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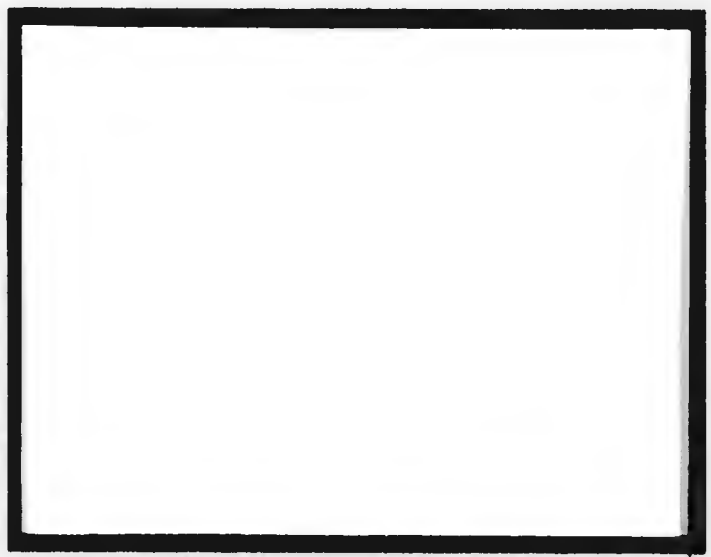
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**MECHANICAL TECHNOLOGY INCORPORATED**  
968 Albany-Shaker Rd.  
Latham, N.Y.

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**ON THE SPIRAL GROOVED,  
SELF ACTING, GAS BEARING**

by

**J. H. Vohr**

**C. H. T. Pan**

**Contract Nonr-3730(00)**

**Task NR 061-131**

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J. H. Vohr  
C. H. T. Pan

*J. H. Vohr C. H. T. Pan*

Author(s)

*Ben Sternlich*  
Approved by

Approved by

Prepared under

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TABLE OF CONTENTS

	Pg.
INTRODUCTION .....	1
SUMMARY AND CONCLUSIONS .....	2
DERIVATION OF EQUATIONS .....	3
Geometry and Coordinate System for Bearing .....	3
Equations of Motion for Flow in Bearing Film ...	6
Mass Flow Continuity in Ridge-Groove Pair .....	9
Relation of Local Ridge and Groove Pressure Gradients to Overall Pressure Gradient .....	12
Differential Equation for $\bar{P}$ .....	17
SOLUTION FOR PRESSURE DISTRIBUTION AROUND SPIRAL-GROOVED, SELF-ACTING, CYLINDRICAL JOURNAL BEARING .....	21
REFERENCES .....	40
APPENDIX .....	41
NOMENCLATURE .....	43

### INTRODUCTION

A hydrodynamic bearing with shallow grooves cut in one of its bearing surfaces at an angle to the direction of motion of the surfaces acts as a viscous pump or compressor; that is, it acts to pump fluid along the grooves or to increase the fluid pressure along the grooves or both. This self-pressurizing effect of grooved bearings has been utilized to support thrust loads of considerable magnitude. The analysis of thrust bearings of this type was first accomplished by Whipple (Ref.1) and more extensive results were provided by Whitley and Williams (Ref.2). To this writer's knowledge, there has been no prior work on the performance of a grooved bearing operating with a variable film thickness, such as an eccentric, spiral-grooved, journal bearing. Grooved self-acting bearings have two desirable features. One is that, with suitable geometry, such bearings can support both radial and thrust loads without external pressurization. A second important feature is that grooving may inhibit the notorious whirl instability common to many self-acting journal bearings.

In this present report a differential equation is derived for the pressure distribution around a grooved self-acting bearing of arbitrary geometry. The differential equation is based on the limiting case of an idealized bearing in which the number of grooves approaches infinity, i.e. the width of each groove and ridge pair becomes infinitesimally small compared to a characteristic dimension of the bearing. It is expected, however, that the analysis will serve quite accurately to describe the overall pressure distribution around a bearing with a large but finite number of grooves.

A sample solution of the differential equation is presented for the case of a spiral-grooved, cylindrical, journal bearing with an incompressible lubricant. The solution is obtained by a perturbation in  $\epsilon$ , the eccentricity ratio, and is valid for small eccentricities only.

SUMMARY AND CONCLUSIONS

A differential equation is derived for the "smoothed" overall pressure distribution around an arbitrarily-shaped, grooved, journal bearing operating with a variable film thickness. The equation is based on the limiting case of an idealized bearing in which the number of grooves approaches infinity, but it is expected that the equation would apply quite well to a bearing with a large but finite number of grooves.

An analytical solution is obtained for the pressure distribution around a spiral-grooved, cylindrical journal bearing valid for small eccentricities and incompressible lubricant. The analysis shows that appropriate grooving of journal bearings can result in a significant improvement in radial stiffness and a significant decrease in attitude angle for the bearing. These results indicate that grooving could be used to enhance the stability of journal bearings.

## DERIVATION OF EQUATIONS

### Geometry and Coordinate System for Bearing

The grooved bearing geometry we will be considering is shown schematically in Fig.1. Basically, this geometry consists of a grooved journal within a smooth outer bearing, the latter indicated in Fig.1 by dotted lines. Actually, it will not matter whether it is the journal or the outer bearing that is grooved. It will be necessary, however, to make a distinction between the velocity of the grooved member and the smooth member. We shall denote the surface velocity of the grooved member by  $\vec{V}$  and the surface velocity of the smooth member by  $\vec{U}$ .

The bearing surfaces to be analyzed will be considered to be surfaces of revolution. Their shape in the axial direction, however, will be arbitrary; e.g. the surfaces may be cylindrical or spherical etc.

The coordinate system that will be used is the  $\xi, \eta, h$  system shown in Fig. 1. The coordinate  $\xi$  is aligned with the direction of rotation while  $\eta$  is aligned with the direction of the grooves. The coordinate  $h$  measures displacements normal to the grooved surface in the bearing clearance while the  $\xi$  and  $\eta$  coordinates lie on the grooved surface. The directions of  $\xi$  and  $\eta$  are shown in Figs. 1 and 2. The use of the skewed  $\xi, \eta, h$  coordinate system greatly simplifies the task of deriving the differential equation for the pressure distribution around a grooved bearing. However, as we shall see later in a sample solution, it is generally more convenient to convert the final differential equation obtained back to a more conventional rectangular coordinate system in order to solve the equation.

There are two sets of base vectors which are conveniently related to the  $\xi, \eta,$  and  $h$  coordinates. One is the set of covariant base vectors,  $\vec{g}_\xi, \vec{g}_\eta$  and  $\vec{g}_h$ . These vectors are tangent, respectively to the coordinate axes for  $\xi, \eta$  and  $h$ .  $\vec{g}_h$ , therefore, is normal to the grooved surface while

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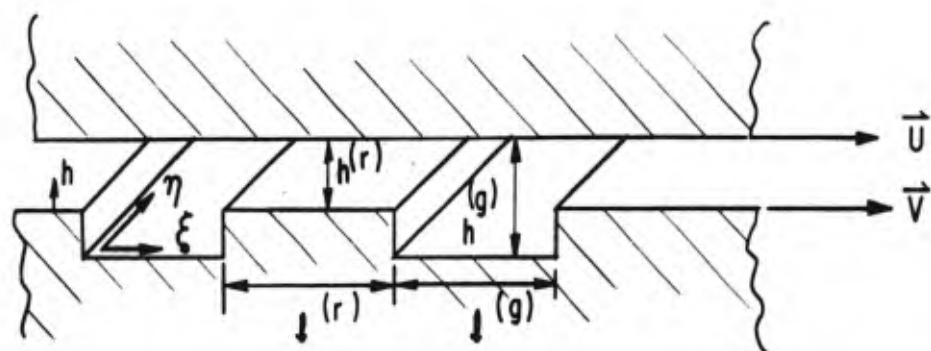
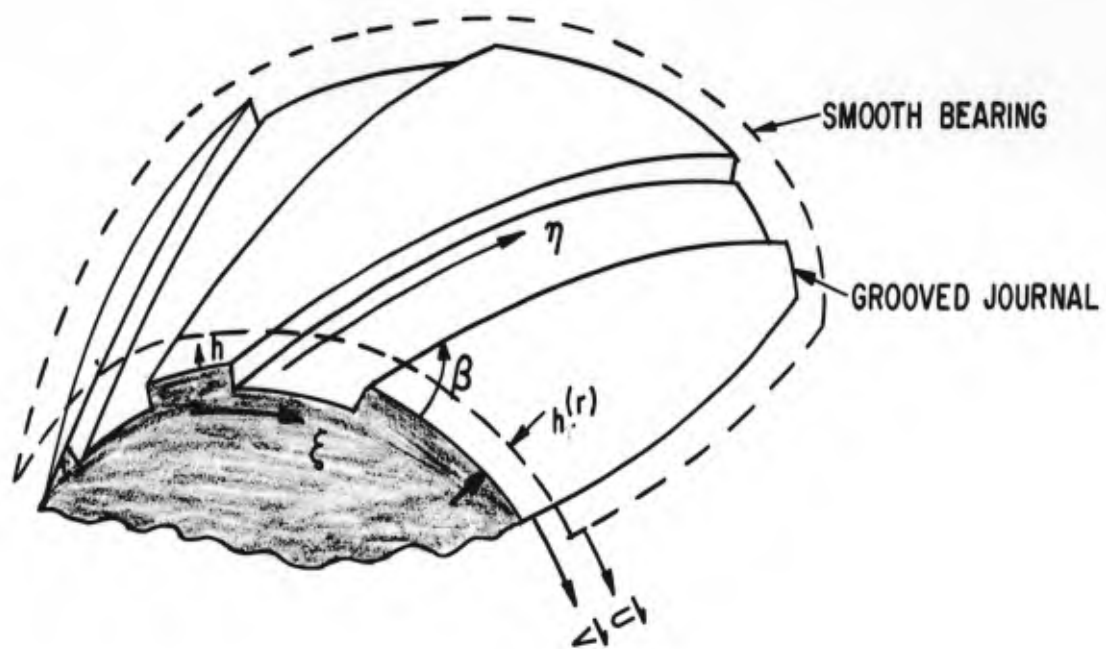


Figure 1. Grooved Bearing Geometry and Coordinate System

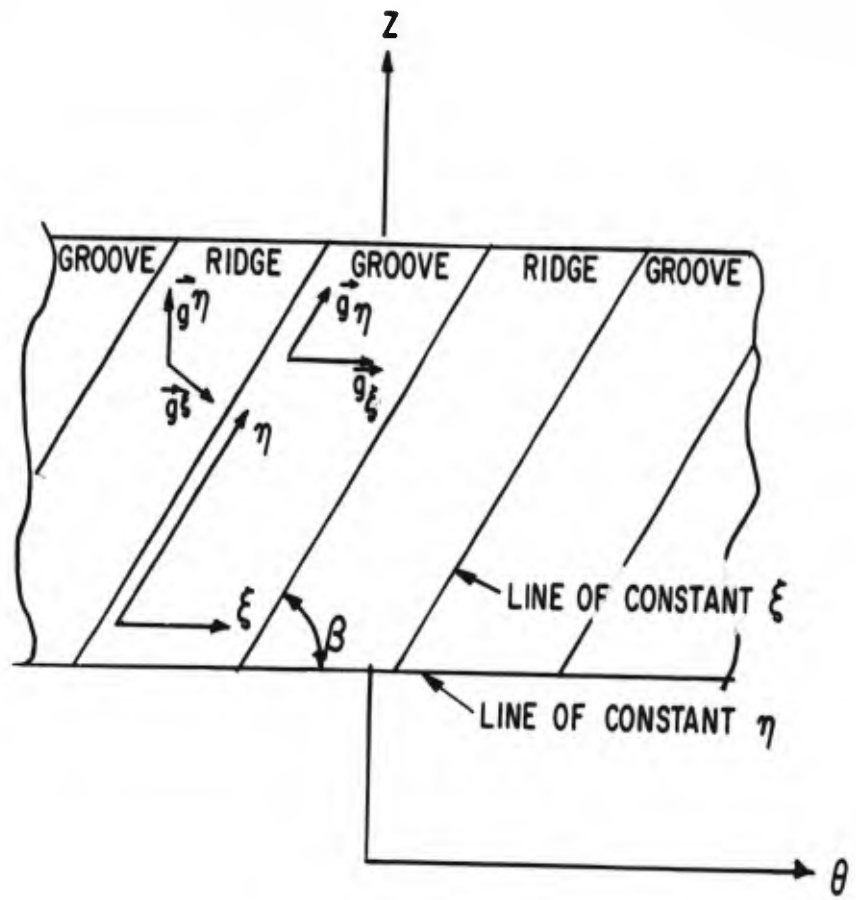


Figure 2.  $\xi$  and  $\eta$  Coordinates and Base Vectors Looking Down on Grooved Surface.

$\vec{g}_\xi$  and  $\vec{g}_\eta$  are tangent to this surface. The directions  $\vec{g}_\xi$  and  $\vec{g}_\eta$  are shown in Fig. 2.

The second set of base vectors for the  $\xi, \eta, h$  coordinate system are the set of contravariant base vectors,  $\vec{g}^\xi, \vec{g}^\eta$ , and  $\vec{g}^h$ . These vectors are each normal to the plane in which their index coordinate is constant, that is,  $\vec{g}^\xi$  is normal to the plane of constant  $\xi$  etc. The directions of  $\vec{g}^\xi$  and  $\vec{g}^\eta$  on the grooved surface are shown in Fig. 2.

An arbitrary vector  $\vec{T}$  expressed in terms of the covariant base vectors  $\vec{g}_\xi$  and  $\vec{g}_\eta$  is written as

$$(1) \quad \vec{T} = T^\xi \vec{g}_\xi + T^\eta \vec{g}_\eta$$

where  $T^\xi$  and  $T^\eta$  are contravariant components of  $\vec{T}$ . Alternately,  $\vec{T}$  expressed in terms of the contravariant base vectors is written as

$$(2) \quad \vec{T} = T_\xi \vec{g}^\xi + T_\eta \vec{g}^\eta$$

where  $T_\xi$  and  $T_\eta$  are the covariant components for the vector  $\vec{T}$ .

In the present analysis of grooved bearings, it will be assumed that the reader has a working knowledge of the mathematics of covariant and contravariant vector and tensor quantities, the treatment of which can be found in many standard texts such as Ref. 3.

#### Equations of Motion for Flow in Bearing Film

In our grooved bearing analysis, we shall make the usual thin film assumptions common to the theory of lubrication. These assumptions are

$$(3) \quad \frac{|\vec{u}|}{V} = 0 \quad (1)$$

$$(4) \quad \frac{|u_h|}{V} = \ll 1$$

$$(5) \quad \frac{\rho V (h^{(1)})^2}{\mu L} \ll 1$$

$$(6) \quad \frac{h^{(i)}}{L} \ll 1$$

$$(7) \quad \frac{h^{(i)}}{h^* K} \ll 1$$

Here,  $V$  and  $L$  are suitable scales for the bearing surface speeds and the bearing size respectively and  $K$  is a measure of the bearing surface curvature.  $h^{(i)}$  is the bearing clearance in the groove region ( $i = g$ ) or in the ridge region ( $i = r$ ). One should note that assumption (4) will generally not be valid in the immediate vicinity of the groove-ridge interface. However, we will further assume that the width of a bearing ridge or groove,  $l^{(r)}$  or  $l^{(g)}$ , is substantially greater than the groove clearance height  $h^{(g)}$  so that assumption (4) should be valid throughout most of the bearing clearance.

By virtue of the above assumptions, the equations of motion governing the flow in the bearing film are

$$(8) \quad \frac{\partial P}{\partial \xi} = \mu \left( \frac{\partial^2 u_\xi}{\partial h^2} \right)$$

$$(9) \quad \frac{\partial P}{\partial \eta} = \mu \left( \frac{\partial^2 u_\eta}{\partial h^2} \right)$$

$$(10) \quad \frac{\partial P}{\partial h} = 0$$

where  $u_\xi$  and  $u_\eta$  are the covariant components of  $\vec{u}$ , the fluid velocity in the bearing film.

For a gas lubricated bearing, it is generally valid to assume that the flow in the bearing clearance is isothermal (Ref.4) and that the fluid obeys the perfect gas law. With these last two additional conditions we can also write

$$(11) \quad \frac{\partial \rho}{\partial h} = 0$$

In the neighborhood of the groove-ridge interface, over a distance of the order of  $h^{(g)}$ ,  $\frac{\partial P}{\partial \xi}$  changes rapidly. Due to the assumption (4), however, this change must be considered to be an abrupt jump. Therefore, in our future notation, we will write  $\frac{\partial P}{\partial \xi}$  as  $\frac{\partial P^{(i)}}{\partial \xi}$  where superscript  $i$  can be either letter  $g$  or  $r$ , referring to either the groove region or the ridge region respectively. The derivative  $\frac{\partial P}{\partial \eta}$ , however, must be continuous across groove-ridge interfaces if  $P$  is to be continuous everywhere in the bearing film.

Introducing the boundary conditions that  $\hat{u} = \hat{v}$  at the grooved surface and  $\hat{u} = \hat{U}$  at the smooth surface, equations (8) and (9) can be integrated twice with respect to  $h$  to yield

$$(12) \quad u_{\xi}^{(i)} = -\frac{1}{2\mu} \frac{\partial P^{(i)}}{\partial \xi} h \left[ 1 - \frac{h}{h^{(i)}} \right] h^{(i)} + v_{\xi} \left[ 1 - \frac{h}{h^{(i)}} \right] + u_{\xi} \frac{h}{h^{(i)}}$$

$$(13) \quad u_{\eta}^{(i)} = -\frac{1}{2\mu} \frac{\partial P}{\partial \eta} h \left[ 1 - \frac{h}{h^{(i)}} \right] h^{(i)} + v_{\eta} \left[ 1 - \frac{h}{h^{(i)}} \right] + u_{\eta} \frac{h}{h^{(i)}}$$

where  $h^{(i)}$  is the height of the bearing clearance in either the groove region ( $i = g$ ) or the ridge region ( $i = r$ ).

Mass Flow Continuity in Ridge-Groove Pair

Consider the control volume,  $\Delta V$ , for one ridge-groove pair shown in Fig.3. We take the control volume to move with the grooved surface at velocity  $\vec{V}$  and we consider that, at the instant shown in Fig.3, the various ridge-groove interfaces are located at  $\xi_n, \xi_{n+1/2}, \xi_{n+1}$  etc., as depicted. The mass flows entering and leaving the moving control volume through the surfaces  $\Delta S_{ridge}^{\xi}$  we denote as  $w_{ridge}^{(\xi)} \Delta \eta$  whereas the mass flows entering and leaving the control volume through the surfaces  $\Delta S^{\eta}$  we denote as  $w^{(\eta)}(\xi_{n+1} - \xi_n)$ . The total time rate of change of the mass,  $\Delta M$  contained in  $\Delta V$  is given by

$$(14) \quad \frac{\partial}{\partial t} (\Delta M) + \vec{v} \cdot \nabla (\Delta M)$$

By the conservation of mass we obtain that

$$(15) \quad \left[ w_{ridge}^{(\xi)} \Big|_{\xi_{n+1}} - w_{ridge}^{(\xi)} \Big|_{\xi_n} \right] \Delta \eta + \left[ w^{(\eta)} \Big|_{\eta+\Delta \eta} - w^{(\eta)} \Big|_{\eta} \right] (\xi_{n+1} - \xi_n) + \frac{\partial}{\partial t} (\Delta M) + \vec{V} \cdot \vec{\nabla} (\Delta M) = 0$$

We next turn to the problem of determining the expressions for  $w_{ridge}^{(\xi)}, w^{(\eta)}$  and  $\Delta M$  in terms of the pressure gradients in the bearing and the geometry of the ridge-groove pair. First, we consider the evaluation of  $w_{ridge}^{(\xi)} \Delta \eta$ , the mass flow through the moving surface  $\Delta S_{ridge}^{\xi}$  at the ridge side of a ridge-groove interface.  $w_{ridge}^{(\xi)}$  is given by the following integral.

$$(16) \quad w_{ridge}^{(\xi)} \Delta \eta = \int_0^{h(r)} \rho (\vec{u} - \vec{V}) \cdot \sqrt{g} \vec{g}^{\xi} \Delta \eta dh$$

where  $\sqrt{g} \vec{g}^{\xi} \Delta \eta dh$  is a vector differential area of height  $dh$  and length  $\Delta \eta$  in the surface  $\Delta S_{ridge}^{\xi}$ . The quantity  $g$  is the determinant of the covariant metric tensor,  $g_{ij}$ , for the  $\xi, \eta, h$  coordinate system. To evaluate integral (16) we write

$$(17) \quad \vec{u}(r) = u_{\xi}^{(r)} \vec{g}^{\xi} + u_{\eta}^{(r)} \vec{g}^{\eta}$$

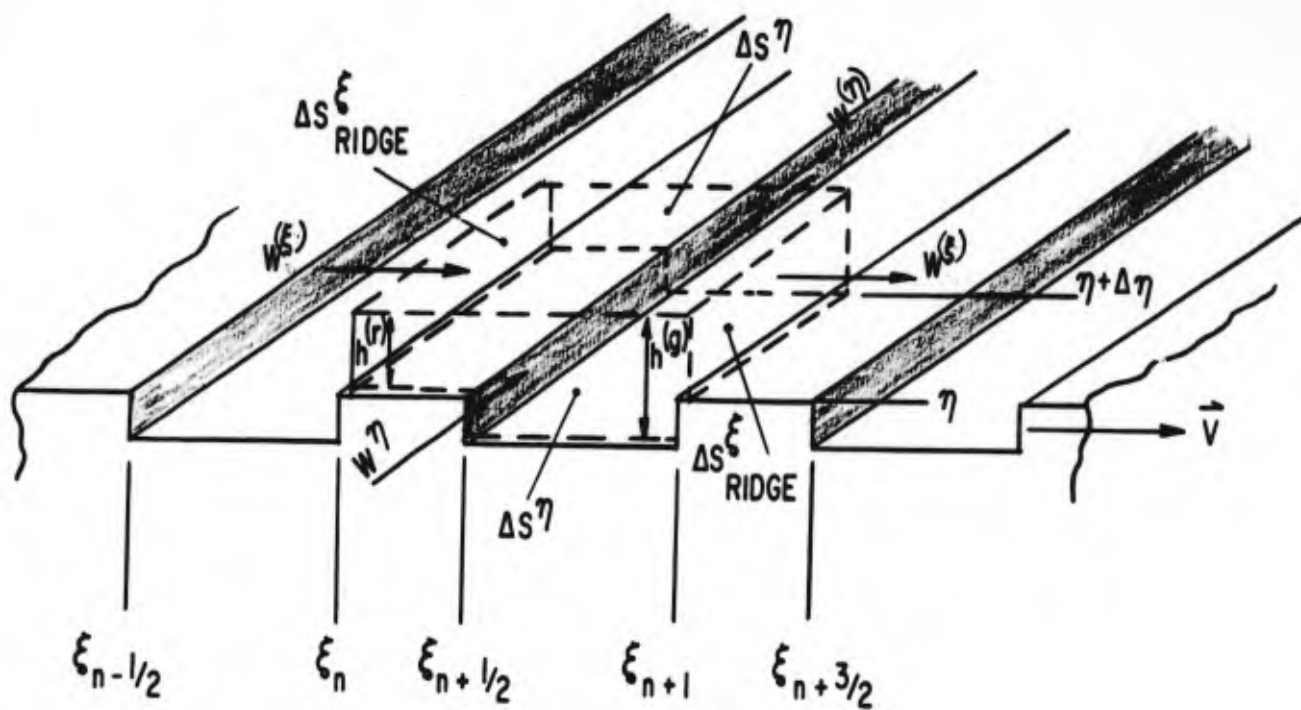


Fig. 3 Control Volume for Mass Flow Continuity Analysis

$$(18) \quad \vec{v} = v_{\xi} \vec{g}^{\xi} + v_{\eta} \vec{g}^{\eta}$$

$$(19) \quad \vec{u} = u_{\xi} \vec{g}^{\xi} + u_{\eta} \vec{g}^{\eta}$$

Substituting (17), (18) and (19) into (16) we get

$$(20) \quad W_{\text{ridge}}^{(\xi)} \Delta\eta = \int_0^{h(r)} \rho \left[ (u_{\xi}^{(r)} - v_{\xi}) g^{\xi\xi} + (u_{\eta}^{(r)} - v_{\eta}) g^{\xi\eta} \right] \sqrt{g} \Delta\eta \, dh$$

where  $g^{\xi\xi}$  and  $g^{\xi\eta}$  are components of the contravariant metric tensor for the  $\xi, \eta, h$  coordinate system.  $g^{\xi\xi}$  and  $g^{\xi\eta}$  are obtained from

$$(21) \quad g^{\xi\xi} = \vec{g}^{\xi} \cdot \vec{g}^{\xi}$$

$$(22) \quad g^{\xi\eta} = \vec{g}^{\xi} \cdot \vec{g}^{\eta}$$

Substituting for  $u_{\xi}^{(r)}$  and  $u_{\eta}^{(r)}$  from equations (22) and (13) and performing the integration with respect to  $h$  we obtain

$$(23) \quad W_{\text{ridge}}^{(\xi)} = \sqrt{g} \left\{ \frac{\rho}{12\mu} (h(r))^3 \left[ g^{\xi\xi} \frac{\partial p}{\partial \xi}(r) + g^{\xi\eta} \frac{\partial p}{\partial \eta} \right] + \frac{\rho h(r)}{2} (u_{\xi}^{\xi} - v_{\xi}^{\xi}) \right\}$$

where

$$(24) \quad v^{\xi} = g^{\xi\xi} v_{\xi} + g^{\xi\eta} v_{\eta}$$

$$(25) \quad u^{\xi} = g^{\xi\xi} u_{\xi} + g^{\xi\eta} u_{\eta}$$

We next determine the expression for  $W^{(\eta)}(\xi_{n+1} - \xi_{\eta})$ , the axial mass flow through the surface  $\Delta S^{(\eta)}$ .  $W^{(\eta)}(\xi_{n+1} - \xi_{\eta})$  is given by

$$(26) \quad W^{(\eta)}(\xi_{n+1} - \xi_{\eta}) = \int_{\xi_n}^{\xi_{n+1/2}} \int_0^{h(r)} \rho \vec{u}^{(r)} \cdot \sqrt{g} \vec{g}^{\eta} \, dh \, d\xi + \int_{\xi_{n+1/2}}^{\xi_{n+1}} \int_0^{h(g)} \rho \vec{u}^{(g)} \cdot \sqrt{g} \vec{g}^{\eta} \, dh \, d\xi$$

where  $\sqrt{g} \vec{g}^\eta dh d\xi$  is a vector differential area in the surface  $\Delta S^\eta$ . Again, substituting for  $u^{(r)}$  and  $u^{(g)}$  in (26) by means of (12) and (13) and performing the integration with respect to  $h$  we get

$$(27) \quad W^{(\eta)}(\xi_{n+1} - \xi_n) = \int_{\xi_n}^{\xi_{n+1/2}} \frac{\rho}{12\mu} (h^{(r)})^3 \left[ g^{\xi\eta} \frac{\partial P^{(r)}}{\partial \xi} + g^{\eta\eta} \frac{\partial P}{\partial \eta} \right] \sqrt{g} d\xi \\ + \int_{\xi_{n+1/2}}^{\xi_{n+1}} \frac{\rho}{12\mu} (h^{(g)})^3 \left[ g^{\xi\eta} \frac{\partial P^{(g)}}{\partial \xi} + g^{\eta\eta} \frac{\partial P}{\partial \eta} \right] \sqrt{g} d\xi$$

Note that in obtaining (27), use has been made of the fact that

$$(28) \quad v^\eta = v_\xi g^{\xi\eta} + v_\eta g^{\eta\eta} = 0$$

$$(29) \quad U^\eta = U_\xi g^{\xi\eta} + U_\eta g^{\eta\eta} = 0$$

$v^\eta$  and  $U^\eta$  are zero because  $\vec{V}$  and  $\vec{U}$  point in the  $\xi$  direction.

Finally, we come to the expression for  $\Delta M$ , the mass contained in the control volume shown in Fig. 3. This mass is

$$(30) \quad \Delta M = \int_{\xi_n}^{\xi_{n+1/2}} \rho h^{(r)} \sqrt{g} \Delta \eta d\xi + \int_{\xi_{n+1/2}}^{\xi_{n+1}} \rho h^{(g)} \sqrt{g} \Delta \eta d\xi$$

#### Relation of Local Ridge and Groove Pressure Gradients to Overall Pressure Gradient

Expressions (23) and (26) for  $W^{(\xi)}$  and  $W^{(\eta)}$  involve the local ridge and groove derivations  $\frac{\partial P^{(r)}}{\partial \xi}$  and  $\frac{\partial P^{(g)}}{\partial \xi}$ . As noted earlier, these local derivatives are discontinuous at the interfaces between the ridge and groove regions. Ultimately, in our grooved bearing analysis, we are interested not in determining the local pressure distribution within a particular groove or ridge but in determining the "overall" "smoothed" pressure distribution around the whole bearing. What is meant by the "overall" "smoothed" pressure distribution is illustrated in Fig. 4 where there is shown schematically the general form of the pressure distribution,  $P(\xi, \eta)$  around a grooved bearing. The local

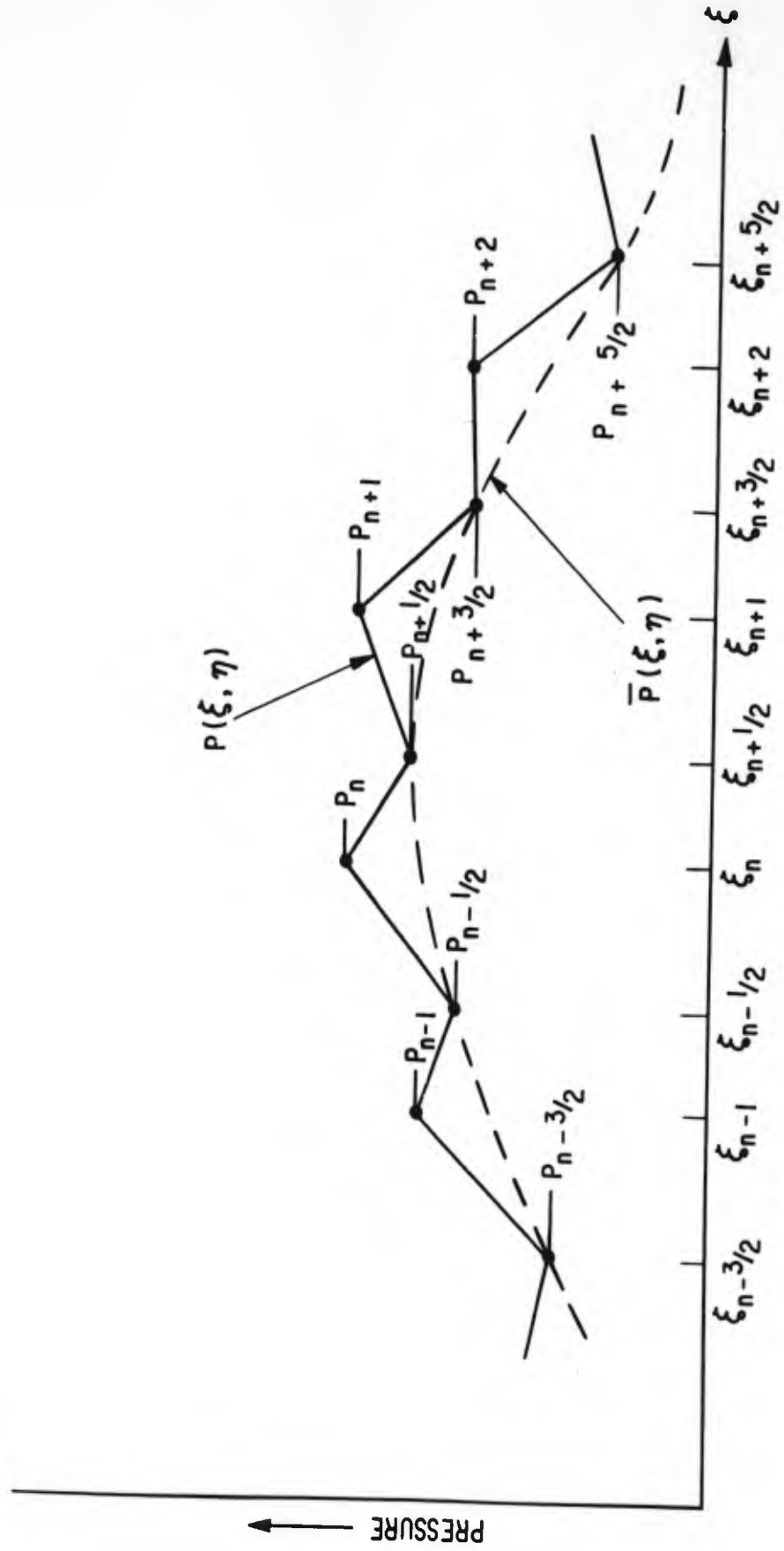


Figure 4. Schematic Representation of Pressure Variation Around Grooved Bearing

"saw toothed" variations in  $P(\xi, \eta)$  are due to the alternating ridges and grooves. The pressures denoted as  $P_n, P_{n+1/2}$  etc., refer to the pressures at  $\xi_n, \xi_{n+1/2}$  etc. Now, in the limit as the number of grooves and ridges around the grooved bearing approaches infinity, i.e., as the width of each ridge-groove pair approaches zero, the saw-toothed pressure distribution  $P(\eta, \xi)$  will approach the limiting smoothed pressure distribution  $\bar{P}(\xi, \eta)$ , shown as the dashed line in Fig. 4. Formally,  $\bar{P}(\xi, \eta)$  is defined as the limiting continuous distribution approached by the discrete pressures  $P_{n+1/2}, P_{n+1/2}, P_{n+3/2}$  etc., as the number of grooves becomes infinite.  $\bar{P}(\xi, \eta)$ , defined in this way, will be continuously differentiable with respect to  $\xi$  with the first derivative being defined by

$$(31) \quad \left. \frac{\partial \bar{P}(\xi, \eta)}{\partial \xi} \right|_{\xi=\xi_n} = \lim_{(\xi_{n+1/2} - \xi_{n-1/2}) \rightarrow 0} \frac{P_{n+1/2} - P_{n-1/2}}{\xi_{n+1/2} - \xi_{n-1/2}}$$

From now on, we shall only be concerned with  $\bar{P}(\xi, \eta)$ . Since  $\bar{P}(\xi, \eta)$  is defined as a limiting pressure distribution, our analysis will strictly apply only to an idealized bearing with an infinite number of grooves. It is expected, however, that the solution for  $P(\xi, \eta)$  would provide a good representation of the overall pressure distribution around a bearing with large but finite number of grooves.

To obtain a differential equation for  $\bar{P}(\xi, \eta)$  our first step will be to eliminate  $\frac{\partial P^{(r)}}{\partial \xi}$  and  $\frac{\partial P^{(g)}}{\partial \xi}$  from expressions (23) and (27) in terms of

$\frac{\partial \bar{P}(\xi, \eta)}{\partial \xi}$  and  $\frac{\partial \bar{P}(\xi, \eta)}{\partial \eta}$ . Let us consider the ridge-groove pair centered at  $\xi_n$  shown in Fig. 5. Now, since the derivatives  $\frac{\partial P^{(g)}}{\partial \xi}$  and  $\frac{\partial P^{(r)}}{\partial \xi}$  are continuous everywhere within a groove or ridge region, we can expand the pressure in a Taylor series about the point  $\xi_n$  in either the ridge or groove region to obtain

$$(32) \quad P_{n-1/2} = P_n + \left. \frac{\partial P^{(g)}}{\partial \xi} \right|_{\xi_n^-} (\xi_{n-1/2} - \xi_n) + \dots$$

$$(33) \quad P_{n+1/2} = P_n + \left. \frac{\partial P^{(r)}}{\partial \xi} \right|_{\xi_n^+} (\xi_{n+1/2} - \xi_n) + \dots$$

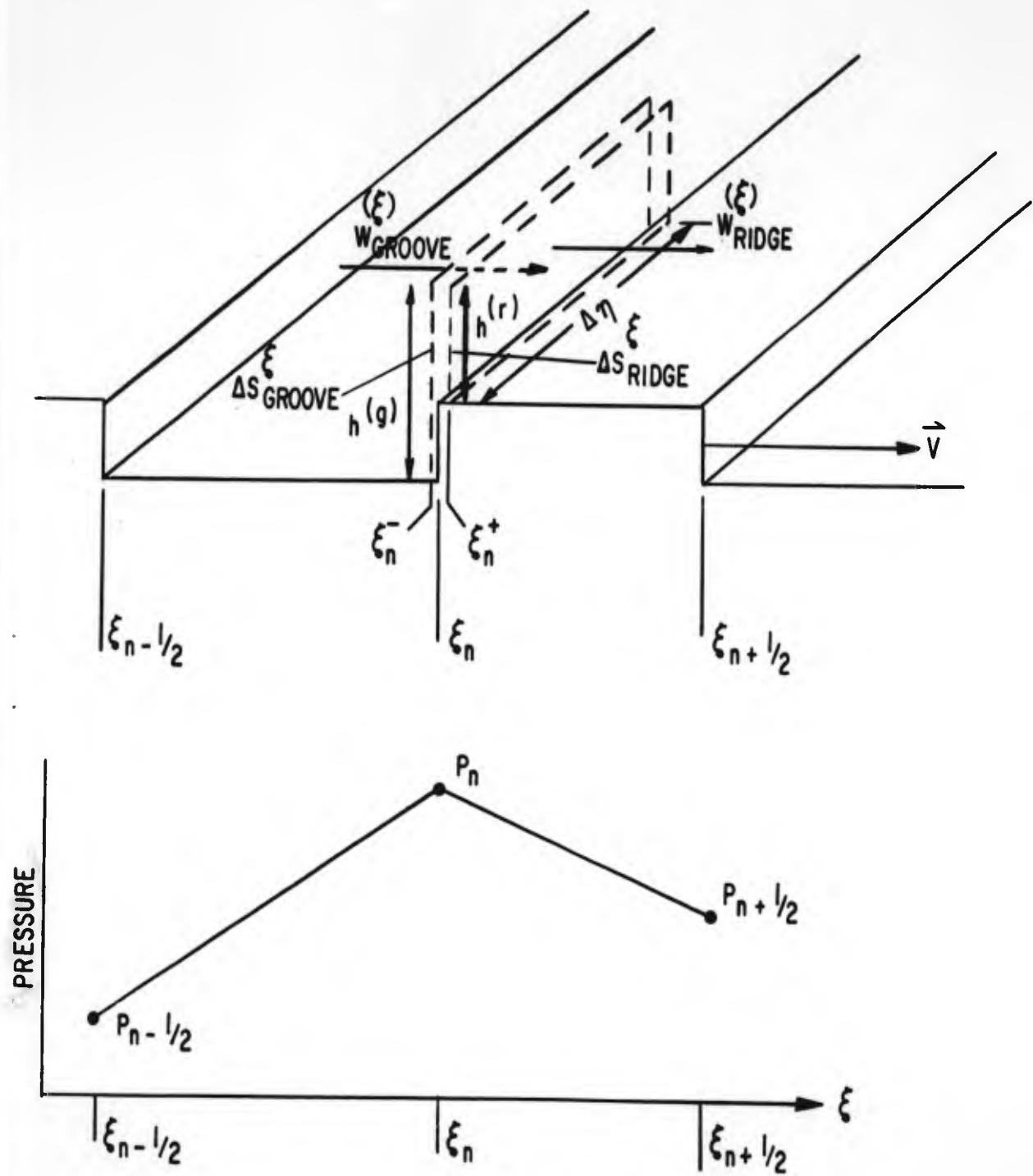


Figure 5. Continuity of Mass Flow across Groove-Ridge Interface and Pressure Variation across Groove-Ridge Interface

where the symbols  $\xi_n^-$  and  $\xi_n^+$  indicate evaluation just to the left or just to the right of  $\xi_n$  respectively. If we define

$$(34) \quad \alpha = \frac{\xi_n - \xi_{n-1/2}}{\xi_{n+1/2} - \xi_{n-1/2}}$$

and subtract expression (32) from (33) and take the limit as  $\xi_{n+1/2} - \xi_{n-1/2}$  goes to zero, we obtain

$$(35) \quad \alpha \left. \frac{\partial P^{(g)}}{\partial \xi} \right|_{\xi_n^-} + (1-\alpha) \left. \frac{\partial P^{(r)}}{\partial \xi} \right|_{\xi_n^+} = \lim_{(\xi_{n+1/2} - \xi_{n-1/2}) \rightarrow 0} \frac{P_{n+1/2} - P_{n-1/2}}{\xi_{n+1/2} - \xi_{n-1/2}}$$

which, by means of (31), gives

$$(36) \quad \alpha \left. \frac{\partial P^{(g)}}{\partial \xi} \right|_{\xi_n^-} + (1-\alpha) \left. \frac{\partial P^{(r)}}{\partial \xi} \right|_{\xi_n^+} = \overline{\frac{\partial P}{\partial \xi}} \Big|_{\xi_n}$$

To obtain a second linear relation between  $\left. \frac{\partial P^{(g)}}{\partial \xi} \right|_{\xi_n^-}$ ,  $\left. \frac{\partial P^{(r)}}{\partial \xi} \right|_{\xi_n^+}$ , and  $\overline{\frac{\partial P}{\partial \xi}} \Big|_{\xi_n}$ , we consider the continuity of mass flow normal to a ridge-groove interface. Referring again to Fig. 5 we note that by continuity of Mass flow,  $W_{ridge}^{(\xi)}$  through the moving surface  $\Delta S_{ridge}^{\xi}$  at the point  $\xi_n^+$  in the ridge region must be equal to  $W_{groove}^{(\xi)}$  through the moving surface  $\Delta S_{groove}^{\xi}$  at the point  $\xi_n^-$  in the groove region. The mass flow through  $\Delta S_{ridge}^{\xi}$  is given by equation (23) with derivatives evaluated at  $\xi = \xi_n^+$ . The mass flow through  $\Delta S_{groove}^{\xi}$  is given by

$$(37) \quad W_{groove}^{(\xi)} = \sqrt{g} \left\{ -\frac{\rho}{12\mu} (h^{(g)})^3 \left[ g \xi \xi \left. \frac{\partial P^{(g)}}{\partial \xi} \right|_{\xi_n^-} + g \xi \eta \left. \frac{\partial P}{\partial \eta} \right|_{\xi_n} \right] + \frac{\rho h^{(g)}}{2} (U^{\xi} - V^{\xi}) \right\}$$

Since  $\frac{\partial P}{\partial \eta}$  is continuous everywhere in the bearing clearance, it follows that, in the limit of an infinite number of grooves,  $\frac{\partial P}{\partial \eta}$  will approach  $\overline{\frac{\partial P}{\partial \eta}}$ . Writing  $\frac{\partial P}{\partial \eta}$  as  $\overline{\frac{\partial P}{\partial \eta}}$  in equations (23) and (37) and setting (23) equal to (37), we obtain the following linear equation relative  $\left. \frac{\partial P^{(g)}}{\partial \xi} \right|_{\xi_n^-}$  and  $\left. \frac{\partial P^{(r)}}{\partial \xi} \right|_{\xi_n^+}$  to  $\overline{\frac{\partial P}{\partial \xi}}$  and  $\overline{\frac{\partial P}{\partial \eta}}$

$$\begin{aligned}
 (38) \quad & \frac{1}{12\mu} g^{\xi\xi} \left[ (h(r))^3 \frac{\partial P^{(r)}}{\partial \xi} \Big|_{\xi_n^+} - (h(g))^3 \frac{\partial P^{(g)}}{\partial \xi} \Big|_{\xi_n^-} \right] \\
 & = - \frac{1}{12\mu} g^{\xi\eta} \left[ (h(r))^3 - (h(g))^3 \right] \frac{\partial \bar{P}}{\partial \eta} \Big|_{\xi_n} \\
 & \quad + \frac{1}{2} (h(r) - h(g)) (U^\xi - V^\xi)
 \end{aligned}$$

Equations (36) and (38) may be solved simultaneously for  $\frac{\partial P^{(r)}}{\partial \xi} \Big|_{\xi_n^+}$  and  $\frac{\partial P^{(g)}}{\partial \xi} \Big|_{\xi_n^-}$ .  
 The solutions are

$$(39) \quad \frac{\partial P^{(g)}}{\partial \xi} \Big|_{\xi_n^-} = \frac{(h(r))^3 \frac{\partial \bar{P}}{\partial \xi} \Big|_{\xi_n} - (1-\alpha) \left\{ [(h(g))^3 - (h(r))^3] g^{\xi\eta} \frac{\partial \bar{P}}{\partial \eta} \Big|_{\xi_n} - 6\mu (U^\xi - V^\xi) (h(g) - h(r)) \right\} \frac{1}{g^{\xi\xi}}}{\left[ \alpha (h(r))^3 + (1-\alpha) (h(g))^3 \right]}$$

$$(40) \quad \frac{\partial P^{(r)}}{\partial \xi} \Big|_{\xi_n^+} = \frac{(h(g))^3 \frac{\partial \bar{P}}{\partial \xi} \Big|_{\xi_n} + \alpha \left\{ [(h(g))^3 - (h(r))^3] g^{\xi\eta} \frac{\partial \bar{P}}{\partial \eta} \Big|_{\xi_n} - 6\mu (U^\xi - V^\xi) (h(g) - h(r)) \right\} \frac{1}{g^{\xi\xi}}}{\left[ \alpha (h(r))^3 + (1-\alpha) (h(g))^3 \right]}$$

Differential Equation for  $\bar{P}$

If we divide equation (15) through by  $\Delta\eta (\xi_{n+1} - \xi_n)$  we obtain

$$\begin{aligned}
 (41) \quad & \frac{w_{ridge}^{(\xi)} \Big|_{\xi_{n+1}} - w_{ridge}^{(\xi)} \Big|_{\xi_n}}{(\xi_{n+1} - \xi_n)} + \frac{w^{(\eta)} \Big|_{\eta+\Delta\eta} - w^{(\eta)} \Big|_{\eta}}{\Delta\eta} \\
 & \quad + \frac{1}{\Delta\eta(\xi_{n+1} - \xi_n)} \left[ \frac{\partial}{\partial t} + \nabla \cdot \nabla \right] (\Delta M) = 0
 \end{aligned}$$

Let us consider the first term of equation (41) in the limit as  $(\xi_{n+1} - \xi_n)$  goes to zero, that is, as the width of the groove-ridge pair goes to zero. First, by substituting equation (40) into equation (23), we find that  $W^{(\xi)}$  (we have dropped the subscript "ridge" since we have required continuity of  $W^{(\xi)}$  at the interface) at any interface  $\xi_n$  in the limit as  $(\xi_{n+1} - \xi_n)$  goes to zero is expressed by

$$(42) \quad W^{(\xi)} \Big|_{\xi_n} = -\frac{\sqrt{g\rho}}{12\mu} \frac{(h^{(g)}h^{(r)})^3}{\alpha(h^{(r)})^3 + (1-\alpha)(h^{(g)})^3} \left\{ g^{\xi\xi} \frac{\partial \bar{P}}{\partial \xi} \Big|_{\xi_n} + g^{\xi\eta} \frac{\partial \bar{P}}{\partial \eta} \Big|_{\xi_n} \right. \\ \left. - 6\mu (U^{\xi} - V^{\xi}) \left[ \frac{\alpha}{(h^{(g)})^2} + \frac{(1-\alpha)}{(h^{(r)})^2} \right] \right\}$$

Since the right hand side of Eq. (42) contains  $\bar{P}$  instead of  $P$ , the first term of Eq. (41) can be formally identified as a derivative in the limit of an infinite number of grooves, i.e.,

$$(43) \quad \frac{\partial W^{(\xi)}}{\partial \xi} \Big|_{\xi_n} = \lim_{(\xi_{n+1} - \xi_n) \rightarrow 0} \frac{W^{(\xi)}_{\text{ridge}} \Big|_{\xi_{n+1}} - W^{(\xi)}_{\text{ridge}} \Big|_{\xi_n}}{(\xi_{n+1} - \xi_n)}$$

Next we consider the second term in equation (41) in the limit as both  $\Delta\eta$  and  $(\xi_{n+1} - \xi_n)$  go to zero. Since  $W^{(\eta)}$  has derivatives with respect to  $\eta$  which are continuous everywhere, it is easily seen that, in the limit as  $\Delta\eta$  goes to zero, the second term in (41) becomes  $\frac{\partial W^{(\eta)}}{\partial \eta}$ . The expression for  $W^{(\eta)}$  in terms of  $\frac{\partial P^{(g)}}{\partial \xi}$ ,  $\frac{\partial P^{(r)}}{\partial \xi}$  and  $\frac{\partial P}{\partial \eta}$  is given by (27). Since  $\frac{\partial P^{(g)}}{\partial \xi}$  and  $\frac{\partial P^{(r)}}{\partial \xi}$  are continuous functions of  $\xi$  within the intervals  $\xi_{n+1/2} < \xi < \xi_{n+1}$  and  $\xi_n < \xi < \xi_{n+1/2}$ , these derivatives can be expanded in a Taylor series in the above mentioned intervals, i.e.,

$$(44) \quad \frac{\partial P^{(r)}}{\partial \xi} = \frac{\partial P^{(r)}}{\partial \xi} \Big|_{\xi_n^+} + \frac{\partial^2 P^{(r)}}{\partial \xi^2} \Big|_{\xi_n^+} (\xi - \xi_n) + \text{etc.}; \quad \xi_n < \xi < \xi_{n+1/2}$$

$$(45) \quad \frac{\partial P^{(g)}}{\partial \xi} = \frac{\partial P^{(g)}}{\partial \xi} \Big|_{\xi_{n+1}^-} + \frac{\partial^2 P^{(g)}}{\partial \xi^2} \Big|_{\xi_{n+1}^-} (\xi - \xi_{n+1}) + \text{etc.}; \quad \xi_{n+1/2} < \xi < \xi_{n+1}$$

Substituting (44) and (45) into (27) and taking the limit as  $(\xi_{n+1} - \xi_n)$  goes to zero we get, neglecting the variation in  $g^{ij}$ ,  $\rho$ , and  $h^{(1)}$  in the interval  $\xi_{n+1} - \xi_n$ .

$$(46) \quad w^{(n)} = \frac{1}{(\xi_{n+1} - \xi_n)} \left( - \frac{\rho \sqrt{g}}{12\mu} \right) \left\{ (h^{(r)})^3 \left[ g^{\xi\eta} \frac{\partial P}{\partial \xi} \Big|_{\xi_n}^{(r)} + g^{\eta\eta} \frac{\partial P}{\partial \eta} \Big|_{\xi_n} \right] (\xi_{n+1/2} - \xi_n) \right. \\ \left. + (h^{(g)})^3 \left[ g^{\xi\eta} \frac{\partial P}{\partial \xi} \Big|_{\xi_{n+1}}^{(g)} + g^{\eta\eta} \frac{\partial P}{\partial \eta} \Big|_{\xi_{n+1}} \right] (\xi_{n+1} - \xi_{n+1/2}) \right\}$$

Substituting for  $\frac{\partial P}{\partial \xi} \Big|_{\xi_{n+1}}^{(g)}$  and  $\frac{\partial P}{\partial \xi} \Big|_{\xi_n}^{(r)}$  in the above equation by means of equations (39) and (40) and noting that in the limit as  $(\xi_{n+1} - \xi_n)$  goes to zero,  $\frac{\partial P}{\partial \xi} \Big|_{\xi_n}$  will approach  $\frac{\partial P}{\partial \xi} \Big|_{\xi_{n+1}}$ , we obtain

$$(47) \quad w^{(n)} = \frac{\sqrt{g\rho}}{12\mu} \frac{(h^{(g)} h^{(r)})^3}{\alpha (h^{(r)})^3 + (1-\alpha) (h^{(g)})^3} \left\{ g^{\xi\eta} \frac{\partial \bar{P}}{\partial \xi} + g^{\eta\eta} \frac{\partial \bar{P}}{\partial \eta} \right. \\ \left. + \alpha (1-\alpha) g^{\eta\eta} \frac{\partial \bar{P}}{\partial \eta} \left[ \frac{(h^{(g)})^3}{(h^{(r)})^3} - 2 + \frac{(h^{(r)})^3}{(h^{(g)})^3} \right] \left[ 1 - \frac{(g^{\xi\eta})^2}{g^{\xi\xi} g^{\eta\eta}} \right] \right. \\ \left. + \frac{g^{\xi\eta}}{g^{\xi\xi}} \alpha (1-\alpha) \left[ \frac{1}{(h^{(r)})^3} - \frac{1}{(h^{(g)})^3} \right] (h^{(g)} - h^{(r)}) 6\mu (U^{\xi} - V^{\xi}) \right\}$$

Finally, from expression (30) we obtain that in the limit as  $(\xi_{n+1} - \xi_n)$  goes to zero, the third term in equation (41) is just

$$(48) \quad \left( \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \left\{ \sqrt{g\rho} \left[ \alpha h^{(g)} + (1-\alpha) h^{(r)} \right] \right\}$$

From all the above relationships we determine that, in the limit as  $(\xi_{n+1} - \xi_n)$  and  $\Delta\eta$  go to zero, Equation (41) becomes

$$(49) \quad \frac{\partial w^{(g)}}{\partial \xi} + \frac{\partial w^{(r)}}{\partial \eta} + \left( \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \left\{ \sqrt{g\rho} \left[ \alpha h^{(g)} + (1-\alpha) h^{(r)} \right] \right\} = 0$$

where  $w^{(\xi)}$  is given by expression (42) with  $\xi_n$  considered as a continuous variable and  $w^{(\eta)}$  is given by expression (47).

Equation (49), combined with Equations (42) and (47), constitutes our final differential equation for  $\bar{P}(\xi, \eta)$ , the pressure distribution around an idealized grooved bearing in which the number of grooves on the bearing is assumed to approach an infinite number. One should note that for gas lubricated bearings, isothermal conditions usually prevail in the bearing film, and  $\rho(\xi, \eta)$  can be replaced by  $\bar{P}(\xi, \eta)$  in Equations (42), (47) and (49). Since Equation (49) is a second order, partial differential equation, its solution will require two boundary conditions. Usually, one of the boundary conditions is the pressure at one edge, while the second is the mass flux at the other edge.

The application of Equation (49) to a spiral-grooved, cylindrical, journal bearing will be considered in the next section.

SOLUTION FOR PRESSURE DISTRIBUTION AROUND SPIRAL-GROOVED, SELF-ACTING, CYLINDRICAL, JOURNAL BEARING

Consider the grooved cylindrical bearing shown in Fig. 6. The journal for this bearing has grooving on both ends arranged in a herringbone pattern as shown so that the journal is symmetrical about the mid-plane. To gain the benefit of some simplification in this example we are presenting, we will assume an incompressible lubricant.

The relations between  $\xi, \eta$  coordinates for the grooved cylinder and the standard cylindrical coordinates  $r, \theta$  and  $z$  are:

(50)  $\xi = \theta - \frac{z \cot \beta}{R}$

(51)  $\eta = z / \sin \beta$

(52)  $z = \eta \sin \beta$

(53)  $\theta = \xi + \frac{\eta \cos \beta}{R}$

(54)  $r = R = \text{Radius of Cylinder}$

The metric tensors for the  $\xi, \eta$  coordinate system are

(55)  $g^{ij} =$ 

$1/R^2 \sin^2 \beta$	$-\cos \beta / R \sin^2 \beta$
$-\cos \beta / R \sin^2 \beta$	$1/\sin^2 \beta$

(56)  $g_{ij} =$ 

$R^2$	$R \cos \beta$
$R \cos \beta$	$1$

(57)  $\sqrt{g} = R \sin \beta$

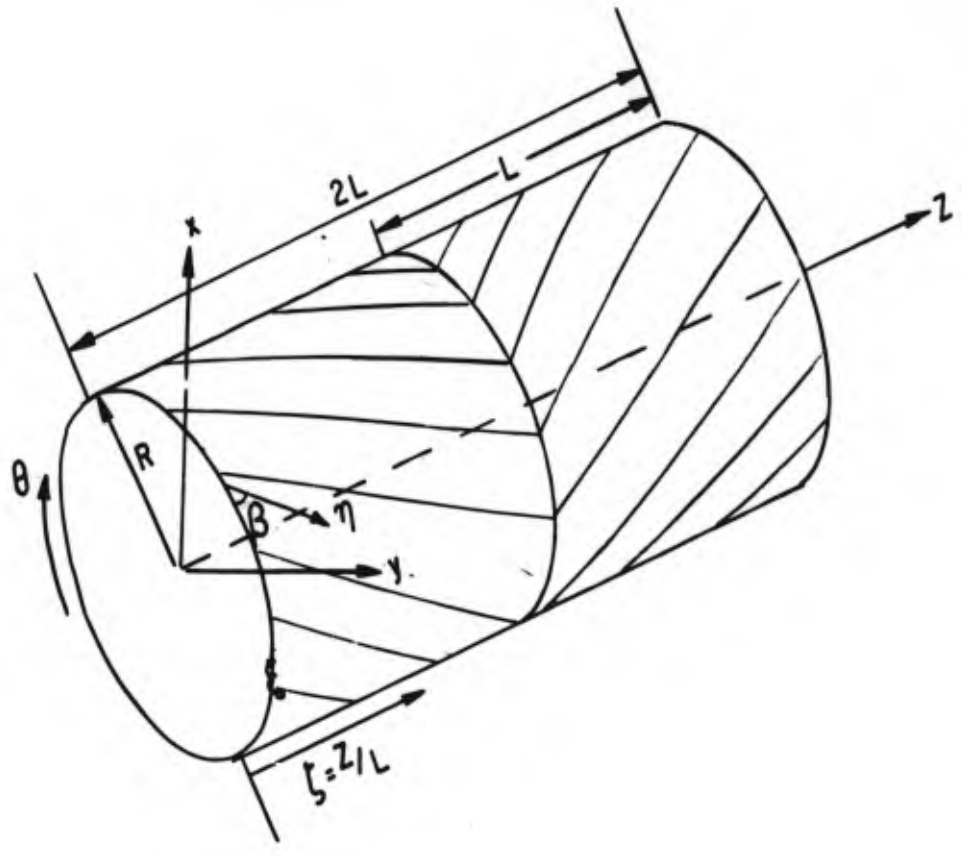


Figure 6. Herringbone Grooved Cylinder

Assuming steady state conditions, Equation (49) for the eccentric bearing becomes

$$(58) \quad \frac{\partial W^{(\xi)}}{\partial \xi} + \frac{\partial W^{(\eta)}}{\partial \eta} + R \sin \beta v^{\xi} \frac{\partial}{\partial \xi} \left[ \rho \alpha h^{(g)} + \rho (1-\alpha) h^{(r)} \right]$$

Now, in the derivation of Equation (58), it proved to be most convenient to use the "skewed"  $\xi, \eta$  coordinate system. To obtain a solution to Equation (58), however, it proves to be convenient to transform Equation (58) to the cylindrical  $r, \theta, z$  coordinate system. This is done by means of the following relationships:

$$(59) \quad \begin{aligned} \frac{\partial W^{(\eta)}}{\partial \eta} &= \frac{\partial W^{(\eta)}}{\partial \theta} \frac{\partial \theta}{\partial \eta} + \frac{\partial W^{(\eta)}}{\partial z} \frac{\partial z}{\partial \eta} \\ &= \frac{\partial W^{(\eta)}}{\partial \theta} \frac{\cos \beta}{R} + \frac{\partial W^{(\eta)}}{\partial z} \sin \beta \end{aligned}$$

$$(60) \quad \begin{aligned} \frac{\partial W^{(\xi)}}{\partial \xi} &= \frac{\partial W^{(\xi)}}{\partial \theta} \frac{\partial \theta}{\partial \xi} + \frac{\partial W^{(\xi)}}{\partial z} \frac{\partial z}{\partial \xi} \\ &= \frac{\partial W^{(\xi)}}{\partial \theta} \end{aligned}$$

$$(61) \quad \frac{\partial h^{(i)}}{\partial \xi} = \frac{\partial h^{(i)}}{\partial \theta}$$

$$(62) \quad v^{\xi} = v^{\theta} \frac{\partial \xi}{\partial \theta} + v^z \frac{\partial \xi}{\partial z} = v^{\theta}$$

Substituting relations (59) through (62) into Equation (58) we get

$$(63) \quad \frac{\partial W^{(\xi)}}{\partial \theta} + \frac{\cos \beta}{R} \frac{\partial W^{(\eta)}}{\partial \theta} + \sin \beta \frac{\partial W^{(\eta)}}{\partial z} + R \sin \beta v^{\theta} \frac{\partial}{\partial \theta} \left[ \rho \alpha h^{(g)} + \rho (1-\alpha) h^{(r)} \right]$$

By the same procedure as above, we can transform the expressions for  $W^{(\xi)}$  and  $W^{(\eta)}$ , i.e., expressions (42) and (47), from the  $\xi, \eta$  coordinates to  $\theta, z$  coordinates. Making these transformations, and substituting for  $g^{\xi\xi}, g^{\xi\eta}$  etc. in expressions (42) and (47) we obtain, after rearranging

$$(64) \quad W^{(g)} = \frac{-\rho}{12\mu} \left[ \frac{(h^{(g)})^3 (h^{(r)})^3}{\alpha(h^{(r)})^3 + (1-\alpha)(h^{(g)})^3} \right] \left\{ \frac{\sin \beta}{R} \frac{\partial \bar{P}}{\partial \theta} \right. \\ \left. - \cos \beta \frac{\partial \bar{P}}{\partial z} - 6\mu R \sin \beta (U^\theta - V^\theta) \left[ \frac{\alpha}{(h^{(g)})^2} + \frac{(1-\alpha)}{(h^{(r)})^2} \right] \right\}$$

$$(65) \quad W^{(r)} = \frac{-\rho}{12\mu} \left[ \frac{1}{\alpha(h^{(r)})^3 + (1-\alpha)(h^{(g)})^3} \right] \left\{ (h^{(g)} h^{(r)})^3 R \frac{\partial \bar{P}}{\partial z} \right. \\ + \alpha (1-\alpha) \sin^2 \beta \left[ (h^{(g)})^3 - (h^{(r)})^3 \right]^2 R \frac{\partial \bar{P}}{\partial z} \\ + \alpha (1-\alpha) \sin \beta \left[ (h^{(g)})^3 - (h^{(r)})^3 \right]^2 \frac{\partial \bar{P}}{\partial \theta} \cos \beta \\ \left. - R^2 \sin \beta \cos \beta \alpha (1-\alpha) \left[ (h^{(g)})^3 - (h^{(r)})^3 \right] \left[ h^{(g)} - h^{(r)} \right] 6\mu (U^\theta - V^\theta) \right\}$$

Now, in equations (63), (64) and (65), the terms  $h^{(g)}$  and  $h^{(r)}$  are functions of  $\theta$ , expressible in the following form

$$(66) \quad h^{(g)} = h_o^{(g)} + e \cos \theta$$

$$(67) \quad h^{(r)} = h_o^{(r)} + e \cos \theta$$

where  $h_o^{(g)}$  and  $h_o^{(r)}$  are the values of  $h^{(g)}$  and  $h^{(r)}$  when the journal is concentric within the bearing and  $e$  is the eccentricity of the bearing.

Making the following definitions

$$(68) \quad C = h_o^{(r)}$$

$$(69) \quad \epsilon = e/C$$

$$(70) \quad A = h_o^{(g)}/h_o^{(r)}$$

we can write

$$(71) \quad h^{(g)} = C (A + \epsilon \cos \theta)$$

$$(72) \quad h^{(r)} = C (1 + \epsilon \cos \theta)$$

The differential equation one obtains by substituting expressions (71) and (72) into Equations (63), (64) and (65) is very difficult to solve in the general case due to the fact that the coefficients of the pressure derivatives and the forcing function in the equation are such complicated functions of  $\theta$ . One can, however, obtain a solution for  $\bar{P}(\theta, z)$  which is valid for vanishingly small values of the eccentricity ratio  $\epsilon$  by means of a perturbation technique, the formulation of which is as follows. We assume that the pressure  $\bar{P}(\theta, z)$  can be expanded in a series in  $\epsilon$  in the following form

$$(73) \quad \bar{P}(\theta, z) = \bar{P}_0(z) + \epsilon \bar{P}_1(\theta, z) + \dots + O(\epsilon^2)$$

We next combine (63), (64), (65), (71), (72) and (73) and collect all the terms of order zero and all the terms of order  $\epsilon$  to obtain two differential equations, one for  $\bar{P}_0(z)$  and one for  $\bar{P}_1(\theta, z)$ . The algebraic manipulations involved in obtaining these two equations are straightforward and are omitted to save space. The two differential equations that are obtained for  $\bar{P}_0(z)$  and  $\bar{P}_1(\theta, z)$  are:

$$(74) \quad \frac{d^2 \bar{P}_0}{dz^2} = 0$$

$$(75) \quad R^2 \frac{\partial^2 \bar{P}_1}{\partial z^2} + 2K_1 R \frac{\partial^2 \bar{P}_1}{\partial \theta \partial z} + K_2 \frac{\partial^2 \bar{P}_1}{\partial \theta^2} = -6\mu\omega \left(\frac{R}{C}\right)^2 K_6 \sin \theta$$

where

$$(76) \quad \omega = |U^\theta| + |V^\theta|$$

$$(77) \quad K_1 = \frac{\alpha(1-\alpha) \sin \beta \cos \beta (A^3 - 1)^2}{A^3 + \alpha(1-\alpha) \sin^2 \beta (A^3 - 1)^2}$$

$$(78) \quad K_2 = \frac{A^3 + \alpha(1-\alpha) \cos^2 \beta (A^3 - 1)^2}{A^3 + \alpha(1-\alpha) \sin^2 \beta (A^3 - 1)^2}$$

$$(79) \quad K_3 = 3R \frac{d\bar{P}_o}{dz} \left[ \frac{\alpha(1-\alpha) \sin\beta \cos\beta (A^3-1)^2}{A^3 + \alpha(1-\alpha) \sin^2\beta (A^3-1)^2} \right] \left[ \frac{\alpha + (1-\alpha)A^2}{\alpha + (1-\alpha)A^3} - 2 \frac{(A^2-1)}{(A^3-1)} \right]$$

$$(80) \quad K_4 = \frac{3(\gamma_2 - \gamma_1) \left[ \frac{\alpha(1-\alpha) \sin^2\beta (A-1)^2}{\alpha + 1(1-\alpha) A^3} \right]}{A^3 + \alpha(1-\alpha) \sin^2\beta (A^3-1)^2}$$

$$(81) \quad K_5 = \frac{(\gamma_2 + \gamma_1) \left[ \alpha + (1-\alpha) A^3 \right]}{A^3 + \alpha(1-\alpha) \sin^2\beta (A^3-1)^2}$$

$$(82) \quad K_6 = K_3 + K_4 + K_5$$

$$(83) \quad \gamma_1 = \frac{V^\theta}{\omega}$$

$$(84) \quad \gamma_2 = \frac{U^\theta}{\omega}$$

Equations (74) and (75) can be put into non-dimensional form by defining

$$(85) \quad P' = \frac{\bar{P} (C/R)^2}{6\mu\omega}$$

$$(86) \quad \zeta = z/L$$

where  $2L$  is the length of the bearing. In non-dimensional form, equations (74) and (75) are

$$(87) \quad \frac{d^2 P'_o}{d\zeta^2} = 0$$

$$(88) \quad (R/L)^2 \frac{\partial^2 P'_1}{\partial \zeta^2} + 2K_1 \left(\frac{R}{L}\right) \frac{\partial^2 P'_1}{\partial \zeta \partial \theta} + K_2 \frac{\partial^2 P'_1}{\partial \theta^2} = -K_6 \sin \theta$$

The solution to Equation (87) is obtained by integrating twice with respect to  $\zeta$

$$(89) \quad P'_o = C_1 \zeta + C_2$$

To determine the constants  $C_1$  and  $C_2$  we impose two conditions on  $P'_o$ . One is that the pressure at  $\zeta = 0$  be 0, which yields

$$(90) \quad C_2 = 0$$

The second condition is that the axial mass flow at the midplane of the bearing ( $\zeta=1$ ) be zero. This last condition results from the symmetry of the bearing at the midplane. To obtain the expression for the axial mass flux, we substitute (71), (72), (73), (85) and (86) into expression (65) and keep only terms of order  $\epsilon$  or lower to get

$$(91) \quad W^{(\eta)} = W_o + \epsilon W_1$$

where

$$(92) \quad W_o = -\frac{\rho}{2} \omega \left(\frac{R}{C}\right)^2 \left[ \frac{C^3}{\alpha + (1-\alpha)A^3} \right] \left\{ \left[ A^3 + \alpha(1-\alpha) \sin^2 \beta (A^3-1)^2 \right] \left(\frac{R}{L}\right) \frac{dP'_o}{d\zeta} - \sin \beta \cos \beta \alpha(1-\alpha) (A^3-1)(A-1)(\gamma_2-\gamma_1) \right\}$$

$$(93) \quad W_1 = -\frac{\rho}{2} \omega \left(\frac{R}{C}\right)^2 C^3 \left\{ \left[ (K_7 + K_8) \left(\frac{R}{L}\right) \frac{dP'_o}{d\zeta} + (K_9 + K_{10}) \right] 3 \cos \theta + K_{11} \left(\frac{R}{L}\right) \frac{\partial P'_1}{\partial \zeta} + K_{12} \frac{\partial P'_1}{\partial \theta} \right\}$$

and where

$$(94) \quad K_7 = -\left[ \frac{\alpha + (1-\alpha) A^2}{\alpha + (1-\alpha) A^3} \right] \left[ \frac{A^3 + \alpha(1-\alpha) \sin^2 \beta (A^3-1)^2}{\alpha + (1-\alpha) A^3} \right]$$

$$(95) \quad K_8 = \frac{A^2 (A+1) + 2\alpha(1-\alpha) \sin^2 \beta (A^3-1) (A^2-1)}{\alpha + (1-\alpha) A^3}$$

$$(96) \quad K_9 = \left[ \frac{\alpha + (1-\alpha) A^2}{\alpha + (1-\alpha) A^3} \right] \left[ \frac{\sin \beta \cos \beta \alpha (1-\alpha) (A^3-1) (A-1) (\gamma_2-\gamma_1)}{\alpha + (1-\alpha) A^3} \right]$$

$$(97) \quad K_{10} = - \frac{\sin \beta \cos \beta \alpha (1-\alpha) (A^2-1) (A-1) (\gamma_2-\gamma_1)}{\alpha + (1-\alpha) A^3}$$

$$(98) \quad K_{11} = \frac{A^3 + \alpha(1-\alpha) \sin^2 \beta (A^3-1)^2}{\alpha + (1-\alpha) A^3}$$

$$(99) \quad K_{12} = \frac{\alpha(1-\alpha) \sin \beta \cos \beta (A^3-1)^2}{\alpha + (1-\alpha) A^3}$$

For  $W^\eta$  to be zero at  $\xi=1$ , both  $W_0$  and  $W_1$  must be zero. Substituting (89) into (92), setting  $W_0$  equal to zero, and solving for  $C_1$  we obtain

$$(100) \quad C_1 = \left( \frac{L}{R} \right) (\gamma_2-\gamma_1) \left[ \frac{\alpha(1-\alpha) \sin \beta \cos \beta (A^3-1) (A-1)}{A^3 + \alpha (1-\alpha) (A^3-1)^2 \sin^2 \beta} \right]$$

One can note that the solution for  $P'_0(\xi)$  given by (89) and (92) is identically the solution obtained by Whipple for a flat grooved plate with incompressible flow and arbitrary Mass Flow  $W_0$  (Ref.1).

Next we consider the solution of equation (88). Noting the form of the non-homogeneous term and considering the nature of the boundary conditions that  $P'_1$  must satisfy, we try a separation of variables solution of the form

$$(101) \quad P'_1(\xi, \theta) = \text{Re} \left\{ f(\xi) e^{i\theta} \right\}$$

Substituting (101) into (88) and dividing through by  $e^{i\theta}$ , we obtain the following ordinary differential equation for  $f(\xi)$

$$(102) \quad \left( \frac{R}{L} \right)^2 f'' + 2i (R/L) K_1 f' - K_2 f = i K_6$$

The solution to (102) is

$$(103) \quad f = \left[ C_3 \cosh \left( \frac{L}{R} K_{13} \xi \right) + C_4 \sinh \left( \frac{L}{R} K_{13} \xi \right) \right] \left[ e^{-iK_1 \left( \frac{L}{R} \right) \xi} \right]^{-i K_6/K_2}$$

where

$$(104) \quad K_{13} = \sqrt{-K_1^2 + K_2} \quad ; \quad -K_1^2 + K_2 > 0 \text{ always}$$

and  $C_3$  and  $C_4$  are arbitrary, complex constants.  $C_3$  and  $C_4$  are evaluated from the following boundary and mass flux conditions

$$(105) \quad \zeta = 0 ; P'_1 = 0$$

$$(106) \quad \zeta = 1 ; W_1 = 0$$

where  $W_1$  would be evaluated by substituting (89) and (101) into (93).

If we write

$$(107) \quad C_3 = \alpha_3 + i \beta_3$$

$$(108) \quad C_4 = \alpha_4 + i \beta_4$$

where  $\alpha_3, \beta_3, \alpha_4$  and  $\beta_4$  are real, we obtain from condition (105)

$$(109) \quad \alpha_3 \cos \theta - \beta_3 \sin \theta + K_6/K_2 \sin \theta = 0$$

which gives

$$(110) \quad \alpha_3 = 0$$

$$(111) \quad \beta_3 = K_6/K_2$$

Now, to determine  $\alpha_4$  and  $\beta_4$  there remains only to apply condition (106) which yields two independent, linear non-homogeneous equations in  $\alpha_4$  and  $\beta_4$ . The solutions obtained for  $\alpha_4$  and  $\beta_4$  are complicated algebraic expressions, which are presented in the appendix. Keeping  $\alpha_4, \beta_4$  and  $C_1$  in symbol form, the final solution for the pressure distribution around the grooved cylinder is

$$(112) \quad P'(\zeta\theta) = C_1 \zeta + \epsilon \left\{ \left[ \begin{aligned} & K_6/K_2 \cosh\left(\frac{L}{R} K_{13} \zeta\right) \sin\left(\frac{L}{R} K_1 \zeta\right) \\ & + \alpha_4 \sinh\left(\frac{L}{R} K_{13} \zeta\right) \cos\left(\frac{L}{R} K_1 \zeta\right) + \beta_4 \sinh\left(\frac{L}{R} K_{13} \zeta\right) \sin\left(\frac{L}{R} K_1 \zeta\right) \end{aligned} \right] \cos \theta \right. \\ & + \left. \left[ \begin{aligned} & -K_6/K_2 \cosh\left(\frac{L}{R} K_{13} \zeta\right) \cos\left(\frac{L}{R} K_1 \zeta\right) + \alpha_4 \sinh\left(\frac{L}{R} K_{13} \zeta\right) \sin\left(\frac{L}{R} K_1 \zeta\right) \\ & - \beta_4 \sinh\left(\frac{L}{R} K_{13} \zeta\right) \cos\left(\frac{L}{R} K_1 \zeta\right) + K_6/K_2 \end{aligned} \right] \sin \theta \right\}$$

Examining the above expression, we find that the pressure distribution around the grooved cylindrical bearing consists of a mean pressure level which increases linearly with  $\zeta$  and a harmonic, circumferential variation in pressure which is given by the  $\cos\theta$  and  $\sin\theta$  terms. The magnitude of this harmonic circumferential pressure variation increases monotonically with  $\zeta$ .

The attitude angle  $\phi$  for the grooved bearing is shown in Figure 7.

$\phi$  is obtained from the relation

$$(113) \quad \phi = \tan^{-1} \left\{ \frac{\int_0^1 \int_0^{2\pi} P'(\zeta, \theta) \sin\theta \, d\theta d\zeta}{-\int_0^1 \int_0^{2\pi} P'(\zeta, \theta) \cos\theta \, d\theta d\zeta} \right\}$$

One can also consider a local attitude angle,  $\phi(\zeta)$ , given by

$$(114) \quad \phi(\zeta) = \tan^{-1} \left\{ \frac{\int_0^{2\pi} P'(\zeta, \theta) \sin\theta \, d\theta}{-\int_0^{2\pi} P'(\zeta, \theta) \cos\theta \, d\theta} \right\}$$

The Sommerfeld number  $S$  for the total bearing is defined as

$$(115) \quad S = \frac{\mu NLD}{W} \left(\frac{R}{C}\right)^2$$

$S$  is given by

$$(116) \quad S = \frac{1}{-6\pi \int_0^1 \int_0^{2\pi} P'(\zeta, \theta) \cos(\theta + \phi) \, d\theta d\zeta}$$

A local Sommerfeld number,  $s(\zeta)$ , can be obtained from

$$(117) \quad s(\zeta) = \frac{1}{-6\pi \int_0^{2\pi} P'(\zeta, \theta) \cos[\theta + \phi(\zeta)] \, d\theta}$$

The radial stiffness for the bearing, that is, the component of force directly opposing a displacement of the journal in a radial direction will be given in dimensionless form by  $\frac{\cos\phi}{\epsilon S}$ . The local radial stiffness at each station  $\zeta$  will be given by  $\frac{\cos\phi(\zeta)}{\epsilon s(\zeta)}$ .

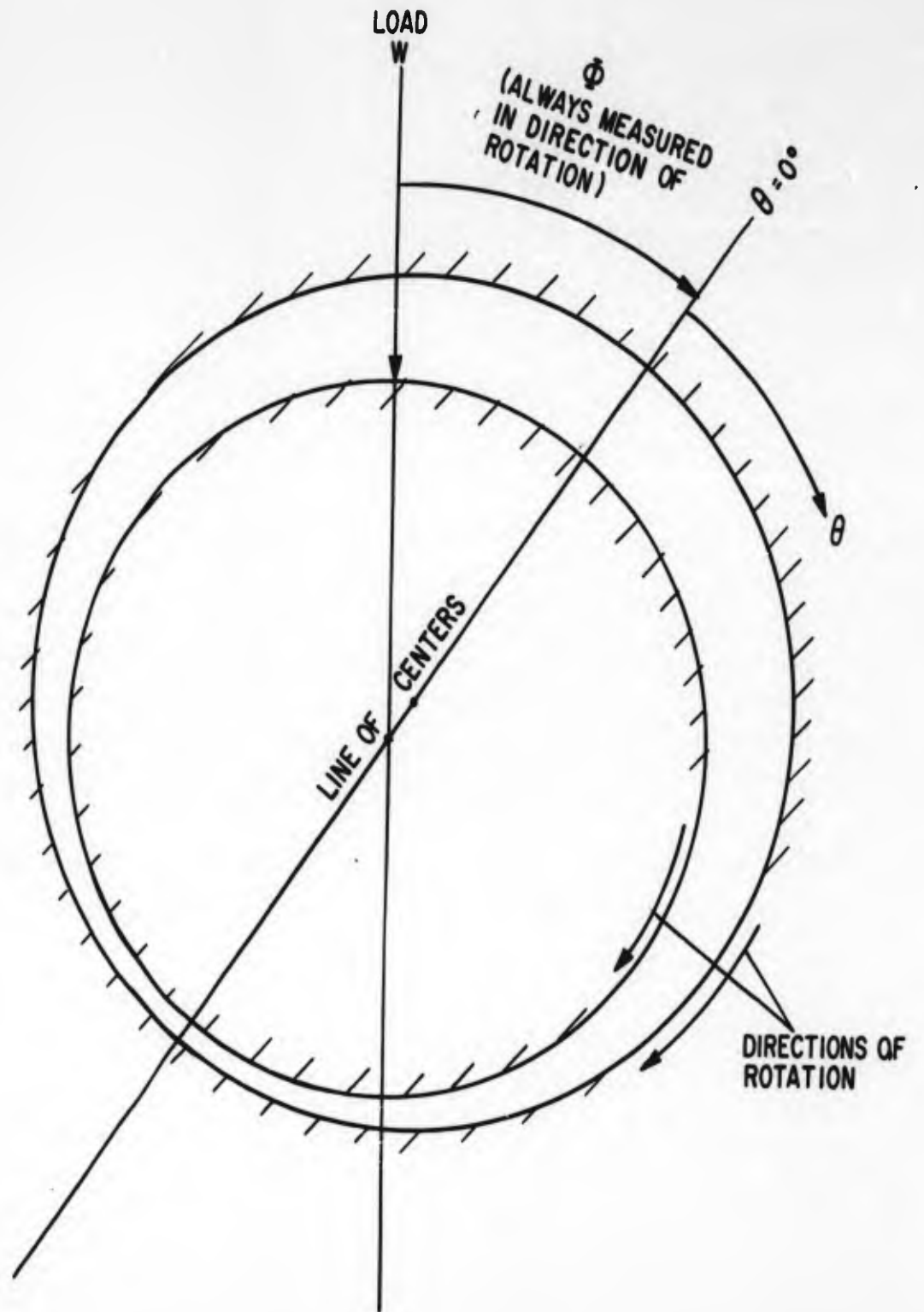


Figure 7. Attitude Angle  $\phi$  for Journal Bearing

In Figures 8,9 and 10 are plotted  $\phi(\zeta)$ ,  $\epsilon s(\zeta)$  and  $\frac{\cos \phi}{\epsilon s}$  for the herringbone bearing with the following values for the groove parameters\*

$$(118) \quad \begin{aligned} \alpha &= 0.5 \\ \beta &= 15^\circ \\ A &= 2.93 \end{aligned}$$

Separate curves are shown for the case where the grooved member is rotating ( $\gamma_1 = -1, \gamma_2 = 0$ ) and the case where the grooved member is stationary ( $\gamma_1 = 0, \gamma_2 = 1$ ). In both cases the rotation is such that the grooves tend to pump fluid into the bearing. For comparison purposes,  $10\epsilon s(\zeta)$  for a smooth bearing of the same geometry and minimum clearance as the grooved bearing is also plotted in Figure 9.

For a smooth bearing with incompressible lubricant, an  $\epsilon$  perturbation analysis yields the well known result that the attitude angle is  $90^\circ$ , i.e. a smooth bearing has zero radial stiffness at vanishingly small  $\epsilon$ . A significant feature of the grooved bearing as shown by Figures 8 and 10 is that the attitude angle is significantly less than  $90^\circ$  and that, consequently, the grooved bearing has substantial radial stiffness at small  $\epsilon$ . This important result indicates that spiral grooves could be used to enhance bearing stability.

An interesting feature of Figures 8 and 10 is the considerable difference in the variation of  $\phi(\zeta)$  and  $\frac{\cos \phi(\zeta)}{\epsilon s(\zeta)}$  depending on whether the grooves are on the rotating member or on the stationary member. The total attitude angle and radial stiffness for the bearing, however, does not depend significantly on whether the grooves are rotating or stationary. This is shown in Table 1, where values of  $\phi$ ,  $\epsilon s$ , and  $\frac{\cos \phi}{\epsilon s}$  are given for the two cases under consideration.

---

\* These values for  $\alpha$ ,  $\beta$  and  $A$  are the ones determined by Whitley and Williams (Ref.2) as providing the maximum axial stiffness for a herringbone-grooved, circular, thrust plate.

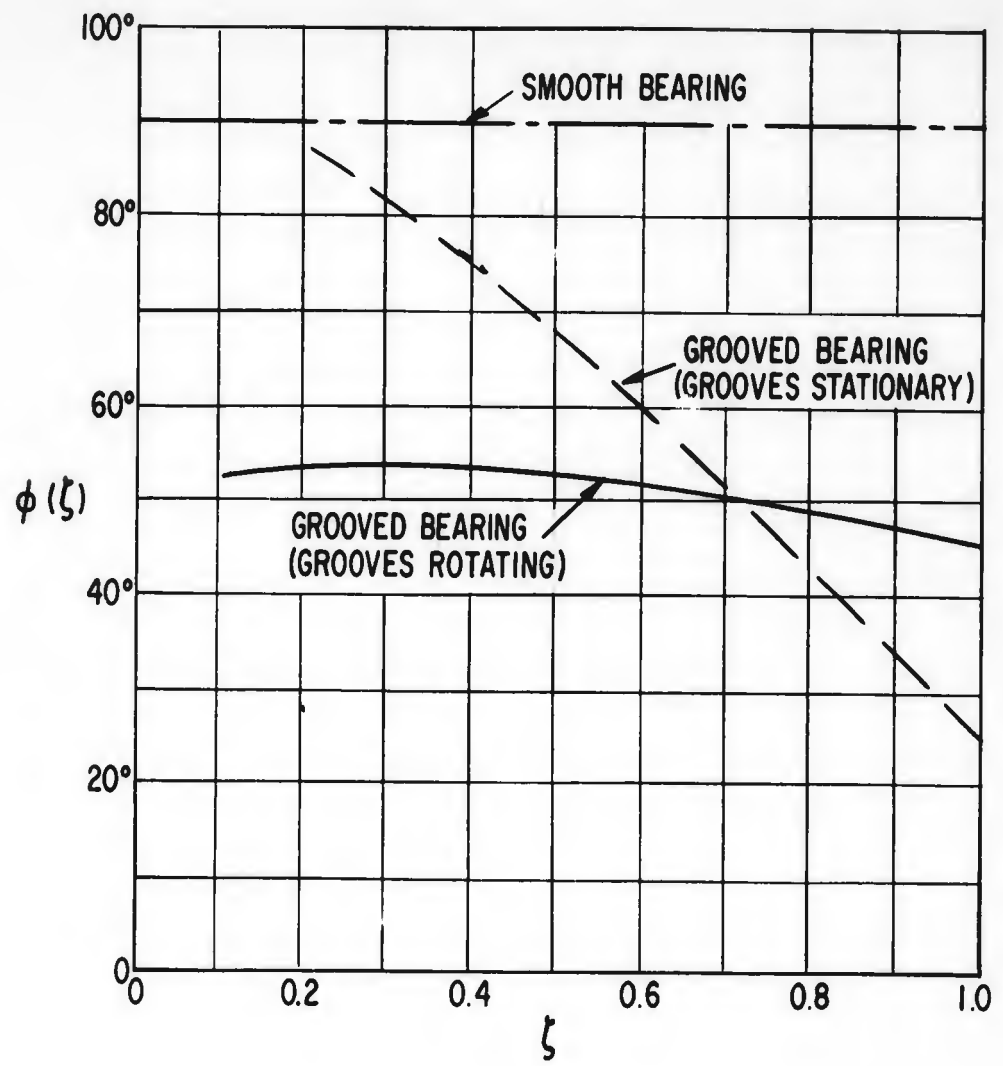


Fig. 8 Local Attitude Angle

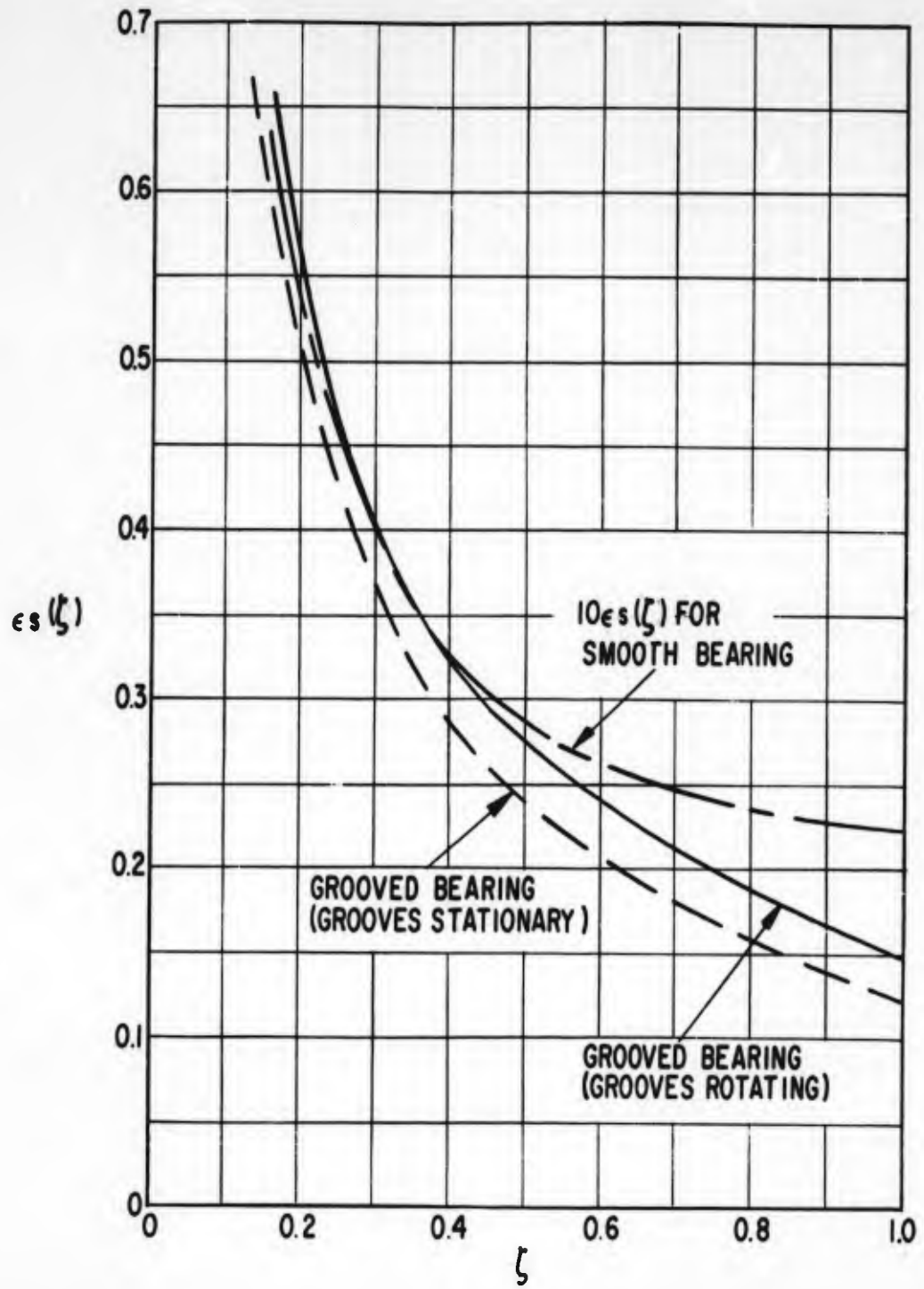


Figure 9. Local Sommerfeld Number

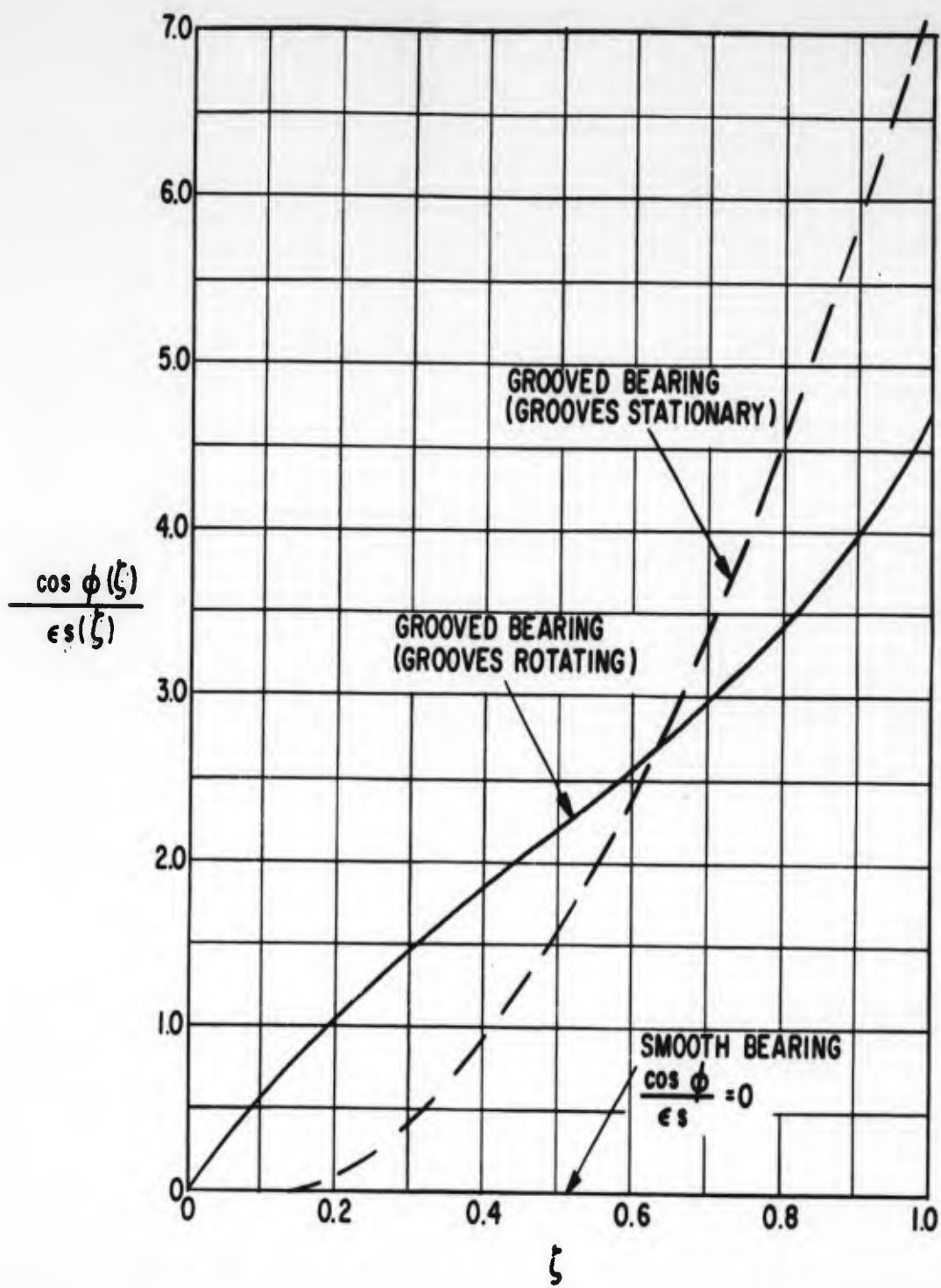


Figure 10. Local Radial Stiffness

TABLE 1

	<u>Grooves Rotating</u>	<u>Grooves Stationary</u>
$\phi$	50.5°	54.5°
$\epsilon S$	.282	.257
$\frac{\cos \phi}{\epsilon S}$	2.26	2.26

The total attitude angle, Sommerfeld number, and Radial Stiffness depend on each of the parameters,  $\alpha$ ,  $\beta$ ,  $A$ ,  $\gamma_1$  and  $\gamma_2$ . Figures 11, 12 and 13 show how  $\phi$ ,  $\epsilon S$  and  $\frac{\cos \phi}{\epsilon S}$  vary with just one of these parameters, namely, the groove angle  $\beta$ . For these figures,  $\alpha$  and  $A$  are fixed at 0.5 and 2.93 respectively and the grooves are stationary i.e.  $\gamma_1 = 0$ ,  $\gamma_2 = 1$ . As can be seen, the attitude angle attains a minimum of just over 46° at  $\beta = 37.5^\circ$ ,  $\epsilon S$  attains a minimum of 0.24 at  $\beta = 23.0^\circ$  and  $\frac{\cos \phi}{\epsilon S}$  attains a sharp maximum at  $\beta = 26.0^\circ$ . One can note that this last value of  $\beta$  for maximum radial stiffness is somewhat larger than the value  $\beta = 15^\circ$  obtained by Whitley and Williams for maximum axial stiffness in a herringbone thrust plate.

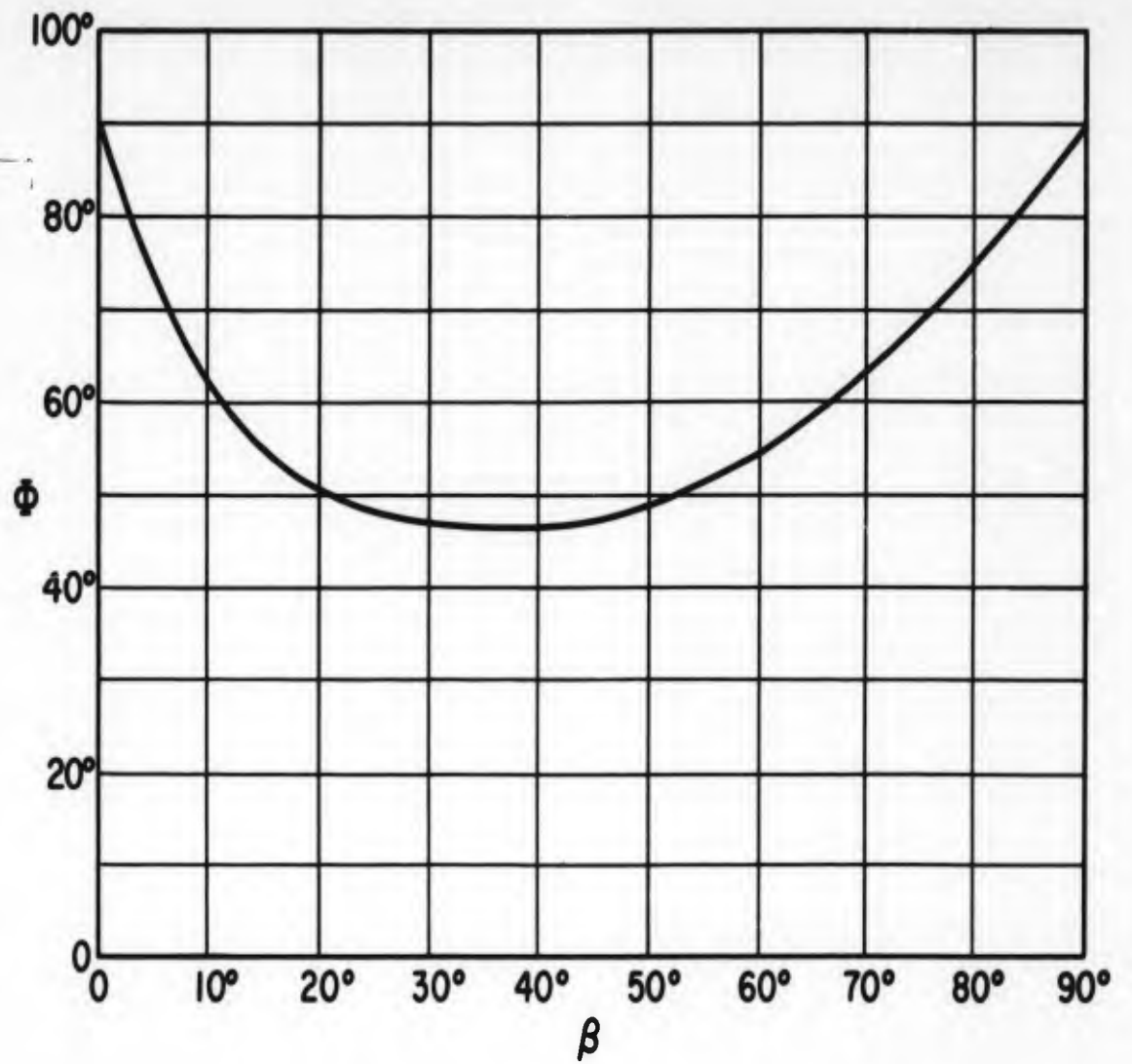


Fig. 11 Attitude Angle vs. Groove Angle

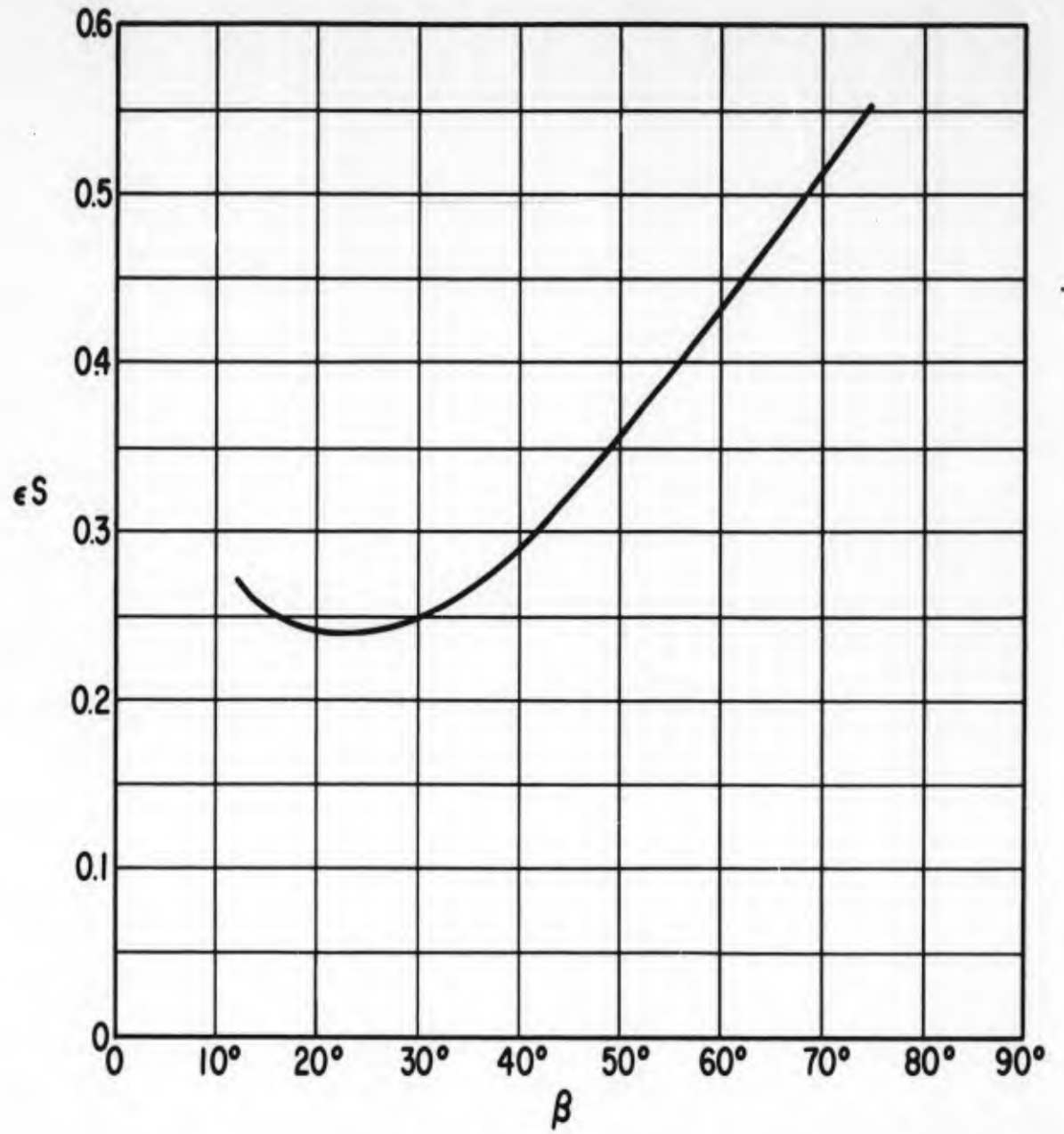


Fig. 12 Sommerfeld Number vs. Groove Angle

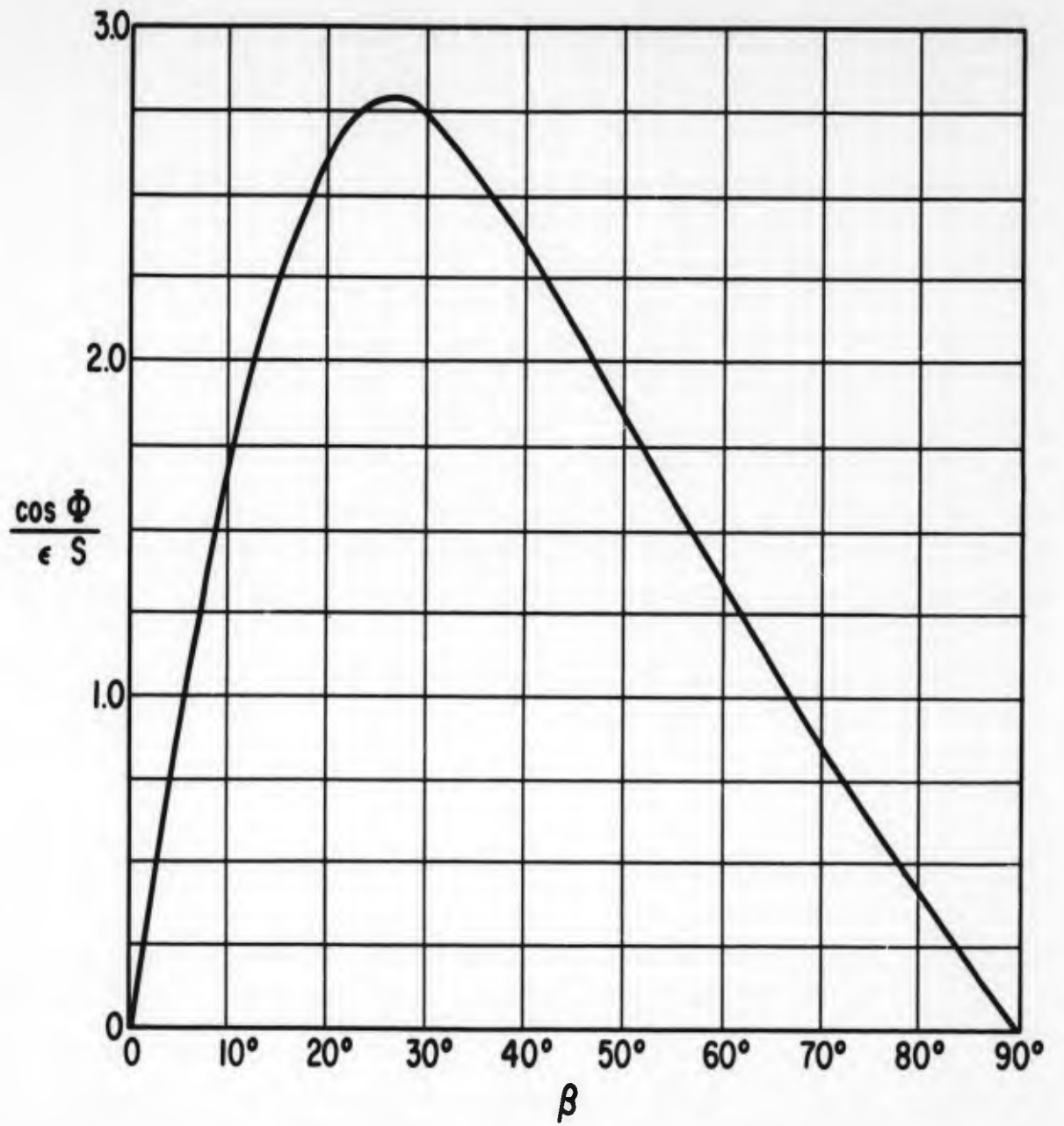


Fig. 13 Radial Stiffness vs. Groove Angle

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APPENDIX

The Solutions for the Coefficients  $\alpha_4$  and  $\beta_4$  referred to on page 29 are given below in determinant form

$$K_{11} \frac{K_6}{K_2} \left[ \begin{array}{c} K_{13} \text{ (SH)(CS)} - K_1 \text{ (CH)(SN)} \\ + K_{12} \frac{K_6}{K_2} \text{ (CS)(SN)} \end{array} \right] \quad \left| \begin{array}{c} K_{11} \left[ -K_{13} \text{ (CH)(CS)} + K_1 \text{ (SH)(SN)} \right] \\ - K_{12} \text{ (SH)(SN)} \end{array} \right|$$

$$K_{11} \frac{K_6}{K_2} \left[ \begin{array}{c} -K_{13} \text{ (SH)(SN)} - K_1 \text{ (CH)(CS)} \\ (CH)(CS) - 1 \end{array} \right] \left| \begin{array}{c} K_{11} \left[ K_{13} \text{ (CH)(SN)} + K_1 \text{ (SH)(CS)} \right] \\ - K_{12} \text{ (SH)(CS)} \end{array} \right|$$

$$+ K_{12} \frac{K_6}{K_2} \left[ \begin{array}{c} -3 \frac{R}{L} C_1 \text{ (K}_7\text{+K}_8\text{)} - 3 \text{ (K}_9\text{+K}_{10}\text{)} \end{array} \right]$$

$\alpha_4 =$

$$K_{11} \left[ \begin{array}{c} K_{13} \text{ (CH)(SN)} + K_1 \text{ (SH)(CS)} \\ - K_{12} \text{ (SH)(CS)} \end{array} \right] \quad \left| \begin{array}{c} K_{11} \left[ -K_{13} \text{ (CH)(CS)} + K_1 \text{ (SH)(SN)} \right] \\ - K_{12} \text{ (SH)(SN)} \end{array} \right|$$

$$K_{11} \left[ \begin{array}{c} K_{13} \text{ (CH)(CS)} - K_1 \text{ (CS)(SN)} \\ + K_{12} \text{ (SH)(SN)} \end{array} \right] \quad \left| \begin{array}{c} K_{11} \left[ K_{13} \text{ (CH)(SN)} + K_1 \text{ (SH)(CS)} \right] \\ - K_{12} \text{ (SH)(CS)} \end{array} \right|$$

$$\begin{array}{l}
K_{11} \left[ K_{13} \text{ (CH)(SN)} + K_1 \text{ (SH)(CS)} \right] \\
\quad - K_{12} \text{ (SH)(CS)} \\
\hline
K_{11} \left[ K_{13} \text{ (CH)(CS)} - K_1 \text{ (SH)(SN)} \right] \\
\quad + K_{12} \text{ (SH)(SN)} \\
\hline
\frac{K_6}{K_{11} K_2} \left[ K_{13} \text{ (SH)(CS)} - K_1 \text{ (CH)(SN)} \right] \\
\quad + K_{12} \frac{K_6}{K_2} \text{ (CH)(SN)} \\
\hline
\frac{K_6}{K_{11} K_2} \left[ -K_{13} \text{ (SH)(SN)} - K_1 \text{ (CH)(CS)} \right] \\
\quad + K_{12} \frac{K_6}{K_2} \left[ \text{(CH)(CS)} - 1 \right] - 3 \frac{R}{L} C_1 (K_7 + K_8) - 3(K_9 + K_{10})
\end{array}$$

$\beta_4 =$

$$\begin{array}{l}
K_{11} \left[ K_{13} \text{ (CH)(SN)} + K_1 \text{ (SH)(CS)} \right] \\
\quad - K_{12} \text{ (SH)(CS)} \\
\hline
K_{11} \left[ -K_{13} \text{ (CH)(CS)} + K_1 \text{ (SH)(SN)} \right] \\
\quad - K_{12} \text{ (SH)(SN)} \\
\hline
K_{11} \left[ K_{13} \text{ (CH)(CS)} - K_1 \text{ (SH)(SN)} \right] \\
\quad + K_{12} \text{ (SH)(SN)} \\
\hline
K_{11} \left[ K_{13} \text{ (CH)(SN)} + K_1 \text{ (SH)(CS)} \right] \\
\quad - K_{12} \text{ (SH)(CS)}
\end{array}$$

where  $\text{CH} = \cosh \left( \frac{L}{R} \right) K_{13}$  ,  $\text{SH} = \sinh \left( \frac{L}{R} \right) K_{13}$   
 $\text{CS} = \cos \left( \frac{L}{R} \right) K_1$  ,  $\text{SN} = \sin \left( \frac{L}{R} \right) K_1$

NOMENCLATURE

- A - Groove clearance ratio,  $A = h_o^{(g)} / h_o^{(r)}$
- C - Mean bearing clearance in ridge region - ft.
- e - Eccentricity of bearing - ft.
- g - Determinant of covariant metric tensor
- $g_{ij}$  - Covariant metric tensor for curvilinear coordinate system
- $g^{ij}$  - Contravariant metric tensor for curvilinear coordinate system
- $\vec{g}_\xi, \vec{g}_\eta, \vec{g}_h, \vec{g}^\xi, \vec{g}^\eta, \vec{g}^h$  - Covariant and contravariant base vectors for  $\xi, \eta, h$  coordinate system
- $h^{(g)}$  - Local bearing clearance in groove region - ft.
- $h^{(r)}$  - Local bearing clearance in ridge region - ft.
- $h_o^{(g)}$  - Mean bearing groove clearance - ft.
- $h_o^{(r)}$  - Mean bearing ridge clearance - ft.
- K - Scale for bearing surface curvature - ft.<sup>-1</sup>
- L - Bearing semi-length. Also, scale for bearing size. - ft.
- $l^{(g)}$  - Length of groove in circumferential direction - ft.
- $l^{(r)}$  - Length of ridge in circumferential direction - ft.
- P - Pressure - lb/ft<sup>2</sup>
- $\bar{P}(\xi, \eta)$  - "overall" pressure distribution - lb/ft<sup>2</sup>
- P' - Dimensionless pressure =  $\frac{\bar{P}(C/R)^2}{6\mu\omega}$
- R - Radius - ft.
- r,  $\theta$ , z - Cylindrical coordinates
- S - Total Sommerfeld Number =  $\frac{2\mu\omega LR}{\pi W} \left(\frac{R}{C}\right)^2$
- s - Local Sommerfeld Number
- t - Time - sec.

- $\bar{U}$  - Surface velocity of smooth bearing - ft/sec.  
 $\bar{u}$  - Lubricant velocity - ft/sec.  
 $V$  - Scale for bearing surface speed - ft/sec.  
 $\bar{V}$  - Surface velocity of grooved journal - ft/sec.  
 $W$  - Bearing load - lbs.  
 $W^{(\eta)}$  - Mass flow normal to constant  $\eta$  line - lb/sec-radian  
 $W^{(\xi)}$  - Mass flow normal to constant  $\xi$  line - lb/sec.ft.  
 $\alpha$  - Fraction of bearing circumference occupied by groove regions  
 $\beta$  - Angle between groove and circumferential direction - radians  
 $\gamma_1$  - Dimensionless rotational speed of bearing grooved member =  $V^\theta/\omega$   
 $\gamma_2$  - Dimensionless rotational speed of bearing smooth member =  $U^\theta/\omega$   
 $\Delta M$  - Differential mass - lb.  
 $\Delta S^\xi, \Delta S^\eta$  - Differential surfaces - ft<sup>2</sup>  
 $\Delta V$  - Differential volume - ft<sup>3</sup>  
 $\epsilon$  - Eccentricity ratio,  $\epsilon = e/C$   
 $\zeta$  - Dimensionless axial coordinate =  $Z/L$   
 $\mu$  - Viscosity - lb/ft-sec.  
 $\xi, \eta, h$  - Curvilinear, grooved-bearing, coordinates  
 $\rho$  - Density - lb/ft<sup>3</sup>  
 $\Phi$  - Total attitude angle - radians  
 $\phi$  - Local attitude angle - radians  
 $\omega$  - Sum of rotational speeds of bearing and journal - radians/sec.

Superscripts

- (g) - Groove region  
 (r) - Ridge region

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Chief Project Engr. 1

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Ampex Corporation  
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Redwood City, Calif. 2

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Ford Motor Company  
Engineering & Research Staff  
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Dearborn, Mich. 1

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Dept., Scientific Lab.  
Ford Motor Co.  
P. O. Box 2053  
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Dr. Calus G. Goetzl, D 53-30  
Bldg. 201, Plant 2, Palo Alto  
Lockheed Missiles & Space Co.  
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Sunnyvale, Calif. 1

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Woodland Hills, Calif. 2

Mr. Don Moors  
Litton Systems  
5500 Canoga Ave.  
Woodland Hills, Calif. 1

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Mortronics  
A Div. of Mortherp Corp.  
500 Bate Angethorpe Ave.  
Anaheim, Calif. 1

Mortronics  
Div. of Mortherp Corp.  
100 Horse St.  
Hirwood, Mass.  
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Precision Products Dept. 1

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Bearing Development & Contract  
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Lockheed Aircraft Corporation  
Missiles & Space Division  
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Bearings & Lubrication Group  
Battelle Memorial Institute  
505 King Ave.  
Columbus 1, Ohio 1

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General Dynamics-Astronautics  
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San Diego 12, Calif.  
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Mr. James Nicol  
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Cryonetics Corporation  
Northwest Industrial Park--  
Burlington, Mass. 1

Mr. Erwin B. Delson  
Manager - Applications  
General Electric Company  
P. O. Box 15132  
Cincinnati 15, Ohio 1

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Rocket Power, Inc. Research Labs.  
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Pasadena, Calif. 1

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Mechanical Technology Inc.  
968 Albany-Shaker Road  
Latham, N. Y. 3

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