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September 1964

INCOMPRESSIBLE FLOW IN CONICAL DIFFUSERS

by

Alan T. McDonald

Robert W. Fox

ARMY RESEARCH OFFICE (DURHAM)
DA Project No. 200 1050 1B700 / ARO(D) Project No. 4332
Contract No. DA-31-124-ARO(D)-138



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Purdue Research Foundation

Lafayette, Indiana

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Work reported herein formed the basis of a Ph. D. thesis in Mechanical Engineering at Purdue University.

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ABSTRACT

There is a definite engineering need for the capability of designing an efficient flow expansion. Although the literature on flows in adverse pressure gradients is extensive, no reliable method exists for predicting conical diffuser performance. A complete literature survey has been included as an appendix to this report; it includes a summary of all previously reported conical diffuser data.

The data reported in the literature were not sufficient to establish reliable design criteria for conical diffusers. Experiments have been performed using water flow in clear plastic diffusers, discharging into a plenum tank, in order to obtain performance and flow regime information for a range of diffuser geometries.

The principal conclusions of the study were:

1. A line of first appreciable stall has been located with sufficient precision that it is useful in conical diffuser design.
2. Performance data have been obtained to allow solution of the four common optimum problems encountered in diffuser design. At present, these solutions can be extrapolated only with caution to other diffuser geometries.
3. The optimums located for conical diffusers are not the same as those suggested by data from tests of plane walled diffusers.

INTRODUCTION

There is a definite engineering need for the capability of designing an efficient flow expansion. Many applications of diffusers are found in practical fluid systems where a need exists to efficiently decrease the kinetic energy of the flow. In automotive and aircraft air handling systems, inlets must be followed by diffusers to decelerate the flow and to provide a uniform stream for the next component. Axial flow and centrifugal compressors are examples of fluid machinery where energy is added to the fluid by increasing its velocity. To recover the maximum amount of this energy it is necessary to decelerate the fluid efficiently.

An adverse pressure gradient must be present in a decelerating flow. The effect of the increasing pressure in the streamwise direction may be to cause separation of low energy fluid near the diffuser walls. This stalled fluid forms eddy regions where recirculation occurs. These eddies cause the effective geometry of the flow passage to change, and they impose large energy losses. Consequently, it is important to reduce the extent of separation as much as possible.

Although flows in adverse pressure gradients have been the subject of a great deal of research, at present there is no reliable way to analytically predict either the behavior of a turbulent boundary layer in an adverse pressure gradient or the performance of a diffusing passage. For conical diffusers, attention has been focused largely on

investigating the effect of auxiliary flow variables on performance, and on auxiliary devices intended to overcome the apparent deficiencies of the diffuser. Such work has demonstrated the complexity of the problem, but it has not led to a general design criterion. There exists a definite need for a systematic research program on the fundamental problems, and the need for reliable data over a wide range of geometries and flow conditions is acute. In addition, the complexity of the problem seems to require more information on the nature of the mechanism of stall in a diffusing passage. Flow visualization techniques can be used to advantage to help illuminate this facet of the problem. The use of visualization will also simplify the problem of data acquisition by clarifying the flow regimes present and by indicating where additional data might be needed.

The extensive literature indicates that the flow regime in a diffuser seems to be chiefly a function of diffuser geometry, while performance appears to depend on other parameters as well. A starting point for an experimental program to explore the overall behavior of a conical diffuser is to hold all variables but geometry and Reynolds number constant.

Sufficient conical diffuser data to establish reliable design criteria are not available; furthermore, the available data sets are not totally consistent. To understand the basic processes which determine diffuser efficiency, additional performance data and flow regime information must be experimentally acquired. In addition to extending currently available information, systematic data for various geometries taken under controlled conditions will be significant in

other respects. These data will provide a firm basis for conical diffuser design, and they will be of use in the development of semi-empirical methods of predicting boundary layer behavior. These data will aid in obtaining correlation among the results of plane walled, annular, and conical diffuser configurations; they will provide a fixed reference for the investigation of the effects of significant non-geometric variables on diffuser performance.

To simplify the problem of flow visualization, water was used as the flowing medium. Diffusers fabricated from clear plastic allowed direct optical visualization of the flow. Static pressure taps and total head traverses were used to measure quantitative data; qualitative flow regime data were noted. Dye injection through hypodermic probes and the wall static pressure taps was used as a flow visualization technique during the study of performance. Inlet flow conditions were controlled as closely as possible to obtain a low turbulence, uniform inlet flow. Although diffuser performance was known to be a function of several variables, the diffuser geometry and the inlet Reynolds number were the only variables considered in the present study. A wide enough range of geometries was covered to completely specify the maps of performance and flow regime.

NOMENCLATURE

A	cross sectional area of channel
AR	area ratio
C_{PR}	pressure recovery coefficient, $\frac{p_2 - p_1}{\frac{1}{2} \rho \bar{U}_1^2}$ (one-dimensional)
C_{PRi}	ideal (frictionless) pressure recovery coefficient, $1 - \frac{1}{AR^2}$ (one dimensional)
D	diameter of circular diffuser
$\mathcal{D}$	Darcy parameter, (reference 81)
H	boundary layer form parameter
L	length of diffuser wall
N	length along diffuser centerline
p	static pressure
P	total pressure
r	radial coordinate
R	local radius of conical diffuser
Re	Reynolds number at diffuser inlet, $\frac{\bar{U}_1 D_1}{\nu}$
U	velocity outside the boundary layer
$\bar{U}$	mean velocity at a section
w	local width of plane walled diffuser
x	axial length coordinate
y	distance perpendicular to wall
α	kinetic energy coefficient, $\frac{1}{A} \int_0^A \left(\frac{u}{\bar{U}}\right)^3 dA$

β	momentum flux coefficient, $\frac{1}{A} \int_0^\delta \left(\frac{u}{U}\right)^2 dA$
γ	ratio of velocities, $\frac{u_\theta}{U}$
δ	disturbance thickness of boundary layer, $\frac{u}{U} = 0.99$ at $y = \delta$
δ^*	displacement thickness of boundary layer, $\int_0^\delta \left(1 - \frac{u}{U}\right) dy$
η	diffuser efficiency as commonly defined, $\frac{C_{PR}}{C_{PRi}}$
η_{ij}^t	diffuser efficiency (definitions for $i = 1, 2, \dots, 7$; $j = 1, 2$, are given in Appendix A)
θ	momentum thickness of boundary layer, $\int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$
λ_{ij}	diffuser loss coefficient (definitions for $i = 1, 2, \dots, 8$; $j = 3, 4$, are given in Appendix A)
ρ	density of fluid
τ	shear stress
ϕ	diffuser half angle
ω	angular velocity

Subscripts

e	equivalent
i	ideal
w	wall
0	plane of entrance of flow channel
1	diffuser entrance plane
2	diffuser exit plane
3	downstream or tailpipe exit plane

LITERATURE SURVEY

A previous literature survey and analysis [52]* of prediction methods for turbulent boundary layers has shown that none of these methods gives completely satisfactory results when applied to calculating the performance of a plane walled diffuser. The limited results obtained for the conical diffuser case indicated that the same conclusion could also be drawn for the latter geometry.

The conventional semi-empirical prediction methods were derived from collections of experimental data for boundary layers in adverse pressure gradients. No data for separated boundary layers have been included in the derivation of any method. Virtually all of the methods employ an integrated form of the boundary layer momentum equation which cannot be applied without modification to separated flows. At present there is no reason to believe that such prediction methods will work for diffusers in which there is separation.

The comprehensive investigations carried out at Stanford University by Kline and his co-workers (summarized in [98]) have shown that for plane walled diffusers, optimum performance is obtained only with geometries in which there is already some separation present. This fact also explains, to some extent, the limited success of empirical methods in predicting the performance of such diffusers.

* Numbers in square brackets, thus, [52], will be used to denote items in the List of References.

That flow visualization is invaluable in exploring areas where knowledge of basic flow behavior is incomplete was demonstrated clearly once again by Kline. Although a number of flow regimes had been predicted and/or experimentally determined before, the correlation between flow regime and performance information had not been set forth. The results of the performance and visual studies for two-dimensional plane walled diffusers are shown in Fig. 1 (taken from [43] and [17]).

As shown in Fig. 1, the line of optimum pressure recovery at constant N/W_1^* is always located above the line of first appreciable stall at constant N/W_1 . This indicates that there is some stall present in diffusers with optimum performance characteristics (only those configurations below line a-a are stall free). Even though variations in other parameters of the diffuser flow, such as inlet boundary layer thickness or turbulence intensity, may cause the absolute values of performance to shift, line α - α always remains above line a-a.

In Fig. 2 the same lines have been cross plotted on the coordinates $(AR - 1)$ and (N/W_1) . These coordinates will be of use in later comparisons with other experimental data. Although the curves presented in Figs. 1 and 2 are not a complete summary of the results of the plane walled diffuser research, they are adequate for the comparisons to be made.

On the basis of the very limited conical diffuser performance data available in 1959, it was suggested that performance results for

* To allow direct comparison with the results of this study, the centerline length N has been used rather than the wall length L . For moderate diffuser angles N and L are essentially equal.

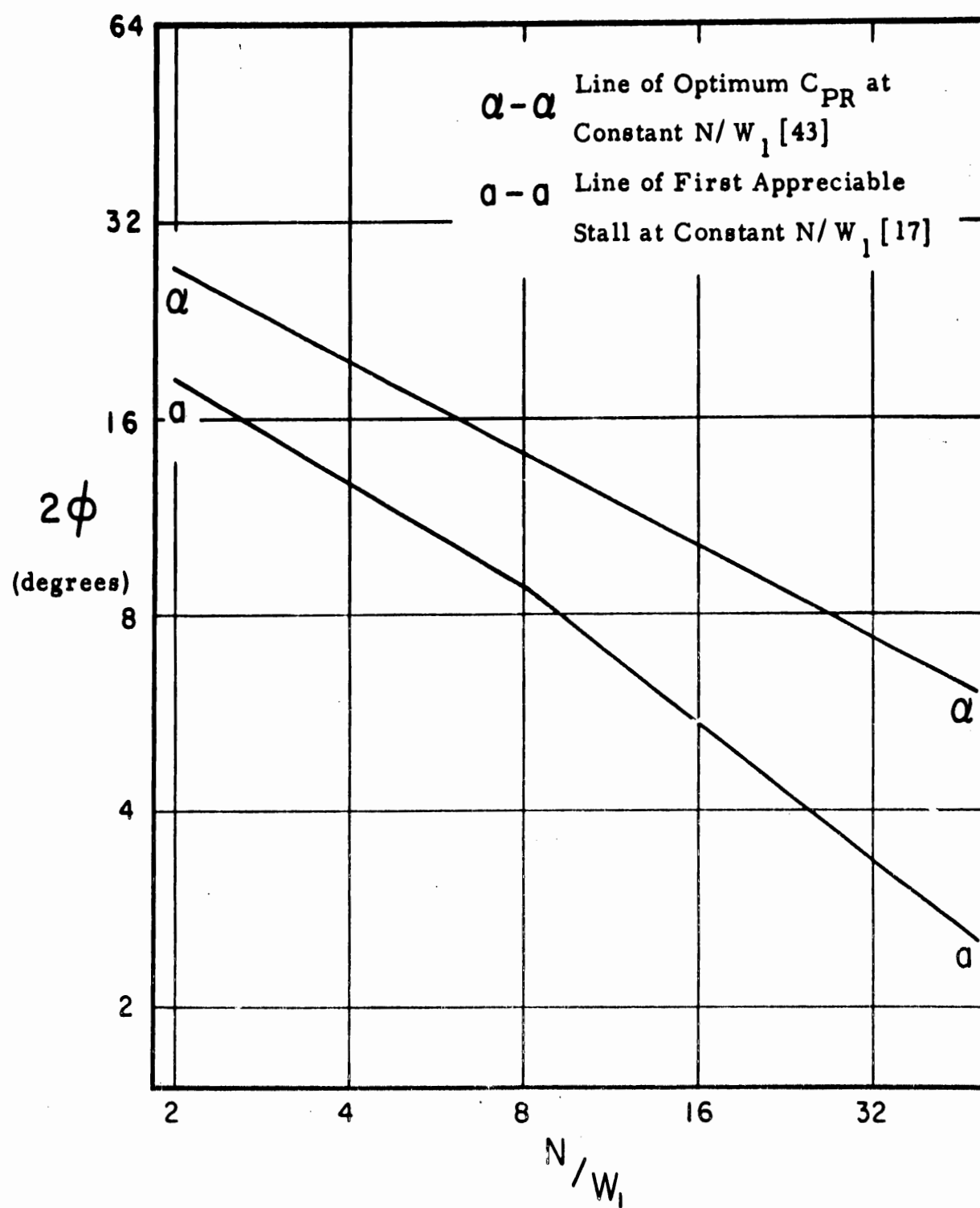


Figure 1. Lines of Optimum C_{PR} and First Appreciable Stall
 (at Constant N/W_1) for Plane Walled Diffusers,
 Plotted on 2ϕ vs. N/W_1 Coordinates.

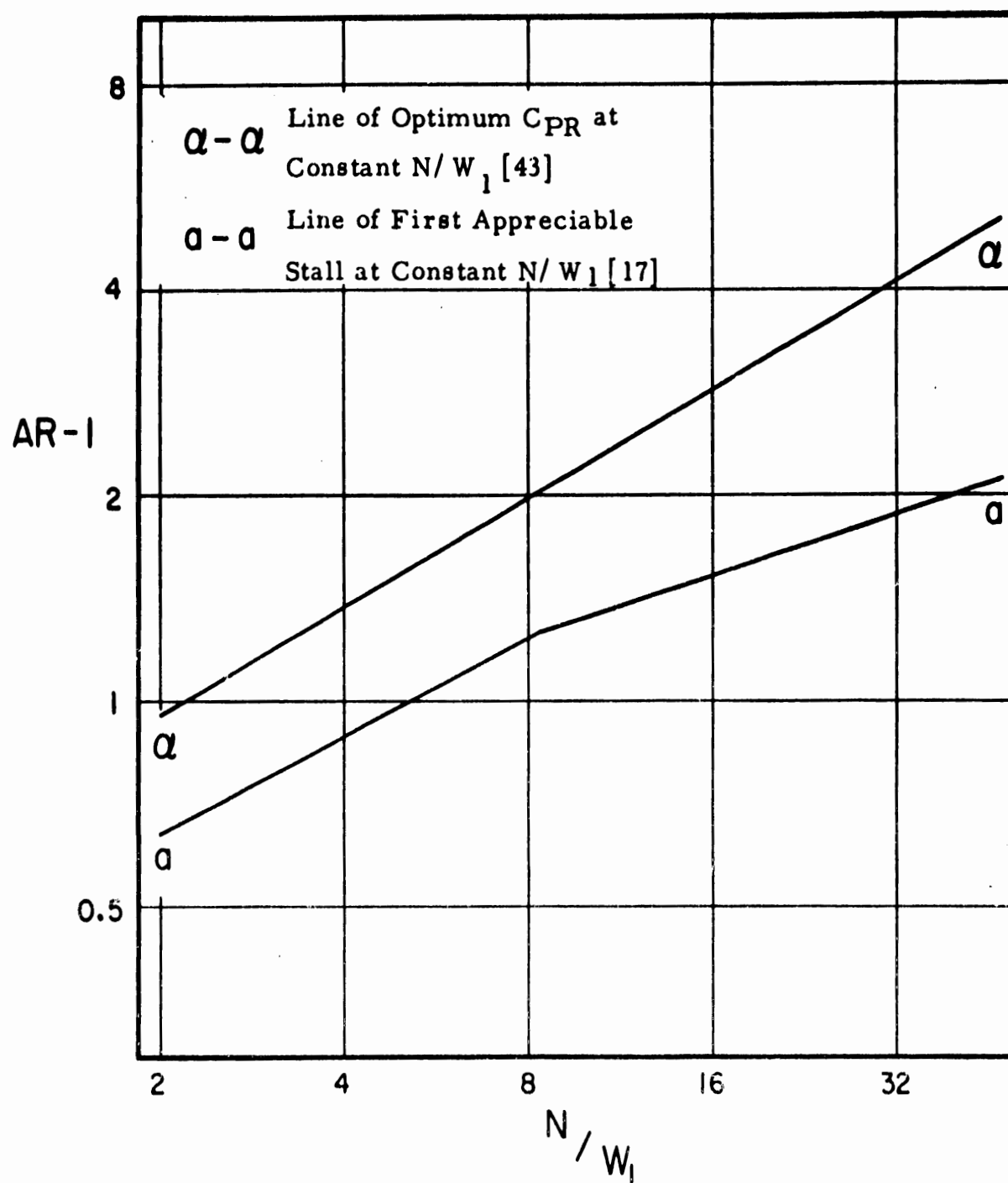


Figure 2. Lines of Optimum C_{PR} and First Appreciable Stall (at Constant N/W_1) for Plane Walled Diffusers, Plotted on $(AR - 1)$ vs. N/W_1 Coordinates.

axisymmetric flows could be correlated on the same basis as the results from plane walled diffusers [43]. The suggested correlation was to transform geometric variables as

Plane walled	→	Conical
2ϕ	→	2ϕ
N/W_1	→	N/R_1

If a correlation is assumed on the basis of the transformed variables, the resulting curves for optimum performance and first appreciable stall, when plotted with 2ϕ and N/R_1 as coordinates, are identical to those shown in Fig. 1 for the plane walled diffuser. However, Fig. 3 illustrates the result of using any coordinate containing the diffuser area ratio. If the diffuser angle and length ratio are maintained in the transformation, then conical and plane walled diffusers must have different area ratios. This is shown by the equations

$$AR)_{\text{plane}} = 1 + 2 \frac{N}{W_1} \tan\phi$$

$$AR)_{\text{conical}} = 1 + 2 \frac{N}{R_1} \tan\phi + \left(\frac{N}{R_1} \tan\phi\right)^2$$

The curves of Fig. 3 show that the differences between locations of the lines for optimum C_{PR} and first appreciable stall are significant. For both lines, the conical diffuser values lie above those for plane walled diffusers.

In addition to reviewing plane walled diffuser information, the voluminous literature on flow in axisymmetric diverging passages has been completely reviewed; most emphasis has been placed on

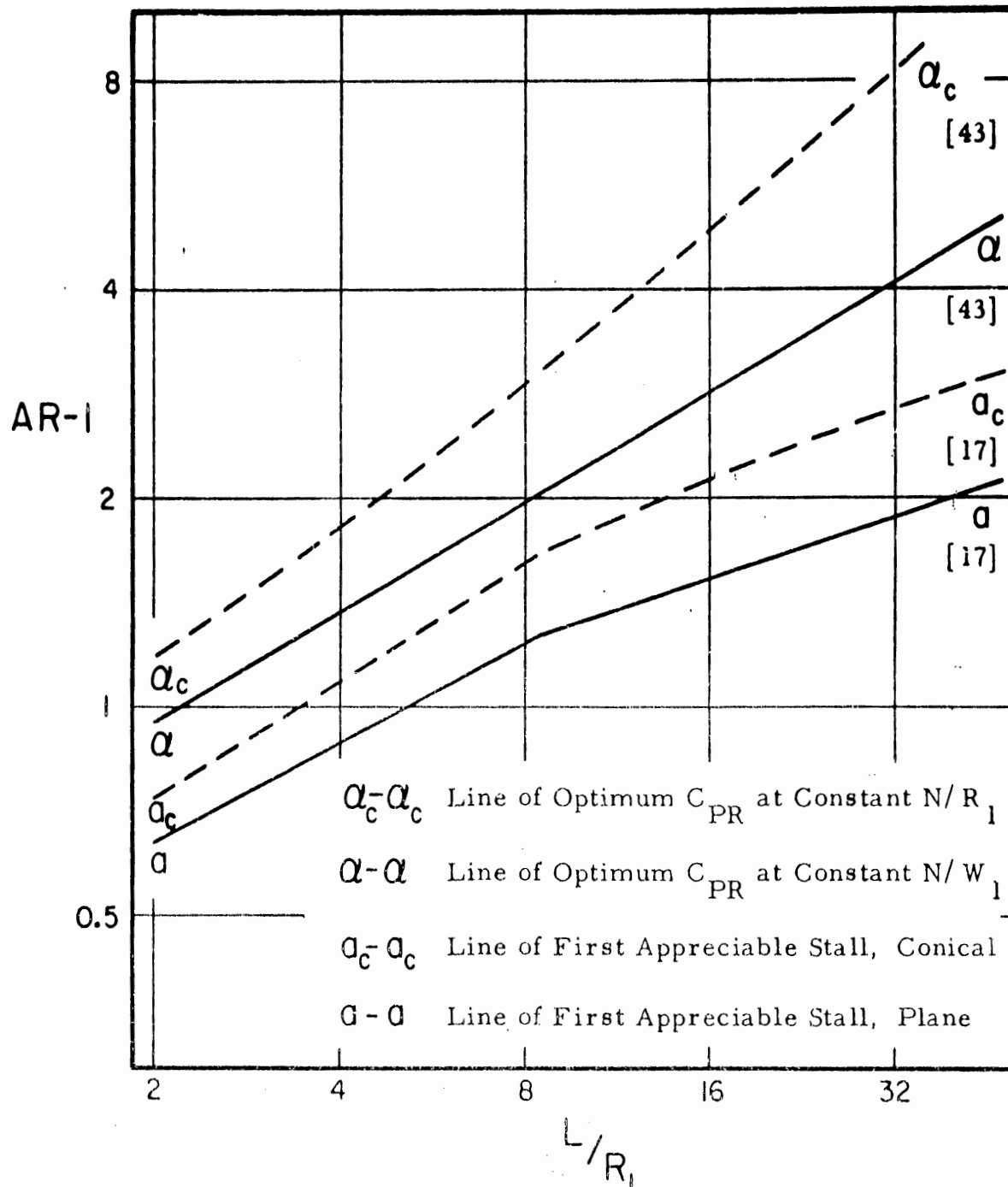


Figure 3. Lines of Optimum C_{PR} at Constant N/W_1 (N/R_1) and Lines of First Appreciable Stall for Plane Walled (Conical) Diffusers, Assuming the Correlation $2\phi_p = 2\phi_c$ and $N/W_1 = N/R_1$, Plotted on $(AR - 1)$ vs. N/W_1 Coordinates.

straight walled conical diffusers. A detailed summary of each source of experimental data is given in Appendix B. Some contributions dealing with prediction methods or flow mechanisms have been included where of significance to the present study. The basic emphasis was on reviewing experimental work, and the data reported by all known investigators of conical diffuser flows are tabulated in detail in Appendix C. Although a great deal of data on flows in diverging passages has been amassed in the last 110 years, very few of these investigations have been in any way systematic. That is, very few studies have been made in which wide ranges of variables were covered.

In the following sections the contributions will be reviewed as they apply to the problems of mapping diffuser performance and of predicting diffuser flow regimes. All data presented as maps have also been cross plotted for use in later comparisons. All performance parameters found in the literature are defined in Appendix A.

The first known performance map for conical diffusers was drawn by Eiffel [13], and is reproduced in Figs. 4 and 5. The curves shown in Figs. 4 and 5 have been recomputed and replotted from the map of losses, λ_{24}^* , given in the only available source of Eiffel's results [99]. The method by which the performance curves were obtained was not completely described. It was intimated that the results were obtained theoretically by accounting separately for the flow losses due to friction and to diffuser shape. The original work is unfortunately not available at this writing.

* See Appendix A for definitions of loss factors and efficiencies.

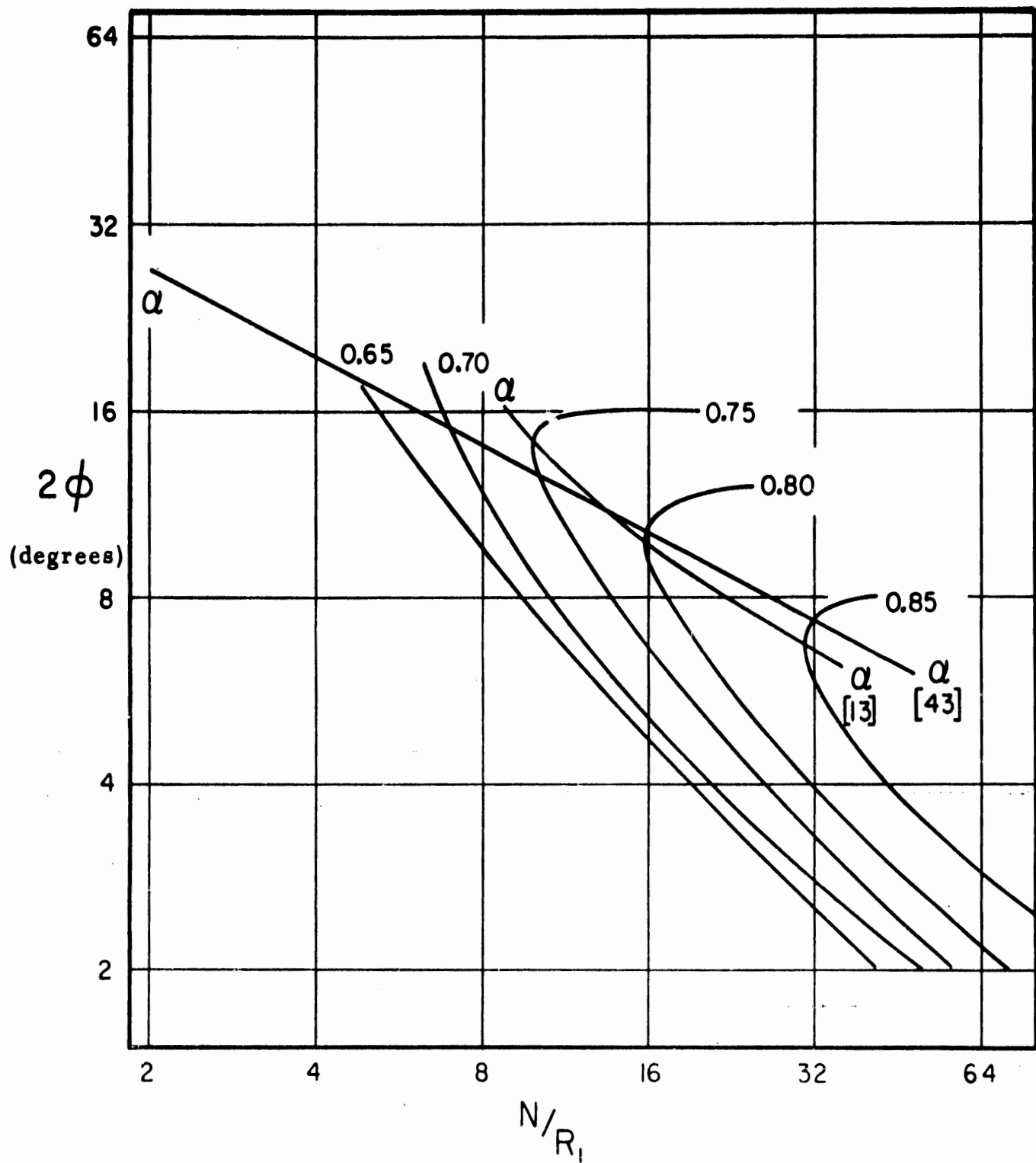


Figure 4. Performance Map of Eiffel (Replotted in Terms of C_{PR}), Compared to Line of Optimum C_{PR} (at Constant N/W_1), Plotted on 2ϕ vs. N/R_1 Coordinates.

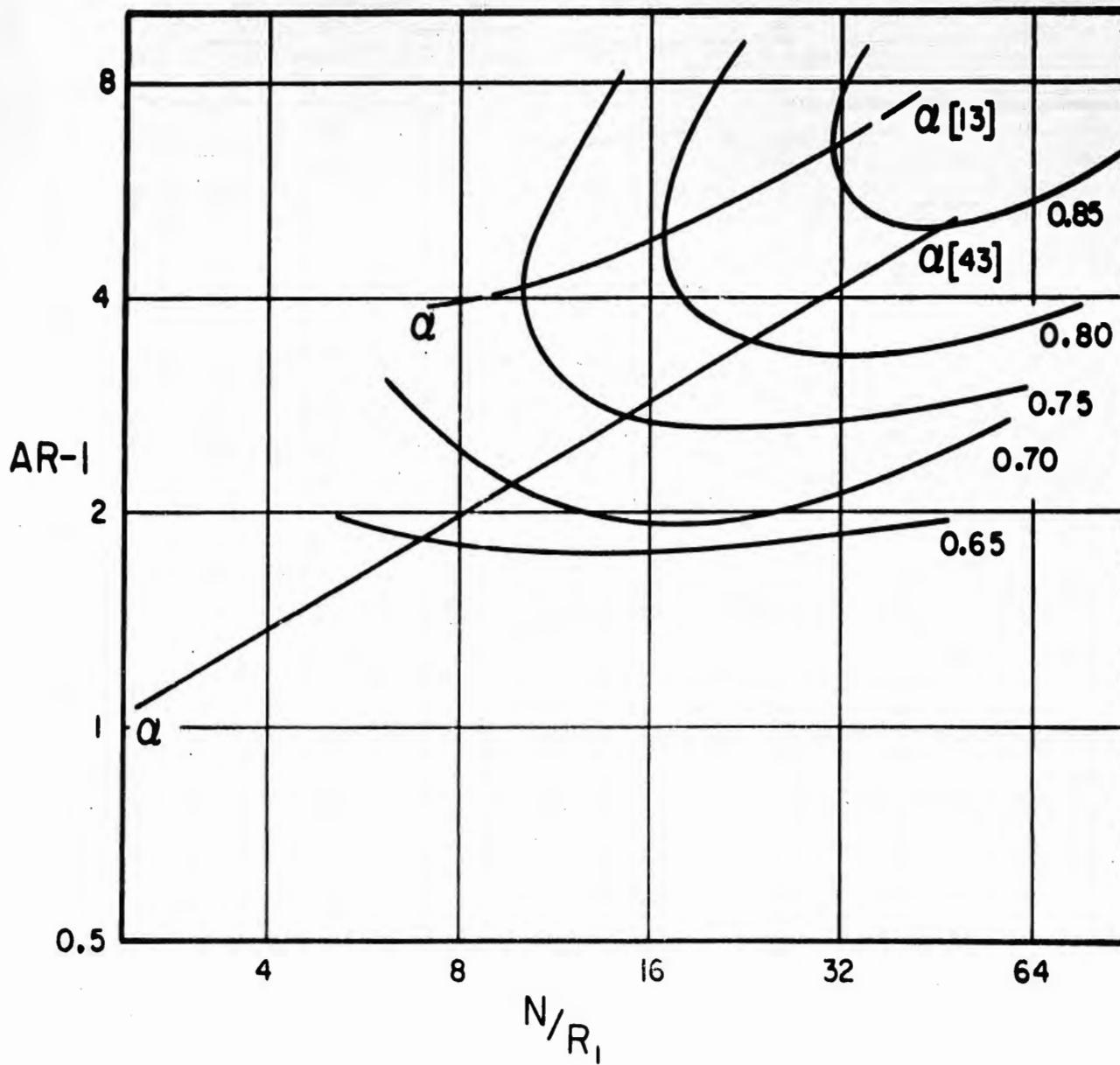


Figure 5. Performance Map of Eiffel (Replotted in Terms of C_{PR}), Compared to Line of Optimum C_{PR} (at Constant N/W_1), Plotted on $(AR - 1)$ vs. N/R_1 Coordinates.

Eiffel suggested that the best compromise diffuser for use in a wind tunnel was one with area ratio 9.0 and included cone angle 7 degrees. Although this geometry does not coincide exactly with optimums based on constant N/R_1 or AR, it falls between them. A line connecting the optimum performance values predicted by Eiffel for constant N/R_1 has been added to Fig. 4. It may be seen that the slope of this line is different from the line of optimums for plane walled diffusers, and also that it lies in a region of higher area ratio than the latter line.

In Gibson's [23,24] classical investigations, diffusers of square, rectangular, and conical shapes were tested over a range of angles from 3 to 180 degrees, for area ratios of 2.25, 4.0 and 9.0. The performance maps of Figs. 6 and 7 have been prepared from the original data of Gibson. Points on the curves indicate the geometries investigated.

Although enough data were not given by Gibson to complete a performance map, a tentative line of optimums at constant N/R_1 has been drawn for comparison to the line from two dimensional work. In common with the line of optimums from Eiffel's work, that of Gibson is seen to have a higher slope than line $\alpha-\alpha$. It lies in nearly the same range of geometries as line $\alpha-\alpha$, but seems to be somewhat lower on the plot of 2ϕ vs. N/R_1 , Fig. 6.

The data of Peters [66] were compared with those of Gibson by Patterson [63]. Peters' investigations were made with an air flow system, while Gibson used water, but the exit conditions were similar for both investigations. Excellent agreement was found between the results for similar area ratios.

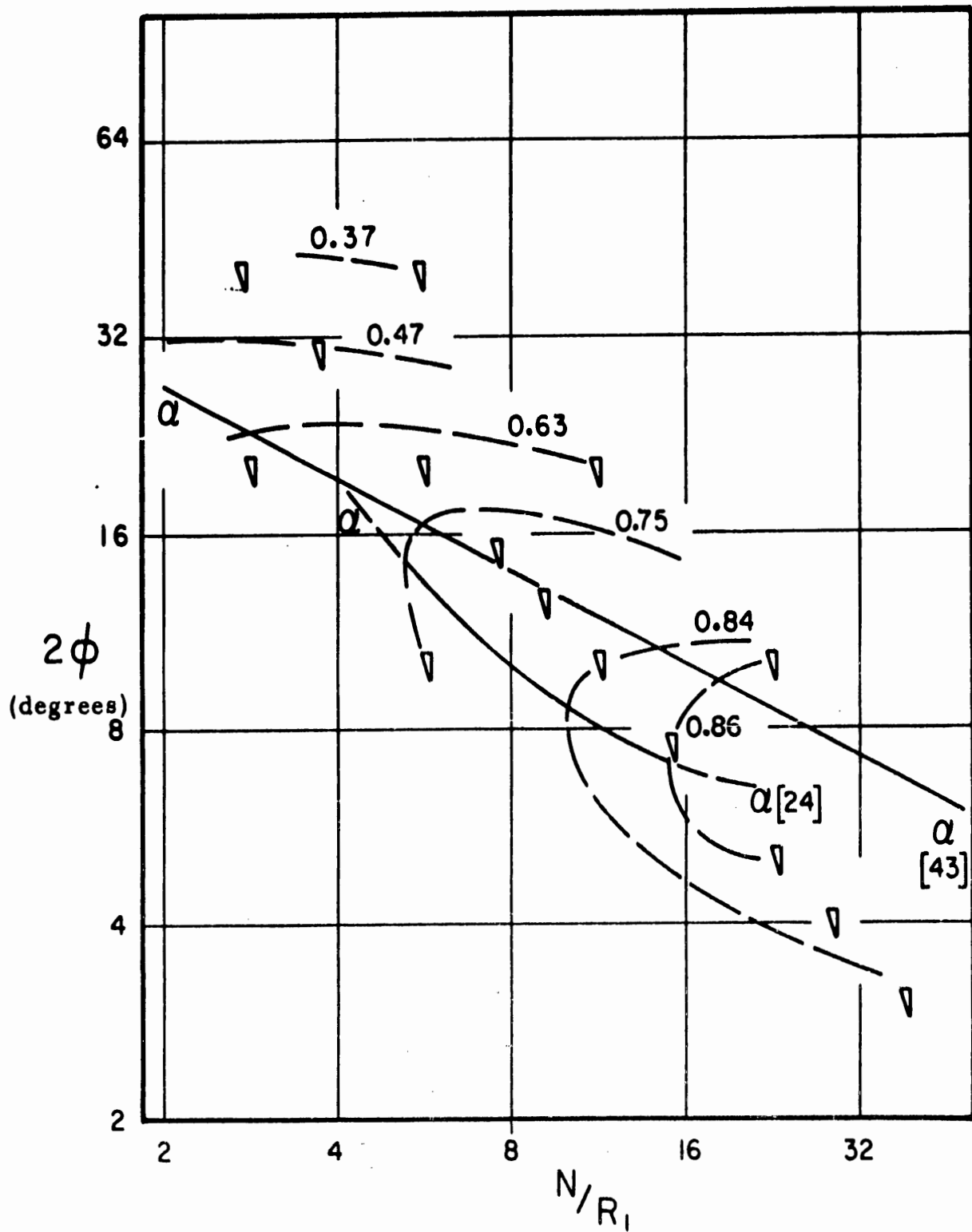


Figure 6. Map from Pressure Recovery Data of Gibson Compared to Line of Optimum C_{PR} (at Constant N/W_1), Plotted on 2ϕ vs. N/R_1 Coordinates.

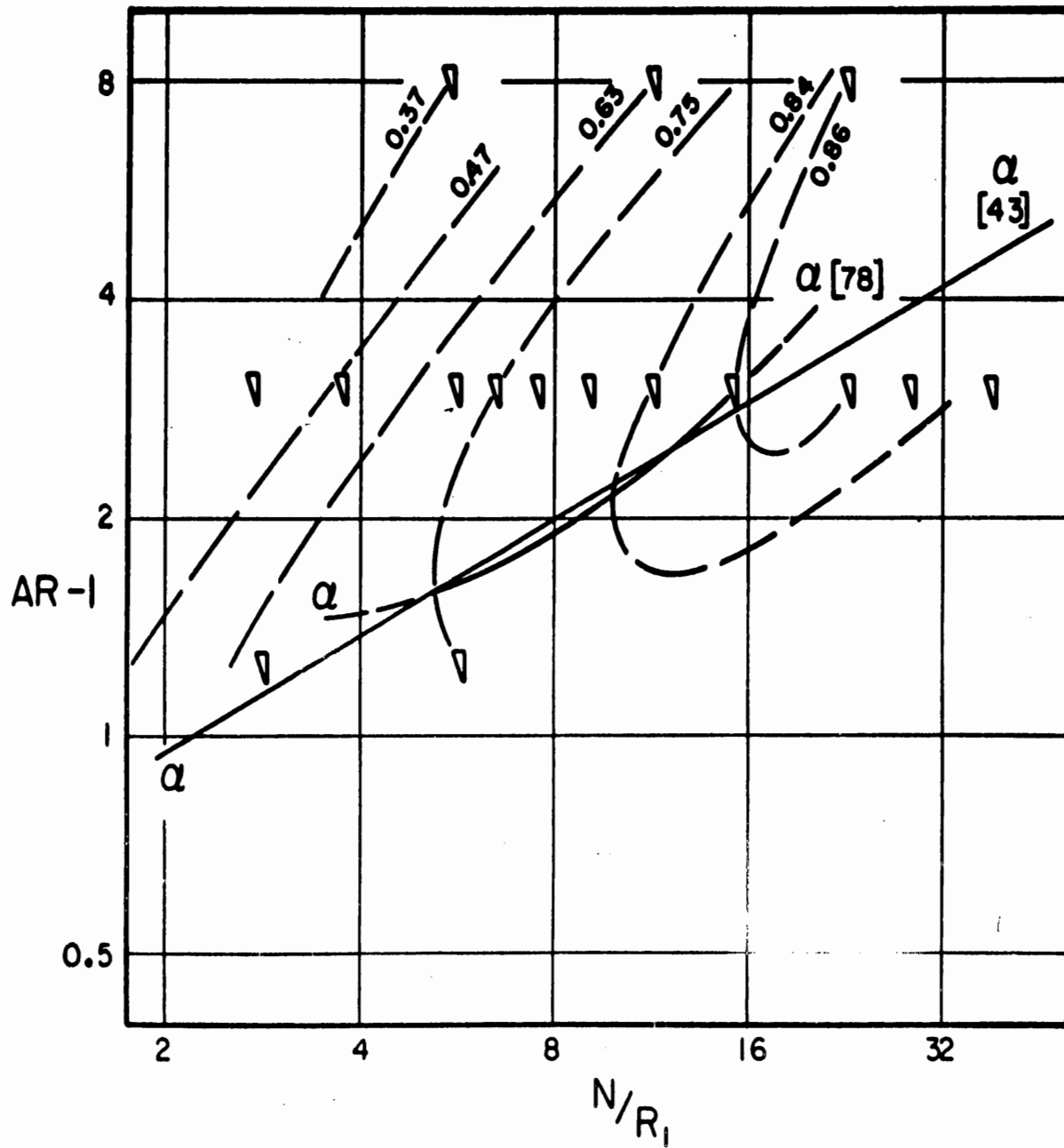


Figure 7. Map from Pressure Recovery Data of Gibson Compared to Line of Optimum C_{PR} (at Constant N/W_1), Plotted on $(AR - 1)$ vs. N/R_1 Coordinates.

The performance map given by Rippl [78] for slim conical diffusers has been replotted in Figs. 8 and 9. Rippl tested a number of diffusers but these included only three different values of divergence angle. Points on the curves indicate the geometries investigated. The performance map was originally presented on coordinates of area ratio vs. length ratio.

The line of optimum performance from Rippl's data is compared with the line from plane walled data in Fig. 8. It is seen that the slopes of the lines are approximately equal, but that the conical data tend to fall below the plane walled line of optimums.

Finally, a performance map was drawn from the performance data given by Cockrell & Markland [7], and it is presented on Figs. 10 and 11. Unfortunately, the tests made by Cockrell & Markland included no diffuser angles under 8 degrees. Optimum values for C_{PR} at constant N/R_1 could not be established from the shapes of the curves.

Line α - α from the plane walled results is included for comparison in Figs. 10 and 11. Although it is not possible to locate the values of optimum C_{PR} from the Cockrell & Markland work, it is possible to tell where they will not fall. The shapes of the curves indicate that the line will fall considerably below that from the plane walled tests. This trend was also indicated by the data of Rippl.

Among the numerous other experimental contributions to the literature there were not nearly enough systematic data to allow performance maps to be established. These data have therefore been presented as isolated C_{PR} values on a geometric map to show the areas of geometric concentration. See Figs. 12, 13 and 14.

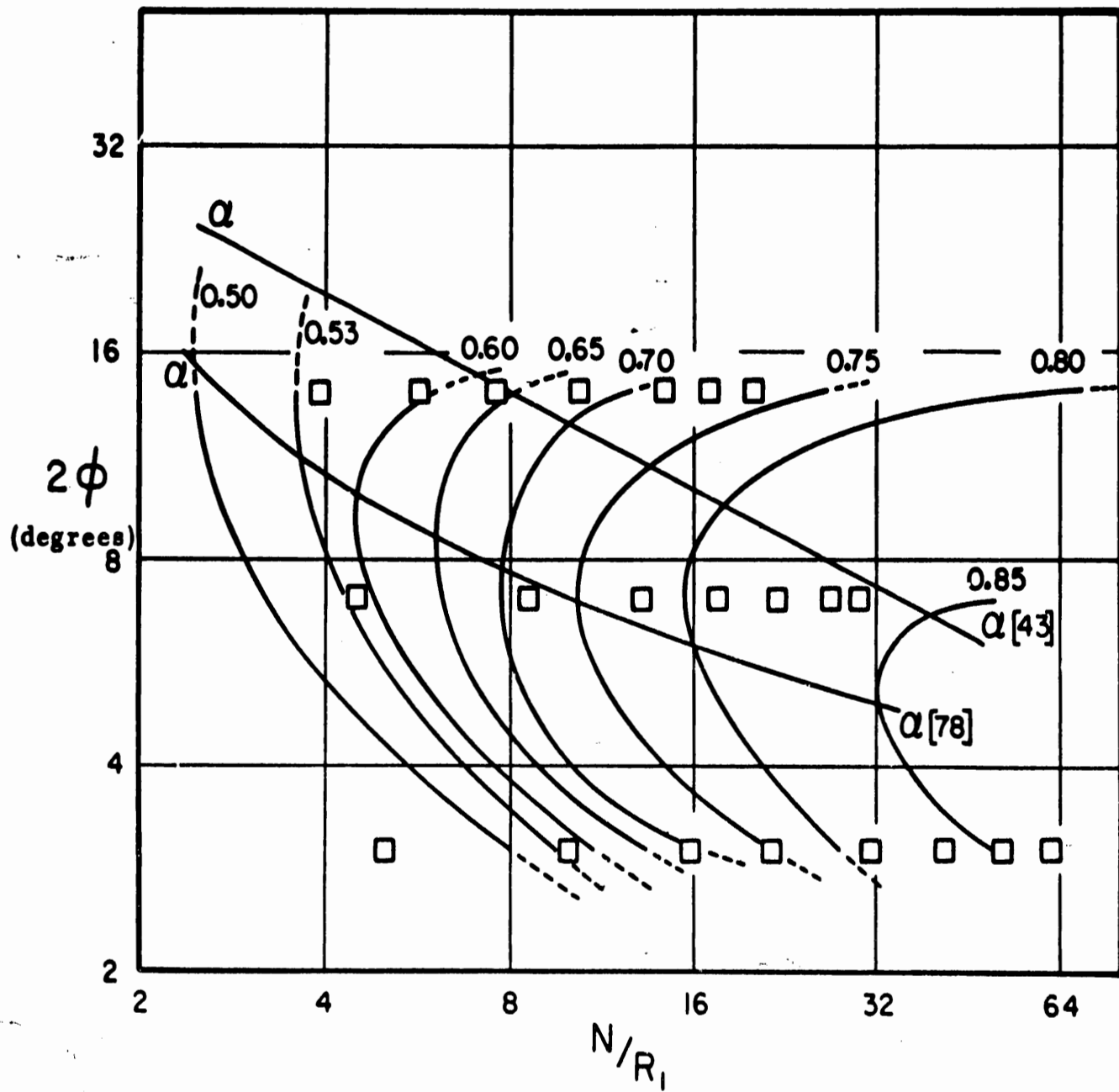


Figure 8. Performance Map of Rippl Compared to Line of Optimum C_{PR} (at Constant N/W_1), Replotted on 2ϕ vs. N/R_1 Coordinates.

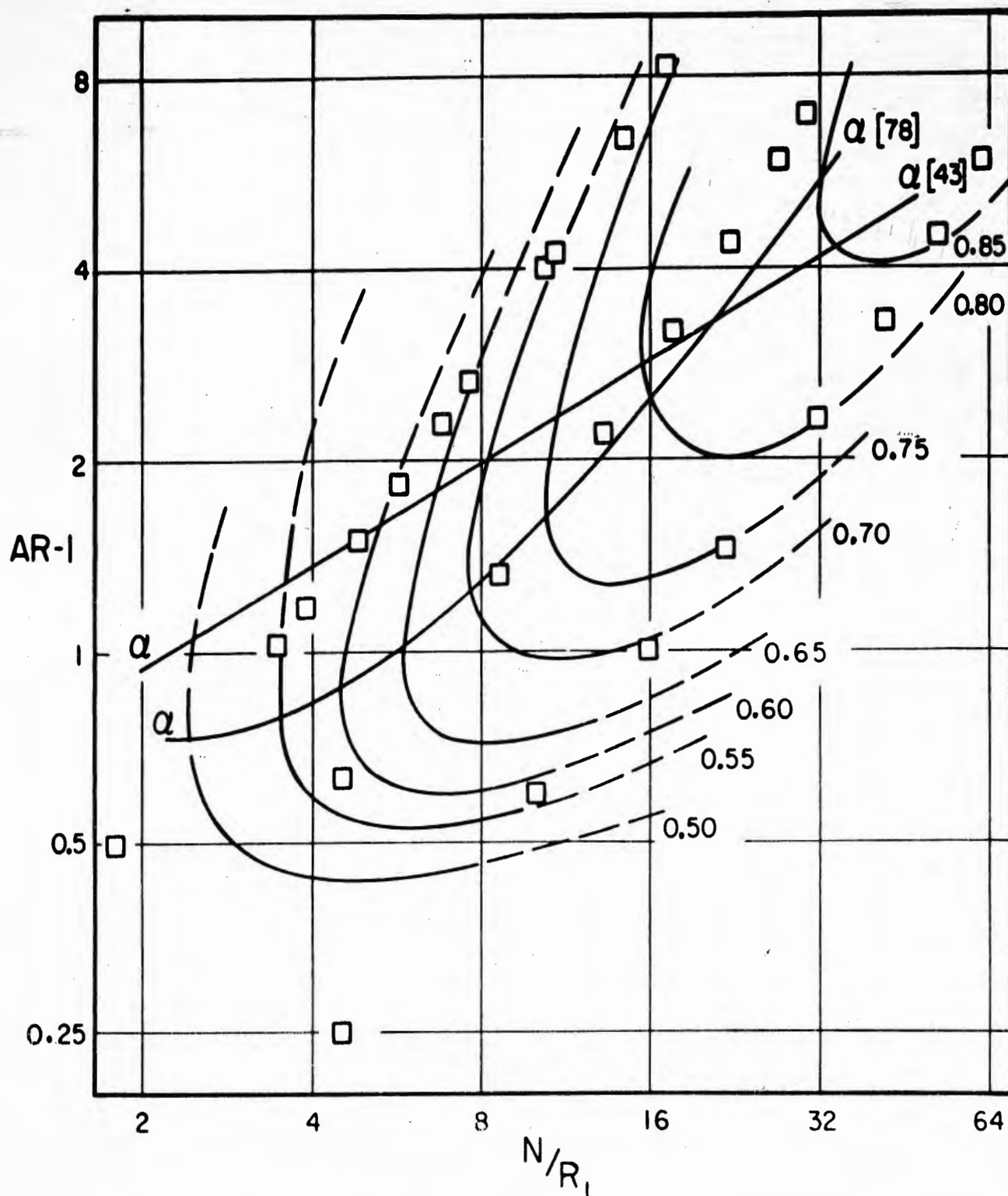


Figure 9. Performance Map of Rippl Replotted on $(AR - 1)$ vs. N/R_1 Coordinates, Comparing Lines of Optimum C_{PR} from Rippl Data and Plane Walled Diffuser Data.

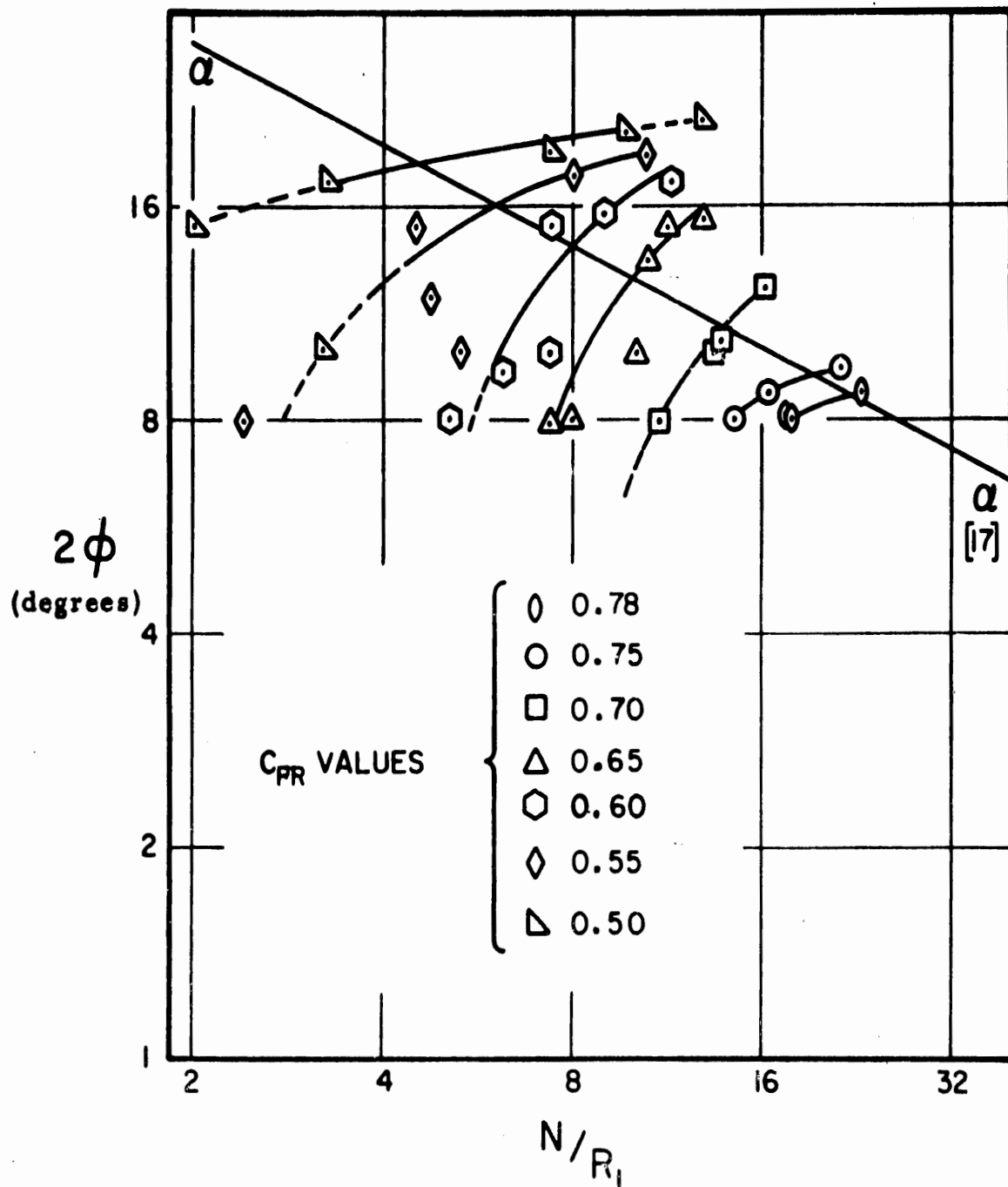


Figure 10. Map from Performance Data of Cockrell & Markland Compared to Line of Optimum C_{PR} (at Constant N/W_1), Plotted on 2ϕ vs. N/R_1 Coordinates.

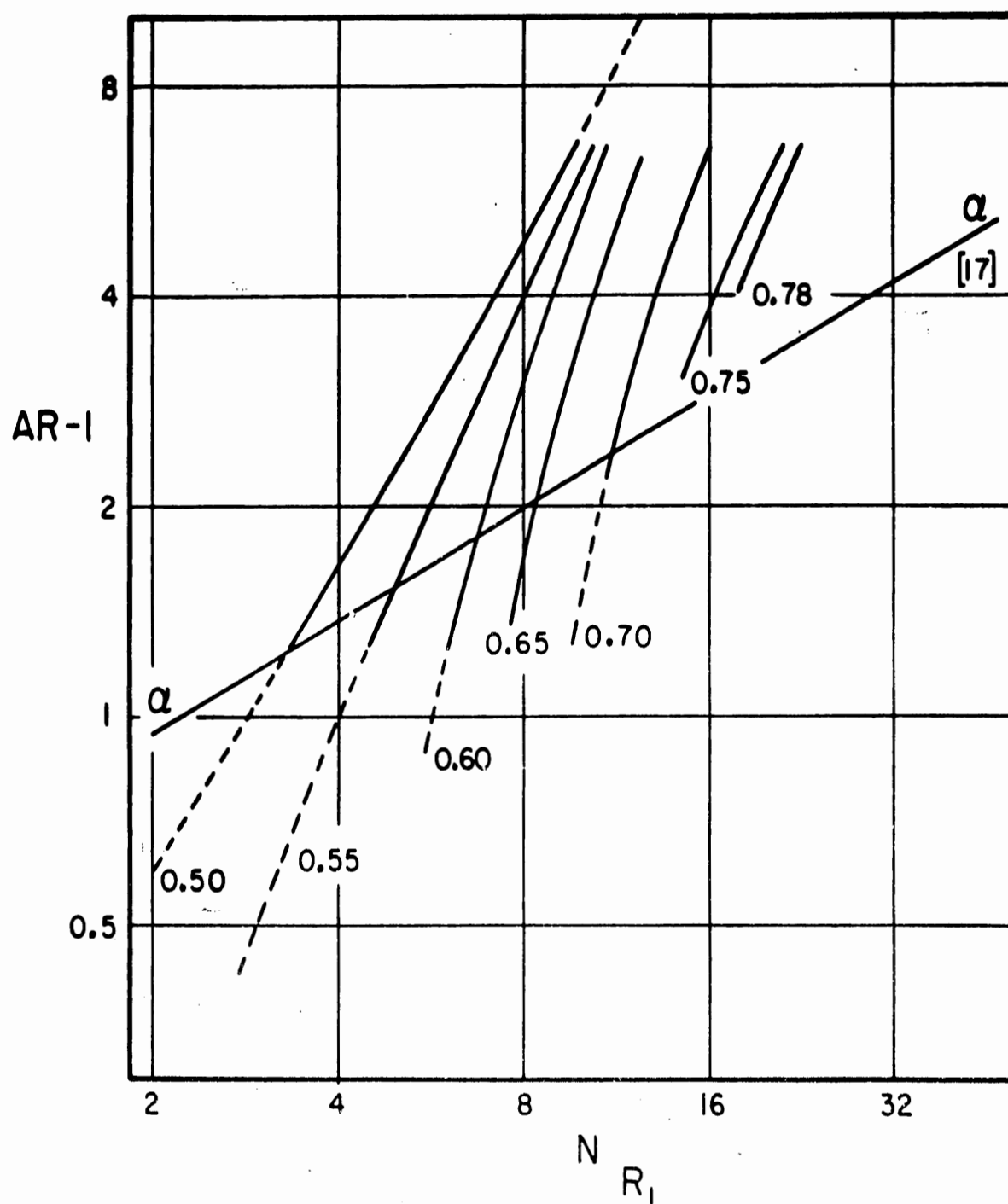


Figure 11. Performance Map Drawn from Data of Cockrell & Markland Compared to Line of Optimum C_{PR} (at Constant N/W_1), Plotted on $(AR - 1)$ vs. N/R_1 Coordinates.

Due to the number of datum points to be plotted, the display has been arbitrarily subdivided into three segments. On Fig. 12 are shown experiments for thin inlet boundary layer with free jet exit conditions; in Fig. 13 the inlet conditions were the same, but exit lengths were included in the flow systems. Fig. 14 displays points for thick inlet boundary layers for all exit conditions. For the purposes of these plots, "thin" inlet boundary layers were defined as those for which $\theta_1/R_1 < 0.025$.

The last three figures (12, 13 and 14) show clearly the lack of agreement among data of various investigators. Within any one set of data the trends show some consistency, but comparisons among investigators are not feasible. These facts have contributed materially to the confusion regarding diffuser performance. This further leads to the conclusion that other parameters besides diffuser geometry and inlet boundary layer conditions exert a definite influence on diffuser performance.

A conscientious attempt has been made in the literature survey for the present work to obtain a complete bibliography of all references reporting data from experiments on conical diffusers. It is hoped that collection of this material in one place will serve a useful purpose.

This review has emphasized that problems in comparing data from various sources are still present. The major difficulties are caused by incomplete reporting of data and by dissimilar geometric conditions under which data have been recorded. In addition, almost no information is available on performance as affected by such "auxiliary"

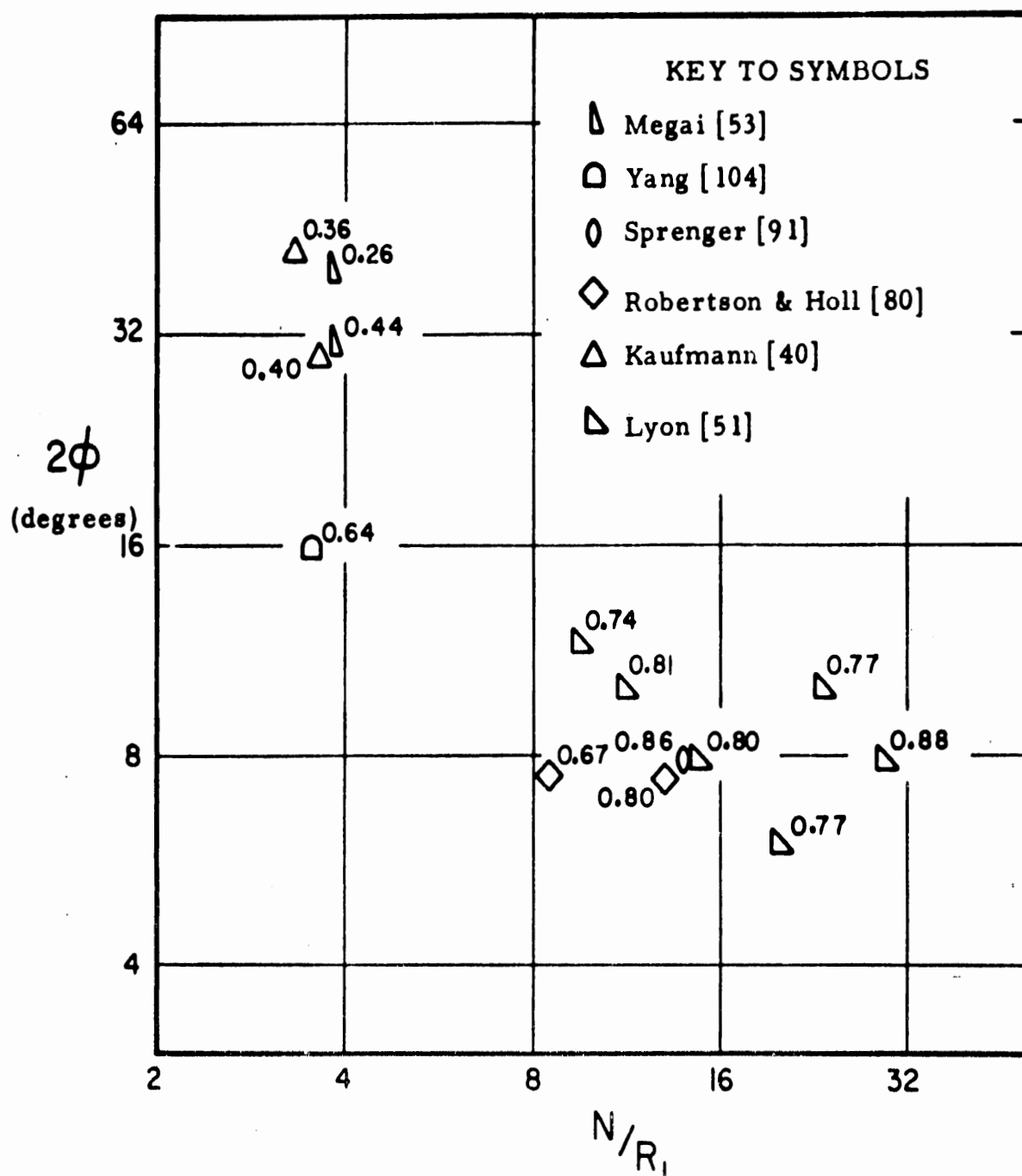


Figure 12. Miscellaneous Recovery Data for Conical Diffusers with Thin Inlet Boundary Layers and Free Jet Exit Flows.

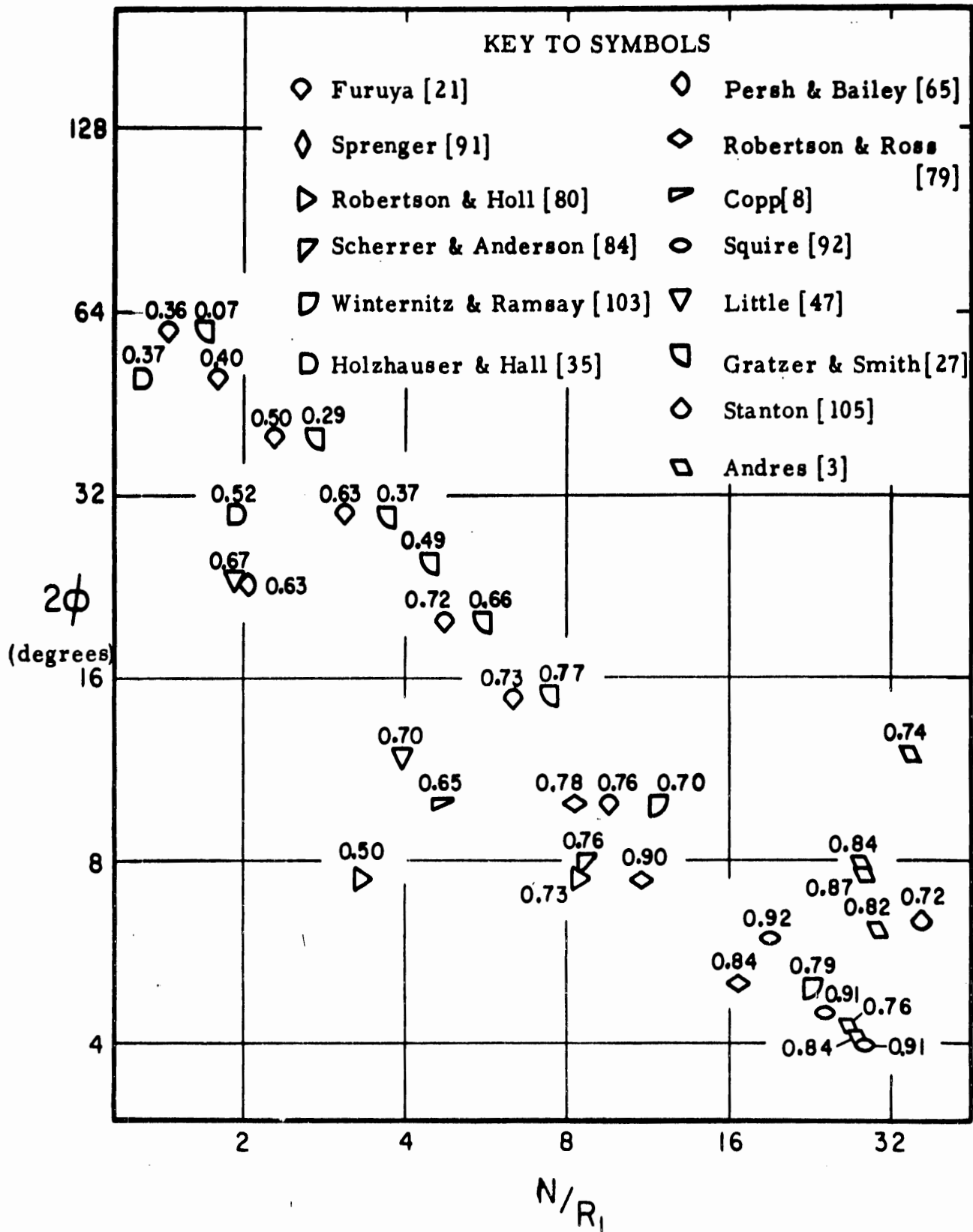


Figure 13. Miscellaneous Recovery Data for Conical Diffusers with Thin Inlet Boundary Layers and Exit Lengths.

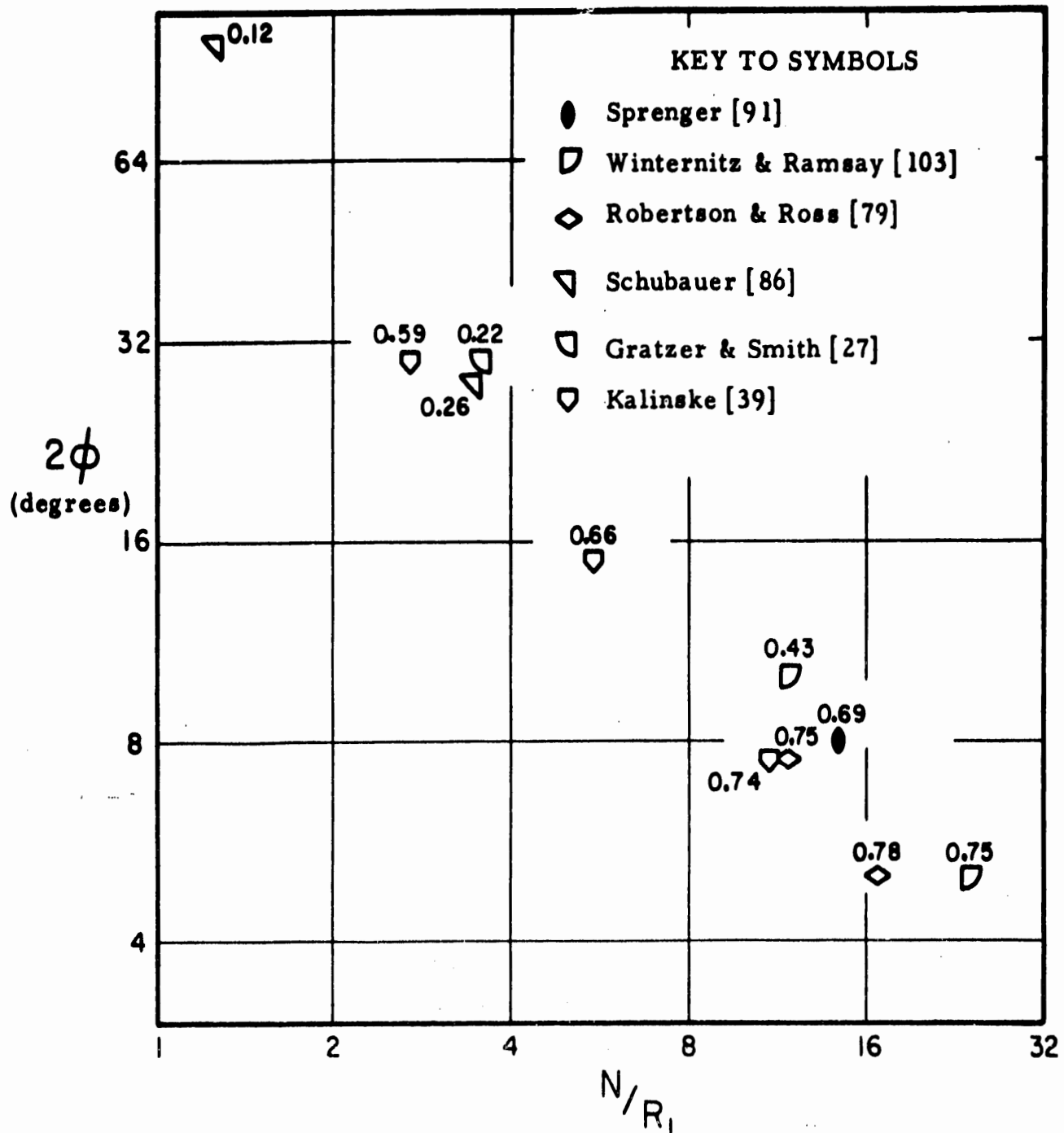


Figure 14. Miscellaneous Recovery Data for Conical Diffusers with Thick Inlet Boundary Layers and All Exit Flow Conditions.

variables as Reynolds number, turbulence intensity, Mach number and diffuser exit conditions. It is impossible to adequately correlate performance on the basis of presently available results.

In the past two years several important pieces of information have become available which have helped to make the task of correlating diffuser performance easier. A study of developing turbulent boundary layers in round tubes [15] has made it possible to estimate values of entrance momentum thickness for some previously reported diffuser flows, providing more information about these earlier investigations.

Recently Cockrell & Markland [7] correlated the effects of exit length on diffuser flow. Utilizing the data of Peters as well as their own, they evaluated the increased static pressure recovery which occurs in an exit length of pipe as it depends on inlet boundary layer parameters, diffuser angle and diffuser area ratio. No systematic data for a wide range of geometries are available comparing performance of diffusers with exit length and those without, but the results of Cockrell & Markland are a step toward helping to correlate some of the data already in the literature.

Several digests or reviews of existing knowledge on diffuser performance prediction have been made, including those of Patterson [63], Reid [75], Henry, Wood & Wilbur [29], and Cockrell & Markland [7]. The survey of Reid covered primarily plane walled diffusers, while the others covered conical units. Patterson did a very competent job of summarizing the state of the art as it stood in 1938, with the exception of his erroneous conclusions on the effects of swirling flow on diffuser

performance [75]. However, he was severely handicapped by lack of data to work with.

Although there have been numerous experimental investigations during the intervening years, there is still insufficient information available to establish completely the dependence of diffuser performance on geometry for any one set of auxiliary variables. The chief failing in most of the sets of data was the narrow range of geometry or other variables which was covered.

Hopefully this need for gathering and reporting more experimental data, while closely controlling the auxiliary variables, has been met by the present study. The experimental equipment for this study has been so designed that it can be used to further extend the ranges of all pertinent variables except Mach number. The data reported are sufficient to completely map conical diffuser performance for one set of auxiliary conditions.

EXPERIMENTAL RESULTS

Equipment Design and Construction

The review of the conical diffuser literature indicated the need for more experimental information. Flow regime data seemed as useful as performance data in mapping an overall experimental program. A water flow system and plexiglas test sections were constructed in order to obtain both flow regime and performance data for a wide range of diffuser geometries. Photographs of the overall flow system and test section are shown as Figs. 15 and 16.

Water flow offered several advantages in carrying out the experimental program. The most important of these was the relatively small system required to obtain a low turbulence flow. Also water made flow visualization simple and reduced instrumentation problems. Low velocities (1 to 10 feet per second) were necessary to cover the desired Reynolds number range from 10,000 to 100,000, based on mean inlet velocity and diffuser diameter. These low velocities were easy to measure using water as a flowing medium due to its large density and consequently large dynamic head for a given velocity.

Water Flow System

The system was designed with sufficient settling tank capacity to give a uniform, swirl-free inlet flow. In designing the system, the approximate design criteria for sizing components given by Rouse

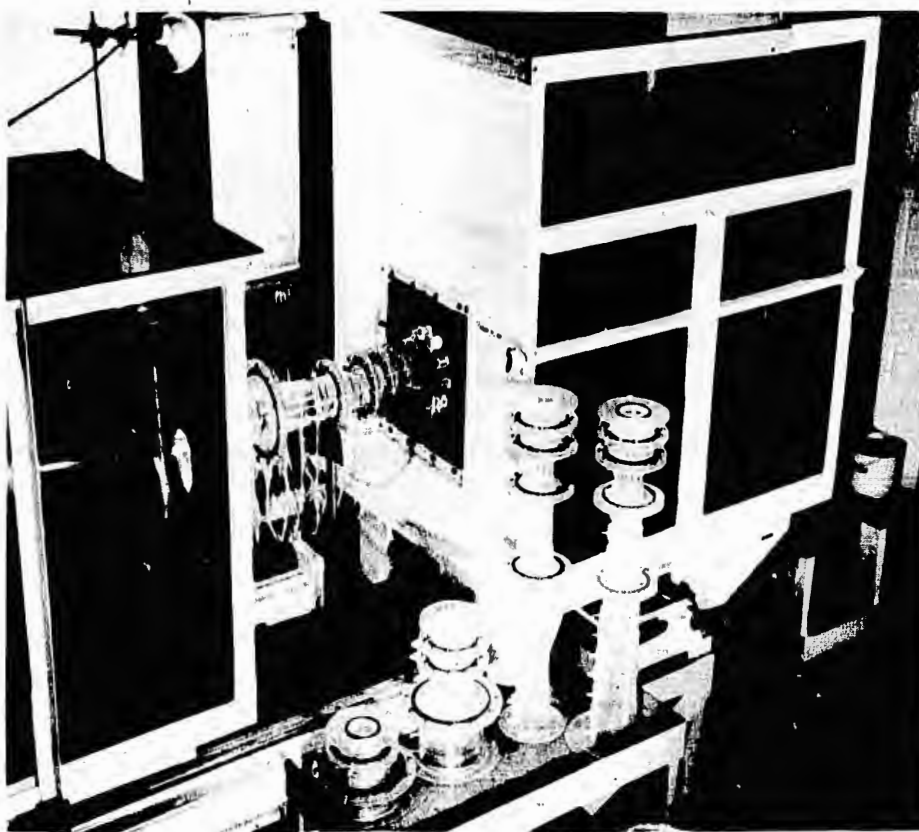


Figure 15. Overall View of Test System.



Figure 16. Closeup View of Test Section and Manometer Board.

[83] were very helpful. The general layout and overall dimensions of the apparatus are shown in the schematic flow diagram, Fig. 17.

The major design problem in building the water flow equipment was allowing for the variations of test section exit diameter and length. Several alternate schemes were considered, including one using a vertical flow direction, but they were rejected in favor of using a horizontal test section with a downstream plenum tank mounted on rollers so that any test section length could be accommodated. To simplify mating the test section exit with the downstream tank, the flanges used to join segments of the diffusers were made in only two sizes. The larger size flange mated directly with the window in the downstream constant head tank; the smaller flange required an adapter rim.

To obtain a uniform velocity at the entrance to the diffuser an inlet cone was designed according to the procedure given by Smith and Wang [90]. This design method was based on the exact analogy between magnetic fields and ideal fluid flow, and on numerical calculations which had been performed for the design of Helmholtz coils for uniform magnetic fields. Of the several possible configurations, the smallest one giving the best velocity distribution (velocity constant across the section within 0.2 percent) was used.

The water supply to the system was filtered using a commercial filter-softener to remove deposit forming impurities and foreign matter. Water from the University supply was piped into the overflow tank after passing through the filter. From there it was pumped to the flow system; the system capacity was about 750

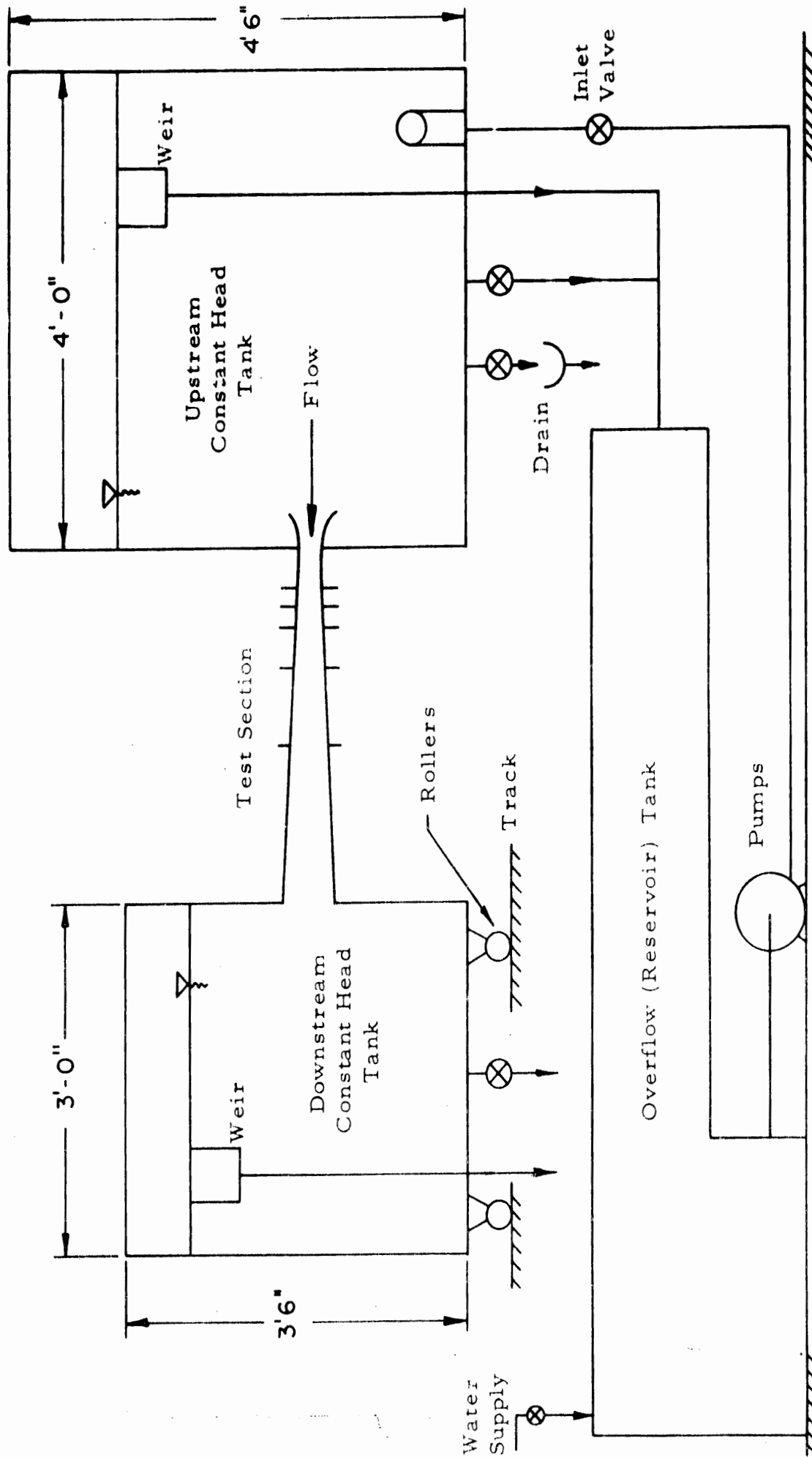


Figure 17. Water Flow System Schematic and Overall Dimensions.

gallons. The water in the flow system was recirculated with two pumps having capacities of about 25 and 80 gallons per minute. The discharge from the pumps was throttled by gate valves in the line leading to the upstream constant head tank. Water entered the tank through a tee leading to horizontal pieces of 4 inch diameter plastic pipe, which had been perforated to give a uniform distribution of inflowing water across the width of the tank. The success of this distributing manifold in reducing the velocity of the water was demonstrated when filling the tank for a test run. As soon as the water covered the manifold to a depth of about 2 inches, there were almost no perceptible ripples on the surface, indicating very little disturbance by the inflow.

The water level in each tank was controlled by an adjustable weir device, as shown in Fig. 17. The sizes of the weirs were determined to give less than 1 percent variation in total head across the test section when there was a total head difference of 6 inches between tanks. It was assumed in making the calculations that the overflow would amount to 10 percent of the flow rate in the test section. The weir sizes were determined on the basis that this 10 percent overflow fluctuated between 5 percent and 15 percent. At each weir a mechanical positioning device provided vertical location and adjusted total head level. Although no special pains were taken to seal the sliding tube (made from telescoping brass tubing) which carried the weir tank, the leakage from this source was very slight.

Provision was made for installing baffles and screens in the settling tank had they become necessary. Immediately after

the system was put into operation a very careful dye study was made in the settling tank to search for regions of swirl and for large eddies in the flow. None were found and the flow in the test section was satisfactory; consequently no baffles were installed.

Test Section Design

The design paper of Kline, Abbott & Fox [43], for straight walled plane diffusers, was used in selecting the range of test section geometries. In the design paper, the conical diffuser data then available were found to correlate well with the two dimensional data on the basis of 2ϕ and N/R_1 . As shown in Fig. 18, the line of optimum performance for plane walled diffusers was selected as the central line in the map of diffuser geometries to be tested in the present study.

As previously discussed in the Literature Survey, the line of optimum performance lies slightly above the line of demarcation between stalled and unstalled flow in a plane diffuser. In the absence of further information on conical diffusers, it was hoped that the geometries chosen to be tested would permit determination of the flow regimes found in axisymmetric diffuser flows.

Altering the geometry in tests of plane walled diffusers had been particularly simple. The diverging walls of the diffuser were mounted on pivots, and the divergence angle was then varied at constant length to entrance width ratio by rotating the side walls on their pivots. The length to width ratio was changed by altering the diffuser inlet width. The conical geometry did not allow such

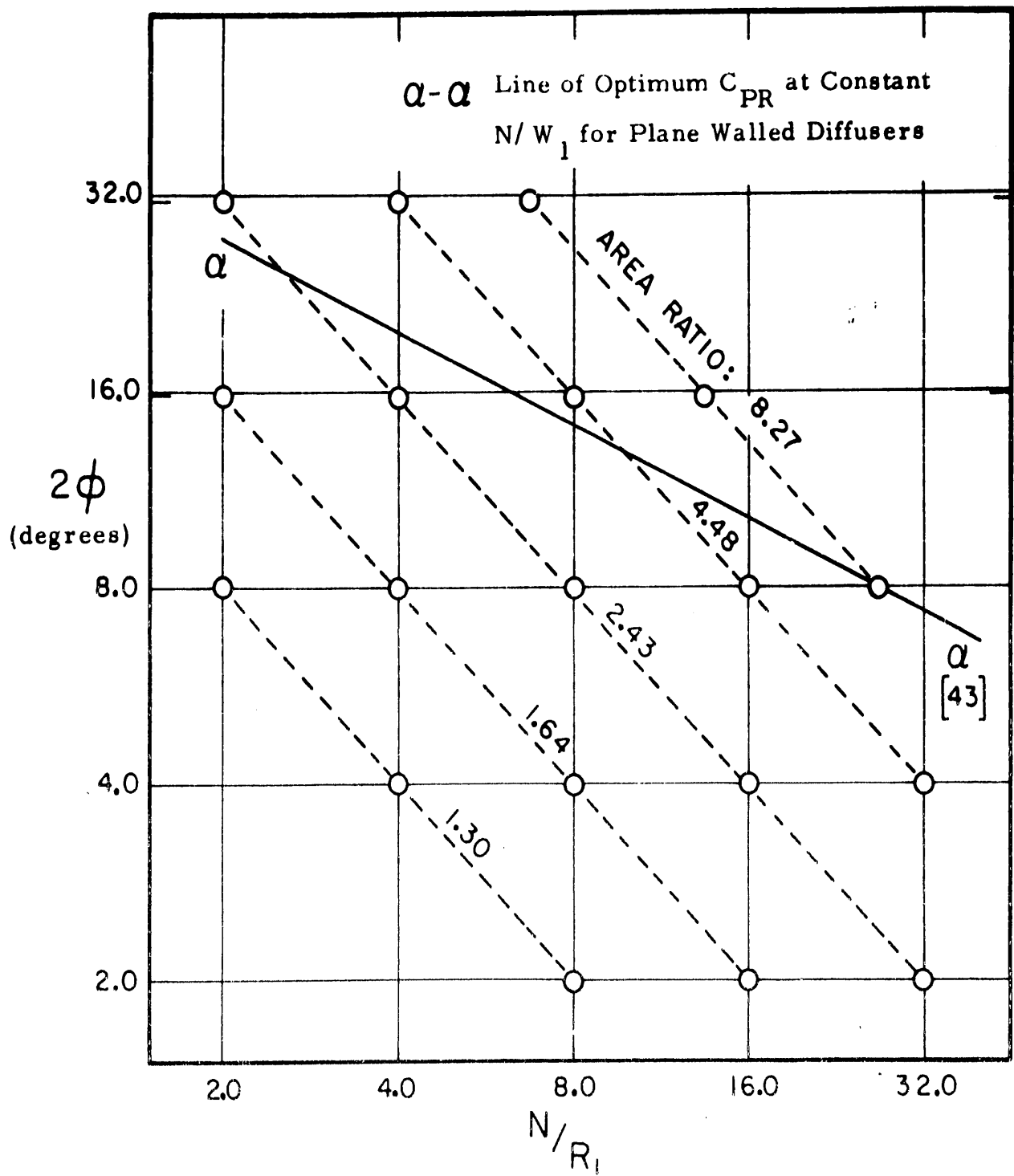


Figure 18. Conical Diffuser Geometries Selected for Study, Showing Relationship to Line of Optimum C_{PR} from Plane Walled Diffuser Data.

simple variation of parameters. Separate units had to be built for each angle tested. Five divergence angles were selected on the basis that they provided a minimum of three points along any line on which N/R_1 or AR was constant. Five different values of length for the longest diffuser were chosen on the same basis. A maximum internal diameter of about 5.75 inches was available in plexiglas tubing; a throat diameter of 2.00 inches was selected to give a range of area ratio from about 1.3 to about 8.3. As shown in Fig. 18, the maximum area ratio available limited the length of the three widest angle diffusers.

The 19 geometries selected for study covered a substantial range of all pertinent variables. Especially well bracketed were the range of area ratio around 3.0 and the range of angle near 7 degrees. These had been suggested for many years as rules of thumb for optimum design. N/R_1 and 2ϕ values were chosen for the test geometries to give equal increments when plotted on log-log coordinates.

The primary criterion in designing and building the test sections was transparency. Glass and plexiglas were the materials considered. Glass was rejected because it was generally difficult to work with and it became expensive when close tolerances were involved. Since the interior of the diffusers required a smoothly machined finish for any method of fabrication from plastic, it was decided to make the diffusers from plexiglas tubing of graduated sizes to reduce the amount of stock to be removed.

As mentioned, each different diffuser angle required that a complete unit be fabricated. To achieve the multiple values of N/R_1 which were required, each angle unit was built up in several sections along the axis. The diffuser sections were connected by flanged joints as shown in Fig. 19. As indicated in the figure, each joint did double duty. In addition to serving as a coupling and water seal between diffuser segments, the flanges were so designed that the inside surface of the downstream tank was completely free from projections when coupled to the flange. This achieved a free jet exit condition for the diffuser.

The insides of the diffuser segments were machined and polished in pairs to assure a uniform transition between sections. Dowel pins in the flanged joints made alignment of mating surfaces positive and preserved the original orientation of the sections.

To monitor the static pressure profile along the diffuser, a series of uniformly spaced (every 2 inches) static pressure taps was drilled into the diffuser wall. The design of the taps is shown in Fig. 20, with the Tygon tubing connection to the measuring device. The assembled diffusers were set up on a horizontal spindle milling machine to drill the pressure tap holes perpendicular to the inside diffuser wall. Indexing from the inside wall allowed the relief holes to be drilled to within 0.050 inch of the inside wall, giving a ratio of hole diameter to length of about 3.0 as recommended by Rayle [73]. A number 78 (0.016 inch diameter) drill was used for the pressure tap holes.

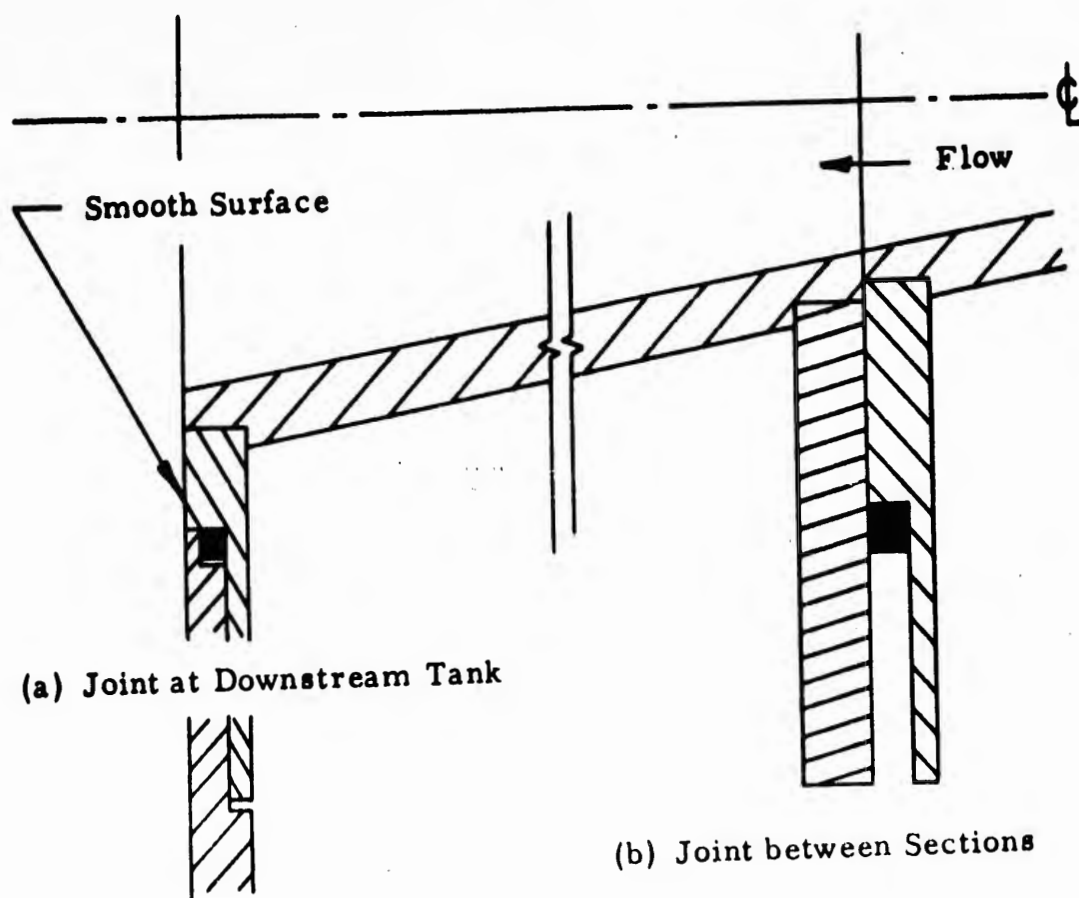


Figure 19. Detail of O-Ring Joints between Diffuser Sections.

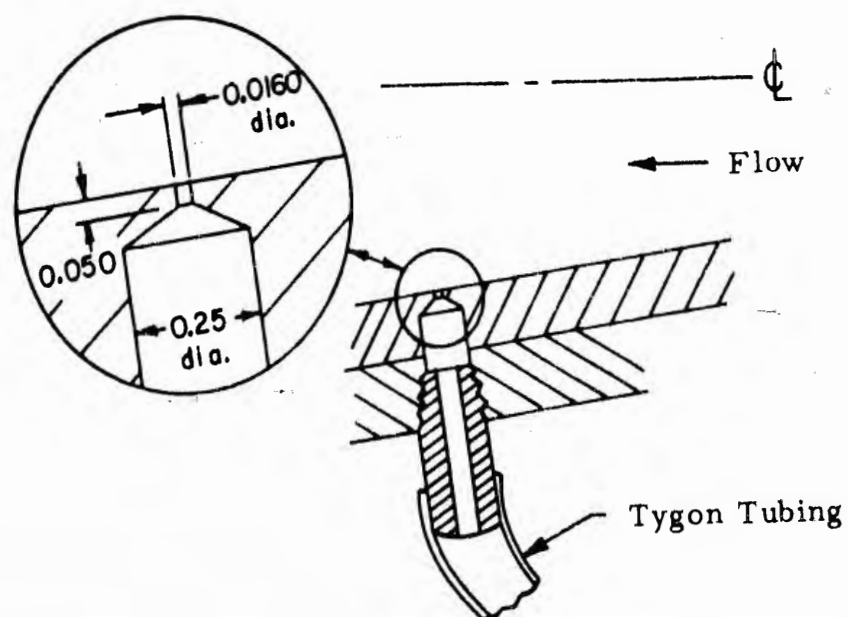


Figure 20. Detail of Static Pressure Tap in Wall of Test Section.

All pressure tap holes were carefully finished on the interior of the diffuser to remove all burrs. The holes were counter-sunk about one fourth to one half diameter using a drill which was several sizes larger than the hole. When test sections were installed in the flow system, pressure taps were located on the bottom of the section to facilitate removal of the last air bubbles in the pressure lines. Clear Tygon tubing was used for pressure lines so that bubbles were visible and could be worked out.

Instrumentation and Procedure

A water filled manometer bank was built for measuring the total and static pressures in the diffuser sections. An inclined tube manometer configuration had been considered to increase the precision of measurement, but this was not a satisfactory solution. Wide variations in total head made it extremely difficult to adapt the inclined manometer to the test system. From the minimum flow condition to the maximum flow condition, there was a total head variation of about two feet. To obtain precision by using a tube bank with a scale factor of 5 would require a manometer length of over 10 feet. The only alternative to such a length would have required that the whole manometer be raised and lowered. In addition, uniformity of the tubing is very important in an inclined tube device, but is not a factor in a vertical tube manometer.

It was much simpler to increase the precision of reading the manometer rather than increasing the precision of the manometer itself. Consequently the vertical tube manometer was used, but pressures were read using a cathetometer graduated in millimeters.

This instrument had a least count of 0.05 mm. The repeatability of measurements with the cathetometer was excellent after the lighting on the manometer board had been arranged to give a sharply defined meniscus. Readings could be duplicated within ± 0.025 mm of water. Other advantages of the vertical tube system became apparent when the testing was under way.

To determine the uniformity of the velocity profile at the diffuser entrance, total head traverses were taken with a 0.0625 inch diameter total head probe across two diameters 90 degrees apart in the short straight section upstream of the diffuser inlet. No change in total head was observed at any point in the stream using a water tube manometer with a least count of 0.05 inch, for a velocity head of about 15 inches of water. It was concluded that the flow could be considered uniform in the diffuser inlet.

While making the inlet velocity profile checks, the head loss in the contraction was found to be negligible. This meant that the dynamic pressure in the inlet section was given by the difference between the total pressure in the upstream head tank and the static pressure in the constant diameter inlet section. Using this value for dynamic pressure made it possible to eliminate the probe from the system while running a performance test. This eliminated any chance of interference from the wake of the probe.

Considerable fluctuation of pressure readings was observed in diffuser configurations with flow separation. Simultaneous readings could not be taken, using the cathetometer setup, of all

the pressures in the diffuser (a traverse of all 20 pressure taps in the longest diffuser took as long as 4 to 5 minutes). A photographic method of taking data was developed which had two advantages over reading pressure directly. In the first place, all readings were recorded simultaneously. Secondly, with the photographic technique it was much easier to take data, and enough data could be taken for each run to compute statistically significant mean values of the measurements.

A 35mm camera with black and white film (Kodak Panatomic X was used for good contrast and small grain size) was used to record the data from the vertical manometer board. To eliminate parallax while filling the entire negative, a 300 mm telephoto lens was used. With this arrangement the camera was located 25 feet from the manometer board.

Pressure data were obtained by projecting the negatives at full size. The water level in each tube was then read with the cathetometer, gaining the precision advantage of that device. To assist in data reduction, (more than 900 pictures were taken) the recorded pressure head data were punched on IBM cards and were analyzed on a 7090 digital computer. Kinematic viscosity values were determined from the temperature noted on the manometer board for each run, and were entered on the IBM cards for the Reynolds number calculation.

Flow Visualization Results

Method of Visualization

The only method of flow visualization used in this study was the injection of water base dye. Other techniques were considered, but they were discarded as being unsatisfactory at the flow velocities used.

The colored dye was made from blue-black Calcoicid* powder dissolved in water. The resulting brilliant blue dye has been previously used in the fluid mechanics laboratories at Purdue University to give excellent definition of flow streamlines. Dye was injected into the water flow system through the static pressure taps along the diffuser and with a hypodermic probe into other regions of the flow. Dye injected through the static pressure holes was injected with very little radial velocity so that it did not penetrate very far into the flow. In this way, the motion of fluid particles close to the wall was observed.

Hypodermic probes were used for dye injection into the main stream, either in front of the diffuser section or in the exit plane. This method of dye injection was useful in checking for the possibility of swirl in the flow and for getting a quantitative idea of the extent of any separated regions present. In the upstream tank a very careful survey of the flow was made at several distances from the entrance plane of the inlet cone to verify that sufficient settling had been attained. The flow proved free from large eddies and from any trace of swirl; consequently no baffles or auxiliary flow straightening devices were installed.

Hypodermic probe injection of dye was also utilized to good effect in the downstream tank. The entrainment action of the

* Calco Chemical Company, Bound Brook, New Jersey.

exhaust jet from the diffuser was effective in bringing fluid toward the diffuser from any radial location on the window of the tank. Dye injected onto the surface of the tank window moved into the jet of water leaving the diffuser. Any separated regions or regions of backflow were easily seen; when separation was present the dye moved upstream along the inside diffuser wall rather than being swept away by the jet.

Method of Flow Regime Description

After injection at the wall static taps, dye trace behavior was essentially analogous to tuft behavior in an air unit. The movements of the dye traces were interpreted exactly as tuft movements had been interpreted by Waitman, Reneau and Kline[98]. The system used for describing tuft movements and the symbols classifying them were concisely set forth by Waitman, et al, and they have been modified only as necessary to apply them to dye traces. The modified definitions have been listed for convenience in Table 1, p. 49. Even though six flow conditions have been defined, the lines of demarcation among the various conditions are far from sharp. In borderline cases the classification was entirely arbitrary, but all results obtained were relative to the same set of standards. All flow regime observations were made by one observer.

Preliminary Investigation

The first experiments with dye injection were carried out with the lowest flow rate possible in order to obtain the most clearly defined dye stream. Preliminary runs were made when the diffuser inlet velocity was approximately 2 feet per second; this corresponded to a throat Reynolds number of about 30,000. For these flow conditions the 31.2 degree and 15.8 degree diffusers exhibited very stable jet flow patterns. It was possible to carefully probe the jet region and to determine exactly the boundaries of the core of high-speed fluid.

The jet at the diffuser exit was approximately 20 percent larger in area than it had been at the diffuser throat.

For the jet flow regime, dye injected with the hypodermic probe in the exit plane of the diffuser flowed smoothly toward the throat along the diffuser wall. When the throat was reached, the dye made the turn of almost 180 degrees and flowed back out along the core of the flow. No swirl was present in the flow even for this highly disrupted flow case. After analyzing the performance results from these low velocity runs, the reasons for the unusual visualization results became apparent.

The effect of low Reynolds numbers on diffuser performance is indicated clearly in Fig. 21, where pressure recovery has been plotted versus inlet Reynolds number. It can be seen that for the diffuser configuration for which the results are presented, a significant effect of low Reynolds numbers is present on performance, but that pressure recovery remains constant for higher values. This drop in performance as Reynolds number was decreased is attributable to at least two features of the flow system: the possibility of transition effects (although the Reynolds number based on the diffuser inlet was approximately 30,000, there was a strong favorable pressure gradient upstream of the diffuser throat), and the impossibility of making precise measurements of the small pressure differentials involved at low flow rates. The low recovery values were due to the presence of jet flow in the diffuser, as indicated by the pressure profiles along the diffuser for a typical low Reynolds number run, Fig. 22. These data gave quantitative agreement to the visually observed result that little diffusion was taking place in the jet flow regime, at low Reynolds numbers.

For moderately high Reynolds numbers, the recovery was considerably improved, and recovery curves looked typically like that shown in Fig. 22 for $Re = 75,000$. To verify that there was no dependence of recovery on Reynolds number for high flow rates,

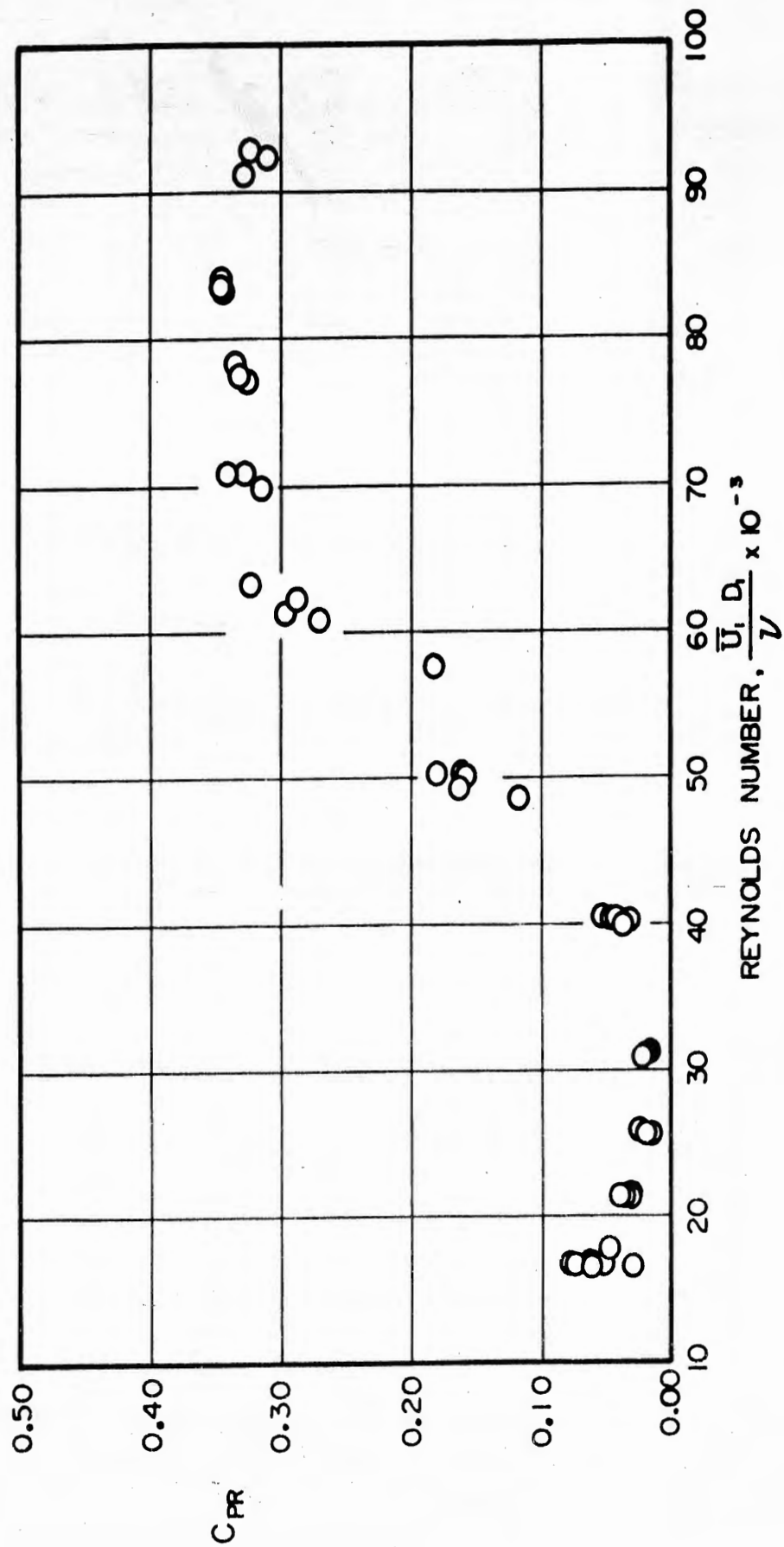


Figure 21. Effect of Low Reynolds Numbers on Conical Diffuser Performance.

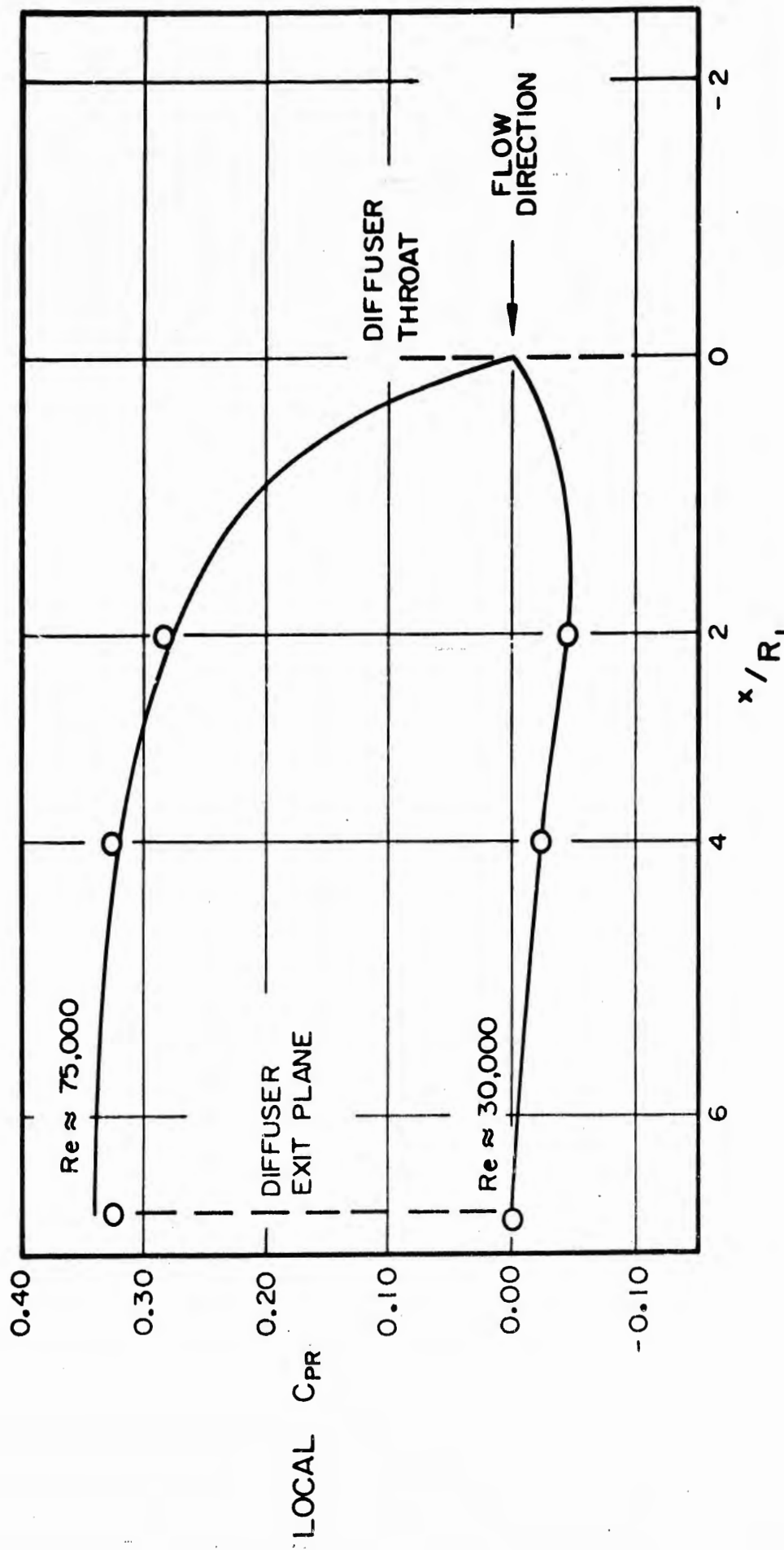


Figure 22. Typical Recovery Profiles for High and Low Reynolds Numbers
 ($2\phi = 31.2$ degrees, $N/R_1 = 6.7$).

linear regression analyses were carried out for the experimentally obtained data. A typical result of such a least squares fit of a straight line through the data showed that C_{PR} varied by at most a few tenths of a percent for a change in Re of 50,000. Since this variation was within the data scatter, recovery was felt to be independent of Reynolds number at moderately high to high flow rates

Flow Regime Results

Although the dye techniques were not nearly so effective at higher Reynolds numbers (due primarily to the much more rapid diffusion of the dye), it was necessary to run all tests at Reynolds numbers above 75,000. For these flow rates, the dye traces from the static pressure taps were visible in unstalled flow for a maximum of about two inches. In the more severely stalled cases, the trace was visible much longer in the low velocity region near the diffuser wall. In most of the stalled cases, it was possible to determine the length over which the stall bubble "communicated" with the surrounding fluid. Due to the low velocities with which water in the downstream plenum tank moved toward the exhaust jet, the hypodermic technique was extremely effective in locating stalled areas at the diffuser exit plane. In most cases where stall occurred at the exit, it was possible to determine the extent of communication of fluid from the exit plane toward the throat of the diffuser.

The results of all flow regime observations are displayed in Fig. 23a on a map of the geometries tested. Flow regime symbols and definitions of the flow regimes are given in Table 1.

The trends of all observations were similar to those expected: the flow condition steadily deteriorated as either diffuser length or diffuser angle was increased. Particularly apparent throughout all the tests was the fact that truly "steady flow" was attainable in the diffuser only for very moderate opening angles

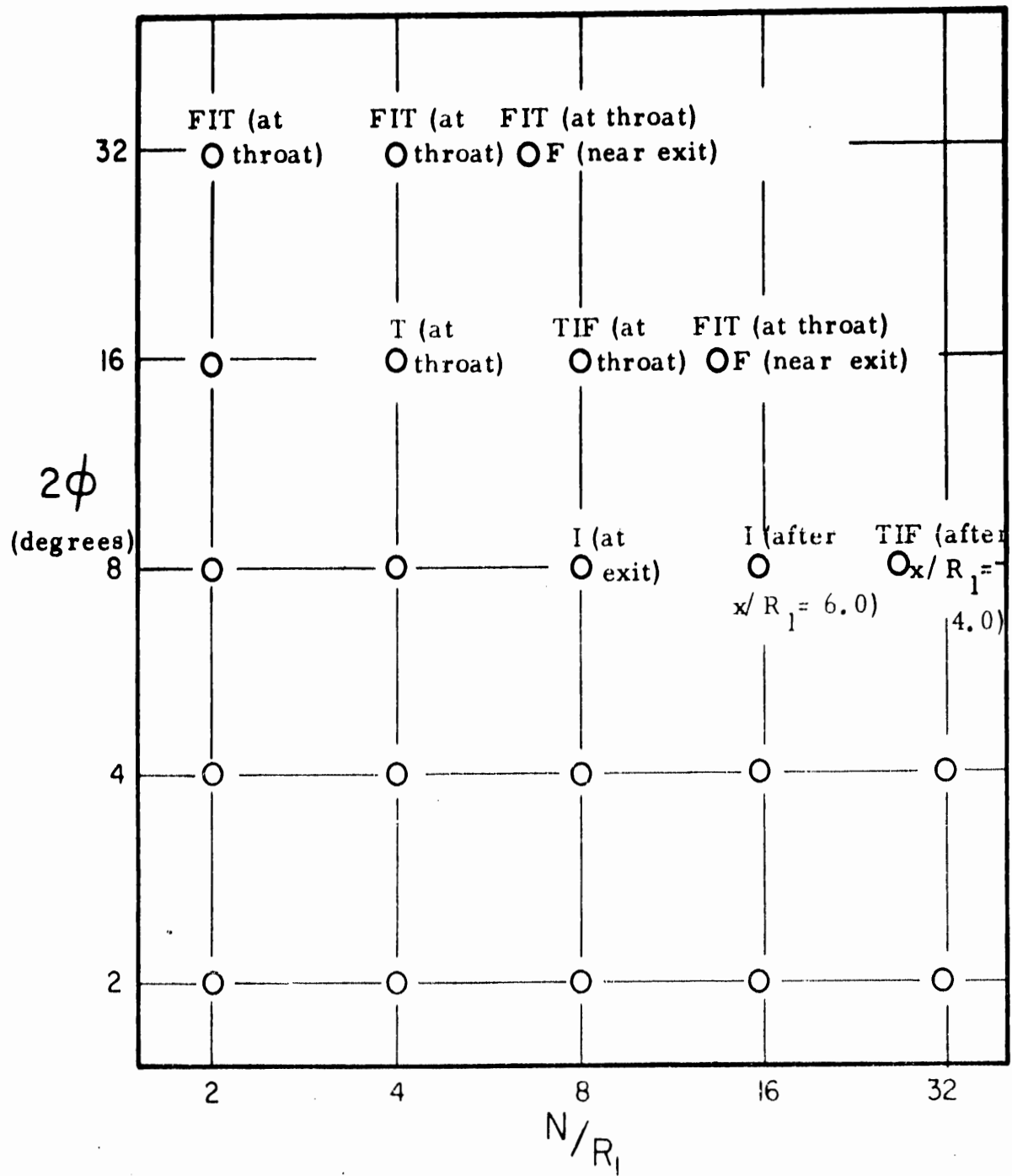


Figure 23a. Flow Regime Observed for Each Diffuser Geometry Investigated (Symbols Defined in Table I).

Table 1. Dye Trace Movements and Their Interpretation [98].

<u>Movement and orientation of trace</u>	<u>Type of flow</u>	<u>Symbol</u>
Dye trace held near wall, pointing downstream, occasionally wiggling	Steady flow with occasional disturbance	-
Major part of time dye trace points downstream wiggling; random flickering (trace quickly points upstream, then downstream again) indicating temporary and local separation	Intermittent transitory stall	I
Dye trace whips continually upstream and downstream, indicating rapid and chaotic occurrence and disappearance of separation	Local transitory stall	T
Dye trace whips upstream and downstream major part of time; trace held in upstream position wiggling at random intervals	Local transitory stall with intermittent fixed stall	TIF
Dye trace held upstream major part of time; temporary whipping at random intervals	Local fixed stall with intermittent transitory stall	FIT
Dye trace held upstream, end wiggling	Local fixed stall	F

and lengths, even though the absolute head difference between the two constant head tanks was closely controlled at all times. For moderate divergence angles, the main flow was steady but the pattern of stall near the wall was not. Only for the most "severe" diffuser configuration tested ($2\phi = 31.2$ degrees, $N/R_1 = 6.7$) was the stall pattern almost completely stationary. All other separated flows were to some extent unsteady.

Analysis of Observations

In the 31.2 degree diffuser with N/R_1 of 6.7, the stalled flow pattern was fairly stable near the exit. That is, the time constant* of the flow was long enough to probe the extent of the separation bubble: about 10 to 15 seconds. The bubble of relatively fixed stall was large enough to block from one-third to one-half the flow area. It covered approximately a third of the circumference of the diffuser, and extended radially inward from the diffuser wall 2 to 3 inches.

As noted on the flow regime summary map, Fig. 23b, the stability of the stall pattern decreased as observations were carried upstream in the diffuser. The time constant became as short as about 2 seconds for dye traces injected through static taps near the throat. At these taps, the dye filament exhibited a slight rotational instability when carried toward the throat of the diffuser. However, the direction of rotation was completely random, and fluctuations were rapid. There was no preferred direction of rotation, and even the small amount of rotation present in the essentially stagnant water very close to the throat disappeared by the time the exit plane was reached.

Moving across the constant angle line, $2\phi = 31.2$ degrees, to the diffuser with $N/R_1 = 4.0$ resulted in a marked reduction in the stability of the flow pattern. The extent of the stall bubble was considerably less than it had been with an, N/R_1 of 6.7. The time constant for the flow in the exit plane of this diffuser was similar to that for the regions near the throat of the diffuser discussed above. No particular variation in flow unsteadiness was noted over the length of the diffuser, but the overall length of this configuration was only 4.0 inches; this led to rather intimate

* Time constant - average period of stable configurations.

"communication" among regions of the flow. In this configuration the stall spots shifted around rapidly enough that it was quite difficult to get an accurate quantitative picture of their extent. The stall bubble at the exit plane extended practically to the diffuser centerline, but it covered only about one-third the circumference of the diffuser exit.

For the same (31.2 degree) diffuser angle, with a length ratio of 2.0, the extent of stall was further decreased and the stability of the flow was greater. The core flow of the diffuser appeared to be steady, but there were intermittently stalled spots on the diffuser wall. In contrast to the diffuser with 4.0 length ratio, these spots did not extend far into the stream. They were only perhaps 1/2 inch thick in the radial direction, but nevertheless they covered a considerable extent of the diffuser wall. Although the predominant flow condition was that of local fixed stall, the percentage of intermittent transitory stall present was considerably increased over that present in the 4.0 length ratio diffuser.

For the diffuser with $2\phi = 15.8$ degrees and $N/R_1 = 13.4$, a stall bubble extending about one-third of the way around the circumference of the diffuser exit was present. The maximum thickness of the bubble was approximately one-half inch, or one-sixth of the exit radius of the diffuser. The area occupied by the bubble was thus about 10 percent of the exit area. This stall bubble was not permanently located at any one point, but moved randomly about. The time constant of the motion was perhaps 10 to 15 seconds.

As the flow regime investigation was carried upstream into the diffuser itself, the time constant of the flow became somewhat shorter. Although the stall patterns were still quasi-stable, observations at the tap nearest the diffuser throat showed that fluctuation occurred at approximately half second intervals. The flow

was reversed at this tap approximately 40 percent of the time. In contrast to this behavior, the flow at the exit plane was approximately 75 percent backflow, but it had a period of as much as 5 seconds.

Several interesting phenomena were observed at the next smaller length ratio. In this ($2\phi = 15.8$, $N/R_1 = 8.0$) diffuser, the local stall bubbles at the exit were about 1/4 inch thick and perhaps 1 inch wide. There was communication of dye injected at the exit with the diffuser throat. Investigation showed that stall bubbles were present about 30 percent of the time at any one circumferential location at the exit. The time constant of the backflow was fairly long, of the order of 4 seconds. The relative amount of backflow at a given station decreased after the second tap from the diffuser exit. At the $x/R_1 = 6.0$ tap, there was backflow present perhaps 75 percent of the time, while at 4.0 and 2.0, respectively, it persisted for approximately 2 seconds and 0.1 second. The amount of stall present at these taps was about 60 and 50 percent respectively.

For $2\phi = 15.8$ and $N/R_1 = 4.0$, there was practically no evidence of flow separation in the diffuser. The exit plane survey showed a few very spotty, thin, transitory stall patches. The dimensions of these patches were no larger than 1/2 inch long, 1/2 inch wide, and 1/4 inch thick. The duration of their persistence was short (time constant $\approx 1/2$ second), and they shifted around rapidly. The static pressure tap just upstream from the exit plane showed approximately the same duration of stall flashes. Stall fluctuations at the exit plane static pressure tap were very much slower than those at the next upstream tap. Stall patterns at the latter section had a time constant of approximately 1/2 seconds, and backflow was present the majority (perhaps 80 percent) of the

time. Backflow persisted about 5 percent of the time at any given circumferential position at the exit plane.

For $2\phi = 15.8$ and $N/R_1 = 2.0$, the flow was entirely steady and unstalled. Nothing of significance was observed at any section of the diffuser.

For diffusers with divergence angle of 8 degrees, only in the three longest tested diffusers ($N/R_1 = 26.8, 16.0, 8.0$) was there evidence of flow separation. For $N/R_1 = 26.8$, the overall flow pattern was characterized by local transitory stall with intermittent fixed stall patches. For the first time (testing had commenced with the largest diffuser angles) a flow regime was observed in which, even when there was backflow, no communication with the throat was achieved. The exit plane survey with the hypodermic probe indicated that the main flow was quite regular and steady, and that, although present, backflow was not a serious enough phenomenon to cause major disruption of the flow pattern. The stall bubbles were very small and erratic in action, and there was little upstream communication. The longest dye travel observed was about 6 inches. Working upstream along the diffuser, the effects of flow separation became less in evidence. Flashes of backflow were occasionally present and backflow persisted momentarily from time to time. Consequently the flow was classified as being composed of local transitory stall with intermittent fixed stall, Fig. 23a. Between the throat and the tap at $x/R_1 = 16.0$, the flow became even less erratic, and began to exhibit similarities to the flow in the 31.2 degree diffuser. However, in this 8 degree unit, the extent of stalled flow was much less. Instead of backflow flashes causing the trace of dye to point toward the diffuser throat, it would often begin to swirl circumferentially. The instantaneous position of the fluid dye filament would rotate as much as 45 degrees from side to side (in the plane of the diffuser wall) before forward flow

took over, snapping the trace in the streamwise direction. Communication with upstream regions was reduced to about 2 inches above each tap. There was no evidence of any backflow between the diffuser throat and $x/R_1 = 4.0$.

Further tests with the 8 degree diffuser at shorter length ratios indicated that nothing more serious than local, intermittent transitory stall occurred. There was no upstream communication of stalled fluid, and no flow disruption could be observed. Even in the end plane, flashes of backflow were sufficiently infrequent that they were hard to spot.

The 4 degree and 2 degree diffuser configurations produced no flow separation of any kind, as indicated in Fig. 23a.

Map of Flow Regimes

The most noticeable feature of the flow visualization study was that separation phenomena had their greatest effect at the diffuser exit. No significant effect of stalled flow was noted in upstream regions of the diffusers until the exit conditions had deteriorated considerably past the level of intermittent transitory stall (I). Even though the presence of intermittent transitory stall (I) at the exit was found not to influence the flow in the diffuser, this condition was considered to form a lower bound for the placement of a line of "first appreciable stall."

In turn, there seemed to be little question that flow in the 31.2 degree diffusers tested was significantly affected by the amount and kind of stall present. The 15.8 degree, 16.0 length ratio configuration (TIF) was also judged sufficiently affected by stall that the effects were noticeable in the mean flow. Consequently, the upper limit to a possible line of first appreciable stall was located near, but below, the onset of TIF stall (see Fig. 23b).

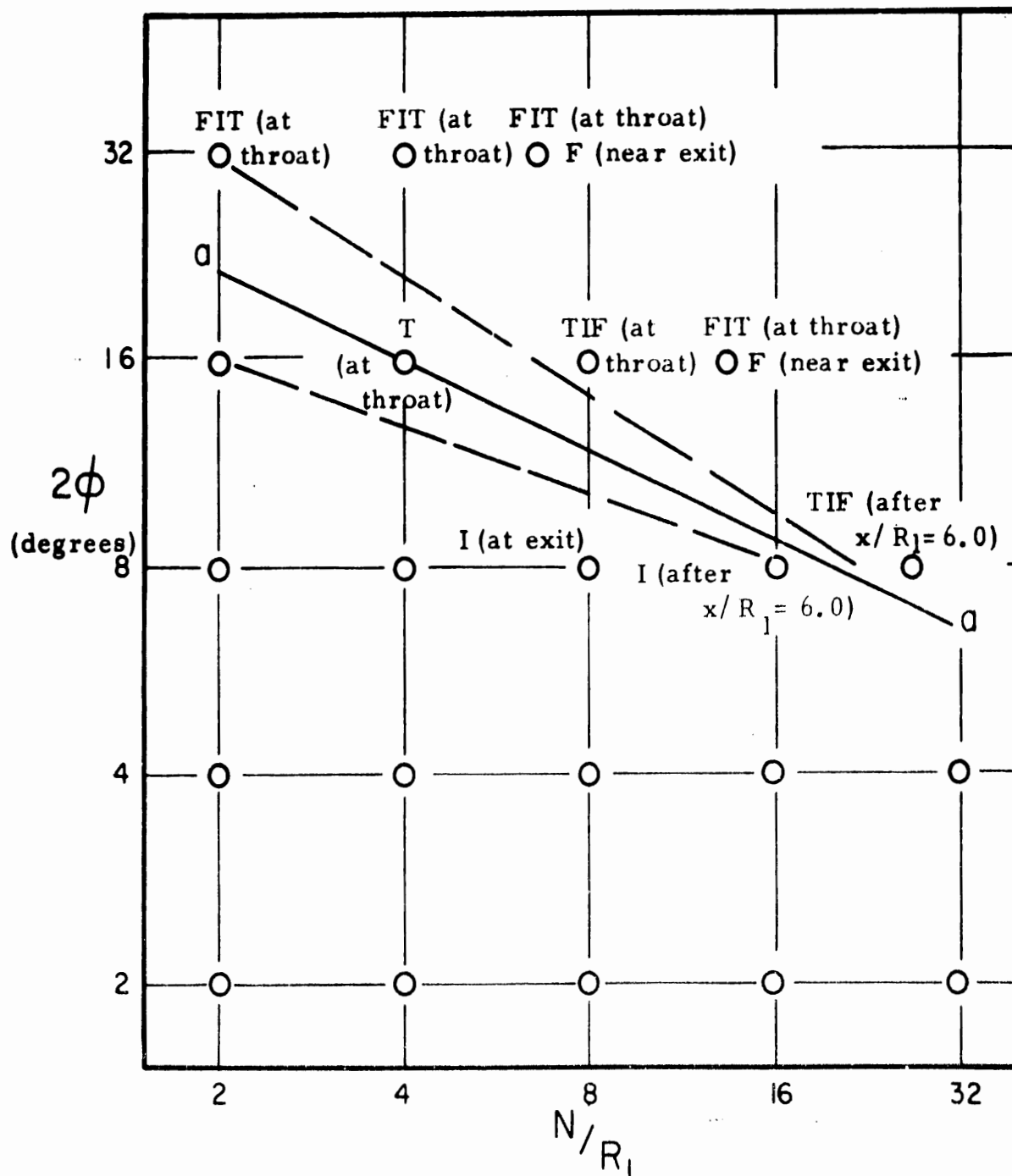


Figure 23b. Flow Condition Observed for Each Diffuser Geometry Investigated (Symbols Defined in Table 1), Showing Location of Line of First Appreciable Stall.

In Fig. 23b, the lower end of the line of appreciable stall for the conical diffusers tested is seen to be located fairly well. At the upper end there is quite a latitude allowed by the upper and lower limits on angle. However, the slope of the line is well defined, and it should be possible to compare it fairly well with that for plane walled diffusers.

Conical Diffuser Performance

Variation of Recovery with Reynolds Number

As previously discussed in connection with flow visualization results, the flow regime and performance in a conical diffuser are affected by Reynolds number variations only when relatively low speed flows are being investigated. In practical installations of diffusers, values of Reynolds numbers are generally at least of the same order as the highest values employed in this study.

In the high Reynolds number ($Re > 75,000$) range of flow, there is no perceptible dependence of either flow regime or pressure recovery on Reynolds number. There is a significant dependence on diffuser geometry, however. All tests for which performance data are reported were conducted in Reynolds number ranges where the effect of that parameter could be neglected.

Variation of Recovery with Geometry

Although diffuser performance can be specified in many ways (see Appendix A for performance definitions), it is convenient for comparison with other results to use the commonly defined pressure recovery coefficient, C_{PR} , and the diffuser efficiency, η . The recovery coefficient is a measure of the completeness of the conversion of kinetic energy to static pressure by a diffuser; the efficiency is a measure of the diffuser head loss. The higher the efficiency, the lower the head loss for a given unit.

In figures 24 and 26, contours of constant recovery and constant efficiency, respectively, are plotted on 2ϕ vs. N/R_1 coordinates. Figures 25 and 27 show the same contours of recovery and efficiency plotted on $(AR - 1)$ vs. N/R_1 coordinates. The pressure recovery contours plotted in Figs. 24 and 25 were independently prepared from the three cross plots of the experimentally obtained performance data.

This was done to check the validity of the choice made for the location of the line of optimum performance at constant N/R_1 , line $\alpha-\alpha$, in Figs. 24 and 25. The two lines agree well, indicating that confidence can be placed in this choice for location of the line of optimums.

The shapes of the recovery contours are such that the location of optimum recovery at constant N/R_1 is quite precisely specified. As shown in Figs. 24 and 25, the fit of computed points to the curves is very satisfactory, particularly in the region near the optimum value at each C_{PR} .

The efficiency contours of Figs. 26 and 27 were again obtained independently of each other by plotting separately the computed points from the cross plots. Except in the low angle and low length ratio regions, the shapes of the efficiency contours are not similar to those for constant recovery. The latter contours have a characteristic "fish-hook" shape. The efficiency curve opens in the opposite direction from that for constant recovery. The method of obtaining the line of optimum efficiency for constant length ratio differed slightly from that used to determine the optimum recovery line. Since the fish-hook shape was not present, it was necessary to interpolate across the "plateau" between the lines for constant efficiency, $\eta = 0.80$, of Fig. 26. The optimum curve was determined for the plateau region, and this was then faired smoothly into the curve from the region on the left of the figure. The result is a line

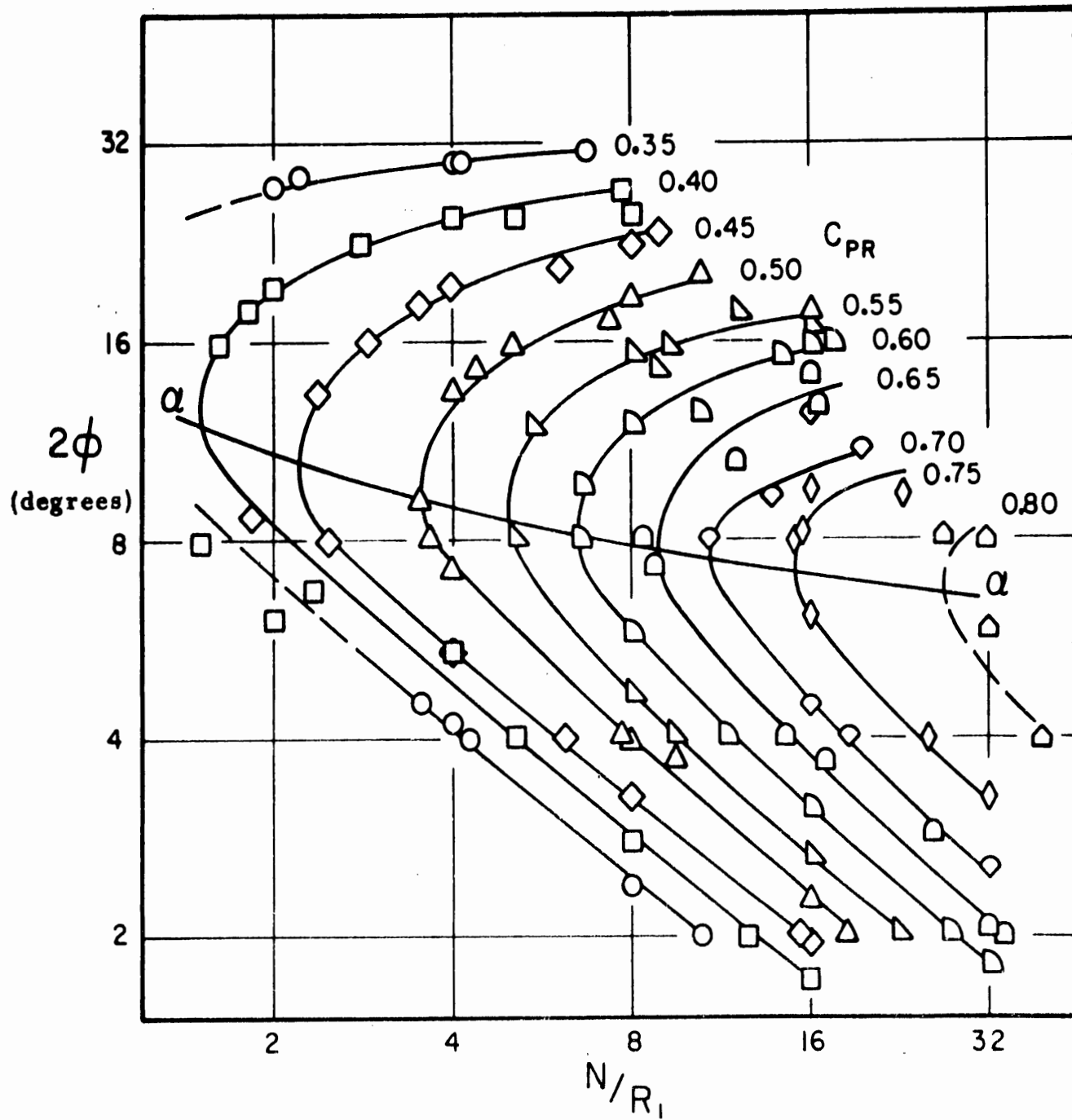


Figure 24. Recovery Contours Cross-Plotted from Conical Diffuser Data, on 2ϕ vs. N/R_1 Coordinates.

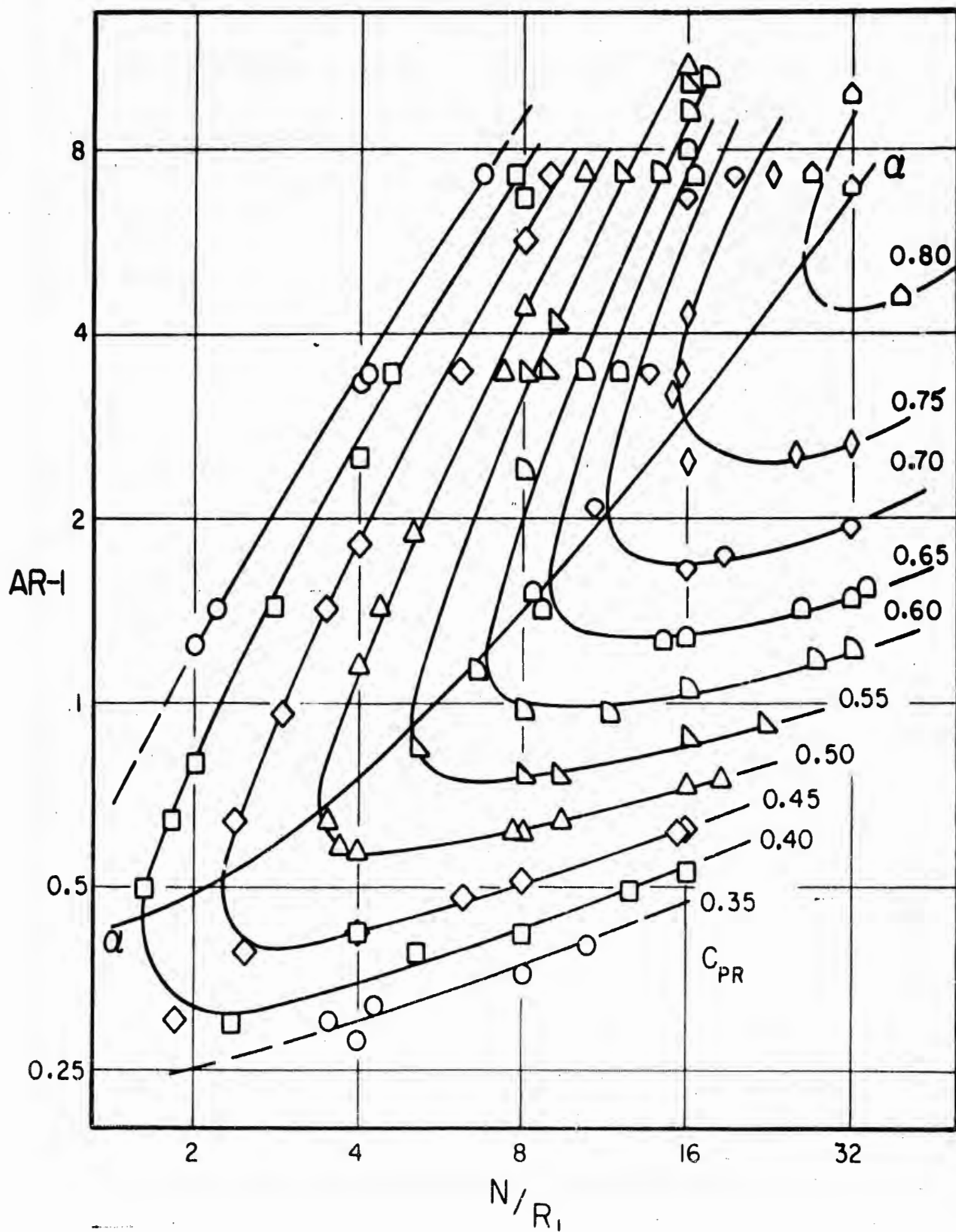


Figure 25. Recovery Contours Cross-Plotted from Conical Diffuser Data, on $(AR - 1)$ vs. N/R_1 Coordinates.

of optimums which lies essentially along a line of constant divergence angle for long diffusers. The angle for optimum efficiency is approximately 4.9 degrees. At its left hand end, for shorter diffusers, the line bends upward considerably. For a length ratio of 2.0, the suggested angle for optimum efficiency becomes almost 8 degrees.

As in the case of the performance curves, a line of optimum efficiency was also independently determined from Fig. 27. The technique for obtaining the curve was the same as that used for Fig. 26: it was drawn utilizing interpolation across the plateau between the contours for 80 percent efficiency. The locations of the independently determined lines show very satisfactory agreement.

The lines of optimum diffuser performance at constant N/R_1 and of first appreciable stall, determined from the results of this study, are shown in Fig. 28. The line of optimum performance falls considerably below that for first appreciable stall for shorter diffuser length ratios, although the lines finally intersect near $N/R_1 = 32$. This indicates that diffusers designed for optimum performance will be free from any stall which affects the flow significantly. The line for optimum diffuser efficiency, determined from Fig. 26, falls even further below the line of first appreciable stall.

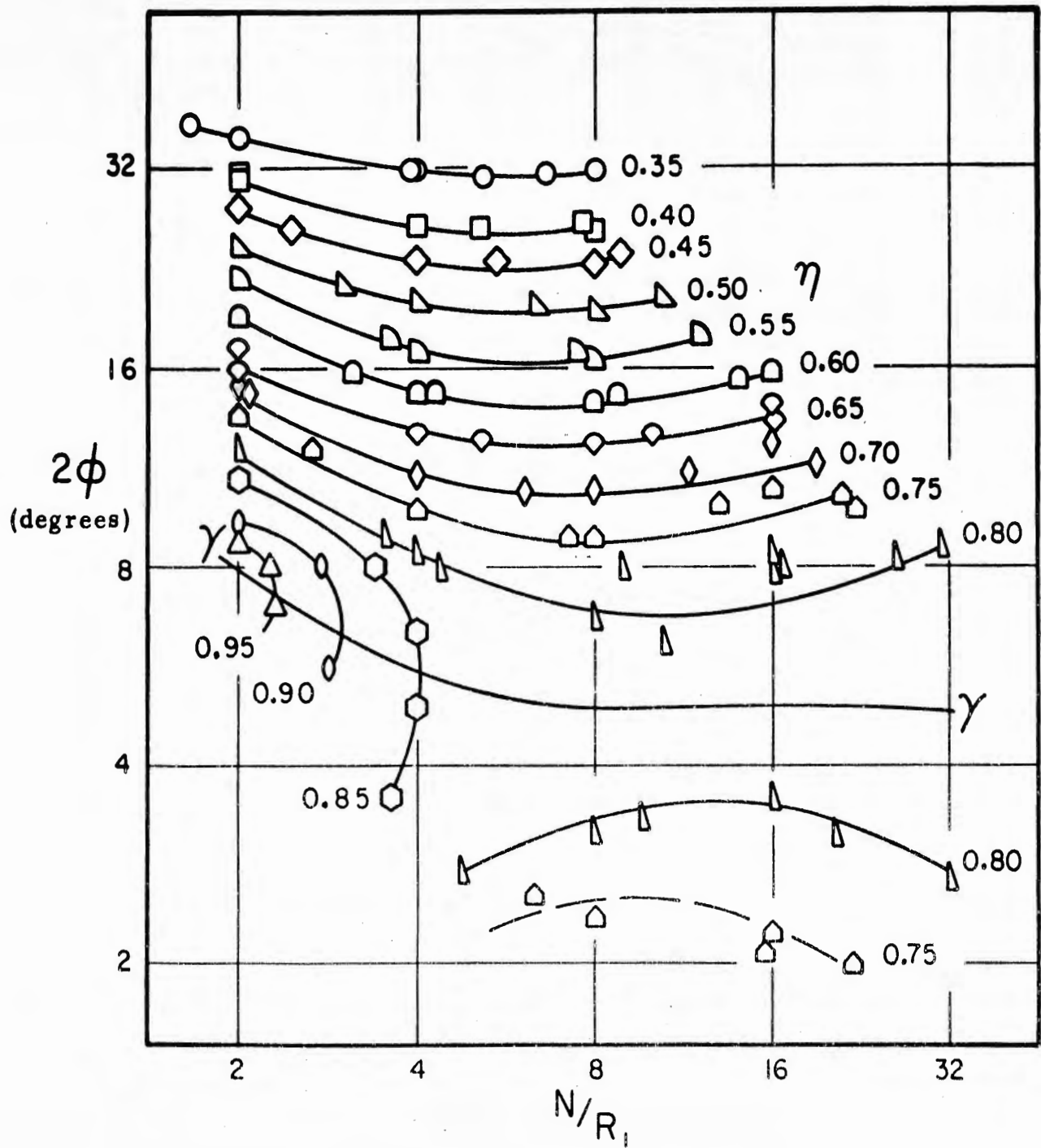


Figure 26. Efficiency Contours Cross-Plotted from Conical Diffuser Data, on 2ϕ vs. N/R_1 Coordinates.

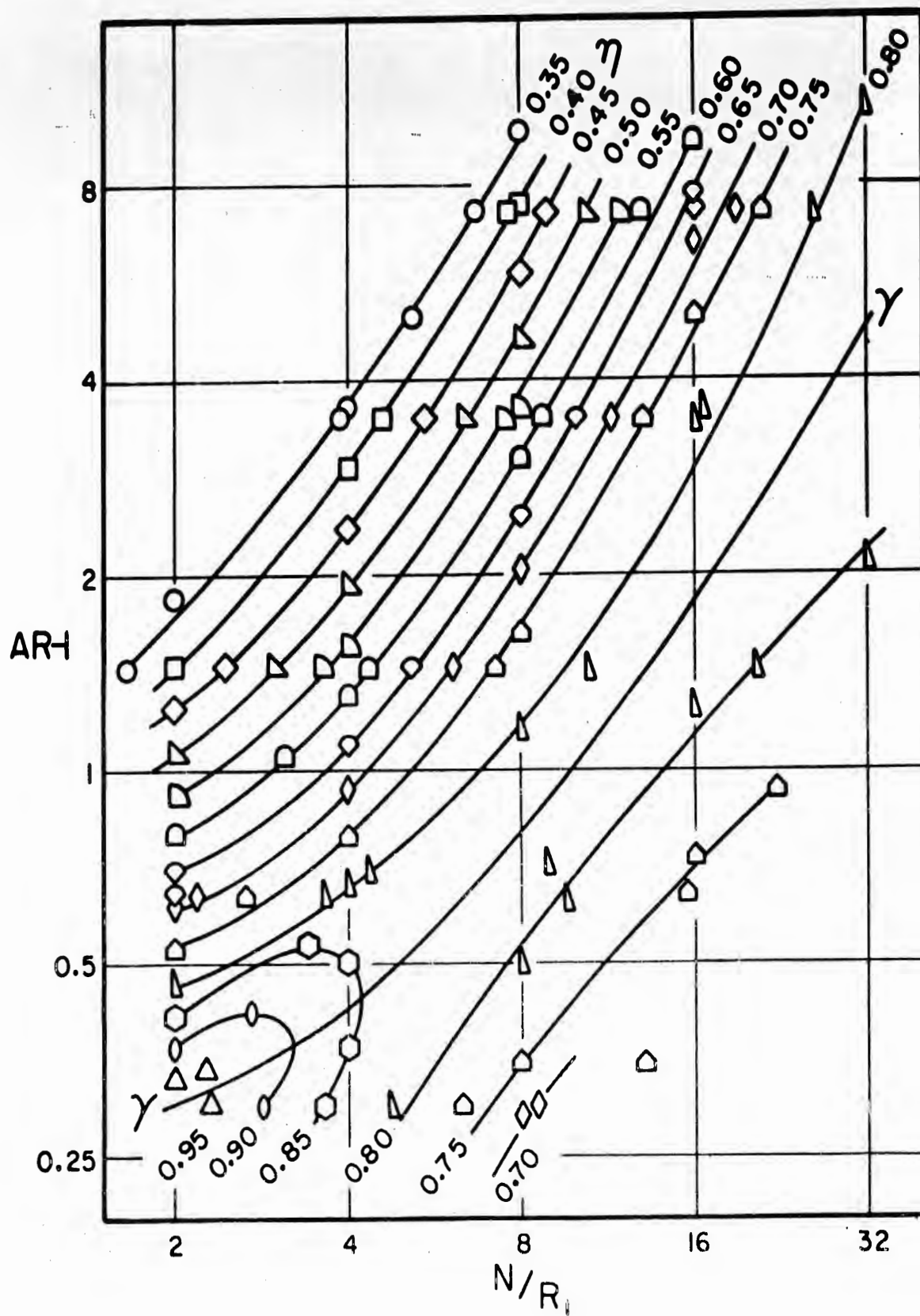


Figure 27. Efficiency Contours Cross-Plotted from Conical Diffuser Data, on $(AR - 1)$ vs. N/R_1 Coordinates.

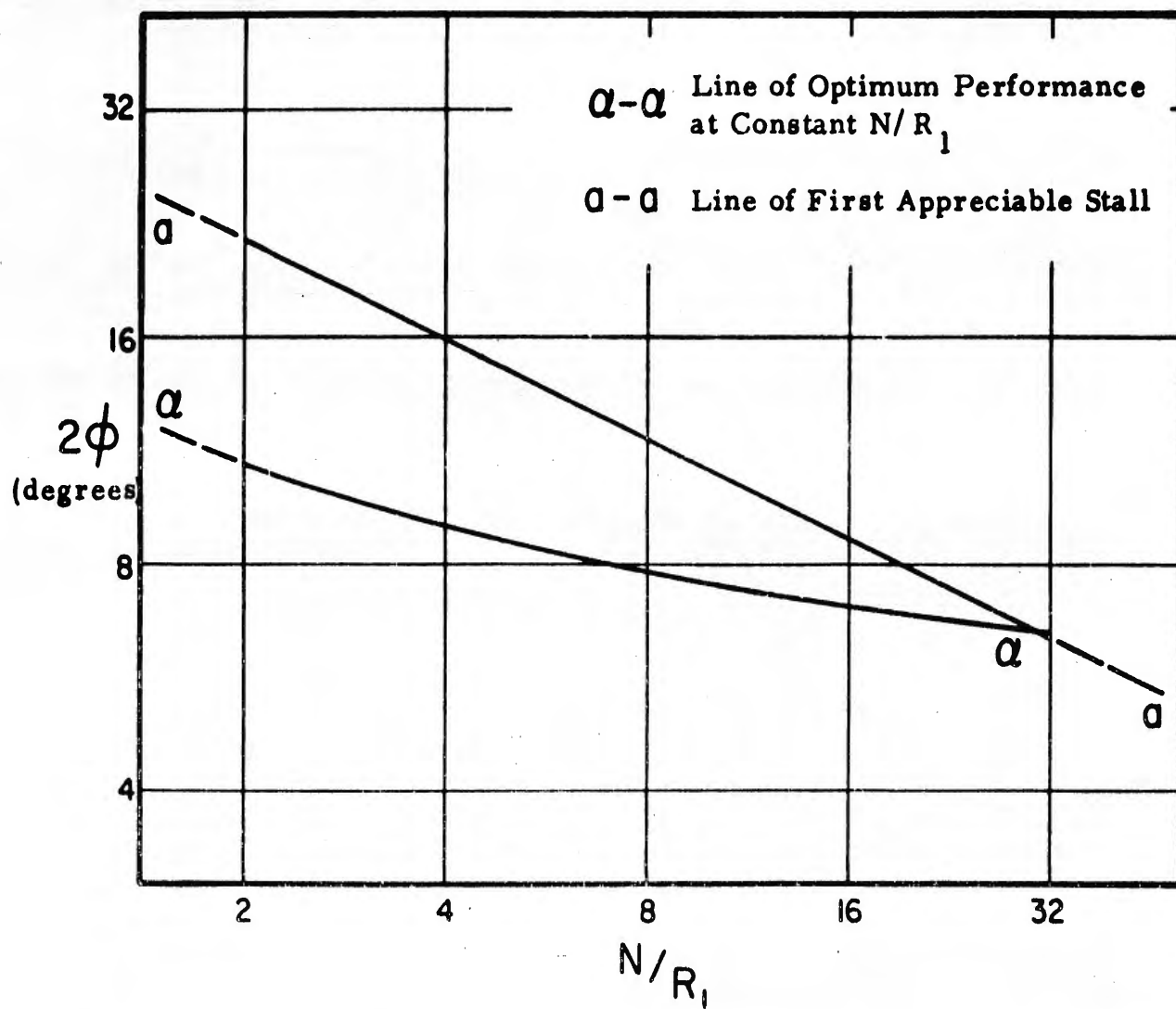


Figure 28. Line of Optimum C_{PR} at Constant N/R_1 Compared with Line of First Appreciable Stall for Conical Diffusers, on 2ϕ vs. N/R_1 Coordinates.

DISCUSSION OF RESULTS

Flow Visualization Results

On Fig. 29 the line of first appreciable stall obtained from the conical diffuser experiments is compared with the corresponding line for plane walled diffusers. When plotted as in Fig. 29, on 2ϕ vs. N/R_1 coordinates, the lines of first appreciable stall differ considerably in location for the two types of diffuser. In the position chosen for the conical line, its slope is different from that for the plane walled diffusers. In addition, the plane walled line is seen to lie somewhat lower. The lower limit for the location of first appreciable stall in conical diffusers and the line suggested by Reference [43] for plane walled diffusers intersect at an N/R_1 of about 3.5. Thus for a given N/R_1 greater than 3.5, first appreciable stall will definitely occur at a lower diffuser angle in plane walled units. Figure 29 indicates that the suggested relationship $2\phi)_p = 2\phi)_c$, $N/W_1)_p = N/R_1)_c$, does not give a good correlation for the line of first appreciable stall.

Lines of first appreciable stall for conical and plane walled units are again compared in Fig. 30. In this plot, the predicted line from two-dimensional results has been re-plotted preserving the expansion angle (i. e., using the relationship suggested in the paragraph above).

As previously pointed out, the latter suggested transformation results in different area ratios for the plane walled and conical diffusers. The correlation shown in Fig. 30 is significantly better than that obtained by letting $AR)_p = AR)_c$ and $N/W_1)_p = N/R_1)_c$. Neither the correlation of stall results based on preservation of angle and length ratio nor that based on preservation of area ratio and length ratio gives satisfactory results.

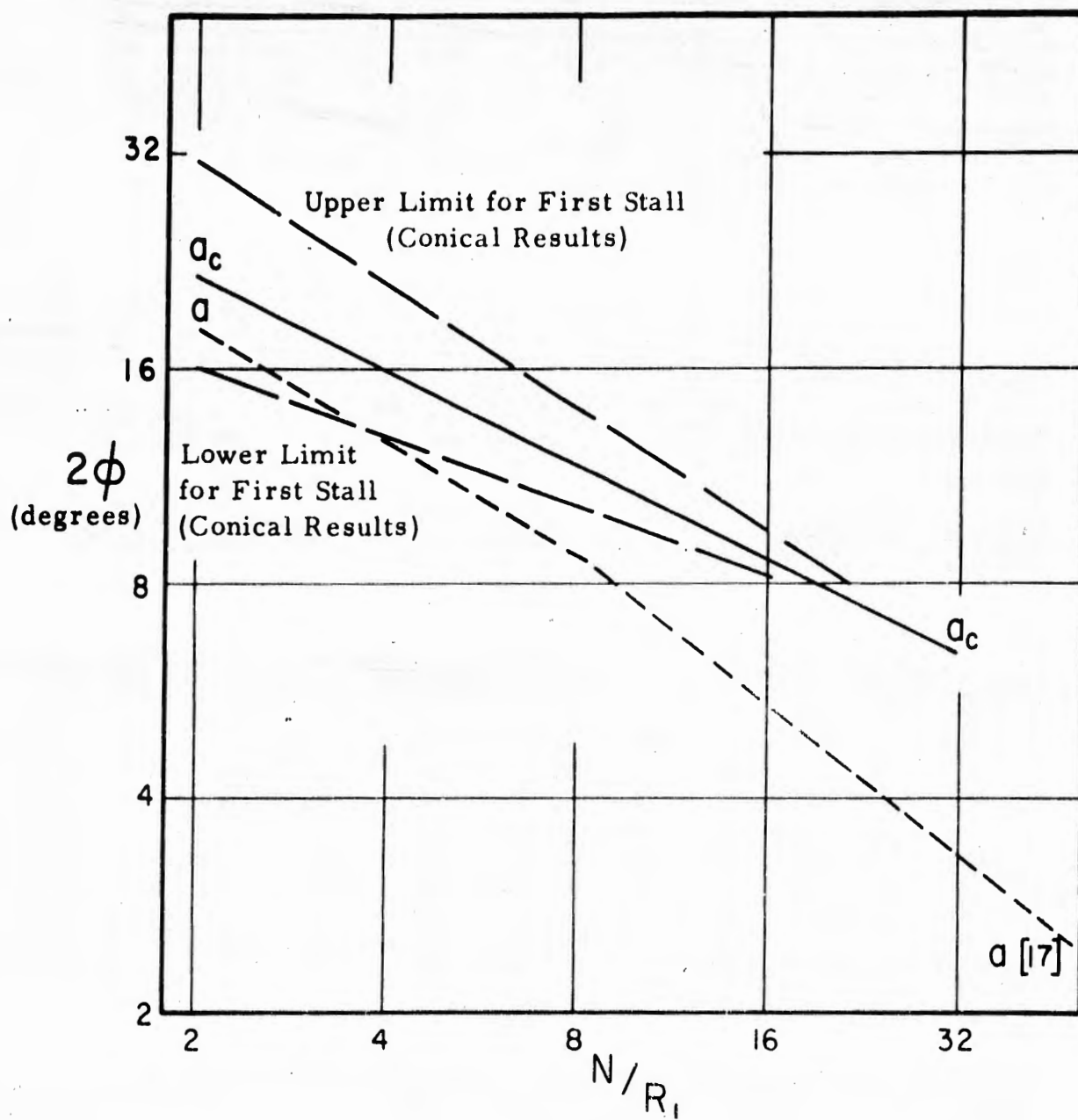


Figure 29. Lines of First Appreciable Stall Compared for Conical and Plane Walled Diffusers, on 2ϕ vs. N/R_1 Coordinates.

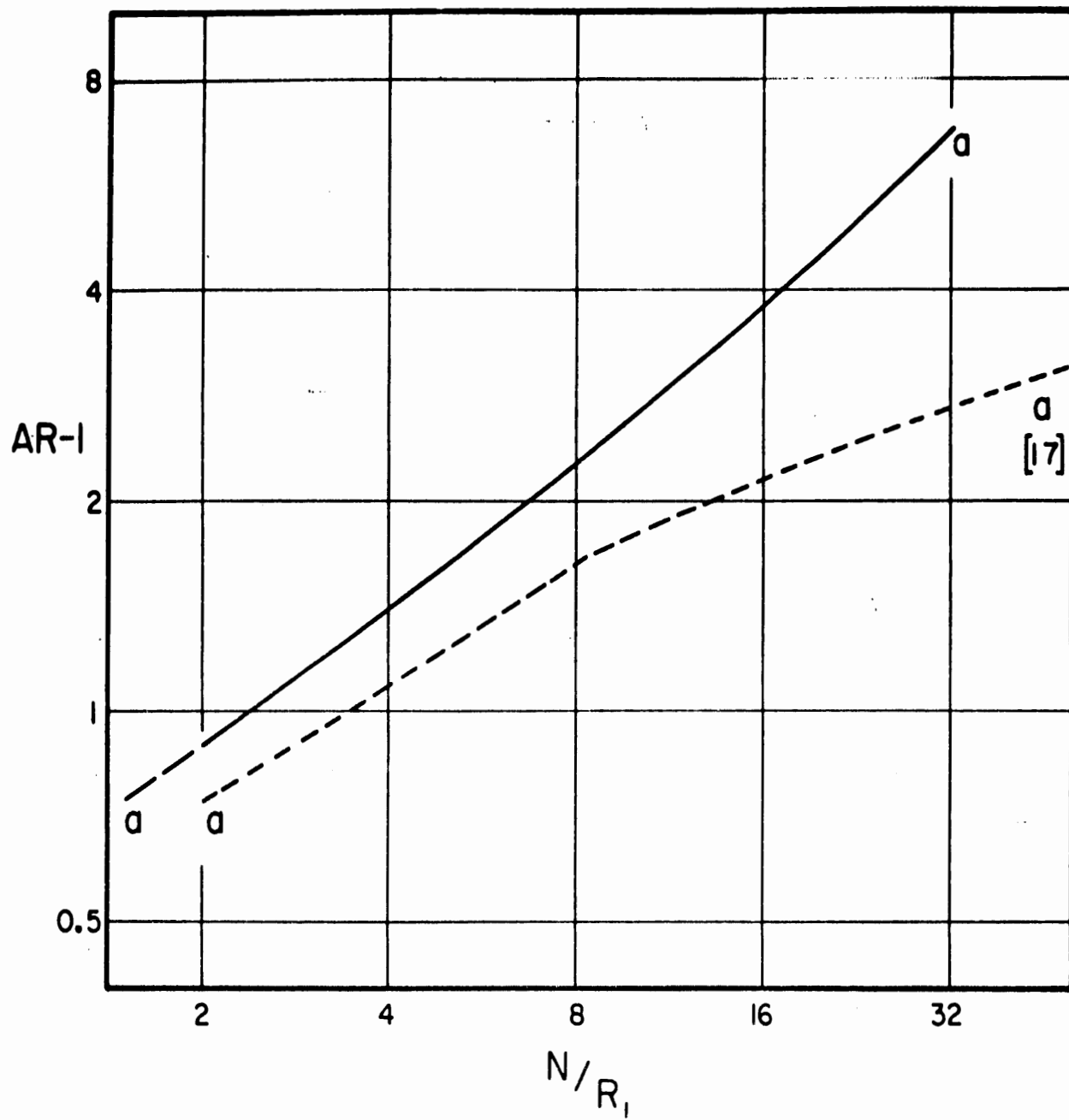


Figure 30. Lines of First Appreciable Stall Compared for Conical and Plane Walled Diffusers, on $(AR - 1)$ vs. N/R_1 Coordinates.

Conical Diffuser Performance

In Fig. 31, lines of optimum diffuser performance at constant length ratio and lines of first appreciable stall are compared for the conical and two-dimensional geometries. On this plot, not only does the line of optimum performance for the conical units lie below that for plane walled diffusers, but the shape is not similar. This indicates that the relationship $2\phi)_p = 2\phi)_c$, $N/W_1)_p = N/R_1)_c$ is not satisfactory to correlate the performance results.

The four lines are compared again in Fig. 32, p. 71, which is plotted on $(AR - 1)$ vs. N/R_1 coordinates. The lines of optimum performance, lines $\alpha - \alpha$, for the two geometries are again seen to differ considerably. Although the line obtained from Reference [43] utilizing the relationship $AR)_p = AR)_c$, $N/W_1)_p = N/R_1)_c$, line $\alpha' - \alpha'$, lies somewhat below the line $\alpha - \alpha$ for planed walled diffusers shown in Fig. 32, the agreement is still not good, since the slopes of the two-dimensional and conical lines are not similar. The latter two lines intersect near $N/R_1 = 15$, and the conical line of optimums is considerably below line $\alpha' - \alpha'$ for N/R_1 values less than 15. The coordinates of Fig. 32 cannot be considered adequate to correlate the performance characteristics of the diffusers of different geometries, either.

Within the boundaries of each diffuser geometry, significantly different relationships between the lines of optimum performance and first appreciable stall exist. Figure 31 illustrates again the well known fact that optimum performance occurs for plane walled diffusers in configurations where some stall is already

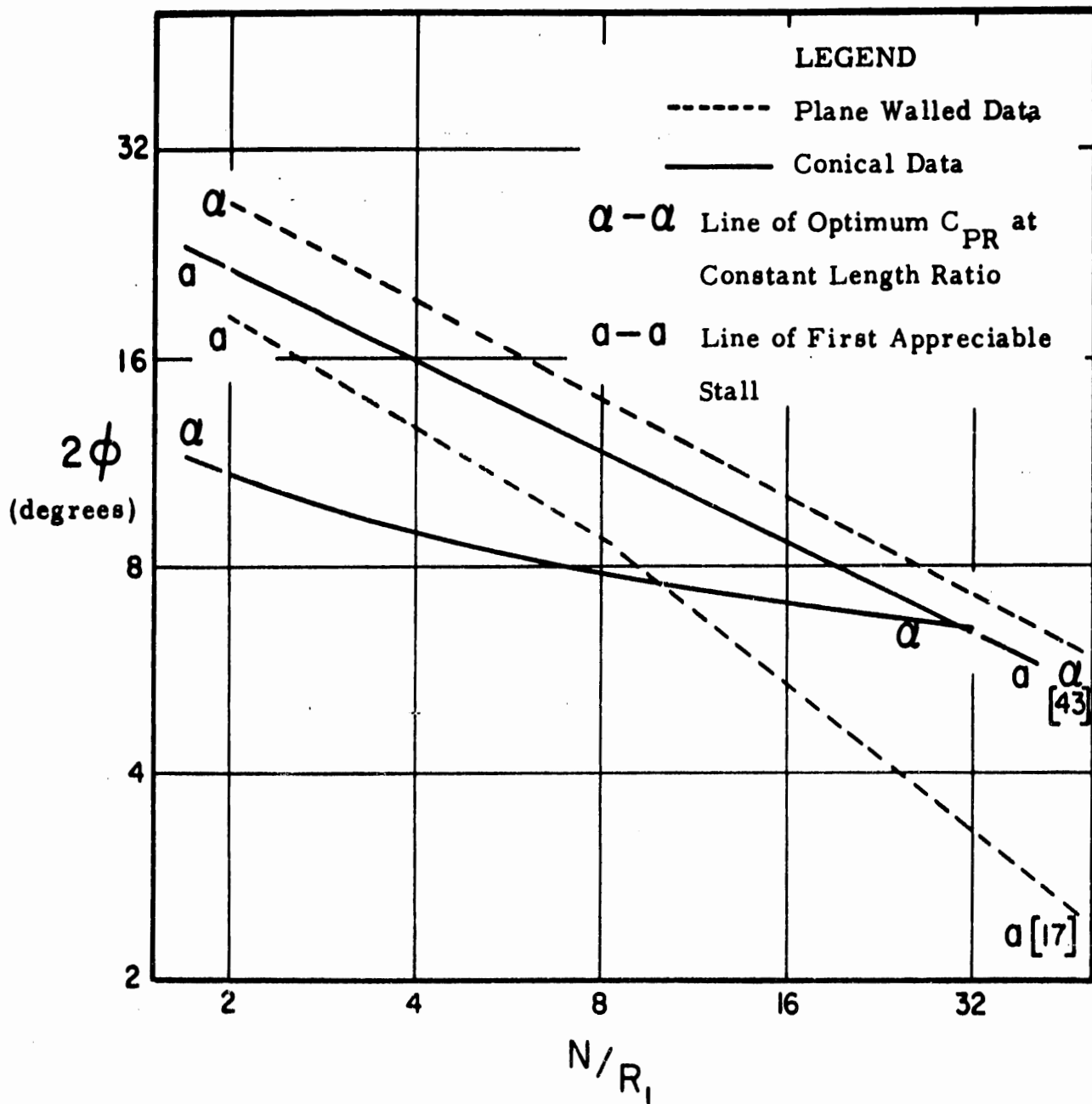


Figure 31. Lines of First Appreciable Stall and Lines of Optimum C_{PR} at Constant Length Ratio Compared for Conical and Plane Walled Diffusers on 2ϕ vs. N/R_1 Coordinates.

present. For conical diffusers the situation is entirely different. All diffusers, designed with moderate length ratios, lying along the line of optimums will be entirely free from flow separation. It is recognized that since the line of demarcation, a-a, depends on qualitative observations, some variation may arise due to interpretation of observed phenomena. The reader is referred back to p. 55 for a discussion of the upper and lower limits of placement of line a-a. In any case, it is not until N/R_1 exceeds about 20 that lines a-a and $\alpha-\alpha$ are within one degree of angle.

The differences within geometries between the conical and plane walled diffuser results indicate that the mechanisms of flow separation involved are different. Intuitively it would seem that flow separation would be delayed in a conical diffuser since there are no corners present, and separation in two-dimensional plane walled diffusers is known to begin in the corners [42]. This type of argument would tend to explain the direction of the shift in the line of first appreciable stall, but it would not necessarily account for the magnitude of the change.

Although the data of Gibson [23,24] were used for comparison in suggesting that the two geometries could be correlated on the basis that $2\phi)_p = 2\phi)_c$ and $N/W_1)_p = N/W_1)_c$, the present data do not substantiate that trend. The exit configuration for the present results (free jet exit conditions) is exactly the same as the condition under which the plane walled results were obtained, whereas the data reported by Gibson were obtained in diffusers with downstream exit lengths. The fact that the present results do not

correlate with the two dimensional results in Fig. 31 indicates that the relationship $2\phi_p = 2\phi_c$ and $N/W_p = N/R_c$ is not suitable to specify similar flow conditions in the different geometries.

In Fig. 32 lines of optimum C_{PR} and of first appreciable stall are again compared on the basis of $(AR - 1)$ vs. N/R . Both of the lines presented for plane walled diffuser correlations were obtained as suggested by relating the variables through preservation of the divergence angle and length ratio, and then computing the equivalent area ratio for the conical geometry. Comparison on the basis of preserving area ratio and length ratio does not give any better correlation.

Finally, lines of optimum efficiency from plane walled and conical diffuser data are compared in Fig. 33. Although the results of the two dimensional diffuser study did not suggest such a correlation, it has been tried on the basis that diffuser angle and length ratio are preserved in going from the plane walled case to the conical case. As shown in Fig. 33, the line of optimum efficiency for the conical units is essentially parallel to that for plane walled units, but it lies considerably below. As previously mentioned, at large length ratios, the conical line of optimums lies along a line of constant angle. This value is 4.9 degrees, while the line of optimums for plane walled diffusers lies along a line of angle where $2\phi = 7$ degrees.

Designing for Optimum Performance

Enough data have been obtained in the present study to allow solution of the four common optimum problems often encountered in diffuser design [43]. These problems are to provide:

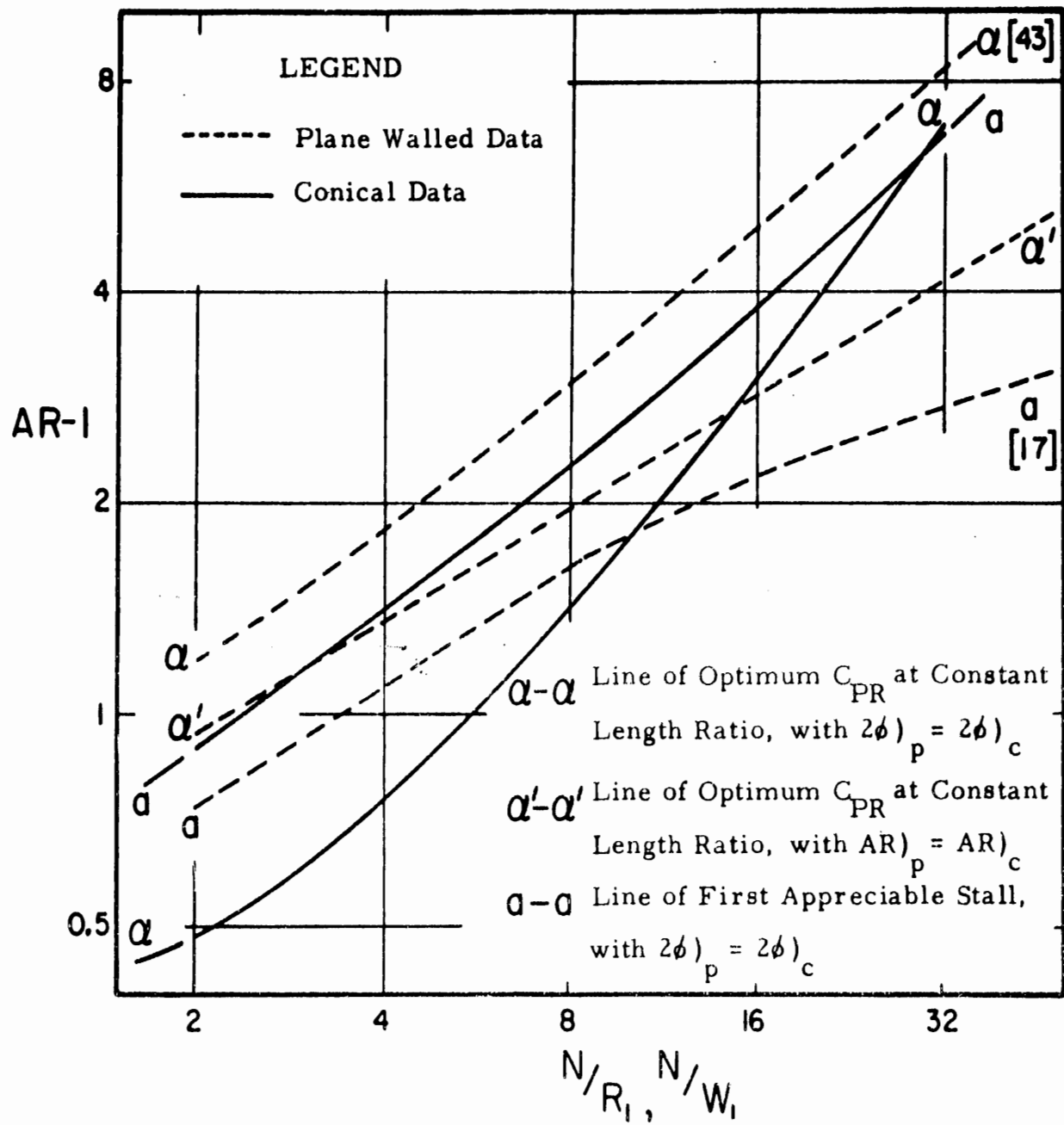


Figure 32. Lines of First Appreciable Stall and Lines of Optimum C_{PR} at Constant Length Ratio Compared for Conical and Plane Walled Diffusers on $(AR - 1)$ Coordinates.

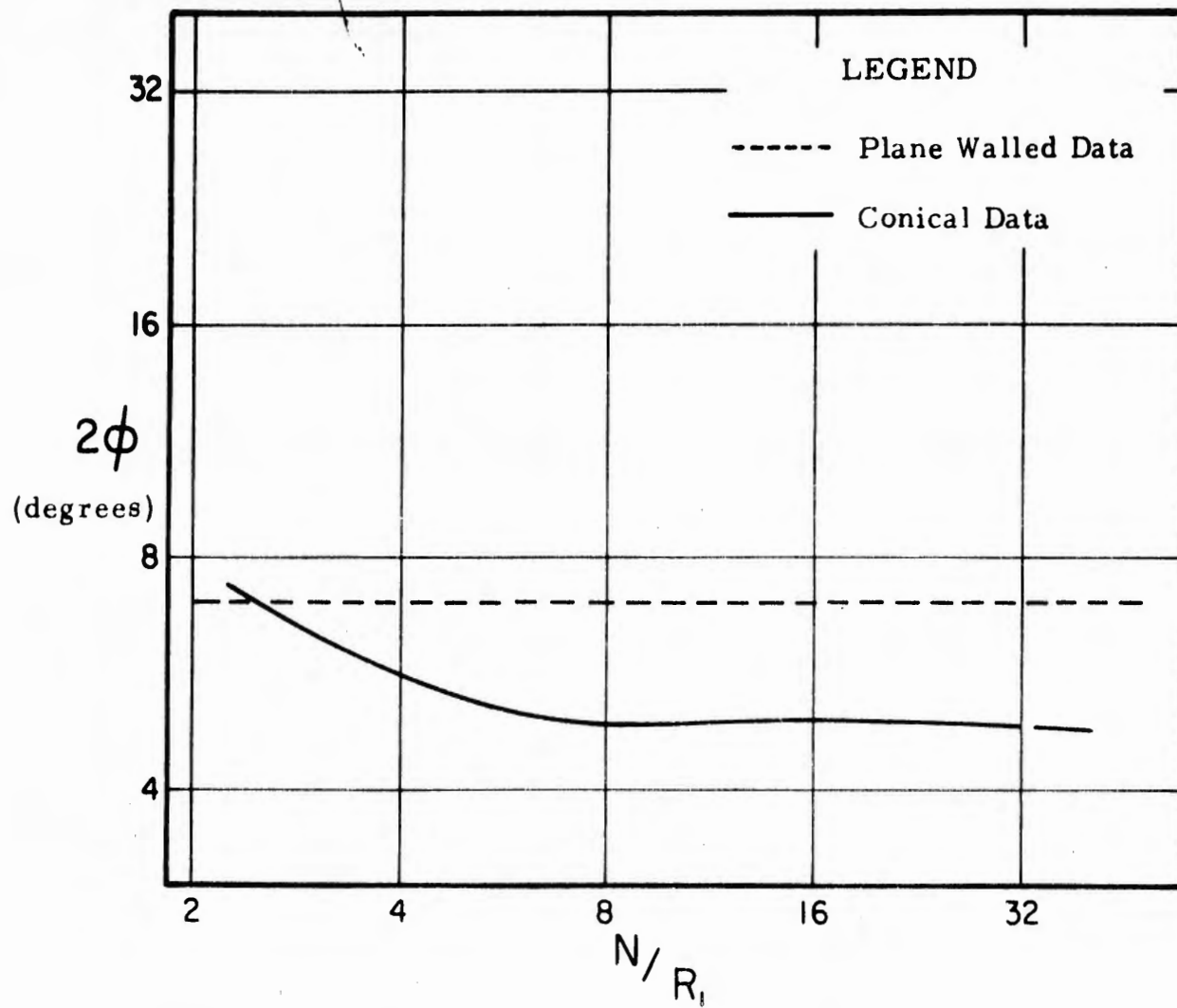


Figure 33. Lines of Optimum Efficiency at Constant Length Ratio Compared for Conical and Plane Walled Diffusers.

- (1) Minimum loss of total pressure for given pressure rise,
- (2) Maximum recovery for a given area ratio,
- (3) Optimum recovery for a given length ratio, and
- (4) Optimum recovery for any possible geometry (best optimum).

The lines of optimum performance and optimum diffuser efficiency obtained from the results of this study are reproduced for convenience in Fig. 34.

As shown in Reference [43], minimum loss of total pressure for a given pressure rise is attained when diffuser efficiency is maximized. The solution for problem (1) has also been given as line $\gamma-\gamma$ on Figs. 26, 27, and 34. The relationship of lines for conical and plane walled units has been shown in Fig. 33. To find the design point, one simply follows the line of optimum efficiency until the desired magnitude of pressure recovery is reached.

Optimum recovery for a given area ratio, problem (2), is most easily solved by referring to Fig. 25. In Fig. 25 the C_{PR} contours are plotted on $(AR - 1)$ vs. N/R_1 coordinates. The horizontal section of each contour represents the optimum point for each area ratio value. The line of optimum C_{PR} obtained in the way just described has been plotted in Fig. 34.

The solution to problem (3) has been given as line $\alpha-\alpha$ in Figs. 24, 25, and 34.

Finally, optimum problem (4) may be solved by referring to Fig. 24. To obtain best possible recovery, one simply follows

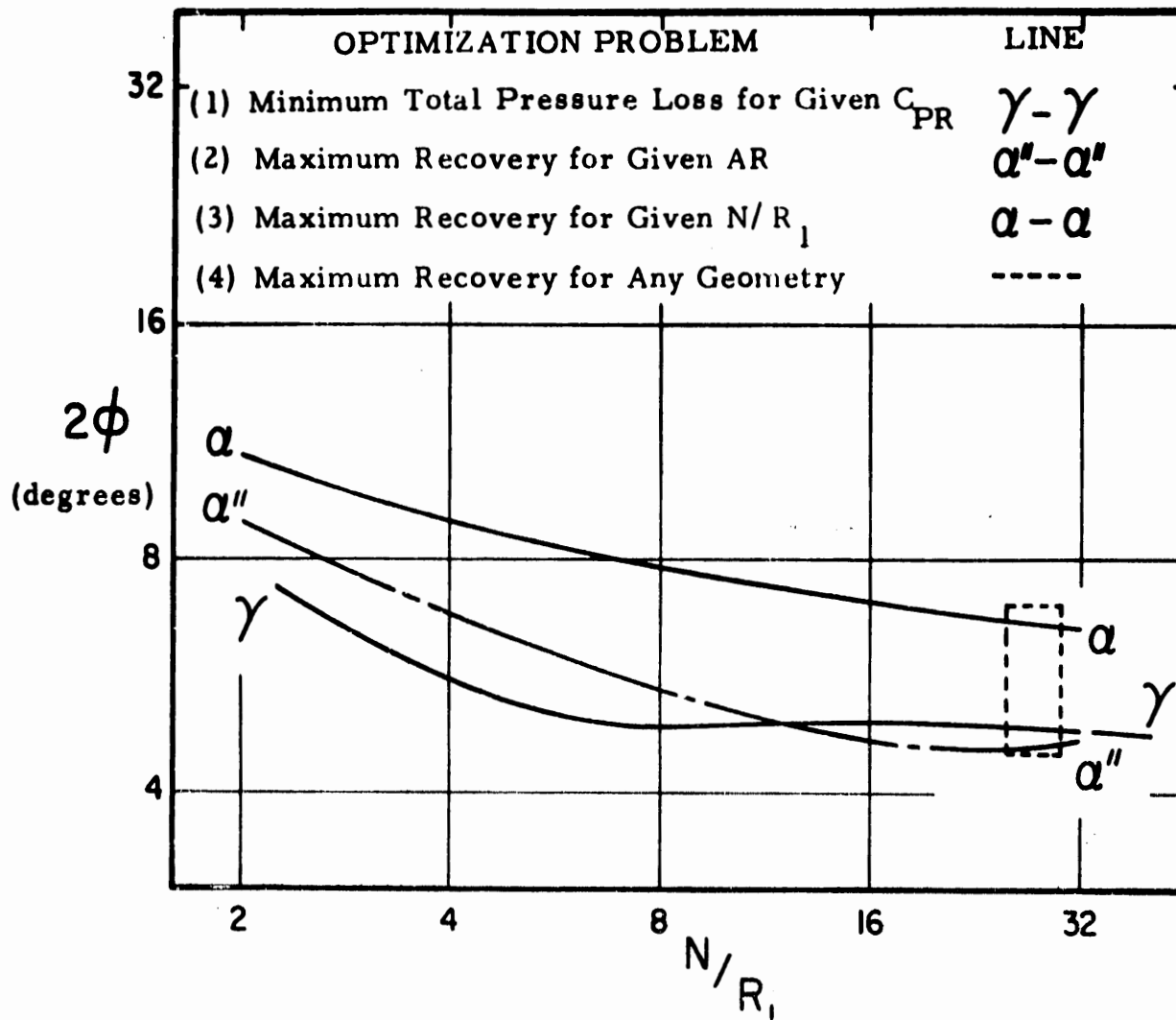


Figure 34. Locations of Solutions to the Four Common Optimization Problems Encountered in Diffuser Design.

the line of optimum C_{PR} toward the right. For the range of geometries tested, the best recovery is 80 percent, and as shown in Fig. 34 it can be obtained with a diffuser configuration having a total divergence angle of from $4\frac{1}{2}$ to 7 degrees and a length of approximately 25 to 30 times its inlet radius. The data tend to indicate that the line of optimum C_{PR} can be extended still further toward higher length ratios, but the numerical value of C_{PR} at higher ratios cannot be determined from the data available.

CONCLUSIONS

Conical diffusers have been investigated in incompressible flow with low turbulence, thin inlet boundary layer entrance conditions and free jet exit conditions. Original experimental data have been presented and the possibility of correlating conical results with previous plane walled diffuser work has been explored.

A literature survey has been presented which includes all previously reported experimental data for incompressible flows in conical diffusers. The limitations of the existing data have been pointed out and the need for further experimental work has been clarified.

The following conclusions form the principal results of the study:

1. For low Reynolds numbers ($Re < 30,000$), recovery in conical diffusers is extremely poor. Complete separation of the fluid from the diffuser wall takes place for even moderate divergence angles.

2. For flows typically found in practical applications of diffusers ($Re > 75,000$), the pressure recovery, efficiency and flow regime are independent of Reynolds number for incompressible flow. These performance parameters are highly dependent on diffuser geometry.

3. The line of first appreciable stall has been located for the diffusers tested. Although some latitude in placing the

line has been allowed, it is located precisely enough to be used in diffuser design work.

4. Sufficient data have been presented to allow solution of the four common optimization problems encountered in diffuser design. These solutions are good for the flow conditions investigated, but at the present time extrapolation of the results to diffuser geometries with exit lengths cannot be accomplished with any degree of assurance.

5. The optimums located for conical diffusers are not the same as those suggested by data from plane walled diffusers. The correlations which preserve the diffuser angle and length ratio, as suggested by plane walled diffuser studies, are not satisfactory.

6. Conical diffusers can be designed for optimum performance; under optimum design conditions these diffusers will be free from flow separation.

BIBLIOGRAPHY

1. Ackeret, J., "Sucking of the Boundary Layer," *Zeitschrift des Vereines deutscher Ingenieure*, vol. 35, 1926, pp. 1153-1158 (translated as NACA TM 395).
2. Allen, C. M., and Winter, I. A., "Comparative Tests on Experimental Draft-Tubes," *Transactions of the American Society of Civil Engineers*, vol. 87, Paper No. 1544, 1924, pp. 893-970.
3. Andres, K., "Versuche uber die Umsetzung von Wassergeschwindigkeit in Druck," *Zeitschrift des Vereines deutscher Ingenieure*, Band 54, Nr. 38, 17 September 1910, pp. 1585-1590, cont'd. in Nr. 39, 24 September 1910, pp. 1637-1641.
4. Archer, W. H., "Experimental Determination of Loss of Head Due to Sudden Enlargement in Circular Pipes," *Transactions of the American Society of Civil Engineers*, vol. 76, 1913, p. 999.
5. Borda, J. C., "Memoir on the Flow of Fluids Through Orifices of Vessels," *Memoirs del' Academie Royale Des Sciences*, Paris, 1769, p. 592.
6. Cockrell, D. J. and Markland, E., "Effects of Inlet Conditions on Incompressible Flow through Conical Diffusers," *Journal of the Royal Aeronautical Society*, vol. 66, January 1962, pp. 51-52.
7. Cockrell, D. J. and Markland, E., "A Review of Incompressible Diffuser Flow," *Aircraft Engineering*, vol. 34, no. 10, October 1963, pp. 286-292.
8. Copp, M. R., "Effects of Inlet Wall Contour on the Pressure Recovery of a 10° 10-inch-inlet-diameter Conical Diffuser," *NACA Research Memorandum L51E11a*, September 1951.
9. Darcy, H., "Recherches experimentales relatives au Mouvement de l'Eau dans les Tuyaux," Paris, 1857.
10. Donch, F., "Divergente und konvergente turbulente Stromungen mit kleinen Offnungswinkeln," *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, Heft 281, 1926.

11. Dubs, H. R., "Method for Designing Concrete Draft Tubes," Engineering Record, August 9, 1913.
12. Eck, W., Technische Stromungslehre, 4th Ed., 1954, p. 235.
13. Eiffel, G., "Note on the Calculation of the Efficiency Coefficients of Air Channels," Paris, 1918 (in French).
14. Eytelwin, Handbuch der Mechanik fester Korper und der Hydraulik, Berlin, 1800.
15. Filippov, G. V., "On Turbulent Flow in the Entrance Length of a Straight Tube of Circular Cross Section," Zhurnal Tekhicheskoi Fiziki, vol. 28, p. 1823, August 1958 (transl. in: Soviet Physics, Technical Physics, vol. 3, no. 8, August 1958, Section B, pp. 1681-1686).
16. Fliegner, A., "Einige Versuche uber das Ausstromen der Atmospharischen Luft durch gut abgerundete Mundungen," Schweizerische Bauzeitung, vol. 31, 1895.
17. Fox, R. W. and Kline, S. J., "Flow Regimes in Curved Subsonic Diffusers," ASME Paper No. 61-WA-191, 1961.
18. Francis, J. B., Lowell Hydraulic Experiments, 3rd Ed., D. van Nostrand Co., New York, 1871 (First Edition published 1855), pp. 1 - 7, 209 - 232.
19. Fraser, H. R., "Study of an Incompressible Turbulent Boundary Layer in a Conical Diffuser," Ph. D. Thesis, Department of Theoretical and Applied Mechanics, University of Illinois, Urbana, July 1956.
20. Fraser, H. R., "The Turbulent Boundary Layer in a Conical Diffuser," Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, vol. 84, no. HY3, June 1958, Paper No. 1684.
21. Furuya, Y., "Experiments and Theory on Flow in the Diffuser," Nagoya University, Japan, Memoirs of the Faculty of Engineering, vol. 10, no. 1, May 1958, pp. 1-41.
22. Furuya, Y. and Sato, T., "Pressure Recovery and Efficiency of Short Conical Diffusers and of Roughened Diffusers," Bulletin of the Japanese Society of Mechanical Engineers, vol. 3, no. 12, 1960, pp. 437-443.

23. Gibson, A. H., "On the Flow of Water through Pipes and Passages Having Converging or Diverging Boundaries," *Proceedings of the Royal Society, London, Series A*, vol. 83, no. A563, March 2, 1910, pp. 366-378.
24. Gibson, A. H., "On the Resistance to Flow of Water through Pipes or Passages Having Divergent Boundaries," *Transactions of the Royal Society of Edinburgh*, vol. 48, Part I, no. 5, 1911, pp. 97-116.
25. Gibson, A. H. and Labrow, S., "The Efficiency of Regain in Straight and Bend Draft-Tubes," *Institution of Civil Engineers, Selected Engineering Papers No. 34*, 1926.
26. Gratzner, L. B. and Smith, R. H., "Boundary Layer Control for Wide Angle Diffusers," Report No. 300, University of Washington Aeronautical Laboratory, November 1948.
27. Heenan and Gilbert, *Minutes of the Proceedings of the Institution of Civil Engineers*, vol. 123, Part I, 1895-1896, pp. 272-294; discussion, pp. 295-325.
28. Henry, J. R., "Design of Power-Plant Installations. Pressure Loss Characteristics of Duct Components." NACA Wartime Report L-208, 1944 (formerly NACA Advance Research Report L4F26).
29. Henry, J. R., Wood, C. C. and Wilbur, S. W., "Summary of Subsonic Diffuser Data," NACA Research Memorandum L56F05, October 1956.
30. Herschel, C., "The Venturi Meter: An instrument making use of a new method of gauging water; applicable to the cases of very large tubes, and of a small value only, of the liquid to be gauged," *Transactions of the American Society of Civil Engineers*, vol. 17, Paper No. 371, November 1887, pp. 228-259, and vol. 18, Paper No. 384, June 1888, pp. 131-141.
31. Herschel, C., The Two Books on the Water Supply of the City of Rome, from Sextus Julius Frontinus (ca. 40-103 a.d.), Boston, 1899, p. 205.
32. Hillberg, A. G., "Design of Turbine Draft Tubes Analyzed, Parts I and II," *Engineering Record*, November 13, 1915, and November 20, 1915.

33. Hochschild, H., "Versuche uber Stromungsvorgange in erweiterten und verengten Kanalen," Forschungsarbeiten auf dem Gebiet des Ingenieurwesens, Heft 114, 1010.
34. Hofmann, A., "Die Energieumsetzung in saugrohranlicherweiten Duzen," Mitteilungen des Hydraulischen Institut der Technischen Hochschule, Munchen, Heft 4, 1931, pp. 44-69.
35. Holzhauser, C. A. and Hall, L. P., "Exploratory Investigation of the Use of Area Suction to Eliminate Airflow Separation in Diffusers Having Large Expansion Angles," NACA Technical Note 3793, October 1956.
36. Hopkins, T., Memoirs of the Literary and Philosophical Society of Manchester, v. 5, Second Series, London, 1931.
37. Hudimoto, B., "On the Turbulent Boundary Layer with Pressure Rise and Fall," Memoirs of the College of Engineering, Kyoto Imperial University, Japan, vol. 9, 1935, pp. 1-17.
38. Jungnitz, G., Rechnerische Untersuchung von Diffusoren," Forschung auf dem Gebiete des Ingenieurwesens, Bild. 16, Heft 2, September/October 1959, pp. 60-62.
39. Kalinske, A. A., "Conversion of Kinetic to Potential Energy in Flow Expansions," Transactions of the ASCE, vol. 111, Paper No. 2273, 1946, pp. 355-390.
40. Kaufmann, K., "Boundary Layer Control on Diffusers of Turbomachines," Voith Forschung & Konstruktion, no. 2, August 1957 (in German).
41. Kerr, S. L., "The Moody Ejector Turbine," Transactions of the ASME vol. 43, no. 1828, 1921, pp. 1201-1217.
42. Kline, S. J., "On the Nature of Stall," Transactions of the ASME, Series D, Journal of Basic Engineering, vol. 81, no. 3, September 1959, pp. 305-320.
43. Kline, S. J., Abbott, D. E. and Fox, R. W., "Optimum Design of Straight Walled Diffusers," Transactions of the ASME Series D, Journal of Basic Engineering, vol. 81, no. 3, September 1959, pp. 321-331.
44. Kratz, A. P. and Fellows, R., "Pressure Losses Resulting from Changes in Cross-Sectional Area in Air Ducts," Bulletin No. 52, Engineering Experiment Station, University of Illinois, Urbana, 1938.

45. Kroner, R., "Versuche über Stromungen in stark erweiterten Kanälen," *Forschungsarbeiten auf dem Gebiet des Ingenieurwesens*, Heft 222, 1920.
46. Lapresle, A., "French Wind-Tunnel Methods," *Journal of the Royal Aeronautical Society*, vol. 33, no. 226, 1929, pp. 838-877.
47. Little, B. H., Jr. and Wilbur, W. S., "Performance and Boundary Layer Data from 12 and 23 deg. Conical Diffusers of Area Ratio 2.0 at Mach Numbers up to Choking and Reynolds numbers to 7.5×10^6 ," *NACA Report 1201*, 1954.
48. Lorenz, H., "Die Energieumsatz divergenter Ströme," *Physikalische Zeitschrift*, Jahrgang 30, Nr. 1, February 1929, pp. 77-80.
49. Lukasiewicz, J., "Diffusers for Supersonic Wind Tunnels," *Journal of the Aeronautical Sciences*, vol. 20, no. 9, September 1953, pp. 617-626.
50. Luksch, A., "An Experimental Study of the Most Favorable Distance of a Water Turbine Draft Tube from the Tailrace," Ph. D. Thesis, State University of Iowa, Iowa City, 1935.
51. Lyon, G. E., "Flow in Conical Draft Tubes of Varying Angles," *Mechanical Engineering*, vol. 44, 1922, pp. 177-180.
52. McDonald, A. T., "Evaluation of Incompressible Turbulent Boundary Layer Prediction Methods Applied to Diffusers," M. S. Thesis, School of Mechanical Engineering, Purdue University, Lafayette, Indiana, June 1962.
53. Megai, V. K., "Investigation of Ribbed Diffusers," *Teploenergetika*, no. 10, 1962, pp. 55-59 (in Russian).
54. Michelotti, G., *Memoires de l'Academie Royale des Sciences de Turin*, 1784-85.
55. Mockmore, C. A., "Flow Characteristics in Elbow Draft Tubes," *Transactions of the American Society of Civil Engineers*, vol. 103, 1938, pp. 402-464.
56. Moody, L. M., discussion to White, W. M., "The White Hydraucone Regainer," *Transactions of the ASME*, 1921, pp. 255-324.

57. Morris, E., "Experiments on the Useful Effect of Turbines in the United States," Franklin Institute. Philadelphia. Journal, December 1843, pp. 377-384.
58. Naumann, K., "Wirkungsgrad von Diffusoren bei hohen Unterschallgeschwindigkeiten," F B Nr. 1705. Deutsche Luftfahrtforschung, Berlin-Aldershof, 1942.
59. Nelson, W. J. and Popp, E. G., "Performance Characteristics of Two 6° and Two 12° Diffusers at High Flow Rates," NACA Research Memorandum L9H09, October 1949.
60. Nikuradse, J., "Untersuchungen über die Geschwindigkeitsverteilung in turbulenten Stromungen," Forschung auf dem Gebiete des Ingenieurwesens, Ausgabe B, Beilage (VDI-Forschungsheft), Heft 281, 1926.
61. Nikuradse, J., "Untersuchungen über die Stromungen des Wassers in konvergenten und divergenten Kanälen," Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, Heft 289, 1929.
62. Nippert, H., "On Flow Losses in Bent Channels," Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, Heft 320, Berlin, 1929 (in German).
63. Patterson, G. N., "Modern Diffuser Design," Aircraft Engineering, vol. 10, 1938, pp. 267-273.
64. Patterson, G. N., "The Design of Aeroplane Ducts: Rules to be Followed for the Reduction of Internal and External Drag," Aircraft Engineering, vol. 11, no. 125, 1939, pp. 263-268.
65. Persh, J., and Bailey, B. M., "Effect of Various Arrangements of Triangular Ledges on the Performance of a 23° Conical Diffuser at Subsonic Mach Numbers," NACA Technical Note 3123, January 1954.
66. Peters, H., "Conversion of Energy in Cross Sectional Divergences under Different Conditions of Inflow," NACA Technical Memorandum No. 737, 1934.
67. Pirie, A. I., "Linear Regression Analysis (Two Variables)," Paper No. 3, Society of Automotive Engineers Special Publication SP-250, April 1964.
68. Pöhlhausen, K., "Zur näherungsgleichen Integration der Differentialgleichung der laminaren Reibungsschicht," Zeitschrift für angewandte Mathematik und Mechanik, vol. 1, 1921.

69. Polzin, J., "Flow Investigations in a Two-Dimensional Diffuser," *Ingenieur-Archiv.*, Bild 11, Heft 5, October 1940, pp. 361-385 (in German) (translated in R. T. P. Translation No. 1286).
70. Prasil, F., "Über Flüssigkeitsbewegung in Rotationsholräumen," *Schweizerische Bauzeitung*, vol. 41, 1903, p. 207.
71. Ramsey, W. K., "A Discussion of Draft-Tube Designs," *Mechanical Engineering*, 1922, pp. 171-176.
72. Rateau, A., *Comptes Rendus de la Soc. de l'Ind.*, 1889, p. 173.
73. Rayle, R. E., "Influence of Orifice Geometry on Static Pressure Measurements," *ASME Paper No. 59-A-234*, 1959.
74. Redtenbacher, Theorie und Bau der Turbinen und Ventilatoren, Mannheim, 1844 and 1860.
75. Reid, E. G., "Performance Characteristics of Plane-Wall Two-Dimensional Diffusers," *NACA Technical Note 2888*, February 1953.
76. Richter, H., "Der Druckabfall in gekrümmten glatten Rohrleitungen," *Forschungsarbeiten auf dem Gebiet des Ingenieurwesens*, Heft 338, 1930.
77. Riffart, A., "Versuche mit Verdichtungsdufen," *Forschungsarbeiten auf dem Gebiet des Ingenieurwesens*, Heft 257, 1922.
78. Rippl, E., "Experimental Investigations Concerning the Efficiency of Slim Conical Diffusers and Their Behavior with Regards to Flow Separation," *Monthly Technical Review*, vol. 2, no. 3, March 1958, pp. 64-70.
79. Robertson, J. M. and Ross, D., "Effects of Entrance Conditions on Diffuser Flow," *Proceedings of the American Society of Civil Engineers*, Separate Volume 78, no. 141, 1952.
80. Robertson, J. M. and Holl, J. W., "Effect of Adverse Pressure Gradients on Turbulent Boundary Layers in Axisymmetric Conduits," *Transactions of the ASME, Series E, Journal of Applied Mechanics*, vol. 24, no. 2, June 1957, pp. 191-196.

81. Rodhouse, T. J., "Effect of Sudden Enlargement on the Flow of Water in Pipes," *Journal of the ASME*, vol. 39, 1917, pp. 169-171.
82. Ross, D., "A New Approach to Turbulent Boundary Layer Problems," *Proceedings of the American Society of Civil Engineers*, vol. 81, Separate No. 604, 1955.
83. Rouse, H., "Laboratory Instruction in the Mechanics of Fluids," *Bulletin, Engineering Experiment Station, State University of Iowa, Iowa City*, 1958.
84. Scherrer, R. and Anderson, W. E., "Preliminary Investigation of a Family of Diffusers Designed for Near Sonic Inlet Velocities," *NACA Technical Note 3668*, February 1956.
85. Schlichting, H., *Boundary Layer Theory*, 4th ed., McGraw-Hill, New York, 1960, pp. 567-584.
86. Schubauer, G. B. and Spangenberg, W. G., "Effect of Screens in Wide-Angle Diffusers," *NACA Technical Report 949*, 1949.
87. Seely, F. B., "The Effect of Mouthpieces on the Flow of Water through a Short Pipe," *Bulletin No. 14, Engineering Experiment Station, University of Illinois, Urbana*, 1917.
88. Shames, I. H., *Mechanics of Fluids*, McGraw-Hill, Inc., New York, 1962, Chapter 5.
89. Shogenji, K., Tanaka, K., Omori, T., and Naramoto, I., "Experiments on the Flow of Water through Diverging Passages of Rectangular Cross Section," *Memoirs of the College of Engineering, Kyushu Imperial University, Fukuoka, Japan*, vol. IV, no. 2, 1927.
90. Smith, R. H., and Wang, Chi-Teh, "Contracting Cones Giving Uniform Throat Speeds," *Journal of the Aeronautical Sciences*, vol. 11, no. 4, October 1944, pp. 356-361.
91. Sprenger, H., "Experimentelle Untersuchungen an geraden und gekrummten Diffusoren," *Institut fur Aerodynamik, Eidgenossischen Technischen Hochschule, Zurich*, Mitteilung Nr. 27, 1959.
92. Squire, H. B., "Experiments on Conical Diffusers," *Great Britain Aeronautical Research Committee, Reports and Memoranda No. 2751*, November 1950.

93. Thurston, R. H., "The Systematic Testing of Turbine Water-Wheels in the United States," Transactions of the ASME, vol. 8, Paper No. 243, 1887, pp. 359-420.
94. Uram, E. M., "The Growth of an Axisymmetric Turbulent Boundary Layer in an Adverse Pressure Gradient," Proceedings of the Second U. S. National Congress of Applied Mechanics, Ann Arbor, Michigan, 1954, pp. 687-695.
95. Valentine, E. F. and Carroll, R. B., "Effects of Some Primary Variables of Rectangular Vortex Generators on the Static Pressure Rise through a Short Diffuser," NACA Research Memorandum L52B13, May 1952.
96. Vedornikoff, A. N., "Luftstromung im flachen, erweiterten Kanal," Kurze Auszuge, Zeitschrift fur Angewandte Mathematik und Mechanik, Band 7, Heft 4, August 1927, pp. 316-317 (translated as NACA Technical Memorandum 1059).
97. Venturi, G. B., "Recherches Experimentales sur le Principe de Communication Lateral dans les Fluides," Paris, 1797 (transl. in Nicholson's Journal of Natural Philosophy, vol. 3, London, 1802, or in Tracts on Hydraulics by Thomas Tredgold, 2d Ed., London, 1836).
98. Waitman, B. A., Reneau, L. R. and Kline, S. J., "Effects of Inlet Conditions on Performance of Two-Dimensional Subsonic Diffusers," ASME Paper No. 60-WA-143, 1960.
99. Warner, E. P., Norton, F. H. and Hebbert, C. M., "The Design of Wind Tunnels and Wind Tunnel Propellers," NACA Technical Report No. 73, 1919.
100. Warren, J., "A Study of Head Loss in Venturi Meter Diffuser Sections," Transactions of the ASME, vol. 73, no. 4, May 1951, pp. 399-402.
101. White, C. M., "Streamline Flow through Curve Pipes," Proceedings of the Royal Society, London, Series A, vol. 123, no. 645, 1929.
102. White, W. M., "The White Hydraucone Regainer," Transactions of the ASME, 1921, pp. 255-324.

103. Winternitz, F. A. L. and Ramsay, W. J., "Effects of Inlet Boundary Layer on Pressure Recovery, Energy Conversion, and Losses in Conical Diffusers," *Journal of the Royal Aeronautical Society*, vol. 61, no. 554, February 1957, pp. 116-124.
104. Yang, T. T., "Splitter Effects in Conical Diffusers," Ph.D. Thesis, Department of Mechanical Engineering, Cornell University, Ithaca, New York, June 1961.
105. Stanton, T. E., "Some Characteristics of the Flow of Water in Channels of Varying Cross-Section," *Engineering*, vol. 74, November 21, 1902, pp. 664, 666.

GENERAL REFERENCES

Ackeret, J., "Boundary Layer in Straight and Curved Diffusers, " Symposium on Boundary Layer Research, H. Gortler, Ed., Springer-Verlag, 1958, pp. 22-40 (in German).

Anonymous, "Draft Tube Tests, " National Electric Light Association, Publication No. 047, 1930.

Anonymous, "Investigation of the Literature on Draft-Tubes for Hydraulic Turbines, Requested by T. V. A., " National Bureau of Standards, U. S. Department of Commerce, 1935.

Baashuus, N., "Loss of Head of Flowing Liquid in Conduits of Continually Varying Section, " *Technische Mechanik und Thermodynamik*, vol. 1, p. 202, 1930 (see VDI Forschungsheft, Ausgabe A).

Ball, J. S., "Discussion of Draft Tubes for Modern Hydraulic Turbines, " *Canadian Engineer*, vol. 64, no. 23, June 6, 1933, p. 5.

Banninger, K., "Studien und Versuche uber Umsetzung von Geschwindigkeit in Druck bei Flussigkeiten unter Berucksichtigung der Diffusoren bei Zentrifugalpumpen, " *Zeitschrift fur des gesamte Turbinenwesen*, January 10, 1906, p. 12.

Bardili, W., Notter, O. and Ibel, G., Bericht des Flugtechnischen Instituts an der Technische Hochschule, Stuttgart: "Wirkungsgrad von Diffusoren, " Jahrbuch der Deutschen versuchanstalt fur Luftfahrt (1939).

Biebel, W. J., "Low Pressure Boundary Layer Control in Diffusers and Bends, " NACA Wartime Report L-84 (formerly NACA Advance Research Report L5C24), 1945.

Bohm, H. and Koppe, M., "The Influence of Friction on Steady Diffuser Flows at High Speed, " Joint Intelligence Objectives Agency, Washington, D. C., July 23, 1946.

Bonnington, S. T., "Some Experiments on Stepped Diffusers," British Hydromechanics Research Association, Research Report 555, March 1957.

Bureau, J. H., "Survey of the Literature on Diffusers," M. S. Thesis, Georgia Institute of Technology, June 1954.

Cochran, D. L. and Kline, S. J., "The Use of Short Flat Vanes for Producing Efficient Wide-Angle Two-Dimensional Subsonic Diffusers," NACA Technical Note 4309, September 1958.

Dryden, H. L., Murnaghan, F. D., and Bateman, H., "Report of the Committee on Hydrodynamics," Bulletin No. 84, National Research Council, 1932, pp. 469-481.

Feil, O. G., "Vane Systems for Very Wide Angle Subsonic Diffusers," ASME Paper No. 64-FE-4, 1964.

Frey, K., "Verminderung des Stromungsverlustes in Kanälen durch Leitflächen," Forschung auf dem Gebiete des Ingenieurwesens, Band 5, Nr. 3, May/June 1934, pp. 105-117.

Furuya, Y. and Suzuki, K., "Experiment on the Efficiency of a Conical Diffuser - The Initial State of Separation of Flow," Transactions of the Japanese Society of Mechanical Engineers, vol. 23, no. 125, January 1957, pp. 7-13.

Gourzhienko, G. A., "The Turbulent Flow in Diffusers of Small Divergence Angle," NACA Technical Memorandum No. 1137, 1939 and 1947 (transl. from Report No. 462, of the Central Aero-Hydrodynamical Institute, Moscow, 1939).

Hall, A. S., "Investigation of Separation in Two-Dimensional Flow," Ph. D. Thesis, School of Mechanical Engineering, Purdue University, Lafayette, Indiana, June 1946.

Henderson, F. D., "Effect of Profile and Length on the Efficiency of Pump Diffusers," Great Britain, Ministry of Supply, Royal Propulsion Establishment, Technical Note No. 181, September 1959.

Herzog, J., "Untersuchungen an Ringdiffusoren," Maschinenbau und Warmewirtschaft, Jahrgang 11, Heft 7, July 1956, pp. 189-191.

Holdhusen, J. S., "Model Experiments for the Design of a 60-inch Water Tunnel; Part IV, Diffuser Studies," University of Minnesota, BuShips Contract NObs-34208, Project Report No. 13, September 1948.

Hudimoto, B., "The Experimental Results of Diverging Flow," Bulletin of the Engineering Institute, Kyoto University, Japan, vol. 2, 1951, pp. 18-23 (in Japanese).

Hunt, H. J., "Progress in Draft Tube Design and Performance," Civil Engineering, vol. 30, no. 3, March 1959, pp. 40-43.

Johnston, I. H., "The Effect of Inlet Conditions on the Flow in Annular Diffusers," Great Britain Aeronautical Research Council, Current Papers, No. 178, January 1953.

Kashkarov, V. P., "Jet Motion of a Viscous Liquid in a Conical Diffuser," Akademiia Nauk Kazakhskoi, S. S. R., Alma-Ata., Izvestiia, Serii Matematiki i Mekhaniki, vol. 6, no. 10, 1957, pp. 20-26.

Kline, S. J., Moore, C. A. and Cockran, D. L., "Wide-Angle Diffusers of High Performance and Diffuser Flow Mechanisms," Readers' Forum, Journal of the Aeronautical Sciences, vol. 24, no. 6, June 1957, pp. 469-470.

Kline, S. J., "Some New Conceptions on the Mechanism of Stall in Turbulent Boundary Layers," Readers' Forum, Journal of the Aeronautical Sciences, vol. 24, no. 6, June 1957, pp. 470-471.

Kmonicek, V., "Unterschallstromung in Kegeldiffusoren," Revue de Mecanique Applique, Bucharest, vol. 6, no. 6, June 1961, pp. 699-706.

Liepe, F., "Wirkungsgrade von schlanken Kegeldiffusoren bei drallbehafteten Stromungen," Maschinenbautechnik, vol. 9, no. 8, 1960, pp. 405-412.

McClellan, C. H. and Nichols, M. R., "An Investigation of Diffuser-Resistance Combinations in Duct Systems," NACA Wartime Report L-329, February 1942.

Magyar, F. and Pollak, E., "An Investigation of a Blower Diffuser," Maschinenbau und Warmewirtschaft, vol. 8, no. 5, May 1963, pp. 121-122 (in German).

Mavis, F. T., Luksch, A., and Chang, H., "Hydraulic Tests of Small Diffusers," Bulletin No. 13, Studies in Engineering, State University of Iowa, Iowa City, 1938.

Migay, V. K., "The Efficiency of a Cross-Ribbed Curvilinear Diffuser," *Energomashinostroyeniye*, No. 1, 1962, pp. 45-46 (in Russian), Translated as FTD-TT-62-1151/1+2+4, 11 September 1962.

Milliat, J. P., "Experimental Study of Turbulent Air Flow in a Divergent Pipe," *La Houille Blanche*, Special No. B, 1956, pp. 497-517 (in French).

Milliat, J. P., "Etude Experimentale de l'Ecoulement Turbulent dans un Divergent Bidimensionnel Parcouru par de l'Air," France, Ministry de l'Air, Publications Scientific et Technique 335, 1957.

Moody, L. F., "The Present Trend in Turbine Development," *Transactions of the ASME*, 1921, pp. 1113-1140.

Moon, I. M., "Distributed Suction Boundary-Layer Control on Conical Diffusers," Report on ONR Contract Nonr 978(01), Aerophysics Department, Mississippi State University, State College, Research Report No. 17, August 1958.

Moore, C. A. and Kline, S. J., "Some Effects of Vanes and of Turbulence in Two-Dimensional Wide-Angle Subsonic Diffusers," NACA Technical Note 4080, June 1958.

Nelson, W. J. and Popp, E. G., "Performance Characteristics of Two 6° and Two 12° Diffusers at High Flow Rates," NACA Research Memorandum L9HO9, October 1949.

Norbury, J. F., "Some Measurements of Boundary Layer Growth in a Two-Dimensional Diffuser," *Transactions of the ASME, Series D, Journal of Basic Engineering*, vol. 81, No. 3, September 1959, pp. 285-296.

Ovchinnikov, O. N., "Effect of Inlet Velocity Profile on the Performance of a Diffuser," *Trudy L. P. I. Energomashinostroenie* 176, pp. 175-188, 1955. (First part of book translated as D. S. I. R. NLL Translation No. M 1841).

Pearsall, I. S. and McNulty, P. J., "Cavitation Inception and Efficiency of a Two-Dimensional Diffuser," Great Britain, Department of Scientific and Industrial Research, Mechanical Engineering Research Laboratory, Fluids Report No. 36, June 1955.

Persh, J., "The Effect of Surface Roughness on the Performance of a 23° Conical Diffuser at Subsonic Mach Numbers, " NACA Research Memorandum L51K09, January 1952.

Persh, J. and Bailey, B. M., "Effect of Surface Roughness over the Downstream Region of a 23° Conical Diffuser, " NACA Technical Note 3066, January 1954.

Ringleb, F. O., "Two-Dimensional Flow with Standing Vortices in Ducts and Diffusers, " Transactions of the ASME, Series D, Journal of Basic Engineering, vol. 82, no. 4, December 1960, pp. 921-928.

Ripkin, J. F., "Design Studies for a Closed-Jet Water Tunnel, Chapter IV, Diffuser Studies, " University of Minnesota, St. Anthony Falls Hydraulics Laboratory, Technical Paper No. 9, Series B, August 1951, pp. 67-82.

Robertson, J. M. and Ross, D., "Water Tunnel Diffuser Flow Studies, Part I: Review of Literature, " Ordnance Research Laboratory, Pennsylvania State University, Report 7958-139, May 1949.

Robertson, J. M. and Fraser, H. R., "Separation Prediction for Conical Diffusers, " Transactions of the ASME, Series D, Journal of Basic Engineering, vol. 82, no. 1, March 1960, pp. 201-209.

Ross, D. and Robertson, J. M., "Water Tunnel Diffuser Flow Studies, Part III: Analytical Research, " Pennsylvania State University, Ordnance Research Laboratory, Report Nord 7958-230, 1952.

Ruden, P., "Wind Tunnel Measurements in a Rotationally Symmetrical Entry Diffuser, " National Research Council of Canada, Technical Translation No. TT-23, 14 August 1947.

Schlichting, H. and Gersten, K., "Calculation of the Flow in Rotationally Symmetric Diffusers with the Aid of Boundary Layer Theory, " Zeitschrift fur Flugwissenschaften, vol. 9, no's. 4/5, April/May 1961, pp. 135-140 (in German).

Scott-Wilson, J. B. and Llewelyn-Davies, D., "The Development of an Improved Diffuser for the 3 x 3 Foot Tunnel, Royal Aircraft Establishment, Bedford, " Royal Aircraft Establishment Technical Notes No. Aero 2446, July 1956.

Sinbel, M. A., "Flow in Conical Passages," Ph. D. Thesis, University of London, 1953.

Sprenger, H., "Messungen an Diffusoren," Verein deutscher Ingenieure, Berichte, vol. 3, 1955, pp. 109-110.

Squire, H. B. and Hogg, H., "Diffuser-Resistance Combinations in Relation to Wind Tunnel Design," Report No. Aero 1933, British Royal Aircraft Establishment, 1944.

Stanitz, J. D., "Aerodynamic Design of Efficient Two-Dimensional Channels," Transactions of the ASME, vol. 75, 1953, pp. 1241-1255.

Stanton, T. E., "Some Characteristics of the Flow of Water in Channels of Varying Cross Section," Engineering, vol. 74, Nov. 21, 1902, pp. 664, 666.

Stratford, B. S., "Turbulent Diffuser Flows," Great Britain, Aeronautical Research Council, Current Papers, No. 307, 1956.

Tults, H., "Wirkungsgradverbesserung der Druckumsetzung in divergenten Stromungen," Patentschrift Nr. 834680, Kl. 88a, Gr. 12, 1952.

Tults, H., "Flow Expansion and Pressure Recovery in Fluids," Transactions of the ASCE, Paper No. 2787, vol. 121, 1956, pp. 65-84.

Vallentine, E. F. and Carroll, R. B., "Effects of Several Arrangements of Rectangular Vortex Generators on the Static-Pressure Rise Through a Short 2:1 Diffuser," NACA Research Memorandum L50L04, 1951.

Vullers, H., "Ausnutzung der kinetischen Austrittsenergie der Luft bei Ventilatoren mittels Diffusoren," Zeitschrift des Vereines deutscher Ingenieure, Band 77, Nr. 31, 5 August 1933, pp. 847-849.

Wilbur, S. W. and Higginbotham, J. T., "Investigation of Two Short Annular Diffuser Configurations Using Suction and Injection as a Means of Boundary Layer Control," NACA Research Memorandum L54K18, January 1955.

Wilbur, S. W. and Higginbotham, J. T., "Investigation of a Short Annular Diffuser Configuration Utilizing Suction as a Means of Boundary Layer Control," NACA Technical Note 3996, June 1957.

Winter, H., "Stromungsverhältnisse in einem Diffusor mit vorgeschalteten Krummer, " *Maschinenbau und Warmewirtschaft*, vol. 8, no. 2, February 1953, pp. 33-35.

Wood, C. C., "Preliminary Investigation of the Effects of Rectangular Vortex Generators on the Performance of a Short 1.9:1 Straight Wall Annular Diffuser, " NACA Research Memorandum L51G09, October 1951.

Wood, C. C. and Higginbotham, J. T., "Performance Characteristics of a 24° Straight-Outer-Wall Annular-Diffuser-Tailpipe Combination Utilizing Rectangular Vortex Generators for Flow Control, " NACA Research Memorandum L53H17a, 1953.

Wood, C. C. and Higginbotham, J. T., "Flow Diffusion in a Constant Diameter Duct Downstream of an Abruptly Terminated Center Body, " NACA Research Memorandum L53D23, 1953.

Wood, C. C., and Higginbotham, J. T., "The Influence of Vortex Generators on the Performance of a Short 1.9:1 Straight Wall Annular Diffuser with a Whirling Inlet Flow, " NACA Research Memorandum L52L01a, February 1953.

Wood, C. C. and Higginbotham, J. T., "Effects of Diffuser and Center Body Length on Performance of Annular Diffusers with Constant Diameter Outer Walls and with Vortex Generator Flow Controls, " NACA Research Memorandum L54G21, September 1954.

Woollett, R. R., "Preliminary Investigation of Short Two-Dimensional Subsonic Diffusers, " NACA Research Memorandum E56C02, May 1956.

Worster, R. C., "The Efficiency of Diffusers and Some Test Results on the Effect of Wall Roughness, " British Hydromechanics Research Association, Research Report 554, March 1957.

Young, A. D. and Green, G. L., "Test of High-Speed Flow in Diffusers of Rectangular Cross-Section, " Great Britain Aeronautical Research Council, Reports and Memoranda No. 2201, July 1944.

APPENDIX A

Applications of Conservation Equations

To illustrate partially why so many different diffuser performance factors have been defined, it is first convenient to set forth the basic mass, momentum and energy conservation equations. Many of the performance definitions were suggested by these equations.

Consider a conical diffuser of inlet and outlet area A_1 and A_2 respectively, as shown in the sectional view of Fig. A1. The

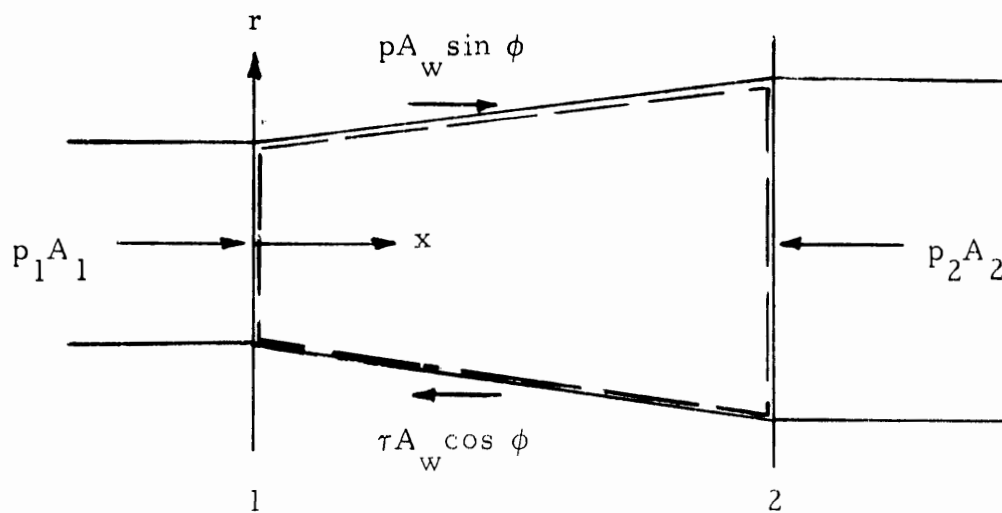


Figure A1. Control Volume and Coordinate System Used for Analysis of Diffuser Flow.

control volume represented by the dashed line in Fig. A1 has been chosen for application of the basic equations for conservation of mass, momentum and energy. The sign convention is that the outward-drawn normal is positive, while velocities are positive or negative according to their orientation in the coordinate system. Surface

forces acting on the control volume are also sketched in the figure.

Conservation of Mass

The equation of conservation of mass as applied to a control volume may be written [88]

$$\oint (\rho \vec{V} \cdot d\vec{A}) + \frac{\partial}{\partial t} \iiint \rho d(\text{vol.}) = 0 \quad (1)$$

For a one-dimensional flow this may be simplified to

$$\rho_1 A_1 \bar{U}_1 = \rho_2 A_2 \bar{U}_2 \quad (1a)$$

and when the further assumption is made that the flow is incompressible, equation (1a) becomes

$$A_1 \bar{U}_1 = A_2 \bar{U}_2 \quad (1b)$$

Conservation of Linear Momentum

The linear momentum equation for the control volume is

$$\sum \vec{F}_s + \iiint \vec{B} \rho d(\text{vol.}) = \oint \vec{U} (\rho \vec{U} \cdot d\vec{A}) + \text{Unsteady Flow Term} \quad (2)$$

which becomes, on substitution of the control volume surface forces,

$$p_1 A_1 + \iint_{A_w} p \sin \phi \, dA - p_2 A_2 - \iint_{A_w} \tau_w \cos \phi \, dA =$$

$$\oint \vec{U} (\rho \vec{U} \cdot d\vec{A})$$

For a one-dimensional flow, this equation immediately simplifies to

$$p_1 A_1 + \iint_{A_w} p \sin \phi \, dA - p_2 A_2 - \iint_{A_w} \tau_w \cos \phi \, dA =$$

$$\rho_2 A_2 \bar{U}_2^2 - \rho_1 A_1 \bar{U}_1^2 \quad (2a)$$

or

$$p_1 A_1 - p_2 A_2 + \iint_{A_w} (p \sin \phi - \tau_w \cos \phi) \, dA =$$

$$\dot{m} (\bar{U}_2 - \bar{U}_1) \quad (2b)$$

Without detailed knowledge of the boundary layer development in the diffuser, p and τ cannot be computed; the momentum equation then contains two unknowns for the case of the diffuser flow. If the flow is frictionless, the wall shear force term is zero. For the frictionless case, combining equations (1b) and (2b) gives an identical result to that of the Bernoulli equation: the pressure and velocity at any cross section depend only upon their values at the inlet of the diffuser and upon the area of the cross section.

However, the momentum equation is useful in the analysis of flow through a sudden expansion, even with friction. If the jet of

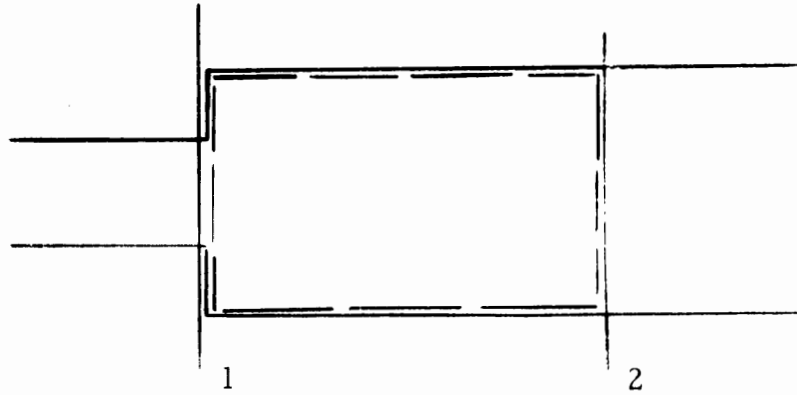


Figure A2. Control Volume for Analysis of Flow in a Sudden Expansion.

fluid issuing from 1 is assumed to enter the larger cross section at p_1 , and if p_2 is assumed to act on the remainder of the control surface, then equation (2b) becomes

$$p_1 A_1 + p_2 (A_2 - A_1) - p_2 A_2 = \rho_2 A_2 \bar{U}_2^2 - \rho_1 A_1 \bar{U}_1^2 \quad (3a)$$

or, making use of equation (1a)

$$(p_1 - p_2) A_1 = \rho_1 A_1 \bar{U}_1 (\bar{U}_2 - \bar{U}_1)$$

$$\text{or } p_2 - p_1 = \rho_1 \bar{U}_1 (\bar{U}_2 - \bar{U}_1)$$

For incompressible flow,

$$\frac{p_2 - p_1}{\rho} = \bar{U}_1 (\bar{U}_2 - \bar{U}_1) \quad (3)$$

Conservation of Energy

The energy equation as applied to a control volume is

$$\dot{Q} - \dot{W}_s = \oint \left(\frac{U^2}{2} + gz + u + \frac{p}{\rho} \right) (\rho \vec{V} \cdot d\vec{A}) + \frac{\partial}{\partial t} \iiint \rho e d(\text{vol}) \quad (4)$$

After simplifying for the diffuser control volume, and assuming that there is no variation of internal energy across either the inlet or outlet sections, equation (4) becomes

$$\dot{Q} + (u_1 - u_2) \dot{m} = \int_{A_2} \left(\frac{U^2}{2} + p/\rho \right) (\rho V) dA - \int_{A_1} \left(\frac{U^2}{2} + \frac{p}{\rho} \right) (\rho V) dA$$

Little error is involved in assuming the quantity p/ρ to be uniform across the entrance and exit sections. When this is done, and the sum of the heat transfer and internal energy changes is recognized as the "head loss" per unit mass of flowing fluid, the equation may be written

$$h_L = \left(\frac{p_2 - p_1}{\rho} \right) \dot{m} + \int_{A_2} \left(\frac{1}{2} U^2 \right) (\rho U) dA - \int_{A_1} \left(\frac{1}{2} U^2 \right) (\rho U) dA \quad (4a)$$

The local value of U can differ appreciably from $\bar{U}$, and a sizeable error can result from assuming one-dimensional flow. To retain the simplicity of the one-dimensional expression, though, define

$$\alpha \equiv \frac{\int_A \left(\frac{1}{2} U^2 \right) (\rho U) dA}{\frac{1}{2} \rho \bar{U}^3 A} \quad (5)$$

so that

$$\int_A \left(\frac{1}{2} U^2 \right) (\rho V) dA = \alpha \frac{1}{2} \rho \bar{U}^3 A = \alpha \dot{m} \frac{1}{2} \bar{U}^2$$

and equation (4a) becomes

$$h_l = \frac{P_2 - P_1}{\rho} + \alpha_2 \frac{1}{2} \bar{U}_2^2 - \alpha_1 \frac{1}{2} \bar{U}_1^2 \quad (4b)$$

Making a final substitution from equation (1b), it is possible to write for the exact energy equation

$$h_l = \frac{P_2 - P_1}{\rho} + \frac{1}{2} \bar{U}_1^2 \left[\alpha_2 \left(\frac{A_1}{A_2} \right)^2 - \alpha_1 \right] \quad (4c)$$

To further simplify equation (4c), it is sometimes possible to assume with little error that $\alpha_1 \approx \alpha_2 \approx 1.0$. Typical observed values for these quantities are $\alpha_1 \approx 1.05$ and $\alpha_2 \approx 1.3$. For large area ratios the kinetic energy term is considerably less at the exit than at the entrance so that, when taking differences, little error results from neglecting the correction factors.

Equation (3b) can be combined with equation (4c) to evaluate the head loss in a frictionless flow through a sudden expansion, with the result

$$h_l = \frac{1}{2} (\bar{U}_1 - \bar{U}_2)^2 \quad (5)$$

Definition of Diffuser Performance Parameters

The performance of a diffuser may be specified in a variety of ways. The most convenient specification in any given situation might be very awkward in another situation: a suitable definition for turbomachine diffusers might not be convenient for wind tunnel diffusers, etc. This situation has led to a proliferation of performance parameters in the diffuser literature.

It is common in the literature to refer to diffuser "efficiency" as the ratio of a desirable characteristic of a diffuser to the amount of energy available for conversion. Each of the many ways of specifying energy suggested by the basic equations has been used, and others have been defined to suit special cases. Thus, although the term

"efficiency" usually represents a standard, a multitude of definitions exists. In order to avoid confusion or ambiguity, each definition must be specified by suitable adjectives. Since any such classification scheme soon becomes entirely too wordy for repetitive use, a convenient index notation has been developed. Its development and use are described in the following sections.

Performance Factors

Depending on the nature of the diffuser application, it is convenient to specify its absolute performance by any of several factors. Factors in common use are shown in table A1. As previously shown, the exact and approximate total pressure losses are obtained from the energy equation upon substituting measured static pressure values.

	Description	Symbol	Equation
Gain Factors	Actual static pressure rise	Δp	$p_2 - p_1$
	Ideal static pressure rise	Δp_i	$\bar{q}_1 \left[1 - \frac{1}{A_R^2} \right]$
Loss Factors	Actual total pressure loss	ΔP	$p_1 + \alpha_1 \bar{q}_1 - p_2 - \alpha_2 \bar{q}_2$
	Approximate total pressure loss	ΔP_a	$p_1 + \bar{q}_1 - p_2 - \bar{q}_2$

Table A1. Diffuser Performance Factors Defined.

Comparison Factors

The basic conservation equations suggest several possible parameters against which the actual performance of a diffuser may be compared. These "standards" for diffuser performance are defined in Table A2.

Diffuser Efficiencies

Fortunately, not all possible combinations of the diffuser performance factors and comparison factors from tables A1 and A2 have been used. In table A3 are given the combinations which have been frequently used and the symbols which have been defined to represent each of them. In the table, efficiencies are represented by η and loss factors by λ . To evaluate the efficiency corresponding to any given loss factor, use the relationship $\eta = 1 - \lambda$.

Description	Symbol	Equation
Actual inlet kinetic energy	$\bar{q}_1$	$\frac{1}{2} \rho \alpha_1 \bar{U}_1^2$
Approximate inlet kinetic energy	$\bar{q}_{1a}$	$\frac{1}{2} \rho \bar{U}_1^2$
Ideal kinetic energy decrease	$\Delta \bar{q}_i$	$\bar{q}_{1a} (1 - \frac{1}{AR})$
Actual kinetic energy decrease	$\Delta \bar{q}$	$\alpha_1 \bar{q}_1 - \alpha_2 \bar{q}_2$
Approximate kinetic energy decrease	$\Delta \bar{q}_a$	$\alpha_1 \bar{q}_1 - \bar{q}_2$
Ideal static pressure increase	Δp_i	$\bar{q}_{1a} (1 - \frac{1}{AR})$
Approximate inlet total pressure	P_{1a}	$p_1 + \bar{q}_{1a}$
Ideal total pressure decrease (Sudden expansion)	ΔP_i	$\frac{1}{2} \rho (\bar{U}_1 - \bar{U}_2)^2$

Table A2. Diffuser Comparison Factors Defined.

To illustrate the use of Table A3, consider the definitions of several commonly used performance factors. Pressure recovery coefficient is the static pressure rise through the diffuser non-dimensionalized on the average entering dynamic pressure (kinetic energy). Referring to Tables A1 and A2 it is seen that Δp = static pressure rise, and $\bar{q}_{1a}$ = average entering kinetic energy. Then from Table A3, η_{21} = static pressure recovery coefficient. C_{PR} is a commonly used symbol for η_{21} .

Ideal or frictionless static pressure recovery coefficient is defined as the ideal static pressure rise non-dimensionalized

Denominator		Numerator			
	Δp	Δp_i	ΔP	ΔP_a	
$\bar{q}_1$	η_{11}		λ_{13}	λ_{14}	
$\bar{q}_{1a}$	η_{21}	η_{22}		λ_{24}	
$\Delta \bar{q}_i$	η_{31}		λ_{33}	λ_{34}	
$\Delta \bar{q}$	η_{41}				
$\Delta \bar{q}_a$	η_{51}				
Δp_i	η_{61}				
P_{1a}				λ_{74}	
ΔP_i				λ_{84}	

Table A3. Combinations of Diffuser Performance Factors.

on the average entering kinetic energy. Again referring to Tables A1 and A2, it is seen that Δp_i = ideal static pressure rise, so from Table A3, η_{22} = ideal ideal static pressure recovery coefficient (= C_{PRi}).

APPENDIX B

Historical Survey of Diffuser Flow Experiments

Early Uses of Diffusers

The earliest recorded practical use of a diffuser in a fluid flow system was by the ancient Romans. Frontinus^[30] * was appointed "Water Commissioner" of Rome in 97 A.D., and he described the situation in his books on the water supply of the City of Rome^[31].

Water rights were granted to individual citizens by the emperor, and consisted of permission to install into the public mains a calibrated, circular, tubular bronze nozzle. The logical assumption was made that the flow rate from each such nozzle would be identical, since it was not realized that the available head varied along the public main.

The citizens were clever enough to take unfair advantage of a law which measured a quantity of flow by such an indirect method. After discovering that a diverging section increased the flow of water from the "official Government issue" nozzle, they put diffusers on the outlet of the nozzle and then conducted the water where they wanted it. To standardize conditions for everyone, a law was passed requiring that the tube attached to the nozzle of each customer be the same diameter for at least 50 lineal feet from the public water main.

* Superscript numbers in brackets are used in this appendix for reference citations to avoid repetitious use of [] and () on the same line.

The Romans did not know why the diffuser increased the flow of water from a nozzle; it was not until 1738 that Bernoulli accounted analytically for the relationship between velocity and static pressure in ideal fluid flow. Although the problem of flow in varying cross sections attracted the attention of various investigators, no practical use was made of the diffuser principle in the 18th century.

Investigations of Converging and Diverging Tubes

Borda^[5] in France, Michelotti^[54] in Italy and Castel (Francis^[18], p. 255) in Germany studied the effects of various contraction shapes on the discharge of water through the side of a reservoir. In addition, Francis mentioned the work of Gravesande on flow coefficients and the better known work of Venturi^[97] on the effects of adding a diverging tube to an orifice.

It was beyond question that a diffuser increased flow through an orifice. With the diverging tube which gave the greatest effect, Venturi's results showed that the flow rate was doubled. After experimenting with many different shaped tubes he suggested a straight conical section with included angle of $4^{\circ} 27'$ as the most effective. Venturi also cautiously predicted that his form of tube would have worked in Rome on the end of a 50 foot pipe, but pessimistically added, "One ought to be suspicious of all hydraulic theories, mine among the rest, so long as experiment has not confirmed them."

Several years later Eytelwein^[14] made some tests to verify Venturi's hypothesis. He agreed that the diverging section increased the flow rate, but he found that the effect diminished as the length of the pipe upstream of the divergent section was increased. Finally, with

20 feet of 1 inch diameter tubing, it made no difference whether the Venturi section was on or off.

Application of Diffusers to Hydraulic Turbines

The industrial revolution focused attention on more practical matters. The rising use of power for mills and other industrial equipment in the early part of the 19th century dictated that power production from natural resources be more widely used. As an illustration of the increasing use of water power, the census of 1880^[93] listed over 55,000 water wheels of various types in the United States. The power production of these wheels exceeded 1.2 million horsepower. It was obvious that the water wheel had received its share of attention from engineers.

Particularly in New England, very large breast water wheels had been the accepted standard of performance. They were attractive because they were powerful and efficient, but could still work on limited head. Francis wrote of wheels of this type 30 feet in diameter with buckets 12 feet wide. Energy conversion efficiencies approached 60 percent. These enormous installations were practical ~~only for~~ large mills; there were also numerous small reaction wheels of various types. These small reaction devices were rather inefficient, but since they were high speed machines, they were very small and compact. These features made them much easier and cheaper to install, and, wrote Francis, "in a country where water power is so much more abundant than capital, the economy of money was generally of greater importance than the saving of water," so their relatively low efficiency could be tolerated.

The design and construction of the widely used small wheels were labors on which considerable care was lavished. They were continually being modified, with the result that several hundred patents were issued for the development of new design features even before 1840. Greater economic pressures became important as more and more installations were made.

The water rights at a given location were usually controlled by a corporation whose authority to rent water power was granted by acts of the state legislature. These companies were directed by good businessmen interested that a variety of manufacturers be located in each spot, and that the available power be used to the best advantage. Power was sold to the user on the basis of "mill-power," which was a variable quantity. A typical value was that at Lowell, Massachusetts: 30 cubic feet per second under 25 feet of head. Since he paid for each cubic foot of "potential power," it was in the best interest of the purchaser to extract as much energy from it as possible. This was the primary motivating force toward developing more efficient water wheels.

In Europe, Fourneyron had begun to study the existing turbines as early as 1823 with an eye toward improving their performance. In 1827 he introduced a turbine of his own design together with a procedure for systematically testing the turbine efficiency by using the Prony brake to measure its power output. This was as significant a contribution to the development of the turbine-building industry as his turbine invention.

In 1844 Redtenbacher^[74] proposed a "centripetal reaction turbine." This was the first European device to use a diffuser. It had a slender spike extending into the conical diffuser to act as a reaction element in deflecting the water flow radially away from the diffuser axis.

Certain aspects of the diffuser principle had been investigated also by Hopkins^[36] (1831). His experiments were performed with air discharging between parallel discs, which formed, in effect, a radial diffuser.

Turbine development also was carried out in the U. S. Several articles appeared in the Journal of the Franklin Institute in the early 1840's, calling the attention of American engineers to the turbine improvements which had been made in Europe. Then in 1843 the Journal published the translation by Ellwood Morris^[57] (1843) of "Experiments on Water Wheels having a Vertical Axis, called Turbines" by Arthur Morin of France. This article described in detail three of the large water turbine installations then operating in France, bringing to light the fact that the best of these had achieved an overall efficiency of 75 percent. The best American overshot water wheels were comparable in efficiency to this turbine, but similar efficiencies were not approached by other designs in this country.

In 1844 Uriah A. Boyden designed and installed a turbine at Lowell which "introduced several improvements of great value,"^[18] and it was found to be 78 percent efficient. This efficiency is more remarkable when it is considered that the losses due to the bearings of two lengths of shafting, and the losses from a set of bevel gears

were automatically deducted from the gross output of the turbine by the experimental arrangement.

Two years later two more turbines were built and installed. These were of roughly three times the size of the first "improved" unit. It was found that the mean efficiency of the two units was 88 percent. Chiefly responsible for this large increase of efficiency was "an apparatus, named by Mr. Boyden, the Diffuser."^[18] Refinements were made in the design of the inlet and blading which improved the overall turbine performance, but tests performed on one of these turbines, with and without the diffuser, showed it alone accounted for an efficiency gain of 3 percent.

Boyden had correctly reasoned that a diverging section after the turbine runner would promote an orderly deceleration of the water flow and would reduce somewhat the amount of kinetic energy lost. This energy loss amounted to 20 or 25 percent of the total head available to the turbine for even the best designs. If the diffuser, in its most crude form, resulted in regaining half of this energy in the form of pressure head, the turbine power could be increased by as much as 10 to 12 percent.

The diffuser used on the first Boyden turbines was a radial flow unit. The water left the turbine runner in the horizontal plane of the runner, and flowed radially into the diffuser. The walls of the diffuser were initially like two flat plates placed parallel to one another with a spacing equal to the height of the outlet orifices of the runner. This area was gradually increased until the diameter at the exit of the diffuser was twice that at the inlet, and the height was also twice as

great. The resulting area ratio of 4.0 gave a theoretical diffuser efficiency of 93.8 percent.

In 1854 Francis calculated the theoretical increase in turbine efficiency due to the addition of the diffuser from its dimensions and an energy balance. According to his calculation, the increase for frictionless flow should have been 5 percent, but it amounted in practice to only 3 percent. The efficiency of the diffuser itself, as a regainer of static pressure head, must have been about 60 percent.

It is very doubtful that in 1844 the inventor had any idea of the controversy his invention would cause. He had no inkling that almost 100 years later publication of a paper on diffusers or "draft tubes"* in the ASCE Transactions would be sufficient to generate a storm of comments and discussions. Bitter debate would ensue among the proponents of the various configurations of draft tubes, while tempers flared over claims of turbine efficiency differences of fractions of 1 percent. But this is ahead of the story of the development of diffusers.

Many American and foreign hydraulic laboratories became very interested in developing efficient designs for draft tubes in the late 1890's and early 1900's. As in any device with a great deal of design leeway, many different forms of the original concept were announced. Each draft tube had a section of minimum area downstream of the turbine runner. This was gradually enlarged in cross section causing the fluid to decelerate along the length of the tube. Space limitations soon began to predicate a swing away from straight conical tubes.

* The name "draft tube" is apparently an analogy to the effect of a chimney in increasing the "draft" of a fireplace.

The first comprehensive series of experiments on conical tubes was carried out by J. B. Francis at Lowell, Massachusetts, in 1850. The tests were made with various lengths of conical tubes attached to the discharge side of a large flume, as shown in fig. B1.

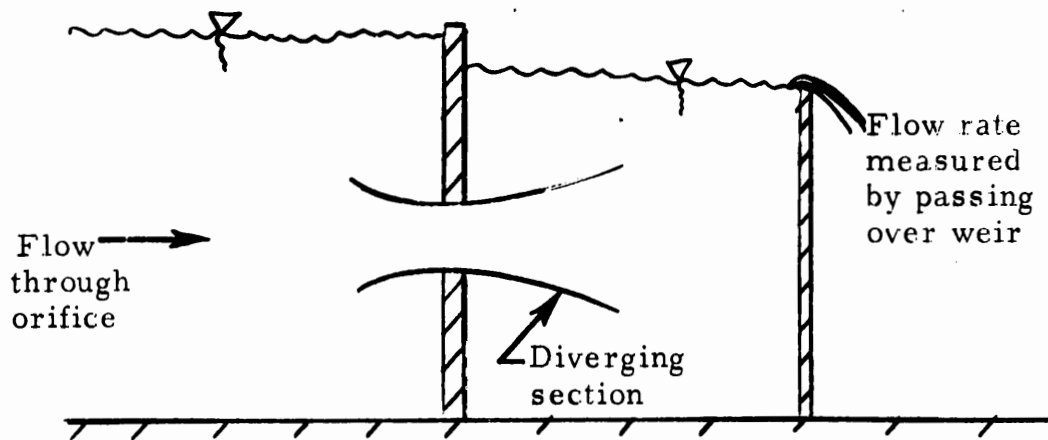


Figure B1. Cross-Sectional View of Flow System Investigated by Francis (1871).

The results of the tests were reported as the flow with the downstream section in place compared to the flow through the orifice alone. Francis had considerable difficulty in rationalizing the results of his experiments. The only analytical tools then available were the Bernoulli theorem and the pioneering work of Darcy^[9] (1857) on the velocity profiles of flow in straight pipes. Francis concluded that the velocity gradients across the channel were responsible for the discrepancy between the measured and calculated results. He wrote, "it is clear that (they) must greatly modify the results deduced from Bernoulli's theory."

Other Uses for Diffusers

In his paper on water wheel testing, Thurston^[93] (1887) commented: that diffusers were very troublesome during turbine testing, that turbines equipped with them were very hard to regulate, and that it was extremely difficult to set them up so as to get all the air out of the draft tube. He implied that the overall effect of the diffuser was not worth the trouble.

Thurston also discussed the accuracy of previous turbine trials. Early tests were frequently not absolutely correct, and they often were not even consistent in any given series to allow comparison of various components. Errors in setting the turbine and in making various measurements were alleged to be the cause of the poor accuracy of efficiency measurements. Thurston regarded as doubtful the spectacular efficiency values recorded earlier for some of the Boyden turbines, among others.

In addition to their use on hydraulic turbines, diffusers found several other practical applications toward the end of the 19th century. Herschel^[30], in his paper describing the use of the Venturi meter for measuring the flow of water in a pipe, mentioned several of these. Included were an aspirator pump which was described in Ingenieur-Archiv in 1873 and the Bourdon Anemometer of 1882. Both devices were similar to the modern Venturi meter, with the former employing the reduced throat pressure in the Venturi to lift water, and the latter its pressure drop for measuring air currents in French mines. Another interesting comment Herschel made in discussing the Boyden turbine

was that the diffuser installed on it was not worth the added cost for so little increase in efficiency. This comment was significant, as Herschel was active in the testing of many turbines at the facility at Holyoke, designed to be the most accurate in the country at that time.

Still other practical arrangements utilizing diffusers were developed. Among these were the use of diffusers in bellows to increase the discharge pressure, and their use in mine ventilating systems. In 1889, Rateau^[72] mentioned the former uses in Europe, and Heenan & Gilbert^[27] reported from England (in 1895) the results of experimental tests of a two-dimensionally diverging duct which was used in a mine ventilating system.

Experimental Research before 1900

Several experimenters reported on facets of the problem of flow in converging or diverging channels before the end of the century. Warner, Norton & Hebbert^[99] credited Fliegner with a formula which adequately related the form loss in a conical diffuser to its divergence angle. Fliegner^[16] (1895) investigated the effects of discharge lengths of various shapes on the flow through an orifice. Warner, et al, also mentioned the correlation for friction losses in diverging ducts due to Fritsch, but unfortunately no reference was given. The last two investigations mentioned were said to be completely summarized by Eiffel^[13] (1908); unfortunately the latter's paper is not available at this writing. The overall results of the study of Eiffel will be discussed later.

Draft Tube Development

During the first 20 years of this century, more than 15 investigations were carried out on various aspects of flow in channels of changing cross section. The importance of the cost of power for individual mills had diminished. With the rise in consumption of electric power, larger and larger hydroelectric plants were being designed and built. Increases in efficiency in these plants could be directly translated into large sums saved through installation of smaller equipment. Research on draft tube designs continued to be important.

In 1903, H. Prasil^[70] of Switzerland applied the basic equations of hydrodynamics to the rational design of a draft tube. He assumed a uniform, non-turbulent, potential inlet flow to a circular draft tube. This allowed him to mathematically determine the optimum flare of the tube walls to secure the desired deceleration of the mean motion required to turn "the water from axial to radial between an outer surface of revolution and a flat bottom surface."

The design method of Dubs^[11] (1913), abstracted in Engineering was originally published in La Houille Blanche. The method was generally of the same form as that proposed by Hillberg (discussed below), but it contained no physical or other justification for the mathematical conditions used.

Hillberg^[32] gave a comprehensive review of design criteria used by him in building draft tubes. He claimed that tubes of this design would eliminate the vibration which had been a recurring problem in hydroelectric stations. The fundamental assumptions of the design

method were a one-dimensional flow and that the change in velocity head was never greater than the elevation change. As a result separation from the walls of the channel was claimed to be eliminated.

As a hydraulic engineer in independent practice, Hillberg was well acquainted with the practical as well as the hydraulic constraints facing the designer. He wrote that the basic geometry of the draft tube was fixed for any given installation. Obviously the inlet diameter was the same as that of the turbine for which the tube was used, but the exit diameter, tube length and curvature were also fixed by economic constraints.

The shape of the exit could be widely varied; the vertical limits were the depth of excavation and the water level required, while the horizontal limits were the thickness of the walls required between draft tubes to support the weight of the power house. The least amount of excavation was required if the exit area was a rectangle and the intervening pier was as thin as permissible in order to carry the load. Hydraulically it was better to have circular cross section. As a circular tube required too much excavation in the tailrace and too much concrete in the powerhouse, a compromise between the structurally and hydraulically best conditions was usually made, and the exit cross section was made elliptical. But, "As elliptical forms are difficult to build, the designer generally uses an area composed of two semicircles with tangents in between..."^[32] For the tube centerline, a segment of a circle was chosen for ease of constructing the form work in the field. The design procedure was completed by specifying the area variation.

Hillberg then went on to show that the area variation along the tube centerline had to be parabolic with depth under the entrance to provide the back pressure discussed above. Powerhouse vibrations were attributed to incomplete filling of the draft tube and to the resulting flow pulsations. The parabolic variation of area along the tube was intended to ensure complete filling and was claimed to eliminate the vibrations.

Experimental Research, 1900-1920

In 1909 Andres^[3] studied the flow of water through a converging-diverging nozzle similar to a modern Venturi flowmeter. Several different cross sections were investigated. These included square and rectangular as well as conical configurations with straight walls and with trumpet-shaped passages. Wall static pressure taps were used at the inlet and outlet planes of each passage tested, but the pressures at other axial locations were measured differently. A very small tube was located on the centerline of the passage, and could be moved axially both upstream and downstream. Static pressure orifices, located along the small tube, could be positioned to determine the pressure variation along the test section. The downstream geometry of the experimental apparatus was not completely clear for all configurations tested. For every configuration there was a 90 degree bend immediately downstream from the test section. The bend radius was about two outlet pipe diameters. Area ratios as high as 21.6 were tested.

In addition to carrying out tests for a uniform axial pipe flow, Andres also investigated several combinations of vanes to produce

swirl and orifice plates and grids to promote turbulent mixing. Screens with two different mesh sizes were installed for some of the tests.

Efficiency data, η_{31}^* , were tabulated from the tests. The results from tests of six of Andres's configurations were of use in the present conical diffuser study.

It should be noted that some of the tests were conducted with the tubes in the un-machined state, while the remainder were finished smoothly. Thus a few data were made available on the effect of roughness on pressure recovery efficiency. It was not possible to compare directly the pressure recovery values obtained with and without swirling flow. The swirl vanes were outside the measuring section. As a consequence, the head loss caused by the vanes was not charged to the diffuser, but was subtracted before the diffuser efficiency was computed.

The results of Andres were overshadowed by the classical results of Gibson^[23,24] reported in 1910 and 1911. The study of Gibson has been called routine by several people (notably Reid^[75]), but it, with the research of Peters^[66] (1934), to be discussed later, has served for many years as the accepted standard. In fact, these were the only two systematic pieces of work carried out on the gross performance of the conical diffuser before 1950.

The study of Gibson had several objectives, the most important of which was to determine the dependence of head loss on cross sectional shape, diffuser, angle and rate of divergence of the diffuser

* See Appendix A for definitions of diffuser performance factors. Hereafter, the conventional symbol, η , will be used for the efficiency, η_{31} .

walls. The experiments reported in 1910 dealt with 25 different test sections, each having an area ratio of 4.0. Included were 15 pipes of conical section which covered a range of total angle between 3 and 180 degrees.

The head loss reported by Gibson was a true loss of total head. The measured values of head loss were compared with the theoretical loss which would occur in a sudden expansion to the larger cross section. For this reason, the values given by Gibson were not directly comparable to those of other investigators; they must be appropriately weighted.

The 1910 results for the area ratio of 4.0 very clearly demonstrated the existence of a minimum in head loss at constant area ratio, and they pointed out two other things, as well. The minimum loss for pipes of circular cross section occurred at a divergence angle of about 5.5 degrees, as shown by the plot of results, fig. B2. The data have been replotted logarithmically to illustrate that optimum performance could be considered to occur over a range of angle. The losses for the rectangular and square cross sections were greater than those for the circular cones.

Gibson^[24] (1911) in his later work reported data for a further 65 different pipes, in a continuation of the first experiments. The objects of the continued investigation were to extend the range of area ratios covered, and to investigate the effect of allowing the smaller pipe to protrude for some distance into the diverging section. Included in these experiments were conical tubes with area ratios of 2.25 and 9.0 in addition to the original 4.0. Again the Reynolds number effect was found to be negligible once some critical value had been exceeded.

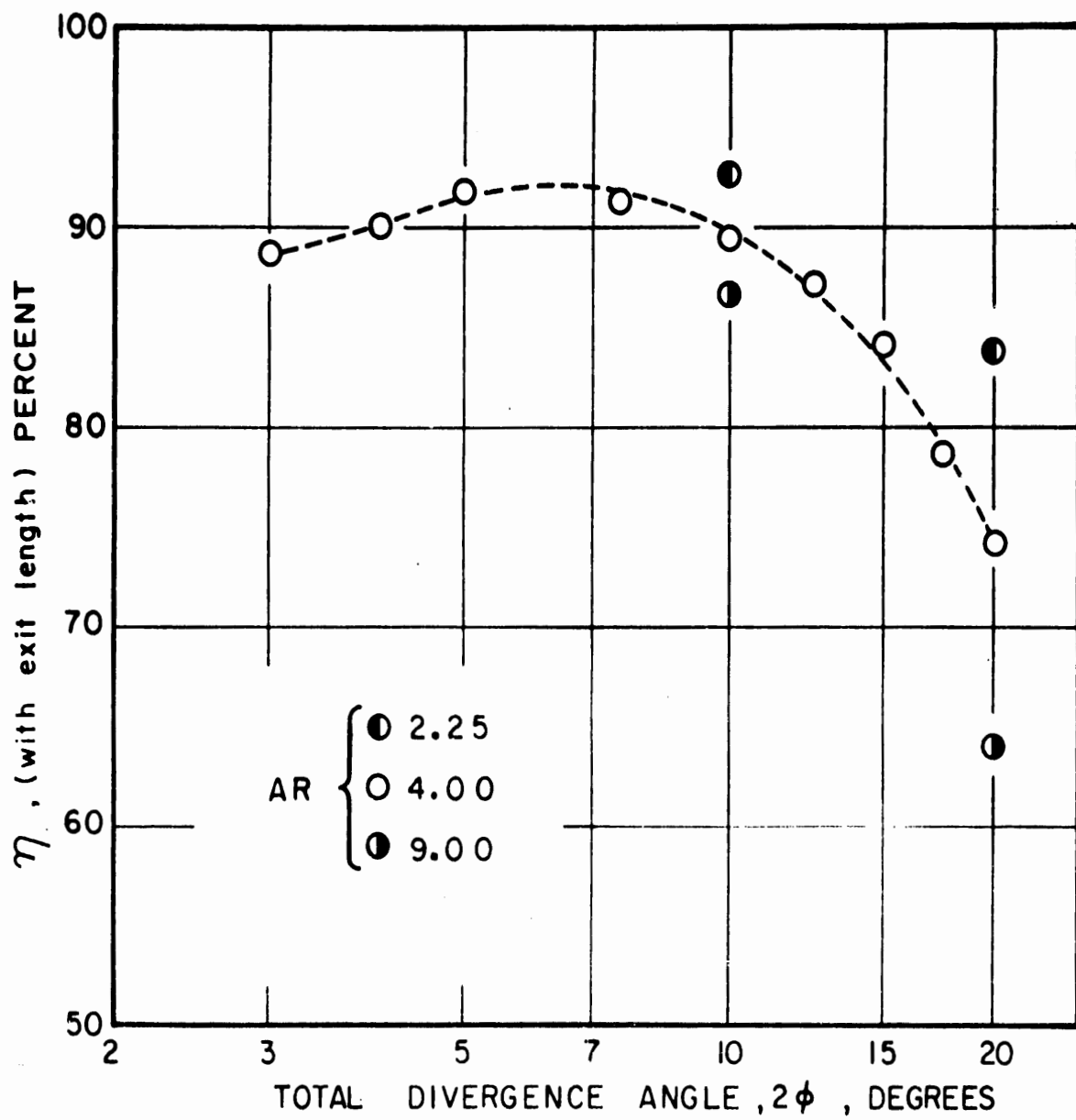


Figure B2. Efficiency Data of Gibson (with Exit Length) vs. Total Divergence Angle.

The effect of area ratio on loss coefficient was negligible at small angles of divergence. The losses in pipes of small area ratio were considerably higher than the 9.0 ratio tests for angles between about 25 and 120 degrees. However, the variation was not entirely consistent. The 4.0 area ratio was found to give the highest loss, with the 2.25 and 9.0 ratios next in order. As in the previous work, maximum head loss occurred in the vicinity of 65 degrees total divergence angle. Near 65 degrees the loss exceeded that of a sudden expansion.

Head loss was satisfactorily represented by an analytical expression over the limited range of divergence angle from 6 to 35 degrees. Projecting the smaller pipe into the diverging cone had no beneficial effect in any of the tests. More careful experiments on the benefits of trumpet-shaped tubes showed that there was a fairly significant decrease in losses as compared to straight cones. The decrease was as large as 20 percent of the head loss of the conical tube.

Hochschild^[33] (1910) carried out investigations in two-dimensional channels which had irregular cross-sections. The basic shape was converging-diverging (similar to a Venturi tube), but the ends were sharply chopped off and reverse curves were faired in to blend the diverging section with the walls of the channel. No pressure recovery data were given; the results of the four configurations tested with water were given as velocity profiles. The information contained in this paper could be used as general background material on the development of turbulent boundary layers, but was of no value to the conical diffuser performance survey.

Wind Tunnel Diffusers

The infant science of aerodynamics received considerable experimental attention during the period 1900-1910. As experimental techniques became more refined, the use of laboratory tests increased, and the advantages of the more accurately controlled conditions offered by wind tunnel testing were apparent. The first open wind tunnels were built by Stanton in 1903 and by Riabouchinsky three years later. These were followed by closed tunnels built by Prandtl in 1907 and Stanton in 1910. None of these tunnels employed diffusers. It remained for Eiffel to incorporate a diffusing section in his first open jet tunnel of 1909. According to Lapresle^[46], who described many of the experimental techniques used in the French work, the use of a diffuser after the test section permitted operating tunnels so equipped with two thirds less power.

As wind tunnel testing became commonplace, more emphasis was given to development of general rules for the design of diffusers in order to reduce the power required to drive the tunnels. After an extensive series of tests on diffusers of straight conical configuration, Eiffel recommended the use of a divergence angle of seven degrees and an area ratio of 9.0 as the optimum values for wind tunnels^[99]. As a result of his experiments, he was also able to map performance as a function of geometry for straight conical diffusers. It was not clear from Warner what entrance conditions or Reynolds number ranges were studied, or how Eiffel defined these efficiencies. Unfortunately, the original work of Eiffel is not available. A little more information can

be obtained from Lapresle^[46] (1929) in "French Wind Tunnel Methods." According to Lapresle, the wind tunnels in use in France at the time were all open circuit, and of the open test section variety. Even in these relatively crude devices the power requirement was reduced two-thirds by the addition of a suitable diffuser.

Of interest were the comparisons given in Warner of the design features of European and American wind tunnels. Even in the earliest designs, such as the Leland Stanford Jr. tunnel, reported in 1918, the exhaust diffusers of some American tunnels were of curved wall shape. It was reasoned in this country that continuity of the fluid deceleration was the most desirable characteristic. Further, it was felt that since the fluid would automatically suffer a considerable instantaneous deceleration upon being discharged into the surrounding quiescent air, there was no need to reduce the acceleration to zero at the exit of the tunnel.

The principal conclusion drawn by Warner et al (1919) was that the energy ratio (energy flux rate of stream in test section divided by input power) of a wind tunnel and the losses in the diffuser were substantially independent of the shape of the exit cone as it developed along the axis. The variables of importance were said to be only the length, mean angle, and total expansion ratio of the diffuser. It was mentioned that the junction between the cylindrical portion of the test section and the diverging exit cone should be faired in with a smooth curve to avoid separation and "turbulence" at that point.

There continued to be tests run to determine the losses of sudden expansions, and of various forms of contractions. Archer

(1913) reported experimental data and computations on losses due to sudden enlargements. His investigation was suggested by work on jet pump phenomena. Rodhouse^[81] (1917) reported on a similar investigation a few years later. In 1917 the flow into various mouthpieces was again investigated by Seely^[87]

Experimental Research, 1920-1929.

Kerr's 1921 paper on the Moody ejector turbine^[41] contained no data on the efficiency of draft tubes alone, as all data were presented in terms of measured turbine output. Some other aspects of diffuser performance discussed were of considerable interest as background to the whole problem. The Moody ejector draft tube was presented as a means of overcoming the difficulty of the decreased net head acting on a hydraulic turbine during flood periods. He first showed that net head really does decrease at flood states due to the increased tailwater depth required to carry the runoff. This draft tube was a modified conical tube with straight taper. Its upper end could be detached from the turbine and moved downward so that an annular slot was exposed. Through this slot water from the surrounding tailrace entered, tending to energize the water at the throat of the draft tube as it came off the turbine wheel. Although the efficiency of the turbine installation under constant head conditions decreased as the ejector slot was opened, the device made it possible to maintain the rated horsepower output of the turbine under conditions when the effective head was lowered. The ejector was of great practical value for low head plants. This seems to be the first recorded practical application of boundary layer control in this country.

The research of Kroner^[45] was another example of the type of test repeatedly carried out during this period. The test channel used by Kroner was square and had inlet and outlet areas smoothly faired into the upstream and downstream sections, which were squares of larger cross section. The overall purpose of the tests was similar to that of Hochschild^[33], and here too the results were presented in the form of velocity profiles. The work contained no information of value in the present conical diffuser study.

After Gibson, Lyon^[51] (1921) was the first to investigate the efficiency of a conically diverging channel. Unlike Gibson, the results of Lyon were for diffusers alone, and they did not rely on an exit length of pipe to increase the apparent pressure recovery. Lyon tested angles between 6 and 12 degrees; he also looked at two area ratios, 4.0 and 9.0. Velocity profiles were determined for several stations along the length of each diffuser, but they were plotted to a very small scale, making it difficult to obtain information from the plots. However, head loss data were also given in tabular form.

Principally apparent from the results in the paper were the facts that for a given area ratio, efficiency was maximum at an angle in the vicinity of 8 degrees total opening, and dependence on Reynolds number was negligible. Lyon also plotted all his efficiency data on one set of coordinates, but this revealed no consistent variation with flow velocity, as it illustrated only the scatter of the experimental points.

The contributions of Moody^[56] and White^[102] (both 1921) were interesting as they both represented the state of the art at the

time they were published. The controversy raging between proponents of the two major types of draft tube* at the time was clearly illustrated by the comments of discussers of these papers. The differences between the Moody and White designs seemed, on looking back over these 40 years, to be more semantic than actual. What did seem to be demonstrated was the efficiency of the axisymmetric type of draft tube as compared with any of the bent types. The amount of swirl remaining in the average turbine's discharge was disputed, but it was conceded that, since the swirl was most pronounced when the turbine was operated at off-design conditions, the bent draft tube carried efficiency penalties under part load operation.

The basic design feature of the White "Hydraucone Regainer" was that the jet leaving the turbine runner impinged on a flat plate approximately normal to the flow direction. Mr. Moody points out the apparent inconsistency in this application since the flow impinging on the plate stagnated and then had to accelerate again. This introduced the possibility of more loss in efficiency. However, the efficiencies of the White and Moody designs were negligibly different. Sometimes one had the edge of performance and sometimes the other had it. Moody sums up his dispute with White in his discussion to the paper of White^[102], where he wrote, "In concluding, the writer would express once more his appreciation of the large amount of research and development work which Mr. White has done in his field, and would also state that he recognizes that the form of draft tube which Mr. White has developed and named 'Hydraucone Regainer' possesses a valuable property in that it is symmetrical about the turbine axis; but the writer feels it to be only

* The two types are: elbow or "bent" tubes, and axisymmetric tubes.

fair that he should take exception to the view that all symmetrical draft tubes are 'Hydraucone Regainers,' and that if a tube is found to be successful it must, of necessity, employ the 'Hydraucone Principle.' "

Ramsey^[71] discussed the general forms of hydraulic passages in water turbines, mentioning for the first time the flow regimes inherent in certain constructions. Professor Daugherty of California Institute of Technology in one of the discussions mentioned the "well known" fact that the efficiency of a conical diffuser was increased by a flat plate inserted normal to the flow discharging from the diffuser (see the discussion of Gibson^[25] 1926 for further information on this point). Of course the basic form of regainer proposed by White^[102] utilized impingement to turn the flow.

The research of Riffart^[77] in 1922 followed closely that of Andres in the layout of the experimental apparatus. The major difference was the direction of the bend following the test section and the use of air instead of water as the flowing medium. Results were presented in several forms, including pressure recovery data and pressure distributions along the test sections. Two sections were tested, one a straight cone and the other with a small divergence angle preceding a sudden increase in angle. The cone angle of the former was 10.6 degrees and the equivalent cone angle of the latter was 7.1 degrees. The measured pressure recovery values were nearly the same for both configurations.

In his paper presented before the ASCE in 1924, Allen^[2] wrote of experimental tests which had been conducted to find the most efficient form of draft tube. Most of these tests were on elbow or

oddly shaped cone draft tubes, and no information contained in the paper was pertinent to the conical diffuser study. However, Herschel (in 1924 a past-President of ASCE) offered an interesting discussion to the paper. Herschel commented on the need for further experiments, and also upon the national need for an endowed hydraulics laboratory to settle once and for all some of the questions that required expensive experimental work.

In a discussion to the same paper, Freeman (also an ASCE past-President) cited the Swain turbine setting of 1871 (approx.) to have been nearly identical to the regainer proposed by White. He also made the important point that the presence of swirl in the exit flow from high speed turbines considerably increased the optimum angle of divergence of a conical passage from the 11 degrees found by Francis. In connection with this idea, the height limitation of approximately 25 feet for a draft tube was based on the inability to raise a column of water higher by atmospheric suction, and also upon the extremely high cost of a deep pit for the exhaust structure. Reference to Francis as the ultimate authority on optimum draft tube angles probably indicated that American designers were still ignorant of the work of Gibson in England.

In 1926 Gibson & Labrow^[25] investigated the pressure recovery and flow characteristics of a number of different draft tubes. The original work of Gibson on diffusers of several cross sections was re-examined in light of the later research. This re-analysis of the earlier data led to somewhat different conclusions on the design of diffusers for maximum efficiency. According to the newer results,

"there is a definite ratio of inlet to outlet area which gives maximum efficiency of regain for a pipe of given inlet diameter and length. This ratio increases with the ratio of length to inlet diameter, and over the range of lengths from 5 to 25 feet and of inlet diameters from 2 to 20 feet, has values between 1.7 and 5.2. The results also show that for a given length of draft tube, the possible efficiency of regain falls off as the diameter of the . . . draft tube is increased." Scale effects were said to be small enough not to invalidate the comparisons among tubes of different proportions.

Additional experimental work covered by the 1926 paper consisted of an extensive series of tests on straight tubes with radial regenerators at their outlets. As in the earliest diffuser tests, the inlet flow was nearly fully developed. Significantly, the efficiency of a radial diffuser configuration showed a definite maximum as the spacing between plates was varied.

The original performance data were given in terms of actual loss compared to theoretical loss for a sudden expansion of the same area ratio. In the later paper, efficiency was computed as the actual static pressure rise as a fraction of the kinetic energy of the entering stream. In writing about the efficiencies of the bent draft tubes investigated, Gibson & Labrow mentioned that unstable flows were observed for flow rates below a certain critical value, but above that value no consistent variation with Reynolds number was observed.

Nikuradse^[60, 61] performed two classic series of experimental investigations in 1926 and 1929. The subject of the first was the velocity distributions for turbulent flow in flat walled channels;

the second dealt with flows in plane walled converging and diverging sections. Unfortunately Nikuradse was completely preoccupied with boundary layer behavior and the results of all his experiments were given only as velocity profiles. Although no pressure data were given for the converging-diverging section tests, some conclusions about the onset of separation were given. The results of these investigations were carefully summarized by Schlichting^[85]. The observed flow regimes were divided into essentially three regions. For total divergence angles below 8 degrees, velocity profiles were completely symmetrical and showed no features associated with separation. For angles above 8 degrees, the profiles were no longer symmetrical, and separation was incipient at 10 degrees. It actually took place for angles larger than 10 degrees. Above 16 degrees, the flow separated from one wall at a time and slowly switched from wall to wall. The frequency of oscillation increased with the divergence angle of the channel.

The experiments of Vedornikoff^[96] were directed toward determining the effect of divergence angle on the efficiency of a two dimensional diffuser. The test section consisted of a square channel, with two divergent walls. This geometry was typical of all "two dimensional" investigations of this period, including those of Hochschild^[33] (1912), Andres^[3] (1910), Donch^[10] (1926) and Shogenji^[89] (1927).

Flow regime data were determined by observing silk tufts through a glass window in the side of the flow channel. Performance information was given in terms of the static pressure at the inlet divided by that at the outlet. No velocity or Reynolds number data

were given, with the result that the performance information was useless, since the values could not be converted into any conventional form of pressure recovery. The valuable flow regime data provided substantial agreement with the observations of Nikuradse.

Fluid motion in axially symmetric diffusing passages was analytically investigated by Lorenz in 1929. Lorenz^[48] made some assumptions about the velocity profiles which existed in such flows. This allowed him to mathematically analyze the flow. In the core flow the velocity profile was assumed to be parabolic, but then the squared term was neglected, leaving essentially a linear profile. More interesting was his equation to account for the wall friction in turbulent flow. By assuming the head loss per unit length to be proportional to that in pipe flow it was possible to derive an analytical expression accounting for the friction loss.

Computed results were presented for two families of diffusers. The first was a straight conical diffuser with linear radius distribution, and the second was the draft tube suggested by Prasil (1903), in which the rate of change of radius along the diffuser was given by a cubic equation. Conical diffuser results were computed for total angles in the range of validity of the analytic expression, from 2 to 20 degrees. Efficiency decreased from 83 to 75 percent as the angle increased.

Information having no bearing on the diffuser problem, but useful in the design of duct systems was generated by Nippert, who in 1929 looked at flow in bent channels. Similar studies were carried out by White^[101] (1929) and Richter^[76] (1930).

Experimental Research, 1930-1939

An extremely significant series of experiments was performed in 1931 by Peters^[66]. The experimental work dealt with conical diffusers with total angles from 5 to 180 degrees. A length of pipe downstream of the diffusers increased the pressure recovery for some tests, but data were also reported for the diffusers alone. All data were taken at one Reynolds number and area ratio, but five different lengths of approach piping produced different inlet boundary layer thicknesses to determine the significance of that parameter.

Peters also looked at swirling flows, but the investigation of their effect on energy conversion efficiency was not complete in all aspects. The upstream pressure measurement was made after the air had passed through the swirl vanes, so any pressure drop occurring in the vanes was not charged to the diffuser, as pointed out by Reid. The flow case investigated corresponded to a situation where some swirl was already present in the flow when it entered the diffuser. Therefore the conclusion that adding swirl to a flow will increase the pressure recovery of a diffusing passage was not necessarily true.

Peters compared his results with Gibson's for 4.0 area ratio to draw some conclusions about the effect of that parameter. Head loss results were compared, after being corrected for frictional losses using the technique of Lorenz. The general trends of both curves were similar, including the behavior at low angles and the peak in head loss around 65.8 degrees included angle. Over most of the range of angles, Peters' head loss values for $AR = 2.34$ were about 15

percent lower than Gibson's for $AR = 4.0$. However, the data of Gibson (1911) for different area ratios showed somewhat the same trend at low angles.

Peters based his diffuser performance on the ideal head loss for a sudden expansion as did Gibson. Peters felt that η was a poor parameter on which to compare diffuser performance for units of different area ratio, as it includes the effect of area ratio. However, as pointed out by Reid, the value of ideal head loss for a sudden expansion is significant as a limit only when dealing with diffusers whose expansion angles are large.

The suitability of η as a performance parameter is open to some debate. It could be argued that the variation of area ratio should be included in any performance factor, since to produce large value of pressure rise in a wide angle diffuser is as stringent a requirement as producing a smaller value in, say, a 7 degree diffuser. The area ratio dependence included in the parameter η tends to equalize the measure of what is being demanded of each unit.

In 1935 Hudimoto^[37] gave an analytically derived criterion for determining the flow regime in a plane walled diffuser. Although Nikuradse had already experimentally determined the same flow regime information in 1929, the work of Hudimoto is significant as the first criterion derived from an analysis of the mathematical aspects of diffuser flow. The prediction criterion of Hudimoto also extended somewhat the flow regimes predicted by Nikuradse.

Hudimoto spoke of three flow regimes: unseparated flow, separated flow on one wall, and fully separated flow. The criterion

for determining the regime depended on the magnitude of the product $2\phi(\text{Re})^{1/4}$. The effect of increasing Reynolds number was to lower the critical angle for separation.

There appeared in 1938 the now classic review of diffuser information by Patterson^[63]. For the first time all the available data were correlated and general rules were laid down for diffuser design. Together with its companion paper on inlet design (Patterson^[64], 1939) this paper formed a textbook on airflow system design. Assembling the data of Gibson (1910, 1911) onto the same plot with the data of Peters indicated that the two sets of results were completely consistent.

The significant conclusions of the Patterson study were that to obtain an efficient diffuser, one should: use a circular section with included angle between 6 and 8 degrees, keep the inlet boundary layer thin, and use a long downstream pipe to promote complete energy conversion. Including the later results of Gibson (1926) might have altered slightly some of these conclusions, but Patterson seems to have been unaware of that work.

Mockmore^[55] (1938) exhaustively investigated different shapes of elbow draft tubes for hydraulic turbines. The results of his determination of pressure recovery and flow regime were reported in the ASCE Transactions. As reported in this paper, the design philosophy for draft tubes had come around full circle since the origin of application of diffusers to turbines. As power plant sizes became larger the structural advantages of the elbow tubes outweighed the relatively small hydraulic advantages of the spreading type. Elbow tubes were fully accepted when further intensive research such as that reported here resulted in their achieving efficiencies comparable with the spreading types.

The tubes tested by Mockmore were made of transparent pyralin, a material like Plexiglas, which could be formed into fairly complex shapes without losing its transparency. Use of this material allowed motion pictures to be taken of all regimes of flow observed. The purpose of the study was to investigate with particular care the bent portion of the elbow draft tube. Flow distributions in pipe bends of uniform cross section were first investigated thoroughly to establish the effect of centerline curvature on a known flow.

The tests of draft tubes indicated that a pipe bend flattened in the plane of the bend offered less resistance to the flow than any other bent configuration. It was also apparent that the distance between the inside and outside walls at the outlet end of the bend should be small compared with the throat diameter of the draft tube. Tests were carried out with whirling flow in an attempt to modify the whirl with a system of splitters or vanes. However, it was not found practical to do this, since large variations in the flow conditions were met by tubes in the field depending on the degree of turbine gate opening.

Although it dealt primarily with elbow draft tubes, the research of Mockmore reached conclusions which were generally applicable to diffuser design. It was concluded that trumpet shaped diffusing sections were preferable to straight conical sections. Although only limited data were available, there was substantial agreement on this point as shown by the results of Andres^[3] (1910), Gibson & Labrow^[25] (1926) and Hofmann^[34] (1931). In addition to its possible efficiency advantage, the trumpet shape was said to be preferable for hydraulic installations for its lessened tendency toward cavitation.

The flattening of the bend as the draft tube curved was considered to be a flow situation analogous to the use of an impact plate near the exit of a draft tube. Results of Luksch^[50] were cited. In 1935, he showed that for direct flow in a trumpet shaped draft tube, maximum efficiency occurred when the plate placed normal to the flow was approximately 0.3 throat diameters downstream from the diffuser exit. The results of Hofmann were also used. Hofmann stated that for a conical draft tube with rounded bottom edges, the most favorable ratio for rounding lay between 0.67 and 1.0 of the throat diameter of the tube.

Kratz and Fellows^[44] (1938) investigated the overall efficiency of various configurations of rectangular air ductwork, such as used in airconditioning, heating and ventilating service. All their data were for configurations of little use to the present study, but they developed an original form of plotting data. After plotting the logarithm of total pressure loss as ordinate vs. the logarithm of mean velocity change as abscissa, it was observed that the values for fairly large angles of divergence were correlated by straight lines. This was illustrated with measured values for 30 and 60 degree divergences, indicating that "large angles" means angles of greater than 30° divergence.

Experimental Research, 1940-1949

Another investigator who looked only at plane walled diffusers was Polzin^[69] (1940). He has been included in this summary since he was the first to suggest the correlation of plane walled diffuser flow regimes on the basis of two variables. The diffuser which

was the subject of the tests reported by Polzin had an inlet of square cross section. The two diverging walls were varied over the range from 0 to 20 degrees total divergence, and the inlet velocity profile was a fully developed turbulent profile as a result of the long entrance length of pipe used.

According to Rippl^[78] (1958, to be discussed later), Polzin combined all diffuser flow regimes into three categories. These were non-separated, "initially" separated, and completely separated. The dividing lines between regions were not sharply defined. They were determined by making hot wire measurements at varying distances from the walls of the "two dimensional" diffuser. The hot wire readings were then extrapolated to zero distance from the wall and the results determined were compared with the theoretical prediction. Unfortunately Polzin did not make measurements of pressure recovery, so the results cannot be compared to those of other investigators.

Rippl and Eck^[12] had further comments to make on the studies of Polzin. Eck (1957) concluded from the results of Polzin that the critical area ratio was a linear function of divergence angle; it was 2.8 for a 6° diffuser and decreased to 2.25 for a total angle of 12° . The experimental results of Rippl were said not to substantiate the latter conclusion, although no mention was made of the method of comparing the "two dimensional" and conical results.

Rippl also questioned the practicality of investigating nearly separated boundary layers with heated wires which might act as sources of instability. The source of heat would have been so minute, however, that its influence would undoubtedly have been negligible.

Although principally interested in high subsonic speeds, Naumann^[58] (1942) gave several pieces of information on pressure recovery in conical diffusers. The configurations covered in his research were of 5 and 6.3 degrees total angle. The original work was not available; the values used were taken from Lukasiewicz^[49] (1953).

Very comprehensive tests were reported in 1946 by Kalinske^[39]. The objects of the experimental investigation had been to obtain data on mean velocity distribution, pressure changes and turbulence in flow expansions in circular pipes. Reported were data for 7.5, 15, 30 and 180 degree total expansions in pipes from 2.75 to 4.75 inches in diameter. The area ratio for all tests was thus approximately 3.0, and two flow rates were investigated. Mean velocity and turbulence measurements were made by photographing the flow in transparent conduits. The above mentioned results were deduced from literally miles of motion picture film by very careful data reduction from measurements of the streaks made by little immiscible droplets in the flow. These allowed determination of the "instantaneous" velocities in two dimensions.

In discussing the fluid mechanics of pressure losses in expanding sections, the author indicated that both analytical and experimental work suggested an angle between 5 and 10 degrees for minimum loss in flow conduits. Furthermore, it was claimed that frictional loss was negligible compared to "form loss," except for expansions with less than 10 degrees included angle.

References in the study included only the data of Gibson (1910) and Peters (1931). The data of Nikuradse were said to show that

"separation occurred at a 5° divergence angle and that, for slightly larger angles, there was a tendency for asymmetrical velocity distributions and a switching of the flow from wall to wall." In the remainder of the section, angular measures had been intended to mean total angle, which definitely led the reader to mis-interpret Nikuradse's results.

Pressure recovery data were reported in two ways. Pressure data along the length of the pipes tested were given, some in tabular form, and some plotted. Pressure recoveries, η , were also given for the same four flow configurations. The pressure rise values were corrected for friction, but the method of correction was not given.

In a discussion to the Kalinske paper, van Driest replotted the data of Peters and Gibson in the form which had been suggested by Patterson^[64] (1938): efficiency vs. divergence angle. The measured efficiency values of Kalinske fell somewhat lower than those of Peters and Gibson for similar area ratios. The general trend was the same: efficiency increased as the angle of divergence was reduced, but the maximum point on the curve was not so clearly defined. The shape near the 10 degree divergence angle was not such as to indicate clearly the presence of a maximum value in that region.

Henry^[28] in his NACA Wartime Report of 1944 significantly extended the work of Patterson by including numerous duct configurations in addition to diffusing passages of various forms. The objectives of the study were similar to those of Patterson: to aid the designer in estimating the pressure loss, determining the flow rate, calculating the exit area and evaluating the airplane drag as a result of a given duct

configuration. To carry out efficiently a review of the multitude of possible duct configurations, the analysis was restricted to a review of effects of each component separately. The assembly of sub-component data into systems data was then carried out using rules established for that purpose. Admittedly this procedure was only an approximation, but it was the only workable method due to the limited data then available.

Data for estimation of diffuser losses were obtained from the experimental results of Gibson and the plotted results of Peters as given by Patterson. Henry indicated that to a limited extent the losses of non-circular (particularly square) diffusers could be estimated from the same information. To secure maximum efficiency in a conical expansion, he advised that the total angle of divergence should lie between 3 and 10 degrees. It was readily admitted that space limitations often precluded use of such a low angle. It was stated that higher angle diffusion could be more efficiently carried out with curved wall diffusers. Gibson was quoted as saying that no gain would result from using divergence angles greater than 40 degrees, and "unpublished results" from other sources gave 60 degrees as the practical limit beyond which large losses occurred.

The investigation of Gratzler & Smith^[26] (1948) was concerned primarily with studying the effect of applying boundary layer suction to improve the performance of wide angle conical diffusers. Several configurations of straight walled conical diffusers with exit lengths were studied. Pressure recovery data were given, but the report did not

make the location of pressure measurement at the diffuser exit entirely clear. It has been assumed that this pressure was measured in the exit plane of the diffuser.

Pressure recovery data given by Gratzer & Smith included measurements on diffusers with angles from 15 to 30 degrees in 5 degree increments, plus angles of 40 and 60 degrees. All tests were made with thin inlet boundary layers except the 30 degree angle unit, which was also tested with three thicker boundary layers. The area ratio for all tests was 4.0.

The report traced the concept of boundary layer suction from its suggestion by Prandtl in 1904 through its first practical demonstration as applied to diffusers by Ackeret^[1] in 1926. The purpose of the experimental work was to determine the characteristics of wide angle diffusers in which boundary layer control was used and to establish criteria for their optimum design. The parameters whose effect on pressure recovery was investigated were diffuser angle, initial boundary layer thickness, location and width of suction slot, together with the suction flow rate.

The conclusions of the experimenters were that performance of wide angle conical diffusers could be significantly improved by incorporating suction slots to bleed off some boundary layer fluid. This performance evaluation was based on a true energy conversion efficiency which accounted for the power required to remove the suction flow. When significant amounts of instability were present in the flow, the addition of suction eliminated them; the results were found to be independent of slot width. The use of a single suction slot was found to

exert sufficient boundary layer control in diffusers of less than 40 degrees included angle at an area ratio of 4.0. No conclusions were drawn for other area ratios or for diffusers of wider angle. Larger suction flow rates were required for adequate control when inlet momentum thickness was increased.

Jungnitz^[38] (1949) reported the results of an analytical investigation of flow in conical diffusers. Included, in addition to the straight walled case, were a configuration with a parabolic radius distribution and one with a fourth power radius distribution. Frictionless flow was assumed, and the paper included calculations of the velocity, acceleration and pressure gradient profiles of the various diffusers.

The straight walled diffuser was said to have the disadvantage of having a very large initial value of fluid deceleration, which was regarded as the cause of early separation and high losses. Jungnitz demonstrated that in the diffuser with parabolic radius distribution the velocity and acceleration both decreased linearly with distance along the diffuser axis. In the case with fourth power radius variation the acceleration remained constant along the diffuser, but the velocity decreased more rapidly with distance toward the diffuser outlet.

To illustrate numerically the effect of contour variation, a straight walled diffuser with 4 degree included angle and area ratio of 1.82 was chosen. This unit was compared with diffusers of parabolic and fourth power radius distribution having the same area ratio and length. For the parabolic diffuser, the divergence was initially 1 degree 52 minutes, which increased to 9 degrees 7 minutes. Comparable values for the fourth power unit were 1 degree 16 minutes and 17 degrees 18 minutes.

Annular diffuser performance characteristics were investigated in 1949 by Nelson & Popp^[59]. Two diffusers, one each of 6 and 12 degrees, were tested at an area ratio of 1.75. Flow rates were varied to cover the range from incompressible to choked flow.

Although the data and results obtained in this study were not directly applicable to the present work, the authors suggested an original correlation technique. They suggested using a basis of wetted area to correlate annular and conical diffuser data for low expansion angles. Good agreement was found with conical data for the 6 degree configuration tested, but wetted area was shown to be unimportant in the case of the 12 degree diffuser.

Schubauer & Spangenberg^[86] reported in 1949 on the use of screens in wide angle diffusers. The object of the study was to investigate the "filling effect" caused by a resistance element placed downstream of a wide angle diffuser and to attempt clarification of the mechanics of the flow. A wide angle diffuser was defined in the report as one in which separation could be expected, and the criterion for such diffusers was that the total included angle be greater than about 8 degrees.

Two of the three diffusers studied were straight walled conical units. These were tested both with and without screens installed, but always with an exit length of pipe. Although none of the screen-diffuser combinations tested produced a higher efficiency than the diffuser alone, it was concluded that the use of screens in wide angle diffusers could have other advantages in certain situations. For badly separated flows it was found possible to obtain a uniform velocity distribution downstream of the diffuser and to reduce the fluctuation in flow by installing a

properly selected screen. This effect could be important in critical diffuser applications, where a uniform flow at the exit is particularly desirable.

Experimental Research, 1950-1959

In 1950, Little & Wilbur^[47] reported the results of tests of 12 and 23 degree conical diffusers with area ratio 2.0. In addition to performance data for two inlet boundary layer thicknesses and a range of Reynolds numbers, complete boundary layer data were presented. Some tentative conclusions about Mach number effects were presented in this paper.

Total pressure losses increased and static pressure recovery decreased with increasing inlet boundary layer thickness in both diffusers tested. Increasing the flow rate caused slight adverse effects on performance for thin inlet boundary layers in the 12 degree diffuser. The performance of the large angle diffuser with thick inlet boundary layer was deteriorated markedly when flow rate was increased.

The 23 degree diffuser was badly separated and the flow was quite unsteady. Addition of roughness elements near the throat of the diffuser was found to eliminate the unsteadiness, but it did not improve the pressure recovery. In none of the tests were the separated flow patterns in the diffusers uniformly distributed around the diffuser periphery, and the extent and location of separation varied with time in an irregular fashion.

Squire^[92] (1950) gave results of a series of tests on smooth walled conical diffusers. Tests were carried out with area ratios of 4.0 and 16.0, with Reynolds numbers of 10^5 and 10^6 , and with diver-

gence angles from 4 to 10 degrees. The results of these tests were presented as measured pressure distributions along the diffuser walls and velocity profiles at the exits. No boundary layer separation was observed for any of the diffusers investigated.

Little variation in efficiency was found as the angle of divergence was varied, but for divergence angles less than 10 degrees there was a slight increase in pressure recovery as Reynolds number was increased from 10^5 to 10^6 . Pressure recoveries obtained when the diffuser was located downstream of the blower were consistently slightly lower than when the diffuser was on the suction side of the blower. In addition, flow stability for the larger angle diffusers was improved when blowing was used.

Two comments made by the author were noteworthy. It was stated that no systematic tests of conical diffusers had been made since the work of Gibson in 1910. This was in spite of the fact that Patterson's review of the work of Peters was included in the bibliography. No correction for compressibility effects was introduced into any of the measurements, although the maximum velocity in the test section was 500 fps.

It should be noted that the suction tests were carried out with an exit length of pipe connecting the diffuser sections to the fan inlet, whereas in the blowing tests the diffuser discharged as a free jet.

Significant results covering a small facet of the conical diffuser problem were reported in 1951 by Copp^[8]. The object of the investigation was to determine the effect, if any, of the curvature of the joint between the cylindrical inlet section and the conical diffuser

segments. Experimental data were obtained for a 10 degree, 19 inch inlet diameter diffuser over a range of Reynolds and Mach numbers. The former was varied over the range from 1.2×10^6 to 3.2×10^6 , while the latter was varied from approximately 0.3 to choking. Data were taken for two inlet boundary layer thicknesses.

It was found that static pressure recovery was entirely independent of inlet wall shape. At flow rates below those at which local choking occurred, the pressure distribution over most of the diffuser was not affected by a change in radius of the inlet junction. It was affected only in the region of the junction of the approach tube and the conical section. Modifying the contour of the diffuser inlet produced no systematic trend in exit velocity profile shape. A slight increase in limiting mass flow was observed as the radius of the inlet junction was increased.

The author felt that testing two inlet boundary layer thicknesses formed insufficient evidence to draw any "broad generalizations," but he suggested that the conclusions were probably independent of inlet boundary layer thickness.

The chief significance of this work seems to be that it pointed up other important features of boundary layer behavior in a conical diffuser. The downstream flow was not affected by sharp curvature at the inlet, so one might conclude, at least for 10 degree diffusers, that no particular care need be given to ensuring a continuous velocity derivative at the inlet. These sort of data are dangerous to extrapolate, and especially so for separating flows, but it might be safely said that no harm will be done to the flow by a sharp inlet break for any diffuser angle less than or equal to 10 degrees.

Flow studies in Venturi discharge sections were reported by Warren^[100] in 1951. The discharge sections were formed as frustrums of right cones; they approximate conical diffusers with bell mouth inlets directly attached to the diverging sections. Data were taken for 5 through 9 degrees, 5 through 17 degrees, and 5 through 20 degree divergences, respectively, for area ratios of 16.0, 4.0 and 1.78. Unfortunately, flow rate and/or Reynolds number data were not specified, nor was the downstream geometry completely defined. It was apparent from the sketch of the apparatus that the data were taken in an exit length, but how far from the diffuser exit the pressure measurement stations were located was not stated.

The data obtained were presented as total head losses, but it was apparent from the discussion in the paper that the "total head" referred to was really static pressure. Although of doubtful value in a survey of diffuser performance with out exit length, the data have been recorded in the table summarizing all available conical diffuser data.

Robertson & Ross^[79] (1952) reported experimental measurements of flow conditions in 5, 7.5 and 10 degree straight walled conical diffusers which were preceded by relatively short lengths of straight pipe. Three different inlet boundary layer thicknesses were produced by using 2 diameter, 5 diameter and 9 diameter lengths of straight pipe preceding the test section. Experimental measurements included pressure recovery data and velocity profiles at several stations. For all tests an exit length of pipe was used to return the water to the return flow system. A 90 degree bend was located immediately downstream from the test section.

The diffuser area ratio was found to be the major geometric parameter governing the shape and extent of the boundary layer. The boundary layer thickness at the entrance to the diffuser, given indirectly by the length to diameter ratio of the preceding straight pipe, was as important in determining the flow as was the divergence angle. Within the range tested, the efficiency was found to be a function of the product of diffuser angle and effective entrance length. Although the energy efficiency, $\eta_{13} = 1 - \lambda_{13}$, decreased with increasing area ratio, it was practically independent of the diffuser angle and entrance conditions. It was finally concluded that the only consideration which should affect the choice of diffuser geometry is the desired factor of safety for freedom from separation.

Some of the velocity traverses in the study indicated that the flow was approaching separation near the diffuser exit. This information allowed the authors to determine the critical values of the well known two dimensional form parameter H , and the Darcy parameter $\mathcal{D}$. A method for estimating the values of these parameters in any diffuser was developed. The study made it possible to predict the mean flow conditions in conical diffusers within the range of test variables.

In their literature review, the authors credited Nikuradse with a semi-rational treatment of the results of his investigations of plane walled diffusers. His result was said to indicate a smaller value of critical angle for the avoidance of separation as the Reynolds number of the flow was increased.

Pressure data were referred to a static pressure tap near the upstream end of the diffuser. All pressure data were converted to

pressure coefficients by normalizing the measured pressure drops with the mean inlet kinetic energy. The original data measured in the experiments were presented in another publication. Pressure coefficient data were treated in a semi-statistical way by grouping them according to wall tap location and according to the Reynolds number range in which they were measured. The reference piezometer value was corrected for frictional losses between its location and the diffuser throat. The method by which this pressure drop was computed was not specified.

Other results computed from the test data were presented in several interesting ways. The pressure recovery values were shown to increase significantly with Reynolds number as the latter was increased from 0.5×10^6 to 2.0×10^6 . The increase was larger for the 10 degree diffuser than for the smaller angle unit. It amounted to about 5 percent in the former case. This behavior was stated to disagree with the implication of the Nikuradse theory.

A cross plot was presented using the effective entrance length and diffuser angle as coordinates. The effective entrance length was computed by correcting the actual length for the boundary layer thickness already present at the end of the contraction nozzle. On this plot the measured recovery curves took the approximate shape of hyperbolas, suggesting that diffuser performance could be characterized by a product of entrance length and diffuser angle. The observed data fell close to a single curve on a plot of efficiency vs. the product of angle and entrance length, but some scatter was apparent for large values of the entrance length.

Several of the two dimensional boundary layer parameters, including momentum thickness and shape factor H , were shown to correlate well with the new variable $AR(L_e/D_1)^{0.2}$, where L_e is the equivalent entrance length computed from the actual entrance length by adding enough length to give the boundary layer thickness present at the exit from the contraction cone. In fact, this parameter correlated both momentum thickness and form parameter better than it did the displacement thickness.

Study of the nearly separated velocity profiles obtained indicated that the commonly accepted value of 2.4 for H near separation was a suitable characterization. Most of the profiles close to separation had H values in the range between $H = 2.3$ and $H = 2.5$.

The last two conclusions were that the Bernoulli energy (total head) remained constant in the central core of the diffuser flow. As mentioned, energy efficiency was found to remain essentially constant with variations in diffuser angle and entrance conditions (over the range of conditions tested), although it decreased slightly with area ratio. The former conclusion pointed up once again that since displacement thickness determines the effective area ratio, knowledge of that parameter completely determines the pressure recovery of a conical diffuser.

In 1951, Valentine & Carroll^[95] considered the use of rectangular vortex generators as a means of increasing the static pressure rise in short diffusers. This was one of a series of NACA publications in the early 1950's dealing with this subject. The diffuser tested was a short, 23 degree conical diffuser of area ratio 2.0. It

was tested both with and without a tailpipe which was about 3.5 diffuser exit diameters in length.

Pressure recovery data with no inlet vortex generators showed decreasing values with increasing inlet boundary layer thickness. As an example of the effect of the tailpipe, the efficiency of the diffuser was increased from about 43 to about 60 percent for a moderately thin inlet boundary layer by adding the pipe. Additional data for the same configuration were also given by Persh (below).

The objectives of the Persh^[65] study were essentially the same as those of the Valentine & Carroll work discussed above, except that thin triangular ledges instead of vortex generators were investigated as a means of flow control in the 23 degree diffuser. Data were reported in the Persh study only for the diffuser with exit duct combination.

The contribution to the conical diffuser data survey made by Uram^[94] in 1954 consisted of reporting two experimental points. The purpose of his paper presented to the Congress on Applied Mechanics was to discuss the state of the art, as it then existed, of calculating the growth of the turbulent boundary layer in adverse pressure gradients. The objectives of the experimental work reported were to gather data on the behavior of boundary layers in conical diffusers for background information to aid in developing a boundary layer prediction method.

The experimental setup was an air flow apparatus, and consisted of the same test section used by Robertson & Ross^[79] (1952), which discharged as a free jet into the atmosphere. As mentioned, the principal results of the study were detailed velocity traverses;

pressure recovery data were provided as incidental information.

Tests were run with two different inlet boundary layer thicknesses.

The experimental pressure recovery results have been tabulated with the rest of the diffuser data in Appendix C. Of possibly more importance was the substantial agreement of the observed data values with previous empirical correlations of Ross. Notable agreement was demonstrated with Ross's hypothesis that the momentum integral equation can be expressed with sufficient accuracy by the equation

$$\frac{\theta}{\theta_1} \frac{R_1}{R} = \left(\frac{U_1}{U} \right)^{2+G}$$

where R is the diffuser radius, and $G = 2.74 = \text{constant}$ for this case.

The author indicated in this paper the desirability of defining three dimensional boundary layer parameters to adequately characterize the boundary layer in adverse pressure gradients.

In 1956, Henry, Wood & Wilbur^[29] reported the results of an intensive effort to gather together and correlate all the existing data for diffusers of three principal configurations: conical, annular and rectangular. A number of data were assembled and a great deal of effort was expended on their analysis and on plotting curves for various combinations of parameters, but the result was still not a design aid.

The most important parameter used in the memorandum was the total head loss factor, or simply loss factor, $K (= \lambda_{33})$. This factor expressed the proportionality of the total pressure loss, in the diffuser-exit duct combination, to the theoretical total pressure loss

in a sudden expansion of the same area ratio. It was selected as a comparison parameter because it remained constant over wide ranges of area ratio, and was thus effective in reducing the number of variables to be considered.

Loss factor values when plotted against expansion angle correlated the data of Gibson (1911) and Peters (1934) very well. The curves for thin inlet boundary layer fell closely on top of one another; the variation in loss factor with inlet boundary layer thickness was toward increased losses for the thicker boundary layers. These curves were faired at their lower angles so they tended toward the value of K_f , the loss factor due to friction alone. This loss factor was based on the assumption that diffuser friction losses are approximated by integrating the differential form of the head loss equation for straight pipes. K_f was given by

$$K_f = \lambda_{33} \lambda_f = \frac{f}{2 \tan \phi} \frac{AR + 1}{AR - 1}$$

The same correlation was plotted on a second figure, and the data of Peters were compared with newer data for higher inlet velocities. All points remained within the incompressible flow regime. The correlation was still satisfactory, with the systematic data showing increased losses as the inlet boundary layer was thickened. However, there were several points which showed significant deviations from the norms, indicating influences on the loss coefficient of factors other than expansion angle. It is possible that the elimination of area ratio caused this effect at the higher flow rates due to other factors which were not controlled.

Loss factors from measured data were correlated with the flow and geometry of the system for angles between 8 and 20 degrees. The correlation was accomplished by breaking the effects of variables on the loss factor into the two parts of a product function. One part was considered the result of the inlet boundary layer, and the other a function of the divergence angle. The product of the two factors successfully predicted the overall loss factor. The correlation based on diffuser angle was found to be a power function of diffuser angle, while the correlation based on inlet boundary layer was not so simple. It consisted essentially of two linear curves having different slopes; the change in slope occurred at a δ^*_1/R_1 value of approximately 0.04.

The same correlations were attempted for the effectiveness loss ($1 - \eta$), with slightly different results. When effectiveness loss was plotted against divergence angle, the results were quite good and they showed the expected trends for incompressible flows. However, higher speed flows produced other effects, and resulted in the data showing a pronounced dependence on AR.

It was again possible, at least for Peters' data, to represent the effectiveness loss as the product of effects of inlet boundary layer conditions and of divergence angle. For this case the dependence on diffuser angle was shown to be linear. The dependence on inlet boundary layer was substantially the same as that previously found for the total pressure loss coefficient.

Additional correlations presented showed the effects of the tail pipe as a regainer of static pressure through alteration of the velocity profile. Although these curves were presented for the data

of Peters only, they are of more than academic interest. Peters ran tests both with and without the exit length in place, measuring pressure recovery values at the section of maximum pressure recovery and at the diffuser exit plane. The conclusion originally drawn by Peters was once again validated: the pressure recovery depends significantly on the inlet boundary layer when the diffuser alone is tested, but much less so when an exit length is used.

Additional results presented were of interest primarily in the case of high speed diffuser flows. Since these were not directly applicable to the work at hand, they will be only briefly reviewed.

Most significant of the high speed flow results was the derivation of an equation representing the effect of Mach number on the effective pressure gradient seen by the flow in a diffuser. This effective pressure gradient was shown to increase rapidly with Mach number, and also to be more severe for the low area ratio diffusers. Total pressure loss coefficients correlated the effects of speed variations on high speed flows quite well, whereas effectiveness loss was not so effective as a correlation variable.

The remainder of the work was devoted to discussing and correlating aspects of limiting Mach number, exit flow distributions (which defied analysis), and boundary layer control applications.

Unfortunately the paper did not include the systematic work of Kalinske^[39], the work of Squire^[92], or that of Warren^[100]. Neither was a correlation presented in such a way that diffuser performance could be optimized for either a given N/R_1 or a given AR value.

The study of Holzhauser & Hall^[35] (1956) had as its primary objective the exploratory investigation of area suction (as opposed to the use of slots) for boundary layer control in wide angle diffusers. Tests were made using 30 and 50 degree conical diffusers with an area ratio of 2.0, and data were taken with and without suction. Tests were made at a mean inlet Mach number of about 0.2.

The results of the experimental work indicated that suction was effective in eliminating separation in large angle conical diffusers. The resulting losses of total and static pressure were less for the diffusers tested with suction than the losses for a normal 10 degree diffuser without boundary layer control.

Due (presumably) to the exploratory nature of the tests, the overall efficiency including pumping losses of the diffusers with area suction was not given. The suction mass flows required were about three percent of the main flow, indicating that excessive amounts of fluid were not removed by the suction apparatus. Of note is the fact that separation was eliminated even for the 50 degree case. Data for the non-suction tests are recorded in the list of data, Appendix C.

Scherrer & Anderson^[84] (1956) reported tests of a family of diffusers designed with variable wall shapes. The shapes were defined by empirical equations with constants related to the inlet boundary layer thickness, and were designed to provide high pressure recoveries for near sonic inlet velocities. To aid in evaluating the results obtained with the variable wall shape diffusers, one diffuser with 8 degree divergence angle and straight conical walls was also tested. The data from this latter diffuser are the only ones of use in the current survey.

The great majority of comments and nearly all the discussion pertained to the high speed flow cases on which the principal efforts of the study were expended. Of interest was the finding that inlet boundary layer thickness was as important a factor in determining diffuser performance with high speed flow as the shape of the diffuser itself.

In 1957, Kaufmann^[40] investigated the effects of boundary layer suction and blowing on the performance of diffusers of a variety of different configurations. Included were three conical geometries, which were tested both with and without suction, with rectilinear flow, and with various degrees of swirling flow. The swirl was produced by vanes installed upstream of the diffuser in a short constant diameter entrance channel. The length of this channel was approximately one diffuser throat diameter. The vanes were tested both with and without streamlined hubs having a diameter of about 0.4 of the diffuser throat diameter.

The measured results of two of the above described tests were applicable to the straight conical diffuser case. The tests were described in the German paper, but there was no apparent comment on them since the bulk of the emphasis was on boundary layer control devices. The data have been recorded in the summary table.

Robertson & Holl^[80] (1957) reported a continuation of the study of turbulent boundary layer development originated by Uram. The apparatus used was the same as had been used earlier, but the geometry was varied by changing the length of the test section. Runs were made at a variety of Reynolds numbers. The flowing medium was air. Three of the configurations reported were free jet discharges,

while the remaining two included exit pipes of two lengths; the exit lengths were short, and the discharge from them was to the atmosphere.

In this paper the previous results of Uram were extended to several additional geometries. The addition of exit lengths significantly altered the performance of the diffusers tested. The conclusions drawn in the former paper were not negated, but considerably less certainty was attached, for example, to the use of Ross's $\mathcal{D}$ parameter for predicting boundary layer growth. It was concluded from the experiments that more experimental data were needed to define properly the relationship between the parameter δ/θ and Ross's $\mathcal{D}$ parameter, whereas the other quantities, θ , γ , and c_f needed to predict the boundary layer were adequately handled by available empirical relations.

The study of Winternitz & Ramsay^[103] (1957) had the objective of determining pressure recovery, energy conversion, and losses as affected by inlet conditions. To study these phenomena, experiments were carried out with conical diffusers of area ratio 4.0, with included angles of 5 and 10 degrees. The tests were made at one Reynolds number for incompressible flows. Inlet boundary layer parameters were varied by two methods: the obvious technique of varying the inlet length of straight, constant diameter pipe, and modifying boundary layer profiles using projecting annular screens. Using the latter technique it was found possible to alter significantly the boundary layer profile while using moderate entrance lengths. According to the authors, no differences existed between the results obtained by the two methods of altering boundary layer profiles.

Energy and pressure coefficients were found to depend on the diffuser angle and the momentum thickness at the inlet. The latter emerged as one of the chief parameters controlling diffuser performance. Variations in the inlet shape parameter H of 20 percent did not significantly affect the pressure recovery in the diffuser. For moderately thick boundary layers, the diffuser angle could be eliminated as a parameter by plotting efficiency against the product of momentum thickness and diffuser angle. These results substantiated the conclusions of Robertson & Ross that diffuser performance could be correlated with a product of effective entrance length and diffuser angle. The results of the Robertson & Ross study were plotted on the same coordinates as the earlier Robertson results, showing substantial agreement, although there was some slight systematic variation between the two sets of data. Substantially better agreement was achieved for static pressure recovery results than for energy efficiency when compared on the basis mentioned above.

Although the gross diffuser performance, that is, the pressure recovery of the diffuser, was substantially the same whether screen rings or approach lengths were used to effect the boundary layer thickening, there were other differences in the flows. The authors noted that boundary layer shape parameters at the inlet to the diffuser were different depending on the method of thickening, and that the inlet kinetic energy was also affected. It was evident from the velocity profiles that the presence of screen rings resulted in higher velocities near the wall of the diffuser. The authors concluded that the presence of screen rings was "believed to have delayed the onset of separation."

The experimental work of Fraser^[19, 20] (1956, 1958) was carried out in continuation of the experimental programs of Uram^[94] and Robertson & Holl^[80]. A prediction method for determining boundary layer growth in axisymmetric diffusers was developed from data which were available from the previous tests. Experimental verification was obtained by measurements made in a 10 degree conical diffuser. Both configurations were of the free jet discharge type, and were diffusers of small area ratio. Velocity profiles were presented, but no pressure data were given.

The data presented in this paper were of considerable value, for separation was known to occur in one of the 10 degree configurations tested, but not in the other. The ΔD parameter was shown to be effective in correlating the occurrence of boundary layer separation, and thus was extended to three dimensional flows. The two datum points have been recorded in the data summary.

The experimental and theoretical analyses of Furuya^[21] (1958) were concerned primarily with two dimensional diffusers, but in the course of the tests some information was also obtained on a 20 degree conical unit. In the experimental setup used the approach pipe length was varied over wide limits to change the thickness of the inlet boundary layer.

Visual observations of separated flow in the diffuser were made using a grid, on which had been tied numerous silk threads, placed at the diffuser exit. In all tests reported there was a backflow region near the wall, and the author noted that the static pressure readings were quite unsteady, showing variations as high as 10

percent of the mean readings. Data for pressure recovery and velocity profiles were presented for each of two Reynolds numbers and each of two boundary layer thicknesses. Inlet boundary layers ranged from very thin to nearly fully developed flow.

The paper of Rippl^[78] was another review of what had been done in axisymmetric diffusers up to 1958. The references listed by the author were all in German. Consequently, the paper could not be considered a complete survey of all available information, but it provided considerable insight into some of the work which had been done in Europe.

As previously mentioned in the discussion of the work of Polzin^[69], Rippl traced the development of boundary layer research in Europe by first looking at the results of Polzin's experiments. These results were given as background to the object of the paper: systematic investigations to determine the influence of divergence angle, relative length and/or area ratio, and Reynolds number, on the performance of small angle conical diffusers. Rippl studied a number of diffuser geometries; he recorded both pressure recovery and flow visualization data. All his tests were conducted with air flow, and the diffusers discharged freely into the atmosphere. Flow visualization results were obtained in two ways. The flow regime at the diffuser exit was surveyed by installing and observing the motion of a grid of woolen tufts. Also used was a china clay drying plate to qualitatively observe the extent of separated flow regions, but the latter technique was found not entirely satisfactory. Where the flow was fully separated it worked well, but in doubtful cases it failed

due to the flow disturbance caused by the plate itself. In addition to this difficulty it was nearly impossible to reproduce the pictures exactly and to get quantitative information from the results.

The range of flow conditions investigated was fairly large. Reynolds numbers for the tests varied from about 75000 to 700,000. The corresponding range of Mach numbers was from about 0.1 to 0.55; this indicated that some of the "Reynolds number effects" could have been compressibility effects. The only pronounced effect of Reynolds number was in the 14 degree diffuser for short length ratios, where a fairly pronounced tendency for efficiency to fall off with increasing Reynolds number was noted. In many cases, the data showed some variability about the curve drawn by the author to fit the experimental points, and sometimes the data scatter was nearly as significant as the parameter change shown by the curve.

Enough data were obtained by Rippl to allow mapping of the performance of conical diffusers. He also found it possible to select optimum performance points both for given area ratio and given diffuser length, each of which is a significant design criterion in practice.

The experimental results for conical diffusers did not agree with the Polzin flow regime plot, although not enough flow regime information was obtained to draw really firm conclusions. It was found by Rippl that separation occurred only for the 14 degree angle among the three diffuser angles tested. However, increasing the Reynolds number caused the area ratio for separation to fall off sharply. For a Reynolds number of 80,000, separation took place at $AR = 2.4$, while for $Re = 200,000$, separation took place at $AR = 1.8$.

Furthermore, the results drawn by Eck, to the effect that separation is inevitable in any diffuser, were not confirmed. It was found impossible to experimentally induce separation in the 3 or 7 degree diffusers tested.

The work of Reid at Stanford was extended and continued by Kline with a systematic investigation of plane walled diffuser flows. Most of the pertinent information produced was summarized in the design paper of Kline, Abbott & Fox^[43], and will not be reviewed again here in detail. The work consisted of investigations to determine flow regime and performance data for plane walled diffusers with thin inlet boundary layers and well rounded entrances, for incompressible flows. The research was further extended when the effects of inlet boundary layer thickness^[98] were investigated.

Two aspects of the work were particularly pertinent to the conical diffuser study. The two dimensional work was successful in establishing the correlation of optimum pressure recovery with geometry. The research also pointed out forcefully the benefits which could be derived from a careful flow visualization study of the basic flow phenomena followed by careful experimental work. Another important conclusion reached was that stall phenomena do not behave as the classical model of stall would predict.^[42] That is, a truly two dimensional stall pattern is a rarity instead of the usual case. Steady two dimensional stall regions are found only after transitory stalls and fluctuating flows have been present for some range of geometry.

Optimum performance for straight walled diffusers (for a constant value of N/W_1) was found to correlate on the basis of divergence angle and diffuser length. These results were tentatively extended to other geometries, when Gibson's data for optimum recovery in a conical diffuser with tailpipe were shown to fall close to the line of two dimensional optimums.

The work of Sprenger^[91] (1959) was the final report summarizing all the work carried on by the group at ETH, Zurich, over a period of more than four years. The principal objective of the investigation had been to obtain data for diffusers of many different cross sections, so the entire effort was not focused on straight conical diffusers. Only one of the many geometries tested was a straight diffuser; the rest were bent diffusers with various cross sections from round to elliptical. However, a significant number of data for conical diffusers with various inlet boundary layer thicknesses were presented.

The general conclusions from the experimental work were that thin inlet boundary layers gave the best energy conversion with all the diffusers tested, with the poorer diffusers suffering steeper drops in efficiency due to thickening the inlet boundary layer. Detailed investigations of the wall pressure profiles, energy distributions at the outlet, turbulence measurements, and data on local pressure conversion were presented in addition to the overall efficiency measurements.

The one straight conical diffuser tested provided information about pressure recovery variation with Reynolds number for a thin inlet

boundary layer. The area ratio for the test was 4.0 and the discharge from the diffuser was a free jet. Additional data were given for energy conversion with four thicknesses of inlet boundary layer at one Reynolds number. The results of varying the Reynolds number for the thin inlet boundary layer tests showed significant variations in efficiency as Reynolds numbers increased, although the rate of change of the effect diminished as soon as a critical value of Reynolds number was passed. Pressure recovery, however, continued to increase. The maximum Reynolds number was 800,000, which corresponded to a Mach number of about 0.35 at the diffuser throat. The flow in the 8 degree conical diffuser was not separated in any of the tests.

Experimental Research, 1960-1963

The research of Furuya^[22] (1960) was carried out in an attempt to develop methods for reducing losses in short diffusers. The flow system consisted of an inlet contraction, variable length 10 degree diffuser, and downstream settling section. An area ratio of about 3.5 existed between the diffuser throat and the downstream settling length. The experimental procedure consisted of testing the 10 degree diffuser with various segments removed from the larger end. This produced an ordinary conical diffuser followed by a sudden expansion to the downstream settling length.

It was concluded as a result of the tests that performance of short diffusers could be improved. The configuration of short diffusers was "improved" by replacing expansions of 20 degrees or more with shorter 10 degree conical diffusers followed by sudden expansions, such that the 10 degree diffuser segment was the same

length as the larger angle diffuser it replaced. It was found possible to further reduce the losses by roughening the walls of the diffuser near the separation point. By this means separation was delayed, with consequent improvement in the pressure recovery. The author claimed that the results of the paper would allow predicting the separation point in a conical diffuser; this would of course be necessary to allow the roughness elements to be properly placed to delay separation. Locating the separation point was found not to be possible.

Yang^[104] (1961) investigated the effects of inserting "splitters" or segments of "ring" airfoils into conical diffusers. Most of the work reported was analytical, but an attempt was made to verify experimentally the results of the analysis. One test was carried out without the splitter in a diffuser of included angle 15.8 degrees at a Reynolds number of about 600,000.

Cockrell & Markland^[6] (1962) investigated again the effects of entrance conditions on diffuser flows. The principal contribution of this paper was that it pointed out that higher values of the diffuser angle-momentum thickness product were of interest than had been considered by Winternitz & Ramsay^[103]. In addition the work of Filippov^[15] (1958) was mentioned, which gave data and a correlation with entrance length for the development of the momentum thickness of a turbulent flow in a circular pipe. Experimental data were given on the variation of performance of a 10 degree diffuser of area ratio 5.05 as the momentum thickness increased. The pressure recovery was shown to pass through a maximum value and then to increase as the momentum thickness was continually increased. (The

experimental data given in this paper were later more completely reported by Cockrell & Markland^[7] in 1963.)

The authors suggested the concept of dividing diffuser performance into "internal" and "external" characteristics in order to rationalize the observed variation of performance. Fairly good results were obtained from applying the methods of Ross^[82] and Fraser^[20] to predicting the "internal" characteristic of the diffuser flow, until θ_1/R_1 became greater than about 0.050.

Megai^[53] (1962) made experimental measurements of loss coefficients in wide angle conical diffusers. The object of the tests was to determine the effectiveness of ribbed walls in preventing separation and in increasing pressure recovery in the diffusers. Data were presented in the paper for diffuser configurations both with and without the ridges. Additional tests were made with added centerbodies of various sizes, but the latter results were not useful in the conical diffuser survey.

The slots in the walls of the diffusers tested were 11 in number, and were machined into the diffuser walls. The diffusers were machined from aluminum castings. The depth of the slots was 14 percent of the inlet radius, and they were spaced on centers equal to 9 percent of the radius. Since each of the diffusers had a length to inlet ratio of about 4.0, the slots extended over slightly more than half the diffuser length.

It was found experimentally that the slots materially increased the performance of these wide angle diffusers. The pressure recovery coefficients were increased from 0.44 to 0.61 and from

0.26 to 0.67 in the 31 and 40 degree diffusers respectively. The pressure recovery of the 31 degree diffuser showed considerable variation with Reynolds number, however. It was suggested that this effect was due to a resonant condition set up by acoustical excitation due to some combination of slot and diffuser geometry. Different slot arrangements were suggested as measures for avoiding the performance decrease, but none were tried.

Only a very rough translation of the original Russian paper was available. According to the figures, however, diffusers of 22 and 60 degrees (with the same length ratio) were also tested, both with and without the circumferential slots. A small increase in performance had been observed for the unit with 22 degree divergence. The performance of the 60 degree unit was not affected by the presence of slots. Unfortunately, it was not possible to compare the absolute performance results from all the tests, since data were given for the 22 and 60 degree tests only as ratios of performance with slots to that without.

The most recent review of subsonic diffuser performance was that of Cockrell & Markland^[7] (1963). It dealt strictly with conical diffusers. New data obtained by the authors for the variation of loss coefficient for conical diffusers exhausting to the atmosphere were presented as functions of inlet boundary layer thickness and area ratio. Other sources of data included (in chronological order) Gibson, Peters, Squire, Sprenger and Cockrell & Markland (1962). The authors pointed out that it was necessary to estimate the effect of the downstream exit length before the data of Peters and Gibson could be compared with those of other investigators.

The results of earlier experiments were presented in essentially unaltered form, with two exceptions. For the first time, enough data were assembled to allow evaluation of the effect of downstream conditions on pressure recovery. A curve was plotted for two values of effective inlet length giving the increase in effectiveness due to additional lengths of straight pipe installed downstream of a diffuser. The primary significance of this was that for the first time the results of Gibson and Peters could be corrected to represent results for free jet flow conditions. The second new contribution of this paper was an attempt to predict analytically the pressure rise in a downstream settling length. Although an assumption of dubious merit was used for the average wall shear stress in the downstream duct, the wall shear force contributed only 2 percent of the effect in the momentum equation, so it was neglected. The results were plotted so that loss coefficient reduction for settling length could be obtained as a function of area ratio for two inlet boundary layer thicknesses.

The original data of the authors were presented in several forms. For diffuser designs a series of charts was presented. In these charts the loss coefficient was given as a function of opening angle for each of four configurations: one combination was tested for each diffuser with and without entrance length and with and without settling length. In addition the reduction in loss coefficient was plotted as a function of opening angle of the diffuser.

From Ackeret (1958) the authors took a figure showing the regions in which the existence of a potential core flow was a valid assumption. The interpretation of this curve was quite vague.

Appendix C.

Summary of Experimental Data for Conical Diffusers

Appendix C

Summary of Experimental Data for Conical Diffusers

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical	Flow						
	2ϕ	N/R_1	AR	Re	θ_1/R_1	Type	R_3/R_2	L_3/R_3
Cockrell & Markland 1963	8.0°	7.18	2.25	1.8×10^5	0.010	FJ	∞	0
	8.0	7.18	2.25	1.8	0.030	FJ		0
	8.0	7.18	2.25	1.8	0.068	FJ		0
	8.0	7.18	2.25	1.8	0.082	FJ		0
	8.0	7.18	2.25	1.8	0.092	FJ		0
	8.0	7.18	2.25	1.8	0.098	FJ		0
	8.0	7.18	2.25	1.8	0.098	FJ		0
	8.0	17.9	5.05	1.8	0.010	FJ		0
	8.0	17.9	5.05	1.8	0.030	FJ		0
	8.0	17.9	5.05	1.8	0.050	FJ		0
	8.0	17.9	5.05	1.8	0.068	FJ		0
	8.0	17.9	5.05	1.8	0.082	FJ		0
	8.0	17.9	5.05	1.8	0.098	FJ		0
	8.0	17.9	5.05	1.8	0.098	FJ		0
	8.0	25.1	7.55	1.8	0.010	FJ		0
	8.0	25.1	7.55	1.8	0.030	FJ		0
	8.0	25.1	7.55	1.8	0.050	FJ		0
	8.0	25.1	7.55	1.8	0.068	FJ		0
	8.0	25.1	7.55	1.8	0.082	FJ		0
	8.0	25.1	7.55	1.8	0.092	FJ		0
	8.0	25.1	7.55	1.8	0.098	FJ		0
	8.0	25.1	7.55	1.8	0.098	FJ		0
	10.0	5.73	2.25	1.8	0.010	FJ		0
	10.0	5.73	2.25	1.8	0.030	FJ		0
	10.0	5.73	2.25	1.8	0.050	FJ		0
	10.0	5.73	2.25	1.8	0.066	FJ		0
	10.0	5.73	2.25	1.8	0.082	FJ		0
	10.0	5.73	2.25	1.8	0.082	FJ		0
	10.0	5.73	2.25	1.8	0.092	FJ		0
	10.0	5.73	2.25	1.8	0.092	FJ		0
	10.0	5.73	2.25	1.8	0.098	FJ		0
	10.0	5.73	2.25	1.8	0.098	FJ		0

Flow Regime
Data

Performance Data

Visual.	Regime	C_{PR}	C_{PRi}	η	Other λ^*	Remarks
None	?	0.638	0.802	0.795	0.205	Outlet con- ditions were inferred, as they were not explicitly stated
		0.622	0.802	0.775	0.225	
		0.565	0.802	0.705	0.295	
		0.577	0.802	0.720	0.280	
		0.581	0.802	0.725	0.275	
		0.589	0.802	0.735	0.265	
		0.593	0.802	0.740	0.260	
		0.783	0.961	0.815	0.185	
		0.716	0.961	0.745	0.255	
		0.711	0.961	0.740	0.260	
		0.711	0.961	0.740	0.260	
		0.721	0.961	0.750	0.250	
		0.754	0.961	0.785	0.215	
		0.745	0.961	0.775	0.225	
		0.815	0.982	0.830	0.170	
		0.761	0.982	0.775	0.225	
		0.741	0.982	0.755	0.245	
		0.756	0.982	0.770	0.230	
		0.766	0.982	0.780	0.220	
		0.781	0.982	0.795	0.205	
		0.791	0.982	0.805	0.195	
		0.800	0.982	0.815	0.185	
		0.561	0.802	0.700	0.300	
		0.485	0.802	0.605	0.395	
		0.473	0.802	0.590	0.410	
		0.477	0.802	0.595	0.405	
		0.485	0.802	0.605	0.395	
		0.493	0.802	0.615	0.385	
		0.485	0.802	0.605	0.395	
		0.501	0.802	0.625	0.375	
		0.497	0.802	0.620	0.380	
		0.513	0.802	0.640	0.360	

* Symbols shown are those used by original sources. These are defined following the table of experimental data, on p. 190.

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical	Flow		Outlet Conditions		Type	R_3/R_2	L_3/R_3
	2ϕ	N/R_1	AR	Re	θ_1/R_1			
Cockrell 1963 (cont'd.)	10.0°	14.4	5.05	1.8×10^5	0.010	FJ	∞	0
	10.0°	14.4	5.05	1.8×10^5	0.030	FJ		0
	10.0°	14.4	5.05	1.8×10^5	0.050	FJ		0
	10.0°	14.4	5.05	1.8×10^5	0.066	FJ		0
	10.0°	14.4	5.05	1.8×10^5	0.082	FJ		0
	10.0°	14.4	5.05	1.8×10^5	0.082	FJ		0
	10.0°	14.4	5.05	1.8×10^5	0.092	FJ		0
	10.0°	14.4	5.05	1.8×10^5	0.092	FJ		0
	10.0°	14.4	5.05	1.8×10^5	0.098	FJ		0
	10.0°	14.4	5.05	1.8×10^5	0.098	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.010	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.010	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.030	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.050	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.066	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.082	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.082	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.092	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.092	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.098	FJ		0
	10.0°	20.1	7.55	1.8×10^5	0.098	FJ		0
	15.0°	3.79	2.25	1.8×10^5	0.010	FJ		0
	15.0°	3.79	2.25	1.8×10^5	0.030	FJ		0
	15.0°	3.79	2.25	1.8×10^5	0.050	FJ		0
	15.0°	3.79	2.25	1.8×10^5	0.066	FJ		0
	15.0°	3.79	2.25	1.8×10^5	0.082	FJ		0
	15.0°	3.79	2.25	1.8×10^5	0.092	FJ		0
	15.0°	9.5	5.05	1.8×10^5	0.010	FJ		0
	15.0°	9.5	5.05	1.8×10^5	0.030	FJ		0
	15.0°	9.5	5.05	1.8×10^5	0.050	FJ		0
	15.0°	9.5	5.05	1.8×10^5	0.066	FJ		0
	15.0°	9.5	5.05	1.8×10^5	0.082	FJ		0
	15.0°	9.5	5.05	1.8×10^5	0.092	FJ		0
	15.0°	13.3	7.55	1.8×10^5	0.010	FJ		0
	15.0°	13.3	7.55	1.8×10^5	0.030	FJ		0
	15.0°	13.3	7.55	1.8×10^5	0.050	FJ		0
	15.0°	13.3	7.55	1.8×10^5	0.066	FJ		0
	15.0°	13.3	7.55	1.8×10^5	0.082	FJ		0
	15.0°	13.3	7.55	1.8×10^5	0.092	FJ		0
	21.2°	2.67	2.25	1.8×10^5	0.010	FJ		0
	21.2°	2.67	2.25	1.8×10^5	0.030	FJ		0
	21.2°	2.67	2.25	1.8×10^5	0.050	FJ		0
	21.2°	2.67	2.25	1.8×10^5	0.066	FJ		0
	21.2°	2.67	2.25	1.8×10^5	0.082	FJ		0
	21.2°	2.67	2.25	1.8×10^5	0.092	FJ		0

Flow Regime
Data

Performance Data

Visual. Regime		C	C	η	Other	Remarks
		PR	PRi		λ	
None	?	0.706	0.961	0.735	0.265	Outlet condi- tions inferred, not stated
		0.615	0.961	0.640	0.360	
		0.596	0.961	0.620	0.380	
		0.605	0.961	0.630	0.370	
		0.610	0.961	0.635	0.365	
		0.620	0.961	0.645	0.355	
		0.629	0.961	0.655	0.345	
		0.639	0.961	0.665	0.335	
		0.644	0.961	0.670	0.330	
		0.649	0.961	0.675	0.325	
		0.737	0.982	0.750	0.250	
		0.756	0.982	0.770	0.230	
		0.663	0.982	0.675	0.325	
		0.643	0.982	0.655	0.345	
		0.643	0.982	0.655	0.345	
		0.658	0.982	0.670	0.330	
		0.663	0.982	0.675	0.325	
		0.673	0.982	0.685	0.315	
		0.697	0.982	0.710	0.290	
		0.678	0.982	0.690	0.310	
		0.692	0.982	0.705	0.295	
		0.537	0.802	0.670	0.330	
		0.469	0.802	0.585	0.415	
		0.461	0.802	0.575	0.425	
		0.449	0.802	0.560	0.440	
		0.445	0.802	0.555	0.445	
		0.441	0.802	0.550	0.450	
		0.629	0.961	0.655	0.345	
		0.538	0.961	0.560	0.440	
		0.533	0.961	0.555	0.445	
		0.524	0.961	0.545	0.455	
		0.519	0.961	0.540	0.460	
		0.524	0.961	0.545	0.455	
		0.668	0.982	0.680	0.320	
		0.574	0.982	0.585	0.415	
		0.565	0.982	0.575	0.425	
		0.560	0.982	0.570	0.430	
		0.550	0.982	0.560	0.440	
		0.565	0.982	0.575	0.425	
		0.373	0.802	0.465	0.535	
		0.325	0.802	0.405	0.595	
		0.321	0.802	0.400	0.600	
		0.325	0.802	0.405	0.595	
		0.357	0.802	0.445	0.555	
		0.361	0.802	0.450	0.550	

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical	Flow				Type	R_3/R_2	L_3/R_3
	2ϕ	N/R_1	AR	Re	θ_1/R_1			
Cockrell & Markland 1963 (cont'd.) ↓	21.2°	6.70	5.05	1.8×10^5	0.010	FJ	∞	0
	21.2	6.70	5.05	1.8	0.030	FJ	↓	0
	21.2	6.70	5.05	1.8	0.050	FJ	↓	0
	21.2	6.70	5.05	1.8	0.066	FJ	↓	0
	21.2	6.70	5.05	1.8	0.066	FJ	↓	0
	21.2	6.70	5.05	1.8	0.082	FJ	↓	0
	21.2	6.70	5.05	1.8	0.092	FJ	↓	0
	21.2	9.36	7.55	1.8	0.010	FJ	↓	0
	21.2	9.36	7.55	1.8	0.030	FJ	↓	0
	21.2	9.36	7.55	1.8	0.050	FJ	↓	0
	21.2	9.36	7.55	1.8	0.066	FJ	↓	0
	21.2	9.36	7.55	1.8	0.066	FJ	↓	0
	21.2	9.36	7.55	1.8	0.082	FJ	↓	0
	21.2	9.36	7.55	1.8	0.092	FJ	↓	0
	21.2	9.36	7.55	1.8	0.092	FJ	↓	0
Megai 1962 ↓	31.0°	3.90	4.00	1.18×10^5	NA	FJ	∞	0
	31.0	3.90	4.00	3.73	NA	FJ	↓	0
	40.0	3.90	5.85	0.83	NA	FJ	↓	0
	40.0	3.90	5.85	2.39	NA	FJ	↓	0
Yang 1961	15.8°	3.50	2.20	6.10×10^5	0.015*	FJ	∞	0
Furuya 1960 ↓	10.0°	0.94	1.17	3.3×10^5	0.015*	Tube	1.73	0
	10.0	0.94	1.17	3.3	0.015*	Tube	1.73	4.0
	10.0	1.20	1.21	3.3	0.015*	Tube	1.70	0
	10.0	1.20	1.21	3.3	0.015*	Tube	1.70	4.0
	10.0	1.63	1.30	3.3	0.015*	Tube	1.63	0
	10.0	1.63	1.30	3.3	0.015*	Tube	1.63	4.0
	10.0	2.49	1.48	3.3	0.015*	Tube	1.55	0
	10.0	2.49	1.48	3.3	0.015*	Tube	1.55	4.0
	10.0	3.33	1.67	3.3	0.015*	Tube	1.45	0
	10.0	3.33	1.67	3.3	0.015*	Tube	1.45	4.0
	10.0°	9.64	3.54	3.3×10^5	0.015*	Tube	Full Recovery*	
	15.0	6.38	3.54	3.3	0.015*	Tube	Full Recovery*	
	20.0	4.76	3.54	3.3	0.015*	Tube	Full Recovery*	
	30.0	3.13	3.54	3.3	0.015*	Tube	Full Recovery*	
	40.0	2.30	3.54	3.3	0.015*	Tube	Full Recovery*	
	50.0	1.80	3.54	3.3	0.015*	Tube	Full Recovery*	
	60.0	1.45	3.54	3.3	0.015*	Tube	Full Recovery*	
	↓							
	↓							

* Estimated value

Flow Regime
Data

Performance Data

Visual	Regime	C_{PR}	C_{PRi}	η	Other	Remarks
					λ	
None	?	0.423	0.961	0.440	0.560	Outlet condi- tions inferred, not stated
↓	↓	0.356	0.961	0.370	0.630	
		0.351	0.961	0.365	0.635	
		0.341	0.961	0.355	0.645	
		0.351	0.961	0.365	0.635	
		0.351	0.961	0.365	0.635	
		0.408	0.961	0.425	0.575	
		0.457	0.982	0.465	0.535	
		0.378	0.982	0.385	0.615	
		0.373	0.982	0.380	0.620	
		0.368	0.982	0.375	0.625	
		0.373	0.982	0.380	0.620	
		0.378	0.982	0.385	0.615	
		0.432	0.982	0.440	0.560	
					η	
None	?	0.440	0.938	0.469	0.47	With rib- bed walls ^{(1)*}
↓	↓	0.616	0.938	0.657	0.66	
		0.260	0.971	0.268	0.29	
		0.670	0.971	0.690	0.71	
					ϕ	
Tufts	F	0.641	0.897	0.717		With rib- bed walls
None	?	0.41	0.918	0.447		Effective area ratio(2)
↓	↓	0.66	0.918	0.719		
		0.49	0.918	0.534		
		0.71	0.918	0.773		
		0.54	0.918	0.588		
		0.76	0.918	0.828		
		0.69	0.918	0.752		
		0.81	0.918	0.882		
		0.75	0.918	0.817		
		0.84	0.918	0.915		
					η	
None	?	0.762	0.918	0.83	0.83	ξ
↓	↓	0.734	0.918	0.80	0.80	
		0.716	0.918	0.78	0.78	
		0.633	0.918	0.69	0.69	
		0.496	0.918	0.54	0.54	
		0.395	0.918	0.43	0.43	
		0.358	0.918	0.39	0.39	

* Numbers in () refer to notes at end of data table,
p. 190.

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical			Flow		Type	R_3/R_2	L_3/R_3
	2ϕ	N/R_1	AR	Re	θ_1/R_1			
Sprenger 1959 ↓	8.0°	14.3	4.00	520,000	0.003	FJ	∞	0
	8.0°	14.3	4.00	520,000	0.003	Tube	1.0	2.0
	8.0°	14.3	4.00	520,000	0.005	FJ	∞	0
	8.0°	14.3	4.00	520,000	0.015	FJ		0
	8.0°	14.3	4.00	520,000	0.025	FJ		0
	8.0°	14.3	4.00	25k	0.060	FJ		0
	8.0°	14.3	4.00	33k	0.060	FJ		0
	8.0°	14.3	4.00	44k	0.060	FJ		0
	8.0°	14.3	4.00	64k	0.060	FJ		0
	8.0°	14.3	4.00	86k	0.060	FJ		0
	8.0°	14.3	4.00	130k	0.060	FJ		0
	8.0°	14.3	4.00	218k	0.060	FJ		0
	8.0°	14.3	4.00	360k	0.060	FJ		0
	8.0°	14.3	4.00	540k	0.060	FJ		0
Rippl 1958 ↓	3.0°	5.00	1.28	150-600k	Thin	FJ	∞	0
	3.0°	10.0	1.59	90-700k	0.01*	FJ		0
	3.0°	15.8	2.00	90-700k	0.01*	FJ		0
	3.0°	21.6	2.47	90-600k	0.01*	FJ		0
	3.0°	31.6	3.34	75-700k	0.01*	FJ		0
	3.0°	41.6	4.35	90-650k	0.01*	FJ		0
	3.0°	51.6	5.53	85-650k	0.01*	FJ		0
	3.0°	61.6	6.83	85-650k	0.01*	FJ		0
	7.0°	4.52	1.63	80-550k	Thin	FJ		0
	7.0°	8.60	2.33	100-700k	Thin	FJ		0
	7.0°	13.2	3.27	98-700k	Thin	FJ		0
	7.0°	17.5	4.28	82-700k	Thin	FJ		0
	7.0°	22.0	5.48	92-750k	Thin	FJ		0
	7.0°	27.0	7.00	75-700k	Thin	FJ		0
	7.0°	30.0	8.00	62-700k	Thin	FJ		0
	14.0°	1.78	1.48	100-700k	Thin	FJ		0
	14.0°	3.92	2.08	90-650k	Thin	FJ		0
	14.0°	5.70	2.89	75-700k	Thin	FJ		0
	14.0°	7.60	3.73	65-650k	Thin	FJ		0
	14.0°	10.4	5.20	75-700k	Thin	FJ		0
	14.0°	14.3	7.56	120-650k	Thin	FJ		0
	14.0°	17.0	9.55	85-650k	Thin	FJ		0
	14.0°	20.0	12.00	95-650k	Thin	FJ		0
	14.0°	3.44	2.01	100-500k	Thin	FJ		0
	14.0°	4.80	2.53	95-500k	Thin	FJ		0
	14.0°	6.80	3.37	90-600k	Thin	FJ		0
	14.0°	10.9	5.47	80-500k	Thin	FJ		0
Furuya 1958	20.0°	1.78	2.63	186k	0.0163	Tube	1.0	3.55
	20.0°	1.78	2.63	195k	0.0163	Tube	1.0	3.55

* Estimated value

Flow Regime
Data

Performance Data

Visual Regime		C_{PR}	C_{PRi}	η	Other	Remarks
					$\bar{\eta}$	
Velocity Profiles ↓	NS	0.857	0.938	0.914	0.914	Average values Assume $\alpha_1 \approx 1.0$ for all readings.
		0.870	0.938	0.928	0.928	
		0.827	0.938	0.882	0.882	
		0.740	0.938	0.789	0.789	
		0.690	0.938	0.736	0.736	
		0.694	0.938	0.740	0.740	
		0.746	0.938	0.795	0.795	
		0.769	0.938	0.820	0.820	
		0.779	0.938	0.831	0.831	
		0.795	0.938	0.848	0.848	
		0.813	0.938	0.867	0.867	
		0.834	0.938	0.889	0.889	
		0.847	0.938	0.903	0.903	
		0.856	0.938	0.913	0.913	
Tufted grid in exit plane ↓	NS	0.374	0.39	0.96		Consider- able scatter
		0.538	0.605	0.89		
		0.652	0.750	0.87		
		0.707	0.836	0.84		
		0.792	0.910	0.87		
		0.824	0.947	0.87		
		0.841	0.967	0.87		
		0.852	0.979	0.87		
Drying plate ↓	NSep	0.583	0.620	0.94		Did not use all data
		0.717	0.815	0.88		
		0.788	0.906	0.87		
		0.813	0.945	0.86		
		0.841	0.967	0.87		
		0.843	0.980	0.86		
		0.856	0.984	0.87		
		0.457	0.544	0.84		
	Sep.	0.561	0.769	0.73		
	Occurs	0.594	0.880	0.67		
		0.640	0.925	0.69		
		0.674	0.963	0.70		
		0.698	0.983	0.71		
		0.722	0.989	0.73		
		0.735	0.993	0.74		
		0.618	0.754	0.82		
		0.629	0.834	0.75		
		0.675	0.912	0.74		
		0.725	0.966	0.75		
Tufts on wire	Sep.	0.59	0.855	0.690		Considerable fluctuation
	Occurs	0.49	0.855	0.573		

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical			Flow		Type	R_3/R_2	L_3/R_3
	2ϕ	N/R_1	AR	Re	θ_1/R_1			
Fraser	10.0°	4.03	1.83	4.20x10 ⁵	0.010*	FJ	∞	0
1958	10.0	3.11	1.61	3.80	0.014*	FJ	∞	0
Winternitz	5.0°	22.9	4.00	2.50x10 ⁵	0.010	Tube	1.00	3.09
& Ramsay	5.0	22.9	4.00	2.50	0.030	Tube	1.00	3.09
1957	5.0	22.9	4.00	2.50	0.050	Tube	1.00	3.09
↓	5.0	22.9	4.00	2.50	0.070	Tube	1.00	3.09
	10.0	11.5	4.00	2.50	0.010	Tube	1.00	3.09
	10.0	11.5	4.00	2.50	0.030	Tube	1.00	3.09
	10.0	11.5	4.00	2.50	0.050	Tube	1.00	3.09
	10.0	11.5	4.00	2.50	0.070	Tube	1.00	3.09
Robertson	7.5°	13.3	3.48	3.00x10 ⁵	0.009	FJ	∞	0
& Hall	7.5	13.3	3.48	2.55	0.027	FJ	∞	0
1957	7.5	3.33	1.48	2.18	0.027	Tube	1.00	3.11
↓	7.5	8.36	2.39	2.45	0.027	Tube	1.00	1.25
	7.5	8.36	2.39	2.37	0.027	FJ	∞	0
Kaufmann	41.6°	3.30	4.70	6.40x10 ⁵	Thin	FJ	∞	0
1957	41.6	3.30	4.70	6.40	Thin	FJ	∞	0
↓	30.0	3.62	3.24	7.01	Thin	FJ	∞	0
	30.0	3.62	3.24	7.08	Thin	FJ	∞	0
	11.0	8.36	3.24	10.30	Thin	FJ	∞	0
Scherrer	8.0°	8.60	2.41	3.85x10 ⁵	0.001	Tube	1.29	Long
1956								
Holzhauser	10.0°	5.13	2.00	2.56x10 ^{5*}	0.003	Tube	1.00	0.00
& Hall	30.0	1.94	2.00	2.56	* 0.003*	Tube	1.00	0.43
↓	50.0	1.29	2.00	2.56	* 0.003*	Tube	1.00	0.57
Uram 1954	7.5°	13.3	3.48	3.00x10 ⁵	0.009	FJ	∞	0
Persh &	23.0°	2.03	2.00	1.90x10 ⁶	0.013	Tube	1.00	0.00
Bailey	23.0	2.03	2.00	3.50	0.011	Tube	1.00	2.49
1954								
Valentine &	23.0°							
Carroll	23.0							
1952	23.0							
↓	23.0							
	23.0							
	23.0							

* Estimated value

Flow Regime
Data

Performance Data

Visual	Regime	C_{PR}	C_{PRI}	η	Other	Remarks
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	Sep.	Unsep.
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 α_2

None ↓	?	0.787	0.938	0.839	1.440
		0.775	0.938	0.826	1.485
		0.763	0.938	0.813	1.530
		0.751	0.938	0.801	1.580
		0.700	0.938	0.746	1.580
		0.610	0.938	0.650	1.580
		0.520	0.938	0.554	1.580
		0.430	0.938	0.458	1.580

None ↓	Unsep.	0.917
		0.797
		0.503
		0.733
		0.673

None ↓	?	0.361	0.955	0.379
			0.955	0.370
		0.403	0.904	0.446
			0.904	0.500
			0.904	0.881

None	?	0.762	0.828
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None ↓	?	0.750	$1 - \eta$	K
	Sep.	0.52	0.750	0.078
	Sep.	0.37	0.750	0.176

None	?	0.80	0.917	0.875
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None	?	0.459	0.75	0.612	K
					0.105

None	?	0.632	0.75	0.843
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Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical			Flow		Type	R_3/R_2	L_3/R_3
	2ϕ	N/R_1	AR	Re	θ_1/R_1			
Robertson & Ross 1952 ↓	5.0°	16.7	3.00	1.2×10^6	0.016*	Tube	1.00	0
	5.0°	16.7	3.00	1.2×10^6	0.024*	Tube	1.00	0
	5.0°	16.7	3.00	1.2×10^6	0.036*	Tube	1.00	0
	7.5°	11.1	3.00	1.2×10^6	0.015*	Tube	1.00	0
	7.5°	11.1	3.00	1.2×10^6	0.025*	Tube	1.00	0
	7.5°	11.1	3.00	1.2×10^6	0.035*	Tube	1.00	0
	10.0°	8.3	3.00	1.2×10^6	0.015*	Tube	1.00	0
	10.0°	8.3	3.00	1.2×10^6	0.025*	Tube	1.00	0
Warren 1951 ↓	5.0°	68.8	16.0	350k*	Thin	Tube	1.00	8-12
	6.0°	57.3	16.0	350k*	Thin	Tube	1.00	8-12
	7.0°	49.2	16.0	350k*	Thin	Tube	1.00	8-12
	8.0°	42.9	16.0	350k*	Thin	Tube	1.00	8-12
	9.0°	38.2	16.0	350k*	Thin	Tube	1.00	8-12
	4.5°	25.4	4.00	350k*	Thin	Tube	1.00	8-12
	5.0°	22.9	4.00	350k*	Thin	Tube	1.00	8-12
	6.0°	19.1	4.00	350k*	Thin	Tube	1.00	8-12
	7.0°	16.4	4.00	350k*	Thin	Tube	1.00	8-12
	8.0°	14.3	4.00	350k*	Thin	Tube	1.00	8-12
	9.0°	12.7	4.00	350k*	Thin	Tube	1.00	8-12
	11.0°	10.5	4.00	350k*	Thin	Tube	1.00	8-12
	14.3°	7.96	4.00	350k*	Thin	Tube	1.00	8-12
	17.0°	6.70	4.00	350k*	Thin	Tube	1.00	8-12
	5.0°	7.67	1.78	350k*	Thin	Tube	1.00	8-12
	6.0°	6.38	1.78	350k*	Thin	Tube	1.00	8-12
	7.0°	5.47	1.78	350k*	Thin	Tube	1.00	8-12
	8.0°	4.79	1.78	350k*	Thin	Tube	1.00	8-12
	9.0°	4.25	1.78	350k*	Thin	Tube	1.00	8-12
	12.0°	3.17	1.78	350k*	Thin	Tube	1.00	8-12
	14.3°	2.66	1.78	350k*	Thin	Tube	1.00	8-12
	20.0°	1.89	1.78	350k*	Thin	Tube	1.00	8-12
Copp 1951 ↓	10.0°	4.72	2.00	1.96×10^6	0.004	Tube	1.00	2.00
	10.0°	4.72	2.00	1.19×10^6	0.022	Tube	1.00	2.00
	10.0°	4.72	2.00	2.15×10^6	0.020	Tube	1.00	2.00
Squire 1950a ↓	4.0°	28.6	4.00	10^5	Thin	Tube	1.00	1.00
	5.0°	22.9	4.00	10^5	Thin	Tube	1.00	1.00
	6.0°	19.0	4.00	10^5	Thin	Tube	1.00	1.00
	8.0°	14.3	4.00	10^5	Thin	Tube	1.00	1.00

* Estimated value

Flow Regime
Data

Performance Data

Visual. Regime		C	C	η	Other	Remarks
		PR	PRi			η_e
None	Nsep	0.836	0.889	0.930		0.965
(vel.		0.808	0.889	0.910		0.955
traverse)		0.777	0.889	0.875		0.915
↓	↓	0.899	0.889	0.900		0.950
		0.768	0.889	0.865		0.940
None	Unsym.	0.753	0.889	0.850		0.950 Turning
(Vel.	Sep.	0.781	0.889	0.880		0.945 vanes
trav.)	↓	0.742	0.889	0.835		0.940 near
						THL
None	No info.	0.857	0.996	0.860		0.140
↓	↓	0.857	0.996	0.861		0.139
		0.851	0.996	0.857		0.143
		0.843	0.996	0.848		0.152
		0.824	0.996	0.828		0.172
		0.822	0.938	0.879		0.121
		0.834	0.938	0.890		0.110
		0.840	0.938	0.897		0.103
		0.842	0.938	0.899		0.101
		0.842	0.938	0.899		0.101
		0.840	0.938	0.892		0.103
		0.831	0.938	0.889		0.111
		0.812	0.938	0.868		0.132
		0.792	0.938	0.847		0.153
		0.599	0.685	0.876		0.124
		0.600	0.685	0.877		0.123
		0.601	0.685	0.879		0.121
		0.602	0.685	0.880		0.120
		0.603	0.685	0.881		0.119
		0.604	0.685	0.882		0.118
		0.601	0.685	0.880		0.120
		0.595	0.685	0.870		0.130
None	-	0.647	0.75	0.861		
	-	0.525	0.75	0.700		
	-	0.525	0.75	0.700		
?	Nsep	0.833	0.938	0.89	μ 0.11	
↓	↓	0.833	0.938	0.89	0.11	
		0.833	0.938	0.89	0.11	
		0.833	0.938	0.89	0.11	

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical	Flow						
	2ϕ	N/R_1	AR	Re	θ_1/R_1	Type	R_3/R_2	L_3/R_3
Squire 1950a (cont'd.) ↓	4.0°	28.6	4.00	10 ⁶	Thin	Tube	1.00	1.00
	5.0°	22.9	4.00	10 ⁶	Thin	Tube	1.00	1.00
	6.0°	19.0	4.00	10 ⁶	Thin	Tube	1.00	1.00
	8.0°	14.3	4.00	10 ⁶	Thin	Tube	1.00	1.00
	10.0°	11.4	4.00	10 ⁶	Thin	Tube	1.00	1.00
	4.0°	82.3	16.0	450k	0.064*	Unspecified		
	5.0°	68.5	16.0	450k	0.064*			
	6.0°	57.2	16.0	450k	0.064*			
	8.0°	42.8	16.0	450k	0.064*			
	10.0°	34.4	16.0	450k	0.064*			
Little 1950 ↓	12.0°	3.93	2.00	1.1x10 ⁶	0.0057	Tube	1.00	2.00
	12.0°	3.93	2.00	1.2x10 ⁶	0.0198			
	12.0°	3.93	2.00	2.4x10 ⁶	0.0025	Tube	1.00	Long
	12.0°	3.93	2.00	3.7x10 ⁶	0.0146	Tube	1.00	Long
	23.0°	2.10	2.00	2.4x10 ⁶	0.0028	Tube	1.00	Long
	23.0°	2.10	2.00	2.5x10 ⁶	0.0168	Tube	1.00	Long
Schubauer 1949	28°	3.48	4.00	950k*	0.034*	Tube	1.00	6.00
	90°	1.26	4.00	950k*	0.034*	Tube	1.00	6.00
Gratzer & Smith 1948 ↓	15°	7.60	4.00	?	0.0038	Tube	1.00	Long
	20°	5.68	4.00	?	0.0038	Tube	1.00	Long
	25°	4.53	4.00	?	0.0038	Tube	1.00	Long
	30°	3.73	4.00	?	0.0038	Tube	1.00	Long
	30°	3.73	4.00	?	0.0077	Tube	1.00	Long
	30°	3.73	4.00	?	0.0197	Tube	1.00	Long
	30°	3.73	4.00	?	0.0333	Tube	1.00	Long
	40°	2.75	4.00	?	0.0038	Tube	1.00	Long
	60°	1.73	4.00	?	0.0038	Tube	1.00	Long
Kalinske 1946 ↓	7.5°	11.1	2.99	66,600	0.10*	Tube	1.00	Max C _p
	7.5°	11.1	2.99	95,000	0.10*	Tube	1.00	Max C _p
	15.0°	5.52	2.99	66,600	0.10*	Tube	1.00	Max C _p
	15.0°	5.52	2.99	95,000	0.10*	Tube	1.00	Max C _p
	30.0°	2.70	2.99	66,600	0.10*	Tube	1.00	Max C _p
	30.0°	2.70	2.99	95,000	0.10*	Tube	1.00	Max C _p
	180°	0	2.99	37,500	0.10*	Tube	1.00	Max C _p
	180°	0	2.99	66,600	0.10*	Tube	1.00	Max C _p
180°	0	2.99	95,000	0.10*	Tube	1.00	Max C _p	

* Estimated value

Flow Regime
Data

Performance Data

Visual. Regime		C	C	η	Other	Remarks
		PR	PRi		μ α_2	
?	NSep	0.852	0.938	0.91	0.09	1.28
			0.938	0.91	0.09	1.43
		0.862	0.938	0.92	0.08	1.51
		0.843	0.938	0.90	0.10	1.68
		-	0.938	-	-	1.80
	NSep	NA	0.996			} Unknown
			0.996			
			0.996			
			0.996			
			0.996			
?	NSep	0.701	0.750	0.933		
		0.520	0.750	0.692		
	Fluct.	0.702	0.750	0.936		
		0.606	0.750	0.808		
	Violent Fluct.	0.670	0.750	0.891		
		0.468	0.750	0.624		
None	Sep.	0.26	0.938			
	Sep.	0.12	0.938			
None	?	0.773	0.938	0.825		
		0.660	0.938	0.703		
		0.488	0.938	0.520		
		0.370	0.938	0.395		
		0.310	0.938	0.330		
		0.215	0.938	0.230		
		0.215	0.938	0.230		
		0.290	0.938	0.310		
		0.070	0.938	0.075		
Dye trace	N.S.	0.735	0.911	0.808		Fully developed inlet flow
	N.S.	0.743	0.911	0.818		
	Slight	0.669	0.911	0.733		
	Sep	0.652	0.911	0.717		
		0.577	0.911	0.631		
		0.603	0.911	0.662		
		0.521	0.911	0.571		
		0.496	0.911	0.544		
		0.578	0.911	0.632		

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical			Flow		Type	R_3/R_2	L_3/R_3
	2ϕ	N/R_1	AR	Re	θ_1/R_1			
Peters 1934	5.2°	11.7	2.34	2×10^5	0.012*	FJ	∞	0
	5.2°	11.7	2.34	2×10^5	0.036*	FJ		0
	5.2°	11.7	2.34	2×10^5	0.082*	FJ		0
	7.8°	7.80	2.34	2×10^5	0.012*	FJ		0
	7.8°	7.80	2.34	2×10^5	0.036*	FJ		0
	7.8°	7.80	2.34	2×10^5	0.082*	FJ		0
	8.8°	6.90	2.34	2×10^5	0.012*	FJ		0
	8.8°	6.90	2.34	2×10^5	0.036*	FJ		0
	8.8°	6.90	2.34	2×10^5	0.082*	FJ		0
	11.4°	5.35	2.34	2×10^5	0.012*	FJ		0
	11.4°	5.35	2.34	2×10^5	0.036*	FJ		0
	11.4°	5.35	2.34	2×10^5	0.082*	FJ		0
	11.4°	5.35	2.34	2×10^5	0.100*	FJ		0
	14.7°	4.10	2.34	2×10^5	0.012*	FJ		0
	14.7°	4.10	2.34	2×10^5	0.036*	FJ		0
	14.7°	4.10	2.34	2×10^5	0.082*	FJ		0
	14.7°	4.10	2.34	2×10^5	0.100*	FJ		0
	28.4°	2.09	2.34	2×10^5	0.012*	FJ		0
	28.4°	2.09	2.34	2×10^5	0.036*	FJ		0
	28.4°	2.09	2.34	2×10^5	0.082*	FJ		0
	28.4°	2.09	2.34	2×10^5	0.100*	FJ		0
	5.2°	11.7	2.34	2×10^5	0.012*	Tube	1.00	Long
	5.2°	11.7	2.34	2×10^5	0.036*	Tube	1.00	Long
	5.2°	11.7	2.34	2×10^5	0.082*	Tube	1.00	Long
	7.8°	7.80	2.34	2×10^5	0.012*	Tube	1.00	Long
	7.8°	7.80	2.34	2×10^5	0.036*	Tube	1.00	Long
	7.8°	7.80	2.34	2×10^5	0.082*	Tube	1.00	Long
	8.8°	6.90	2.34	2×10^5	0.012*	Tube	1.00	Long
	8.8°	6.90	2.34	2×10^5	0.036*	Tube	1.00	Long
	8.8°	6.90	2.34	2×10^5	0.082*	Tube	1.00	Long
	11.4°	5.35	2.34	2×10^5	0.012*	Tube	1.00	Long
	11.4°	5.35	2.34	2×10^5	0.036*	Tube	1.00	Long
	11.4°	5.35	2.34	2×10^5	0.082*	Tube	1.00	Long
	11.4°	5.35	2.34	2×10^5	0.100*	Tube	1.00	Long
	14.7°	4.10	2.34	2×10^5	0.012*	Tube	1.00	Long
	14.7°	4.10	2.34	2×10^5	0.036*	Tube	1.00	Long
	14.7°	4.10	2.34	2×10^5	0.082*	Tube	1.00	Long
	14.7°	4.10	2.34	2×10^5	0.100*	Tube	1.00	Long
	28.4°	2.09	2.34	2×10^5	0.012*	Tube	1.00	Long
	28.4°	2.09	2.34	2×10^5	0.036*	Tube	1.00	Long
	28.4°	2.09	2.34	2×10^5	0.082*	Tube	1.00	Long
	28.4°	2.09	2.34	2×10^5	0.100*	Tube	1.00	Long

* Estimated value

Flow Regime
Data

Performance Data

Visual. Regime		C	C	η	Other		Remarks
		PR	PRi		A	B	
None	?	0.719	0.817	0.88		1.03	Various
		0.678	0.817	0.83		1.03	inlet
		0.644	0.817	0.79		1.03	moment-
		0.701	0.817	0.86		1.03	um
		0.644	0.817	0.79		1.03	thick-
		0.596	0.817	0.73		Typ-	ness
		0.693	0.817	0.85		ical	
		0.629	0.817	0.77		val-	
		0.579	0.817	0.71		ues	
		0.678	0.817	0.83	1.007		
		0.596	0.817	0.73	1.020		
		0.530	0.817	0.65	1.033		
		0.539	0.817	0.66	1.042		
		0.629	0.817	0.77	Typ-		
		0.539	0.817	0.66	ical		
		0.473	0.817	0.58	val-		
		0.482	0.817	0.59	ues		
		0.441	0.817	0.54			
		0.335	0.817	0.41			
		0.269	0.817	0.33			
		0.285	0.817	0.35			
		0.742	0.817	0.91			
		0.734	0.817	0.90			
		0.734	0.817	0.90			
		0.759	0.817	0.93			
		0.750	0.817	0.92			
		0.734	0.817	0.90			
		0.759	0.817	0.93			
		0.734	0.817	0.90			
		0.710	0.817	0.87			
		0.750	0.817	0.92			
		0.730	0.817	0.90			
		0.720	0.817	0.88			
		0.710	0.817	0.87			
		0.730	0.817	0.89			
		0.700	0.817	0.86			
		0.690	0.817	0.85			
		0.710	0.817	0.87			
		0.620	0.817	0.76			
		0.620	0.817	0.76			
		0.610	0.817	0.75			
		0.610	0.817	0.75			

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical	Flow						
	2ϕ	N/R_1	AR	Re	θ_1/R_1	Type	R_3/R_2	L_3/R_3
Peters 1934 (cont'd.) ↓	55.8°	1.00	2.34	2×10^5	0.012*	Tube	1.00	Long
	55.8°	1.00	2.34	2×10^5	0.036*	Tube	1.00	Long
	55.8°	1.00	2.34	2×10^5	0.082*	Tube	1.00	Long
	66.9°	0.80	2.34	2×10^5	0.012*	Tube	1.00	Long
	66.9°	0.80	2.34	2×10^5	0.036*	Tube	1.00	Long
	66.9°	0.80	2.34	2×10^5	0.082*	Tube	1.00	Long
	91.6°	0.51	2.34	2×10^5	0.012*	Tube	1.00	Long
	91.6°	0.51	2.34	2×10^5	0.036*	Tube	1.00	Long
	91.6°	0.51	2.34	2×10^5	0.082*	Tube	1.00	Long
	180°	0	2.34	2×10^5	0.012*	Tube	1.00	Long
	180°	0	2.34	2×10^5	0.036*	Tube	1.00	Long
	180°	0	2.34	2×10^5	0.082*	Tube	1.00	Long
	180°	0	2.34	2×10^5	0.100*	Tube	1.00	Long
Lyon 1921 ↓	4.0°	28.7	4.0	-	Thin	FJ(?)	$\infty(?)$	0
	6.0°	20.1	4.0	195k	Thin	FJ(?)		0
	6.0°	20.1	4.0	309k	Thin	FJ(?)		0
	6.0°	20.1	4.0	410k	Thin	FJ(?)		0
	8.0°	14.5	4.0	195k	Thin	FJ(?)		0
	8.0°	14.5	4.0	309k	Thin	FJ(?)		0
	8.0°	14.5	4.0	410k	Thin	FJ(?)		0
	10.0°	11.6	4.0	195k	Thin	FJ(?)		0
	10.0°	11.6	4.0	309k	Thin	FJ(?)		0
	10.0°	11.6	4.0	410k	Thin	FJ		0
	12.0°	9.7	4.0	195k	Thin	FJ		0
	12.0°	9.7	4.0	309k	Thin	FJ		0
	12.0°	9.7	4.0	403k	Thin	FJ		0
	8.0°	29.0	9.0	195k	Thin	FJ		0
	8.0°	29.0	9.0	309k	Thin	FJ		0
	8.0°	29.0	9.0	410k	Thin	FJ		0
	10.0°	23.1	9.0	195k	Thin	FJ		0
	10.0°	23.1	9.0	309k	Thin	FJ		0
	10.0°	23.1	9.0	410k	Thin	FJ		0
Gibson 1910 ↓	3.0°	38.2	4.00	50,800	Thick*	Tube	1.00	P_2 max
	4.0°	28.6	4.00	to		Tube	1.00	P_2 max
	5.0°	23.0	4.00	233,000	abt. 0.06	Tube	1.00	P_2 max
	7.5°	15.2	4.00			Tube	1.00	P_2 max
	10.0°	11.4	4.00			Tube	1.00	P_2 max
	12.5°	9.15	4.00			Tube	1.00	P_2 max
	15.0°	7.60	4.00			Tube	1.00	P_2 max
	17.5°	6.50	4.00			Tube	1.00	P_2 max

* Estimated value

Flow Regime
Data

Performance Data

Visual	Regime	C _{PR}	C _{PR1}	η	Other	Remarks
None	?	0.47	0.817	0.57		
		0.47	0.817	0.58		
		0.49	0.817	0.60		
		0.46	0.817	0.56		
		0.47	0.817	0.57		
		0.47	0.817	0.57		
		0.47	0.817	0.57		
		0.47	0.817	0.57		
		0.49	0.817	0.60		
		0.47	0.817	0.58		
		0.48	0.817	0.59		
		0.48	0.817	0.59		
		0.47	0.817	0.58		
None	?	-	0.938	-		
		0.756	0.938	0.806		
		0.797	0.938	0.850		
		0.765	0.938	0.816		
		0.791	0.938	0.845		
		0.838	0.938	0.893		
		0.757	0.938	0.807		
		0.809	0.938	0.862		
		0.816	0.938	0.870		
		0.793	0.938	0.845		
		0.735	0.938	0.784		
		0.760	0.938	0.810		
		0.736	0.938	0.785		
		0.857	0.938	0.914		
		0.896	0.988	0.907		
		0.904	0.988	0.915		
		0.769	0.988	0.778		
		0.773	0.988	0.782		
		0.761	0.988	0.770		
None	?	0.831	0.938	0.888	%H. L.	
		0.845	0.938	0.901	0.190	Losses
		0.860	0.938	0.919	0.165	stated
		0.858	0.938	0.913	0.135	to be
		0.839	0.938	0.895	0.145	"sensibly
		0.818	0.938	0.871	0.175	constant"
		0.790	0.938	0.841	0.215	with
		0.738	0.938	0.787	0.265	Reynolds
					0.355	number

Investigator	Inlet Conditions					Outlet Conditions		
	Geometrical		Flow					
	2ϕ	N/R_1	AR	Re	θ_1/R_1	Type	R_3/R_2	L_3/R_3
Gibson 1910 (cont'd.) ↓	20.0°	5.68	4.00	50,800	Thick*	Tube	1.00	P_2 max
	30.0°	3.74	4.00	to	abt. 0.06	Tube	1.00	P_2 max
	40.0°	2.75	4.00	233,000	↓	Tube	1.00	P_2 max
	50.0°	2.15	4.00	↓	↓	Tube	1.00	P_2 max
	60.0°	1.73	4.00	↓	↓	Tube	1.00	P_2 max
	90.0°	1.00	4.00	↓	↓	Tube	1.00	P_2 max
	180.0°	0.00	4.00	↓	↓	Tube	1.00	P_2 max
Gibson 1911 ↓	10.0°	22.9	9.00	6200	Thick	Tube	1.00	P_2 max
	20.0°	11.3	9.00	to	abt. 0.06*	Tube	1.00	P_2 max
	40.0°	5.50	9.00	71,400	↓	Tube	1.00	P_2 max
	60.0°	3.47	9.00	↓	↓	Tube	1.00	P_2 max
	90.0°	2.00	9.00	↓	↓	Tube	1.00	P_2 max
	180.0°	0	9.00	↓	↓	Tube	1.00	P_2 max
	20.0°	11.3	9.00	12,400	↓	Tube	1.00	P_2 max
	40.0°	5.50	9.00	to	↓	Tube	1.00	P_2 max
	60.0°	3.47	9.00	143,000	↓	Tube	1.00	P_2 max
	90.0°	2.00	9.00	↓	↓	Tube	1.00	P_2 max
	180.0°	0	9.00	↓	↓	Tube	1.00	P_2 max
	10.0°	5.75	2.25	24,800	↓	Tube	1.00	P_2 max
	20.0°	2.83	2.25	to	↓	Tube	1.00	P_2 max
	40.0°	1.37	2.25	286,000	↓	Tube	1.00	P_2 max
	60.0°	0.87	2.25	↓	↓	Tube	1.00	P_2 max
	90.0°	0.50	2.25	↓	↓	Tube	1.00	P_2 max
	180.0°	0	2.25	↓	↓	Tube	1.00	P_2 max
Stanton 1902 ↓	6.4°	36.0	9.00	20.1k	?	Tube	1.00	P_2 max
	6.4°	36.0	9.00	23.3k	?	Tube	1.00	P_2 max
	6.4°	36.0	9.00	29.2k	?	Tube	1.00	P_2 max
	6.4°	36.0	9.00	53.0k	?	Tube	1.00	P_2 max
Andres 1910 ↓	12.0°	34.2	21.1	131k	Thin*	Tube	1.00	P_2 at
	7.7°	28.3	8.50	to	Thin*	Tube	1.00	end
	7.9°	28.1	8.65	588k	Thin*	Tube	1.00	plane
	6.2°	30.4	6.90	↓	Thin*	Tube	1.00	↓
	4.1°	28.3	4.00	↓	Thin*	Tube	1.00	↓
	4.3°	26.6	3.94	↓	Thin*	Tube	1.00	↓

Flow Regime
Data

Performance Data

Visual	Regime	C_{PR}	C_{PRi}	η	Other	Remarks
None ↓	? ↓	0.697	0.938	0.742	0.430	% H. L.
		0.473	0.938	0.505	0.825	
		0.353	0.938	0.376	1.020	
		0.300	0.938	0.320	1.135	
		0.260	0.938	0.277	1.205	
		0.310	0.938	0.331	1.115	
		0.367	0.938	0.392	1.015	
		None ↓	? ↓	0.858	0.988	0.867
0.632	0.988			0.640	0.450	
0.339	0.988			0.343	0.820	
0.175	0.988			0.177	1.028	
0.175	0.988			0.177	1.026	
0.175	0.988			0.177	1.028	
0.640	0.988			0.648	0.440	
0.350	0.988			0.354	0.808	
0.188	0.988			0.190	1.013	
0.166	0.988			0.168	1.041	
0.181	0.988			0.183	1.021	
0.745	0.803			0.928	0.186	
0.674	0.803			0.839	0.419	
0.529	0.803			0.659	0.887	
0.431	0.803			0.537	1.203	
0.456	0.803			0.569	1.121	
0.497	0.803	0.619	0.992			
None ↓	? ↓	0.712	0.988	0.721	0.58	THL % THL
		0.713	0.988	0.722	0.78	26.1
		0.732	0.988	0.741	1.18	26.2
		0.729	0.988	0.739	3.92	25.3
None ↓	? ↓	0.744	1.000	0.744		25.8
		0.870	0.986	0.883		
		0.842	0.987	0.854		
		0.820	0.979	0.838		
		0.839	0.938	0.893		
		0.762	0.936	0.812		

Definitions of Symbols Used by Original Sources for Appendix C.

Investigator	Symbol	Defined in Nomenclature of Appendix A
Cockrell & Markland	λ	λ_{31} or λ_{61}
	η	η_{31}
Megai	φ	$(1 - \eta_{31}) \left(\frac{AR + 1}{AR - 1} \right)$
Furuya	ξ	$(1 - \eta_{31}) \left(\frac{AR + 1}{AR - 1} \right)$
Sprenger	$\bar{\eta}$	η_{51}
Winternitz & Ramsay	α_2	Energy coefficient at diffuser exit
Holzhauser & Hall	K	η_{61}
Robertson & Ross	η_e	$\frac{\eta_{31}}{\alpha_1} + \frac{\alpha_2 - \eta_{31}}{\alpha_1} \frac{1}{AR^2}$
Squire	μ	$1 - \eta_{31}$
	α_2	Energy coefficient at diffuser exit
Peters	A	α_1
	B	α_2
Gibson	H. L.	$(1 - \eta_{31}) \left(\frac{AR + 1}{AR - 1} \right)$
Stanton	THL	λ_{74}

Numbered Notes from Appendix C.

- (1) See Appendix B, pp. 164-169, for discussion of method of fabrication of "ribbed" diffuser.

Numbered Notes (cont'd.)

(2) Effective area ratio = $AR \times \left(\frac{R_3}{R_2}\right)^2.$

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