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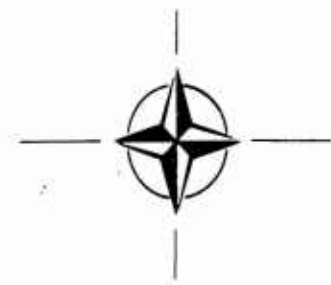
REPORT 444

**INVESTIGATIONS ON THE DYNAMIC
STABILITY OF PERSONNEL GUIDE
SURFACE PARACHUTES**

by

R. LUDWIG and W. HEINS

APRIL 1963



NORTH ATLANTIC TREATY ORGANIZATION

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REPORT 444

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ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT

INVESTIGATIONS ON THE DYNAMIC STABILITY OF
PERSONNEL GUIDE SURFACE PARACHUTES

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R. Ludwig and W. Heins

This Report was presented at the Twenty-Second Meeting of the Flight Mechanics Panel,
held in Torino, Italy, 16-19 April 1963

SUMMARY

In order to investigate the dynamic stability of a parachute the system of the non-linear equations of motion is derived. In contrast to previous contributions this system of differential equations is not linearized, but is integrated by means of electronic computers for a number of examples of personnel guide surface parachutes. The influence of the suspension line length, the effective porosity of the canopy, the weight of the load and the apparent mass on the dynamic behaviour are studied for the examples chosen. The results show that the oscillations of parachutes cannot be described by linearized equations of motion. For instance, only the non-linear procedure enables the complete description of the velocity oscillations. Furthermore, it becomes evident that frequency and damping depend on amplitude. For some amplitudes the square of the oscillation period is proportional to the length of the suspension lines. With increasing porosity of the canopy the damping increases and the oscillation period decreases.

SOMMAIRE

Afin d'enquêter sur la stabilité dynamique d'un parachute, on a établi le système des équations non-linéaires du mouvement. Contrairement à des contributions précédentes, ce système d'équations différentielles n'est pas linéarisé, mais il est intégré au moyen de calculatrices électroniques pour plusieurs exemples de parachutes de surface guides de personnel. L'influence de la longueur de la ligne de suspension, la porosité effective de la calotte, le poids de la charge et la masse apparente sur le comportement dynamique sont étudiés comme exemples choisis. Les résultats montrent que les oscillations des parachutes ne peuvent être décrites par des équations de mouvement linéarisées. Par exemple, seule la procédure non-linéaire permet la description complète des oscillations de vitesse. De plus, il devient évident que la fréquence et l'amortissement dépendent de l'amplitude. Pour certaines amplitudes, le carré de la période d'oscillation est proportionnel à la longueur des lignes de suspension. Avec une porosité accrue de la calotte, l'amortissement augmente et la période d'oscillation décroît.

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NOTATION

\vec{V}, v, v_x, v_y	velocity vector, resultant velocity, velocity components (m/sec)
v_0	rate of descent (m/sec)
$\vec{\omega}, \omega$	angular velocity (sec^{-1})
W_l	suspended load ($= m_l g$) (kp)
m_l	mass of the load ($\text{kp} \cdot \text{sec}^2/\text{m}$)
m_c	apparent air mass ($\text{kp} \cdot \text{sec}^2/\text{m}$)
μ_c	ratio of apparent mass to mass of the load ($= m_c/m_l$) (1)
γ	angle between parachute axis and the vertical (1)
β	angle between velocity vector and the vertical (1)
α	angle of attack (1)
I_c	moment of inertia of canopy ($\text{kp} \cdot \text{m} \cdot \text{sec}^2$)
i_c	radius of inertia (m)
s	distance from load point to centre of gravity of canopy
$\vec{F}^{(i)}$	internal force (kp)
$\vec{F}^{(e)}$	external force (kp)
D	drag (kp)
L	lift (kp)
M	moment (m.kp)
C_D, C_L, C_M	coefficients of drag, lift and moment of canopy (1)
S	projected area of inflated canopy ($= R^2\pi$) (m^2)
R	radius of inflated canopy (m)
k	damping (sec^{-1})
T	oscillation period (sec)
x', y'	coordinates fixed in earth (m)

ρ air density (kp. sec²/m⁴)
 g acceleration due to gravity (= 9.81) (m/sec²)

Time derivatives are designated by a dot.

Subscripts

c canopy
 l load
 x, y in direction of x -, y -axis of parachute

INVESTIGATIONS ON THE DYNAMIC STABILITY OF PERSONNEL GUIDE SURFACE PARACHUTES

R. Ludwig* and W. Heins*

1. INTRODUCTION - PROBLEMS AND PURPOSE OF THE INVESTIGATIONS

The dynamic stability of parachutes is of great importance in many respects. Insufficient stability of a man-carrying parachute may lead to injuries while landing; a lack of stability in deceleration parachutes can strongly disturb the control of the aeroplane during the landing run; and instability of a stabilizing parachute of a missile or a space vehicle would be contrary to the real purpose of such a parachute.

Advances in aeronautical engineering and the growing field of application of parachutes have in the last decades led to the development of parachutes of high dynamic stability. In particular, favourable canopy shapes with an appropriate porosity have been developed experimentally. Ribbon parachutes and guide surface parachutes are typical examples having good stability characteristics.

The theoretical investigation of the dynamic stability had been undertaken quite early; that is, already by 1918, as can be seen in a paper by Brodetzky¹. But in more recent times there have only been a few papers on this subject, for example by Henn², and more recently by Riabokin³, of the Department of Aeronautical Engineering of the University of Minnesota, Minneapolis (USA); by J.F. Reagan and F.J. Stimler⁴ (USA), and, very recently, by W.G.S. Lester⁷ (Great Britain). All these papers are confined to the linearized theory and the assumption of small disturbances. Sample calculations can only be found in the paper by Henn².

It is the purpose of the investigations described in this Report to calculate a greater number of cases for the personnel guide surface parachute by means of electronic computers after having derived the general non-linear equations of motion. The results will be discussed with a view to understanding the different modes of behaviour in dependence on certain parameters, which will again be useful with regard to experimental investigations. The prototype of the guide surface parachute, on which is based the personnel type, is an invention during World War II of Prof. Dr. H.G. Heinrich, at that time member of the Flugtechnisches Institut Stuttgart (Germany), under the direction of Prof. Dr. Madelung. Three-component measurements carried out by H.G. Heinrich and E.L. Haak have greatly contributed to these investigations⁵.

The principal reasons which have led to these theoretical investigations were:

- (a) The conditions for a linearized theory - the existence of small disturbances - are not satisfied in practice.

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- (b) The linearization of the equations of motion leads to a decomposition of the system of differential equations; the equation governing the velocity disturbance splits off and yields a monotone but not a periodic decrease with the frequency of the parachute oscillations, as would be expected.
- (c) The curves for the aerodynamic coefficients C_D , C_L , and C_M in dependence on the angle of attack show that a linearization does not meet the true conditions, for $\partial C_M / \partial \alpha_c$ sometimes even takes a negative value in the environment of $\alpha_c = 0$.

The oscillation behaviour of a given type of parachute depends on numerous parameters; the most important, which are to be varied here, are the length of the suspension lines, the weight of the suspended load, the assumptions on the apparent mass of the canopy, and the porosity of the parachute cloth. The oscillation behaviour is characterized by frequency (i.e. oscillation period) and damping, as generally known of linear systems. Detailed calculations show that discrepancies occur due to non-linear behaviour and also that different initial conditions lead to a different oscillation behaviour. All these results can be considered to be useful for experimental investigations.

Even though comparative measurements on the personnel guide surface parachute were not possible, observations on other types show qualitative agreement with the calculations.

2. EQUATIONS OF MOTION

To begin with, the equations of motion on which the calculations are based will be indicated. The detailed procedure is given in Appendix 1. The equations are valid under the following conditions:

- (a) The system canopy - load is assumed to be rigid.
- (b) The mass of air associated with the inflated canopy must be considered an inertial mass, but not a heavy mass⁶. It is designated as apparent mass. The weight of the canopy can be neglected.
- (c) The aerodynamic forces which act on the load are assumed to be negligibly small. This assumption is not satisfied with loads of special shape, as, for instance, with recovery systems of rockets or with stabilization systems of torpedos.

Figure 1 shows the geometric quantities and the forces acting. The differential equations of forces result of the equilibrium of the mass forces with the internal and external forces acting in the direction of the tangent and of the perpendicular to the trajectory.

For the canopy we have

$$m_c \left(\dot{\vec{v}}_c + [\vec{\omega} \vec{v}_c] \right) = \vec{F}_c^{(i)} + \vec{F}_c^{(e)} \quad (1)$$

and for the load

$$m_l \left(\dot{\vec{v}} + [\vec{\omega} \vec{v}] \right) = -\vec{F}_l^{(1)} + \vec{F}_l^{(e)} . \quad (2)$$

The equation for the moments of the canopy in the system of coordinates fixed in the axis of the parachute and referred to the load point is

$$I_c \dot{\omega} - m_c s \dot{v}_{cy} - m_c s v_{cx} \omega = -M_c . \quad (3)$$

If we introduce the condition of rigidity, the geometrical relationship between the angles α , β , γ , the external forces with the aerodynamic forces in the usual form, where the coefficients depend on the angle of attack α_c of the canopy, we obtain a system of four differential equations of the 1st order (see Fig.2):

$$\begin{aligned} (m_l + m_c)(\dot{v}_x - \omega v_y) + m_c s \omega^2 \\ = m_l g \cos \gamma - \frac{\rho}{2} S v_c^2 [C_D(\alpha_c) \cos \alpha_c + C_L(\alpha_c) \sin \alpha_c] \end{aligned} \quad (11)^*$$

$$\begin{aligned} (m_l + m_c)(\dot{v}_y + \omega v_x) - m_c s \dot{\omega} \\ = -m_l g \sin \gamma + \frac{\rho}{2} S v_c^2 [C_D(\alpha_c) \sin \alpha_c - C_L(\alpha_c) \cos \alpha_c] \end{aligned} \quad (12)^*$$

$$m_c (i_c^2 + s^2) \dot{\omega} - m_c s (\dot{v}_y + \omega v_x) = -\frac{\rho}{2} S 2R v_c^2 C_M(\alpha_c) \quad (13)^*$$

$$\dot{\gamma} = \omega . \quad (14)^*$$

The parachute trajectories come from a further integration of the velocity components in the system of coordinates fixed in the ground. Thus the motion of load and canopy is completely described by the following 15 quantities:

for the load:

$$v, v_x, v_y, \gamma, \omega, \beta, \alpha, x', y' ;$$

and, for the canopy, as $v_x = v_{cx}$:

$$v_c, v_{cy}, \alpha_c, \beta_c, x'_c, y'_c .$$

3. DATA FOR CALCULATIONS ON THE PERSONNEL GUIDE SURFACE PARACHUTE

The calculations on the personnel guide surface parachute (see Fig.3) are based on the following dimensions:

* Numbered as in Appendix

$$R = 3.0 \text{ m}, \quad s = 9.1 \text{ m}, \quad i_c^2 = 0.344 \text{ m}^2.$$

To determine the apparent air mass the volume of the canopy is first substituted by an ellipsoid of revolution with a volume equal to that of the canopy. The centre C (Fig.1) of the ellipsoid coincides with that of the canopy. The ellipsoid is then enlarged to take into account the apparent air mass. The moment of inertia I_c or the radius of inertia i_c can then be determined from known relations. For a usual type of man-carrying parachute with a normal load (≈ 100 kp) it can be assumed, according to von Kármán⁶ that the apparent mass is of approximately the same order as the mass of the load, i.e. $\mu_c = m_c/m_l \approx 1$.

As initial conditions we assume that the parachute which is steadily descending at a rate v_0 is deflected at an angle γ_0 , as in the case of a gust. Hence, for

$$t = 0 : \quad v = v_0, \quad \gamma = \gamma_0, \quad \omega = \beta = 0,$$

where v_0 results from the equilibrium condition $W_l = D$:

$$v_0 = \sqrt{2W_l/\rho S C_{D0}}.$$

The value 0.25 has generally been chosen for γ_0 , corresponding to 14.3° . The wind-tunnel measurements⁵ which have already been mentioned comprise the aerodynamic coefficients C_x , C_y and C_M referred to the axes x , y in the system of coordinates fixed in the parachute (see Fig.4). The coefficients C_D , C_L and C_M (for different lengths of the suspension lines) were obtained from C_x , C_y , C_M . All aerodynamic coefficients apply to a parachute of the same geometric shape, but are given for different effective porosities. The effective porosity η , introduced by H.G. Heinrich⁸ is a non-dimensional quantity providing the ratio of the mean flow velocity through the parachute fabric to the velocity of the free air stream. The curves plotted for C_M show at once that the parachute is statically stable when the greatest porosity $\eta = 0.096$ is used, as $\partial C_M/\partial a_c > 0$. But when $\eta = 0.042$ or $\eta = 0$ (non-porous canopy) the parachute is statically unstable, for in the environment of $a_c = 0$: $\partial C_M/\partial a_c < 0$.

4. RESULTS AND DISCUSSION

Considering first the statically stable case ($\eta = 0.096$), oscillations of a certain frequency are expected to be damped with time (Fig.5). The following is of interest here:

The velocities v , v_c , and the component v_x show a two-fold frequency compared to the velocity components v_y , v_{cy} , to the angular velocity ω and to the angles γ , β , β_c , α , and a_c . From the physical point of view this is easy to understand when the motion of the parachute is considered to be that of a pendulum. From the mathematical point of view this can be explained from the terms of ω^2 and ωv_y of the differential equations, which must immediately lead to an oscillation of a doubled frequency.

The small amplitudes of oscillation of the canopy compared to the oscillation of the load are remarkable. This can be seen from the angles β_c and β shown in Figure 5. This means that *the motion of the load is similar to that of a pendulum with a length s*.

The two frequencies of the velocity oscillations, as illustrated in Figure 5, are confirmed by cine-theodolite measurements of a parachute trajectory. The measurements were evaluated in the D.F.L.⁹. Such a case, though for a different type of parachute, is shown in Figure 6, giving the vertical and the horizontal velocity components v'_y and v'_x . As a comparison the corresponding calculated quantities are plotted for the personnel guide surface parachute.

At first sight, the quantities plotted versus time do not show a remarkable difference with respect to the oscillation behaviour of a linear system. They become more distinct, however, by representation in the phase plane (Fig.7), where $\omega = \dot{\gamma}$ is plotted versus γ as is generally done for a technical oscillation process. The time scale of the curve shows that *the oscillation period increases with decreasing amplitude*. Of the amplitude ratios it becomes evident that *damping decreases with amplitude*. The representation in the phase plane shows the oscillation period T and the damping k referred to the mean time t_m of a complete oscillation (Fig.8). At the same time, the load is varied, while the apparent mass is kept constant. It can be stated that *the oscillation period hardly depends on the mass of the load*. The oscillation period of a mathematical pendulum, for instance, is independent of the pendulum mass, but this is not valid any more for the physical pendulum. The variation of period with amplitude can, however, be approximately 10 per cent. This is, all the same, enough to be taken into account in experimental investigation, at least, when appreciating the scattering of results for a drop test disturbed by wind and gusts. Damping is rather strongly influenced by amplitude (up to about 50 per cent), a bigger load has, of course, a stronger damping effect.

The trajectory curves of canopy and load (Fig.9) will be considered next. In these curves the parachute is plotted for time intervals of 1 second. On the one hand, the porosity is varied (trajectories a, b, c). We see that *with increasing porosity there is an increase in damping and a decrease in the oscillation period*. On the other hand, the length of the suspension lines is varied, while the porosity is kept constant, and it appears that *the oscillation period increases with increasing length of the suspension lines* (C_{D0} as well decreases with increasing porosity).

The time history for the velocities and angles (Fig.10) shows that a more precise statement can be made on the oscillation period in dependence on the pendulum length; this, of course, pre-supposes that the oscillation period refers to the same amplitude, hence to the initial value γ_0 . It can be seen (Fig.11, upper part) that *the square of the oscillation period is proportional to the distance canopy-load*. This suggests basing an empiric formula on that for the mathematical pendulum, where a factor of proportionality must be introduced. The calculation of a number of cases for three different porosities shows that the drag coefficient C_{D0} can be introduced as a factor of proportionality and, substituting this factor by the rate of descent, we obtain

$$T \approx C_{D0} \cdot 2\pi \sqrt{\frac{s}{g}} = \frac{4\pi W_l}{\rho S v_0^2} \sqrt{\frac{s}{g}}.$$

The oscillation periods determined from this approximate formula (dashed lines in Fig. 11) agree quite well with those of exact calculations. The damping is scarcely affected by the length of the suspension lines (Fig. 11, lower part); in the case $\eta = 0.096$ a flat maximum appears for the usual length of the suspension lines. It is open to question, however, as to how far this statement can be generalized, especially when $\partial C_M / \partial \alpha_c$ takes on a negative value, and the oscillation becomes indifferent.

Up to now, the apparent mass had been set up so that $\mu_c = 1$ for $W_l = 100$ kp. As an assumption, the influence of a change of μ_c (Fig. 12) will still be demonstrated. According to the conception gained that motion of the load, essentially, is similar to that of a pendulum of length s , it can easily be realized that the *oscillation period is practically independent of the apparent mass*, apart from the already known dependence of amplitudes. It is remarkable, as can also be seen from the trajectories, that at equal initial position the canopy will be more deflected with the smaller apparent mass on account of its smaller inertial forces; at larger apparent mass, of course, the damping will become larger. The amplitudes of load, on the contrary, will be enlarged with increasing μ_c only in a very small degree.

In all cases considered up to now, the air density $\rho = \rho_0$ on the ground was the basis of the calculations and it was kept constant for the calculated section of trajectory. Now, cases with different air densities or, in other words, with different altitudes, will be considered, but the respective air density within the calculated section of trajectory is again assumed constant.

The following points, however, are pre-supposed:

- (a) the aerodynamic coefficients remain constant, i. e. the porosity does not change;
- (b) since the apparent mass decreases at lower air densities, we assume that the apparent air volume is kept constant, which means that

$$m_c = \frac{\rho}{\rho_0} m_{c0}, \quad \mu_c = \frac{\rho}{\rho_0} \mu_{c0};$$

The subscript 0 relates the respective values to the altitude 0 .

Comparing the time histories (Fig. 13), say β_c , for air densities corresponding to the altitudes between 0 and 10 km, it can be seen that, in the first instance, the same phenomena occur as when varying the apparent mass. Since also the aerodynamic forces decrease in correspondence with the altitude, the effect will be enlarged. This means that *the lateral deflection is larger than for the same μ_c on the ground; also, the damping is somewhat larger, and the oscillating period smaller, with increasing altitude.*

In a linear oscillating system, the character of the oscillation is not influenced by initial conditions, whereas in a non-linear system this problem is much more complicated. This is particularly significant in those cases in which no restoring moment occurs in the environment of $\alpha_c = 0$ (Fig. 14). Starting from a small initial

deflection, this deflection increases at first until the canopy, on account of the oscillating load, inverses its direction of movement. Figure 14a shows that this is the reason for the lateral drift of the parachute. Starting from a fairly large deflection (Fig. 14b), an almost indifferent oscillation without lateral drift is obtained on account of the restoring moment.

5. CONCLUSION

Of the numerous examples which have been treated, it becomes evident that parachute oscillations show many typical modes of non-linear oscillation, thus confirming that the true behaviour of a parachute is not correctly represented by a linear procedure. Phenomena such as the dependency of amplitude on frequency and damping cannot be ignored in connection with experimental investigations. By experiment on other types of parachutes the occurrence of two frequencies of the ratio 1:2 could be qualitatively confirmed as well as the order of magnitude of these frequencies. It had already been supposed that the relation between oscillation period and length of suspension lines could well be understood by application of the pendulum formula, but up to now this could not be derived from exact calculations. The indicated empiric formula can also be obtained under certain simplifying assumptions from the system of the linearized equations. The results of the calculations show that the effect of the apparent mass is not very important.

As it principally affects damping, it should be rather difficult to obtain more accurate estimations from drop tests which are never carried out in undisturbed air. That the initial conditions are of special interest for the time history of the trajectory curves is particularly easy to understand with regard to the complicated relation between the moment coefficient and the angle of attack. In practice it seems to be of importance that under certain initial conditions a drift may occur for the same parachute, while this would not be the case under different initial conditions.

The authors feel these investigations may make some practical contribution to the problem. Whether the assumptions made are satisfied in practice should be examined by test.

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APPENDIX 1

Equations of Motion

We introduce the internal forces $\vec{F}_c^{(i)}$ and $\vec{F}_l^{(i)}$ which act between canopy and load, and the vector of the angular velocity $\vec{\omega}$ (perpendicular to the vertical plane in which the motion takes place).

The equations of forces are considered separately for the centre C of the canopy volume and the load point L. Therefore the tensions in the suspension lines $\vec{F}_c^{(i)}$ and $\vec{F}_l^{(i)}$ act as internal forces. The following vector equations yield the equilibrium of the mass forces with the internal and external forces in the direction of the tangent and of the perpendicular to the trajectory:

$$m_c \left(\dot{\vec{v}}_c + [\vec{\omega} \vec{v}_c] \right) = \vec{F}_c^{(i)} + \vec{F}_c^{(e)} \quad (\text{A.1})$$

$$m_l \left(\dot{\vec{v}} + [\vec{\omega} \vec{v}] \right) = -\vec{F}_l^{(i)} + \vec{F}_l^{(e)} . \quad (\text{A.2})$$

To these equations we add the moment equation referred to the load point, viz.,

$$I_c \dot{\omega} - m_c s (\dot{v}_{cy} + v_{cx} \omega) = -M_c . \quad (\text{A.3})$$

the vectors being split in components of the system of coordinates fixed in the axis of the parachute.

As the system canopy-load is assumed to be rigid, the relation

$$\vec{F}_c^{(i)} = \vec{F}_l^{(i)} \quad (\text{A.4})$$

must be applied.

For the velocities we can deduce:

$$v_{cx} = v_x \quad (\text{A.5})$$

$$v_{cy} - v_y = -s \omega . \quad (\text{A.6})$$

Figure 1 gives the angular relations

$$\left. \begin{aligned} \gamma &= \beta + \alpha = \beta_c + \alpha_c \\ \tan \alpha &= -\frac{v_y}{v_x} \\ \tan \alpha_c &= -\frac{v_{cy}}{v_x} \end{aligned} \right\} \quad (\text{A.7})$$

The external forces are the aerodynamic forces and the load:

$$\left. \begin{aligned} F_{cx}^{(e)} &= -D \cos \alpha_c - L \sin \alpha_c \\ F_{cy}^{(e)} &= D \sin \alpha_c - L \cos \alpha_c \end{aligned} \right\} \quad (\text{A. 8})$$

$$\left. \begin{aligned} F_{lx}^{(e)} &= m_l g \cos \gamma \\ F_{ly}^{(e)} &= -m_l g \sin \gamma \end{aligned} \right\} \quad (\text{A. 9})$$

The equations of the aerodynamic forces and the aerodynamic moment are formulated in the usual way:

$$\left. \begin{aligned} D &= \frac{\rho}{2} S v_c^2 C_D(\alpha_c) \\ L &= \frac{\rho}{2} S v_c^2 C_L(\alpha_c) \\ M_c &= \frac{\rho}{2} S 2R v_c^2 C_M(\alpha_c) \end{aligned} \right\} \quad (\text{A. 10})$$

Eliminating the internal forces and considering that by differentiation

$$\dot{v}_{cy} = \dot{v}_y - s \dot{\omega},$$

the results of equation (A.6), we obtain the following four differential equations:

$$\begin{aligned} (m_l + m_c)(\dot{v}_x - \omega v_y) + m_c s \omega^2 \\ = m_l g \cos \gamma - \frac{\rho}{2} S v_c^2 [C_D(\alpha_c) \cos \alpha_c + C_L(\alpha_c) \sin \alpha_c] \end{aligned} \quad (\text{A. 11})$$

$$\begin{aligned} (m_l + m_c)(\dot{v}_y + \omega v_x) - m_c s \dot{\omega} \\ = -m_l g \sin \gamma + \frac{\rho}{2} S v_c^2 [C_D(\alpha_c) \sin \alpha_c - C_L(\alpha_c) \cos \alpha_c] \end{aligned} \quad (\text{A. 12})$$

$$m_c (i_c^2 + s^2) \dot{\omega} - m_c s (\dot{v}_y + \omega v_x) = -\frac{\rho}{2} S 2R v_c^2 C_M(\alpha_c) \quad (\text{A. 13})$$

$$\dot{\gamma} = \omega. \quad (\text{A. 14})$$

In order to determine the coordinates of the trajectory of the load two more differential equations must be considered:

$$\dot{x}' = v \cos \beta \quad (\text{A. 15})$$

$$\dot{y}' = -v \sin \beta . \quad (\text{A. 16})$$

Furthermore, to carry out the calculation, nine algebraic relations are necessary:

$$v_{cy} = v_y - s \omega \quad (\text{A. 17})$$

$$v_c = \sqrt{v_x^2 + v_{cy}^2} \quad (\text{A. 18})$$

$$v = \sqrt{v_x^2 + v_y^2} \quad (\text{A. 19})$$

$$\alpha_c = - \arctan \left(\frac{v_{cy}}{v_x} \right) \quad (\text{A. 20})$$

$$\alpha = - \arctan \left(\frac{v_y}{v_x} \right) \quad (\text{A. 21})$$

$$\beta_c = \gamma - \alpha_c \quad (\text{A. 22})$$

$$\beta = \gamma - \alpha \quad (\text{A. 23})$$

$$x'_c = x' - s \cos \gamma \quad (\text{A. 24})$$

$$y'_c = y' + s \sin \gamma \quad (\text{A. 25})$$

Equations (A. 11) to (A. 25) are used for the complete description of the behaviour of canopy and load; by integration of the system under given initial conditions one obtains in dependence on time:

for the load: $v, v_x, v_y, \gamma, \omega, \beta, \alpha, x', y'$

and, for the canopy: $v_c, v_{cy}, \beta_c, \alpha_c, x'_c, y'_c$.

If we substitute $\cos \alpha_c = v_x/v_c$ and $\sin \alpha_c = -v_{cy}/v_c$ in Equations (A. 11) and (A. 12), and if these two equations are solved for \dot{v}_x and \dot{v}_y , while \dot{v}_y in Equation (A. 13) is replaced by the corresponding expression, we obtain:

$$\dot{v}_x = \omega v_y - C_1 \omega^2 + C_2 \cos \gamma - C_3 v_c (v_x C_D - v_{cy} C_L) , \quad (\text{A. 26})$$

$$\dot{v}_y = \omega v_x - [C_4 \sin \gamma + C_5 v_c (v_{cy} C_D + v_x C_L) + C_6 v_c^2 C_M] , \quad (\text{A. 27})$$

$$\dot{\omega} = - [C_7 \sin \gamma + C_8 v_c (v_{cy} C_D + v_x C_L) + C_9 v_c^2 C_M] , \quad (\text{A. 28})$$

with the coefficients:

$$\begin{array}{ll}
 C_1 = \frac{\mu_c s}{1 + \mu_c}, & C_2 = \frac{g}{1 + \mu_c}, \\
 C_3 = \frac{\rho s}{2m_l(1 + \mu_c)}, & C_4 = \frac{1}{N}g(i_c^2 + s^2), \\
 C_5 = \frac{\rho s(i_c^2 + s^2)}{2m_l N}, & C_6 = \frac{\rho SRs}{m_l N}, \\
 C_7 = \frac{sg}{N}, & C_8 = \frac{\rho Ss}{2m_l N}, \\
 C_9 = \frac{1 + \mu_c}{\mu_c} \frac{\rho SR}{m_l N}, & N = (1 + \mu_c)i_c^2 + s^2.
 \end{array} \quad (A. 29)$$

Equations (A.26) to (A.28), together with Equations (A.14) to (A.25) now represent the system on which are based the sample calculations. The integration of this system of differential equations has been carried out on a digital computer as well as on an analogue computer. The numerical operations have been programmed according to the integration method provided by Runge-Kutta for the digital computer Siemens 2002 in the Centre of Calculation of the D.F.L. At steps of 0.1 second for an integration range of sometimes 60 seconds, the required accuracy was reached. Only every second step was printed. Parallel operations were carried out on an analogue computer PACE 231 R in the D.F.L.

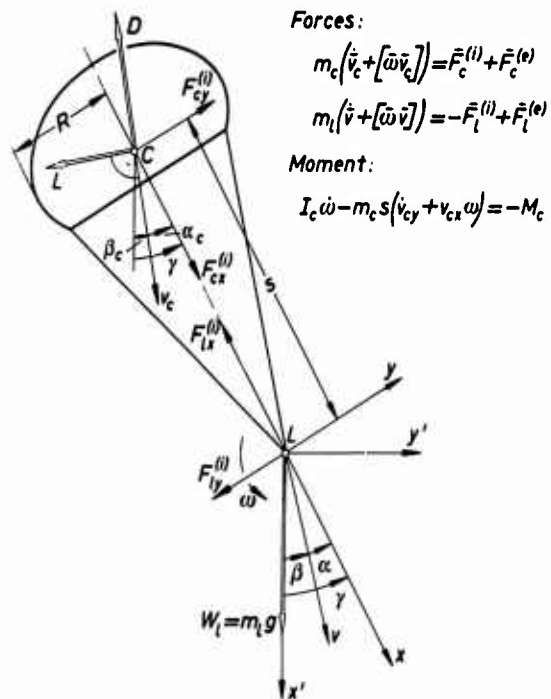


Fig. 1 Symbols and equations of motion I

$$(m_l + m_c)(\dot{v}_x - \omega v_y) + m_c s \omega^2$$

$$= m_l g \cos \gamma - \frac{\rho}{2} S v_c^2 [C_D(\alpha_c) \cos \alpha_c + C_L(\alpha_c) \sin \alpha_c]$$

$$(m_l + m_c)(\dot{v}_y + \omega v_x) - m_c s \dot{\omega}$$

$$= m_l g \sin \gamma + \frac{\rho}{2} S v_c^2 [C_D(\alpha_c) \sin \alpha_c - C_L(\alpha_c) \cos \alpha_c]$$

$$m_c (j_c^2 + s^2) \dot{\omega} - m_c s (\dot{v}_y + \omega v_x)$$

$$= -\frac{\rho}{2} S 2 R v_c^2 C_M(\alpha_c)$$

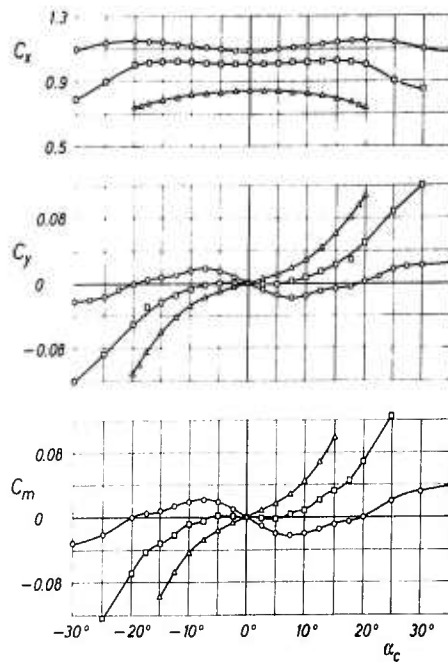
$$\dot{\gamma} = \omega$$

Fig. 2 Equations of motion II



$R=3.0m$
 $s=9.1m$
 $i_c^2=0.344 m^2$

Fig.3 Personnel guide surface parachute



see H.G. Heinrich and E.L. Hoak, ASD-TDR-62-100 (1962)

Fig.4 Aerodynamic coefficients
 0: $\eta = 0$; \square : $\eta = 0.042$; \triangle : $\eta = 0.09$

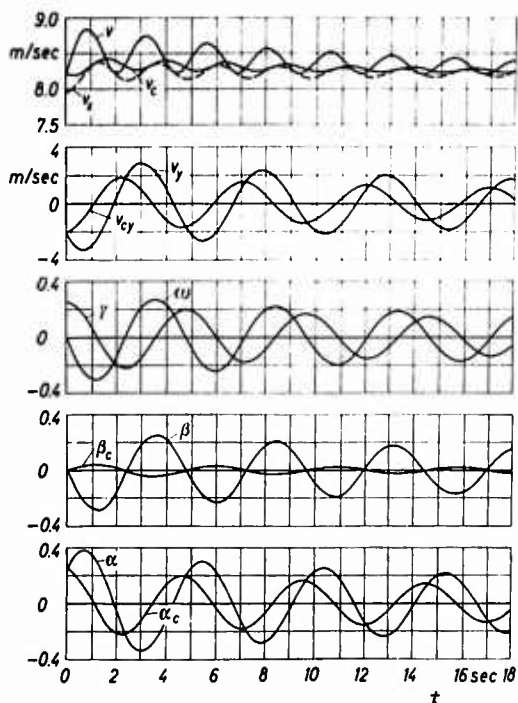
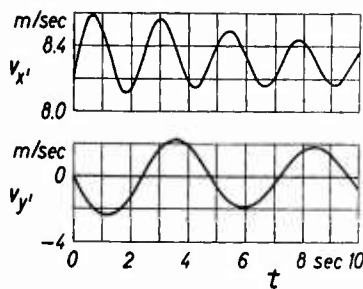


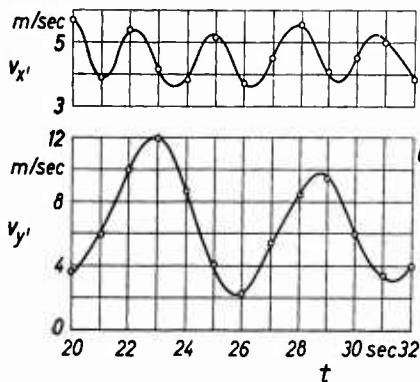
Fig. 5 Time histories

$W_l = 100 \text{ kp}$; $\gamma_0 = 0.25$; $\eta = 0.096$



Calculated results
(personnel guide surface parachute)

$W_l = 100 \text{ kp}$
 $\gamma_0 = 0.25$
 $\eta = 0.096$



Experimental results
obtained from cinethedolite measurements
(flat circular parachute)

$W_l = 100 \text{ kp}$

based on a paper by O. Weber,
WGL-Meeting 1962

Fig. 6 Comparison between calculated and experimental results

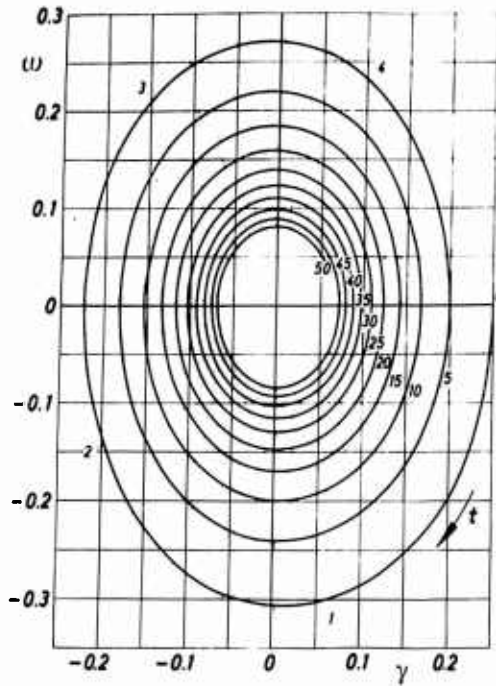


Fig.7 Diagram of ω and γ in the phase plane
 $W_l = 100 \text{ kp}$, $\eta = 0.096$

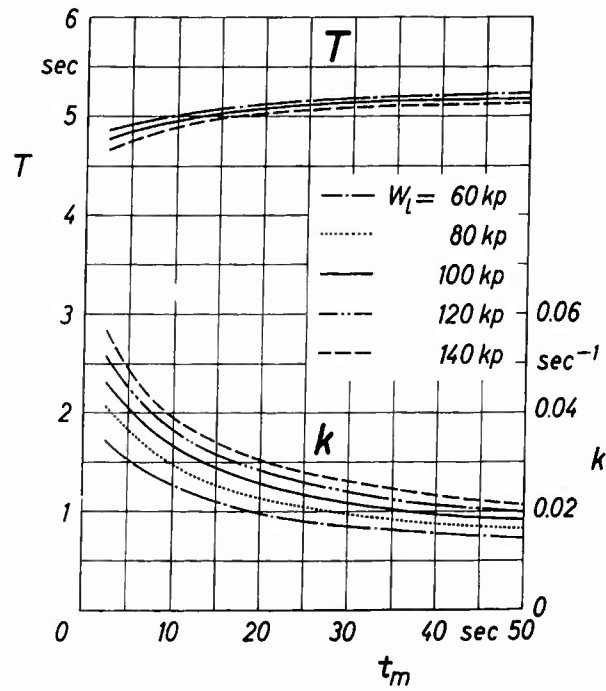


Fig.8 Oscillation period T and damping k for different loads
 $\eta = 0.096$; $\gamma_0 = 0.25$

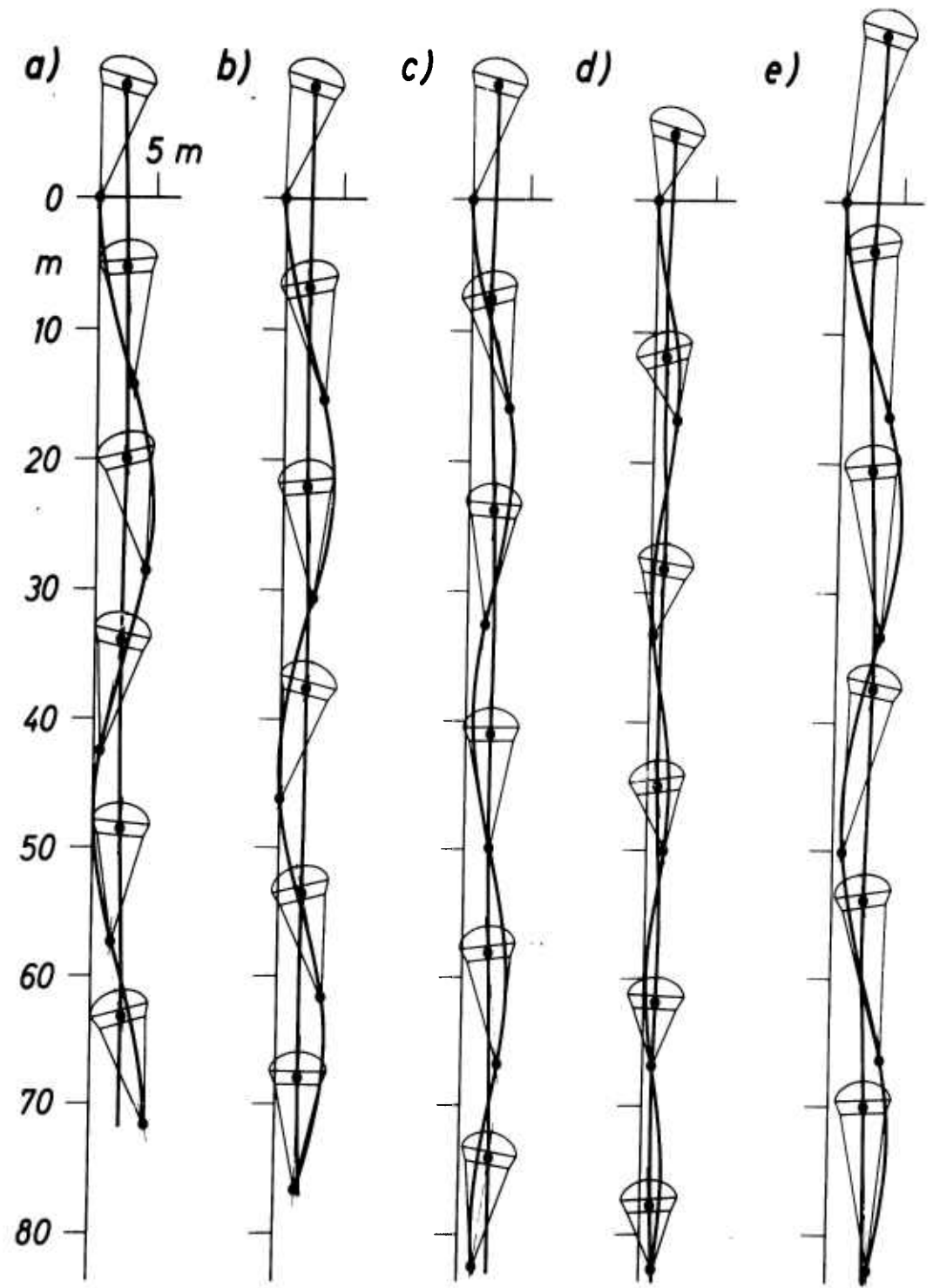


Fig.9 Trajectories
 $W_1 = 100 \text{ kp} , \gamma_0 = 0.25$

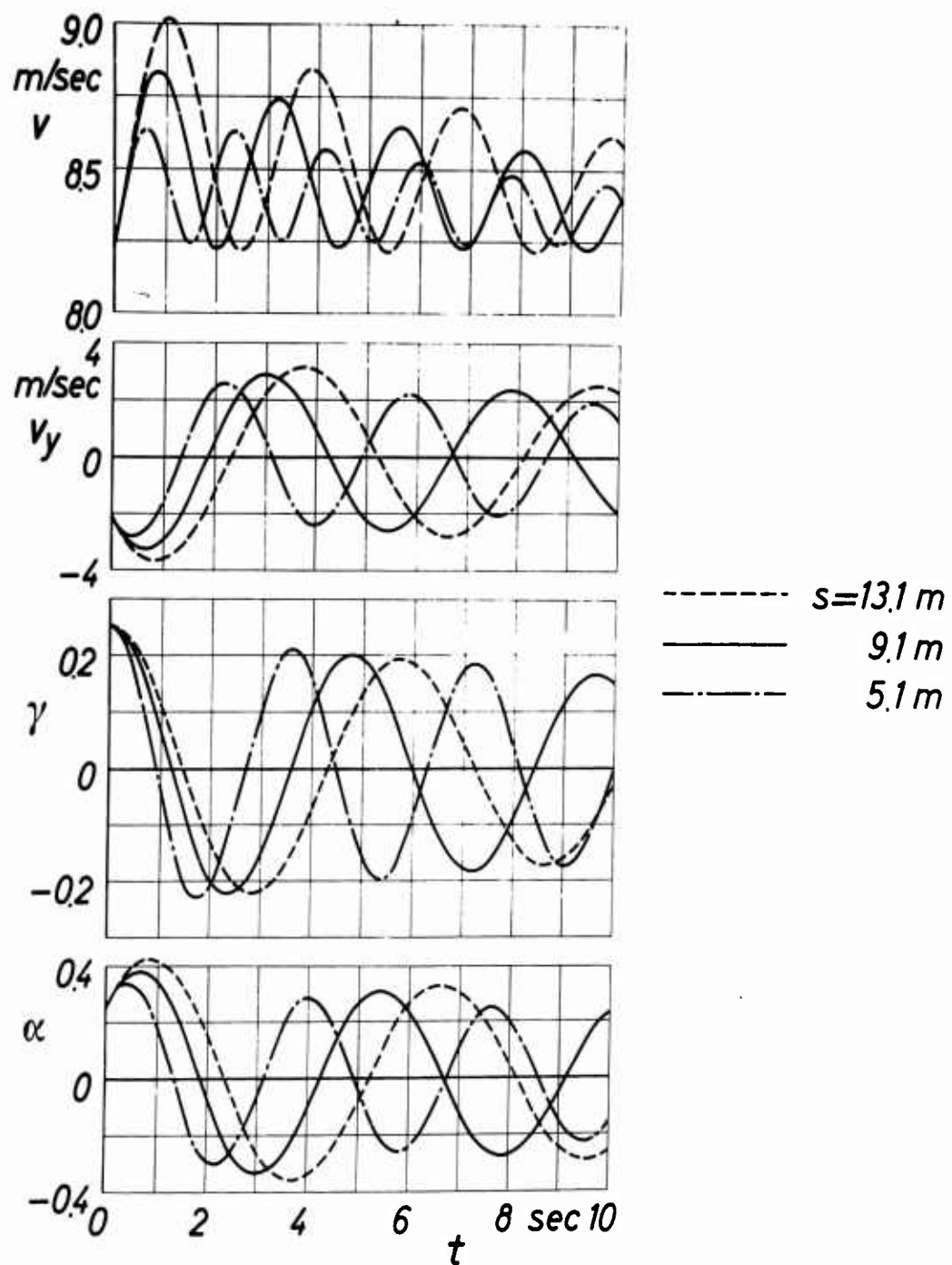


Fig. 10 Time histories for different suspension line lengths

$$W_l = 100 \text{ kp}, \quad \eta = 0.096$$

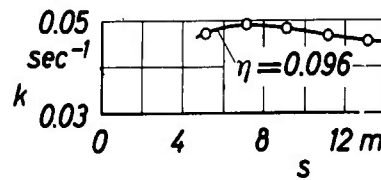
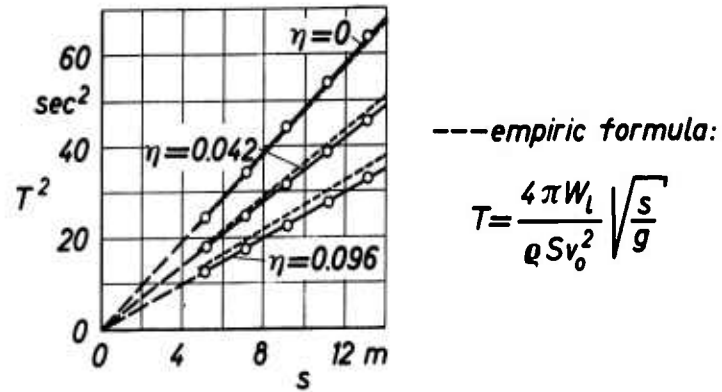


Fig. 11 Oscillation period T and damping k
 $W_l = 100 \text{ kp}$

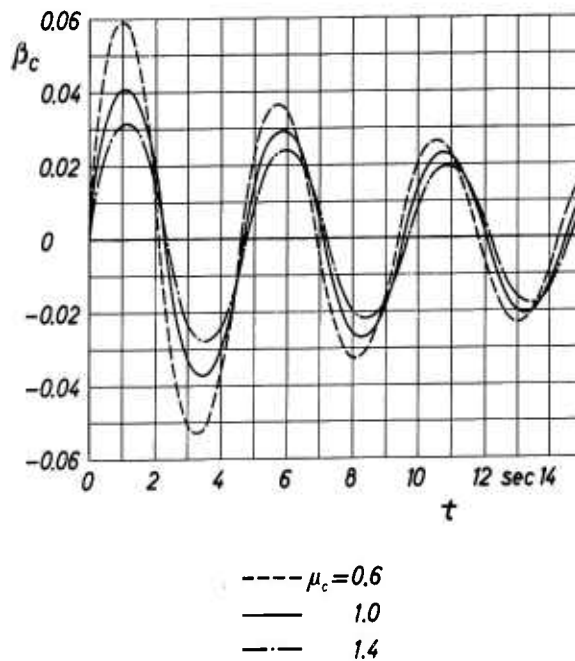


Fig. 12 Angle between the velocity vector and the vertical of the canopy, the apparent mass being varied
 $W_l = 100 \text{ kp}$, $\gamma_0 = 0.25$, $\eta = 0.096$

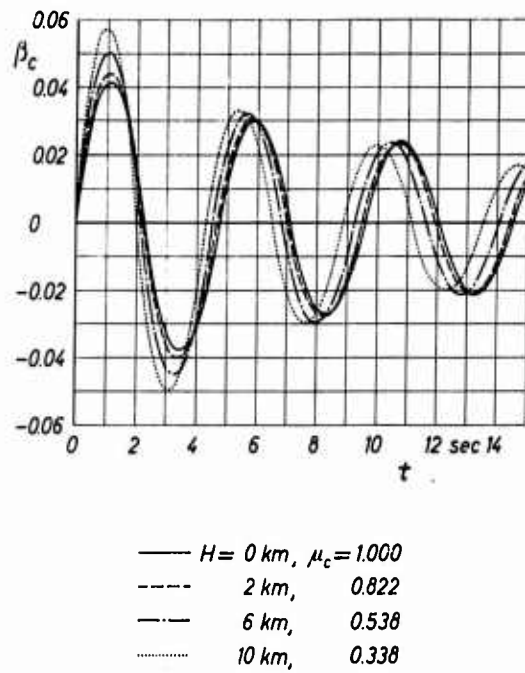


Fig. 13 Angle between the velocity vector and the vertical of the canopy, the altitude being varied

$$W_l = 100 \text{ kp}, \gamma_0 = 0.25, \eta = 0.096$$

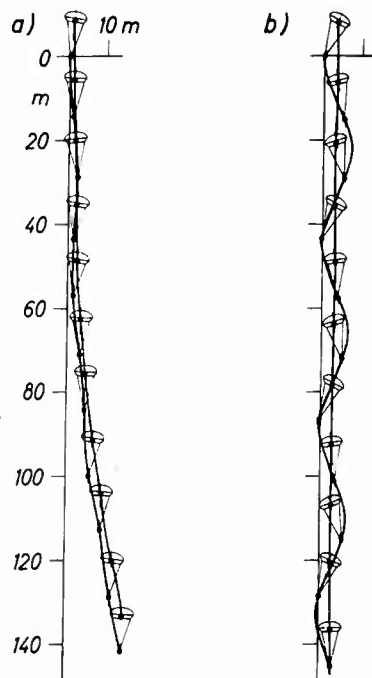


Fig. 14 Trajectories

$$W_l = 100 \text{ kp}, \eta = 0; \text{ (a) } \gamma_0 = 0.10, \text{ (b) } \gamma_0 = 0.40$$

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