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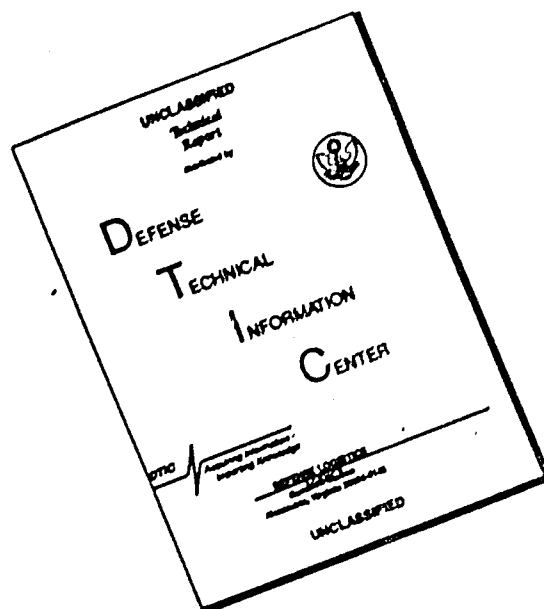
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Report No. 8391 LL 95

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STATISTICAL STUDIES
OF STABLE GROUND ROUGHNESS

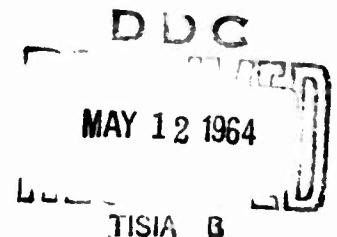
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A method of estimating statistical characteristics of stable ground roughness for use in off-road land locomotion is developed. Computer programs for obtaining these estimates from survey data have been prepared. Several estimated power spectral densities obtained from survey data taken at military installations are presented.

Pertinent features of these estimated power spectral densities are interpretable in terms of visual features of plotted ground profiles. The estimated power spectral densities may be approximated by similar and simple form.

OBJECT

Develop a statistical method of estimating characteristics of stable ground roughness for use in off-road land locomotion. Apply method to data obtained from surveys.

RESULTS

Computer programs have been constructed to estimate stable ground roughness characteristics. Several estimated power spectral densities are presented.

CONCLUSIONS

Power spectral density methods are useful for characterizing quantitatively stable ground roughness. Features of the estimated p.s.d.'s are interpretable in terms of visual features of plotted ground profiles. Estimated p.s.d.'s may be approximated by similar and simple form in cases considered.

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GLOSSARY

Stable ground roughness is the variation in elevation of hard ground whose influence on vehicle dynamics remains reasonably constant over distance, and is free of obstacles.

Statistical estimation is the technique of estimating average properties from sample data.

Power spectral density (p. s.d.) of a profile measures the amount of variation, by frequency bands, of the profile height.

Ground profile is the plot of survey height vs. distance.

SECTION I

INTRODUCTION

The Midwest Applied Science Corp. was asked by the Land Locomotion Laboratory, Detroit Arsenal, to identify and study the several areas which determine the speed of military vehicles under off-road hard ground conditions. Three areas have been identified as pertinent and are now under study; they are specification or description of stable ground roughness, comfort or tolerance limits of humans to vehicle oscillation, and dynamical characteristics of a military vehicle. Although results are usually presented separately, the separate studies in these areas must be synthesized into vehicle suspension system concepts which will permit increases in vehicle speeds.

This report is concerned only with the description of stable ground roughness.

The problem of describing the roughness of fluctuations in quantities which vary with time or distance is an old one. It has been and still is of interest in diverse areas such as physics, economics, mathematics, engineering, neurology, etc. Questions of the type: What is the composition of white light?, Will the stockmarket fall next week?, Is a function smooth?, Will noise in a telephone mask the message?, How rough is the road a car can travel safely at 60 m.p.h., etc., all relate to the roughness of quantities which change with time or distance.

Early ways of describing roughness are contained in the motion of bounded variation, magnitude of the Fourier coefficients of a periodic motion, power in frequency bands of light as determined by spectrographs, electrical power in frequency bands of fluctuating electrical current, etc. Some of these concepts are familiar to all of us.

That the designer of military vehicle suspension systems is vitally interested in ground roughness, or more particularly, stable ground roughness, follows from the substantial number of test courses at the various military installations around the country which are supposed to simulate "rough ground".

A military vehicle operating off roads encounters two types of hard ground roughness. One type consists of obstacles such as large rocks, tree stumps, stream beds, abrupt hills, etc. The other type consists of variations in elevation which are stable over reasonably large areas and change but gradually with distance. The latter type is what we have called stable ground roughness.

In general, obstacles require special driver attention dictated by the capabilities of the vehicle. What is an obstacle to one vehicle is not an obstacle to another.

Rapid traversability of the stable roughness depends upon the vehicle, suspension system, and cargo. Again, what is rough to one vehicle may not be rough to another vehicle or to the same vehicle at another speed. Thus, stable ground roughness has meaning in two contexts: in one, only geometry is involved; in the other, geometry coupled with vehicle-driver-cargo and speed is of importance. Hence, for the vehicle-driver-cargo system, it is impossible to identify the roughest terrain without knowledge of speed and system characteristics.

Environmental roughness exists and is now being studied for other than land vehicles. Descriptions of the roughness of the sea, atmospheric turbulence, and airport runways are needed to predict the vibrational state in ships and airplanes. Starts have been made in obtaining descriptions in terms of power spectral density and some of the preliminary results obtained from such descriptions appear useful to the designer. We note, of course, the lack of obstacles in air and sea travel, and the lack of long unimportant trends such as gradual hills. Here, as with land vehicles, what is rough to an airplane or ship depends upon speed and dynamical characteristics.

It is worth explicitly noting that there are no prior references to be cited in connection with statistical descriptions of stable off-road hard ground roughness.

OBJECT

The object of this report is to present statistical methods of describing or estimating quantitatively the characteristics of stable roughness of hard ground of interest in vehicle dynamics, and to present estimates of roughness obtained from survey data. Smoothing techniques must be developed to eliminate features of roughness unimportant in vehicle dynamics, such as long gradual hills. Further, the assumption that the underlying roughness is reasonably stable over lengths large in comparison with vehicle lengths must be verified.

RESULTS

Figures 1-4 present line profiles obtained from elevation surveys taken at the indicated places. Horizontal distance was measured with a tape; elevation was measured with level and rod.

In each of the first three figures, the complete profile is shown; each data point (represented by a dot) is shown, and the data points are two feet apart. Figure 4 shows every tenth data point and the data points are one foot apart. The vertical scale is substantially different from the horizontal; a twenty foot interval is marked to give an idea of the horizontal scale. In Figure 4, a complete section has been added at each end to indicate details.

The effect of ground roughness upon a vehicle is dependent on the vehicle's size and speed. It is obvious that in the context of today's military vehicles some of the variations in ground elevation in these figures are of such a scale as not to be of interest in vehicle dynamics. For example, in Figure 4, the profile has an upward sloping trends which indicates the ascent of a hill. Superimposed upon this hill are the variations in elevation or roughness which a vehicle will feel. In the other figures, such a striking trend is not present although there are large scale elevation variations which might be termed hills (in these figures) and are not of interest in this study. However, if one mentally eliminates these slowly varying elevation changes, one notes that the remaining variations or roughness patterns appear to be fairly constant. What is needed is a quantitative description of the roughness pattern.

Whatever the description adopted for roughness measurement, it is essential that it be usable by the suspension system designer. Moreover, it must classify as having the same or different roughness (two roads or fields classified by experience as having the same or different roughnesses). We shall, in this report, be concerned with the power spectral density description of ground roughness.

A complete description of power spectral density analysis of roughness on a line is given below for the interested reader. Briefly and loosely speaking, the power spectral density of an oscillating function or ground elevation along a line provides a measure of the power or intensity in frequency (cycles/ft.) bands; all oscillating functions having the same p. s. d. are classified as having the same roughness.

Figures 5 to 8 present the p. s. d. 's (power spectral densities) estimated from the profiles given in Figures 1 to 4, respectively. The ordinate has units of feet³ and the abscissa is frequency with unit feet⁻¹.

The general features of these p. s. d. 's are much the same. There is a sharp spike at the origin with a more or less rapid drop as frequency increases, and a long low tail. In two cases, Figure 5 and Figure 6, there is a bump in the rapid drop-off at approximately $\lambda = .025$ and $\lambda = .035$, respectively. The tails show irregularities at a very low level, in all cases below 0.30. Some of the irregularities in the tails are linked to inconsistencies in the profile data. However, the main features of interest in vehicle dynamics lie in the frequency range where the p. s. d. is descending to the tails.

Let us return to Figure 1. (Aberdeen) This profile shows no prominent hills. All elevations lie between ± 4 feet. This is more or less characteristic of the area within the Perryman Mud Course where the profile was taken. Qualitatively, the roughness seems to consist of changes in elevation of approximately 2 feet within distances of 20 to 30 feet. In the first third, there is a fairly periodic oscillation of ground elevation with a period of 30 to 40 feet; in the middle third, they are almost gone. One notices occasionally in the profile sharp irregularities in the elevation between adjacent neighboring points; these are due to ruts, holes, rocks, errors in data recording, etc.

The Aberdeen p.s.d., Figure 5, indicates just a hint of a bump at $\lambda = .025$, corresponding to the more or less regular oscillation of approximately 40 ft. wave length in the profile noted earlier. We see below that the p.s.d. of the first half has a much more pronounced bump at $\lambda = 0.025$ while the second half does not have this bump.

The profile from Ft. Knox, Figure 2, also does not exhibit any substantial long range variation, except after 1600 feet. As the profile was taken in bottom land bordering the Ohio River, this is reasonable. The roughness has amplitudes which are about half that observed in the Aberdeen profile. There is a pronounced oscillation between 400 ft. and 1600 ft. with a wave length of approximately 27 feet; this explains the presence of the bump in the corresponding p.s.d. estimate, Figure 6, at $\lambda = 0.035$. The minor irregularities in the profile are distributed through it as in the Aberdeen profile.

The profile taken at Yuma, Figure 3, exhibits major long range changes in elevation, varying between +12 feet and -10 feet. These are represented by hills of approximately 400 feet length; throughout the profile there are smaller hills of about 50 feet length; the pertinent roughness, from the point of view of vehicle dynamics, exhibits no significant periodicities. The scale of this latter roughness appears smaller than present in either Aberdeen or Knox. The p.s.d. estimate, in Figure 7, indicates a much higher value near the origin, corresponding to the smaller hills, and a sharper drop due to less roughness. The ordinate of the p.s.d. at $\lambda = 0.03$ is 1.0 whereas Aberdeen and Knox are substantially higher, confirming this observation.

All 6201 data points of the Battlefield Day profile (taken at Fort Knox) could not be shown on the main profile, the spacing between points shown is 10 ft.; consequently the relevant roughness is largely masked. Two sections at either end have been plotted completely to

give samples of the type of roughness met. The main profile exhibits one long rising trend with superimposed minor hills. The scale of the roughness of interest, indicated only in part in the detailed portions, appears to be smaller even than in Yuma. Rarely do these roughness variations exceed 1/4 feet. The p.s.d., Figure 8, drops faster than in the Yuma p.s.d., as would be expected. The fluctuation in the tail of this p.s.d., as well as its elevation, will be discussed below; essentially it is traceable to inconsistencies in the data.

CONCLUSIONS

Our main conclusion is that the principal problems of characterizing ground roughness by p. s. d. methods are behind us. We must now turn to the more difficult task of relating the measurements to the suspension system design.

The raw data for the one and two dimensional cases are available in IBM card decks. Anyone interested in these data may obtain them by writing to MASC. These data will also be tabulated and presented in a later report.

The following practical conclusions appear to be consistent with the results obtained to date:

One-dimensional

- a. Visual observed roughness characteristics of the profile are related to characteristics of the estimated p. s. d.
- b. The estimated p. s. d. 's may be approximated with reasonable accuracy by a relatively simple class of functions.
- c. Abnormal height and periodic oscillations in the tails of the p. s. d. estimates are traceable to inconsistencies in the survey data.
- d. Sections of ground which appear visually to have constant roughness characteristics satisfy reasonably well the stationarity assumption on intervals of 1000-2000 feet length.
- e. The normality assumption for the smoothed data is reasonably well satisfied although there is a consistent deviation from it.

- f. There may be considerable difficulty in removing unwanted hills (long range trends) from the data. Running average methods have proved satisfactory with present data.
- g. The one-dimensional p.s.d. estimate computer program is almost ready for distribution.

Two-dimensional

- h. P.s.d. estimates obtained to date, although not completely satisfactory, display general features observed in one-dimensional results.

In general, successes to date in obtaining p.s.d. estimates are quite gratifying. There are details, however, mostly related to smoothing and interpretation for two-dimensional p.s.d.'s, which must be worked out. We anticipate no substantial difficulties in cleaning up these details and arriving at programs available for distribution.

RECOMMENDATIONS

More surveys are necessary to provide p.s.d. estimates for a wider variety of ground types.

Smoothing procedures must be examined for their effects on characteristics relevant to vehicle dynamics.

Cross-correlation effects from the two-dimensional data must be ascertained.

Section II

One-dimensional Power Spectral Densities

1. Introduction, Interpretation of Spectra

The spectral analysis of random functions of one variable is the subject of a number of books and articles in the engineering literature. We will review in this chapter the principal ideas with emphasis on the descriptive aspects.

It is often the case that a random function is of interest as an input to a linear or almost linear system. Its effect in such a problem is conveniently separated into the sum of effects due to frequency components. The effect of terrain roughness on the vibration of a traveling vehicle is such a problem. As a first approach we will consider a single track vehicle. The profile heights on the path of the vehicle will be a function, $h(x)$, of the distance, x along the track. The roughness of the profile will be a matter of the variation of $h(x)$ about some smooth medial terrain consisting of the large hills etc. The problem of obtaining this smooth terrain is a practical one and is discussed in a later section. Let us denote the deviations of the profile height by $d(x)$. These will average zero and their variation is what we mean by roughness. As a drastic simplification of separation into frequency components, we might have

$$d(x) = \sum_{k=0}^M (a_k \sin \omega_k x + b_k \cos \omega_k x)^* . \quad (1.1)$$

It is very unrealistic to consider, as in (1.1), that only a few frequencies are present, we should admit all frequencies each in an infinitesimal amount. These ideas are familiar to most engineers as an intro-

*Generally we will distinguish between frequencies in radians per foot (ω) and cycles per foot (λ).

duction to the Fourier integral. Here, however, the fact of randomness complicates matters. If $d(x)$ with fixed frequencies is random, then it is the coefficients a_k, b_k that are random. How to deal with infinitesimal random quantities in the case of continuous frequencies, presents a special problem in probability theory which we cannot hope to deal with here. The unrealistic, fixed frequency, case, does "converge to" the realistic case and it is worthwhile to consider it in some detail.

If a function of the form (1.1) is used as an input to a linear system, the output also has this form

$$a(x) = \sum_K (a'_k \sin \omega_k x + b'_k \cos \omega_k x) \quad (1.2)$$

The coefficients a'_k, b'_k of the k 'th pair depend only on a_k, b_k of the k 'th pair of (1.1). The form of dependence is stated most clearly if we change the form of (1.1), (1.2)

$$a_n \sin \omega_n x + b_n \cos \omega_n x = A_n \cos(\omega_n x + \phi_n) \quad (1.3)$$

$$A_n = \sqrt{a_n^2 + b_n^2} \quad \phi_n = \arctan(a_n/b_n)$$

Then with the general term of (1.2) in the same form it may shown that

$$\begin{aligned} A'_n &= A_n P(\omega_n) \\ \phi'_n &= \phi_n + \Phi(\omega_n) \end{aligned} \quad (1.4)$$

The functions $P(\omega)$ and $\Phi(\omega)$ combine to form what is called the transfer function of the linear system. The phase of the input does not affect the amplitude of the output, nor does the amplitude of the input affect the phase of the output. In many problems the ultimate

effect does not depend on the phase of the output so that only A'_n and hence only A_n is of interest. The various A_n arranged according to the sizes of the ω_n form what is called the frequency spectrum of the input. Their sizes give the amounts of the frequencies present. For a continuum of frequencies this concept becomes that of a spectral density.

This is a quick account of the standard method of analyzing the effect of an input $d(x)$ on a linear system. We have not mentioned that $d(x)$ is random -- by random we mean that $d(x)$ may turn out to be any one of an "ensemble" of functions and we have no hope of determining which one, but we can determine or estimate some ensemble averages. We will assume that only the coefficients a_n, b_n are random in the expression (1.1). This is no restriction, for we can assume that all the possible frequencies are represented among the ω 's and that their absence or presence is a matter of which coefficients are or are not zero.

It will frequently be convenient to use the exponential form of the trigonometric series (1.1). If the ω_k are given for $k=0, 1, 2, \dots, M$ and $\omega_0=0$) this is

$$d(x) = \sum_{k=-M}^M \alpha_k e^{i\omega_k x}$$

where

$$\alpha_k = \frac{1}{2} (b_k - i a_k) \quad k > 0 \quad (1.5)$$

$$\alpha_{-k} = \alpha_k^* = \frac{1}{2} (b_k + i a_k) \quad k > 0$$

$$\alpha_0 = b_0$$

Complex quantities appear on the right, but $d(x)$ is real.

We assume two main properties for the randomness of $d(x)$. The first is that the ensemble average at any point x is zero. From the linear property of the expected value (ensemble average) operation,

$$E \{ d(x) \} = \sum_k E \{ a_k \} e^{i\omega_k x} = 0 \quad (1.6)$$

The series above, being identically zero as a function of x , must have zero coefficients, i.e.

$$E \{ a_k \} = \frac{1}{2} (E \{ b_k \} \pm i E \{ a_k \}) = 0$$

$$E \{ a_k \} = E \{ b_k \} = 0$$

Our second assumption is that of stationarity. A consequence of the assumption is that any expected value concerning the d 's must remain unchanged for different placements of the origin of x . In particular the following does not depend on x :

$$\begin{aligned} E \{ d(x+s) d(x) \} &= E \left\{ \sum_k \sum_j a_k a_j^* e^{i\omega_k(x+s) - i\omega_j x} \right\} \\ &= \sum_k \sum_j E \{ a_k a_j^* \} e^{i(\omega_k - \omega_j)x} e^{i\omega_k s} \end{aligned}$$

In this expression we have made use of the fact that $d(x) = d^*(x)$ for convenience. The series on the right, which must be identically constant for all x , falls into two parts,

$$\sum_k E \{ a_k a_k^* \} e^{i\omega_k s} + \sum_{k \neq j} E \{ a_k a_j^* \} e^{i(\omega_k - \omega_j)x} e^{i\omega_k s}$$

If the ω_k are all different, which we may assume from the start, then the second series must be identically constant as a function of x hence its coefficients must be zero;

$$E \{ \alpha_k \alpha_j^* \} = 0 \quad k \neq j$$

Thus if $d(x)$ is to be stationary, it is necessary that the complex coefficients be mutually uncorrelated. In this case

$$E \{ d(x+s) d(x) \} = R(s) = \sum_k E \{ \alpha_k \alpha_k^* \} e^{i\omega_k s} \quad (1.7)$$

$R(s)$ is called the covariance function of $d(x)$. From (1.3)

$$|\alpha_k|^2 = \alpha_k \alpha_k^* = \frac{1}{4} (a_k^2 + b_k^2) = \frac{1}{4} A_k^2 \quad (\text{taking } A_{-k} = A_k)$$

$$|\alpha_0|^2 = \alpha_0^2 = A_0^2$$

This gives the important relation

$$R(s) = E \{ A_0^2 \} + \sum_{\substack{k=-M \\ \neq 0}}^M E \left\{ \frac{1}{4} A_k^2 \right\} e^{i\omega_k s} \quad (1.8)$$

which links the covariance function with the spectrum. Note that the components are all in phase, in fact we can write

$$R(s) = E \{ A_0^2 \} + \frac{1}{2} \sum_{k=1}^M E \{ A_k^2 \} \cos \omega_k s \quad (1.8')$$

Therefore $R(0)$ is a maximum (positive) value of $R(s)$. The coefficients in the trigonometric series (1.8) are ensemble averages and are possible to know or to estimate. When they are arranged in order of the sizes of the ω_k , they form what is called the power spectrum of the random function $d(x)$. From (1.8) The power spectrum is the frequency spectrum of the covariance function.

To summarize what we have discussed to now, we will deal with random inputs to linear systems. The spectrum will characterize the effect. However, the spectrum is random and cannot be known except for its ensemble averages. The power spectrum is a collection of such averages. The different spectral values are uncorrelated. The power spectrum is the spectrum of the covariance function.

To interpret the power spectrum, let us imagine that the frequency values ω_k are very close together. We have from (1.8) that

$$R(0) = E \{A_0^2\} + \sum_{k=1}^M E \left\{ \frac{1}{2} A_k^2 \right\} \quad (1.9)$$

From (1.7)

$$R(0) = E \{d^2(x)\} \quad (1.10)$$

Assuming that $d(x)$ is approximately normal and has a mean of zero. $R(0)$ is its variance and the probability is about .003 that $d(x)$ will exceed in absolute value $3\sqrt{R(0)}$. Thus $R(0)$ is a measure of the approximate extremes of sizes in profile height. The usual values of $d(x)$ lie between $\pm 2\sqrt{R(0)}$. From (1.9) we see that the left side is limited by the range of oscillation of $d(x)$, so that the terms of the sum of the right side are also limited in size. If there are many frequencies, then most of them have small values for $E \{A_k^2\}$.

Equation (1.10) has another interpretation through the ergodic hypothesis which says that ensemble averages may be found as limits of time averages. In particular

$$\lim_T \frac{1}{T} \int_0^T (d(x))^2 dx = E \{(d(x))^2\} = R(0)$$

The left side is a measure of the average variability of $d(x)$ over a long stretch. By equation (1.9) this is divided into averages of uncorrelated components each of which has a separate effect on the linear system. In our work we will usually be interested more in frequency bands, and it is natural to consider that the total variance can be partitioned into partial variances due to frequency bands. In fact, suppose we have several random functions $d_1(x)$, $d_2(x)$, etc. each of which may be represented as a trigonometric series of form (1.5) with random coefficients. Since the frequencies are different we will assume the coefficients to be uncorrelated. Thus $d_1(x)$, $d_2(x)$, etc. are also uncorrelated so that the variance of the sum is the sum of the variances

$$\text{Var} \{d_1(x) + d_2(x) + \dots\} = R_1(0) + R_2(0) + \dots$$

Calling the sum $d(x)$, we have

$$* \quad \text{Var} \{d(x)\} = \sum_1 E \left\{ \frac{1}{2} A_k^2 \right\} + \sum_2 E \left\{ \frac{1}{2} A_k^2 \right\} + \dots \quad (1.12)$$

which shows that the variance of $d(x)$ can be partitioned into the sums of variances of various frequency components each of which may be thought of as a separate random function the sum of which is $d(x)$ itself.

Sometimes a power spectrum has a bump which we can "isolate" by sketching a "normal" spectrum on which it is superimposed. The power in the bump is the added area over the normal

* Σ_1 , for example, means sum over those ω_k 's which are frequency components of $d_1(x)$, etc..

spectrum. We may visualize this power as being manifested approximately by a sine wave of the frequency at the center of the bump and amplitudes which may change but which average about the square root of twice the area. With pronounced bumps such oscillations are often visible in the profile data.

2. Relation of the theory to the estimation problem.

In estimating spectral values from profile data we will be motivated principally by two relations. The first is (1.8) which links the power spectrum to the covariance function, and the second is another aspect of the ergodic hypothesis (1.11),

$$R(s) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [d(x+s) d(x)] dx \quad (2.1)$$

These equations suggest that the values of $R(s)$ might be estimated by taking mean lagged products of the data, and then the values of the power spectrum might be gotten by determining the Fourier coefficients in the expansion of $R(s)$. Before going into details we must make some things definite that have, to now, been left vague.

We will assume a continuous power spectrum given by a spectral density function so that (1.8) becomes

$$R(s) = \int_{-\infty}^{\infty} f(\lambda) e^{2\pi i \lambda s} d\lambda = \int_{-\infty}^{\infty} f(\lambda) \cos 2\pi \lambda s d\lambda \quad (2.2)$$

$$(f(-\lambda) = f(\lambda)).$$

The quantity $f(\omega_k)d\lambda$ may be associated with the value of the sum of $E\{\frac{1}{4} A_n^2\}$ for those ω_n within a band of width $d\lambda$ about ω_k . The dimension of the values of $f(\lambda)$ is (distance)³ since $d\lambda$ is in cycles per foot or (dist.)⁻¹ and A^2 is (dist.)².

We must also take into account the form of the data that we will have to work with. This will be n values of an outcome of $d(x)$ taken at evenly spaced intervals:

$$d(x_0 + k\Delta), k = 0, 1, 2, \dots, n-1.$$

We shall define a mean lagged product as

$$r_j = \begin{cases} \frac{1}{n-j} \sum_{k=0}^{n-j-1} d(x_0 + k\Delta + j\Delta) d(x_0 + k\Delta) & \text{if } j \geq 0 \\ \frac{1}{n-|j|} \sum_{k=j}^{n-1} d(x_0 + k\Delta + j\Delta) d(x_0 + k\Delta) & \text{if } j < 0 \end{cases}$$

then

$$r_j = r_{-j}$$

It is apparent from the definition of $R(s)$, (1.7) that

$$E \{r_j\} = R(j\Delta)$$

By the ergodic hypothesis, the probability limit, as $n \rightarrow \infty$ of r_j is $R(j\Delta)$. It is also evident that we cannot hope to estimate any values of the covariance function other than $R(j\Delta)$ and that this limitation is imposed by the fact that we have only observed evenly spaced data. Our data is also limited to a finite number of values so that the number of covariance values is also limited. We should limit the number of lags so that the number in terms $(n - |j|)$ in the mean (2.3) is sufficient to make it a good estimate. Let us denote the maximum lag by m : r_j is calculated for $j=0, \pm 1, \pm 2, \dots, \pm m$.

The above indicates how we shall estimate the covariance function. The determination of its Fourier coefficients is accomplished by taking a linear combination of its values with cosine co-

efficients. We will investigate the properties of linear combinations of the r_j 's with any sort of coefficients w_j

$$\tilde{W} = \sum_{j=-m}^m r_j w_j \quad (2.6)$$

The expected value of this estimate is

$$\begin{aligned} E \{ \tilde{W} \} &= \sum_{j=-m}^m R(j\Delta) w_j \\ &= \sum_{j=-m}^m \int_{-\infty}^{\infty} f(\lambda) w_j \cos 2\pi \lambda j \Delta d\lambda \\ &= \int_{-\infty}^{\infty} f(\lambda) \left[\sum w_j \cos 2\pi \lambda j \Delta \right] d\lambda \\ &= \int f(\lambda) W(\lambda) d\lambda \end{aligned} \quad (2.7)$$

where

$$W(\lambda) = \sum_{j=-m}^m w_j \cos 2\pi \lambda j \Delta \quad (2.8)$$

Ideally if in (2.7) $E \{ \tilde{W} \}$ were to be the value $f(\lambda_0)$ then $W(\lambda)$ should be a Dirac impulse or "delta" function centered at λ_0 . However $W(\lambda)$ is a finite Fourier series and cannot be a Dirac Function --in fact it is periodic with a period of $1/\Delta$ and cannot be zero except at isolated points.

The function $W(\lambda)$ of (2.8) is called the "spectral window". The estimates $\tilde{W}$ obtained by linear combinations of the covariance estimates are not estimates of the values of the power spectrum, but of averages of those values of $f(\lambda)$ admitted by the window as in (2.7). By making $W(\lambda)$ have a narrow peak of unit area near λ_0 and very low values elsewhere we will estimate very nearly the power over an interval of frequencies near λ_0 . This corresponds to one of the sums in the partition (1.12) of the total power.

The properties of $W(\lambda)$ arising from its form as a finite Fourier series (2.8) give several limitations of the linear estimate.

From the fact that $W(\lambda)$ is even and periodic, $W(\lambda_0) = W(1/\Delta - \lambda_0)$. This means that while trying to estimate $f(\lambda_0)$ we will unavoidably pick up what power there is in $f(\lambda)$ around $1/\Delta - \lambda_0$ since there is also a peak in the window around this point. All the power beyond the frequency $1/2\Delta$ must unavoidably be confused with power in the range $0, 1/2\Delta$. There is also another peak at $-\lambda_0$ which picks up the power around $f(-\lambda_0)$, but since the spectral density function is even, this is not a bad feature; it means, however, the $E\{\tilde{W}\}$ approximates $f(\lambda_0) + f(-\lambda_0)$ unless the total area of the two peaks is unity. The window has an infinity of other peaks, all of them contributing spurious power to the estimate, in the same manner as the peak at $1/\Delta - \lambda_0$. These spurious contributions are called "aliasing", and their exact nature depends on the sampling interval Δ . In designing the sampling method, Δ must be chosen so that the total power in the frequencies beyond $1/2\Delta$ is not appreciable.

From the fact that $W(\lambda)$ can be zero only at isolated points (actually $z^m W(\frac{1}{2\pi i \Delta} \log z)$ is a polynomial in z of degree $2m + 1$ and cannot have more than $2m + 1$ zeros) it follows that in trying to estimate $f(\lambda_0)$ we will unavoidably pick up some power in $f(\lambda)$ near the non-zero parts of $W(\lambda)$. If the spectrum has a high spot at one of the off center non-zero places of $W(\lambda)$ the power picked up may be considerable. If $W(\lambda)$ is negative, this spurious power is subtracted from the total and may make the expected spectral estimate go negative. These non-zero off center parts of the window are called "side lobes". To make them small one must at the same time make the peak broader so that $E\{\tilde{W}\}$ becomes an average of a wider band of frequencies. The band width is inversely proportional to m and, though the exact definition of band width is not fixed (the window cannot be a rectangle) it is usually taken to be about $1/2m\Delta$. In this way the size of the sample is related, through the maximum lag number, m , to the resolution or band width of the estimate.

These limitations on the exactness of the estimate are not of a statistical nature. The statistical errors are random deviations of the estimate $\tilde{W}$ from its expected value $E\{\tilde{W}\}$ (2.6), (2.7). What we have considered so far are properties of $E\{\tilde{W}\}$; it is an average of spectral power over a band of frequencies with unavoidable contamination from aliasing and from the side lobes of the window. Considerable attention has been paid to the shape of the window (see, for example, Blackman and Tukey, "The Measurement of the Power Spectra", Dover 1958, Parzen, "Mathematical Considerations in the Estimation of Spectra", Technometrics Vol. 3, No. 2, May 1961, 167-190). Most of the considerations in the selection of a window are more a matter of art than of hard-eyed analysis, but some of the aspects of calculation are based on simple mathematical identities which we will now present.

The first window that comes to mind is the finite Fourier series whose coefficients are the first m Fourier coefficients of the Dirac function centered at λ_0 . In terms of (2.8)

$$\bar{w}_j = \Delta \int_{-\frac{1}{2\Delta}}^{\frac{1}{2\Delta}} \delta(\lambda - \lambda_0) \cos 2\pi \lambda j \Delta d\lambda = \Delta \cos 2\pi \lambda_0 j \Delta \quad (2.9)$$

which gives the window function

$$\bar{W}(\lambda) = \Delta \sum_{-m}^m \cos 2\pi \lambda_0 j \Delta \cos 2\pi \lambda j \Delta \quad (2.10)$$

This sum may be expressed in closed form by writing the cosines in exponential form and summing the resulting geometric series, or by using the formula $\sum_{-m}^m \cos j\theta = \sin(m+1/2)\theta / \sin \theta/2$:

$$\bar{W}(\lambda) = \frac{\Delta}{2} \left(\frac{\sin \pi(2m+1)(\lambda + \lambda_0)\Delta}{\sin \pi(\lambda + \lambda_0)\Delta} + \frac{\sin \pi(2m+1)(\lambda - \lambda_0)\Delta}{\sin \pi(\lambda - \lambda_0)\Delta} \right) \quad (2.11)$$

This function has high, sharp peaks at λ_0 and $-\lambda_0$ as well as the aliasing peaks at $\pm\lambda_0 + k/\Delta$. It also has large side lobes including negative ones (Graphs may be seen in Blackman and Tukey, referred to above, or in Jenkins, "General Considerations in the Analysis of Spectra, Technometrics Vol. 3, No. 2, May 1961, 133-166). Both of these are bad features of this window.--We have already discussed the side lobes, the narrowness has the effect of increasing the statistical errors as we shall see later. Integrating (2.10) term by term over $(-1/2\Delta, 1/2\Delta)$, all but the 0 term vanishes and the result is unity. This means that the two peaks of $\bar{W}$ in this interval have a total area of unity and for $\lambda = 0$ the single peak has unit area. Thus $\bar{W}$ always estimates an approximation to $f(\lambda_0)$.

All of the spectral windows presented in the literature are modifications of this simple window--actually of these simple windows since there are different ones for each λ_0 . The modification does not depend on λ_0 and consists in multiplying the coefficients $\bar{w}_j$ of the simple window (2.9) by constants c_j , ($c_{-j}=c_j$). These may be applied to the covariances r_j before the calculations (2.6) are made with the simple window coefficients so that the simple windows for each λ_0 are all applied to the same modified covariances. The modified window function may be calculated in terms of the simple window. We will use the notation $\bar{w}$ and $\bar{W}$ to refer only to the simple window defined by (2.9) and (2.10). We will express (2.10) in exponential form:

Let us denote the cosine transform of the constants c_k by

$$C(\lambda) = \sum_{-m}^m c_k \cos 2\pi \lambda k \Delta = \sum_{-m}^m c_k e^{2\pi i \lambda k \Delta}$$

Then the convolution of C and $\bar{W}$,

$$\begin{aligned} W(\lambda) &= \Delta \int_{-\frac{1}{2\Delta}}^{\frac{1}{2\Delta}} \bar{W}(\omega) C(\lambda - \omega) d\omega \\ &= \Delta \sum_j \sum_k \bar{w}_j c_k e^{2\pi i \lambda k \Delta} \int_{-\frac{1}{2\Delta}}^{\frac{1}{2\Delta}} e^{2\pi i (j-k)\omega \Delta} d\omega \\ &= \sum_k c_k \bar{w}_k e^{2\pi i \lambda k \Delta} \end{aligned}$$

is the modified window. This equation allows $W(\lambda)$ to be calculated directly in terms of the simple window. It also furnishes a basis for selection of $C(\lambda)$ or c_k to attain a desired $W(\lambda)$. We have shown, however, that we do not have complete freedom in selecting a window, and (2.12) can only be a guide in these matters.

The modified windows in usual use are wider and have smaller side lobes than does the simple window. Graphs of some of them may be seen in Blackmand and Tukey, or in Jenkins (Dover, and Technometrics, v. 3, 2 referred to above). In general they compare to an ideal rectange centered at λ_0 and of width $1/2m \Delta$. If this ideal were the case and we strung out such rectangles without overlapping we could put about $2m + 1$ in the interval $(-1/2\Delta, 1/2\Delta)$. Each rectangular window would "view" a separate part of the power spectrum and give estimates of $f(\lambda)$ at $2m + 1$ evenly spaced points in the interval where it is to be estimated. If we take

the spacing of the points and the width of the rectangles to be $1/(2m+1)$ then the rectangles are somewhat narrower but they do not go outside the interval where $f(\lambda)$ is to be estimated. Let us use the notation (see (2.6) and (2.9))

$$f_k = \Delta \sum_{-m}^m r_j \cos 2\pi (k\delta) j \Delta = \Delta \sum_{-m}^m r_j \cos 2\pi \frac{kj}{2m+1} \quad (2.13)$$

for the spectral estimates at evenly spaced points using the simple window. Then by using either the exponential form of this series or the formula cited in connection with (2.10), we have the inverse relation

$$r_j = \delta \sum_{-m}^m f_k \cos 2\pi \frac{kj}{2m+1} \quad (2.14)$$

The values f_k are sometimes called the "raw spectrum". The modified window gives the spectral estimates

$$\hat{f}_k = \sum_{-m}^m r_j c_j \cos 2\pi \frac{jk}{2m+1} \quad (2.15)$$

The convolution theorem gives (taking f_k as periodic and even)

$$\hat{f}_k = \sum_{-m}^m f_{k-j} C(j\delta) \quad (2.16)$$

where $C(j\delta)$ is given by (2.11):

$$C(j\delta) = \sum_{-m}^m c_k \cos 2\pi \frac{jk}{2m+1} \quad (2.17)$$

With evenly spaced spectral estimates, therefore, the modification of the window may be done by a running average smoothing of the raw spectrum, using as coefficients the finite Fourier transform of the c_j 's.

The inverse relation (2.13), (2.14) and the convolution relation (2.15), (2.16), (2.17) are to be used later in a discussion of filtering.

In this section we have discussed the limitations imposed by the finiteness and discreteness of the data on what we must try to estimate. We shall go on now to discuss the random deviations from this.

3. Statistical problem

We have been considering the character of the expected value of linear spectral estimates. The actual outcome of the estimates is random, the ultimate randomness being in the smoothed profile data itself. We will assume that this is approximately Gaussian. Evidence in support of this assumption will be found in the following section. We discuss here the main ideas of the statistical problem without giving detailed proofs. These may be found in Blackman and Tukey (Dover, cited above) or in Grenander and Rosenblatt (Statistical Analysis of Stationary Time Series, Wiley 1957). In Chapter 3 of this report, the section corresponding to this one has similar proofs worked out for the two dimensional case.

The estimate $\tilde{W}$ (2.6) considered as a function of the n data points, is a homogeneous quadratic polynomial (a quadratic form). In practice such polynomials in Gaussian random variables are found to have distributions that are well approximated by a gamma distribution, i.e.

$$P \{ \tilde{W} < z \} = \int_0^z \frac{A^k}{\Gamma(k)} t^{k-1} e^{-At} dt \quad (3.1)$$

For purposes of using tabulated values it is convenient to recognize this as a generalization of a chi-squared of χ^2 -distribution. In fact (3.1) would be stated equivalently as, $\tilde{W}/2A$ has a χ^2 distribution with $2k$ degrees of freedom, except that $2k$ need not be an integer.

The gamma distribution is fitted by the method of moments, that is the values of A and k are chosen so that the first two moments of the gamma distribution match those of W .

$$4Ak = E\{\tilde{W}\}$$

$$16A^2k = \text{Var}\{\tilde{W}\}$$

from which

$$k \frac{[E\{\tilde{W}\}]^2}{\text{Var}\{\tilde{W}\}} \quad A = \frac{E\{\tilde{W}\}}{4k} \quad (3.2)$$

Using a χ^2 table we may find a 95% confidence interval for $E\{\tilde{W}\}$ by getting numbers L and R such that

$$P\{L < \chi^2 < R\} = .95$$

Then since $\tilde{W}/2A = 2kW/E\{\tilde{W}\}$ has a χ^2 distribution

$$P\left\{\frac{2k}{R} \tilde{W} < E\{\tilde{W}\} < \frac{2k}{L} \tilde{W}\right\} = .95$$

The factors $2k/R$ and $2k/L$ give the ends of a confidence interval for the random deviations of the observed W from the expected W . A short table will illustrate how the width of the interval depends on the degrees of freedom, $2k$.

2k = deg. f.d.m.	$\frac{2k}{R}$	$\frac{2k}{L}$
5	.39	6.02
10	.495	3.08
15	.546	2.40
20	.622	2.09
25	.615	1.91
30	.640	1.79
40	.675	1.64
50	.700	1.54
60	.720	1.48

The degrees of freedom, $2k$, measure the amount of statistical error. Various equivalent measures of statistical error are listed in the paper by Jenkins cited above (Technometrics v. 3, No. 2). They all depend ultimately on the first two moments of $\tilde{W}$. The first moment of $\tilde{W}$ is given in equation (2.7) which we repeat now

$$E \{ \tilde{W} \} = \int_{-\infty}^{\infty} f(\lambda) W(\lambda) d\lambda \quad (2.7 \text{ rep})$$

The variance of $\tilde{W}$ may be approximated in a similar form by making use of the assumption of approximate normality.

$$\text{Var} \{ \tilde{W} \} = \frac{1}{\Delta(n-m)} \int_{-\infty}^{\infty} f^2(\lambda) W^2(\lambda) d\lambda \quad (3.3)$$

The proof of this formula, which is long and tedious, may be found in Blackman and Tukey (Dover 1958, cited above) and will be given in the next chapter for the two dimension case. With these two formulas, the degrees of freedom formula (3.2) becomes

$$2k = 2\Delta(n-m) \frac{\left[\int f(\lambda) W(\lambda) d\lambda \right]^2}{\int f^2(\lambda) W^2(\lambda) d\lambda}$$

If the power spectral density function, $f(\lambda)$, does not have a steep slope in the region where the window is large, and if it vanishes substantially outside the interval $(-1/2\Delta, 1/2\Delta)$ then the integrals may be approximated by taking an average value of $f(\lambda)$ out and these average values will approximately cancel giving

$$2k = (n-m) \frac{\left[\int_{-\frac{1}{2\Delta}}^{\frac{1}{2\Delta}} W(\lambda) d\lambda \right]^2}{\int_{-\frac{1}{2\Delta}}^{\frac{1}{2\Delta}} W^2(\lambda) d\lambda}$$

The numerator of this should be unity for a properly normalized window function, so the size of the denominator gives the dependence of the statistical error on the window. Generally it is larger for the sharper windows. For an ideal window, a rectangle of width $1/2m\Delta$ and height $m\Delta$ (to make a unit area for the two peaks; for the window at $\lambda=0$ where the two peaks coincide, the height should be $2m\Delta$.) we have a final approximation

$$2k = 2\Delta(n-m) \frac{1}{\frac{2m^2\Delta^2}{2\Delta^2}} = 2 \frac{n-m}{m}$$

This approximation is on the optimistic or large side but it has the advantage of simplicity.

The magnitude of the statistical error may be indicated by confidence intervals. These may be found from the degrees of freedom as in the table given in this section. From the table one can see that precision increases with an increase in degrees of freedom. Formula (3.6) suggests that degrees of freedom increases as the ratio $g = m/n$ decreases. To decrease this we must either raise the number, n , of observations or lower the number, m , of lags. The former almost always increases the expense of the investigation, and the latter decreases the resolution of the spectral estimates.

4. Smoothing the Profile

In the first section we immediately passed from actual heights $h(x)$ to differences $d(x)$ of the actual heights from smoothed heights, eqn. (1.1). We postponed discussion of this until the present section so that we could relate this discussion to the ideas of the intervening sections. Our mathematical assumption concerning the profile of ground heights, is that it is stationary--of the same character throughout the region of measurement. In most areas this property is clearly untrue for any possible region, though it might be "made true" for suitable regions if the hills could be flattened. In their turn the hills might appear as roughness in very large regions in which general statewide level changes were removed and so on. Our problem is to carry out the investigation on a scale relevant to our purpose.

The first part of limiting the scale of the investigation is to limit the extent of the region to be measured so that the roughness--other than "hills" is of a uniform character. The second part is to

remove the hills in some way. In the limited region they are not usually stationary. (In the Battlefield Day data the ground rises steadily so that the heights at the end are not nearly the same as those at the beginning.) They are also not relevant as vibration producing roughness. Nevertheless the variations in height due to these "hills" are ordinarily much larger than those variations we want to study. If we were to leave them in the data the spectrum would consist almost exclusively of a central value at zero frequency. The other values would be lost in the round-off of the computation.

We have used two methods of smoothing the profile. One method consists in constructing a smooth profile by fitting paraboloids to segments of the data. The second--so far in use only with the linear data--is a running average scheme.

The first method consists in building up a smoothed profile by fitting second degree polynomials $H(x)$ to the data points $h(x_i)$. Actually, several polynomials may be fitted each to a separate segment. We may imagine these to be graphed along with the data so that the data or actual ground profile cuts back and forth across the smooth parabolical curve. It is clear that a parabola can remove only one hill from each segment so that in a profile with several hills there should be several segments fitted by parabolas. This was done, but it was found that the smoothed profiles obtained in this way had steps where the segments came together. In some of the data the steps, though few in number, were about ten times the values of the $d(x)$ values and an approximate analysis indicated that they could affect the resulting spectra. (See Appendix Y of this chapter.

The theoretical effect of parabolic smoothing is difficult to work out. Computations were made which indicated that this type of smoothing has several advantages over the running average type of smoothing. We have experienced some difficulties in the analysis of our data which suggest that parabolic smoothing works well when the parabolas fit closely but stronger smoothing is needed on hilly ground.

We have, therefore, done some calculations using another spectral program which has been equipped with a smoothing subroutine of the running average type. A theoretical discussion of the effects of this type of smoothing is, fortunately, very easy and we will present it for the linear case.

Let us denote the heights of the ground above points evenly spaced on a straight line of the datum plane by

$$h_{-a}, h_{-a+1}, \dots, h_0, h_1, \dots, h_n, h_{n+1}, \dots, h_{n+a} \quad (4.1)$$

We have added points on each end for a purpose which will be clear later. Running average smoothing requires that we select some coefficients

$$b_{-a}, b_{-a+1}, \dots, b_0, \dots, b_a \quad (4.2)$$

The smoothed values will be calculated by the formula

$$y_k = \sum_{\alpha=-a}^a b_{\alpha} h_{k+\alpha} \quad k = 1, 2, \dots, n \quad (4.3)$$

(Because of our foresight the numbers here run from 1 to n. Similar foresight in the computations, accomplished by faking in a few values at the beginning and end of the data accomplishes the same end.) The deviations used for spectral calculation and called "the smoothed profile" are:

$$d_k = h_k - y_k = h_k - \sum_{\alpha=-a}^a b_{\alpha} h_{k+\alpha} = \sum_{\alpha=-a}^a b'_{\alpha} h_{k+\alpha} \quad (4.4)$$

where

$$b'_{\alpha} = \begin{cases} -b_{\alpha} & \text{if } \alpha \neq 0 \\ 1-b_0 & \text{for } \alpha = 0 \end{cases}$$

To show what this type of smoothing can accomplish, let us suppose that the ground heights (5.1) are composed of a smooth "hilly" part P_k plus a rough part δ_k

$$h_k = P_k + \delta_k$$

Then

$$d_k = \sum_{\alpha} b'_{\alpha} P_{k+\alpha} + \sum_{\alpha} b'_{\alpha} \delta_{k+\alpha} \quad (4.5)$$

Now if the P_k are values of a polynomial on integer points

$$P_k = C_0 + C_1 k + C_2 k^2 + \dots + C_m k^m$$

Then the coefficients b (4.2) may be selected so that the first term of (4.5) is zero. Suppose, for example, the P_k is of 2nd degree. Then

$$\begin{aligned} \sum_{\alpha=-a}^a b'_\alpha P_{k+\alpha} &= \sum_{\alpha=-a}^a b'_\alpha [C_0 + C_1(k+\alpha) + C_2(k+\alpha)^2] \\ &= \sum_{\alpha} b'_\alpha [(C_0 + C_1 k + C_2 k^2) + \alpha (C_1 + 2C_2 k) + \alpha^2 C_2] \quad (4.6) \\ &= (C_0 + C_1 k + C_2 k^2) \sum_{\alpha} b'_\alpha + (C_1 + 2C_2 k) \sum_{\alpha} \alpha b'_\alpha + C_2 \sum_{\alpha} \alpha^2 b'_\alpha \end{aligned}$$

If we select the coefficients so that

$$\begin{aligned} \sum_{\alpha} b'_\alpha &= 0 \\ \sum_{\alpha} \alpha b'_\alpha &= 0 \\ \sum_{\alpha} \alpha^2 b'_\alpha &= 0 \end{aligned} \quad (4.7)$$

Then the right side of (4.6) will be zero. In this sense the running average smoothing can remove a parabolic trend. Note that the smoothing is very strong in that the polynomial P may be different at each point. Since (4.7) is a set of three homogenous linear equations with coefficients

$$\begin{aligned} &1, 1, \dots, 1, 1, 1, \dots, 1, 1 \\ &-a, -a+1, \dots, -1, 0, 1, \dots, a-1, a \\ &a^2, (a-1)^2, \dots, 1, 0, 1, \dots, (a-1)^2, a^2. \end{aligned}$$

For $a = 0$ or 1 the matrix is non singular so there is only the trivial--all zero--solution. Thus a must be at least 2 . The solutions are not unique and for larger a 's there is a large variety of them.

The removal of trend is not the only effect of this smoothing. In (4.5) the second term on the right is the smoothed time series whose spectrum we want. When we estimate the spectrum of the smoothed series we will not be estimating what we want. Suppose we denote

$$\gamma_k = \sum_{\alpha=-a}^a b'_\alpha \delta_{k+\alpha} \quad k = 1, 2, \dots, n$$

The covariance function for γ_k is

$$\begin{aligned} \tilde{r}_d &= E \{ \gamma_k \gamma_{k+d} \} = E \left\{ \sum_{\alpha=-a}^a b'_\alpha \delta_{k+\alpha} \sum_{\beta=-a}^a b'_\beta \delta_{k+d+\beta} \right\} \\ &= \sum_{\alpha} \sum_{\beta} b'_\alpha b'_\beta E (\delta_{k+\alpha} \delta_{k+d+\beta}) \\ &= \sum_{\alpha} \sum_{\beta} b'_\alpha b'_\beta r_{d+\beta+\alpha} \end{aligned}$$

where $r_a = E\{\delta_k \delta_{k+a}\}$ is the covariance function of the δ 's. If we change variables in the summation so that $\beta - \alpha = \mu$ and consider the b 's to be defined for all α , taking the value zero for $\alpha < -a$, $\alpha > a$ then the covariance function of the smoothed series may be written

$$\tilde{r}_d = \sum_{\mu} \left(\sum_{\alpha} b'_\alpha b'_{\mu+\alpha} \right) r_{d+\mu} \quad (4.8)$$

which is a running average smoothing of the γ 's using the coefficients

$$B_{\mu} = \sum_{\alpha} b'_{\alpha} b'_{\alpha+\mu} \quad (4.9)$$

A running average smoothing of the stationary part of a profile results in a profile whose covariances are a running average smoothing of the original covariances. The coefficients are the "covariances" of the coefficients used to smooth the profile.

The running average (4.8) is a convolution. If we rewrite the convolution relations (2.15), (2.16), (2.17) using the same conventions as to periodicity, we have

$$\tilde{r}_d = \sum_{j=-m}^m r_{d+j} B_j \quad (2.16 \text{ rep})$$

$$\tilde{r}_d = \sum_{j=-m}^m f_j \beta_j \cos 2\pi \frac{jk}{2m+1} \quad (2.15 \text{ rep})$$

where

$$\beta_j = \sum_{k=-m}^m B_k \cos 2\pi \frac{jk}{2m+1}$$

and f_j is the r 's given by (2.14).

A running average smoothing of the stationary part of a profile results in a profile whose spectral estimates are multiples of those of the original profile. The factors are the "power spectral estimates" of the smoothing coefficients. This is so since (2.15 rep) shows the spectrum of $\tilde{r}_d$ to be $f_j \beta_j$. If we calculate $\tilde{f}_j$, the spectral estimates of the smoothed data, then we may correct it using as factors $1 / \beta_j$, given by (2.17 rep).

Some work remains to be done on this type of smoothing that needs experience more than mathematical analysis. The smoothing coefficients, b' , which we have tried, have a spectrum which is zero at $\lambda = 0$, and very close to zero for low frequencies. The extent of the frequency range for which these spectral values are zero indicates the severity of smoothing by indicating how much of the stationary part of the profile is removed. Correcting may not give it back since it also exaggerates round-off error as well as the low spectral value. Therefore, some care must be exercised in choosing the b' so as to leave in the important part of the desired spectrum. We may choose from the variety of smoothing coefficients, available to us as solutions of equations such as (5.7), those which give a desirable spectrum--equivalently a desirable covariance function (5.8). In general there may be many solutions of (5.7) giving exactly the same covariances. Each of these will act the same according to our mathematical model; they will remove the appropriate polynomial trend and affect the spectrum of the stationary part in the prescribed way. What is the actual effect in practice depends on how our actual problem differs from this mathematical problem. This will require an investigation by experience rather than by logic.

The problem of finding coefficients that have a prescribed spectrum i.e., prescribed correction factors seems to be casier to solve by computation than by analysis. This is another area for experience. As yet, we have used only one set of coefficients so these matters are for future investigation.

Inasmuch as the coefficients b'_j may be chosen so as to make the spectrum f_k have smoother slopes, it will help in the estimation problem. There will be no sharp bumps that leak in

through the side bands and so on. Smoothing for this purpose has been called "prewhitening". Our first computer program has an elaborate prewhitening section in which the spectrum was first estimated and then coefficients were calculated to correct it. Our experience indicates that the main bad part of the spectrum occurs at zero frequency so that simpler methods of prewhitening are more effective.

5. The Computing Programs

We have used two computer programs to estimate line spectra. A complete description of the first program is no longer available. The general operation of that program is as follows:

- 1) The data is "smoothed" by taking deviations of the profile heights from heights of quadratic polynomials fitted to segments by least squares.

- 2) Covariances (mean lagged products) are computed.

- 3) A preliminary spectrum is calculated by making a cosine transform of the covariances and smoothing it by the Hamming coefficients.

- 4) A trigonometric polynomial of a given number of terms is fitted to the spectrum by least squares. The coefficients of this are used in a series of equations whose solutions are running average coefficients to be used on the profile data. This running average affects the spectrum as outlined in Section 4 and the computations are made so that it tends to flatten out steep places of the spectrum.

This is the "Prewhitening" referred to in Section 4.

- 5) The profile is smoothed by running average.

- 6) New covariances are computed and also their spectrum (cosine transform).

7) Correction coefficients for the prewhitening running average are computed.

8) A corrected spectrum is computed.

9) The corrected spectrum is smoothed by the Hamming coefficients.

This program recently became inoperable and we have replaced it. We have obtained a spectral program made for the IBM 7090. The complete details of the program are available. Generally it performs these operations:

1) Subtracts the average of the entire profile from each profile value.

2) Computes covariances of any desired segment of the profile data.

3) Computes the spectrum of the covariances using the Hamming W function.

The first step is a sort of smoothing but it is not strong enough for our purposes. We have modified the program by inserting a subroutine which, previous to the above steps

(0) smooths the profile by running average using specified coefficients.

This modification is only temporary and it is lacking in several respects. The most serious respect is that the correction factors must be calculated and applied by hand. Since this type of smoothing seems to be very satisfactory, we are including a new subroutine which will

(0.0) smooth the profile by running average using specified coefficients.

(0.1) indicate any profile values that are inconsistent with their neighbors.

(0.2) compute and store the correction factors

(0.3) correct the spectral estimates.

6. Discussion of One-dimensional Spectral Results

In this section, we shall comment in some detail upon the one-dimensional p.s.d.'s, how they were obtained, and their corresponding profiles.

One of the significant problems from the point of view of subsequent use is the taking out or elimination of trends, hills, etc. which are not considered relevant. When we began the survey data analysis, we estimated trends by means of quadratic polynomials fit to segments of the data by least squares. These polynomials were subtracted from the profile heights to produce smoothed data (deviations from fitted polynomials). The roughness of these smoothed data were then analyzed in the spectral analysis program. The spectral estimated for the Yuma data contained anomalies such as high tail values and occasional negative values. In the case of Aberdeen and Knox, these anomalies were not observed. Moreover, in the Yuma data, spectral estimates of the halves and whole were different in character. It was not possible to interpret these anomalies in terms of the data. This was somewhat surprising in view of the consistency of the results with Aberdeen and Knox and from highway survey data.

The origin of these anomalies remained a mystery for some time, but attention was gradually directed to the smoothing procedure. We next obtained another p.s.d. estimate program which only removed the general average; it produced with the smoothed Yuma data precisely the same p.s.d. estimates which established beyond doubt the troubles in the smoothing procedure. We then made a preliminary routine for running average smoothing which added to the program produced p.s.d. estimates free of most of the anomalies noted above. Moreover, the remaining anomalies were immediately traceable to inconsistent raw data points not previously detected. The results presented here are those obtained from this revised program.

The general features were already discussed in the Introduction and that discussion will not be repeated. We must describe however the significance of data processing and analysis.

As explained in Section II, Part 4, smoothing throws away part of the data and partially restores it by numerical correction. The nature and significance in vehicle dynamics of the part thrown away is not completely known to us and the control of this part by choice of smoothing coefficients is not yet entirely complete. It is important to understand that smoothing must bare a direct relationship to p.s.d. estimate use. Whether hills are removed or not depends upon vehicle size, speed and the presence and nature of the hills. Thus, use of the p.s.d. estimate procedures and the estimates without regard to the method of estimation employed may in unusual conditions lead to error.

Figure 5 shows the p.s.d. estimates of the full 750 data points, the first 350 data points, and the second 350 data points from Aberdeen. The raw data, the smoothed data, the covariance of the smoothed data, the p.s.d. estimates of the smoothed data, and the corrected p.s.d. estimates are given in Tables 1 to 1D. The plots are of the corrected p.s.d. estimates. The uncorrected p.s.d. estimates, calculated from the smoothed data, are distorted by the smoothing and are corrected by multiplication as explained in Section II, Part 4.

In general, the p.s.d. of the full set of data should be approximately the average of the p.s.d.'s of the two halves. This feature is noticeable upon Figure 5. The matching in the significant range $\lambda = .005$, to $\lambda = 0.075$ between the halves is substantial except in the interval containing the bump. The bump is due to a periodic component in the profile, (Figure 1) which was most visible in the first half of the profile. This is confirmed by the presence of a strong bump in the first half p.s.d. and the absence of any bump in the second half. The generally high level in the low tails is due to inconsistencies in the data as evidenced by the readings at

112 feet and 1,254 feet in the smoothed data (Table 1B). As these tails are not of interest, these inconsistencies are not important to us at this time. The other significant difference between halves is in the $\lambda = .09$ to $\lambda = .10$, where the second half seems to have about twice as much power as the first. Examination of the profile to confirm this by inspection indicates that visual methods are not capable of explaining this difference, but since the power involved is low this is not surprising.

Figure 6 gives the p.s.d. estimates of the full and two half sets of data points from Ft. Knox. The tabulated results are given in Tables 2 to 2D.

The three spectra match better, in general, than those of Aberdeen. In particular, the tails fall off far more rapidly and to a lower value than in Aberdeen. The broad bump in the first half p.s.d. is spread over the wave lengths from 25 feet to 50 feet whereas in the second half this power is concentrated more in the wave length around 30 feet. Again these differences are not noticeable in scanning the profile (Figure 2) by eye.

Figure 7 gives the 3 p.s.d. estimates from the Yuma data. The tabulated results are given in Tables 3 to 3D.

The matching in Yuma is quite remarkable. If anything, the first half p.s.d. estimate indicates less roughness in the higher frequencies than the second half; this is evident in the profile (Figure 3). The drop off to a very low level of power in the tails is also very satisfactory.

Figure 8 gives the 5 p.s.d. estimates from the Battlefield Day data. Numerical data and computed results are tabulated in Tables 4 to 4D. In this figure, we give the p.s.d. estimates of the four quarters as well as of the whole. The estimates match very well up to $\lambda = .08$. There is no noticeable bump or any other feature

of interest in the range from $\lambda = 0.00$ to $\lambda = .08$. Two of the quarters have comparatively high tails and one exhibits a distinct periodicity. These features are transmitted to the spectra estimate of the full set of data. They are adequately explained by the presence of inconsistent data points, as mentioned in Appendix Y, Section 2.

The differences between p. s. d. estimates of the different tracts of ground are much greater than the differences encountered with estimates from sections of the same tract. This indicates that the assumption of stationarity over reasonable lengths is a good one and justifies the use of p. s. d. techniques in the measurement of ground roughness. It must be expected that the assumption of stationarity is not ideally fulfilled in any tract of ground. However, in ground selected with reasonable care, our data reveals that the assumption is reasonably well fulfilled over lengths of 1,000 feet or more. This indicates the possibility of adjusting suspension for slowly varying roughness.

These p.s.d. estimates have similar shapes though they are from widely different areas of the country. The fact that the spectral shapes are similar and simple in form is a matter of some importance. First, it might be possible to character ground roughness (spectral shape) in terms of three or four parameters. This may simplify classification of ground roughness types; it would permit easy simulation of roughness types; and it would define ranges of suspension system parameters for adjustment to changing ground roughness.

Figures 9, 10, and 11 show cumulative diagrams of the smoothed data from Aberdeen, Knox, and Yuma plotted on probability paper. The scale is the same in each; the horizontal scale is in percentage of the data, the vertical in feet of deviation. The points are plotted to indicate the percentages of data smaller than

various values. The arrangement of values on the horizontal scale is such that a theoretical Gaussian distribution plots as a straight line passing through the points

$$(15.87, \mu - \sigma) , (0, \mu) , (84.13, \mu + \sigma)$$

where μ is its mean and σ^2 its variance. Gaussian data will also be straight except for random deviations. The values of μ and σ may be estimated by fitting a straight line and reading the ordinates of the points 15.87, 0, 84.13. Interpretation of deviations from straightness requires some experience but is not difficult.

For a random function satisfying the ergodic hypothesis, the time average of an indicator function, $I(A)$, which is unity if the event A occurs and zero if not, equals the ensemble average or probability of A . This means for profile heights that the fraction of heights smaller than a number, z , should approximate the probability that a height is smaller than z . If the profile is a Gaussian random function, the data should therefore be Gaussian data. The curves for our data indicate systematic departures from straightness. In our opinion the amount of departure is not enough to invalidate the Gaussian approximations we must use.

The curves suggest that the distribution of the data is symmetric, as is the Gaussian distribution, but that it has a higher percentage of its probability in the region of small deviations than does the Gaussian. This might possibly be a result of the effects of erosion to round off peaks and fill in holes. The amount of this departure from the Gaussian is more for Yuma than for Aberdeen or Knox. It is possible that we may find in the future that the deviation from Gaussian for some locations is large enough that we should inquire into its effect on the Gaussian approximations. This effect will be confined to the formulas concerning the statistical errors.

Appendix Y

Effect of Certain Errors on Spectral Calculations.

In this appendix we will present the results of two investigations we carried out in an attempt to explain some anomalous features of the spectral calculations for Yuma. The first of these was done after one of the profile heights was found to be some thirty feet lower than its neighbors. It was extended somewhat in the light of similar errors found in other profiles. The second was concerned with the effect of a bad joint between parts of a profile that has been smoothed by fitting parabolas to two segments. The result of the latter indicated that the unruly behavior of the Yuma spectrum could not be due principally to this type of error so the investigation was not developed further.

For the first investigation we will suppose that the proper ground profile data $d_1 d_2 \dots d_n$ is actually presented to the computer as $d'_1 d'_2 \dots d'_n$ where

$$\begin{aligned} d'_k &= d_k & k \neq a \\ d'_k &= d_a + \Delta & k = a \end{aligned} \quad (Y1)$$

The covariances actually computed are

$$\begin{aligned} r'_t &= \frac{1}{n-t} \sum_k d'_k d'_{k+t} \\ &= \frac{1}{n-t} \sum_k d_k d_{k+t} + \frac{\Delta}{n-t} (d_{a+t} + d_{a-t}) \end{aligned} \quad (Y2)$$

for $t \neq 0$, and

$$\begin{aligned} r'_0 &= \frac{1}{n} \left[\sum_k d_k^2 + 2 \Delta d_a + \Delta^2 \right] \\ &= r_0 + \frac{2 \Delta d_a}{n} + \frac{\Delta^2}{n} \end{aligned} \quad (Y3)$$

For large Δ the difference between γ'_0 and the neighboring γ'_1, γ'_2 etc. (on the order of Δ^2/n) is noticeable. If a is near one end of the data, the point where $d_a \pm t$ cuts out of Y_2 may be noticeable (to avoid complicated limits of summation we take $d_k = 0$ for $k < 0$ or $k > n$.)

The computed spectral values are

$$f'_u = f_u + \frac{2\Delta d_a}{n} + \frac{\Delta^2}{n} + 2\Delta \sum_{t=1}^m \frac{d_{a+t} + d_{a-t}}{n-t} \cos 2\pi \frac{ut}{2m+1} \quad (Y4)$$

In this formula the terms

$$\frac{2d_a}{n} + \sum_{t=1}^m \frac{d_{a+t} + d_{a-t}}{n-t} \cos 2\pi \frac{ut}{2m+1}$$

are a Fourier series of a small segment of the data and may be quite erratic. The dominant term in (Y.4) is Δ^2/n which is a constant or "white noise" term. This was a very prominent feature of the spectrum obtained from the Yuma data when it contained a single large error in the profile.

A single small error will have its size divided by n , the length of the profile record, in the Fourier series and its square divided by n in the white noise term. Since n is usually large, a small error will not have much effect. The effect of several errors may, however, accumulate to add noticeable white noise to the calculated spectrum.

The third quarter of the Battlefield Day profile data presented a spectrum that was affected, seemingly, by both a white noise and a periodic component. The latter had a period of almost exactly six stations, or about .06 cycles per foot. Since $1/.06 = 16.6$ feet one can expect a pip on the covariances near station 17. Such a pip is indeed to be found there as well as a pip at zero to account for the white noise. The full run spectrum also had a similar shape and has similar pips so these are not machine errors.

The data was searched for bad points to explain the pip at the zero covariances and the white noise part of the spectrum. A list of those found is

No. 3175	2.0 ft above adjacent points
4474	5.0 ft above adjacent points
4540	3.0 ft above adjacent points
4557	10.0 ft above adjacent points

The last two are exactly 17 stations apart and are without doubt the cause of the pip at the 17th lag. The white noise parts of the errors add together, as we shall show, and adding up the values Δ^2/n for the above we have

$$(4 + 25 + 9 + 100) / 1550 = .09$$

The height of the zero'th lag for the third quarter data is .0916, while the first lag is -.0112 making a pip of .0928 which is quite close to the above estimate.

If two errors are farther apart than the maximum lag then their effects in (Y2) and (Y3) are additive. However, if two errors, say of Δ_1 and Δ_2 are separated by a distance of $t_0 < m$ stations in the data, say

$$y'_0 = y'_0 + \Delta_1, \quad y'_{0+t_0} = y_{0+t_0} + \Delta_2$$

Then the effects of these errors on the covariances are

$$r'_t = r_t + \frac{1}{n-|t|} \left\{ \Delta_1 (y_{a+t} + y_{a-t}) + \Delta_2 (y_{a+t_0+t} + y_{a+t_0-t}) \right\}$$

$$r'_{t_0} = r_{t_0} + \frac{1}{n-|t_0|} \left\{ \Delta_1 (y_{a+t_0} + y_{a-t_0}) + \Delta_2 (y_{a+t_0+t} + y_{a+t_0-t}) \right\} + \frac{\Delta_1 \Delta_2}{n-|t_0|}$$

$$r'_0 = r_0 + \frac{1}{n} (2 \Delta_1 y_a + 2 \Delta_2 y_{a+t_0}) + \frac{\Delta_1^2 + \Delta_2^2}{n}$$

The effects are additive as for widely separated errors except at a lag of t_0 where there is a pip of

$$\frac{\Delta_1 \Delta_2}{n-|t_0|}$$

For the Battlefield Day data third quarter errors, this is

$$3 \times 10 / (1550 - 17) = .0195$$

In the covariances the 17th is .0178 while the neighbors are -.0020 making .0198. These anomalies in the spectrum are therefore explained adequately by the errors or inconsistencies in the data.

The spectra for Ft. Knox and Yuma seem to be affected somewhat by periodic components and the spectra for Aberdeen seems to be rather high on the high frequency end. The compilation of this data for the purpose of plotting on probability paper shows several outlying in each profile, but none as spectacular as those found in the Battlefield Day data. The cumulative effect of these

could explain the behavior of the spectra. In these cases the actual amount of the bad behavior is small and seems to affect mostly those frequencies that are unimportant in our work.

The second investigation is concerned with a smoothed profile $d_1 d_2 \dots d_n d_{n+1} \dots d_{2n}$ which are deviations of the actual profile heights from parabolas fitted to each half. We will suppose that the fitted parabolas do not meet in the middle--thus the d 's also do not meet--and we will try to estimate the effect of this error. Our attention will be on the relation between the spectrum $f'(u)$ of the first half, $f''(u)$ of the second, and $f(u)$ of the entire profile. In theory the average of the two parts should give the whole. To show this, let us recall

$$r_t = \frac{1}{2n-t} \sum_{k=1}^{2n-t} d_k d_{k+t} \quad t \geq 0$$

We will take $t \geq 0$ for the sake of having definite limits on the sum. This sum may be partitioned

$$\begin{aligned} r_t &= \frac{1}{2n-t} \left[\sum_{k=1}^{n-t} d_k d_{k+t} + \sum_{k=n-t+1}^n d_k d_{k+t} + \sum_{k=n+1}^{2n-t} d_k d_{k+t} \right] \\ &= \frac{1}{2n-t} \left[(n-t) r_t' + \sum_{k=n-t+1}^n d_k d_{k+t} + (n-t) r_t'' \right] \quad (Y5) \\ &= \frac{1}{2} \left[\frac{n - \frac{1}{2} \tau}{n - \frac{1}{2} \tau} \right] \frac{r_t' + r_t''}{2} + \frac{1}{2n-t} \sum_{k=n-t+1}^n d_k d_{k+t} \end{aligned}$$

If t is a small fraction of $n-t$ and $2n-t$ is large with respect to the sizes of the data near the joint, then

$$r_t = \frac{r'_t + r''_t}{2} \quad (Y6)$$

Since the cosine transform is a linear operation this equation leads to

$$f_u \doteq \frac{1}{2} (f'_u + f''_u) \quad (Y7)$$

The approximation to the average in the first term on the right of (Y.5) may be made somewhat better by choosing a value of t near, say, $1/3m$ since the r 's are largest near zero. With this (Y.7) may be modified a little

$$f_u = \left(1 - \frac{1}{6n}\right) \frac{1}{2} (f'_u + f''_u)$$

The central values, however, are not as predictable. The sum is the same kind as used to estimate p_t and if it weren't for the error we are discussing we might expect the second term to have the approximate size

$$\frac{t}{2n-t} r_t \approx \frac{1}{6u} r_t$$

which would cancel out the $1/6n$ part of (Y.8) on transformation.

(Of course this is approximate and the amounts $1/6n$ are very small --no doubt the other approximations make larger effects than this.)

The error in matching the two halves of the profile affects this central part only.

We will assume that the values d_k are roughly constant on each side of the half mark and are separated by a distance D

$$d_k = \begin{cases} d & n-m \leq k \leq n \\ d+D & n+1 \leq k \leq n+m \end{cases}$$

where m is the maximum lag. Then for any value of t $0 \leq t \leq m$ the central term is

$$\frac{1}{2^{n-t}} \sum_{k=n-t+1}^n d_k d_{k+t} = \frac{t}{2^{n+t}} d(d+D) = \frac{d(d+D)}{2^n} t$$

Since r_t is an even function we have for $-m \leq t \leq m$

$$r_t = \frac{r'_t + r''_t}{2} + \frac{d(d+D)}{2^n} |t|$$

The cosine transform of $|t|$ maybe gotten in closed form in formula (428) of Jolley's "Summation of Series" (Dover, 1961)

$$\begin{aligned} \sum_{t=-m}^m |t| \cos 2\pi \frac{ut}{2^{m+1}} &= \frac{(m+1) \sin \pi u}{\sin \frac{\pi u}{2^{m+1}}} - \frac{1 - \cos \frac{m+1}{2^{m+1}} 2\pi u}{2 \sin^2 \frac{\pi u}{2^{m+1}}} \\ &= (m+1) \frac{\sin \pi u}{\sin \frac{\pi u}{2^{m+1}}} - \left(\frac{\sin \pi \frac{m+1}{2^{m+1}} u}{\sin \frac{\pi u}{2^{m+1}}} \right)^2 \end{aligned} \quad (Y8)$$

The denominators both terms rise to near unity as u goes from 0 to m for most values of u they may be approximated by the argument. In fact for

$$0 < \frac{\pi u}{2m+1} < \frac{1}{2}$$

$$\frac{2m+1}{\pi u} < \frac{1}{\sin \frac{\pi u}{2m+1}} < 1.05 \frac{2m+1}{\pi u}$$

Thus we may approximate the right side of (Y.8) for $0 < u < 1/3 m$ by

$$\frac{(m+1)(2m+1)}{\pi u} \sin \pi u - \frac{(2m+1)^2}{\pi^2 u^2} \sin^2 \left(\frac{1}{2} + \frac{1}{4m+2} \right) \pi u \quad (Y.9)$$

For $u = 0$ the left side of (Y.8) is at its maximum

$$m(m+1) \approx \left(m + \frac{1}{2} \right)^2 = \frac{(2m+1)^2}{4}$$

which is the limit as $u > 0$ of Y.9. Using (Y.8), (Y.9) to approximate the cosine transform of (Y.7) we obtain

$$f_u = \frac{1}{2} (f'_u + f''_u) + \frac{d(d+D)}{n} \frac{(2m+1)^2}{4} \left[2 \frac{\sin \pi u}{\pi u} \left(\frac{\sin \frac{\pi u}{2}}{\frac{\pi u}{2}} \right)^2 \right]$$

Thus the total spectrum will deviate from the average of its parts by a term whose maximum size is approximately

$$d(d+D) \frac{m^2}{n}$$

which oscillates with a period of about 2 and which damps out as u increases.

The approximations in this second investigation are so rough as to render the result of aid only in determining the order of magnitude of the effect of this type of error on the behavior of the spectrum. Since we will not smooth by parabolic fit to segments in the future, more exact work does not seem justified.

Section III

Two-dimensional Power Spectral Density

I. Introduction

Spectral analysis for random functions of one variable is common in engineering work. Problems involving random functions of two variables have appeared seldom in the literature. Indeed we are aware of but one article in which results of a two-dimensional spectrum were presented, "The Directional Spectrum of a Wind Generated Sea as Determined from Data Obtained by the Stereo Wave Observation Project", Willard J. Pierson ed. (Meteorological Papers, N.Y.U. College of Engineering, Vol. 2, No. 6, June 1960). The material in the present section is a further development of work done by one of the authors in connection with the above article and appears in full detail for the first time here. The motivations and interpretations connected with this work will be given even though they are in many cases repetitions of similar ideas appearing in the preceeding sections. Thus the section will be self-contained.

To be able to speak mathematically about the height irregularities of a piece of ground, let us suppose that we have established a horizontal datum and a coordinate system on it. The coordinates (x, y) (Measurements in feet) locate a point on the datum and $h(x, y)$ will denote the height in feet of the ground surface above the datum. The pattern of irregularity is given completely by $h(x, y)$, but, in a sense, it is given in too much detail. For another piece of ground having the same kind of irregularity the function $h(x, y)$ will in general be different. In order to extract the common character of the irregularity we will expand on the common mathematical model of a stationary random function.

The actual heights above datum $h(x,y)$ include variations that are not of interest--gentle slopes, large hills, etc. We will invent a variable datum called the "smoothed heights" which consists of the slopes and hills together with the general level of $h(x,y)$ above the actual datum. The differences

$$d(x,y) = h(x,y) \left[\text{smoothed height of } (x,y) \right] \quad (1.1)$$

are the variations of ground height that affect the vehicle. We will conceive of $d(x,y)$ as being random--that is, this plot of ground gives us one of an "ensemble" of possible roughnesses selected at random according to probabilities that characterize the roughness type. When we say that the general level has been taken out we mean that the expected value or ensemble average, denoted $E \{d(x,y)\}$, of $d(x,y)$ at any point is zero, thus, unchanging or stationary from point to point. More than this we assume that all the characteristics of the probabilities are unchanging from point to point. This is the meaning of "stationary random functions".

Stationary random functions of one variable have been used in engineering for more than 20 years. Problems concerning the output of a linear system with a stationary random input are nowadays routine. A brief summary is given in Section II, Part 1 of this report. In our work, such a problem would be to determine the variable force on the driver of a motorcycle which is proceeding at a uniform rate across our ground on a straight line through (x_0, y_0) at an angle to the x -axis. The input is the random function of distance, s .

$$g(s) = d(x_0 + s \cos \alpha, y_0 + s \sin \alpha) \quad (1.2)$$

For a two track vehicle our input must be regarded as two correlated random functions (one for each track); the formulas are readily worked out as above. In any case, the output--variable force acting on the driver (or the variable displacement of the driver)--is also a random function. In engineering practice, the effects of such random functions have been found to be related to the covariance function or its spectrum. This is true in electrical engineering problems and in those problems of mechanical engineering which deal with vibration. In these problems frequency of oscillation is of obvious importance. However, the spectrum of frequencies has also been useful in the study of such "non-oscillatory" phenomena as atmospheric turbulence.

In this discussion of random functions of two variables, we will attempt to use standard notation and wording as far as possible and to make a development which is parallel to the usual one for functions of one variable. This may be found in Grenander & Rosenblatt, "Statistical Analysis of Stationary Time Series" (Wiley, 1957) and Blackman & Tukey, "The Measurement of Power Spectra", (Dover, 1958).

We will begin with the definitions of covariance and the spectrum.

$$R(a, b) = D \left\{ d(x, y) d(x + a, y + b) \right\} \quad (1.3)$$

In this the symbol E denotes the expected value operator or the ensemble average. Because of the stationarity or probabilistic sameness of one location to another, the average does not depend on the location (x, y) but only on the "lag" (a, b) . The covariance function is not random--it is a fixed characteristic of the roughness.

$$a) \quad R(0, 0) = E \left\{ (d(x, y))^2 \right\} \geq 0$$

$$b) \quad |R(a, b)| \leq R(0, 0) \quad (1.4)$$

$$c) \quad R(-a, -b) = R(a, b)$$

These properties are related to the connection between covariance and correlation

$$\text{corr.} (x, y) = \frac{\text{cov.} (x, y)}{\sqrt{\text{var}(x), \text{var}(y)}}$$

The first property, in other terms, states that the variance is the same for any point x, y . The second property is that of the correlation coefficient $-1 \leq \text{corr}(x, y) \leq 1$. And the third is $\text{corr}(x, y) = \text{corr}(y, x)$. Since the correlation is a measure of the relationship between the outcomes of two random variables, we may expect $R(a, b)$ to be large and positive if the height at a displacement (a, b) from a point tends to be about the same as the heights at the point itself. R will be negative when large height at (x, y) is associated generally with small heights at $x + a, y + b$. The absolute size of $R(a, b)$ will indicate the strength of the relationship. From continuity of the ground height $h(x, y)$, $R(a, b)$ should be near $R(0, 0)$ for small (a, b) --the correlation between $d(x, y)$ and immediately adjacent points should be close to unity.

In many applications the covariance function has been useful mostly in the form of its Fourier transform

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(xu + yv)} R(x, y) dx dy \quad (1.5)$$

Since $e^{i\theta} = \cos \theta + i \sin \theta$, the quantity $2\pi (xu + yv)$ must be in radians. Since x and y are in feet, $2\pi u$ and $2\pi v$ are in radians/ft., or u, v in cycles/ft.

As with one variable transform this can be inverted

$$R(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(xu + yv)} F(u, v) du dv \quad (1.6)$$

The fact that $R(x, y)$ is an even function, (1.4c) implies that $F(u, v)$ is real valued and in fact

$$\begin{aligned} F(u, v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(xu + yv)} \frac{1}{2} [R(x, y) + R(-x, -y)] dx dy \\ &= \frac{1}{2} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(xu + yv)} R(x, y) dx dy \right. \\ &\quad \left. + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(-xu - yv)} R(x, y) dx dy \right] \end{aligned}$$

The second integral is obtained by transforming $x' = -x$, $y' = -y$ then omitting the primes. The two may then be averaged to give

$$\begin{aligned} F(u, v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(xu + yv)} \frac{1}{2} [R(x, y) + R(-x - y)] dx dy \\ &= \frac{1}{2} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(xu + yv)} R(x, y) dx dy \right. \\ &\quad \left. + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(-xu - yv)} R(x, y) dx dy \right] \quad (1.7) \end{aligned}$$

which corresponds to the usual relation between even functions and cosine transforms in one variable Fourier analysis. From (1.7) it easily follows that $F(u, v)$ is an even function so that (1.6) may be written

$$R(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos 2\pi(xu + yv) F(u, v) du dv \quad (1.8)$$

Some properties of $S(uv)$ are

$$a) F(-u, -v) = F(u, v)$$

$$b) F(u, v) \geq 0 \quad (1.9)$$

$$c) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) du dv = R(0, 0)$$

(Actually (1.9a) is proved from the fact that $R(a, b)$ is real valued while (1.9b) stems from a generalization of (1.4a, b) called the "positive semi-definite" property of R to wit, for any $C_1, C_2, \dots, C_n$ and points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ $\sum_i \sum_j C_i C_j R(x_i - x_j, y_i - y_j) \geq 0$. The derivation of (1.9b) from this is from a simple extension of Bochner's theorem.)

Properties b) and c) suggest that the total variance, $R(0, 0)$ may be divided up into components $F(u, v)$ at the frequency values (u, v) . This is indeed the case and one can derive generalizations of the various representations of a stationary stochastic process of one variable. We will interpret the spectral density function $F(u, v)$ as giving the "power" of the roughness in the frequency region u cycles/ft. parallel to the x -axis, and v cycles/ft. parallel to the y -axis. An elementary wave at this frequency will be a cosine wave $\cos 2\pi (ux + vy)$ which has the appearance of corrugated iron on the xy plane with the crests parallel to the line $ux + vy = 0$ and having crests evenly spaced every $\frac{1}{\sqrt{u^2 + v^2}}$ feet. A representation theorem makes us conceive of $d(x, y)$ as a sum of such elementary waves at different frequencies, amplitudes and phases. For purposes of power those waves of the same frequencies are classified together. The power is analogous to the sum of the squared amplitudes. For a more detailed explanation, see Section II, Part 1 of this report.

A vehicle which runs over the rough ground is affected only by the roughness under its wheels. If its track is straight across a stationary field, the roughness is stationary. The simplest case deals only with one track but more realistically we will have two parallel tracks. Let us derive first the formulas for the single track. Equation (1.2) gives the random function involved. The covariance function for this is

$$\begin{aligned} C(\tau) &= E \left\{ g(s) g(s + \tau) \right\} \\ &= E \left\{ d(x_0 + s \cos \alpha, y_0 + s \sin \alpha) \quad d(x_0 + s \cos \alpha + \tau \cos \alpha, \right. \\ &\quad \left. y_0 + s \sin \alpha + \tau \sin \alpha) \right\} \quad (1.10) \\ &= R(\tau \cos \alpha, \tau \sin \alpha) \end{aligned}$$

The spectral function for $C(\tau)$ is the function $D(\lambda)$ that satisfies

$$C(\tau) = \int_{-\infty}^{\infty} e^{2\pi i \lambda \tau} D(\lambda) d\lambda$$

But we have from (1.6)

$$R(\tau \cos \alpha, \tau \sin \alpha) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i (u\tau \cos \alpha + v\tau \sin \alpha)} F(u, v) du dv$$

and if we take

$$\begin{aligned} u \cos \alpha + v \sin \alpha &= \lambda \\ -u \sin \alpha + v \cos \alpha &= \lambda \end{aligned} \tag{1.11}$$

whence

$$\lambda \cos \alpha - u \sin \alpha = u$$

$$\lambda \sin \alpha + u \sin \alpha = v$$

and

$$|J| = \frac{\partial(\lambda, u)}{\partial(u, v)} = 1$$

the integral may be transformed to

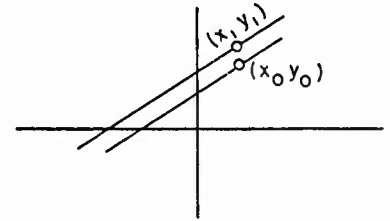
$$\int_{-\infty}^{\infty} e^{2\pi i \lambda \tau} \left[\int_{-\infty}^{\infty} F(\lambda \cos \alpha - \mu \sin \alpha, \lambda \sin \alpha + \mu \cos \alpha) d\mu \right] d\lambda$$

from which we have

$$D(\lambda) = \int_{-\infty}^{\infty} F(\lambda \cos \alpha - \mu \sin \alpha, \lambda \sin \alpha + \mu \cos \alpha) d\mu \quad (1.12)$$

relating the linear spectra in various directions to the two dimensional spectrum.

For parallel tracks separated by a distance b the spectrum of each track is given by (1.12). The cross covariance of the two profiles is



$$\begin{aligned} H(\tau) &= E \left\{ d(x_0 + s \cos \alpha, y_0 + s \sin \alpha) d(x_1 + s \cos \alpha + \tau \cos \alpha, y_1 + s \sin \alpha + \tau \sin \alpha) \right\} \\ &= R(x_1 - x_0 + \tau \cos \alpha, y_1 - y_0 + \tau \sin \alpha) \end{aligned}$$

If (x_1, y_1) is opposite (x_0, y_0) on a line perpendicular to the track $x_1 - x_0 = -b \sin \alpha$ $y_1 - y_0 = b \cos \alpha$. Since $H(\tau)$ is not an even function, its Fourier transform has both a real and an imaginary part. These are, respectively, its cosine and its sine transform. They give the frequency--phase difference relationship between the two tracks. As above, this complex spectrum is the function $K(\lambda)$ such that

$$H(\tau) = \int_{-\infty}^{\infty} e^{2\pi i \lambda \tau} K(\lambda) d\lambda$$

and (1.12) and (1.6) gives us

$$H(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i [(-b \sin \alpha + \tau \cos \alpha) u + (b \cos \alpha + \tau \sin \alpha) v]} F(u, v) du dv$$

Making use again of the transformation (1.11)

$$H(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i (bu + \tau \lambda)} F(\lambda \cos \alpha - \mu \sin \alpha, \lambda \sin \alpha + \mu \cos \alpha) du dv$$

whence

$$K(\lambda) = \int_{-\infty}^{\infty} e^{2\pi i b\mu} F(\cos \alpha - \mu \sin \alpha, \lambda \sin \alpha + \mu \cos \alpha) d\mu$$

Formula (1.12) is a special case of this where $b = 0$. For each λ the integral is a line integral of the two dimensional spectrum along lines perpendicular to the direction (α) of the track and at a distance from the origin.

This work gives two possible derivations that may be made from the two dimensional spectrum. In both, the track of the vehicle is straight. When the track curves, the resulting roughness is not stationary unless the roughness of the field is the same in all directions. These spectra (1.14) for various α will give the conditions met by the vehicle in various directions and may be of aid in studying the effects of slowly varying non-stationarity.

In this introduction we have dealt with the theoretical quantities associated with the stochastic process. We have described them and indicated how they can be used. Our present problems are more

concerned with estimating these quantities, particularly the spectral function. Estimation of a two-dimensional spectrum is a simple extension of one dimensional spectral estimation. Since the formulas are not available in the literature we will present a fairly complete account of them here.

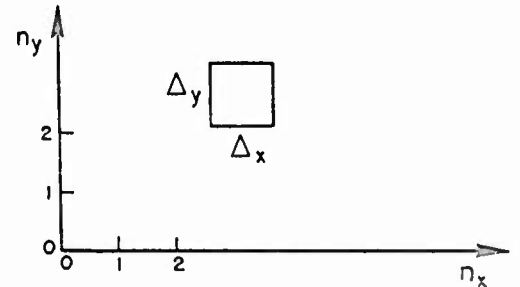
2. Relation of the Theory to the Practical Problem.

As in all statistical work we will use data from outcomes of the random scheme to make estimates of the desired characteristics of the probabilities. In our case the desired characteristic is the spectral density function of the covariance. Statistical estimation makes use of the fact that when repeated outcomes are averaged, the random part of the average tends to be small. This repetition may be accomplished with $d(x,y)$ by taking the values at different places; the stationary property is, in effect, the property that the randomness repeats itself in different places. Actually a somewhat more stringent property, the ergodic property, is necessary so that averages over (x,y) tend to ensemble averages. We will assume as do most practitioners that this property holds.

It is most convenient for computation and for measurement to take data at intersection points of a rectangular lattice. Though the data we have taken has the separations equal in both directions, it is just as easy to deal with different separations Δ_x, Δ_y . Let us suppose that we make n_x by n_y observations, as indicated in the figure, at the points

$$(k\Delta_x, j\Delta_y)$$

$$k = 0, 1, 2 \dots n_x - 1 ; j = 0, 1, 2 \dots n_y - 1$$



Supposing the smoothing of the profile to have been done, a topic we will discuss in Section 5, our data will consist of n_x by n_y values

$$d(k \Delta_x, j \Delta_y)$$

Only a part of the randomness is involved in this data. The covariances which underlie it are of the form

$$E \left\{ d(k \Delta_x, j \Delta_y) d((k+a) \Delta_x, (j+b) \Delta_y) \right\} = R(a \Delta_x, b \Delta_y) \quad (2.1)$$

for

$$a = 0, \pm 1, \dots, \pm n_x \quad b = 0, \pm 1, \dots, \pm n_y$$

No other values of the covariance function are used to "produce" the data and we cannot, therefore, hope to make direct estimates of any but these.

Supposing that we knew these values of $R(x, y)$, what can we find out about the spectrum? This is a preliminary question and we must later refine the answer because we will not have the values of $R(x, y)$ but only estimates of them. This, of course, will affect the way we answer the preliminary question.

The spectrum is related to these values of $R(x, y)$ by equation (1.8)

$$R(a \Delta_x, b \Delta_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos 2\pi (a \Delta_x u + b \Delta_y v) F(u, v) du dv \quad (1.8)$$

In analogy to equation (1.7) we will later multiply the $R(a \Delta_x, b \Delta_y)$ by cosines and add. But we want to investigate the matter generally so instead of using cosines to obtain

$$\sum_a \sum_b \cos 2\pi (a \Delta_x u_0 + b \Delta_y v_0) R(a \Delta_x, b \Delta_y) \Delta_x \Delta_y \quad (2.2)$$

let us use general coefficients to obtain

$$\begin{aligned} F^*(u_0, v_0) &= \sum_a \sum_b w(a, b; u_0, v_0) R(a \Delta_x, b \Delta_y) \\ &= \sum_a \sum_b w(a, b; u_0, v_0) \iint \cos 2\pi (a \Delta_x u + b \Delta_y v) F(u, v) du dv \\ &= \iint \left\{ \sum_a \sum_b w(a, b; u_0, v_0) \cos 2\pi (a \Delta_x u + b \Delta_y v) \right\} F(u, v) du dv \end{aligned} \quad (2.3)$$

From this we see that $F^*(u_0, v_0)$ is a weighted average of spectral values,

$$F^*(u_0, v_0) = \iint W(u_0, v_0; u, v) F(u, v) du dv \quad (2.4)$$

Our purpose is to obtain $F(u_0, v_0)$, therefore

$$W(u_0, v_0; u, v) = \sum_a \sum_b w(a, b; u_0, v_0) \cos 2\pi (a \Delta_x u + b \Delta_y v) \quad (2.5)$$

is ideally a Dirac or delta function of (u, v) centered at (u_0, v_0) . It is, of course, impossible to attain this ideal since $W(\cdot)$ is a finite Fourier series. An obvious $W(\cdot)$ to try is the finite part of the Fourier series for the Dirac function

$$\sum_{a=-n_x}^{n_x} \sum_{b=-n_y}^{n_y} \Delta_x \Delta_y \cos 2\pi(a \Delta_x u_0 + b \Delta_y v_0) \cos 2\pi(a \Delta_x u + b \Delta_y v) \quad (2.6)$$

which corresponds to (2.2).

We will not go into detail here about the form of $W(u_0, v_0; u, v)$. Let us note, however, from (2.5) that it is even and periodic with a period of $1/\Delta_x$ in the u direction and $1/\Delta_y$ in the v direction so that if it has a sharp peak at (u_0, v_0) it has identical peaks at $(u_0 + r/\Delta_x, v_0 + s/\Delta_y)$ for integers r, s . Referring to formula (2.4) we see that the sharp peak of $W(\cdot)$ at (u_0, v_0) picks up the values of $F(u, v)$ near (u_0, v_0) while the other peaks get the values near their positions. $F^*(u_0, v_0)$ is therefore made up of an average of the values of $F(u, v)$ near (u_0, v_0) with the addition of similar averages for values near the other peaks of $W(\cdot)$. These other averages are called "aliases". Their presence is, of course, unwanted. It is due to the fact that the data is taken at equally spaced points. The smaller the spacing (Δ_x, Δ_y) the farther apart are the peaks of $W(\cdot)$. The usual spectrum "dies out" for large (u, v) and if Δ_x, Δ_y are small enough, the aliased values of $F(u, v)$ will be at large enough frequencies that they do not affect the estimates. The periodicity of $W(\cdot)$ means that the function $F^*(u, v)$ of (2.3) is itself periodic. Actually the values outside the rectangle

$$\left(-\frac{1}{2\Delta_x} \leq u \leq \frac{1}{2\Delta_x} ; -\frac{1}{2\Delta_y} \leq v \leq \frac{1}{2\Delta_y} \right) \quad (2.7)$$

are repetitions of those inside this rectangle. Moreover, the spectral function is even (1.9a). Consequently all the values of the estimated spectrum may be found from those within the rectangle $(-1/2\Delta_x \leq u \leq 1/2\Delta_x ; 0 \leq v \leq 1/2\Delta_y)$ and these need be the only values reported. (For a one-variable spectral estimate the only values ordinarily presented are for positive frequencies out to the Nyquist frequency of $1/2\Delta$.)

The phenomenon of aliasing is a limitation on accuracy resulting from sampling at intervals. It arises from the form of Eqn. (2.5) and not from the specific coefficients $w(a,b; u_0, v_0)$. Another limitation of accuracy arises from our having only a finite number of data points. Equation (2.5) is a Fourier expansion with all coefficients zero for $a \geq n_x$, $b \geq n_y$. Since the coefficients may be calculated from $W(\cdot)$ by

$$w(a,b,u_0,v_0) = \Delta_x \Delta_y \int_{-\frac{1}{2\Delta_x}}^{\frac{1}{2\Delta_x}} \int_{-\frac{1}{2\Delta_y}}^{\frac{1}{2\Delta_y}} W(u_0,v_0;u,v) \cos 2\pi(a\Delta_x u + b\Delta_y v) du dv \quad (2.8)$$

$$W(u_0,v_0;u,v) \cos 2\pi(a\Delta_x u + b\Delta_y v) du dv$$

it is clear that the ideal form for $W(\cdot)$, (Dirac "delta" function), cannot be realized, otherwise all $w(\cdot)$ would be non-zero. In fact if $W(\cdot)$ is to be large near both positive and negative parts of $\cos 2\pi(a\Delta_x u + b\Delta_y v)$ for $a \geq n_x$, $b \geq n_y$ for the integrals to be zero. This indicates that the non-zero area of $w(\cdot)$ should be at least a half period wide and a half period high or

$$\frac{1}{2n_x \Delta_x} \quad \text{by} \quad \frac{1}{2n_y \Delta_y} \quad (2.9)$$

As a check on this we note that the top line of (2.3) connects a finite number of R values, by a linear relation, to the values of F^* . Under any but abnormal conditions an equal number of values of F^* determine all the R 's and hence also the other values of F^* . If we space these values evenly, in the relevant area of frequencies, $(-1/2\Delta_x \leq u \leq 1/2\Delta_x, 0 \leq v \leq 1/2\Delta_y)$, the spacing will be $1/2n_x\Delta_x$ by $1/2n_y\Delta_y$ as before,

3. Statistical Problem

In Section 2 we acted as though the values of $R(a\Delta_x, b\Delta_y)$ were known for $a = 0, \pm 1, \dots, \pm n_x$ $b = 0, \pm 1, \dots, \pm n_y$. Actually, we will estimate these values from the height data $d(k\Delta_x, j\Delta_y)$. The usual estimate for an expected value (2.1) is an average

$$\frac{1}{(n_x - |a|)(n_y - |b|)} \sum_k \sum_j d(k\Delta_x, j\Delta_y) d(k+a)\Delta_x, (j+b)\Delta_y = r_{a,b} \quad (3.1)$$

where k and j range over all the pairs on hand. The actual limits of summation depend on the signs of a and b . Under the ergodic hypothesis this converges, as n_x and n_y grow larger, to $R(a\Delta_x, b\Delta_y)$. For values of a, b close to n_x, n_y , the sum has only a few terms and the estimate is not good. Therefore we must be content with estimates $r_{a,b}$ for $-m_x \leq a \leq m_x, -m_y \leq b \leq m_y$ where m_x and m_y are considerably smaller than n_x and n_y . The expected value of $r_{a,b}$ is $R(a\Delta_x, b\Delta_y)$. The different $r_{a,b}$ are averages of different numbers of terms, those of larger a and b have fewer terms and thus less precision. In general, also, the different r 's are correlated.

In accordance with our findings in Section 2, we will try to estimate values of the spectral function which are evenly spaced throughout the relevant range. Since we will have covariance values only up to m_x, m_y , the spacing of spectral values is correspondingly larger. For mathematical convenience we will use a spacing of $1/(2m+1)\Delta$ rather than $1/2m\Delta$, where m is m_x in the x-direction and m_y in the y-direction. Thus we will try to estimate (see Eqn. (2.3))

$$F^* \left(\frac{\alpha}{(2m_x+1)} \frac{\beta}{(2m_y+1)} \right) \equiv F_{\alpha,\beta}^* \quad (3.2)$$

$$\alpha = 0, \pm 1, \pm 2, \dots, \pm m_x, \quad \beta = 0, \pm 1, \pm 2, \dots, \pm m_y$$

Using the coefficients $w(a,b; u_0, v_0)$ of Eqn. (2.2) we can calculate the estimates

$$f_{\alpha,\beta} = \sum_{a=-m_x}^{m_x} \sum_{b=-m_y}^{m_y} w \left(a,b; \frac{\alpha}{(2m_x+1)\Delta_x}, \frac{\beta}{(2m_y+1)\Delta_y} \right) r_{a,b} \quad (3.3)$$

whose expected values are $F_{\alpha,\beta}^*$.

To approximate the distribution of the random variable $f_{\alpha,\beta}$ we will assume the Gamma distribution which has been found to fit well in this kind of work. The parameters A, K of the distribution will be estimated by the method of moments.

The following is the mathematical expression of our first assumption:

$$P \{ f_{\alpha, \beta} < \Delta \} = \frac{A^K}{\Gamma(K)} \int_0^{\Delta} t^{K-1} e^{-At} dt \quad (A1)$$

As in Section II, Part 3 above, we have corresponding to (3.2)

$$K = \frac{(F_{\alpha, \beta}^*)}{\text{var} \{ f_{\alpha, \beta}^* \}} \quad (3.4)$$

$$A = \frac{F_{\alpha, \beta}^*}{4K}$$

Confidence limits may be found as in Section II, Part 3 and interpreted by means of the table there.

The derivation of the formula for $\text{Var} \{ f_{\alpha, \beta} \}$ which also gives $\text{cov} \{ f_{\alpha, \beta}, f_{\alpha', \beta} \} = 0$ is somewhat tedious and will be presented in an appendix to this section. Several simplifying assumptions are necessary and since these are relevant in practice we will discuss them below. The result is a formula for the degrees of freedom.

$$\text{deg. fdm.} = 2K \approx 2 \left(\frac{1-g}{g} \right)^2 \quad (3.5)$$

where $g = m_x/n_x = m_y/n_y$ is the ratio of the number of R values in each direction the number of profile values in each direction.

The assumptions made in the approximate calculation of $\text{var } \{f_{\alpha, \beta}\}$ and $\text{cov } \{f_{\alpha, \beta}, f_{\alpha', \beta'}\}$ are as follows

A.2 The profile height deviations $d(x, y)$ of (1.1) are approximately Gaussian with means 0.

A.3 The values of m_x and m_y are small in comparison to n_x and n_y so that the numbers of terms in the sums for different covariance estimates (3.1) are approximately the same.

A.4 $F(u, v)$ is essentially zero for (u, v) outside the range (2.8) so that aliasing does not occur. It is relatively constant within any of the small intervals (2.10).

A.5 The non-zero parts of the functions $W(\cdot)$ of Eqn. (2.4) are no wider than $(-\frac{1}{2n_x\Delta_x}, \frac{1}{2n_x\Delta_x})$ by $(-\frac{1}{2n_y\Delta_y}, \frac{1}{2n_y\Delta_y})$, so that their "overlap" is negligible.

We have little control over assumptions A.1 and A.2. Assumption A.5 is somewhat in conflict with the discussion of Eqn. (2.9) which says that the non-zero part of $W(\cdot)$ must be at least that wide. To come near a compromise between A.5 and this discussion we must use a relatively long Fourier expansion (2.5) and choose the coefficients with care. This means a fairly large m_x, m_y -- also desirable for good resolution or closeness of spectral estimates. But A.3 requires an even larger (n_x, n_y) . The ratio g , of m to n should also be small to obtain a large degree of freedom (3.5). Assumption A.4 relates to the size of the spacings Δ_x, Δ_y in the profile data.

The desire for good resolution, statistical precision, and a wide range for the values of $F(u, v)$ require a large number of profile values that are closely spaced. The expense of measuring the profile and handling the data must be balanced with these desires. Even if large amounts of data could be obtained and processed cheaply, the lack of stationarity and the inherent errors of measurement, which have not been taken into account here, would limit the accuracy and precision of the results in other ways so we will always be faced with the above dilemma. The principles outlined in this section will help us to deal rationally with it.

Appendix to Section 3

Derivation of degrees of freedom formula.

Let us consider two quantities of the form (3.3)

$$f_1 = \sum_{a=-m_x}^{m_x} \sum_{b=-m_y}^{m_y} g_{ab} r_{ab}$$

$$f_2 = \sum_{a=-m_x}^{m_x} \sum_{b=-m_y}^{m_y} h_{ab} r_{ab}$$

$$\text{cov} \{f_1, f_2\} = \sum_a \sum_b \sum_{a'} \sum_{b'} g_{ab} h_{a'b'} \text{cov}(r_{ab}, r_{a'b'}) \quad (\text{X1})$$

In this sum the terms $\text{cov}(r_{ab}, r_{a'b'})$ are readily worked out using (3.1)

$$\text{cov}(r_{ab}, r_{a'b'}) = \frac{1}{(n_x - |a|)(n_y - |b|)} \frac{1}{(n_x - |a'|)(n_y - |b'|)} \quad (\text{X2})$$

$$\sum_i \sum_j \sum_{i'} \sum_{j'} C(i, j; i', j')$$

where the limits of the sums depend on a, b, a', b' and

$$C(i, j; i', j') = \text{cov} \left\{ d(i\Delta_x, j\Delta_y) d[(i+a)\Delta_x, (j+b)\Delta_y] , \right. \\ \left. d(i'\Delta_x, j'\Delta_y) \cdot d[(i'+a')\Delta_x, (j'+b')\Delta_y] \right\} \quad (\text{X3})$$

Using the Gaussian assumption A.2 we can evaluate this covariance from the formula

$$\begin{aligned} \text{cov}(MN, ST) &= E \left\{ \left[MN - E(MN) \right] \left[ST - E(ST) \right] \right\} \\ &= E \{ MNST \} - E \{ MN \} E \{ ST \} \\ &= E \{ MS \} E \{ NT \} + E \{ MT \} E \{ NS \} \end{aligned}$$

In this the Gaussian assumption is used only to evaluate the fourth moment.

Using this, (X.3) becomes

$$\begin{aligned} C(ij; i'j') &= R \left[(i'-i)\Delta_x, (j'-j)\Delta_y \right] R \left[(i'-i+a'-a)\Delta_x, (j'-j+b'-b)\Delta_y \right] \\ &\quad + R \left[(i'+a'-i)\Delta_x, (j'+b'-j)\Delta_y \right] R \left[(i'-i-a)\Delta_x, (j'-j-b)\Delta_y \right] \end{aligned} \quad (X4)$$

We need, as our result, (X.1) in terms of F , so we will apply (1.6) to express the product of R 's as an integral

$$R(ab) R(a'b') = \int \int \int \int e^{2\pi i(au + bv + a'u' + b'v')} F(uv) F(u'v') du dv du' dv' \quad (X5)$$

Using (X.5) in (X.4) we obtain

$$C(ij; i'j') = \int \int \int \int E F(uv) F(u'v') du dv du' dv' \quad (X6)$$

where

$$\begin{aligned}
 E &= \exp 2\pi i \left[(i'-i) \Delta_x u + (j'-j) \Delta_y v + (i'-i+a'-a) \Delta_x u' + (j'-j+b'-b) \Delta_y v' \right] \\
 &= \exp 2\pi i \left[(i'+a'-i) \Delta_x u + (j'+b'-j) \Delta_y v + (i'-i-a) \Delta_x u' + (j'-j-a) \Delta_y v' \right] \\
 &= \exp 2\pi i \left[(i'-i) \Delta_x (u+u') + (j'-j) \Delta_y (v+v') - a \Delta_x u' - b \Delta_y v' \right] \times \\
 &\times \left\{ \exp 2\pi i \left[a' \Delta_x u' + b' \Delta_y v' \right] + \exp 2\pi i \left[a' \Delta_x u + b' \Delta_y v \right] \right\} \\
 &= 2 \exp 2\pi i \left[(i'-i + \frac{a'}{2}) \Delta_x (u'+u) + (j'-j + \frac{b'}{2}) \Delta_y (v'+v) - a \Delta_x u' - b \Delta_y v' \right] \\
 &\quad \cos 2\pi \left(a' \Delta_x \frac{u'-u}{2} + b' \Delta_y \frac{v'-v}{2} \right)
 \end{aligned}$$

Because $F(u, v)$ is an even function, the integral (X.6) with u interchanged with v and u' with v' also equals $C(i, j; i', j')$. It has in place of E , a function identical with E except for the last two terms of the exponential. The average of the two integrals also equals $C(i, j; i', j')$ and is like (X.6) except that in place of E it has

$$\begin{aligned}
 &\exp 2\pi i \left\{ (i'-i + \frac{a'}{2}) \Delta_x (u'+u) + (j'-j + \frac{b'}{2}) \Delta_y (v'+v) \right\} \cos 2\pi \left(a' \Delta_x \frac{u'-u}{2} + b' \Delta_y \frac{v'-v}{2} \right) \\
 &\quad \left[\exp 2\pi i (-a \Delta_x u' - b \Delta_y v') + \exp 2\pi i (-a \Delta_x u - b \Delta_y v) \right] \\
 &= 2 \exp 2\pi i \left\{ (i'-i + \frac{a'-a}{2}) \Delta_x (u'+u) + (j'-j + \frac{b'-b}{2}) \Delta_y (v'+v) \right\} \quad (X.7) \\
 &\quad \cos 2\pi \left(a \Delta_x \frac{u'-u}{2} + b \Delta_y \frac{v'-v}{2} \right) \cos 2\pi \left(a' \Delta_x \frac{u'-u}{2} + b' \Delta_y \frac{v'-v}{2} \right)
 \end{aligned}$$

We will imagine this substituted into (X.6) and thence into (X.2) where we must sum the integrals. The sums are finite and can be taken into the integral where they are found to affect only the exponential factor of (X.7). This sum falls into the product of four sums of the form

$$\sum_{i'} e^{2\pi i \Delta_x (u' + u) i'} \quad (X.8)$$

and a factor

$$e^{2\pi i \left[\frac{a' - a}{2} \Delta_x (u' + u) + \frac{b' - b}{2} \Delta_y (v' + v) \right]} \quad (X.9)$$

The sum (X.8) is a geometrical series and its limits depend upon the value of a . It can be put into the closed form

$$\begin{aligned} \sum_{i'} e^{2\pi i \Delta_x (u + u') i} &= e^{2\pi i \Delta_x \frac{u + u'}{2} (n_x - a' + 1)} \\ &= \frac{\sin 2\pi \Delta_x \frac{u + u'}{2} (n_x - |a|)}{\sin 2\pi \Delta_x \frac{u + u'}{2}} \end{aligned}$$

The other sums may be worked out similarly. Their product consists of four factors that are sine ratios and an exponential factor which will be found to be the reciprocal of (X.9). Thus, the formula (X.2) for the covariance of two γ values becomes

$$\begin{aligned} & \frac{2}{(n_x - |a|)(n_x - |a'|)(n_y - |b|)(n_y - |b'|)} \iiint \iiint \\ & \frac{\sin \pi \Delta_x (u + u') (n_x - |a'|)}{\sin \pi \Delta_x (u + u')} \frac{\sin \pi \Delta_x (u + u') (n_x - |a|)}{\sin \pi \Delta_x (u + u')} \quad (X.10) \end{aligned}$$

$$\frac{\sin \pi \Delta_y (v + v') (u_y - |b'|)}{\sin \pi \Delta_y (v + v')} \quad \frac{\sin \pi \Delta_y (v + v') (u_y - |b|)}{\sin \pi \Delta_y (v + v')} \quad (\times 10 \text{ con.})$$

$$\cos 2\pi (a \Delta_x \frac{u'-u}{2} + b \Delta_y \frac{v'-v}{2})$$

$$\cos 2\pi (a' \Delta_x \frac{u'-u}{2} + b' \Delta_y \frac{v'-v}{2}) F(u, v) F(u', v') du dv du' dv'$$

Before substituting this long expression into (X.1) where it must be multiplied by coefficients and summed, let us examine the integrand. The sine ratios may be combined with the first factor to give four functions of the form

$$\frac{\sin(A\xi E)}{N \sin A\xi}$$

Each of these has a spike at $\xi = 0$ of height 1. and also spikes at $\xi = \frac{K\pi}{A}$ for the integer K. The factor N affects only the width of the spikes. If we adopt assumption A.3 the dependence of the sine ratio terms on a, b, a', b' largely disappears. The product of four factors is approximately a product of two squares, and the indices of summation in (X.1) affect only the cosine terms of (X.10). Let us take the sum and the factors $g_{ab} h_{a'b'}$ inside the integral whence we have the sum

$$\sum_a \sum_b g_{ab} \cos 2\pi (a \Delta_x \frac{u'-u}{2} + b \Delta_y \frac{v'-v}{2}) \sum_{a'} \sum_{b'} h_{a'b'} \cos 2\pi (a' \Delta_x \frac{u'-u}{2} + b' \Delta_y \frac{v'-v}{2})$$

$$= G(\frac{u'-u}{2}, \frac{v'-v}{2}) H(\frac{u'-u}{2}, \frac{v'-v}{2})$$

where G and H are two of the functions W defined in (2.4),
i.e., W for two different pairs (u, v) .

We may now express (X.1) as an integral of the spectral
function. Let us denote the "common value" of

$$n_x - |a| = n_x - |a'| = N_x$$

$$n_y - |b| = n_y - |b'| = N_y$$

$$\text{cov}(f_1, f_2) = 2 \iiint \left[\frac{\sin \pi \Delta_x (u+u') N_x}{N_x \sin \pi \Delta_x (u+u')} \right]^2 \left[\frac{\sin \pi \Delta_y (v+v') N_y}{N_y \sin \pi \Delta_y (v+v')} \right]^2$$

$$G\left(\frac{u'-u}{2}, \frac{v'-v}{2}\right) H\left(\frac{u'-u}{2}, \frac{v'-v}{2}\right) F(u, v) F(u', v') du dv du' dv'$$

Let us change variables so

$$\begin{cases} \frac{1}{2}(u'+u) & = w \\ \frac{1}{2}(u'-u) & = w' \\ \frac{1}{2}(v'+v) & = z \\ \frac{1}{2}(v'-v) & = z' \end{cases} \quad \begin{cases} w+w' & = u' \\ w-w' & = u \\ z+z' & = v' \\ z-z' & = v \end{cases}$$

$$\frac{\partial(w, w', z, z')}{\partial(u, u', v, v')} = |J| = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = \left| -\frac{1}{4} \right| = \frac{1}{4}$$

$$\text{cov}(f_1, f_2) = 2 \iiint \left(\frac{\sin 2\pi \Delta_x N_x w}{N_x \sin 2\pi \Delta_x w} \right)^2 \left(\frac{\sin 2\pi \Delta_y N_y z}{N_y \sin 2\pi \Delta_y z} \right)^2 G(w', z') H(w', z')$$

$$\times F(w-w', z-z') F(w+w', z+z') 4 dw dw' dz dz'$$

Carrying out the integration first with respect to w and z we note that the squared sine ratios have spikes for w (or z) equal to $0, \pm \frac{1}{2\Delta_x}$ (or $0, \pm \frac{1}{2\Delta_y}$) etc. and that except for these spikes whose width is about $2 \cdot \frac{1}{2N_x \Delta_x}$ (or $2 \cdot \frac{1}{2N_y \Delta_y}$) the values are quite small. Using assumption A.4, F is relatively constant inside the peak at 0 and has very small values in the region of the other peaks. The first two integrations

$$\delta \iint G(w'z') H(w'z') [F(w'z')]^2 \left\{ \int_{-\frac{1}{4\Delta_x}}^{\frac{1}{4\Delta_x}} \left(\frac{\sin 2\pi \Delta_x N_x w}{N_x \sin 2\pi \Delta_x w} \right)^2 dw \right. \\ \left. \int_{-\frac{1}{4\Delta_y}}^{\frac{1}{4\Delta_y}} \left(\frac{\sin 2\pi \dots}{N_y \sin 2\pi \Delta_y z} \right) dz \right\} dw' dz'$$

The two integrals of the sine ratio may be evaluated from Fejer's integral (Titchmarsh "Theory of Functions", p. 413). The result is

$$\text{cov}(f_1, f_2) = \frac{2}{\Delta_x \Delta_y N_x N_y} \iint G(w'z') [F(w'z')]^2 dw' dz' \quad (X 12)$$

From assumption A.5 when G and H are two different w functions their non zero parts do not overlap and so the covariance of two spectral estimates is zero.

The variance of f is the covariance of f with itself and formula (X.11)

$$\text{var}(f) = \frac{2}{\Delta_x \Delta_y N_x N_y} \iint [G(w'z') F(w'z')]^2 dv' dz' \quad (\text{X } 13)$$

Let us recall that our purpose is to compute the degrees of freedom using formula (3.5). Our result (X.12) will be in the denominator and the numerator will have

$$[E\{f\}]^2 = \left[\iint G(w', z') F(w', z') dw' dz' \right]^2 \quad (\text{X } 14)$$

This follows from taking the expected value on both sides of (3.3) then using (2.2). In both (X.12) and (X.13) we will take $G(w'z')$ to be an abbreviation of $w(u_0, v_0; w', z')$ of (2.4)

To attain approximations of the two integrals we will make strong use of assumption A.5 and also assume an idealized rectangular form for G . This form proceeds from our discussion at the end of Section 2; it is that G is 0 in the relevant region of frequencies (w', z') except in two rectangles of dimensions $\frac{1}{m_x \Delta_x} \times \frac{1}{m_y \Delta_y}$ centered at (u_0, v_0) and $(-u_0, -v_0)$. The total volume of these rectangles should be unity so the height of G will be $\frac{1}{2} (m_x m_y \Delta_x \Delta_y)$. With this (X.12) and (X.13) work out to be approximately

$$\begin{aligned} \text{var}\{f\} &= \frac{2}{\Delta_x \Delta_y N_x N_y} [F(u_0, v_0)]^2 (m_x m_y \Delta_x \Delta_y) \\ [E\{f\}]^2 &= [F(u_0, v_0)]^2 \end{aligned} \quad (\text{X } 15)$$

The degrees of freedom formula (3.5)

$$\text{d. f.} = 2K = \frac{2 [F(u_0 v_0)]^2}{[F(u_0 v_0)]^2 \frac{2 m_x m_y}{N_x N_y}} = \frac{N_x N_y}{M_x M_y}$$

Now if we recall the definition (X.11) of N_x and N_y we have as an approximation

$$\text{d. f.} = \frac{n_x - m_x}{m_x} \frac{n_y - m_y}{m_y}$$

which, when we use the fixed fraction g for both m_x and m_y we get the formula (3.6). This approximation is probably as good as the assumptions justify but more exact results may be found by approximating more closely (X.13) and (X.14) using the actual W function instead of the rectangular idealization.

4. Filtering

The assumptions at the end of Section 3 are used in both the statistical work and, as we pointed out in the discussion of them, in relating the theoretical work to the practical. We will discuss here assumptions A.4 and A.5 beginning with the latter.

The shape of the function $W(\cdot)$ should be two symmetrically placed peaks on a field of zeros. Though considerations in Section 2 indicate that the peaks should be as narrow and high as possible around the central point, the statistical considerations expounded in equations (X.14) and (X.15) indicate that the variance of f increases with the narrowness of the peak. Thus we are led to seek a box shaped $W(\cdot)$ function. The choice of this function is the subject of a good deal of literature. Here we will follow some of the early work--that given in Blackman and Tukey, "Measurement of Power Spectra". This will take the direction of using as standard the $W(\cdot)$ given by (2.6) and modifying it by filtering. Let us note first several formulas concerning finite Fourier expansions.

Suppose we have an expansion $C(u, v)$ with coefficients $c_{a,b}$

$$C(u, v) = \sum_{a=-m_x}^{m_x} \sum_{b=-m_y}^{m_y} c_{ab} e^{2\pi i (a \Delta_x u + b \Delta_y v)}$$

and we want to select evenly spaced values of (u, v) to "represent" $W(u, v)$. Let us take

$$u = 0, \pm \delta_x, \pm 2\delta_x, \dots, \pm m_x \delta_x$$

$$v = 0, \pm \delta_y, \pm 2\delta_y, \dots, \pm m_y \delta_y$$

$$C_{j,k} \equiv C(j\delta_x, k\delta_y) = \sum_{a=-m_x}^{m_x} \sum_{b=-m_y}^{m_y} c_{a,b} e^{2\pi i(\Delta_x \delta_x a j + \Delta_y \delta_y b k)}$$

Multiplying both sides by

$$e^{2\pi i(\Delta_x \delta_x \bar{a} j + \Delta_y \delta_y \bar{b} k)}$$

and summing on

$$(j, k)$$

gives

$$\sum_{j=-m_x}^{m_x} \sum_{k=-m_y}^{m_y} C_{j,k} e^{2\pi i(\Delta_x \delta_x \bar{a} j + \Delta_y \delta_y \bar{b} k)}$$

$$= \sum_a \sum_b c_{a,b} \sum_j \sum_k e^{2\pi i(\Delta_x \delta_x j(a-\bar{a}) + \Delta_y \delta_y k(b-\bar{b}))}$$

The second sums are easy to work out. The exponential is readily separated into a product and gives a product of two geometric series, each of the form

$$\begin{aligned} \sum_{j=-m_x}^{m_x} e^{2\pi i \Delta_x \delta_x j(a-\bar{a})} &= \frac{e^{2\pi i \Delta_x \delta_x m_x(a-\bar{a})} - e^{2\pi i \Delta_x \delta_x (m_x+1)(a-\bar{a})}}{1 - e^{2\pi i \Delta_x \delta_x (a-\bar{a})}} \\ &= \frac{\sin \pi \Delta_x \delta_x (2m_x+1)(a-\bar{a})}{\sin \pi \Delta_x \delta_x (a-\bar{a})} \end{aligned}$$

This formula holds for $a \neq \bar{a}$. For $a = \bar{a}$ the sum is evidently $(2m_x+1)$.

Now if we take $\Delta_x \delta_x = \frac{1}{2m_x+1}$ the numerator is zero for all the (integral) values of $(a - \bar{a})$ while the denominator is not. From this

$$\begin{aligned} \sum_j \sum_k e^{2\pi i [\Delta_x \delta_x j(a-\bar{a}) + \Delta_y \delta_y k(b-\bar{b})]} \\ &= \begin{cases} (2m_x+1)(2m_y+1) & \text{if } a=\bar{a}, b=\bar{b} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{4.1}$$

This means we can solve (4.1) for the $c_{a,b}$ in terms of the $C_{j,k}$ when

$$C_{j,k} = C \left[\frac{j}{(2m_x+1)\Delta_x}, \frac{k}{(2m_y+1)\Delta_y} \right]$$

Let us display the two formulas

$$C_{j,k} = \sum_{a=-m_x}^{m_x} \sum_{b=-m_y}^{m_y} c_{a,b} e^{2\pi i \left(\frac{aj}{2m_x+1}, \frac{bk}{2m_y+1} \right)} \quad (4.2)$$

$$= \sum_a \sum_b c_{a,b} \cos 2\pi \left(\frac{aj}{2m_x+1}, \frac{bk}{2m_y+1} \right)$$

$$\begin{aligned} c_{a,b} &= \frac{1}{(2m_x+1)(2m_y+1)} \sum_j \sum_k C_{j,k} e^{-2\pi i \left(\frac{aj}{2m_x+1} + \frac{bk}{2m_y+1} \right)} \\ &= \frac{1}{(2m_x+1)(2m_y+1)} \sum_j \sum_k C_{j,k} \cos 2\pi \left(\frac{aj}{2m_x+1} + \frac{bk}{2m_y+1} \right) \end{aligned} \quad (4.3)$$

These relations are analogous to the usual reciprocal relations for Fourier series, e.g., (2.5) and (2.8). More important to the problem of filtering is the following convolution formula. Suppose

C and $\bar{C}$ are given by c and $\bar{c}$ through formula (4.2), then their convolution is defined as

$$\frac{1}{(2m_x+1)(2m_y+1)} \sum_{\alpha=-m_x}^{m_x} \sum_{\beta=-m_y}^{m_y} C_{j-\alpha, k-\beta} \bar{C}_{\alpha\beta} = T_{jk} \quad (4.4)$$

where C is regarded here as a periodic function having values outside the range $(-m_x, m_x; -m_y, m_y)$. To be specific, $C_{m_x+1, k} = C_{-m_x, k}$; $C_{m_x+2, k} = C_{-m_x+1, k}$; etc. If we transform T_{jk} by a formula such as (4.3)

$$\begin{aligned} t_{ab} &= \frac{1}{(2m_x+1)^2(2m_y+1)^2} \sum_j \sum_k \sum_{\alpha} \sum_{\beta} C_{j-\alpha, k-\beta} \bar{C}_{\alpha\beta} e^{2\pi i \left(\frac{aj}{2m_x+1} + \frac{bk}{2m_y+1} \right)} \\ &= \frac{1}{(2m_x+1)^2(2m_y+1)^2} \sum_{\alpha} \sum_{\beta} \bar{C}_{\alpha\beta} e^{2\pi i \left(\frac{a\alpha}{2m_x+1} + \frac{b\beta}{2m_y+1} \right)} \times \\ &\quad \times \sum_j \sum_k C_{j-\alpha, k-\beta} e^{2\pi i \left(\frac{a(j-\alpha)}{2m_x+1} + \frac{b(k-\beta)}{2m_y+1} \right)} \end{aligned}$$

Since in the second sum both "C" and the exponential term are periodic with period $(2m_x+1)$ by $(2m_y+1)$ we can rearrange the summands to achieve the sum that could be gotten by erasing α and β . This gives us, from (4.3),

$$t_{ab} = c_{ab} \bar{c}_{ab} \quad (4.5)$$

So the Fourier coefficients of a convolution are the products of the coefficients of the two functions being convolved.

If we define a convolution of coefficients as

$$\sum_{\alpha=-m_x}^{m_x} \sum_{\beta=-m_y}^{m_y} c_{a-\alpha, b-\beta} \bar{c}_{\alpha\beta} = y_{ab} \quad (4.6)$$

then we may derive similarly,

$$\sum_a \sum_b y_{ab} e^{2\pi i \left(\frac{aj}{2m_x+1} + \frac{bk}{2m_y+1} \right)} = c_{jk} \bar{c}_{jk} = y_{jk} \quad (4.7)$$

so that the expansion whose coefficients are convolutions is the product of two expansions.

We will apply (4.4) and (4.5) to the situation where $\bar{c}_{a,b} = r_{a,b}$ of (3.1) so that $c_{j,k}$ are spectral estimates of the type (2.2) whose filter function is (2.6). With this we may take $c_{a,b}$ so that

$$c_{ab} \cos 2\pi \left(\frac{aj}{2m_x+1} + \frac{bk}{2m_y+1} \right) = w(a, b; j \delta_x, k \delta_y) \quad (4.9)$$

where w are the coefficients for the spectral estimates (3.3), and correspond to a desirable filter function $W(\cdot)$, (2.5). Then from (4.4) and (4.5) and (3.3) we have,

$$\begin{aligned} f_{\alpha, \beta} &= \sum_a \sum_b w(a, b; \alpha \delta_x, \beta \delta_y) r_{ab} = \sum_a \sum_b c_{ab} \cos 2\pi \left(\frac{a\alpha}{2m_x+1} + \frac{b\beta}{2m_y+1} \right) r_{ab} \\ &= \frac{1}{2m_x+1} + \frac{1}{2m_y+1} \sum_a \sum_b c_{\alpha-a, \beta-b} \bar{c}_{ab} \end{aligned} \quad (4.10)$$

From this we have a fundamental relation. $\bar{C}$ is the "raw spectrum" of the covariances $r_{a,b}$ and the coefficients C are used to "smooth" it by taking a running average. By this we may achieve a variety of $W(\cdot)$ functions. Note that the actual $W(\cdot)$ function is a transform (2.5) of the product of the C 's with cosines and is therefore a convolution of the type in (4.8). All the filters used in practice are given either in terms of the running average weights

C or in terms of the coefficients c which modify the covariances before making a Fourier transformation. Corresponding to the two sides of the convolution theorem, we have two ways of computing the spectrum. We may first compute a "raw spectrum" or cosine transformation of $r_{a,b}$ and then smooth it by a running average. This is convenient when only a few of the running average coefficients $C_{j,k}$ are non-zero. Or we may modify the $r_{a,b}$ by multiplying them by the factors $c_{a,b}$ then take the cosine transformation. This is done when the c 's are easy to compute.

An application of the convolution relations is to smoothing the profile which was discussed for the one-dimensional case in Section II, Part 4. We should note that the work of the section deals primarily with the data and not with the mathematical model of randomness. The various sequences r_{ab} , $f_{\alpha\beta}$, are handled as even, periodic sequences. In actual computation of the sums we do not go back to the beginning after exhausting the table so that the numerical results are affected by "end effects" which cause them to depart somewhat from the formulas presented here.

Section 5. The two-dimensional spectral computer program.

The program operates using data as outlined in the preceding sections. The general operation of the program involves the steps:

- (1) The area data is smoothed by taking differences from one fitted quadratic polynomial.
- (2) The covariances of the smoothed data are computed.
- (3) A preliminary spectrum of these covariances is computed.
- (4) A trigonometric polynomial is fitted to the reciprocals of the spectral values, its coefficients are used to make a running average smoothing of the covariances.
- (5) A new spectrum of the smoothed covariances is computed.
- (6) The spectrum is smoothed by the Hamming method.
- (7) The spectrum is corrected using the reciprocals of the polynomial (4) as factors.

The program was checked by running some sea surface data from the report by Pierson mentioned in Section I. The results checked very well except for the latter part of the program.

Recently this part has been made to operate correctly.

The model of this program was the one (Section II, Part 5) which we were using on the linear data at the time the specifications were sent to the programmer. Our later experience with the linear program indicates that the quadratic polynomial method of smoothing is not enough for our purposes, and that the prewhitening steps may be accomplished better by using an improved method of smoothing. In order to complete the two-dimensional analysis, the program is being modified.

6. Preliminary Spectral Estimate

Table 5 contains a preliminary spectral estimate from our present program using the Aberdeen data. These results are reproduced to illustrate some of the comments we have made regarding two-dimensional results. Complete studies of this two-dimensional data will be presented in a later report.

TABLE 1

ABERDEEN - 1500 Ft.

TWO FOOT SPACING

	0	2	4	6	8	10	12	14	16	18
0	0.19	0.65	0.51	0.70	0.36	0.04	-0.09	-0.73	-0.80	-1.17
20	-1.25	-1.31	-1.17	-1.06	-0.80	-0.30	0.28	0.72	0.72	0.55
	0.40	0.23	0.16	-0.01	-0.07	-0.02	0.07	-0.04	-0.28	-0.71
	-1.22	-1.58	-1.95	-2.21	-1.98	-1.90	-1.50	-1.04	-0.43	0.38
	0.81	1.18	1.40	1.61	1.83	2.03	2.04	2.04	1.94	2.01
	1.01	0.49	0.28	-0.05	-0.10	-0.17	-0.28	-0.14	-0.01	0.32
	0.81	1.09	0.95	0.74	0.33	0.16	0.04	-0.08	-0.18	-0.10
	0.03	0.14	0.47	1.06	1.57	1.90	2.12	2.10	1.88	1.86
	1.69	1.39	1.08	0.99	0.87	0.58	0.58	0.59	0.77	1.00
	1.27	1.62	2.02	2.14	1.96	1.63	2.19	0.87	0.93	0.82
200	0.75	0.75	0.63	0.26	-0.05	-0.36	-0.69	-0.53	-0.12	0.23
	1.55	1.64	1.43	1.37	1.51	1.25	0.94	0.73	0.68	0.60
	0.62	1.03	1.00	1.16	1.25	1.11	0.91	0.68	0.36	-0.02
	-0.25	-0.56	-0.86	-0.89	-0.92	-0.92	-0.92	-0.92	-0.92	-0.92
	-0.92	-0.92	-0.92	-0.92	-0.95	-0.71	-0.47	-0.22	0.35	0.80
	1.07	1.49	1.85	1.91	1.79	1.55	1.20	0.89	0.64	0.60
	0.31	0.50	1.07	1.36	1.60	1.74	2.14	2.03	2.01	1.36
	2.03	1.37	0.85	0.52	0.40	0.25	0.07	-0.05	-0.25	-0.30
400	-0.31	-0.25	0.10	0.89	1.13	1.39	1.83	1.92	1.83	1.35
	0.92	0.93	0.77	0.35	0.09	-0.05	-0.19	0.11	0.04	0.06
	0.06	0.83	1.18	1.27	1.35	1.44	1.57	1.55	1.68	1.75
	1.83	1.95	1.86	2.12	2.21	2.32	2.29	2.24	2.21	2.12
	2.11	2.26	2.33	2.48	2.38	2.43	2.44	2.51	2.58	2.64
	2.71	2.76	2.89	3.00	3.07	3.32	3.53	3.57	3.40	2.96
	2.92	2.70	2.28	1.64	1.37	1.19	1.15	1.10	1.23	1.52
	1.83	1.83	2.11	2.01	1.69	1.50	1.40	1.22	1.15	1.19
	1.35	1.71	2.16	2.22	2.55	2.71	3.43	3.24	3.05	2.99
	2.93	2.92	3.11	3.15	3.24	3.46	3.55	3.76	4.03	3.83
	3.71	3.41	3.13	3.22	3.26	3.19	3.08	3.27	3.49	3.15
600	3.03	2.81	2.59	2.60	2.46	2.48	2.40	2.31	2.19	2.11
	2.25	2.25	2.25	2.30	2.12	2.24	2.07	2.00	1.89	1.83
	1.75	1.35	1.28	1.53	1.43	1.48	1.93	2.79	3.22	3.29
	3.36	3.32	3.35	3.37	3.22	3.21	3.14	3.04	2.65	2.32
	2.00	1.93	1.72	1.67	1.44	1.10	1.26	1.42	2.00	2.14
	2.13	1.70	1.38	1.38	0.87	0.82	0.79	0.88	0.72	0.74
	0.30	0.30	0.64	1.18	1.61	2.30	2.58	2.70	2.70	2.48
	2.26	2.14	1.72	1.71	1.65	1.55	1.35	1.26	1.37	1.60
	1.67	1.72	1.67	-1.70	-1.64	1.52	-1.14	0.90	0.64	0.63
	0.50	0.26	0.45	0.30	0.45	0.97	1.60	1.93	2.00	1.98
	2.02	2.00	2.10	2.03	2.00	2.10	1.89	1.84	1.76	1.64

TABLES AND CHARTS

Each table of results consists of five parts. The first part is the raw profile heights given in feet and hundredths above the surveyors' datum. The heights are presented in reading order, left to right and down. The numbers of feet from the beginning for certain of the heights in the first column are given on the left margin. On the top margin are given the numbers of feet to add to the position of an item in the left column so as to obtain the positions of the heights in other columns of the same row. The second parts of the tables, labeled A, are indexed similarly. These parts contain smoothed heights.

The third tables, labeled B, give autocorrelations of the smoothed data in ft.^2 for the various lag numbers, k . The fourth tables, labeled C, give the spectral density estimates of the smoothed data in ft.^2 for various frequencies, λ , in cycles/ft. The last tables, D, give the corrected spectral estimates, and are labeled similarly.

After making the calculations, several small errors were found in the raw data of Aberdeen, Knox, and Yuma. The number of the errors was small, at most fifteen in the Yuma data, and the sizes were on the order of a few tenths of a foot. The effects on the calculations were deemed to be minor so they were not done over. The raw data portion of each table is presented in a correct version, but the other parts are as calculated.

TABLE 1A
SMOOTHED ABERDEEN - 1500 Ft. TWO FOOT SPACING

	0	2	4	6	8	10	12	14	16	18
0	-0.19556	0.28111	0.18889	0.48111	0.26778	0.09889	0.18000	-0.25778	-0.12000	-0.33222
20	-0.31889	-0.35556	-0.32778	-0.38667	-0.33667	-0.03667	0.35333	0.63778	0.50222	0.24444
	0.06889	-0.06778	-0.06556	-0.15111	-0.11889	0.05444	0.30556	0.38889	0.36444	0.17222
	-0.12000	-0.26111	-0.46889	-0.64444	-0.44556	-0.54333	-0.40889	-0.29667	-0.08778	0.32333
	0.33889	0.31567	0.19444	0.13000	0.17667	0.24333	0.27222	0.37333	0.31000	0.53889
	-0.17444	-0.44889	0.31444	-0.78444	-0.61000	-0.60333	1.81111	-0.58778	-0.56889	-0.33222
	0.10222	0.61778	0.45778	0.25556	-0.09889	-0.16778	-0.17000	-0.20000	-0.27000	-0.27111
	-0.29778	-0.39444	-0.30889	0.02778	0.31778	0.44444	0.49222	0.37000	0.14778	0.19222
	0.13667	0.00778	-0.13333	-0.08000	-0.07889	-0.29222	-0.27889	-0.32889	-0.26333	-0.17444
	-0.05778	0.17556	0.39778	0.50667	0.33444	0.05444	0.71111	-0.46778	-0.24000	-0.16111
	-0.04444	0.23889	0.29222	0.08444	-0.12111	-0.37333	-0.79222	-0.74444	-0.46444	-0.27222
200	0.84000	0.71444	0.34111	0.18667	0.27667	0.12222	-0.07444	-0.24000	-0.24889	-0.29000
	-0.27000	0.12111	0.07111	0.23111	0.34778	0.27889	0.22111	0.16444	0.06889	-0.07333
	-0.07778	-0.18444	-0.30667	-0.19444	-0.12444	-0.05000	-0.01000	-0.00333	-0.00000	-0.00000
	0.00333	-0.02000	-0.07000	-0.14778	-0.31889	-0.27000	-0.25111	-0.26889	-0.00567	0.12556
	0.11778	0.31333	0.51556	0.51556	0.41333	0.22556	0.00667	-0.15333	-0.31000	-0.30222
	-0.59778	-0.46778	-0.03667	0.09889	0.18222	0.20556	0.43556	0.29222	0.32889	-0.20111
	0.61778	0.16778	-0.13444	-0.23556	-0.17667	-0.06778	-0.06111	-0.05889	-0.21222	-0.31667
	-0.42444	-0.51111	-0.37000	0.17889	0.18222	0.25778	0.56778	0.56556	0.48889	0.29556
400	-0.19000	0.02889	0.10333	-0.12556	-0.24000	-0.28444	-0.32778	-0.03444	-0.19667	-0.30778
	-0.46333	0.12556	0.31333	0.23556	0.13556	0.03778	0.05667	-0.04889	0.01556	0.00000
	-0.00556	0.03111	-0.14111	0.05667	0.09556	0.17333	0.12556	0.03111	-0.02222	-0.14222
	-0.15889	-0.02444	0.02333	0.14000	-0.01111	-0.02000	-0.06000	-0.03778	-0.01333	-0.02222
	-0.02333	-0.07111	-0.05444	-0.05444	-0.06889	0.15333	0.34556	0.40667	0.31667	0.03556
	0.21222	0.25222	0.10111	-0.28333	-0.36111	-0.38556	-0.32889	-0.32889	-0.25111	-0.03222
	0.22222	0.18333	0.43000	0.33111	0.05222	-0.06667	-0.11333	-0.24889	-0.33556	-0.35444
	-0.31111	-0.09667	0.10778	-0.06444	0.05889	0.03667	0.62111	0.34667	0.05778	-0.06889
	-0.18778	-0.20111	-0.04556	-0.08444	-0.11000	0.01000	0.01222	0.18889	0.46111	0.26333
	0.16556	-0.09444	-0.29889	-0.12444	-0.04667	-0.05444	-0.12222	0.10333	0.39333	0.12667
	0.08778	-0.06556	0.18889	-0.04778	0.08111	0.04111	0.02333	-0.02889	-0.11000	-0.17222
600	-0.00778	0.02556	-0.05222	0.12333	-0.03222	0.13444	0.02000	-0.05000	0.05333	0.05889
	0.06889	-0.26556	-0.32778	-0.17778	-0.43222	-0.55333	-0.32667	0.30567	0.53444	0.34889
	0.26556	0.06333	0.07444	0.11444	0.03556	0.14111	0.21778	0.27556	0.06889	-0.09289
	-0.21222	-0.05556	-0.06778	0.01889	-0.17556	-0.53111	-0.39333	-0.23111	0.38111	0.52778
	0.54333	0.16222	-0.08778	0.03667	-0.31556	-0.21111	-0.08556	0.12444	0.04667	0.03222
	-0.49556	-0.66333	-0.51222	-0.19222	0.02000	0.46778	0.53000	0.48333	0.42333	0.19222
	0.04444	-0.03889	-0.23111	-0.08111	-0.01778	-0.04444	-0.19222	-0.28222	-0.16778	-0.56667
	0.11667	0.14778	0.11111	0.19333	0.24000	0.23556	-0.00889	-0.09222	-0.21333	-0.07444
	-0.08556	-0.30667	-0.19444	-0.48778	-0.49000	-0.13444	0.30000	0.45778	0.32778	-0.13222
	0.05778	-0.01778	0.08667	0.03444	0.02889	0.17111	-0.00778	-0.02556	-0.06111	-0.17556

TABLE 1A CONT.

800

-0.06778	0.02333	0.01444	-0.00778	-0.01222	0.03444	0.07222	0.02889	0.03333	0.05333
0.09667	0.03556	0.10222	0.02333	0.09000	0.10333	-0.08000	-0.32000	-0.26778	-0.16667
-0.25556	0.01667	0.25444	0.61111	0.35333	0.14778	0.00000	-0.09889	-0.42556	-0.41444
-0.42667	-0.31556	-0.15333	0.32444	0.51556	0.32000	0.11111	-0.09222	-0.15222	-0.17333
-0.23000	-0.25222	-0.21444	-0.06000	0.23000	0.41444	0.37889	0.32333	0.00111	-0.05889
-0.05667	-0.02111	0.01000	0.03222	0.02222	-0.05667	0.00000	0.07667	0.13444	0.18778
0.06000	-0.03444	-0.05667	-0.07778	-0.39889	-0.35222	-0.18333	-0.16222	-0.18333	-0.06222
-0.13111	-0.09000	-0.09556	0.29444	0.27000	0.15889	0.25889	0.25000	-0.58222	0.34889
0.06444	0.24667	0.13667	0.01222	0.20556	0.07778	-0.05222	-0.27333	-0.95000	0.29333
0.23111	0.20222	0.02444	-0.19889	-0.11111	-0.01222	0.01000	-0.01889	-0.25889	0.13667
0.26889	0.34444	0.13778	-0.05778	-0.30778	-0.33556	0.12778	-0.11222	-0.38778	-0.25667
0.16333	-0.13111	0.00222	-0.00333	0.01889	0.06556	-0.00778	0.07556	0.13444	0.05667
0.00000	0.09222	0.09222	0.08778	0.05778	0.05000	-0.05111	-0.07667	0.03889	0.05333
-0.01111	0.04222	0.12000	0.43000	0.73889	0.33222	-0.41333	-0.59000	-0.39333	-0.24444
-0.23111	-0.13556	0.14222	-0.58222	-0.06333	0.49667	0.42444	0.29444	0.13333	-0.00778
0.01111	0.14889	0.05333	0.03556	-0.04333	-0.06222	-0.07333	-0.01889	-0.15556	-0.09556
-0.11667	-0.11889	-0.15222	-0.22333	-0.38889	-0.37111	-0.10444	0.23444	0.50889	0.20222
0.20222	0.07222	0.01889	0.09778	0.02444	0.03111	0.04667	-0.51111	0.10889	0.08333
0.07333	0.10333	0.00667	0.02667	-0.00667	-0.02889	-0.04889	-0.07667	0.02556	0.00111
-0.00333	-0.30111	-0.02556	-0.01778	0.02778	0.01333	0.03000	0.00778	-0.01000	-0.01000
-0.01444	-0.03333	0.01111	-0.01889	0.00222	0.01444	0.03111	0.00444	-0.02333	-0.04333
-0.00889	0.09222	0.09667	0.09000	-0.00444	-0.00889	-0.00889	-0.06222	-0.05444	0.02111
0.04222	0.13778	0.16222	0.44778	0.38222	0.32778	0.36444	-3.07333	0.28556	0.17889
0.19444	0.17333	-0.10000	0.11333	0.15222	0.13000	0.07667	-0.02000	-0.01333	0.04000
0.07889	0.09333	-0.01556	-0.03556	-0.01333	-0.03444	-0.03556	-0.09778	-0.07222	0.07778
0.07667	0.04889	0.01333	-0.03222	-0.04667	0.00667	0.02333	-0.02000	0.05556	0.12444
0.15778	0.05444	-0.06556	-0.16667	-0.23333	0.45444	0.50778	-0.53667	-0.40778	0.72556
0.79556	0.36556	-0.75889	-0.81889	-0.65778	-0.30889	-0.08889	-0.23333	-0.21444	-0.15667
0.00333	0.74556	0.21778	0.15000	0.07111	-0.05111	-0.05444	-0.08000	-0.15778	-0.24778
-0.26889	-0.19444	-0.11778	0.02667	0.94000	0.80667	0.76667	-0.13889	-0.46444	-0.54444
-0.19667	0.01778	-0.06333	0.00889	-0.01778	0.13333	0.20444	-0.49444	-0.33111	-0.26333
-0.16667	0.10778	0.04778	0.04333	0.21778	0.21000	0.30111	0.31667	0.22333	0.10444
-0.01222	-0.02333	-0.42000	-0.02889	0.02333	-0.23778	-0.29333	-0.20889	-0.16222	-0.08111
0.16556	0.31222	0.35444	0.27667	-0.07778	0.15556	-0.34444	-0.43778	-0.39333	-0.29556
-0.06778	0.09667	0.13111	0.20778	0.14222	0.05444	0.06111	0.08778	0.13000	0.19000

1000

1200

1400

TABLE 1B
AUTOCORROLATION ABERDEEN
SMOOTHED DATA

k	1-1500 ft.	1-750 ft.	750-1500 ft.
1	0.0280	0.0291	0.0268
2	0.0379	0.0489	0.0268
3	0.0135	0.0244	0.0026
4	0.0073	0.0005	0.0141
5	0.0149	0.0118	0.0181
6	0.0171	0.0252	0.0092
7	0.0225	0.0318	0.0135
8	0.0219	0.0276	0.0162
9	0.0203	0.0263	0.0142
10	0.0132	0.0180	0.0079
11	0.0029	0.0138	0.0086
12	0.0002	0.0084	0.0093
13	0.0026	0.0018	0.0073
14	0.0031	0.0041	0.0020
15	0.0040	0.0101	0.0027
16	0.0062	0.0118	0.0003
17	0.0057	0.0117	0.0016
18	0.0044	0.0112	0.0036
19	0.0023	0.0082	0.0044
20	0.0003	0.0045	0.0041
21	0.0020	0.0046	0.0000
22	0.0018	0.0040	0.0004
23	0.0001	0.0018	0.0011
24	0.0015	0.0047	0.0031
25	0.0018	0.0073	0.0055
26	0.0014	0.0074	0.0068
27	0.0042	0.0092	0.0024
28	0.0044	0.0068	0.0011
29	0.0042	0.0048	0.0036
30	0.0021	0.0030	0.0022
31	0.0002	0.0001	0.0019
32	0.0007	0.0005	0.0054
33	0.0015	0.0016	0.0053
34	0.0002	0.0010	0.0022
35	0.0044	0.0034	0.0048
36	0.0090	0.0084	0.0111
37	0.0087	0.0089	0.0123
38	0.0076	0.0097	0.0067
39	0.0026	0.0101	0.0042
40	0.0008	0.0069	0.0054
41	0.0048	0.0062	0.0045
42	0.0065	0.0093	0.0034
43	0.0156	0.0130	0.0180
44	0.0147	0.0127	0.0162
45	0.0116	0.0082	0.0135
46	0.0013	0.0102	0.0092
47	0.0023	0.0063	0.0109
48	0.0049	0.0029	0.0107
49	0.0062	0.0011	0.0082
50	0.0057	0.0051	0.0037
51	0.0099	0.0060	0.0117

TABLE 1C

POWER SPECTRAL DENSITY

SMOOTHED DATA

ABERDEEN

λ	1-1500 ft.	1-750 ft.	750-1500 ft.
0.00000	0.00239	0.01560	0.00479
0.00500	0.02014	0.02946	0.01434
0.01000	0.10575	0.10283	0.10202
0.01500	0.27612	0.30310	0.23916
0.02000	0.53145	0.69460	0.30944
0.02500	1.07736	1.69098	0.49692
0.03000	1.50238	2.20492	0.70204
0.03500	1.28241	1.71860	0.79920
0.04000	1.33669	1.44279	1.30112
0.04500	1.07281	1.02973	1.15177
0.05000	0.84050	0.89131	0.77151
0.05500	0.69880	0.70279	0.72529
0.06000	0.50667	0.49875	0.50718
0.06500	0.50175	0.57522	0.39657
0.07000	0.41796	0.50602	0.34092
0.07500	0.37456	0.32400	0.42598
0.08000	0.35928	0.23250	0.48397
0.08500	0.44999	0.21263	0.69028
0.09000	0.43022	0.24646	0.60723
0.09500	0.42739	0.23562	0.60828
0.10000	0.45689	0.25063	0.67066
0.10500	0.27400	0.19130	0.35643
0.11000	0.23433	0.20663	0.25778
0.11500	0.23103	0.27404	0.18538
0.12000	0.20347	0.20156	0.21707
0.12500	0.19745	0.17218	0.22566
0.13000	0.15222	0.14506	0.15994
0.13500	0.16626	0.14854	0.16353
0.14000	0.17335	0.13139	0.18790
0.14500	0.17077	0.16407	0.17863
0.15000	0.16942	0.13023	0.21114
0.15500	0.12592	0.08670	0.16351
0.16000	0.14245	0.11308	0.16873
0.16500	0.13742	0.10667	0.17185
0.17000	0.11710	0.08909	0.15352
0.17500	0.12579	0.07409	0.17735
0.18000	0.10680	0.06178	0.14961
0.18500	0.13301	0.05656	0.21701
0.19000	0.15245	0.03813	0.26533
0.19500	0.15763	0.05646	0.23216
0.20000	0.22056	0.11009	0.32025
0.20500	0.18520	0.12716	0.24235
0.21000	0.17471	0.11782	0.21327
0.21500	0.16536	0.15753	0.17378
0.22000	0.16584	0.15678	0.17666
0.22500	0.15657	0.17531	0.13980
0.23000	0.14259	0.13922	0.14534
0.23500	0.14512	0.12477	0.16547
0.24000	0.15285	0.12623	0.18039
0.24500	0.15049	0.11010	0.18736
0.25000	0.14910	0.13104	0.16608

TABLE 1D
POWER SPECTRAL DENSITY ABERDEEN

λ	0-1500 ft	0-750 ft	750-1500 ft
0.000	-----	-----	-----
0.005	116.19168	169.96063	82.73032
0.010	39.36861	38.28155	37.98000
0.015	21.11489	23.17805	18.28856
0.020	13.57748	17.74564	10.20489
0.025	12.09444	18.98294	5.57842
0.030	8.86158	13.00153	4.13964
0.035	4.56807	6.12182	2.84718
0.040	3.10900	3.35578	3.02627
0.045	1.78086	1.70935	1.91193
0.050	1.06373	1.12804	.97642
0.055	.71340	.71747	.74044
0.060	.43838	.43153	.43882
0.065	.38445	.44074	.30385
0.070	.29503	.35719	.24070
0.075	.25244	.21837	.28710
0.080	.23881	.15454	.32169
0.085	.30367	.14349	.46582
0.090	.30238	.17322	.42679
0.095	.31971	.17625	.45502
0.100	.37008	.21111	.54323
0.105	.24321	.16980	.31638
0.110	.22928	.20222	.25154
0.115	.24893	.29527	.19974
0.120	.24534	.23721	.25546
0.125	.24989	.21791	.28584
0.130	.20277	.19323	.21305
0.135	.22719	.20297	.25086
0.140	.23721	.24750	.22909
0.145	.22681	.21791	.23725
0.150	.21441	.16481	.26721
0.155	.14953	.10295	.19416
0.160	.15732	.13040	.18634
0.165	.14081	.10315	.17609
0.170	.11172	.06592	.15601
0.175	.11285	.06636	.15838
0.180	.09087	.05171	.12730
0.185	.10908	.04638	.16978
0.190	.12246	.03062	.21317
0.195	.13490	.04516	.22572
0.200	.17865	.09646	.26669
0.205	.15426	.10591	.20186
0.210	.15188	.11967	.18519
0.215	.15227	.14424	.15906
0.220	.16123	.15242	.17175
0.225	.16222	.18163	.14484
0.230	.15723	.15351	.16026
0.235	.16916	.14544	.19283
0.240	.18621	.15390	.21976
0.245	.18860	.13798	.23732
0.250	.18870	.16584	.08363

TABLE 2
FORT KNOX - 1800 Ft. TWO FOOT SPACING

	0	2	4	6	8	10	12	14	16	18
0	0.	0.02	0.08	0.17	0.26	-0.08	0.44	0.40	0.68	0.74
	0.65	0.40	0.56	0.88	0.66	0.55	0.13	0.36	0.57	0.48
	0.24	0.25	0.15	-0.02	-0.12	0.48	0.84	0.83	1.26	1.78
	1.48	1.02	1.06	1.35	0.71	1.14	1.38	1.58	1.28	1.23
	1.04	0.52	0.33	0.80	0.26	0.12	0.44	0.26	0.18	0.28
	-0.	0.09	0.06	-0.16	0.08	-0.12	-0.27	-0.32	-0.18	-0.22
	-0.24	-0.28	-0.12	-0.20	-0.27	-0.30	-0.27	-0.32	-0.30	0.03
	-0.40	-0.42	-0.41	-0.46	-0.82	-0.90	-0.72	-0.64	-0.48	-0.34
	-0.42	-0.42	-0.74	-0.70	-0.72	-0.69	-0.46	-0.53	-0.64	-0.68
	-0.67	-0.57	-0.72	-0.67	-0.62	-0.52	-0.52	-0.54	-0.56	-0.56
	-0.62	-0.62	-0.55	-0.74	-0.73	-0.64	-0.72	-0.67	-0.18	-0.94
	-0.65	-1.16	-1.22	-0.92	-0.94	-0.93	-0.92	-0.96	-0.92	-0.76
	-0.92	-1.05	-0.98	-0.93	-0.72	-0.74	-0.80	-0.72	-0.71	-0.72
	-0.92	-1.03	-0.86	-0.86	-0.92	-0.82	-0.79	-0.62	-0.62	-0.56
	-0.66	-0.72	-0.67	-0.57	-0.57	-0.60	-0.70	-0.62	-0.66	-0.82
	-0.88	-0.82	-0.78	-0.72	-0.57	-0.44	-0.70	-0.73	-0.92	-1.02
	-1.20	-1.27	-1.37	-1.12	-1.30	-1.32	-1.32	-1.29	-1.30	-1.32
	-1.27	-1.27	-1.34	-1.33	-1.34	-1.37	-1.40	-1.42	-1.42	-1.34
	-1.04	-1.37	-1.32	-1.47	-1.45	-1.47	-1.48	-1.53	-1.52	-1.60
	-1.52	-1.58	-1.57	-1.60	-1.62	-1.47	-1.50	-1.52	-1.36	-1.27
	-0.20	-0.15	-0.18	-0.37	-0.49	-0.51	-0.52	-0.72	-0.27	-0.35
	-0.21	0.04	0.05	0.21	0.13	0.09	-0.05	-0.16	-0.25	-0.21
	-0.15	-0.11	0.39	0.23	0.28	0.51	0.69	0.69	0.72	0.77
	0.79	0.76	0.65	0.54	0.47	0.37	0.29	0.45	0.49	0.39
	0.23	0.29	0.42	0.15	0.15	0.40	0.65	0.33	0.19	-0.41
	-0.64	-0.81	-0.82	-0.85	-0.93	-1.23	-1.02	-0.61	-0.88	-0.83
	-0.76	-0.64	-0.51	-0.43	-0.67	-0.75	-0.76	-0.78	-0.82	-0.82
	-0.86	-0.91	-0.95	-1.11	-1.13	-1.13	-1.05	-0.95	-0.86	-0.81
	-0.71	-0.49	-0.50	-0.51	-0.41	-0.61	-0.73	-0.78	-0.79	-0.75
	-0.71	-0.72	-0.71	-0.35	-0.39	-0.38	-0.51	-0.17	-0.51	-1.11
	-1.46	-1.61	-1.66	-1.59	-1.45	-1.39	-1.31	-1.20	-1.09	-1.14
	-1.25	-1.33	-1.41	-1.57	-1.55	-1.53	-1.52	-1.46	-1.18	-1.04
	-0.95	-0.84	-0.81	-0.85	-1.18	-1.41	-1.76	-2.01	-1.84	-1.81
	-1.82	-1.06	-0.99	-0.71	-0.33	-0.31	-0.84	-0.91	-1.14	-1.51
	-1.61	-1.61	-1.61	-1.51	-1.27	-0.98	-0.61	-0.41	-0.26	-0.22
	-0.61	-0.76	-1.08	-1.25	-1.33	-1.31	-1.19	-1.01	-0.83	-0.69
	-0.45	-0.43	-0.44	-0.52	-0.71	-0.81	-0.84	-0.87	-0.94	-0.89
	-0.84	-0.62	-0.43	-0.39	-0.41	-0.52	-0.71	-0.91	-1.04	-1.11
	-1.01	-0.71	-0.53	-0.39	-0.21	-0.15	-0.11	-0.22	-0.33	-0.46
	-0.53	-0.61	-0.79	-0.81	-0.73	-0.62	-0.54	-0.45	-0.42	-0.41

200

400

600

TABLE 2 CONT.

800	-0.77	-1.06	-1.34	-1.62	-1.72	-1.79	-1.72	-1.63	-1.45	-1.30
	-1.11	-0.96	-0.90	-0.87	-0.90	-1.02	-1.08	-1.22	-1.25	-1.22
	-1.04	-0.86	-0.62	-0.47	-0.42	-0.43	-0.54	-0.70	-0.87	-1.02
	-1.22	-1.13	-0.87	-0.72	-0.66	-0.59	-0.54	-0.55	-0.62	-0.70
	-0.84	-0.90	-0.95	-0.96	-1.06	-1.02	-0.95	-0.87	-0.82	-0.74
	-0.67	-0.64	-0.72	-0.66	-0.72	-0.78	-0.92	-1.10	-1.17	-1.30
	-1.32	-1.22	-1.11	-0.93	-0.82	-0.72	-0.55	-0.44	-0.37	-0.40
	-0.52	-0.68	-0.74	-0.86	-0.92	-0.97	-0.92	-0.92	-0.96	-1.00
	-1.02	-0.97	-1.02	-1.08	-1.12	-1.19	-1.21	-1.20	-1.24	-1.23
	-1.19	-1.05	-0.92	-0.77	-0.70	-0.67	-0.66	-0.68	-0.69	-0.74
	-0.92	-0.90	-0.97	-1.00	-0.76	-0.46	-0.37	-0.21	-0.13	-0.10
1000	0.10	0.03	0.01	-0.02	-0.22	-0.42	-0.65	-0.76	-0.77	-0.72
	-0.57	-0.66	-0.56	-0.35	-0.12	-0.04	0.08	0.03	0.01	-0.11
	-0.15	-0.12	-0.12	-0.04	0.05	0.13	0.18	-0.30	-0.28	-0.27
	-0.28	-0.32	-0.29	-0.25	-0.24	-0.24	-0.19	-0.17	-0.13	-0.12
	-0.22	-0.31	-0.32	-0.30	-0.32	-0.07	0.08	0.28	0.38	0.48
	0.53	0.42	0.18	-0.03	-0.14	-0.24	-0.22	-0.22	-0.07	0.00
	-0.02	0.10	0.12	0.06	-0.12	-0.20	-0.32	-0.45	-0.35	-0.28
	-0.28	-0.28	-0.22	-0.15	-0.17	-0.19	-0.20	-0.22	-0.22	-0.24
	-0.30	-0.40	-0.56	-0.66	-0.80	-0.90	-1.02	-1.41	-1.42	-1.42
1200	-1.34	-1.24	-0.89	-0.74	-0.53	-0.38	-0.34	-0.37	-0.38	-0.38
	-0.43	-0.46	-0.44	-0.37	-0.34	-0.44	-0.54	-0.61	-0.64	-0.67
	-0.72	-0.68	-0.59	-0.66	-0.68	-0.60	-0.55	-0.59	-0.65	-0.66
	-0.64	-0.59	-0.68	-0.54	-0.66	-0.62	-0.63	-0.61	-0.70	-0.77
	-0.84	-0.94	-0.94	-0.94	-0.92	-1.11	-0.92	-0.84	-0.82	-0.84
	-0.99	-1.24	-1.29	-1.12	-1.02	-0.89	-0.82	-0.73	-0.72	-0.66
	-0.61	-0.82	-0.92	-0.84	-0.78	-0.84	-0.86	-0.80	-0.77	-0.76
	-0.74	-0.70	-0.69	-0.59	-0.54	-0.50	-0.38	-0.34	-0.40	-0.44
	-0.40	-0.35	-0.31	-0.24	-0.37	-0.36	-0.44	-0.46	-0.44	-0.40
	-0.34	-0.29	-0.19	-0.08	-0.02	0.04	0.10	0.16	0.15	0.21

TABLE 2 CONT.

1400	0.18	0.06	-0.04	-0.14	-0.34	-0.44	-0.59	-0.69	-0.55	-0.35
	-0.23	-0.04	-0.03	0.01	-0.01	-0.16	-0.32	-0.49	-0.54	-0.41
	-0.28	-0.26	-0.25	-0.35	-0.37	-0.42	-0.44	-0.50	-0.44	-0.46
	-0.56	-0.64	-0.75	-0.76	-0.85	-1.11	-1.14	-1.04	-0.69	-0.59
	-0.57	-0.44	-0.37	-0.24	-0.32	-0.34	-0.54	-0.81	-0.91	-0.84
	-0.48	-0.41	-0.49	-0.58	-0.57	-0.76	-0.72	-0.72	-0.64	-0.44
	-0.24	-0.40	-0.44	-0.40	-0.54	-0.73	-0.82	-0.89	-1.08	-0.79
	-0.86	-0.82	-0.70	-0.76	-0.85	-0.94	-0.98	-1.06	-1.19	-1.12
	-1.45	-1.38	-1.47	-1.56	-1.69	-1.74	-2.44	-2.44	-2.09	-2.19
	-1.74	-2.52	-2.46	-2.36	-1.64	-1.99	-1.64	-1.46	-1.44	-1.72
	-2.93	-3.05	-3.19	-3.09	-2.97	-2.72	-2.64	-2.46	-2.41	-2.29
1600	-2.29	-2.37	-2.44	-2.49	-2.64	-2.69	-2.34	-2.37	-2.39	-2.42
	-2.54	-2.67	-2.59	-2.69	-2.59	-2.79	-2.74	-2.64	-2.69	-2.74
	-2.73	-2.69	-2.72	-2.94	-3.23	-3.36	-3.54	-3.63	-3.66	-3.74
	-3.76	-3.89	-3.79	-3.49	-3.49	-3.39	-3.41	-3.42	-3.43	-3.39
	-3.66	-3.69	-3.62	-3.54	-3.21	-3.27	-3.39	-3.39	-3.59	-3.69
	-3.92	-3.97	-4.06	-3.94	-4.04	-3.74	-4.25	-3.69	-3.97	-4.17
	-4.02	-4.32	-4.39	-4.59	-4.56	-4.47	-4.27	-4.01	-4.06	-3.88
	-3.87	-3.20	-3.79	-3.45	-3.59	-3.77	-3.84	-4.41	-4.24	-4.29
	-4.39	-4.49	-4.34	-4.42	-4.46	-4.41	-4.44	-4.59	-4.57	-4.64
	-4.59	-4.79	-5.09	-5.19	-5.09	-5.04	-5.02	-5.34	-5.42	-5.34
1800	-5.03	-4.49	-4.03	-4.00	-3.69	-3.91	-3.89	-3.99	-3.97	-3.79
	-3.92	-3.99	-3.87	-3.56	-3.59	-3.57	-3.79	-4.17	-4.33	-4.56
	-4.74	-5.22	-5.21	-5.19	-5.09	-4.69	-4.29	-3.78	-3.48	-3.59
	-3.84	-4.13	-4.34	-4.39	-4.79	-4.41	-4.29	-3.84	-3.61	-3.67

TABLE 2A
SMOOTHED KNOX - 1900 Ft. TWO FOOT SPACING

	0	2	4	6	8	10	12	14	16	18
0	0.00778	0.01000	0.00111	0.03333	0.04111	-0.38111	0.06889	-0.00667	0.23000	0.22111
	0.04889	-0.21333	-0.02333	0.33222	0.13111	0.04000	-0.36222	-0.09778	0.19333	0.17889
	0.01333	-0.01556	-0.16889	-0.36778	-0.55444	-0.12556	0.09778	-0.03889	0.30111	0.65778
	0.33222	-0.16111	-0.18222	0.07222	-0.51222	-0.05444	0.19333	0.44333	0.25667	0.19567
	0.10444	-0.27556	-0.33889	0.24444	-0.17889	-0.23444	0.14333	-0.01000	-0.00778	0.13889
	-0.13667	0.01556	0.04444	-0.12000	0.17111	-0.00444	-0.11778	-0.13000	0.00556	-0.00333
	-0.00667	-0.04333	0.11111	0.04667	-0.01444	-0.07444	-0.03111	-0.04778	-0.00444	0.34867
	-0.02556	0.02444	0.07889	0.06667	-0.23667	-0.32333	-0.14333	-0.06222	0.12889	0.25556
	0.15556	0.15222	-0.18778	-0.14222	-0.12889	-0.07000	0.18778	0.09889	-0.00889	-0.05444
	-0.05222	0.05444	-0.09667	-0.05778	-0.02111	0.06667	0.07222	0.04111	0.00778	0.02111
	-0.01556	-0.00222	0.08778	-0.09000	-0.12222	0.00333	-0.07333	0.04444	0.58778	-0.15111
200	0.17222	-0.31444	-0.34667	0.04000	0.01778	0.04000	0.02333	-0.03556	0.01111	0.17000
	-0.01333	-0.16333	-0.11111	-0.08333	0.12111	0.07889	0.00444	0.09000	0.09222	0.09778
	-0.08222	-0.19000	-0.01222	-0.02222	-0.09333	-0.03333	-0.04444	0.11000	0.08889	0.11000
	-0.01778	-0.09889	-0.04000	0.06000	0.07111	0.05889	-0.02333	0.07333	0.05667	-0.06667
	-0.15000	-0.11889	-0.07000	-0.00222	0.15889	0.30444	0.08667	0.11111	-0.00667	-0.04556
	-0.13000	-0.13111	-0.16556	0.12556	-0.02333	-0.03000	-0.03000	-0.01111	0.00333	-0.01333
	0.03889	0.04444	-0.01333	0.01000	0.01000	-0.01222	-0.05778	-0.08444	-0.07556	0.00889
	0.31778	-0.00444	-0.05222	-0.08444	-0.04444	-0.03222	0.00444	-0.01657	0.00444	-0.05889
	0.03778	-0.02333	-0.01667	-0.04667	-0.03333	0.02889	-0.15444	-0.33222	0.33000	-0.27889
	0.58222	0.52222	0.38111	0.12000	-0.11111	-0.11444	-0.11778	-0.34222	0.06111	-0.09667
400	-0.02778	0.15444	0.09000	0.23778	0.14667	0.10667	-0.01222	-0.13444	-0.21444	-0.18556
	-0.14667	-0.16889	0.23667	-0.02778	-0.08111	0.04667	0.12667	0.08556	0.06889	0.09000
	0.11444	0.12000	0.05444	-0.02556	-0.06444	-0.12000	-0.14111	0.05889	0.11222	0.04778
	-0.08778	-0.04000	0.06778	-0.18444	-0.16222	0.15889	0.51222	0.32889	0.29667	-0.19222
	-0.27444	-0.23556	-0.09556	-0.03667	-0.06444	-0.34333	-0.13889	0.25111	-0.05667	-0.06222
	-0.05444	0.03556	0.18222	0.25111	0.01000	-0.06333	-0.04889	-0.02444	-0.00667	0.04222
	0.04444	0.03556	0.02556	-0.12000	-0.13556	-0.14111	-0.08333	-0.03444	-0.01222	-0.03111
	-0.01111	0.16000	0.12556	0.10667	0.20444	0.00889	-0.08667	-0.11222	0.10000	-0.06667
	-0.05111	-0.10000	-0.12000	0.17111	0.10444	0.15889	0.11111	0.55111	0.35667	-0.11000
600	-0.34111	-0.39333	-0.31667	-0.17000	-0.03222	-0.00778	0.03222	0.10556	0.19556	0.15889
	0.06667	0.01111	-0.03333	-0.15222	-0.12778	-0.13111	-0.16333	-0.16667	0.02889	0.09111
	0.14222	0.24000	0.30333	0.35556	0.11444	-0.02000	-0.26111	-0.48333	-0.29778	-0.32000
	-0.45000	0.14889	0.08889	0.26556	0.57111	0.55667	0.08778	0.08667	-0.04333	-0.28222
	-0.27556	-0.26000	-0.29333	-0.27444	-0.17333	-0.03778	0.22111	0.32667	0.42889	0.46667
	0.11556	0.04333	-0.19000	-0.27667	-0.28889	-0.26000	-0.17444	-0.06667	0.02333	0.07333
	0.24667	0.22444	0.19556	0.12000	-0.04222	-0.09333	-0.07778	-0.08778	-0.16778	-0.15333
	-0.14778	0.03667	0.20889	0.24556	0.24222	0.16222	0.01556	-0.15333	-0.26778	-0.34000
	-0.27444	-0.03667	0.05444	0.10333	0.19667	0.19556	0.21556	0.11444	0.04889	-0.01444
	-0.02000	-0.04333	-0.18778	-0.19333	-0.11778	-0.02111	0.07667	0.18667	0.28556	0.39444

TABLE 2A CONT.

1500	0.14556	0.24000	0.15000	0.03889	0.02667	-0.16778	-0.14667	-0.15657	-0.09222	0.08889
	0.26444	0.10556	0.07667	0.14444	0.07556	-0.05333	-0.09222	-0.12000	-0.27667	0.03778
	-0.01889	0.03444	0.16444	0.10222	0.05667	-0.00444	0.02556	0.02111	-0.03000	0.11889
	-0.12778	0.02667	0.09000	0.13889	0.11667	0.14889	-0.51111	-0.39444	0.05556	0.03000
1600	0.46889	-0.36111	-0.39000	-0.36000	0.27667	-0.07556	0.32000	0.56556	0.67778	0.55889
	-0.54222	-0.54222	-0.55111	-0.33778	-0.14111	0.03778	0.03333	0.12222	0.10000	0.16667
	0.15778	0.08333	-0.00000	-0.05444	-0.19333	-0.22889	0.14000	0.13556	0.12667	0.10222
	-0.02889	-0.10889	0.01222	-0.06000	0.07000	-0.10778	-0.05111	0.06000	0.01333	0.00222
1800	0.06111	0.17000	0.24000	0.12444	-0.06333	-0.08111	-0.14222	-0.10222	-0.03778	-0.08889
	-0.09444	-0.24111	-0.16556	0.10778	0.07333	0.13222	0.08667	0.05556	0.07000	0.11556
	-0.17444	-0.22000	-0.15333	-0.07778	0.27444	0.21778	0.12333	0.16222	0.02000	0.00111
	-0.14333	-0.15444	-0.14889	-0.01778	-0.08667	0.24111	-0.26333	0.32556	0.09556	-0.04333
	0.19778	-0.07778	-0.08333	-0.27889	-0.26111	-0.18667	-0.03667	0.09111	-0.04778	0.00889
	-0.07889	0.53556	-0.07333	0.30556	0.20556	0.07222	0.13444	-0.35778	-0.08889	-0.04667
	-0.07000	-0.10667	0.04667	0.00556	-0.00333	0.07444	0.05556	-0.04444	0.05000	0.06111
	0.18667	0.05333	-0.19889	-0.21333	-0.02667	0.10667	0.15333	-0.23333	-0.39778	-0.43689
	-0.27889	0.13778	0.03667	0.30778	0.46556	0.10778	0.06444	-0.08444	-0.07889	0.08667
	-0.07889	-0.18444	-0.08667	0.24556	0.27556	0.36667	0.23000	-0.00000	0.02333	-0.02889
	-0.04000	-0.42000	-0.39667	-0.43778	-0.45778	-0.18556	0.06111	0.45111	0.65667	0.46889
	-0.23000	-0.04667	-0.20000	-0.21000	-0.60778	-0.24667	-0.20556	0.11444	0.18111	-0.11444

TABLE 2B
AUTOCORROLATION KNOX

k	1-1900 ft.	1-950 ft.	950-1900 ft.
0	0.0298	0.0313	0.0282
1	0.0168	0.0176	0.0159
2	0.0062	0.0063	0.0062
3	-	0.0027	-
4	-	0.0108	-
5	-	0.0143	-
6	-	0.0142	-
7	-	0.0122	-
8	-	0.0074	-
9	-	0.0022	-
10	0.0028	0.0040	0.0018
11	0.0063	0.0084	0.0045
12	0.0062	0.0081	0.0044
13	0.0065	0.0079	0.0053
14	0.0061	0.0082	0.0040
15	0.0032	0.0059	0.0005
16	0.0000	0.0014	-
17	-	0.0027	-
18	-	0.0070	-
19	-	0.0091	-
20	-	0.0087	-
21	-	0.0076	-
22	-	0.0037	-
23	0.0010	0.0002	0.0020
24	0.0032	0.0050	0.0016
25	0.0051	0.0092	0.0012
26	0.0047	0.0103	-
27	0.0034	0.0078	-
28	0.0012	0.0037	-
29	-	0.0013	-
30	-	0.0032	-
31	-	0.0071	-
32	-	0.0092	-
33	-	0.0085	-
34	-	0.0055	-
35	0.0003	0.0018	-
36	0.0021	0.0029	-
37	0.0037	0.0067	-
38	0.0034	0.0067	-
39	0.0030	0.0073	-
40	0.0021	0.0071	-
41	0.0004	0.0039	-
42	-	0.0011	-
43	-	0.0049	-
44	-	0.0076	-
45	-	0.0081	-
46	-	0.0071	-
47	-	0.0050	-
48	-	0.0017	-
49	0.0005	0.0017	-
50	0.0021	0.0072	-

TABLE 2C

POWER SPECTRAL DENSITY

KNOX

SMOOTHED DATA

λ	-1-1900 ft.	1-750 ft.	750-1900 ft.
-0.00000	0.00042	0.00671	0.00370
0.00500	0.00720	0.01029	0.00320
0.01000	0.01298	0.00370	0.01912
0.01500	0.04368	0.04129	0.04414
0.02000	0.07294	0.05083	0.08900
0.02500	0.15443	0.10686	0.20543
0.03000	0.30829	0.25217	0.36722
0.03500	0.70377	0.91519	0.47923
0.04000	0.95972	1.23784	0.70308
0.04500	0.62758	0.55708	0.70781
0.05000	0.37234	0.31496	0.42663
0.05500	0.32086	0.34175	0.29879
0.06000	0.25758	0.29263	0.22290
0.06500	0.20449	0.19727	0.20981
0.07000	0.17224	0.13977	0.20400
0.07500	0.14001	0.09833	0.18329
0.08000	0.11015	0.09414	0.12397
0.08500	0.09245	0.09207	0.09176
0.09000	0.08590	0.08895	0.08302
0.09500	0.09092	0.11834	0.06334
0.10000	0.10691	0.12719	0.08727
0.10500	0.09625	0.06292	0.10923
0.11000	0.05753	0.03763	0.07664
0.11500	0.04477	0.03153	0.05721
0.12000	0.04182	0.03970	0.04418
0.12500	0.02707	0.03119	0.02227
0.13000	0.03201	0.04644	0.01699
0.13500	0.04835	0.07903	0.01702
0.14000	0.04166	0.07103	0.01261
0.14500	0.03602	0.04927	0.01036
0.15000	0.03273	0.05482	0.01078
0.15500	0.02732	0.03717	0.01696
0.16000	0.02926	0.03036	0.02822
0.16500	0.02874	0.03079	0.02643
0.17000	0.03466	0.04867	0.02006
0.17500	0.04625	0.06493	0.02706
0.18000	0.05175	0.06003	0.04419
0.18500	0.04985	0.04807	0.05188
0.19000	0.04212	0.04421	0.04014
0.19500	0.03042	0.03048	0.02969
0.20000	0.02984	0.03509	0.02445
0.20500	0.03110	0.03580	0.02669
0.21000	0.02781	0.02533	0.02995
0.21500	0.03494	0.01424	0.05535
0.22000	0.04699	0.02498	0.06947
0.22500	0.04260	0.03038	0.05490
0.23000	0.03358	0.03105	0.03609
0.23500	0.02569	0.02256	0.02850
0.24000	0.02368	0.02878	0.01833
0.24500	0.02117	0.02772	0.01434
0.25000	0.01932	0.02633	0.01209

TABLE 2D

POWER SPECTRAL DENSITY

KNOX

λ	0-950 ft	9-475 ft	476-950 ft
0.000	----	----	----
0.005	41.53824	59.36506	18.46144
0.010	4.83219	1.37743	7.11799
0.015	3.34020	3.15744	3.37538
0.020	1.86347	1.29860	2.27377
0.025	1.73363	1.19961	2.30615
0.030	1.81786	1.48694	2.16534
0.035	2.50689	3.25999	1.70706
0.040	2.23221	2.87909	1.63529
0.045	1.04178	.92475	1.17496
0.050	.47123	.39861	.53994
0.055	.32756	.34889	.30503
0.060	.22286	.25319	.19285
0.065	.15668	.15115	.16076
0.070	.12158	.09866	.14400
0.075	.09436	.06627	.12353
0.080	.07321	.06257	.08240
0.085	.06238	.06213	.06192
0.090	.06037	.06251	.05835
0.095	.06801	.08852	.04238
0.100	.08659	.10302	.06068
0.105	.07655	.05585	.09695
0.110	.05629	.03681	.07498
0.115	.04823	.03397	.06164
0.120	.04921	.04672	.05199
0.125	.03425	.03947	.02818
0.130	.04264	.08186	.02263
0.135	.06607	.10799	.02448
0.140	.05684	.09692	.01720
0.145	.03987	.06544	.01376
0.150	.04142	.06938	.01364
0.155	.03244	.04413	.02014
0.160	.03231	.03352	.03116
0.165	.02944	.03155	.02702
0.170	.03306	.04643	.01913
0.175	.04142	.05815	.02495
0.180	.04403	.05108	.03760
0.185	.04088	.03942	.04254
0.190	.03383	.03351	.03224
0.195	.02433	.02438	.02375
0.200	.02417	.02842	.01980
0.205	.02590	.02981	.02223
0.210	.02411	.02199	.02600
0.215	.03198	.01303	.05066
0.220	.04568	.02428	.06754
0.225	.04413	.03147	.05688
0.230	.03702	.03423	.03979
0.235	.02994	.02629	.03322
0.240	.02884	.03506	.02233
0.245	.02653	.03474	.01797
0.250	.02445	.03332	.01530

TABLE 3
YUMA - 1500 Ft. TWO FOOT SPACING

	0	2	4	6	8	10	12	14	16	18
0	5.28	5.15	4.90	4.81	4.76	4.48	4.27	3.73	3.84	4.01
20	4.44	4.66	4.86	5.29	5.06	5.07	5.11	5.20	4.89	4.86
	5.36	5.76	6.38	6.80	7.16	7.50	7.83	8.22	8.61	8.94
	9.23	9.76	10.18	10.64	11.20	11.60	11.87	12.00	12.15	12.22
	12.06	12.10	12.06	12.00	12.06	12.01	12.06	12.24	12.40	12.53
	12.67	12.83	13.08	13.29	13.39	13.48	13.52	13.62	13.62	13.66
	13.76	13.76	13.86	13.97	14.08	14.18	14.35	14.44	14.54	14.59
	14.50	14.75	14.61	14.78	14.53	14.68	14.68	14.56	14.49	14.35
	14.25	14.14	14.03	13.92	13.85	13.73	13.67	13.67	13.57	13.45
	13.35	13.25	13.09	13.02	12.83	12.50	12.33	12.16	11.88	11.73
200	11.61	11.50	11.39	11.23	11.18	11.22	11.30	11.38	11.38	11.49
	11.66	11.71	11.85	12.06	12.11	12.23	12.36	12.50	12.61	12.77
	12.96	13.12	13.23	13.38	13.59	13.75	13.79	13.91	13.85	13.66
	13.41	13.22	12.93	12.65	12.31	12.08	11.79	11.47	11.17	10.91
	10.50	10.19	9.73	9.35	8.91	8.63	8.18	7.90	7.55	7.23
	7.02	6.68	6.48	6.30	6.10	6.00	5.83	5.68	5.54	5.45
	5.28	5.17	5.01	4.95	4.91	4.76	4.65	4.45	4.48	4.17
	4.19	4.12	4.28	4.34	4.40	4.27	4.43	4.41	4.42	4.47
	4.50	4.51	4.56	4.56	4.52	4.63	4.65	4.67	4.61	4.66
	4.62	4.63	4.65	4.66	4.75	4.71	4.72	4.77	4.79	4.83
400	4.61	4.44	4.24	4.03	3.78	3.52	3.34	3.12	2.91	2.78
	2.58	2.29	2.01	1.74	1.47	1.04	1.16	0.88	0.61	0.29
	0.11	-0.20	-0.73	-0.89	-0.99	-1.31	-1.53	-1.57	-1.66	-1.69
	-1.66	-1.67	-1.38	-0.92	-0.45	0.07	0.43	0.91	1.18	1.42
	1.64	1.76	1.89	2.00	2.12	2.15	2.23	2.20	2.27	2.26
	2.25	2.25	2.21	2.16	2.20	2.10	2.00	1.90	1.81	1.66
	1.52	1.37	1.17	0.83	0.85	1.09	1.27	1.53	1.80	2.00
	2.05	2.11	2.14	2.16	2.22	2.29	2.21	2.18	2.16	2.13
	2.12	2.11	2.13	2.09	2.08	2.08	2.07	2.06	2.07	2.04
	2.03	1.94	1.97	1.98	1.97	1.95	1.97	1.92	2.00	2.04
	2.00	2.02	1.90	1.80	1.63	1.49	1.37	1.24	1.14	1.03
600	0.82	0.61	0.38	0.17	-0.04	-0.26	-0.42	-0.60	-0.78	-0.91
	-1.05	-1.10	-1.17	-1.35	-1.41	-1.47	-1.57	-1.67	-1.74	-1.82
	-1.91	-1.96	-2.07	-2.21	-2.22	-2.37	-2.59	-2.73	-3.04	-3.00
	-2.93	-2.86	-2.88	-2.91	-2.99	-3.05	-3.11	-3.20	-3.24	-3.32
	-3.44	-3.54	-3.60	-3.62	-3.73	-3.77	-3.78	-3.80	-3.86	-3.85
	-3.92	-3.97	-3.93	-4.08	-4.16	-4.11	-4.20	-4.21	-4.27	-4.15
	-4.15	-4.18	-4.27	-4.29	-4.34	-4.35	-4.42	-4.48	-4.41	-4.52
	-4.53	-4.54	-4.58	-4.60	-4.55	-4.46	-4.32	-4.26	-4.25	-4.22
	-4.17	-4.31	-4.16	-4.12	-4.18	-4.11	-4.01	-3.90	-3.82	-3.65

TABLE 3 CONT.

	0	2	4	6	8	10	12	14	16	18
800	-3.49	-3.30	-3.05	-2.89	-2.67	-2.58	-2.11	-2.05	-1.69	-1.42
	-1.18	-0.92	-0.65	-0.45	-0.35	-0.24	0.07	0.06	0.05	0.10
	0.18	0.18	0.20	0.19	0.07	-0.07	-0.23	-0.42	-0.65	-0.84
	-1.05	-1.10	-1.18	-1.45	-1.51	-1.54	-1.51	-1.50	-1.45	-1.38
	-1.31	-1.21	-1.20	-1.07	-1.11	-1.10	-1.09	-1.10	-1.18	-1.25
	-1.46	-1.23	-1.71	-1.51	-1.91	-2.05	-2.40	-2.48	-2.91	-3.06
	-3.16	-3.18	-3.55	-3.93	-4.08	-3.84	-4.33	-4.66	-4.89	-5.26
	-5.55	-5.82	-6.12	-6.49	-6.68	-7.00	-7.18	-7.49	-7.68	-8.66
	-8.90	-8.92	-8.43	-7.80	-7.85	-7.90	-8.00	-7.94	-8.12	-8.06
	-7.90	-8.27	-8.31	-8.31	-8.42	-8.77	-8.80	-8.86	-8.52	-8.44
	-8.58	-8.60	-8.96	-9.06	-9.00	-9.14	-9.16	-9.20	-9.17	-9.20
1000	-9.13	-9.14	-9.20	-9.27	-9.04	-8.74	-9.06	-9.10	-9.07	-8.69
	-8.98	-9.32	-9.43	-9.54	-9.60	-9.67	-9.72	-9.83	-9.88	-9.81
	-9.83	-9.84	-9.66	-9.26	-9.57	-9.65	-9.67	-9.68	-9.54	-9.72
	-9.93	-9.77	-9.55	-9.20	-9.31	-9.40	-9.73	-9.93	-10.02	-10.07
	-9.97	-9.92	-10.10	-10.32	-10.37	-10.52	-10.75	-10.66	-10.68	-10.54
	-10.29	-10.27	-10.21	-10.17	-10.41	-10.41	-10.15	-9.91	-9.70	-9.91
	-9.90	-10.51	-10.45	-10.27	-10.06	-10.22	-10.11	-10.46	-10.03	-10.98
	-10.91	-10.94	-10.73	-10.29	-10.31	-10.37	-10.30	-10.06	-10.10	-10.60
	-10.47	-10.67	-10.77	-10.83	-10.68	-10.61	-10.39	-10.23	-10.44	-10.61
	-10.79	-10.77	-10.72	-11.14	-11.20	-11.05	-10.53	-10.18	-10.30	-10.33
	-10.31	-10.34	-10.43	-10.38	-10.32	-10.38	-10.37	-10.42	-10.47	-10.46
	-10.47	-10.50	-10.61	-10.77	-10.83	-10.88	-10.91	-10.93	-10.95	-10.95
	-10.95	-10.91	-10.88	-10.77	-10.70	-10.61	-10.53	-10.51	-10.50	-10.55
	-10.54	-10.60	-10.67	-10.71	-10.73	-10.76	-10.73	-10.68	-10.68	-10.56
	-10.78	-10.95	-10.47	-10.42	-10.12	-10.30	-10.27	-10.17	-10.11	-10.55
	-9.90	-9.72	-9.65	-9.55	-9.42	-9.35	-9.23	-9.18	-9.05	-8.97
	-8.91	-8.85	-8.82	-8.77	-8.74	-8.71	-8.69	-8.69	-8.67	-8.67
	-8.66	-8.65	-8.63	-8.67	-8.68	-8.67	-8.66	-8.69	-8.71	-8.73
	-8.79	-8.85	-8.85	-8.85	-8.93	-8.97	-9.02	-9.05	-9.15	-9.17
	-9.18	-9.19	-9.26	-9.32	-9.38	-9.49	-9.45	-9.42	-9.46	-9.54
	-9.52	-9.63	-9.57	-9.53	-9.56	-9.53	-9.55	-9.52	-9.52	-9.18
	-9.40	-9.36	-9.10	-9.05	-9.18	-9.45	-9.65	-9.68	-9.72	-9.75
	-9.81	-9.79	-9.81	-9.87	-9.89	-9.92	-9.90	-9.93	-9.97	-10.00
	-10.05	-10.07	-10.09	-10.15	-10.18	-10.18	-10.59	-11.05	-11.42	-11.50

TABLE 3A
SMOOTHED YUMA - 1500 Ft. TWO FOOT SPACING

	0	2	4	6	8	10	12	14	16	18
0	0.0000	0.02556	-0.06778	0.04778	0.17444	0.03556	-0.04556	-0.60889	-0.50444	-0.19333
	-0.02778	0.10889	0.15556	0.43444	0.10667	0.07000	0.03222	0.02222	-0.40889	-0.63222
	-0.36444	-0.23000	0.09778	0.14778	0.09111	0.03333	-0.02222	-0.00778	0.00667	-0.05000
	-0.17111	-0.06000	-0.05111	0.03222	0.23556	0.30333	0.36778	0.23444	0.22667	0.20778
	-0.00333	0.02667	-0.02000	-0.09000	-0.05000	-0.15222	-0.16556	-0.07111	-0.03111	-0.03778
	-0.05111	-0.04889	0.05889	0.13333	0.11222	0.09222	0.02889	0.05333	-0.01000	-0.04000
	-0.00667	-0.08000	-0.06111	0.00778	-0.03000	-0.02222	0.06556	0.05667	0.09111	0.06333
	-0.06555	0.14778	-0.01889	0.14889	-0.09000	0.07667	0.13222	0.06445	0.07778	0.00556
	-0.00222	-0.00667	-0.01778	-0.03667	-0.02000	-0.05111	-0.02333	0.06333	0.05556	0.02778
	0.02778	0.05778	0.04667	0.13333	0.11778	-0.03222	-0.02000	-0.01333	-0.11222	-0.08444
	-0.05778	-0.04444	-0.05889	-0.16333	-0.17444	-0.12111	-0.05889	-0.01444	-0.08333	-0.07111
	0.00000	-0.05333	-0.02222	0.06333	-0.01111	-0.01444	-0.02333	-0.02444	-0.04444	-0.02556
	0.01333	0.01889	-0.01444	-0.00889	0.08111	0.16333	0.17111	0.29222	0.28222	0.19667
	0.10667	0.10667	0.05222	0.03667	-0.02667	0.02111	0.03333	0.01778	0.04222	0.11111
	0.05333	0.09444	-0.00000	-0.04778	-0.18444	-0.17555	-0.33222	-0.05444	0.20444	0.17222
	0.06111	-0.05000	-0.02889	-0.12667	-0.09667	-0.06889	-0.08111	-0.00667	-0.02111	-0.02556
	-0.02222	0.01556	-0.03333	-0.02444	-0.07000	-0.00889	0.05889	-0.03222	-0.03111	-0.07000
	0.03444	-0.21222	-0.15222	-0.18000	-0.01778	0.05000	0.08222	-0.08000	0.03667	-0.01444
	-0.02889	0.01778	-0.00778	0.03333	-0.00444	-0.05667	0.03333	-0.04111	-0.05333	-0.01667
	0.01778	-0.03556	-0.03222	-0.01778	-0.02556	0.05000	-0.01333	-0.00111	0.07222	0.13889
	0.25889	0.14222	0.10556	0.06444	0.04000	0.00333	-0.05333	-0.02667	-0.03000	-0.01556
	0.08111	0.10889	0.07444	0.01222	-0.03222	-0.06111	-0.23667	0.12556	0.09111	0.09556
	0.03778	0.08333	-0.04778	-0.21444	-0.13222	-0.01556	-0.13556	-0.19333	-0.12889	-0.16444
	-0.20222	-0.26778	-0.45556	-0.38778	-0.32444	-0.17333	0.00444	-0.00333	0.12778	0.08556
	0.05333	0.04556	-0.02556	0.06889	0.06556	0.09111	0.05222	0.07778	0.01222	0.06444
	0.04556	0.04111	0.02667	0.01000	0.10111	0.06778	0.04889	0.04222	0.06222	0.06444
	0.06333	0.01444	-0.11556	-0.42444	-0.42000	-0.23333	-0.12889	-0.02667	-0.15111	-0.20556
	0.13000	0.07667	0.03111	0.00889	0.05111	0.11222	0.03111	0.00444	-0.01222	-0.02778
	-0.01444	-0.01000	-0.02222	-0.00667	-0.01000	-0.00111	-0.00222	-0.00889	-0.03222	-0.11222
	0.11444	0.03778	0.07778	-0.078556	0.09889	0.07778	0.09111	0.03556	0.03556	0.09444
	0.09000	0.16333	0.01444	0.08889	0.01889	-0.01333	0.00000	-0.00333	-0.06111	-0.11333
	0.07333	0.04444	-0.00111	-0.01778	-0.02667	-0.05444	-0.03000	-0.04556	-0.07667	-0.06111
	-0.07333	-0.00667	-0.03111	-0.05000	-0.01778	0.00778	-0.00222	-0.01444	-0.00444	-0.00444
	-0.00222	0.03667	0.02889	-0.00111	0.12444	0.09556	-0.01667	-0.06889	-0.30444	-0.18778
	-0.04889	0.07222	-0.09444	0.08222	0.02889	0.01222	0.01667	-0.00000	0.03667	0.02667
	-0.01778	-0.04444	-0.04000	0.00222	-0.04778	-0.04222	-0.01000	0.01111	-0.01444	0.03444
	-0.00778	-0.00556	0.07889	-0.03222	-0.06556	-0.01000	-0.06000	-0.04222	-0.08111	-0.05333
	0.07889	0.06556	-0.00111	0.00222	-0.01889	0.01222	-0.01889	-0.04889	0.05333	-0.02778
	-0.01556	-0.02111	-0.07889	-0.11556	-0.09556	-0.04000	0.05889	-0.08689	0.05000	0.03222
	0.05111	-0.11222	0.01000	0.01111	-0.09333	-0.08111	-0.07222	-0.05778	-0.03667	-0.07000

TABLE 3B
AUTOCORRELATION YUMA

k	1-1500 ft.	1-750 ft.	750-1500 ft.
0	0.0222	0.0199	0.0245
1	0.0110	0.0104	0.0117
2	0.0033	0.0061	0.0005
3	0.0038	0.0008	0.0084
4	0.0056	0.0026	0.0086
5	0.0040	0.0036	0.0044
6	0.0027	0.0042	0.0012
7	0.0007	0.0023	0.0009
8	0.0004	0.0016	0.0024
9	0.0013	0.0001	0.0025
10	0.0014	0.0009	0.0019
11	0.0002	0.0017	0.0013
12	0.0003	0.0017	0.0023
13	0.0010	0.0007	0.0028
14	0.0010	0.0011	0.0009
15	0.0014	0.0030	0.0001
16	0.0004	0.0021	0.0011
17	0.0006	0.0019	0.0004
18	0.0008	0.0012	0.0004
19	0.0014	0.0004	0.0026
20	0.0019	0.0001	0.0038
21	0.0005	0.0004	0.0014
22	0.0002	0.0007	0.0013
23	0.0015	0.0008	0.0039
24	0.0014	0.0015	0.0045
25	0.0001	0.0023	0.0021
26	0.0016	0.0024	0.0009
27	0.0022	0.0020	0.0025
28	0.0023	0.0018	0.0030
29	0.0014	0.0011	0.0019
30	0.0001	0.0002	0.0005
31	0.0005	0.0003	0.0007
32	0.0013	0.0003	0.0029
33	0.0014	0.0002	0.0032
34	0.0006	0.0001	0.0013
35	0.0004	0.0001	0.0008
36	0.0010	0.0003	0.0024
37	0.0005	0.0009	0.0021
38	0.0007	0.0012	0.0026
39	0.0004	0.0012	0.0005
40	0.0005	0.0014	0.0003
41	0.0000	0.0005	0.0005
42	0.0001	0.0000	0.0002
43	0.0004	0.0002	0.0010
44	0.0002	0.0012	0.0014
45	0.0008	0.0012	0.0028
46	0.0005	0.0020	0.0008
47	0.0004	0.0013	0.0004
48	0.0009	0.0011	0.0005
49	0.0003	0.0004	0.0000
50	0.0004	0.0004	0.0007

TABLE 3C
POWER SPECTRAL DENSITY

λ	1-1500 ft.	1-750 ft.	YUMA 750-150 ft.
-0.	0.03108	0.03456	0.02306
0.00500	0.07247	0.10978	0.03372
0.01000	0.13516	0.20851	0.06286
0.01500	0.14861	0.20158	0.09640
0.02000	0.13675	0.14153	0.13135
0.02500	0.14893	0.19571	0.10132
0.03000	0.15961	0.22116	0.09656
0.03500	0.17715	0.22175	0.12855
0.04000	0.24567	0.27879	0.21640
0.04500	0.32252	0.33020	0.32251
0.05000	0.27873	0.31742	0.24195
0.05500	0.23495	0.18960	0.27628
0.06000	0.28751	0.10343	0.47389
0.06500	0.26737	0.03711	0.45017
0.07000	0.16216	0.03261	0.24132
0.07500	0.11908	0.08066	0.15674
0.08000	0.13259	0.09052	0.17507
0.08500	0.13532	0.07840	0.19329
0.09000	0.12021	0.05484	0.18576
0.09500	0.03172	0.03633	0.12576
0.10000	0.07369	0.02887	0.11815
0.10500	0.07263	0.02206	0.12353
0.11000	0.06451	0.02919	0.09947
0.11500	0.03259	0.04391	0.06105
0.12000	0.04383	0.04288	0.04437
0.12500	0.03369	0.03544	0.03113
0.13000	0.02457	0.03030	0.01783
0.13500	0.02263	0.01989	0.02712
0.14000	0.02496	0.01866	0.03149
0.14500	0.02687	0.03298	0.02077
0.15000	0.03062	0.03970	0.02139
0.15500	0.02299	0.02607	0.01969
0.16000	0.02263	0.02317	0.02124
0.16500	0.02875	0.02536	0.02232
0.17000	0.02305	0.02109	0.02520
0.17500	0.01729	0.02008	0.01443
0.18000	0.02132	0.02507	0.01648
0.18500	0.03152	0.03344	0.02931
0.19000	0.03777	0.03681	0.03860
0.19500	0.02999	0.02959	0.02791
0.20000	0.03989	0.02759	0.03398
0.20500	0.03366	0.02995	0.03741
0.21000	0.04219	0.04606	0.03887
0.21500	0.04794	0.05372	0.04212
0.22000	0.04089	0.04846	0.03335
0.22500	0.02683	0.03541	0.01791
0.23000	0.02347	0.02730	0.01889
0.23500	0.02575	0.02803	0.02322
0.24000	0.03009	0.02571	0.03422
0.24500	0.03508	0.02217	0.04830
0.25000	0.03063	0.01836	0.04387

TABLE 3D			
POWER SPECTRAL DENSITY			YUMA
λ	0-750 ft	0-375 ft	376-750 ft
0.000	----	----	----
0.005	418.09392	633.34277	194.53742
0.010	50.31736	77.62410	23.40152
0.015	11.36420	15.41482	7.37170
0.020	3.49368	3.61580	3.35572
0.025	1.67188	2.19704	1.13741
0.030	.94115	1.30409	.56937
0.035	.63102	.78989	.45790
0.040	.57140	.64843	.50332
0.045	.53538	.54813	.53536
0.050	.35276	.40172	.30621
0.055	.23986	.19356	.28205
0.060	.24876	.08949	.41002
0.065	.20486	.06674	.34492
0.070	.11446	.05831	.17039
0.075	.08025	.05436	.10564
0.080	.08813	.06016	.11636
0.085	.09131	.05290	.13043
0.090	.08448	.03854	.13056
0.095	.06128	.02717	.09422
0.100	.05968	.02338	.09570
0.105	.06446	.01958	.10965
0.110	.06312	.02856	.09732
0.115	.05666	.04731	.06578
0.120	.05158	.05046	.05221
0.125	.04263	.04485	.03939
0.130	.03272	.04036	.02375
0.135	.03229	.02717	.03705
0.140	.03405	.02546	.04296
0.145	.03568	.04380	.02758
0.150	.03875	.05024	.02707
0.155	.02730	.03095	.02338
0.160	.02499	.02558	.02345
0.165	.02946	.02598	.03311
0.170	.02199	.02012	.02404
0.175	.01548	.01798	.01292
0.180	.01814	.02133	.01402
0.185	.02585	.02742	.02403
0.190	.03009	.02956	.03100
0.195	.02327	.02367	.02232
0.200	.02494	.02234	.02752
0.205	.02803	.02494	.03116
0.210	.03663	.03999	.03375
0.215	.04388	.04917	.03855
0.220	.03975	.04711	.03242
0.225	.02779	.03668	.01855
0.230	.02588	.03010	.02083
0.235	.03001	.03267	.02706
0.240	.03665	.03132	.04169
0.245	.04396	.02778	.06053
0.250	.03901	.02323	.05552

TABLE 4

BATTLEFIELD DAY - 6201 FT.

ONE FOOT SPACING

	0	1	2	3	4	5	6	7	8	9	10	11
	00003	00009	00016	00022	00027	00035	00042	00048	00053	00059	00064	00069
	00074	00079	00081	00084	00098	00109	00116	00122	00130	00139	00146	00152
	00165	00174	00180	00189	00195	00201	00206	00214	00224	00233	00239	00245
	00252	00253	00255	00264	00271	00277	00286	00294	00302	00309	00319	00326
	00331	00335	00344	00350	00356	00371	00394	00413	00429	00433	00439	00445
	00459	00478	00491	00498	00513	00527	00534	00540	00549	00557	00568	00578
	00589	00597	00605	00618	00635	00648	00664	00682	00700	00709	00719	00730
	00736	00737	00728	00718	00711	00701	00692	00687	00683	00678	00674	00671
	00666	00660	00657	00648	00642	00638	00637	00636	00637	00630	00626	00623
	00617	00610	00606	00602	00596	00591	00586	00580	00576	00572	00564	00561
	00546	00543	00539	00528	00517	00510	00509	00512	00518	00517	00518	00519
	00520	00520	00518	00512	00512	00510	00507	00506	00505	00540	00513	00527
	00533	00545	00560	00573	00588	00608	00634	00653	00666	00669	00660	00651
	00640	00627	00613	00595	00572	00557	00551	00548	00540	00535	00535	00533
	00526	00517	00502	00484	00468	00451	00432	00409	00390	00376	00367	00347
	00339	00332	00327	00324	00324	00324	00327	00324	00320	00317	00314	00308
	00307	00300	00295	00292	00287	00283	00279	00275	00273	00268	00266	00268
	00269	00275	00281	00285	00293	00296	00296	00296	00294	00299	00303	00302
	00294	00290	00288	00288	00288	00286	00286	00290	00291	00295	00297	00297
298	00298	00300	00302	00305	00303	00300	00296	00297	00298	00297	00296	00297
	00301	00302	00302	00302	00302	00304	00307	00309	00309	00311	00315	00316
	00320	00323	00324	00324	00327	00330	00332	00335	00335	00334	00331	00330
	00328	00326	00326	00325	00335	00335	00335	00334	00336	00337	00339	00341
	00340	00334	00332	00336	00340	00342	00343	00346	00340	00336	00340	00341
	00341	00340	00338	00336	00335	00336	00338	00338	00344	00346	00344	00343
	00345	00349	00353	00349	00350	00351	00352	00355	00360	00364	00365	00375
	00388	00398	00404	00410	00415	00418	00421	00426	00428	00426	00422	00416
	00414	00411	00411	00412	00402	00393	00385	00381	00378	00374	00370	00368
336	00365	00365	00367	00368	00371	00374	00376	00381	00385	00391	00398	00403
	00406	00406	00403	00401	00400	00395	00388	00381	00375	00371	00365	00363
	00364	00363	00364	00364	00367	00374	00381	00388	00391	00390	00391	00389
	00392	00391	00388	00381	00377	00375	00378	00377	00381	00386	00393	00401
	00419	00432	00443	00453	00467	00480	00482	00479	00476	00475	00470	00460
	00451	00442	00434	00432	00432	00450	00445	00445	00446	00440	00435	00433
	00431	00430	00428	00431	00432	00433	00437	00448	00457	00469	00477	00490
	00505	00505	00510	00511	00513	00516	00523	00527	00528	00530	00537	00540
	00541	00551	00567	00572	00582	00585	00587	00588	00591	00597	00600	00603
	00609	00616	00622	00628	00633	00636	00636	00646	00659	00668	00678	00688
456	00694	00700	00706	00712	00723	00732	00727	00717	00712	00709	00707	00705
	00709	00713	00717	00721	00722	00727	00725	00726	00728	00733	00740	00743
	00747	00752	00757	00758	00759	00761	00759	00756	00758	00763	00769	00779
	00783	00791	00801	00811	00822	00833	00842	00849	00856	00857	00854	00850
	00845	00834	00823	00816	00816	00813	00812	00812	00814	00816	00820	00825
	00825	00826	00826	00821	00816	00810	00804	00804	00800	00820	00817	00817
	00821	00834	00840	00838	00835	00841	00844	00851	00860	00868	00876	00883
	00895	00909	00917	00921	00921	00920	00913	00898	00877	00858	00841	00832
	00831	00832	00830	00836	00846	00858	00869	00885	00898	00901	00902	00897
564	00889	00881	00863	00848	00838	00832	00829	00828	00828	00833	00841	00844
	00845	00846	00844	00850	00855	00854	00856	00860	00863	00866	00866	00866
	00873	00879	00881	00883	00885	00887	00887	00883	00872	00865	00860	00857
	00852	00846	00843	00842	00848	00851	00857	00869	00879	00887	00894	00897
	00898	00900	00898	00892	00887	00884	00878	00876	00878	00880	00883	00885
	00888	00892	00896	00899	00904	00906	00906	00898	00895	00896	00896	00898
	00904	00907	00909	00912	00914	00917	00918	00921	00923	00923	00920	00914
	00908	00908	00902	00900	00895	00910	00921	00922	00921	00926	00930	00935
	00938	00944	00948	00952	00952	00954	00956	00959	00960	00964	00970	00976
	00982	00992	01001	01008	01017	01025	01030	01031	01028	01024	01021	01017

TABLE 4 CONT.

	0	1	2	3	4	5	6	7	8	9	10	11
684	01013	01011	01012	01020	01030	01040	01045	01058	01069	01079	01096	01105
	01107	01103	01099	01097	01091	01084	01074	01063	01058	01055	01053	01053
	01052	01052	01055	01058	01082	01069	01078	01089	01106	01130	01149	01163
	01176	01185	01190	01192	01194	01191	01189	01185	01180	01179	01181	01187
	01194	01198	01201	01203	01203	01204	01203	01202	01203	01205	01207	01209
	01214	01220	01234	01246	01254	01262	01269	01277	01284	01294	01301	01307
	01312	01315	01318	01320	01321	01321	01320	01320	01319	01316	01321	01323
	01332	01342	01350	01354	01361	01369	01372	01377	01379	01381	01383	01390
	01392	01398	01405	01409	01417	01426	01437	01446	01453	01458	01464	01467
	01466	01465	01465	01464	01463	01466	01470	01476	01479	01487	01507	01522
804	01535	01580	01562	01572	01581	01582	01585	01588	01586	01583	01578	01579
	01580	01587	01596	01603	01606	01602	01593	01591	01585	01584	01585	01588
	01593	01601	01611	01628	01647	01663	01680	01693	01700	01705	01709	01713
	01714	01712	01706	01701	01694	01687	01682	01683	01684	01685	01685	01686
	01688	01690	01695	01700	01705	01715	01723	01730	01737	01740	01742	01746
	01751	01754	01755	01756	01757	01760	01760	01759	01761	01762	01765	01769
	01771	01772	01770	01768	01789	01792	01798	01801	01804	01804	01806	01814
	01821	01824	01832	01847	01868	01893	01911	01925	01939	01955	01963	01962
	01956	01946	01939	01931	01922	01911	01909	01925	01931	01936	01944	01947
912	01952	01958	01964	01971	01998	02006	02027	02036	02034	02032	02030	02026
	02018	02017	02018	02013	02014	02016	02017	02017	02016	02013	02008	02010
	02013	02019	02028	02035	02040	02044	02052	02063	02073	02083	02095	02103
	02110	02117	02126	02134	02141	02144	02146	02148	02149	02155	02160	02165
	02173	02186	02196	02207	02218	02227	02237	02241	02245	02249	02257	02265
	02274	02285	02294	02305	02310	02317	02326	02340	02349	02357	02366	02376
	02386	02397	02412	02423	02432	02444	02458	02468	02477	02487	02497	02505
	02512	02520	02526	02536	02551	02565	02587	02609	02629	02645	02647	02644
	02639	02631	02621	02606	02593	02585	02576	02569	02562	02560	02559	02562
	02567	02575	02588	02591	02593	02596	02598	02602	02605	02605	02606	02607
1032	02617	02624	02634	02647	02662	02684	02707	02722	02732	02744	02752	02753
	02750	02750	02748	02743	02741	02739	02738	02743	02753	02767	02781	02797
	02811	02821	02828	02828	02821	02812	02804	02791	02771	02754	02743	02737
	02734	02733	02730	02726	02725	02728	02728	02735	02744	02750	02760	02761
	02766	02775	02788	02801	02817	02834	02845	02853	02859	02863	02866	02867
	02865	02862	02860	02859	02861	02862	02859	02856	02863	02865	02870	02881
	02886	02893	02904	02911	02921	02930	02940	02950	02955	02960	02966	02972
	02977	02989	03000	03011	03017	03026	03037	03043	03050	03055	03058	03060
	03062	03066	03069	03071	03075	03078	03076	03075	03080	03087	03091	03097
1140	03111	03123	03131	03141	03153	03165	03173	03185	03189	03196	03199	03209
	03214	03214	03221	03225	03231	03236	03242	03251	03258	03264	03271	03277
	03282	03292	03294	03300	03305	03310	03309	03310	03314	03317	03325	03334
	03347	03360	03372	03377	03384	03399	03409	03416	03423	03427	03431	03436
	03445	03446	03452	03455	03459	03464	03465	03467	03468	03469	03474	03478
	03479	03491	03483	03476	03471	03467	03461	03452	03448	03444	03443	03447
	03447	03451	03463	03475	03485	03493	03498	03500	03500	03496	03491	03487
	03484	03482	03480	03477	03480	03483	03487	03495	03504	03516	03529	03536
	03546	03551	03555	03554	03549	03540	03535	03535	03533	03532	03532	03530
	03529	03527	03529	03534	03538	03540	03543	03553	03561	03568	03581	03599
1260	03601	03606	03610	03609	03610	03611	03613	03615	03618	03626	03634	03633
	03639	03647	03652	03656	03664	03670	03674	03675	03675	03671	03667	03664
	03661	03662	03661	03659	03664	03664	03665	03670	03673	03679	03687	03693
	03698	03702	03709	03711	03713	03713	03712	03715	03713	03713	03714	03718
	03723	03730	03735	03738	03740	03746	03746	03748	03749	03749	03753	03755
	03762	03765	03769	03777	03787	03792	03794	03794	03798	03800	03802	03805
	03807	03810	03817	03826	03830	03838	03851	03872	03884	03887	03893	03899
	03903	03908	03913	03911	03916	03921	03928	03934	03936	03944	03953	03963
	03973	03984	03995	04004	04016	04029	04036	04039	04039	04043	04052	04054

TABLE 4 CONT.

	0	1	2	3	4	5	6	7	8	9	10	11
1368	04059	04067	04066	04093	04112	04125	04134	04141	04145	04156	04167	04176
	04174	04174	04177	04179	04193	04200	04204	04212	04217	04226	04246	04251
	04258	04271	04281	04287	04288	04292	04301	04309	04315	04323	04330	04340
	04352	04360	04371	04381	04389	04394	04400	04405	04415	04425	04434	04438
	04449	04459	04469	04477	04486	04495	04512	04524	04535	04541	04551	04562
	04575	04586	04594	04599	04609	04613	04615	04618	04628	04635	04643	04653
	04658	04669	04685	04693	04702	04709	04716	04728	04737	04748	04752	04766
	04776	04787	04802	04815	04827	04841	04852	04864	04878	04886	04895	04903
	04910	04915	04920	04922	04928	04937	04941	04953	04957	04966	04977	04987
	04993	05001	05010	05018	05023	05030	05037	05041	05046	05052	05064	05071
1488	05083	05102	05110	05114	05115	05113	05110	05107	05104	05095	05086	05082
	05083	05091	05101	05118	05132	05144	05151	05154	05154	05151	05145	05137
	05132	05127	05125	05128	05132	05138	05148	05159	05168	05173	05172	05175
	05178	05181	05176	05168	05164	05159	05154	05154	05152	05146	05129	05111
	05103	05101	05098	05094	05095	05099	05097	05093	05084	05075	05069	05066
	05060	05056	05054	05055	05060	05062	05063	05065	05070	05075	05082	05093
	05095	05091	05089	05088	05088	05085	05081	05076	05073	05069	05064	05057
	05055	05050	05041	05035	05027	05014	05002	04992	04983	04971	04962	04951
	04942	04938	04938	04937	04944	04952	04967	04963	04959	04955	04953	04950
	04953	04952	04951	04954	04950	04946	04945	04942	04940	04935	04938	04940
1608	04926	04923	04918	04914	04919	04912	04909	04907	04916	04918	04914	04914
	04910	04906	04901	04905	04903	04908	04902	04897	04893	04888	04882	04876
	04871	04866	04866	04875	04883	04898	04908	04909	04911	04906	04901	04910
	04918	04914	04910	04906	04904	04901	04894	04893	04887	04883	04879	04878
	04876	04873	04872	04874	04881	04901	04916	04914	04914	04916	04914	04911
	04915	04930	04927	04933	04934	04935	04939	04949	04986	04987	04986	04980
	04997	04997	04993	04989	04983	04981	04976	04970	04966	04962	04964	04966
	04965	04962	04960	04959	04958	04957	04977	04971	04969	04969	04983	04985
	04985	04987	04991	04994	04994	04997	04999	04999	04998	04993	04990	04988
1716	04987	04985	04990	04994	04992	05000	05009	05015	05023	05024	05020	05021
	05020	05020	05017	05010	05007	05004	05003	05007	05011	05015	05019	05029
	05035	05044	05052	05053	05053	05048	05047	05044	05044	05061	05057	05054
	05051	05049	05048	05058	05063	05062	05061	05060	05058	05055	05051	05047
	05043	05034	05031	05046	05045	05036	05037	05039	05031	05026	05028	05030
	05027	05032	05044	05048	05049	05068	05088	05097	05103	05110	05116	05119
	05122	05120	05134	05131	05125	05123	05129	05118	05116	05119	05121	05124
	05122	05119	05115	05109	05109	05105	05098	05105	05108	05105	05107	05113
	05125	05134	05137	05154	05176	05196	05203	05218	05226	05227	05235	05244
	05253	05258	05262	05265	05268	05270	05276	05284	05291	05296	05303	05310
1836	05324	05345	05356	05364	05371	05382	05394	05405	05413	05428	05452	05470
	05475	05482	05491	05503	05515	05525	05532	05538	05550	05583	05598	05605
	05614	05635	05648	05655	05658	05670	05681	05690	05701	05716	05730	05740
	05752	05762	05771	05782	05800	05815	05826	05839	05848	05858	05865	05867
	05869	05886	05889	05886	05880	05875	05870	05863	05857	05854	05859	05869
	05893	05899	05904	05916	05923	05927	05932	05935	05937	05938	05954	05961
	05967	05978	05983	05990	05997	06001	06007	06015	06031	06037	06054	06061
	06066	06070	06073	06095	06107	06110	06114	06116	06121	06127	06137	06145
	06150	06158	06167	06173	06176	06182	06189	06193	06191	06190	06192	06195
1944	06199	06210	06211	06217	06240	06247	06253	06260	06271	06279	06304	06308
	06313	06317	06322	06333	06340	06349	06354	06364	06371	06376	06382	06392
	06399	06404	06408	06414	06414	06416	06422	06427	06435	06442	06456	06482
	06501	06511	06515	06520	06535	06545	06550	06552	06553	06553	06552	06550
	06548	06553	06556	06570	06572	06577	06588	06603	06614	06620	06627	06634
	06638	06653	06663	06665	06682	06692	06694	06696	06701	06706	06708	06709
	06709	06710	06710	06710	06712	06715	06718	06770	06724	06731	06748	06757
	06765	06771	06784	06790	06792	06794	06793	06790	06785	06779	06775	06778
	06778	06781	06782	06783	06783	06786	06786	06797	06808	06806	06804	06802
	06795	06792	06786	06781	06779	06779	06779	06773	06777	06774	06772	06769

TABLE 4 CONT.

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	06739	06737	06734	06730	06724	06719	06714	06313	06708	06702	06698	06694
	06694	06690	06682	06676	06672	06666	06659	06652	06643	06644	06638	06632
	06626	06622	06620	06619	06630	06630	06636	06648	06648	06650	06654	06665
	06683	06699	06712	06723	06736	06750	06761	06771	06778	06785	06795	06807
	06818	06831	06847	06856	06866	06880	06892	06907	06923	06940	06953	06963
	06978	06989	06996	07005	07015	07025	07049	07079	07095	07120	07152	07185
	07205	07225	07237	07265	07286	07293	07306	07319	07328	07335	07351	07369
	07381	07393	07407	07415	07431	07450	07465	07471	07477	07483	07487	07487
	07486	07488	07507	07516	07523	07529	07535	07558	07569	07585	07604	07613
2184	07618	07626	07632	07637	07643	07651	07656	07660	07667	07674	07676	07682
	07685	07685	07694	07704	07711	07727	07740	07743	07741	07742	07749	07756
	07758	07759	07762	07765	07783	07801	07805	07808	07810	07823	07824	07825
	07826	07831	07835	07840	07841	07846	07856	07872	07877	07883	07887	07897
	07913	07914	07917	07935	07946	07950	07951	07954	07955	07957	07960	07963
	07967	07970	07975	07977	07980	07982	07987	07988	07990	07995	07997	08000
	08002	08009	08025	08031	08032	08029	08027	08021	08014	08008	08002	07999
	08005	08002	07998	08000	08004	08008	08021	08034	08058	08060	08067	08075
	08081	08086	08094	08105	08119	08125	08134	08141	08157	08167	08176	08188
	08204	08616	08233	08250	08261	08275	08294	08314	08332	08349	08392	08421
2304	08449	08475	08503	08536	08565	08590	08612	08630	08649	08664	08694	08716
	08730	08745	08770	08794	08812	08829	08849	08868	08886	08910	08939	08973
	08993	09013	09038	09072	09105	09129	09154	09184	09216	09241	09268	09294
	09322	09347	09368	09390	09419	09451	09481	09498	09515	09534	09551	09573
	09593	09612	09638	09651	09667	09688	09713	09734	09755	09776	09798	09812
	09836	09856	09881	09910	09936	09952	09971	09994	10011	10038	10062	10081
	10099	10109	10130	10147	10167	10174	10195	10210	10219	10230	10242	10254
	10259	10269	10279	10295	10314	10324	10344	10360	10382	10412	10434	10466
	10488	10508	10540	10569	10568	10567	10564	10560	10552	10545	10561	10551
	10538	10525	10512	10497	10485	10471	10454	10440	10420	10410	10394	10379
2436	10372	10365	10358	10350	10344	10340	10334	10326	10321	10310	10320	10331
	10326	10320	10313	10304	10292	10280	10269	10258	10244	10230	10216	10198
	10180	10169	10159	10148	10132	10123	10113	10099	10078	10058	10047	10031
	10008	09993	09980	09966	09949	09938	09927	09915	09902	09895	09884	09868
	09856	09847	09828	09819	09829	09838	09831	09819	09806	09792	09779	09772
	09760	09748	09745	09736	09723	09712	09703	09693	09685	09675	09678	09671
	09665	09661	09664	09657	09650	09643	09631	09624	09614	09607	09601	09597
	09592	09583	09578	09570	09559	09549	09543	09538	09532	09524	09516	09507
	09498	09488	09495	09502	09505	09490	09474	09463	09451	09440	09430	09418
	09403	09389	09378	09366	09352	09342	09333	09325	09315	09312	09307	09303
	09296	09288	09295	09296	09287	09278	09271	09261	09250	09240	09230	09218
2556	09205	09195	09187	09179	09171	09164	09156	09148	09144	09141	09138	09136
	09135	09135	09135	09137	09138	09140	09145	09158	09163	09165	09166	09167
	09165	09169	09181	09183	09197	09198	09198	09210	09218	09218	09227	09228
	09227	09225	09228	09227	09227	09229	09232	09235	09238	09242	09245	09245
	09248	09255	09255	09257	09261	09267	09271	09280	09285	09286	09288	09295
	09301	09303	09308	09310	09318	09320	09318	09317	09325	09333	09334	09337
	09341	09342	09346	09347	09345	09338	09330	09335	09335	09340	09344	09347
	09349	09357	09360	09362	09365	09368	09368	09375	09371	09368	09374	09378
	09379	09365	09364	09368	09374	09380	09388	09397	09410	09422	09445	09463
	09488	09499	09503	09505	09506	09515	09516	09522	09525	09527	09529	09531

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	09539	09535	09537	09541	09549	09550	09553	09555	09560	09569	09574	09580
	09585	09588	09594	09597	09602	09608	09627	09625	09625	09625	09621	09619
	09616	02613	09610	09608	09612	09617	09620	09622	09619	09620	09624	09628
	09631	09638	09640	09635	09647	09645	09647	09646	09648	09648	09646	09648
	09655	09658	09653	09650	09648	09648	09646	09643	09642	09639	09636	09635
	09636	09635	09637	09642	09644	09651	09652	09653	09651	09653	09654	09654
	09660	09662	09661	09667	09668	09668	09670	09673	09679	09684	09685	09685
	09688	09689	09703	09705	09706	09705	09708	09714	09710	09711	09718	09725
	09730	09733	09739	09740	09742	09711	09739	09738	09755	09756	09759	09761
2796	09761	09763	09769	09778	09790	09800	09805	09810	09819	09822	09828	09832
	09838	09842	09841	09849	09855	09860	09870	09882	09893	09903	09912	09919
	09924	09932	09934	09948	09956	09963	09975	09981	09985	09995	10003	10008
	10013	10020	10024	10024	10024	10040	10043	10042	10044	10044	10048	10049
	10047	10050	10052	10062	10069	10080	10084	10090	10097	10103	10107	10113
	10110	10114	10121	10127	10135	10142	10159	10168	10184	10209	10214	10217
	10219	10219	10218	10210	10208	10205	10203	10206	10205	10222	10225	10225
	10227	10227	10228	10230	10230	10231	10228	10229	10223	10215	10214	10224
	10226	10231	10236	10239	10265	10267	10284	10295	10295	10294	10292	10292
	10293	10291	10289	10303	10313	10317	10316	10318	10319	10320	10327	10345
2916	10345	10344	10345	10345	10344	10341	10336	10324	10314	10305	10299	10312
	10320	10337	10344	10352	10361	10370	10378	10384	10390	10397	10396	10396
	10402	10403	10397	10395	10390	10389	10389	10385	10388	10387	10388	10388
	10385	10384	10384	10383	10381	10380	10380	10377	10375	10371	10369	10367
	10366	10366	10365	10363	10364	10362	10360	10359	10355	10353	10352	10351
	10348	10351	10349	10346	10342	10337	10333	10321	10316	10312	10308	10307
	10306	10308	10313	10320	10327	10337	10344	10350	10352	10356	10358	10358
	10358	10354	10352	10349	10346	10346	10349	10351	10356	10363	10368	10373
	10382	10391	10399	10411	10418	10437	10451	10460	10467	10471	10480	10494
	10512	10527	10546	10565	10576	10585	10592	10591	10584	10573	10561	10554
3036	10548	10539	10530	10526	10523	10521	10521	10522	10522	10523	10526	10528
	10537	10542	10553	10564	10570	10574	10573	10567	10562	10554	10548	10544
	10540	10540	10541	10541	10544	10546	10549	10551	10552	10552	10546	10540
	10535	10527	10520	10513	10508	10506	10501	10495	10493	10495	10503	10508
	10508	10505	10503	10504	10502	10499	10495	10491	10485	10476	10469	10463
	10458	10454	10453	10449	10447	10448	10451	10455	10459	10465	10468	10470
	10473	10473	10473	10471	10468	10465	10465	10464	10464	10464	10478	10483
	10485	10491	10495	10495	10495	10497	10498	10495	10492	10490	10488	10485
	10486	10487	10488	10490	10498	10507	10512	10518	10524	10516	10506	10497
	10484	10472	10460	10439	10426	10417	10411	10413	10405	10395	10388	10377
3156	10365	10356	10342	10329	10314	10303	10289	10275	10262	10253	10245	10237
	10232	10229	10229	10230	10231	10233	10232	10432	10230	10226	10218	10212
	10209	10206	10202	10194	10186	10179	10173	10169	10165	10157	10154	10157
	10162	10165	10167	10168	10172	10175	10181	10186	10192	10196	10203	10211
	10223	10229	10246	10254	10263	10267	10271	10278	10279	10282	10287	10294
	10305	10312	10317	10322	10330	10335	10335	10331	10329	10327	10321	10316
	10315	10320	10326	10333	10342	10350	10360	10372	10384	10395	10403	10412
	10418	10426	10429	10432	10439	10446	10456	10468	10477	10485	10494	10508
	10515	10524	10538	10547	10558	10574	10588	10595	10605	10607	10610	10612
	10614	10620	10623	10625	10633	10644	10651	10659	10669	10677	10683	10687

TABLE 4 CONT.

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	10751	10756	10762	10770	10776	10780	10783	10786	10790	10797	10803	10807
	10812	10817	10819	10821	10824	10828	10832	10838	10847	10855	10864	10870
	10881	10887	10897	10904	10912	10919	10927	10937	10951	10959	10970	10978
	10988	10996	10999	10999	10993	10982	10967	10955	10946	10938	10927	10916
	10919	10917	10933	10934	10935	10939	10938	10937	10938	10942	10950	10950
	10945	10944	10947	10947	10948	10947	10945	10948	10948	10950	10952	10954
	10957	10958	10958	10959	10960	10964	10967	10969	10977	10981	10981	10978
	10977	10976	10974	10968	10960	10949	10944	10937	10932	10926	10917	10911
	10908	10901	10885	10878	10882	10894	10900	10902	10904	10900	10896	10891
	10887	10886	10886	10884	10880	10874	10887	10884	10881	10879	10878	10876
3402	10870	10864	10863	10864	10864	10865	10868	10875	10884	10891	10997	10910
	10927	10938	10949	10959	10968	10973	10977	10978	10977	10974	10966	10960
	10953	10946	10940	10937	10934	10932	10928	10943	10944	10946	10948	10951
	10954	10973	10987	11000	11002	11006	11008	11012	11016	11021	11027	11032
	11032	11035	11037	11040	11043	11053	11067	11069	11071	11072	11084	11085
	11085	11083	11084	11084	11083	11077	11074	11069	11064	11065	11063	11062
	11060	11061	11064	11065	11069	11074	11086	11096	11104	11107	11109	11109
	11106	11104	11099	11095	11092	11091	11091	11092	11098	11104	11122	11142
	11149	11154	11157	11160	11160	11154	11148	11145	11145	11144	11144	11146
	11149	11149	11150	11154	11152	11154	11151	11147	11147	11148	11150	11153
3528	11148	11158	11157	11158	11158	11157	11153	11148	11146	11144	11141	11138
	11142	11148	11153	11161	11164	11171	11176	11182	11191	11195	11193	11191
	11188	11187	11187	11187	11183	11179	11175	11177	11179	11182	11184	11186
	11187	11188	11188	11183	11170	11176	11176	11175	11172	11170	11169	11168
	11167	11167	11173	11179	11182	11183	11179	11175	11170	11161	11157	11152
	11148	11142	11138	11134	11132	11129	11127	11123	11119	11117	11112	11107
	11103	11104	11107	11106	11100	11095	11090	11085	11080	11077	11072	11065
	11061	11057	11035	11049	11041	11035	11026	11017	11007	10993	10982	10973
	10972	10962	10953	10944	10937	10926	10921	10915	10906	10898	10894	10890
	10886	10883	10878	10871	10865	10862	10853	10841	10830	10820	10810	10799
3648	10791	10784	10779	10770	10763	10760	10756	10748	10746	10752	10740	10738
	10738	10738	10731	10730	10726	10720	10719	10716	10709	10703	10700	10697
	10694	10693	10692	10689	10687	10686	10686	10685	10683	10680	10678	10675
	10674	10676	10676	10674	10677	10677	10676	10672	10672	10668	10667	10664
	10660	10657	10656	10655	10655	10655	10654	10651	10654	10655	10653	10655
	10660	10659	10661	10667	10675	10677	10674	10677	10683	10689	10694	10702
	10711	10716	10723	10733	10737	10740	10738	10739	10738	10739	10742	10745
	10747	10751	10757	10762	10771	10778	10785	10793	10798	10804	10814	10821
	10829	10839	10845	10850	10855	10859	10865	10870	10875	10881	10890	10907
	10912	10920	10926	10934	10942	10952	10961	10969	10981	10991	11001	11011
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	11060	11065	11070	11076	11084	11096	11113	11128	11140	11152	11161	11164
	11165	11168	11171	11174	11178	11185	11197	11207	11220	11229	11234	11239
	11245	11244	11240	11237	11234	11230	11228	11229	11228	11229	11232	11243
	11252	11258	11262	11266	11267	11267	11268	11272	11277	11283	11290	11296
	11299	11306	11322	11338	11338	11337	11345	11350	11355	11360	11367	11372
	11379	11388	11399	11406	11413	11420	11422	11425	11428	11433	11436	11439
	11440	11442	11445	11449	11454	11449	11464	11472	11481	11489	11495	11504
	11513	11521	11526	11532	11537	11543	11551	11557	11563	11569	11578	11593
	11607	11620	11631	11641	11649	11658	11662	11667	11672	11677	11678	11679
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TABLE 4 CONT.

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	12084	12097	12109	12117	12126	12134	12142	12149	12156	12165	12173	12178
	12186	12195	12205	12219	12232	12248	12260	12277	12288	12299	12311	12322
	12333	12343	12353	12364	12376	12386	12395	12404	12411	12420	12428	12435
	12442	12448	12456	12464	12471	12478	12486	12494	12503	12513	12517	12522
	12526	12534	12542	12549	12556	12564	12571	12578	12588	12597	12609	12624
	12648	12667	12679	12686	12685	12676	12661	12638	12612	12585	12569	12558
	12544	12534	12528	12527	12529	12531	12535	12540	12556	12550	12553	12556
	12558	12559	12565	12565	12565	12564	12562	12560	12559	12559	12560	12561
4020	12558	12555	12557	12560	12563	12579	12585	12588	12592	12595	12599	12601
	12603	12606	12609	12611	12614	12610	12625	12627	12633	12641	12652	12662
	12668	12673	12675	12679	12676	12672	12668	12662	12658	12655	12650	12644
	12642	12638	12638	12637	12638	12637	12635	12633	12633	12630	12626	12624
	12620	12615	12612	12605	12598	12592	12586	12582	12576	12570	12564	12558
	12551	12544	12536	12528	12520	12513	12502	12494	12487	12474	12471	12465
	12457	12450	12438	12437	12419	12412	12404	12396	12389	12383	12382	12384
	12380	12373	12368	12364	12360	12357	12355	12351	12348	12347	12344	12338
	12333	12325	12321	12317	12310	12307	12302	12314	12310	12305	12303	12299
	12296	12293	12300	12297	12295	12294	12292	12292	12292	12291	12292	12292
4152	12294	12295	12294	12292	12288	12286	12285	12283	12281	12278	12277	12277
	12275	12277	12278	12278	12282	12285	12287	12287	12288	12287	12285	12284
	12286	12285	12295	12302	12304	12304	12307	12308	12306	12304	12303	12300
	12310	12317	12316	12318	12325	12333	12343	12350	12353	12354	12354	12351
	12349	12347	12348	12350	12354	12359	12365	12371	12379	12386	12395	12400
	12405	12409	12410	12411	12412	12420	12422	12426	12429	12433	12435	12435
	12437	12438	12438	12440	12442	12445	12469	12480	12489	12497	12506	12512
	12520	12525	12529	12532	12532	12532	12532	12532	12530	12526	12525	12524
	12523	12527	12525	12523	12523	12523	12523	12523	12524	12524	12524	12524
4260	12524	12526	12539	12541	12544	12549	12554	12560	12565	12569	12576	12580
	12583	12586	12590	12591	12592	12593	12593	12592	12590	12591	12590	12590
	12591	12592	12594	12604	12607	12611	12613	12615	12615	12615	12617	12618
	12624	12624	12625	12634	12646	12653	12657	12661	12665	12667	12667	12663
	12659	12651	12641	12633	12624	12616	12607	12600	12595	12591	12589	12587
	12585	12586	12583	12584	12582	12582	12581	12581	12581	12581	12580	12582
	12580	12581	12580	12580	12579	12578	12577	12576	12574	12574	12573	12580
	12584	12583	12602	12599	12598	12597	12593	12592	12589	12587	12586	12584
	12579	12578	12577	12575	12573	12572	12570	12569	12569	12570	12568	12567
	12566	12565	12563	12563	12563	12563	12563	12561	12561	12559	12559	12558
	12557	12554	12551	12548	12544	12541	12537	12532	12528	12532	12529	12524
	12521	12517	12514	12511	12509	12509	12508	12501	12498	12494	12492	12493
4392	12493	12494	12495	12497	12498	12502	12502	12502	12502	12503	12504	12503
	12501	12499	12498	12496	12491	12487	12482	12475	12482	12483	12478	12471
	12460	12456	12452	12449	12447	12442	12436	12430	12427	12423	12418	12412
	12404	12395	12386	12381	12375	12369	12360	12350	12343	12338	12333	12323
	12318	12312	12309	12309	12307	12304	12302	12303	12309	12310	12313	12317
	12325	12331	12336	12342	12348	12353	12360	12367	12373	12382	12390	12400
	12410	12417	12425	12437	12435	12442	12452	12554	12462	12480	12993	12503
	12516	12532	12540	12552	12564	12576	12590	12602	12614	12630	12644	12656
	12669	12687	12704	12720	12734	12753	12760	12782	12799	12811	12818	12824
	12832	12841	12847	12852	12860	12867	12879	12894	12911	12925	12943	12965

TABLE 4 CONT.

	0	1	2	3	4	5	6	7	8	9	10	11
4512	12983	13010	13030	13051	13071	13087	13103	13126	13143	13160	13174	13197
	13219	13229	13242	13247	13258	13272	13284	13298	13309	13320	13333	13349
	13363	13374	13389	13403	13717	13431	13445	13458	13472	13493	13509	13520
	13533	13546	13551	13563	13585	13604	13623	13639	13652	14673	13690	13706
	13724	13738	13751	13760	13773	13778	13783	13791	13795	13826	13841	13855
	13874	13898	13913	13926	13943	13956	13963	13978	13991	13991	13998	14007
	14016	14023	14034	14035	14031	14049	14057	14071	14087	14095	14111	14122
	14135	14148	14167	14179	14192	14200	14210	14220	14229	14239	14244	14247
	14255	14267	14276	14284	14291	14303	14315	14328	14347	14358	14370	14381
	14392	14406	14416	14425	14433	14442	14453	14461	14468	14475	14481	14486
	14494	14501	14508	14515	14522	14533	14543	14557	14570	14581	14594	14608
4644	14621	14631	14644	14657	14663	14674	14689	14701	14712	14726	14736	14741
	14759	14766	14781	14790	14799	14810	14819	14833	14841	14852	14860	14866
	14873	14883	14897	14910	14925	14931	14937	14938	14947	14955	14960	14966
	14974	14982	14988	14996	15008	15018	15030	15041	15047	15052	15069	15071
	15082	15095	15105	15112	15116	15124	15135	15142	15149	15166	15174	15181
	15195	15220	15231	15248	15264	15284	15303	15319	15333	15351	15362	15373
	15385	15403	15412	15421	15432	15449	15461	15466	15482	15495	15503	15509
	15523	15532	15543	15551	15558	15572	15574	15582	15608	15618	15634	15645
	15659	15672	15681	15683	15692	15709	15733	15740	15755	15765	15776	15785
	15802	15823	15844	15860	15880	15900	15910	15923	15933	15943	15946	15946
	15949	15952	15956	15963	15971	15982	15989	15996	16007	16017	16020	16023
4776	16027	16030	16034	16038	16045	16051	16061	16071	16086	16099	16106	16117
	16127	16134	16143	16154	16163	16171	16179	16188	16195	16202	16214	16220
	16223	16228	16235	16238	16237	16242	16248	16252	16253	16254	16254	16252
	16251	16252	16253	16256	16256	16259	16266	16278	16297	16314	16328	16344
	16367	16395	16413	16421	16424	16424	16423	16422	16420	16416	16423	16435
	16448	16459	16472	16479	16488	16503	16513	16524	16531	16542	16551	16553
	16562	16568	16584	16697	16606	16616	16623	16625	16627	16628	16632	16638
	16647	16661	16670	16690	16687	16691	16697	16704	16710	16711	16710	16713
	16719	16727	16734	16740	16749	16757	16763	16770	16776	16783	16787	16793
	16797	16800	16809	16810	16828	16833	16844	16854	16866	16874	16883	16891
4896	16896	16905	16911	16917	16924	16932	16939	16945	16954	16965	16969	16979
	16989	16999	17008	17016	17026	17034	17043	17057	17067	17077	17085	17095
	17104	17112	17119	17126	17133	17140	17149	17157	17165	17174	17182	17197
	17209	17218	17234	17242	17250	17254	17260	17265	17269	17272	17277	17283
	17303	17311	17324	17335	17343	17355	17361	17368	17376	17383	17393	17403
	17407	17410	17417	17422	17428	17434	17440	17447	17450	17456	17463	17468
	17473	17472	17454	17488	17491	17491	17499	17513	17521	17533	17536	17541
	17549	17560	17575	17594	17602	17612	17625	17624	17628	17638	17642	17645
	17652	17657	17665	17666	17670	17685	17692	17700	17706	17717	17724	17731
	17738	17749	17756	17762	17775	17786	17794	17799	17808	17814	17821	17827
5016	17839	17844	17850	17855	17857	17860	17863	17873	17881	17886	17893	17899
	17906	17912	17921	17936	17945	17957	17969	17977	17986	17995	18003	18011
	18018	18027	18035	18040	18047	18054	18059	18065	18073	18083	18092	18104
	18118	18135	18150	18166	18185	18203	18217	18234	18251	18265	18278	18295
	18316	18333	18352	18370	18383	18393	18407	18420	18429	18439	18452	18466
	18478	18491	18509	18522	18538	18556	18571	18583	18599	18617	18633	18645
	18659	18675	18689	18704	18720	18735	18752	18765	18788	18814	18835	18855

TABLE 4 CONT.

	0	1	2	3	4	5	6	7	8	9	10	11
5100	18870	18891	18906	18928	18947	18962	18974	18995	18986	18992	19003	19016
	19030	19044	19060	19078	19098	19115	19126	19138	19146	19149	19151	19152
	19153	19153	19150	19139	19129	19121	19117	19120	19126	19126	19122	19122
	19125	19127	19120	19115	19113	19113	19113	19118	19130	19149	19172	19185
	19200	19219	19235	19240	19239	19235	19234	19235	19229	19226	19225	19224
	19227	19231	19235	19234	19226	19217	19204	19207	19191	19179	19164	19148
	19132	19120	19113	19109	19106	19105	19104	19106	19104	19102	19100	19092
	19087	19077	19064	19058	19052	19044	19034	19024	19013	19002	18994	18983
	18970	18953	18939	18925	18908	18887	18873	18859	18839	18824	18810	18793
	18772	18759	18749	18729	18709	18701	18685	18671	18659	18641	18621	18598
5220	18583	18564	18546	18529	18511	18498	18485	18466	18447	18432	18416	18406
	18422	18408	18395	18386	18374	18361	18345	18329	18313	18288	18294	18286
	18278	18266	18256	18249	18247	18224	18208	18202	18195	18194	18188	18184
	18182	18183	18169	18061	18172	18171	18168	18165	18160	18150	18147	18141
	18129	18124	18120	18112	18106	18101	18098	18094	18089	18079	18076	18070
	18074	18068	18068	18078	18088	18085	18081	18079	18076	18071	18066	18060
	18056	18055	18056	18063	18058	18056	18057	18055	18053	18045	18043	18038
	18028	18021	18017	18005	17993	17988	17975	17963	17950	17938	17923	17936
	17939	17932	17926	17917	17912	17912	17908	17896	17894	17889	17885	17871
	17853	17843	17845	17846	17847	17850	17849	17844	17838	17830	17828	17832
5340	17839	17837	17840	17841	17838	17845	17850	17851	17845	17832	17822	17817
	17812	17813	17827	17853	17852	17832	17815	17815	17814	17809	17802	17801
	17799	17790	17784	17779	17780	17780	17777	17773	17767	17762	17757	17759
	17751	17747	17739	17737	17732	17728	17723	17716	17714	17706	17704	17704
	17705	17704	17703	17703	17704	17705	17706	17714	17715	17713	17708	17706
	17703	17697	17692	17686	17679	17676	17669	17666	17662	17665	17666	17662
	17662	17660	17651	17645	17640	17630	17641	17644	17645	17647	17656	17672
	17669	17664	17668	17662	17656	17653	17657	17660	17663	17665	17668	17671
	17675	17678	17680	17683	17684	17685	17681	17679	17684	17681	17678	17670
	17673	17676	17680	17682	17684	17686	17689	17687	17692	17702	17716	17712
	17706	17704	17702	17701	17697	17687	17676	17665	17665	17682	17683	17679
5472	17663	17683	17688	17695	17715	17711	17710	17719	17711	17708	17706	17704
	17701	17700	17700	17709	17715	17723	17733	17739	17739	17732	17726	17714
	17710	17712	17716	17714	17715	17722	17737	17750	17766	17788	17799	17807
	17817	17824	17827	17828	17824	17817	17813	17811	17804	17800	17797	17795
	17788	17780	17773	17766	17758	17751	17742	17744	17740	17756	17750	17745
	17750	17751	17750	17750	17748	17743	17736	17736	17733	17727	17725	17720
	17725	17726	17738	17747	17754	17759	17765	17768	17770	17769	17766	17763
	17753	17742	17735	17730	17723	17720	17720	17721	17724	17729	17731	17734
	17737	17739	17740	17739	17736	17735	17732	17733	17734	17738	17741	17743
	17746	17747	17746	17743	17740	17739	17738	17740	17742	17745	17756	17761
	17763	17765	17766	17768	17775	17778	17779	17781	17783	17783	17783	17783
5604	17781	17779	17776	17775	17769	17768	17768	17761	17763	17763	17763	17765
	17754	17755	17760	17772	17771	17774	17782	17795	17805	17814	17825	17832
	17838	17842	17844	17841	17836	17828	17825	17824	17821	17819	17824	17831
	17839	17854	17867	17875	17882	17890	17895	17897	17901	17905	17906	17904
	17902	17900	17896	17908	17906	17905	17904	17908	17920	17930	17936	17938
	17942	17942	17948	17950	17951	17949	17951	17956	17957	17957	17957	17960
	17961	17963	17966	17972	17979	17987	18003	18015	18028	18032	18036	18037
	18031	18026	18021	18016	18009	18000	17992	17988	17986	17988	17990	17991
	17998	18006	18016	18029	18045	18063	18078	18091	18098	18098	18091	18075
	18057	18039	18013	17990	17972	17956	17952	17946	17949	17949	17959	17972
5724	17980	17992	17998	18005	18010	18011	18008	18001	17988	17970	17956	17947
	17933	17916	17911	17916	17930	17940	17943	17947	17966	17983	17992	17997

TABLE 4 CONT.

	0	1	2	3	4	5	6	7	8	9	10	11
	17992	17981	17972	17961	17955	17957	17950	17944	17944	17948	17951	17952
	17953	17956	17959	17960	17961	17962	17966	17971	17973	17978	17989	17993
	17989	17987	17986	17983	17980	17981	17980	17980	17980	17985	17993	18006
	18018	18027	18036	18043	18051	18064	18072	18080	18085	18083	18083	18081
	18085	18081	18078	18076	18077	18082	18088	18098	18113	18128	18145	18154
	18169	18173	18173	18172	18171	18170	18170	18170	18170	18171	18173	18175
	18179	18186	18191	18197	18204	18212	18218	18226	18239	18259	18266	18274
	18279	18281	18282	18283	18284	18284	18283	18296	18293	18291	18291	18289
5856	18288	18286	18285	18285	18284	18284	18284	18280	18279	18280	18283	18284
	18285	18289	18292	18293	18295	18299	18306	18312	18318	18325	18333	18337
	18341	18344	18346	18345	18344	18344	18343	18340	18338	18339	18339	18340
	18345	18348	18349	18351	18353	18352	18353	18354	18355	18359	18361	18358
	18353	18351	18350	18350	18351	18351	18351	18352	18361	18360	18376	18380
	18384	18401	18403	18406	18406	18404	18404	18407	18412	18419	18429	18433
	18442	18448	18450	18451	18451	18452	18448	18445	18441	18439	18431	18431
	18430	18426	18428	18429	18428	18428	18429	18429	18431	18437	18448	18463
	18475	18488	18501	18510	18517	18522	18524	18522	18520	18516	18512	18512
	18512	18513	18516	18522	18526	18529	18528	18526	18522	18517	18513	18509
	18506	18505	18503	18504	18508	18512	18516	18519	18520	18530	18529	18527
5988	18522	18516	18512	18509	18510	18513	18517	18526	18530	18535	18544	18552
	18559	18570	18584	18600	18610	18623	18636	18646	18651	18657	18654	18655
	18647	18638	18631	18626	18620	18619	18620	18620	18626	18647	18650	18653
	18657	18659	18660	18662	18664	18666	18669	18677	18682	18689	18696	18703
	18708	18710	18709	18705	18700	18692	18684	18675	18670	18669	18670	18675
	18689	18707	18730	18749	18768	18791	18807	18816	18822	18822	18819	18813
	18799	18788	18771	18761	18751	18741	18738	18738	18745	18754	18764	18772
	18784	18791	18796	18805	18809	18815	18818	18821	18831	18841	18851	18859
	18867	18873	18881	18886	18890	18889	18891	18894	18900	18907	18916	18921
	18928	18934	18940	18941	18937	18924	18931	18944	18942	18937	18932	18929
	18925	18925	18925	18926	18927	18932	18937	18940	18942	18945	18956	18957
6120	18961	18965	18969	18975	18979	18981	18982	18983	18981	18973	18966	18955
	18955	18953	18954	18954	18948	18943	18941	18942	18940	18938	18941	18950
	18964	18983	18996	19003	19010	19018	19020	19020	19019	19014	19010	19000
	18990	18986	18985	18988	18995	18996	19000	19006	19015	19020	19026	19033
	19044	19054	19060	19065	19070	19072	19078	19083	19088	19096	19109	19119
	19124	19127	19129	19135	19140	19141	19146	19148	19150	19158	19168	19174
	19185	19201	19212	19220	19224	19232	19235	19238	19241	19245	19245	19250
	19257	19263	19264	19269	19278	19285	19293	19300	19308			

TABLE 4A
SMOOTHED BATTLEFIELD DAY -6201 Feet

	0	1	2	3	4	5	6	7	8	9
-0.00222	-0.00444	0.00222	-0.00111	-0.01333	0.03444	0.01333	0.01444	0.02556	0.02111	0.00889
0.00778	0.01111	0.00556	-0.00667	-0.05000	-0.08444	-0.01222	0.02556	0.02111	0.02222	0.02222
-0.00778	-0.00222	-0.01111	-0.03222	0.01667	0.02778	0.01333	0.02778	0.02778	0.02778	-0.00778
-0.03000	-0.02222	0.00778	0.03333	0.03333	0.02889	0.03556	-0.01333	-0.05222	-0.02333	-0.02333
-0.01667	-0.02000	-0.00333	-0.00222	0.00333	0.03222	0.02778	0.02657	0.00778	-0.02889	-0.02889
-0.03333	-0.07778	-0.13222	-0.09556	0.01889	0.09657	0.13556	0.04000	-0.03333	-0.03333	-0.08889
-0.06000	0.02111	0.03889	-0.00333	0.03111	0.05222	0.03222	-0.00444	-0.01556	-0.02889	-0.02889
-0.00556	0.00111	0.00556	-0.02444	-0.06333	-0.05000	-0.02556	-0.02889	-0.00444	0.03657	0.03657
0.08556	0.06222	0.07333	0.12333	0.15111	0.15000	0.08889	0.02444	0.00557	-0.02889	-0.02889
-0.04889	-0.03556	-0.01778	-0.01111	-0.03222	0.01667	0.01567	0.02657	0.02222	-0.02556	-0.02556
-0.04778	-0.04778	-0.02000	0.00778	0.05222	0.01778	0.01333	0.02222	0.00557	-0.01222	-0.01222
-0.00333	0.00778	0.00000	0.00000	0.00111	-0.00889	0.01444	-0.04444	0.02444	0.01111	0.01111
-0.03556	0.00778	0.03778	-0.01444	-0.07667	-0.11444	-0.09567	-0.04444	-0.05557	-0.05557	-0.05557
0.01222	0.01889	0.02889	0.03778	0.02889	-0.01778	-0.00222	-0.04444	-0.05557	-0.05557	-0.05557
-0.12000	0.19333	-0.13222	-0.06556	-0.09667	-0.09111	-0.04556	-0.07111	-0.07356	-0.02657	-0.02657
0.10556	0.19444	0.25000	0.23667	0.14111	0.09444	0.07444	0.05556	0.05557	0.05557	0.05557
-0.10556	-0.13889	-0.09667	0.03778	-0.04111	-0.03000	0.03111	0.08556	0.10444	0.10444	0.10444
0.07778	0.03778	0.03667	0.03333	0.01000	-0.04778	-0.07567	-0.05556	-0.01778	-0.01778	-0.01778
-0.08333	-0.08000	-0.07556	-0.05778	-0.02778	-0.03333	0.04567	0.03778	0.01557	0.01557	0.01557
0.01556	-0.00556	0.02556	-0.00333	-0.01111	0.03222	-0.00889	-0.00556	-0.00778	-0.00778	-0.00778
-0.01222	-0.04889	-0.06667	-0.05333	-0.06333	-0.02889	0.00000	0.00657	0.05778	0.05778	0.05778
0.02333	0.00000	-0.03000	0.02333	0.07222	0.07111	0.00000	-0.03111	-0.03557	-0.03557	-0.03557
-0.01000	-0.03111	-0.03889	-0.00889	-0.01333	0.01657	0.01889	-0.00222	-0.00557	-0.00557	-0.00557
0.02222	0.05222	0.03111	0.00222	-0.03333	-0.01778	-0.00333	-0.01222	-0.02444	-0.02444	-0.02444
0.01333	0.01667	0.00556	-0.00889	-0.02222	-0.01657	-0.00778	0.00657	0.01778	0.01778	0.01778
0.00111	-0.00778	0.01222	0.01889	0.00556	-0.01657	-0.00778	0.00657	0.01778	0.01778	0.01778
0.03667	0.02778	0.00222	0.00000	-0.02000	-0.04000	-0.04111	-0.05444	0.03889	0.03889	0.03889
0.01444	-0.01222	-0.00889	0.00222	0.02556	0.04444	0.02778	-0.03889	-0.03556	-0.03556	-0.03556
0.00778	0.03222	0.03556	0.05556	-0.01000	-0.05000	0.03000	0.03444	0.02444	0.02444	0.02444
-0.00333	-0.02111	-0.03444	-0.03000	-0.01444	-0.02000	-0.03000	0.03444	-0.03444	-0.03444	-0.03444
-0.02000	0.01222	0.04556	-0.00667	-0.01556	-0.02657	-0.03778	-0.03222	-0.02556	-0.02556	-0.02556
-0.05778	-0.05222	0.01111	0.04667	0.04333	0.03889	0.03000	0.01778	0.02111	0.02111	0.02111
0.07333	0.05778	0.02556	-0.02444	-0.01778	-0.00889	0.03667	0.09222	0.03444	0.03444	0.03444
-0.04556	-0.03778	-0.01556	-0.01444	-0.02556	-0.02657	-0.04556	-0.04111	-0.02333	-0.02333	-0.02333
-0.01444	-0.01333	-0.03000	-0.02000	-0.02222	-0.03111	-0.03567	0.05839	0.06778	0.06778	0.06778
0.03000	0.02889	0.05000	0.03889	0.01444	-0.01111	-0.03000	-0.02839	-0.05444	-0.05444	-0.05444
-0.02222	0.03111	-0.03222	-0.05778	-0.05889	-0.01778	0.02111	0.05333	0.06222	0.06222	0.06222
0.02000	0.00000	0.04222	0.05000	0.03333	-0.02111	-0.05222	-0.05556	-0.03378	-0.03378	-0.03378
-0.06444	-0.07556	-0.03111	-0.08444	-0.00444	0.01556	0.01889	0.02333	0.08000	0.08000	0.08000

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
3.12556	0.07667	0.04889	0.06667	0.06778	0.02333	-0.01444	-0.07556	-0.12222	-0.11444
-0.09889	0.04333	0.05111	0.05222	0.06333	0.00556	-0.02000	-0.02444	-0.03000	-0.02556
-0.04222	-0.02667	-0.04333	-0.07556	-0.06778	-0.04667	-0.03889	-0.03000	-0.00556	-0.04222
0.12000	0.05444	0.04444	-0.00111	-0.02333	-0.02111	0.01333	0.02000	-0.00333	-0.02556
-0.01222	-0.03667	-0.08778	-0.05111	0.04556	0.03889	0.08222	0.05000	0.01556	-0.01444
-0.02556	-0.00333	-0.01444	-0.03000	-0.02000	0.00000	0.01567	0.02556	0.01333	-0.02222
-0.09111	-0.06444	-0.00778	0.00778	0.03000	0.04556	0.02000	-0.00111	-0.00557	0.01000
0.09333	0.16567	0.10889	0.01000	-0.03667	-0.05556	-0.05989	-0.07222	-0.03778	-0.01444
0.00778	0.02667	0.01111	0.03444	-0.01556	-0.03444	-0.04333	-0.02657	0.01000	0.03333
0.00667	0.02000	0.04111	0.03333	-0.02444	-0.01333	0.01111	-0.03839	0.05389	0.07444
-0.04222	0.00000	-0.03333	-0.03667	-0.02444	-0.01333	0.01111	0.03839	0.05389	0.07444
0.10667	0.10333	0.08444	0.07333	0.06000	-0.00222	-0.06222	-0.08556	-0.04556	-0.04333
-0.03778	-0.04000	-0.03000	-0.02111	0.00444	0.04444	0.04000	0.05444	0.06778	0.03556
0.01333	-0.04111	-0.09111	-0.08111	-0.12111	0.05889	-0.00444	-0.04222	-0.03557	0.04778
0.08111	0.02333	-0.05444	-0.04667	-0.06333	-0.04111	-0.01444	-0.01657	-0.02111	-0.03667
0.00556	0.07889	0.10889	0.12444	0.13111	0.15222	0.16778	0.11222	0.00222	-0.08889
-0.15889	-0.16333	-0.11556	-0.08444	-0.11667	-0.10556	-0.07889	-0.03657	-0.00444	0.08111
0.15222	0.14333	0.14778	0.12111	0.09333	0.08667	-0.01333	-0.09111	-0.10444	-0.10222
-0.08778	-0.07667	-0.07333	-0.03222	0.03444	0.04111	0.02111	0.02222	-0.04333	-0.03444
0.02444	-0.00889	-0.01111	0.00444	0.00889	0.01222	-0.01778	-0.04778	-0.00555	0.02778
0.02444	0.02556	0.03889	0.06778	0.08889	0.07556	0.00000	-0.02657	-0.02778	-0.03778
-0.01889	-0.05556	-0.07667	-0.09667	-0.06111	-0.07000	-0.06333	-0.03333	0.03444	0.05657
0.07444	0.06556	0.05556	0.07000	0.06000	0.02000	-0.00889	-0.01839	-0.06000	-0.05556
-0.04111	-0.02667	-0.01000	-0.01333	-0.01444	-0.00556	0.00556	0.01839	0.05778	0.06889
0.06444	-0.01778	0.05333	-0.04667	-0.05000	-0.03667	0.00556	0.01111	0.00557	0.03889
0.00111	0.01000	0.00556	0.03000	0.05444	0.06111	0.04778	0.03778	-0.02333	-0.03889
-0.06667	-0.08889	-0.14667	-0.01667	0.06889	0.04222	-0.01000	-0.01444	-0.01557	-0.03111
-0.00444	0.01889	0.02556	0.03333	0.00556	-0.00333	-0.01222	-0.01333	-0.03557	-0.04111
-0.03333	-0.03111	-0.03556	-0.00778	0.00889	0.01111	0.04333	0.07657	0.09444	0.08667
0.05111	0.01778	0.00222	-0.02667	-0.06555	-0.09889	-0.11222	-0.07333	-0.03111	-0.03444
-0.04889	-0.02222	-0.00889	0.01000	0.11444	0.14667	0.13000	0.07333	0.03339	0.05556
0.04778	0.03556	-0.00889	-0.06778	-0.06778	-0.05444	-0.04222	-0.02444	-0.05555	-0.05778
-0.06333	-0.07333	0.10778	-0.10889	-0.12667	-0.13667	-0.09778	0.02778	0.08333	0.09667
0.11000	0.10556	0.09000	0.07000	0.07111	0.03778	0.02222	-0.01444	-0.06557	-0.08111
-0.02222	-0.02444	-0.03778	-0.06555	-0.07333	0.03556	0.01000	0.03222	0.02389	0.02000
-0.02555	-0.02667	-0.00444	0.02778	0.03556	0.03889	0.04000	0.02889	0.03000	0.02889
0.00000	-0.00111	-0.00444	-0.00111	-0.02444	-0.07778	-0.06000	-0.07778	-0.03333	0.01111
0.02556	0.02111	0.00444	-0.00111	0.02444	-0.07778	-0.06000	-0.07778	-0.03333	0.01111
0.02889	0.00667	0.01444	0.04000	0.02445	0.03000	0.00778	0.01556	0.00444	-0.02333
-0.02889	-0.02111	-0.01333	-0.04333	-0.03333	-0.01667	0.02000	0.03222	0.02389	0.02000
0.06111	0.06111	0.03222	-0.00778	-0.00555	-0.02889	-0.05222	-0.04555	-0.05222	-0.05555
-0.10444	-0.15444	-0.06111	-0.02444	-0.01111	0.32445	0.03555	0.04556	0.06445	0.02111
0.05333	0.06445	0.03556	-0.00111	-0.06667	-0.07667	-0.03867	-0.03444	0.04445	0.10000

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
0.12333	0.07889	-0.00889	-0.02000	-0.06889	-0.07333	-0.07333	-0.08222	-0.09444	-0.10111
-0.10778	-0.05778	0.00778	0.04333	0.09333	0.11000	0.08445	0.06222	0.05445	0.07111
0.08000	0.07445	0.04000	0.01889	-0.01889	-0.05667	-0.07567	-0.04444	-0.02000	-0.00555
-0.01444	-0.02444	-0.02889	-0.04333	-0.03555	-0.03555	-0.04222	0.00000	0.02222	0.03556
0.04889	0.02445	0.00000	0.00333	0.02333	0.02778	0.01556	0.00657	0.00000	0.01778
0.00556	-0.02000	-0.01567	-0.02333	-0.00444	0.02667	0.01333	-0.01111	-0.07111	-0.13111
0.04000	0.03333	0.05556	0.03667	0.00778	-0.03111	-0.05555	-0.03000	-0.03444	-0.10333
-0.14222	-0.12444	-0.05333	0.04778	0.07333	0.05889	0.08778	0.15111	0.19000	0.15778
0.10111	0.03222	0.01333	-0.02444	-0.08000	-0.15778	-0.18555	-0.03444	0.00222	0.01222
0.03333	-0.00555	-0.03667	-0.06000	-0.10111	-0.13333	0.04000	0.03111	0.16111	0.18222
0.11000	0.06889	0.03556	0.01111	-0.04444	-0.03444	-0.07778	-0.04333	-0.02222	0.00333
0.02333	0.03222	0.02222	-0.01333	-0.07667	-0.07667	-0.07778	-0.02556	0.00333	0.01222
-0.00778	-0.04555	-0.05000	-0.02333	-0.00567	0.00778	0.03567	0.02556	0.00889	0.00000
0.02000	0.04111	0.06000	0.04000	0.01222	-0.01111	-0.04444	-0.03444	-0.04222	-0.05000
-0.05778	-0.01444	-0.00555	0.01445	0.03556	0.04111	0.06222	0.02556	-0.00889	-0.04333
-0.03778	-0.03333	-0.02000	0.01000	-0.02444	0.03222	-0.01111	-0.03333	-0.03333	0.01556
0.01556	-0.00111	-0.01667	-0.02444	-0.02667	0.02222	0.01555	0.01222	-0.01000	-0.00222
0.02667	0.02333	0.01445	0.01667	0.02556	0.01889	-0.00333	-0.02111	-0.07222	-0.03657
-0.08444	-0.09222	-0.01333	0.07556	0.16111	0.23222	0.19000	0.13833	0.10557	0.07555
0.05222	-0.01111	-0.05000	-0.04222	-0.05222	-0.05667	-0.08333	-0.03333	-0.09557	-0.08333
-0.06000	-0.01778	0.07000	0.05222	0.02445	0.01222	-0.00222	0.01657	0.01778	-0.01667
0.04889	-0.09333	-0.06000	-0.07778	-0.09111	-0.09000	-0.07889	0.00000	0.06778	0.10556
0.09111	0.11333	0.12222	0.09222	0.04111	0.03333	0.02000	-0.02000	-0.04000	-0.07889
-0.12333	-0.12778	-0.10333	-0.05222	-0.01111	-0.04889	0.10222	0.13657	0.16555	0.15445
0.11333	0.08667	0.09333	0.06445	-0.03111	-0.10333	-0.12222	-0.09555	-0.05222	-0.01444
-0.01555	-0.04667	-0.06444	-0.05222	-0.08222	-0.04667	-0.00111	0.00333	0.03557	-0.03444
-0.07555	-0.08555	-0.06111	-0.03444	0.01667	0.07889	0.08778	0.09000	0.06889	0.05889
0.06000	0.05445	0.02556	-0.00778	-0.02333	-0.02222	0.00000	0.01222	-0.02557	-0.03000
-0.04000	-0.05555	-0.05222	0.00000	-0.02222	-0.02667	0.00000	-0.01889	-0.00111	0.00657
0.02556	0.05000	0.02667	0.00111	-0.01667	-0.03555	-0.06000	-0.01889	0.00555	0.03000
0.00333	0.00667	0.04000	0.03333	0.04667	0.04222	0.02445	0.00657	-0.00889	0.00000

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
0.00667	0.00778	0.02556	0.02778	-0.02000	-0.05111	-0.05555	-0.03889	-0.05778	-0.07000
-0.01667	0.00889	-0.00667	-0.01111	0.00667	0.03222	0.02778	0.05111	0.02000	0.02222
-0.01000	0.03222	0.03111	-0.02111	-0.00222	-0.02000	-0.01444	-0.02000	-0.02333	0.00445
0.01111	0.00333	0.00889	0.00445	-0.00555	0.03667	0.00567	0.02333	0.03222	0.04333
-0.00333	-0.03778	-0.05000	-0.08111	-0.07000	-0.05555	-0.00778	0.02778	0.04555	-0.00555
-0.03444	0.02667	0.04778	0.04667	0.04111	0.01222	-0.00567	-0.00778	0.03445	-0.00111
0.01667	0.00667	0.01111	0.03445	0.01333	0.03445	-0.01222	-0.03778	0.00889	0.01889
0.02445	0.01556	0.07445	0.02889	0.01222	0.01111	0.00445	-0.04555	-0.05333	-0.07111
-0.07666	-0.05222	-0.08889	-0.09889	-0.03889	0.01778	0.05889	0.08445	0.09000	0.08334
0.07334	0.03667	0.00111	-0.01555	-0.02333	-0.02444	-0.03444	-0.05889	-0.05778	-0.05333
-0.07555	-0.05778	-0.04444	-0.00333	0.04667	0.04222	0.08222	0.09222	0.11111	0.09445
0.04778	-0.02667	-0.05555	-0.02778	-0.02000	-0.00555	0.00567	-0.01222	-0.02555	-0.05333
-0.04555	-0.01889	-0.01333	-0.03667	-0.06667	-0.04444	-0.03889	-0.04444	0.00778	0.11445
0.07111	0.06556	0.05556	0.00778	-0.00333	-0.02111	-0.03222	-0.03778	-0.04111	-0.03222
0.03222	-0.02555	-0.02000	0.00222	-0.00111	-0.00666	0.02567	0.05111	0.06889	0.05556
0.06000	0.02222	-0.00778	-0.02111	-0.03889	-0.01666	-0.02000	-0.04333	-0.00333	-0.02333
-0.04111	-0.02667	-0.04000	-0.02222	0.00778	0.01667	0.01889	0.01445	0.04778	0.03667
0.03445	0.01778	-0.00555	0.01445	-0.01889	-0.03778	-0.05222	-0.04111	-0.01889	0.01445
0.02778	0.02000	0.00556	0.03667	0.01111	0.03889	-0.00778	-0.03555	-0.02111	-0.03555
-0.00889	-0.02667	-0.03666	-0.00222	0.05000	0.05778	0.03567	-0.00333	0.00333	-0.00222
-0.01000	-0.01555	-0.03555	-0.05000	-0.03666	-0.02444	-0.07222	-0.08111	-0.04333	0.07555
0.11000	0.05334	0.03000	0.02333	0.01445	0.02334	0.02778	-0.03778	-0.02889	-0.02444
-0.00444	0.00000	-0.04889	-0.04444	-0.03565	-0.02111	-0.01222	-0.00555	0.00222	-0.00333
0.03222	0.08445	0.07889	0.04333	-0.01778	-0.03444	0.01445	-0.02889	-0.06000	-0.07555
-0.18666	-0.01555	0.07334	0.09556	0.07445	0.02222	-0.02778	0.01334	0.06556	0.10556
0.02778	-0.03333	-0.05666	-0.08666	0.00778	0.02000	-0.02000	-0.02222	-0.06000	-0.05666
0.05334	0.01111	-0.00333	0.04334	0.06000	0.05000	-0.01111	-0.04333	-0.01889	-0.00444
-0.01666	-0.01566	-0.03444	-0.02333	0.00778	0.00000	0.02445	0.04111	0.03778	0.00667
-0.01555	-0.04000	-0.01555	0.00667	0.01334	-0.03222	-0.01222	-0.00111	-0.00222	-0.01778
-0.03555	-0.04778	0.02000	0.03667	0.03778	-0.01333	0.02333	-0.01000	0.02556	0.04889
0.04667	0.02222	0.04889	0.02222	-0.02111	-0.05666	-0.02222	-0.01889	-0.03189	-0.00555
-0.04889	-0.02889	0.04111	0.02667	0.02334	-0.00666	-0.02889	0.00111	-0.00111	0.01445
-0.04889	-0.01889	-0.02889	-0.03444	0.00000	0.00556	-0.00111	0.01889	0.00333	0.01667
0.05111	0.03334	0.03556	0.03778	0.03667	0.02111	0.01000	-0.03444	-0.03444	-0.00666
-0.03555	0.01000	-0.02889	-0.02000	0.00889	0.02334	0.00556	0.00445	0.01556	0.02445
0.00889	0.01334	0.01334	-0.01444	-0.03566	-0.05444	-0.03333	-0.04889	-0.01111	0.10445
0.12000	0.11222	0.08556	0.05222	0.04000	0.04111	0.04556	-0.01778	-0.03444	-0.14333

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
-0.16111	-0.12555	-0.08778	0.00667	0.06667	0.11111	0.12111	0.11111	0.09555	0.07111
0.03222	-0.02222	-0.04778	-0.08000	-0.09666	-0.08222	-0.07666	-0.05222	-0.01222	0.04222
0.07567	0.07222	0.02000	0.02778	0.05223	0.03222	0.06334	0.03333	-0.01111	-0.02555
-0.01778	0.05445	0.10667	0.11667	0.01445	-0.09889	-0.11333	-0.07444	-0.05000	-0.05000
-0.01000	0.06111	0.07667	0.07223	0.02000	-0.02666	-0.03666	-0.02000	-0.04333	-0.05889
-0.06555	-0.05111	-0.00555	-0.00222	-0.02111	-0.04444	-0.03889	-0.02333	0.01567	0.09889
0.09334	0.03667	0.01000	0.00667	0.02889	0.02778	0.01778	0.00334	0.01000	-0.01222
0.01111	-0.00778	0.02667	0.04222	0.02667	0.04667	0.04889	0.01222	-0.01000	-0.01000
0.00334	-0.01778	-0.02333	-0.06111	-0.09778	-0.10333	-0.09889	-0.11000	-0.04889	0.01667
0.15000	0.09667	0.03889	-0.01000	-0.02889	-0.04444	0.00000	0.03445	0.00555	0.04778
0.01889	-0.00111	0.00445	-0.01333	-0.00222	-0.02222	0.03889	0.03334	-0.02111	-0.02000
-0.04111	-0.04666	0.03000	-0.03111	-0.05111	-0.06666	0.02778	0.05222	0.03445	0.03889
0.00334	-0.02778	-0.05000	-0.00111	0.00222	0.07667	0.04334	0.02111	0.01889	0.01000
-0.00333	-0.03333	-0.06778	-0.12333	-0.14555	-0.08555	-0.04444	0.06667	0.12778	0.08889
0.06111	-0.02333	-0.08566	0.00556	0.09111	0.05222	0.03556	0.03445	0.01000	0.01889
-0.01222	0.01334	-0.01333	-0.01889	-0.02666	-0.01444	-0.02111	-0.05666	-0.11333	-0.13222
-0.10222	0.05334	0.15778	0.09445	0.04889	0.01445	-0.03444	-0.08333	-0.06555	0.06111
0.00556	0.02567	-0.04666	-0.11666	-0.13889	-0.09778	0.20111	0.14111	0.06557	-0.04889
0.08333	0.08889	0.05111	0.03889	-0.00555	0.01334	0.00000	-0.03000	-0.04333	-0.06000
-0.01666	0.02222	0.02556	0.00556	-0.03111	-0.04889	-0.06222	-0.07666	0.10000	0.01222
-0.03666	-0.06889	0.03334	0.03445	0.00889	-0.02222	0.00445	0.01657	0.02222	0.02333
0.04000	0.04333	0.04111	-0.00111	-0.02111	-0.03555	-0.03778	-0.06000	-0.02778	0.01555
-0.07444	-0.03555	0.01556	0.04111	0.09222	0.07111	0.01222	0.02111	0.02000	0.04111
0.03445	-0.02111	-0.04000	-0.05444	-0.07333	-0.04666	-0.03444	-0.03555	-0.04889	-0.03444
0.00445	0.05333	0.09778	0.08000	0.06334	-0.01555	-0.04000	-0.07222	-0.07000	0.10445
0.06445	0.02222	-0.02889	-0.06889	-0.07889	0.01778	0.06333	0.04889	0.03567	0.02778
0.02445	0.02667	0.02111	-0.00222	-0.02555	-0.03111	-0.10111	0.05222	0.07000	-0.00111
0.01556	0.03667	-0.02222	-0.05778	-0.04666	-0.03889	-0.08000	-0.07111	-0.02000	-0.05666
-0.12778	-0.03000	0.07667	0.08334	0.06111	0.05222	0.03889	0.02111	0.02000	-0.02222
0.09667	0.06445	0.00778	-0.00889	0.05000	-0.04889	-0.05889	-0.02222	0.00557	0.05889
0.04889	0.03111	0.01445	-0.02778	-0.01000	-0.03111	-0.08778	-0.01555	-0.03033	-0.06111
-0.07666	-0.07889	-0.03778	-0.04555	-0.12444	-0.07778	0.01567	0.10334	0.06111	0.09222
0.06222	-0.01889	-0.01222	0.03889	0.04334	0.04445	0.03000	0.00556	-0.01566	-0.04444
-0.03444	-0.00778	-0.00333	-0.03889	-0.06444	-0.09222	-0.04889	0.06000	0.06111	0.02778

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
-0.01666	-0.02222	-0.02111	-0.03778	-0.08111	-0.05444	0.06445	0.12334	0.05111	-0.00333
-0.02889	-0.03444	0.02667	0.00667	-0.05222	-0.11889	-0.12222	0.07445	0.08778	0.02111
-0.02222	0.05445	0.07556	0.04334	-0.03333	-0.02666	-0.02222	-0.03444	-0.03222	0.00222
0.03000	0.01778	0.01556	-0.01111	-0.04333	-0.05444	0.00556	0.03778	0.03334	0.05557
0.05000	0.05445	0.04222	-0.00444	-0.03000	0.11000	0.12567	0.09839	0.05000	0.01657
-0.00333	-0.05111	-0.11889	-0.17000	-0.15222	-0.10333	0.07000	0.05223	0.01556	0.05111
0.04556	0.03556	0.02445	-0.00889	-0.04555	-0.09666	0.00111	0.00657	-0.00222	0.03667
0.01000	0.01222	0.00445	-0.03333	-0.05778	-0.06444	0.01111	-0.01000	0.08000	0.00222
0.00300	-0.04778	-0.10333	0.04778	0.10111	0.06334	0.02889	-0.03111	-0.04222	-0.03889
-0.00222	0.01222	-0.00444	0.00778	0.02889	0.02667	0.00555	0.02111	0.05334	0.05222
0.01334	-0.03444	-0.04666	-0.04777	-0.06000	-0.01222	-0.07222	-0.08778	0.05778	0.03889
-0.00555	-0.04333	-0.04000	-0.04555	0.12111	0.07223	0.03334	-0.01333	-0.04555	-0.00333
-0.00333	0.01567	-0.00555	0.01667	0.01334	-0.00778	-0.01333	0.02000	0.03445	0.03445
0.02334	0.03334	-0.01444	-0.04222	-0.04000	-0.07222	-0.08889	-0.12655	-0.09556	0.05445
0.12445	0.10223	0.02223	-0.03444	0.03667	0.07889	0.08334	0.05445	0.04334	0.02334
0.00111	-0.04111	-0.08333	-0.06000	-0.06889	0.01445	-0.03556	-0.05656	-0.03889	0.02445
0.05889	0.02889	0.00334	-0.01222	-0.06000	0.00334	0.02111	-0.03555	0.06000	0.03445
0.04334	0.01223	0.01334	0.03223	0.03223	0.02445	0.00567	0.00111	-0.01222	-0.08111
-0.07778	-0.07222	-0.08444	0.03834	-0.13777	-0.13333	-0.04000	-0.03000	0.02556	0.00776
0.06889	0.08223	0.07111	0.07556	0.06111	0.03778	0.00111	-0.04656	-0.07333	-0.03222
-0.02444	0.00445	0.00667	-0.00777	-0.04111	-0.04222	-0.06777	0.02000	0.11557	0.03657
0.06667	0.05223	0.00223	0.00445	-0.02555	-0.04666	-0.03889	-0.01555	0.00557	0.01556
0.02000	0.00445	0.00667	0.00445	0.02778	0.04222	0.00445	-0.01444	-0.04556	-0.01889
-0.02222	-0.01333	0.00334	0.00556	0.00889	0.02556	0.01778	0.02556	0.02778	0.04545
0.44223	0.43334	0.42667	-3.53889	0.45111	0.42889	0.43000	0.43222	0.03334	0.04000
0.00778	-0.00111	0.01556	0.01111	-0.00111	-0.01555	-0.05000	0.01556	0.00557	-0.00889
-0.04444	-0.07000	-0.08111	-0.10222	-0.01000	-0.03666	-0.01222	0.05778	-0.01333	-0.07000
-0.12111	-0.10778	-0.02555	0.02111	0.02778	0.00778	0.01223	0.03889	0.04222	0.03657
0.00111	-0.03444	-0.04222	-0.02777	-0.02333	-0.00666	0.03445	0.00000	-0.02889	-0.02444
-0.04000	-0.01888	0.00556	0.03889	0.04000	0.01445	0.04445	0.04111	-0.01000	-0.05000
-0.10666	-0.16444	-0.10555	-0.01555	-0.07778	-0.06111	-0.02334	-0.11334	0.08334	0.05334
-0.02333	0.07111	0.12223	0.04778	0.03778	0.02111	-0.01777	-0.05656	0.03333	0.02556
0.02111	0.00556	0.00111	-0.05222	-0.01222	0.05445	0.11000	0.09111	0.06223	0.05889
0.03556	-0.02111	-0.08889	-0.12666	0.00556	0.01657	-0.00444	-0.05444	-0.12333	-0.01111
-0.01444	0.03111	0.10667	0.08334	0.03889	0.02778	0.00889	-0.00333	-0.00333	0.01445

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
0.00889	-0.00666	0.01000	0.03334	0.00556	0.01223	-0.01444	-0.08111	-0.06444	-0.03888
-0.03444	0.06223	0.12111	0.08222	0.00223	-0.04111	-0.01000	0.03223	0.00778	-0.03889
-0.08889	-0.12444	-0.00444	0.10334	0.07111	0.03111	-0.01565	0.05000	0.03223	0.00334
-0.02333	-0.01333	-0.01000	-0.01333	-0.06111	-0.07444	-0.03665	0.05445	0.02334	0.00223
-0.03666	-0.02444	0.05334	-0.01777	-0.06333	0.04223	0.08778	0.07839	0.03778	0.01657
-0.00889	-0.01555	-0.01333	-0.01222	-0.00111	-0.03111	0.01555	0.03445	0.00445	-0.00666
0.01334	-0.00444	-0.01222	0.00556	-0.02222	-0.04111	-0.07000	-0.04333	0.08111	0.11445
0.10889	0.07223	0.06000	0.02889	-0.01222	-0.03889	-0.06444	-0.05444	0.01445	-0.00889
-0.06333	-0.07989	-0.10444	-0.12555	-0.06777	-0.02333	0.12667	0.05556	0.03000	0.01667
-0.01777	-0.04222	-0.04444	-0.01666	0.03223	-0.03333	-0.01333	-0.04777	0.02223	-0.05000
-0.48000	-0.48389	-0.46222	3.52667	-0.46444	-0.42777	-0.47777	-0.49838	-0.06000	-0.05888
-0.11000	-0.17777	-0.00111	0.02000	0.02112	-0.00555	-0.01777	0.04778	0.08445	0.09556
0.07223	0.01556	-0.00999	-0.05000	0.04000	0.05778	-0.00444	-0.05444	-0.01000	0.03667
0.02773	-0.00222	-0.01777	-0.05333	-0.09444	-0.07777	-0.02000	0.07223	0.00339	-0.05111
-0.08222	-0.01444	0.04556	0.01000	-0.02333	-0.00777	0.03445	0.01556	0.02000	0.01778
0.03667	0.02556	-0.03111	-0.06666	-0.02222	0.05223	0.13555	0.07778	0.02223	-0.00222
-0.04000	-0.00888	0.00334	0.00112	-0.06222	-0.01111	-0.05333	-0.04555	-0.00333	0.01334
0.01778	0.01778	0.02334	-0.05555	-0.04000	-0.05888	-0.02555	0.04557	0.08556	0.02112
-0.01777	-0.01000	-0.05000	0.02778	0.07000	0.05445	0.05223	-0.02888	0.00557	0.01223
0.05889	-0.01666	0.04556	0.05778	0.02334	0.02000	0.02334	0.03223	-0.03333	-0.05000
-0.07666	-0.04777	0.00000	-0.07000	-0.05333	-0.10111	-0.09555	-0.01111	-0.03111	0.03889
0.02778	0.02223	0.17334	0.02334	0.21778	0.14445	0.05555	0.00334	-0.04222	-0.05444
-0.05000	-0.01666	-0.05111	-0.08555	0.01555	0.00000	0.02555	0.02778	0.00334	0.01112
0.00334	-0.04566	-0.07000	-0.15333	-0.02333	0.12000	0.10778	0.03334	0.06389	0.04778
0.02445	0.01112	0.01667	0.03445	0.03223	0.02889	0.02334	-0.02222	-0.05222	-0.03777
-0.00777	0.01223	-0.01444	0.03112	0.06567	0.05889	0.01445	-0.03111	0.01778	0.02112
-0.04222	-0.03666	-0.02111	-0.01444	-0.04111	-0.02555	-0.01444	-0.01000	0.01777	0.02556
0.03778	-0.00222	-0.02665	-0.04555	-0.16444	-0.18222	-0.01333	0.14778	0.15334	0.09555
0.03112	-0.01888	-0.04555	-0.00999	-0.02333	-0.03888	0.03001	0.03555	0.00223	-0.03223
-0.02555	-0.04333	-0.04444	-0.07555	0.00778	-0.01111	-0.02333	-0.01655	0.06223	0.05223
0.04556	0.04000	-0.01333	-0.03889	-0.03666	-0.03222	-0.02000	0.00778	0.03000	0.01223
0.03334	0.02334	-0.01444	-0.03889	-0.02444	0.03445	0.02445	0.02334	0.00334	-0.04111
-0.09444	-0.14777	-0.02222	0.10667	0.19889	0.11334	0.01778	-0.00656	-0.01555	0.00223
0.02667	0.02667	0.00000	-0.01889	-0.01000	-0.01333	-0.03889	-0.03777	-0.03555	-0.03333
-0.05555	-0.01444	-0.01222	-0.01111	-0.03889	-0.07778	0.03778	0.09839	0.06778	0.04000
0.03445	0.02000	0.01111	0.01334	0.01445	-0.00333	-0.03333	-0.03777	-0.02444	-0.01333
-0.01111	-0.01000	-0.02656	-0.05000	-0.04111	-0.03111	-0.02889	-0.02777	-0.02556	-0.02222
-0.02666	-0.02888	-0.04889	-0.05222	-0.04566	0.04778	0.06567	0.05223	0.01567	-0.01555
-0.07889	-0.07777	0.00556	-0.02333	0.06000	0.01112	-0.05333	0.01445	0.04555	0.01445
0.07112	0.04889	0.02000	-0.01222	0.00223	-0.01656	-0.02777	-0.02444	-0.01555	-0.00555

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
0.00111	0.01000	0.01112	-0.01666	-0.01555	0.02223	-0.01000	-0.02838	-0.03333	-0.01555
-0.01222	0.03334	0.03445	-0.00222	-0.02777	-0.00111	0.01567	-0.03222	0.01223	0.03000
0.04567	0.03112	-0.02333	-0.06555	-0.02000	0.03334	0.01445	0.01223	0.02112	0.01667
0.06000	0.06889	0.05112	-0.01777	-0.10000	-0.05111	-0.05333	-0.01555	-0.00111	-0.03666
-0.02000	0.02334	0.02223	0.00778	0.01112	0.02000	0.00111	0.05112	-0.00777	-0.03777
0.02667	0.06567	0.07778	-0.07222	-0.10444	-0.09000	-0.06555	-0.05223	-0.06222	-0.08222
-0.08555	-0.10444	-0.01111	0.03889	0.16778	0.15112	0.09567	0.03112	-0.02777	0.01889
-0.00444	0.02445	0.02556	0.01667	0.01778	0.01556	-0.00111	-0.03656	-0.01555	0.01667
0.02112	0.03556	0.00000	-0.01889	-0.01222	-0.01000	0.00223	0.02657	0.00111	-0.05444
-0.05333	-0.03333	0.02445	0.00112	-0.01222	-0.04000	-0.03889	0.00778	0.00889	0.02000
0.01778	-0.00555	-0.01000	-0.03666	-0.03565	-0.02111	0.13223	0.01334	0.03111	0.03111
0.00889	0.01000	-0.00555	-0.02666	-0.05111	-0.07222	-0.03222	-0.03777	0.05000	0.01112
-0.02444	-0.04333	-0.02888	-0.00666	-0.00444	0.03657	0.02567	-0.03333	-0.02444	-0.04444
0.02223	0.00334	0.01000	0.00000	0.00556	-0.02222	0.04556	0.07556	0.02778	0.01112
-0.01222	0.00556	0.00555	0.04556	0.03445	0.02556	-0.01444	-0.03333	-0.01555	-0.03222
-0.03888	-0.00777	-0.00555	0.02223	0.00445	-0.02222	-0.02777	-0.02555	0.01112	0.03778
0.01111	0.01223	-0.01666	0.02223	0.00445	-0.02556	0.02889	-0.00656	-0.00888	0.02667
0.00889	-0.02000	-0.03666	-0.05555	0.05778	0.04556	0.08223	0.07000	0.05657	-0.02822
-0.04111	-0.06111	0.02888	0.00556	0.02445	0.05334	0.03223	0.03477	-0.03722	-0.03888
-0.03111	-0.06555	0.08111	0.05778	0.03334	0.01000	-0.01567	-0.03777	-0.03722	0.01223
0.01667	0.04889	0.02667	0.00667	0.03000	-0.03223	0.01567	0.03778	0.01778	0.02667
-0.05111	-0.03111	-0.03889	-0.06111	-0.03888	-0.03222	0.03889	0.01657	-0.01555	0.01334
-0.00555	-0.00333	-0.05333	0.00000	0.00667	-0.02444	-0.06444	0.05112	0.06001	0.02223
0.02556	0.02112	0.02334	0.03223	0.03334	-0.02444	-0.08111	-0.02777	-0.01111	0.03667
0.01334	0.00556	0.02223	0.03223	0.02667	0.03889	-0.04111	-0.05111	-0.04333	-0.05111
-0.05000	-0.09000	-0.03111	-0.04777	0.01000	0.15657	0.13223	0.0556	0.08112	0.05778
0.05445	-0.01666	-0.02333	-0.05666	-0.08333	-0.05111	-0.09000	0.05839	0.06334	0.03334
0.02667	-0.00222	0.03112	0.01667	0.01889	0.04223	0.02567	0.04111	-0.01444	-0.03555
-0.11111	0.02333	-0.04333	-0.04222	-0.06888	-0.12889	0.05223	-0.03333	0.09889	0.14667
0.08667	0.04778	0.00334	-0.01777	-0.02777	-0.07222	-0.11666	-0.03555	0.06445	0.07445
0.02445	-0.01777	-0.05444	-0.07889	-0.04000	0.13778	0.07889	0.04445	0.03667	0.04000
0.06445	0.07889	0.07889	-0.00444	-0.07666	-0.15889	-0.22222	-0.13939	-0.07111	0.03667
0.02556	0.01112	0.01445	0.01889	0.03334	0.03556	0.04000	0.06334	0.02334	0.00445
0.05778	0.06889	0.01778	0.01000	-0.03111	-0.02444	-0.00777	-0.03777	0.00334	0.00001
0.01556	0.02223	-0.00333	-0.00444	0.00334	0.00556	0.00000	0.00556	0.02223	0.01112
0.01000	-0.01333	-0.01666	-0.01777	-0.01333	0.00112	0.00334	-0.00555	0.01778	0.01223
0.00778	0.01334	-0.01000	-0.01555	-0.01111	-0.00555	-0.03334	0.03556	0.03555	0.04000
0.03889	0.02889	0.03667	-0.03666	-0.04222	-0.04444	-0.05777	-0.05333	-0.07000	-0.07333
-0.05888	-0.03555	-0.01555	0.02889	0.04334	0.05334	0.03112	0.04111	0.04445	0.03889
0.04334	0.01000	-0.00222	-0.02444	-0.05222	-0.05777	-0.04333	-0.04656	-0.03333	-0.01333

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
-0.0222	-0.04111	-0.02555	-0.02555	-0.04333	-0.02555	-0.06000	0.03112	0.07223	0.05667
0.01445	-0.06666	-0.09777	-0.08444	-0.03333	-0.01444	0.04112	0.13778	0.11778	0.14000
0.17223	0.15334	0.10223	0.03334	-0.02555	-0.02222	-0.00566	-0.02656	-0.05889	-0.05555
-0.05000	-0.04222	-0.02777	-0.01555	-0.02777	-0.03888	-0.04444	-0.07222	-0.03555	-0.04333
0.01112	0.07556	0.09778	0.11889	0.10223	0.05223	0.02889	-0.01777	0.04111	-0.04555
-0.05999	-0.04222	-0.02666	-0.02999	-0.00888	-0.00222	0.02112	0.04223	0.05889	0.07778
0.04667	0.02667	0.02445	-0.00444	-0.01777	-0.03111	-0.02888	-0.03444	-0.02777	-0.07444
-0.08889	-0.06555	0.01778	0.06445	0.05667	0.02000	0.00000	0.02334	0.02889	0.03445
0.03445	0.03889	0.03000	-0.00666	-0.02555	-0.03444	-0.03555	-0.03444	-0.01566	-0.04111
-0.05666	-0.05444	-0.04000	-0.01888	-0.00555	0.02556	0.02778	0.02556	0.04112	0.03445
0.03445	0.01889	-0.00444	-0.02444	-0.02999	-0.05111	0.06665	-0.03222	0.01445	0.03111
0.01667	0.04000	0.04222	0.02334	0.01334	0.02778	0.04111	0.02223	0.00222	-0.03889
-0.01889	-0.04000	-0.03333	-0.04000	-0.05444	-0.05777	-0.03111	0.02556	0.05445	0.10445
0.17112	0.12000	0.07223	0.06334	0.03556	0.03445	0.03112	-0.07555	-0.10333	-0.09444
-0.06111	0.05112	0.05334	0.03111	0.04445	0.02556	0.01555	0.03839	0.01557	0.01223
-0.01000	0.00445	-0.01222	-0.03555	-0.05777	-0.05333	-0.05111	-0.05555	-0.06556	-0.05444
-0.04111	-0.23888	-0.22111	-0.19444	-0.19222	1.82667	-0.17000	-0.18222	-0.22777	-0.24555
-0.00222	0.02445	0.04334	0.01778	-0.01000	-0.02222	-0.02444	-0.01444	-0.01888	-0.07555
-0.09222	-0.05666	-0.00999	0.03889	0.00223	-0.02333	-0.02222	-0.02999	-0.01222	-0.01111
-0.01222	-0.03555	-0.04444	-0.04555	-0.01111	-0.03444	0.05223	0.04839	0.06334	0.03778
0.01334	0.03000	-0.01666	-0.03667	0.04666	-0.03333	0.01889	0.02657	0.01778	0.01889
0.06000	0.08556	0.07556	0.03667	0.02445	0.01556	-0.03444	-0.09222	-0.10444	-0.07777
-0.05444	-0.04111	-0.02666	-0.03555	-0.02777	-0.03333	0.02223	0.03839	0.03111	0.04112
0.02667	0.03778	0.00001	-0.04222	-0.04444	-0.04888	-0.02444	0.03778	0.00556	-0.03888
-0.02111	0.01778	-0.01222	-0.02999	-0.00444	-0.02666	-0.02444	0.03334	0.07778	0.06556
0.09112	0.04223	0.01778	-0.00333	-0.02555	-0.03888	-0.02777	-0.05222	-0.04555	-0.03555
-0.00555	0.00334	0.03778	0.06556	0.08667	0.09445	0.03555	-0.03000	-0.06777	-0.09777
-0.07111	-0.04777	-0.01000	-0.00333	-0.01222	-0.03111	0.02112	0.03556	0.02223	0.02223
-0.00111	0.02334	0.03334	0.02223	0.00001	-0.01999	-0.02666	-0.03222	0.01445	0.01223
0.02001	0.02778	0.00889	-0.01000	-0.02444	-0.03222	-0.04444	-0.04111	-0.01777	-0.03777
0.00556	-0.01444	0.01334	-0.00666	0.01334	0.03223	0.00777	-0.02444	-0.03555	-0.02655
0.02000	0.00667	0.02778	0.02778	0.06556	0.11112	0.13223	0.14839	0.12445	0.07001
-0.00333	-0.03111	-0.03222	-0.02777	-0.08333	-0.15666	-0.10444	-0.11656	0.04334	0.04223
0.02778	0.04223	-0.00444	-0.03333	-0.03555	-0.03555	0.06555	0.05556	-0.03055	-0.02666
0.00000	0.03223	0.01445	-0.00111	-0.02999	-0.03777	-0.01888	-0.03999	0.00222	0.03223
0.01889	0.01112	-0.03777	0.01666	-0.03222	0.01888	-0.01444	-0.01655	0.04334	0.05556
0.05445	0.02334	0.02334	0.04445	0.06556	0.05445	0.02555	-0.02777	-0.01222	-0.01222
0.00445	0.01000	-0.00888	0.00445	0.03556	0.03778	-0.12333	-0.17656	-0.12888	0.00000
0.06556	0.07889	0.08889	0.04445	0.01334	-0.01888	-0.03444	-0.01111	0.03334	-0.03333
-0.03222	-0.08333	0.05556	0.03667	0.02223	0.02000	0.02223	0.02778	-0.12111	-0.15333
-0.16111	-0.14777	0.84334	-0.17000	-0.28777	-0.25999	-0.25000	-0.15111	0.81556	-0.15555
-0.08888	-0.07777	-0.06333	0.05778	0.07334	0.07112	0.08000	0.07778	0.07445	0.06889

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
0.02556	0.01300	-0.03111	-0.03111	-0.03999	-0.04444	-0.05565	-0.05888	-0.11111	0.02667
0.01778	-0.00555	-0.04666	-0.09666	-0.13222	-0.01111	0.06000	0.11889	0.06557	0.03223
-0.00777	-0.01777	-0.01333	0.00000	0.02556	0.04001	0.00555	-0.00555	-0.03555	-0.05333
-0.06666	-0.01111	0.07445	0.04112	0.01112	-0.02333	0.06223	0.05334	0.03778	0.01112
0.01889	0.03556	0.04889	0.01112	0.00334	-0.02222	-0.04555	-0.01111	-0.01556	-0.01655
-0.03666	-0.03777	-0.03111	-0.05777	-0.06444	-0.06666	0.00301	0.05020	0.08445	0.07556
0.06778	0.05778	0.03223	0.02667	-0.00555	-0.02666	-0.04444	-0.05222	0.07222	-0.10999
-0.10999	-0.11888	-0.01222	0.11112	0.10556	0.09334	0.07445	0.07889	0.07556	0.02112
-0.02777	-0.04555	-0.03333	-0.03111	-0.02666	-0.01333	0.00889	-0.03111	0.00112	0.03778
0.01667	0.03778	0.00667	-0.03666	-0.03000	-0.02666	-0.00999	0.01223	-0.04999	0.03890
0.02334	0.03556	0.04334	0.03778	0.01667	-0.01222	-0.01444	-0.02333	0.04888	-0.08777
-0.06555	-0.03333	-0.01888	0.01556	-0.01333	-0.00222	-0.00222	0.01556	0.07556	0.09000
-0.05223	0.02001	-0.01111	-0.00777	0.01445	0.03223	0.00555	-0.02777	-0.06444	-0.04333
-0.02333	0.00112	0.01112	0.02223	0.04001	0.05334	0.06001	0.02001	-0.09444	-0.01555
0.00556	0.01778	0.00556	-0.01111	-0.01777	-0.03111	-0.04888	-0.06111	-0.01111	0.04223
0.07001	0.08667	0.05778	0.04112	0.02556	-0.02000	-0.01000	-0.00999	-0.00222	-0.01555
-0.01888	-0.02111	-0.00444	0.00001	0.01334	0.00778	0.00223	0.01334	-0.01222	-0.03880
-0.05333	-0.01666	0.04334	0.06334	0.03334	0.01223	-0.00222	-0.00555	-0.00555	0.01223
0.02889	0.00445	0.01334	0.02334	-0.14000	0.06112	0.04556	0.06112	0.05445	0.03334
0.01889	-0.03333	-0.05222	-0.05111	0.02778	0.01778	0.00778	-0.00778	-0.00333	-0.03111
-0.00555	0.00445	-0.02111	-0.04111	-0.02777	-0.01222	0.00334	0.02223	0.02223	0.01112
0.01778	0.06112	0.05223	0.02001	-0.00111	-0.01111	-0.01888	-0.03656	-0.02999	-0.02222
-0.00111	-0.02222	-0.03333	-0.02000	-0.01111	-0.04555	-0.03000	0.05778	-0.02999	-0.02111
0.00334	0.03223	-0.00111	0.01556	0.00778	-0.01333	0.01889	0.02657	-0.00333	-0.02655
-0.02555	-0.02222	-0.02000	-0.00444	0.00445	-0.00889	-0.01333	-0.00777	0.00389	0.01778
0.01445	-0.00333	-0.01222	-0.02888	-0.03000	-0.00333	0.00111	-0.01222	0.02112	0.02776
0.02778	0.00112	0.01667	-0.00111	0.01223	0.00556	-0.01555	-0.02656	-0.02111	-0.01333
-0.00222	0.00334	-0.00222	-0.03111	-0.00666	-0.00111	-0.02777	-0.02222	0.00111	-0.03444
-0.03555	-0.00222	0.04667	0.03445	-0.03444	-0.05000	-0.03883	-0.02444	-0.02555	-0.01111
0.01223	-0.00111	0.01445	0.06445	0.06445	0.06334	0.01445	0.00000	-0.02555	-0.03111
-0.02000	-0.01666	-0.03222	-0.03666	-0.02777	-0.03444	-0.00333	0.00334	0.00334	0.01223
-0.01222	-0.02777	-0.00222	-0.00444	0.00667	-0.03889	0.03112	0.01933	0.00999	-0.03885
-0.00555	-0.02444	-0.04333	-0.05555	-0.04000	0.05334	0.02334	0.01778	-0.01111	-0.01888
-0.02111	-0.00888	-0.00888	-0.02333	0.00112	0.00657	0.01223	0.01334	0.00556	-0.01444
0.00556	0.06556	0.07890	0.08445	0.06223	0.05112	0.02001	-0.00888	-0.05777	-0.07222
-0.05444	-0.04222	-0.05111	-0.07111	-0.08444	-0.06666	-0.00333	0.04223	0.06334	0.09001
0.09667	0.05889	0.01334	-0.00666	-0.02566	-0.04777	-0.07000	-0.07111	-0.02444	0.03001

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
0.05112	0.06778	0.05667	0.06223	0.09223	0.07112	0.03223	0.03778	-0.00399	-0.03222
-0.03888	-0.03222	-0.05888	-0.07555	-0.08111	-0.01333	0.03445	0.05112	0.04778	0.04334
0.01555	-0.01888	-0.04444	-0.04222	-0.02888	-0.01222	-0.00333	-0.02111	-0.06444	-0.05111
0.03000	0.12334	0.05778	-0.01999	0.00777	-0.01333	-0.00888	-0.01444	-0.01333	-0.03111
-0.03111	-0.01333	0.02778	0.03334	0.04112	0.05112	0.01778	0.03334	-0.00444	0.01334
0.01556	0.01556	-0.00666	-0.01000	-0.01444	-0.01444	-0.01111	-0.11555	-0.02444	-0.01000
0.00889	0.01445	-0.01110	0.00334	0.02112	0.03223	0.01334	0.00445	-0.01111	-0.01333
0.00334	-0.01111	-0.03444	-0.06666	-0.07444	-0.02444	0.01334	0.03778	0.04445	0.04556
0.03778	0.05001	0.02556	0.02223	0.02556	0.03001	-0.00333	-0.04444	-0.06222	-0.05333
-0.07111	-0.05110	-0.04666	0.00334	-0.01444	-0.03111	-0.01999	0.00890	0.01445	0.03334
0.03890	0.05445	0.06223	0.04223	0.00889	-0.01888	-0.03888	0.03112	0.01445	0.01334
-0.02333	-0.01777	-0.07222	-0.07555	0.01556	0.02778	0.03001	-0.01555	-0.01444	0.03667
-0.00222	-0.00222	0.00334	-0.00555	0.01112	0.03112	0.04555	0.02667	0.02223	0.01223
0.00778	0.00112	-0.00555	0.00778	0.00889	-0.02666	-0.03888	-0.05111	-0.05666	-0.03222
-0.02444	0.01000	0.00112	0.04112	0.02445	0.01112	0.01445	0.00839	0.00390	0.03001
-0.00666	0.00300	0.02112	0.02445	0.02001	0.01890	0.00223	0.01223	0.01445	0.03778
0.00334	-0.01111	-0.00444	0.00333	-0.00333	-0.01222	-0.00888	-0.00222	0.01889	0.04889
0.01778	-0.00222	-0.03111	-0.01888	-0.00333	-0.00110	-0.00444	-0.00333	-0.01666	-0.03777
-0.04777	-0.08111	-0.08888	-0.06666	0.05445	0.14667	0.19556	0.23334	0.23557	0.21657
0.17556	0.08300	-0.02222	-0.12444	-0.12000	-0.08111	-0.03999	0.13999	0.11444	-0.03222
-0.06999	-0.05666	-0.03777	-0.01888	0.10667	-0.01334	0.00556	-0.00223	-0.00555	-0.03444
0.04223	0.03445	0.03112	0.02000	-0.00111	-0.01666	-0.01888	-0.00777	0.01000	0.02223
-0.01111	-0.06333	-0.07222	-0.07333	-0.07777	0.04112	0.05223	0.03334	0.02555	0.03778
0.01445	0.03556	-0.00333	0.00667	0.00334	-0.00777	-0.01333	-0.02555	0.00334	-0.03555
-0.03888	-0.02444	0.01334	0.05334	0.05889	0.06556	0.05556	0.08445	0.05889	0.03334
0.01890	-0.00666	-0.00555	0.00667	-0.00555	-0.03111	-0.02444	-0.04112	-0.01888	-0.03999
0.01223	0.01556	0.00889	0.00445	0.02334	0.01889	0.00667	0.02001	0.01890	0.01445
0.03334	0.01223	-0.00444	-0.00888	-0.01222	0.03778	0.03778	0.03778	0.01000	0.01445
0.01334	0.01334	0.00889	0.00667	0.00556	0.02112	-0.00777	-0.00888	0.00001	-0.05222
0.00112	-0.01334	0.01667	0.03001	-0.01222	0.05112	-0.03444	-0.02222	0.02666	-0.04565
-0.05333	-0.06222	-0.02333	0.04112	0.04112	0.00667	-0.01222	-0.01777	-0.01111	-0.01111
0.00112	-0.03555	-0.00111	0.02778	0.03778	0.02000	0.01555	-0.01889	-0.00398	-0.01555
-0.05444	-0.05333	-0.07888	0.06556	0.04889	0.01778	0.00555	-0.02888	-0.03777	-0.04999
0.03445	0.01667	0.00445	0.00001	-0.01888	-0.03999	-0.00666	-0.01666	-0.00566	-0.00666
0.01778	0.03445	0.03112	0.02112	-0.00666	-0.03888	0.00112	0.00001	0.00111	-0.01888
-0.01999	-0.01222	-0.03111	-0.01555	-0.01555	-0.02666	0.00111	-0.01778	0.02334	0.02223
0.02334	0.01000	-0.02111	-0.04777	-0.04666	-0.07444	0.00334	0.04778	0.04334	0.02334
0.03334	0.03778	0.00889	-0.02555	-0.04888	-0.09110	-0.03999	0.03000	-0.02333	-0.05555

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
-0.04444	-0.01333	0.04556	0.07667	0.07223	0.05778	0.04112	0.03334	-0.02111	-0.04111
-0.04999	-0.04388	-0.04000	-0.03111	-0.02444	-0.02222	-0.03033	0.03556	0.03389	0.03777
0.04223	0.03667	0.00667	-0.01777	-0.03999	0.00889	0.00000	0.01223	0.01334	0.02444
0.02445	0.00445	0.00667	-0.02333	-0.06333	-0.09333	-0.13333	0.03001	-0.00555	0.02223
0.02334	0.01112	0.03001	0.02001	0.04223	0.04445	0.04556	0.04557	0.02557	0.02001
0.02001	0.02556	0.01556	-0.01888	-0.02111	-0.02111	-0.02111	0.02657	0.01000	-0.00777
-0.00777	-0.00888	-0.00555	-0.00444	0.00445	0.00112	-0.01666	-0.03556	-0.05999	-0.05777
0.02889	0.00889	-0.00666	-0.00666	-0.01222	0.00223	0.00556	-0.03111	0.02334	0.02223
0.01667	0.01556	0.02889	0.02112	0.02001	0.02112	0.01667	0.00657	-0.01333	-0.00333
-0.01444	-0.02666	-0.03333	-0.04666	-0.05111	0.02112	0.02334	0.03557	0.02389	0.02223
0.00001	-0.01888	-0.01444	-0.02777	-0.00222	-0.04444	-0.08111	-0.04000	0.02778	0.05000
0.04223	0.04000	0.05223	0.06667	0.08001	0.06667	0.06778	0.04223	0.00889	0.00334
-0.01111	-0.01555	-0.03666	-0.04666	-0.04333	-0.04111	-0.02444	-0.01888	-0.01888	-0.01555
-0.01333	0.00556	-0.00777	-0.00333	-0.00666	-0.00555	-0.00111	0.00000	-0.00777	0.01334
-0.00444	0.00390	0.00334	0.00778	0.00667	0.00334	0.00223	-0.00777	-0.03222	-0.03666
-0.07333	-0.02777	-0.01222	-0.04777	0.12112	0.07001	0.05000	0.03657	-0.00556	-0.00334
-0.00444	-0.00222	0.01001	0.01001	-0.01888	-0.00999	-0.00111	-0.00222	-0.00555	-0.00777
-0.01444	-0.01333	-0.00333	0.01556	0.00556	0.00334	0.00001	-0.00333	-0.01555	-0.00777
-0.00111	0.00667	0.01334	-0.00111	0.00556	-0.00444	0.00889	0.01556	0.02445	0.01667
0.01112	0.01112	0.00445	0.00223	-0.00999	-0.02999	-0.03999	0.03001	0.03001	0.00889
0.00445	-0.01444	-0.01777	-0.01666	-0.00777	0.02223	0.04001	-0.00655	-0.01555	-0.03999
-0.04444	-0.02222	-0.01888	-0.01333	-0.01222	-0.00333	-0.00333	0.02556	0.01445	0.00556
0.00112	0.01000	0.02445	0.02112	0.01334	0.01000	0.02334	0.03556	0.00889	-0.01111
-0.03777	-0.07777	0.03223	0.08112	0.07001	0.03667	-0.04222	-0.03777	-0.02555	-0.00222
0.02667	0.01778	0.00000	-0.01555	0.00445	0.02223	0.03445	0.03556	0.01557	-0.00889
-0.02888	-0.00333	0.01334	0.02667	0.00556	-0.02444	-0.02444	-0.00444	0.01223	-0.03111
-0.03333	-0.04999	-0.04000	-0.00666	-0.01111	-0.03222	-0.05333	-0.05222	-0.01000	-0.02666
-0.03222	-0.03666	-0.00666	0.00445	-0.00111	-0.03111	-0.00333	0.01656	-0.01222	-0.01333
-0.02888	-0.01555	-0.01555	-0.00110	0.02334	0.01667	0.01890	-0.04333	-0.01322	-0.01000
-0.07999	0.02534	-0.07544	-0.08222	0.03889	-0.07222	-0.05333	-0.01999	-0.06222	-0.00777
-0.01111	-0.01777	-0.00222	-0.01111	-0.02111	0.00223	0.00000	-0.02444	-0.04111	-0.01555
0.01001	0.01667	-0.00222	0.03000	-0.04555	0.04112	0.08667	0.09778	0.05334	0.01112
0.00445	0.01890	0.00334	-0.03111	-0.04777	-0.03111	-0.03444	0.05555	-0.03111	-0.05777
-0.05888	-0.02999	-0.04666	0.02778	0.03001	0.03667	0.03889	0.00223	-0.01999	0.02445
0.00778	0.00223	-0.02999	0.04001	0.11334	0.07001	0.06223	-0.02555	-0.03999	-0.01222
-0.00777	0.01334	-0.00555	-0.02444	-0.02444	0.00334	-0.03222	-0.03477	-0.03556	-0.03555
0.06778	-0.03666	-0.03666	-0.03622	-0.03666	0.03334	0.06001	0.03889	0.02779	0.01112
-0.08333	-0.10777	-0.03444	-0.11110	-0.08110	-0.09333	-0.14222	8.83778	-0.09555	-0.03777
-1.05666	-1.05666	0.06223	0.04000	0.07112	0.00778	-0.05665	-0.00322	-0.01788	-0.00777

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
-0.00777	-0.02666	-0.00555	0.05556	0.05334	0.03112	0.05000	0.05001	0.00889	0.03445
0.08445	-0.00444	-0.02110	-0.01110	0.02001	0.02556	0.06223	-0.00838	-0.01377	-0.03555
-0.06333	-0.02111	0.02778	-0.02222	0.00567	-0.01888	-0.02333	-0.01838	0.04334	0.04223
0.05334	0.01778	0.01112	0.02223	0.02778	0.04445	0.01001	-0.04222	-0.04111	-0.03333
0.00223	-0.01111	-0.05222	-0.04666	-0.04111	-0.02777	0.04223	0.02445	0.01889	0.03657
0.00000	0.03445	0.02890	0.01778	0.00112	-0.00111	0.02555	0.02778	0.02112	0.01556
0.00223	-0.01666	-0.00444	-0.00666	-0.01221	-0.02666	-0.04999	-0.03656	-0.03999	-0.01110
0.00112	-0.00999	-0.00333	0.01001	0.02223	0.00667	0.01667	0.02779	-0.02777	-0.03444
-0.00110	0.01112	0.00778	0.03334	0.01445	-0.04777	0.02334	-0.01555	0.03112	0.01334
-0.00777	-0.00110	-0.01555	0.03001	0.01779	0.03445	0.01778	-0.02333	-0.05555	-0.05555
-0.00999	0.03334	0.09334	0.06223	0.03667	-0.02999	-0.01110	0.03556	-0.00777	-0.01332
-0.01110	-0.00999	-0.03333	-0.04333	-0.01332	0.03001	0.02334	0.04112	0.00555	-0.04111
0.03223	-0.03888	-0.01222	0.03223	0.04001	0.02889	-0.01777	-0.03110	-0.00888	-0.02333
-0.04555	0.00890	-0.02999	-0.08555	-0.08110	0.01889	-0.02333	-0.01444	-0.02332	0.03334
0.03556	0.03778	0.02556	0.05112	0.01890	-0.00221	-0.00777	0.04334	0.01112	-0.01444
-0.02555	0.02223	0.03112	-0.02666	0.02001	0.03889	0.01445	-0.02555	0.01223	0.00223
0.02445	0.01667	-0.02332	0.01112	-0.08221	-0.11555	0.02445	-0.00221	0.03557	0.02556
0.04334	0.05112	0.02334	-0.07444	-0.10665	-0.05444	0.07001	0.02445	0.04223	-0.03332
-0.04333	-0.09444	-0.07999	-0.03110	0.01779	0.01445	0.05000	0.03334	0.05557	0.07334
0.07445	0.09445	0.06223	0.00334	-0.01999	-0.04444	-0.05555	-0.04111	-0.02888	0.05556
0.00001	-0.00444	0.03445	0.06890	0.04112	0.01668	0.00223	-0.01656	-0.02555	-0.04221
-0.04221	-0.06221	-0.04666	-0.03888	0.01223	0.04334	0.01112	0.01779	0.01555	-0.03888
-0.00777	0.01112	0.01445	0.01112	0.00223	0.00667	0.00001	-0.00221	0.04657	0.04112
0.01668	0.01445	0.03334	0.02112	-0.02555	-0.00999	0.02112	0.04223	0.03778	0.03112
0.01890	-0.00999	-0.02444	-0.02110	-0.02444	0.02110	-0.07110	-0.11110	-0.12555	-0.10655
-0.03999	-0.02444	-0.05555	-0.06777	0.00001	0.13890	0.19779	0.17334	0.11890	0.05445
0.02334	-0.01110	-0.06111	-0.13999	-0.12333	-0.06555	-0.00888	0.00889	0.03112	-0.01110
-0.02777	0.01778	0.01556	0.03556	0.01334	0.03445	0.03445	-0.14999	-0.15110	-0.18555
-0.11555	0.93223	-0.05999	-0.03333	-0.03444	-0.07444	0.00112	-0.04999	-0.06999	-0.08444
-0.06333	0.00556	0.01890	0.13890	0.02889	-0.00221	0.00334	0.02556	0.05334	0.01890
-0.03888	-0.05666	-0.04666	-0.01888	-0.00666	-0.01332	0.00668	0.01555	0.00890	0.01334
0.01001	0.02334	0.00556	0.01334	-0.01110	-0.04444	-0.02221	-0.03656	0.01223	-0.02332
-0.00555	0.00334	0.02779	0.02223	0.02556	0.02445	-0.00333	0.01334	0.00112	-0.00777
-0.00777	-0.03443	-0.00555	-0.02110	0.01556	-0.02888	-0.01444	-0.01444	-0.00444	0.03658
0.01001	-0.00777	-0.00555	-0.02332	-0.02888	0.01445	0.01667	0.02112	0.00657	0.01445
0.02001	0.01890	0.00890	-0.00110	-0.00888	-0.01666	-0.00444	-0.01110	-0.02332	-0.02777
-0.05221	-0.00555	0.01112	0.00223	0.06667	0.05445	0.05445	0.02445	0.01890	0.01445
-0.01333	-0.05110	-0.07888	-0.10221	0.01112	-0.00444	0.02567	0.03556	0.01223	0.04334
0.01223	-0.00555	-0.00555	-0.00999	0.02112	0.05334	0.02567	-0.00777	-0.00110	-0.03110
-0.00332	0.00223	0.00334	0.01667	-0.00999	0.00112	0.04890	0.04556	0.04657	-0.03888

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
-0.23666	0.04778	0.01890	-0.04777	-0.03888	0.00445	0.01567	0.05001	-0.00333	-0.05888
-0.07777	-0.06888	-0.02110	0.07112	0.05445	0.05556	0.09445	0.00558	-0.01777	0.02112
0.00223	-0.01333	0.00556	-0.00777	0.01223	-0.04221	-0.06999	0.00778	0.00334	0.01001
-0.00999	0.01223	0.00334	-0.00444	-0.01777	0.00334	-0.01221	-0.03555	0.00890	0.03445
0.03445	0.00557	0.01001	-0.00666	-0.00777	-0.01555	0.04001	0.03223	0.03779	0.03001
-0.00999	-0.03221	-0.05665	-0.01110	0.01223	0.00112	0.00334	-0.01777	-0.02779	0.05221
-0.05444	0.00223	-0.00444	0.01657	0.03556	0.01556	0.01445	0.01334	0.00558	0.00779
0.00001	0.01445	0.02334	0.00445	0.00556	0.00334	-0.01888	-0.03555	-0.04221	-0.03999
-0.05666	-0.05555	-0.04888	-0.02332	-0.02221	-0.01999	0.00658	0.02334	0.00445	0.01334
0.01667	-0.00777	-0.04333	-0.04333	0.00112	0.01334	0.04556	0.05779	0.04330	0.01223
0.02001	0.02334	-0.00666	-0.02656	-0.02555	0.01332	-0.02444	-0.03555	-0.00221	0.01777
-0.00555	0.02001	0.01223	-0.01888	-0.01110	0.01657	0.02890	0.00112	-0.01110	-0.00221
-0.01221	-0.00889	-0.00777	-0.02999	-0.03777	-0.04221	-0.04665	0.02334	0.04334	0.04779
-0.00444	0.01223	-0.01555	0.02658	0.07112	0.04556	0.08112	0.15890	-0.03444	-0.03221
-0.08110	-0.06566	-0.04110	-0.04444	-0.03333	-0.00332	0.05223	0.09001	0.08112	0.09890
0.09556	0.06446	0.04555	0.04112	0.06112	0.08890	0.09445	0.01890	-0.05222	-0.10221
-0.10777	-0.04666	0.02890	0.03112	-0.00777	-0.00555	0.03223	0.05657	0.01112	-0.03444
-0.06332	-0.08999	-0.13999	-0.15221	-0.13665	-0.05443	0.03001	0.01890	0.03445	0.10779
0.17334	0.15334	0.09445	0.02556	0.00890	0.03112	-0.01444	-0.03555	-0.04555	-0.05555
-0.01555	0.03779	0.10223	0.11223	0.06890	0.03223	-0.02332	0.10334	0.05658	0.05445
0.02001	-0.03444	-0.08221	-0.10666	-0.09332	-0.05888	-0.02332	-0.02556	-0.01444	0.02890
0.03334	0.04556	0.07112	0.04223	0.05223	0.01890	-0.03555	-0.01110	0.01557	0.03112
0.02334	0.01334	0.00112	0.00112	0.03779	0.04890	0.04778	0.01779	0.02112	0.03112
0.02112	-0.02666	-0.00777	0.01445	-0.01555	0.00001	0.01334	-0.00334	-0.03999	-0.01665
0.03779	-0.00777	-0.05888	0.00667	0.00001	0.02778	0.07001	0.05112	0.02334	-0.03333
-0.00555	-0.01666	-0.02332	-0.02110	-0.03333	0.00445	0.03889	0.00445	-0.02223	-0.01221
-0.14777	-0.13777	0.12445	0.08001	0.04667	0.05334	0.03567	0.05556	0.02223	-0.01665
-0.05566	-0.18666	-0.00999	0.01658	0.02779	0.00667	-0.00444	0.02779	0.10890	-0.02777
-0.10110	-0.08110	-0.07666	-0.01555	-0.01444	0.10890	0.12223	0.15890	0.04779	-0.00665
0.13001	0.15556	0.16556	0.16658	0.04112	-0.00555	0.02112	-0.02334	-0.03111	-0.01555
0.00223	-0.01888	-0.02110	-0.01555	0.00779	0.02334	0.01557	-0.04221	-0.03555	-0.03733
-0.02666	-0.08221	-0.08444	0.01223	0.10557	0.07890	0.04112	0.03001	0.02445	0.01112
-0.00666	-0.04666	-0.06333	-0.05111	-0.02555	0.05667	0.01445	0.00657	0.03001	0.03001
0.04890	0.01001	0.03334	0.04112	0.01001	0.01223	0.05001	0.01990	-0.03032	0.04657
0.02556	-0.00444	-0.06110	-0.11333	0.19444	0.00001	0.08667	0.05890	0.03223	-0.02777
-0.03111	0.02445	0.03668	-0.02221	0.02890	0.05556	0.09001	0.01890	-0.10555	-0.15777
-0.09332	-0.03777	0.00890	0.06446	0.07112	0.03556	-0.01555	-0.03555	-0.09444	-0.04555
0.03112	0.00334	0.01112	-0.00443	-0.04888	0.02890	0.09557	0.13112	0.10334	0.00112
-0.07888	-0.13221	-0.18332	-0.15888	0.00001	0.25779	0.26112	0.05446	-0.09332	-0.05444

TABLE 4A CONT.

0	1	2	3	4	5	6	7	8	9
-0.01443	0.00445	-0.01221	0.01779	0.03668	-0.01555	-0.03999	-0.05777	-0.00999	0.03112
0.03779	0.02556	-0.00333	-0.01656	-0.02110	0.04334	0.00890	0.01223	-0.02444	0.00112
0.00112	0.01112	0.00890	-0.02221	-0.00666	-0.05555	-0.04777	-0.02555	-0.00221	-0.00221
-0.01221	-0.02332	-0.02555	-0.02444	-0.01888	0.05779	0.06779	0.05556	0.02001	0.02223
0.03112	0.01445	0.01334	0.00001	-0.02110	-0.03888	-0.04444	-0.04110	0.05444	-0.00332
0.03445	0.02112	0.05001	0.06555	0.00223	-0.03333	-0.06444	-0.01477	-0.03333	-0.02666
-0.04333	-0.04999	-0.00221	0.01344	0.09112	0.03223	0.06112	-0.03332	-0.05333	-0.07888
-0.04333	-0.01566	-0.00110	-0.00555	-0.00555	-0.03444	0.00890	0.01445	0.01558	0.03445
0.03001	0.03334	-0.00666	-0.01555	0.04556	0.02445	0.00001	-0.08110	-0.05555	-0.02888
0.02445	0.01001	0.03779	-0.00444	-0.01888	-0.07444	-0.05110	0.02557	0.014390	0.03555
0.07001	0.00890	-0.02666	0.05777	0.07778	0.03445	-0.08221	-0.01556	0.012444	0.05112
0.01556	0.00223	-0.00555	-0.02444	-0.04999	-0.07332	-0.01010	-0.04777	-0.02556	0.01890
0.09001	0.01345	0.01334	0.06668	0.01445	-0.08443	-0.09777	-0.05838	-0.02444	-0.07110
-0.01188	-0.01355	-0.08221	-0.05332	-0.00777	0.09112	0.08445	0.05334	0.08112	0.03445
0.09668	0.09334	0.05668	0.00556	-0.00443	0.01112	-0.01443	-0.03555	0.01334	0.04556
0.03445	0.01334	0.00779	-0.00332	-0.02221	-0.05666	-0.01133	-0.05221	-0.08444	0.03834
0.02445	-0.03444	0.01112	0.01778	0.03001	0.04556	0.03890	0.01445	-0.02655	0.03667
0.00445	-0.03110	-0.04555	-0.01777	-0.07777	-0.09666	-0.01888	0.02334	0.03779	0.03889
0.05445	0.05567	0.07001	0.07334	0.07001	0.07890	0.02890	-0.02555	-0.04110	-0.04110
-0.06777	-0.07110	-0.05888	-0.04777	-0.02555	0.00667	0.00445	0.01334	0.02557	0.03445
0.04112	0.02889	-0.00110	-0.01221	-0.04444	-0.03777	-0.03555	-0.03777	0.01001	0.01778
0.04001	0.04445	0.03445	0.00556	-0.02332	-0.03221	-0.05221	-0.04888	-0.05110	-0.04688
0.03112	0.04779	0.02890	0.00890	-0.01888	-0.02666	0.01890	0.02657	0.01557	0.01778
0.02334	0.01390	0.02112	0.02556	0.01890	0.01556	0.00223	0.01658	-0.02110	-0.01110
0.00667	-0.05110	-0.03777	0.00779	0.01668	0.02212	0.03888	-0.03910	-0.02222	0.02223
-0.03222	-0.06888	-0.06666	-0.01666	0.01001	0.02112	0.05334	0.05779	0.07223	0.03657
0.09445	0.06557	0.02779	-0.03110	-0.04110	-0.01666	-0.06444	-0.04444	-0.03977	-0.08333
-0.06777	0.03556	0.05112	0.05001	0.04223	0.04890	0.04112	0.02001	0.03001	0.05001
0.05334	0.01390	-0.01110	-0.03555	-0.07444	0.04334	0.00555	-0.03555	-0.08555	-0.04221
-0.00999	0.05001	0.05223	0.03112	0.02334	-0.00888	0.02778	0.02555	0.01445	-0.02221
-0.01888	0.01779	0.01556	0.00223	-0.01666	-0.00999	-0.02555	-0.03888	-0.05999	-0.05444
-0.06999	-0.05888	0.01001	0.05112	0.01556	0.01334	0.01055	0.01012	0.04778	0.02890
0.02334	0.02667	0.01334	-0.02888	-0.06888	-0.07555	-0.07555	-0.05221	-0.04999	-0.08110
-0.07444	-0.07999	-0.07999	-0.06222	-0.02110	0.04779	0.01334	0.01579	0.02557	0.021334
0.19890	0.01368	0.08890	0.06668	-0.03110	-0.09999	-0.01399	-0.01999	-0.01310	-0.01555
-0.01443	-0.01266	-0.07333	-0.00221	0.00667	0.05779	0.05223	0.07556	0.01079	0.012890
0.13890	0.012556	0.07556	0.00001	-0.02888	-0.01666	-0.03777	-0.01444	-0.02144	-0.015444
-0.03555	0.00890	-0.04555	-0.01010	0.00445	0.01179	0.01723	0.02223	0.014334	0.04334
-0.00999	-0.05566	-0.06777	0.00112	-0.03555	-0.07333	-0.06444	-0.02555	0.00223	0.00112
-0.00777	0.00223	0.01223	0.00001	-0.01333	-0.03110	-0.02777	-0.01555	-0.02777	-0.03555
0.07668	0.09778	0.04779	0.01890	0.00668	-0.01332	-0.02888	-0.01443	-0.03310	-0.03332

TABLE 4B

AUTOCORRELATION BATTLEFIELD DAY

k	1-6201 ft.	10-1551 ft.	1551-3101 ft.
0	0.0031	0.0212	0.0277
1	0.0024	-	0.0018
2	0.0014	-	0.0030
3	0.0005	-	0.0032
4	0.0001	-	0.0049
5	0.0005	-	0.0009
6	0.0009	-	0.0003
7	0.0012	-	0.0002
8	0.0013	-	0.0004
9	0.0013	-	0.0007
10	0.0011	-	0.0007
11	0.0009	-	0.0006
12	0.0005	-	0.0001
13	0.0002	-	0.0010
14	0.0002	-	0.0007
15	0.0005	-	0.0004
16	0.0006	-	0.0001
17	0.0007	-	0.0048
18	0.0006	-	0.0003
19	0.0005	-	0.0005
20	0.0003	-	0.0007
21	0.0001	-	0.0007
22	0.0000	-	0.0004
23	0.0002	-	0.0003
24	0.0004	-	0.0001
25	0.0005	-	0.0002
26	0.0004	-	0.0002
27	0.0004	-	0.0001
28	0.0002	-	0.0001
29	0.0001	-	0.0000
30	0.0000	-	0.0001
31	0.0001	-	0.0002
32	0.0002	-	0.0000
33	0.0003	-	0.0002
34	0.0004	-	0.0002
35	0.0004	-	0.0001
36	0.0003	-	0.0000
37	0.0001	-	0.0001
38	0.0000	-	0.0000
39	0.0001	-	0.0000
40	0.0002	-	0.0000
41	0.0003	-	0.0000
42	0.0003	-	0.0000
43	0.0003	-	0.0000
44	0.0002	-	0.0000
45	0.0001	-	0.0000
46	0.0000	-	0.0000
47	0.0000	-	0.0000
48	0.0001	-	0.0000
49	0.0001	-	0.0001
50	0.0002	-	0.0001

TABLE 4B CONT.

k	3101-4651 ft.	4651-6201 ft.
0	0.0816	
1	- 0.0112	0.0049
2	- 0.0120	0.0023
3	- 0.0101	0.0008
4	- 0.0150	0.0001
5	0.0039	0.0007
6	0.0026	0.0005
7	0.0014	0.0007
8	0.0010	0.0010
9	- 0.0002	0.0011
10	- 0.0002	0.0011
11	- 0.0004	0.0010
12	0.0010	0.0008
13	- 0.0034	0.0004
14	- 0.0031	0.0001
15	- 0.0025	0.0001
16	- 0.0018	0.0003
17	0.0178	0.0004
18	- 0.0024	0.0003
19	- 0.0025	0.0004
20	- 0.0031	0.0002
21	- 0.0030	0.0003
22	0.0015	0.0002
23	0.0012	0.0003
24	0.0004	0.0003
25	- 0.0002	0.0001
26	- 0.0003	0.0000
27	0.0000	0.0001
28	- 0.0002	0.0003
29	0.0004	0.0001
30	- 0.0003	0.0001
31	- 0.0007	0.0001
32	- 0.0000	0.0001
33	0.0003	0.0002
34	0.0003	0.0003
35	- 0.0000	0.0002
36	- 0.0001	0.0001
37	0.0001	0.0001
38	- 0.0003	0.0001
39	- 0.0004	0.0002
40	- 0.0002	0.0002
41	0.0004	0.0002
42	0.0006	0.0001
43	0.0006	0.0000
44	0.0002	0.0000
45	- 0.0002	0.0001
46	- 0.0001	0.0002
47	- 0.0002	0.0002
48	- 0.0003	0.0001
49	0.0002	0.0001
50	- 0.0001	0.0001
		0.0000

TABLE 4C

POWER SPECTRAL DENSITY

BATTLEFIELD DAY

λ	1-6201 ft.	10-1551 ft.	1551-3101 ft.
-0.	0.00168	0.00091	0.00298
0.01000	0.00229	0.00258	0.00301
0.02000	0.00577	0.00675	0.00683
0.03000	0.01280	0.01474	0.01380
0.04000	0.02512	0.02694	0.01762
0.05000	0.04145	0.04695	0.02527
0.06000	0.04944	0.05605	0.03493
0.07000	0.04344	0.03540	0.03616
0.08000	0.03536	0.01792	0.03427
0.09000	0.03611	0.01177	0.03671
0.10000	0.04979	0.00838	0.04440
0.11000	0.07341	0.00832	0.05582
0.12000	0.08966	0.00805	0.06461
0.13000	0.08512	0.00783	0.06755
0.14000	0.07005	0.00775	0.06719
0.15000	0.06500	0.00582	0.07004
0.16000	0.07923	0.00456	0.06452
0.17000	0.10050	0.00518	0.06391
0.18000	0.10250	0.00509	0.06829
0.19000	0.07870	0.00359	0.06284
0.20000	0.05382	0.00284	0.05513
0.21000	0.04873	0.00244	0.05126
0.22000	0.05753	0.00177	0.04638
0.23000	0.06602	0.00107	0.04360
0.24000	0.06240	0.00080	0.03823
0.25000	0.04714	0.00074	0.03280
0.26000	0.03495	0.00089	0.03166
0.27000	0.03483	0.00113	0.03148
0.28000	0.04390	0.00086	0.03150
0.29000	0.05434	0.00054	0.03269
0.30000	0.05738	0.00064	0.03414
0.31000	0.05063	0.00075	0.03673
0.32000	0.04386	0.00075	0.03946
0.33000	0.04926	0.00088	0.04090
0.34000	0.06720	0.00088	0.04246
0.35000	0.08321	0.00079	0.04709
0.36000	0.08240	0.00088	0.05018
0.37000	0.06740	0.00100	0.04949
0.38000	0.05504	0.00107	0.04902
0.39000	0.05984	0.00100	0.05012
0.40000	0.07740	0.00100	0.05197
0.41000	0.08781	0.00088	0.05212
0.42000	0.07777	0.00064	0.04995
0.43000	0.05662	0.00061	0.04706
0.44000	0.04233	0.00063	0.04430
0.45000	0.04396	0.00057	0.04153
0.46000	0.05436	0.00058	0.03921
0.47000	0.05931	0.00050	0.03717
0.48000	0.05170	0.00046	0.03446
0.49000	0.03803	0.00051	0.03204
0.50000	0.03093	0.00050	0.03053

TABLE 4C CONT.

λ	3101-4651 ft.	4651-6201 ft.
-0.	0.00141	0.00121
0.01000	0.00174	0.00152
0.02000	0.00440	0.00472
0.03000	0.01044	0.01184
0.04000	0.02629	0.02973
0.05000	0.04910	0.04447
0.06000	0.06513	0.04186
0.07000	0.06949	0.03319
0.08000	0.06423	0.02500
0.09000	0.07831	0.01763
0.10000	0.13129	0.01474
0.11000	0.21706	0.01247
0.12000	0.27288	0.01366
0.13000	0.24698	0.01862
0.14000	0.18810	0.01691
0.15000	0.16925	0.01425
0.16000	0.23372	0.01404
0.17000	0.32036	0.01326
0.18000	0.32592	0.01141
0.19000	0.23869	0.00965
0.20000	0.14728	0.00948
0.21000	0.13192	0.00887
0.22000	0.17423	0.00764
0.23000	0.21311	0.00652
0.24000	0.20546	0.00538
0.25000	0.15049	0.00433
0.26000	0.10292	0.00381
0.27000	0.10256	0.00370
0.28000	0.13951	0.00355
0.29000	0.18088	0.00334
0.30000	0.19149	0.00341
0.31000	0.16048	0.00439
0.32000	0.12967	0.00507
0.33000	0.14996	0.00483
0.34000	0.22070	0.00477
0.35000	0.28059	0.00482
0.36000	0.27510	0.00382
0.37000	0.21533	0.00364
0.38000	0.16497	0.00455
0.39000	0.18257	0.00528
0.40000	0.25152	0.00528
0.41000	0.29454	0.00426
0.42000	0.25698	0.00381
0.43000	0.17453	0.00399
0.44000	0.12038	0.00341
0.45000	0.13044	0.00289
0.46000	0.17513	0.00256
0.47000	0.19765	0.00214
0.48000	0.16954	0.00238
0.49000	0.11634	0.00289
0.50000	0.08904	0.00309

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TABLE 4D

POWER SPECTRAL DENSITY			BATTLEFIELD DAY		
λ	1-6201 ft	10-1551 ft	1551-3101 ft	3101-4651 ft	4651-6201 ft
.00	---	---	---	---	---
.01	13.21146	14.88454	17.36529	10.03841	8.76918
.02	2.14806	2.51289	2.54267	1.63803	1.75716
.03	.97882	1.12717	1.05529	.79835	.90540
.04	.64177	.68826	.45016	.67166	.75954
.05	.46532	.52706	.28368	.55120	.49922
.06	.29168	.33067	.20607	.38424	.28412
.07	.15317	.12482	.12750	.24502	.11703
.08	.08224	.04168	.07971	.14939	.05815
.09	.05994	.01954	.06094	.12999	.02927
.10	.06301	.01053	.05619	.16616	.01865
.11	.07494	.00849	.05699	.22160	.02173
.12	.07758	.00697	.05590	.23610	.01182
.13	.06522	.00600	.05176	.18924	.01427
.14	.04945	.00547	.04743	.13278	.01194
.15	.04381	.00392	.04721	.12678	.01147
.16	.05266	.00303	.04289	.15535	.00933
.17	.06782	.00350	.04313	.21619	.00895
.18	.07204	.00358	.04800	.22907	.00802
.19	.05887	.00269	.04701	.17855	.00722
.20	.04359	.00230	.04466	.11930	.00768
.21	.04325	.00217	.04550	.11710	.00787
.22	.05629	.00173	.04538	.17048	.00748
.23	.07114	.00115	.04698	.22963	.00703
.24	.07344	.00094	.04499	.24181	.00633
.25	.05966	.00094	.04151	.19046	.00548
.26	.04656	.00119	.04217	.13710	.00508
.27	.04760	.00154	.04302	.14015	.00506
.28	.05990	.00117	.04298	.19036	.00484
.29	.07217	.00072	.04342	.24024	.00444
.30	.07262	.00081	.04321	.24235	.00432
.31	.06012	.00089	.04362	.19057	.00521
.32	.04844	.00083	.04358	.14321	.00560
.33	.05048	.00090	.04191	.15366	.00495
.34	.06412	.00084	.04051	.21057	.00455
.35	.07453	.00071	.04218	.25133	.00432
.36	.07012	.00075	.04270	.23409	.00325
.37	.05528	.00082	.04059	.17661	.00299
.38	.04421	.00086	.03938	.13252	.00365
.39	.04787	.00080	.04010	.14606	.00422
.40	.06269	.00081	.04210	.20373	.00428
.41	.07314	.00073	.04341	.24534	.00355
.42	.06753	.00056	.04337	.22315	.00331
.43	.05183	.00056	.04308	.15976	.00365
.44	.04115	.00061	.04307	.11794	.00332
.45	.04555	.00059	.04303	.13515	.00299
.46	.05994	.00064	.04324	.19312	.00282
.47	.06914	.00058	.04333	.23040	.00249
.48	.06299	.00056	.04198	.20655	.00290
.49	.04766	.00064	.04016	.14581	.00362
.50	.03915	.00063	.03864	.11269	.00391

NOTE:

After making the calculations, several small errors were found in the raw data of Aberdeen, Knox and Yuma. Since their effects on the results would be minor, the calculations were not done over. The raw data table in each case is corrected, but the smoothed data tables are as computed.

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Analysis of a Linear Vehicle Using
Measured Ground Power Spectral Density

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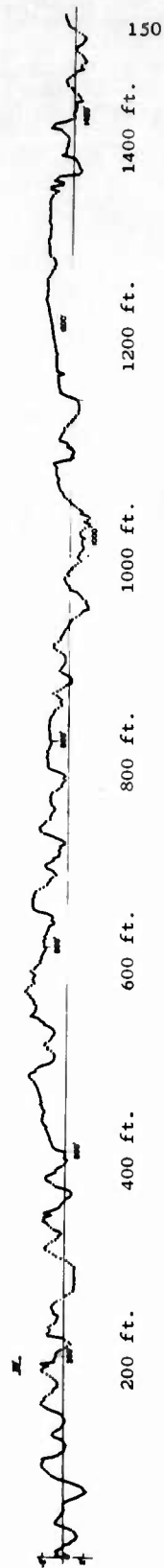


Figure 1 - PROFILE - ABERDEEN

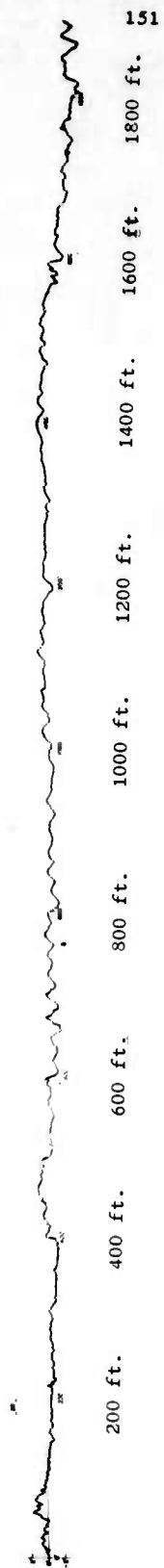


Figure 2 - PROFILE - FT. KNOX

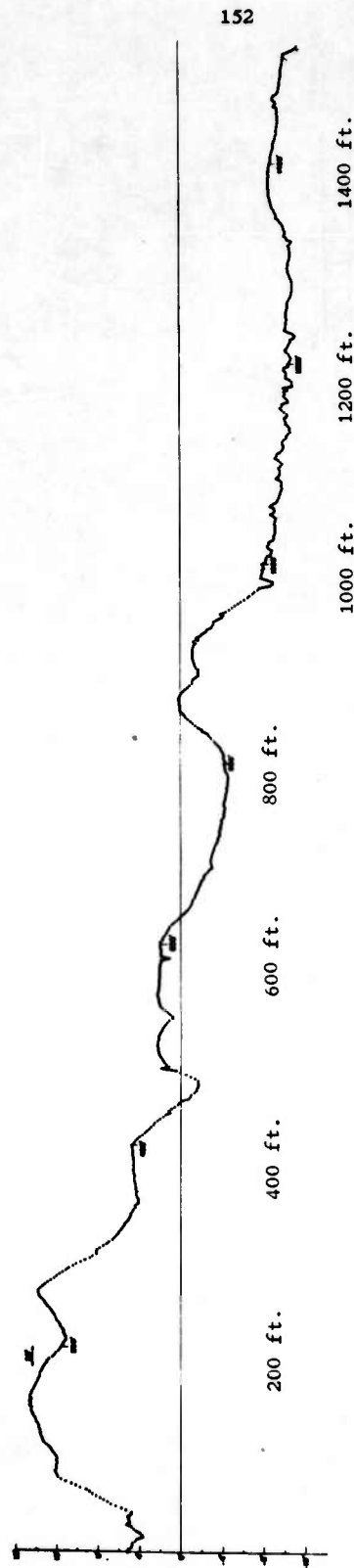


Figure 3 - PROFILE - YUMA

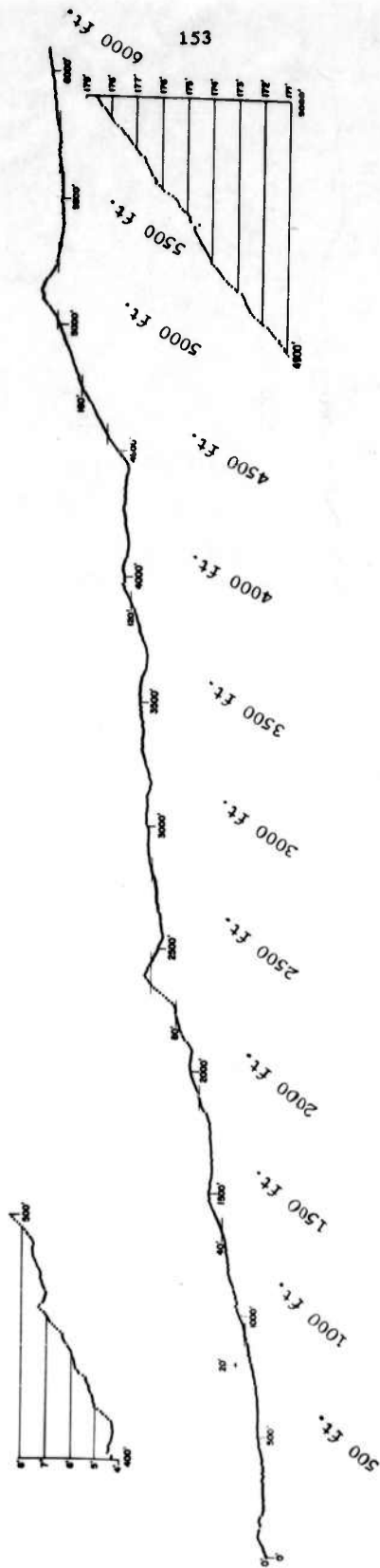


Figure 4 - PROFILE - BATTLEFIELD DAY

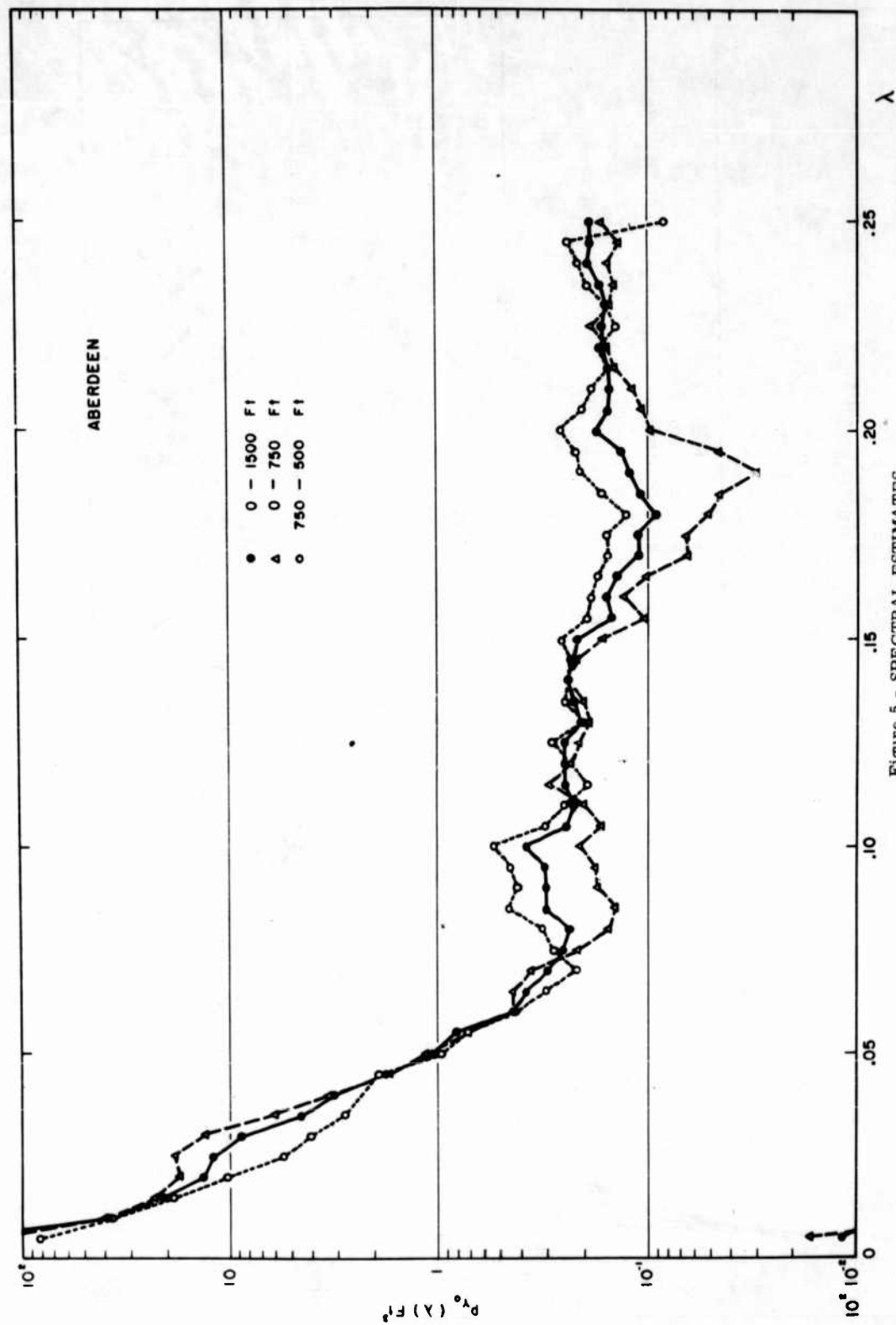


Figure 5 - SPECTRAL ESTIMATES
ABERDEEN

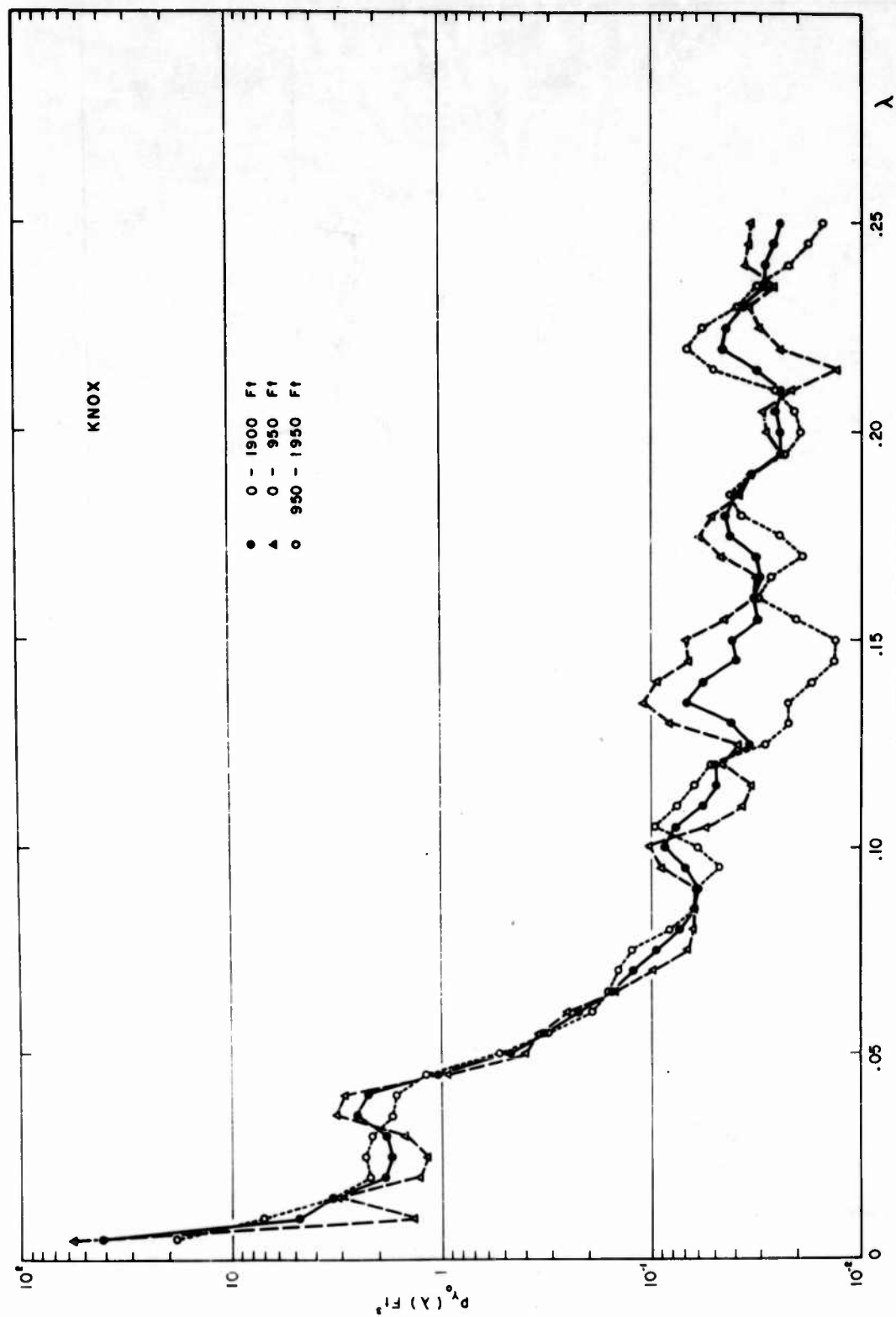


Figure 6 - SPECTRAL ESTIMATES
FT. KNOX

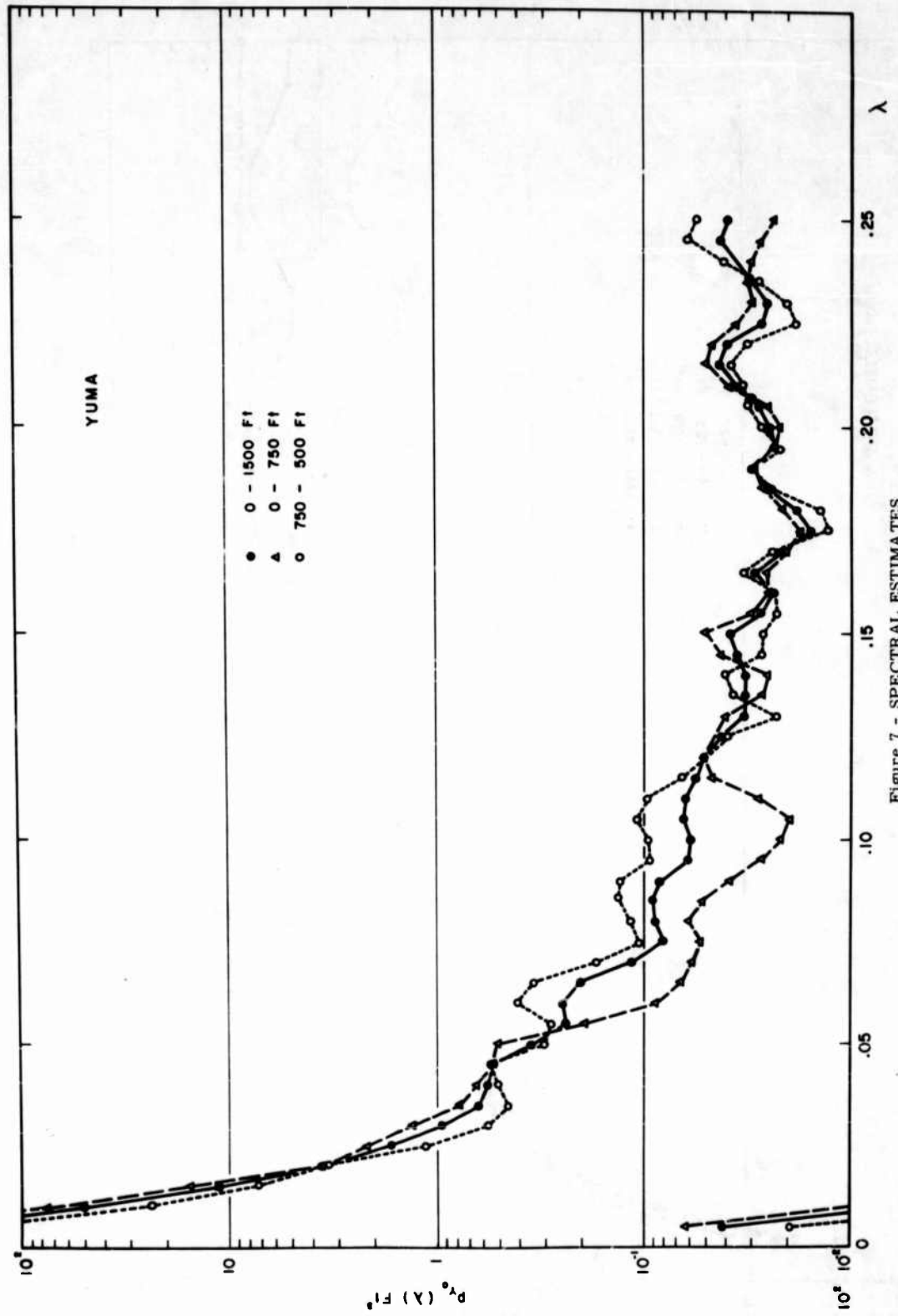
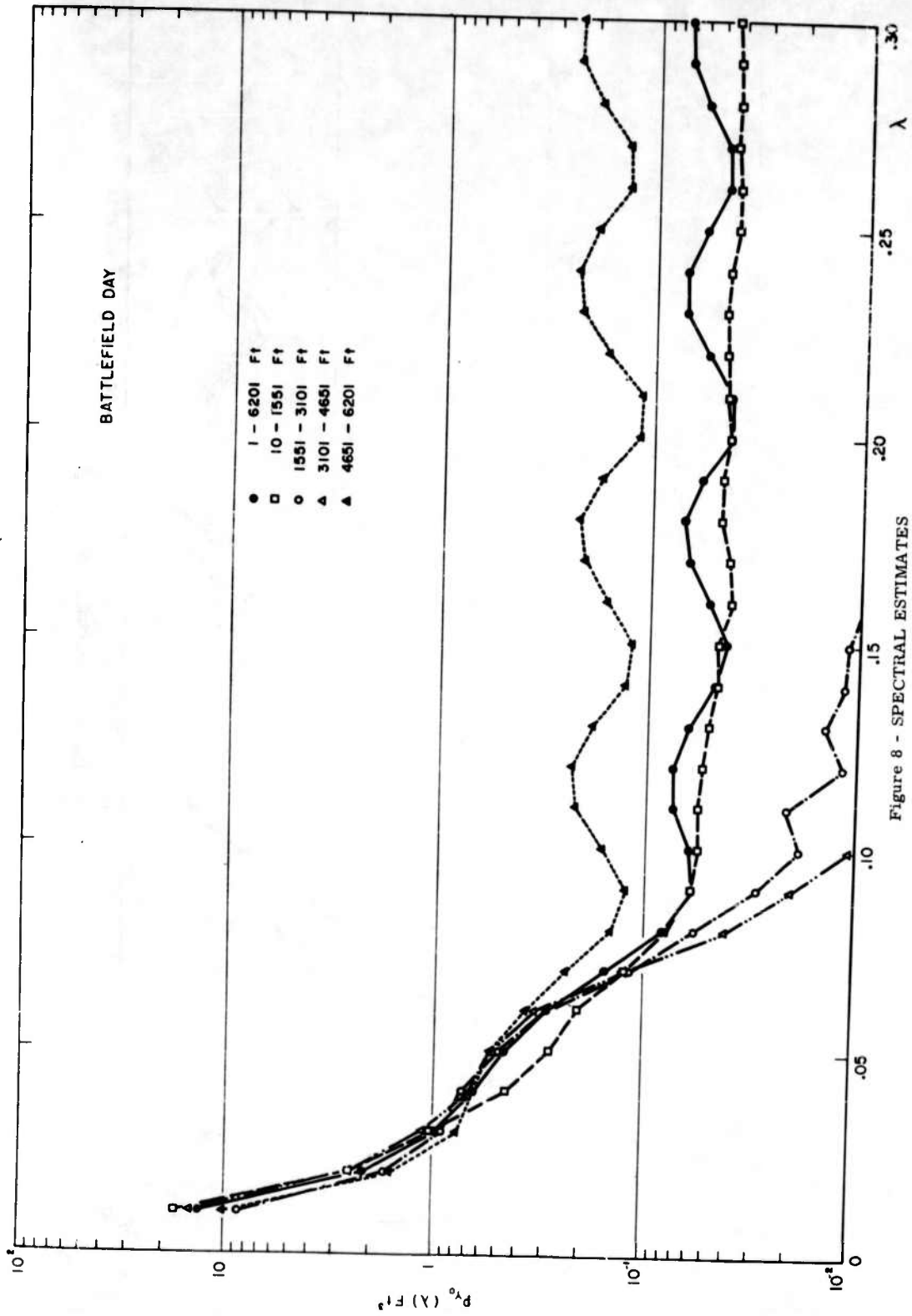


Figure 7 - SPECTRAL ESTIMATES
YUMA



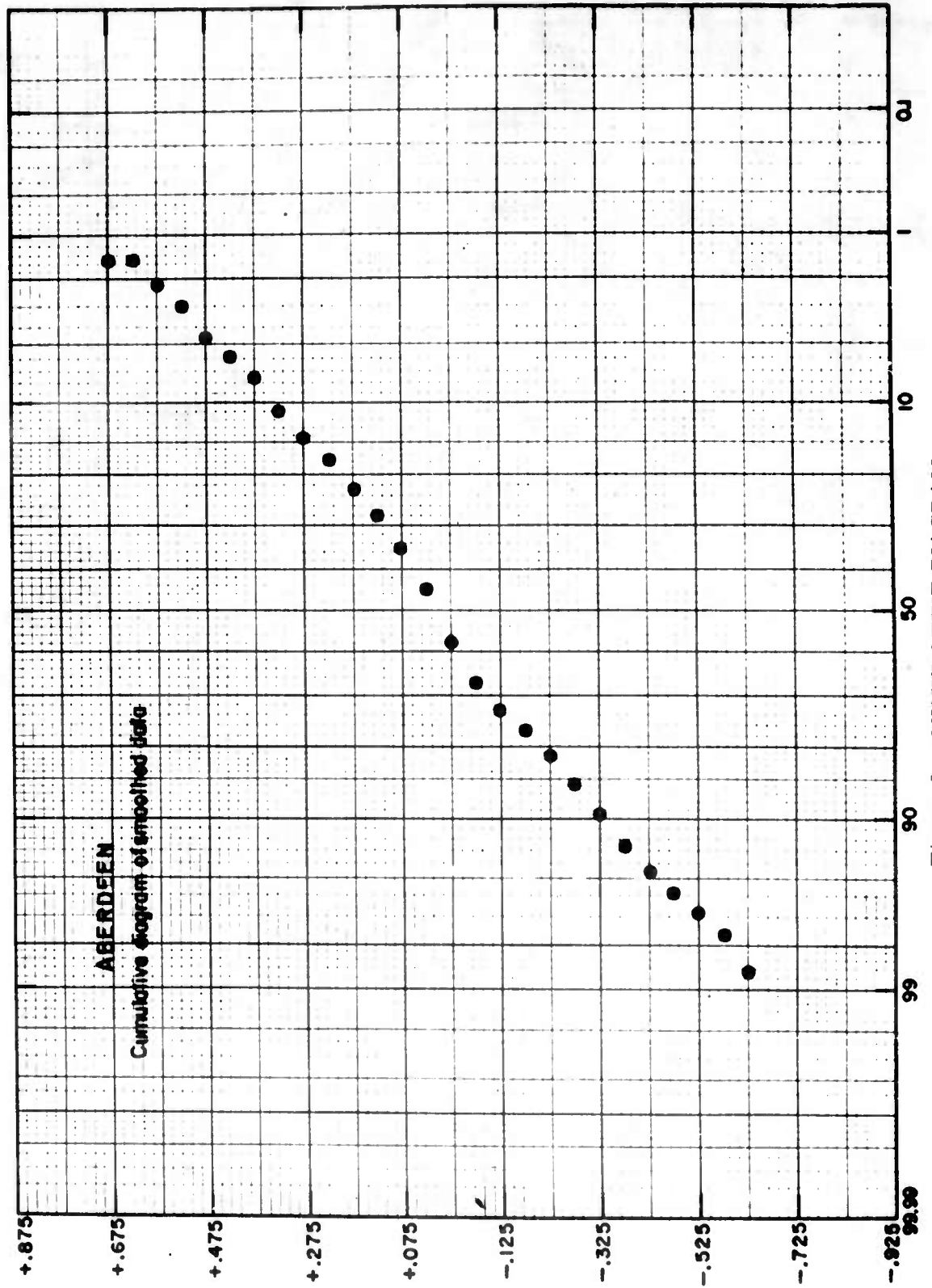


Figure 9 - CUMULATIVE DIAGRAM

ABERDEEN

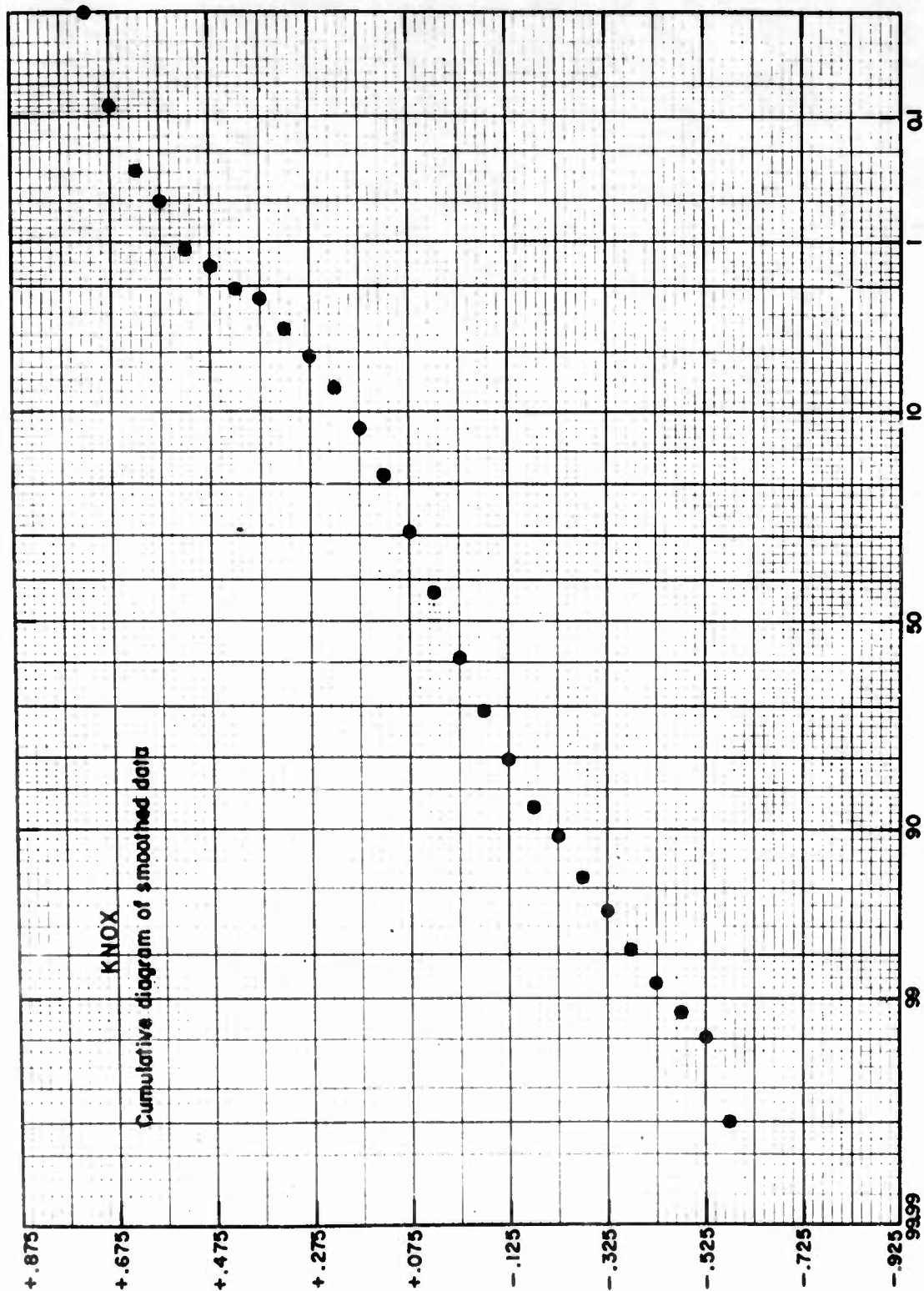


Figure 10 - CUMULATIVE DIAGRAM

FT. KNOX

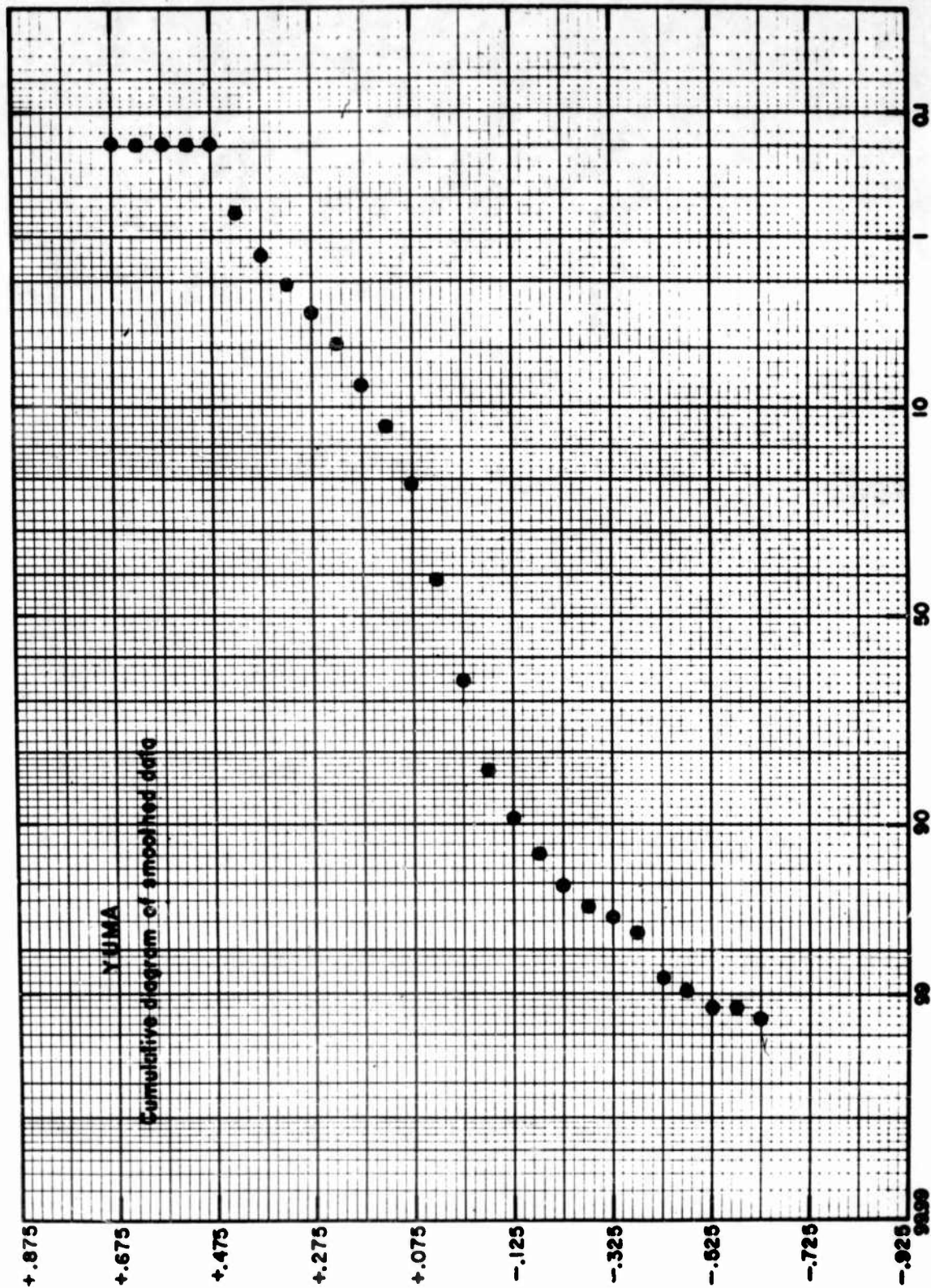


Figure 11 - CUMULATIVE DIAGRAM

YUMA

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Report No. 8391 (Final), November 1963, 149 pp., 11 figures
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A method of estimating statistical characteristics of stable ground roughness for use in off-road road locomotion is developed. Computer programs for obtaining these estimates from survey data have been prepared. Several estimated power spectral densities obtained from survey data taken at military installations are presented.

Pertinent features of these estimated power spectral densities are interpretable in terms of visual features of plotted ground profiles. The estimated power spectral densities may be approximated by similar and simple form

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