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AeroChem Research Laboratories, Inc.
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P. O. BOX 12, PRINCETON, N. J.
PHONE: WAlnut 1-7070

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Dissociating Gases as Natural Convection Coolants[1]

DANIEL E. ROSNER[2]

Introduction

The use of liquids near their critical point has been investigated by F. Schmidt [1] for cooling turbine blades by natural convection. This method takes advantage of (i) buoyancy induced-convection currents due to the very large accelerations associated with turbine blade rotation, and (ii) the augmented thermal expansion coefficient and thermal conductivity of liquids near their critical point. Now, it is also known that the thermal dissociation of polyatomic gases greatly increases their expansion coefficient [2] and thermal conductivity [3-6]. It therefore seems relevant to inquire as to whether equilibrium dissociating gases might not offer some advantages in natural convection cooling applications.[3] For example, in the

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[2] Aeronautical Research Scientist

[3] The only available theoretical study of natural convection in a dissociating gas appears to be that of Scheller and Dryden [7]. In the present note the feasibility of this basic scheme is briefly investigated for this class of coolants in, for example, rotating machinery.

case of a turbine one might ask whether comparable cooling effects can be achieved with less total mass of coolant in the blade, thereby reducing centrifugal stress levels. Our purpose here then is to present and illustrate a method to examine the possible merits of equilibrium dissociating gases for this general class of natural convection cooling applications.

Conventional Natural Convection Cooling Systems

A typical physical configuration envisioned here is shown in Fig. 1. The coolant is confined in one or more holes drilled (in the spanwise direction) in each turbine blade and closed at the outer end. The centrifugal acceleration, a, for such blading is of the order of 10^7 to 10^8 cm/sec^2 and is thus some 10^4 to 10^5 times greater than the gravitational acceleration, g.

In the presence of temperature gradients produced by contact with the convectively heated blade material, the coolant expands and experiences a large body force directed toward the blade root. Heat is removed at this location by a separate cooling system and the heavier (coolant) medium now experiences a body force driving it centrally toward the blade tip. Being closed at this end, the fluid is then heated once again, turns the corner and repeats the cycle. In the thin thermal boundary layer limit[4] attainable heat transfer coefficients have been estimated by Schmidt [1] for the case of water as the coolant by invoking a correlation formula of the Nusselt type Nu = Nu(Gr, Pr), developed from observations of heated vertical plates in air. Here Gr is the familiar Grashof number, defined by

$$Gr \equiv a\,\beta \cdot \Delta T \cdot x^3/\nu^2 \qquad (1)$$

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[4] *i. e.*, non-interacting boundary layers, small curvature effects.

where a is the effective acceleration, β is the thermal expansion coefficient of the fluid, ΔT is the imposed temperature difference, x is the distance from the origin of the thermal boundary layer, and ν is the kinematic viscosity. As pointed out by Ellerbrock [8] Grashof numbers in the holes of free convection turbines based on the length of the holes ($x = L$) are in the approximate range 10^{10} to 10^{15}. Since transition to turbulence is ordinarily associated with values of $Gr \cdot Pr$ of the order of 10^9 (the so-called critical Rayleigh number, Ra_{crit}) this indicates that in these applications the boundary layers must be turbulent. Hence, the correlation formula suggested by Eckert and Jackson [9] would be appropriate,[5] viz.

$$\overline{Nu} = 2.46 \times 10^{-2} (Gr)^{2/5} (Pr)^{7/15} \left[1 + 0.494 (Pr)^{2/3} \right]^{-2/5} \tag{2}$$

Equation (2) is expected to be most accurate for Prandtl numbers of order unity, if the property variations across the boundary layer are small. In a variable property situation reasonable predictions are possible if one introduces all property values evaluated at a reference temperature about midway[6] between the highest and lowest temperatures appearing in the system.

If in this class of applications one is interested in ranking fluids as to their ability to transfer heat at high rates in a system of prescribed a, L, ΔT, then Eq. (2) indicates that the relevant property group to consider is

$$\bar{\dot{q}}''_{ref} \equiv 2.46 \times 10^{-2} \lambda \left(\frac{g \beta}{\nu^2} \right)^{2/5} \left(\frac{\nu}{\alpha} \right)^{7/15} \left[1 + 0.494 \left(\frac{\nu}{\alpha} \right)^{2/3} \right]^{-2/5} \tag{3}$$

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[5] The exponent of the Prandtl number is erroneously given as 7/5 in the second edition of Ref. 10.

[6] Actually, the work of Sparrow quoted in Ref. 10 indicates that T_* should be chosen somewhat closer to the hot-side temperature.

which may be regarded as a reference heat flux, i. e., the heat flux pertaining to an acceleration of one g, unit length and unit ΔT, in any consistent set of units.[7] It is therefore of interest to compute values of this group over the temperature and pressure range of interest, for both liquids and gases, with and without chemical reaction. Some initial results of this type will discussed below.

Effect of Dissociation on the Thermal Conductivity and Expansion Coefficient of Gases

According to the simple kinetic theory of ideal, inert, gases: $\lambda \propto T^{1/2}$, $\beta \propto T^{-1}$, $\nu \propto T^{3/2}p^{-1}$ and $\alpha \propto T^{3/2}p^{-1}$, so that for such gases one expects the approximate behavior

$$\bar{\dot{q}}''_{ref} \propto p^{4/5}\, T^{-11/10} \tag{4}$$

Therefore $\bar{\dot{q}}''_{ref}$ will ordinarily be largest at high pressures[8] and low temperatures, being almost directly proportional to the density. However, for gases which dissociate (e. g., $H_2 \rightarrow 2H$, $N_2O_4 \rightarrow 2NO_2$, $Cl_2 \rightarrow 2Cl$, $2HI \rightarrow H_2 + I_2$, $(HF)_6 \rightarrow 6HF$) large departures from this simple behavior are known to occur. This is particularly true in the case of the heat conductivity λ and expansion coefficient β. By comparison, dissociation introduces relatively small changes in the kinematic viscosity ν, thermal diffusivity, α, and, hence the Prandtl number $Pr \equiv \nu/\alpha$. A rough estimate of the thermochemical augmentation in $\bar{\dot{q}}''_{ref}$ is then provided by the simple approximate

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[7] Alternatively, $\bar{\dot{q}}''_{ref}$ may be regarded as the average heat transfer coefficient for a configuration with unit length and unit temperature difference.

[8] Stress considerations will, of course, set an upper limit on the allowable internal pressure. Calculations below have not been made for pressures above 10^2 atm.

result

$$\frac{\bar{\dot{q}}''_{ref,\ dissoc}}{\bar{\dot{q}}''_{ref,\ inert}} \approx \left(\frac{\lambda_{eq}}{\lambda_f}\right)\left(\frac{\beta_{eq}}{\beta_f}\right)^{2/5} \qquad (5)$$

For example, consider Figs. 2 and 3, which were prepared for the case of equilibrium dissociating hydrogen [5, 6]. At p = 0.1 atm, $\lambda_{eq}/\lambda_f \approx 9.5$ and $\beta_{eq}/\beta_f \approx 3.1$ so that $\bar{\dot{q}}''_{ref,eq}/\bar{\dot{q}}''_{ref,f} \approx 15$ at a temperature level of about 3250°K. At p = 1 atm, $\lambda_{eq}/\lambda_f \approx 7.5$, $\beta_{eq}/\beta_f \approx 2.8$ so that $\bar{\dot{q}}''_{ref,eq}/\bar{\dot{q}}''_{ref,f} \approx 11$ at a temperature level of 3750°K. Thus, dissociation can increase the reference heat flux by more than one order of magnitude, with the greatest augmentation being realized at low pressures. Of course the temperature level at which these large effects occur (for any specified pressure) will depend on the strength of the chemical bond being broken. Indeed it can be shown that for any dissociating diatomic gas the maximum enhancement will occur at the temperature at which the degree of dissociation, α_{eq}, is near 0.50. In terms of the equilibrium constant $K_p(T)$, this will occur at the "optimum" temperature T_{opt} defined implicitly by $K_p(T_{opt}) \approx (4/3)\ p$. Conversely, if one desires operation at some prescribed temperature level T then the maximum advantage of dissociation will be realized at the pressure level $p_{opt} \approx (3/4)\ K_p(T)$.

In the light of this background information and the detailed data given in Refs. 3, 4, 11, and 12, one can now compute the "figure of merit" $\bar{\dot{q}}''_{ref}$ for various coolants, over a range of pressures and temperatures.

Relative Effectiveness of Dissociating Gases as Heat Transfer Media in the Turbulent Natural Convection Regime

Values of $\bar{\dot{q}}''_{ref}$ defined by Eq. (3) are shown plotted in Fig. 4 over the temperature range 300°K < T < 5000°K for the particular dissociating gases H_2, N_2O_4 and air.

For both H_2 and N_2O_4 the expansion coefficient was computed from chemical contribution, $C_{p,\,chem}$, to the molar heat capacity, with the help of the relation [5]

$$\beta = \frac{1}{T}\left[1 + \frac{\left(C_{p,\,chem}/R\right)}{(\Delta H/RT)}\right] \tag{6}$$

Relevant data was obtained from Refs. 11, 3, 4, and 12, respectively. For dissociating air, use was made of the compressibility, Z, tabulated by Hansen [12], together with the relation

$$\beta = \frac{1}{T}\left[1 + \left(\frac{\partial \ln Z}{\partial \ln T}\right)_{p\,=\,const}\right] \tag{7}$$

derived from the definitions of β and Z. The trends anticipated in the discussion of the previous section are borne out (cf. Fig. 4). In particular, it is interesting to note that at $p = 1$ atm and $T = 300^{\circ}K$, dissociating nitrogen tetroxide outperforms either air or hydrogen by more than one order of magnitude on a heat flux basis Also, as expected, both hydrogen and air exhibit their maximum thermochemical contribution at much higher temperature levels, reflecting the higher strengths of the bonds being broken [H_2, (103 Kcal/mole), O_2 (118 Kcal/mole) and N_2 (225 Kcal/mole) as compared with N_2O_4 (~ 14 Kcal/mole)]. While data for N_2O_4 at higher pressures was not readily available, it appears that N_2O_4 could be used to obtain values of $\bar{q}''_{ref}$ larger than 10^{-3} cal$(cm)^{-2}(sec)^{-1}$ at pressures less than 10^2 atm. However, it is also clear from Fig. 4 that liquids outperform pressurized dissociating gases on a heat flux basis (cf. data [10] for saturated Hg, NH_3 and H_2O). However, additional data is required at higher temperature levels; and the role of stress limitations in properly assessing the relative merits of liquid coolants in a particular application remain to be investigated. It would also be of interest to consider saturated vapor/liquid mixtures in this regard, with some attention paid to cases in which the vapor dissociates [13]. Finally, additional study of phenomena near the critical point is necessary.

Required Values of the Reference Heat Flux

An estimate of the value of $\bar{\dot{q}}''_{ref}$ required in any particular case can be obtained on the basis of the following illustrative heat balance considerations. Suppose it is desired to have a turbine blade operate at a temperature ΔT_g below the turbine inlet gas total temperature. Then, if $\bar{h}_g$ is the mean heat transfer coefficient for the outer area, A_o of the entire blade, the total forced convective heat transfer rate will be

$$\dot{Q} = \bar{h}_g A_o \cdot \Delta T_g \tag{8}$$

In the steady state, $\dot{Q}$ must be equal to the net rate of heat transfer by natural convection inside the blade where the wetted area is A_i. Using the definition of $\bar{\dot{q}}''_{ref}$ this may be written

$$Q = \bar{\dot{q}}''_{ref} A_i \left[\left(\frac{\Delta T}{\Delta T_{ref}}\right)^{7/5} \left(\frac{a}{g}\right)^{2/5} \left(\frac{L}{L_{ref}}\right)^{1/5} \right] \tag{9}$$

where, as noted earlier, we have adopted $\Delta T_{ref} \equiv 1^{\circ}K$, $L_{ref} \equiv 1$ cm. Combining Eqs. (8) and (9) gives the immediate result

$$\bar{\dot{q}}''_{ref} = \bar{h}_g \Delta T_g \left[\left(A_o/A_i\right) \left(\Delta T/\Delta T_{ref}\right)^{-7/5} (a/g)^{-2/5} \left(L/L_{ref}\right)^{-1/5} \right] \tag{10}$$

For example, consider a case for which $L = 5$ cm, $\bar{h}_g = 10^{-1}$ cal(cm)$^{-2}$(sec)$^{-1}$($^{\circ}$K)$^{-1}$, $A_o/A_i = 2$, and $a/g = 10^5$. Then, if one desires a cooling effect of $\Delta T_g \approx 300^{\circ}$K and one can maintain the blade root at a temperature such that $\Delta T \approx \Delta T_g$ one finds that $\bar{\dot{q}}''_{ref}$ must be of the order of 10^{-4}cal(cm)$^{-2}$(sec)$^{-1}$. As suggested by Fig. 4, values of this order of magnitude can be achieved with dissociating gases at internal pressures below 10^2 atm.

Conclusions

On the basis of the considerations outlined above the following conclusions have been drawn:

a. Molecular dissociation increases the effectiveness of gases as natural convection heat transfer media largely by virtue of the attendent increase in thermal expansion coefficient β and thermal conductivity λ, with the latter effect being the more important (cf. Eq. 5). Consequently, in temperature limited systems an optimum pressure level, strongly dependent on the strength of the bond being broken, will exist for maximum coolant effectiveness.

b. Coolants in the dissociation region can be ranked in accord with a figure of merit derived from the thermophysical property groupings which enter the expression for the relevent convective heat transfer coefficient (cf. Eq. 3).

c. Fully enclosed, pressurized dissociating gases may be capable of transferring heat away from turbomachinery components at rates sufficient to enable the use of higher inlet total temperatures. Even at 1 atm, 1g and temperatures not far from 300°K, easily dissociated gaseous coolants can be found which are more than ten times as effective as air.

d. A simple ranking technique of the type presented can be used to show when dissociating gases are competitive for a given natural convection cooling application and, hence, when more accurate boundary layer calculations or experiments are required. As already indicated, very few accurate natural convection solutions for dissociating gases have been reported in the open literature to date.

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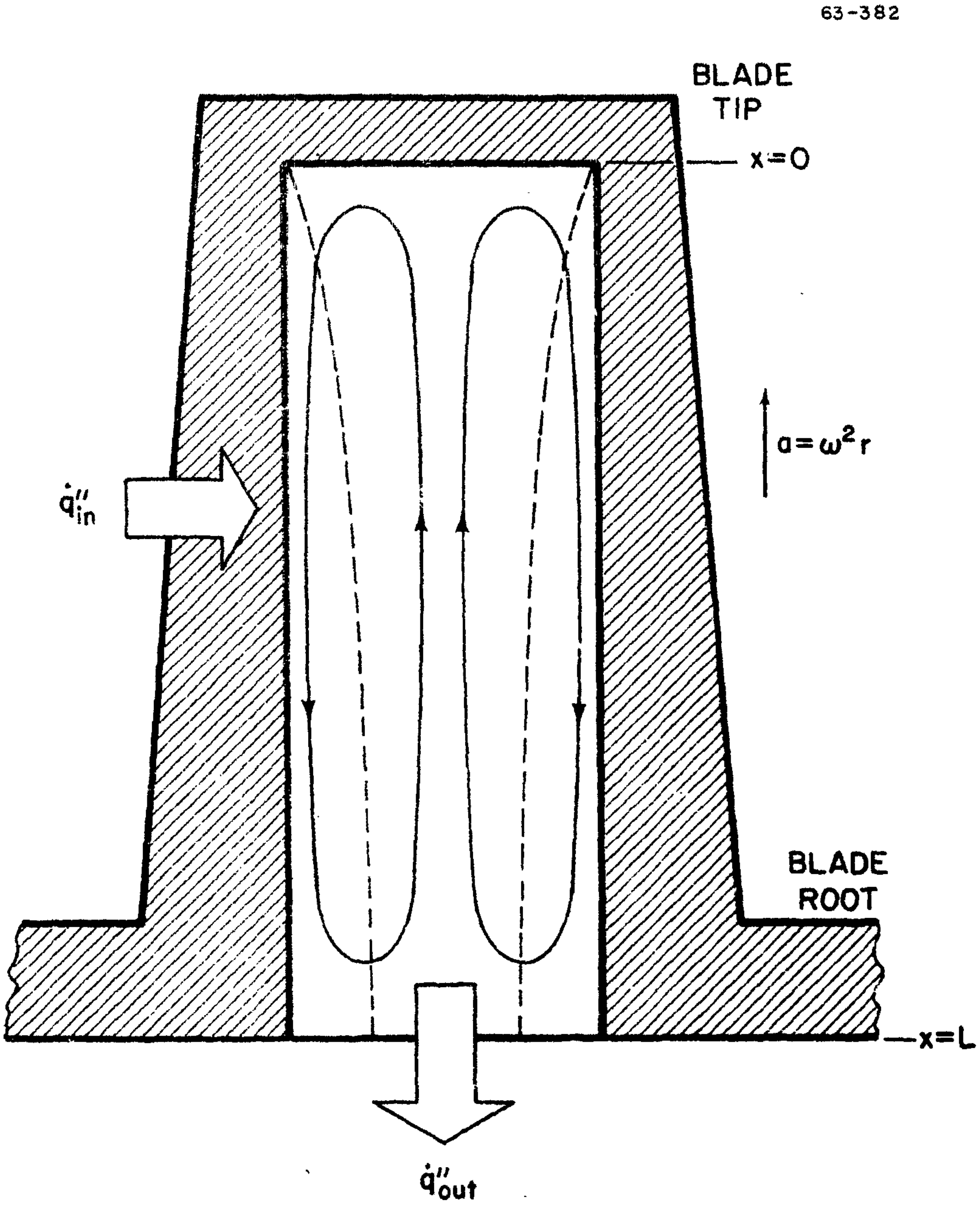

FIG. 1 SCHEMATIC OF A NATURAL CONVECTION COOLED GAS TURBINE ROTOR BLADE

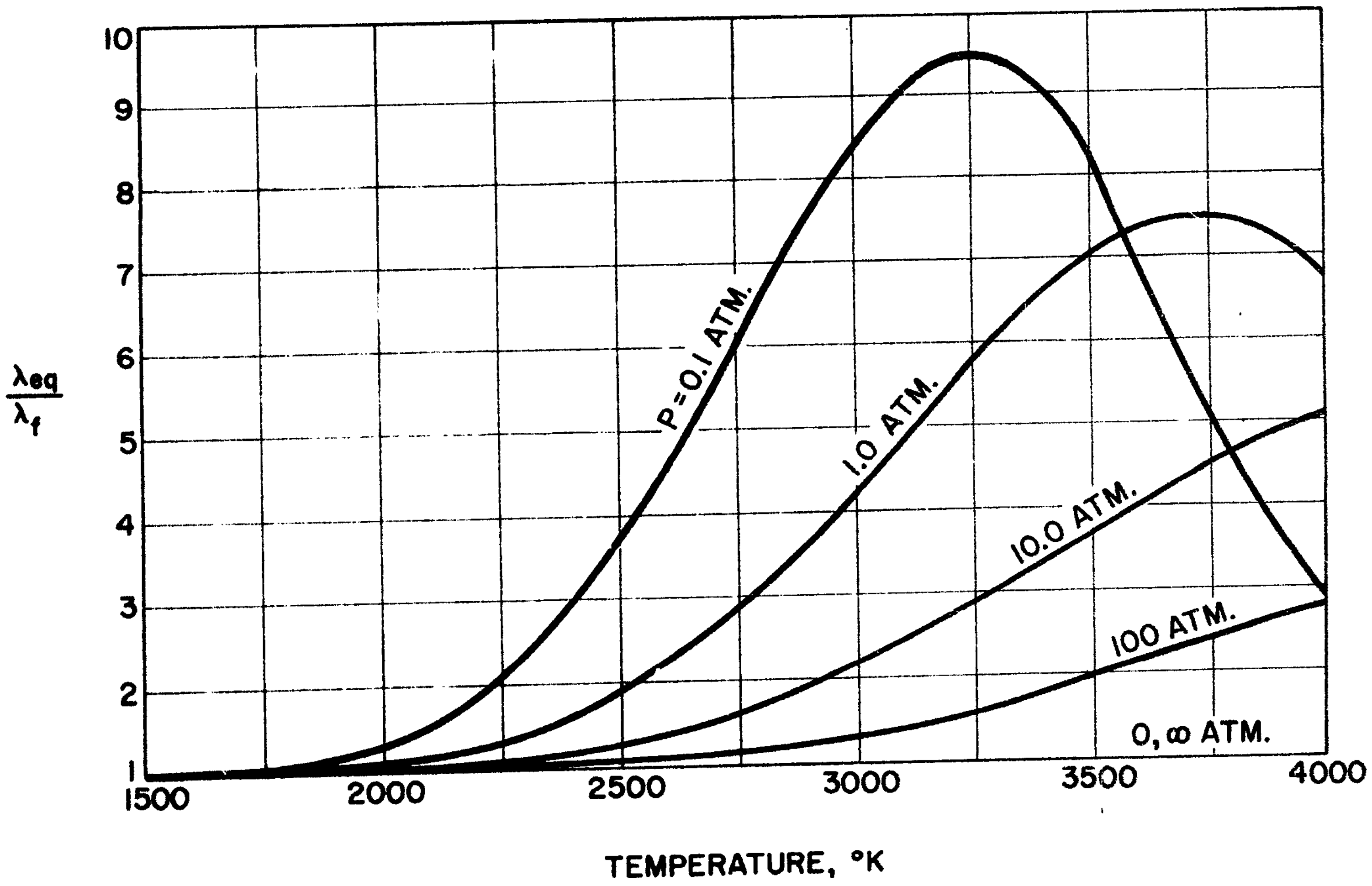

FIG. 2 EFFECT OF DISSOCIATION ON THE HEAT CONDUCTIVITY OF EQUILIBRIUM HYDROGEN

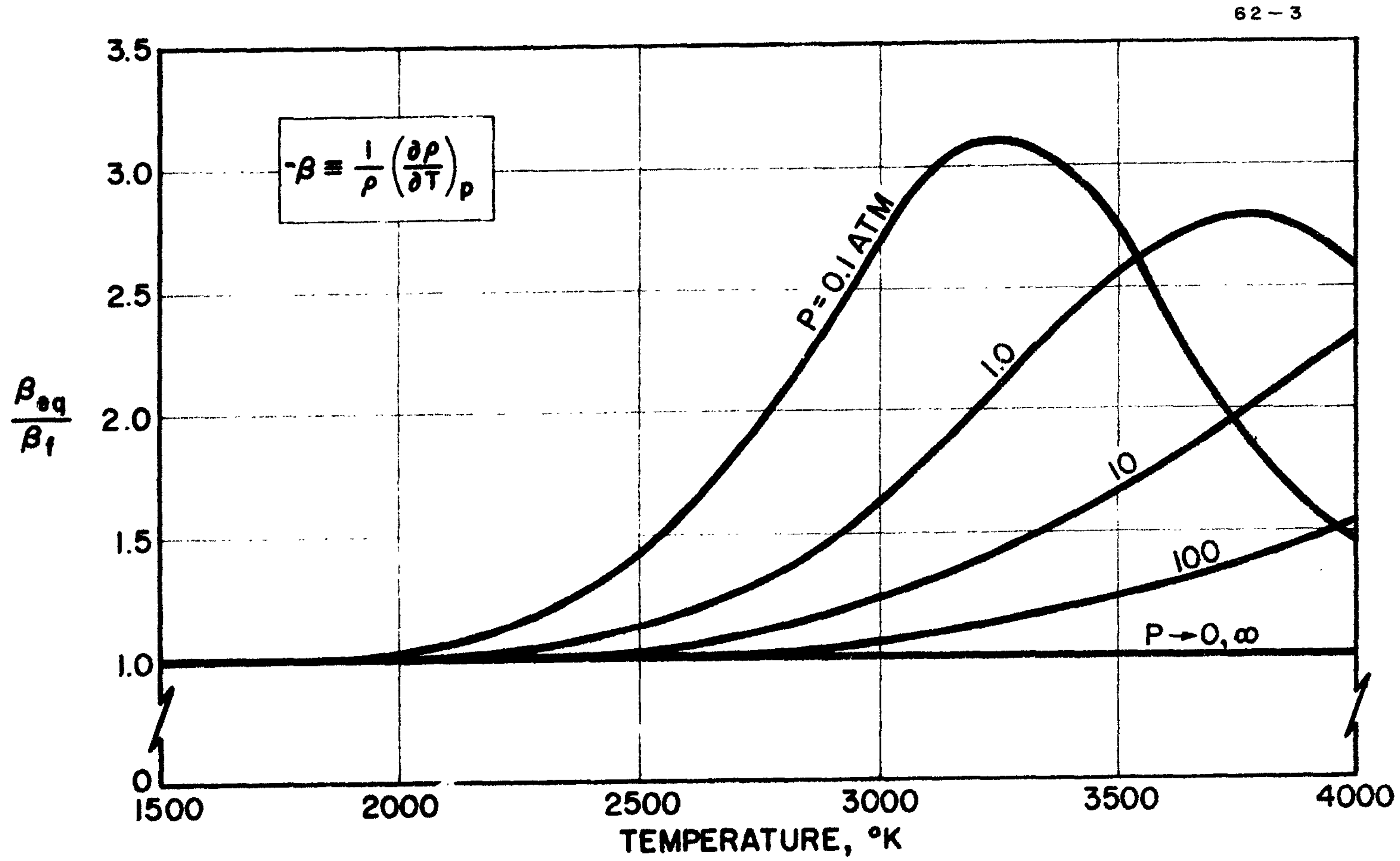

FIG. 3 EFFECT OF DISSOCIATION ON THE THERMAL EXPANSION COEFFICIENT OF EQUILIBRIUM HYDROGEN

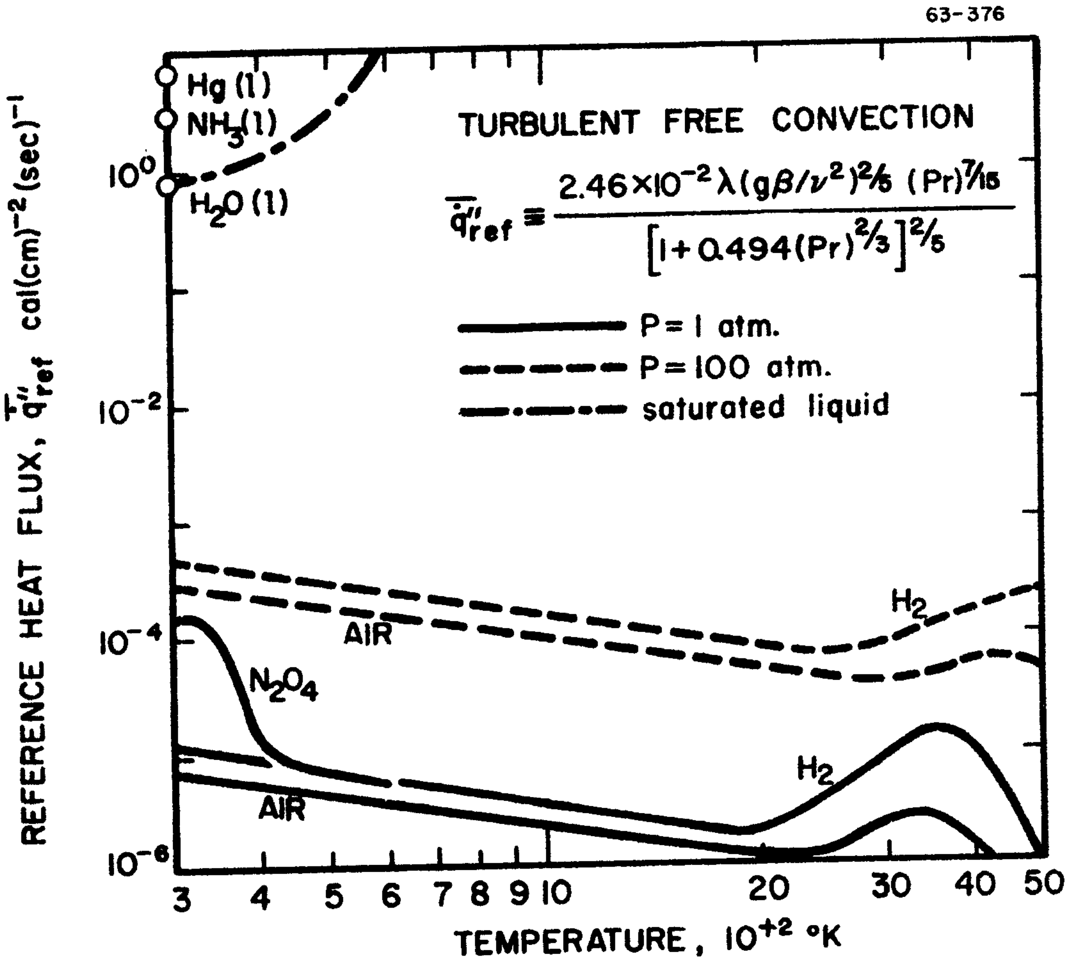

FIG. 4 COMPARISON OF LIQUIDS WITH DISSOCIATING GASES AS NATURAL CONVECTION COOLANTS