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GRAPHICAL METHOD FOR DETERMINING MAXIMUM STRESSES IN RING-STIFFENED CYLINDERS UNDER EXTERNAL HYDROSTATIC PRESSURE

by

Martin A. Krenzke and Robert D. Short

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

October 1959 Report 1348

PRNC-TMB-648 (Rev. 3-58)

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NS 731-038

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ABSTRACT

Calculation of the maximum stresses in a ring-stiffened cylinder subjected to external hydrostatic pressure as obtained from the analysis of Salerno and Pulos is facilitated by curves presented in this report. Stresses obtained by means of these curves do not deviate from those obtained by a precise calculation by more than 0.2 percent.

INTRODUCTION

The effect of secondary moments produced by the end load on the elastic axisymmetric stress distribution in a ring-stiffened cylinder subjected to external hydrostatic pressure was investigated by Salerno and Pulos.* They considered small deflections of a cylindrical shell with "equally spaced circular ring frames" composed of a material which adheres to the well-known laws of linear elasticity.

Unfortunately, the numerical solution of the analysis of Salerno and Pulos is lengthy and, therefore, is not often used by the designer. Results of calculations of Salerno and Pulos stress functions performed on the IBM 704 high-speed computer are presented in graphical form in this report. These curves enable the rapid calculation of maximum stresses with sufficient accuracy for design purposes.

METHOD

The axisymmetric stresses in a stiffened cylinder are functions of the following non-dimensional parameters:

$$\alpha = A_f / hL_f$$

$$\beta = b / L_f$$

$$\gamma = \frac{P}{2E}\left(\frac{R}{h}\right)^2 \sqrt{3(1-\nu^2)}$$

$$\theta = \frac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{Rh}} L$$

*Salerno, V.L. and Pulos, J.G., "Stress Distribution in a Circular Cylindrical Shell under Hydrostatic Pressure Supported by Equally Spaced Circular Ring Frames," Polytechnic Institute of Brooklyn Aeronautical Laboratory Report No. 171-A (Jun 1951).

where A_f is the area of the stiffener, assumed to be concentrated at the median surface of the shell,

h is the shell thickness,

L_f is the center-to-center spacing of the stiffeners,

b is the effective width of a stiffener in contact with the shell,

$L = L_f - b$,

R is the radius to the median surface of the shell,

ν is Poisson's ratio,

E is Young's modulus, and

P is external pressure.

The following expressions may be obtained in terms of these parameters from Equations [47] and [43] of the Salerno and Pulos report.

$$1 - \frac{\sigma_{\phi mf}}{\sigma_u} = \frac{\left(1 - \frac{\nu}{2}\right) \alpha}{\alpha + \beta + F_1 (1 - \beta)} \qquad [1]$$

$$1 - \frac{\sigma_{\phi mm}}{\sigma_u} = \left(1 - \frac{\sigma_{\phi mf}}{\sigma_u}\right) F_2 \qquad [2]$$

$$\sqrt{\frac{1 - \nu^2}{0.91}}\, \frac{\sigma_{xbf}}{\sigma_u} = \left(1 - \frac{\sigma_{\phi mf}}{\sigma_u}\right) F_3 \qquad [3]$$

$$\sqrt{\frac{1 - \nu^2}{0.91}}\, \frac{\sigma_{xbm}}{\sigma_u} = \left(1 - \frac{\sigma_{\phi mf}}{\sigma_u}\right) F_4 \qquad [4]$$

where

$$F_1 = \left(\frac{4}{\theta}\right) \left[\frac{\cosh^2 \eta_1 \theta - \cos^2 \eta_2 \theta}{\dfrac{\cosh \eta_1 \theta \sinh \eta_1 \theta}{\eta_1} + \dfrac{\cos \eta_2 \theta \sin \eta_2 \theta}{\eta_2}}\right] \qquad [5]$$

$$F_2 = \frac{\dfrac{\cosh \eta_1 \theta \sin \eta_2 \theta}{\eta_2} + \dfrac{\sinh \eta_1 \theta \cos \eta_2 \theta}{\eta_1}}{\dfrac{\cosh \eta_1 \theta \sinh \eta_1 \theta}{\eta_1} + \dfrac{\cos \eta_2 \theta \sin \eta_2 \theta}{\eta_2}} \qquad [6]$$

$$F_3 = \sqrt{\frac{3}{0.91}} \left[\frac{\dfrac{\cos\eta_2\theta\sin\eta_2\theta}{\eta_2} - \dfrac{\cosh\eta_1\theta\sinh\eta_1\theta}{\eta_1}}{\dfrac{\cosh\eta_1\theta\sinh\eta_1\theta}{\eta_1} + \dfrac{\cos\eta_2\theta\sin\eta_2\theta}{\eta_2}} \right] \qquad [7]$$

$$F_4 = \sqrt{\frac{3}{0.91}} \left[\frac{\dfrac{\cosh\eta_1\theta\sin\eta_2\theta}{\eta_2} - \dfrac{\sinh\eta_1\theta\cos\eta_2\theta}{\eta_1}}{\dfrac{\cosh\eta_1\theta\sinh\eta_1\theta}{\eta_1} + \dfrac{\cos\eta_2\theta\sin\eta_2\theta}{\eta_2}} \right] \qquad [8]$$

$$\eta_1 = \frac{1}{2}\sqrt{1-\gamma}$$

$$\eta_2 = \frac{1}{2}\sqrt{1+\gamma}$$

and $\sigma_u = -\frac{PR}{h}$ is the circumferential stress in an infinite unstiffened cylinder,

$\sigma_{\phi mf}$ is the circumferential membrane stress in the shell at a stiffener,

$\sigma_{\phi mm}$ is the circumferential membrane stress at midbay,

σ_{xbf} is the longitudinal bending stress in the shell at a stiffener,

σ_{xbm} is the longitudinal bending stress at midbay.

The circumferential stress $\sigma_{\phi_i^o m}$ and the longitudinal stress $\sigma_{x_i^o m}$ on the shell surface at midbay may be obtained by the following relations

$$\sigma_{\phi_i^o m} = \sigma_{\phi mm} \pm \nu\sigma_{xbm} \qquad [9]$$

$$\sigma_{x_i^o m} = \frac{1}{2}\sigma_u \pm \sigma_{xbm} \qquad [10]$$

while the circumferential stresses $\sigma_{\phi_i^o f}$ and the longitudinal stresses $\sigma_{x_i^o f}$ on the shell surface at stiffener are given by

$$\sigma_{\phi_i^o f} = \sigma_{\phi mf} \pm \nu\sigma_{xbf} \qquad [11]$$

$$\sigma_{x_i^o f} = \frac{1}{2}\sigma_u \pm \sigma_{xbf} \qquad [12]$$

where the subscripts o and i represent the outside and inside surfaces of the shell, respectively. The circumferential stress $\sigma_{\phi f}$ in a stiffener may be obtained from

$$\sigma_{\phi f} = \sigma_{\phi m f} - \frac{\nu}{2} \sigma_u \qquad [13]$$

Figures 1, 2, 3, and 4 are plots of the stress functions F_1, F_2, F_3, and F_4, respectively, for values of γ from 0 to 1 and for values of θ from 0 to 8 covering the region of practical interest. It should be noted that $\sigma_{\phi mm}$ is not the maximum circumferential membrane stress when F_4 is negative and that σ_{xbm} is not the maximum negative bending stress when

$$\eta_1 (1 + 2\gamma) \cosh \eta_1 \theta \sin \eta_2 \theta < \eta_2 (1 - 2\gamma) \sinh \eta_1 \theta \cos \eta_2 \theta \qquad [14]$$

Equation [14] will never be satisfied for values of θ less than 4.74.

NUMERICAL EXAMPLE

A numerical example is provided to illustrate the use of Figures 1 through 4. The scantlings and material properties are:

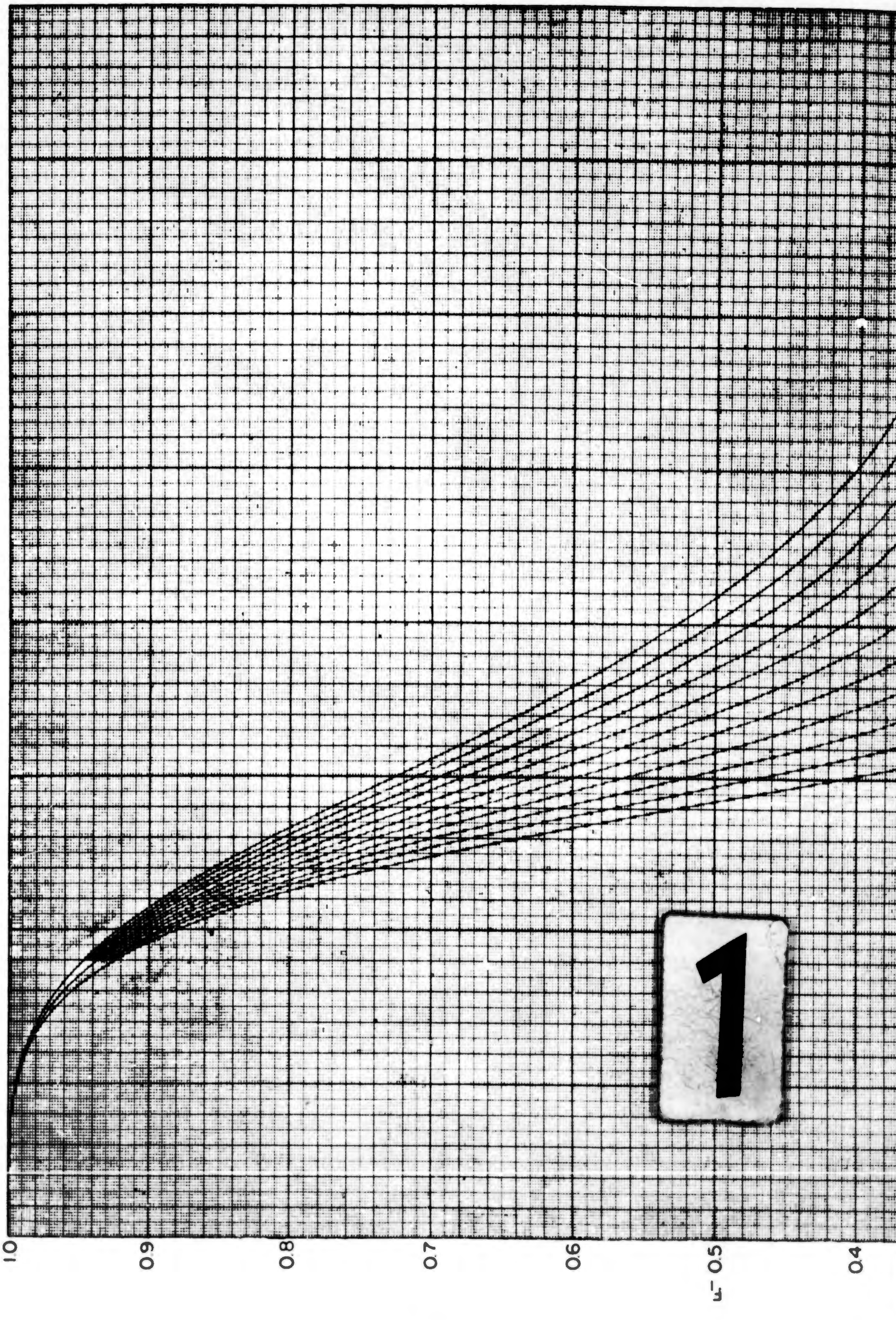
R = 13.000 in.
h = 0.169 in.
L_f = 3.692 in.
b = 0.467 in.
L = 3.225 in.
A_f = 0.312 sq in.
E = 30×10^6 psi
ν = 0.3

From the above, values of the nondimensional parameters at a pressure of 1800 psi are

α = 0.5000
β = 0.1265
γ = 0.2933
θ = 2.7967

From Figures 1, 2, 3, and 4, respectively,

F_1 = 0.735
F_2 = 0.497
F_3 = −1.841
F_4 = 0.940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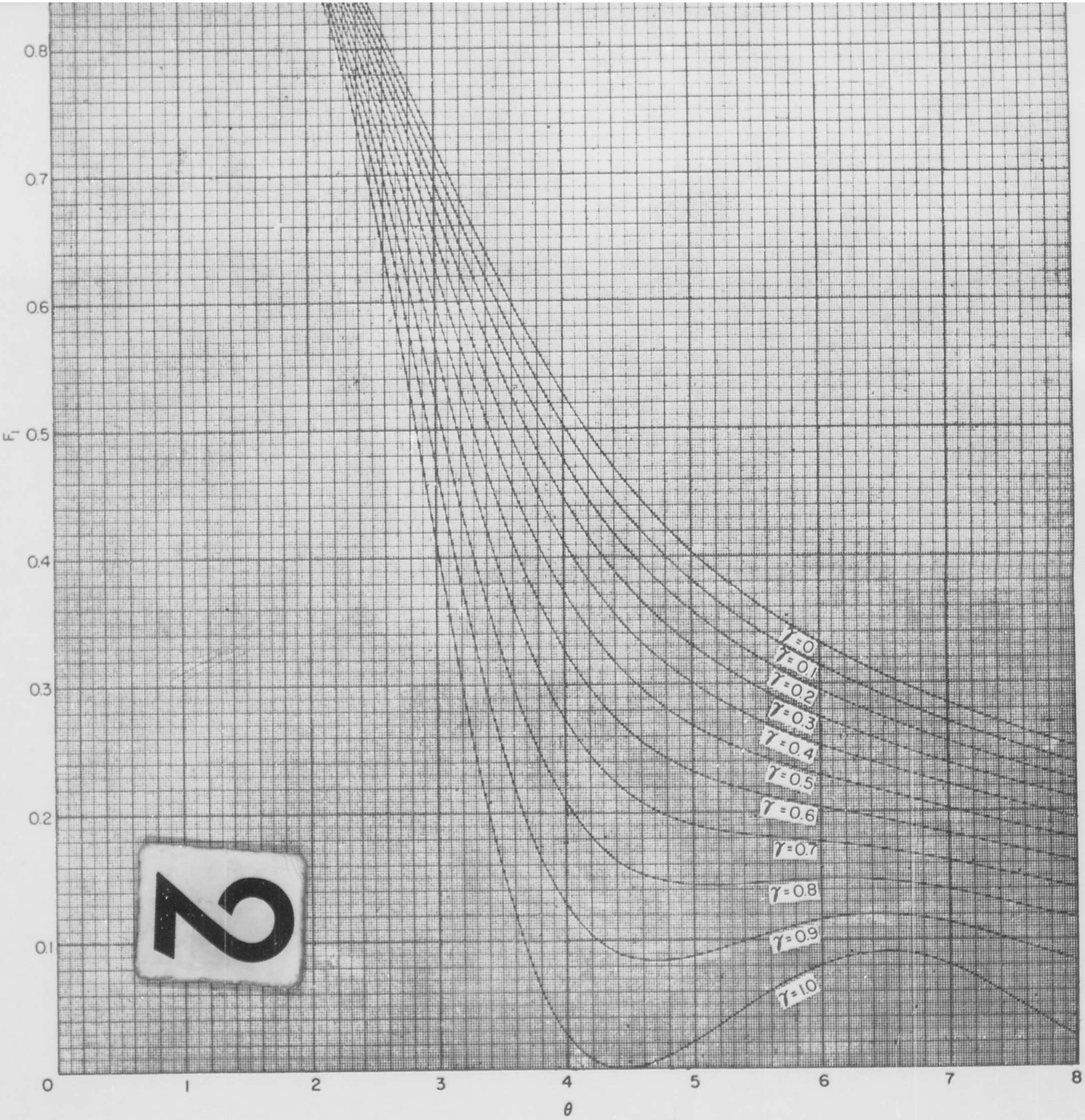
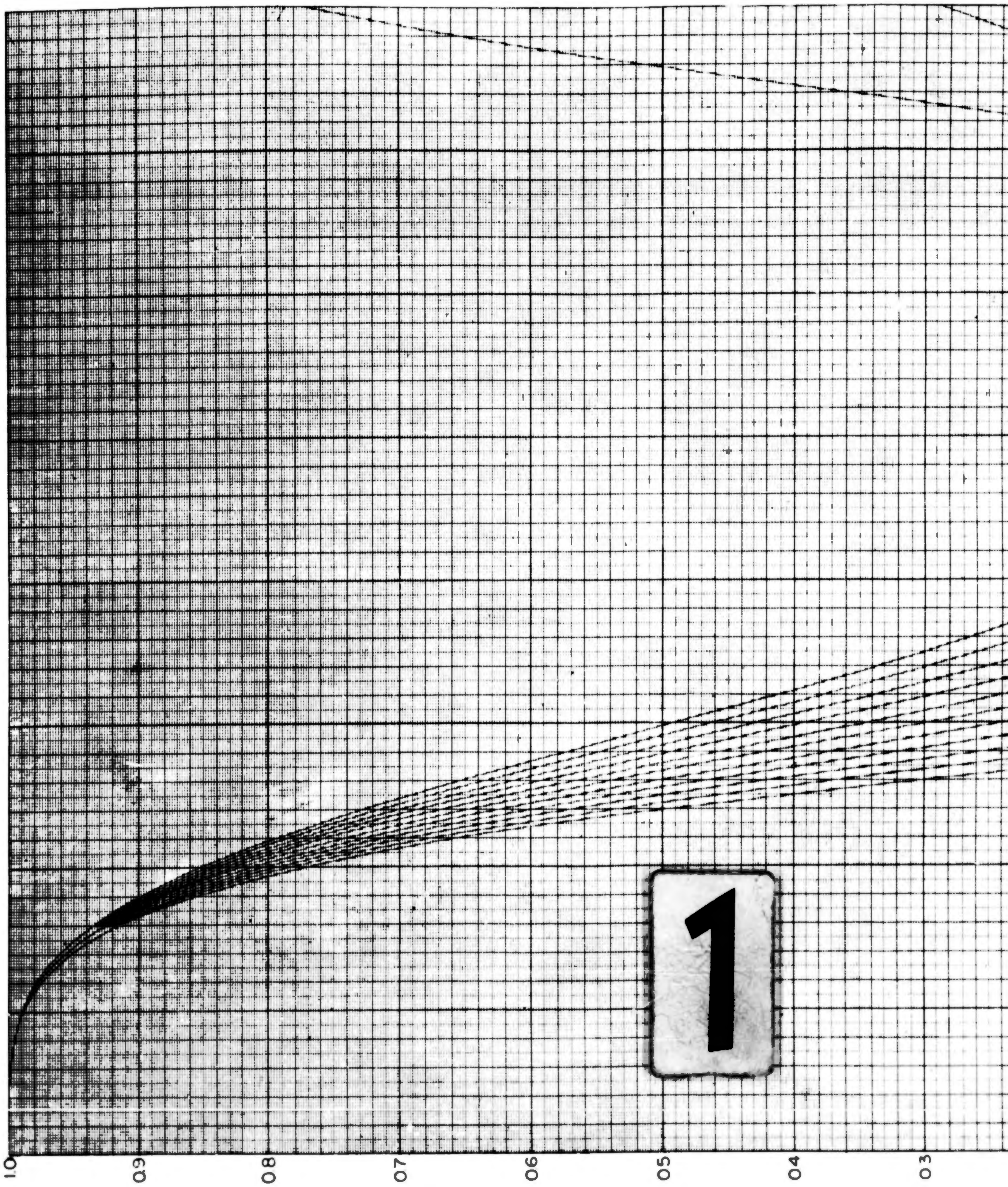


Figure 1 – Stress Function F_1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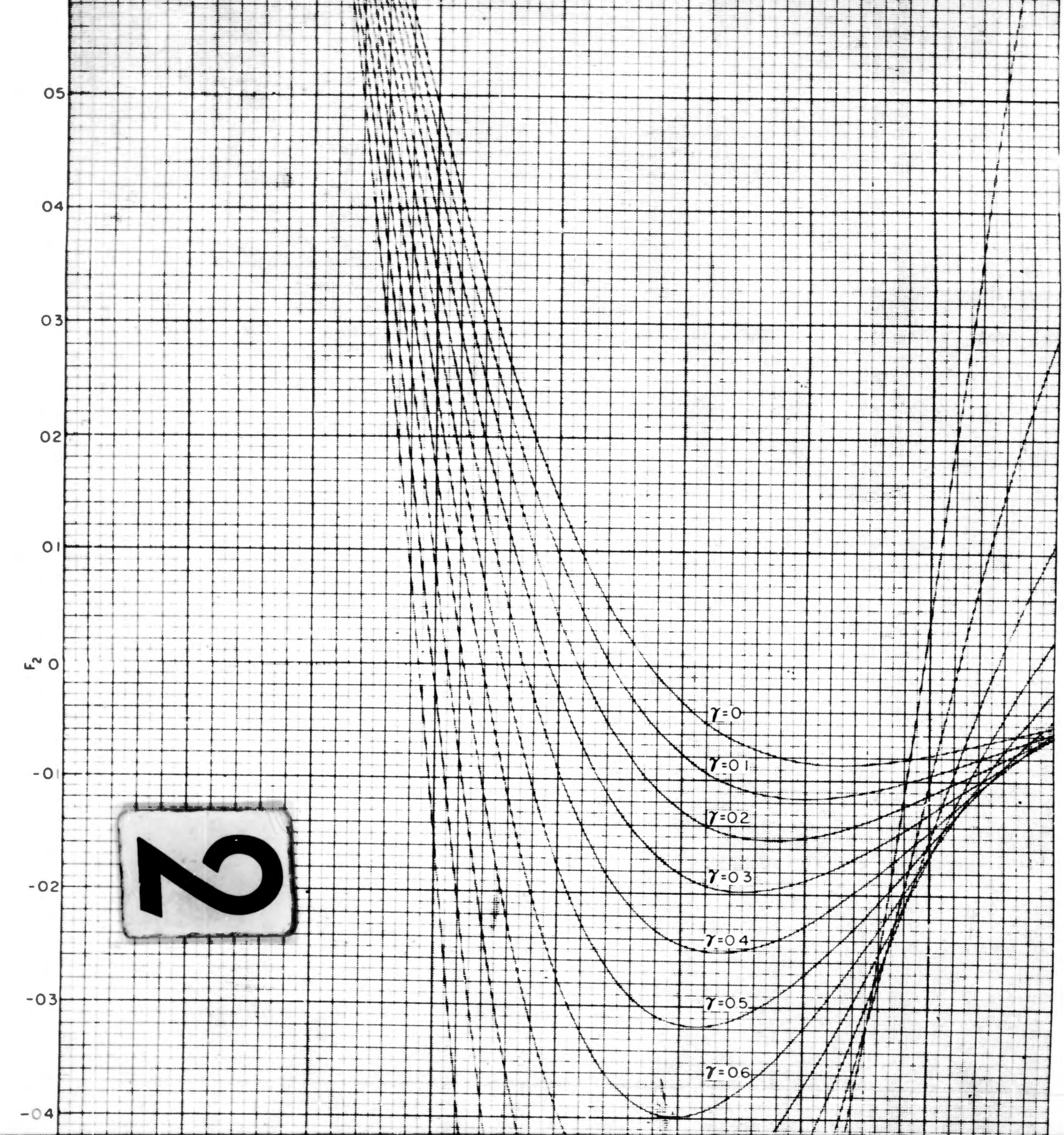
0.3
1



05
04
03
02
01
F_2 0
-01
-02
-03
-04
γ=0
γ=01
γ=02
γ=03
γ=04
γ=05
γ=06

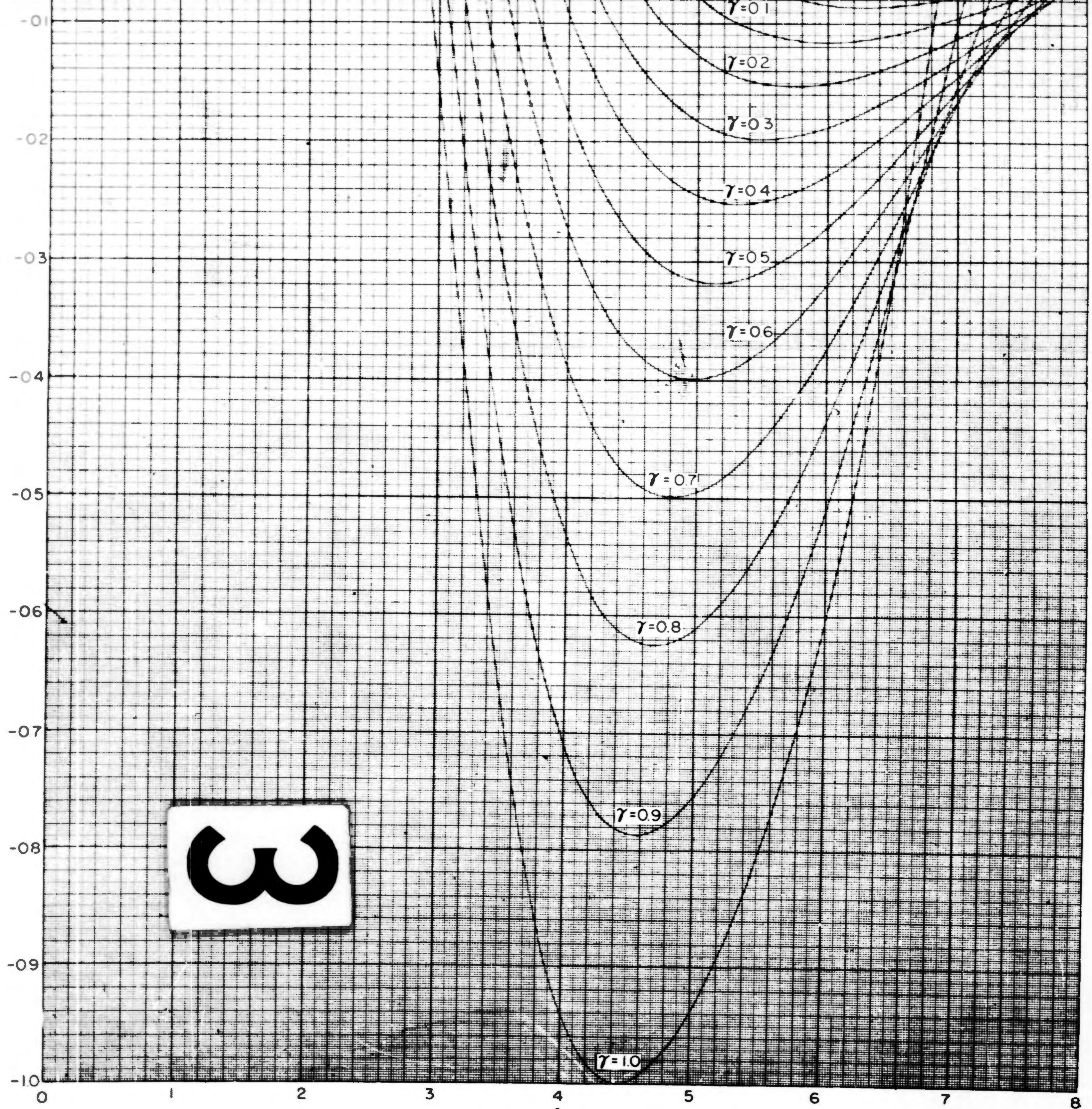
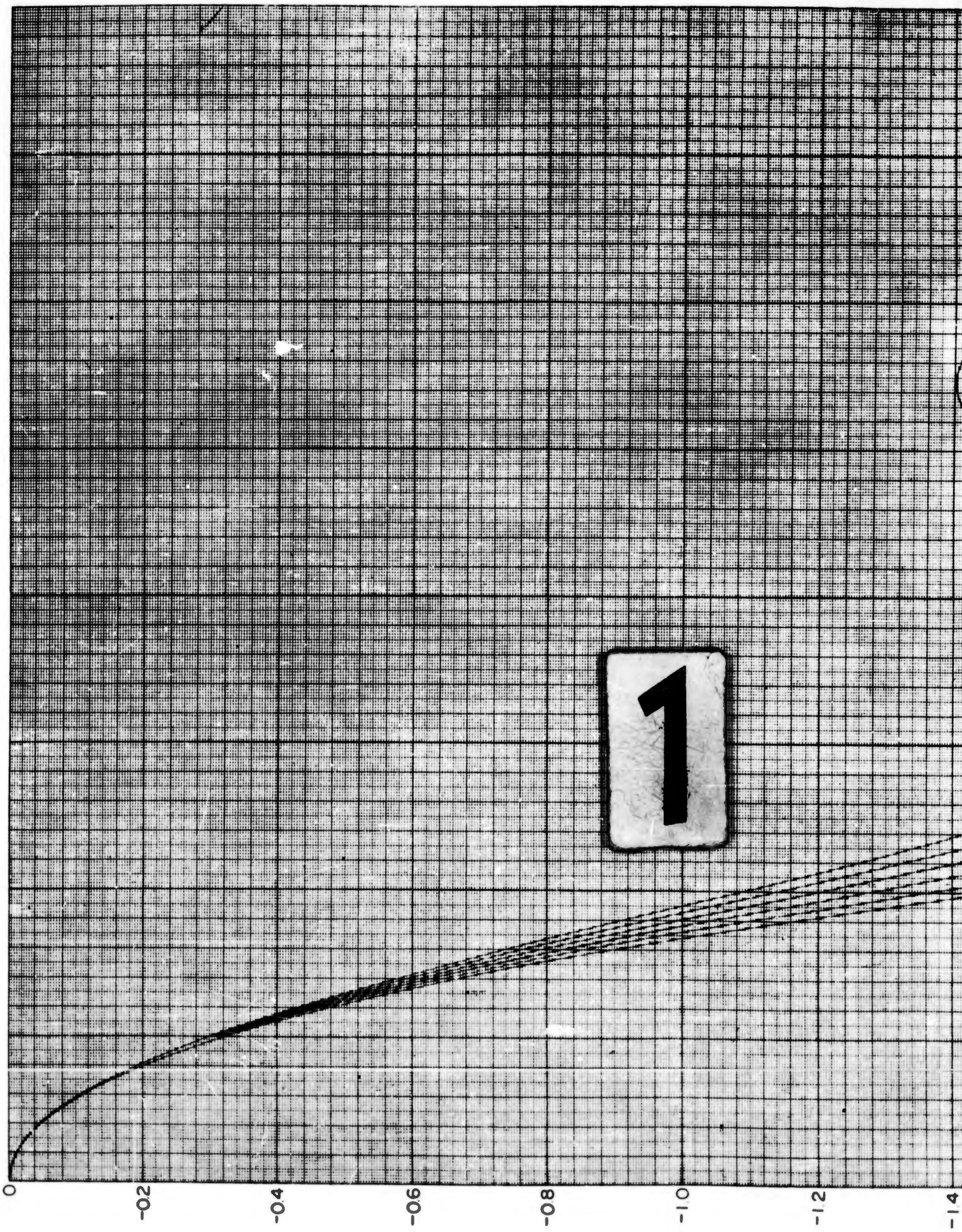


Figure 2 — Stress Function F_2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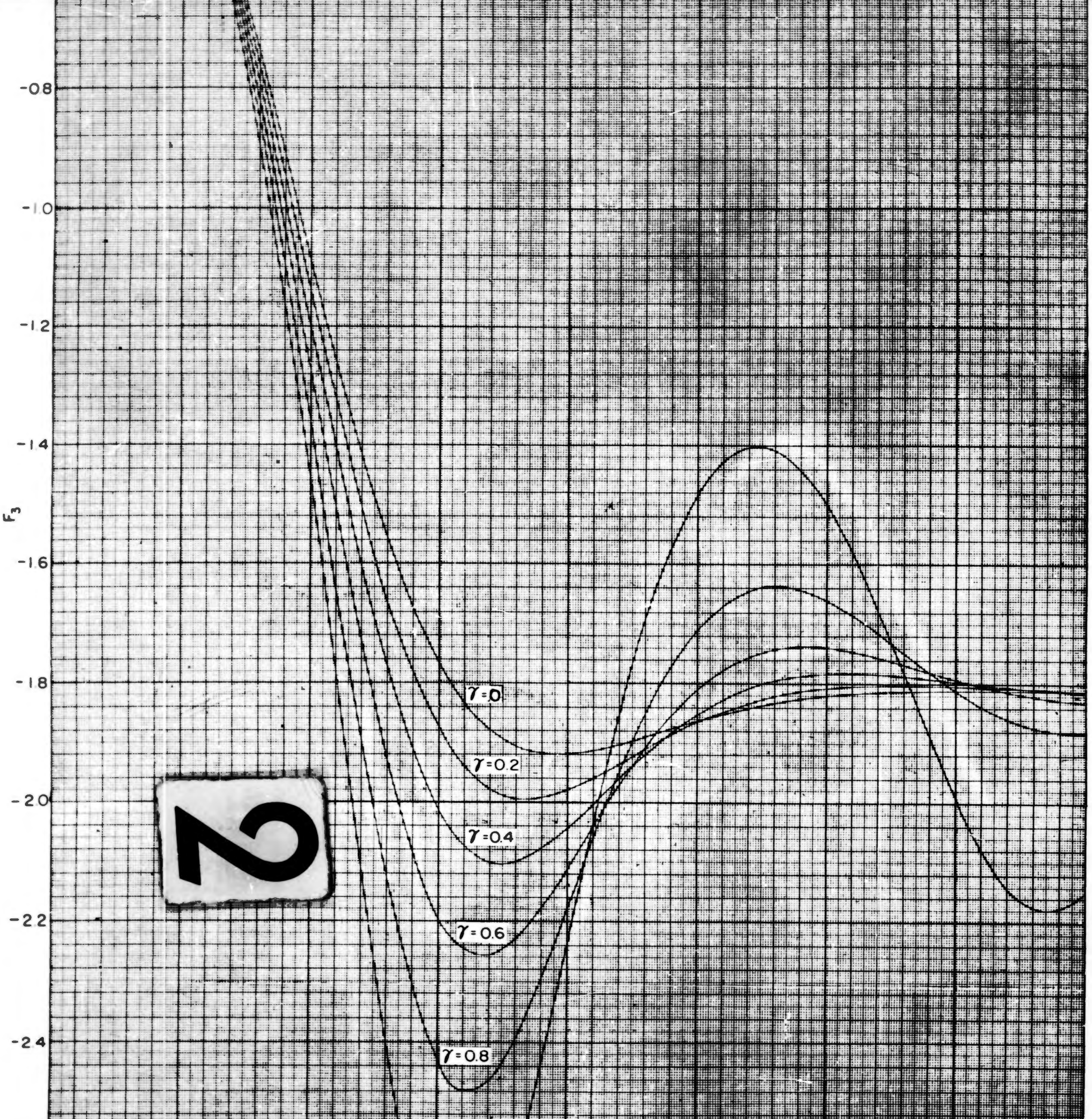
0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4


-0.8
-1.0
-1.2
-1.4
-1.6
-1.8
-2.0
-2.2
-2.4
F_3
γ=0
γ=0.2
γ=0.4
γ=0.6
γ=0.8
2

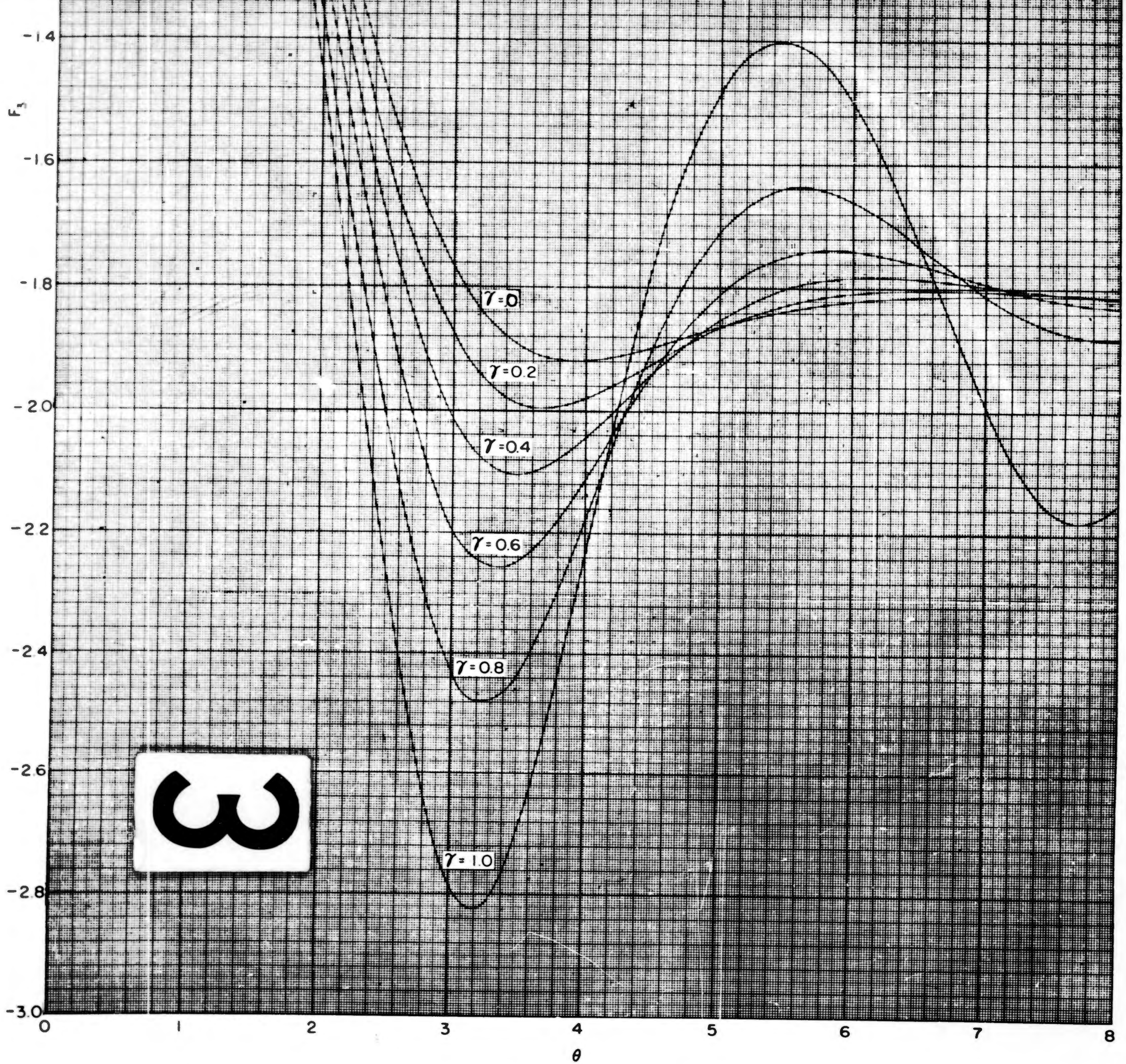
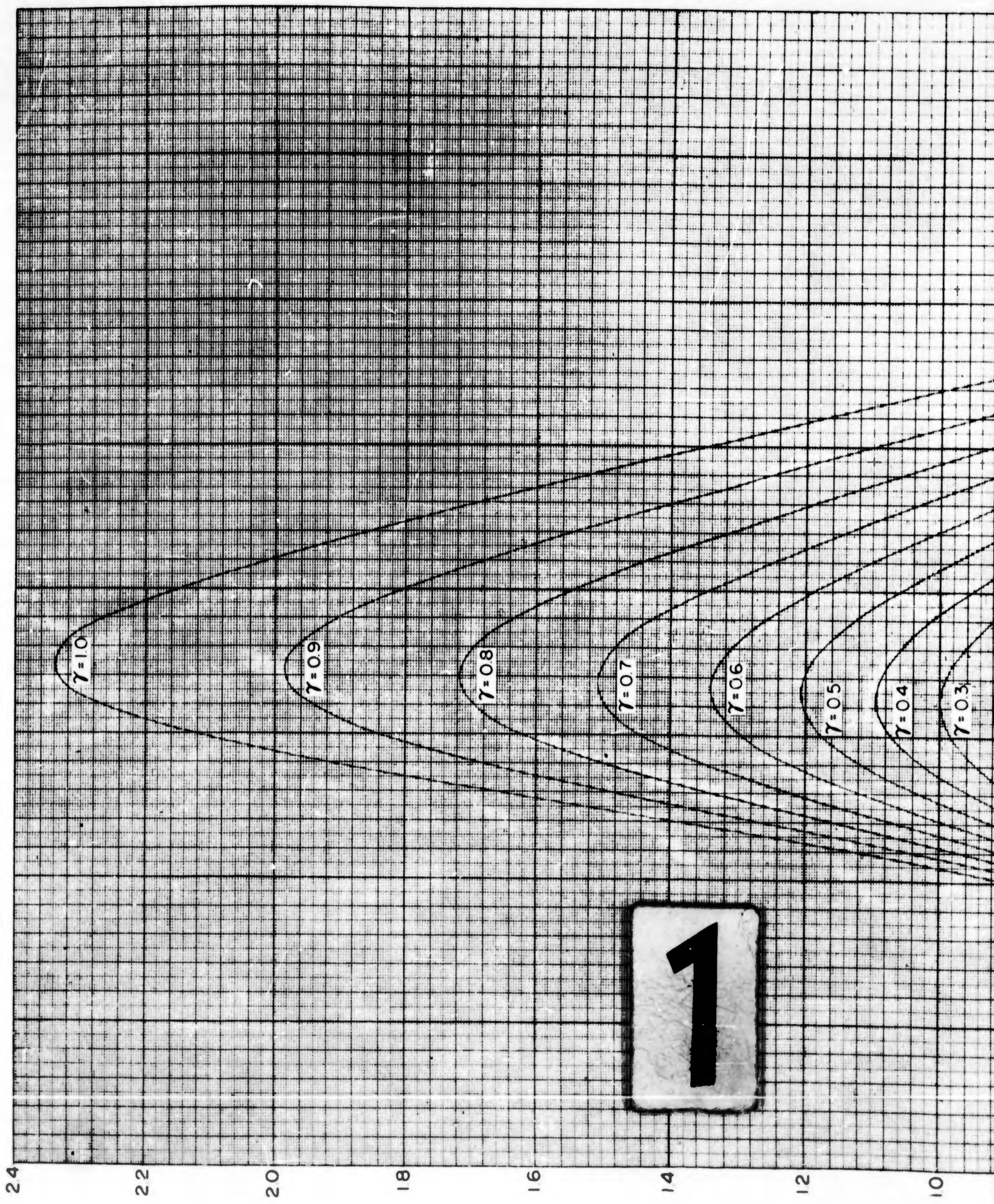
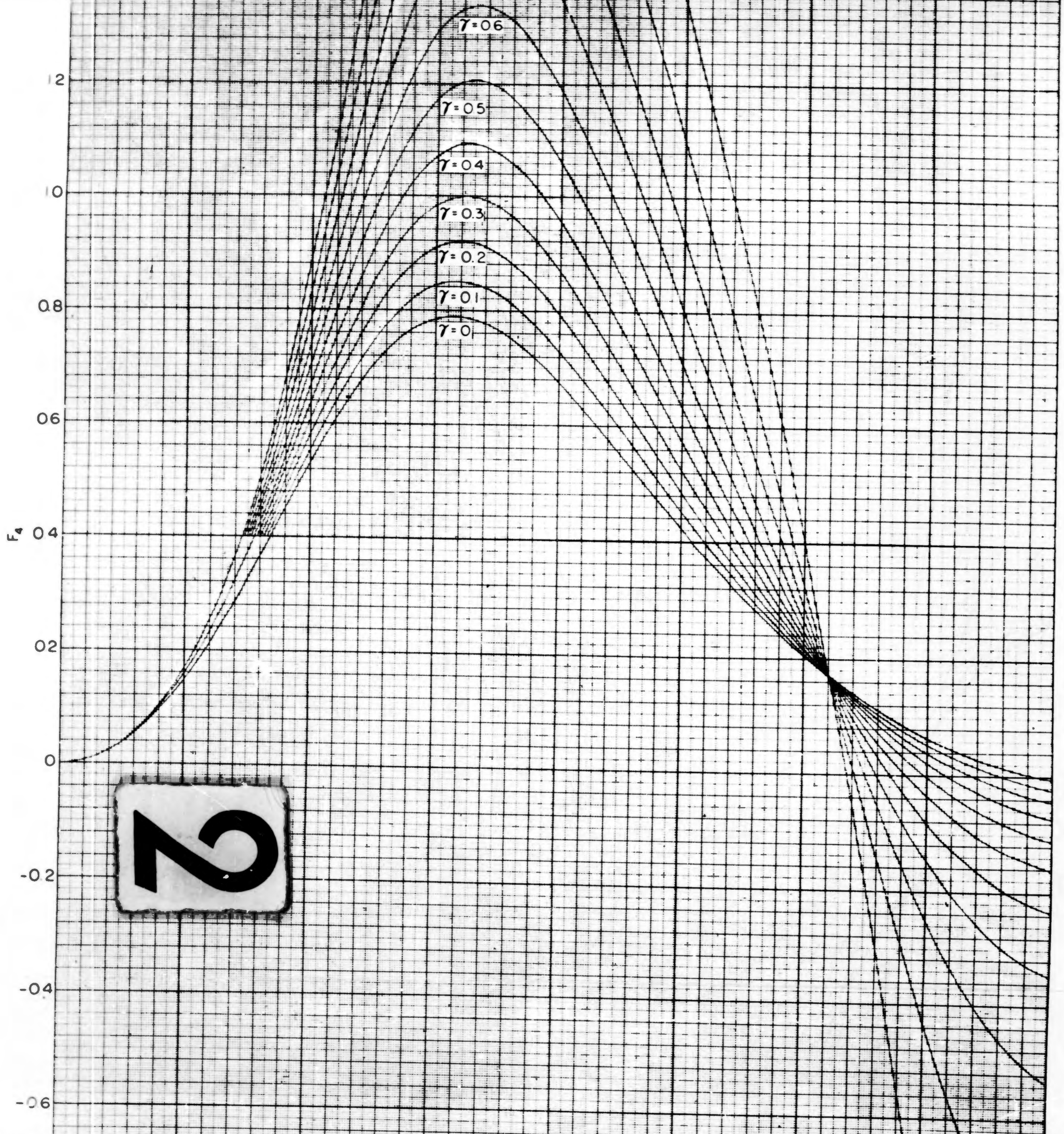


Figure 3 – Stress Function F_3



γ=1.0
γ=0.9
γ=0.8
γ=0.7
γ=0.6
γ=0.5
γ=0.4
γ=0.3
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
1


γ=0.6
γ=0.5
γ=0.4
γ=0.3
γ=0.2
γ=0.1
γ=0
1.2
1.0
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F4 0.4
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-0.4
-0.6
2

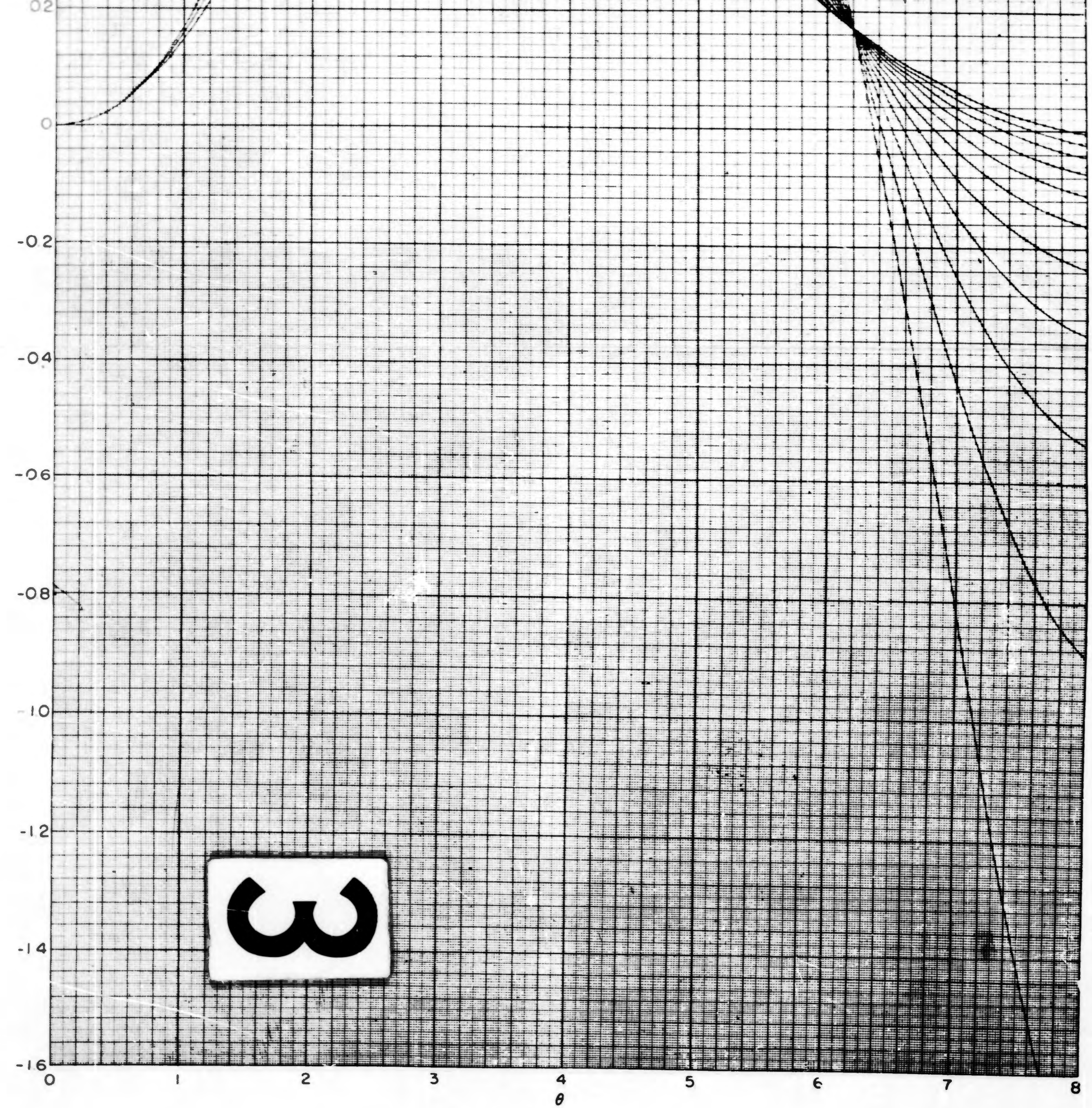

Figure 4 – Stress Function F_4

Then, using Equations [1] through [4],

$$1 - \frac{\sigma_{\phi mf}}{\sigma_u} = 0.3350$$

$$1 - \frac{\sigma_{\phi mm}}{\sigma_u} = 0.1665$$

$$\frac{\sigma_{xbf}}{\sigma_u} = -0.6168$$

$$\frac{\sigma_{xbm}}{\sigma_u} = 0.3149$$

and

$$\sigma_u = -\frac{PR}{h} = -138,460 \text{ psi}$$

Therefore

$$\sigma_{\phi mf} = -92,100 \text{ psi}$$

$$\sigma_{\phi mm} = -115,400 \text{ psi}$$

$$\sigma_{xbf} = 85,400 \text{ psi}$$

$$\sigma_{xbm} = -43,600 \text{ psi}$$

Using Equation [9]

$$\sigma_{\phi om} = -115,400 + 0.3(-43,600) = -128,500 \text{ psi}$$

$$\sigma_{\phi im} = -115,400 - 0.3(-43,600) = -102,300 \text{ psi}$$

Using Equation [10]

$$\sigma_{xom} = \frac{1}{2}(-138,460) + (-43,600) = -112,800 \text{ psi}$$

$$\sigma_{xim} = \frac{1}{2}(-138,460) - (-43,600) = -25,600 \text{ psi}$$

Using Equation [11]

$$\sigma_{\phi of} = -92,100 + 0.3\,(85,400) = -66,500 \text{ psi}$$

$$\sigma_{\phi if} = -92,100 - 0.3\,(85,400) = -117,700 \text{ psi}$$

Using Equation [12]

$$\sigma_{xof} = \frac{1}{2}(-138{,}460) + (85{,}400) = +16{,}200 \text{ psi}$$

$$\sigma_{xif} = \frac{1}{2}(-138{,}460) - (85{,}400) = -154{,}600 \text{ psi}$$

Using Equation [13]

$$\sigma_{\phi f} = -92{,}100 - \frac{0.3}{2}(-138{,}460) = -71{,}300 \text{ psi}$$

ACKNOWLEDGMENT

The authors wish to acknowledge the assistance of Mrs. Sharon E. Good of the Applied Mathematics Laboratory in obtaining the data presented in this report.

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1. Cylindrical shells (Stiffened) – Stresses – Graphical analysis
2. Cylindrical shells (Stiffened) – Pressure – Measurement

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