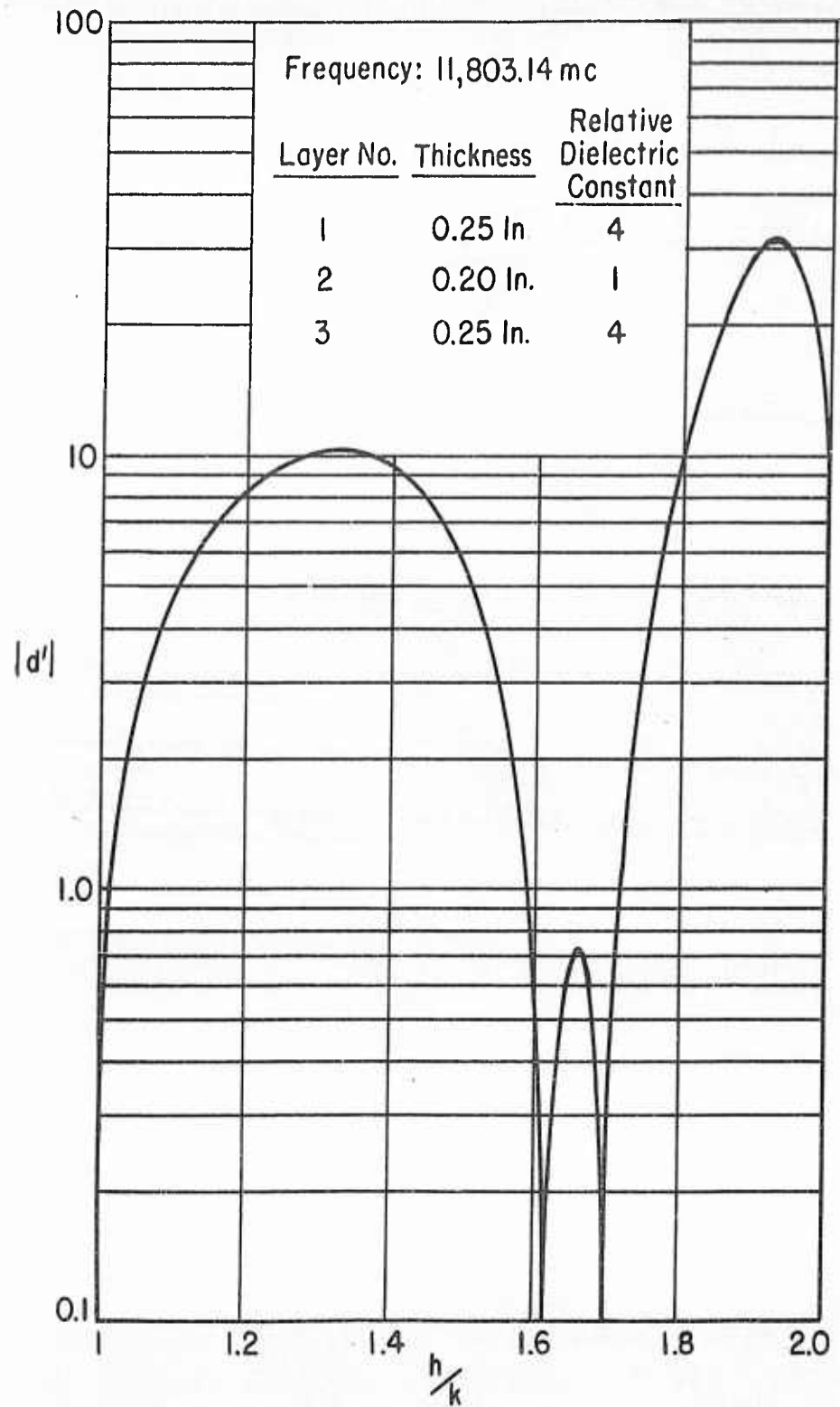


Fig. 9. Matrix element d' versus h/k for three-layer sandwich.

when $Y_n = 0$ or

$$(72) \quad h/k = \sqrt{\mu_n \epsilon_n}.$$

These zeros of $|d'|$ are not significant with respect to surface waves. They are easily identified, thus permitting the significant zeros to be recognized. For example, in Fig. 9 the zeros at $h/k = 1$ and 2 are to be ignored.

TM surface waves on a plane multilayer can be analyzed by means of a similar program using the expression for the interface reflection coefficient for parallel polarization instead of Eq. (69).

IV. TRANSMISSION COEFFICIENT OF AN INHOMOGENEOUS LAYER

Consider a plane wave incident on a plane inhomogeneous layer immersed in free space, as in Fig. 10. If the permittivity is a function of the normal coordinate only

$$(73) \quad \epsilon = \epsilon(z)$$

and

$$(74) \quad \mu = \mu_0.$$

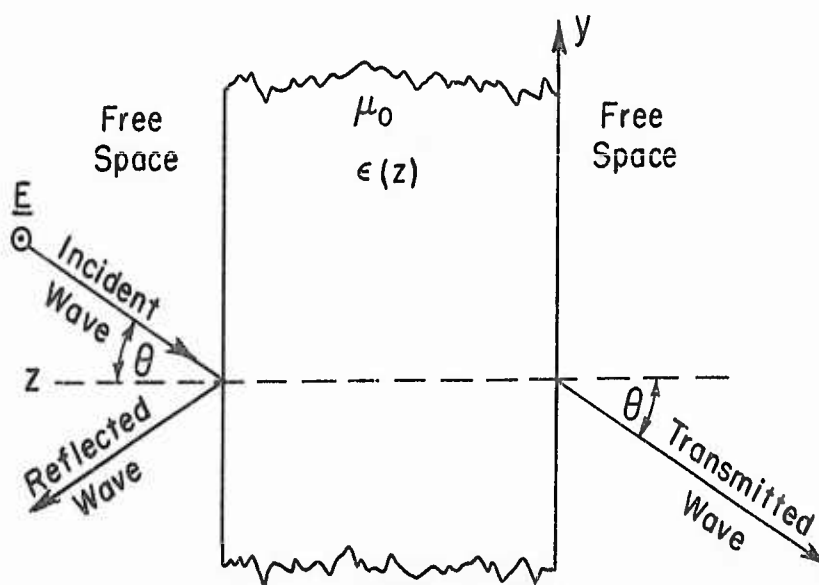


Fig. 10. Plane wave incident on inhomogeneous plane layer.

Using step-by-step numerical integration, equations are developed in Report 1180-4⁴ for the electric field intensity distribution $E_x(z)$ and the transmission coefficient of such a layer. These equations are listed below for perpendicular polarization.

If the electric field intensity is set equal to unity ($E_0 = 1$) at the coordinate origin, the field at the point $(x, y, z) = (0, 0, h)$ is given by

$$(75) \quad E_1 = 1 - k^2 h^2 (\epsilon_+ - \sin^2 \theta) / 2 \\ + j \left[kh \cos \theta + \frac{k^2 h^2}{2} \epsilon_+ \tan \delta_+ - \frac{k^3 h^3}{6} (\epsilon_+ - \sin^2 \theta) \cos \theta \right]$$

where ϵ_+ and $\tan \delta_+$ represent the relative permittivity and the loss tangent just inside the inhomogeneous layer at $z = 0+$.

Let E_{n-1} , E_n , and E_{n+1} represent the electric field intensities E_x at the points $z = (n-1)h$, nh , and $(n+1)h$. Further, let R and S represent the real and imaginary parts of the electric field intensity; for example,

$$(76) \quad E_n = R_n + j S_n .$$

The recursion formulas for the real and imaginary parts of the electric field intensity in the inhomogeneous layer are

$$(77) \quad R_{n+1} = [2 - k^2 h^2 (\epsilon_n - \sin^2 \theta)] R_n - k^2 h^2 \epsilon_n S_n \tan \delta_n - R_{n-1}$$

and

$$(78) \quad S_{n+1} = [2 - k^2 h^2 (\epsilon_n - \sin^2 \theta)] S_n + k^2 h^2 \epsilon_n R_n \tan \delta_n - S_{n-1}$$

where ϵ_n and $\tan \delta_n$ represent the relative dielectric constant and the loss tangent at the point $(x, y, z) = (0, 0, nh)$.

After Eq. (75) is used to calculate the field E_1 at $z = h$, Eqs. (77) and (78) may be employed to calculate the field (R_2 and S_2) at $z = 2h$. Following this, Eqs. (77) and (78) are used again to calculate the field at the next point $z = 3h$, and so on until the inhomogeneous layer has been traversed.

Finally, the transmission coefficient is determined by the following expression:

$$(79) \quad T = \frac{4 kh \cos \theta}{2E_N kh \cos \theta - j(E_{N+1} - E_{N-1})}$$

where E_{N-1} , E_N , and E_{N+1} are the electric field intensities at the points $z = d-h$, $z = d$, and $z = d+h$, with d representing the thickness of the inhomogeneous layer.

The step size h must be small to obtain accurate results with these equations.

Figure 11 shows a computer program based on the above equations. Table IV defines the symbols used in this program.

TABLE IV
Symbols Used in Computer Program in Fig. 11

N	=	number of field points to be calculated
H	=	kh
TH	=	increment in θ , degrees
THETA	=	angle of incidence θ , radians
K	=	angle of incidence θ , degrees
S	=	$\sin^2 \theta$
C	=	$\cos \theta$
H2	=	$(kh)^2$
E(I)	=	relative dielectric constant at point I
R1, R2, R3	=	real parts of electric field intensity at points 1, 2, and 3
S1, S2, S3	=	imaginary parts of electric field intensity at points 1, 2, and 3
EM	=	$ E $
TR + j TI	=	complex transmission coefficient
TS	=	power transmission coefficient
T	=	voltage transmission coefficient magnitude
PH	=	phase of normal transmission coefficient

Some of the results obtained with this digital computer program are listed in Tables V through VIII. These results are found to agree very well with the exact solutions for homogeneous and exponentially-inhomogeneous layers.

```

C      J RICHMOND, LOSSLESS INHOMOGENEOUS PLANE LAYER, PERPENDICULAR
C      ELECTRIC FIELD DISTRIBUTION AND TRANSMISSION COEFFICIENT
      DIMENSION E(16)
      2  FORMAT (15,F15.8,F15.8,F15.8,F15.8,F15.8,F15.8)
      3  FORMAT (F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6,F10.6)
      READ 2,N,H,TH
      H2 = H**H
      READ 3,E(1),E(2),E(3),E(4),E(5),E(6),E(7),E(8)
      READ 3,E(9),E(10),E(11),E(12),E(13),E(14),E(15),E(16)
      M = 90./TH
      TH = .01745329*TH
      THETA = 0.
      DO 26 J = 1,M
      S = SIN(THETA)*SIN(THETA)
      C = COS(THETA)
      R1 = 1.
      S1 = 0.
      I = 1
      PUNCH 2,I,R1,S1,R1
      A = E(1) - S
      R2 = 1. - H2*A/2. - H2*(E(2)-E(1))/0.
      S2 = H*C-H2*H*A*C/0.
      I = 2
      EM = SQRT(R2*R2+S2*S2)
      PUNCH 2,I,R2,S2,EM
      DO 10 I = 2,N
      II = I+1
      A = E(I)-S
      B = 2.*H2*A
      R3 = B*R2-R1
      S3 = B*S2-S1
      EM = SQRT(R3*R3+S3*S3)
      PUNCH 2,II,R3,S3,EM
      IF(N-I)30,15,5
      5  R1 = R2
      S1 = S2
      R2 = R3
      10  S2 = S3
      15  DR = 2.*H*C*R2+S3-S1
      DI = 2.*H*C*S2-R3+R1
      TR = 4.*H*C*DR/(DR*DR+DI*DI)
      TI = -4.*H*C*DI/(DR*DR+DI*DI)
      TS = TR*TR+TI*TI
      T = SQRT(TS)
      PH = ATAN(TI/TR)
      IF(TR)20,25,25
      20  PH = PH*3.1415926
      25  PH = PH*57.29578
      K = 57.2956*THETA
      PUNCH 2,K,TR,TI,TS,T,PH
      PUNCH 2
      26  THETA = THETA + TH
      30  STOP
      END

```

Fig. 11. Computer program for field distribution and transmission coefficient of inhomogeneous plane layer for perpendicular polarization.

TABLE V
Electric Field Intensity Distribution
in Homogeneous Layer

<u>kz</u>	<u>ϵ_r</u>	<u> E </u>	
		<u>$\theta = 0$</u>	<u>$\theta = 80^\circ$</u>
0	4	1.0000	1.0000
.1	4	.9850	.9850
.2	4	.9411	.9405
.3	4	.8717	.8678
.4	4	.7827	.7693
.5	4	.6835	.6482
.6	4	.5888	.5086
.7	4	.5199	.3561
.8	-	.4999	.2011

TABLE VI
Electric Field Intensity Distribution
in Inhomogeneous Layer

$\epsilon_r = 4 \exp(-.3077 \text{ kz})$

<u>kz</u>	<u>ϵ_r</u>	<u> E </u>	
		<u>$\theta = 0$</u>	<u>$\theta = 80^\circ$</u>
0	4.0000	1.0000	1.0000
.1	3.8787	.9852	.9852
.2	3.7612	.9427	.9420
.3	3.6473	.8768	.8730
.4	3.5367	.7938	.7810
.5	3.4296	.7030	.6696
.6	3.3257	.6171	.5428
.7	3.2249	.5533	.4052
.8	-----	.5297	.2636

TABLE VII
Voltage Transmission Coefficient of Homogeneous Layer
for Perpendicular Polarization
 $\epsilon_r = 4$, $d = .111408 \lambda_0$

<u>θ</u>	<u> T </u>	
	<u>Computed</u>	<u>Exact</u>
0°	.8076	.8042
10	.8023	.7990
20	.7859	.7827
30	.7560	.7530
40	.7087	.7060
50	.6379	.6356
60	.5354	.5335
70	.3934	.3921
80	.2106	.2099

TABLE VIII
Voltage Transmission Coefficient of Inhomogeneous Layer
for Perpendicular Polarization
 $\epsilon_r = 4 \exp(-.3077 \text{ kz})$
 $d = .111408 \lambda_0$

<u>θ</u>	<u> T Computed</u>
0°	.8357
10	.8310
20	.8161
30	.7889
40	.7450
50	.6774
60	.5761
70	.4298
80	.2331

While the calculated results listed in these tables are sufficiently accurate for most radome design purposes, more accurate data can be obtained quite readily by using the program shown in Fig. 11 with a

smaller value of the step size h or kh . With an IBM 1620 digital computer, a total running time of six minutes was required to obtain the data listed in Tables V through VIII.

V. CONCLUSIONS

Digital computer programs are given for the transmission and reflection coefficients of low-loss, multilayer, radome sandwiches. Using matrix multiplication, these programs yield design data for parallel and perpendicular polarization in the most efficient manner. Typical data are included for a five-layer sandwich for normal and oblique incidence at frequencies from zero to 40 gigacycles.

Matrix multiplication is also employed to calculate the phase velocity of the surface waves which may propagate on a plane multilayer sandwich. Typical results are given for three-layer and five-layer sandwiches.

The transmission coefficient of an inhomogeneous plane layer is determined with step-by-step numerical integration of the wave equation. A computer program is included for this problem, along with typical computed data for an exponentially-inhomogeneous layer.

VI. REFERENCES

1. Speer, W., "Matrix Method of Radome Transmission Computation," Memorandum Report EG-MR-3, Douglas Aircraft Co., El Segundo, California, 4 January 1951 (ATI-178476).
2. Henschke, R.A., "Application of Matrices to Problem of Transmission through a Multilayered Dielectric Wall," Hughes Aircraft Co., Culver City, California.
3. Tice, T. E., (Editor), "Techniques for Airborne Radome Design," WADC Technical Report 57-67, September, 1957, (AD 142 001).
4. Richmond, J.H., "Transmission through Inhomogeneous Plane Layers," Report 1180-4, 15 July 1961, Antenna Laboratory, The Ohio State University Research Foundation; prepared under Contract AF 33(616)-7614, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio.

ELECTROMAGNETIC WINDOWS
DISTRIBUTION LIST

Project 1751

Contract Number AF 33(615)-1081

<u>Copies</u>	<u>Agencies at WPAFB</u>
1	ASNRR
3	AVWE-3 (Richard Ireland)
	<u>Other Department of Defense Agencies</u>
20	DDC Cameron Station Alexandria, Virginia
1	Chief Technical Library Office of Asst. Secretary of Defense (R&D) Room 3E1065 The Pentagon Washington 25, D. C.
	<u>Army</u>
1	Commanding Officer USASRDL Attn: Technical Documents Center Fort Monmouth, New Jersey
2	Commanding General Redstone Arsenal Attn: Technical Library-EC Redstone Arsenal, Alabama

Project 1751 distribution list - p. 2

<u>Copies</u>	<u>Navy</u>
2	Chief Bureau of Naval Weapons Attn: RRMA-31 Department of the Navy Washington 25, D. C.
1	Chief Bureau of Aeronautics Avionics Division Material Coordination Unit Department of the Navy Washington 25, D. C.
1	Chief Bureau of Ships Code 816-E Department of the Navy Washington 25, D. C.
1	NADC Attn: Jerry Guarini Johnsville, Pennsylvania
	<u>Air Force</u>
1	Commander Air Force Cambridge Research Center Attn: Documents Unit, CROOTR-2 L. G. Hanscom Field Bedford, Massachusetts
1	Hq. ESD (ESRDW) Attn: Major John J. Hobson L. G. Hanscom Field Bedford, Massachusetts
1	Narmco Industries, Inc. Attn: Roger Long 8125 Aero Drive San Diego 11, California