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REPORT 470

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ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT

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REPORT 470

MODELS FOR HELICOPTER DYNAMIC STABILITY INVESTIGATIONS

by

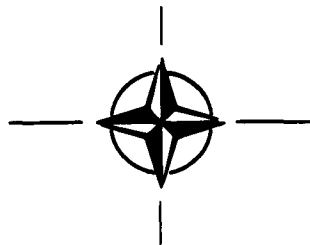
L. R. LUCASSEN and F. J. STERK

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**REPORT 470**

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**MODELS FOR HELICOPTER DYNAMIC  
STABILITY INVESTIGATIONS**

by

**L. R. Lucassen and F. J. Sterk**

**This Report was presented at the Twenty-Third Meeting of the Flight Mechanics Panel  
held in Athens, Greece, 8-10 July 1963**

#### SUMMARY

A mathematical model is described, which has been devised for improving the physical understanding of helicopter dynamic instability in hovering (two degrees of freedom). A demonstration model has been built according to this principle. The influence of parameters, artificial stabilization and sling load on the dynamic characteristics is shown.

A short 16 mm film is available.

#### SOMMAIRE

On décrit un modèle mathématique qui a été conçu pour améliorer la compréhension physique de l'instabilité dynamique des hélicoptères en planant (deux degrés de liberté). On a construit un modèle pour démonstration conformément à ce principe. L'influence des paramètres, de la stabilisation artificielle et de la charge élinguée sur les caractéristiques dynamiques est montrée ici.

Un court film 16 mm est disponible

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## NOTATION

See Figure 1 for positive directions

$\alpha$	angle between thrust and rotor shaft centreline
$a_q$	damping in pitch or roll = change in $\alpha$ per unit in angular velocity of the rotor head, including the effect of artificial rate stabilization
$a'_q$	$a_q + ha_u$
$a_u$	speed stability = change in $\alpha$ per unit in linear velocity of the rotor head
$g$	acceleration due to gravity
$h$	distance of the rotor head above the helicopter centre of gravity
$I$	helicopter moment of inertia about axis through centre of gravity
$M$	helicopter mass
$q$	$= \dot{\theta}$
$u$	horizontal velocity of helicopter centre of gravity
$\theta$	angular attitude of fuselage
$\lambda$	linear scale of mechanical model with respect to mathematical model
$\mu$	mass scale of mechanical model with respect to mathematical model

## MODELS FOR HELICOPTER DYNAMIC STABILITY INVESTIGATIONS

L.R. Lucassen\* and F.J. Sterk\*

### 1. INTRODUCTION

It is known from experience that the motion of a normal helicopter without special provisions is dynamically unstable. This is also proved by several theoretical considerations, in which it is pointed out that certain criteria are not met or unstable roots appear. Only relatively seldom is a mechanical approach used in order to improve the physical understanding of the behaviour.

The main object of this Report is to give a contribution in this direction, firstly by making use of a rather simple mathematical helicopter model. Thereafter it is explained how this model can be built at a reduced size for demonstration purposes.

### 2. THEORETICAL ASPECTS

#### 2.1 General

Before dealing with these models, it is necessary to recall some important theoretical aspects of helicopter and rotor dynamics. In order not to confuse the derivations with too many details, it has been attempted to consider only essential quantities. This has resulted in the adoption of some rather drastic simplifications:

- (a) hovering helicopter.
- (b) small deviations of the helicopter in two degrees of freedom (roll and side slip or pitch and forward speed).

Other assumptions are mentioned in the next two sections.

#### 2.2 The Helicopter

The helicopter quantities which are important for the analysis are:

- (a) the helicopter mass  $M$ ,
- (b) the distance  $h$  of the centre of gravity below the rotor head,
- (c) the moment of inertia  $I$  about the axis through the centre of gravity about which the aircraft is free to rotate.

Some additional assumptions are:

- (i) helicopter centre of gravity on the shaft centreline,
- (ii) rotor thrust equal to weight.

---

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### 2.3 The Rotor

An important parameter in the theory of dynamic stability is the angle  $\alpha$  between the centreline of the shaft and the thrust. This angle is generally assumed to be linearly dependent on the angular velocity of the rotor head and its linear velocity:

$$\alpha = a_q \dot{\theta} + a_u (u + h\dot{\theta}) . \quad (1)$$

(See Fig.1(a).) The damping in pitch  $a_q$  and the speed stability  $a_u$  depend on geometric, mass, and operational characteristics of the rotor blades. The sign convention, as indicated in Equation (1) and Figure 1(a), leads to positive values for  $a_q$  and  $a_u$  in the normal case.

In Equation (1), two terms are proportional to  $\dot{\theta}$ . Taking these together, then

$$\alpha = a'_q \dot{\theta} + a_u u , \quad (2)$$

where

$$a'_q = a_q + ha_u . \quad (3)$$

As  $a'_q$  depends on  $h$ , it is strictly speaking no longer a rotor quantity.

However,  $ha_u$  is generally small in comparison with  $a_q$ , so that  $a'_q \approx a_q$  is still mainly determined by the rotor.

The eccentricity of the blade flapping hinges is assumed to be zero. This leads to zero moments from the rotor on the shaft.

### 2.4 Helicopter Dynamics

The equations of motion for the helicopter in two degrees of freedom are, according to Figure 1(a):

$$\text{translation: } M\dot{u} = Mg(\theta - \alpha) , \quad (4)$$

$$\text{rotation: } I\ddot{\theta} = -hMga . \quad (5)$$

After substitution of Equation (2), these become:

$$M\dot{u} = Mg(\theta - a'_q \dot{\theta} - a_u u) , \quad (6)$$

$$I\ddot{\theta} = -hMg(a'_q \dot{\theta} + a_u u) . \quad (7)$$

These equations, although based on several assumptions as mentioned before, give very reasonable approximations for the type of motion, the period and the damping time of a hovering helicopter.

Dynamic models, representing the helicopter, should also obey these equations.

### 2.5 Mathematical Models

In order to improve the physical understanding of the helicopter dynamic behaviour, it has been attempted to devise a mathematical model which is simpler than that shown in Figure 1(a). This model is indicated in Figure 1(b). An important feature is that it does no longer include the rotor. On the shaft centreline, a point has been indicated at a distance  $\frac{a'_q}{a_u}$  above the helicopter centre of gravity. This point is, for reasons which will soon become clear, called the *damper point*. It moves with the helicopter fuselage and, because of the chosen distance, its horizontal velocity components due to rotation and translation are  $\frac{a'_q \dot{\theta}}{a_u}$  and  $u$  respectively. The ratio between these velocities is exactly equal to the ratio of the rotor forces  $Mga'_q \dot{\theta}$  and  $Mga_u u$ , these being two of the three horizontal components of the thrust. If therefore a horizontal damper is assumed to act at the damper point, this will exert forces dependent on  $\dot{\theta}$  and  $u$  in the right proportion. It is important to note that the distance between the damper point and the helicopter centre of gravity is by definition constant and almost fully determined by rotor quantities (if the small correction  $ha_u$  on  $a_q$  is disregarded).

The damper point should not be confused with the so-called neutral point above a rotor, which is sometimes used in rotor theory, particularly of German origin. The neutral point is related to the moment which a rotor exerts on the shaft, for instance due to eccentric flapping hinges. Such moments are not considered here.

For normal helicopters,  $\frac{a'_q}{a_u}$  may be of the order of 100 feet, so the damper point is considerably above the rotor.

The damper constant, being the ratio of the damper force and velocity, should be equal to  $Mga_u$  in order to produce a force of the right magnitude.

The third horizontal component  $Mg\theta$  of the rotor thrust in the mathematical model is assumed to act at the helicopter centre of gravity. This guarantees the absence of a moment dependent on  $\theta$ , as is in fact required by Equation (7).

The equation for the horizontal motion of the mathematical model appears to be:

$$M\dot{u} = Mg\theta - Mga_u \left( \frac{a'_q}{a_u} \dot{\theta} + u \right) \quad (8)$$

and this is equivalent to Equation (6). In order to fulfil the moment equation (Eqn. (7)), it is necessary to scale the moment of inertia with the factor  $\frac{a'_q}{ha_u}$ . The reason for this scaling can also be explained in other terms. Consider the distance  $\frac{a'_q}{a_u}$ , which is larger than  $h$ . However, the damper force is equal to the two horizontal components of the rotor thrust. Therefore the moment will be larger and, in order to obtain the same angular acceleration, the moment of inertia should be scaled up.

The corresponding equation is

$$\frac{a'_d}{h a_u} I \ddot{\theta} = - \frac{a'_d}{a_u} M g a_u \left( \frac{a'_d}{a_u} \dot{\theta} + u \right) \quad (9)$$

which is identical to Equation (7). The mathematical model will therefore have the same dynamic characteristics as the helicopter. The time scale of the motions is equal to one.

These considerations show that the action of a helicopter rotor is equivalent to the combined action of a physically more understandable damper at the damper point and a force  $Mg\theta$  at the centre of gravity, both forces being horizontal.

Some general results may be obtained by paying closer attention to this model. It is, however, preferred first to proceed to the description of the mechanical model. The discussion may be found in Section 4.

### 3. MECHANICAL MODEL

In developing the mathematical model, the questions did arise whether it would be possible to materialize this model and for which purposes it could be used.

The first question is probably best answered by referring to Figures 3 and 4, showing a diagram and a picture of the mechanical model. This is similar to the mathematical model, but constructed at a length scale  $\lambda$  and a mass scale  $\mu$ . It consists of a horizontal rail, along which a car is allowed to move. This car represents the centre of gravity of the helicopter. A bar, representing the moment of inertia of the helicopter, is mounted on pivots in this car. The upper end of the bar, corresponding to the damper point, is equipped with an air damper. In order to exert the force  $Mg\theta$  at the centre of gravity, the car is connected to a second car on a sloping rail. Its gradient is varied by a servomechanism. This receives an input from a potentiometer in the first car, measuring the attitude angle  $\theta$  of the bar with respect to the vertical. The weights of cars and bar together correspond to the helicopter weight (scale  $\mu$ ). As only the second car is on the sloping rail, its gradient is somewhat larger than, but proportional to, the attitude angle of the bar.

With regard to the model scales, the following remarks apply (Fig. 2). All lengths of the mathematical model are reduced by the linear scale  $\lambda$ . As  $\frac{a'_d}{a_u}$  is large,  $\lambda$  must be chosen rather small in order to obtain reasonable model dimensions.

For the time scale, one must remember that  $g$ , having the dimension  $[l/t^2]$ , is equal for the mathematical and mechanical models. So a time scale of  $\sqrt{\lambda}$  must be accepted. Independent of these scales is the mass scale  $\mu$ . Scales for other quantities such as  $I$ , damping constant, etc., are combinations of  $\lambda$  and  $\mu$ .

The model as shown in Figure 4 has been built from a universal construction system (Swedish FAC X-2), a servo-component kit (English Feedback Ltd.) and some model railway elements.

## 4. DISCUSSION

### 4.1 General

Having described the mathematical and mechanical models, it is possible to go into some more detail. The discussion will be given in terms applying to the mathematical model (Fig.1(b)); for the mechanical model, the scales have to be taken into account (Fig.2).

### 4.2 Period

The length  $\frac{a'_q}{a_u}$  of the mathematical model may for a moment be considered as the length of a simple pendulum. Its period of oscillation would then be  $T = 2\pi \sqrt{\left(\frac{a'_q}{ga_u}\right)}$ . This equation for helicopters was first derived by Hohenemser in 1944 by starting from the equations of motion and neglecting the helicopter moment of inertia. This approximation is, however, seldom appropriate. The mathematical model shows two reasons. First, the helicopter moment of inertia (compound pendulum instead of simple pendulum) leads anyhow to larger periods and it had to be scaled up with the factor  $\frac{a'_q}{ha_u}$ . Secondly, the upper point of the length  $\frac{a'_q}{a_u}$  is not fixed to space but attached to the damper. This also leads to an increase in period.

### 4.3 Helicopter Instability

The action of the rotor thrust and weight on the helicopter may, according to the models, be considered as being equivalent to that of two horizontal forces (Fig.1(b)). The upper force, depending on  $a'_q$  and  $a_u$ , is purely damping. Energy is permanently withdrawn from the system at the damper point. The only reason for instability must therefore be sought in the force  $Mg\theta$ . If  $a'_q$  and  $a_u$  were zero, then the damper would exert no moment about the centre of gravity and the angular velocity  $\dot{\theta}$  would be constant. The translational motion, which is then only influenced by the force  $Mg\theta$  at the centre of gravity, will have a linearly increasing acceleration, which is obviously unstable. The introduction of the aforementioned damping leads to a statically stable motion, but the dynamic instability remains in the form of diverging oscillations.

### 4.4 Artificial Stabilization

In its simplest form, artificial stabilization is obtained by cyclic control inputs to the rotor which make the thrust angle  $a$  dependent on fuselage attitude  $\theta$ :

$$a = a'_q \dot{\theta} + a_u u + a_\theta \theta . \quad (10)$$

Substitution in the equation of motion (Eqn.(5)), shows that artificial stabilization leads to an extra moment on the helicopter of  $-hMga_\theta \theta$  about the centre of gravity. In terms of the mathematical model, this means that the force  $Mg\theta$ , or more precisely  $Mg\theta (1-a_\theta)$ , should act at a point  $\frac{a'_q}{a_u} a_\theta$  below the centre of gravity.

It is now obvious that such a force tends to decrease the angle  $\theta$ , thus having a stabilizing effect on the motion. As the moment arm is proportional to  $a_p$ , larger values of this coefficient will naturally be more favourable.

#### 4.5 Sling Load

Helicopters are often used for the transportation of external loads. In several cases these may have an unfavourable influence on the flying qualities, thus restricting the operational possibilities.

In order to avoid such restrictions, different methods of load suspension have been devised, their common idea being to avoid moments of the load about the helicopter centre of gravity.

Some preliminary tests on the mechanical model have been made to demonstrate the influence of the sling load on the motion. Figure 5 shows this model with the sling load attached. Because the bar represents the fuselage, the load may be directly suspended from this bar.

#### 5. CONCLUSIONS

Theoretical considerations and experiments on models, representing the dynamic characteristics of a hovering helicopter with two degrees of freedom, with and without artificial stabilization and sling load, have led to the following conclusions:

- (i) The *damper point* on the rotor shaft centreline at a distance  $\frac{a'_d}{a_u}$  above the helicopter centre of gravity is a concept which may be used for improving the physical understanding of helicopter instability.
- (ii) The complicated action of rotor thrust and weight on a helicopter may for stability considerations be replaced by that from two *horizontal forces*: one at the damper point and the other at the centre of gravity, proportional to the attitude angle.
- (iii) It has appeared possible to construct a simple *mechanical model*, on the basis of the mathematical model, which can be used for demonstrating dynamic characteristics and, amongst others, the influence of artificial stabilization and/or sling load on the motion.

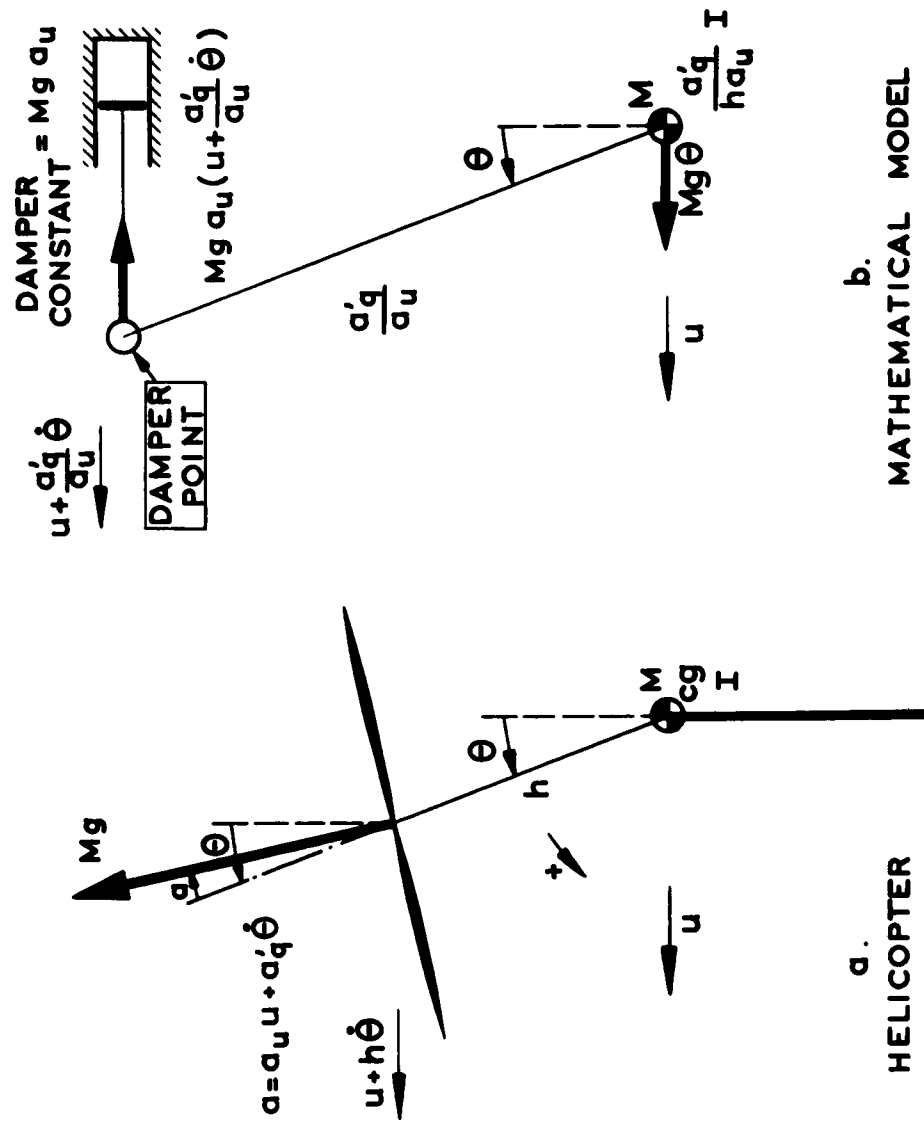
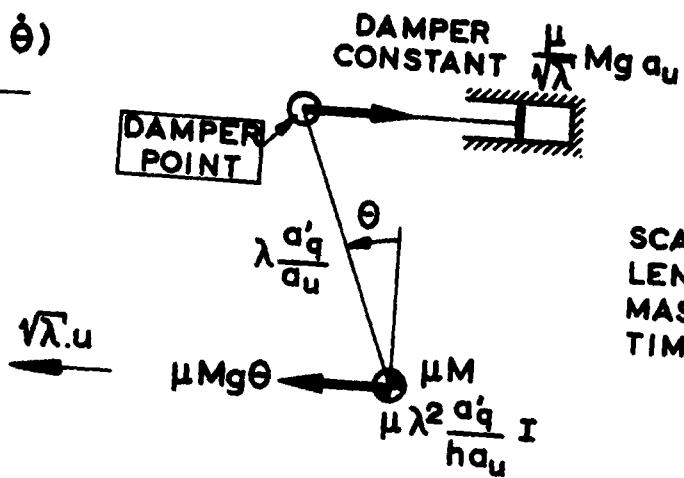


Fig. 1 Comparison between helicopter and mathematical model

$$\sqrt{\lambda} \left( u + \frac{a'q}{a_u} \dot{\theta} \right)$$



SCALES :  
 LENGTH  $\lambda$   
 MASS  $\mu$   
 TIME  $\sqrt{\lambda}$

Fig. 2 Mechanical model

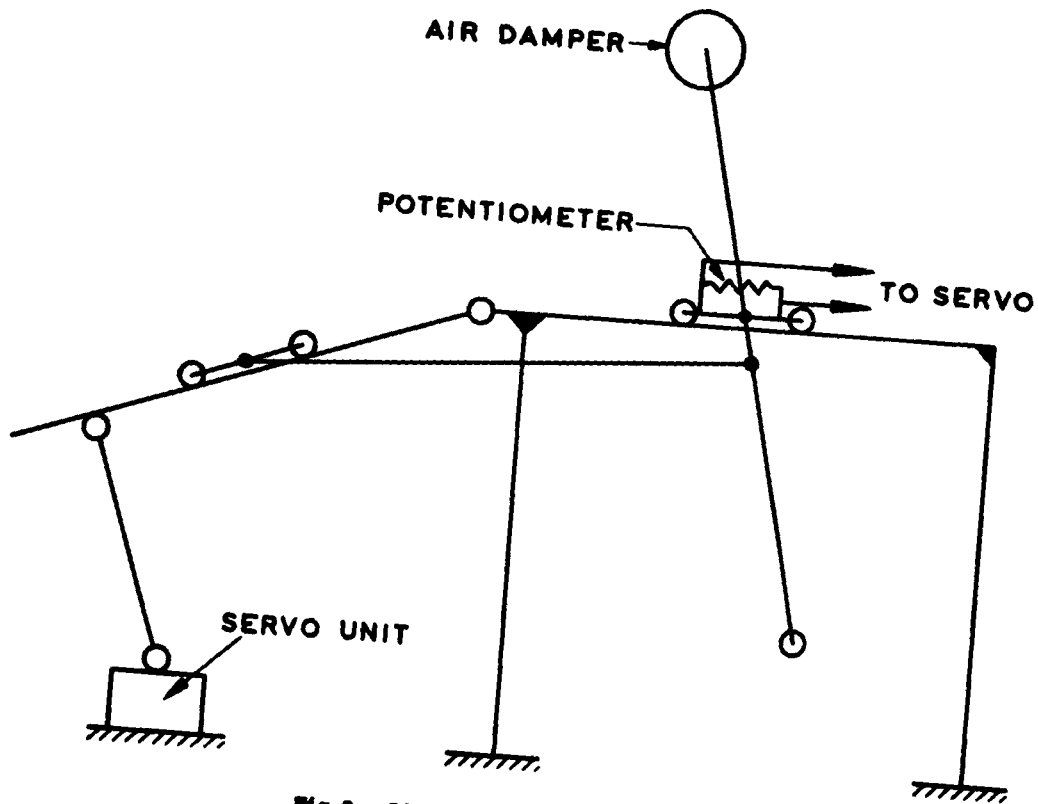


Fig. 3 Diagram of mechanical model

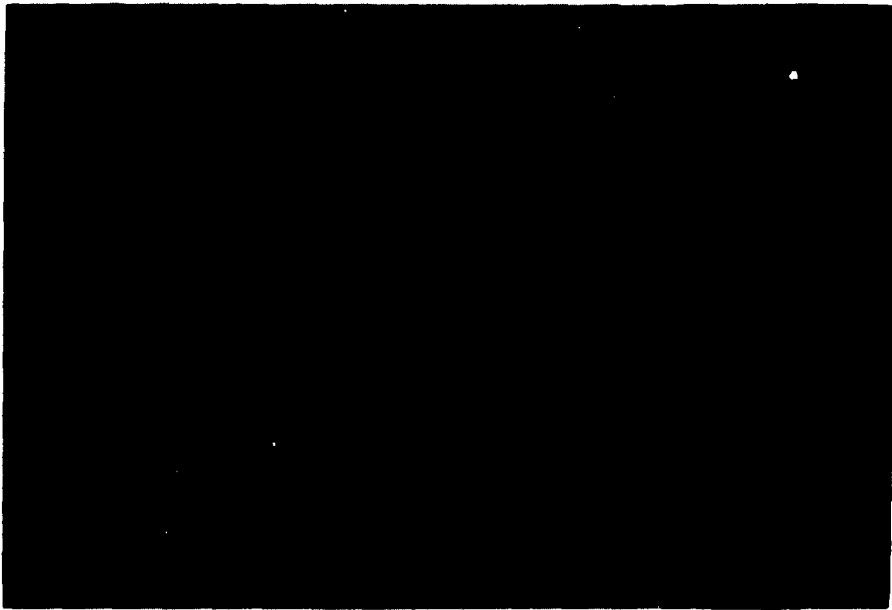


Fig. 4 Mechanical model

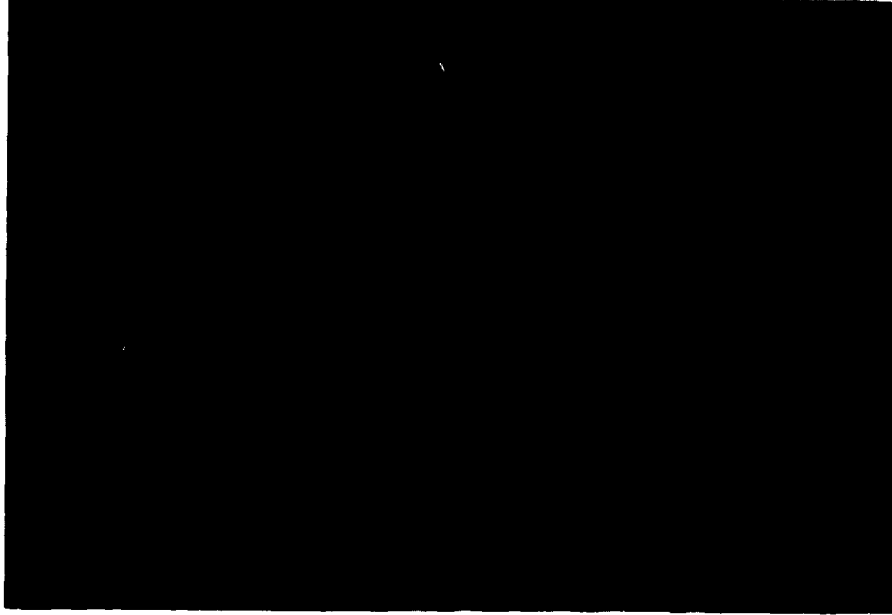


Fig. 5 Sling load on mechanical model

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