

UNCLASSIFIED

AD 427641

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA

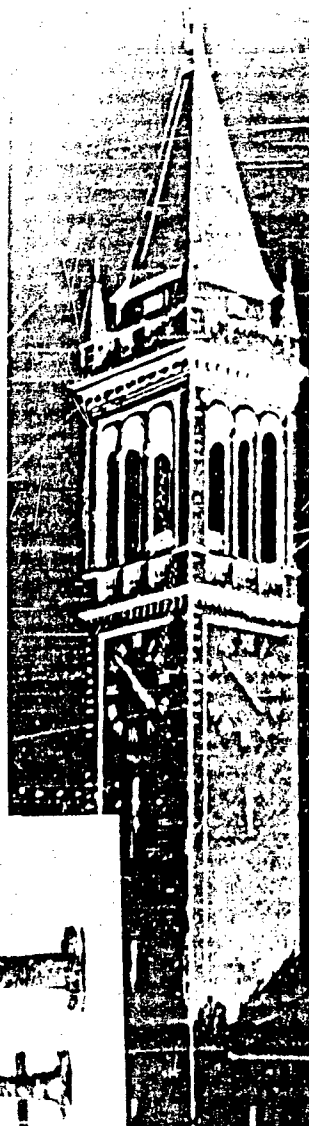
19990708210

Reproduced From
Best Available Copy



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.



AS AD NO. 42764

A Solution to the Frequency-Independent Antenna Problem

by

B. R. Cheo

V. H. Rumsey

W. J. Welch

JAN 26 1962
RECEIVED
TISIA B

Series No. 60 Issue No. 428

Contract No. AF 49(638)-1043

January 8, 1962

ELECTRONICS RESEARCH LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA

"Qualified requestors may obtain copies of this report from the Armed Services Technical Information Agency (ASTIA). Department of Defense contractors must be established for ASTIA services, or have their 'need-to-know' certified by the military agency cognizant of their contract."

"This report has been released to the Office of Technical Services, Department of Commerce, Washington 25, D.C., for sale to the general public."

Electronics Research Laboratory
University of California
Berkeley, California

A SOLUTION TO THE FREQUENCY-INDEPENDENT ANTENNA PROBLEM

by

B. R. Cheo
V. H. Rumsey
W. J. Welch

Institute of Engineering Research
Series No. 60, Issue No. 428

Air Force Office of Scientific Research
of the Air Research and Development Command;
Department of the Navy, Office of Naval Research;
and Department of the Army
Contract No. AF 49(638)-1043

January 8, 1962

A Solution to the Frequency-Independent Antenna Problem*

B. R.-S. CHEO†, MEMBER, IRE, V. H. RUMSEY‡, FELLOW, IRE, AND W. J. WELCH‡, MEMBER, IRE

Summary—A solution of Maxwell's equations is obtained for an antenna consisting of an infinite number of equally spaced wires in the form of coplanar equiangular spirals. Radiation amplitude patterns obtained from this solution agree closely with measurements on two-element spiral antennas. The phase pattern shows the approximate validity of a phase center at a distance behind the antenna which decreases with the tightness of the spiral. The current distribution clearly shows increased attenuation with increase in the tightness of the spiral, thus showing how the frequency-independent mode depends on the curvature. A remarkable feature of the solution is that the current consists of an inward traveling wave at infinity when the antenna is excited in π -sense which produces an outward wave at the center.

I. INTRODUCTION

IT has been found in recent years that there is a large class of antennas which are independent of frequency in essentially all their characteristics such as impedance, pattern, polarization and so on.¹⁻³ The equiangular spiral antenna is one of the basic types: that illustrated in Fig. 1 consists of two conductors cut out of a plane metal sheet. Let us consider how this antenna scales with the wavelength. The shape of the antenna is given by the formula (in polar coordinates r and ϕ)

$$r = e^{-a\phi} \quad (a \text{ is a constant}). \quad (1)$$

Therefore,

$$\frac{r}{\lambda} = e^{-a(\phi - \phi_0)}, \quad (2)$$

where

$$\phi_0 = \frac{1}{a} \ln \lambda. \quad (3)$$

This shows that a change of wavelength λ is equivalent to turning the antenna through the angle ϕ_0 , except for the scaling of the radius r_0 shown in Fig. 1. Now the remarkable property of these antennas is that, so long as the wavelength is shorter than about $2r_0$, the performance is independent of frequency, except for the rotation described in (2) and (3), and therefore it is the same as if r_0 were infinite. Evidently this means that the current distribution must decrease with distance from the input much more rapidly than it does for conventional antennas.

To bring out this point let us compare it with the bi-conical antenna, shown in Fig. 2. The field, represented by the vectors E and H , decreases as $1/r$ for large values of r , and therefore the surface current J (which equals tangential H) also decreases as $1/r$. The total current I is $2\pi r \sin \alpha J$, where α is the angle of the cone shown in Fig. 2. Thus I remains constant with increasing r . The peculiarity of frequency-independent antennas is then that the field at the surface of the antenna must decrease more rapidly than $1/r$, or alternatively, the total current must decrease fast enough, so that the infinite antenna can be truncated with practically no effect on the radiation pattern.

The theoretical problem posed by the equiangular spiral antenna is to solve Maxwell's equations subject to the vanishing of tangential E on the metal surface, the radiation condition at infinity and the input condition at $r=0$. For the two-element antenna of Fig. 1, this has so far proved intractable even for the infinite case.⁴ We are therefore driven to consider some simpler problem which, while retaining the frequency-independent feature, is amenable to theoretical solution. The problem we shall consider in this paper is such a simplification. It can be described by taking an antenna with many elements, as in Fig. 3, the space between the ele-

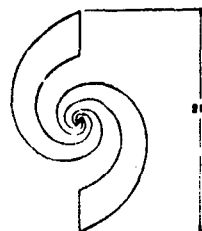


Fig. 1.

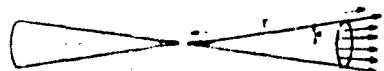


Fig. 2.



Fig. 3.

* Received by the PGAP, May 3, 1961. This research was supported by the U. S. Army Signal Corps under Contract DA 36-039 SC-81923.

† Bell Telephone Labs.; formerly with the University of California, Berkeley, Calif.

‡ Elec. Engrg. Dept., University of California, Berkeley, Calif.

1 V. H. Rumsey, "Frequency-independent antennas," 1957 IRE NATIONAL CONVENTION RECORD, pt. 1, pp. 114-118.

2 R. H. Duffin and D. E. Isbell, "Logarithmically periodic antennas," 1957 NATIONAL CONVENTION RECORD, pt. 1, pp. 119-128.

3 E. D. Dyson, "Equiangular spiral antennas," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-7, pp. 181-187; April, 1959.

4 P. E. Mast, "A Theoretical Study of the Equiangular Spiral Antenna," Elec. Engrg. Res. Lab., University of Illinois, Urbana, Tech. Rept. No. 35; September, 1958.

ments being the same as the space occupied by an element, so that the antenna is "self-complementary" in the sense of Rumsey.¹ We now suppose that the number of elements is infinite, so that the antenna takes the form of a smooth anisotropic sheet which is perfectly conducting in the direction of the spiral lines and perfectly transparent in the perpendicular direction.

This is the kind of problem which can be solved² by putting $E = j\eta H$ on one side of the antenna and $E = -j\eta H$ on the other side, where E and H are complex vectors defined according to the $e^{j\omega t}$ time convention, and η is the intrinsic impedance of space. The boundary conditions at the surface are that tangential E be continuous, tangential H be discontinuous by the amount of the surface-current density, E parallel to the spirals be zero, and H parallel to the spirals be continuous. All of these conditions are met if we make E parallel to the wires vanish and tangential E continuous, with $E = j\eta H$ above the surface, and $E = -j\eta H$ below the surface.

The source of fields on this antenna is located at its center. Recognizing that the structure is essentially uniform in azimuth, we assume that the fields of the antenna will have the same dependence on the coordinate ϕ as the source. Thus, we shall take the ϕ variation of the field to be everywhere $e^{jn\phi}$, where n is an integer. This corresponds to the excitation arrangement shown in Fig. 3, in which each generator has the same magnitude as its neighbor and differs infinitesimally from its neighbor in phase. The case $n=1$ corresponds approximately to the excitation of the balanced two-arm antenna, shown in Fig. 1.

II. FORMAL SOLUTION

Suppose that the antenna lies in the plane $z=0$ of the cylindrical coordinate system of Fig. 4. Let $E_1 = j\eta H_1$ for $z>0$ and $E_2 = -j\eta H_2$ for $z<0$. Then we have³

$$E_1 = -\beta \nabla \times (\nabla U_1) + \nabla \times \nabla \times (\nabla U_1), \quad (4)$$

$$E_2 = \beta \nabla \times (\nabla U_2) + \nabla \times \nabla \times (\nabla U_2). \quad (5)$$

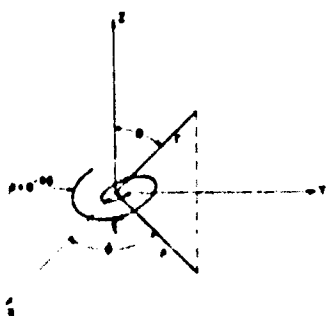


Fig. 4.

¹ V. H. Rumsey, "A New Way of Solving Maxwell's Equations," Electronics Res. Lab., University of California, Berkeley, Series No. 60, Issue No. 43, December 19, 1960. Also to be published in IRE TRANS. ON ANTENNAS AND PROPAGATION.

The functions U_1 and U_2 satisfy the scalar wave equation

$$\nabla^2 U + \beta^2 U = 0, \quad \beta = \omega/c. \quad (6)$$

We can express a general solution of (6) which varies as $e^{jn\phi}$ by using the Hankel transform formula:

$$U_1 = e^{jn\phi} \int_0^\infty g_1(\lambda) J_n(\lambda \rho) e^{+j\sqrt{\beta^2 - \lambda^2} z} \lambda d\lambda, \quad (7)$$

$$U_2 = e^{jn\phi} \int_0^\infty g_2(\lambda) J_n(\lambda \rho) e^{+j\sqrt{\beta^2 - \lambda^2} z} \lambda d\lambda, \quad (8)$$

in which $g_1(\lambda)$ and $g_2(\lambda)$ are arbitrary functions. The Bessel function of the first kind, namely J_n , has been chosen in order that the field be regular at $\rho=0$ for $z \neq 0$. In order that the fields radiate away from the structure, the negative sign must be taken in the exponential factor in the integrand of (7), and the positive sign in the integrand of (8). Then the continuity of tangential electric field at $z=0$ is satisfied if we put $g_1(\lambda) = g_2(\lambda) = g(\lambda)$, as can be verified by direct substitution into (4) and (5). Then,

$$U_1 = e^{jn\phi} \int_0^\infty g(\lambda) J_n(\lambda \rho) e^{-j\sqrt{\beta^2 - \lambda^2} z} \lambda d\lambda, \quad (9)$$

$$U_2 = e^{jn\phi} \int_0^\infty g(\lambda) J_n(\lambda \rho) e^{+j\sqrt{\beta^2 - \lambda^2} z} \lambda d\lambda. \quad (10)$$

The remaining condition, $E_1 \cdot l = 0$, l being tangential to the spiral wires, will determine $g(\lambda)$. From (1) we find $E_1 \cdot l = 0$ implies that

$$a E_{1\phi} = E_{1\phi}. \quad (11)$$

Substitution into this equation from (4) leads to the following expression for the boundary condition:

$$a \left(\frac{\partial U_1}{\partial z} - \frac{\beta}{\rho} \frac{\partial U_1}{\partial \phi} \right) = \frac{1}{\rho} \frac{\partial^2 U_1}{\partial \phi \partial z} + \beta \frac{\partial U_1}{\partial \rho} \Big|_{z=0}. \quad (12)$$

Then, substituting (9) into (12), we find

$$\int_0^\infty g(\lambda) \left\{ (jn a \beta + n \sqrt{\beta^2 - \lambda^2}) \frac{\lambda J_n(\lambda \rho)}{\rho} + (j a \sqrt{\beta^2 - \lambda^2} + \beta) \lambda^2 J_n'(\lambda \rho) \right\} d\lambda = 0. \quad (13)$$

Then term containing the derivative of the Bessel function may be integrated by parts, so that (13) becomes

$$\int_0^\infty \left\{ g(\lambda) (jn a \beta + n \sqrt{\beta^2 - \lambda^2}) \lambda \right. \\ \left. - \frac{d}{d\lambda} \left[g(\lambda) \lambda^2 (j a \sqrt{\beta^2 - \lambda^2} + \beta) \right] \right\} \frac{J_n(\lambda \rho)}{\rho} d\lambda \\ + \left. \frac{g(\lambda) [j a \sqrt{\beta^2 - \lambda^2} + \beta] \lambda^2 J_n(\lambda \rho)}{\rho} \right|_0^\infty = 0. \quad (14)$$

Suppose that $g(\lambda) [ja\sqrt{\beta^2 - \lambda^2} + \beta] \lambda^n J_n(\lambda\rho)$ vanishes for λ equal to zero or infinity. Then the boundary terms in (14) may be discarded. (We shall see later that this assumption is justified.) Applying the inverse Hankel transform to (14) with the boundary terms set equal to zero yields an ordinary differential equation for $g(\lambda)$:

$$(\beta\lambda + j\lambda a\sqrt{\beta^2 - \lambda^2})g'(\lambda) + \left[\beta(z - jna) - (n - 2ja)\sqrt{\beta^2 - \lambda^2} - \frac{j\lambda^2}{\sqrt{\beta^2 - \lambda^2}} \right] g(\lambda) = 0. \quad (15)$$

For convenience let $\lambda = y\beta$ and $g(y\beta) = f(y)$. In terms of $f(y)$ the solution to (15) is

$$g(\lambda) = g(y\beta) = f(y) = k \left(\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}} \right)^{n/2} \cdot y^{-2} (1 + aj\sqrt{1 - y^2})^{-1-j(n/a)}. \quad (16)$$

Notice that $f(y)$ is independent of β , exhibiting the frequency-independent nature of the solution explicitly.

For $n > 0$, the behavior of $f(y)$ is such that the integral (9) exists, and the assumption that the boundary terms in (14) vanish is valid. For $n < 0$, $f(y)$ becomes infinite at $y = 0$ or $\lambda = 0$ and (9) diverges. It turns out that we can obtain a solution for $n < 0$ only if we begin with the assumption that $E_1 = -j\eta I_1$ and $E_2 = j\eta I_2$. There appears to be a simple explanation for this. With the radiation condition fixed, the choice of the plus or minus sign in the equation $E = \pm j\eta I$ determines the sense of polarization of the far field. At the same time, the sign of n specifies the polarization sense of the source. The interpretation of the situation described above is that the field must have the same sense of polarization as the source.

The complete expressions for U_1 are (taking $n > 0$)

$$U_1 = ke^{jn\phi} \int_0^\infty \left(\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}} \right)^{n/2} \frac{(1 + aj\sqrt{1 - y^2})^{-1-j(n/a)}}{y} \cdot e^{-\sqrt{1 - y^2} \beta r \cos \theta} J_n(\beta \rho y) dy, \quad (17)$$

or, for $n < 0$,

$$U_1 = ke^{-jn\phi} \int_0^\infty \left(\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}} \right)^{n/2} \frac{(1 - ja\sqrt{1 - y^2})^{-1-j(n/a)}}{y} \cdot e^{-\sqrt{1 - y^2} \beta r \cos \theta} J_n(\beta \rho y) dy, \quad (18)$$

where k is a constant which is to be adjusted according to the source strength. Notice that the integrand contains a branch point at $y = +1$ in the complex y plane. The branch cut must be taken in the fourth quadrant, and the path of integration must pass over the branch point in order that $(1 - y^2)^{1/2} \rightarrow -j(y^2 - 1)^{1/2}$ for $y > 1$. This completes the formal solution to the boundary-value problem.

III. LIMITING CASES

In this section we shall evaluate the integral for several limiting cases to find the behavior of the field near the input terminals, the radiation pattern, and the behavior of the antenna current at large distances from the input terminals.

A. The Field Near the Input Terminals

The requirement that the behavior of the field approach the static field distribution near the input terminals was never actually employed in the derivation of the preceding section, and it must be verified that this condition is in fact satisfied by (17) and (18). Let us consider the behavior of the electric field as $\beta r \rightarrow 0$. According to (4) and (6),

$$E_1 = -\beta \nabla \times (\partial U_1) + \nabla \times \nabla \times (\partial U_1) \\ = -\beta \nabla \times (\partial U_1) + \nabla \left(\frac{\partial U_1}{\partial z} \right) + \beta^2 \partial U_1. \quad (19)$$

In the limit as $\beta r \rightarrow 0$, the second term of (19) dominates.

$$\lim_{\beta r \rightarrow 0} E_1 = \nabla \left(\frac{\partial U_1}{\partial z} \right). \quad (20)$$

This implies that as $\beta r \rightarrow 0$, $\partial U / \partial z$ must approach the static potential distribution, which is

$$V = r^{j(n/a)} e^{jn\phi} P_{j(n/a)}^n(\cos \theta) = (re^{-a\phi})^{j(n/a)} P_{j(n/a)}^n(\cos \theta) \quad (21)$$

The function V satisfies Laplace's equation and is constant along the wires: it is the standard form $r^m P_m^n(\cos \theta) e^{jn\phi}$ with $m = j(n/a)$.

From (17) we find that

$$\frac{\partial U_1}{\partial z} = ke^{jn\phi} \int_0^\infty \left(\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}} \right)^{n/2} \frac{(1 + aj\sqrt{1 - y^2})^{-1-j(n/a)}}{y} \cdot (-j\sqrt{1 - y^2}) e^{-\sqrt{1 - y^2} \beta r \cos \theta} J_n(\beta \rho y \sin \theta) dy, \quad (22)$$

where we have put $z = r \cos \theta$ and $\rho = r \sin \theta$. For small values of βr , the Bessel function is small except where y is very large, because $J_n(x) \rightarrow x^n$ as $x \rightarrow 0$. Since the other part of the integrand is well behaved in the neighborhood of $y = 0$, the entire integrand contributes very little, except where y is large, in this limit. Therefore, it is reasonable to approximate the part of the integrand other than the Bessel function by its behavior for large y and consider the resulting integral. Hence,

$$\lim_{\beta r \rightarrow 0} \frac{\partial U_1}{\partial z} = ke^{jn\phi} \int_0^\infty y^{-1-j(n/a)} e^{-\beta r \cos \theta} J_n(\beta r \sin \theta y) dy \\ = \frac{k \Gamma\left(n - j\frac{n}{a}\right) \Gamma\left(j\frac{n}{a} - n + 1\right)}{\Gamma\left(j\frac{n}{a} + n + 1\right)} \cdot e^{jn\phi} (\beta r)^{j(n/a)} P_{j(n/a)}^n(\cos \theta). \quad (23)$$

Apart from the constant multiplier, this agrees precisely with (21).

Furthermore, (23) shows that the magnitude of the current flowing into a sector of the antenna from the source is constant. Thus, if I_s is the current per unit angle at the input, $I_s = \rho J$, where J is the surface current density, and

$$J = 2(H_\phi + aH_\theta)/(1 + a^2)^{1/2} = (2/a)(1 + a^2)^{1/2}H_\theta.$$

According to (19) and (23),

$$H_\theta|_{r=1/2} \propto E_\theta|_{r=1/2} \propto \frac{1}{\rho} e^{jn\phi} (\beta\rho)^{1/(n/a)} P_{j(n/a)}^n(0),$$

and ρ times this quantity has a constant magnitude.

B. Radiation Patterns

In order to investigate the radiation properties of the antenna, we need only consider the asymptotic behavior of the field at large distances from the structure. We shall see that the method of stationary phase readily lends itself to the asymptotic evaluation of (17) for large values of both ρ and z . However, before the integral can be approximated, the differentiations indicated in (5) for the electric field must first be performed. Of interest are the components of the electric field with respect to the spherical coordinate system (r, θ, ϕ) of Fig. 4. Because the distant field is circularly polarized,⁵ we need only work out E_ϕ , a component which is common to both the cylindrical and spherical systems. Using (4), we find

$$E_\phi = k\beta^2 e^{jn\phi} \int_0^\pi f(y) \left\{ \beta n \sqrt{1 - y^2} \frac{J_n(\beta y \rho)}{\rho} + \beta^2 y \left[J_{n-1}(\beta \rho y) - \frac{n}{\beta \rho y} J_n(\beta \rho y) \right] \right\} e^{-j\beta z \sqrt{1 - y^2}} y dy, \quad (24)$$

where $\rho = r \sin \theta$ and $z = r \cos \theta$. Except at $\theta = 0$, both ρ and z are large when r is large. With ρ large, the leading term in the integrand of (24) is the term containing the factor $J_{n-1}(\beta \rho y)$. Furthermore, the Bessel function may be replaced by its asymptotic value for large argument,⁶

$$\lim_{x \rightarrow \infty} J_n(x) = \sqrt{\frac{2}{\pi x}} \left\{ \frac{e^{j(x - (n+1/2)\pi/2)}}{2} + \frac{e^{-j(x - (n+1/2)\pi/2)}}{2} \right\}, \quad (25)$$

Using this in the integrand of (24) causes only a second-order error even in the neighborhood of $y = 0$, because $f(y)$ tends to zero as y^{n-2} and therefore the integrand tends to zero as y^{n-1} . Using these approximations and substituting r for $\rho/\sin \theta$ and $z/\cos \theta$, we obtain the fol-

lowing approximation for (24) for large r :

$$E_\phi \approx k\beta^2 e^{jn\phi} \int_0^\pi \frac{f(y) y^{n-2}}{\sqrt{\pi r} \sin \theta} \cdot \left\{ e^{-j\beta r \sqrt{1 - y^2} \cos \theta} \frac{1 - y^2 + y \sin \theta + j(\pi/2)(n+1/2)}{\sqrt{1 - y^2} + y \sin \theta + j(\pi/2)(n+1/2)} \right. \\ \left. + e^{j\beta r \sqrt{1 - y^2} \cos \theta} \frac{1 - y^2 + y \sin \theta - j(\pi/2)(n+1/2)}{\sqrt{1 - y^2} + y \sin \theta - j(\pi/2)(n+1/2)} \right\} dy. \quad (26)$$

This integral contains two terms of the following form: an exponential phase term with a large factor r , multiplied by a relatively slowly-varying function of the variable of integration. According to the principle of stationary phase, the main contribution to this integral comes from the neighborhood of the stationary points of the phase function. In general,⁷

$$\int_a^b g(t) e^{jxh(t)} dt \approx \left[\frac{2\pi}{x |h''(\tau)|} \right]^{1/2} g(\tau) e^{jxh(\tau)} e^{\pm j(\pi/4)}, \quad (27)$$

where x is the large parameter, $h'(\tau) = 0$, and the plus or minus sign is to be taken according to whether the stationary point is a minimum or maximum. Only the second of the two terms in (26) has a real stationary point, and its value is $y = \sin \theta$. Applying formula (27), we find

$$E_\phi \approx k\beta^2 e^{jn\phi} \cos \theta f(\sin \theta) \frac{e^{-j\beta r}}{r} e^{jn(\pi/2)}. \quad (28)$$

Furthermore, from (16),

$$f(\sin \theta) = \frac{(1 + aj \cos \theta)^{-1-j(n/a)} \left(\tan \frac{\theta}{2} \right)^n}{\sin^2 \theta}. \quad (29)$$

Finally, the far-zone electric field is

$$E_\phi \approx k\beta^2 \frac{\cos \theta (1 + aj \cos \theta)^{-1-j(n/a)} \left(\tan \frac{\theta}{2} \right)^n}{\sin^2 \theta} \frac{e^{jn(\pi/2) - \beta r}}{r}, \quad n > 0, \quad (30)$$

If we express the field in terms of magnitude and phase

$$E_\phi \approx A(\theta) e^{-j\psi(\theta)} \frac{e^{jn(\pi/2) - \beta r}}{r}, \quad (31)$$

we have

$$A(\theta) = \frac{\cos \theta \left(\tan \frac{\theta}{2} \right)^n e^{(n/a) \tan^{-1}(a \cos \theta)}}{\sin \theta \sqrt{1 + a^2 \cos^2 \theta}}, \quad (32)$$

and

$$\psi(\theta) = \frac{n}{2a} \ln |1 + a^2 \cos^2 \theta| + \tan^{-1} a \cos \theta. \quad (33)$$

⁵ J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Co., Inc., New York, N. Y., ch. 6, p. 359, (18); 1941.

⁷ A. Erdelyi, "Asymptotic Expansions," Dover Publications, Inc., New York, N. Y., p. 51, 2.9(2); 1956.

For $e^{-jn\phi}$ excitation $A(\theta)$ is the same but the sign of $\psi(\theta)$ is reversed. The pattern $A(\theta)$ is plotted in Figs. 5 and 6 for various values of n and a . As is typical with frequency-independent antennas, there is no radiation along the surface of the structure. The patterns predicted by (33) agree remarkably well with measurements made by Dyson³ on two-arm spiral antennas. According to (32), making a small decreases the beamwidth, but only up to a point. For the case $n=1$, the minimum beamwidth attainable is approximately 70° .

Before leaving the discussion of the radiation field, we shall consider the question of whether the antenna has a phase center. The total phase of the far field, apart from the ϕ dependence and some constants, is given by

$$\beta r + \psi(\theta). \quad (34)$$

Because of the complicated form of (33), (34) does not, in general, describe a circular phase front. However, when a is small, $\psi(\theta) \approx a \cos \theta$. In this case, the phase fronts are approximately circular, and, according to the diagram of Fig. 7, the antenna has a phase center located $a/2\pi$ wavelengths behind its center.

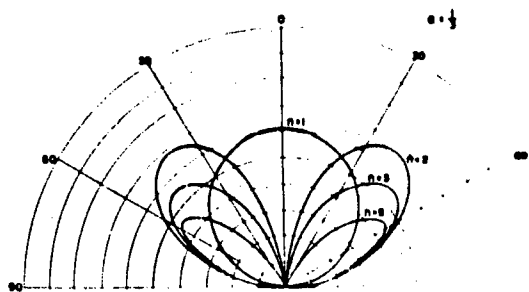


Fig. 5.

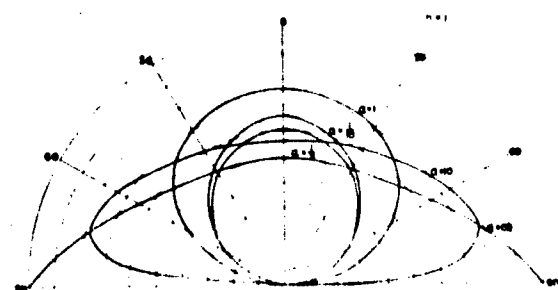


Fig. 6.

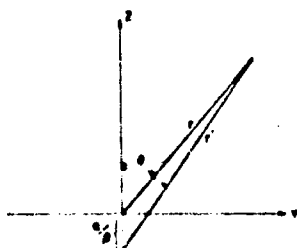


Fig. 7.

C. The Current Distribution

As we saw in Section 1, the current distribution is one of the peculiar features of frequency-independent antennas. In the present case, it is obtainable from the field at the plane $z=0$. Since E is proportional to H and E_ϕ is proportional to E_r at $z=0$, the surface-current density is proportional to E_ϕ . The current density per unit of angle ϕ corresponds to the total antenna current I ; it varies as ρE_ϕ . Unfortunately it has not been possible to work out the current for all values of ρ . However, fairly simple expressions have been obtained for small values of ρ and alternatively for large values of ρ . For small values of ρ we have already found that

$$I \approx e^{jn(\phi + (1/a) \ln \rho)},$$

which has constant amplitude and rapidly varying phase as a function of ρ . Note however that the phase is constant if we move along a spiral as it ought to be for the steady-current case.

Turning now to the case where ρ is large, according to (4) and (9) and (16), we find that

$$I \propto E_\phi = k\beta^2 e^{jn\phi} \int_0^\infty f(y) \left[n\sqrt{1-y^2} - \frac{J_n(\beta y \rho)}{\rho} + \beta y J_n'(\beta y \rho) \right] y dy. \quad (35)$$

Upon integrating (35) by parts and substituting for $f(y)$ from (16), we obtain, with $n > 0$,

$$I \propto e^{jn\phi} \int_0^\infty \left(\frac{1 - \sqrt{1-y^2}}{1 + \sqrt{1-y^2}} \right)^{n/2} \left(n + \frac{1}{\sqrt{1-y^2}} \right) \cdot (1 + aj\sqrt{1-y^2})^{-2-j(n/a)} J_n(\beta y \rho) y dy. \quad (36)$$

For $n < 0$ the correct formula is the conjugate of (36), not the result of reversing the sign of n —see (18). We express the integral as the sum of two integrals over the intervals $(0, 1)$ and $(1, \infty)$, and treat the two parts separately. Consider first the integration over $(0, 1)$:

$$I_1 = \int_0^1 \left(\frac{1 - \sqrt{1-y^2}}{1 + \sqrt{1-y^2}} \right)^{n/2} \left(n + \frac{1}{\sqrt{1-y^2}} \right) \cdot (1 + aj\sqrt{1-y^2})^{-2-j(n/a)} J_n(\beta y \rho) y dy. \quad (37)$$

The singularity at $y=1$ makes the major contribution to the integral. This is especially true for large $\beta\rho$, in which case the Bessel function oscillates very rapidly as a function of y and cancels all contributions to the integral, except from those regions where the rest of the integrand is also a rapidly varying function of y . We will expand the integrand, excepting the Bessel function, in ascending powers of $(1-y^2)^{1/2}$, beginning with $1/(1-y^2)^{1/2}$, and integrate term by term. Each term in

the resulting series will have successively less importance for large $\beta\rho$ because of the relative smoothness of the successive powers of $[(1-y^2)^{1/2}]^n$. We rewrite (37) slightly and then perform the expansion according to Maclaurin's formula:

$$I_1 = \int_0^1 \left\{ \frac{1}{y^n} \left(\frac{1 - \sqrt{1-y^2}}{1 + \sqrt{1-y^2}} \right)^{n/2} \left(n + \frac{1}{\sqrt{1-y^2}} \right) \cdot (1 + aj\sqrt{1-y^2})^{-2-j(n/a)} \right\} y^{n+1} J_n(\beta\rho y) dy$$

$$= \int_0^1 \sum_{m=1}^{\infty} a_m (\sqrt{1-y^2})^m y^{n+1} J_n(\beta\rho y) dy. \quad (38)$$

Each term in the series may be integrated by means of Sonine's first formula.⁹

$$J_{\mu+(v/2)+1}(z) = \frac{z^{(v/2)+1}}{2^{(v/2)} \Gamma\left(\frac{v}{2} + 1\right)} \int_0^{\pi/2} J_n(z \sin \theta) \sin^{v+1} \theta \cos^{v+1} \theta d\theta. \quad (39)$$

Substituting $y = \sin \theta$ in (38) and using (39), we find

$$I_1 = \sum_{m=1}^{\infty} \frac{a_m 2^{m/2} \Gamma\left(1 + \frac{m}{2}\right)}{(\beta\rho)^{(m/2)+1}} J_{n+(m/2)+1}(\beta\rho). \quad (40)$$

Consider next the integration over $(1, \infty)$, which we write in the following form:

$$I_2 = \int_1^{\infty} \left\{ \left(\frac{1 + j\sqrt{y^2-1}}{1 - j\sqrt{y^2-1}} \right)^{n/2} (1 + \sqrt{y^2-1})^{-2-j(n/a)} \cdot \left(n + \frac{j}{\sqrt{y^2-1}} \right) \right\} J_n(\beta\rho y) y dy. \quad (41)$$

Following the same reasoning as before, we expand the term in the braces of (41) in a series such that each term can be integrated and, furthermore, such that each term has successively less importance for large $\beta\rho$. In this case, the expansion is in powers of $(y^2-1)^{1/2}/y$.

$$I_2 = j \int_1^{\infty} \sum_{n=0}^{\infty} b_n \left(\frac{\sqrt{y^2-1}}{y} \right)^n \frac{J_n(\beta\rho y) dy}{\sqrt{y^2-1} y^{n+1}}. \quad (42)$$

There appears to be no single integral formula which can be applied to every term in (42), so that each term must be treated separately. In what follows, we shall work out only the first four terms of the series obtaining an

asymptotic expansion in $\beta\rho$ up to terms which behave as $O(\beta\rho^{-1})$. The first is

$$I_{20} = j b_0 \int_1^{\infty} \frac{J_n(\beta\rho y) dy}{\sqrt{y^2-1} y^{n+1}} = j b_0 \int_0^{\pi} \frac{J_n(\beta\rho \sqrt{1+\xi^2}) d\xi}{(1+\xi^2)^{n/2}}, \quad (43)$$

where $y = (1+\xi^2)^{1/2}$. This may be evaluated by means of Sonine's second formula.⁹

$$\int_0^{\pi} \frac{J_n(a\sqrt{t^2+z^2}) t^{2\mu+1} dt}{(t^2+z^2)^{\mu/2}} = \frac{2^{\mu} \Gamma(\mu+1)}{a^{\mu+1} z^{\mu-1}} J_{\mu-1}(az). \quad (44)$$

Let $z=1$, $\beta\rho=a$, and $\mu = -(\frac{1}{2})$. Then, applying (44) to (43) we obtain

$$I_{20} = \frac{j b_0 \Gamma(\frac{1}{2})}{\sqrt{2\beta\rho}} J_{n-1/2}(\beta\rho). \quad (45)$$

The second term is

$$I_{21} = j b_1 \int_1^{\infty} \frac{J_n(\beta\rho y) dy}{y^n}. \quad (46)$$

An asymptotic expansion of this integral may be obtained by repeated integration by parts. In general,

$$- \int_1^{\infty} \frac{J_n(\beta\rho y) dy}{y^n} = \frac{1}{\beta\rho} J_{n+1}(\beta\rho) + \frac{n+n+1}{(\beta\rho)^2} J_{n+2}(\beta\rho) + O\left(\frac{1}{\beta\rho}\right)^3, \quad (47)$$

where $p > \frac{1}{2}$. In principle, one could carry out (47) to as many terms as desired. Thus, for the second term,

$$- I_{21} = j b_1 \left\{ \frac{J_{n+1}(\beta\rho)}{\beta\rho} + (2n+1) \frac{J_{n+2}(\beta\rho)}{(\beta\rho)^2} \right\} + O\left(\frac{1}{\beta\rho}\right)^3. \quad (48)$$

The third term is

$$I_{22} = j b_2 \int_1^{\infty} \frac{\sqrt{y^2-1}}{y} \frac{J_n(\beta\rho y) dy}{y^n}. \quad (49)$$

After one integration by parts we find

$$I_{22} = -j b_2 \left\{ \frac{1}{\beta\rho} \int_1^{\infty} \frac{J_{n+1}(\beta\rho y) dy}{y^n \sqrt{y^2-1}} - \frac{2n+2}{\beta\rho} \int_1^{\infty} \frac{J_{n+1}(\beta\rho y) \sqrt{y^2-1} dy}{y^{n+2}} \right\}. \quad (50)$$

⁹ G. N. Watson, "A Treatise on the Theory of Bessel Functions," Cambridge University Press, Cambridge, Eng., 2nd ed., ch. 12, p. 473, 1952.

¹⁰ *Ibid.*, ch. 13, p. 417.

The first term of (50) may be evaluated by means of Sonine's second formula (44). Repeated integration by parts shows that the second term of (50) is $O(\beta\rho)^{-3}$ and may be discarded. Thus,

$$I_{22} = \frac{-jb_2\Gamma(\frac{1}{2})}{\sqrt{2}(\beta\rho)^{3/2}} J_{n+(1/2)}(\beta\rho) + O\left(\frac{1}{\beta\rho}\right)^2. \quad (51)$$

The fourth term of the series is

$$\begin{aligned} I_{24} &= jb_4 \int_1^\infty \left(\frac{y^2 - 1}{y^2} \right) \frac{J_n(\beta\rho y) dy}{y^n} \\ &= jb_4 \int_1^\infty \frac{J_n(\beta\rho y) dy}{y^n} - jb_4 \int_1^\infty \frac{J_n(\beta\rho y) dy}{y^{n+2}}. \end{aligned} \quad (52)$$

We may apply the result of (47) to the two terms in (52) to obtain

$$I_{24} = jb_4 \frac{2J_{n+2}(\beta\rho)}{(\beta\rho)^2} + O\left(\frac{1}{\beta\rho}\right)^2. \quad (53)$$

It is possible to show that the next term in (42) contributes only $O(\beta\rho)^{-3}$ to the series. Let the input current per unit angle be I_0 . Then taking the first four terms of (40), adding them to (45), (48), (51), and (53), and adjusting the constant of proportionality to the input current, we find

Straight wires are represented by $a = \infty$, but our integral representation (17) fails in this case which therefore has to be considered separately. The solution is fairly simple and gives a distribution of $|I|$ which is constant with ρ^2 , and a phase velocity equal to that of light, as illustrated on the graphs.

The phase characteristic is perhaps the most interesting feature of these results. For $n > 0$ it consists of an inward slow wave when r is very small, changing to a fast wave as r increases, which becomes infinitely fast at the point where the phase is a maximum in Fig. 9. Passing beyond this point, we find a fast outward wave which slows down to the velocity of light when $r \rightarrow \infty$. For $n < 0$ we find the same sequence of changes, except that the direction of the phase velocity is reversed everywhere. The extraordinary feature is that we now have an inward wave at infinity. At first sight this might appear to be physically inadmissible because certainly the power must flow outward at infinity. However, in this case we are not dealing with the ordinary radiation field, namely the field which varies as $1/r$, for this is zero on the antenna when $r = \infty$. Indeed, that such a reversal of the phase velocity is necessary with reversal of n can be quickly seen for small r by working from the requirement that the current along any individual wire must be constant in the quasi-static approximation. Also, when $r = \infty$, the curvature of the

$$\begin{aligned} I &= I_0 \frac{a^n}{n} \left| \frac{\Gamma\left(j\frac{n}{a} + n + 1\right)}{\Gamma\left(n - j\frac{n}{a}\right) \Gamma\left(j\frac{n}{a} - n + 1\right) P_{n,n,a}^n(0)} \right| e^{jn\phi} \left(\frac{e^{-j\beta\rho + jn(\pi/2)}}{\beta\rho} \left(j + \frac{n^2 + j5an + 6a^2}{2\beta\rho} \right) \right. \\ &\quad \left. - \sqrt{\frac{2}{\pi\beta\rho}} \left(\frac{n}{3} + 3n^2 - \frac{26n^2n}{3} - 2ja - 6jan - \frac{2jn^2a}{3} + 8ja^2 \right) \cos\left(\beta\rho - \frac{n\pi}{2} - \frac{\pi}{4}\right) \right\} \\ &\quad + O(\beta\rho)^{-3}. \end{aligned} \quad (54)$$

For the case $n=1$, this expression reduces to

$$\begin{aligned} I &= I_0 a \sqrt{1 + a^2} e^{j\phi} \left\{ \frac{e^{-j\beta\rho}}{\beta\rho} \left(1 + \frac{5a - j - 6ja^2}{2\beta\rho} \right) - \sqrt{\frac{2}{\pi\beta\rho}} \frac{(10 - 26a^2 - 26ja + 24ja^2) \sin\left(\beta\rho - \frac{\pi}{4}\right)}{3(\beta\rho)^2} \right\} \\ &\quad + O(\beta\rho)^{-3}. \end{aligned} \quad (55)$$

In (54) and (55) the Bessel functions have been replaced by their asymptotic expansions.

The current distribution has also been worked out directly from the integral by using a digital computer for the cases $a=0.1, 0.5$ and 1.0 with $n=1$. The results are plotted in Figs. 8-11 (next page). The salient feature of these graphs is the marked increase in attenuation of the current with increase in the curvature of the spiral.

spiral becomes negligible and the waves become essentially plane. By using the results of Rumsey,⁴ it will be found that solutions for straight wires can be constructed in which the phase velocity is inward on the wires but outward some distance away. It is thus possible to see how the inward wave on the antenna is connected to the outward wave in the radiation field, and to the mode of excitation.

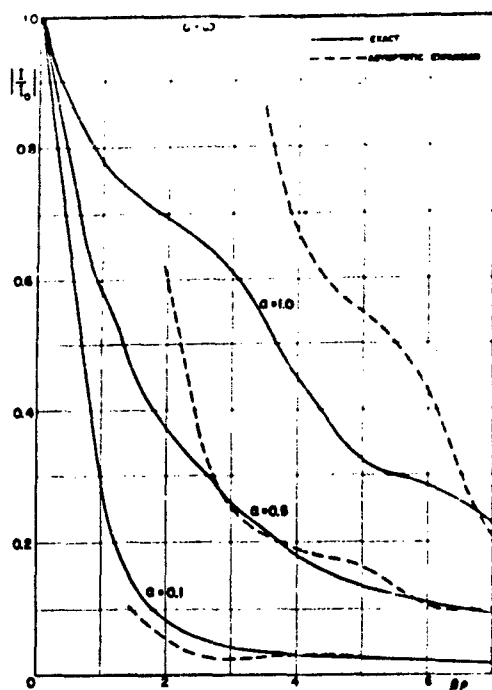


Fig. 8.

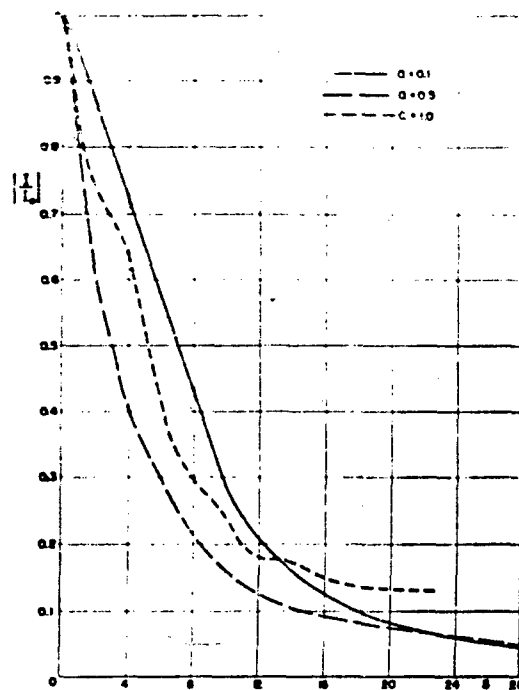


Fig. 10.

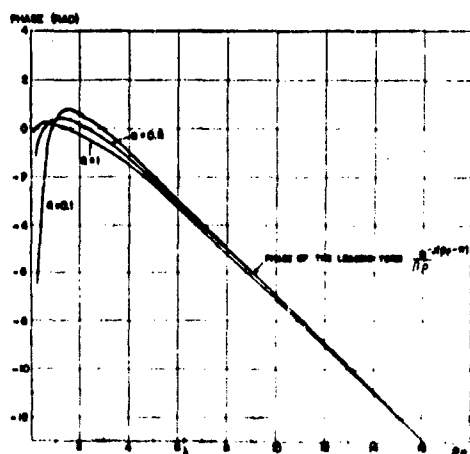


Fig. 9—Phase variation of current distribution as computed by numerical integration.

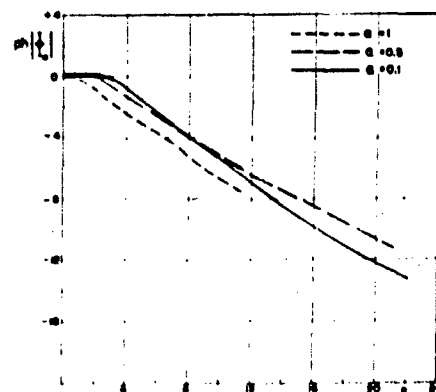


Fig. 11.

Reprinted from IRE TRANSACTIONS
ON ANTENNAS AND PROPAGATION
Volume AP 9, Number 6, November, 1961

PRINTED IN THE U.S.A.

DISTRIBUTION LIST
Contract No. AF 49(508)-1043

ORGANIZATION	NO. COPIES	ORGANIZATION	NO. COPIES	ORGANIZATION	NO. COPIES
Commander AF Office of Scientific Research Washington 25, D. C. Attn: SRY	3	Langley Research Center (NASA) Langley Air Force Base Virginia Attn: Technical Library	1	Dr. E. G. Whiting Deputy Director of Research and Development Lab. Department of the Army Room 3F, 190, Pentagon Washington 25, D. C.	1
Commander Aeronautical Systems Division (ASD) Wright-Patterson Air Force Base Ohio	4	APOS (SRLTL) Holloman Air Force Base New Mexico	1	Dr. E. M. Reilly Director, Institute for Exploratory Research U. S. Army Signal Research and Development Lab. Fort Monmouth, New Jersey	1
P. O. Box AA Wright-Patterson Air Force Base Ohio	1	Lt. Richard M. Elser, USAF Integration Technique Section Molecular Electronics Branch Electronics Technology Laboratory Aeronautical Systems Division Wright-Patterson Air Force Base Ohio Attn: ASRNEM-2	3	Mr. H. F. Lamer Department of Defense Office of Electronics Washington 25, D. C.	1
Armed Services Technical Information Agency Arlington Hall Arlington 12, Virginia Attn: TIPCR	10	Captain M. I. Hooper AF Office of Scientific Research Office of Aerospace Research USAF Washington 25, D. C.	5	Dr. J. P. Ruina Office of Research and Engineering Department of Defense Washington 25, D. C.	1
Office of Naval Research Department of the Navy Washington 25, D. C. Attn: Code 420	1	Dr. Arnold Shostak, Code 427 Head, Electronics Branch Department of the Navy Washington 25, D. C.	3	Chief of Research and Development OSC, Department of the Army Washington 25, D. C.	1
Chief of Research and Development Department of the Army Washington 25, D. C. Attn: Scientific Information Branch	1	Mr. W. C. Eppers, Jr. Ossonge Electronic Section Aeronautical Systems Division AF Systems Command U. S. Air Force Wright-Patterson Air Force Base Ohio	3	Director U. S. Naval Research Laboratory Washington 25, D. C. Attn: Code 2817	1
U. S. Atomic Energy Commission Technical Information Extension P. O. Box 64 Oak Ridge, Tennessee	1	Air Force Cambridge Laboratories HQ Electronic Systems Division Air Force Systems Command USAF Lawrence G. Hanscom Field Bedford, Massachusetts Attn: Sheldon B. Moravetz	1	Commander Wright Air Development Division Wright-Patterson Air Force Base Ohio Attn: WCOS-1	2
Physics Program National Science Foundation Washington 25, D. C.	1	Dr. Hermann Rehl Director, Physical Sciences U. S. Army Research Box CM, Duke Station Durham, North Carolina	1	Commander Rome Air Development Griffith Air Force Base New York Attn: RCOL-2	1
AEDC (ADGEM) Arnold Air Force Station Tullahoma, Tennessee Attn: AEDC Library	1	Aeronautical Systems Division Wright-Patterson Air Force Base, Ohio Attn: ASRNEA-1, Contract AF 33(657)7164	3	Commanding Officer Diamond Ordnance Fuse Laboratories Washington 25, D. C. Attn: Library, Room 311, Building 91	1
Commander AF Missile Development Center Holloman Air Force Base, New Mexico Attn: EDOE	1	Dr. Seth Washburn Bell Telephone Laboratories, Inc. Whippany, New Jersey	1	Adj. Dir. (ident) U. S. Army Security Agency Bldg. Arlington Hall Station Arlington 12, Virginia	1
Commandant AF Institute of Technology (AIU) Library MCL-Lib, Building 115, Area B Wright-Patterson Air Force Base Ohio	1	Dr. R. Weiss Chief Scientist Army Research Office Arlington Hall Washington 25, D. C.	3	Marine Corps Liaison U. S. Army Signal Research and Development Lab. Fort Monmouth, New Jersey Attn: SIGRA/SL-LNR	1
Commander AF Office of Scientific Research Washington 25, D. C. Attn: SROL	4	Dr. Shirlough Silverman Director, Naval Research Group Office of Naval Research Code 404 Washington 25, D. C.	1	Commanding Officer U. S. Army Signal Research and Development Lab. Fort Monmouth, New Jersey Attn: Technical Documents Center	1
Commander AF Cambridge Research Laboratories L. G. Hanscom Field Bedford, Massachusetts Attn: CRREA	1	Col. R. E. Kimball USAROL Signal Corps Fort Monmouth, New Jersey	1	Advisory Group on Electron Devices 144 Broadway New York 15, New York	1
Aeronautical Research Laboratories Building 450 Wright-Patterson Air Force Base Ohio Attn: Technical Library	1	Major R. I. McPadden and Dr. Robert Watson Army Research Office Arlington Hall Department of the Army Washington 25, D. C.	1	Chief of Ordnance Washington 25, D. C. Attn: ORDTX-AR	1
Director of Research and Development Headquarters, USAF Washington 25, D. C. Attn: AFDRD	1	UASD (R and S) Room 5104A The Pentagon Washington 25, D. C. Attn: Technical Library	1	Commander Army Rocket and Guided Missile Agency Redstone Arsenal, Alabama Attn: Technical Library	1
Director Naval Research Laboratory Washington 25, D. C. Attn: Technical Information Officer	1	Chief, Signal Office, Department of the Army Washington 25, D. C. Attn: SIGRD	1	Stanford University Electronic Research Laboratory Palo Alto, California Attn: Prof. D. A. Watkins	1
Chief, Physics Branch Division of Research U. S. Atomic Energy Commission Washington 25, D. C.	1	Commanding Officer and Director U. S. Navy Electronics Laboratory San Diego 32, California	1	Hughes Aircraft Company Culver City, California Attn: Dr. Mendel, Microwave Tube Laboratory	1
National Bureau of Standards Library Room 201, Northwest Building Washington 25, D. C.	1	Commander Air Force Command and Control Development Division Air Research and Development Command USAF Lawrence G. Hanscom Field Bedford, Massachusetts Attn: CROTL	1	Raytheon Manufacturing Company Microwave and Power Tube Operations Waltham 14, Massachusetts Attn: W. C. Brown	1
Commanding Officer Army Research Office (Durham) Box CM, Duke Station Durham, North Carolina	1	Commanding Officer 440th TSU U. S. Army Signal Electronics Research Unit P. O. Box 204 Mountain View, California	1	Radio Corporation of America Laboratories Princeton, New Jersey Attn: Dr. L. S. Nergard	1
Commander AF Flight Test Center Edwards Air Force Base California Attn: FTOTL	1	Leads Research Center (NASA) 2100 Brookpark Road Cleveland 11, Ohio Attn: Technical Library	1	Sylvania Electric Products Physics Laboratory Bayville, Long Island, New York Attn: L. B. Bloom	1
Commander Army Rocket and Guided Missile Agency Redstone Arsenal Alabama Attn: ORDER-OTL	1	Mr. R. E. Olliver, APOS 61-1 University Grants Division AF Office of Scientific Research Washington 25, D. C.	1	Commanding Officer U. S. Army Signal Material Support Agency Fort Monmouth, New Jersey Attn: SIGMS-ADJ	1
Commanding General U. S. Army Signal Corps Research and Development Lab. Fort Monmouth, New Jersey Attn: SIGPM/SL-RPO	1			Commanding Officer U. S. Army Signal Research and Development Lab. Fort Monmouth, New Jersey Attn: SIGRA/SL-PRM (Records File Copy)	1
High Speed Flight Station (NASA) Edwards Air Force Base California Attn: Technical Library	1			Commanding Officer Frankford Arsenal Philadelphia 17, Pennsylvania Attn: ORDBA-FEL	1

ORGANIZATION	NO COPIES	ORGANIZATION	NO COPIES	ORGANIZATION	NO COPIES
Chief, Bureau of Ships Department of the Navy Washington 25, D. C. Attn: 67-46	1	University of Illinois Urbana, Illinois Attn: D. A. Apple, Control Systems Laboratory	1	East Corporation 717 Main Street Santa Monica, California Attn: Margaret Anderson, Librarian	1
University of Michigan Electron Tube Laboratory Ann Arbor, Michigan Attn: Prof. J. E. Rowe	1	Electrical Engineering Department Illinois Institute of Technology Technology Center Chicago 90, Illinois	1	Diamond Ordnance Pulse Laboratory C. S. Ordway Drive Washington 25, D. C. Attn: ORDTL-610-638, Mr. Raymond W. Conner	1
California Institute of Technology Research Laboratory of Electronics Cambridge, Massachusetts Attn: Prof. L. Samuels	1	Brooklyn Polytechnic Institute Microwave Research Institute 55 Johnson Street Brooklyn 1, New York Attn: Dr. A. Ginzler	1	Dr. Herbert K. Lofter Manager, Basic Physics Fairchild Semiconductor 844 Charleston Road Palo Alto, California	1
General Electric Research Laboratory Electron Tube Division The Knolls Schenectady, New York Attn: E. D. MacArthur	1	New York University Mathematical Research Group 15 Waverly Place New York, New York Attn: Dr. M. Eliaz	1	Varian Associates Technical Library 611 Menlo Way Palo Alto, California	1
Rail Telephone Laboratories Murray Hill, New Jersey Attn: Dr. W. Kluwer	1	Stanford University Stanford, California Attn: Applied Electronics Laboratory Document Library	1	Caltech Radio Company Engineering Building Cedar Rapids, Iowa Attn: Dr. Jon M. Mayer, Director of Research Dr. John Crandall, Technical Staff Dr. Frank Kottwitz, Chemical Engineer	1
Westinghouse Electric Corporation Research Laboratory Boulah Road, Churchill Boro Pittsburg 14, Pennsylvania	1	Mr. Julian H. Baglio Institute for Advanced Study Princeton, New Jersey	1	Dr. Dwight L. Wannerson Chief, General Physics Division AF Office of Scientific Research Office of Aerospace Research USAF Washington 25, D. C. Attn: BRPP/DLW	1
Research Division Library Raytheon Company 28 Boyer Street Waltham 54, Massachusetts	1	Texas Technological College Lubbock, Texas Attn: Paul G. Griffin Department of Electrical Engineering	1	Dr. Marshall Vortis Head, Information Systems Branch Department of the Navy Office of Naval Research Washington 25, D. C.	1
Stanford Research Institute Southern California Laboratories 820 Mission Street South Pasadena, California Attn: Document Librarian	1	California Institute of Technology Pasadena, California Attn: Department of Electrical Engineering	1	Mr. Jay Franzen Office of Naval Research Department of the Navy 1000 Oyster San Francisco, California	1
Prof. Sarah Kaprielian University of Southern California Department of Electrical Engineering University Park Los Angeles 7, California	1	Columbia University 510 West 120 Street New York 27, New York Attn: Librarian, Radiation Laboratories	1	Mr. A. W. Sengo Naval Service Force Washington 25, D. C.	1
Lincoln Laboratory Massachusetts Institute of Technology Lexington 73, Massachusetts Attn: Library	1	Ohio State University Columbus, Ohio Attn: Electrical Engineering Department	1	Dr. Lee A. Shinn Head, Biochemistry Branch Office of Naval Research Department of the Navy Washington 25, D. C.	1
University of Wisconsin 180 Prioleau Building Ann Arbor, Michigan Attn: J. E. Rowe Electron Tube Laboratory	1	Harvard University Cambridge, Massachusetts Attn: Technical Reports Collection 101A Pierce Hall	1	Headquarters Electronic Systems Division Air Force Systems Command U. S. Air Force Laurence G. Hanscom Field Bedford, Massachusetts Attn: ESKR/J. J. Cronin/2164	1
Prof. Arvin A. Dougal Department of Electrical Engineering University of Texas Austin 12, Texas	1	General Electric Research Laboratory The Knolls Schenectady, New York	1	National Aeronautics and Space Administration Washington 25, D. C.	1
George Washington University Washington, D. C. Attn: Prof. H. Oriskany	1	The Mitre Corporation P. O. Box 208 Lexington, Massachusetts	1	Advanced Research Projects Agency Washington 25, D. C.	1
University of Illinois Department of Electrical Engineering Urbana, Illinois	1	Hughes Aircraft Company Calver City, California Attn: Document Library Microwave Laboratory Dr. Mendel	1	Ames Research Center (NASA) Moffett Field, California Attn: Technical Library	1
Electronics Division Denver Research Institute University of Denver Denver 10, Colorado Attn: Mr. Carl A. Hedberg, Head	1	Liton Systems, Inc. Advanced Development Laboratory 221 Crescent Street Waltham, Massachusetts Attn: R. Turyn	1		
Prof. Samuel Seely, Head Department of Electrical Engineering Case Institute of Technology University Circle Cleveland 6, Ohio	1	Stanford University Stanford, California Attn: Librarian W. Hansen Library of Physics	1		
Watkins-Johnson Company 311 Milliken Avenue Stanford Industrial Park Palo Alto, California	1	Johns Hopkins University Applied Physics Laboratory 841 Georgia Avenue Silver Spring, Maryland Attn: Supervisor of Technical Reports	1		
Stanford Research Institute Menlo Park, California Attn: M. D. Crane, Computer Laboratory	1	Yale University Department of Electrical Engineering New Haven, Connecticut	1		
Stanford Research Institute Menlo Park, California Attn: Dr. D. E. Schuch, Assistant Director Division of Engineering Research	1	Carnegie Institute of Technology Pittsburgh, Pennsylvania Attn: Director, Computation Center Alan J. Peritz	1		
University of California Los Angeles 24, California Attn: Department of Engineering Prof. Gerald Selvin	1	University of Pennsylvania Moore School of Electrical Engineering 200 South 33rd Street Philadelphia 4, Pennsylvania Attn: Miss Fane L. Compton	1		
Massachusetts Institute of Technology Cambridge 39, Massachusetts Attn: Mr. J. H. Smith, Document Room 26-517 Research Laboratory of Electronics	1	Ohio State University Columbus, Ohio Attn: Antenna Laboratory, Dr. Tol	1		
University of Michigan 180 Prioleau Building Ann Arbor, Michigan Attn: Gordon E. Peterson Communications, Science Laboratory	1	Dr. R. W. Gelline ARCON Incorporated 401 Massachusetts Avenue Lexington, Massachusetts	1		
Department of Electrical Engineering Cornell University Ithaca, New York	1	Raytheon Manufacturing Company Microwave and Power Tube Operations Waltham 54, Massachusetts Attn: Research Division Library	1		
		Amper Computer Products P. O. Box 519 Calver City, California Attn: E. Tomash	1		