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Systematic Errors in Ultrasonic Propagation Parameter Measurements

Part 1 - Effect of Free-Field Diffraction

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Appreciable errors in sound speed and sound absorption determinations may be attributed to a neglect in applying appropriate corrections to situations closely approximating free-field conditions. The errors are shown to be more severe for the lower frequencies and shorter paths. For long paths the errors are generally such as to indicate an excessive loss in absorption measurements. In sound speed determinations over relatively long paths, the error leads to group velocities lower than the plane-wave phase velocity and to phase velocities, obtained by noniterative techniques, higher than the plane-wave phase velocity.

A reduction in the size of such errors (which are not errors if corrected for) may be made by a judicious selection of a particular differential path, although certain intervals may result in an increased error.

This report contains, principally, graphs and tables of relative pressure and phase measurements as a function of the distance perpendicular to the transducer element. These data have been compiled for various transducer-crystal radii and for various wavelengths of operation. The tables, especially, are set up to permit accurate reading of diffraction corrections, particularly for absorption measurements in relatively low loss liquids or for sound speed measurements over relatively short paths or path differences.

INTRODUCTION

In an earlier Naval Research Laboratory (NRL) investigation of the accuracies of measurements of ultrasonic propagation parameters, it was stated that the results of sound speed determinations by the interferometer "indicate that all sources of error other than diffraction are smaller than the claimed accuracy of one part in 30,000" (1). The interferometer which was employed was constructed deliberately devoid of cylindrical symmetry in order to preclude the generation and synthesis of Rayleigh modes. The unusual cross section of this cell was considered to result in a slight perturbation of the results expected for a rigid cylinder, which would establish an ordering of the predicted modes and facilitate their experimental exploration and eventual negation. It was further stated that "The experimental data as well as a theoretical wave field ... indicate that the total error, including diffraction, is not above that claimed, provided that no measurements are made within 15 $\lambda/2$ of the 1-inch 1-Mc source."

NRL Problem S01-02; Project RF 001-03-45-5251. This is an interim report on the problem; work is continuing. Manuscript submitted October 9, 1963.

Granting the well-established interest in accurate sound propagation measurements and the presence of ubiquitous equipment for determining the same, we still note unfortunate discrepancies in the results of different observers, qualified and otherwise. Therefore, an additional investigation into the least-well-practiced facet of such measurements is highly desirable. At this advanced stage of the art of physical measurement, we can surely preclude the problems associated with the control and measurement of the more common variables used to denote the physical condition of the sample and the conditions of measurement, *e.g.*, temperature, frequency, time, distance, pressure, sample purity, impedance, etc. The dispersion of velocity with frequency attributable to the intrinsic properties of a medium is also generally well understood. But there is an apparent uncertainty among some investigators concerning dispersion that is created by the geometrical configuration of a particular test setup. The effect of such configurationally induced dispersion leads to a differentiation between group, phase, and signal velocities.

Prior to the detailed analysis of the experimental work, and concurrent with its later stages, it was

decided to report the theoretical predictions appropriate to such measurements in a manner designed to be of maximum benefit to the many others engaged in similar endeavors.

This report is concerned with the effects of free-field diffraction, that is, with measurements for which the approximation of radiation into a semi-infinite half-space may be safely assumed. This means that either there is no reflection from side walls or termination, or that the effects of such reflection may be time-separated. Included is a short discourse on the several concepts of sound velocity and also some elementary examples and descriptions of predictable errors. Immediately subsequent reports will deal with (a) guided mode (dispersion) effects, (b) iterative reflection effects involving both free-field diffraction and guided mode dispersion, and (c) correlation with experimental findings.

Many references may be quoted that are relevant to this investigation (1-6). While several of these deal with amplitude (absorption) effects, only one other (2) discusses free-field diffraction effects on sound speed measurement.

CONCEPTS OF SOUND VELOCITIES – PHASE, GROUP, AND SIGNAL

The usual connotation of the term sound velocity involves properties of the familiar linear hyperbolic partial differential equation of second order. By the use of this equation we derive our concept of phase velocity—that velocity pertinent to interference phenomena, the definition of wavelength, the calculation of phase difference, and in general, most acoustical phenomena. Thus, the usual meaning of sound velocity is just this phase velocity; indeed, if one considers an unterminated sinusoidal wave, the definition of any other velocity is not possible. However, nothing has been propagated in such a situation, and if one wishes to discuss the propagation of some quantity, the obvious choice is to impress some modulation on this carrier. Group velocity then refers to the propagation of this modulation or group, while the phase velocity refers to the motion of the elementary wavelets in the carrier. In the presence of velocity dispersion, attributable either to an intrinsic property of the medium or to the experimental configuration, these two velocities are distinct.

In discussing the propagation of a well-defined signal, one arrives at the concept of a signal velocity pertaining to the clear arrival, after the forerunners, of the signal. (The forerunners propagate, in the absence of absorption, with the plane-wave phase velocity.) The signal may be distorted, however, even without absorption, because of dispersion. Thus, the definition of signal velocity is arbitrary and ambiguous and even depends upon the sensitivity of a detector. The wavefront velocity, however, propagates with plane-wave phase velocity (which is the same as the group velocity in the absence of dispersion). The discrepancies among these various velocities are further enhanced by the presence of intrinsic absorption of the medium; then all signals are even further distorted, the forerunners may disappear, and the signal velocity becomes even more unclear. However, the concept of group velocity remains meaningful, and there is obviously no modification required in the idea of phase velocity.

RELATIONSHIP BETWEEN PHASE AND GROUP VELOCITY

Though this report is primarily concerned with free-field (no lateral boundary) diffraction effects, the free-field conditions will be approached (only in this section) by proceeding from the Fourier Series of a discrete family of modes relevant to a cylindrical lateral boundary (for simplicity of calculation) to the Fourier Integral of a continuum of modes by allowing the cylinder radius to become infinitely large. Borrowing terminology from the next report to be published concerning these discrete modes, we will now establish the relationship between phase velocity and group velocity in the presence of dispersion but not absorption.

We will use C for phase velocity and V for group velocity. The Characteristic Function X_{om} obtained by assuming both cylindrical symmetry and harmonic vibrations of the plane circular piston transducer is expressed by*

$$X_{om} = b\sqrt{k^2 - q_{om}^2} \qquad (1a)$$

*This nomenclature is consistent with that to appear in ensuing reports in which this equation, among others, is fully developed; this particular equation, however, may be found in numerous references, including Ref. 1.

or

$$q_{om} = \sqrt{k^2 - \left(\frac{X_{om}}{b}\right)^2} \tag{1b}$$

where k is the ordinary wave number for plane waves and q_{om} is that for a particular mode, the subscript o referring to the invoked cylindrical symmetry and the subscript m to the numbering or ordering of the mode; thus, q_{oo} is equivalent to k, q_{o1} is the next mode, etc. Also, we have

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \text{ and } q_{om} = \frac{\omega}{c_{om}} = \frac{2\pi}{\lambda_{om}} \tag{1c}$$

where ω is the angular frequency, c is the sound velocity, and λ is the wavelength. Here, c_{00} is equivalent to c and λ_{00} is equivalent to λ. (For outgoing waves and circular symmetry there is an additional term $e^{-iq_{om}z}$.)

The well-known cutoff frequencies $\omega_{c.o.}$ are predicted by substitution of Eq. (1c) into Eq. (1b):

$$\omega = \sqrt{k^2 c_{om}^2 - \left(\frac{X_{om}}{b}\right)^2 c_{om}^2} \tag{2a}$$

where

$$\omega_{c.o.} = \frac{cX_{om}}{b}. \tag{2b}$$

When $X_{om}/b > k$, we have a nonpropagating or evanescent mode where q_{om} is imaginary (λ_{om} is imaginary).

The phase velocity is given by

$$C = \frac{\omega}{k} \text{ or } C_{oo} = \frac{\omega}{q_{oo}}$$

for plane waves and by

$$C_{om} = \frac{\omega}{q_{om}}$$

for any particular mode. The group velocity

$$V \equiv \frac{dw}{dk}$$

becomes

$$V_{om} \equiv \frac{d\omega}{dq_{om}}.$$

From Eq. (1b) we have

$$k^2 = q_{om}^2 + \left(\frac{X_{om}}{b}\right)^2 \tag{3}$$

and, by substitution in Eq. (2a),

$$\omega^2 = q_{om}^2 C^2 + \frac{X_{om}^2}{b^2} C^2$$

whereby

$$\frac{d\omega}{dq_{om}} = \frac{q_{om}C^2}{\omega} = \frac{C^2}{C_{om}} \tag{4a}$$

and

$$C^2 = V_{om}C_{om}. \tag{4b}$$

We have assumed that phase velocity is a function of frequency. In the configurationally induced dispersion this amounts to recognizing that the particular summation of permitted modes is frequency-dependent. Thus, for the individual modes Eqs. (3), (4a), and (4b) hold, and for the effective value we have

$$C^2 = V_{eff}C_{eff} \equiv \overline{V}\,\overline{C}. \tag{4c}$$

We further obtain

$$\overline{V} = \overline{C} + \overline{q}\,\frac{\partial \overline{C}}{\partial \overline{q}}, \tag{5a}$$

or, in terms of $\overline{\lambda}$ instead of $\overline{q}$,

$$\overline{V} = \overline{C} - \overline{\lambda}\,\frac{\partial \overline{C}}{\partial \overline{q}}. \tag{5b}$$

We note that Eq. (5b) seems to indicate that the same effective group velocity observed over a wide range of carrier frequencies does not imply that the effective phase velocity is constant, but only that it varies linearly with $\overline{\lambda}$ over a wide range (and vice versa). This is misleading, however, because from Eq. (4c) we note that if $\overline{V}$ is constant over a range, so also is $\overline{C}$.

ELEMENTARY RAY ANALYSIS AND TRANSIENT EFFECT

An elementary ray analysis of the transient effect in pulsing may be demonstrated. Figure 1 shows a finite transducer (crystal) radiating to the right, with each element of the transducer radiating into a 90-degree solid angle. Assuming the crystal parameter $ka >> 1$, we may neglect the edge effect. From the hemispherical pattern, it is

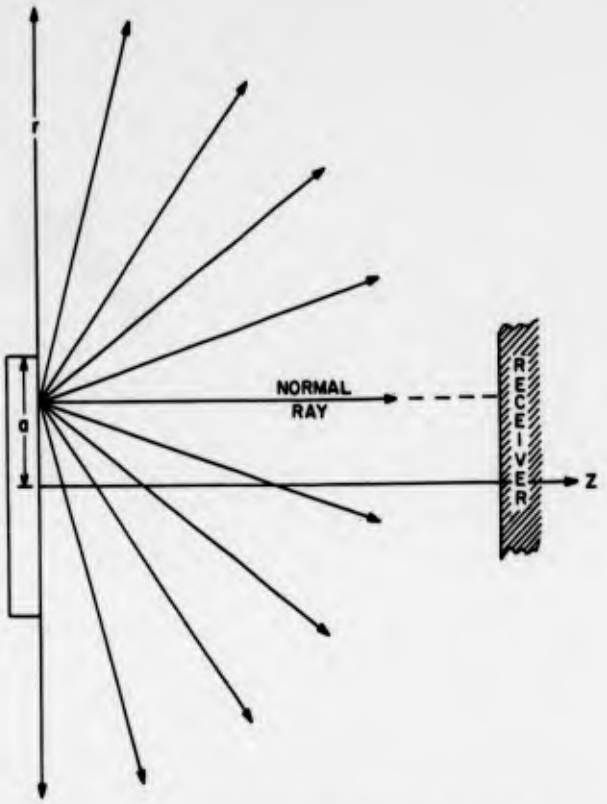


Fig. 1 – Finite transducer radiating into semi-infinite half-space

apparent that only one ray from each element is directed normal to the surface of the transducer. These normal rays travel the shortest distance to a parallel finite receiver and are the only ones which would yield the unconfined plane-wave phase velocity; their arrival at the receiver marks the beginning of the forerunners. Rays which have traveled an increasingly greater distance commence arriving as time advances. Eventually a "full-blown" signal is received, and the signal velocity refers to this somewhat arbitrary point. The group velocity refers to the propagation of the disturbance (pulse). Obviously, any reasonable criterion that an experimenter might establish to determine the time-of-arrival of a signal would result in an error if he hoped to measure the phase velocity, unless of course he had some unknown means of determining the time of the first zero or axis crossing (beginning). However, the experimenter might decide that the error in a particular technique, for which a maximum bound is rather easily calculated, is acceptable in his work.

Obviously, the result of this addition of successive rays impinging at a common point, after having traveled successively greater distances, is a frequency modulation of the received signal, that is, the first maximum would occur at a later time after the initial zero than if all rays traveled in a normal direction. Consequently, one would expect the period of the first received half cycle to be larger than that of the second, etc. This is just a way of demonstrating that, from the beginning of the received signal to the eventual attainment of steady-state conditions, an increasing time delay in the observance of any identifiable characteristic of the pulse will be observed. This delay, of course, results in too low a sound speed determination.

Figure 2 is an attempt to show this effect for the relatively simple case of neither intrinsic (much less anomalous) absorption nor intrinsic dispersion. Time t refers to the arrival of the normal rays. Times t_1 and t_2 refer to the contribution of rays from discrete conical surfaces, the t_2 rays making a greater angle with the normal than the

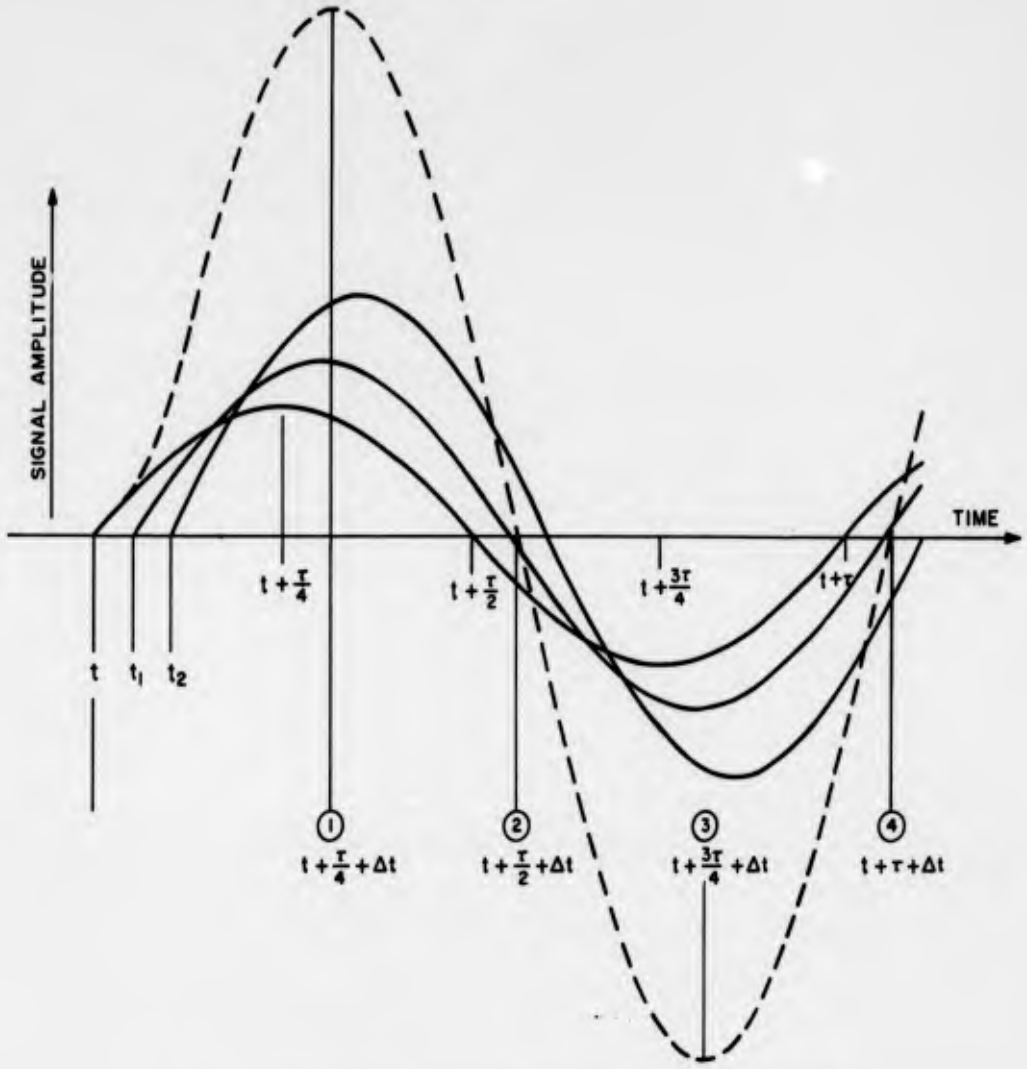


Fig. 2 – Summation effect of individual rays from elements of transducer surface

t_1 rays and, hence, being delayed a greater time. The summation of the rays over the pertinent diverging angle is represented by the dashed curve. It is apparent that the intervals (1) to (2), (2) to (3), and (3) to (4) are equal to quarter periods ($\tau/4$) of the driving frequency, but the interval t to (1) is greater than a quarter period by an amount Δt. So the sound speed observed is equal to the distance traveled (d) divided by the elapsed time, *i.e.*,

$$c_{obs} = \frac{d}{\left(t + \frac{\tau}{4} + \Delta t\right) - \left(t_o + \frac{\tau}{4}\right)}$$

where t_o is the starting time at the source. Clearly, c_{obs} is less than

$$c = \frac{d}{t - t_o}.$$

It should be noted that other reference points of the observed signal could be used with the same result, *i.e.*,

$$c_{obs} = \frac{d}{\left(t + \frac{\tau}{2} + \Delta t\right) - \left(t_o + \frac{\tau}{2}\right)}, \text{ etc.}$$

Both Polk and Mayo (7, 8) discuss the transient behavior of aperture antennas. They reiterate

that, while there is no transient effect for a point source, increasing *ka* to increase "planarity" increases the transient effect. Obviously, this states that by increasing either or both the frequency and the source radius, the duration of the transient effect is increased. An important result of Polk's analysis is that the steady-state main lobe is established within one-half period of the carrier frequency (at a point in space this refers to the time after the signal is first received). The significance of this is that a cycle-by-cycle search for variations in a received pulse is fruitless even in the absence of excess absorption; the "frequency modulation" has been completed before the first maximum, and even this maximum often cannot be located. Of course, excessive electronic rise-time delay due to insufficient broadband response will confuse the issue by operating in the fashion of a low-pass filter, in much the same way as that intrinsic absorption of the medium which increases with frequency. And if there happens to be a relaxation process within the frequencies of interest, the picture is further beclouded. With straightforward dispersion the particular configuration could act as a selective filter by sorting out the various frequency components of the signal and yielding a stretched-out, phase-modulated disturbance. The frequency components themselves would depend on the type of signal (*i.e.*, whether the drive is a gated carrier, square wave, spike transient, step, etc.), on the bandwidth of the electronics and transducer, and on the acoustic properties of the medium and the configuration.

The foregoing discussion obviously refers only to those methods employing pulses or other defined signals and indicates that all measurements derivable in this manner will result in apparent sound velocities intermediate between the unconfined phase (signal) velocity and the group velocity, that is, lower than free-field phase velocity. In a later report it will be shown that most attempts to measure true phase velocity result in apparent sound speeds intermediate between unconfined phase (signal) velocity and waveguide mode phase velocity, that is, higher than free-field phase velocity. Invoking the concept of modes for ease of discussion, the relationship between the straightforward waveguide mode (summation) phase velocity and waveguide mode (summation) group velocity was shown in Eq. (4c) to be

$$C^2 = \overline{V}\,\overline{C}$$

where C is the plane-wave phase velocity, $\overline{V}$ is the waveguide summation group velocity, and $\overline{C}$ is the waveguide summation phase velocity. Actually, as indicated earlier in this section, it is possible to approach the signal velocity value by an intelligent pulse-type measurement. Similarly, by a judicious application of the techniques of iterative reflection (interferometry), it is possible to effectively and selectively sharpen the beam or bundle of rays from the transducer and to obtain a phase velocity result much closer to the plane-wave value than predictable from nonterminated waveguide calculations. The choice of the type of measurement to be employed in a given investigation would then depend upon whether in practice it is more possible to modify a standard waveguide group-velocity technique to a signal velocity technique, or whether an iterative reflection technique modifies a standard waveguide phase-velocity technique more closely to plane-wave results. Of course, if maximum accuracy is not desired, then the simplest technique may be preferable.

It is to be noted that the term accuracy as used herein refers to just that and is not to be confused with precision, repeatability, or any other statistical manipulation.

FORMULATION OF FREE-FIELD DIFFRACTION EXPRESSIONS

In Ref. 1 use was made of King's (9) expression for the velocity potential in cylindrical coordinates in order to map the radiation field of a 1-Mc, 1-in.-diam quartz crystal. King's expression

$$\varphi = v_0 a \int_0^\infty \mu^{-1} e^{-\mu z} J_0(\alpha r) J_1(\alpha a)\, d\alpha, \quad (6)$$

where

$$\mu \equiv (\alpha^2 - k^2)^{1/2},$$

is not affected by the error found later in his excellent article. Following the method used by Williams (4), we derive the average of φ over a circle of area πa^2 centered on the z axis and then divide this average by the plane-wave value. The

average rms pressure relative to a plane wave is then given by

$$\langle p\rangle_{rel} = \sqrt{(Re\langle\varphi\rangle_{rel})^2 + (Im\ \langle\varphi_{rel}\rangle)^2}, \quad (7a)$$

and the phase difference relative to the plane-wave phase is given by

$$\langle\theta\rangle_{rel} = \tan^{-1}\frac{Im\langle\varphi\rangle_{rel}}{Re\langle\varphi\rangle_{rel}} \quad (7b)$$

where the real and imaginary parts of the average velocity potential relative to the plane-wave value are given by

$$Re\langle\varphi\rangle_{rel} = 1 - B\cos A - C\sin A$$
$$Im\langle\varphi\rangle_{rel} = B\sin A - C\cos A \quad (7c)$$

and the following definitions hold:

$$A = M + N$$
$$M = \frac{ka^2}{z} - \frac{ka^4}{z^3}$$
$$N = \frac{ka^4}{4z^3}$$
$$B = J_0 - \frac{N^2}{4}J_0 - \frac{N}{2}J_1 + \frac{N}{2}J_3 - \frac{N^2}{4}J_4$$
$$C = J_1 - \frac{N^2}{4}J_1 - NJ_2 + \frac{N^2}{8}J_3 - \frac{N^2}{8}J_5. \quad (7d)$$

The J's are Bessel Functions of the lst kind of the order indicated and argument M, z is one of the cylindrical coordinates, a is the transducer crystal radius, and k is the ordinary wave number. (The foregoing expression was obtained by neglecting all terms beyond the cube in two separate expansions of exponential functions. But even then, the expressions are more accurate than claimed by Williams.) From Figs. 3 and 4 it is apparent that the smooth character does not break, except for distances closer than 4.5 cm. It will be shown later that the expression is indeed valid for this short a distance which corresponds to $z = 3a$ for $a = 10\lambda$.

It is unfortunate that Williams did not plot his resulting calculations sufficiently to determine the presence of the successive maxima and minima, which escaped his notice. However, neither he nor anyone else appears to have taken the logical step of applying these types of calculations to predictions of sound speed or phase errors. Seki, Granato, and Truell (3) did point out the amplitude variations that escaped Williams' attention. Bass (6), obviously believing Williams' results to be poorer than they are, recalculated the average pressure by utilizing another (more valid) approximation. His 1st-order approximation may be transformed into our notation as follows:

$$Re\langle p\rangle_{rel} = 1 - [J_0\cos x + J_1\sin x]\left[1 - \frac{1}{2}\left(\frac{x}{ka}\right)^2\right] - \frac{J_1\sin x}{x}\left(\frac{x}{ka}\right)^2, \quad (8a)$$

and

$$Im\langle p\rangle_{rel} = [J_0\sin x - J_1\cos x]\left[1 - \frac{1}{2}\left(\frac{x}{ka}\right)^2\right] - \frac{J_1\cos x}{x}\left(\frac{x}{ka}\right)^2 \quad (8b)$$

where the argument of the Bessel Functions is

$$x \equiv \frac{k}{2}[(z^2 + 4a^2)^{1/2} - z].$$

By a straightforward application of this method we obtain a 2nd-order approximation, *viz.*,

$$Re\langle p\rangle_{rel} = 1 - [J_0\cos x + J_1\sin x]\left[1 - \frac{1}{2}\left(\frac{x}{ka}\right)^2 - \frac{1}{8}\left(\frac{x}{ka}\right)^4\right] - J_1\frac{\sin x}{x}\left[\left(\frac{x}{ka}\right)^2 + \frac{1}{4}\left(\frac{x}{ka}\right)^4\right], \quad (9a)$$

and

$$Im\langle p\rangle_{rel} = [J_0\sin x - J_1\cos x]\left[1 - \frac{1}{2}\left(\frac{x}{ka}\right)^2 - \frac{1}{8}\left(\frac{x}{ka}\right)^4\right] - \frac{J_1\cos x}{x}\left[\left(\frac{x}{ka}\right)^2 + \frac{1}{4}\left(\frac{x}{ka}\right)^4\right] \quad (9b)$$

A comparison of the foregoing three equations for $\langle p \rangle_{rel}$ plotted as the ordinate versus z as the abscissa is shown in Fig. 5. The actual values used were $\lambda = 0.15$ cm and $a = 1.5$ cm ($ka = 20\pi$). It is apparent that the Williams' approximation agrees with the others to 0.5 percent at $z = 4.5$ cm and almost coincides for greater distances. It should be noted that there is a discrepancy of less than 1 percent between the 1st- and 2nd-order approximations at the closest calculated point ($z = 0.075$ cm).

Figure 6 is a comparison of $\langle \theta \rangle_{rel}$ calculated by the above three approximations. Here again it is obvious that the phase discrepancy between Williams' approximation and the other two is less than 2 percent at $z = 4.5$ cm and is even closer at greater distances. The 1st- and 2nd-order approximations, which differed by only 1 percent in magnitude at the closest point, disagree by over 30 percent in phase at this same point (needless to say the Williams' approximation is useless here). It should be recalled, however, that neither Williams nor Bass calculated the quantity $\langle \theta \rangle_{rel}$.

To consider the effect of attenuation in the liquid (intrinsic absorption) we replace the wave number k by $k^* \equiv k_0 - j\alpha$. We also define

$$A \equiv \frac{k_0 Y}{2}$$

$$B \equiv \frac{\alpha Y}{2}$$

$$C = A - jB \text{ (arg of } J_0 \text{ and } J_1)$$

$$D = \frac{1}{A^2 + B^2}$$

$$E = 1 - \frac{1}{2}\left(\frac{Y}{2a}\right)^2 - \frac{1}{8}\left(\frac{Y}{2a}\right)^4$$

$$F = \left(\frac{Y}{2a}\right)^2 + \frac{1}{4}\left(\frac{Y}{2a}\right)^4$$

$$Y = (z^2 + 4a^2)^{1/2} - z$$

so that we obtain

$$\begin{aligned} Re\langle p \rangle_{rel} = \\ & 1 - E\cos A\,(-\sinh B + \cosh B)\,Re\,J_0 \\ & - E\sin A\,(-\sinh B + \cosh B)\,Re\,J_1 \\ & - FDA\sin A\,(-\sinh B + \cosh B)\,Re\,J_1 \\ & + FDB\cos A\,(\sinh B + \cosh B)\,Re\,J_1 \\ & + E\cos A\,(-\sinh B + \cosh B)\,Im\,J_1 \\ & - E\sin A\,(-\sinh B + \cosh B)\,Im\,J_0 \\ & + FDA\cos A\,(-\sinh B + \cosh B)\,Im\,J_1 \\ & + FDB\sin A\,(-\sinh B + \cosh B)\,Im\,J_1 \end{aligned} \tag{10a}$$

and

$$\begin{aligned} Im\langle p \rangle_{rel} = \\ & - E\cos A\,(-\sinh B + \cosh B)\,Im\,J_0 \\ & - E\sin A\,(-\sinh B + \cosh B)\,Im\,J_1 \\ & - FDA\sin A\,(-\sinh B + \cosh B)\,Im\,J_1 \\ & + FDB\cos A\,(\sinh B + \cosh B)\,Im\,J_1 \\ & - E\cos A\,(-\sinh B + \cosh B)\,Re\,J_1 \\ & + E\sin A\,(-\sinh B + \cosh B)\,Re\,J_0 \\ & - FDA\cos A\,(-\sinh B + \cosh B)\,Re\,J_1 \\ & - FDB\sin A\,(-\sinh B + \cosh B)\,Re\,J_1. \end{aligned} \tag{10b}$$

The effect of intrinsic absorption is shown in Figs. 7a, 7b, 8a, 8b. Figure 7a is a plot of $\langle p \rangle_{rel}$ versus z for the attenuation parameters $\alpha = 0.01$, 0.10, 1.00, and 10.0 nepers/cm. Figure 7b is the same plot continued to larger distances (300 cm). Figure 8a and 8b show $\langle \theta \rangle_{rel}$ versus z for the same parameters plotted to a distance of 15 cm and 300 cm, respectively. Note that for these calculations $k_0 a = 20\pi$ or $k_0 \simeq 42$ cm^{-1}, which is to be compared to the values of α between 0.01 cm^{-1} and 10.0 cm^{-1}.

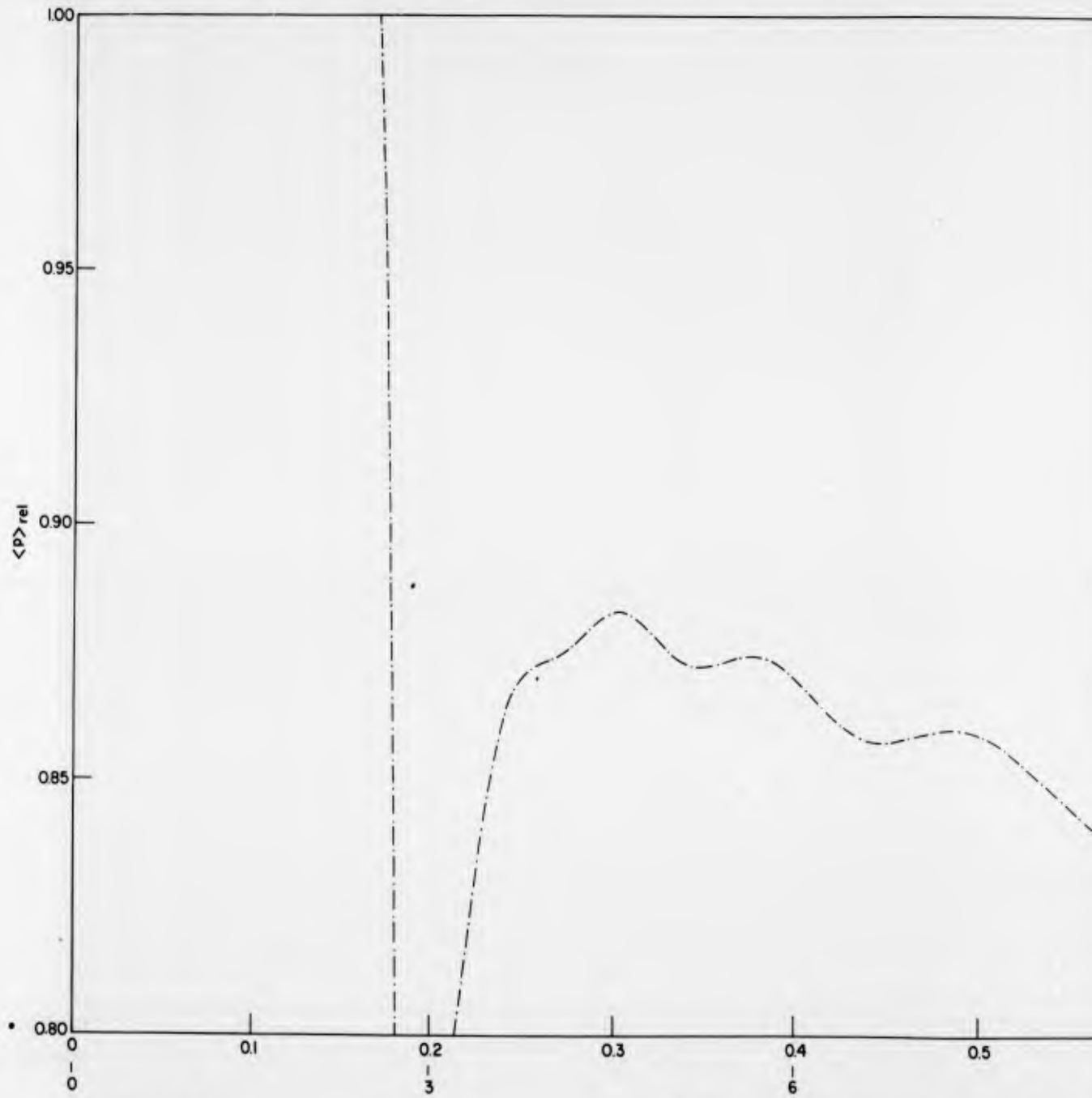


Fig. 3 – Average relative sound pressure $(p)_{rel}$ vs the distance z
calculated from Williams' velocity potential approxima

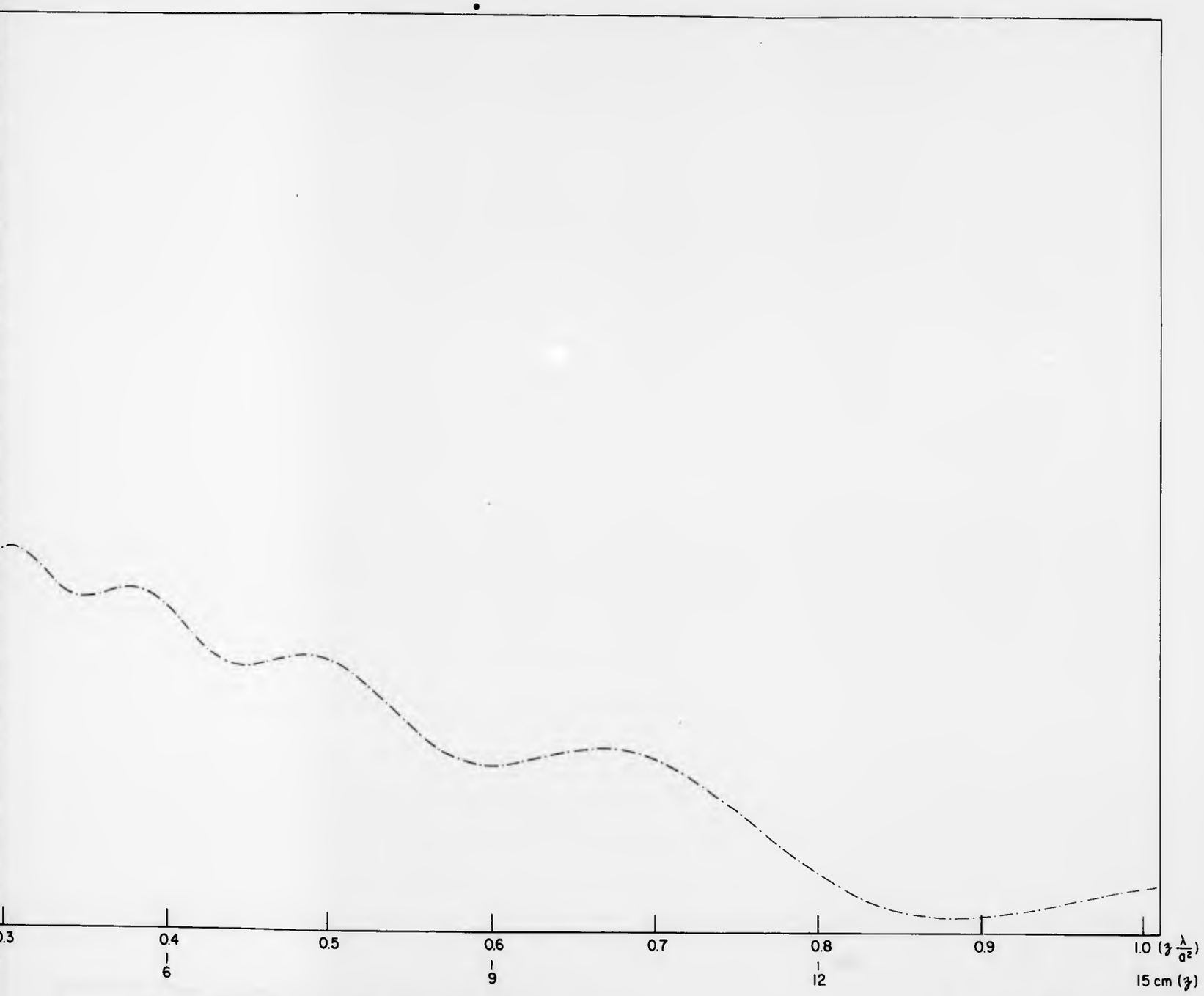


Fig. 3 – Average relative sound pressure $\langle p \rangle_{rel}$ vs the distance z along the transducer axis as calculated from Williams' velocity potential approximation for $ka = 20\pi$

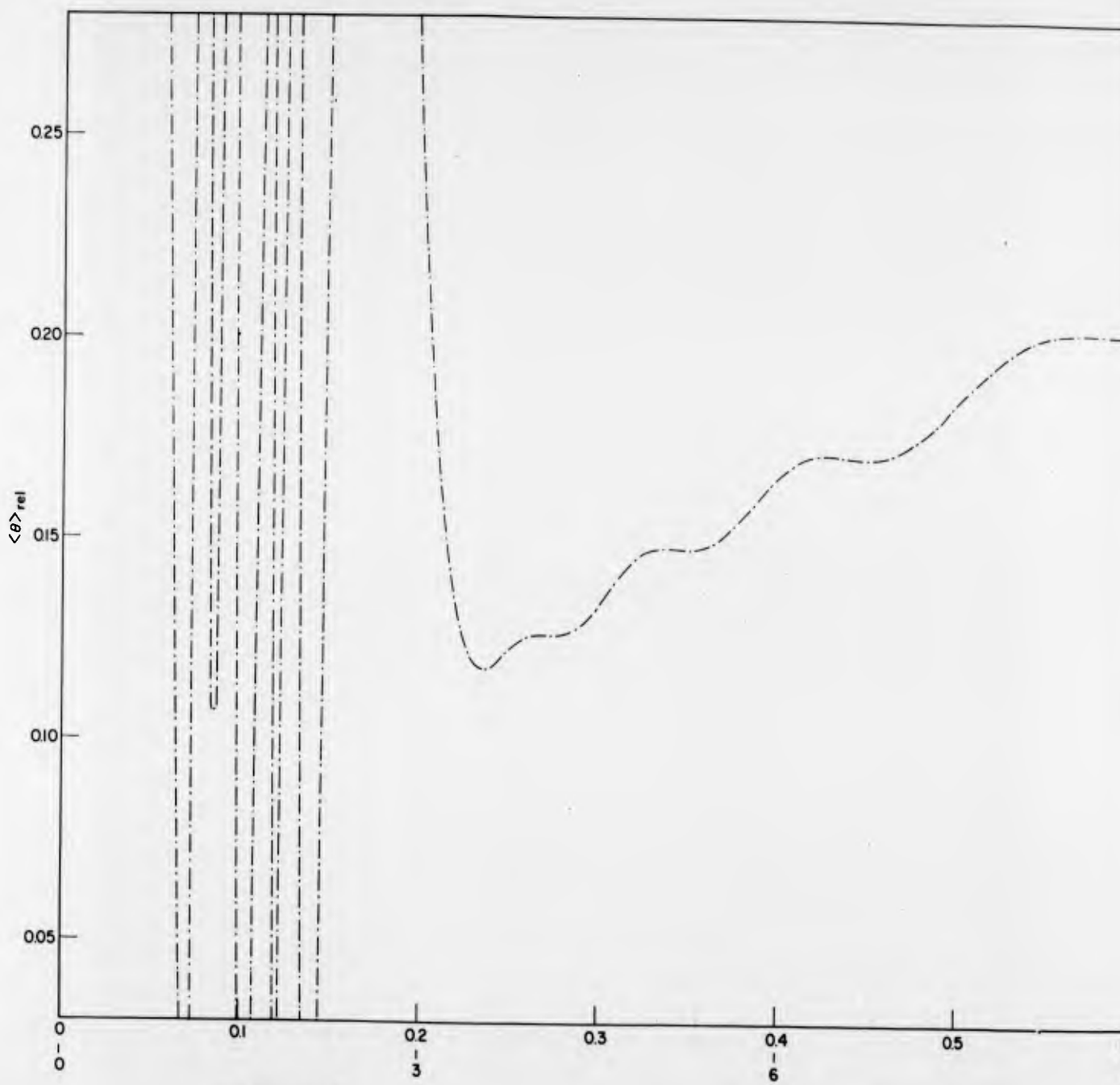
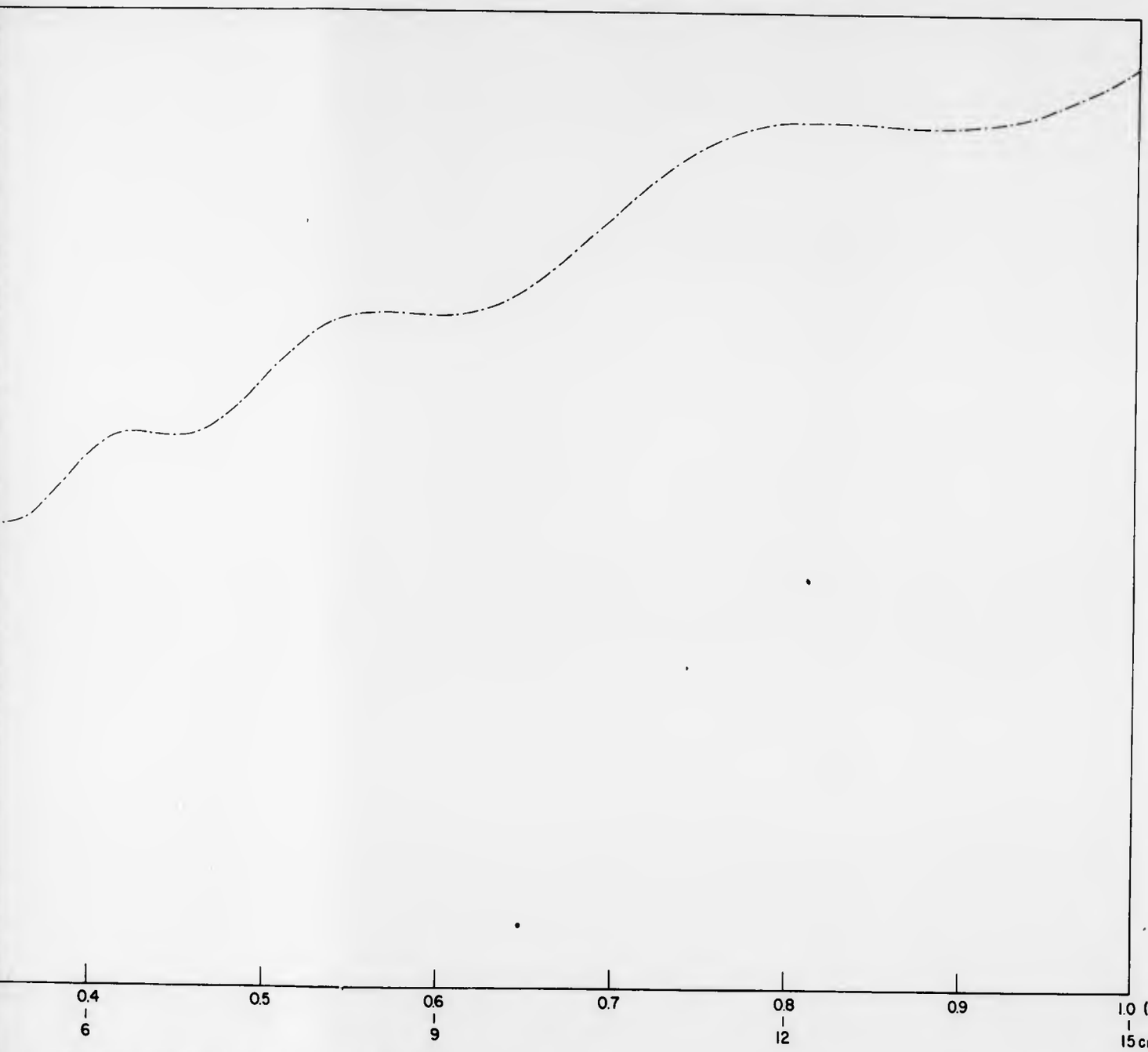


Fig. 4 – Average relative phase $\langle\theta\rangle_{rel}$ vs the distance z along the trans
from Williams' velocity potential approximation for $ka =$



4 – Average relative phase $\langle\theta\rangle_{rel}$ vs the distance z along the transducer axis as derived from Williams' velocity potential approximation for $ka = 20\pi$

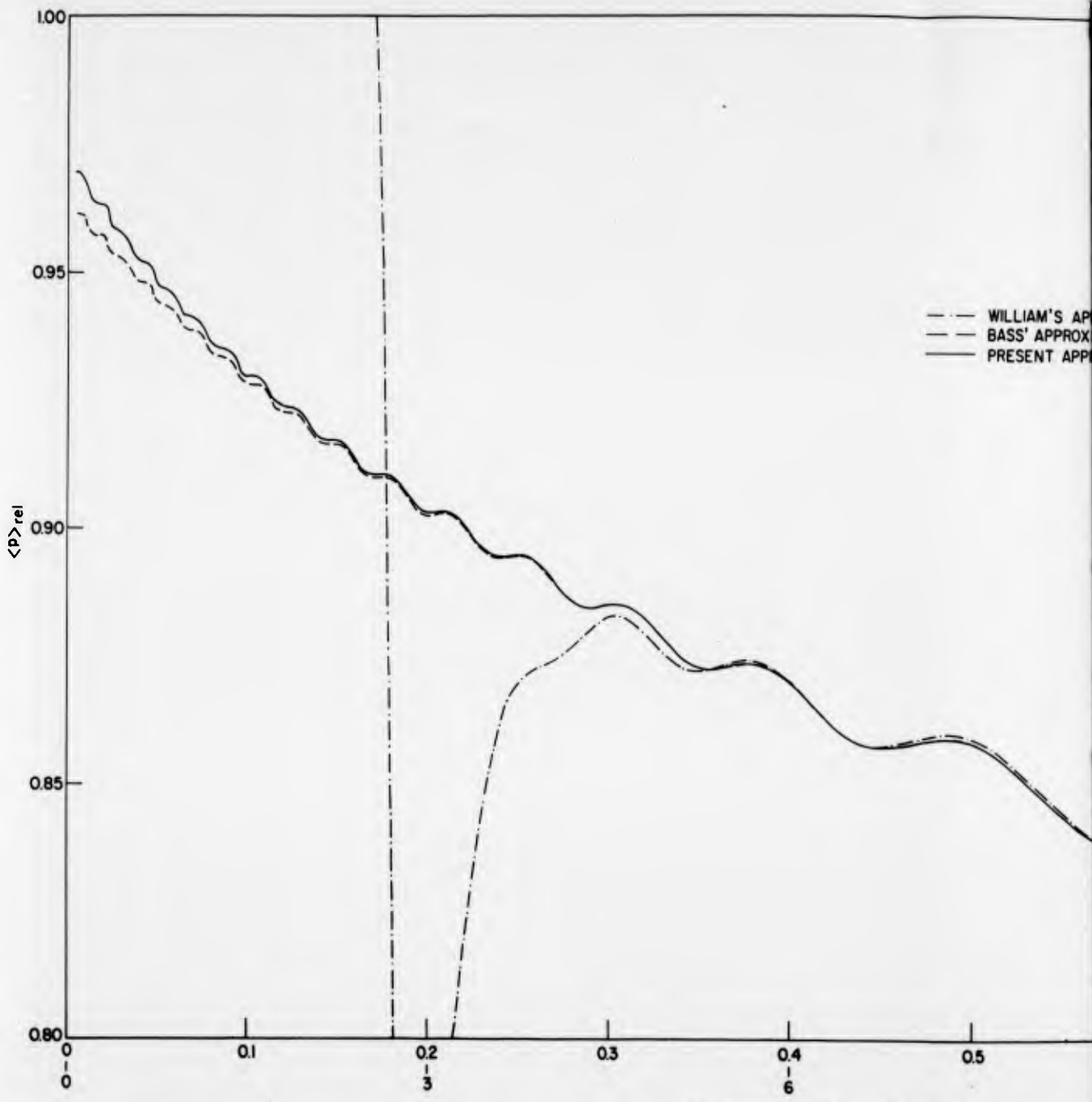


Fig. 5 – Comparison of Williams', Bass', and the present approxi
average relative pressure $\langle p \rangle_{rel}$ vs the distance z along t

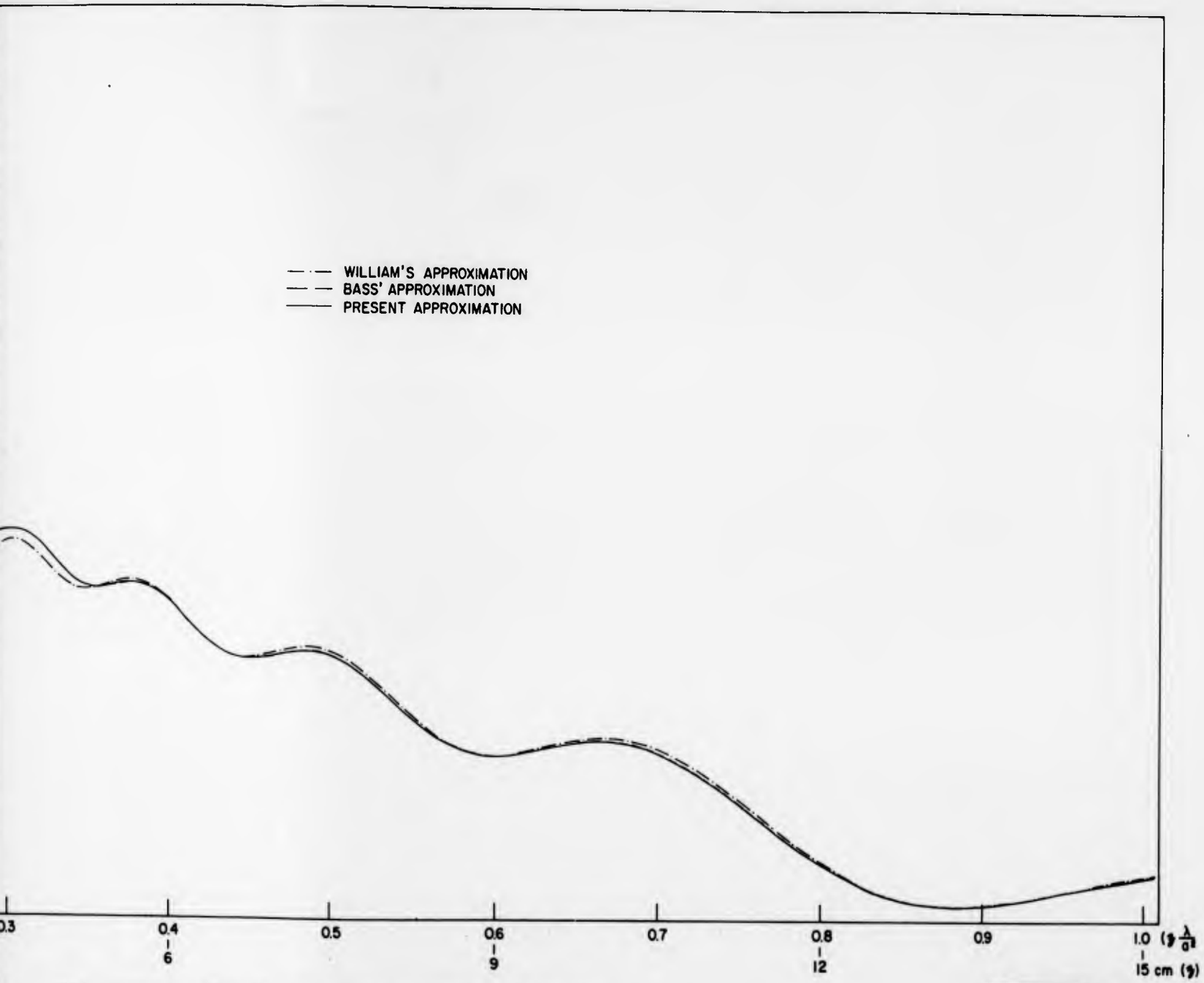


Fig. 5 — Comparison of Williams', Bass', and the present approximation (for $ka = 20\pi$) for the average relative pressure $\langle p \rangle_{rel}$ vs the distance z along the transducer axis

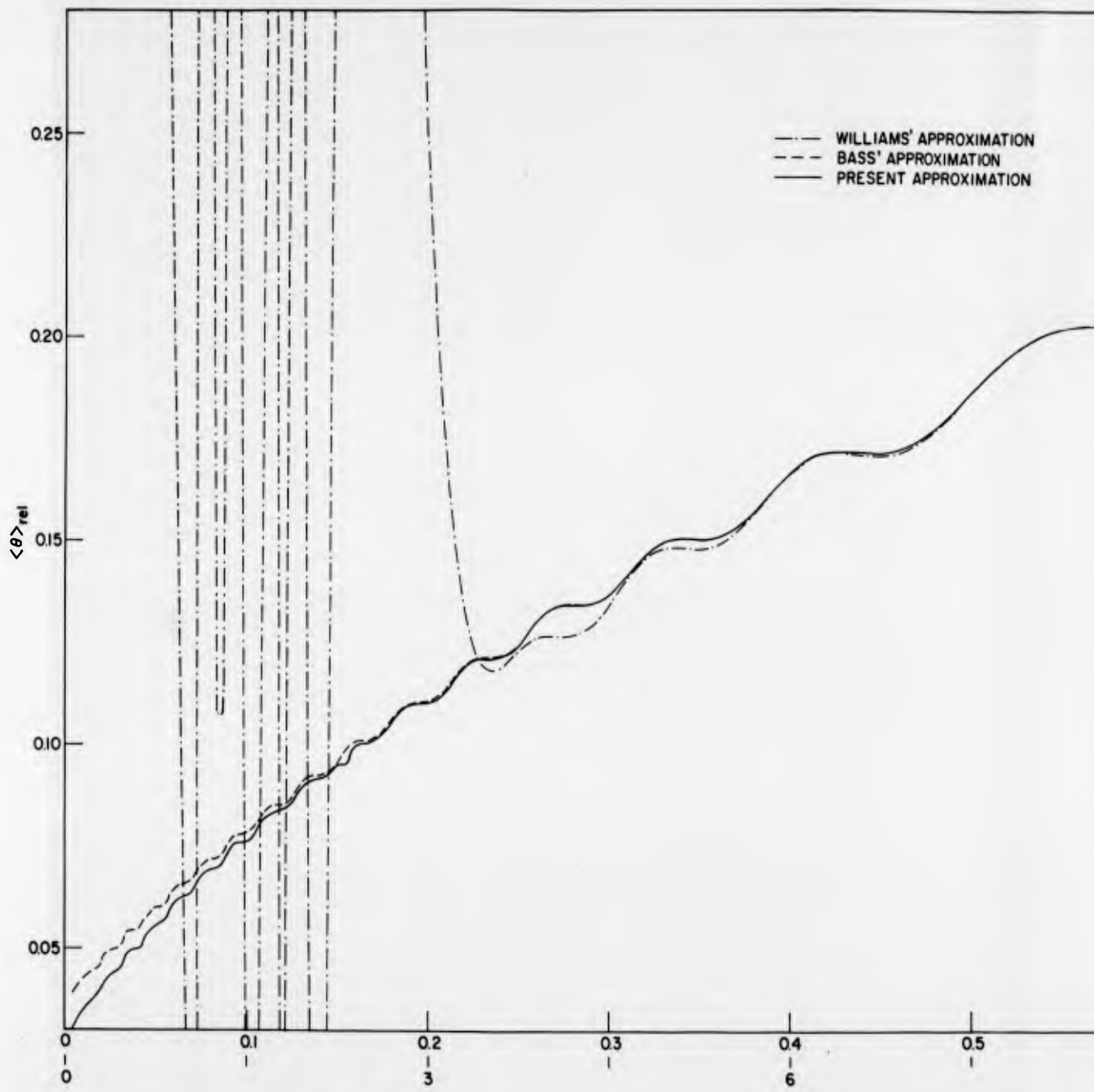
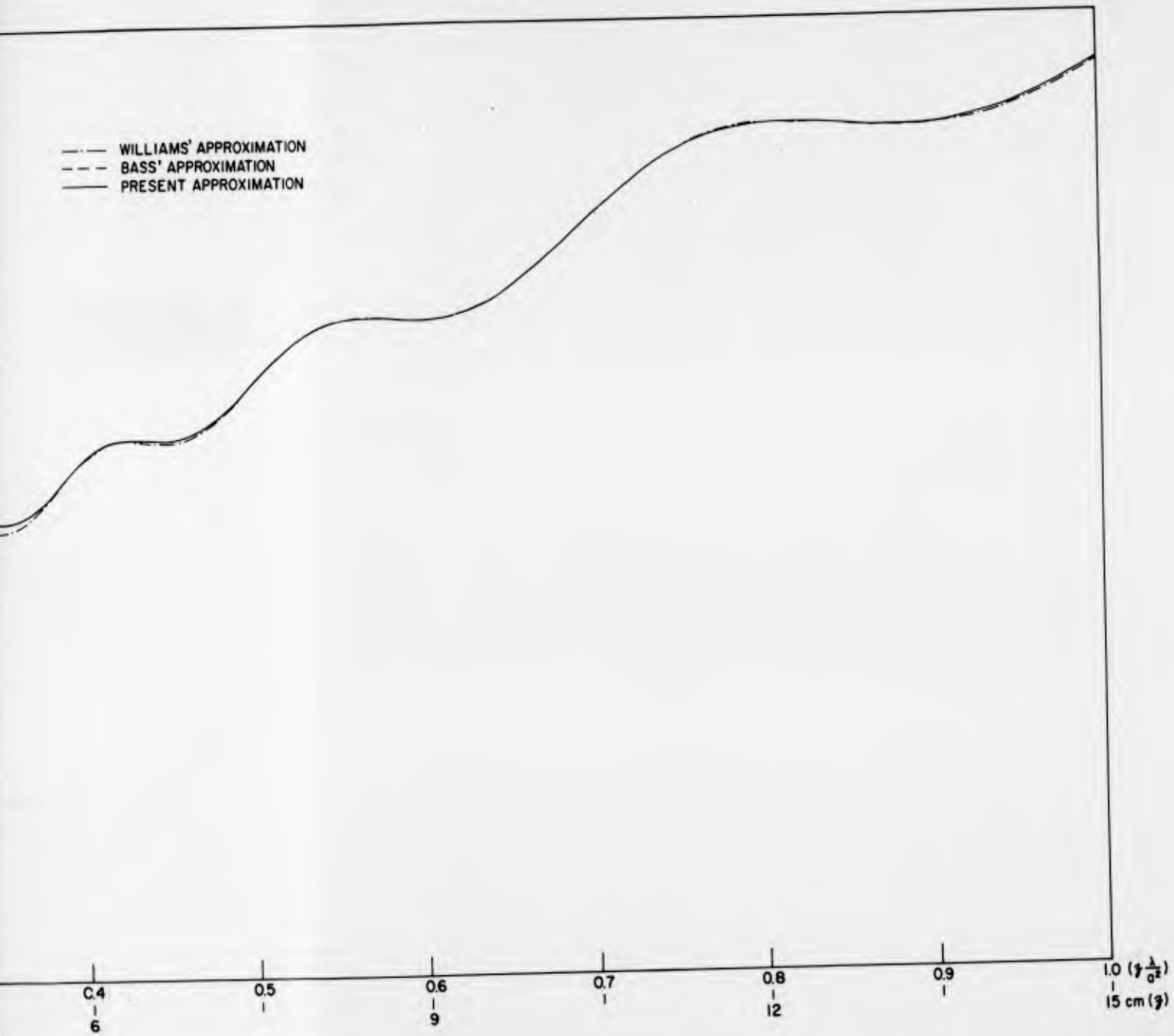


Fig. 6 – Comparison of Williams', Bass', and the present approxi
average relative phase $\langle\theta\rangle_{rel}$ vs the distance z along the



6 – Comparison of Williams', Bass', and the present approximation (for $ka = 20\pi$) for the average relative phase $\langle\theta\rangle_{rel}$ vs the distance z along the transducer axis

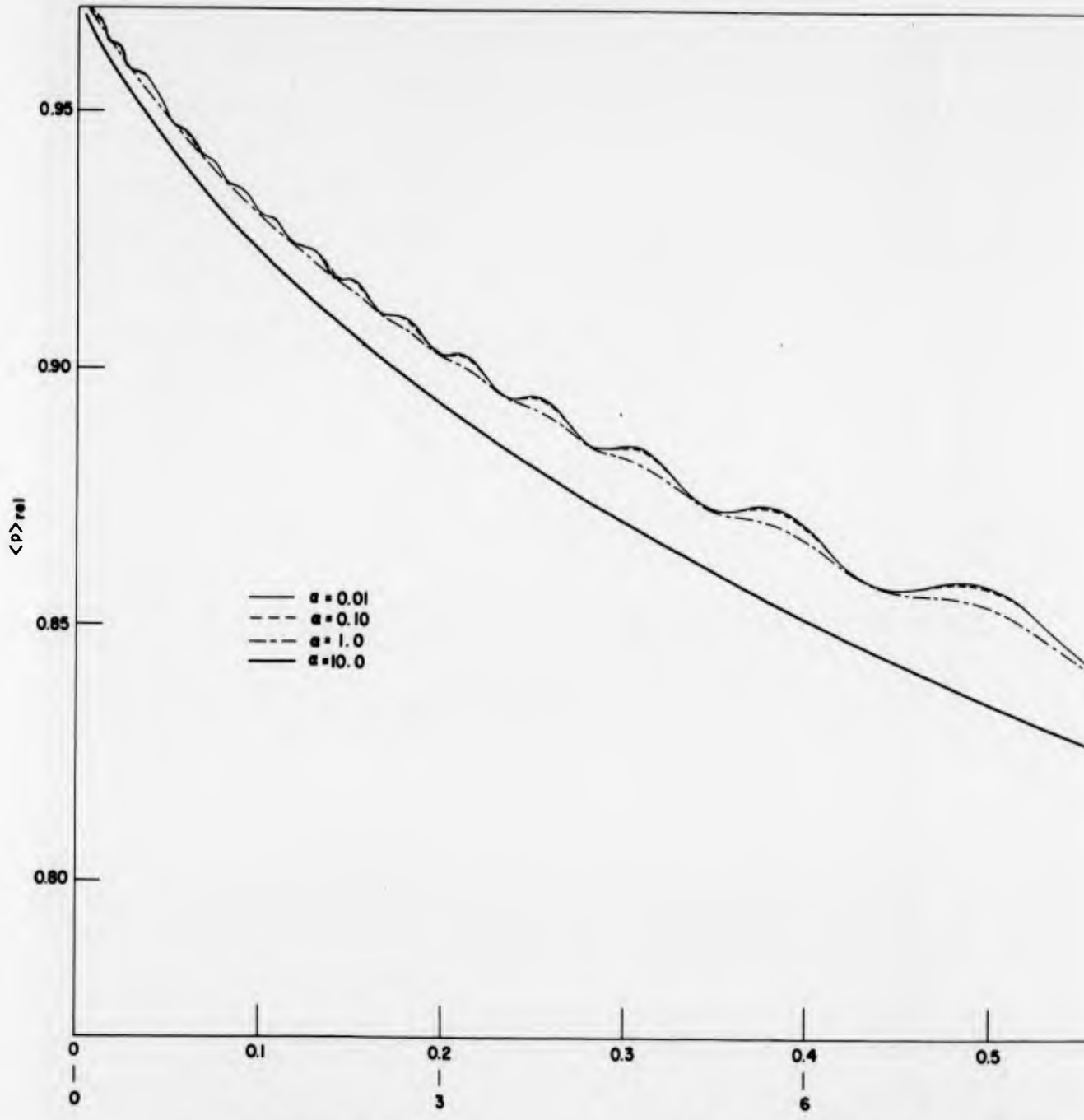


Fig. 7a – Effect of absorption on $\langle p \rangle_{rel}$ over the interval 0
coefficients indicated and for a transducer-crystal p

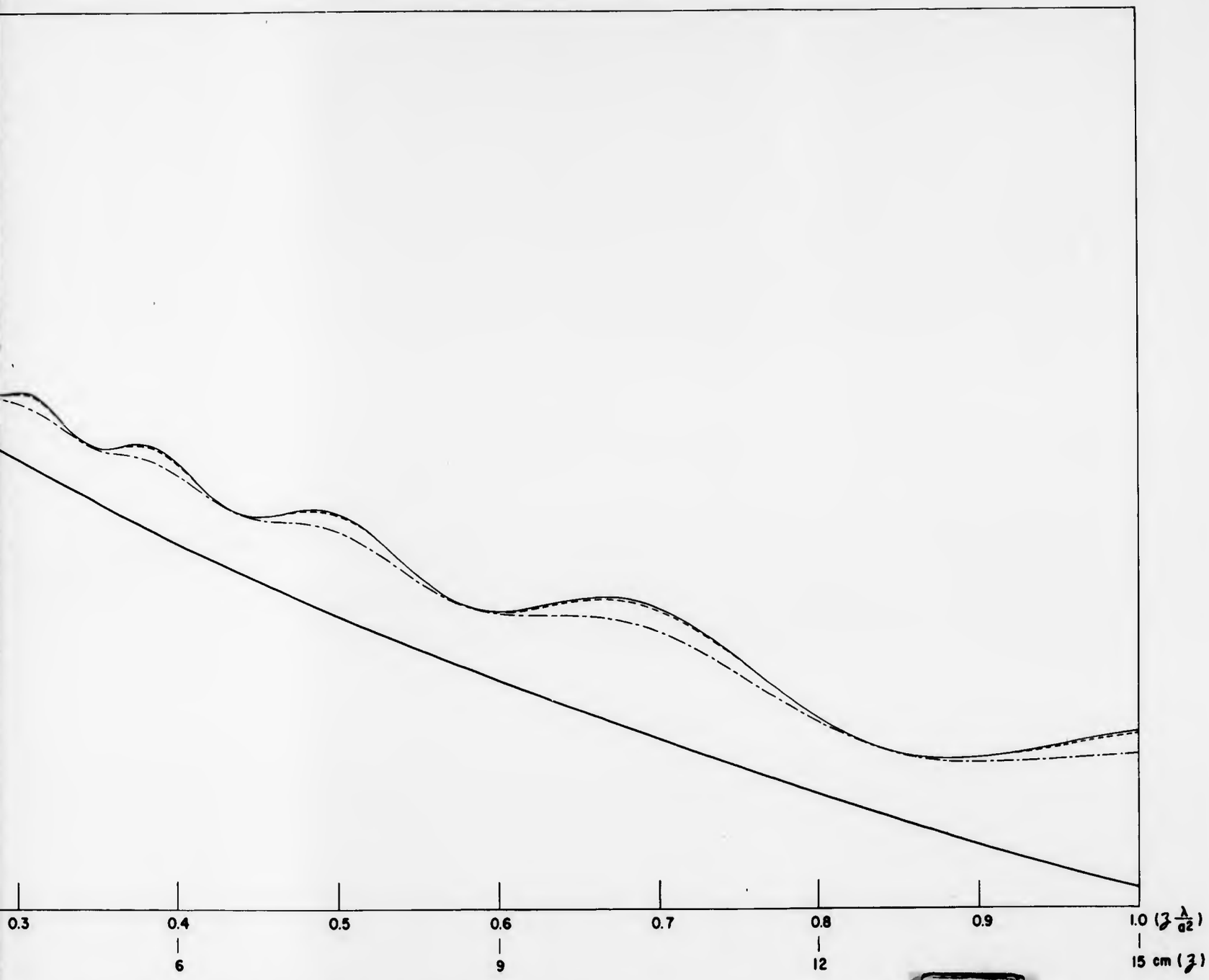


Fig. 7a — Effect of absorption on $\langle p \rangle_{rel}$ over the interval $0 < z \leq 15$ cm for the attenuation coefficients indicated and for a transducer-crystal parameter $ka = 20\pi$

2

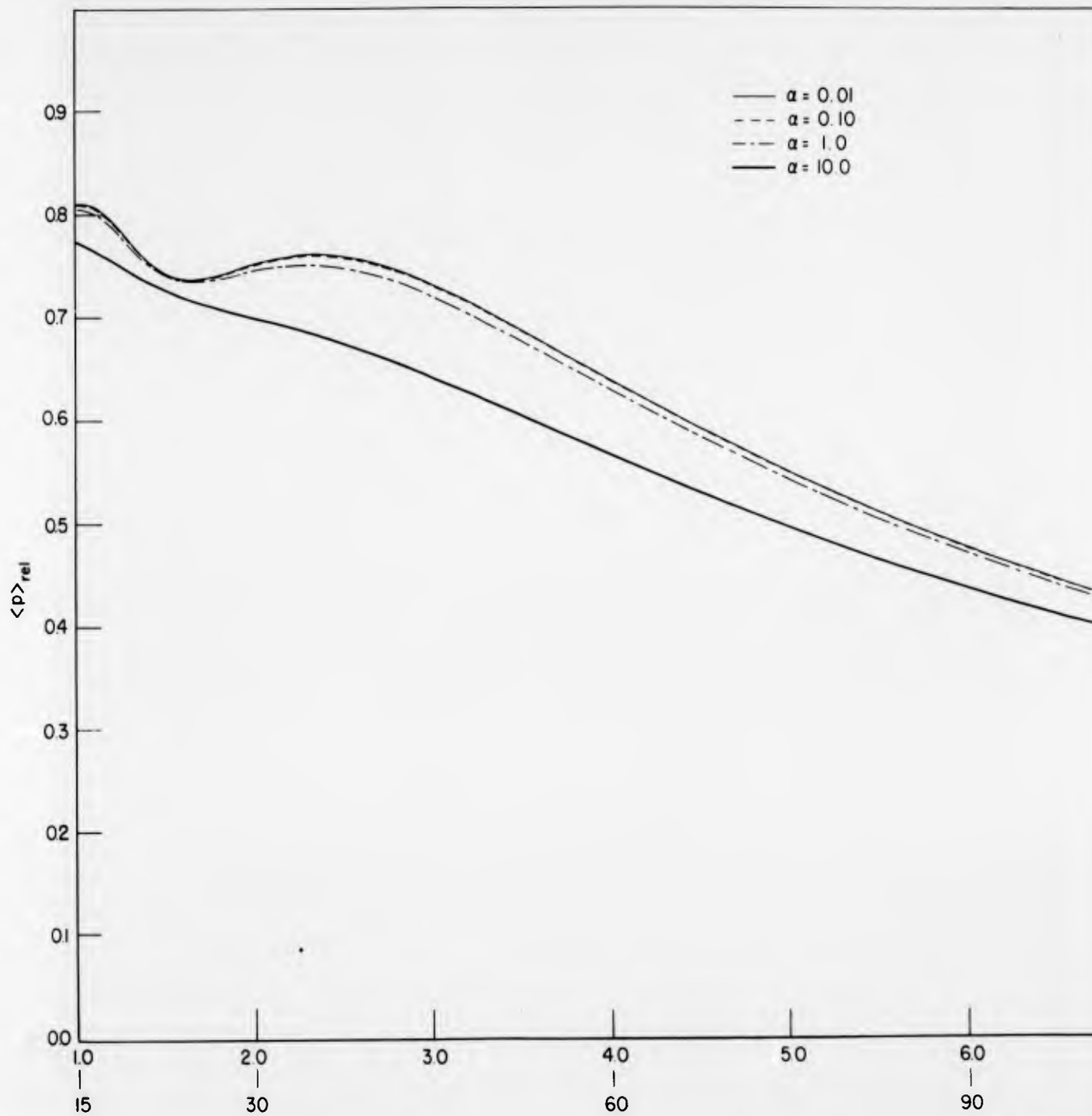
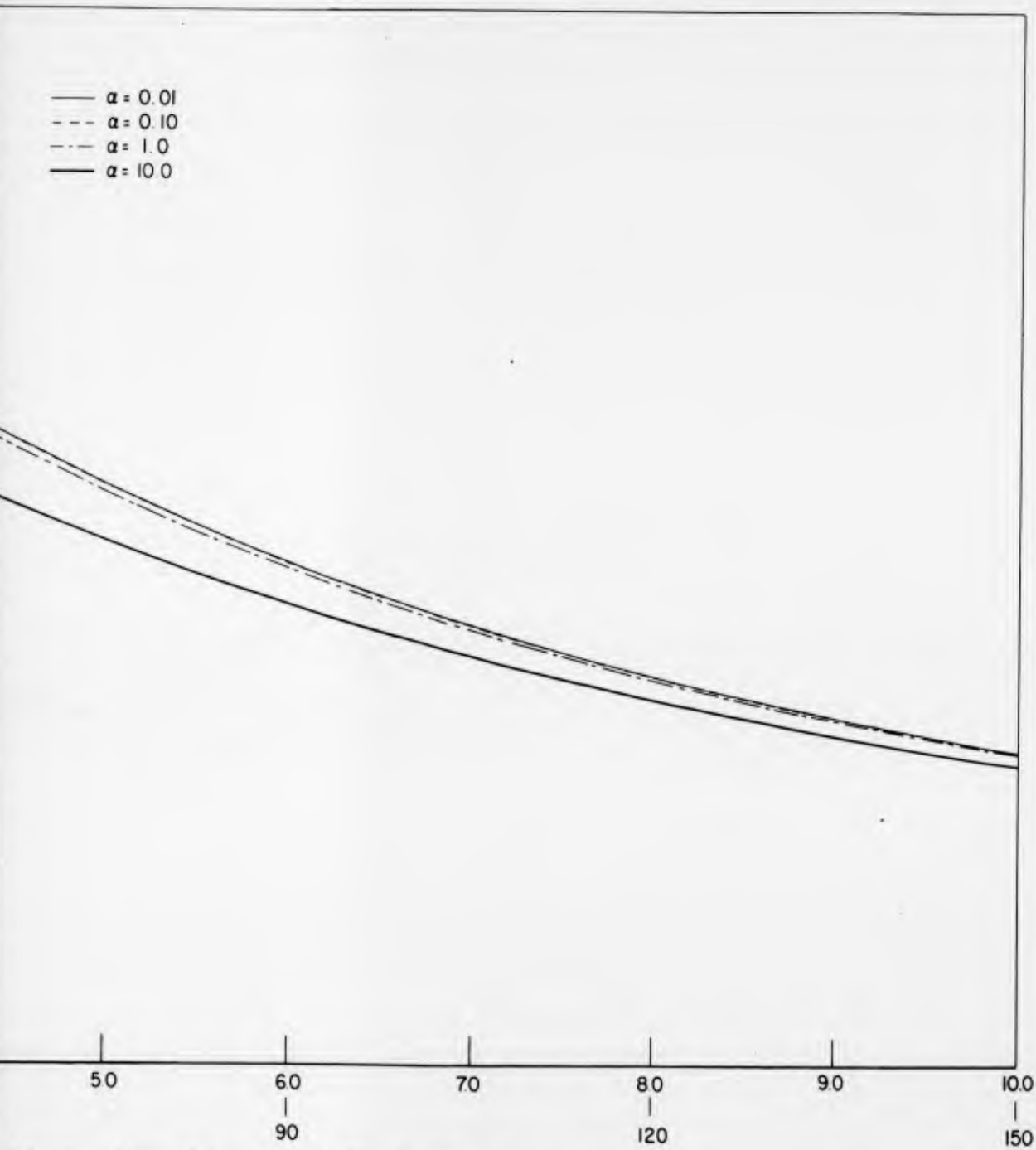


Fig. 7b – Effect of absorption on $\langle p \rangle_{rel}$ over the interval 15 cm $\leq z \leq$ 300 cm fo
coefficients indicated and for a transducer-crystal parameter $ka =$



of absorption on $\langle p \rangle_{rel}$ over the interval 15 cm $\leq z \leq$ 300 cm for the attenuation
ficients indicated and for a transducer-crystal parameter $ka = 20\pi$

2

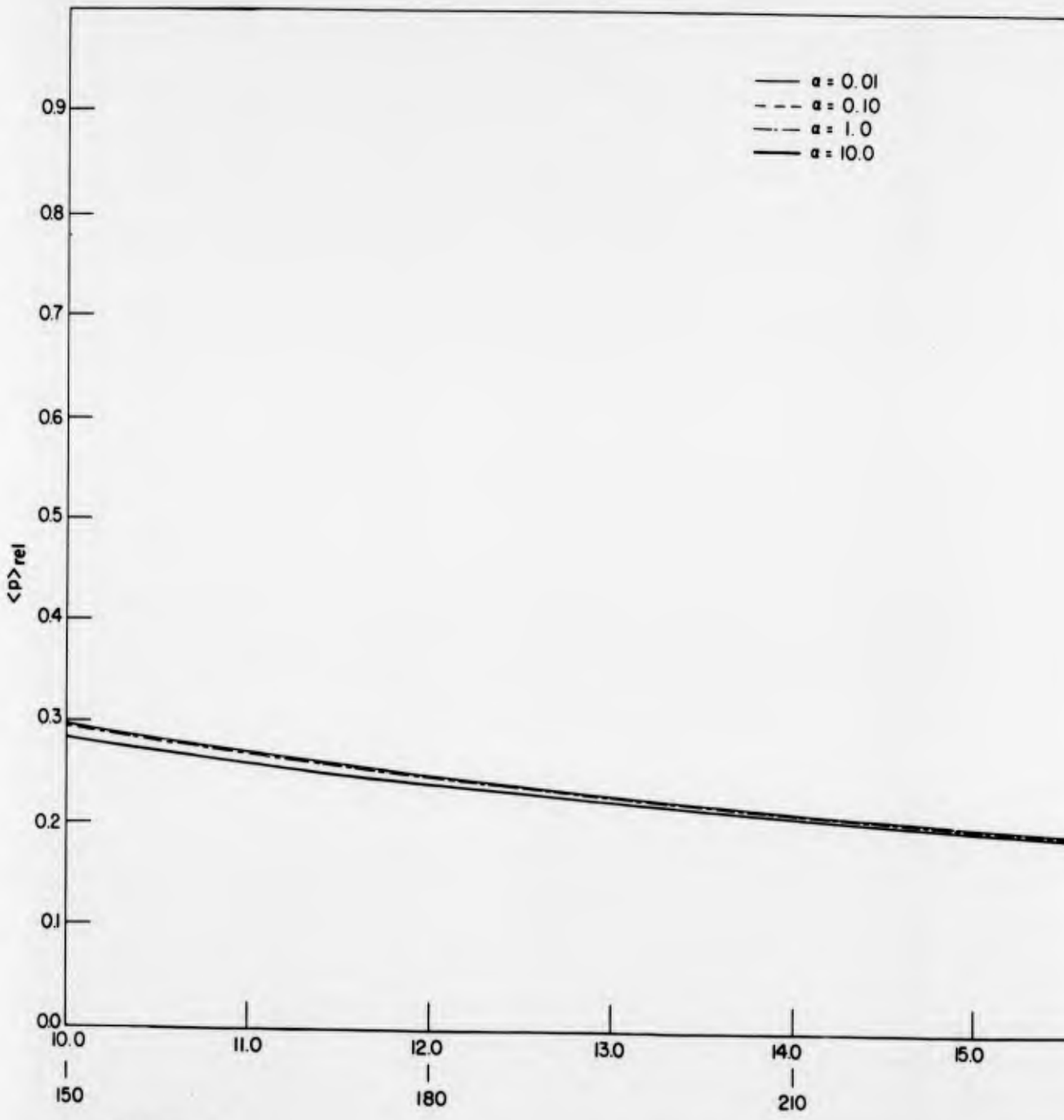


Fig. 7b (continued) – Effect of absorption on $\langle p \rangle_{rel}$ over the interva
coefficients indicated and for a transducer-crysta

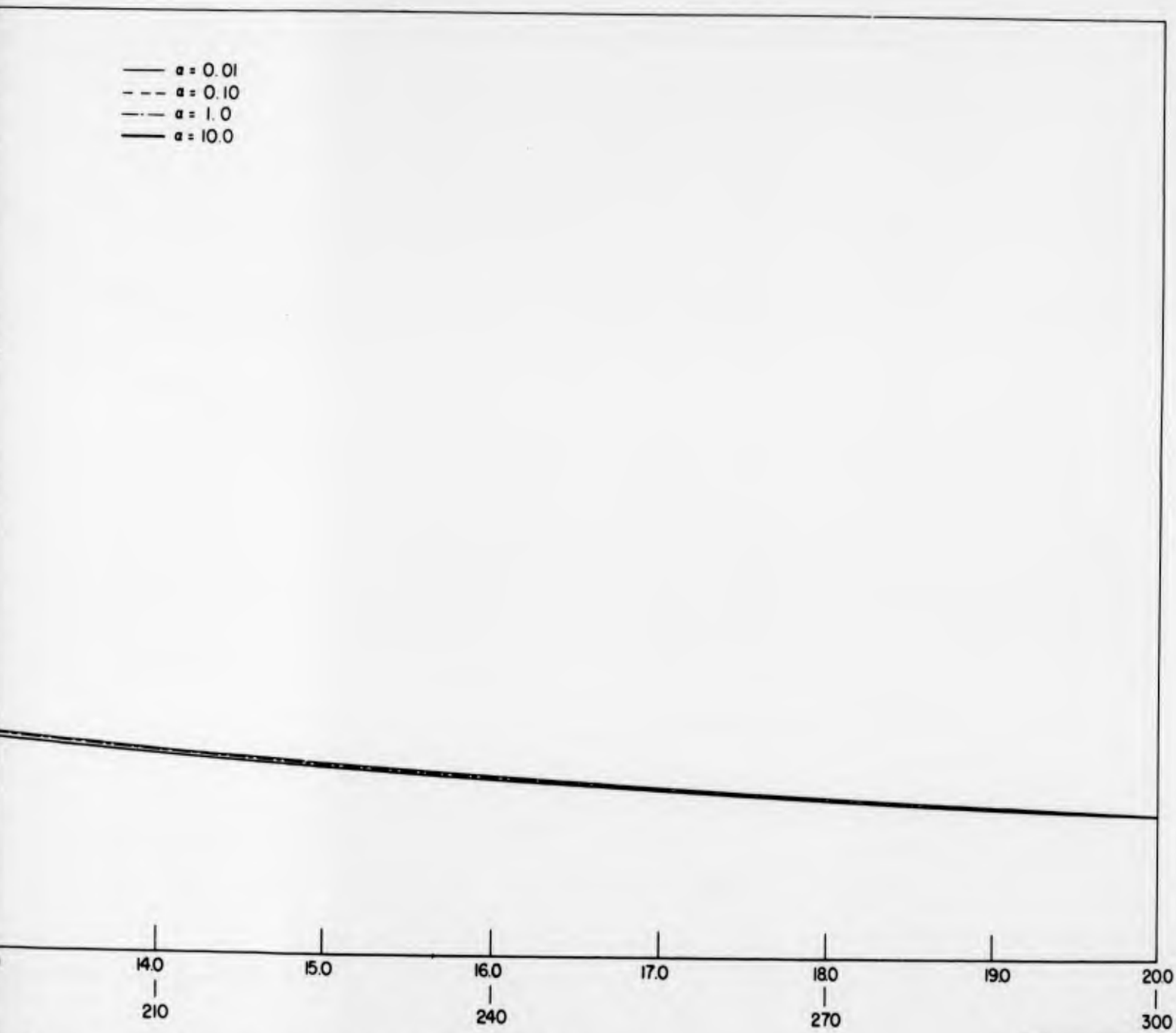


Fig. 7b (continued) – Effect of absorption on $\langle p \rangle_{rel}$ over the interval 15 cm $\leq z \leq$ 300 cm for the attenuation coefficients indicated and for a transducer-crystal parameter $ka = 20\pi$

2

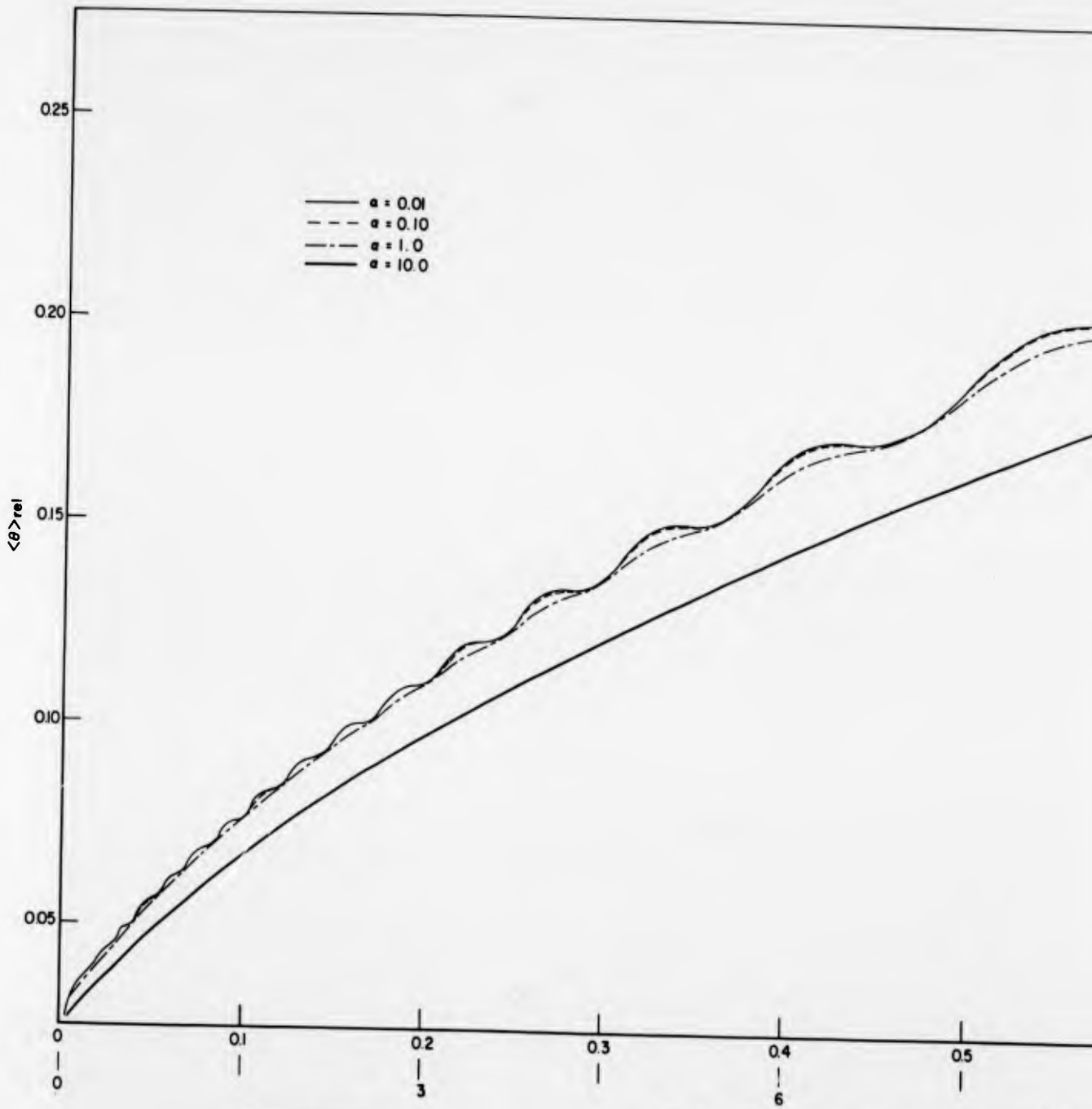


Fig. 8a – Effect of absorption on $\langle\theta\rangle_{rel}$ over the interval $0 < z$ coefficients indicated and for a transducer-crystal par

1

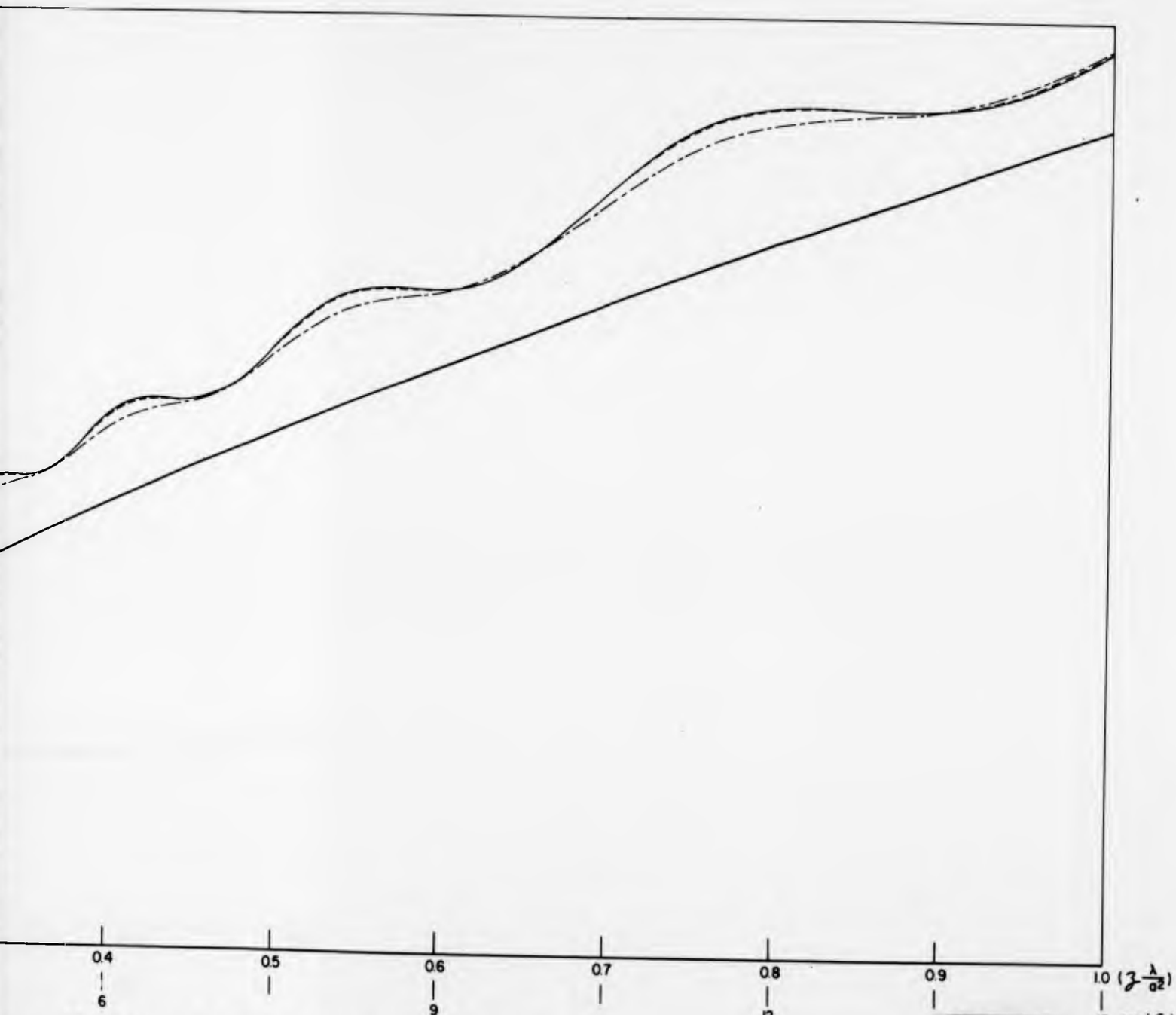


Fig. 8a — Effect of absorption on $\langle\theta\rangle_{rel}$ over the interval $0 < z \leq 15$ cm for the attenuation coefficients indicated and for a transducer-crystal parameter $ka = 20\pi$

2

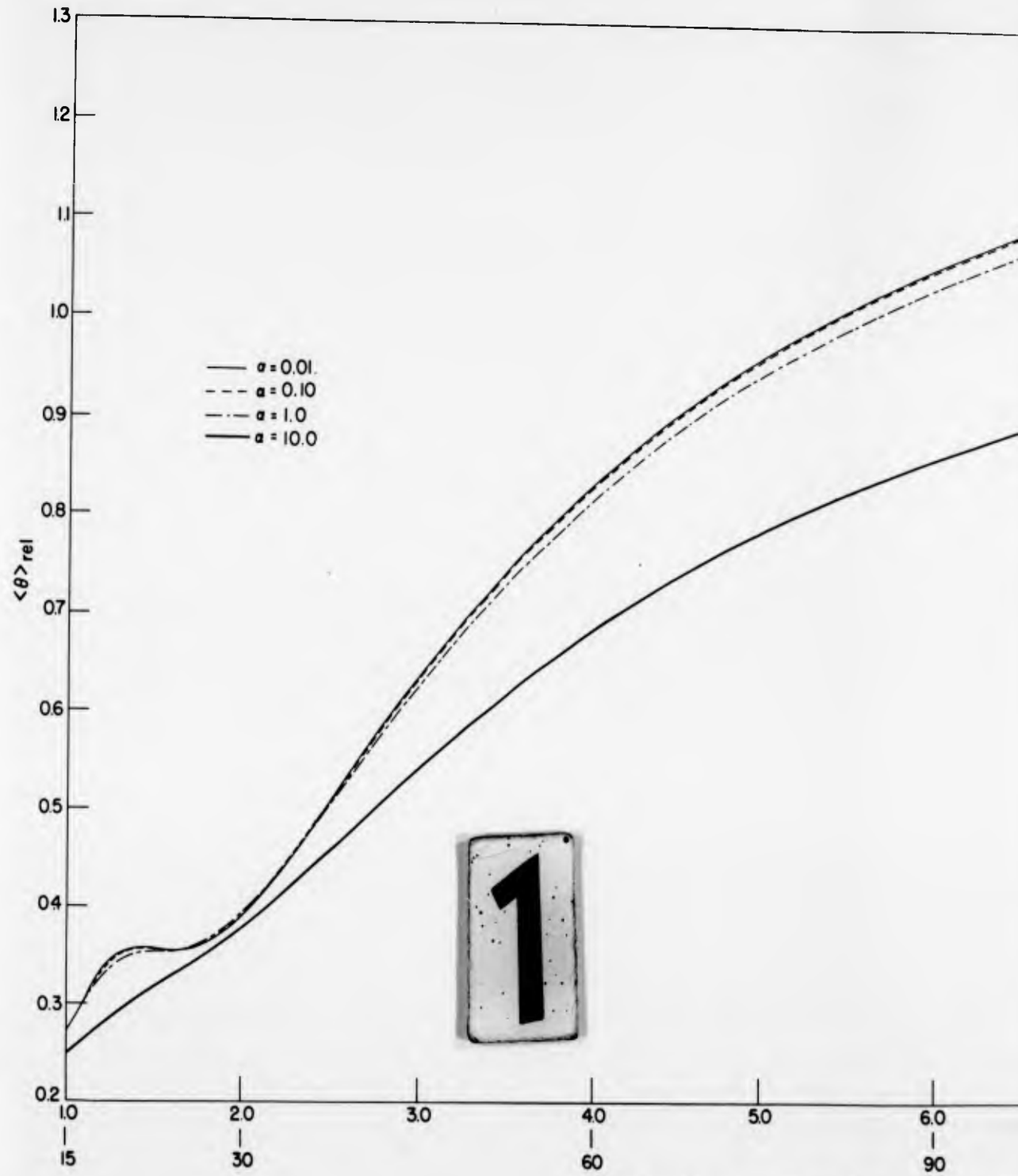


Fig. 8b – Effect of absorption on $\langle\theta\rangle_{rel}$ over the interval 15 cm ≤ z ≤ 300
coefficients indicated and for a transducer-crystal paramete

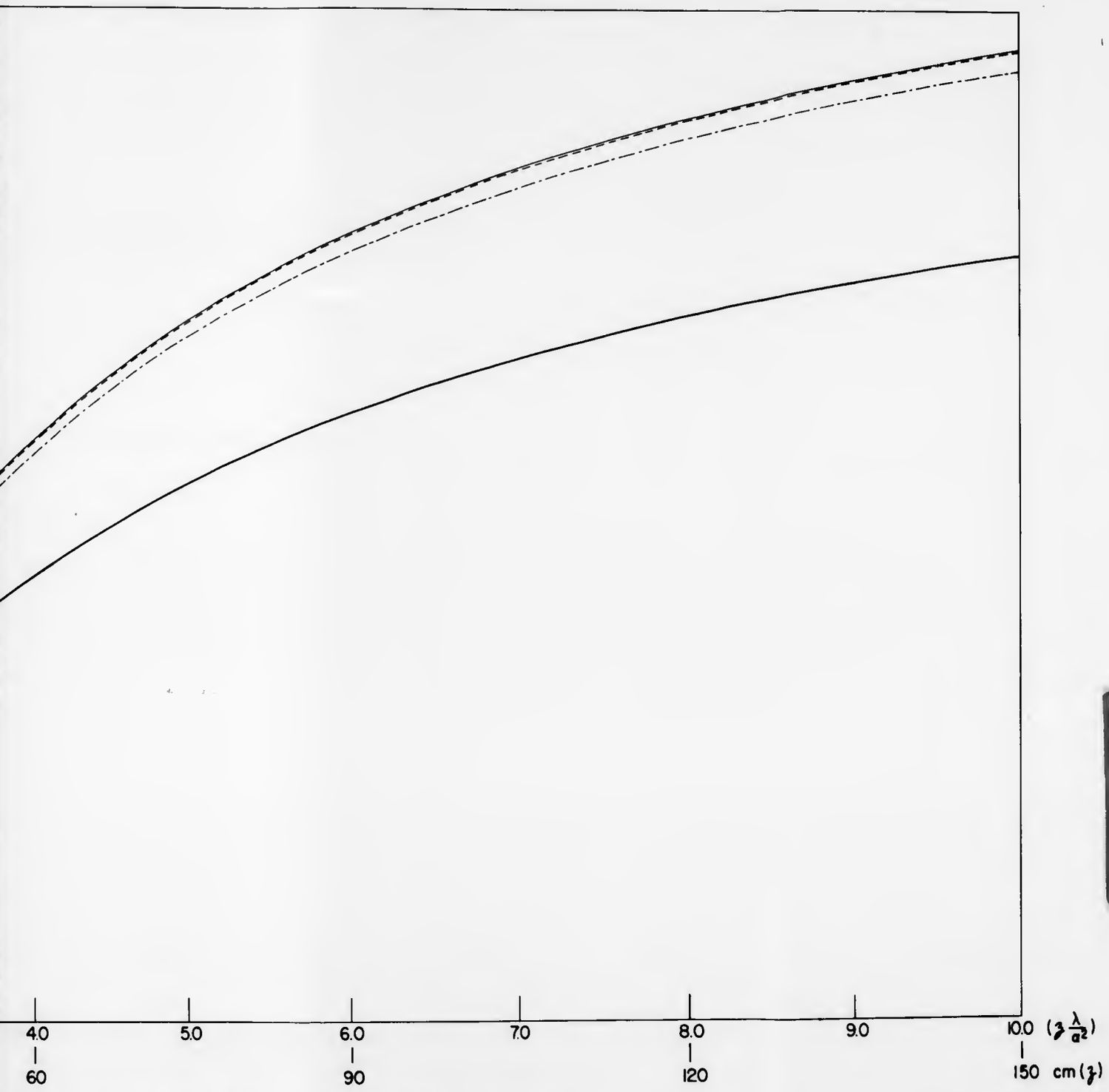


Fig. 8b – Effect of absorption on $\langle\theta\rangle_{rel}$ over the interval 15 cm $\leq z \leq$ 300 cm for the attenuation coefficients indicated and for a transducer-crystal parameter $ka = 20\pi$

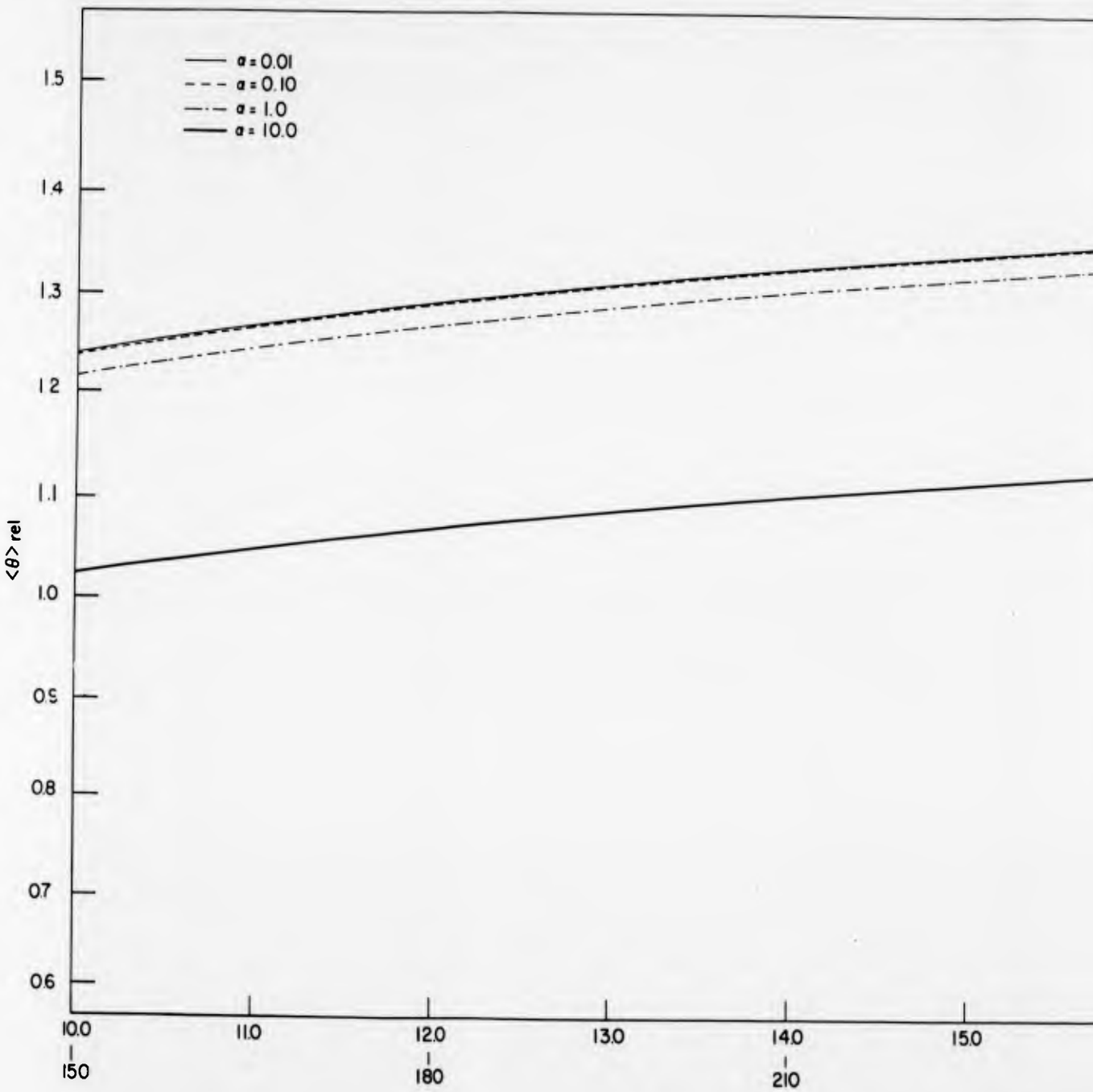
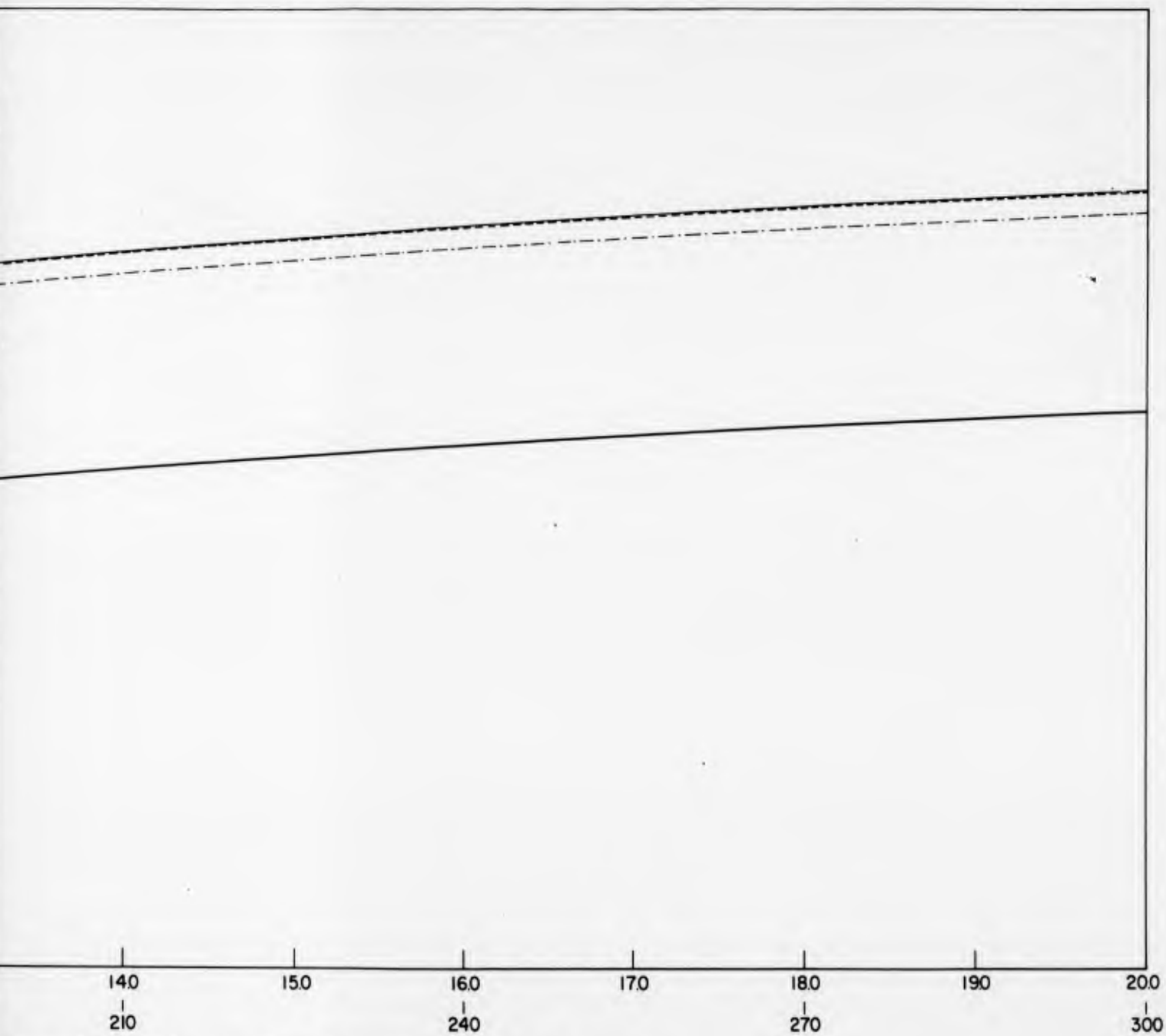


Fig. 8b (continued) – Effect of absorption on $\langle\theta\rangle_{rel}$ over the interval 15
coefficients indicated and for a transducer-crystal par



8b (continued) – Effect of absorption on $\langle \theta \rangle_{rel}$ over the interval 15 cm $\leq z \leq$ 300 cm for the attenuation coefficients indicated and for a transducer-crystal parameter $ka = 20\pi$

CALCULATIONS

Williams indicated that, at least for $ka > 50$, the plots of $\langle p \rangle_{rel}$ for various ka would practically superimpose if the abscissa were $z\lambda/a^2$ rather than z. For Figs. 3 through 8b, calculations were made for $ka = 20\pi$, with $a = 10\lambda$ and $\lambda = 0.15$ cm so that $\lambda/a^2 = 1/15$ cm^{-1}. Thus, the values of z must be divided by 15 to convert them to $z\lambda/a^2$.

Calculations were made, using the NAREC computer, for values of $\langle p \rangle_{rel}$ and $\langle \theta \rangle_{rel}$ versus $z\lambda/a^2$, in increments of 0.005 for $z\lambda/a^2 = 0.005$ up to 1 and in increments of 0.05 for $z\lambda/a^2 = 1$ up to 10, and for ka parameter values between 4π and 100π. The values of $z\lambda/a^2$ for the various ka correspond to different values of z, depending on the actual values of a and λ used. A given value of ka, it should be noted, determines the value of the ratio a/λ since

$$ka = 2\pi\frac{a}{\lambda},$$

but it does not determine the value of either a or λ. For the purpose of these calculations, a and λ values were selected as indicated in Table 1. But note that other possible combinations also apply with a corresponding change in z.

The results of these calculations are shown in Figs. 9 and 10. Figure 9a is a plot of $\langle p \rangle_{rel}$ for $ka = 100\pi$ and 20π and for $z\lambda/a^2$ between 0 and 1; Fig. 9b is an extension to $z\lambda/a^2 = 10$. As Williams indicated, the graphs obviously do almost coincide. Since we are interested in lower frequencies for experiment, even with the extrapolations permitted by the application of pertinent theoretical predictions of, say, chemical relaxation theory, we have also run calculations on NAREC for smaller values of ka. Figure 9c is for $ka = 4\pi$.

TABLE 1
Specific a and λ Values Used for ka Parameter Calculations

ka ($= 2\pi a/\lambda$)	a (cm)	λ (cm)	a^2/λ (cm)	z range* (cm)	z increment (cm)
4π	0.5	0.25	1	0.005-1 1-10	0.005 0.05
6π	0.75	0.25	2.25	0.01125-2.25 2.25-22.5	0.01125 0.1125
8π	0.9	0.225	3.6	0.018-3.6 3.6-36	0.018 0.18
10π	1.0	0.20	5	0.025-5 5-50	0.025 0.25
15π	1.2	0.16	9	0.045-9 9-90	0.045 0.45
20π	1.5	0.15	15	0.075-15 15-150	0.075 0.75
30π	1.8	0.12	27	0.135-27 27-270	0.135 1.35
40π	2.0	0.10	40	0.2-40 40-400	0.2 2.0
60π	2.4	0.08	72	0.360-72 72-720	0.36 3.6
80π	2.4	0.06	96	0.480-96 96-960	0.48 4.8
100π	2.5	0.05	125	0.625-125 125-1250	0.625 6.25

*Short ranges correspond to $z\lambda/a^2$ from 0.005 to 1 in increments of 0.005. Long ranges correspond to $z\lambda/a^2$ from 1 to 10 in steps of 0.05.

6π, 8π, 10π, and 15π for $0 < z\lambda/a^2 \leq 1$ and Fig. 9d is the same for $1 \leq z\lambda/a^2 \leq 10$. Obviously for these values applicable to small values of a/λ (small crystal radii and/or long wavelength), it becomes necessary to have a separate calculation for each small range of ka.

Figure 10a is a plot of $\langle\theta\rangle_{rel}$ for $0 < z\lambda/a^2 \leq 1$ for $ka = 20\pi$ and 100π, and Fig. 10b is the same for $1 \leq z\lambda/a^2 \leq 10$. Here we find that the prediction of Williams for $\langle p\rangle_{rel}$ is borne out for our $\langle\theta\rangle_{rel}$. Figure 10c is a plot of $\langle\theta\rangle_{rel}$ for $0 < z\lambda/a^2 \leq 1$ for $ka = 4\pi$, 6π, 8π, 10π, and 15π, and Fig. 10d is the same for $1 \leq z\lambda/a^2 \leq 10$. Again we note that separate calculations are required for each small incremental variation in ka in this range of the parameter. It should be noted that while the curve separations appear comparable for both the $\langle p\rangle_{rel}$ and $\langle\theta\rangle_{rel}$ plots, because the former begin at unity and the latter at zero, the percentage variation in ordinate for a change in the parameter is appreciably greater for the $\langle\theta\rangle_{rel}$ plots than for the $\langle p\rangle_{rel}$ plots. For example, at $z\lambda/a^2 = 0.005$ the percentage variation between $ka = 4\pi$ and $ka = 100\pi$ is 4 percent for $\langle p\rangle_{rel}$, but 33 percent for $\langle\theta\rangle_{rel}$ (100π is used as the base since this is essentially the ∞ value). Described another way, the $\langle p\rangle_{rel}$ value at $ka = 4\pi$ is 0.96 of the 100π value but the $\langle\theta\rangle_{rel}$ value at 4π is 4.37 times the 100π value.

APPLICATION TO SOUND ABSORPTION MEASUREMENTS

The application of these results to absorption measurements has been made for some time at NRL. Dunn and Breyer (10) refer to a 1-Mc diffraction-corrected reading of ours which fits in nicely with higher frequency measurements of theirs. Indeed, in an earlier report (1) it was pointed out that the reaction curve maxima, which yield large errors in absorption measurements and display superimposed maxima and minima (see Fig. 11, which is Fig. 26 of Ref. 1) rather than a simple exponential or hyperbolic cotangent form, must be explained on the basis of diffraction.

The method of application of diffraction corrections to absorption measurements is almost intuitive and can be derived quite simply. An amplitude absorption coefficient α is defined by the simple differential equation relating a change in amplitude A to a change in distance x

$$dA = -\alpha A\, dx \tag{11a}$$

which yields

$$A_2 = A_1 e^{-\alpha(x_2 - x_1)} \tag{11b}$$

Similarly, for sufficiently small distance intervals, we may define a diffraction loss and coefficient

$$D_2 = D_1 e^{-\delta(x_2 - x_1)} \tag{12}$$

and a measured loss and coefficient

$$N_2 = N_1 e^{-\nu(x_2 - x_1)}. \tag{13}$$

Assuming no interaction, we write

$$\nu = \alpha + \delta, \tag{14}$$

which directly yields

$$\left(\frac{N_2}{N_1}\right)\left(\frac{D_1}{D_2}\right) = \frac{A_2}{A_1}, \tag{15a}$$

and α is calculated simply from

$$\alpha = \frac{1}{(x_2 - x_1)} \ln\left(\frac{A_1}{A_2}\right). \tag{15b}$$

In our earlier absorption work, the crystal mount employed did not permit an adequate approximation to a flush crystal which was both closely fit in a hole in an infinite baffle and yet free to slip; so the results indicated an effective crystal radius somewhat smaller than the actual one. This crystal radius was determined experimentally by utilizing a liquid (distilled water) of known absorption coefficient to determine the proper ka curve for diffraction corrections. Actually, however, we note that

$$\left(\frac{N_1}{N_2}\right)\left(\frac{D_2}{D_1}\right) = \text{constant} \tag{16a}$$

or

$$\frac{N_1}{N_2} = \text{constant} \times \left(\frac{D_1}{D_2}\right) \tag{16b}$$

so that both the diffraction curve and the measured curve maxima and minima should coincide when plotted versus distance. This fact was utilized for identifying the proper a or λ/a^2 factor.

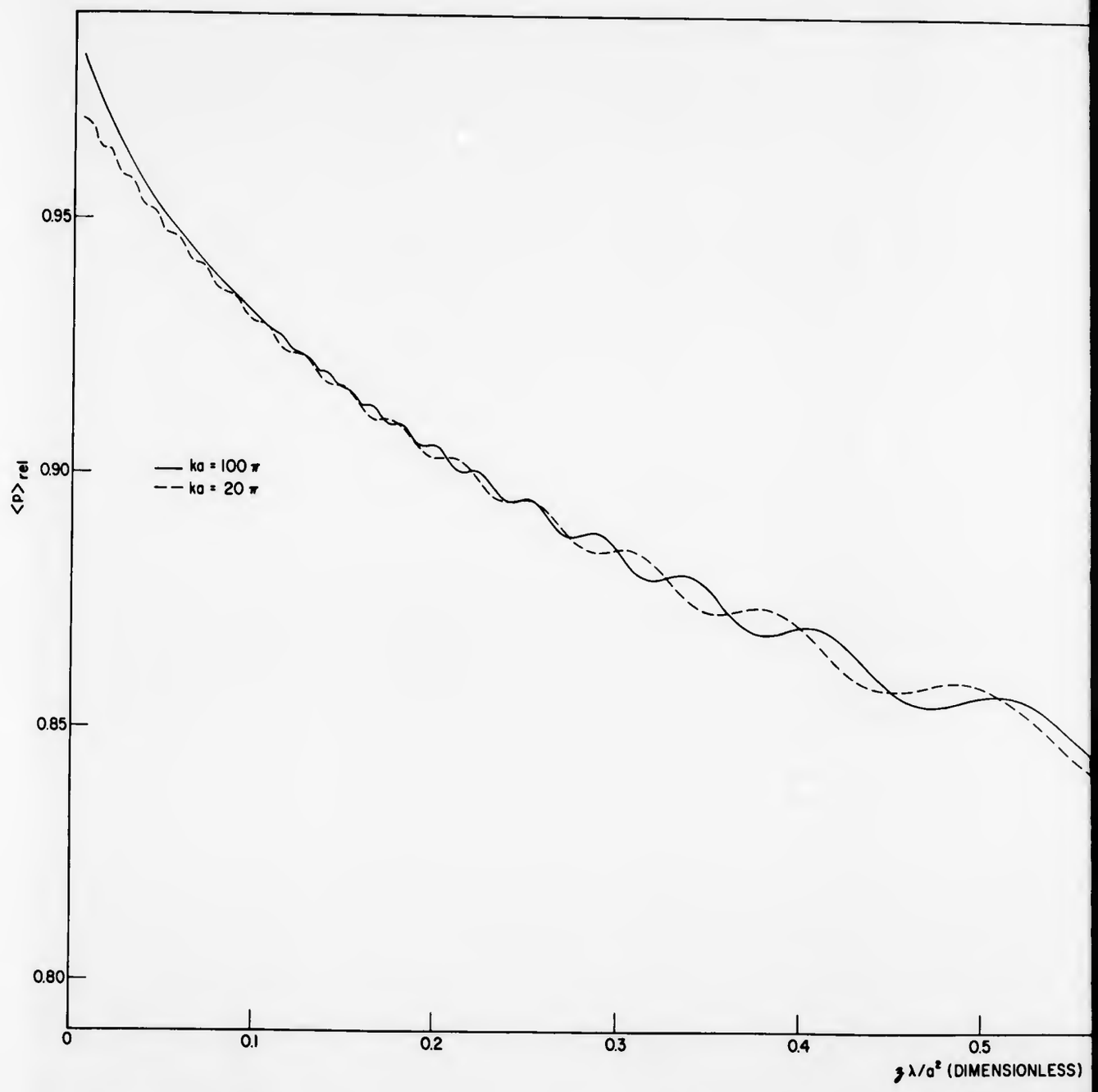


Fig. 9a – Average relative pressure $\langle p \rangle_{rel}$ as a function $z\lambda/a^2$ ov
for the transducer-crystal parameters $ka = 20\pi$

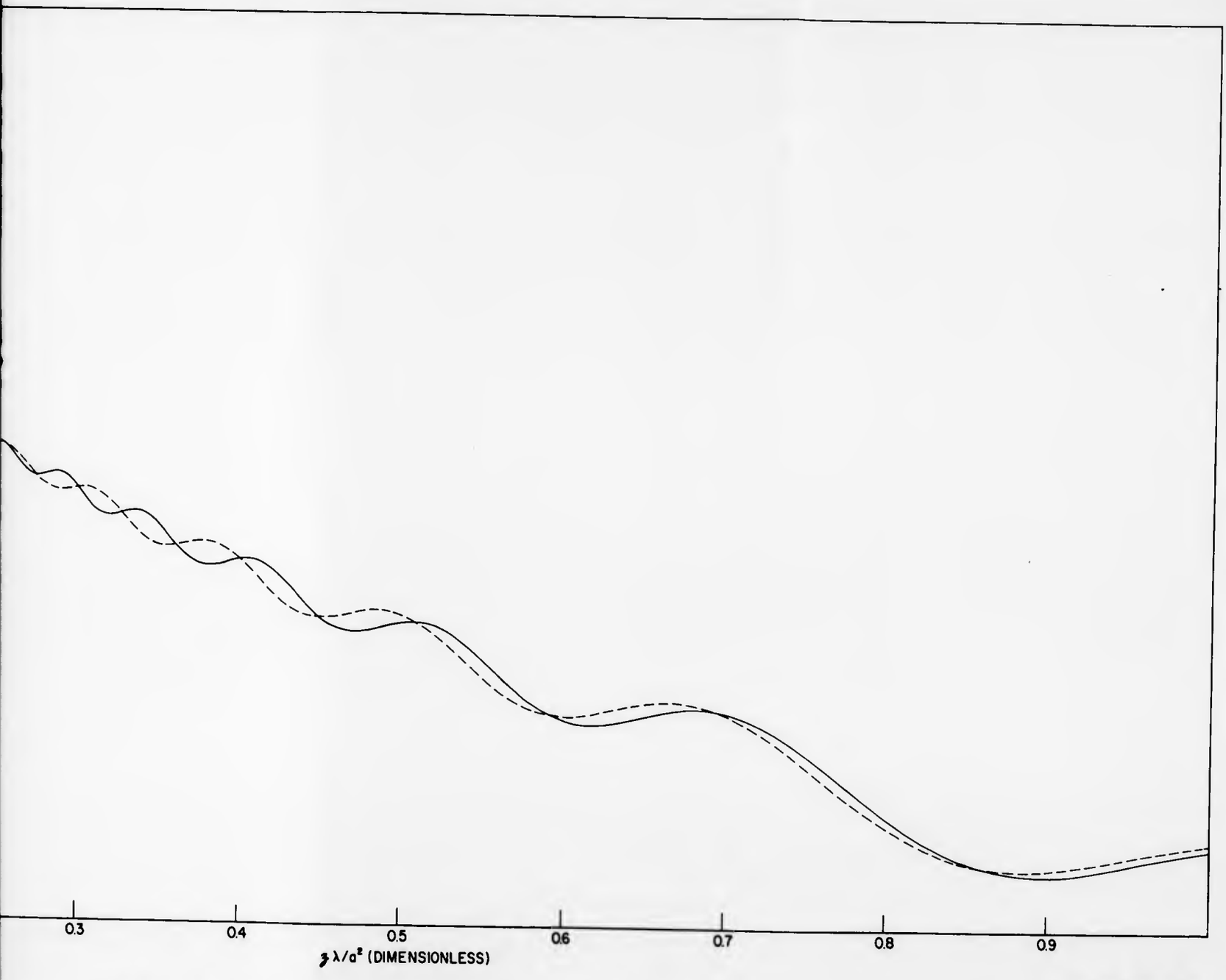


Fig. 9a — Average relative pressure $\langle p \rangle_{rel}$ as a function $z\lambda/a^2$ over the range $0 < z\lambda/a^2 \leq 1$ for the transducer-crystal parameters $ka = 20\pi$ and 100π

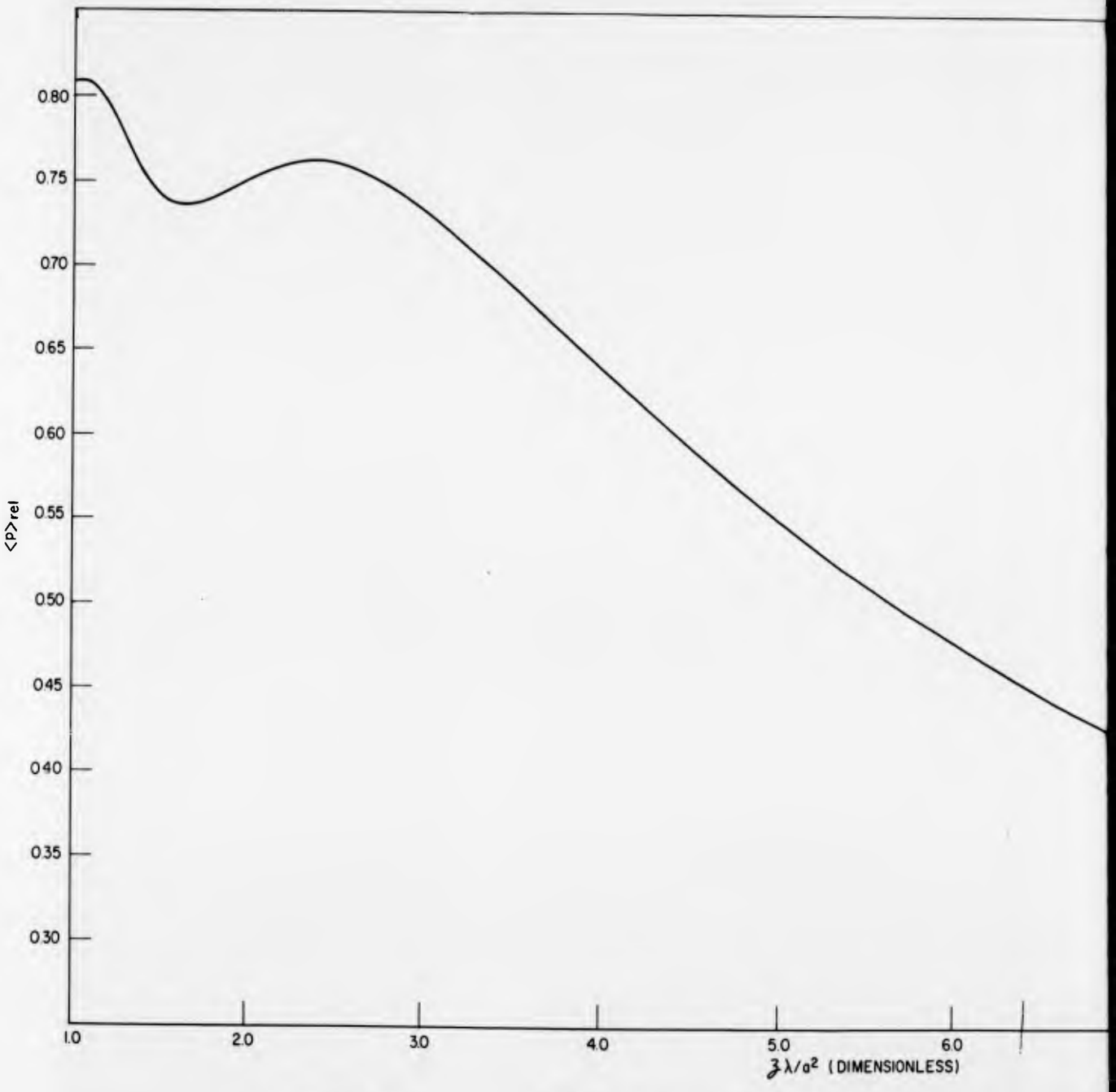
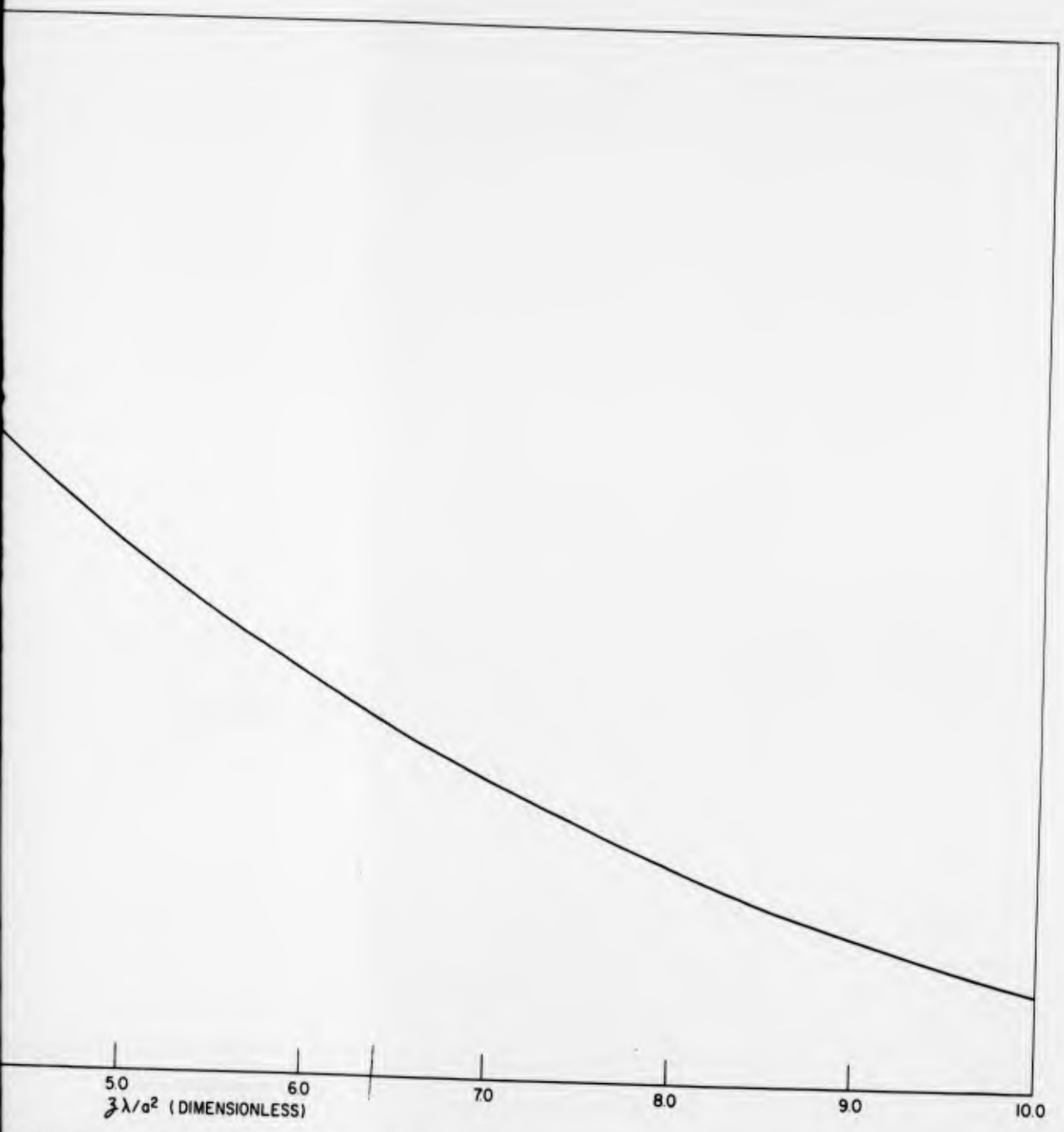


Fig. 9b — Average relative pressure $\langle p \rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $1 \leq z\lambda/a^2$ for the transducer-crystal parameters $ka = 20\pi$ and 100π. Over this range of values the curves are indistinguishable.



elative pressure $\langle p \rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $1 \leq z\lambda/a^2 \leq 10$ ystal parameters $ka = 20\pi$ and 100π. Over this range of values the two ishable.

2

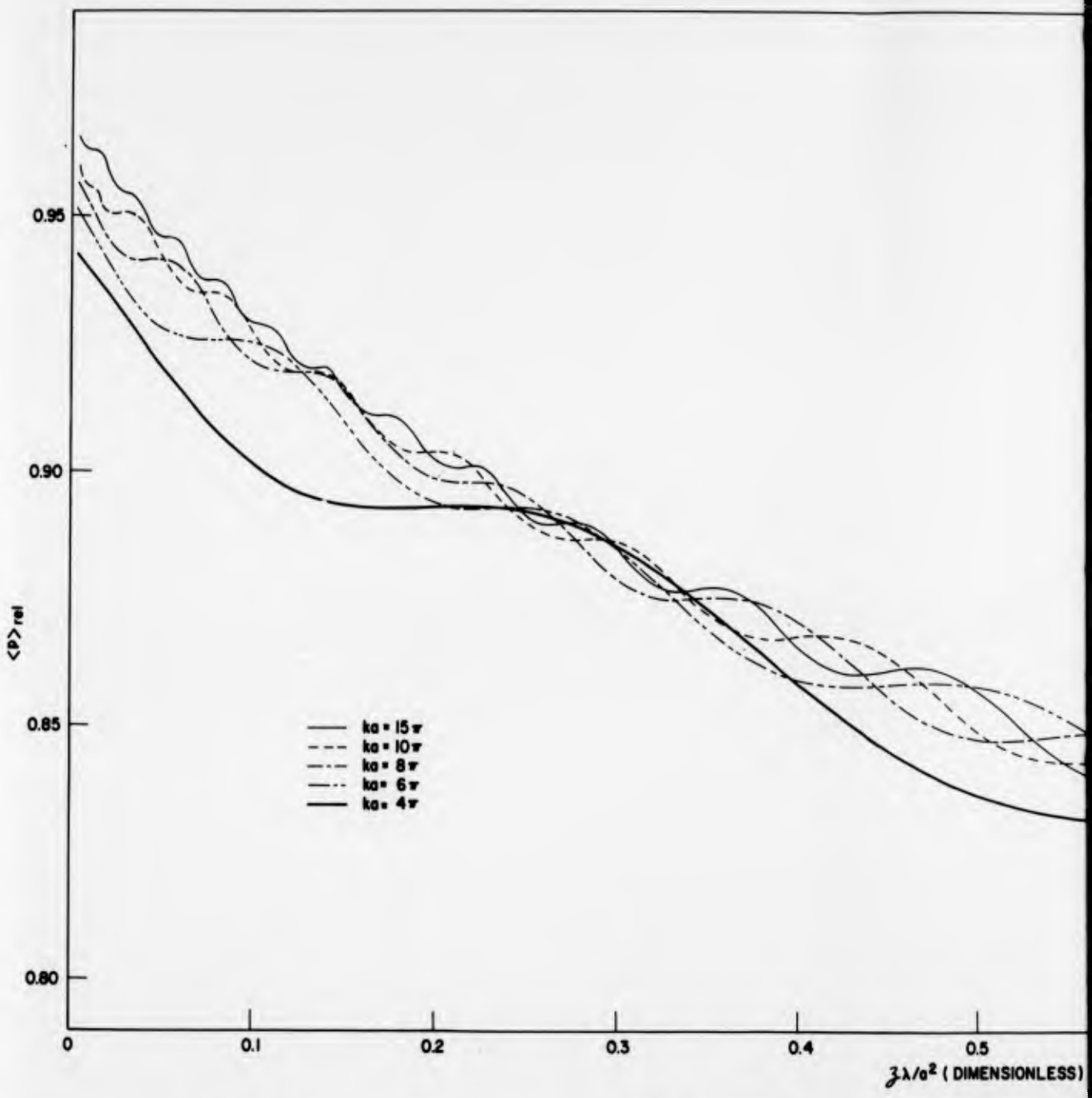


Fig. 9c — Average relative pressure $\langle p \rangle_{rel}$ as a function of $z\lambda/a^2$ ov
for the transducer-crystal parameters $ka = 4\pi, 6\pi, 8\pi,$

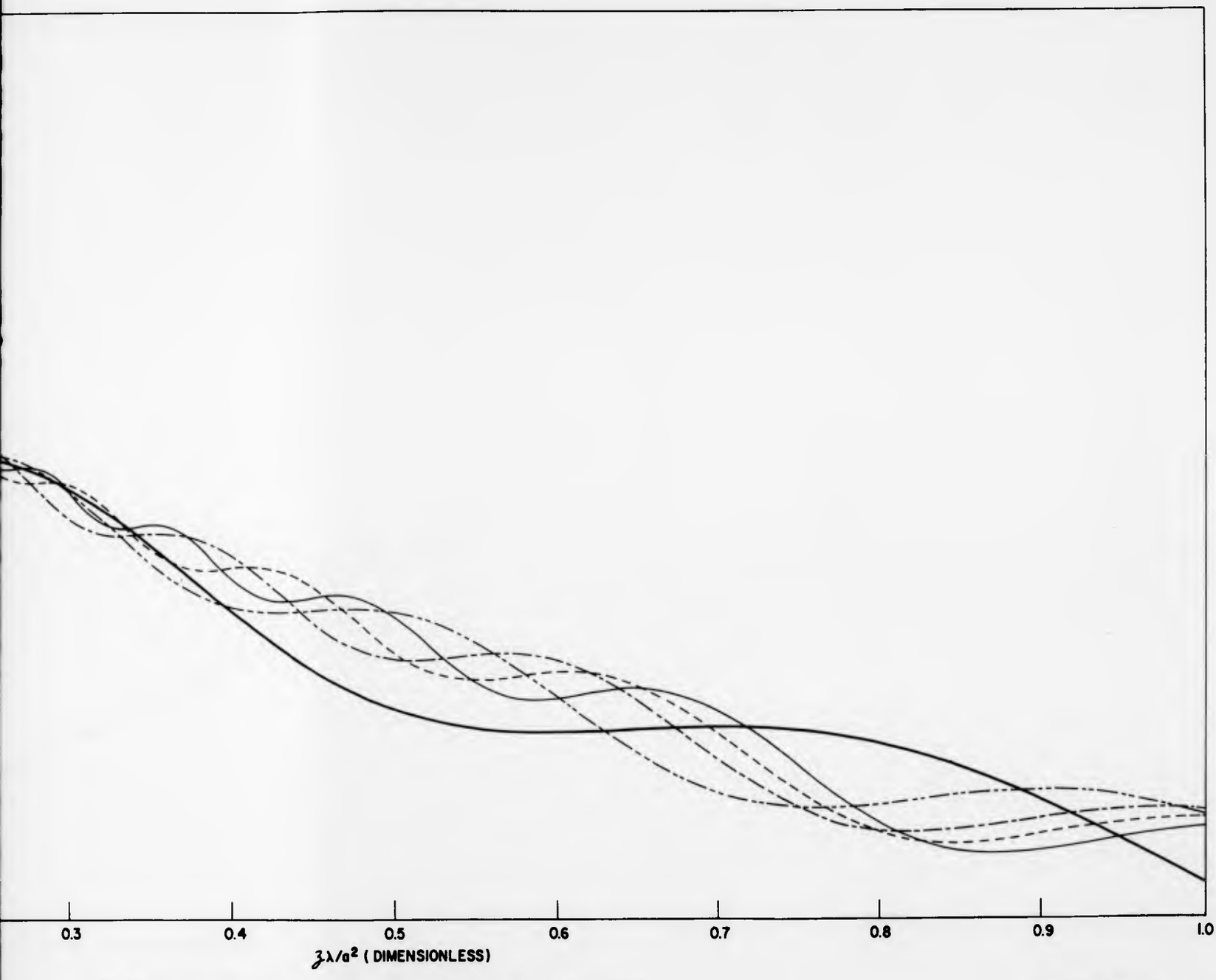


Fig. 9c – Average relative pressure $\langle p \rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $0 < z\lambda/a^2 \leq 1$ for the transducer-crystal parameters $ka = 4\pi$, 6π, 8π, 10π, and 15π

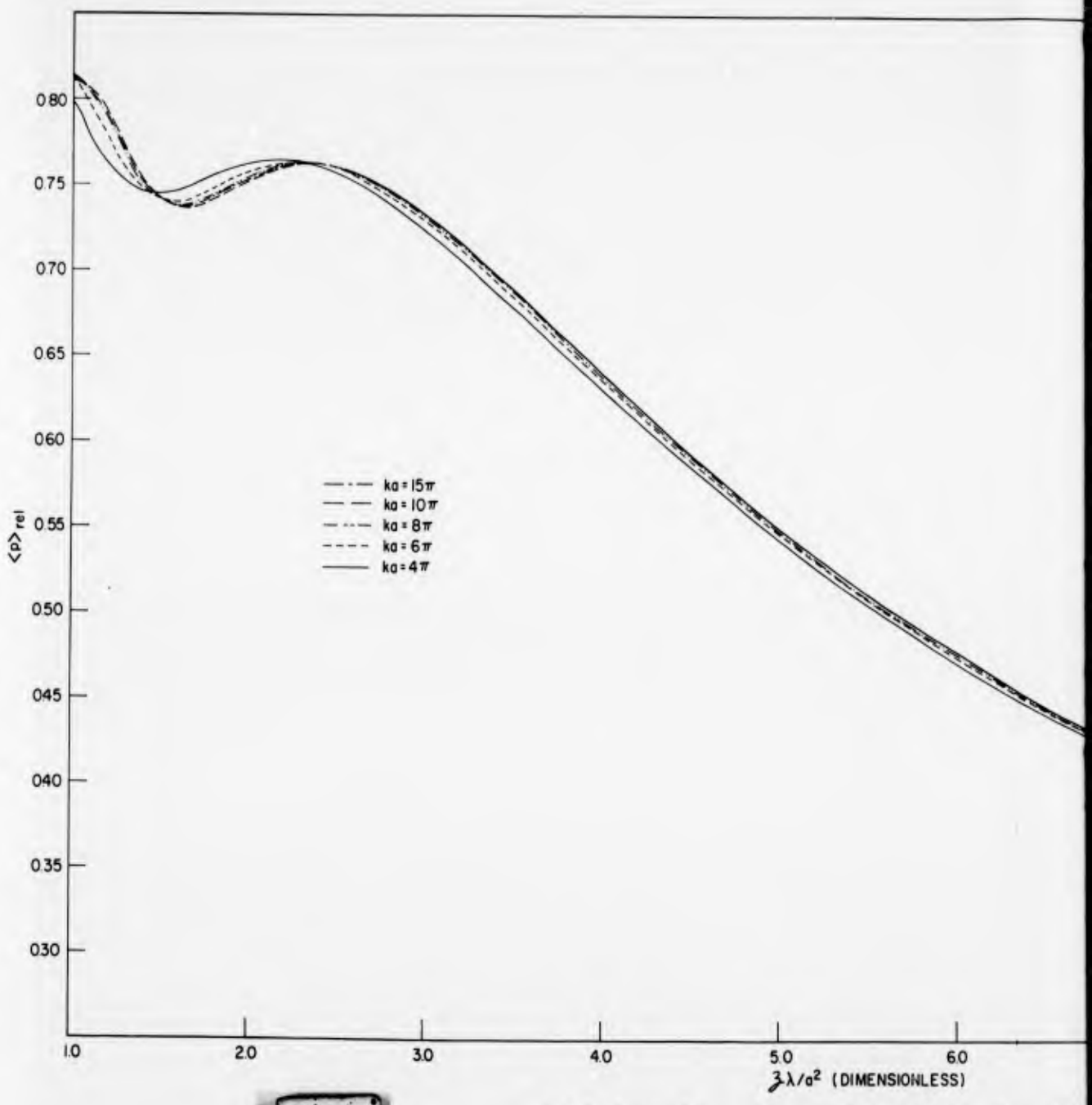
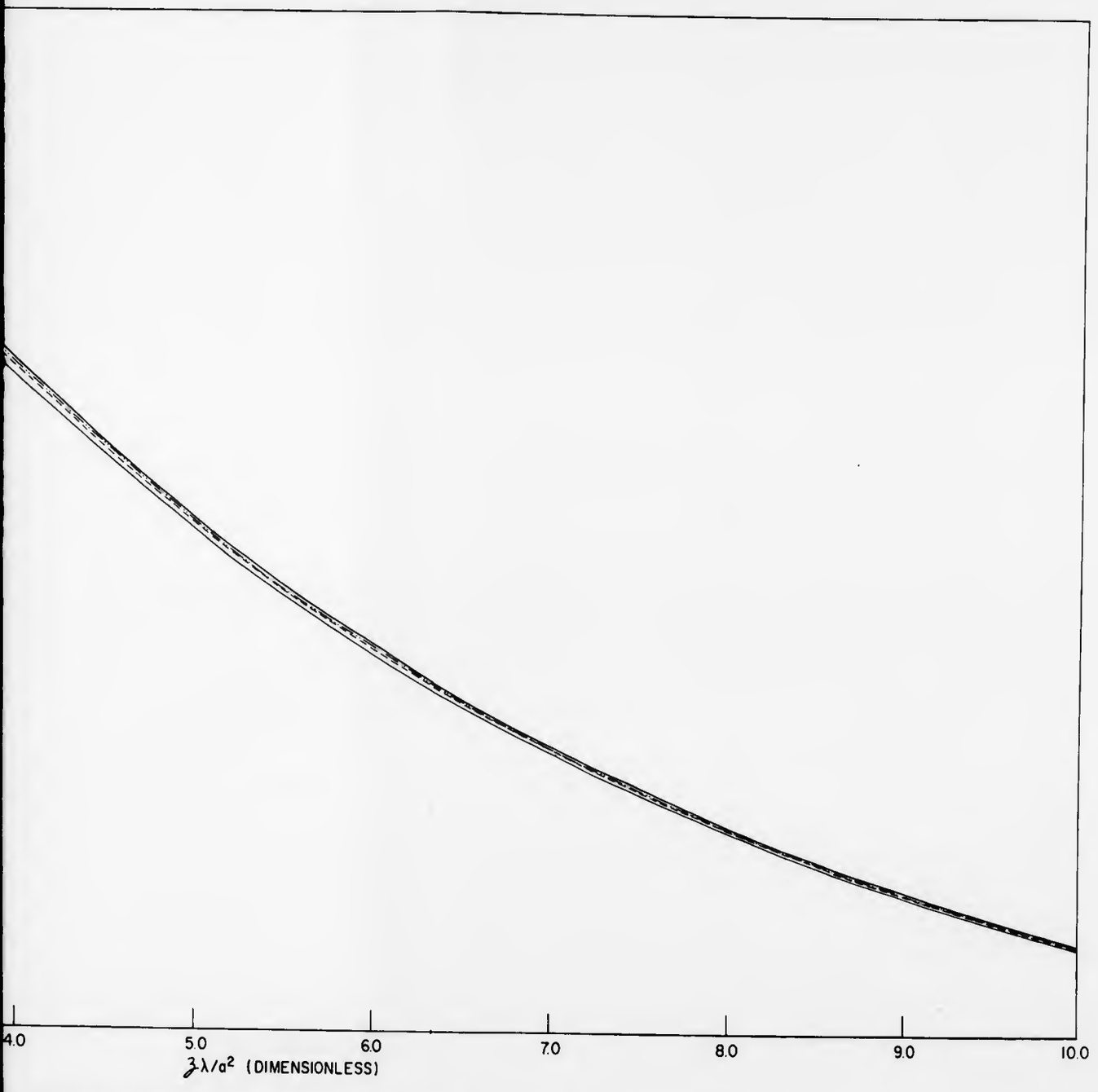


Fig. 9d – Average relative pressure $\langle p \rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $1 \leq$
for the transducer-crystal parameters $ka = 4\pi$, 6π, 8π, 10π, and 15π.



– Average relative pressure $\langle p \rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $1 \leq z\lambda/a^2 \leq 10$ for the transducer-crystal parameters $ka = 4\pi$, 6π, 8π, 10π, and 15π.

2

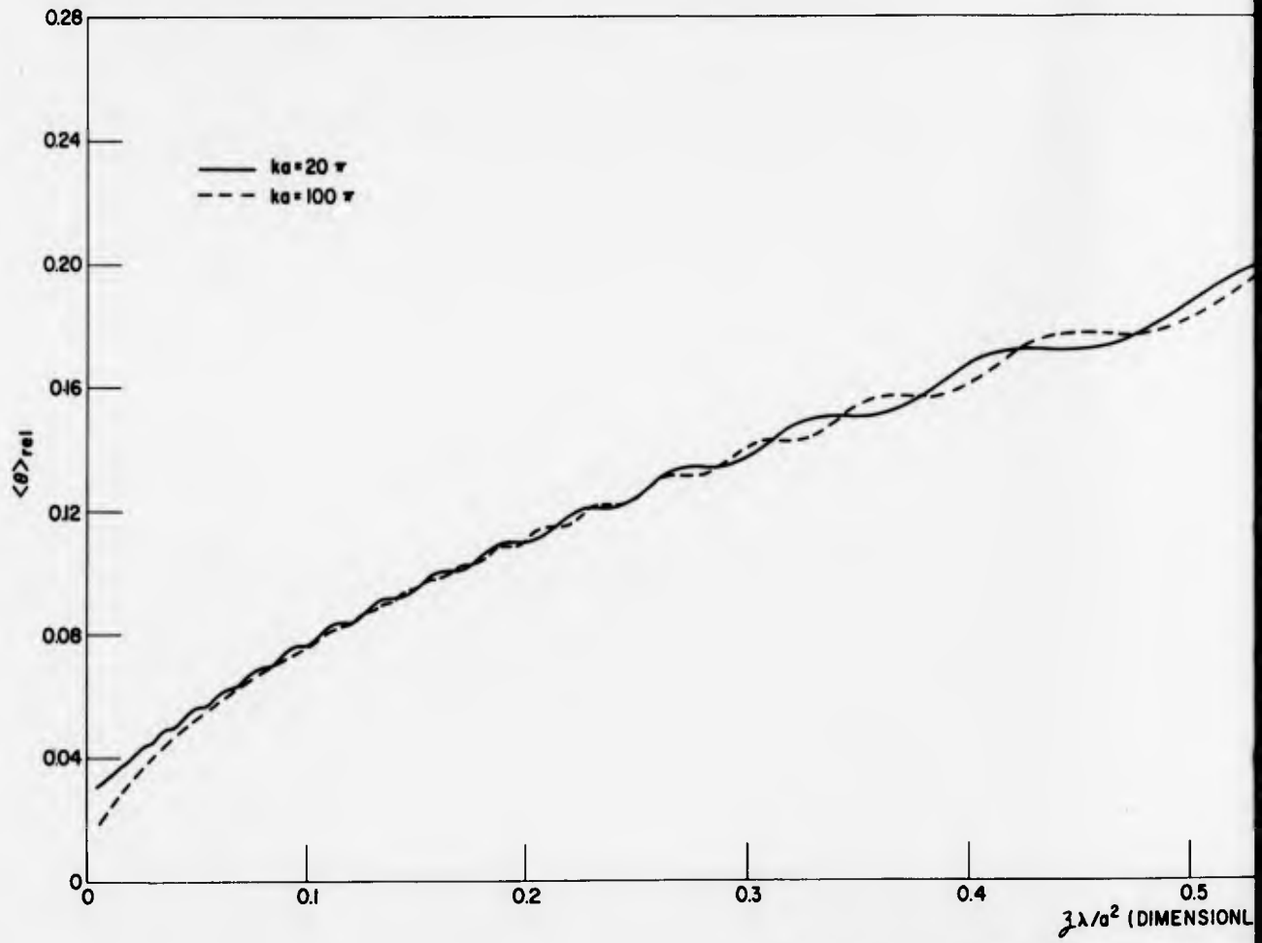


Fig. 10a – Average relative phase $\langle\theta\rangle_{rel}$ as a function of z for the transducer-crystal parameters ka

1

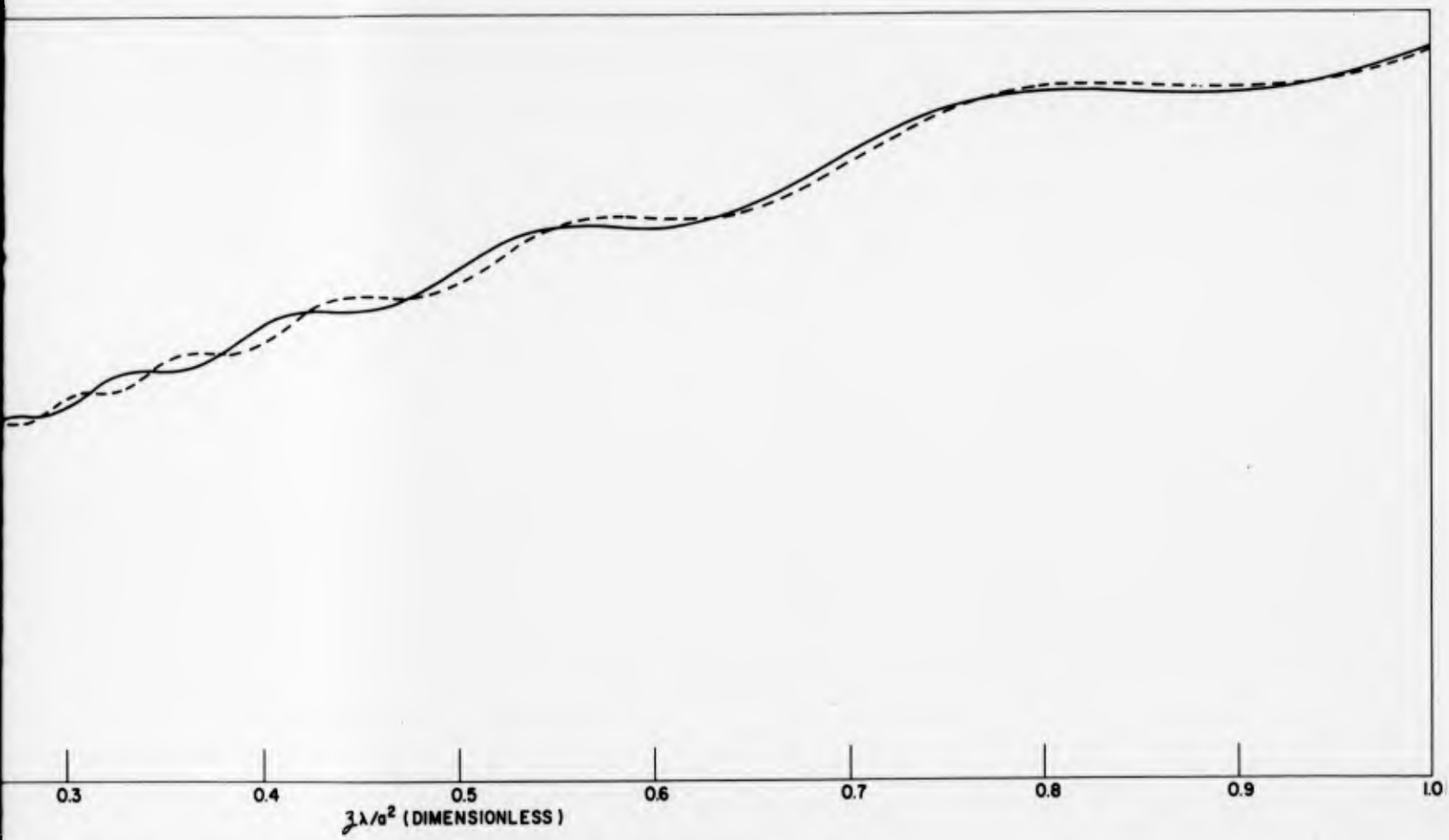


Fig. 10a – Average relative phase $\langle\theta\rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $0 < z\lambda/a^2 \leq 1$ for the transducer-crystal parameters $ka = 20\pi$ and 100π

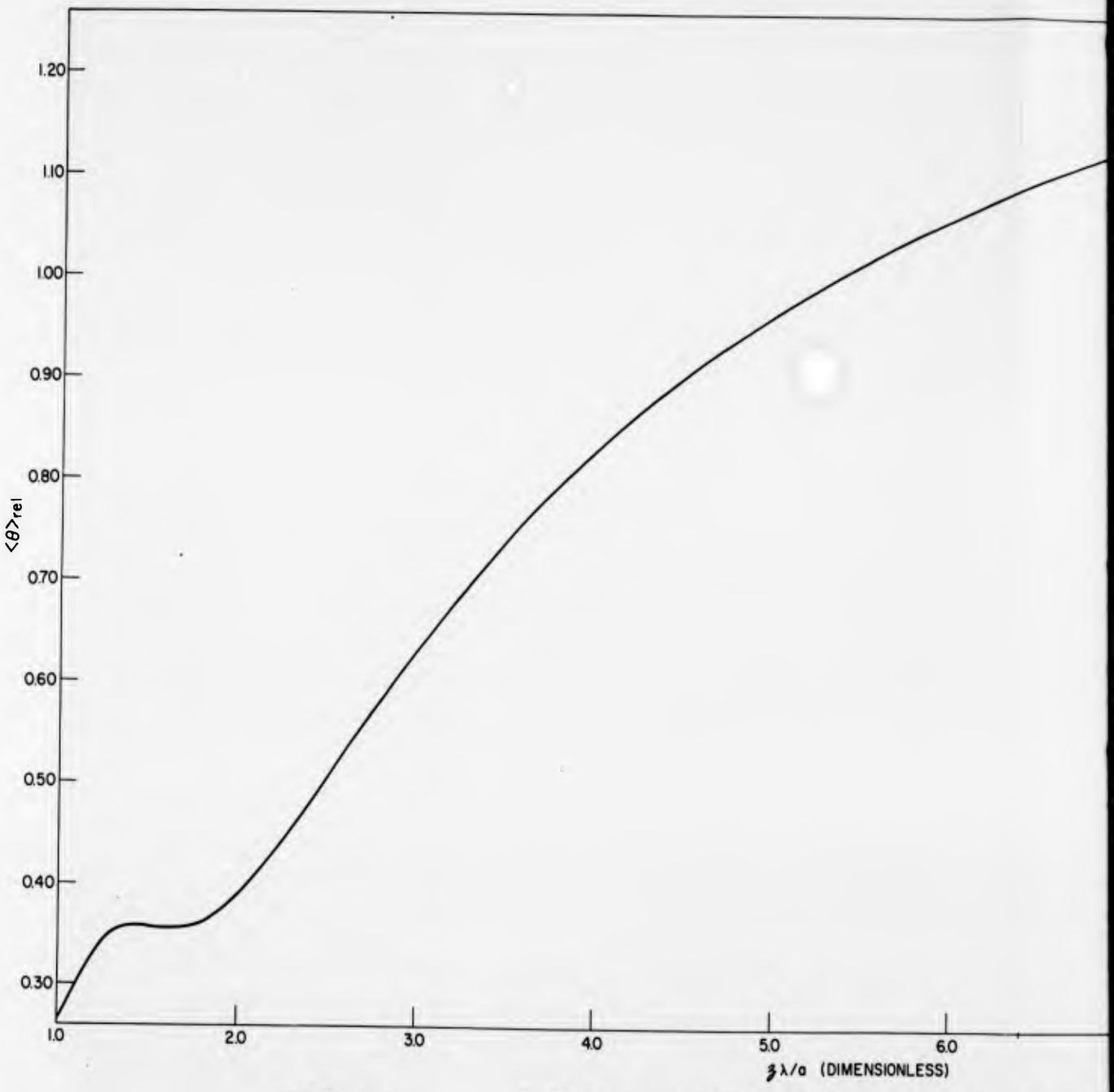
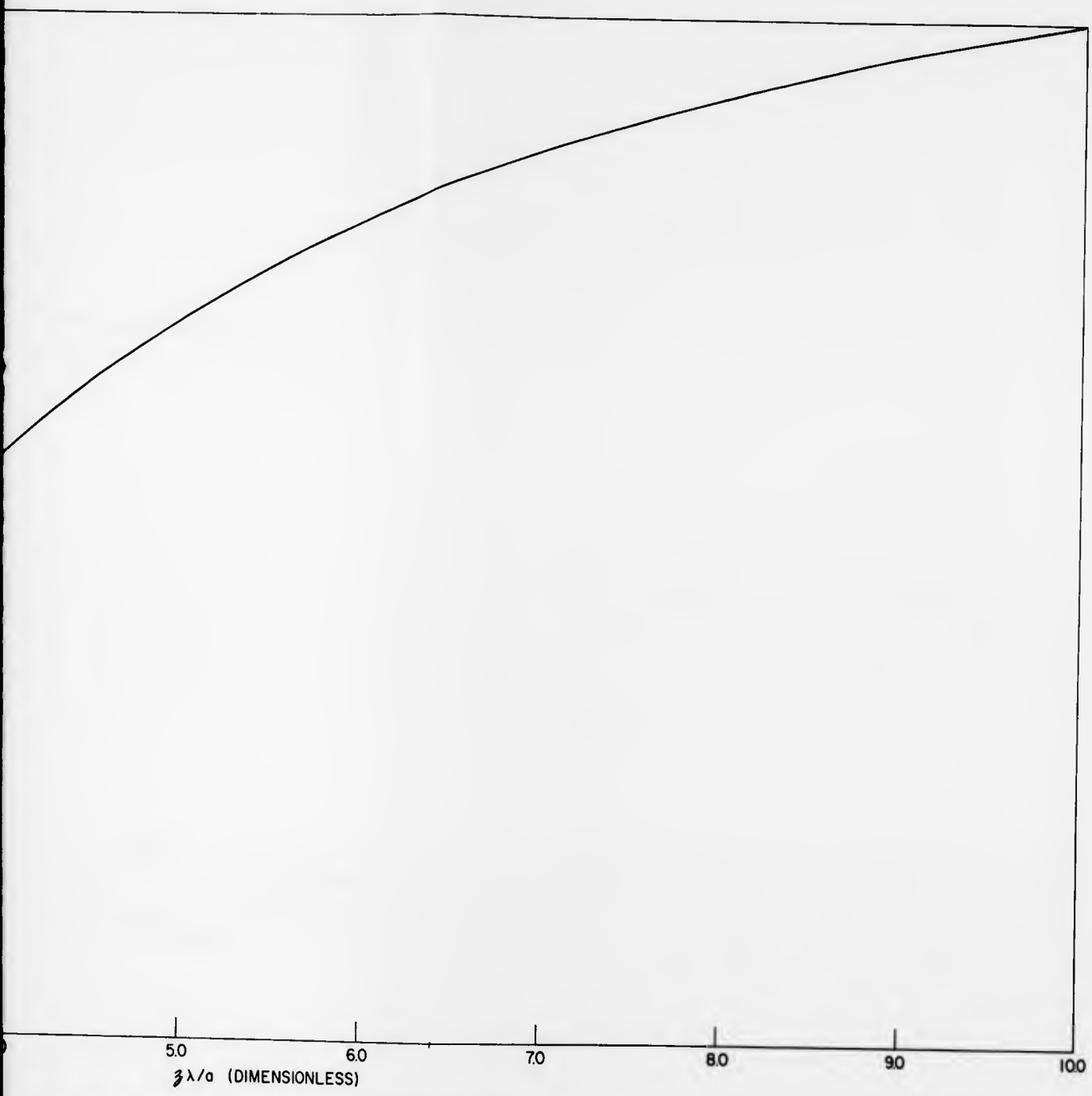


Fig. 10b – Average relative phase $\langle\theta\rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $1 \leq z\lambda/a^2 \leq$ for the transducer-crystal parameters $ka = 20\pi$ and 100π. Over this range of values the 20 and 100π curves are indistinguishable.



Average relative phase $\langle\theta\rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $1 \leq z\lambda/a^2 \leq 10$
ducer-crystal parameters $ka = 20\pi$ and 100π. Over this range of values the 20π
rves are indistinguishable.

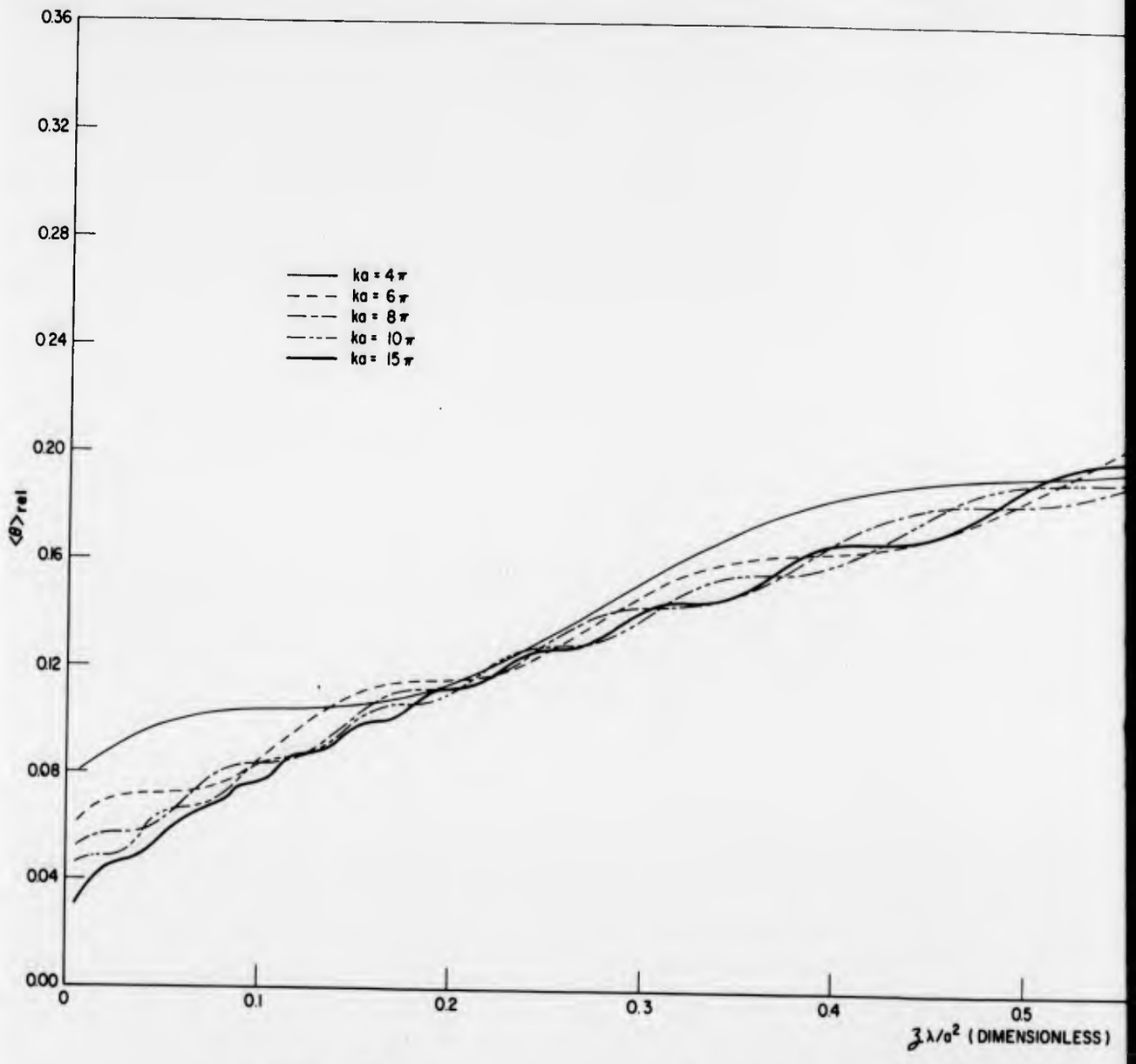


Fig. 10c – Average relative phase $\langle\theta\rangle_{rel}$ as a function of $z\lambda/a^2$ c
for the transducer-crystal parameters $ka = 4\pi, 6\pi, 8$

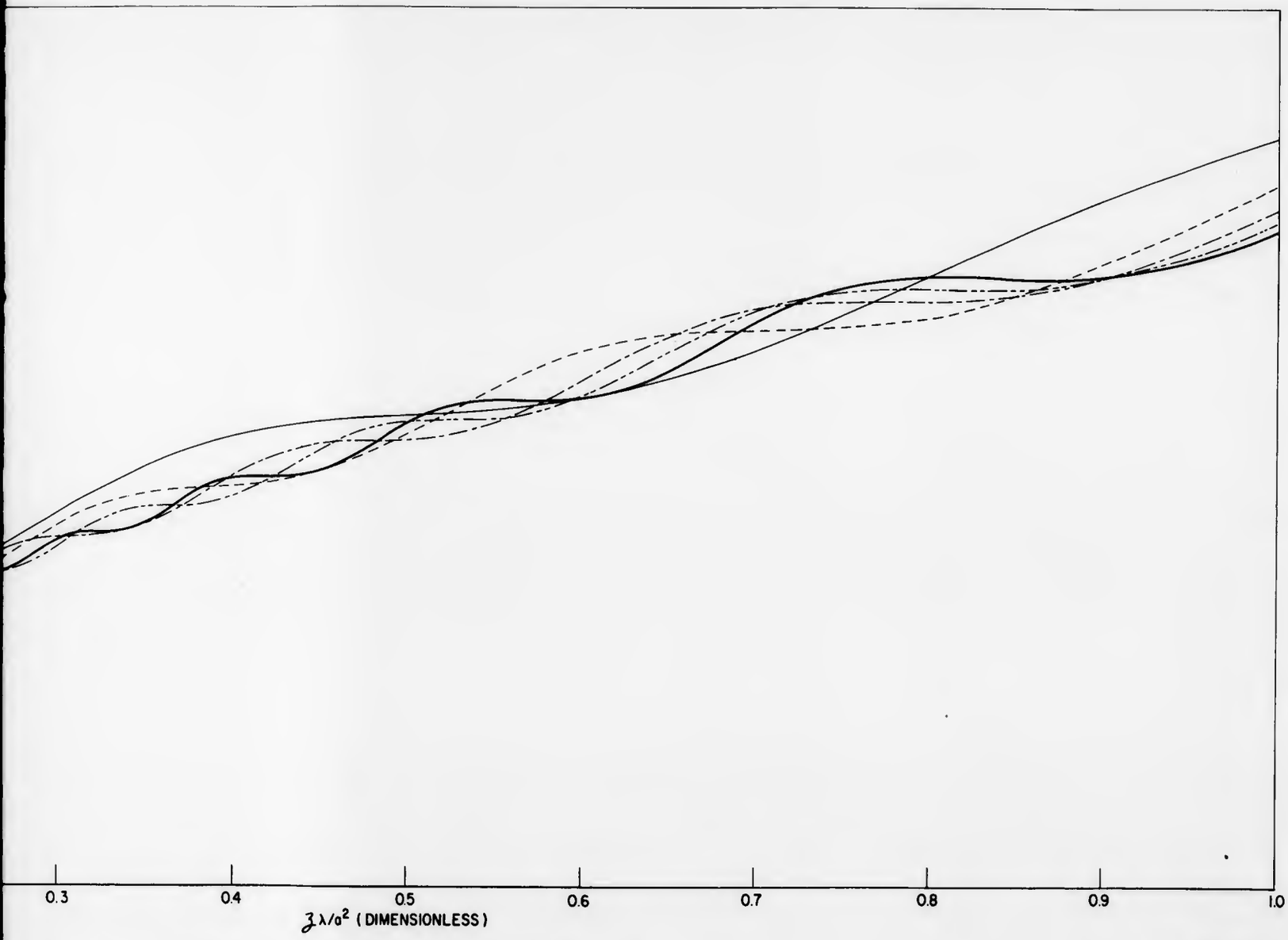


Fig. 10c – Average relative phase $\langle\theta\rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $0 < z\lambda/a^2 \leq 1$ for the transducer-crystal parameters $ka = 4\pi$, 6π, 8π, 10π, and 15π

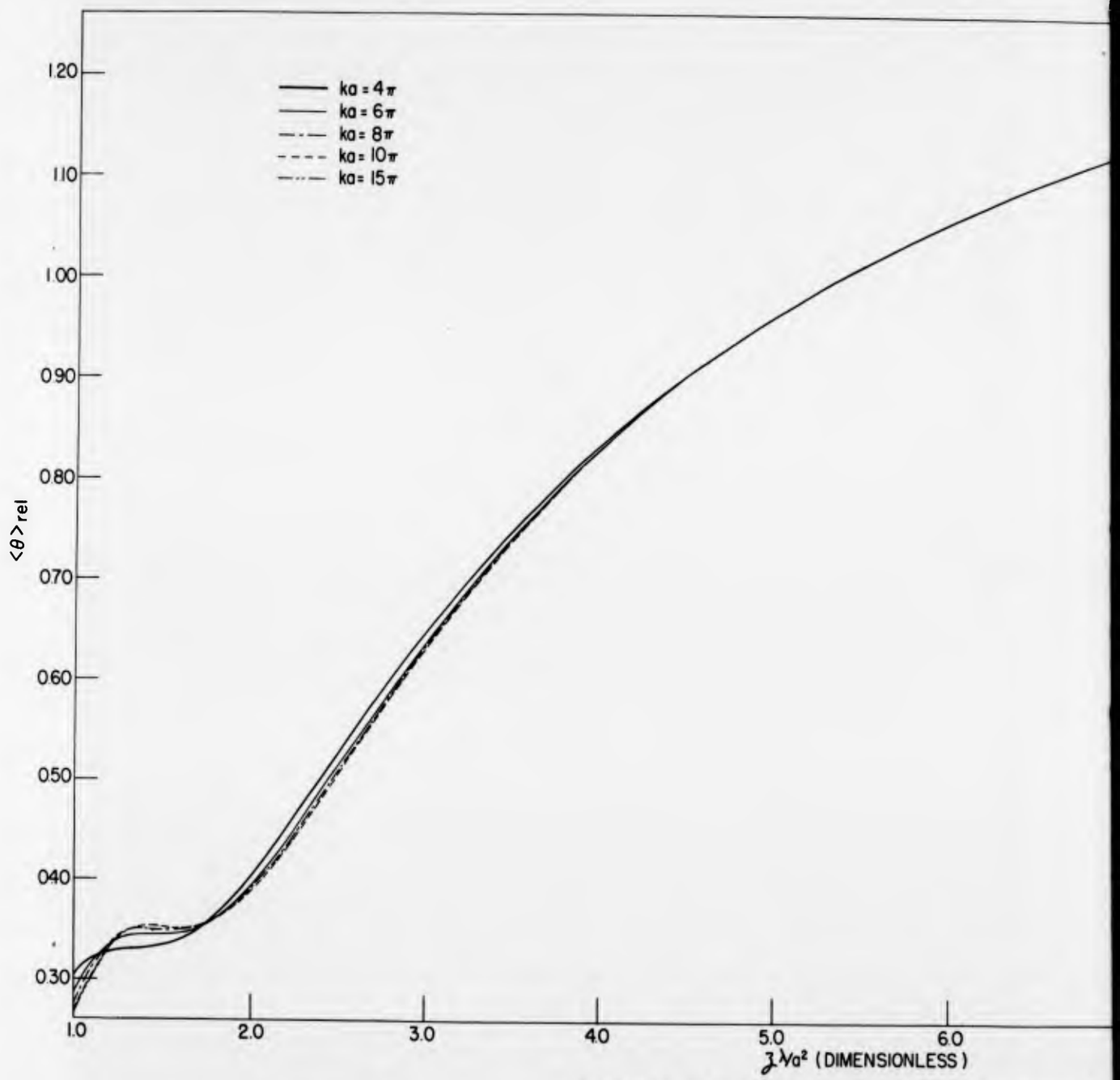
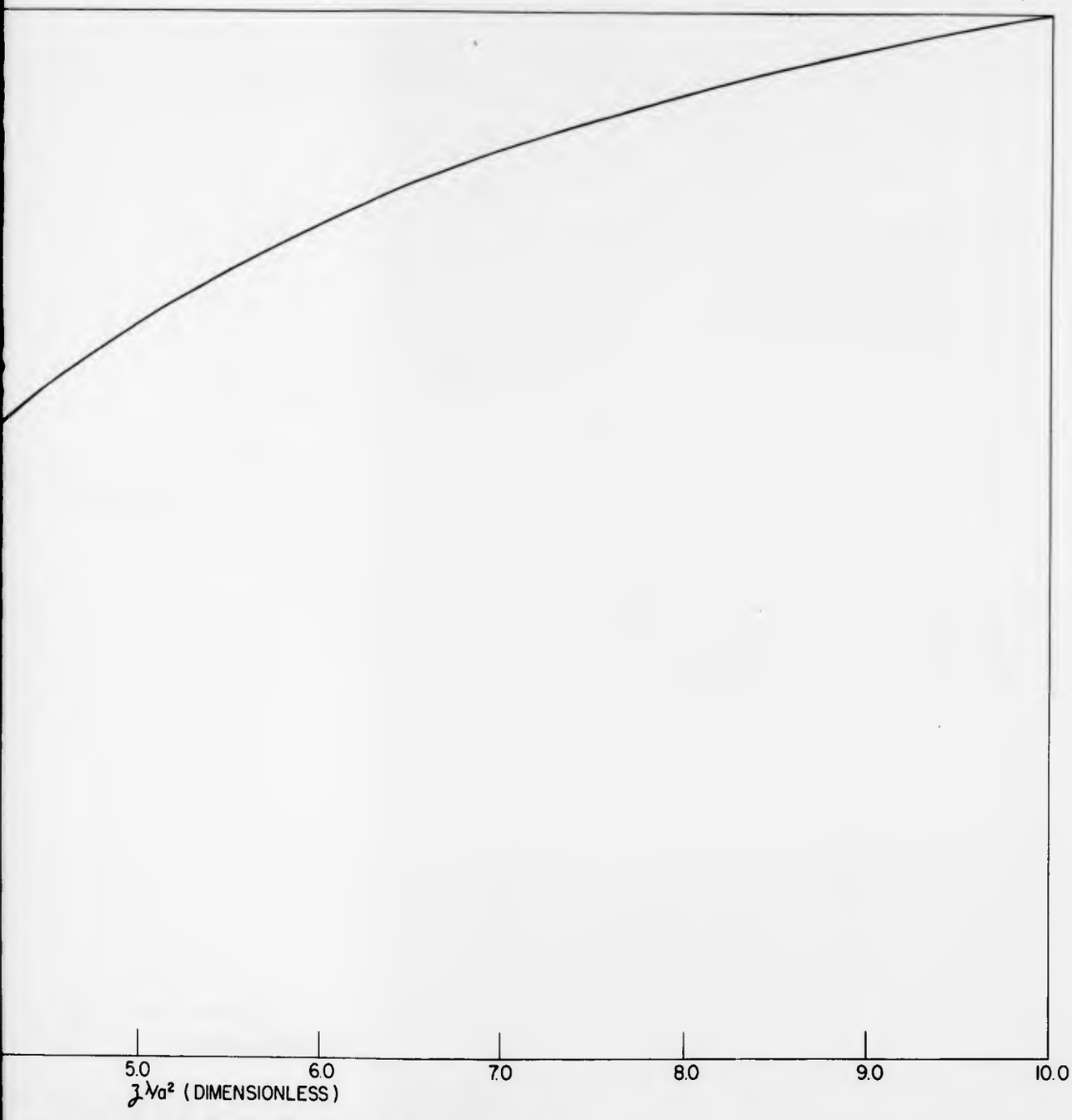


Fig. 10d – Average relative phase $\langle\theta\rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $1 \leq z$ for the transducer-crystal parameters $ka = 4\pi$, 6π, 8π, 10π, and 15π.



Average relative phase $\langle\theta\rangle_{rel}$ as a function of $z\lambda/a^2$ over the range $1 \leq z\lambda/a^2 \leq 10$ for the transducer-crystal parameters $ka = 4\pi$, 6π, 8π, 10π, and 15π.

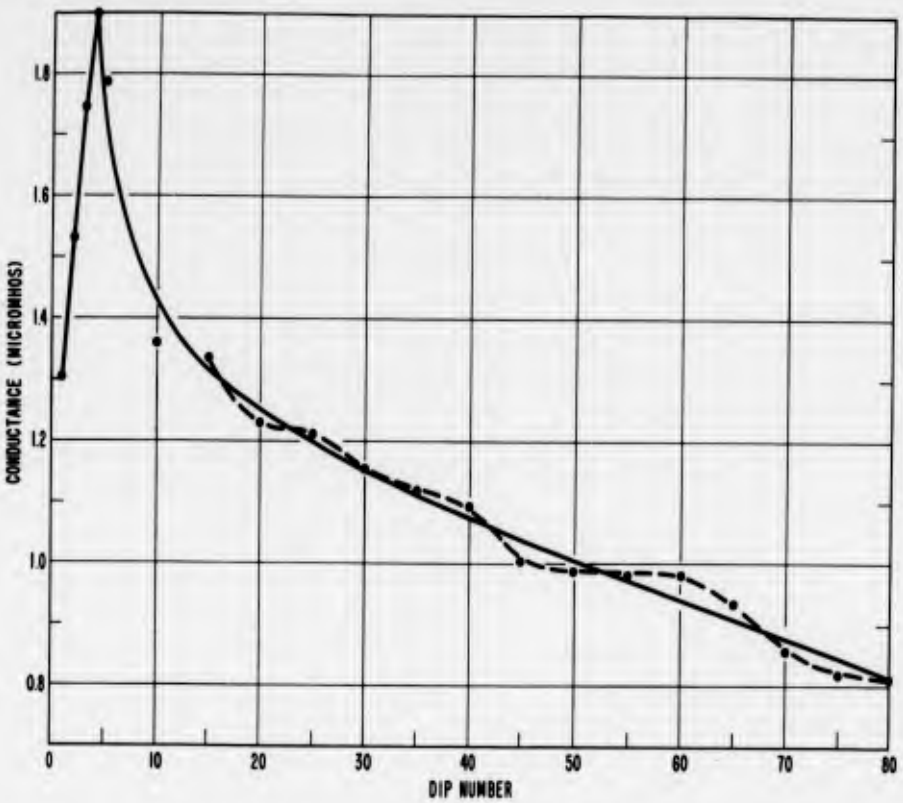


Fig. 11 – Reaction curve maxima for distilled water

While the calculations in this report deal with nonreflection cases of cw or the true steady-state portion of pulses, an interesting observation is that the attempt to destroy cylindrical symmetry in the NRL 1954 interferometer was apparently successful inasmuch as the positions of the maxima and minima observed in the reaction curve do match surprisingly well with those predicted on the basis of these free-field diffraction expressions. Of course, in the light of advantages accruing from the process of iterative reflection, it is true that there is considerable damping of the oscillatory nature of the predicted curve (this will be detailed in a forthcoming report on interferometry). For the cell with which Fig. 11 was obtained, where the actual $a = 1.27$ cm, the effective $a = 1.12$ cm, and the effective $ka \simeq 15\pi$, we read the positions of maxima and minima from Figs. 9c and 9d as in Table 2. From Fig. 11 we read the positions of the turning points as in Table 3.

The agreement between these tables is amazingly close when one considers the limited number of points used to obtain the oscillatory nature of the dashed line in Fig. 11 (points every 0.75 cm) and the fact that this curve was obtained experimentally for an iterative reflection situation in a limited enclosure while the apparent agreement is with a nonterminated free-field situation.

APPLICATION TO SOUND SPEED MEASUREMENTS

With respect to the correction of sound speed determinations for free-field diffraction effects, it is well to reiterate that these corrections do not apply to interferometer (iterative reflection) techniques because of the ensuing beam collimation even though the magnitude variation apparently follows (in a much suppressed manner) the predictions of these calculations. It should be

TABLE 2
Calculated Turning Points z from Figs. 9c and 9d for $ka = 15\pi$ and Effective Transducer Radius $a = 1.12$ cm

Kind of Turning Point	$z\lambda/a^2$ (dimensionless)	z (cm)
min	.43	3.4
max	.465	3.7
min	.585	4.7
max	.65	5.2
min	.875	7.0
max	1.0	8.0
min	1.6	12.8

TABLE 3
Experimental Maxima and Minima Turning Points z from Fig. 11

Kind of Turning Point	Dip No.	z (cm)
min	20	3.0
max	25	3.8
min	31	4.7
max	36	5.4
min	47	7.0
max	60	9.0
min	78	11.7

expected that the phase variations would likewise be suppressed. Figure 12 (Fig. 27 of Ref. 1) is a representation of the constant phase surfaces or wave field of the interferometer as used in Ref. 1; the data were determined by combined graphical and analytical methods and do not involve reflection. The averaged relative phase of these present calculations indicates the cumulative amount of nonplanarity of these fronts. As an example of the size of the sound speed correction, let us assume a transducer of $ka = 20\pi$, $a = 10\pi$, and $\lambda = 0.15$ cm transmitting over a path of 150 cm (1000λ). From Fig. 10b we find the phase error to be 1.26 radians. Since we are assuming a pulse measurement, this represents a lag with respect to plane wave phase at this distance, equivalent to an error of 1.26/1000 (2π) or one part in 5000 low. This has been calculated by taking the entire error into account, including the maximum effect occurring near the source. A method recommended to eliminate this source region would be to employ a differential path (this advantage automatically accrues to interferometry since the absolute distance between crystal and reflector is of small concern and only the intervals between selected reflector positions are actually used). An example of this differential distance pulse measurement may be found in the recent literature (11). Here we find a source frequency of 2 Mc, a first distance of 40.5 in., and a second distance of 81 in. being used. We will assume that $ka = 20\pi$ and $a = 1.5$ cm. Since water was the medium, we take $\lambda = 0.075$ cm. Referring to Fig. 10a, after conversion to $z\lambda/a^2$ values of 3.429 and 6.858 with respect to the two distances we obtain phase lags of 0.7245 and 1.1208 radians, respectively; this gives a phase lag difference of 0.3963 radians for a distance difference of 102.86 cm. For this distance interval the plane-wave phase is (102.86/0.075)2π or 8613 radians so that the proportional error in phase (or sound speed) is 0.3963/8613 = 1/40,000. Had the entire distance to the first point alone been used, the error would have been 0.7245/8613 = 1/12,000 and for the entire distance to the second point it would have been 1.1208/17,226 = 1/15,000.

The particular experimental setup used for the above, therefore, did not result in an appreciable free-field diffraction error, that is, if we were correct in assuming the transducer size used. This is unfortunately another instance of the plethora of articles suffering from a dearth of relevant data so we cannot be certain of the source size, but in any event the timing delay measured by Brooks with respect to Greenspan's measurements must be due to some effect other than diffraction, such as, for example, rise time delay. We mention only in passing that Brook's measurements are about as much below Greenspan and Tschiegg's (12) as the latter's are below ours (13).

As another example of the error that could be caused by neglect of free-field diffraction corrections when appropriate, we will calculate using the following: a 1-Mc(λ = 0.15 cm) crystal with $ka = 20\pi$, $a = 1.5$ cm, and a path length of 15 cm (or 100λ, which is 200π radians). The value of

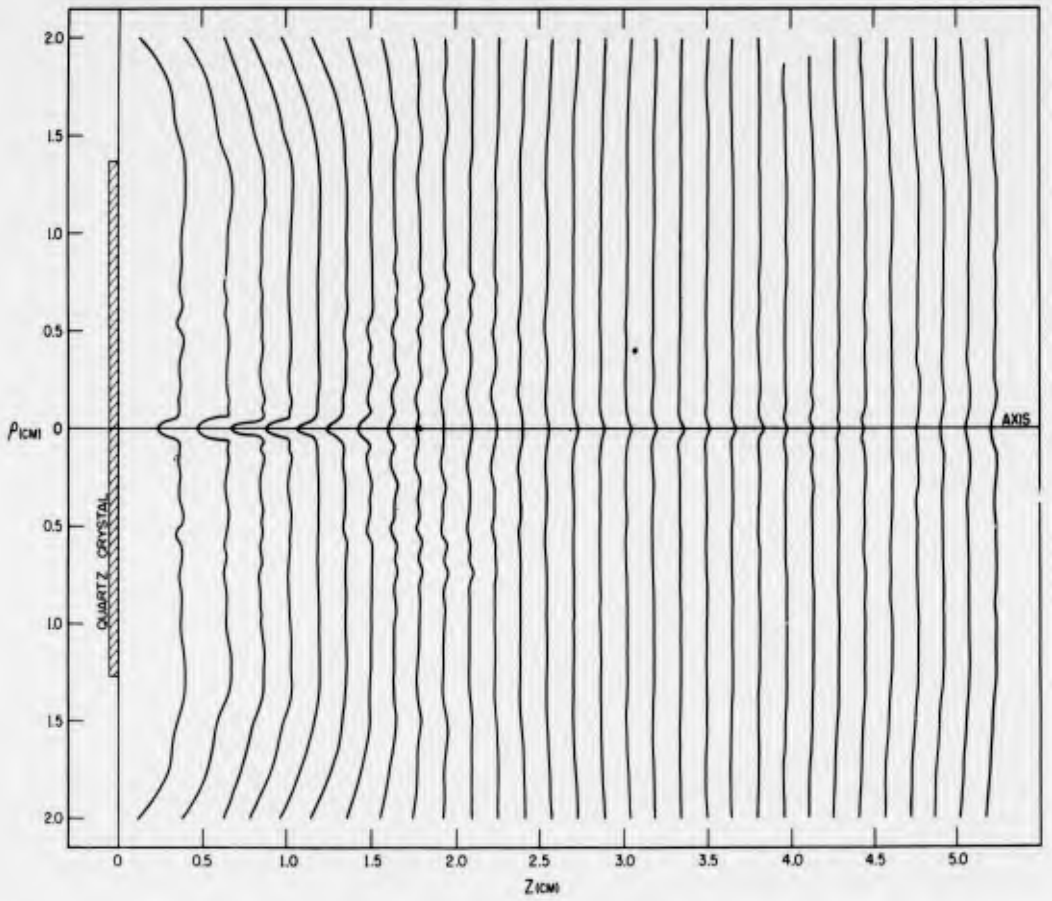


Fig. 12 – Constant-phase surfaces – wave field of the NRL interferometer

$\langle\theta\rangle_{rel}$ from Fig. 10a is 0.28 radians, so the proportional error in phase is $0.28/200\pi = 1/2243$ which corresponds to a sound speed error of 0.67 m/s low. If the frequency in the above example were changed to 2 Mc and the same size transducer were used, then $ka = 40\pi$ and the path length of 15 cm would be 400π radians since $\lambda = 0.075$ cm. The path length now would correspond to a $z\lambda/a^2$ of 0.5 instead of the 1.0 in the previous example, so $\langle\theta\rangle_{rel} = 0.19$ and the proportional error becomes $0.19/400\pi = 1/7137$, corresponding to a sound speed error of 0.21 m/s low

APPLICATION TO FIXED PATH OR SMALL PATH VARIATION TECHNIQUES

A particularly serious type of error occurs in those situations for which a free-field representation is appropriate when measurements are made over a small range of $z\lambda/a^2$. This small range can be due to small deliberate variations in z pertinent to any changing path measurement or to small changes in λ naturally occurring in a fixed path measurement (now commonly employed in field-type instruments). The error alluded to may not reveal itself since the phase or amplitude anomaly may be monotonic or smooth and involve no inflections or turning points in the small $z\lambda/a^2$ range involved An inspection of the relevant graphs over the appropriate distances will indicate the magnitude of this error. Without giving specific examples, it may be pointed out that the differential anomaly in absorption may be either negative or positive and thus cause either an additional loss or an apparent gain. Particularly in low-loss liquids such as water, and for the lower frequencies, we have often noticed experimentally an

increase in the size of the received echo in pulse amplitude absorption work when the path distance was *increased.* Similarly, it is possible for the far greater portion of the observed loss to be due to diffraction rather than to absorption.

In sound speed measurements, likewise, over a small increment in $z\lambda/a^2$ the differential phase anomaly may either add to or subtract from the total plane-wave phase. It should be pointed out that if this anomaly remained constant over the range it would result in sound speed differences being correct, although absolute values would still be in error even in this differential-type measurement. However, if it increased over the $z\lambda/a^2$ range involved, it would introduce an error even in the differences such that those indicated for pulse measurements would be too low. Conversely, a decrease in the phase anomoly over the pertinent $z\lambda/a^2$ variation would result in differences in observed group velocities being too high. These situations would of course be reversed for noniterative cw phase velocities. Whether the errors in the differences (as opposed to the absolute values which have been shown to always be in error) would be constant or varying would depend upon whether the anomoly over the pertinent $z\lambda/a^2$ increment was a straight line or merely a smooth curve.

From an inspection of the current literature, apparently too much has been made of observed smooth relationships between indicated sound speeds and physical parameter changes. From the foregoing discussion it should be apparent that such indications may not serve as proof that true sound speed differences, much less absolute values, are being measured in a particular test setup. Indeed, in an experimental configuration for which the $z\lambda/a^2$ change is small, it should be expected that the functional relationships should be smooth even if not correct. Another fallacy apparently in favor concerns the calibration of a particular instrument by means of data obtained from a similar technique and then utilizing a comparison of later results between the two as evidence that true phase velocity is somehow being measured. The reference here is to configurations whose path is not amenable to direct measurement but which must be specified by calibration against another configuration whose path is by comparison rather easily defined. It would be a surprise if the calibrated instrument did not rather faithfully reproduce the results of the one against which it was calibrated, irrespective of whether this was done directly or indirectly through tables relating apparent sound speed as determined by the "standard" instrument to certain specified physical conditions of the medium. Obviously, the calibrated instrument can do no more than reproduce the diffraction errors of the so-called standard. Indeed, its own diffraction errors might be calibrated out, but the advantage of this is somewhat nebulous.

CONCLUSIONS

Appreciable errors in sound speed and sound absorption determinations may be attributed to a neglect in applying appropriate corrections to situations closely approximating free-field conditions. The errors are shown to be more severe for the lower frequencies and shorter paths. For long paths the errors are generally such as to indicate an excessive loss in absorption measurements. In sound speed determinations over relatively long paths, the error leads to group velocities lower than the plane-wave phase velocity and to phase velocities, obtained by noniterative techniques, higher than the plane-wave phase velocity. A reduction in the size of the error (which is not an error if corrected for) may be made by a judicious selection of a particular differential path, although certain intervals may result in an increased error.

Because the graphs in this report are not sufficiently delineated to permit accurate reading of diffraction corrections, particularly for absorption measurements in relatively low loss liquids or sound speed measurements over relatively short paths or path differences, Tables A1a through A1ld in Appendix A show values of $\langle p \rangle_{rel}$ and $\langle \theta \rangle_{rel}$ as obtained on the NAREC computer for ka values from 4π to 100π, and all for common values of $z\lambda/a^2$ as indicated. Table A1a, for example, is of $\langle p \rangle_{rel}$ for $z\lambda/a^2$ values between 0 and 1 and for $ka = 4\pi$; Table A1b is an extension of the same for $z\lambda/a^2$ values between 1 and 10. Tables A1c and A1d are for $\langle \theta \rangle_{rel}$ for $ka = 4\pi$ over the short range and long range, respectively, of $z\lambda/a^2$.

FUTURE WORK

In the next report of this series, as indicated in the introduction of this one, the formulation of

guided mode (waveguide dispersion) effects will be detailed, and relevant calculations will be presented for the nonterminated situations of a plane circular crystal radiating into a concentric cylinder with respective lateral boundary conditions appropriate to absolutely rigid walls, infinitely flexible walls, liquid walls, and elastic walls (the latter of infinite thickness to avoid the ring resonances). Comparisions will be made with the free-field situation detailed herein, including the approximation of free-field conditions as the ratio of cylinder radius to crystal radius increases.

The third and final theoretical report of this series will add iterative reflection to all the foregoing, while the fourth will deal with recent experimental findings.

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APPENDIX A

NAREC Computer Values of Average Relative Pressure $\langle p \rangle_{rel}$ and Average Relative Phase $\langle \theta \rangle_{rel}$ vs $z\lambda/a^2$ for Transducer-Crystal Values $ka = 4\pi$, 6π, 8π, 10π, 10π, 15π, 20π, 30π, 40π, 60π, 80π, and 100π

TABLE A1A
$\langle p \rangle_{rel}$ for $ka = 4\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 0.50\ cm$ $\lambda = 0.25\ cm$ $0 < z \leq 1\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
.005	+941776 +00	.340	+875561 +00	.675	+832315 +00
.010	+939659 +00	.345	+874172 +00	.680	+832379 +00
.015	+937451 +00	.350	+872756 +00	.685	+832434 +00
.020	+935168 +00	.355	+871319 +00	.690	+832476 +00
.025	+932826 +00	.360	+869863 +00	.695	+832506 +00
.030	+930443 +00	.365	+868394 +00	.700	+832522 +00
.035	+928034 +00	.370	+866915 +00	.705	+832522 +00
.040	+925618 +00	.375	+865431 +00	.710	+832507 +00
.045	+923210 +00	.380	+863946 +00	.715	+832475 +00
.050	+920827 +00	.385	+862464 +00	.720	+832426 +00
.055	+918483 +00	.390	+860990 +00	.725	+832358 +00
.060	+916195 +00	.395	+859526 +00	.730	+832270 +00
.065	+913974 +00	.400	+858078 +00	.735	+832163 +00
.070	+911834 +00	.405	+856647 +00	.740	+832036 +00
.075	+909786 +00	.410	+855238 +00	.745	+831888 +00
.080	+907838 +00	.415	+853854 +00	.750	+831718 +00
.085	+906001 +00	.420	+852498 +00	.755	+831526 +00
.090	+904279 +00	.425	+851173 +00	.760	+831313 +00
.095	+902679 +00	.430	+849881 +00	.765	+831077 +00
.100	+901204 +00	.435	+848624 +00	.770	+830818 +00
.105	+899856 +00	.440	+847405 +00	.775	+830536 +00
.110	+898636 +00	.445	+846227 +00	.780	+830231 +00
.115	+897544 +00	.450	+845089 +00	.785	+829902 +00
.120	+896577 +00	.455	+843995 +00	.790	+829551 +00
.125	+895733 +00	.460	+842946 +00	.795	+829176 +00
.130	+895006 +00	.465	+841941 +00	.800	+828778 +00
.135	+894391 +00	.470	+840983 +00	.805	+828357 +00
.140	+893882 +00	.475	+840073 +00	.810	+827913 +00
.145	+893472 +00	.480	+839209 +00	.815	+827445 +00
.150	+893152 +00	.485	+838394 +00	.820	+826956 +00
.155	+892914 +00	.490	+837626 +00	.825	+826443 +00
.160	+892749 +00	.495	+836906 +00	.830	+825909 +00
.165	+892647 +00	.500	+836233 +00	.835	+825353 +00
.170	+892600 +00	.505	+835608 +00	.840	+824775 +00
.175	+892597 +00	.510	+835028 +00	.845	+824176 +00
.180	+892630 +00	.515	+834495 +00	.850	+823556 +00
.185	+892687 +00	.520	+834006 +00	.855	+822915 +00
.190	+892760 +00	.525	+833560 +00	.860	+822255 +00
.195	+892840 +00	.530	+833157 +00	.865	+821575 +00
.200	+892918 +00	.535	+832796 +00	.870	+820877 +00
.205	+892986 +00	.540	+832473 +00	.875	+820159 +00
.210	+893035 +00	.545	+832190 +00	.880	+819424 +00
.215	+893059 +00	.550	+831942 +00	.885	+818671 +00
.220	+893050 +00	.555	+831730 +00	.890	+817902 +00
.225	+893002 +00	.560	+831551 +00	.895	+817116 +00
.230	+892910 +00	.565	+831404 +00	.900	+816314 +00
.235	+892768 +00	.570	+831287 +00	.905	+815498 +00
.240	+892573 +00	.575	+831197 +00	.910	+814667 +00
.245	+892319 +00	.580	+831133 +00	.915	+813822 +00
.250	+892004 +00	.585	+831093 +00	.920	+812963 +00
.255	+891626 +00	.590	+831075 +00	.925	+812092 +00
.260	+891183 +00	.595	+831078 +00	.930	+811209 +00
.265	+890672 +00	.600	+831098 +00	.935	+810315 +00
.270	+890094 +00	.605	+831135 +00	.940	+809410 +00
.275	+889448 +00	.610	+831186 +00	.945	+808495 +00
.280	+888734 +00	.615	+831249 +00	.950	+807571 +00
.285	+887953 +00	.620	+831323 +00	.955	+806638 +00
.290	+887106 +00	.625	+831405 +00	.960	+805697 +00
.295	+886196 +00	.630	+831494 +00	.965	+804749 +00
.300	+885223 +00	.635	+831588 +00	.970	+803793 +00
.305	+884190 +00	.640	+831685 +00	.975	+802832 +00
.310	+883101 +00	.645	+831784 +00	.980	+801866 +00
.315	+881958 +00	.650	+831882 +00	.985	+800894 +00
.320	+880765 +00	.655	+831979 +00	.990	+799919 +00
.325	+879524 +00	.660	+832071 +00	.995	+798940 +00
.330	+878241 +00	.665	+832160 +00	1.000	+797958 +00
.335	+876918 +00	.670	+832241 +00		

TABLE A1B

$\langle p \rangle_{rel}$ for $ka = 4\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 0.50\ cm$ $\lambda = 0.25\ cm$ $1 \leq z \leq 10\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$	
1.00	+797958	+00	4.05	+626537	+00	7.05	+413315	+00
1.05	+788117	+00	4.10	+621882	+00	7.10	+410815	+00
1.10	+778627	+00	4.15	+617255	+00	7.15	+408342	+00
1.15	+769949	+00	4.20	+612658	+00	7.20	+405895	+00
1.20	+762422	+00	4.25	+608091	+00	7.25	+403475	+00
1.25	+756249	+00	4.30	+603558	+00	7.30	+401081	+00
1.30	+751508	+00	4.35	+599058	+00	7.35	+398713	+00
1.35	+748179	+00	4.40	+594594	+00	7.40	+396370	+00
1.40	+746164	+00	4.45	+590166	+00	7.45	+394052	+00
1.45	+745310	+00	4.50	+585776	+00	7.50	+391759	+00
1.50	+745435	+00	4.55	+581423	+00	7.55	+389490	+00
1.55	+746343	+00	4.60	+577110	+00	7.60	+387245	+00
1.60	+747840	+00	4.65	+572836	+00	7.65	+385023	+00
1.65	+749745	+00	4.70	+568602	+00	7.70	+382825	+00
1.70	+751891	+00	4.75	+564409	+00	7.75	+380650	+00
1.75	+754137	+00	4.80	+560256	+00	7.80	+378497	+00
1.80	+756362	+00	4.85	+556145	+00	7.85	+376367	+00
1.85	+758465	+00	4.90	+552074	+00	7.90	+374259	+00
1.90	+760367	+00	4.95	+548045	+00	7.95	+372173	+00
1.95	+762009	+00	5.00	+544058	+00	8.00	+370108	+00
2.00	+763346	+00	5.05	+540111	+00	8.05	+368065	+00
2.05	+764345	+00	5.10	+536207	+00	8.10	+366042	+00
2.10	+764988	+00	5.15	+532343	+00	8.15	+364040	+00
2.15	+765265	+00	5.20	+528521	+00	8.20	+362058	+00
2.20	+765173	+00	5.25	+524740	+00	8.25	+360096	+00
2.25	+764715	+00	5.30	+521000	+00	8.30	+358154	+00
2.30	+763900	+00	5.35	+517301	+00	8.35	+356231	+00
2.35	+762740	+00	5.40	+513642	+00	8.40	+354328	+00
2.40	+761249	+00	5.45	+510024	+00	8.45	+352443	+00
2.45	+759444	+00	5.50	+506445	+00	8.50	+350577	+00
2.50	+757342	+00	5.55	+502906	+00	8.55	+348730	+00
2.55	+754961	+00	5.60	+499407	+00	8.60	+346901	+00
2.60	+752320	+00	5.65	+495946	+00	8.65	+345090	+00
2.65	+749438	+00	5.70	+492524	+00	8.70	+343296	+00
2.70	+746334	+00	5.75	+489141	+00	8.75	+341520	+00
2.75	+743024	+00	5.80	+485795	+00	8.80	+339761	+00
2.80	+739527	+00	5.85	+482487	+00	8.85	+338020	+00
2.85	+735858	+00	5.90	+479216	+00	8.90	+336295	+00
2.90	+732034	+00	5.95	+475982	+00	8.95	+334586	+00
2.95	+728071	+00	6.00	+472785	+00	9.00	+332894	+00
3.00	+723981	+00	6.05	+469623	+00	9.05	+331218	+00
3.05	+719780	+00	6.10	+466497	+00	9.10	+329558	+00
3.10	+715478	+00	6.15	+463406	+00	9.15	+327914	+00
3.15	+711089	+00	6.20	+460350	+00	9.20	+326285	+00
3.20	+706623	+00	6.25	+457328	+00	9.25	+324671	+00
3.25	+702090	+00	6.30	+454340	+00	9.30	+323073	+00
3.30	+697500	+00	6.35	+451386	+00	9.35	+321489	+00
3.35	+692863	+00	6.40	+448465	+00	9.40	+319920	+00
3.40	+688186	+00	6.45	+445576	+00	9.45	+318366	+00
3.45	+683476	+00	6.50	+442720	+00	9.50	+316826	+00
3.50	+678742	+00	6.55	+439895	+00	9.55	+315300	+00
3.55	+673989	+00	6.60	+437102	+00	9.60	+313788	+00
3.60	+669223	+00	6.65	+434341	+00	9.65	+312289	+00
3.65	+664450	+00	6.70	+431609	+00	9.70	+310805	+00
3.70	+659676	+00	6.75	+428908	+00	9.75	+309334	+00
3.75	+654904	+00	6.80	+426237	+00	9.80	+307876	+00
3.80	+650138	+00	6.85	+423595	+00	9.85	+306431	+00
3.85	+645383	+00	6.90	+420983	+00	9.90	+304999	+00
3.90	+640643	+00	6.95	+418399	+00	9.95	+303579	+00
3.95	+635920	+00	7.00	+415843	+00	10.00	+302173	+00
4.00	+631217	+00						

TABLE A1C

$\langle\theta\rangle_{rel}$ for $ka = 4\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 0.50\ cm$ $\lambda = 0.25\ cm$ $0 < z \leq 1\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+804258 -01	.340	+168395 +00	.675	+213609 +00
.010	+829670 -01	.345	+170084 +00	.680	+214806 +00
.015	+853771 -01	.350	+171709 +00	.685	+216035 +00
.020	+876457 -01	.355	+173270 +00	.690	+217294 +00
.025	+897643 -01	.360	+174765 +00	.695	+218583 +00
.030	+917267 -01	.365	+176192 +00	.700	+219900 +00
.035	+935286 -01	.370	+177551 +00	.705	+221245 +00
.040	+951680 -01	.375	+178840 +00	.710	+222616 +00
.045	+966449 -01	.380	+180061 +00	.715	+224012 +00
.050	+979611 -01	.385	+181213 +00	.720	+225432 +00
.055	+991207 -01	.390	+182297 +00	.725	+226874 +00
.060	+100129 +00	.395	+183313 +00	.730	+228337 +00
.065	+100995 +00	.400	+184264 +00	.735	+229820 +00
.070	+101726 +00	.405	+185150 +00	.740	+231322 +00
.075	+102333 +00	.410	+185973 +00	.745	+232840 +00
.080	+102828 +00	.415	+186735 +00	.750	+234375 +00
.085	+103223 +00	.420	+187438 +00	.755	+235924 +00
.090	+103532 +00	.425	+188086 +00	.760	+237486 +00
.095	+103768 +00	.430	+188680 +00	.765	+239060 +00
.100	+103948 +00	.435	+189223 +00	.770	+240644 +00
.105	+104084 +00	.440	+189719 +00	.775	+242238 +00
.110	+104192 +00	.445	+190170 +00	.780	+243839 +00
.115	+104286 +00	.450	+190580 +00	.785	+245447 +00
.120	+104381 +00	.455	+190952 +00	.790	+247060 +00
.125	+104490 +00	.460	+191289 +00	.795	+248678 +00
.130	+104626 +00	.465	+191596 +00	.800	+250298 +00
.135	+104801 +00	.470	+191875 +00	.805	+251919 +00
.140	+105027 +00	.475	+192130 +00	.810	+253541 +00
.145	+105313 +00	.480	+192365 +00	.815	+255162 +00
.150	+105669 +00	.485	+192584 +00	.820	+256781 +00
.155	+106103 +00	.490	+192789 +00	.825	+258397 +00
.160	+106621 +00	.495	+192984 +00	.830	+260009 +00
.165	+107229 +00	.500	+193173 +00	.835	+261615 +00
.170	+107932 +00	.505	+193359 +00	.840	+263215 +00
.175	+108733 +00	.510	+193546 +00	.845	+264807 +00
.180	+109634 +00	.515	+193736 +00	.850	+266391 +00
.185	+110636 +00	.520	+193932 +00	.855	+267965 +00
.190	+111739 +00	.525	+194138 +00	.860	+269529 +00
.195	+112942 +00	.530	+194357 +00	.865	+271082 +00
.200	+114244 +00	.535	+194590 +00	.870	+272623 +00
.205	+115641 +00	.540	+194841 +00	.875	+274151 +00
.210	+117130 +00	.545	+195113 +00	.880	+275664 +00
.215	+118706 +00	.550	+195406 +00	.885	+277164 +00
.220	+120366 +00	.555	+195724 +00	.890	+278647 +00
.225	+122103 +00	.560	+196068 +00	.895	+280115 +00
.230	+123912 +00	.565	+196441 +00	.900	+281566 +00
.235	+125786 +00	.570	+196844 +00	.905	+282999 +00
.240	+127719 +00	.575	+197278 +00	.910	+284415 +00
.245	+129705 +00	.580	+197745 +00	.915	+285811 +00
.250	+131736 +00	.585	+198245 +00	.920	+287188 +00
.255	+133805 +00	.590	+198781 +00	.925	+288546 +00
.260	+135905 +00	.595	+199353 +00	.930	+289883 +00
.265	+138029 +00	.600	+199961 +00	.935	+291199 +00
.270	+140169 +00	.605	+200607 +00	.940	+292494 +00
.275	+142320 +00	.610	+201291 +00	.945	+293768 +00
.280	+144474 +00	.615	+202012 +00	.950	+295019 +00
.285	+146624 +00	.620	+202772 +00	.955	+296248 +00
.290	+148764 +00	.625	+203571 +00	.960	+297454 +00
.295	+150888 +00	.630	+204407 +00	.965	+298638 +00
.300	+152990 +00	.635	+205283 +00	.970	+299797 +00
.305	+155064 +00	.640	+206196 +00	.975	+300934 +00
.310	+157104 +00	.645	+207146 +00	.980	+302047 +00
.315	+159106 +00	.650	+208134 +00	.985	+303135 +00
.320	+161066 +00	.655	+209159 +00	.990	+304200 +00
.325	+162978 +00	.660	+210219 +00	.995	+305240 +00
.330	+164839 +00	.665	+211315 +00	1.000	+306257 +00
.335	+166646 +00	.670	+212445 +00		

TABLE A1D
$\langle\theta\rangle_{rel}$ for $ka = 4\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 0.50\ cm$ $\lambda = 0.25\ cm$ $1 \leq z \leq 10\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+306257	+00	4.05	+843341	+00	7.05	+113172	+01
1.05	+315078	+00	4.10	+850930	+00	7.10	+113466	+01
1.10	+321538	+00	4.15	+858360	+00	7.15	+113756	+01
1.15	+325890	+00	4.20	+865694	+00	7.20	+114043	+01
1.20	+328530	+00	4.25	+872874	+00	7.25	+114325	+01
1.25	+329931	+00	4.30	+879924	+00	7.30	+114605	+01
1.30	+330591	+00	4.35	+886847	+00	7.35	+114880	+01
1.35	+330984	+00	4.40	+893645	+00	7.40	+115153	+01
1.40	+331529	+00	4.45	+900321	+00	7.45	+115421	+01
1.45	+332568	+00	4.50	+906879	+00	7.50	+115687	+01
1.50	+334358	+00	4.55	+913320	+00	7.55	+115949	+01
1.55	+337074	+00	4.60	+919647	+00	7.60	+116208	+01
1.60	+340817	+00	4.65	+925863	+00	7.65	+116464	+01
1.65	+345626	+00	4.70	+931971	+00	7.70	+116716	+01
1.70	+351491	+00	4.75	+937972	+00	7.75	+116966	+01
1.75	+358367	+00	4.80	+943870	+00	7.80	+117213	+01
1.80	+366185	+00	4.85	+949667	+00	7.85	+117456	+01
1.85	+374859	+00	4.90	+955364	+00	7.90	+117697	+01
1.90	+384297	+00	4.95	+960965	+00	7.95	+117935	+01
1.95	+394403	+00	5.00	+966471	+00	8.00	+118170	+01
2.00	+405084	+00	5.05	+971885	+00	8.05	+118402	+01
2.05	+416249	+00	5.10	+977209	+00	8.10	+118632	+01
2.10	+427816	+00	5.15	+982444	+00	8.15	+118859	+01
2.15	+439708	+00	5.20	+987593	+00	8.20	+119083	+01
2.20	+451853	+00	5.25	+992658	+00	8.25	+119305	+01
2.25	+464190	+00	5.30	+997640	+00	8.30	+119524	+01
2.30	+476663	+00	5.35	+100254	+01	8.35	+119741	+01
2.35	+489221	+00	5.40	+100736	+01	8.40	+119956	+01
2.40	+501823	+00	5.45	+101211	+01	8.45	+120168	+01
2.45	+514430	+00	5.50	+101678	+01	8.50	+120377	+01
2.50	+527010	+00	5.55	+102138	+01	8.55	+120584	+01
2.55	+539534	+00	5.60	+102590	+01	8.60	+120789	+01
2.60	+551978	+00	5.65	+103036	+01	8.65	+120992	+01
2.65	+564323	+00	5.70	+103474	+01	8.70	+121193	+01
2.70	+576549	+00	5.75	+103906	+01	8.75	+121391	+01
2.75	+588644	+00	5.80	+104331	+01	8.80	+121587	+01
2.80	+600595	+00	5.85	+104750	+01	8.85	+121781	+01
2.85	+612392	+00	5.90	+105162	+01	8.90	+121973	+01
2.90	+624026	+00	5.95	+105568	+01	8.95	+122163	+01
2.95	+635493	+00	6.00	+105969	+01	9.00	+122351	+01
3.00	+646786	+00	6.05	+106363	+01	9.05	+122537	+01
3.05	+657902	+00	6.10	+106751	+01	9.10	+122721	+01
3.10	+668839	+00	6.15	+107134	+01	9.15	+122903	+01
3.15	+679594	+00	6.20	+107511	+01	9.20	+123083	+01
3.20	+690167	+00	6.25	+107883	+01	9.25	+123261	+01
3.25	+700557	+00	6.30	+108249	+01	9.30	+123438	+01
3.30	+710766	+00	6.35	+108610	+01	9.35	+123613	+01
3.35	+720794	+00	6.40	+108966	+01	9.40	+123785	+01
3.40	+730641	+00	6.45	+109317	+01	9.45	+123957	+01
3.45	+740311	+00	6.50	+109663	+01	9.50	+124126	+01
3.50	+749804	+00	6.55	+110004	+01	9.55	+124294	+01
3.55	+759123	+00	6.60	+110341	+01	9.60	+124460	+01
3.60	+768271	+00	6.65	+110673	+01	9.65	+124624	+01
3.65	+777250	+00	6.70	+111000	+01	9.70	+124787	+01
3.70	+786062	+00	6.75	+111323	+01	9.75	+124948	+01
3.75	+794711	+00	6.80	+111642	+01	9.80	+125108	+01
3.80	+803200	+00	6.85	+111956	+01	9.85	+125266	+01
3.85	+811531	+00	6.90	+112266	+01	9.90	+125422	+01
3.90	+819707	+00	6.95	+112572	+01	9.95	+125577	+01
3.95	+827733	+00	7.00	+112874	+01	10.00	+125731	+01
4.00	+835609	+00						

TABLE A2A

$\langle p \rangle_{rel}$ for $ka = 6\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 0.75\ cm$ $\lambda = 0.25\ cm$ $0.01125 \leq z \leq 2.25\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$	
.005	+950631	+00	.340	+871840	+00	.675	+821593	+00
.010	+947740	+00	.345	+870243	+00	.680	+820749	+00
.015	+944694	+00	.350	+868708	+00	.685	+819958	+00
.020	+941607	+00	.355	+867248	+00	.690	+819222	+00
.025	+938593	+00	.360	+865876	+00	.695	+818541	+00
.030	+935757	+00	.365	+864601	+00	.700	+817915	+00
.035	+933187	+00	.370	+863431	+00	.705	+817344	+00
.040	+930951	+00	.375	+862371	+00	.710	+816827	+00
.045	+929094	+00	.380	+861425	+00	.715	+816364	+00
.050	+927632	+00	.385	+860594	+00	.720	+815955	+00
.055	+926560	+00	.390	+859878	+00	.725	+815597	+00
.060	+925846	+00	.395	+859274	+00	.730	+815289	+00
.065	+925440	+00	.400	+858778	+00	.735	+815030	+00
.070	+925277	+00	.405	+858384	+00	.740	+814819	+00
.075	+925281	+00	.410	+858086	+00	.745	+814652	+00
.080	+925372	+00	.415	+857874	+00	.750	+814528	+00
.085	+925470	+00	.420	+857739	+00	.755	+814444	+00
.090	+925500	+00	.425	+857671	+00	.760	+814399	+00
.095	+925393	+00	.430	+857660	+00	.765	+814390	+00
.100	+925097	+00	.435	+857695	+00	.770	+814414	+00
.105	+924567	+00	.440	+857764	+00	.775	+814468	+00
.110	+923779	+00	.445	+857856	+00	.780	+814550	+00
.115	+922719	+00	.450	+857959	+00	.785	+814657	+00
.120	+921391	+00	.455	+858064	+00	.790	+814787	+00
.125	+919811	+00	.460	+858160	+00	.795	+814936	+00
.130	+918006	+00	.465	+858237	+00	.800	+815103	+00
.135	+916014	+00	.470	+858286	+00	.805	+815283	+00
.140	+913880	+00	.475	+858299	+00	.810	+815475	+00
.145	+911653	+00	.480	+858268	+00	.815	+815676	+00
.150	+909386	+00	.485	+858185	+00	.820	+815884	+00
.155	+907129	+00	.490	+858047	+00	.825	+816095	+00
.160	+904934	+00	.495	+857846	+00	.830	+816308	+00
.165	+902844	+00	.500	+857580	+00	.835	+816519	+00
.170	+900898	+00	.505	+857245	+00	.840	+816727	+00
.175	+899130	+00	.510	+856839	+00	.845	+816930	+00
.180	+897562	+00	.515	+856361	+00	.850	+817125	+00
.185	+896209	+00	.520	+855809	+00	.855	+817310	+00
.190	+895078	+00	.525	+855184	+00	.860	+817484	+00
.195	+894166	+00	.530	+854487	+00	.865	+817644	+00
.200	+893464	+00	.535	+853720	+00	.870	+817788	+00
.205	+892955	+00	.540	+852885	+00	.875	+817916	+00
.210	+892617	+00	.545	+851985	+00	.880	+818026	+00
.215	+892422	+00	.550	+851024	+00	.885	+818116	+00
.220	+892340	+00	.555	+850004	+00	.890	+818184	+00
.225	+892339	+00	.560	+848932	+00	.895	+818230	+00
.230	+892384	+00	.565	+847810	+00	.900	+818253	+00
.235	+892444	+00	.570	+846646	+00	.905	+818251	+00
.240	+892487	+00	.575	+845443	+00	.910	+818224	+00
.245	+892484	+00	.580	+844207	+00	.915	+818171	+00
.250	+892408	+00	.585	+842944	+00	.920	+818090	+00
.255	+892237	+00	.590	+841660	+00	.925	+817983	+00
.260	+891952	+00	.595	+840359	+00	.930	+817847	+00
.265	+891540	+00	.600	+839049	+00	.935	+817682	+00
.270	+890990	+00	.605	+837735	+00	.940	+817489	+00
.275	+890296	+00	.610	+836422	+00	.945	+817267	+00
.280	+889458	+00	.615	+835115	+00	.950	+817015	+00
.285	+888478	+00	.620	+833820	+00	.955	+816734	+00
.290	+887362	+00	.625	+832542	+00	.960	+816423	+00
.295	+886119	+00	.630	+831285	+00	.965	+816083	+00
.300	+884763	+00	.635	+830055	+00	.970	+815714	+00
.305	+883307	+00	.640	+828855	+00	.975	+815316	+00
.310	+881768	+00	.645	+827689	+00	.980	+814889	+00
.315	+880165	+00	.650	+826561	+00	.985	+814433	+00
.320	+878515	+00	.655	+825474	+00	.990	+813949	+00
.325	+876839	+00	.660	+824432	+00	.995	+813437	+00
.330	+875155	+00	.665	+823436	+00	1.000	+812898	+00
.335	+873483	+00	.670	+822489	+00			

TABLE A2B
$\langle p \rangle_{rel}$ for $ka = 6\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 0.75\ cm$ $\lambda = 0.25\ cm$ $2.25 \leq z \leq 22.5\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
1.00	+812898 +00	4.05	+631695 +00	7.05	+415428 +00
1.05	+806133 +00	4.10	+626981 +00	7.10	+412898 +00
1.10	+797400 +00	4.15	+622294 +00	7.15	+410395 +00
1.15	+787564 +00	4.20	+617634 +00	7.20	+407920 +00
1.20	+777490 +00	4.25	+613005 +00	7.25	+405472 +00
1.25	+767919 +00	4.30	+608407 +00	7.30	+403050 +00
1.30	+759412 +00	4.35	+603843 +00	7.35	+400655 +00
1.35	+752336 +00	4.40	+599313 +00	7.40	+398285 +00
1.40	+746869 +00	4.45	+594820 +00	7.45	+395941 +00
1.45	+743036 +00	4.50	+590364 +00	7.50	+393622 +00
1.50	+740743 +00	4.55	+585946 +00	7.55	+391328 +00
1.55	+739817 +00	4.60	+581567 +00	7.60	+389058 +00
1.60	+740039 +00	4.65	+577228 +00	7.65	+386812 +00
1.65	+741171 +00	4.70	+572929 +00	7.70	+384590 +00
1.70	+742978 +00	4.75	+568671 +00	7.75	+382392 +00
1.75	+745241 +00	4.80	+564454 +00	7.80	+380216 +00
1.80	+747765 +00	4.85	+560279 +00	7.85	+378063 +00
1.85	+750383 +00	4.90	+556146 +00	7.90	+375933 +00
1.90	+752957 +00	4.95	+552054 +00	7.95	+373825 +00
1.95	+755376 +00	5.00	+548005 +00	8.00	+371739 +00
2.00	+757554 +00	5.05	+543998 +00	8.05	+369674 +00
2.05	+759428 +00	5.10	+540033 +00	8.10	+367630 +00
2.10	+760952 +00	5.15	+536110 +00	8.15	+365608 +00
2.15	+762096 +00	5.20	+532230 +00	8.20	+363606 +00
2.20	+762844 +00	5.25	+528391 +00	8.25	+361625 +00
2.25	+763188 +00	5.30	+524594 +00	8.30	+359663 +00
2.30	+763131 +00	5.35	+520838 +00	8.35	+357721 +00
2.35	+762680 +00	5.40	+517124 +00	8.40	+355799 +00
2.40	+761846 +00	5.45	+513451 +00	8.45	+353897 +00
2.45	+760647 +00	5.50	+509819 +00	8.50	+352013 +00
2.50	+759099 +00	5.55	+506228 +00	8.55	+350148 +00
2.55	+757224 +00	5.60	+502677 +00	8.60	+348301 +00
2.60	+755040 +00	5.65	+499165 +00	8.65	+346473 +00
2.65	+752569 +00	5.70	+495693 +00	8.70	+344663 +00
2.70	+749832 +00	5.75	+492261 +00	8.75	+342870 +00
2.75	+746850 +00	5.80	+488867 +00	8.80	+341095 +00
2.80	+743643 +00	5.85	+485511 +00	8.85	+339337 +00
2.85	+740230 +00	5.90	+482194 +00	8.90	+337597 +00
2.90	+736629 +00	5.95	+478914 +00	8.95	+335873 +00
2.95	+732859 +00	6.00	+475671 +00	9.00	+334165 +00
3.00	+728936 +00	6.05	+472466 +00	9.05	+332474 +00
3.05	+724875 +00	6.10	+469296 +00	9.10	+330800 +00
3.10	+720693 +00	6.15	+466163 +00	9.15	+329141 +00
3.15	+716402 +00	6.20	+463065 +00	9.20	+327498 +00
3.20	+712015 +00	6.25	+460002 +00	9.25	+325870 +00
3.25	+707545 +00	6.30	+456974 +00	9.30	+324258 +00
3.30	+703003 +00	6.35	+453980 +00	9.35	+322661 +00
3.35	+698400 +00	6.40	+451020 +00	9.40	+321078 +00
3.40	+693744 +00	6.45	+448094 +00	9.45	+319511 +00
3.45	+689045 +00	6.50	+445200 +00	9.50	+317958 +00
3.50	+684310 +00	6.55	+442339 +00	9.55	+316419 +00
3.55	+679549 +00	6.60	+439510 +00	9.60	+314895 +00
3.60	+674766 +00	6.65	+436713 +00	9.65	+313384 +00
3.65	+669970 +00	6.70	+433947 +00	9.70	+311887 +00
3.70	+665165 +00	6.75	+431212 +00	9.75	+310404 +00
3.75	+660358 +00	6.80	+428507 +00	9.80	+308934 +00
3.80	+655552 +00	6.85	+425833 +00	9.85	+307478 +00
3.85	+650752 +00	6.90	+423188 +00	9.90	+306035 +00
3.90	+645963 +00	6.95	+420573 +00	9.95	+304604 +00
3.95	+641189 +00	7.00	+417986 +00	10.00	+303186 +00
4.00	+636431 +00				

TABLE A2C

$\langle\theta\rangle_{rel}$ for $ka = 6\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 0.75\ cm$ $\lambda = 0.25\ cm$ $0.01125 \leq z \leq 2.25\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+621369 -01	.340	+159788 +00	.675	+227971 +00
.010	+648763 -01	.345	+160621 +00	.680	+228129 +00
.015	+671772 -01	.350	+161322 +00	.685	+228258 +00
.020	+690123 -01	.355	+161903 +00	.690	+228365 +00
.025	+703867 -01	.360	+162378 +00	.695	+228453 +00
.030	+713363 -01	.365	+162760 +00	.700	+228528 +00
.035	+719238 -01	.370	+163067 +00	.705	+228595 +00
.040	+722334 -01	.375	+163315 +00	.710	+228658 +00
.045	+723635 -01	.380	+163522 +00	.715	+228722 +00
.050	+724195 -01	.385	+163706 +00	.720	+228792 +00
.055	+725061 -01	.390	+163882 +00	.725	+228873 +00
.060	+727201 -01	.395	+164069 +00	.730	+228968 +00
.065	+731451 -01	.400	+164282 +00	.735	+229081 +00
.070	+738460 -01	.405	+164535 +00	.740	+229216 +00
.075	+748675 -01	.410	+164843 +00	.745	+229377 +00
.080	+762319 -01	.415	+165216 +00	.750	+229567 +00
.085	+779402 -01	.420	+165666 +00	.755	+229789 +00
.090	+799733 -01	.425	+166201 +00	.760	+230046 +00
.095	+822951 -01	.430	+166829 +00	.765	+230340 +00
.100	+848554 -01	.435	+167554 +00	.770	+230674 +00
.105	+875941 -01	.440	+168380 +00	.775	+231050 +00
.110	+904445 -01	.445	+169310 +00	.780	+231470 +00
.115	+933374 -01	.450	+170343 +00	.785	+231934 +00
.120	+962044 -01	.455	+171480 +00	.790	+232444 +00
.125	+989813 -01	.460	+172716 +00	.795	+233002 +00
.130	+101610 +00	.465	+174050 +00	.800	+233608 +00
.135	+104043 +00	.470	+175475 +00	.805	+234263 +00
.140	+106240 +00	.475	+176986 +00	.810	+234966 +00
.145	+108176 +00	.480	+178577 +00	.815	+235719 +00
.150	+109836 +00	.485	+180239 +00	.820	+236520 +00
.155	+111218 +00	.490	+181966 +00	.825	+237371 +00
.160	+112331 +00	.495	+183748 +00	.830	+238269 +00
.165	+113195 +00	.500	+185576 +00	.835	+239215 +00
.170	+113840 +00	.505	+187442 +00	.840	+240208 +00
.175	+114301 +00	.510	+189337 +00	.845	+241247 +00
.180	+114622 +00	.515	+191251 +00	.850	+242331 +00
.185	+114849 +00	.520	+193175 +00	.855	+243458 +00
.190	+115028 +00	.525	+195100 +00	.860	+244628 +00
.195	+115208 +00	.530	+197017 +00	.865	+245839 +00
.200	+115431 +00	.535	+198919 +00	.870	+247089 +00
.205	+115739 +00	.540	+200796 +00	.875	+248378 +00
.210	+116167 +00	.545	+202641 +00	.880	+249702 +00
.215	+116744 +00	.550	+204448 +00	.885	+251062 +00
.220	+117491 +00	.555	+206209 +00	.890	+252453 +00
.225	+118423 +00	.560	+207919 +00	.895	+253876 +00
.230	+119549 +00	.565	+209572 +00	.900	+255329 +00
.235	+120869 +00	.570	+211163 +00	.905	+256808 +00
.240	+122378 +00	.575	+212687 +00	.910	+258313 +00
.245	+124064 +00	.580	+214142 +00	.915	+259841 +00
.250	+125911 +00	.585	+215524 +00	.920	+261391 +00
.255	+127899 +00	.590	+216831 +00	.925	+262960 +00
.260	+130005 +00	.595	+218061 +00	.930	+264547 +00
.265	+132202 +00	.600	+219213 +00	.935	+266150 +00
.270	+134464 +00	.605	+220286 +00	.940	+267766 +00
.275	+136761 +00	.610	+221280 +00	.945	+269395 +00
.280	+139067 +00	.615	+222197 +00	.950	+271033 +00
.285	+141355 +00	.620	+223037 +00	.955	+272680 +00
.290	+143599 +00	.625	+223801 +00	.960	+274333 +00
.295	+145774 +00	.630	+224492 +00	.965	+275991 +00
.300	+147861 +00	.635	+225113 +00	.970	+277651 +00
.305	+149840 +00	.640	+225667 +00	.975	+279313 +00
.310	+151696 +00	.645	+226156 +00	.980	+280974 +00
.315	+153416 +00	.650	+226585 +00	.985	+282633 +00
.320	+154992 +00	.655	+226957 +00	.990	+284287 +00
.325	+156418 +00	.660	+227277 +00	.995	+285937 +00
.330	+157692 +00	.665	+227550 +00	1.000	+287579 +00
.335	+158813 +00	.670	+227779 +00		

TABLE A2D

$\langle\theta\rangle_{rel}$ for $ka = 6\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 0.75\ cm$ $\quad \lambda = 0.25\ cm$ $\quad 2.25 \leq z \leq 22.5\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
1.00	+287579 +00	4.05	+839527 +00	7.05	+113227 +01
1.05	+303327 +00	4.10	+847294 +00	7.10	+113523 +01
1.10	+317092 +00	4.15	+854915 +00	7.15	+113816 +01
1.15	+328188 +00	4.20	+862393 +00	7.20	+114104 +01
1.20	+336389 +00	4.25	+869732 +00	7.25	+114389 +01
1.25	+341836 +00	4.30	+876935 +00	7.30	+114670 +01
1.30	+344934 +00	4.35	+884005 +00	7.35	+114947 +01
1.35	+346252 +00	4.40	+890945 +00	7.40	+115221 +01
1.40	+346425 +00	4.45	+897757 +00	7.45	+115492 +01
1.45	+346075 +00	4.50	+904446 +00	7.50	+115759 +01
1.50	+345756 +00	4.55	+911014 +00	7.55	+116022 +01
1.55	+345920 +00	4.60	+917463 +00	7.60	+116283 +01
1.60	+346903 +00	4.65	+923796 +00	7.65	+116540 +01
1.65	+348930 +00	4.70	+930017 +00	7.70	+116794 +01
1.70	+352127 +00	4.75	+936128 +00	7.75	+117045 +01
1.75	+356537 +00	4.80	+942130 +00	7.80	+117293 +01
1.80	+362145 +00	4.85	+948028 +00	7.85	+117538 +01
1.85	+368888 +00	4.90	+953823 +00	7.90	+117780 +01
1.90	+376678 +00	4.95	+959518 +00	7.95	+118019 +01
1.95	+385410 +00	5.00	+965115 +00	8.00	+118255 +01
2.00	+394971 +00	5.05	+970616 +00	8.05	+118489 +01
2.05	+405247 +00	5.10	+976023 +00	8.10	+118719 +01
2.10	+416131 +00	5.15	+981340 +00	8.15	+118947 +01
2.15	+427520 +00	5.20	+986567 +00	8.20	+119173 +01
2.20	+439318 +00	5.25	+991707 +00	8.25	+119395 +01
2.25	+451442 +00	5.30	+996762 +00	8.30	+119616 +01
2.30	+463814 +00	5.35	+100173 +01	8.35	+119833 +01
2.35	+476366 +00	5.40	+100662 +01	8.40	+120048 +01
2.40	+489041 +00	5.45	+101144 +01	8.45	+120261 +01
2.45	+501785 +00	5.50	+101617 +01	8.50	+120472 +01
2.50	+514554 +00	5.55	+102083 +01	8.55	+120680 +01
2.55	+527310 +00	5.60	+102541 +01	8.60	+120885 +01
2.60	+540020 +00	5.65	+102992 +01	8.65	+121089 +01
2.65	+552655 +00	5.70	+103436 +01	8.70	+121290 +01
2.70	+565192 +00	5.75	+103873 +01	8.75	+121489 +01
2.75	+577611 +00	5.80	+104303 +01	8.80	+121686 +01
2.80	+589896 +00	5.85	+104727 +01	8.85	+121880 +01
2.85	+602032 +00	5.90	+105144 +01	8.90	+122073 +01
2.90	+614010 +00	5.95	+105555 +01	8.95	+122263 +01
2.95	+625820 +00	6.00	+105959 +01	9.00	+122452 +01
3.00	+637454 +00	6.05	+106358 +01	9.05	+122638 +01
3.05	+648908 +00	6.10	+106750 +01	9.10	+122823 +01
3.10	+660176 +00	6.15	+107137 +01	9.15	+123005 +01
3.15	+671261 +00	6.20	+107518 +01	9.20	+123186 +01
3.20	+682154 +00	6.25	+107893 +01	9.25	+123365 +01
3.25	+692858 +00	6.30	+108263 +01	9.30	+123542 +01
3.30	+703372 +00	6.35	+108628 +01	9.35	+123717 +01
3.35	+713697 +00	6.40	+108987 +01	9.40	+123890 +01
3.40	+723833 +00	6.45	+109341 +01	9.45	+124062 +01
3.45	+733783 +00	6.50	+109691 +01	9.50	+124231 +01
3.50	+743547 +00	6.55	+110035 +01	9.55	+124399 +01
3.55	+753129 +00	6.60	+110374 +01	9.60	+124566 +01
3.60	+762531 +00	6.65	+110709 +01	9.65	+124730 +01
3.65	+771756 +00	6.70	+111039 +01	9.70	+124893 +01
3.70	+780806 +00	6.75	+111365 +01	9.75	+125055 +01
3.75	+789684 +00	6.80	+111686 +01	9.80	+125215 +01
3.80	+798394 +00	6.85	+112002 +01	9.85	+125373 +01
3.85	+806938 +00	6.90	+112315 +01	9.90	+125530 +01
3.90	+815320 +00	6.95	+112623 +01	9.95	+125685 +01
3.95	+823544 +00	7.00	+112927 +01	10.00	+125839 +01
4.00	+831612 +00				

TABLE A3A

$\langle p \rangle_{rel}$ for $ka = 8\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 0.9\ cm$ $\lambda = 0.225\ cm$ $0.018 \leq z \leq 3.6\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$	
.005	+955955	+00	.340	+874537	+00	.675	+831747	+00
.010	+952359	+00	.345	+874635	+00	.680	+830419	+00
.015	+948809	+00	.350	+874743	+00	.685	+829092	+00
.020	+945707	+00	.355	+874824	+00	.690	+827771	+00
.025	+943336	+00	.360	+874844	+00	.695	+826464	+00
.030	+941814	+00	.365	+874772	+00	.700	+825175	+00
.035	+941080	+00	.370	+874582	+00	.705	+823913	+00
.040	+940918	+00	.375	+874256	+00	.710	+822681	+00
.045	+941020	+00	.380	+873780	+00	.715	+821485	+00
.050	+941053	+00	.385	+873147	+00	.720	+820329	+00
.055	+940723	+00	.390	+872355	+00	.725	+819219	+00
.060	+939828	+00	.395	+871408	+00	.730	+818157	+00
.065	+938283	+00	.400	+870316	+00	.735	+817147	+00
.070	+936128	+00	.405	+869092	+00	.740	+816192	+00
.075	+933507	+00	.410	+867754	+00	.745	+815294	+00
.080	+930636	+00	.415	+866321	+00	.750	+814455	+00
.085	+927763	+00	.420	+864816	+00	.755	+813676	+00
.090	+925125	+00	.425	+863264	+00	.760	+812959	+00
.095	+922909	+00	.430	+861687	+00	.765	+812305	+00
.100	+921230	+00	.435	+860111	+00	.770	+811712	+00
.105	+920116	+00	.440	+858559	+00	.775	+811181	+00
.110	+919515	+00	.445	+857052	+00	.780	+810712	+00
.115	+919305	+00	.450	+855611	+00	.785	+810303	+00
.120	+919321	+00	.455	+854253	+00	.790	+809953	+00
.125	+919378	+00	.460	+852994	+00	.795	+809660	+00
.130	+919304	+00	.465	+851846	+00	.800	+809423	+00
.135	+918953	+00	.470	+850818	+00	.805	+809239	+00
.140	+918229	+00	.475	+849916	+00	.810	+809107	+00
.145	+917086	+00	.480	+849143	+00	.815	+809022	+00
.150	+915533	+00	.485	+848500	+00	.820	+808984	+00
.155	+913629	+00	.490	+847983	+00	.825	+808988	+00
.160	+911468	+00	.495	+847588	+00	.830	+809032	+00
.165	+909170	+00	.500	+847308	+00	.835	+809113	+00
.170	+906863	+00	.505	+847132	+00	.840	+809228	+00
.175	+904671	+00	.510	+847050	+00	.845	+809372	+00
.180	+902701	+00	.515	+847049	+00	.850	+809544	+00
.185	+901032	+00	.520	+847117	+00	.855	+809740	+00
.190	+899710	+00	.525	+847240	+00	.860	+809956	+00
.195	+898748	+00	.530	+847402	+00	.865	+810189	+00
.200	+898125	+00	.535	+847590	+00	.870	+810437	+00
.205	+897789	+00	.540	+847790	+00	.875	+810695	+00
.210	+897671	+00	.545	+847988	+00	.880	+810961	+00
.215	+897685	+00	.550	+848170	+00	.885	+811232	+00
.220	+897740	+00	.555	+848325	+00	.890	+811505	+00
.225	+897750	+00	.560	+848441	+00	.895	+811777	+00
.230	+897637	+00	.565	+848508	+00	.900	+812045	+00
.235	+897338	+00	.570	+848517	+00	.905	+812306	+00
.240	+896806	+00	.575	+848461	+00	.910	+812559	+00
.245	+896018	+00	.580	+848332	+00	.915	+812801	+00
.250	+894967	+00	.585	+848126	+00	.920	+813029	+00
.255	+893669	+00	.590	+847838	+00	.925	+813242	+00
.260	+892152	+00	.595	+847467	+00	.930	+813437	+00
.265	+890461	+00	.600	+847009	+00	.935	+813612	+00
.270	+888649	+00	.605	+846466	+00	.940	+813767	+00
.275	+886773	+00	.610	+845838	+00	.945	+813898	+00
.280	+884894	+00	.615	+845126	+00	.950	+814005	+00
.285	+883069	+00	.620	+844333	+00	.955	+814087	+00
.290	+881352	+00	.625	+843464	+00	.960	+814141	+00
.295	+879786	+00	.630	+842522	+00	.965	+814168	+00
.300	+878406	+00	.635	+841512	+00	.970	+814166	+00
.305	+877236	+00	.640	+840440	+00	.975	+814134	+00
.310	+876285	+00	.645	+839312	+00	.980	+814072	+00
.315	+875555	+00	.650	+838134	+00	.985	+813978	+00
.320	+875035	+00	.655	+836914	+00	.990	+813853	+00
.325	+874703	+00	.660	+835658	+00	.995	+813696	+00
.330	+874533	+00	.665	+834373	+00	1.000	+813506	+00
.335	+874490	+00	.670	+833067	+00			

TABLE A3B

$\langle p \rangle_{rel}$ for $ka = 8\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 0.9\ cm$ $\lambda = 0.225\ cm$ $3.6 \leq z \leq 36\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
1.00	+813506 +00	4.05	+633514 +00	7.05	+416172 +00
1.05	+809834 +00	4.10	+628780 +00	7.10	+413631 +00
1.10	+803212 +00	4.15	+624072 +00	7.15	+411118 +00
1.15	+794408 +00	4.20	+619391 +00	7.20	+408633 +00
1.20	+784407 +00	4.25	+614739 +00	7.25	+406175 +00
1.25	+774174 +00	4.30	+610119 +00	7.30	+403743 +00
1.30	+764520 +00	4.35	+605532 +00	7.35	+401338 +00
1.35	+756044 +00	4.40	+600980 +00	7.40	+398959 +00
1.40	+749114 +00	4.45	+596464 +00	7.45	+396606 +00
1.45	+743895 +00	4.50	+591985 +00	7.50	+394278 +00
1.50	+740384 +00	4.55	+587544 +00	7.55	+391975 +00
1.55	+738456 +00	4.60	+583142 +00	7.60	+389696 +00
1.60	+737907 +00	4.65	+578779 +00	7.65	+387442 +00
1.65	+738494 +00	4.70	+574457 +00	7.70	+385211 +00
1.70	+739959 +00	4.75	+570177 +00	7.75	+383004 +00
1.75	+742055 +00	4.80	+565937 +00	7.80	+380821 +00
1.80	+744556 +00	4.85	+561739 +00	7.85	+378660 +00
1.85	+747265 +00	4.90	+557584 +00	7.90	+376522 +00
1.90	+750016 +00	4.95	+553470 +00	7.95	+374406 +00
1.95	+752674 +00	5.00	+549399 +00	8.00	+372312 +00
2.00	+755133 +00	5.05	+545370 +00	8.05	+370240 +00
2.05	+757313 +00	5.10	+541384 +00	8.10	+368189 +00
2.10	+759155 +00	5.15	+537440 +00	8.15	+366160 +00
2.15	+760621 +00	5.20	+533539 +00	8.20	+364151 +00
2.20	+761684 +00	5.25	+529679 +00	8.25	+362162 +00
2.25	+762334 +00	5.30	+525862 +00	8.30	+360194 +00
2.30	+762568 +00	5.35	+522087 +00	8.35	+358246 +00
2.35	+762390 +00	5.40	+518353 +00	8.40	+356317 +00
2.40	+761813 +00	5.45	+514661 +00	8.45	+354408 +00
2.45	+760850 +00	5.50	+511010 +00	8.50	+352518 +00
2.50	+759519 +00	5.55	+507400 +00	8.55	+350646 +00
2.55	+757842 +00	5.60	+503830 +00	8.60	+348794 +00
2.60	+755838 +00	5.65	+500301 +00	8.65	+346959 +00
2.65	+753529 +00	5.70	+496811 +00	8.70	+345143 +00
2.70	+750938 +00	5.75	+493361 +00	8.75	+343345 +00
2.75	+748086 +00	5.80	+489950 +00	8.80	+341564 +00
2.80	+744994 +00	5.85	+486578 +00	8.85	+339800 +00
2.85	+741683 +00	5.90	+483244 +00	8.90	+338054 +00
2.90	+738172 +00	5.95	+479948 +00	8.95	+336325 +00
2.95	+734479 +00	6.00	+476689 +00	9.00	+334612 +00
3.00	+730623 +00	6.05	+473468 +00	9.05	+332916 +00
3.05	+726620 +00	6.10	+470283 +00	9.10	+331236 +00
3.10	+722486 +00	6.15	+467135 +00	9.15	+329572 +00
3.15	+718236 +00	6.20	+464022 +00	9.20	+327924 +00
3.20	+713883 +00	6.25	+460945 +00	9.25	+326291 +00
3.25	+709440 +00	6.30	+457902 +00	9.30	+324674 +00
3.30	+704919 +00	6.35	+454894 +00	9.35	+323072 +00
3.35	+700331 +00	6.40	+451921 +00	9.40	+321485 +00
3.40	+695686 +00	6.45	+448981 +00	9.45	+319913 +00
3.45	+690993 +00	6.50	+446074 +00	9.50	+318356 +00
3.50	+686261 +00	6.55	+443200 +00	9.55	+316812 +00
3.55	+681499 +00	6.60	+440358 +00	9.60	+315284 +00
3.60	+676713 +00	6.65	+437549 +00	9.65	[illegible]
3.65	+671910 +00	6.70	+434771 +00	9.70	+312268 +00
3.70	+667096 +00	6.75	+432024 +00	9.75	+310780 +00
3.75	+662277 +00	6.80	+429307 +00	9.80	+309306 +00
3.80	+657458 +00	6.85	+426621 +00	9.85	+307846 +00
3.85	+652644 +00	6.90	+423965 +00	9.90	+306398 +00
3.90	+647839 +00	6.95	+421338 +00	9.95	+304964 +00
3.95	+643046 +00	7.00	+418741 +00	10.00	+303542 +00
4.00	+638270 +00				

TABLE A3C

$\langle\theta\rangle_{rel}$ for $ka = 8\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 0.9\ cm$ $\lambda = 0.225\ cm$ $0.018 \leq z \leq 3.6\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+525178 -01	.340	+146431 +00	.675	+232430 +00
.010	+550346 -01	.345	+147480 +00	.680	+233485 +00
.015	+565867 -01	.350	+148738 +00	.685	+234449 +00
.020	+573035 -01	.355	+150199 +00	.690	+235324 +00
.025	+574862 -01	.360	+151847 +00	.695	+236110 +00
.030	+575303 -01	.365	+153660 +00	.700	+236810 +00
.035	+578331 -01	.370	+155611 +00	.705	+237426 +00
.040	+587092 -01	.375	+157670 +00	.710	+237963 +00
.045	+603331 -01	.380	+159802 +00	.715	+238423 +00
.050	+627179 -01	.385	+161972 +00	.720	+238812 +00
.055	+657291 -01	.390	+164145 +00	.725	+239135 +00
.060	+691237 -01	.395	+166288 +00	.730	+239396 +00
.065	+726040 -01	.400	+168369 +00	.735	+239601 +00
.070	+758732 -01	.405	+170358 +00	.740	+239755 +00
.075	+786849 -01	.410	+172232 +00	.745	+239866 +00
.080	+808800 -01	.415	+173970 +00	.750	+239939 +00
.085	+824060 -01	.420	+175556 +00	.755	+239980 +00
.090	+833182 -01	.425	+176978 +00	.760	+239995 +00
.095	+837639 -01	.430	+178230 +00	.765	+239991 +00
.100	+839526 -01	.435	+179311 +00	.770	+239973 +00
.105	+841184 -01	.440	+180223 +00	.775	+239949 +00
.110	+844828 -01	.445	+180975 +00	.780	+239923 +00
.115	+852222 -01	.450	+181577 +00	.785	+239901 +00
.120	+864464 -01	.455	+182043 +00	.790	+239889 +00
.125	+881901 -01	.460	+182392 +00	.795	+239891 +00
.130	+904147 -01	.465	+182643 +00	.800	+239914 +00
.135	+930213 -01	.470	+182816 +00	.805	+239962 +00
.140	+958685 -01	.475	+182934 +00	.810	+240038 +00
.145	+987933 -01	.480	+183020 +00	.815	+240148 +00
.150	+101632 +00	.485	+183096 +00	.820	+240294 +00
.155	+104239 +00	.490	+183183 +00	.825	+240480 +00
.160	+106501 +00	.495	+183303 +00	.830	+240709 +00
.165	+108348 +00	.500	+183474 +00	.835	+240985 +00
.170	+109755 +00	.505	+183713 +00	.840	+241308 +00
.175	+110744 +00	.510	+184036 +00	.845	+241682 +00
.180	+111378 +00	.515	+184454 +00	.850	+242107 +00
.185	+111752 +00	.520	+184979 +00	.855	+242586 +00
.190	+111981 +00	.525	+185617 +00	.860	+243119 +00
.195	+112185 +00	.530	+186375 +00	.865	+243707 +00
.200	+112481 +00	.535	+187254 +00	.870	+244351 +00
.205	+112969 +00	.540	+188256 +00	.875	+245051 +00
.210	+113724 +00	.545	+189379 +00	.880	+245806 +00
.215	+114796 +00	.550	+190619 +00	.885	+246616 +00
.220	+116201 +00	.555	+191972 +00	.890	+247482 +00
.225	+117928 +00	.560	+193430 +00	.895	+248401 +00
.230	+119941 +00	.565	+194985 +00	.900	+249374 +00
.235	+122182 +00	.570	+196628 +00	.905	+250398 +00
.240	+124581 +00	.575	+198348 +00	.910	+251474 +00
.245	+127058 +00	.580	+200135 +00	.915	+252598 +00
.250	+129531 +00	.585	+201978 +00	.920	+253771 +00
.255	+131925 +00	.590	+203864 +00	.925	+254989 +00
.260	+134170 +00	.595	+205781 +00	.930	+256252 +00
.265	+136209 +00	.600	+207718 +00	.935	+257556 +00
.270	+138002 +00	.605	+209663 +00	.940	+258901 +00
.275	+139523 +00	.610	+211605 +00	.945	+260285 +00
.280	+140764 +00	.615	+213532 +00	.950	+261704 +00
.285	+141735 +00	.620	+215435 +00	.955	+263157 +00
.290	+142458 +00	.625	+217302 +00	.960	+264642 +00
.295	+142969 +00	.630	+219126 +00	.965	+266156 +00
.300	+143316 +00	.635	+220897 +00	.970	+267698 +00
.305	+143550 +00	.640	+222607 +00	.975	+269264 +00
.310	+143727 +00	.645	+224251 +00	.980	+270853 +00
.315	+143905 +00	.650	+225822 +00	.985	+272463 +00
.320	+144136 +00	.655	+227314 +00	.990	+274090 +00
.325	+144468 +00	.660	+228724 +00	.995	+275733 +00
.330	+144942 +00	.665	+230048 +00	1.000	+277391 +00
.335	+145589 +00	.670	+231284 +00		

TABLE A3D

$\langle\theta\rangle_{rel}$ for $ka = 8\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 0.9\ cm$ $\quad \lambda = 0.225\ cm$ $\quad 3.6 \leq z \leq 36\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+277391	+00	4.05	+838146	+00	7.05	+113246	+01
1.05	+294251	+00	4.10	+845977	+00	7.10	+113543	+01
1.10	+310358	+00	4.15	+853660	+00	7.15	+113836	+01
1.15	+324346	+00	4.20	+861198	+00	7.20	+114125	+01
1.20	+335449	+00	4.25	+868594	+00	7.25	+114411	+01
1.25	+343440	+00	4.30	+875852	+00	7.30	+114692	+01
1.30	+348509	+00	4.35	+882975	+00	7.35	+114970	+01
1.35	+351138	+00	4.40	+889966	+00	7.40	+115245	+01
1.40	+351969	+00	4.45	+896828	+00	7.45	+115516	+01
1.45	+351704	+00	4.50	+903564	+00	7.50	+115783	+01
1.50	+351011	+00	4.55	+910177	+00	7.55	+116048	+01
1.55	+350468	+00	4.60	+916670	+00	7.60	+116309	+01
1.60	+350534	+00	4.65	+923046	+00	7.65	+116566	+01
1.65	+351537	+00	4.70	+929307	+00	7.70	+116821	+01
1.70	+353685	+00	4.75	+935457	+00	7.75	+117072	+01
1.75	+357083	+00	4.80	+941498	+00	7.80	+117321	+01
1.80	+361752	+00	4.85	+947432	+00	7.85	+117566	+01
1.85	+367654	+00	4.90	+953262	+00	7.90	+117808	+01
1.90	+374711	+00	4.95	+958990	+00	7.95	+118048	+01
1.95	+382820	+00	5.00	+964620	+00	8.00	+118285	+01
2.00	+391863	+00	5.05	+970152	+00	8.05	+118518	+01
2.05	+401720	+00	5.10	+975590	+00	8.10	+118749	+01
2.10	+412272	+00	5.15	+980935	+00	8.15	+118978	+01
2.15	+423408	+00	5.20	+986191	+00	8.20	+119204	+01
2.20	+435021	+00	5.25	+991358	+00	8.25	+119427	+01
2.25	+447018	+00	5.30	+996439	+00	8.30	+119647	+01
2.30	+459313	+00	5.35	+100144	+01	8.35	+119865	+01
2.35	+471830	+00	5.40	+100635	+01	8.40	+120081	+01
2.40	+484504	+00	5.45	+101118	+01	8.45	+120294	+01
2.45	+497276	+00	5.50	+101594	+01	8.50	+120504	+01
2.50	+510097	+00	5.55	+102062	+01	8.55	+120713	+01
2.55	+522923	+00	5.60	+102522	+01	8.60	+120919	+01
2.60	+535718	+00	5.65	+102975	+01	8.65	+121122	+01
2.65	+548450	+00	5.70	+103421	+01	8.70	+121324	+01
2.70	+561092	+00	5.75	+103860	+01	8.75	+121523	+01
2.75	+573624	+00	5.80	+104292	+01	8.80	+121720	+01
2.80	+586026	+00	5.85	+104718	+01	8.85	+121915	+01
2.85	+598283	+00	5.90	+105137	+01	8.90	+122108	+01
2.90	+610383	+00	5.95	+105549	+01	8.95	+122298	+01
2.95	+622315	+00	6.00	+105955	+01	9.00	+122487	+01
3.00	+634073	+00	6.05	+106355	+01	9.05	+122674	+01
3.05	+645649	+00	6.10	+106749	+01	9.10	+122858	+01
3.10	+657039	+00	6.15	+107137	+01	9.15	+123041	+01
3.15	+668240	+00	6.20	+107520	+01	9.20	+123222	+01
3.20	+679249	+00	6.25	+107896	+01	9.25	+123401	+01
3.25	+690067	+00	6.30	+108267	+01	9.30	+123578	+01
3.30	+700692	+00	6.35	+108633	+01	9.35	+123753	+01
3.35	+711124	+00	6.40	+108994	+01	9.40	+123926	+01
3.40	+721365	+00	6.45	+109349	+01	9.45	+124098	+01
3.45	+731417	+00	6.50	+109699	+01	9.50	+124268	+01
3.50	+741280	+00	6.55	+110045	+01	9.55	+124436	+01
3.55	+750958	+00	6.60	+110385	+01	9.60	+124603	+01
3.60	+760452	+00	6.65	+110721	+01	9.65	+124767	+01
3.65	+769765	+00	6.70	+111052	+01	9.70	+124931	+01
3.70	+778901	+00	6.75	+111379	+01	9.75	+125092	+01
3.75	+787863	+00	6.80	+111701	+01	9.80	+125252	+01
3.80	+796653	+00	6.85	+112018	+01	9.85	+125410	+01
3.85	+805275	+00	6.90	+112331	+01	9.90	+125567	+01
3.90	+813731	+00	6.95	+112640	+01	9.95	+125723	+01
3.95	+822027	+00	7.00	+112945	+01	10.00	+125876	+01
4.00	+830164	+00						

TABLE A4A

$\langle p \rangle_{rel}$ for $ka = 10\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 1.0\ cm$ $\lambda = 0.2\ cm$ $0.025 \leq z \leq 5\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$	
.005	+959514	+00	.340	+875024	+00	.675	+837151	+00
.010	+955485	+00	.345	+873319	+00	.680	+836019	+00
.015	+952315	+00	.350	+871757	+00	.685	+834833	+00
.020	+950607	+00	.355	+870385	+00	.690	+833599	+00
.025	+950195	+00	.360	+869236	+00	.695	+832326	+00
.030	+950301	+00	.365	+868328	+00	.700	+831023	+00
.035	+949979	+00	.370	+867662	+00	.705	+829696	+00
.040	+948572	+00	.375	+867227	+00	.710	+828355	+00
.045	+945982	+00	.380	+866999	+00	.715	+827007	+00
.050	+942651	+00	.385	+866941	+00	.720	+825661	+00
.055	+939321	+00	.390	+867013	+00	.725	+824323	+00
.060	+936691	+00	.395	+867168	+00	.730	+823002	+00
.065	+935135	+00	.400	+867358	+00	.735	+821703	+00
.070	+934582	+00	.405	+867536	+00	.740	+820433	+00
.075	+934589	+00	.410	+867657	+00	.745	+819199	+00
.080	+934550	+00	.415	+867684	+00	.750	+818005	+00
.085	+933932	+00	.420	+867584	+00	.755	+816858	+00
.090	+932456	+00	.425	+867332	+00	.760	+815760	+00
.095	+930165	+00	.430	+866912	+00	.765	+814717	+00
.100	+927373	+00	.435	+866314	+00	.770	+813731	+00
.105	+924541	+00	.440	+865537	+00	.775	+812806	+00
.110	+922115	+00	.445	+864587	+00	.780	+811943	+00
.115	+920394	+00	.450	+863476	+00	.785	+811144	+00
.120	+919452	+00	.455	+862222	+00	.790	+810411	+00
.125	+919140	+00	.460	+860848	+00	.795	+809744	+00
.130	+919152	+00	.465	+859379	+00	.800	+809144	+00
.135	+919123	+00	.470	+857843	+00	.805	+808610	+00
.140	+918731	+00	.475	+856268	+00	.810	+808141	+00
.145	+917771	+00	.480	+854685	+00	.815	+807737	+00
.150	+916199	+00	.485	+853122	+00	.820	+807396	+00
.155	+914118	+00	.490	+851605	+00	.825	+807116	+00
.160	+911743	+00	.495	+850158	+00	.830	+806895	+00
.165	+909345	+00	.500	+848803	+00	.835	+806731	+00
.170	+907186	+00	.505	+847558	+00	.840	+806621	+00
.175	+905461	+00	.510	+846437	+00	.845	+806563	+00
.180	+904272	+00	.515	+845450	+00	.850	+806552	+00
.185	+903608	+00	.520	+844604	+00	.855	+806587	+00
.190	+903363	+00	.525	+843902	+00	.860	+806664	+00
.195	+903365	+00	.530	+843342	+00	.865	+806778	+00
.200	+903410	+00	.535	+842921	+00	.870	+806928	+00
.205	+903303	+00	.540	+842630	+00	.875	+807109	+00
.210	+902890	+00	.545	+842460	+00	.880	+807317	+00
.215	+902078	+00	.550	+842399	+00	.885	+807549	+00
.220	+900842	+00	.555	+842433	+00	.890	+807802	+00
.225	+899225	+00	.560	+842546	+00	.895	+808072	+00
.230	+897324	+00	.565	+842722	+00	.900	+808355	+00
.235	+895273	+00	.570	+842945	+00	.905	+808649	+00
.240	+893218	+00	.575	+843197	+00	.910	+808949	+00
.245	+891300	+00	.580	+843462	+00	.915	+809252	+00
.250	+889636	+00	.585	+843725	+00	.920	+809556	+00
.255	+888305	+00	.590	+843969	+00	.925	+809857	+00
.260	+887344	+00	.595	+844181	+00	.930	+810153	+00
.265	+886744	+00	.600	+844348	+00	.935	+810440	+00
.270	+886455	+00	.605	+844458	+00	.940	+810717	+00
.275	+886399	+00	.610	+844501	+00	.945	+810979	+00
.280	+886477	+00	.615	+844470	+00	.950	+811227	+00
.285	+886584	+00	.620	+844356	+00	.955	+811456	+00
.290	+886618	+00	.625	+844154	+00	.960	+811665	+00
.295	+886490	+00	.630	+843861	+00	.965	+811852	+00
.300	+886134	+00	.635	+843474	+00	.970	+812015	+00
.305	+885509	+00	.640	+842993	+00	.975	+812153	+00
.310	+884598	+00	.645	+842417	+00	.980	+812264	+00
.315	+883411	+00	.650	+841748	+00	.985	+812347	+00
.320	+881983	+00	.655	+840990	+00	.990	+812401	+00
.325	+880363	+00	.660	+840146	+00	.995	+812424	+00
.330	+878616	+00	.665	+839221	+00	1.000	+812416	+00
.335	+876812	+00	.670	+838221	+00			

TABLE A4B
$\langle p \rangle_{rel}$ for $ka = 10\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 1.0\ cm$ $\lambda = 0.2\ cm$ $5 \leq z \leq 50\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
1.00	+812416 +00	4.05	+634359 +00	7.05	+416517 +00
1.05	+810502 +00	4.10	+629616 +00	7.10	+413971 +00
1.10	+805281 +00	4.15	+624897 +00	7.15	+411454 +00
1.15	+797351 +00	4.20	+620206 +00	7.20	+408964 +00
1.20	+787692 +00	4.25	+615545 +00	7.25	+406501 +00
1.25	+777359 +00	4.30	+610914 +00	7.30	+404065 +00
1.30	+767287 +00	4.35	+606317 +00	7.35	+401655 +00
1.35	+758201 +00	4.40	+601754 +00	7.40	+399272 +00
1.40	+750578 +00	4.45	+597227 +00	7.45	+396914 +00
1.45	+744668 +00	4.50	+592737 +00	7.50	+394582 +00
1.50	+740526 +00	4.55	+588285 +00	7.55	+392275 +00
1.55	+738062 +00	4.60	+583873 +00	7.60	+389992 +00
1.60	+737088 +00	4.65	+579500 +00	7.65	+387734 +00
1.65	+737361 +00	4.70	+575167 +00	7.70	+385499 +00
1.70	+738617 +00	4.75	+570876 +00	7.75	+383289 +00
1.75	+740595 +00	4.80	+566626 +00	7.80	+381101 +00
1.80	+743055 +00	4.85	+562417 +00	7.85	+378937 +00
1.85	+745783 +00	4.90	+558251 +00	7.90	+376795 +00
1.90	+748601 +00	4.95	+554128 +00	7.95	+374676 +00
1.95	+751360 +00	5.00	+550046 +00	8.00	+372578 +00
2.00	+753943 +00	5.05	+546007 +00	8.05	+370503 +00
2.05	+756263 +00	5.10	+542011 +00	8.10	+368448 +00
2.10	+758253 +00	5.15	+538058 +00	8.15	+366415 +00
2.15	+759869 +00	5.20	+534147 +00	8.20	+364403 +00
2.20	+761083 +00	5.25	+530278 +00	8.25	+362411 +00
2.25	+761878 +00	5.30	+526451 +00	8.30	+360440 +00
2.30	+762251 +00	5.35	+522667 +00	8.35	+358489 +00
2.35	+762205 +00	5.40	+518924 +00	8.40	+356557 +00
2.40	+761750 +00	5.45	+515223 +00	8.45	+354645 +00
2.45	+760901 +00	5.50	+511563 +00	8.50	+352752 +00
2.50	+759676 +00	5.55	+507944 +00	8.55	+350877 +00
2.55	+758094 +00	5.60	+504366 +00	8.60	+349022 +00
2.60	+756178 +00	5.65	+500828 +00	8.65	+347185 +00
2.65	+753948 +00	5.70	+497330 +00	8.70	+345366 +00
2.70	+751427 +00	5.75	+493872 +00	8.75	+343565 +00
2.75	+748638 +00	5.80	+490453 +00	8.80	+341781 +00
2.80	+745602 +00	5.85	+487073 +00	8.85	+340015 +00
2.85	+742341 +00	5.90	+483731 +00	8.90	+338266 +00
2.90	+738873 +00	5.95	+480428 +00	8.95	+336535 +00
2.95	+735218 +00	6.00	+477162 +00	9.00	+334819 +00
3.00	+731395 +00	6.05	+473933 +00	9.05	+333121 +00
3.05	+727420 +00	6.10	+470741 +00	9.10	+331438 +00
3.10	+723310 +00	6.15	+467586 +00	9.15	+329772 +00
3.15	+719080 +00	6.20	+464466 +00	9.20	+328122 +00
3.20	+714744 +00	6.25	+461382 +00	9.25	+326487 +00
3.25	+710314 +00	6.30	+458333 +00	9.30	+324867 +00
3.30	+705804 +00	6.35	+455319 +00	9.35	+323263 +00
3.35	+701223 +00	6.40	+452339 +00	9.40	+321674 +00
3.40	+696584 +00	6.45	+449392 +00	9.45	+320100 +00
3.45	+691895 +00	6.50	+446480 +00	9.50	+318540 +00
3.50	+687165 +00	6.55	+443599 +00	9.55	+316995 +00
3.55	+682402 +00	6.60	+440752 +00	9.60	+315464 +00
3.60	+677615 +00	6.65	+437936 +00	9.65	+313947 +00
3.65	+672809 +00	6.70	+435153 +00	9.70	+312444 +00
3.70	+667991 +00	6.75	+432400 +00	9.75	+310955 +00
3.75	+663167 +00	6.80	+429678 +00	9.80	+309479 +00
3.80	+658342 +00	6.85	+426987 +00	9.85	+308016 +00
3.85	+653521 +00	6.90	+424325 +00	9.90	+306567 +00
3.90	+648709 +00	6.95	+421694 +00	9.95	+305131 +00
3.95	+643908 +00	7.00	+419091 +00	10.00	+303707 +00
4.00	+639124 +00				

TABLE A4C

$\langle\theta\rangle_{rel}$ for $ka = 10\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 1.0\ cm$ $\lambda = 0.2\ cm$ $0.025 \leq z \leq 5\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
.005	+463445	-01	.340	+154412	+00	.675	+228755	+00
.010	+482072	-01	.345	+155176	+00	.680	+230424	+00
.015	+486921	-01	.350	+155688	+00	.685	+232013	+00
.020	+487505	-01	.355	+155996	+00	.690	+233518	+00
.025	+494713	-01	.360	+156164	+00	.695	+234933	+00
.030	+515164	-01	.365	+156259	+00	.700	+236254	+00
.035	+548430	-01	.370	+156350	+00	.705	+237479	+00
.040	+588044	-01	.375	+156503	+00	.710	+238607	+00
.045	+625067	-01	.380	+156775	+00	.715	+239636	+00
.050	+652231	-01	.385	+157215	+00	.720	+240566	+00
.055	+666873	-01	.390	+157860	+00	.725	+241400	+00
.060	+671663	-01	.395	+158731	+00	.730	+242139	+00
.065	+673054	-01	.400	+159839	+00	.735	+242785	+00
.070	+678308	-01	.405	+161181	+00	.740	+243343	+00
.075	+692546	-01	.410	+162743	+00	.745	+243816	+00
.080	+717022	-01	.415	+164500	+00	.750	+244209	+00
.085	+749108	-01	.420	+166423	+00	.755	+244528	+00
.090	+783658	-01	.425	+168472	+00	.760	+244778	+00
.095	+815013	-01	.430	+170607	+00	.765	+244965	+00
.100	+838896	-01	.435	+172786	+00	.770	+245095	+00
.105	+853641	-01	.440	+174965	+00	.775	+245176	+00
.110	+860491	-01	.445	+177103	+00	.780	+245214	+00
.115	+862964	-01	.450	+179164	+00	.785	+245215	+00
.120	+865590	-01	.455	+181114	+00	.790	+245188	+00
.125	+872482	-01	.460	+182923	+00	.795	+245138	+00
.130	+886212	-01	.465	+184571	+00	.800	+245073	+00
.135	+907270	-01	.470	+186040	+00	.805	+245000	+00
.140	+934171	-01	.475	+187321	+00	.810	+244924	+00
.145	+964047	-01	.480	+188409	+00	.815	+244852	+00
.150	+993485	-01	.485	+189308	+00	.820	+244791	+00
.155	+101937	+00	.490	+190025	+00	.825	+244746	+00
.160	+103955	+00	.495	+190573	+00	.830	+244722	+00
.165	+105324	+00	.500	+190969	+00	.835	+244725	+00
.170	+106106	+00	.505	+191235	+00	.840	+244760	+00
.175	+106478	+00	.510	+191393	+00	.845	+244830	+00
.180	+106690	+00	.515	+191471	+00	.850	+244940	+00
.185	+107003	+00	.520	+191493	+00	.855	+245094	+00
.190	+107641	+00	.525	+191488	+00	.860	+245295	+00
.195	+108750	+00	.530	+191481	+00	.865	+245545	+00
.200	+110379	+00	.535	+191498	+00	.870	+245847	+00
.205	+112483	+00	.540	+191563	+00	.875	+246204	+00
.210	+114941	+00	.545	+191697	+00	.880	+246616	+00
.215	+117580	+00	.550	+191918	+00	.885	+247086	+00
.220	+120210	+00	.555	+192242	+00	.890	+247614	+00
.225	+122650	+00	.560	+192683	+00	.895	+248201	+00
.230	+124757	+00	.565	+193249	+00	.900	+248848	+00
.235	+126440	+00	.570	+193948	+00	.905	+249554	+00
.240	+127671	+00	.575	+194784	+00	.910	+250319	+00
.245	+128478	+00	.580	+195756	+00	.915	+251143	+00
.250	+128946	+00	.585	+196864	+00	.920	+252025	+00
.255	+129195	+00	.590	+198102	+00	.925	+252964	+00
.260	+129364	+00	.595	+199465	+00	.930	+253958	+00
.265	+129594	+00	.600	+200943	+00	.935	+255008	+00
.270	+130010	+00	.605	+202528	+00	.940	+256110	+00
.275	+130707	+00	.610	+204208	+00	.945	+257264	+00
.280	+131745	+00	.615	+205971	+00	.950	+258467	+00
.285	+133143	+00	.620	+207803	+00	.955	+259717	+00
.290	+134882	+00	.625	+209691	+00	.960	+261013	+00
.295	+136911	+00	.630	+211622	+00	.965	+262353	+00
.300	+139154	+00	.635	+213582	+00	.970	+263733	+00
.305	+141520	+00	.640	+215557	+00	.975	+265152	+00
.310	+143910	+00	.645	+217533	+00	.980	+266608	+00
.315	+146230	+00	.650	+219498	+00	.985	+268097	+00
.320	+148394	+00	.655	+221440	+00	.990	+269618	+00
.325	+150333	+00	.660	+223347	+00	.995	+271169	+00
.330	+151996	+00	.665	+225208	+00	1.000	+272746	+00
.335	+153357	+00	.670	+227014	+00			

TABLE A4D

$\langle\theta\rangle_{rel}$ for $ka = 10\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 1.0\ cm$ $\lambda = 0.2\ cm$ $5 \leq z \leq 50\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+272746	+00	4.05	+837499	+00	7.05	+113255	+01
1.05	+289449	+00	4.10	+845360	+00	7.10	+113552	+01
1.10	+306304	+00	4.15	+853071	+00	7.15	+113845	+01
1.15	+321549	+00	4.20	+860637	+00	7.20	+114135	+01
1.20	+334085	+00	4.25	+868060	+00	7.25	+114421	+01
1.25	+343440	+00	4.30	+875344	+00	7.30	+114703	+01
1.30	+349647	+00	4.35	+882492	+00	7.35	+114981	+01
1.35	+353104	+00	4.40	+889507	+00	7.40	+115256	+01
1.40	+354437	+00	4.45	+896392	+00	7.45	+115527	+01
1.45	+354365	+00	4.50	+903150	+00	7.50	+115795	+01
1.50	+353609	+00	4.55	+909785	+00	7.55	+116059	+01
1.55	+352808	+00	4.60	+916298	+00	7.60	+116321	+01
1.60	+352483	+00	4.65	+922694	+00	7.65	+116579	+01
1.65	+353019	+00	4.70	+928974	+00	7.70	+116833	+01
1.70	+354671	+00	4.75	+935142	+00	7.75	+117085	+01
1.75	+357577	+00	4.80	+941201	+00	7.80	+117334	+01
1.80	+361782	+00	4.85	+947152	+00	7.85	+117579	+01
1.85	+367264	+00	4.90	+952998	+00	7.90	+117822	+01
1.90	+373951	+00	4.95	+958742	+00	7.95	+118061	+01
1.95	+381743	+00	5.00	+964387	+00	8.00	+118298	+01
2.00	+390520	+00	5.05	+969934	+00	8.05	+118532	+01
2.05	+400160	+00	5.10	+975386	+00	8.10	+118763	+01
2.10	+410540	+00	5.15	+980745	+00	8.15	+118992	+01
2.15	+421542	+00	5.20	+986013	+00	8.20	+119218	+01
2.20	+433056	+00	5.25	+991193	+00	8.25	+119441	+01
2.25	+444983	+00	5.30	+996286	+00	8.30	+119662	+01
2.30	+457234	+00	5.35	+100129	+01	8.35	+119880	+01
2.35	+469728	+00	5.40	+100622	+01	8.40	+120095	+01
2.40	+482396	+00	5.45	+101107	+01	8.45	+120309	+01
2.45	+495177	+00	5.50	+101583	+01	8.50	+120519	+01
2.50	+508018	+00	5.55	+102052	+01	8.55	+120728	+01
2.55	+520874	+00	5.60	+102514	+01	8.60	+120934	+01
2.60	+533706	+00	5.65	+102968	+01	8.65	+121138	+01
2.65	+546482	+00	5.70	+103414	+01	8.70	+121339	+01
2.70	+559173	+00	5.75	+103854	+01	8.75	+121539	+01
2.75	+571756	+00	5.80	+104287	+01	8.80	+121736	+01
2.80	+584213	+00	5.85	+104713	+01	8.85	+121931	+01
2.85	+596526	+00	5.90	+105133	+01	8.90	+122124	+01
2.90	+608682	+00	5.95	+105546	+01	8.95	+122314	+01
2.95	+620672	+00	6.00	+105953	+01	9.00	+122503	+01
3.00	+632487	+00	6.05	+106354	+01	9.05	+122690	+01
3.05	+644119	+00	6.10	+106748	+01	9.10	+122875	+01
3.10	+655566	+00	6.15	+107137	+01	9.15	+123057	+01
3.15	+666822	+00	6.20	+107520	+01	9.20	+123238	+01
3.20	+677887	+00	6.25	+107897	+01	9.25	+123417	+01
3.25	+688757	+00	6.30	+108269	+01	9.30	+123594	+01
3.30	+699434	+00	6.35	+108636	+01	9.35	+123770	+01
3.35	+709917	+00	6.40	+108997	+01	9.40	+123943	+01
3.40	+720208	+00	6.45	+109353	+01	9.45	+124115	+01
3.45	+730307	+00	6.50	+109703	+01	9.50	+124285	+01
3.50	+740217	+00	6.55	+110049	+01	9.55	+124453	+01
3.55	+749939	+00	6.60	+110390	+01	9.60	+124620	+01
3.60	+759477	+00	6.65	+110726	+01	9.65	+124785	+01
3.65	+768832	+00	6.70	+111058	+01	9.70	+124948	+01
3.70	+778009	+00	6.75	+111385	+01	9.75	+125109	+01
3.75	+787009	+00	6.80	+111707	+01	9.80	+125269	+01
3.80	+795837	+00	6.85	+112025	+01	9.85	+125428	+01
3.85	+804495	+00	6.90	+112339	+01	9.90	+125585	+01
3.90	+812986	+00	6.95	+112648	+01	9.95	+125740	+01
3.95	+821315	+00	7.00	+112954	+01	10.00	+125894	+01
4.00	+829485	+00						

TABLE A5A

$\langle p \rangle_{rel}$ for $ka = 15\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 1.2\ cm$ $\lambda = 0.16\ cm$ $0.045 \leq z \leq 9\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
.005	-01 +964998	.340	+00 +876333	.675	+839693 +00
.010	-01 +962688	.345	+00 +876598	.680	+839153 +00
.015	-01 +962568	.350	+00 +876828	.685	+838510 +00
.020	-01 +959824	.355	+00 +876894	.690	+837766 +00
.025	-01 +955576	.360	+00 +876694	.695	+836928 +00
.030	-01 +953983	.365	+00 +876166	.700	+835999 +00
.035	-01 +953840	.370	+00 +875289	.705	+834988 +00
.040	-01 +951437	.375	+00 +874080	.710	+833901 +00
.045	-01 +947554	.380	+00 +872591	.715	+832746 +00
.050	-01 +945572	.385	+00 +870898	.720	+831532 +00
.055	-01 +945470	.390	+00 +869092	.725	+830268 +00
.060	-01 +944260	.395	+00 +867272	.730	+828963 +00
.065	-01 +941013	.400	+00 +865531	.735	+827626 +00
.070	-01 +937998	.405	+00 +863951	.740	+826267 +00
.075	-01 +937013	.410	+00 +862597	.745	+824895 +00
.080	-01 +936936	.415	+00 +861510	.750	+823519 +00
.085	-01 +935585	.420	+00 +860712	.755	+822148 +00
.090	-01 +932646	.425	+00 +860200	.760	+820789 +00
.095	-01 +929816	.430	+00 +859952	.765	+819452 +00
.100	-01 +928561	.435	+00 +859928	.770	+818143 +00
.105	-01 +928476	.440	+00 +860078	.775	+816870 +00
.110	-01 +927961	.445	+00 +860343	.780	+815639 +00
.115	-01 +926031	.450	+00 +860659	.785	+814455 +00
.120	-01 +923210	.455	+00 +860966	.790	+813324 +00
.125	-01 +920848	.460	+00 +861206	.795	+812250 +00
.130	-01 +919792	.465	+00 +861331	.800	+811237 +00
.135	-01 +919704	.470	+00 +861299	.805	+810289 +00
.140	-01 +919490	.475	+00 +861081	.810	+809408 +00
.145	-01 +918267	.480	+00 +860659	.815	+808595 +00
.150	-01 +916004	.485	+00 +860026	.820	+807853 +00
.155	-01 +913439	.490	+00 +859183	.825	+807182 +00
.160	-01 +911479	.495	+00 +858145	.830	+806582 +00
.165	-01 +910570	.500	+00 +856930	.835	+806053 +00
.170	-01 +910470	.505	+00 +855565	.840	+805595 +00
.175	-01 +910476	.510	+00 +854083	.845	+805205 +00
.180	-01 +909891	.515	+00 +852517	.850	+804882 +00
.185	-01 +908407	.520	+00 +850905	.855	+804625 +00
.190	+00 +906220	.525	+00 +849282	.860	+804431 +00
.195	+00 +903875	.530	+00 +847683	.865	+804296 +00
.200	+00 +901959	.535	+00 +846142	.870	+804219 +00
.205	+00 +900826	.540	+00 +844688	.875	+804196 +00
.210	+00 +900467	.545	+00 +843344	.880	+804223 +00
.215	+00 +900551	.550	+00 +842132	.885	+804297 +00
.220	+00 +900599	.555	+00 +841068	.890	+804414 +00
.225	+00 +900188	.560	+00 +840160	.895	+804570 +00
.230	+00 +899100	.565	+00 +839415	.900	+804762 +00
.235	+00 +897371	.570	+00 +838833	.905	+804985 +00
.240	+00 +895254	.575	+00 +838409	.910	+805236 +00
.245	+00 +893113	.580	+00 +838135	.915	+805510 +00
.250	+00 +891296	.585	+00 +838001	.920	+805805 +00
.255	+00 +890038	.590	+00 +837990	.925	+806115 +00
.260	+00 +889402	.595	+00 +838087	.930	+806437 +00
.265	+00 +889280	.600	+00 +838273	.935	+806768 +00
.270	+00 +889445	.605	+00 +838529	.940	+807103 +00
.275	+00 +889613	.610	+00 +838834	.945	+807441 +00
.280	+00 +889527	.615	+00 +839168	.950	+807776 +00
.285	+00 +889004	.620	+00 +839513	.955	+808106 +00
.290	+00 +887971	.625	+00 +839849	.960	+808428 +00
.295	+00 +886466	.630	+00 +840158	.965	+808739 +00
.300	+00 +884621	.635	+00 +840425	.970	+809036 +00
.305	+00 +882623	.640	+00 +840636	.975	+809318 +00
.310	+00 +880677	.645	+00 +840776	.980	+809580 +00
.315	+00 +878970	.650	+00 +840837	.985	+809821 +00
.320	+00 +877634	.655	+00 +840808	.990	+810040 +00
.325	+00 +876738	.660	+00 +840683	.995	+810233 +00
.330	+00 +876275	.665	+00 +840458	1.000	+810400 +00
.335	+00 +876179	.670	+00 +840128		

TABLE A5B
$\langle p \rangle_{rel}$ for $ka = 15\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 1.2\ cm$ $\lambda = 0.16\ cm$ $9 \leq z \leq 90\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$	
1.00	+810400	+00	4.05	+635195	+00	7.05	+416858	+00
1.05	+810330	+00	4.10	+630442	+00	7.10	+414308	+00
1.10	+806752	+00	4.15	+625715	+00	7.15	+411785	+00
1.15	+799966	+00	4.20	+621014	+00	7.20	+409291	+00
1.20	+790885	+00	4.25	+616342	+00	7.25	+406823	+00
1.25	+780626	+00	4.30	+611701	+00	7.30	+404383	+00
1.30	+770246	+00	4.35	+607093	+00	7.35	+401969	+00
1.35	+760605	+00	4.40	+602520	+00	7.40	+399581	+00
1.40	+752303	+00	4.45	+597983	+00	7.45	+397219	+00
1.45	+745687	+00	4.50	+593482	+00	7.50	+394883	+00
1.50	+740883	+00	4.55	+589020	+00	7.55	+392571	+00
1.55	+737844	+00	4.60	+584596	+00	7.60	+390285	+00
1.60	+736404	+00	4.65	+580213	+00	7.65	+388022	+00
1.65	+736326	+00	4.70	+575870	+00	7.70	+385784	+00
1.70	+737339	+00	4.75	+571568	+00	7.75	+383570	+00
1.75	+739172	+00	4.80	+567307	+00	7.80	+381379	+00
1.80	+741569	+00	4.85	+563089	+00	7.85	+379210	+00
1.85	+744300	+00	4.90	+558912	+00	7.90	+377065	+00
1.90	+747172	+00	4.95	+554778	+00	7.95	+374942	+00
1.95	+750023	+00	5.00	+550687	+00	8.00	+372841	+00
2.00	+752726	+00	5.05	+546638	+00	8.05	+370762	+00
2.05	+755182	+00	5.10	+542632	+00	8.10	+368705	+00
2.10	+757318	+00	5.15	+538669	+00	8.15	+366668	+00
2.15	+759084	+00	5.20	+534748	+00	8.20	+364653	+00
2.20	+760447	+00	5.25	+530870	+00	8.25	+362658	+00
2.25	+761389	+00	5.30	+527034	+00	8.30	+360683	+00
2.30	+761902	+00	5.35	+523241	+00	8.35	+358729	+00
2.35	+761989	+00	5.40	+519489	+00	8.40	+356794	+00
2.40	+761659	+00	5.45	+515779	+00	8.45	+354879	+00
2.45	+760925	+00	5.50	+512110	+00	8.50	+352983	+00
2.50	+759807	+00	5.55	+508483	+00	8.55	+351106	+00
2.55	+758322	+00	5.60	+504896	+00	8.60	+349248	+00
2.60	+756494	+00	5.65	+501350	+00	8.65	+347408	+00
2.65	+754344	+00	5.70	+497844	+00	8.70	+345586	+00
2.70	+751896	+00	5.75	+494378	+00	8.75	+343782	+00
2.75	+749171	+00	5.80	+490951	+00	8.80	+341996	+00
2.80	+746192	+00	5.85	+487563	+00	8.85	+340228	+00
2.85	+742980	+00	5.90	+484214	+00	8.90	+338476	+00
2.90	+739557	+00	5.95	+480903	+00	8.95	+336742	+00
2.95	+735941	+00	6.00	+477629	+00	9.00	+335024	+00
3.00	+732151	+00	6.05	+474393	+00	9.05	+333323	+00
3.05	+728206	+00	6.10	+471195	+00	9.10	+331638	+00
3.10	+724120	+00	6.15	+468032	+00	9.15	+329970	+00
3.15	+719910	+00	6.20	+464906	+00	9.20	+328317	+00
3.20	+715591	+00	6.25	+461815	+00	9.25	+326680	+00
3.25	+711175	+00	6.30	+458759	+00	9.30	+325058	+00
3.30	+706676	+00	6.35	+455739	+00	9.35	+323452	+00
3.35	+702104	+00	6.40	+452752	+00	9.40	+321861	+00
3.40	+697470	+00	6.45	+449800	+00	9.45	+320284	+00
3.45	+692785	+00	6.50	+446881	+00	9.50	+318722	+00
3.50	+688057	+00	6.55	+443995	+00	9.55	+317175	+00
3.55	+683295	+00	6.60	+441141	+00	9.60	+315642	+00
3.60	+678506	+00	6.65	+438320	+00	9.65	+314123	+00
3.65	+673697	+00	6.70	+435531	+00	9.70	+312618	+00
3.70	+668876	+00	6.75	+432773	+00	9.75	+311127	+00
3.75	+664047	+00	6.80	+430045	+00	9.80	+309649	+00
3.80	+659217	+00	6.85	+427349	+00	9.85	+308185	+00
3.85	+654389	+00	6.90	+424682	+00	9.90	+306734	+00
3.90	+649570	+00	6.95	+422045	+00	9.95	+305296	+00
3.95	+644761	+00	7.00	+419437	+00	10.00	+303871	+00
4.00	+639969	+00						

TABLE A5C

$\langle\theta\rangle_{rel}$ for $ka = 15\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 1.2\ cm$ $\lambda = 0.16\ cm$ $0.045 \leq z \leq 9\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+367378 -01	.340	+146367 +00	.675	+221523 +00
.010	+370128 -01	.345	+147485 +00	.680	+223547 +00
.015	+399538 -01	.350	+149042 +00	.685	+225559 +00
.020	+447098 -01	.355	+150984 +00	.690	+227548 +00
.025	+466361 -01	.360	+153213 +00	.695	+229498 +00
.030	+470032 -01	.365	+155605 +00	.700	+231398 +00
.035	+499158 -01	.370	+158024 +00	.705	+233237 +00
.040	+542868 -01	.375	+160341 +00	.710	+235004 +00
.045	+564034 -01	.380	+162443 +00	.715	+236691 +00
.050	+567047 -01	.385	+164243 +00	.720	+238290 +00
.055	+585868 -01	.390	+165689 +00	.725	+239794 +00
.060	+625247 -01	.395	+166762 +00	.730	+241199 +00
.065	+656890 -01	.400	+167477 +00	.735	+242500 +00
.070	+665882 -01	.405	+167880 +00	.740	+243696 +00
.075	+670772 -01	.410	+168037 +00	.745	+244784 +00
.080	+693785 -01	.415	+168034 +00	.750	+245765 +00
.085	+730580 -01	.420	+167961 +00	.755	+246640 +00
.090	+759206 -01	.425	+167909 +00	.760	+247410 +00
.095	+769151 -01	.430	+167963 +00	.765	+248077 +00
.100	+772426 -01	.435	+168194 +00	.770	+248647 +00
.105	+787295 -01	.440	+168656 +00	.775	+249123 +00
.110	+817584 -01	.445	+169385 +00	.780	+249510 +00
.115	+850593 -01	.450	+170399 +00	.785	+249815 +00
.120	+871740 -01	.455	+171696 +00	.790	+250043 +00
.125	+878585 -01	.460	+173257 +00	.795	+250201 +00
.130	+881690 -01	.465	+175051 +00	.800	+250297 +00
.135	+893848 -01	.470	+177035 +00	.805	+250338 +00
.140	+919062 -01	.475	+179161 +00	.810	+250332 +00
.145	+950487 -01	.480	+181373 +00	.815	+250286 +00
.150	+976991 -01	.485	+183617 +00	.820	+250208 +00
.155	+991695 -01	.490	+185840 +00	.825	+250106 +00
.160	+996272 -01	.495	+187990 +00	.830	+249987 +00
.165	+998900 -01	.500	+190024 +00	.835	+249859 +00
.170	+100827 +00	.505	+191903 +00	.840	+249729 +00
.175	+102820 +00	.510	+193599 +00	.845	+249605 +00
.180	+105598 +00	.515	+195088 +00	.850	+249493 +00
.185	+108474 +00	.520	+196359 +00	.855	+249399 +00
.190	+110751 +00	.525	+197408 +00	.860	+249329 +00
.195	+112074 +00	.530	+198237 +00	.865	+249289 +00
.200	+112565 +00	.535	+198859 +00	.870	+249285 +00
.205	+112716 +00	.540	+199290 +00	.875	+249320 +00
.210	+113131 +00	.545	+199554 +00	.880	+249399 +00
.215	+114250 +00	.550	+199678 +00	.885	+249526 +00
.220	+116194 +00	.555	+199693 +00	.890	+249705 +00
.225	+118750 +00	.560	+199630 +00	.895	+249938 +00
.230	+121496 +00	.565	+199523 +00	.900	+250228 +00
.235	+123970 +00	.570	+199405 +00	.905	+250576 +00
.240	+125827 +00	.575	+199306 +00	.910	+250986 +00
.245	+126936 +00	.580	+199257 +00	.915	+251457 +00
.250	+127405 +00	.585	+199283 +00	.920	+251991 +00
.255	+127517 +00	.590	+199407 +00	.925	+252588 +00
.260	+127637 +00	.595	+199649 +00	.930	+253249 +00
.265	+128102 +00	.600	+200023 +00	.935	+253973 +00
.270	+129130 +00	.605	+200542 +00	.940	+254760 +00
.275	+130784 +00	.610	+201212 +00	.945	+255610 +00
.280	+132971 +00	.615	+202038 +00	.950	+256520 +00
.285	+135486 +00	.620	+203019 +00	.955	+257490 +00
.290	+138063 +00	.625	+204151 +00	.960	+258519 +00
.295	+140441 +00	.630	+205430 +00	.965	+259605 +00
.300	+142414 +00	.635	+206846 +00	.970	+260745 +00
.305	+143862 +00	.640	+208388 +00	.975	+261939 +00
.310	+144765 +00	.645	+210043 +00	.980	+263183 +00
.315	+145201 +00	.650	+211798 +00	.985	+264476 +00
.320	+145319 +00	.655	+213638 +00	.990	+265815 +00
.325	+145311 +00	.660	+215546 +00	.995	+267199 +00
.330	+145375 +00	.665	+217508 +00	1.000	+268623 +00
.335	+145685 +00	.670	+219505 +00		

TABLE A5D
$\langle\theta\rangle_{rel}$ for $ka = 15\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 1.2\ cm$ $\lambda = 0.16\ cm$ $9 \leq z \leq 90\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+268623	+00	4.05	+836854	+00	7.05	+113263	+01
1.05	+284566	+00	4.10	+844745	+00	7.10	+113561	+01
1.10	+301801	+00	4.15	+852485	+00	7.15	+113855	+01
1.15	+318142	+00	4.20	+860079	+00	7.20	+114144	+01
1.20	+332097	+00	4.25	+867529	+00	7.25	+114430	+01
1.25	+342892	+00	4.30	+874838	+00	7.30	+114713	+01
1.30	+350360	+00	4.35	+882011	+00	7.35	+114991	+01
1.35	+354785	+00	4.40	+889049	+00	7.40	+115266	+01
1.40	+356750	+00	4.45	+895957	+00	7.45	+115538	+01
1.45	+356985	+00	4.50	+902738	+00	7.50	+115806	+01
1.50	+356251	+00	4.55	+909394	+00	7.55	+116071	+01
1.55	+355250	+00	4.60	+915928	+00	7.60	+116332	+01
1.60	+354571	+00	4.65	+922343	+00	7.65	+116590	+01
1.65	+354658	+00	4.70	+928642	+00	7.70	+116845	+01
1.70	+355816	+00	4.75	+934829	+00	7.75	+117097	+01
1.75	+358224	+00	4.80	+940905	+00	7.80	+117346	+01
1.80	+361954	+00	4.85	+946872	+00	7.85	+117592	+01
1.85	+367001	+00	4.90	+952735	+00	7.90	+117835	+01
1.90	+373303	+00	4.95	+958495	+00	7.95	+118075	+01
1.95	+380762	+00	5.00	+964155	+00	8.00	+118312	+01
2.00	+389260	+00	5.05	+969716	+00	8.05	+118546	+01
2.05	+398671	+00	5.10	+975182	+00	8.10	+118777	+01
2.10	+408867	+00	5.15	+980555	+00	8.15	+119006	+01
2.15	+419726	+00	5.20	+985836	+00	8.20	+119232	+01
2.20	+431133	+00	5.25	+991028	+00	8.25	+119455	+01
2.25	+442984	+00	5.30	+996134	+00	8.30	+119676	+01
2.30	+455185	+00	5.35	+100115	+01	8.35	+119894	+01
2.35	+467651	+00	5.40	+100609	+01	8.40	+120110	+01
2.40	+480309	+00	5.45	+101095	+01	8.45	+120323	+01
2.45	+493096	+00	5.50	+101572	+01	8.50	+120534	+01
2.50	+505955	+00	5.55	+102042	+01	8.55	+120743	+01
2.55	+518839	+00	5.60	+102505	+01	8.60	+120949	+01
2.60	+531708	+00	5.65	+102960	+01	8.65	+121153	+01
2.65	+544526	+00	5.70	+103407	+01	8.70	+121355	+01
2.70	+557264	+00	5.75	+103848	+01	8.75	+121554	+01
2.75	+569898	+00	5.80	+104282	+01	8.80	+121751	+01
2.80	+582408	+00	5.85	+104709	+01	8.85	+121946	+01
2.85	+594776	+00	5.90	+105129	+01	8.90	+122139	+01
2.90	+606989	+00	5.95	+105543	+01	8.95	+122330	+01
2.95	+619035	+00	6.00	+105951	+01	9.00	+122519	+01
3.00	+630907	+00	6.05	+106352	+01	9.05	+122706	+01
3.05	+642597	+00	6.10	+106747	+01	9.10	+122891	+01
3.10	+654099	+00	6.15	+107137	+01	9.15	+123074	+01
3.15	+665411	+00	6.20	+107521	+01	9.20	+123255	+01
3.20	+676529	+00	6.25	+107899	+01	9.25	+123434	+01
3.25	+687453	+00	6.30	+108271	+01	9.30	+123611	+01
3.30	+698182	+00	6.35	+108638	+01	9.35	+123786	+01
3.35	+708716	+00	6.40	+109000	+01	9.40	+123960	+01
3.40	+719055	+00	6.45	+109356	+01	9.45	+124132	+01
3.45	+729202	+00	6.50	+109707	+01	9.50	+124302	+01
3.50	+739157	+00	6.55	+110054	+01	9.55	+124470	+01
3.55	+748924	+00	6.60	+110395	+01	9.60	+124636	+01
3.60	+758505	+00	6.65	+110732	+01	9.65	+124801	+01
3.65	+767903	+00	6.70	+111064	+01	9.70	+124965	+01
3.70	+777119	+00	6.75	+111391	+01	9.75	+125126	+01
3.75	+786159	+00	6.80	+111714	+01	9.80	+125286	+01
3.80	+795024	+00	6.85	+112032	+01	9.85	+125445	+01
3.85	+803718	+00	6.90	+112346	+01	9.90	+125602	+01
3.90	+812245	+00	6.95	+112656	+01	9.95	+125757	+01
3.95	+820607	+00	7.00	+112962	+01	10.00	+125911	+01
4.00	+828809	+00						

TABLE A6A

$\langle p \rangle_{rel}$ for $ka = 20\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 1.5\ cm$ $\lambda = 0.15\ cm$ $0.075 \leq z \leq 15\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
.005	+969218 +00	.340	+874732 +00	.675	+839417 +00
.010	+968232 +00	.345	+873610 +00	.680	+839154 +00
.015	+963727 +00	.350	+872983 +00	.685	+838783 +00
.020	+963067 +00	.355	+872804 +00	.690	+838302 +00
.025	+958845 +00	.360	+872958 +00	.695	+837712 +00
.030	+957796 +00	.365	+873290 +00	.700	+837017 +00
.035	+954926 +00	.370	+873628 +00	.705	+836220 +00
.040	+952264 +00	.375	+873817 +00	.710	+835328 +00
.045	+951446 +00	.380	+873732 +00	.715	+834347 +00
.050	+947673 +00	.385	+873292 +00	.720	+833283 +00
.055	+946733 +00	.390	+872468 +00	.725	+832147 +00
.060	+944996 +00	.395	+871276 +00	.730	+830946 +00
.065	+941668 +00	.400	+869773 +00	.735	+829689 +00
.070	+941165 +00	.405	+868049 +00	.740	+828387 +00
.075	+939382 +00	.410	+866308 +00	.745	+827049 +00
.080	+936215 +00	.415	+864361 +00	.750	+825685 +00
.085	+935547 +00	.420	+862613 +00	.755	+824304 +00
.090	+934552 +00	.425	+861054 +00	.760	+822917 +00
.095	+931483 +00	.430	+859752 +00	.765	+821531 +00
.100	+929810 +00	.435	+858748 +00	.770	+820157 +00
.105	+929669 +00	.440	+858057 +00	.775	+818803 +00
.110	+927898 +00	.445	+857670 +00	.780	+817476 +00
.115	+925009 +00	.450	+857553 +00	.785	+816185 +00
.120	+923767 +00	.455	+857657 +00	.790	+814935 +00
.125	+923706 +00	.460	+857920 +00	.795	+813733 +00
.130	+922334 +00	.465	+858275 +00	.800	+812584 +00
.135	+919601 +00	.470	+858650 +00	.805	+811493 +00
.140	+917688 +00	.475	+858981 +00	.810	+810464 +00
.145	+917412 +00	.480	+859209 +00	.815	+809501 +00
.150	+917209 +00	.485	+859285 +00	.820	+808607 +00
.155	+915564 +00	.490	+859172 +00	.825	+807783 +00
.160	+912962 +00	.495	+858845 +00	.830	+807031 +00
.165	+911042 +00	.500	+858292 +00	.835	+806352 +00
.170	+910556 +00	.505	+857515 +00	.840	+805746 +00
.175	+910658 +00	.510	+856523 +00	.845	+805213 +00
.180	+909983 +00	.515	+855338 +00	.850	+804752 +00
.185	+908047 +00	.520	+853988 +00	.855	+804362 +00
.190	+905579 +00	.525	+852505 +00	.860	+804041 +00
.195	+903727 +00	.530	+850929 +00	.865	+803786 +00
.200	+903058 +00	.535	+849298 +00	.870	+803596 +00
.205	+903182 +00	.540	+847652 +00	.875	+803468 +00
.210	+903171 +00	.545	+846029 +00	.880	+803398 +00
.215	+902276 +00	.550	+844465 +00	.885	+803383 +00
.220	+900400 +00	.555	+842992 +00	.890	+803420 +00
.225	+898069 +00	.560	+841636 +00	.895	+803504 +00
.230	+896045 +00	.565	+840419 +00	.900	+803633 +00
.235	+894866 +00	.570	+839358 +00	.905	+803801 +00
.240	+894585 +00	.575	+838463 +00	.910	+804006 +00
.245	+894807 +00	.580	+837738 +00	.915	+804242 +00
.250	+894936 +00	.585	+837185 +00	.920	+804506 +00
.255	+894472 +00	.590	+836796 +00	.925	+804794 +00
.260	+893213 +00	.595	+836564 +00	.930	+805102 +00
.265	+891297 +00	.600	+836475 +00	.935	+805425 +00
.270	+889106 +00	.605	+836512 +00	.940	+805761 +00
.275	+887099 +00	.610	+836657 +00	.945	+806105 +00
.280	+885636 +00	.615	+836891 +00	.950	+806453 +00
.285	+884877 +00	.620	+837191 +00	.955	+806802 +00
.290	+884748 +00	.625	+837537 +00	.960	+807148 +00
.295	+884998 +00	.630	+837908 +00	.965	+807489 +00
.300	+885288 +00	.635	+838282 +00	.970	+807821 +00
.305	+885302 +00	.640	+838641 +00	.975	+808142 +00
.310	+884818 +00	.645	+838965 +00	.980	+808448 +00
.315	+883756 +00	.650	+839240 +00	.985	+808736 +00
.320	+882177 +00	.655	+839449 +00	.990	+809005 +00
.325	+880252 +00	.660	+839581 +00	.995	+809253 +00
.330	+878216 +00	.665	+839625 +00	1.000	+809476 +00
.335	+876309 +00	.670	+839572 +00		

TABLE A6B

$\langle p \rangle_{rel}$ for $ka = 20\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 1.5\ cm$ $\quad \lambda = 0.15\ cm$ $\quad 15 \leq z \leq 150\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
1.00	+809476 +00	4.05	+635487 +00	7.05	+416978 +00
1.05	+810053 +00	4.10	+630732 +00	7.10	+414426 +00
1.10	+807104 +00	4.15	+626001 +00	7.15	+411902 +00
1.15	+800789 +00	4.20	+621297 +00	7.20	+409405 +00
1.20	+791974 +00	4.25	+616621 +00	7.25	+406936 +00
1.25	+781787 +00	4.30	+611977 +00	7.30	+404494 +00
1.30	+771330 +00	4.35	+607366 +00	7.35	+402079 +00
1.35	+761509 +00	4.40	+602789 +00	7.40	+399690 +00
1.40	+752973 +00	4.45	+598247 +00	7.45	+397326 +00
1.45	+746106 +00	4.50	+593743 +00	7.50	+394988 +00
1.50	+741062 +00	4.55	+589277 +00	7.55	+392675 +00
1.55	+737811 +00	4.60	+584850 +00	7.60	+390387 +00
1.60	+736197 +00	4.65	+580463 +00	7.65	+388124 +00
1.65	+735985 +00	4.70	+576116 +00	7.70	+385884 +00
1.70	+736905 +00	4.75	+571810 +00	7.75	+383668 +00
1.75	+738680 +00	4.80	+567546 +00	7.80	+381476 +00
1.80	+741049 +00	4.85	+563324 +00	7.85	+379306 +00
1.85	+743777 +00	4.90	+559144 +00	7.90	+377160 +00
1.90	+746665 +00	4.95	+555006 +00	7.95	+375035 +00
1.95	+749546 +00	5.00	+550912 +00	8.00	+372933 +00
2.00	+752289 +00	5.05	+546859 +00	8.05	+370853 +00
2.05	+754793 +00	5.10	+542850 +00	8.10	+368794 +00
2.10	+756980 +00	5.15	+538883 +00	8.15	+366757 +00
2.15	+758799 +00	5.20	+534959 +00	8.20	+364740 +00
2.20	+760215 +00	5.25	+531078 +00	8.25	+362744 +00
2.25	+761208 +00	5.30	+527239 +00	8.30	+360769 +00
2.30	+761771 +00	5.35	+523442 +00	8.35	+358813 +00
2.35	+761905 +00	5.40	+519687 +00	8.40	+356877 +00
2.40	+761620 +00	5.45	+515974 +00	8.45	+354961 +00
2.45	+760927 +00	5.50	+512302 +00	8.50	+353064 +00
2.50	+759846 +00	5.55	+508671 +00	8.55	+351186 +00
2.55	+758397 +00	5.60	+505082 +00	8.60	+349327 +00
2.60	+756600 +00	5.65	+501533 +00	8.65	+347486 +00
2.65	+754479 +00	5.70	+498024 +00	8.70	+345663 +00
2.70	+752056 +00	5.75	+494555 +00	8.75	+343858 +00
2.75	+749354 +00	5.80	+491125 +00	8.80	+342071 +00
2.80	+746396 +00	5.85	+487735 +00	8.85	+340302 +00
2.85	+743202 +00	5.90	+484383 +00	8.90	+338550 +00
2.90	+739794 +00	5.95	+481069 +00	8.95	+336814 +00
2.95	+736192 +00	6.00	+477793 +00	9.00	+335096 +00
3.00	+732415 +00	6.05	+474555 +00	9.05	+333394 +00
3.05	+728479 +00	6.10	+471353 +00	9.10	+331708 +00
3.10	+724403 +00	6.15	+468188 +00	9.15	+330039 +00
3.15	+720200 +00	6.20	+465060 +00	9.20	+328385 +00
3.20	+715887 +00	6.25	+461967 +00	9.25	+326747 +00
3.25	+711476 +00	6.30	+458909 +00	9.30	+325125 +00
3.30	+706980 +00	6.35	+455886 +00	9.35	+323518 +00
3.35	+702412 +00	6.40	+452897 +00	9.40	+321926 +00
3.40	+697780 +00	6.45	+449942 +00	9.45	+320349 +00
3.45	+693096 +00	6.50	+447021 +00	9.50	+318786 +00
3.50	+688369 +00	6.55	+444133 +00	9.55	+317238 +00
3.55	+683607 +00	6.60	+441278 +00	9.60	+315704 +00
3.60	+678818 +00	6.65	+438455 +00	9.65	+314185 +00
3.65	+674009 +00	6.70	+435663 +00	9.70	+312679 +00
3.70	+669186 +00	6.75	+432903 +00	9.75	+311187 +00
3.75	+664356 +00	6.80	+430174 +00	9.80	+309709 +00
3.80	+659523 +00	6.85	+427475 +00	9.85	+308244 +00
3.85	+654693 +00	6.90	+424807 +00	9.90	+306792 +00
3.90	+649871 +00	6.95	+422168 +00	9.95	+305353 +00
3.95	+645060 +00	7.00	+419558 +00	10.00	+303928 +00
4.00	+640265 +00				

TABLE A6c

$\langle\theta\rangle_{rel}$ for $ka = 20\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 1.5\ cm$ $\lambda = 0.15\ cm$ $0.075 \leq z \leq 15\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
.005	+306829	-01	.340	+150708	+00	.675	+218663	+00
.010	+346055	-01	.345	+150636	+00	.680	+220662	+00
.015	+369830	-01	.350	+150536	+00	.685	+222692	+00
.020	+398581	-01	.355	+150629	+00	.690	+224734	+00
.025	+432509	-01	.360	+151089	+00	.695	+226772	+00
.030	+446554	-01	.365	+152020	+00	.700	+228793	+00
.035	+490496	-01	.370	+153445	+00	.705	+230780	+00
.040	+498356	-01	.375	+155318	+00	.710	+232720	+00
.045	+534536	-01	.380	+157535	+00	.715	+234601	+00
.050	+559873	-01	.385	+159955	+00	.720	+236412	+00
.055	+570612	-01	.390	+162424	+00	.725	+238143	+00
.060	+610870	-01	.395	+164789	+00	.730	+239785	+00
.065	+625756	-01	.400	+166920	+00	.735	+241332	+00
.070	+639779	-01	.405	+168720	+00	.740	+242778	+00
.075	+678049	-01	.410	+170130	+00	.745	+244118	+00
.080	+692978	-01	.415	+171133	+00	.750	+245350	+00
.085	+702550	-01	.420	+171751	+00	.755	+246471	+00
.090	+731191	-01	.425	+172040	+00	.760	+247481	+00
.095	+761093	-01	.430	+172083	+00	.765	+248380	+00
.100	+765383	-01	.435	+171977	+00	.770	+249171	+00
.105	+786241	-01	.440	+171826	+00	.775	+249856	+00
.110	+820542	-01	.445	+171730	+00	.780	+250439	+00
.115	+836553	-01	.450	+171780	+00	.785	+250925	+00
.120	+840204	-01	.455	+172049	+00	.790	+251317	+00
.125	+859867	-01	.460	+172588	+00	.795	+251624	+00
.130	+892843	-01	.465	+173427	+00	.800	+251850	+00
.135	+913613	-01	.470	+174575	+00	.805	+252004	+00
.140	+917552	-01	.475	+176020	+00	.810	+252092	+00
.145	+925648	-01	.480	+177732	+00	.815	+252123	+00
.150	+951047	-01	.485	+179668	+00	.820	+252103	+00
.155	+982052	-01	.490	+181777	+00	.825	+252043	+00
.160	+100035	+00	.495	+183998	+00	.830	+251948	+00
.165	+100414	+00	.500	+186273	+00	.835	+251828	+00
.170	+100831	+00	.505	+188540	+00	.840	+251690	+00
.175	+102600	+00	.510	+190744	+00	.845	+251543	+00
.180	+103518	+00	.515	+192834	+00	.850	+251393	+00
.185	+108268	+00	.520	+194766	+00	.855	+251249	+00
.190	+109772	+00	.525	+196508	+00	.860	+251116	+00
.195	+110099	+00	.530	+198034	+00	.865	+251002	+00
.200	+110273	+00	.535	+199329	+00	.870	+250913	+00
.205	+111346	+00	.540	+200388	+00	.875	+250855	+00
.210	+113612	+00	.545	+201215	+00	.880	+250833	+00
.215	+116502	+00	.550	+201821	+00	.885	+250852	+00
.220	+119076	+00	.555	+202225	+00	.890	+250916	+00
.225	+120655	+00	.560	+202453	+00	.895	+251030	+00
.230	+121187	+00	.565	+202574	+00	.900	+251196	+00
.235	+121204	+00	.570	+202601	+00	.905	+251419	+00
.240	+121467	+00	.575	+202660	+00	.910	+251700	+00
.245	+122544	+00	.580	+202236	+00	.915	+252041	+00
.250	+124566	+00	.585	+202075	+00	.920	+252445	+00
.255	+127222	+00	.590	+201939	+00	.925	+252912	+00
.260	+129954	+00	.595	+201860	+00	.930	+253443	+00
.265	+132215	+00	.600	+201865	+00	.935	+254040	+00
.270	+133672	+00	.605	+201978	+00	.940	+254701	+00
.275	+134306	+00	.610	+202219	+00	.945	+255427	+00
.280	+134383	+00	.615	+202603	+00	.950	+256217	+00
.285	+134330	+00	.620	+203141	+00	.955	+257071	+00
.290	+134582	+00	.625	+203840	+00	.960	+257988	+00
.295	+135445	+00	.630	+204702	+00	.965	+258965	+00
.300	+137023	+00	.635	+205726	+00	.970	+260002	+00
.305	+139219	+00	.640	+206907	+00	.975	+261096	+00
.310	+141784	+00	.645	+208238	+00	.980	+262247	+00
.315	+144396	+00	.650	+209708	+00	.985	+263451	+00
.320	+146748	+00	.655	+211305	+00	.990	+264706	+00
.325	+148606	+00	.660	+213015	+00	.995	+266011	+00
.330	+149858	+00	.665	+214822	+00	1.000	+267365	+00
.335	+150516	+00	.670	+216711	+00			

TABLE A6D

$\langle\theta\rangle_{rel}$ for $ka = 20\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 1.5\ cm$ $\quad \lambda = 0.15\ cm$ $\quad 15 \leq z \leq 150\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+267363	+00	4.05	+836627	+00	7.05	+113266	+01
1.05	+282869	+00	4.10	+844528	+00	7.10	+113564	+01
1.10	+300128	+00	4.15	+852279	+00	7.15	+113858	+01
1.15	+316801	+00	4.20	+859882	+00	7.20	+114148	+01
1.20	+331243	+00	4.25	+867342	+00	7.25	+114434	+01
1.25	+342560	+00	4.30	+874660	+00	7.30	+114716	+01
1.30	+350500	+00	4.35	+881842	+00	7.35	+114995	+01
1.35	+355301	+00	4.40	+888889	+00	7.40	+115270	+01
1.40	+357521	+00	4.45	+895805	+00	7.45	+115542	+01
1.45	+357892	+00	4.50	+902593	+00	7.50	+115810	+01
1.50	+357189	+00	4.55	+909256	+00	7.55	+116075	+01
1.55	+356134	+00	4.60	+915797	+00	7.60	+116336	+01
1.60	+355339	+00	4.65	+922219	+00	7.65	+116595	+01
1.65	+355272	+00	4.70	+928526	+00	7.70	+116850	+01
1.70	+356258	+00	4.75	+934718	+00	7.75	+117102	+01
1.75	+358489	+00	4.80	+940800	+00	7.80	+117350	+01
1.80	+362049	+00	4.85	+946774	+00	7.85	+117596	+01
1.85	+366939	+00	4.90	+952643	+00	7.90	+117839	+01
1.90	+373101	+00	4.95	+958408	+00	7.95	+118079	+01
1.95	+380439	+00	5.00	+964073	+00	8.00	+118316	+01
2.00	+388836	+00	5.05	+969640	+00	8.05	+118550	+01
2.05	+398163	+00	5.10	+975111	+00	8.10	+118782	+01
2.10	+408291	+00	5.15	+980488	+00	8.15	+119011	+01
2.15	+419097	+00	5.20	+985774	+00	8.20	+119237	+01
2.20	+430465	+00	5.25	+990970	+00	8.25	+119460	+01
2.25	+442287	+00	5.30	+996080	+00	8.30	+119681	+01
2.30	+454469	+00	5.35	+100110	+01	8.35	+119899	+01
2.35	+466924	+00	5.40	+100605	+01	8.40	+120115	+01
2.40	+479578	+00	5.45	+101091	+01	8.45	+120329	+01
2.45	+492366	+00	5.50	+101569	+01	8.50	+120540	+01
2.50	+505231	+00	5.55	+102039	+01	8.55	+120748	+01
2.55	+518124	+00	5.60	+102502	+01	8.60	+120954	+01
2.60	+531005	+00	5.65	+102957	+01	8.65	+121158	+01
2.65	+543838	+00	5.70	+103405	+01	8.70	+121360	+01
2.70	+556592	+00	5.75	+103846	+01	8.75	+121559	+01
2.75	+569244	+00	5.80	+104280	+01	8.80	+121757	+01
2.80	+581773	+00	5.85	+104707	+01	8.85	+121952	+01
2.85	+594160	+00	5.90	+105128	+01	8.90	+122145	+01
2.90	+606393	+00	5.95	+105542	+01	8.95	+122336	+01
2.95	+618459	+00	6.00	+105950	+01	9.00	+122525	+01
3.00	+630351	+00	6.05	+106352	+01	9.05	+122711	+01
3.05	+642061	+00	6.10	+106747	+01	9.10	+122896	+01
3.10	+653583	+00	6.15	+107137	+01	9.15	+123079	+01
3.15	+664914	+00	6.20	+107521	+01	9.20	+123260	+01
3.20	+676052	+00	6.25	+107899	+01	9.25	+123439	+01
3.25	+686994	+00	6.30	+108271	+01	9.30	+123617	+01
3.30	+697741	+00	6.35	+108639	+01	9.35	+123792	+01
3.35	+708292	+00	6.40	+109001	+01	9.40	+123966	+01
3.40	+718649	+00	6.45	+109357	+01	9.45	+124137	+01
3.45	+728813	+00	6.50	+109709	+01	9.50	+124307	+01
3.50	+738785	+00	6.55	+110055	+01	9.55	+124476	+01
3.55	+748567	+00	6.60	+110397	+01	9.60	+124642	+01
3.60	+758163	+00	6.65	+110734	+01	9.65	+124807	+01
3.65	+767575	+00	6.70	+111066	+01	9.70	+124971	+01
3.70	+776806	+00	6.75	+111393	+01	9.75	+125132	+01
3.75	+785859	+00	6.80	+111716	+01	9.80	+125292	+01
3.80	+794738	+00	6.85	+112035	+01	9.85	+125451	+01
3.85	+803445	+00	6.90	+112349	+01	9.90	+125608	+01
3.90	+811984	+00	6.95	+112659	+01	9.95	+125763	+01
3.95	+820358	+00	7.00	+112965	+01	10.00	+125917	+01
4.00	+828571	+00						

TABLE A7A

$\langle p \rangle_{rel}$ for $ka = 30\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 1.8\ cm$ $\quad \lambda = 0.12\ cm$ $\quad 0.135 \leq z \leq 27\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
.005	+974738 +00	.340	+878345 +00	.675	+838778 +00
.010	+971257 +00	.345	+876277 +00	.680	+838720 +00
.015	+968088 +00	.350	+874269 +00	.685	+838554 +00
.020	+965209 +00	.355	+872567 +00	.690	+838278 +00
.025	+962587 +00	.360	+871339 +00	.695	+837889 +00
.030	+960100 +00	.365	+870651 +00	.700	+837387 +00
.035	+957464 +00	.370	+870463 +00	.705	+836774 +00
.040	+954493 +00	.375	+870654 +00	.710	+836054 +00
.045	+951741 +00	.380	+871050 +00	.715	+835231 +00
.050	+950019 +00	.385	+871462 +00	.720	+834312 +00
.055	+948234 +00	.390	+871712 +00	.725	+833302 +00
.060	+945272 +00	.395	+871661 +00	.730	+832212 +00
.065	+943644 +00	.400	+871223 +00	.735	+831049 +00
.070	+941996 +00	.405	+870369 +00	.740	+829822 +00
.075	+939266 +00	.410	+869124 +00	.745	+828542 +00
.080	+938473 +00	.415	+867559 +00	.750	+827219 +00
.085	+935557 +00	.420	+865776 +00	.755	+825862 +00
.090	+934972 +00	.425	+863893 +00	.760	+824482 +00
.095	+932081 +00	.430	+862032 +00	.765	+823089 +00
.100	+931642 +00	.435	+860306 +00	.770	+821692 +00
.105	+928940 +00	.440	+858805 +00	.775	+820302 +00
.110	+928066 +00	.445	+857595 +00	.780	+818926 +00
.115	+926568 +00	.450	+856711 +00	.785	+817574 +00
.120	+924167 +00	.455	+856159 +00	.790	+816253 +00
.125	+924045 +00	.460	+855917 +00	.795	+814970 +00
.130	+921801 +00	.465	+855940 +00	.800	+813733 +00
.135	+919863 +00	.470	+856168 +00	.805	+812548 +00
.140	+919867 +00	.475	+856528 +00	.810	+811419 +00
.145	+917957 +00	.480	+856947 +00	.815	+810351 +00
.150	+915541 +00	.485	+857350 +00	.820	+809349 +00
.155	+915324 +00	.490	+857669 +00	.825	+808415 +00
.160	+914801 +00	.495	+857848 +00	.830	+807552 +00
.165	+912305 +00	.500	+857841 +00	.835	+806761 +00
.170	+910345 +00	.505	+857617 +00	.840	+806045 +00
.175	+910294 +00	.510	+857158 +00	.845	+805403 +00
.180	+910059 +00	.515	+856462 +00	.850	+804835 +00
.185	+908037 +00	.520	+855535 +00	.855	+804341 +00
.190	+905549 +00	.525	+854398 +00	.860	+803920 +00
.195	+904517 +00	.530	+853079 +00	.865	+803570 +00
.200	+904755 +00	.535	+851613 +00	.870	+803290 +00
.205	+904505 +00	.540	+850038 +00	.875	+803075 +00
.210	+902758 +00	.545	+848397 +00	.880	+802925 +00
.215	+900260 +00	.550	+846732 +00	.885	+802836 +00
.220	+898484 +00	.555	+845084 +00	.890	+802804 +00
.225	+898088 +00	.560	+843491 +00	.895	+802826 +00
.230	+898432 +00	.565	+841989 +00	.900	+802898 +00
.235	+898318 +00	.570	+840605 +00	.905	+803017 +00
.240	+897035 +00	.575	+839363 +00	.910	+803177 +00
.245	+894813 +00	.580	+838282 +00	.915	+803376 +00
.250	+892524 +00	.585	+837373 +00	.920	+803609 +00
.255	+891016 +00	.590	+836641 +00	.925	+803871 +00
.260	+890582 +00	.595	+836086 +00	.930	+804160 +00
.265	+890873 +00	.600	+835703 +00	.935	+804469 +00
.270	+891182 +00	.605	+835481 +00	.940	+804796 +00
.275	+890864 +00	.610	+835407 +00	.945	+805137 +00
.280	+889642 +00	.615	+835465 +00	.950	+805487 +00
.285	+887676 +00	.620	+835633 +00	.955	+805843 +00
.290	+885433 +00	.625	+835891 +00	.960	+806201 +00
.295	+883458 +00	.630	+836218 +00	.965	+806557 +00
.300	+882148 +00	.635	+836589 +00	.970	+806909 +00
.305	+881629 +00	.640	+836984 +00	.975	+807253 +00
.310	+881745 +00	.645	+837380 +00	.980	+807585 +00
.315	+882155 +00	.650	+837758 +00	.985	+807904 +00
.320	+882457 +00	.655	+838099 +00	.990	+808206 +00
.325	+882322 +00	.660	+838385 +00	.995	+808488 +00
.330	+881568 +00	.665	+838601 +00	1.000	+808750 +00
.335	+880192 +00	.670	+838736 +00		

TABLE A7B
$\langle p \rangle_{rel}$ for $ka = 30\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 1.8\ cm$ $\lambda = 0.12\ cm$ $27 \leq z \leq 270\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
1.00	+808750 +00	4.05	+635697 +00	7.05	+417063 +00
1.05	+809782 +00	4.10	+630939 +00	7.10	+414510 +00
1.10	+807299 +00	4.15	+626205 +00	7.15	+411985 +00
1.15	+801342 +00	4.20	+621499 +00	7.20	+409487 +00
1.20	+792737 +00	4.25	+616821 +00	7.25	+407017 +00
1.25	+782620 +00	4.30	+612174 +00	7.30	+404574 +00
1.30	+772118 +00	4.35	+607560 +00	7.35	+402157 +00
1.35	+762175 +00	4.40	+602981 +00	7.40	+399767 +00
1.40	+753474 +00	4.45	+598437 +00	7.45	+397403 +00
1.45	+746427 +00	4.50	+593930 +00	7.50	+395064 +00
1.50	+741208 +00	4.55	+589461 +00	7.55	+392750 +00
1.55	+737802 +00	4.60	+585031 +00	7.60	+390461 +00
1.60	+736060 +00	4.65	+580641 +00	7.65	+388196 +00
1.65	+735750 +00	4.70	+576292 +00	7.70	+385955 +00
1.70	+736600 +00	4.75	+571984 +00	7.75	+383738 +00
1.75	+738331 +00	4.80	+567717 +00	7.80	+381545 +00
1.80	+740678 +00	4.85	+563492 +00	7.85	+379375 +00
1.85	+743403 +00	4.90	+559310 +00	7.90	+377227 +00
1.90	+746301 +00	4.95	+555169 +00	7.95	+375102 +00
1.95	+749203 +00	5.00	+551072 +00	8.00	+372999 +00
2.00	+751975 +00	5.05	+547017 +00	8.05	+370918 +00
2.05	+754511 +00	5.10	+543005 +00	8.10	+368859 +00
2.10	+756735 +00	5.15	+539036 +00	8.15	+366820 +00
2.15	+758592 +00	5.20	+535110 +00	8.20	+364803 +00
2.20	+760046 +00	5.25	+531226 +00	8.25	+362806 +00
2.25	+761076 +00	5.30	+527385 +00	8.30	+360830 +00
2.30	+761675 +00	5.35	+523585 +00	8.35	+358873 +00
2.35	+761843 +00	5.40	+519828 +00	8.40	+356937 +00
2.40	+761589 +00	5.45	+516113 +00	8.45	+355020 +00
2.45	+760927 +00	5.50	+512439 +00	8.50	+353122 +00
2.50	+759873 +00	5.55	+508806 +00	8.55	+351243 +00
2.55	+758448 +00	5.60	+505215 +00	8.60	+349383 +00
2.60	+756674 +00	5.65	+501663 +00	8.65	+347542 +00
2.65	+754574 +00	5.70	+498153 +00	8.70	+345718 +00
2.70	+752169 +00	5.75	+494681 +00	8.75	+343913 +00
2.75	+749484 +00	5.80	+491250 +00	8.80	+342125 +00
2.80	+746540 +00	5.85	+487857 +00	8.85	+340355 +00
2.85	+743360 +00	5.90	+484504 +00	8.90	+338602 +00
2.90	+739963 +00	5.95	+481188 +00	8.95	+336866 +00
2.95	+736371 +00	6.00	+477910 +00	9.00	+335147 +00
3.00	+732602 +00	6.05	+474670 +00	9.05	+333445 +00
3.05	+728674 +00	6.10	+471467 +00	9.10	+331758 +00
3.10	+724604 +00	6.15	+468300 +00	9.15	+330088 +00
3.15	+720407 +00	6.20	+465170 +00	9.20	+328434 +00
3.20	+716098 +00	6.25	+462075 +00	9.25	+326796 +00
3.25	+711691 +00	6.30	+459016 +00	9.30	+325173 +00
3.30	+707198 +00	6.35	+455991 +00	9.35	+323565 +00
3.35	+702631 +00	6.40	+453001 +00	9.40	+321973 +00
3.40	+698002 +00	6.45	+450044 +00	9.45	+320395 +00
3.45	+693319 +00	6.50	+447122 +00	9.50	+318832 +00
3.50	+688592 +00	6.55	+444232 +00	9.55	+317283 +00
3.55	+683830 +00	6.60	+441375 +00	9.60	+315749 +00
3.60	+679041 +00	6.65	+438551 +00	9.65	+314229 +00
3.65	+674231 +00	6.70	+435758 +00	9.70	+312723 +00
3.70	+669407 +00	6.75	+432996 +00	9.75	+311230 +00
3.75	+664576 +00	6.80	+430266 +00	9.80	+309751 +00
3.80	+659742 +00	6.85	+427566 +00	9.85	+308286 +00
3.85	+654911 +00	6.90	+424896 +00	9.90	+306834 +00
3.90	+650087 +00	6.95	+422256 +00	9.95	+305395 +00
3.95	+645274 +00	7.00	+419645 +00	10.00	+303969 +00
4.00	+640476 +00				

TABLE A7c

$\langle\theta\rangle_{rel}$ for $ka = 30\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 1.8\ cm$ $\lambda = 0.12\ cm$ $0.135 \leq z \leq 27\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+269040 -01	.340	+152039 +00	.675	+216728 +00
.010	+305004 -01	.345	+153493 +00	.680	+218642 +00
.015	+339120 -01	.350	+154275 +00	.685	+220619 +00
.020	+371853 -01	.355	+154496 +00	.690	+222641 +00
.025	+402606 -01	.360	+154372 +00	.695	+224689 +00
.030	+430196 -01	.365	+154168 +00	.700	+226747 +00
.035	+454806 -01	.370	+154142 +00	.705	+228797 +00
.040	+480867 -01	.375	+154500 +00	.710	+230824 +00
.045	+513384 -01	.380	+155365 +00	.715	+232814 +00
.050	+544642 -01	.385	+156769 +00	.720	+234752 +00
.055	+562956 -01	.390	+158661 +00	.725	+236625 +00
.060	+586468 -01	.395	+160923 +00	.730	+238424 +00
.065	+619384 -01	.400	+163395 +00	.735	+240137 +00
.070	+632967 -01	.405	+165903 +00	.740	+241758 +00
.075	+662430 -01	.410	+168281 +00	.745	+243278 +00
.080	+682101 -01	.415	+170389 +00	.750	+244693 +00
.085	+703629 -01	.420	+172125 +00	.755	+245999 +00
.090	+726211 -01	.425	+173436 +00	.760	+247192 +00
.095	+746511 -01	.430	+174313 +00	.765	+248272 +00
.100	+764644 -01	.435	+174792 +00	.770	+249239 +00
.105	+791336 -01	.440	+174944 +00	.775	+250093 +00
.110	+799315 -01	.445	+174867 +00	.780	+250837 +00
.115	+832638 -01	.450	+174672 +00	.785	+251474 +00
.120	+841017 -01	.455	+174473 +00	.790	+252008 +00
.125	+859544 -01	.460	+174374 +00	.795	+252445 +00
.130	+889700 -01	.465	+174468 +00	.800	+252790 +00
.135	+893931 -01	.470	+174822 +00	.805	+253050 +00
.140	+912417 -01	.475	+175482 +00	.810	+253231 +00
.145	+943816 -01	.480	+176468 +00	.815	+253341 +00
.150	+951359 -01	.485	+177777 +00	.820	+253389 +00
.155	+958545 -01	.490	+179385 +00	.825	+253381 +00
.160	+988352 -01	.495	+181250 +00	.830	+253326 +00
.165	+101257 +00	.500	+183319 +00	.835	+253234 +00
.170	+101546 +00	.505	+185532 +00	.840	+253111 +00
.175	+102266 +00	.510	+187824 +00	.845	+252966 +00
.180	+105011 +00	.515	+190128 +00	.850	+252808 +00
.185	+107857 +00	.520	+192384 +00	.855	+252643 +00
.190	+108835 +00	.525	+194535 +00	.860	+252481 +00
.195	+108813 +00	.530	+196533 +00	.865	+252328 +00
.200	+109928 +00	.535	+198339 +00	.870	+252191 +00
.205	+112679 +00	.540	+199925 +00	.875	+252076 +00
.210	+115570 +00	.545	+201271 +00	.880	+251991 +00
.215	+117064 +00	.550	+202372 +00	.885	+251940 +00
.220	+117179 +00	.555	+203228 +00	.890	+251928 +00
.225	+117260 +00	.560	+203851 +00	.895	+251962 +00
.230	+118550 +00	.565	+204261 +00	.900	+252044 +00
.235	+121142 +00	.570	+204483 +00	.905	+252178 +00
.240	+124077 +00	.575	+204549 +00	.910	+252368 +00
.245	+126224 +00	.580	+204494 +00	.915	+252617 +00
.250	+127087 +00	.585	+204355 +00	.920	+252927 +00
.255	+127052 +00	.590	+204168 +00	.925	+253299 +00
.260	+127020 +00	.595	+203973 +00	.930	+253736 +00
.265	+127805 +00	.600	+203804 +00	.935	+254238 +00
.270	+129691 +00	.605	+203693 +00	.940	+254806 +00
.275	+132367 +00	.610	+203670 +00	.945	+255440 +00
.280	+135169 +00	.615	+203760 +00	.950	+256140 +00
.285	+137431 +00	.620	+203983 +00	.955	+256905 +00
.290	+138768 +00	.625	+204356 +00	.960	+257735 +00
.295	+139200 +00	.630	+204888 +00	.965	+258629 +00
.300	+139085 +00	.635	+205587 +00	.970	+259586 +00
.305	+138953 +00	.640	+206456 +00	.975	+260603 +00
.310	+139288 +00	.645	+207492 +00	.980	+261680 +00
.315	+140379 +00	.650	+208689 +00	.985	+262814 +00
.320	+142252 +00	.655	+210041 +00	.990	+264004 +00
.325	+144705 +00	.660	+211534 +00	.995	+265246 +00
.330	+147395 +00	.665	+213157 +00	1.000	+266539 +00
.335	+149944 +00	.670	+214893 +00		

TABLE A7D
$\langle\theta\rangle_{rel}$ for $ka = 30\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 1.8\ cm$ $\lambda = 0.12\ cm$ $27 \leq z \leq 270\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+266539	+00	4.05	+836464	+00	7.05	+113268	+01
1.05	+281673	+00	4.10	+844373	+00	7.10	+113566	+01
1.10	+298909	+00	4.15	+852131	+00	7.15	+113860	+01
1.15	+315798	+00	4.20	+859742	+00	7.20	+114150	+01
1.20	+330583	+00	4.25	+867208	+00	7.25	+114436	+01
1.25	+342276	+00	4.30	+874533	+00	7.30	+114719	+01
1.30	+350564	+00	4.35	+881720	+00	7.35	+114997	+01
1.35	+355643	+00	4.40	+888773	+00	7.40	+115273	+01
1.40	+358058	+00	4.45	+895695	+00	7.45	+115544	+01
1.45	+358536	+00	4.50	+902489	+00	7.50	+115813	+01
1.50	+357862	+00	4.55	+909157	+00	7.55	+116078	+01
1.55	+356773	+00	4.60	+915704	+00	7.60	+116339	+01
1.60	+355899	+00	4.65	+922131	+00	7.65	+116598	+01
1.65	+355724	+00	4.70	+928442	+00	7.70	+116853	+01
1.70	+356587	+00	4.75	+934639	+00	7.75	+117105	+01
1.75	+358691	+00	4.80	+940726	+00	7.80	+117354	+01
1.80	+362128	+00	4.85	+946704	+00	7.85	+117600	+01
1.85	+366905	+00	4.90	+952576	+00	7.90	+117842	+01
1.90	+372965	+00	4.95	+958346	+00	7.95	+118082	+01
1.95	+380216	+00	5.00	+964014	+00	8.00	+118320	+01
2.00	+388538	+00	5.05	+969585	+00	8.05	+118554	+01
2.05	+397804	+00	5.10	+975059	+00	8.10	+118785	+01
2.10	+407883	+00	5.15	+980440	+00	8.15	+119014	+01
2.15	+418651	+00	5.20	+985729	+00	8.20	+119240	+01
2.20	+429989	+00	5.25	+990929	+00	8.25	+119464	+01
2.25	+441790	+00	5.30	+996041	+00	8.30	+119685	+01
2.30	+453958	+00	5.35	+100107	+01	8.35	+119903	+01
2.35	+466405	+00	5.40	+100601	+01	8.40	+120119	+01
2.40	+479056	+00	5.45	+101088	+01	8.45	+120332	+01
2.45	+491844	+00	5.50	+101566	+01	8.50	+120543	+01
2.50	+504713	+00	5.55	+102036	+01	8.55	+120752	+01
2.55	+517613	+00	5.60	+102499	+01	8.60	+120958	+01
2.60	+530503	+00	5.65	+102955	+01	8.65	+121162	+01
2.65	+543346	+00	5.70	+103403	+01	8.70	+121364	+01
2.70	+556112	+00	5.75	+103844	+01	8.75	+121563	+01
2.75	+568776	+00	5.80	+104279	+01	8.80	+121761	+01
2.80	+581318	+00	5.85	+104706	+01	8.85	+121956	+01
2.85	+593719	+00	5.90	+105127	+01	8.90	+122149	+01
2.90	+605966	+00	5.95	+105541	+01	8.95	+122340	+01
2.95	+618047	+00	6.00	+105949	+01	9.00	+122529	+01
3.00	+629953	+00	6.05	+106351	+01	9.05	+122716	+01
3.05	+641677	+00	6.10	+106747	+01	9.10	+122900	+01
3.10	+653213	+00	6.15	+107137	+01	9.15	+123083	+01
3.15	+664558	+00	6.20	+107521	+01	9.20	+123264	+01
3.20	+675710	+00	6.25	+107899	+01	9.25	+123443	+01
3.25	+686665	+00	6.30	+108272	+01	9.30	+123621	+01
3.30	+697425	+00	6.35	+108639	+01	9.35	+123796	+01
3.35	+707990	+00	6.40	+109001	+01	9.40	+123970	+01
3.40	+718359	+00	6.45	+109358	+01	9.45	+124142	+01
3.45	+728534	+00	6.50	+109710	+01	9.50	+124312	+01
3.50	+738518	+00	6.55	+110056	+01	9.55	+124480	+01
3.55	+748312	+00	6.60	+110398	+01	9.60	+124647	+01
3.60	+757919	+00	6.65	+110735	+01	9.65	+124811	+01
3.65	+767341	+00	6.70	+111067	+01	9.70	+124975	+01
3.70	+776582	+00	6.75	+111395	+01	9.75	+125136	+01
3.75	+785645	+00	6.80	+111718	+01	9.80	+125297	+01
3.80	+794533	+00	6.85	+112036	+01	9.85	+125455	+01
3.85	+803249	+00	6.90	+112351	+01	9.90	+125612	+01
3.90	+811797	+00	6.95	+112661	+01	9.95	+125767	+01
3.95	+820179	+00	7.00	+112967	+01	10.00	+125921	+01
4.00	+828401	+00						

TABLE A8A

$\langle p \rangle_{rel}$ for $ka = 40\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 2.0\ cm$ $\lambda = 0.1\ cm$ $0.2 \leq z \leq 40\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
.005	+977372 +00	.340	+879581 +00	.675	+838467 +00
.010	+973556 +00	.345	+877790 +00	.680	+838479 +00
.015	+969597 +00	.350	+875721 +00	.685	+838386 +00
.020	+966538 +00	.355	+873669 +00	.690	+838183 +00
.025	+963645 +00	.360	+871901 +00	.695	+837867 +00
.030	+960569 +00	.365	+870608 +00	.700	+837437 +00
.035	+957702 +00	.370	+869869 +00	.705	+836894 +00
.040	+955595 +00	.375	+869655 +00	.710	+836239 +00
.045	+953035 +00	.380	+869845 +00	.715	+835478 +00
.050	+950422 +00	.385	+870261 +00	.720	+834616 +00
.055	+948041 +00	.390	+870703 +00	.725	+833658 +00
.060	+945882 +00	.395	+870987 +00	.730	+832614 +00
.065	+943949 +00	.400	+870964 +00	.735	+831491 +00
.070	+942230 +00	.405	+870545 +00	.740	+830298 +00
.075	+940424 +00	.410	+869698 +00	.745	+829046 +00
.080	+938081 +00	.415	+868449 +00	.750	+827744 +00
.085	+935926 +00	.420	+866873 +00	.755	+826402 +00
.090	+934933 +00	.425	+865074 +00	.760	+825031 +00
.095	+932894 +00	.430	+863178 +00	.765	+823641 +00
.100	+930893 +00	.435	+861309 +00	.770	+822242 +00
.105	+930071 +00	.440	+859553 +00	.775	+820844 +00
.110	+927464 +00	.445	+858063 +00	.780	+819455 +00
.115	+927047 +00	.450	+856904 +00	.785	+818086 +00
.120	+924397 +00	.455	+856050 +00	.790	+816743 +00
.125	+923954 +00	.460	+855534 +00	.795	+815436 +00
.130	+921845 +00	.465	+855331 +00	.800	+814171 +00
.135	+920400 +00	.470	+855393 +00	.805	+812954 +00
.140	+919921 +00	.475	+855659 +00	.810	+811791 +00
.145	+917156 +00	.480	+856044 +00	.815	+810688 +00
.150	+916722 +00	.485	+856481 +00	.820	+809649 +00
.155	+915928 +00	.490	+856891 +00	.825	+808677 +00
.160	+913187 +00	.495	+857207 +00	.830	+807775 +00
.165	+912480 +00	.500	+857371 +00	.835	+806945 +00
.170	+912432 +00	.505	+857339 +00	.840	+806189 +00
.175	+910167 +00	.510	+857083 +00	.845	+805508 +00
.180	+907990 +00	.515	+856585 +00	.850	+804901 +00
.185	+907933 +00	.520	+855846 +00	.855	+804370 +00
.190	+907810 +00	.525	+854876 +00	.860	+803912 +00
.195	+905748 +00	.530	+853697 +00	.865	+803527 +00
.200	+903286 +00	.535	+852340 +00	.870	+803212 +00
.205	+902485 +00	.540	+850840 +00	.875	+802966 +00
.210	+902856 +00	.545	+849241 +00	.880	+802786 +00
.215	+902404 +00	.550	+847583 +00	.885	+802668 +00
.220	+900360 +00	.555	+845912 +00	.890	+802610 +00
.225	+897885 +00	.560	+844267 +00	.895	+802608 +00
.230	+896514 +00	.565	+842688 +00	.900	+802658 +00
.235	+896537 +00	.570	+841207 +00	.905	+802757 +00
.240	+896920 +00	.575	+839854 +00	.910	+802900 +00
.245	+896426 +00	.580	+838651 +00	.915	+803084 +00
.250	+894666 +00	.585	+837614 +00	.920	+803303 +00
.255	+892262 +00	.590	+836753 +00	.925	+803555 +00
.260	+890242 +00	.595	+836073 +00	.930	+803834 +00
.265	+889289 +00	.600	+835571 +00	.935	+804137 +00
.270	+889356 +00	.605	+835240 +00	.940	+804460 +00
.275	+889806 +00	.610	+835069 +00	.945	+804797 +00
.280	+889861 +00	.615	+835043 +00	.950	+805147 +00
.285	+889026 +00	.620	+835144 +00	.955	+805503 +00
.290	+887293 +00	.625	+835350 +00	.960	+805864 +00
.295	+885062 +00	.630	+835640 +00	.965	+806224 +00
.300	+882917 +00	.635	+835992 +00	.970	+806582 +00
.305	+881358 +00	.640	+836382 +00	.975	+806933 +00
.310	+880617 +00	.645	+836788 +00	.980	+807274 +00
.315	+880613 +00	.650	+837188 +00	.985	+807602 +00
.320	+881025 +00	.655	+837563 +00	.990	+807914 +00
.325	+881428 +00	.660	+837894 +00	.995	+808209 +00
.330	+881444 +00	.665	+838164 +00	1.000	+808483 +00
.335	+880843 +00	.670	+838359 +00		

TABLE A8B
$\langle p \rangle_{rel}$ for $ka = 40\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 2.0\ cm$ $\lambda = 0.1\ cm$ $40 \leq z \leq 400\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
1.00	+808483 +00	4.05	+635770 +00	7.05	+417093 +00
1.05	+809673 +00	4.10	+631011 +00	7.10	+414539 +00
1.10	+807356 +00	4.15	+626277 +00	7.15	+412014 +00
1.15	+801529 +00	4.20	+621570 +00	7.20	+409516 +00
1.20	+793002 +00	4.25	+616891 +00	7.25	+407045 +00
1.25	+782912 +00	4.30	+612243 +00	7.30	+404602 +00
1.30	+772396 +00	4.35	+607628 +00	7.35	+402185 +00
1.35	+762412 +00	4.40	+603048 +00	7.40	+399794 +00
1.40	+753653 +00	4.45	+598503 +00	7.45	+397429 +00
1.45	+746543 +00	4.50	+593995 +00	7.50	+395090 +00
1.50	+741262 +00	4.55	+589525 +00	7.55	+392776 +00
1.55	+737802 +00	4.60	+585095 +00	7.60	+390486 +00
1.60	+736014 +00	4.65	+580704 +00	7.65	+388221 +00
1.65	+735669 +00	4.70	+576354 +00	7.70	+385980 +00
1.70	+736494 +00	4.75	+572044 +00	7.75	+383763 +00
1.75	+738209 +00	4.80	+567777 +00	7.80	+381569 +00
1.80	+740548 +00	4.85	+563551 +00	7.85	+379399 +00
1.85	+743271 +00	4.90	+559367 +00	7.90	+377251 +00
1.90	+746173 +00	4.95	+555227 +00	7.95	+375126 +00
1.95	+749083 +00	5.00	+551128 +00	8.00	+373022 +00
2.00	+751864 +00	5.05	+547073 +00	8.05	+370941 +00
2.05	+754412 +00	5.10	+543060 +00	8.10	+368881 +00
2.10	+756649 +00	5.15	+539090 +00	8.15	+366842 +00
2.15	+758519 +00	5.20	+535163 +00	8.20	+364825 +00
2.20	+759986 +00	5.25	+531278 +00	8.25	+362828 +00
2.25	+761029 +00	5.30	+527436 +00	8.30	+360851 +00
2.30	+761641 +00	5.35	+523636 +00	8.35	+358894 +00
2.35	+761821 +00	5.40	+519878 +00	8.40	+356958 +00
2.40	+761578 +00	5.45	+516162 +00	8.45	+355040 +00
2.45	+760926 +00	5.50	+512487 +00	8.50	+353142 +00
2.50	+759882 +00	5.55	+508854 +00	8.55	+351263 +00
2.55	+758466 +00	5.60	+505261 +00	8.60	+349403 +00
2.60	+756700 +00	5.65	+501709 +00	8.65	+347561 +00
2.65	+754606 +00	5.70	+498198 +00	8.70	+345738 +00
2.70	+752209 +00	5.75	+494726 +00	8.75	+343932 +00
2.75	+749529 +00	5.80	+491294 +00	8.80	+342144 +00
2.80	+746590 +00	5.85	+487900 +00	8.85	+340374 +00
2.85	+743415 +00	5.90	+484546 +00	8.90	+338621 +00
2.90	+740022 +00	5.95	+481230 +00	8.95	+336884 +00
2.95	+736434 +00	6.00	+477951 +00	9.00	+335165 +00
3.00	+732668 +00	6.05	+474710 +00	9.05	+333462 +00
3.05	+728742 +00	6.10	+471507 +00	9.10	+331776 +00
3.10	+724674 +00	6.15	+468339 +00	9.15	+330106 +00
3.15	+720479 +00	6.20	+465208 +00	9.20	+328451 +00
3.20	+716172 +00	6.25	+462113 +00	9.25	+326813 +00
3.25	+711766 +00	6.30	+459053 +00	9.30	+325190 +00
3.30	+707274 +00	6.35	+456028 +00	9.35	+323582 +00
3.35	+702708 +00	6.40	+453037 +00	9.40	+321989 +00
3.40	+698079 +00	6.45	+450080 +00	9.45	+320411 +00
3.45	+693397 +00	6.50	+447157 +00	9.50	+318848 +00
3.50	+688670 +00	6.55	+444267 +00	9.55	+317299 +00
3.55	+683908 +00	6.60	+441409 +00	9.60	+315765 +00
3.60	+679119 +00	6.65	+438584 +00	9.65	+314244 +00
3.65	+674309 +00	6.70	+435791 +00	9.70	+312738 +00
3.70	+669485 +00	6.75	+433029 +00	9.75	+311245 +00
3.75	+664653 +00	6.80	+430298 +00	9.80	+309766 +00
3.80	+659819 +00	6.85	+427598 +00	9.85	+308301 +00
3.85	+654987 +00	6.90	+424927 +00	9.90	+306848 +00
3.90	+650162 +00	6.95	+422287 +00	9.95	+305409 +00
3.95	+645349 +00	7.00	+419676 +00	10.00	+303983 +00
4.00	+640550 +00				

TABLE A8C

$\langle\theta\rangle_{rel}$ for $ka = 40\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 2.0\ cm$ $\lambda = 0.1\ cm$ $0.2 \leq z \leq 40\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+238227 -01	.340	+150908 +00	.675	+216095 +00
.010	+275130 -01	.345	+153117 +00	.680	+217965 +00
.015	+312991 -01	.350	+154684 +00	.685	+219910 +00
.020	+354504 -01	.355	+155549 +00	.690	+221910 +00
.025	+382217 -01	.360	+155808 +00	.695	+223949 +00
.030	+420868 -01	.365	+155678 +00	.700	+226009 +00
.035	+445854 -01	.370	+155434 +00	.705	+228071 +00
.040	+476796 -01	.375	+155351 +00	.710	+230120 +00
.045	+509220 -01	.380	+155648 +00	.715	+232139 +00
.050	+537344 -01	.385	+156464 +00	.720	+234114 +00
.055	+563329 -01	.390	+157838 +00	.725	+236032 +00
.060	+588970 -01	.395	+159718 +00	.730	+237881 +00
.065	+614221 -01	.400	+161984 +00	.735	+239650 +00
.070	+637246 -01	.405	+164471 +00	.740	+241329 +00
.075	+656087 -01	.410	+166997 +00	.745	+242911 +00
.080	+675463 -01	.415	+169390 +00	.750	+244390 +00
.085	+703703 -01	.420	+171504 +00	.755	+245761 +00
.090	+726900 -01	.425	+173235 +00	.760	+247020 +00
.095	+739129 -01	.430	+174528 +00	.765	+248165 +00
.100	+769424 -01	.435	+175377 +00	.770	+249196 +00
.105	+782214 -01	.440	+175821 +00	.775	+250112 +00
.110	+806699 -01	.445	+175937 +00	.780	+250917 +00
.115	+822199 -01	.450	+175826 +00	.785	+251611 +00
.120	+846201 -01	.455	+175604 +00	.790	+252199 +00
.125	+856630 -01	.460	+175388 +00	.795	+252687 +00
.130	+887326 -01	.465	+175286 +00	.800	+253078 +00
.135	+890670 -01	.470	+175388 +00	.805	+253380 +00
.140	+919811 -01	.475	+175765 +00	.810	+253599 +00
.145	+936678 -01	.480	+176458 +00	.815	+253742 +00
.150	+941813 -01	.485	+177484 +00	.820	+253818 +00
.155	+973558 -01	.490	+178838 +00	.825	+253834 +00
.160	+989406 -01	.495	+180491 +00	.830	+253798 +00
.165	+991084 -01	.500	+182399 +00	.835	+253720 +00
.170	+101694 +00	.505	+184506 +00	.840	+253606 +00
.175	+104467 +00	.510	+186748 +00	.845	+253467 +00
.180	+104829 +00	.515	+189057 +00	.850	+253309 +00
.185	+105338 +00	.520	+191368 +00	.855	+253142 +00
.190	+108068 +00	.525	+193617 +00	.860	+252972 +00
.195	+110911 +00	.530	+195750 +00	.865	+252808 +00
.200	+111703 +00	.535	+197717 +00	.870	+252657 +00
.205	+111611 +00	.540	+199482 +00	.875	+252526 +00
.210	+112963 +00	.545	+201018 +00	.880	+252420 +00
.215	+115922 +00	.550	+202309 +00	.885	+252346 +00
.220	+118621 +00	.555	+203348 +00	.890	+252310 +00
.225	+119646 +00	.560	+204142 +00	.895	+252317 +00
.230	+119503 +00	.565	+204704 +00	.900	+252370 +00
.235	+119821 +00	.570	+205055 +00	.905	+252475 +00
.240	+121626 +00	.575	+205224 +00	.910	+252634 +00
.245	+124533 +00	.580	+205244 +00	.915	+252850 +00
.250	+127287 +00	.585	+205150 +00	.920	+253127 +00
.255	+128861 +00	.590	+204982 +00	.925	+253467 +00
.260	+129156 +00	.595	+204777 +00	.930	+253870 +00
.265	+128933 +00	.600	+204574 +00	.935	+254338 +00
.270	+129203 +00	.605	+204407 +00	.940	+254873 +00
.275	+130582 +00	.610	+204309 +00	.945	+255474 +00
.280	+133008 +00	.615	+204308 +00	.950	+256141 +00
.285	+135881 +00	.620	+204429 +00	.955	+256875 +00
.290	+138441 +00	.625	+204690 +00	.960	+257674 +00
.295	+140133 +00	.630	+205106 +00	.965	+258538 +00
.300	+140820 +00	.635	+205687 +00	.970	+259465 +00
.305	+140781 +00	.640	+206439 +00	.975	+260454 +00
.310	+140547 +00	.645	+207361 +00	.980	+261504 +00
.315	+140670 +00	.650	+208452 +00	.985	+262612 +00
.320	+141526 +00	.655	+209703 +00	.990	+263777 +00
.325	+143218 +00	.660	+211107 +00	.995	+264996 +00
.330	+145583 +00	.665	+212650 +00	1.000	+266267 +00
.335	+148280 +00	.670	+214318 +00		

TABLE A8D

$\langle\theta\rangle_{rel}$ for $ka = 40\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 2.0\ cm$ $\quad \lambda = 0.1\ cm$ $\quad 40 \leq z \leq 400\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+266267	+00	4.05	+836407	+00	7.05	+113269	+01
1.05	+281258	+00	4.10	+844319	+00	7.10	+113567	+01
1.10	+298477	+00	4.15	+852080	+00	7.15	+113861	+01
1.15	+315438	+00	4.20	+859692	+00	7.30	+114151	+01
1.20	+330341	+00	4.25	+867161	+00	7.25	+114437	+01
1.25	+342167	+00	4.30	+874488	+00	7.30	+114720	+01
1.30	+350578	+00	4.35	+881678	+00	7.35	+114998	+01
1.35	+355758	+00	4.40	+888733	+00	7.40	+115274	+01
1.40	+358243	+00	4.45	+895657	+00	7.45	+115545	+01
1.45	+358760	+00	4.50	+902452	+00	7.50	+115814	+01
1.50	+358098	+00	4.55	+909123	+00	7.55	+116079	+01
1.55	+356999	+00	4.60	+915671	+00	7.60	+116340	+01
1.60	+356097	+00	4.65	+922100	+00	7.65	+116599	+01
1.65	+355885	+00	4.70	+928413	+00	7.70	+116854	+01
1.70	+356705	+00	4.75	+934611	+00	7.75	+117106	+01
1.75	+358764	+00	4.80	+940699	+00	7.80	+117355	+01
1.80	+362158	+00	4.85	+946679	+00	7.85	+117601	+01
1.85	+366895	+00	4.90	+952553	+00	7.90	+117844	+01
1.90	+372920	+00	4.95	+958324	+00	7.95	+118084	+01
1.95	+380139	+00	5.00	+963994	+00	8.00	+118321	+01
2.00	+388435	+00	5.05	+969565	+00	8.05	+118555	+01
2.05	+397679	+00	5.10	+975041	+00	8.10	+118787	+01
2.10	+407741	+00	5.15	+980423	+00	8.15	+119015	+01
2.15	+418495	+00	5.20	+985713	+00	8.20	+119241	+01
2.20	+429823	+00	5.25	+990914	+00	8.25	+119465	+01
2.25	+441617	+00	5.30	+996028	+00	8.30	+119686	+01
2.30	+453779	+00	5.35	+100106	+01	8.35	+119904	+01
2.35	+466223	+00	5.40	+100600	+01	8.40	+120120	+01
2.40	+478873	+00	5.45	+101086	+01	8.45	+120334	+01
2.45	+491661	+00	5.50	+101565	+01	8.50	+120545	+01
2.50	+504532	+00	5.55	+102036	+01	8.55	+120753	+01
2.55	+517434	+00	5.60	+102499	+01	8.60	+120959	+01
2.60	+530326	+00	5.65	+102954	+01	8.65	+121163	+01
2.65	+543173	+00	5.70	+103403	+01	8.70	+121365	+01
2.70	+555943	+00	5.75	+103844	+01	8.75	+121565	+01
2.75	+568612	+00	5.80	+104278	+01	8.80	+121762	+01
2.80	+581159	+00	5.85	+104706	+01	8.85	+121957	+01
2.85	+593565	+00	5.90	+105127	+01	8.90	+122150	+01
2.90	+605816	+00	5.95	+105541	+01	8.95	+122341	+01
2.95	+617902	+00	6.00	+105949	+01	9.00	+122530	+01
3.00	+629813	+00	6.05	+106351	+01	9.05	+122717	+01
3.05	+641542	+00	6.10	+106747	+01	9.10	+122902	+01
3.10	+653084	+00	6.15	+107137	+01	9.15	+123085	+01
3.15	+664434	+00	6.20	+107521	+01	9.20	+123266	+01
3.20	+675590	+00	6.25	+107899	+01	9.25	+123445	+01
3.25	+686550	+00	6.30	+108272	+01	9.30	+123622	+01
3.30	+697315	+00	6.35	+108639	+01	9.35	+123798	+01
3.35	+707883	+00	6.40	+109001	+01	9.40	+123971	+01
3.40	+718257	+00	6.45	+109358	+01	9.45	+124143	+01
3.45	+728436	+00	6.50	+109710	+01	9.50	+124313	+01
3.50	+738424	+00	6.55	+110057	+01	9.55	+124481	+01
3.55	+748222	+00	6.60	+110398	+01	9.60	+124648	+01
3.60	+757833	+00	6.65	+110735	+01	9.65	+124813	+01
3.65	+767259	+00	6.70	+111068	+01	9.70	+124976	+01
3.70	+776504	+00	6.75	+111395	+01	9.75	+125138	+01
3.75	+785570	+00	6.80	+111718	+01	9.80	+125298	+01
3.80	+794461	+00	6.85	+112037	+01	9.85	+125457	+01
3.85	+803180	+00	6.90	+112351	+01	9.90	+125613	+01
3.90	+811731	+00	6.95	+112661	+01	9.95	+125769	+01
3.95	+820117	+00	7.00	+112967	+01	10.00	+125923	+01
4.00	+828341	+00						

TABLE A9A

$\langle p \rangle_{rel}$ for $ka = 60\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 2.4\ cm$ $\lambda = 0.08\ cm$ $0.36 \leq z \leq 72\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$	
.005	+979562	+00	.340	+880047	+00	.675	+838218	+00
.010	+975218	+00	.345	+878691	+00	.680	+838278	+00
.015	+971569	+00	.350	+876826	+00	.685	+838237	+00
.020	+967648	+00	.355	+874724	+00	.690	+838086	+00
.025	+964291	+00	.360	+872692	+00	.695	+837823	+00
.030	+961405	+00	.365	+870996	+00	.700	+837446	+00
.035	+958451	+00	.370	+869811	+00	.705	+836953	+00
.040	+955965	+00	.375	+869197	+00	.710	+836348	+00
.045	+953399	+00	.380	+869100	+00	.715	+835633	+00
.050	+951005	+00	.385	+869378	+00	.720	+834813	+00
.055	+948359	+00	.390	+869839	+00	.725	+833895	+00
.060	+946341	+00	.395	+870277	+00	.730	+832886	+00
.065	+943981	+00	.400	+870510	+00	.735	+831794	+00
.070	+942406	+00	.405	+870399	+00	.740	+830628	+00
.075	+940445	+00	.410	+869869	+00	.745	+829397	+00
.080	+938435	+00	.415	+868904	+00	.750	+828113	+00
.085	+936628	+00	.420	+867547	+00	.755	+826784	+00
.090	+935027	+00	.425	+865884	+00	.760	+825422	+00
.095	+933387	+00	.430	+864033	+00	.765	+824036	+00
.100	+931257	+00	.435	+862123	+00	.770	+822637	+00
.105	+929234	+00	.440	+860280	+00	.775	+821235	+00
.110	+928464	+00	.445	+858615	+00	.780	+819839	+00
.115	+926573	+00	.450	+857216	+00	.785	+818459	+00
.120	+924753	+00	.455	+856139	+00	.790	+817102	+00
.125	+924023	+00	.460	+855407	+00	.795	+815778	+00
.130	+921630	+00	.465	+855012	+00	.800	+814494	+00
.135	+921182	+00	.470	+854920	+00	.805	+813255	+00
.140	+919021	+00	.475	+855074	+00	.810	+812070	+00
.145	+917946	+00	.480	+855404	+00	.815	+810942	+00
.150	+917117	+00	.485	+855832	+00	.820	+809876	+00
.155	+914520	+00	.490	+856279	+00	.825	+808877	+00
.160	+914635	+00	.495	+856671	+00	.830	+807947	+00
.165	+912995	+00	.500	+856942	+00	.835	+807089	+00
.170	+910620	+00	.505	+857040	+00	.840	+806304	+00
.175	+910769	+00	.510	+856926	+00	.845	+805595	+00
.180	+909848	+00	.515	+856575	+00	.850	+804961	+00
.185	+907117	+00	.520	+855978	+00	.855	+804402	+00
.190	+906101	+00	.525	+855140	+00	.860	+803918	+00
.195	+906486	+00	.530	+854076	+00	.865	+803507	+00
.200	+905253	+00	.535	+852812	+00	.870	+803167	+00
.205	+902594	+00	.540	+851384	+00	.875	+802898	+00
.210	+901110	+00	.545	+849831	+00	.880	+802695	+00
.215	+901369	+00	.550	+848195	+00	.885	+802556	+00
.220	+901394	+00	.555	+846520	+00	.890	+802479	+00
.225	+899729	+00	.560	+844851	+00	.895	+802459	+00
.230	+897165	+00	.565	+843227	+00	.900	+802493	+00
.235	+895463	+00	.570	+841685	+00	.905	+802577	+00
.240	+895303	+00	.575	+840258	+00	.910	+802707	+00
.245	+895780	+00	.580	+838971	+00	.915	+802878	+00
.250	+895505	+00	.585	+837844	+00	.920	+803088	+00
.255	+893889	+00	.590	+836891	+00	.925	+803331	+00
.260	+891479	+00	.595	+836118	+00	.930	+803604	+00
.265	+889369	+00	.600	+835527	+00	.935	+803901	+00
.270	+888341	+00	.605	+835113	+00	.940	+804220	+00
.275	+888400	+00	.610	+834867	+00	.945	+804555	+00
.280	+888894	+00	.615	+834775	+00	.950	+804903	+00
.285	+888994	+00	.620	+834819	+00	.955	+805260	+00
.290	+888172	+00	.625	+834982	+00	.960	+805622	+00
.295	+886418	+00	.630	+835239	+00	.965	+805985	+00
.300	+884161	+00	.635	+835570	+00	.970	+806346	+00
.305	+882016	+00	.640	+835950	+00	.975	+806701	+00
.310	+880497	+00	.645	+836358	+00	.980	+807048	+00
.315	+879827	+00	.650	+836769	+00	.985	+807383	+00
.320	+879899	+00	.655	+837164	+00	.990	+807703	+00
.325	+880359	+00	.660	+837523	+00	.995	+808005	+00
.330	+880764	+00	.665	+837828	+00	1.000	+808288	+00
.335	+880732	+00	.670	+838064	+00			

TABLE A9B

$\langle p \rangle_{rel}$ for $ka = 60\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 2.4\ cm$ $\lambda = 0.08\ cm$ $72 \leq z \leq 720\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
1.00	+808288 +00	4.05	+635822 +00	7.05	+417115 +00
1.05	+809590 +00	4.10	+631063 +00	7.10	+414561 +00
1.10	+807392 +00	4.15	+626328 +00	7.15	+412035 +00
1.15	+801659 +00	4.20	+621620 +00	7.20	+409536 +00
1.20	+793189 +00	4.25	+616941 +00	7.25	+407066 +00
1.25	+783120 +00	4.30	+612293 +00	7.30	+404622 +00
1.30	+772596 +00	4.35	+607677 +00	7.35	+402205 +00
1.35	+762582 +00	4.40	+603096 +00	7.40	+399814 +00
1.40	+753783 +00	4.45	+598550 +00	7.45	+397448 +00
1.45	+746627 +00	4.50	+594042 +00	7.50	+395109 +00
1.50	+741303 +00	4.55	+589572 +00	7.55	+392794 +00
1.55	+737803 +00	4.60	+585140 +00	7.60	+390505 +00
1.60	+735982 +00	4.65	+580749 +00	7.65	+388239 +00
1.65	+735612 +00	4.70	+576398 +00	7.70	+385998 +00
1.70	+736419 +00	4.75	+572088 +00	7.75	+383781 +00
1.75	+738122 +00	4.80	+567819 +00	7.80	+381587 +00
1.80	+740455 +00	4.85	+563593 +00	7.85	+379416 +00
1.85	+743177 +00	4.90	+559409 +00	7.90	+377268 +00
1.90	+746081 +00	4.95	+555267 +00	7.95	+375142 +00
1.95	+748996 +00	5.00	+551168 +00	8.00	+373039 +00
2.00	+751785 +00	5.05	+547112 +00	8.05	+370957 +00
2.05	+754341 +00	5.10	+543099 +00	8.10	+368897 +00
2.10	+756587 +00	5.15	+539128 +00	8.15	+366858 +00
2.15	+758466 +00	5.20	+535200 +00	8.20	+364840 +00
2.20	+759943 +00	5.25	+531315 +00	8.25	+362843 +00
2.25	+760996 +00	5.30	+527472 +00	8.30	+360866 +00
2.30	+761616 +00	5.35	+523672 +00	8.35	+358909 +00
2.35	+761805 +00	5.40	+519913 +00	8.40	+356972 +00
2.40	+761570 +00	5.45	+516196 +00	8.45	+355055 +00
2.45	+760925 +00	5.50	+512521 +00	8.50	+353157 +00
2.50	+759888 +00	5.55	+508887 +00	8.55	+351278 +00
2.55	+758479 +00	5.60	+505294 +00	8.60	+349417 +00
2.60	+756718 +00	5.65	+501742 +00	8.65	+347575 +00
2.65	+754630 +00	5.70	+498230 +00	8.70	+345751 +00
2.70	+752237 +00	5.75	+494757 +00	8.75	+343946 +00
2.75	+749561 +00	5.80	+491325 +00	8.80	+342158 +00
2.80	+746626 +00	5.85	+487931 +00	8.85	+340387 +00
2.85	+743454 +00	5.90	+484576 +00	8.90	+338634 +00
2.90	+740065 +00	5.95	+481259 +00	8.95	+336897 +00
2.95	+736478 +00	6.00	+477981 +00	9.00	+335178 +00
3.00	+732715 +00	6.05	+474739 +00	9.05	+333475 +00
3.05	+728791 +00	6.10	+471535 +00	9.10	+331788 +00
3.10	+724725 +00	6.15	+468367 +00	9.15	+330118 +00
3.15	+720531 +00	6.20	+465236 +00	9.20	+328464 +00
3.20	+716225 +00	6.25	+462140 +00	9.25	+326825 +00
3.25	+711820 +00	6.30	+459080 +00	9.30	+325201 +00
3.30	+707329 +00	6.35	+456054 +00	9.35	+323593 +00
3.35	+702763 +00	6.40	+453063 +00	9.40	+322001 +00
3.40	+698134 +00	6.45	+450106 +00	9.45	+320422 +00
3.45	+693452 +00	6.50	+447182 +00	9.50	+318859 +00
3.50	+688726 +00	6.55	+444292 +00	9.55	+317310 +00
3.55	+683964 +00	6.60	+441434 +00	9.60	+315776 +00
3.60	+679175 +00	6.65	+438608 +00	9.65	+314255 +00
3.65	+674364 +00	6.70	+435815 +00	9.70	+312749 +00
3.70	+669540 +00	6.75	+433052 +00	9.75	+311256 +00
3.75	+664708 +00	6.80	+430321 +00	9.80	+309777 +00
3.80	+659873 +00	6.85	+427620 +00	9.85	+308311 +00
3.85	+655041 +00	6.90	+424950 +00	9.90	+306859 +00
3.90	+650216 +00	6.95	+422309 +00	9.95	+305420 +00
3.95	+645402 +00	7.00	+419697 +00	10.00	+303993 +00
4.00	+640603 +00				

TABLE A9c

$\langle\theta\rangle_{rel}$ for $ka = 60\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 2.4\ cm$ $\lambda = 0.08\ cm$ $0.36 \leq z \leq 72\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+209800 -01	.340	+149686 +00	.675	+215663 +00
.010	+256565 -01	.345	+152289 +00	.680	+217496 +00
.015	+298979 -01	.350	+154416 +00	.685	+219412 +00
.020	+339085 -01	.355	+155860 +00	.690	+221394 +00
.025	+375577 -01	.360	+156591 +00	.695	+223422 +00
.030	+411359 -01	.365	+156734 +00	.700	+225478 +00
.035	+440230 -01	.370	+156533 +00	.705	+227545 +00
.040	+473112 -01	.375	+156275 +00	.710	+229606 +00
.045	+503398 -01	.380	+156238 +00	.715	+231644 +00
.050	+529793 -01	.385	+156632 +00	.720	+233643 +00
.055	+555232 -01	.390	+157575 +00	.725	+235591 +00
.060	+585903 -01	.395	+159083 +00	.730	+237473 +00
.065	+606114 -01	.400	+161084 +00	.735	+239280 +00
.070	+630895 -01	.405	+163438 +00	.740	+241000 +00
.075	+657889 -01	.410	+165967 +00	.745	+242626 +00
.080	+681978 -01	.415	+168486 +00	.750	+244150 +00
.085	+704340 -01	.420	+170821 +00	.755	+245568 +00
.090	+724814 -01	.425	+172836 +00	.760	+246874 +00
.095	+741903 -01	.430	+174438 +00	.765	+248066 +00
.100	+759230 -01	.435	+175587 +00	.770	+249143 +00
.105	+785680 -01	.440	+176292 +00	.775	+250105 +00
.110	+807865 -01	.445	+176607 +00	.780	+250953 +00
.115	+817497 -01	.450	+176620 +00	.785	+251690 +00
.120	+847603 -01	.455	+176442 +00	.790	+252319 +00
.125	+856649 -01	.460	+176192 +00	.795	+252843 +00
.130	+883654 -01	.465	+175989 +00	.800	+253269 +00
.135	+892239 -01	.470	+175938 +00	.805	+253603 +00
.140	+922132 -01	.475	+176124 +00	.810	+253850 +00
.145	+924715 -01	.480	+176607 +00	.815	+254019 +00
.150	+956654 -01	.485	+177421 +00	.820	+254117 +00
.155	+966005 -01	.490	+178575 +00	.825	+254151 +00
.160	+977261 -01	.495	+180051 +00	.830	+254131 +00
.165	+100984 +00	.500	+181816 +00	.835	+254064 +00
.170	+101504 +00	.505	+183817 +00	.840	+253959 +00
.175	+102372 +00	.510	+185994 +00	.845	+253824 +00
.180	+105634 +00	.515	+188280 +00	.850	+253669 +00
.185	+107217 +00	.520	+190605 +00	.855	+253500 +00
.190	+107081 +00	.525	+192905 +00	.860	+253327 +00
.195	+108828 +00	.530	+195118 +00	.865	+253156 +00
.200	+112062 +00	.535	+197190 +00	.870	+252996 +00
.205	+113682 +00	.540	+199077 +00	.875	+252853 +00
.210	+113527 +00	.545	+200746 +00	.880	+252734 +00
.215	+114198 +00	.550	+202174 +00	.885	+252645 +00
.220	+116846 +00	.555	+203351 +00	.890	+252592 +00
.225	+119856 +00	.560	+204274 +00	.895	+252579 +00
.230	+121306 +00	.565	+204955 +00	.900	+252613 +00
.235	+121229 +00	.570	+205410 +00	.905	+252696 +00
.240	+121278 +00	.575	+205665 +00	.910	+252833 +00
.245	+122804 +00	.580	+205752 +00	.915	+253027 +00
.250	+125644 +00	.585	+205704 +00	.920	+253281 +00
.255	+128521 +00	.590	+205561 +00	.925	+253596 +00
.260	+130248 +00	.595	+205361 +00	.930	+253975 +00
.265	+130599 +00	.600	+205144 +00	.935	+254420 +00
.270	+130325 +00	.605	+204946 +00	.940	+254930 +00
.275	+130508 +00	.610	+204802 +00	.945	+255507 +00
.280	+131835 +00	.615	+204742 +00	.950	+256151 +00
.285	+134263 +00	.620	+204793 +00	.955	+256862 +00
.290	+137164 +00	.625	+204977 +00	.960	+257638 +00
.295	+139734 +00	.630	+205311 +00	.965	+258480 +00
.300	+141396 +00	.635	+205808 +00	.970	+259386 +00
.305	+142021 +00	.640	+206474 +00	.975	+260355 +00
.310	+141918 +00	.645	+207313 +00	.980	+261385 +00
.315	+141654 +00	.650	+208324 +00	.985	+262474 +00
.320	+141801 +00	.655	+209500 +00	.990	+263621 +00
.325	+142728 +00	.660	+210835 +00	.995	+264823 +00
.330	+144512 +00	.665	+212316 +00	1.000	+266078 +00
.335	+146954 +00	.670	+213931 +00		

TABLE AGD

$\langle\theta\rangle_{rel}$ for $ka = 60\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 2.4\ cm$ $\quad \lambda = 0.08\ cm$ $\quad 72 \leq z \leq 720\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+266078	+00	4.05	+836367	+00	7.05	+113270	+01
1.05	+280964	+00	4.10	+844280	+00	7.10	+113568	+01
1.10	+298168	+00	4.15	+852043	+00	7.15	+113861	+01
1.15	+315178	+00	4.20	+859657	+00	7.20	+114151	+01
1.20	+330166	+00	4.25	+867127	+00	7.25	+114438	+01
1.25	+342087	+00	4.30	+874456	+00	7.30	+114720	+01
1.30	+350586	+00	4.35	+881647	+00	7.35	+114999	+01
1.35	+355838	+00	4.40	+888704	+00	7.40	+115274	+01
1.40	+358374	+00	4.45	+895629	+00	7.45	+115546	+01
1.45	+358920	+00	4.50	+902426	+00	7.50	+115814	+01
1.50	+358266	+00	4.55	+909098	+00	7.55	+116079	+01
1.55	+357160	+00	4.60	+915648	+00	7.60	+116341	+01
1.60	+356240	+00	4.65	+922078	+00	7.65	+116599	+01
1.65	+356001	+00	4.70	+928392	+00	7.70	+116855	+01
1.70	+356790	+00	4.75	+934592	+00	7.75	+117107	+01
1.75	+358817	+00	4.80	+940681	+00	7.80	+117356	+01
1.80	+362180	+00	4.85	+946661	+00	7.85	+117601	+01
1.85	+366888	+00	4.90	+952536	+00	7.90	+117844	+01
1.90	+372887	+00	4.95	+958308	+00	7.95	+118084	+01
1.95	+380084	+00	5.00	+963979	+00	8.00	+118322	+01
2.00	+388361	+00	5.05	+969552	+00	8.05	+118556	+01
2.05	+397590	+00	5.10	+975028	+00	8.10	+118787	+01
2.10	+407639	+00	5.15	+980411	+00	8.15	+119016	+01
2.15	+418384	+00	5.20	+985702	+00	8.20	+119242	+01
2.20	+429704	+00	5.25	+990904	+00	8.25	+119466	+01
2.25	+441493	+00	5.30	+996018	+00	8.30	+119687	+01
2.30	+453651	+00	5.35	+100105	+01	8.35	+119905	+01
2.35	+466093	+00	5.40	+100599	+01	8.40	+120121	+01
2.40	+478742	+00	5.45	+101086	+01	8.45	+120334	+01
2.45	+491531	+00	5.50	+101564	+01	8.50	+120545	+01
2.50	+504402	+00	5.55	+102035	+01	8.55	+120754	+01
2.55	+517306	+00	5.60	+102498	+01	8.60	+120960	+01
2.60	+530201	+00	5.65	+102954	+01	8.65	+121164	+01
2.65	+543050	+00	5.70	+103402	+01	8.70	+121366	+01
2.70	+555823	+00	5.75	+103843	+01	8.75	+121566	+01
2.75	+568495	+00	5.80	+104278	+01	8.80	+121763	+01
2.80	+581045	+00	5.85	+104705	+01	8.85	+121958	+01
2.85	+593454	+00	5.90	+105126	+01	8.90	+122151	+01
2.90	+605710	+00	5.95	+105541	+01	8.95	+122342	+01
2.95	+617799	+00	6.00	+105949	+01	9.00	+122531	+01
3.00	+629714	+00	6.05	+106351	+01	9.05	+122718	+01
3.05	+641446	+00	6.10	+106747	+01	9.10	+122903	+01
3.10	+652991	+00	6.15	+107137	+01	9.15	+123086	+01
3.15	+664344	+00	6.20	+107521	+01	9.20	+123267	+01
3.20	+675504	+00	6.25	+107899	+01	9.25	+123446	+01
3.25	+686468	+00	6.30	+108272	+01	9.30	+123623	+01
3.30	+697236	+00	6.35	+108640	+01	9.35	+123799	+01
3.35	+707807	+00	6.40	+109002	+01	9.40	+123972	+01
3.40	+718184	+00	6.45	+109358	+01	9.45	+124144	+01
3.45	+728367	+00	6.50	+109710	+01	9.50	+124314	+01
3.50	+738357	+00	6.55	+110057	+01	9.55	+124482	+01
3.55	+748158	+00	6.60	+110399	+01	9.60	+124649	+01
3.60	+757772	+00	6.65	+110736	+01	9.65	+124814	+01
3.65	+767200	+00	6.70	+111068	+01	9.70	+124977	+01
3.70	+776448	+00	6.75	+111396	+01	9.75	+125139	+01
3.75	+785516	+00	6.80	+111719	+01	9.80	+125299	+01
3.80	+794410	+00	6.85	+112037	+01	9.85	+125458	+01
3.85	+803131	+00	6.90	+112352	+01	9.90	+125614	+01
3.90	+811684	+00	6.95	+112662	+01	9.95	+125770	+01
3.95	+820072	+00	7.00	+112968	+01	10.00	+125924	+01
4.00	+828299	+00						

TABLE A10A

$\langle p \rangle_{rel}$ for $ka = 80\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 2.4\ cm$ $\quad \lambda = 0.06\ cm$ $\quad 0.48 \leq z \leq 96\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$	
.005	+981090	+00	.340	+880098	+00	.675	+838126	+00
.010	+976214	+00	.345	+878933	+00	.680	+838203	+00
.015	+971955	+00	.350	+877190	+00	.685	+838179	+00
.020	+968316	+00	.355	+875120	+00	.690	+838047	+00
.025	+964666	+00	.360	+873032	+00	.695	+837802	+00
.030	+961546	+00	.365	+871214	+00	.700	+837443	+00
.035	+958600	+00	.370	+869874	+00	.705	+836969	+00
.040	+955751	+00	.375	+869105	+00	.710	+836381	+00
.045	+953259	+00	.380	+868882	+00	.715	+835682	+00
.050	+950921	+00	.385	+869083	+00	.720	+834878	+00
.055	+948759	+00	.390	+869523	+00	.725	+833974	+00
.060	+946555	+00	.395	+869993	+00	.730	+832978	+00
.065	+944144	+00	.400	+870301	+00	.735	+831897	+00
.070	+942170	+00	.405	+870293	+00	.740	+830741	+00
.075	+940100	+00	.410	+869875	+00	.745	+829519	+00
.080	+938645	+00	.415	+869019	+00	.750	+828241	+00
.085	+936787	+00	.420	+867751	+00	.755	+826917	+00
.090	+934949	+00	.425	+866152	+00	.760	+825558	+00
.095	+933320	+00	.430	+864332	+00	.765	+824174	+00
.100	+931780	+00	.435	+862420	+00	.770	+822776	+00
.105	+929881	+00	.440	+860547	+00	.775	+821372	+00
.110	+927676	+00	.445	+858829	+00	.780	+819974	+00
.115	+926631	+00	.450	+857360	+00	.785	+818590	+00
.120	+925473	+00	.455	+856205	+00	.790	+817229	+00
.125	+923032	+00	.460	+855395	+00	.795	+815900	+00
.130	+922771	+00	.465	+854928	+00	.800	+814609	+00
.135	+920107	+00	.470	+854774	+00	.805	+813363	+00
.140	+920039	+00	.475	+854883	+00	.810	+812169	+00
.145	+917292	+00	.480	+855185	+00	.815	+811033	+00
.150	+917238	+00	.485	+855603	+00	.820	+809958	+00
.155	+915177	+00	.490	+856056	+00	.825	+808949	+00
.160	+913617	+00	.495	+856470	+00	.830	+808009	+00
.165	+913665	+00	.500	+856775	+00	.835	+807142	+00
.170	+911038	+00	.505	+856916	+00	.840	+806347	+00
.175	+909834	+00	.510	+856850	+00	.845	+805628	+00
.180	+910083	+00	.515	+856551	+00	.850	+804984	+00
.185	+907970	+00	.520	+856005	+00	.855	+804416	+00
.190	+905707	+00	.525	+855215	+00	.860	+803922	+00
.195	+905768	+00	.530	+854194	+00	.865	+803502	+00
.200	+905629	+00	.535	+852967	+00	.870	+803154	+00
.205	+903376	+00	.540	+851567	+00	.875	+802876	+00
.210	+901043	+00	.545	+850033	+00	.880	+802665	+00
.215	+900648	+00	.550	+848408	+00	.885	+802519	+00
.220	+901094	+00	.555	+846736	+00	.890	+802434	+00
.225	+900230	+00	.560	+845060	+00	.895	+802408	+00
.230	+897838	+00	.565	+843423	+00	.900	+802436	+00
.235	+895574	+00	.570	+841862	+00	.905	+802515	+00
.240	+894803	+00	.575	+840410	+00	.910	+802640	+00
.245	+895213	+00	.580	+839094	+00	.915	+802807	+00
.250	+895396	+00	.585	+837936	+00	.920	+803013	+00
.255	+894309	+00	.590	+836950	+00	.925	+803253	+00
.260	+892081	+00	.595	+836145	+00	.930	+803523	+00
.265	+889720	+00	.600	+835522	+00	.935	+803819	+00
.270	+888238	+00	.605	+835078	+00	.940	+804136	+00
.275	+887953	+00	.610	+834804	+00	.945	+804470	+00
.280	+888403	+00	.615	+834688	+00	.950	+804818	+00
.285	+888742	+00	.620	+834712	+00	.955	+805175	+00
.290	+888280	+00	.625	+834857	+00	.960	+805537	+00
.295	+886817	+00	.630	+835102	+00	.965	+805901	+00
.300	+884659	+00	.635	+835424	+00	.970	+806263	+00
.305	+882401	+00	.640	+835800	+00	.975	+806620	+00
.310	+880630	+00	.645	+836206	+00	.980	+806969	+00
.315	+879682	+00	.650	+836620	+00	.985	+807306	+00
.320	+879553	+00	.655	+837021	+00	.990	+807628	+00
.325	+879948	+00	.660	+837389	+00	.995	+807934	+00
.330	+880427	+00	.665	+837706	+00	1.000	+808219	+00
.335	+880570	+00	.670	+837956	+00			

TABLE A10B

$\langle p \rangle_{rel}$ for $ka = 80\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 2.4\ cm$ $\lambda = 0.06\ cm$ $96 \leq z \leq 960\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
1.00	+808219 +00	4.05	+635841 +00	7.05	+417122 +00
1.05	+809561 +00	4.10	+631081 +00	7.10	+414568 +00
1.10	+807404 +00	4.15	+626346 +00	7.15	+412042 +00
1.15	+801705 +00	4.20	+621638 +00	7.20	+409544 +00
1.20	+793255 +00	4.25	+616958 +00	7.25	+407073 +00
1.25	+783193 +00	4.30	+612310 +00	7.30	+404629 +00
1.30	+772666 +00	4.35	+607694 +00	7.35	+402211 +00
1.35	+762642 +00	4.40	+603113 +00	7.40	+399820 +00
1.40	+753828 +00	4.45	+598567 +00	7.45	+397455 +00
1.45	+746657 +00	4.50	+594058 +00	7.50	+395115 +00
1.50	+741317 +00	4.55	+589588 +00	7.55	+392801 +00
1.55	+737803 +00	4.60	+585156 +00	7.60	+390511 +00
1.60	+735971 +00	4.65	+580764 +00	7.65	+388246 +00
1.65	+735592 +00	4.70	+576413 +00	7.70	+386004 +00
1.70	+736392 +00	4.75	+572103 +00	7.75	+383787 +00
1.75	+738092 +00	4.80	+567834 +00	7.80	+381593 +00
1.80	+740423 +00	4.85	+563608 +00	7.85	+379422 +00
1.85	+743145 +00	4.90	+559423 +00	7.90	+377274 +00
1.90	+746049 +00	4.95	+555282 +00	7.95	+375148 +00
1.95	+748966 +00	5.00	+551182 +00	8.00	+373045 +00
2.00	+751757 +00	5.05	+547126 +00	8.05	+370963 +00
2.05	+754316 +00	5.10	+543112 +00	8.10	+368903 +00
2.10	+756565 +00	5.15	+539142 +00	8.15	+366864 +00
2.15	+758448 +00	5.20	+535214 +00	8.20	+364846 +00
2.20	+759928 +00	5.25	+531328 +00	8.25	+362848 +00
2.25	+760984 +00	5.30	+527485 +00	8.30	+360872 +00
2.30	+761607 +00	5.35	+523684 +00	8.35	+358915 +00
2.35	+761799 +00	5.40	+519926 +00	8.40	+356978 +00
2.40	+761567 +00	5.45	+516209 +00	8.45	+355060 +00
2.45	+760925 +00	5.50	+512533 +00	8.50	+353162 +00
2.50	+759890 +00	5.55	+508899 +00	8.55	+351283 +00
2.55	+758483 +00	5.60	+505306 +00	8.60	+349422 +00
2.60	+756725 +00	5.65	+501753 +00	8.65	+347580 +00
2.65	+754638 +00	5.70	+498241 +00	8.70	+345756 +00
2.70	+752246 +00	5.75	+494769 +00	8.75	+343950 +00
2.75	+749573 +00	5.80	+491336 +00	8.80	+342162 +00
2.80	+746639 +00	5.85	+487942 +00	8.85	+340392 +00
2.85	+743468 +00	5.90	+484587 +00	8.90	+338638 +00
2.90	+740079 +00	5.95	+481270 +00	8.95	+336902 +00
2.95	+736494 +00	6.00	+477991 +00	9.00	+335182 +00
3.00	+732731 +00	6.05	+474749 +00	9.05	+333479 +00
3.05	+728808 +00	6.10	+471545 +00	9.10	+331793 +00
3.10	+724742 +00	6.15	+468377 +00	9.15	+330122 +00
3.15	+720549 +00	6.20	+465245 +00	9.20	+328468 +00
3.20	+716243 +00	6.25	+462150 +00	9.25	+326829 +00
3.25	+711838 +00	6.30	+459089 +00	9.30	+325206 +00
3.30	+707348 +00	6.35	+456063 +00	9.35	+323598 +00
3.35	+702782 +00	6.40	+453072 +00	9.40	+322005 +00
3.40	+698154 +00	6.45	+450115 +00	9.45	+320427 +00
3.45	+693472 +00	6.50	+447191 +00	9.50	+318863 +00
3.50	+688746 +00	6.55	+444300 +00	9.55	+317314 +00
3.55	+683984 +00	6.60	+441442 +00	9.60	+315780 +00
3.60	+679194 +00	6.65	+438617 +00	9.65	+314259 +00
3.65	+674384 +00	6.70	+435823 +00	9.70	+312753 +00
3.70	+669560 +00	6.75	+433060 +00	9.75	+311260 +00
3.75	+664727 +00	6.80	+430329 +00	9.80	+309781 +00
3.80	+659892 +00	6.85	+427628 +00	9.85	+308315 +00
3.85	+655060 +00	6.90	+424957 +00	9.90	+306863 +00
3.90	+650235 +00	6.95	+422316 +00	9.95	+305423 +00
3.95	+645421 +00	7.00	+419705 +00	10.00	+303997 +00
4.00	+640622 +00				

TABLE A1OC

$\langle\theta\rangle_{rel}$ for $ka = 80\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 2.4\ cm$ $\lambda = 0.06\ cm$ $0.48 \leq z \leq 96\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+192811 -01	.340	+149214. +00	.675	+215515 +00
.010	+245373 -01	.345	+151905 +00	.680	+217335 +00
.015	+291125 -01	.350	+154205 +00	.685	+219240 +00
.020	+332155 -01	.355	+155860 +00	.690	+221214 +00
.025	+371039 -01	.360	+156785 +00	.695	+223237 +00
.030	+405835 -01	.365	+157067 +00	.700	+225292 +00
.035	+441649 -01	.370	+156925 +00	.705	+227360 +00
.040	+471653 -01	.375	+156644 +00	.710	+229424 +00
.045	+502964 -01	.380	+156514 +00	.715	+231468 +00
.050	+527732 -01	.385	+156770 +00	.720	+233475 +00
.055	+556538 -01	.390	+157558 +00	.725	+235433 +00
.060	+581910 -01	.395	+158921 +00	.730	+237327 +00
.065	+605291 -01	.400	+160807 +00	.735	+239146 +00
.070	+634803 -01	.405	+163090 +00	.740	+240881 +00
.075	+652674 -01	.410	+165597 +00	.745	+242522 +00
.080	+677166 -01	.415	+168140 +00	.750	+244062 +00
.085	+702657 -01	.420	+170540 +00	.755	+245495 +00
.090	+725329 -01	.425	+172646 +00	.760	+246818 +00
.095	+745990 -01	.430	+174356 +00	.765	+248027 +00
.100	+763651 -01	.435	+175614 +00	.770	+249120 +00
.105	+779295 -01	.440	+176419 +00	.775	+250099 +00
.110	+802063 -01	.445	+176815 +00	.780	+250962 +00
.115	+828655 -01	.450	+176884 +00	.785	+251714 +00
.120	+837909 -01	.455	+176734 +00	.790	+252357 +00
.125	+862044 -01	.460	+176484 +00	.795	+252895 +00
.130	+879964 -01	.465	+176255 +00	.800	+253333 +00
.135	+895746 -01	.470	+176157 +00	.805	+253678 +00
.140	+915697 -01	.475	+176280 +00	.810	+253936 +00
.145	+934267 -01	.480	+176691 +00	.815	+254114 +00
.150	+944463 -01	.485	+177430 +00	.820	+254220 +00
.155	+975317 -01	.490	+178510 +00	.825	+254261 +00
.160	+975983 -01	.495	+179921 +00	.830	+254246 +00
.165	+100100 +00	.500	+181629 +00	.835	+254184 +00
.170	+102481 +00	.505	+183587 +00	.840	+254082 +00
.175	+102393 +00	.510	+185736 +00	.845	+253949 +00
.180	+104617 +00	.515	+188008 +00	.850	+253795 +00
.185	+107589 +00	.520	+190334 +00	.855	+253626 +00
.190	+107911 +00	.525	+192647 +00	.860	+253452 +00
.195	+108354 +00	.530	+194884 +00	.865	+253279 +00
.200	+111222 +00	.535	+196990 +00	.870	+253116 +00
.205	+113933 +00	.540	+198918 +00	.875	+252969 +00
.210	+114361 +00	.545	+200632 +00	.880	+252845 +00
.215	+114298 +00	.550	+202109 +00	.885	+252751 +00
.220	+116102 +00	.555	+203334 +00	.890	+252692 +00
.225	+119259 +00	.560	+204305 +00	.895	+252673 +00
.230	+121534 +00	.565	+205029 +00	.900	+252700 +00
.235	+121951 +00	.570	+205523 +00	.905	+252776 +00
.240	+121696 +00	.575	+205811 +00	.910	+252905 +00
.245	+122525 +00	.580	+205923 +00	.915	+253091 +00
.250	+124955 +00	.585	+205894 +00	.920	+253336 +00
.255	+128005 +00	.590	+205762 +00	.925	+253644 +00
.260	+130276 +00	.595	+205567 +00	.930	+254014 +00
.265	+131100 +00	.600	+205347 +00	.935	+254450 +00
.270	+130916 +00	.605	+205140 +00	.940	+254952 +00
.275	+130800 +00	.610	+204981 +00	.945	+255521 +00
.280	+131668 +00	.615	+204901 +00	.950	+256157 +00
.285	+133755 +00	.620	+204929 +00	.955	+256859 +00
.290	+136591 +00	.625	+205087 +00	.960	+257628 +00
.295	+139369 +00	.630	+205393 +00	.965	+258462 +00
.300	+141382 +00	.635	+205860 +00	.970	+259360 +00
.305	+142334 +00	.640	+206496 +00	.975	+260322 +00
.310	+142405 +00	.645	+207306 +00	.980	+261345 +00
.315	+142112 +00	.650	+208288 +00	.985	+262428 +00
.320	+142062 +00	.655	+209437 +00	.990	+263568 +00
.325	+142714 +00	.660	+210747 +00	.995	+264764 +00
.330	+144240 +00	.665	+212206 +00	1.000	+266013 +00
.335	+146519 +00	.670	+213800 +00		

TABLE A10D

$\langle\theta\rangle_{rel}$ for $ka = 80\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 2.4\ cm$ $\quad \lambda = 0.06\ cm$ $\quad 96 \leq z \leq 960\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+266013	+00	4.05	+836352	+00	7.05	+113270	+01
1.05	+280861	+00	4.10	+844267	+00	7.10	+113568	+01
1.10	+298060	+00	4.15	+852030	+00	7.15	+113862	+01
1.15	+315087	+00	4.20	+859645	+00	7.20	+114152	+01
1.20	+330104	+00	4.25	+867115	+00	7.25	+114438	+01
1.25	+342058	+00	4.30	+874445	+00	7.30	+114720	+01
1.30	+350588	+00	4.35	+881637	+00	7.35	+114999	+01
1.35	+355866	+00	4.40	+888694	+00	7.40	+115275	+01
1.40	+358419	+00	4.45	+895620	+00	7.45	+115546	+01
1.45	+358976	+00	4.50	+902417	+00	7.50	+115815	+01
1.50	+358325	+00	4.55	+909090	+00	7.55	+116080	+01
1.55	+357217	+00	4.60	+915639	+00	7.60	+116341	+01
1.60	+356290	+00	4.65	+922070	+00	7.65	+116600	+01
1.65	+356041	+00	4.70	+928384	+00	7.70	+116855	+01
1.70	+356819	+00	4.75	+934585	+00	7.75	+117107	+01
1.75	+358836	+00	4.80	+940674	+00	7.80	+117356	+01
1.80	+362188	+00	4.85	+946655	+00	7.85	+117602	+01
1.85	+366886	+00	4.90	+952531	+00	7.90	+117845	+01
1.90	+372876	+00	4.95	+958303	+00	7.95	+118085	+01
1.95	+380065	+00	5.00	+963974	+00	8.00	+118322	+01
2.00	+388336	+00	5.05	+969547	+00	8.05	+118556	+01
2.05	+397559	+00	5.10	+975024	+00	8.10	+118788	+01
2.10	+407604	+00	5.15	+980407	+00	8.15	+119017	+01
2.15	+418345	+00	5.20	+985698	+00	8.20	+119243	+01
2.20	+429663	+00	5.25	+990900	+00	8.25	+119466	+01
2.25	+441449	+00	5.30	+996015	+00	8.30	+119687	+01
2.30	+453607	+00	5.35	+100104	+01	8.35	+119905	+01
2.35	+466048	+00	5.40	+100599	+01	8.40	+120121	+01
2.40	+478696	+00	5.45	+101085	+01	8.45	+120335	+01
2.45	+491485	+00	5.50	+101564	+01	8.50	+120546	+01
2.50	+504357	+00	5.55	+102035	+01	8.55	+120754	+01
2.55	+517261	+00	5.60	+102498	+01	8.60	+120961	+01
2.60	+530157	+00	5.65	+102953	+01	8.65	+121165	+01
2.65	+543007	+00	5.70	+103402	+01	8.70	+121366	+01
2.70	+555781	+00	5.75	+103843	+01	8.75	+121566	+01
2.75	+568454	+00	5.80	+104278	+01	8.80	+121763	+01
2.80	+581005	+00	5.85	+104705	+01	8.85	+121958	+01
2.85	+593416	+00	5.90	+105126	+01	8.90	+122151	+01
2.90	+605672	+00	5.95	+105541	+01	8.95	+122342	+01
2.95	+617763	+00	6.00	+105949	+01	9.00	+122531	+01
3.00	+629679	+00	6.05	+106351	+01	9.05	+122718	+01
3.05	+641412	+00	6.10	+106747	+01	9.10	+122903	+01
3.10	+652959	+00	6.15	+107137	+01	9.15	+123086	+01
3.15	+664313	+00	6.20	+107521	+01	9.20	+123267	+01
3.20	+675474	+00	6.25	+107899	+01	9.25	+123446	+01
3.25	+686439	+00	6.30	+108272	+01	9.30	+123624	+01
3.30	+697208	+00	6.35	+108640	+01	9.35	+123799	+01
3.35	+707781	+00	6.40	+109002	+01	9.40	+123973	+01
3.40	+718158	+00	6.45	+109359	+01	9.45	+124144	+01
3.45	+728342	+00	6.50	+109710	+01	9.50	+124314	+01
3.50	+738334	+00	6.55	+110057	+01	9.55	+124483	+01
3.55	+748136	+00	6.60	+110399	+01	9.60	+124649	+01
3.60	+757750	+00	6.65	+110736	+01	9.65	+124814	+01
3.65	+767180	+00	6.70	+111068	+01	9.70	+124978	+01
3.70	+776428	+00	6.75	+111396	+01	9.75	+125139	+01
3.75	+785498	+00	6.80	+111719	+01	9.80	+125299	+01
3.80	+794392	+00	6.85	+112038	+01	9.85	+125458	+01
3.85	+803114	+00	6.90	+112352	+01	9.90	+125615	+01
3.90	+811668	+00	6.95	+112662	+01	9.95	+125770	+01
3.95	+820057	+00	7.00	+112968	+01	10.00	+125924	+01
4.00	+828284	+00						

TABLE A11A

$\langle p \rangle_{rel}$ for $ka = 100\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 2.5\ cm$ $\lambda = 0.05\ cm$ $0.625 \leq z \leq 125\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$	$z\lambda/a^2$	$\langle p \rangle_{rel}$
.005	+982022 +00	.340	+880100 +00	.675	+838082 +00
.010	+976926 +00	.345	+879028 +00	.680	+838167 +00
.015	+972374 +00	.350	+877350 +00	.685	+838151 +00
.020	+968476 +00	.355	+875304 +00	.690	+838028 +00
.025	+964822 +00	.360	+873198 +00	.695	+837792 +00
.030	+961786 +00	.365	+871329 +00	.700	+837441 +00
.035	+958608 +00	.370	+869919 +00	.705	+836975 +00
.040	+955818 +00	.375	+869076 +00	.710	+836395 +00
.045	+953475 +00	.380	+868791 +00	.715	+835705 +00
.050	+950989 +00	.385	+868951 +00	.720	+834908 +00
.055	+948443 +00	.390	+869376 +00	.725	+834010 +00
.060	+946215 +00	.395	+869857 +00	.730	+833020 +00
.065	+944051 +00	.400	+870196 +00	.735	+831944 +00
.070	+942085 +00	.405	+870234 +00	.740	+830793 +00
.075	+940464 +00	.410	+869869 +00	.745	+829575 +00
.080	+938310 +00	.415	+869063 +00	.750	+828300 +00
.085	+936205 +00	.420	+867840 +00	.755	+826978 +00
.090	+934563 +00	.425	+866272 +00	.760	+825621 +00
.095	+932882 +00	.430	+864469 +00	.765	+824238 +00
.100	+931059 +00	.435	+862559 +00	.770	+822840 +00
.105	+929299 +00	.440	+860674 +00	.775	+821436 +00
.110	+928136 +00	.445	+858933 +00	.780	+820037 +00
.115	+927111 +00	.450	+857433 +00	.785	+818652 +00
.120	+924759 +00	.455	+856242 +00	.790	+817289 +00
.125	+923580 +00	.460	+855395 +00	.795	+815956 +00
.130	+922474 +00	.465	+854893 +00	.800	+814662 +00
.135	+920329 +00	.470	+854711 +00	.805	+813413 +00
.140	+919809 +00	.475	+854797 +00	.810	+812216 +00
.145	+917676 +00	.480	+855085 +00	.815	+811075 +00
.150	+916733 +00	.485	+855496 +00	.820	+809996 +00
.155	+915738 +00	.490	+855952 +00	.825	+808983 +00
.160	+913301 +00	.495	+856374 +00	.830	+808039 +00
.165	+913558 +00	.500	+856695 +00	.835	+807167 +00
.170	+911518 +00	.505	+856855 +00	.840	+806368 +00
.175	+909522 +00	.510	+856812 +00	.845	+805644 +00
.180	+909878 +00	.515	+856536 +00	.850	+804995 +00
.185	+908407 +00	.520	+856014 +00	.855	+804422 +00
.190	+905770 +00	.525	+855247 +00	.860	+803924 +00
.195	+905369 +00	.530	+854246 +00	.865	+803500 +00
.200	+905603 +00	.535	+853037 +00	.870	+803148 +00
.205	+903762 +00	.540	+851650 +00	.875	+802866 +00
.210	+901182 +00	.545	+850126 +00	.880	+802651 +00
.215	+900357 +00	.550	+848506 +00	.885	+802502 +00
.220	+900839 +00	.555	+846835 +00	.890	+802414 +00
.225	+900360 +00	.560	+845158 +00	.895	+802385 +00
.230	+898174 +00	.565	+843515 +00	.900	+802410 +00
.235	+895729 +00	.570	+841945 +00	.905	+802486 +00
.240	+894637 +00	.575	+840482 +00	.910	+802609 +00
.245	+894924 +00	.580	+839153 +00	.915	+802775 +00
.250	+895267 +00	.585	+837981 +00	.920	+802979 +00
.255	+894445 +00	.590	+836980 +00	.925	+803218 +00
.260	+892360 +00	.595	+836159 +00	.930	+803486 +00
.265	+889933 +00	.600	+835521 +00	.935	+803781 +00
.270	+888248 +00	.605	+835063 +00	.940	+804097 +00
.275	+887776 +00	.610	+834777 +00	.945	+804431 +00
.280	+888164 +00	.615	+834649 +00	.950	+804778 +00
.285	+888586 +00	.620	+834663 +00	.955	+805135 +00
.290	+888288 +00	.625	+834800 +00	.960	+805497 +00
.295	+886976 +00	.630	+835039 +00	.965	+805862 +00
.300	+884889 +00	.635	+835357 +00	.970	+806225 +00
.305	+882601 +00	.640	+835730 +00	.975	+806582 +00
.310	+880723 +00	.645	+836136 +00	.980	+806932 +00
.315	+879643 +00	.650	+836551 +00	.985	+807270 +00
.320	+879408 +00	.655	+836955 +00	.990	+807594 +00
.325	+879758 +00	.660	+837327 +00	.995	+807900 +00
.330	+880258 +00	.665	+837649 +00	1.000	+808187 +00
.335	+880474 +00	.670	+837905 +00		

TABLE A11B

$\langle p \rangle_{rel}$ for $ka = 100\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 2.5\ cm$ $\quad \lambda = 0.05\ cm$ $\quad 125 \leq z \leq 1250\ cm$

$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$		$z\lambda/a^2$	$\langle p \rangle_{rel}$	
1.00	+808187	+00	4.05	+635849	+00	7.05	+417126	+00
1.05	+809547	+00	4.10	+631090	+00	7.10	+414571	+00
1.10	+807409	+00	4.15	+626355	+00	7.15	+412045	+00
1.15	+801725	+00	4.20	+621646	+00	7.20	+409547	+00
1.20	+793285	+00	4.25	+616966	+00	7.25	+407076	+00
1.25	+783227	+00	4.30	+612318	+00	7.30	+404632	+00
1.30	+772698	+00	4.35	+607702	+00	7.35	+402215	+00
1.35	+762670	+00	4.40	+603120	+00	7.40	+399823	+00
1.40	+753849	+00	4.45	+598575	+00	7.45	+397458	+00
1.45	+746671	+00	4.50	+594066	+00	7.50	+395118	+00
1.50	+741324	+00	4.55	+589595	+00	7.55	+392804	+00
1.55	+737803	+00	4.60	+585163	+00	7.60	+390514	+00
1.60	+735966	+00	4.65	+580771	+00	7.65	+388249	+00
1.65	+735583	+00	4.70	+576420	+00	7.70	+386007	+00
1.70	+736380	+00	4.75	+572110	+00	7.75	+383790	+00
1.75	+738078	+00	4.80	+567841	+00	7.80	+381596	+00
1.80	+740408	+00	4.85	+563615	+00	7.85	+379425	+00
1.85	+743129	+00	4.90	+559430	+00	7.90	+377276	+00
1.90	+746034	+00	4.95	+555288	+00	7.95	+375151	+00
1.95	+748952	+00	5.00	+551189	+00	8.00	+373047	+00
2.00	+751744	+00	5.05	+547132	+00	8.05	+370965	+00
2.05	+754305	+00	5.10	+543119	+00	8.10	+368905	+00
2.10	+756555	+00	5.15	+539148	+00	8.15	+366866	+00
2.15	+758439	+00	5.20	+535220	+00	8.20	+364848	+00
2.20	+759921	+00	5.25	+531334	+00	8.25	+362851	+00
2.25	+760978	+00	5.30	+527491	+00	8.30	+360874	+00
2.30	+761603	+00	5.35	+523690	+00	8.35	+358917	+00
2.35	+761797	+00	5.40	+519931	+00	8.40	+356980	+00
2.40	+761566	+00	5.45	+516214	+00	8.45	+355062	+00
2.45	+760925	+00	5.50	+512539	+00	8.50	+353164	+00
2.50	+759891	+00	5.55	+508905	+00	8.55	+351285	+00
2.55	+758485	+00	5.60	+505311	+00	8.60	+349424	+00
2.60	+756728	+00	5.65	+501759	+00	8.65	+347582	+00
2.65	+754642	+00	5.70	+498246	+00	8.70	+345758	+00
2.70	+752251	+00	5.75	+494774	+00	8.75	+343953	+00
2.75	+749578	+00	5.80	+491341	+00	8.80	+342164	+00
2.80	+746645	+00	5.85	+487947	+00	8.85	+340394	+00
2.85	+743474	+00	5.90	+484592	+00	8.90	+338640	+00
2.90	+740086	+00	5.95	+481275	+00	8.95	+336904	+00
2.95	+736501	+00	6.00	+477996	+00	9.00	+335185	+00
3.00	+732739	+00	6.05	+474754	+00	9.05	+333482	+00
3.05	+728816	+00	6.10	+471550	+00	9.10	+331795	+00
3.10	+724750	+00	6.15	+468382	+00	9.15	+330124	+00
3.15	+720557	+00	6.20	+465250	+00	9.20	+328470	+00
3.20	+716252	+00	6.25	+462154	+00	9.25	+326831	+00
3.25	+711847	+00	6.30	+459093	+00	9.30	+325208	+00
3.30	+707356	+00	6.35	+456067	+00	9.35	+323599	+00
3.35	+702791	+00	6.40	+453076	+00	9.40	+322007	+00
3.40	+698163	+00	6.45	+450119	+00	9.45	+320428	+00
3.45	+693481	+00	6.50	+447195	+00	9.50	+318865	+00
3.50	+688755	+00	6.55	+444304	+00	9.55	+317316	+00
3.55	+683993	+00	6.60	+441446	+00	9.60	+315781	+00
3.60	+679203	+00	6.65	+438621	+00	9.65	+314261	+00
3.65	+674393	+00	6.70	+435827	+00	9.70	+312754	+00
3.70	+669569	+00	6.75	+433064	+00	9.75	+311262	+00
3.75	+664736	+00	6.80	+430333	+00	9.80	+309783	+00
3.80	+659901	+00	6.85	+427632	+00	9.85	+308317	+00
3.85	+655069	+00	6.90	+424961	+00	9.90	+306864	+00
3.90	+650244	+00	6.95	+422320	+00	9.95	+305425	+00
3.95	+645429	+00	7.00	+419708	+00	10.00	+303998	+00
4.00	+640630	+00						

TABLE A11C

$\langle\theta\rangle_{rel}$ for $ka = 100\pi$ and $0 < z\lambda/a^2 \leq 1$

$a = 2.5\ cm$ $\lambda = 0.05\ cm$ $0.625 \leq z \leq 125\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	$z\lambda/a^2$	$\langle\theta\rangle_{rel}$
.005	+184071 -01	.340	+148994 +00	.675	+215448 +00
.010	+239536 -01	.345	+151713 +00	.680	+217261 +00
.015	+287005 -01	.350	+154088 +00	.685	+219161 +00
.020	+331741 -01	.355	+155839 +00	.690	+221131 +00
.025	+369760 -01	.360	+156858 +00	.695	+223152 +00
.030	+405816 -01	.365	+157211 +00	.700	+225206 +00
.035	+439186 -01	.370	+157105 +00	.705	+227274 +00
.040	+471109 -01	.375	+156820 +00	.710	+229340 +00
.045	+500024 -01	.380	+156653 +00	.715	+231386 +00
.050	+527780 -01	.385	+156847 +00	.720	+233397 +00
.055	+557751 -01	.390	+157564 +00	.725	+235359 +00
.060	+580960 -01	.395	+158857 +00	.730	+237258 +00
.065	+606259 -01	.400	+160687 +00	.735	+239084 +00
.070	+634152 -01	.405	+162933 +00	.740	+240825 +00
.075	+653329 -01	.410	+165426 +00	.745	+242473 +00
.080	+681304 -01	.415	+167976 +00	.750	+244020 +00
.085	+699665 -01	.420	+170403 +00	.755	+245461 +00
.090	+718673 -01	.425	+172550 +00	.760	+246791 +00
.095	+739195 -01	.430	+174309 +00	.765	+248008 +00
.100	+760600 -01	.435	+175618 +00	.770	+249109 +00
.105	+784645 -01	.440	+176470 +00	.775	+250095 +00
.110	+808722 -01	.445	+176906 +00	.780	+250966 +00
.115	+822181 -01	.450	+177003 +00	.785	+251724 +00
.120	+837389 -01	.455	+176868 +00	.790	+252374 +00
.125	+866887 -01	.460	+176620 +00	.795	+252918 +00
.130	+873328 -01	.465	+176381 +00	.800	+253363 +00
.135	+902346 -01	.470	+176263 +00	.805	+253713 +00
.140	+909322 -01	.475	+176357 +00	.810	+253976 +00
.145	+939321 -01	.480	+176736 +00	.815	+254158 +00
.150	+941571 -01	.485	+177440 +00	.820	+254267 +00
.155	+974305 -01	.490	+178486 +00	.825	+254312 +00
.160	+980492 -01	.495	+179865 +00	.830	+254299 +00
.165	+995449 -01	.500	+181546 +00	.835	+254239 +00
.170	+102639 +00	.505	+183484 +00	.840	+254139 +00
.175	+102765 +00	.510	+185618 +00	.845	+254007 +00
.180	+104165 +00	.515	+187882 +00	.850	+253853 +00
.185	+107453 +00	.520	+190207 +00	.855	+253684 +00
.190	+108368 +00	.525	+192526 +00	.860	+253510 +00
.195	+108375 +00	.530	+194773 +00	.865	+253336 +00
.200	+110777 +00	.535	+196894 +00	.870	+253171 +00
.205	+113844 +00	.540	+198841 +00	.875	+253023 +00
.210	+114739 +00	.545	+200577 +00	.880	+252897 +00
.215	+114507 +00	.550	+202076 +00	.885	+252800 +00
.220	+115843 +00	.555	+203323 +00	.890	+252738 +00
.225	+118901 +00	.560	+204316 +00	.895	+252717 +00
.230	+121514 +00	.565	+205060 +00	.900	+252740 +00
.235	+122258 +00	.570	+205573 +00	.905	+252813 +00
.240	+121970 +00	.575	+205876 +00	.910	+252938 +00
.245	+122491 +00	.580	+206001 +00	.915	+253121 +00
.250	+124661 +00	.585	+205981 +00	.920	+253362 +00
.255	+127714 +00	.590	+205855 +00	.925	+253666 +00
.260	+130212 +00	.595	+205662 +00	.930	+254033 +00
.265	+131288 +00	.600	+205441 +00	.935	+254465 +00
.270	+131202 +00	.605	+205230 +00	.940	+254963 +00
.275	+130986 +00	.610	+205065 +00	.945	+255528 +00
.280	+131646 +00	.615	+204977 +00	.950	+256160 +00
.285	+133548 +00	.620	+204994 +00	.955	+256858 +00
.290	+136318 +00	.625	+205140 +00	.960	+257623 +00
.295	+139165 +00	.630	+205432 +00	.965	+258454 +00
.300	+141334 +00	.635	+205886 +00	.970	+259349 +00
.305	+142448 +00	.640	+206508 +00	.975	+260307 +00
.310	+142621 +00	.645	+207304 +00	.980	+261327 +00
.315	+142337 +00	.650	+208273 +00	.985	+262406 +00
.320	+142211 +00	.655	+209410 +00	.990	+263544 +00
.325	+142737 +00	.660	+210708 +00	.995	+264737 +00
.330	+144138 +00	.665	+212156 +00	1.000	+265983 +00
.335	+146328 +00	.670	+213741 +00		

TABLE A11D

$\langle\theta\rangle_{rel}$ for $ka = 100\pi$ and $1 \leq z\lambda/a^2 \leq 10$

$a = 2.5\ cm$ $\lambda = 0.05\ cm$ $125 \leq z \leq 1250\ cm$

$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$		$z\lambda/a^2$	$\langle\theta\rangle_{rel}$	
1.00	+265983	+00	4.05	+836346	+00	7.05	+113270	+01
1.05	+280814	+00	4.10	+844260	+00	7.10	+113568	+01
1.10	+298009	+00	4.15	+852024	+00	7.15	+113862	+01
1.15	+315044	+00	4.20	+859639	+00	7.20	+114152	+01
1.20	+330075	+00	4.25	+867110	+00	7.25	+114438	+01
1.25	+342044	+00	4.30	+874440	+00	7.30	+114720	+01
1.30	+350589	+00	4.35	+881632	+00	7.35	+114999	+01
1.35	+355879	+00	4.40	+888689	+00	7.40	+115275	+01
1.40	+358440	+00	4.45	+895615	+00	7.45	+115546	+01
1.45	+359002	+00	4.50	+902413	+00	7.50	+115815	+01
1.50	+358353	+00	4.55	+909086	+00	7.55	+116080	+01
1.55	+357243	+00	4.60	+915636	+00	7.60	+116341	+01
1.60	+356313	+00	4.65	+922067	+00	7.65	+116600	+01
1.65	+356060	+00	4.70	+928381	+00	7.70	+116855	+01
1.70	+356833	+00	4.75	+934581	+00	7.75	+117107	+01
1.75	+358845	+00	4.80	+940671	+00	7.80	+117356	+01
1.80	+362192	+00	4.85	+946652	+00	7.85	+117602	+01
1.85	+366885	+00	4.90	+952528	+00	7.90	+117845	+01
1.90	+372871	+00	4.95	+958300	+00	7.95	+118085	+01
1.95	+380057	+00	5.00	+963972	+00	8.00	+118322	+01
2.00	+388324	+00	5.05	+969545	+00	8.05	+118556	+01
2.05	+397544	+00	5.10	+975022	+00	8.10	+118788	+01
2.10	+407587	+00	5.15	+980405	+00	8.15	+119017	+01
2.15	+418327	+00	5.20	+985696	+00	8.20	+119243	+01
2.20	+429643	+00	5.25	+990898	+00	8.25	+119466	+01
2.25	+441429	+00	5.30	+996013	+00	8.30	+119687	+01
2.30	+453586	+00	5.35	+100104	+01	8.35	+119906	+01
2.35	+466027	+00	5.40	+100599	+01	8.40	+120122	+01
2.40	+478675	+00	5.45	+101085	+01	8.45	+120335	+01
2.45	+491464	+00	5.50	+101564	+01	8.50	+120546	+01
2.50	+504336	+00	5.55	+102035	+01	8.55	+120755	+01
2.55	+517240	+00	5.60	+102498	+01	8.60	+120961	+01
2.60	+530136	+00	5.65	+102953	+01	8.65	+121165	+01
2.65	+542987	+00	5.70	+103402	+01	8.70	+121367	+01
2.70	+555761	+00	5.75	+103843	+01	8.75	+121566	+01
2.75	+568435	+00	5.80	+104278	+01	8.80	+121763	+01
2.80	+580986	+00	5.85	+104705	+01	8.85	+121959	+01
2.85	+593398	+00	5.90	+105126	+01	8.90	+122152	+01
2.90	+605655	+00	5.95	+105541	+01	8.95	+122343	+01
2.95	+617746	+00	6.00	+105949	+01	9.00	+122532	+01
3.00	+629663	+00	6.05	+106351	+01	9.05	+122718	+01
3.05	+641397	+00	6.10	+106747	+01	9.10	+122903	+01
3.10	+652944	+00	6.15	+107137	+01	9.15	+123086	+01
3.15	+664299	+00	6.20	+107521	+01	9.20	+123267	+01
3.20	+675460	+00	6.25	+107899	+01	9.25	+123446	+01
3.25	+686426	+00	6.30	+108272	+01	9.30	+123624	+01
3.30	+697195	+00	6.35	+108640	+01	9.35	+123799	+01
3.35	+707769	+00	6.40	+109002	+01	9.40	+123973	+01
3.40	+718147	+00	6.45	+109359	+01	9.45	+124145	+01
3.45	+728331	+00	6.50	+109710	+01	9.50	+124315	+01
3.50	+738323	+00	6.55	+110057	+01	9.55	+124483	+01
3.55	+748125	+00	6.60	+110399	+01	9.60	+124650	+01
3.60	+757740	+00	6.65	+110736	+01	9.65	+124815	+01
3.65	+767170	+00	6.70	+111068	+01	9.70	+124978	+01
3.70	+776419	+00	6.75	+111396	+01	9.75	+125140	+01
3.75	+785489	+00	6.80	+111719	+01	9.80	+125300	+01
3.80	+794383	+00	6.85	+112038	+01	9.85	+125458	+01
3.85	+803106	+00	6.90	+112352	+01	9.90	+125615	+01
3.90	+811660	+00	6.95	+112662	+01	9.95	+125770	+01
3.95	+820049	+00	7.00	+112968	+01	10.00	+125924	+01
4.00	+828277	+00						

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A reduction in the size of such errors (which are not errors if corrected for) may be made by a judicious selection of a particular differential path, although certain intervals may result in an increased error.

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1. Sound — Propagation — Meas.

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