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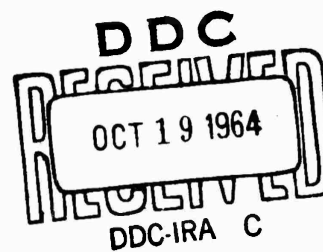
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# **APPENDIX TO MANUAL CONTROL**

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APPENDIX

A SURVEY OF HUMAN OPERATOR MODELS  
FOR MANUAL CONTROL

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## GLOSSARY OF SYMBOLS

|                          |                                                                                                                                     |
|--------------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| $b_j$                    | The coefficient of the $j$ th mimic filter                                                                                          |
| $C(s)$                   | The Laplace transform of the transfer function of a closed loop system in which the human operator is an element                    |
| $e^{-\tau s}$            | A time delay; it represents, for the human operator model, man's reaction time function                                             |
| $f(t), g(t)$             | Symbols signifying a time function (in the context $f(t)$ is often an input time function and $g(t)$ represents the output)         |
| $F(j\omega), G(j\omega)$ | The Fourier transforms of $f(t)$ and $g(t)$ , respectively                                                                          |
| $F(s)$                   | The Laplace transform of $f(t)$                                                                                                     |
| $h(t)$                   | The human operator transfer function (or "describing" function) in the time domain $o_h(t)/i_h(t)$                                  |
| $h(\tau)$                | The linear human weighting function which combines with $n_c(t)$ to form the time-dependent transfer function of the human operator |
| $H(j\omega)$             | The Fourier transform of $h(t)$                                                                                                     |
| $H(s)$                   | The Laplace transform of $h(t)$                                                                                                     |
| $i$                      | Input to the simple system model                                                                                                    |
| $i(t)$                   | Conventionally, a system forcing function; the input as a function of time                                                          |
| $i_h$                    | Input to the human in a man-machine system                                                                                          |
| $I_h(s)$                 | The Laplace transform of $i_h$                                                                                                      |
| $i_S$                    | Input or forcing function to a man-machine tracking system                                                                          |
| $j$                      | A subscript representing an integer, generally 1, 2, 3, ..., N                                                                      |
| $k$                      | A constant                                                                                                                          |
| $K$                      | In general, a gain symbol                                                                                                           |

|              |                                                                                                    |
|--------------|----------------------------------------------------------------------------------------------------|
| $K_p$        | Gain for the human operator model                                                                  |
| $M(j\omega)$ | Mimic transfer function                                                                            |
| $n_c(t)$     | The remnant time function which includes the non-linear human characteristic not covered by $h(t)$ |
| $n_i$        | A partial number of input elements (see context)                                                   |
| $N$          | Represents integral number, e.g., the number of filters for mimicking                              |
| $N_i$        | Total number of input elements                                                                     |
| $o$          | Output of the simple system model                                                                  |
| $o( )$       | Output as a function of ( )                                                                        |
| $o_c$        | Output of the controlled element in a man-machine system                                           |
| $o_h$        | Output of the human in a man-machine system                                                        |
| $O_h(s)$     | The Laplace transform of $o_h$                                                                     |
| $o_s$        | Tracking system output                                                                             |
| $p( )$       | Probability of ( )                                                                                 |
| $P$          | Parameters of the simple system model                                                              |
| $S_{o_h}^2$  | Variance of the operator's output                                                                  |
| $S_e$        | Mean-square error of the operator-mimic output difference                                          |
| $t$          | A symbol representing the variable, time                                                           |
| $T$          | (See $\varphi_{ff}(\tau):/T/>>/\tau/$ ) A specified time or a generalized time constant            |
| $T_I$        | A lag compensation time constant characteristic of the human operator                              |
| $T_L$        | A lead compensation time constant characteristic of the human operator                             |

|                               |                                                                                                                                                               |
|-------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $T_N$                         | The neuromuscular system time constant characteristic of the human operator                                                                                   |
| $T_\sigma$                    | Sampling period for the sampled data system                                                                                                                   |
| $V_i$                         | Input voltage                                                                                                                                                 |
| $V_o$                         | Output voltage                                                                                                                                                |
| $w$                           | The human operator's linear weighting function                                                                                                                |
| $z(t)$                        | The output of a single filter                                                                                                                                 |
| $Z(t)$                        | The output of the sum of weighted filter outputs                                                                                                              |
| $\epsilon(t)$                 | In mimicking, the difference between the human operator's output and the output of the mimic model                                                            |
| $\zeta$                       | The damping ratio for a damped system or system element                                                                                                       |
| $\tau$                        | An interval of time from some reference time; ( $\tau \ll T$ )                                                                                                |
| $\varphi_{\Delta c \Delta c}$ | Autocorrelation function of the time-dependent portion in $C(s)$                                                                                              |
| $\varphi_{ff}(\tau)$          | The general symbol for the autocorrelation function in which every instant $f(t)$ is correlated with its associated $f(t + \tau)$ in the interval $-T$ to $T$ |
| $\varphi_{ii}$                | Autocorrelation function of the input forcing function                                                                                                        |
| $\varphi_{io}(\tau)$          | The cross correlation function which every instant input $i(t)$ is correlated with its time associated output $o(t + \tau)$ in the interval $-T$ to $T$       |
| $\varphi_{nn}$                | Autocorrelation function of the remnant                                                                                                                       |
| $\varphi_{oo}$                | Autocorrelation function of the output function                                                                                                               |
| $\Phi_{ff}(j\omega)$          | The power spectrum (or spectral density) and Fourier transform of $\varphi_{ff}(\tau)$                                                                        |
| $\Phi_{ii}$                   | The power spectrum (or spectral density) and Fourier transform of $\varphi_{ii}(\tau)$                                                                        |



$$\Phi_{io}(j\omega)$$

The cross-power spectrum (or spectral density) and Fourier transform of  $\varphi_{io}(\tau)$

$$\Phi_{hi}$$

The human operator output power spectrum

$$\Phi_{nn}$$

The output power spectrum of the remnant

$$\Phi_{oo}$$

The power spectrum (or spectral density) and Fourier transform of  $\varphi_{oo}(\tau)$

$$\sum$$

Symbol representing a sum

$$\sum_{j=1}^N b_j$$

The sum of the coefficients of all mimic filters from the first ( $j=1$ ) to the last ( $j=N$ )

$$\omega$$

Frequency in radians per second ( $\omega = 2\pi$  times the frequency in cycles per second)

$$\omega_n$$

The undamped natural frequency of a system or a system parameter

$$\triangleq$$

Symbol meaning "is defined as"

$$\cong$$

Symbol meaning "is approximately equal to"

# A SURVEY OF HUMAN OPERATOR MODELS FOR MANUAL CONTROL

Meredith B. Mitchell<sup>1</sup>

## A. Introduction

When we speak of a "model," in our modern world of technology and advertising we must usually specify whether we are concerned with an artist's companion, a cover girl, a motor driven airplane one carries in his briefcase, a small plan for a stage set, a topological representation of the brain or what we will term an analytical model. This section is concerned with the latter type.

An analytical model may be defined as a symbolic representation of the functional relationships between the pertinent variables, characteristics, and parameters of a bounded system. While it is possible to portray this type of model in the form of a block or logic diagram -- such as an engineer might do -- theoretically all analytical models can ultimately be represented as an equation or set of equations. The equations, then, describe a system by symbolically defining the interactions between pertinent inputs, outputs, and system constraints and properties so as to meet the system output requirements.

A model<sup>2</sup> may be considered complete if no term can be removed without destroying the model's representation significantly and if either (a) no other system factors can be added that would alter the adequacy or the solution of the equations, (b) additional non-measurable, complex or insignificant features may affect the solution, but only negligibly, or (c) other influencing factors exist but are undefinable or uncontrollable; however, probabilities associated with the solutions of the equations can be defined (stochastic model).

A system may be simply represented as in Figure 1, with a measurable input or input configuration (i), a measurable output (o),

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<sup>1</sup>Mr. Mitchell, an Associate Scientist for Dunlap and Associates, Inc., is an electrical engineer and psychologist.

<sup>2</sup>Unless otherwise defined, "model" will henceforth refer to the analytical variety.

and pertinent system parameters (P), some or all of which may or may not be measurable. If the system is a linear one<sup>1</sup> with constant

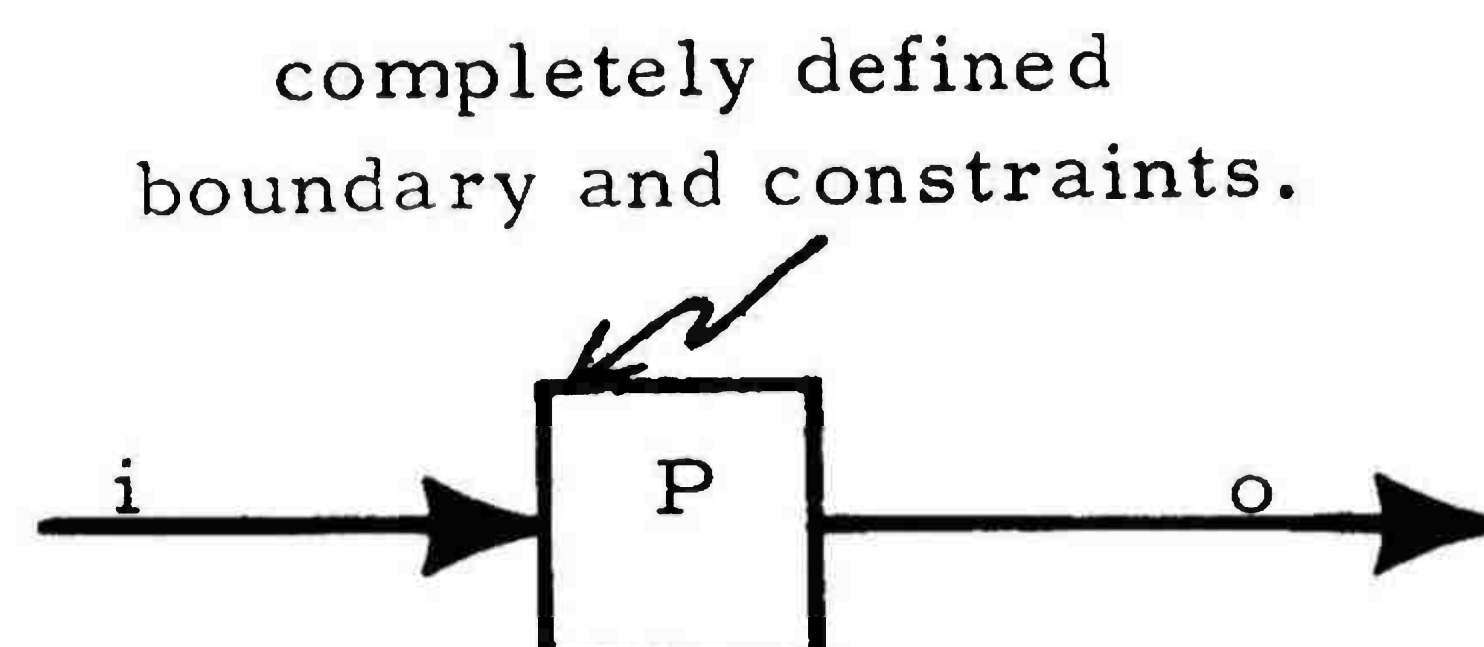


Figure 1. Schematic diagram of a system.

parameters, so that its input and output functions are mathematically relatable, the system operation can be described by its transfer function. Very briefly, the transfer function is the ratio of the output to the input, each expressed as a Laplace transform. (For a discussion of Laplace transforms, see pages 10 and 11.)

On a simple conceptual level, if, within the system constraints,  $o = f(i, P)$  then (a) above holds; i.e.,  $o$  is completely defined by  $f(i, P)$ , and no other datum can change this relationship. However, (b) might be represented by the symbolic statement,  $o = g(i, i', P, P', \text{etc.}) \cong f(i, P)$ . For example, an amplifier operating within voltage and temperature limits is functionally represented by  $V_o = k V_i$ , where  $V_o$  is the output voltage,  $V_i$  is the input voltage and  $k$  is the amplification constant. It is implied that  $k = f(i, P)$  and that variation in  $i$  results in a precise compensatory variation in  $P$  to maintain a constant function. But, since some of the amplifier's components are non-linear,  $f(i, P)$  is not a simple differential equation, and  $k$  cannot be a true constant; however,  $V_o \approx k V_i$  is acceptable for practical purposes if  $k$  does not vary beyond the system-specified tolerances.

Stochastic models are frequently found in the field of psychology. These are probability models based on the fact that uncontrollable or nonmeasurable variabilities are inherent in some aspect of the system. Stochastic models of elements of human behavior are numerous in the field of psychological learning theory. For example, Estes and Burke<sup>2</sup>

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<sup>1</sup> A linear system is one which is describable by a linear differential equation.

<sup>2</sup> see Bush, R. R., 1960, pp. 130-131



derived a simple relationship between a response probability and sampled stimuli from conditioning experiments with humans. In our terminology, their model -- which equates the probability of the response and the probability of particular stimuli -- can be represented thus:

$$p(o_\lambda) = \frac{n_{i_\lambda}}{N_i} = p(i_\lambda)$$

where  $p( )$  represents the probability of whatever is contained in the parentheses;  $N_i$  is the total number of stimuli presented to a subject;  $i_\lambda$  are the "sampled" stimuli to which are conditioned the response,  $o_\lambda$ ; and  $n_{i_\lambda}$  is the number of  $i_\lambda$  within  $N_i$ . For example, if a subject is randomly visually exposed to the 26 letters of the alphabet ( $N_i = 26$ ), and he is conditioned to blink ( $o_\lambda$ ) when three ( $n_{i_\lambda}$ ) particular letters ( $i_\lambda$ ) appear, the probability that blinking will occur to randomly presented letters is  $p(o_\lambda) = \frac{3}{26} \cong .12$ . This simple model makes only a rudimentary assumption concerning the human in the system, i.e., that in this particular kind of activity, his response is solely a function of the number of stimuli, and no other human parameter influences the relationship significantly. It is of interest to note that although  $p(i_\lambda)$  can be precisely measured,  $p(o_\lambda)$  would be expected to vary within a population of human subjects (and has been found to do so<sup>1</sup>); therefore, a more accurate model -- though not necessarily a more useful one -- would state the probability density, i.e.,

how  $p(o_\lambda) = \frac{n_{i_\lambda}}{N_i}$  varies with the probability due to population variation, thus at least accounting for non-measurable P's.

### 1. Limitations Due to Man's Complexity

It is this inability to discern and measure all the parameters of man that makes human behavior modelling so difficult. The data collector is unable to control (let alone observe) every possible external and internal stimulus impinging upon his experimental subject. Therefore, mathematically expressible models of human behavior must -- at least for the present -- be limited to relatively simple, well defined man-included systems in which (1) all the pertinent inputs to and outputs from the human operators are objectively measurable (either directly or indirectly) and (2) the equations describing all other elements in the system are known or readily derivable. With this information, and provided a mathematical technique is available, the input-output relationships of experimental subjects can, theoretically, be manipulated to yield a meaningful transfer function (i.e., output function/input function) for the

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<sup>1</sup> see Bush, R.R., 1960, p. 131

human performing the task under study.<sup>1</sup>

For purposes of modelling, the task generally required of the subjects in these studies has been that of tracking. Investigators have used either pursuit or compensatory tracking or both to determine  $h(t)$ , the human's time-dependent transfer function. The system output is generally a measure of the output of the controlled element,  $o_c$  (such as the position of a "joy stick" in simulating aircraft pitch control), and the input to the human,  $i_h$ , is usually the visual display showing a target and the output of the controlled element (pursuit tracking) or an indication of the difference -- e.g., via a meter reading -- between the target and the output (compensatory tracking). (See Figure 2.)

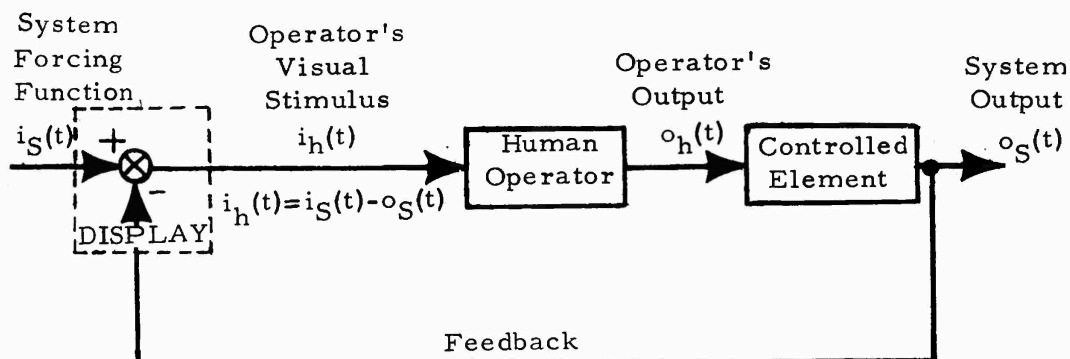


Figure 2. A general representation of a closed loop compensatory tracking system (after Bekey, 1962, p. 44)

Two of the characteristics of the target stimulus which have been studied are (1) the way in which it changes in time and (2) the degree to

<sup>1</sup>Ham, J. M., 1958, points out that for tasks in which the operator must control several system stimuli (e.g., when "he acts as a time-multiplexed feedback link"), he is "not representable by an elementary transfer function." Ham does not, however, indicate that a model cannot be constructed; he seems only to imply that no one has come up with a model and possibly not even with a technique for deriving one.

which it changes (amplitude). Regarding (1), since input configurations must be measurable and amenable to mathematical analyses, most investigators have restricted themselves to one or more of the following: (a) a step function, (b) a square wave, (c) a ramp function, (d) a saw tooth wave, (e) a superposition of non-synchronous, inharmonic square wave, saw tooth, and periodic impulse functions, (f) a sine wave, (g) a superimposition of several, pre-established inharmonic, "random-appearing" sinusoids (see Figure 3), or (h) a random input signal describable statistically in terms of its frequency spectrum. In general,

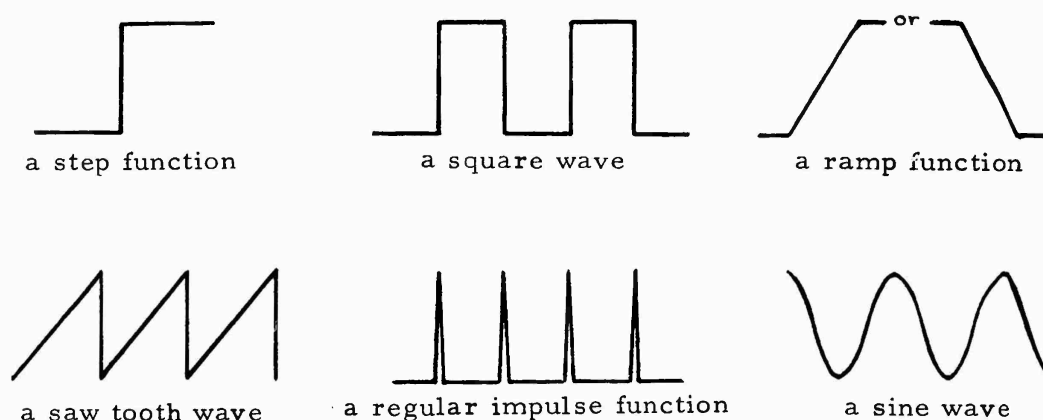


Figure 3. Types of inputs (or components of complex inputs).

then, the input can be clearly and completely described mathematically as a function of time. Similarly, the output can be measured with regard to variation and amplitude as a function of time.

## 2. Some Mathematics

In order to determine the dynamic relationships between the input to and output of a human in a control system so as to be able to state man's transfer function characteristics, some quite complex mathematical techniques are available and have been extensively applied to various data by such mathematically oriented investigators as Tustin (1947), McRuer and Krendel (1957), Ornstein (1961), Sheridan (1962) and Fogel (1957).

A lucid description of these techniques is offered by Licklider (1960, pp. 178-199). He explains that if we are given any temporally dependent waveform -- such as would be characteristic of an input signal for tracking -- we can find its frequency spectrum by analyzing the original wave into independent sinusoids, since "any physically measurable time function is the sum of just one (usually infinite) set of sinusoids." (p. 184). While the totality of the spectrum is equivalent to the original time function, the component set of sinusoids is easier to visualize and to handle mathematically. This frequency spectrum is defined as the Fourier transform of the waveform,

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt,$$

where  $F(j\omega)$  is the Fourier transform;

$f(t)$  is the time-dependent function to be transformed;  
and  $\omega$  represents frequency in radians per second. It may be noted that to find  $F(j\omega)$ , all possible values of  $\omega$  are used in the integration; only those corresponding to frequencies in  $f(t)$  will yield a non-zero product of  $f(t)e^{-j\omega t}$ .

Conversely, if we know the set of sinusoids in a signal, we can compute the inverse Fourier transform to find the time function of the waveform,

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega)e^{j\omega t} d\omega$$

Closely related to the Fourier transform is the Laplace transform. Essentially, they operate in the same way, in that they both convert a temporal waveform into its frequency spectrum. However, the Laplace transform has the form,

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

where  $s = \sigma + j\omega t$  and both  $\sigma$  and  $\omega$  are real.

Essentially, then, the integrand of the Fourier transform equation is simply multiplied by a real exponential factor to become the integrand of a Laplace transform equation. While  $j\omega t$  is a frequency varying characteristic,  $\sigma$  represents the rate of amplitude increase or decrease of its associated sinusoid.

Therefore, we can conclude that the two basic differences between Fourier and Laplace transforms are that (1) the components of the function are generally constant amplitude sinusoids for the former and

exponentially increasing or decaying oscillations for the Laplace and (2) the Fourier transformed process is assumed always to have been operating (i. e., since  $t = -\infty$ ) while the customarily used one-sided Laplace transform has the components of the spectrum starting at some definable reference time,  $t = 0$ .

Transforms are applicable when the signals (e. g., target movements) or any other time-varying function (e. g., human output) can be specified as a function of time,  $f(t)$ . If, however, only certain characteristics of the function are known, but the time functions themselves cannot be specified, other techniques must be used. For example, if we know the inertia of a system and its maximum capabilities along measurable continua, and if from observation we are able to extrapolate future performance, we do not operate according to  $f(t)$ ; rather, we are attempting to estimate a correlation between the historical  $f(t)$  and  $f(t+\tau)$ , where  $\tau$  is some interval in the future.

Mathematically, integration allows us to repeat this intra-function correlation an infinite number of times across the range of  $\tau$ , and over a selected period of time,  $T$ , making it possible to determine the nature of any regular or constant pattern existing within the total complex signal. Such an integration process is called autocorrelation:

$$\varphi_{ff}(\tau) = \frac{1}{2T} \int_{-T}^T f(t) f(t+\tau) dt, *$$

where  $\varphi_{ff}(\tau)$  = the autocorrelation function;

$T = 1/2$  the finite time period within which the correlation is being computed;

$f(t)$  = the original time function; and

$f(t+\tau)$  = the original time function shifted forward  $\tau$  seconds.

The interval,  $-T$  to  $T$ , represents the relevant time over which the observation is made. Ideally,  $T \rightarrow \infty$ . If this were a simple integral of spectral energy, the autocorrelation function would grow as  $T$  increased. However, it should be noted that  $\varphi_{ff}(\tau)$  is expressed as an average of the integral, so that as  $T$  increases the integral will not grow to unwieldy proportions. By taking the average, we examine not the energy of the  $f(t)$  spectra, but the power; doing this only alters the units and does not change the forms of the functions.

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\* Note that for our purposes all the  $f$ 's in the equation can be replaced by either  $i$  (input) or  $o$  (output).



The symbol  $\tau$  represents a time interval, ideally anywhere from zero to the longest period within the  $f(t)$  spectrum. To obtain the autocorrelation functions,  $\tau$  is increased either continuously or incrementally (e.g., for digital computation) and the integration repeated for each value of  $\tau$ . If  $f(t)$  contains a frequency whose period equals  $\tau$ ,  $\rho_{ff}(t)$  has a non-zero value. If  $\tau$  is far from any oscillatory period within  $f(t)$ ,  $\rho_{ff}(t) = 0$ . (See Figure 4, D and E.)

The autocorrelation function tells about the frequency spectrum in a given signal, but not about phase vs. frequency relationships nor the amplitude vs. frequency characteristics of that signal. The amplitude-frequency relationships, as well as the related phase patterns, can be derived from taking the Fourier transform of the autocorrelation function. The resulting transform is called the power spectrum,  $\Phi_{ff}$ :

$$\Phi_{ff}(j\omega) = F(j\omega) F(-j\omega) \triangleq |F(j\omega)|^2 = \int_{-\infty}^{\infty} \rho_{ff}(\tau) e^{-j\omega\tau} d\tau^*$$

So far, we have discussed the nature of analyzing a complex, single input and/or output signal. When a signal,  $i(t)$ , is fed into a network, (e.g., a human component) the resulting output has its own characteristics which we will represent as  $o(t)$ , and which presumably has some relation to  $i(t)$ . To correlate the output function with its associated input function, a cross-correlation function can be calculated from

$$\rho_{io}(\tau) = \frac{1}{2T} \int_{-T}^T i(t) o(t+\tau) dt,$$

where  $f(t)$  represents the input function and  $o(t+\tau)$  the subsequent, contiguous output function. For the cross-power spectrum (amplitude vs. frequency as well as phase shift characteristics of the output relative to the input), we can then compute

$$\Phi_{io}(j\omega) = \int_{-\infty}^{\infty} \rho_{io}(\tau) e^{-j\omega\tau} d\tau$$

If the input and output signals are those measured in relation to a human operator, then we can describe the frequency response function of the human as follows:

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\*Note that all subscript f's in the equation can be replaced by either i or o, and correspondingly, all F's by either I or O.

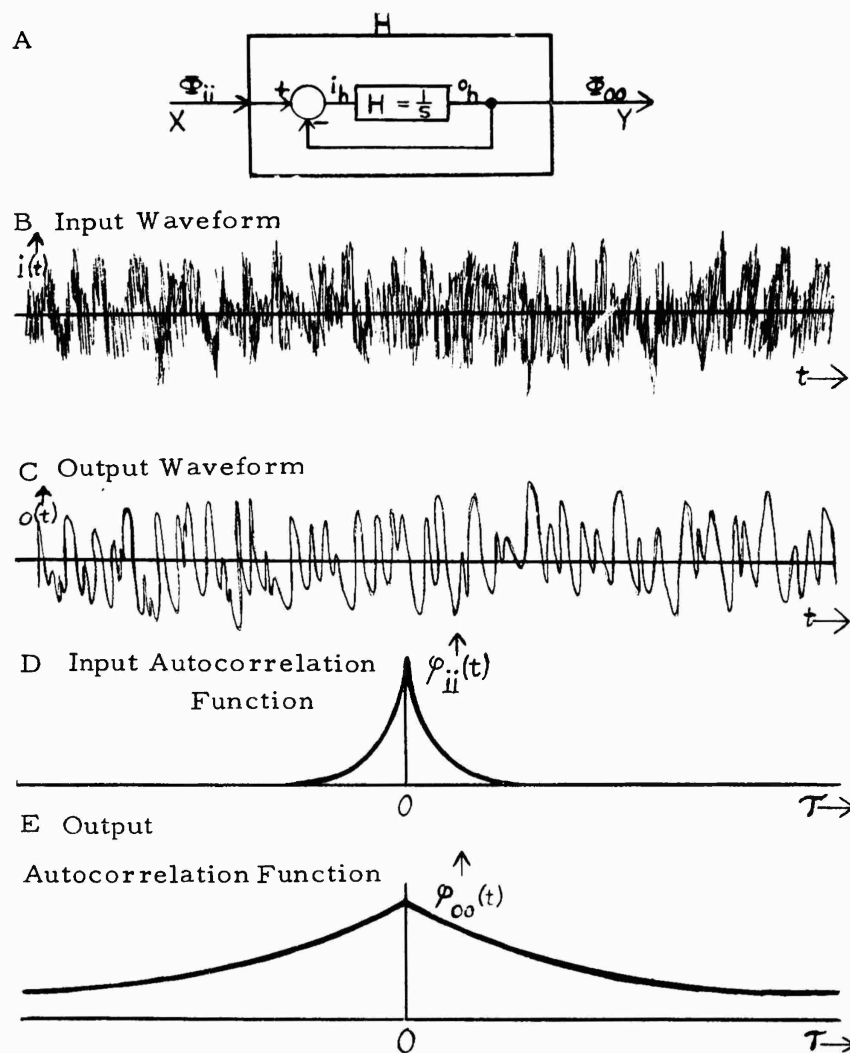
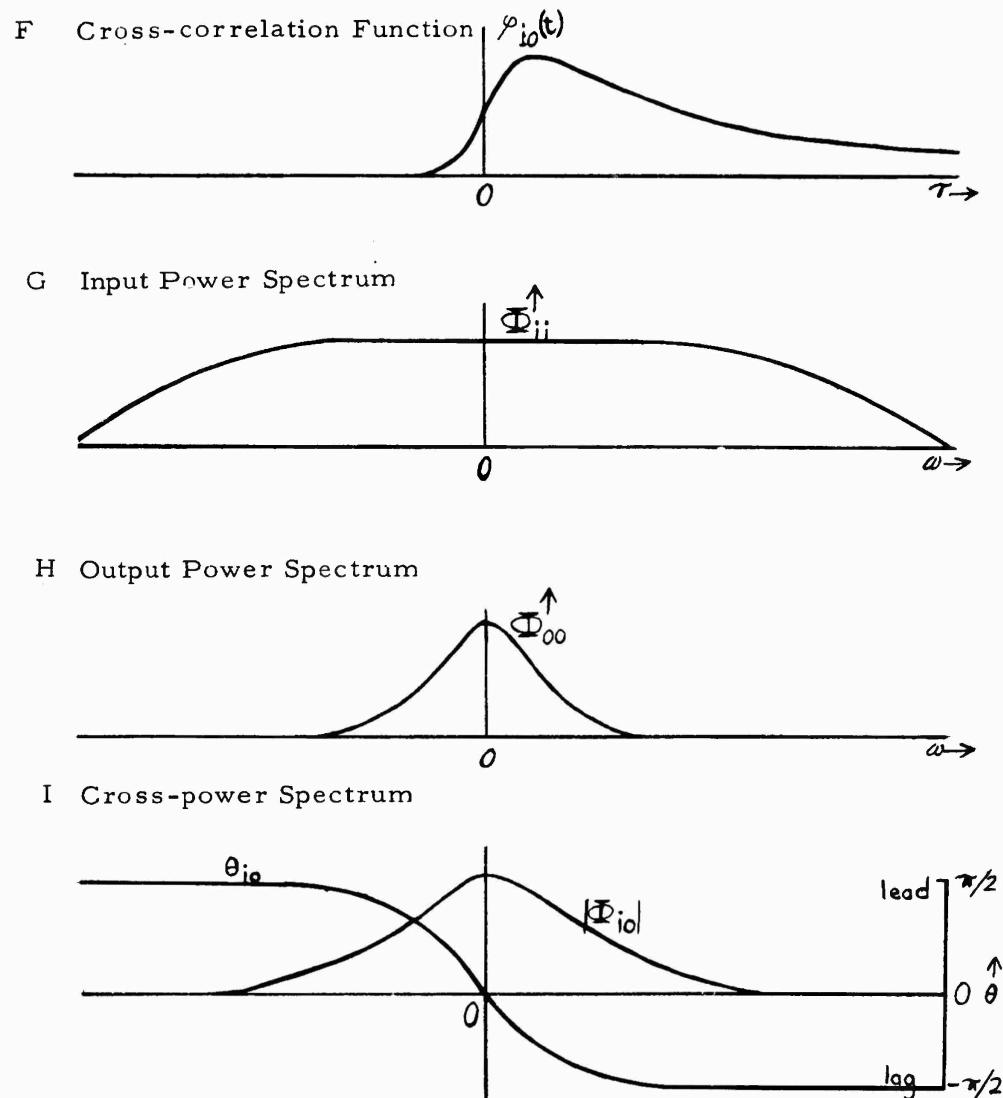


Figure 4. Illustrations of a correlational and spectral approaches to determination of the transfer function of a linear network. At A, the "black box" is shown with its cover off. B is the input waveform. C is the corresponding output waveform. With an autocorrelator, we determine D from B and E from C. With a cross-correlator, we determine F from B and C together. Since D dies out fairly rapidly as  $\tau$  increases, F is a fair approximation of the impulse response of the network. To find the true impulse response, which is a declining exponential, we would have to "deconvolve" F against D. Alternatively, in the frequency domain, we use a power-spectrum analyzer to



(Figure 4 continued)

determine G and H from B and C, respectively, and a cross-power-spectrum analyzer to determine I from B and C together. The magnitude of the frequency-response function is the ratio of the magnitudes of I and G, which obviously is nearly the same as the magnitude of I. The phase shift of the frequency response functions is exactly the phase shift of I. (Taken from Licklider, pp. 195-196)

$$H(j\omega) = \frac{\Phi_{io}(j\omega)}{\Phi_{ii}(j\omega)},$$

where  $H(j\omega)$  is the Fourier transform of the human time function,  $h(t)$ .

This equation is a transfer function statement similar to  $H(s) = \frac{O_h(s)}{I_h(s)}$ .

$H(j\omega)$  is the Fourier transform of  $h(t)$ ;  $\Phi_{io}(j\omega)$  is the output spectrum characteristics solely correlated with the defined input; and  $\Phi_{ii}(j\omega)$  is the discernible non-random input spectrum characteristics. (See Figure 4, F and I)

The techniques summarized above form "only a part of the highly developed apparatus associated with linear network theory." (Licklider, p. 199). But they are adequate to understand quite generally how human transfer functions have been derived and analyzed.

### 3. Accounting for Man's Nonlinearity

In spite of the sophistication characterizing the mathematical approach, the assumption of linearity still raises a question as to the reliability of the outcome. Tustin (1947), one of the first publishing non-psychologists interested in the mathematical nature of human response, pointed to implications in his data that, in spite of nonlinearities, there seems to be an "approximate linear law" of response which exists. He, therefore, adopted the procedure to hypothesize a linear function "which accounts for the main trends of the response, and subsequently to show the general nature of the departure from the linear relationship, in respect both of nonlinearity and of superimposed haphazard variations." (p. 192) McRuer and Krendel (1957, p. 33) also use this procedure and assign the former (the approximate linear term) the weighting function symbol,  $h(\tau)$ , and the latter "remnant" term, the symbol,  $n_c(t)$ . Thus, the mathematical techniques discussed earlier have generally been applied to experimental data to determine both the "nearest linear law" and the superimposed deviation due to nonlinearities and haphazard additions.

If the human component in the system were truly linear, it could be represented as shown in Figure 5. The mathematics for determining the

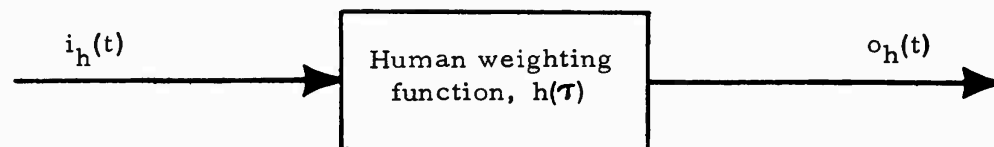


Figure 5. An idealized human operator model.



output function as a result of  $i(t)$  is simply the solution of the "convolution" integral,

$$o_h(t) = \int_{-\infty}^{\infty} h(\tau) i_h(t-\tau) d\tau,$$

whose Fourier transformation is

$$O(j\omega) = \int_{-\infty}^{\infty} o_h(t) e^{-j\omega t} dt.$$

Note that the equation for  $o_h(t)$  is not too different from the earlier equation for an amplifier, where the output is simply a constant multiple of the input; here, however,  $h(\tau)$  is a weighting function rather than a constant weighting term, and the integral describes the total picture of the interaction between  $h(\tau)$  and  $i_h(t-\tau)$  -- the excitation  $\tau$  seconds in the past -- rather than an instantaneous or static value.

Special task requirements and limited input variability sometimes make it possible to derive human operator models from just these mathematical manipulations which do not include a remnant term. They are, as stated, only linear models. They are described by linear differential equations, which means that the superposition principle is applicable, i.e., inputs or outputs are additive; for example, the total input function is a simple sum of all its component functions. If the linear system is assumed to have inherent properties which do not vary with time, the coefficients of the linear differential equations are constant.

Since human beings are consistent neither temporally nor in their mode of performance, the assumption of linearity may not appear to be applicable, except for short periods of time under fairly constant operational conditions. For certain tasks, such as tracking, it has been found that a "quasi-linear" model provides a very good approximation of the human operator. (See Figures 6 and 7.) Parameters may drift, noise may exist within the system (or in its input or output), or discrete changes may occur periodically, but all of these nonlinearities can be lumped into a remnant term. "The quasi-linear approach rests on the assumption that the part of the output not linearly related to the input is not likely to be readily usable, anyway, and that it may therefore be lumped together with random noise." (Licklider, p. 177)

The basic principle underlying quasi-linear modelling is that within restricted control system limits the parameters are essentially invariant and the system behavior is a linear function. Experimental data seems to support the conclusion:

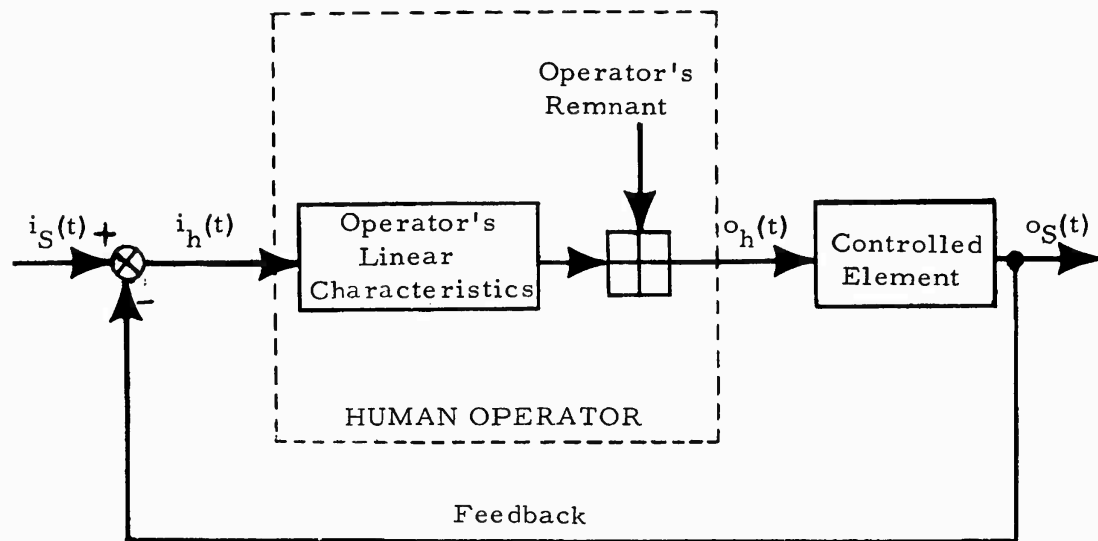


Figure 6. A "quasi-linear" model (after McRuer and Krendel, 1959, p.10)

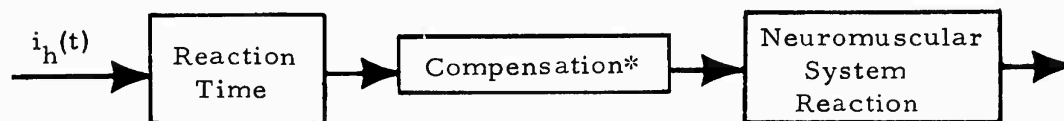


Figure 7. The generally accepted fundamental linear characteristics of the human operator.

\* Compensation (sometimes called "equalization") refers to the human's need to make adjustments or interpretations of the control-display relationships in order to stabilize the system, i.e., to be able to respond in a controlled manner to the input.

that the characteristics of the linear part of the human operator are approximately invariant over a range of input amplitudes or display scale factors, and that -- within that range -- the magnitude of the noise introduced by the human operator is roughly proportional to the magnitude of the response. (Licklider, p. 243)

The latter finding makes it possible to describe the nonlinear remnant term as a meaningful, unobscure mathematical function.

The quasi-linear approach has been adopted by many human operator model makers. It allows for a relatively simple symbolic description of transfer characteristics, particularly in comparison with the nonlinear mathematical model which might resemble an awkward dictionary of stimulus-response pairs.<sup>1</sup> Since, however, human operator characteristics have been found to be dependent upon type of input (step functions, sinusoids, etc.), close approximations of describing functions<sup>2</sup> can be derived for each category of input. That is, the output-input relationships tend to be linear under a fixed set of conditions, even though the general system is nonlinear. These describing functions, then, are the linear portions of quasi-linear models.

#### 4. Recent Approaches

Very recently, quite a different approach to designing a quasi-linear model has been developed and applied by Elkind, et al (1961 and 1963). Starting with the general model shown in Figure 6, they express the operator's linear characteristics and remnant in terms of filters which can be combined to approximate these functions. The operator's linear characteristics are defined by a "time-invariant linear filter,"  $w(t-\tau)$ , and a "time-variant linear filter,"  $w(t, t-\tau)$ . (1963, p. 10). They demonstrate that either of these two weighting functions ( $w$ ) can be derived by a special form of parallel model adjustment technique, where the difference between the human operator output and the filter system output is analyzed. (See Figure 8) This technique is called "mimicking."

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<sup>1</sup> Elkind, J. I., & Green, D. M. (1961) suggest a different method for measuring nonlinear systems and thereby establishing a model (p. 53ff). They explain that a selected set of filters can be found to represent a nonlinear system so as to approximate the system by a piecewise-linear construction. At this writing, no such model has been derived. (The filter technique is presented later in the text.)

<sup>2</sup> A describing function is the linear equivalent of a nonlinear element within specified restrictions. In this case, the response of the nonlinear element is related to a particular class or type of input.



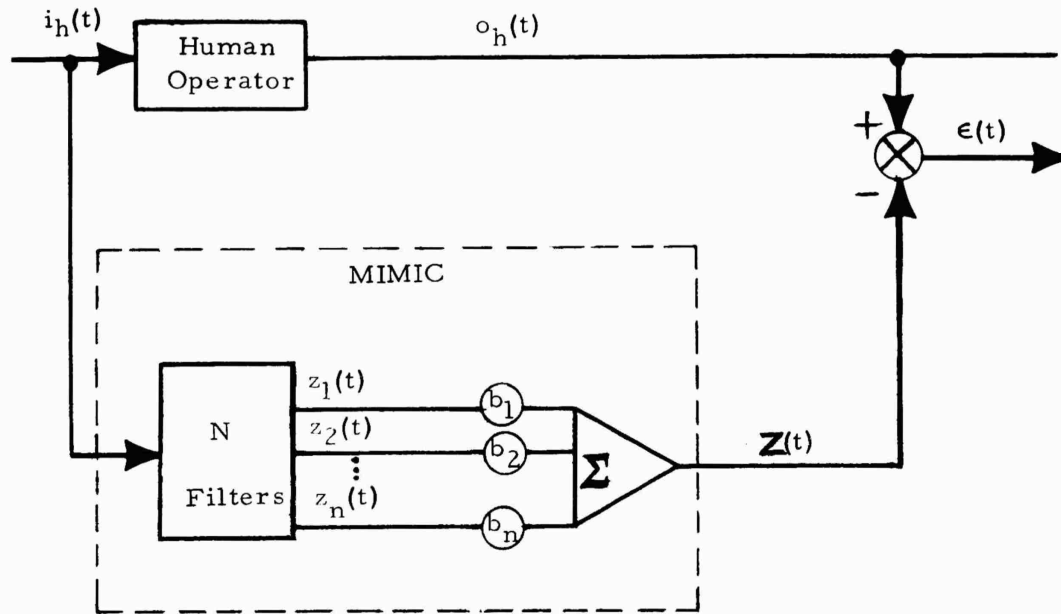


Figure 8. Measurement by the mimicking technique.  
(after Elkind and Green, 1961, p. 10)

For the human operator, the input-output relation can be written in terms of the convolution integral,

$$o_h(t) = \int_{-\infty}^{\infty} i_h(\tau) w(t-\tau) d\tau + n_c(t)$$

From Figure 8, it can be seen that the mimic output is

$$Z(t) = \sum_{j=1}^N b_j z_j(t),$$

where  $Z(t)$  is the mimic output function,  $z_j(t)$  is the output of the  $j$ th filter, and  $b_j$  is the weighting coefficient for the output of the  $j$ th filter.

To get the equation for  $o_h(t)$  into a form comparable to that of  $Z(t)$  requires explanations too extensive to be included here; however, with



certain assumptions  $o_h(t)$  can be set into the form

$$o_h(t) = \sum_{j=1}^{\infty} w_j z_j(t) + n_c(t)$$

"The measurement problem now becomes one of finding the mimic coefficients  $[b_j]$  that give the least mean-square difference (MSD) between the mimic output" and the output of the human operator (1963, p. 13). That is, since the difference,  $\epsilon(t) = o_h(t) - Z(t)$ , over the period from  $t = 0$  to  $t = T$ , the mean-square is

$$1/T \int_0^T \epsilon(t)^2 dt \triangleq \overline{E^2} = \overline{[o_h(t) - Z(t)]^2}$$

The  $b_j$  values which minimize the MSD can be derived by setting the derivative of  $\overline{E^2}$  equal to zero for each  $b_j$ .

This method not only supplies information for determining the linear characteristics of the operator's output, but simplifies the extraction of the remnant which falls out as a residual term after  $w(t-\tau)$  is shown to be approximated by the mimic, with very little error, by the expression

$$S_{\epsilon}^2 = \overline{E^2} = S_{o_h}^2 - \sum_{j=1}^N \overline{z_j} \overline{o_h}$$

where  $S_{\epsilon}^2$  is "the mean-square error of approximation to the [operator] characteristics" and  $S_{o_h}^2$  is the variance of the operator's output (1963, p. 20). Knowing the mimic transfer function,  $M(j\omega)$ , and the input-output power spectra,  $\Phi_{ii}(j\omega)$  and  $\Phi_{oo}(j\omega)$  respectively, the power spectrum of  $\epsilon(t)$  is

$$\Phi_{\epsilon\epsilon}(j\omega) = \Phi_{hh}(j\omega) - |M(j\omega)|^2 \Phi_{ii}(j\omega)$$

Note that the second term on the right is the power spectrum of the mimic output. Actually, then, if the mimic is a good approximation of the operator,  $\Phi_{\epsilon\epsilon}(j\omega)$  is the power spectrum of the residual or remnant.

The above discussion applies to time-invariant functions of the operator and mimic. For time-varying systems, the approach is the same as long as it can be assumed that the system characteristics do not

change too rapidly. The equation for the time-varying operator's output is

$$o_h(t) = \sum_{j=1}^{\infty} w_j(t) z_j(t) + n(t)$$

Here,  $w_j(t)$  is a function of time. But by partitioning this weighting function into a non-infinite set of time-invariant filters,  $w_j(t)$  can be closely approximated -- as long as periods of time can be defined during which the system is essentially invariant.

Using this technique, experimenters should be able to reverse the conclusion drawn by Sheridan in 1960 that at that time "experimental results permit no analytic generalizations regarding patterns of time-variation of the quasi-linear human operator parameters as forced by changes in environmental parameters," (p. 97) since Sheridan's analytic techniques were less sensitive than Elkind's. The most striking difference between Sheridan's harmonic analysis and Elkind's filter model technique is that while Sheridan took a sample length of two minutes and required a sample interval of 15 seconds (p. 59), Elkind, et al used a sample of 48 seconds and could reduce their sampling interval to 0.1 seconds (which they could have reduced even further by selecting different filters). In addition, the method of Elkind permits easy calculations of the confidence limits for the measured values of the coefficients. Elkind's development appears to be a definite advantage for analyzing time-variant characteristics of quasi-linear model.

It is clear that the approach of Elkind, et al is a quasi-linear one since the weighting function "approximates the relation between (operator) input and output for only a single situation" (p. 9) and a remnant term is included to account for nonlinearities.

In recent years, three other categories of models have been investigated: Adaptive Control System, Nonmathematical Nonlinear, and Sampled-Data Models. The Adaptive Control System Model takes into consideration the "dither" displayed by pilots when tracking (see Figure 9). "Dither" is the term given to a low amplitude, oscillatory "succession of impulses to test the response of the system." It appears to be a subtle attempt of man to see what will happen before taking a definitive action. "Based on selective filtering of the response to the test signals, the loop gain can be adjusted." (Bekey, 1959, p. 27)

Earlier, it was mentioned that a mathematical-type nonlinear model would essentially be a dictionary of stimulus-response pairs. However, some investigators have deviated from the analytical treatment of a model

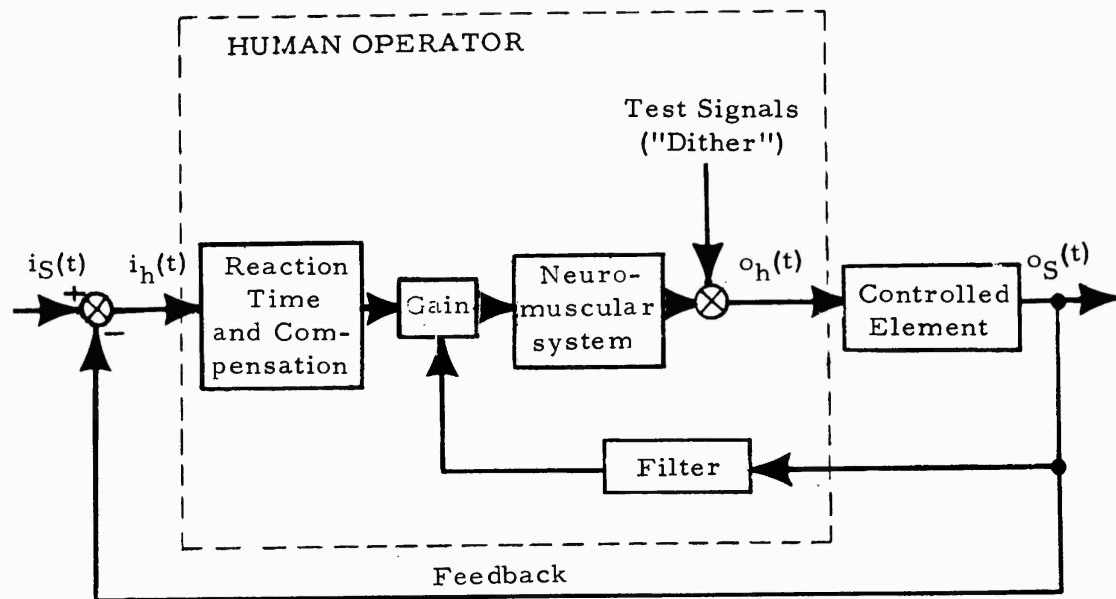


Figure 9. Adaptive model of the human operator (after Bekey, 1959, p. 28)

and have used analog computer simulation directly, which greatly restricts the general usefulness of the model. However, in this way, a typical nonlinear model, as shown in Figure 10, can include several functions whose individual parameters can be independently adjusted to obtain the best possible simulation of particular human characteristics (Bekey, 1959, pp. 28-29):

- a. The threshold of the operator (no response below a minimal level)
- b. The saturation rate above which an operator cannot respond
- c. The compensation or equalization of the operator, i.e., the leadlag characteristics necessary to stabilize the system
- d. The operator's reaction time
- e. His neuromuscular system with proprioceptive feedback
- f. The operator's adaptive "dither"
- g. The effect of anticipation

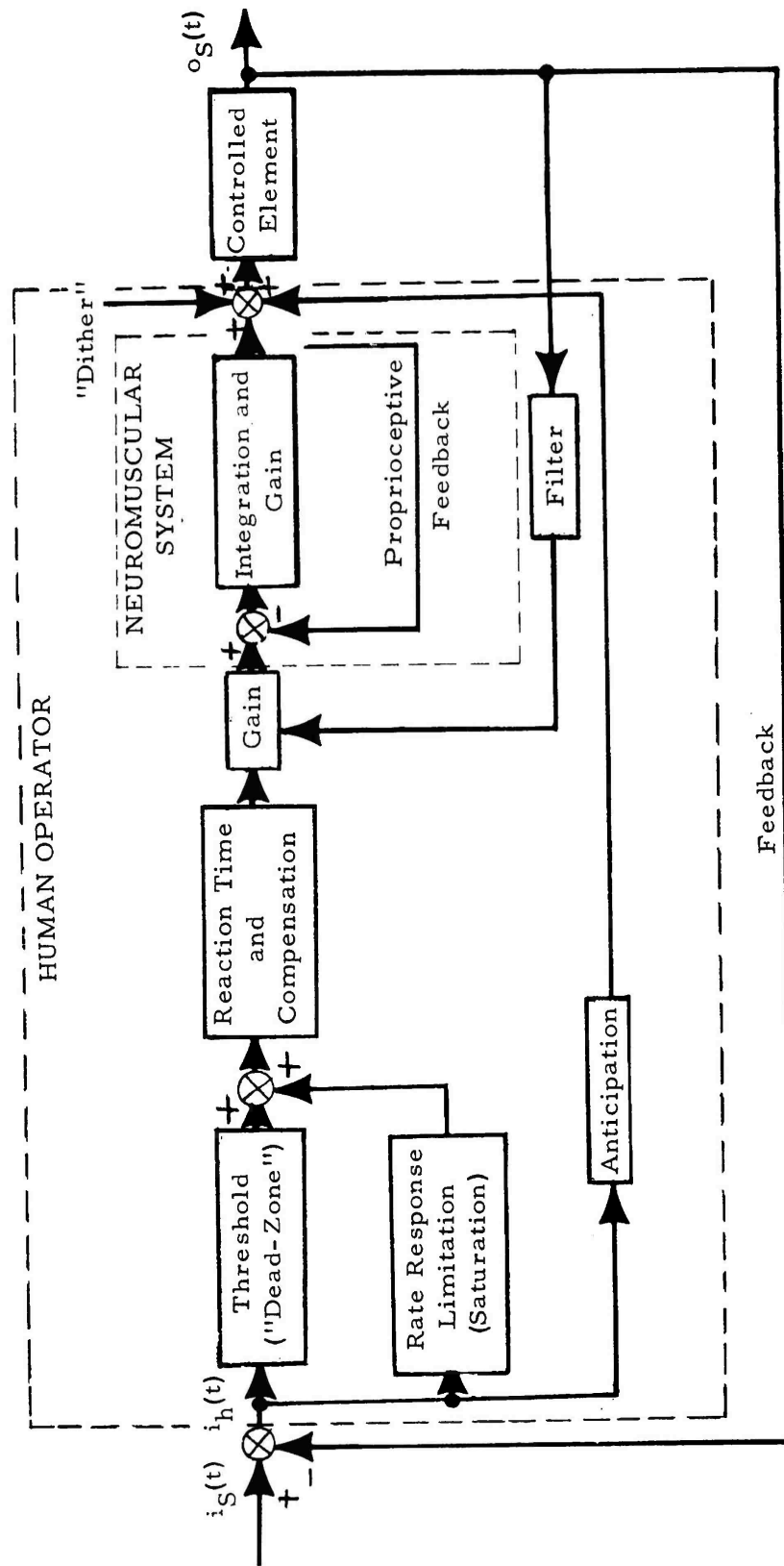


Figure 10. Computerized nonlinear model (after Bekey, 1959, p. 29)

Sampled-data models take into consideration the human's refractory period and deny the implicit assumption of other models "that over the the frequencies of interest the system can be assumed continuous." (Bekey, 1959, p. 29) Sampled-data models include (in addition to basic parameters, e.g., reaction time) sampling and hold functions which together represent the human operator's momentary acceptance of the stimulus followed immediately by an interval of refraction or insensitivity (see Figure 11).

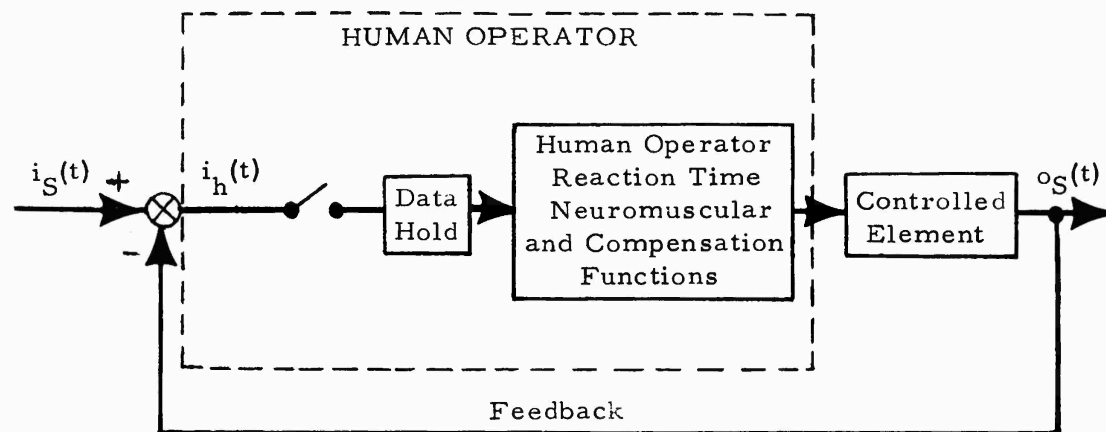


Figure 11. Simple sampled-data model (after Bekey, 1962, p. 44)

Bekey (1962), who supports the sampled-data (intermittency) model, lists some drawbacks of the quasi-linear model which, he admits, "gives impressive evidence of the nearly linear behavior of the human operator" tracking low frequency signals. However, in addition to the frequency limitation, quasi-linear models have the following drawbacks:

- "1. Being linear and continuous, the output of the model cannot contain frequencies not present in the input signal. (Such frequencies are known to exist in human operator outputs.)
- "2. The model cannot account for the substantial body of experimental evidence (cited above) which suggests that the human operator acts in an intermittent manner.

"3. The model does not account for the known ability of the human operator to extrapolate his response even when the input stimulus temporarily vanishes. For example, if a target disappears momentarily a human tracker will continue to respond at nearly constant velocity.

"On the basis of the above considerations a new mathematical model has been formulated to include the features of the continuous quasi-linear representations as well as the intermittent operation of a sampling switch." (p. 45)

Implementing his sampled-data concept, Bekey tested the performance of a computer model against data he obtained from subjects performing a compensatory tracking task "with a random appearance input signal." He found excellent correspondence between model and human responses. Also he noted that his model's input-output behavior more closely approximated, the "experimental results than that which results from linear continuous models." (p. 44)

In summary, we have discussed to this point an overview of various concepts of particular human characteristics, task limitations, and applicable mathematical techniques for constructing bounded human operator models. These concepts will undoubtedly be developed, expanded and revised in the future, as we learn more about human behavior, as we look at more different types of tasks, and as we discover more readily interpretable and broadly applicable mathematical methods of handling nonlinear data. Meanwhile, it is the purpose here to bring together the bulk of findings currently available.

## B. The Structure of Models

### 1. Introduction

Tables 1, 2, and 3, taken directly from McRuer and Krendel (1957), summarize quite succinctly the human describing and remnant functions found by Goodyear (1950, 1952, 1953), Searle and Taylor (1948), Cheatham (1954), Mayne (1951), Ellson and Hill (1948), Ellson and Wheeler (1949), Slack (1953), Russell (1951), Elkind (1956), Tustin (1947), and the Franklin Institute (Krendel, 1956, 1960). In brief, it will be noted that the tables for the describing functions, Tables 1 and 2, include (1) "Type of forcing function," (2) "General control task," (3) "Controlled element transfer function," (4) "Best fit human operator transfer function," (5) "Frequency range of human operator measurements," (6) "Average linear correlation," and (7) "Investigators and remarks." For each of the human operator transfer functions, numerical values of the gains and time constraints are listed as functions of the controlled element transfer function. And the values differ too, depending upon the input forcing function which was used in the experiment whose resulting data yielded the derived values.

### 2. The Conventional, General Linear Model

Most of the mathematical models of the linear approximation of the human transfer function (generally expressed in Laplace transform) include a gain term,  $K$ , a reaction time function,  $e^{-\tau s}$ , and one or more of the following:  $(T_L s + 1)^*$ , a lead term acting like a kind of anticipation and contributing especially to high frequency stability;  $(T_I s + 1)$ , a lag term contributing particularly to low frequency system stability; and  $(T_N s + 1)$ , a neuromuscular lag term due to the human body's inertia. In the earlier discussion,  $\frac{T_L s + 1}{T_I s + 1}$  has been called "compensation," while  $e^{-\tau s}$  has been "reaction time," and  $\frac{K}{T_N s + 1}$  was the "neuromuscular system" describing

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\*  $T_L$ ,  $T_I$  and  $T_N$  are time constants. A time constant is a system parameter which expresses the characteristic rapidity with which an oscillation tends to die out. The time constant is inversely related to the time it takes an exponential function to fall a given amount. When the amplitude of an exponential function decreases to 0.368 times its initial value,  $T = 1/\zeta\omega_n$  where  $T$  is the time constant,  $\zeta$  is the damping ratio and  $\omega_n$  is the natural undamped frequency of the function.

Table 1 Summary of Operator Describing Functions in Compensatory Tasks,  $Y_c = 1$ .

| TYPE OF FORCING FUNCTION                                                                                                             | GENERAL CONTROL TASK                                                                     | CONTROLLED ELEMENT TRANSFER FUNCTION (GAIN NOT SHOWN) | BEST FIT HUMAN OPERATOR TRANSFER FUNCTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    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| ① STEP FUNCTIONS<br>                                                                                                                 | SIMPLE FOLLOWING WITH PENCIL, STICK, OR WHEEL                                            | 1                                                     | CLOSED LOOP TRANSFER FUNCTION<br>$\frac{e^{-Ts}}{T^2 s^3 + \frac{2\sigma T s^2}{K} + \frac{\sigma}{K} + 1}$<br>$\approx \frac{e^{-Ts}}{T_N s + 1}$<br>$\sigma = 0.5 \quad K = 6.8-7.8 \text{ sec}^{-1}$<br>$T = 0.25 \text{ sec} \quad T = 0.042 \text{ sec} \quad T_N = 1/K$                                                                                                                                                                                                                                                                                                                                                                                                                                                                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TAYLOR [1948]<br>CHEATHAM [1954]<br>MAYNE [1957]<br>ELLISON & HILL [1948] |    |     |     |     |      |      |      |      |       |       |       |        |        |        |         |         |         |         |          |          |          |           |           |           |            |            |            |            |             |             |             |              |              |              |               |               |               |               |                |                |                |                 |                 |                 |                  |                  |                  |                  |                   |                   |                   |                    |                    |                    |                     |                     |                     |                     |                      |                      |                      |                       |                       |   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| ② SINE WAVES AND SQUARE WAVES<br>                                                                                                    | SIMPLE FOLLOWING WITH PENCIL OR WHEEL                                                    | 1                                                     | "NONSYNCHRONOUS" RESONANCE, CLOSED LOOP<br>$\frac{1}{\frac{-2s}{K} + \frac{2\sigma T s}{K} + \frac{\sigma}{K} + 1}$<br>WITH CONSTANTS AS ABOVE<br>"SYNCHRONOUS" RESONANCE, CLOSED LOOP<br>$K(\omega)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        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[1948]<br>CHEATHAM [1954]                                                  |    |     |     |     |      |      |      |      |       |       |       |        |        |        |         |         |         |         |          |          |          |           |           |           |            |            |            |            |             |             |             |              |              |              |               |               |               |               |                |                |                |                 |                 |                 |                  |                  |                  |                  |                   |                   |                   |                    |                    |                    |                     |                     |                     |                     |                      |                      |                      |                       |                       |  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                                                         |
| ③ SEQUENCE OF STEPS<br>                                                                                                              | SIMPLE FOLLOWING WITH PENCIL, STICK, OR WHEEL                                            | 1                                                     | SAME AS FOR SINGLE STEPS<br>THOUGH T CAN APPROACH 0.20 SEC AS STEPS GET CLOSER TOGETHER AND APPROX MORE RANDOM                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                |                            | GOODYEAR (MAYNE) [1950, 1952, 1953]<br>SMALL RANGE EFFECT:<br>TAYLOR [1948]<br>ELLISON & WHEELER [1949]<br>SLACK [1953]          |    |     |     |     |      |      |      |      |       |       |       |        |        |        |         |         |         |         |          |          |          |           |           |           |            |            |            |            |             |             |             |              |              |              |               |               |               |               |                |                |                |                 |                 |                 |                  |                  |                  |                  |                   |                   |                   |                    |                    |                    |                     |                     |                     |                     |                      |                      |                      |                       |                       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|                                                           |
| ④ RANDOM APPEARING; WHITE NOISE THROUGH 3RD ORDER BANDPASS FILTER GIVING AVAILABLE CORNER FREQUENCIES OF 1/2, 1, 2, 4, 8 rad/sec<br> | SIMPLE TRACKER WITH SPRING RESTRAINED ANGSTROM CONTROL STICK IN ANGSTROM COORDINATE MODE | 1                                                     | $\frac{K e^{-Ts} (T_N s + 1)}{(T_N s + 1) (T_N s + 1)}$<br>$T_N = 0.3 \quad T_N \text{ EFFECT INCLUDED IN } T$<br>CORNER FREQUENCY TABLE:<br><table border="1"> <tr><th>CORNER FREQ</th><th>1</th><th>2</th><th>4</th><th>8</th><th>16</th><th>32</th><th>64</th><th>128</th><th>256</th><th>512</th><th>1024</th><th>2048</th><th>4096</th><th>8192</th><th>16384</th><th>32768</th><th>65536</th><th>131072</th><th>262144</th><th>524288</th><th>1048576</th><th>2097152</th><th>4194304</th><th>8388608</th><th>16777216</th><th>33554432</th><th>67108864</th><th>134217728</th><th>268435456</th><th>536870912</th><th>1073741824</th><th>2147483648</th><th>4294967296</th><th>8589934592</th><th>17179869184</th><th>34359738368</th><th>68719476736</th><th>137438953472</th><th>274877906944</th><th>549755813888</th><th>1099511627776</th><th>2199023255552</th><th>4398046511104</th><th>8796093022208</th><th>17592186044416</th><th>35184372088832</th><th>70368744177664</th><th>140737488355328</th><th>281474976710656</th><th>562949953421312</th><th>1125899906842624</th><th>2251799813685248</th><th>4503599627370496</th><th>9007199254740992</th><th>18014398509481984</th><th>36028797018963968</th><th>72057594037927936</th><th>144115188075855872</th><th>288230376151711744</th><th>576460752303423488</th><th>1152921504606846976</th><th>2305843009213693952</th><th>4611686018427387904</th><th>9223372036854775808</th><th>18446744073709551616</th><th>36893488147419103232</th><th>73786976294838206464</th><th>147573952589676412928</th><th>295147905179352825856</th><th>590295810358705651712</th><th>1180591620717411303424</th><th>2361183241434822606848</th><th>4722366482869645213696</th><th>9444732965739290427392</th><th>18889465931478580854784</th><th>37778931862957161709568</th><th>75557863725914323419136</th><th>151115727451828646838272</th><th>302231454903657293676544</th><th>604462909807314587353088</th><th>1208925819614629174706176</th><th>2417851639229258349412352</th><th>4835703278458516698824704</th><th>9671406556917033397649408</th><th>19342813113834066795298816</th><th>38685626227668133590597632</th><th>77371252455336267181195264</th><th>154742504910672534362390528</th><th>309485009821345068724781056</th><th>618970019642690137449562112</th><th>1237940039285380274899124224</th><th>2475880078570760549798248448</th><th>4951760157141521099596496896</th><th>9903520314283042199192993792</th><th>19807040628566084398385987584</th><th>39614081257132168796771975168</th><th>79228162514264337593543950336</th><th>158456325028528675187087900672</th><th>316912650057057350374175801344</th><th>633825300114114700748351602688</th><th>1267650600228229401496703205376</th><th>2535301200456458802993406410752</th><th>5070602400912917605986812821504</th><th>10141204801825835211973625643008</th><th>20282409603651670423947251286016</th><th>40564819207303340847894502572032</th><th>81129638414606681695789005144064</th><th>162259276829213363391578010288128</th><th>324518553658426726783156020576256</th><th>649037107316853453566312041152512</th><th>1298074214633706907132624082305024</th><th>2596148429267413814265248164610048</th><th>5192296858534827628530496329220096</th><th>10384593717069655257060992658440192</th><th>20769187434139310514121985316880384</th><th>41538374868278621028243970633760768</th><th>83076749736557242056487941267521536</th><th>166153499473114484112975882535043072</th><th>332306998946228968225951765070086144</th><th>664613997892457936451903530140172288</th><th>1329227995784915872903807060280344576</th><th>2658455991569831745807614120560689152</th><th>5316911983139663491615228241121378304</th><th>10633823966279326983230456482242756608</th><th>21267647932558653966460912964485513216</th><th>42535295865117307932921825928971026432</th><th>85070591730234615865843651857942052864</th><th>170141183460469231731687303715884105728</th><th>340282366920938463463374607431768211456</th><th>680564733841876926926749214863536422912</th><th>1361129467683753853853498429727072845824</th><th>272225893536750770770699685945414569152</th><th>544451787073501541541399371890829138304</th><th>1088903574147003083082798743781658276608</th><th>2177807148294006166165597487563316553216</th><th>4355614296588012332331194975126633106432</th><th>8711228593176024664662389950253266212864</th><th>17422457186352049329324779900506532425728</th><th>34844914372704098658649559801013064851456</th><th>69689828745408197317299119602026129702912</th><th>139379657490816394634598239204052259405824</th><th>278759314981632789269196478408104518811648</th><th>557518629963265578538392956816209037623296</th><th>1115037259926531157076785913632418075246592</th><th>2230074519853062314153571827264836150493184</th><th>4460149039706124628307143654529672300986304</th><th>8920298079412249256614287309059344601972608</th><th>17840596158824498513228574618118689203945216</th><th>35681192317648997026457149236237378407890432</th><th>71362384635297994052914298472474756815780864</th><th>142724769270595988105828596944949513631561728</th><th>285449538541191976211657193889899027263123552</th><th>570899077082383952423314387779798054526247104</th><th>1141798154164767904846628775559596109052494208</th><th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| CORNER FREQ                                                                                                                          | 1                                                                                        | 2                                                     | 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            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          |                                                           |                                                            |                                                           |



0.7 TO 0.8

0.4 TO 4.0 RAD/SEC

| CORNER<br>FREQ | $1/2$ | $1/4$ | $1/8$ | $1/16$ | $1/32$ | $1/64$ | $1/128$ | $1/256$ | $1/512$ | $1/1024$ | $1/2048$ | $1/4096$ | $1/8192$ | $1/16384$ | $1/32768$ | $1/65536$ | $1/131072$ | $1/262144$ | $1/524288$ | $1/1048576$ | $1/2097152$ | $1/4194304$ | $1/8388608$ | $1/16777216$ | $1/33554432$ | $1/67108864$ | $1/134217728$ | $1/268435456$ | $1/536870912$ | $1/1073741824$ | $1/2147483648$ | $1/4294967296$ | $1/8589934592$ | $1/17179869184$ | $1/34359738368$ | $1/68719476736$ | $1/137438953472$ | $1/274877906944$ | $1/549755813888$ | $1/1099511627776$ | $1/2199023255552$ | $1/4398046511104$ | $1/8796093022208$ | $1/17592186044416$ | $1/35184372088832$ | $1/70368744177664$ | $1/140737488355328$ | $1/281474976710656$ | $1/562949953421312$ | $1/1125899906842624$ | $1/2251799813685248$ | $1/4503599627370496$ | $1/9007199254740992$ | $1/18014398509481984$ | $1/36028797018963968$ | $1/72057594037927936$ | $1/144115188075855872$ | $1/288230376151711744$ | $1/576460752303423488$ | 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| <p>6) RANDOM ADDRESSING;<br/>FUNCTION MADE UP<br/>OF 40-120 SINUSOIDS,<br/>GIVING ANY DESIRABLE<br/>RECTANGULAR SPECTRA<br/>1 RMS AMPLITUDE</p> <p><math>f_{c0} = 40, 24, 40, 64, 96, 120</math><br/>or<br/><math>\omega_{c0} = 1, 1.5, 2.5, 4, 6, 10, 15 \text{ rad/sec}</math></p>                                                 | <p>I</p> <p>SIMPLE FOLLOWING<br/>WITH<br/>PID TRAPPER</p> | <p><math>K e^{-\tau s}</math><br/><math>(\tau s + 1)(\tau s + 1)</math><br/>where: <math>K = 0.22 \frac{1}{\text{sec}}</math>, <math>K = 0.42 \frac{1}{\text{sec}}</math></p> <p>ACTUAL FITTED DATA:</p> <table><tr><th>FORCING FUNCTION</th><th>K (sec)</th><th><math>\tau</math></th><th><math>1/\tau</math></th><th><math>1/\tau</math></th></tr><tr><td>R.16</td><td>34.5</td><td>110</td><td>2.2</td><td>1885</td></tr><tr><td>R.24</td><td>31.5</td><td>104</td><td>3.14</td><td>622</td></tr><tr><td>R.40</td><td>22.5</td><td>153</td><td>7.85</td><td>123</td></tr><tr><td>R.64</td><td>15.0</td><td>150</td><td>173</td><td>303</td></tr><tr><td>R.96</td><td>6.5</td><td>139</td><td>3.65</td><td>90</td></tr><tr><td>R.16</td><td>-0.6</td><td>122</td><td>3.77</td><td>90</td></tr><tr><td>R.24</td><td>-3.0</td><td>116</td><td>1885</td><td>90</td></tr></table>                                                                                                                                                                                                                                                                                               | FORCING FUNCTION          | K (sec)  | $\tau$   | $1/\tau$ | $1/\tau$      | R.16 | 34.5 | 110 | 2.2           | 1885 | R.24  | 31.5 | 104           | 3.14 | 622  | R.40 | 22.5                                                                                                                                                                                                                                                               | 153    | 7.85            | 123    | R.64      | 15.0     | 150  | 173       | 303      | R.96 | 6.5       | 139     | 3.65 | 90                   | R.16 | -0.6 | 122 | 3.77 | 90   | R.24 | -3.0 | 116 | 1885 | 90  | <p>HIGHEST FREQUENCY<br/>EQUAL TO FORCING<br/>FUNCTION BANDWIDTH,<br/>LOWEST FREQUENCIES<br/>GIVEN BELOW IN <math>\text{rad/sec}</math></p> <p>R.16 - .985<br/>R.24 - .99<br/>R.40 - .995<br/>R.64 - .98<br/>R.96 - .92<br/>R.16 - .75<br/>R.24 - .58</p> | <p>ELKIND [1956]</p> |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
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| FORCING FUNCTION                                                                                                                                                                                                                                                                                                                     | K (sec)                                                   | $\tau$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $1/\tau$                  | $1/\tau$ |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| R.16                                                                                                                                                                                                                                                                                                                                 | 34.5                                                      | 110                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 2.2                       | 1885     |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| R.24                                                                                                                                                                                                                                                                                                                                 | 31.5                                                      | 104                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 3.14                      | 622      |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| R.40                                                                                                                                                                                                                                                                                                                                 | 22.5                                                      | 153                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 7.85                      | 123      |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| R.64                                                                                                                                                                                                                                                                                                                                 | 15.0                                                      | 150                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 173                       | 303      |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| R.96                                                                                                                                                                                                                                                                                                                                 | 6.5                                                       | 139                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 3.65                      | 90       |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| R.16                                                                                                                                                                                                                                                                                                                                 | -0.6                                                      | 122                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 3.77                      | 90       |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| R.24                                                                                                                                                                                                                                                                                                                                 | -3.0                                                      | 116                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 1885                      | 90       |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| <p>7) RANDOM ADDRESSING;<br/>APPROXIMATE WHITE NOISE<br/>THROUGH 1ST, 2ND, 3RD ORDER<br/>BANDPASS FILTERS AS SHOWN<br/>BELOW. HIGHEST FREQUENCY<br/>IN FORCING FUNCTIONS WAS<br/>2 EBERS 1 RMS AMPLITUDE</p> <p><math>f_{c0} = 20, 40, 60</math><br/>or<br/><math>\omega_{c0} = 1, 1.5, 2.5, 4, 6, 10, 15 \text{ rad/sec}</math></p> | <p>I</p> <p>SIMPLE FOLLOWING<br/>WITH<br/>PID TRAPPER</p> | <p><math>K e^{-\tau s}</math><br/><math>(\tau s + 1)</math><br/>where: <math>K = \frac{3.8}{\tau}</math>, <math>K = 0.42 \frac{1}{\text{sec}}</math><br/>or <math>K = 0.39 \frac{1}{\text{sec}}</math>; <math>K = 0.42 \frac{1}{\text{sec}}</math><br/><math>\tau = \left[ \int_0^\infty \frac{1}{f^2} df \right]^{1/2}</math><br/><math>\tau = \left[ \int_0^\infty \frac{1}{f^3} df \right]^{1/3}</math><br/><math>\tau = \left[ \int_0^\infty \frac{1}{f^4} df \right]^{1/4}</math></p> <p>ACTUAL FITTED DATA:</p> <table><tr><th>FORCING FUNCTION &amp; FILTER</th><th>K (sec)</th><th><math>1/\tau</math></th><th><math>\tau</math></th></tr><tr><td>F1; 1ST ORDER</td><td>10.5</td><td>113</td><td>139</td></tr><tr><td>F2; 2ND ORDER</td><td>25.0</td><td>0.314</td><td>126</td></tr><tr><td>F3; 3RD ORDER</td><td>33.0</td><td>1885</td><td>178</td></tr></table>                                                                                                                                                                                                                                                                                                     | FORCING FUNCTION & FILTER | K (sec)  | $1/\tau$ | $\tau$   | F1; 1ST ORDER | 10.5 | 113  | 139 | F2; 2ND ORDER | 25.0 | 0.314 | 126  | F3; 3RD ORDER | 33.0 | 1885 | 178  | <table><tr><th>FILTER</th><th>RANGE (rad/sec)</th><th><math>\rho</math></th></tr><tr><td>1ST ORDER</td><td>25 TO 18</td><td>0.9</td></tr><tr><td>2ND ORDER</td><td>25 TO 12</td><td>0.96</td></tr><tr><td>3RD ORDER</td><td>25 TO 6</td><td>0.97</td></tr></table> | FILTER | RANGE (rad/sec) | $\rho$ | 1ST ORDER | 25 TO 18 | 0.9  | 2ND ORDER | 25 TO 12 | 0.96 | 3RD ORDER | 25 TO 6 | 0.97 | <p>ELKIND [1956]</p> |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| FORCING FUNCTION & FILTER                                                                                                                                                                                                                                                                                                            | K (sec)                                                   | $1/\tau$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | $\tau$                    |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| F1; 1ST ORDER                                                                                                                                                                                                                                                                                                                        | 10.5                                                      | 113                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 139                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| F2; 2ND ORDER                                                                                                                                                                                                                                                                                                                        | 25.0                                                      | 0.314                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 126                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| F3; 3RD ORDER                                                                                                                                                                                                                                                                                                                        | 33.0                                                      | 1885                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 178                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| FILTER                                                                                                                                                                                                                                                                                                                               | RANGE (rad/sec)                                           | $\rho$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| 1ST ORDER                                                                                                                                                                                                                                                                                                                            | 25 TO 18                                                  | 0.9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| 2ND ORDER                                                                                                                                                                                                                                                                                                                            | 25 TO 12                                                  | 0.96                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| 3RD ORDER                                                                                                                                                                                                                                                                                                                            | 25 TO 6                                                   | 0.97                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| <p>8) RANDOM ADDRESSING;<br/>24 TO 144 SINUSOIDS<br/>COMBINATION GIVING FORCING<br/>FUNCTION SPECTRA SHOWN<br/>(NO OF COMPONENTS SHOWN<br/>WITH PARTICULAR SPECTRA<br/>IN PARENTHESES), 1 RMS</p> <p><math>f_{c0} = 24, 144</math><br/>or<br/><math>\omega_{c0} = 1, 1.5, 2.5, 4, 6, 10, 15 \text{ rad/sec}</math></p>               | <p>I</p> <p>SIMPLE FOLLOWING<br/>WITH<br/>PID TRAPPER</p> | <p><math>K e^{-\tau s}</math><br/><math>(\tau s + 1)(\tau s + 1)</math><br/>where: <math>K = 0.42 \frac{1}{\text{sec}}</math>, <math>K = \frac{3.8}{\tau}</math><br/><math>\tau = \left[ \int_0^\infty \frac{1}{f^2} df \right]^{1/2}</math><br/><math>\tau = \left[ \int_0^\infty \frac{1}{f^3} df \right]^{1/3}</math><br/><math>\tau = \left[ \int_0^\infty \frac{1}{f^4} df \right]^{1/4}</math></p> <p>ACTUAL FITTED DATA:</p> <table><tr><th>FORCING FUNCTION</th><th>K (sec)</th><th><math>1/\tau</math></th><th><math>\tau</math></th></tr><tr><td>S1</td><td>3.0</td><td>4.76</td><td>123</td></tr><tr><td>S2</td><td>1.8</td><td>5.03</td><td>107</td></tr><tr><td>S3</td><td>1.0</td><td>12.6</td><td>90</td></tr><tr><td>S4</td><td>0.3</td><td>12.6</td><td>128</td></tr><tr><td>S5</td><td>1.1</td><td>1.89</td><td>100</td></tr><tr><td>S6</td><td>1.27</td><td>1.00</td><td>140</td></tr><tr><td>S7</td><td>2.82</td><td>1.18</td><td>178</td></tr><tr><td>S8</td><td>3.0</td><td>3.14</td><td>100</td></tr><tr><td>S9</td><td>0.4</td><td>12.6</td><td>128</td></tr><tr><td>S10</td><td>1.10</td><td>2.82</td><td>114</td></tr></table> <p>B APPROXIMATE</p> | FORCING FUNCTION          | K (sec)  | $1/\tau$ | $\tau$   | S1            | 3.0  | 4.76 | 123 | S2            | 1.8  | 5.03  | 107  | S3            | 1.0  | 12.6 | 90   | S4                                                                                                                                                                                                                                                                 | 0.3    | 12.6            | 128    | S5        | 1.1      | 1.89 | 100       | S6       | 1.27 | 1.00      | 140     | S7   | 2.82                 | 1.18 | 178  | S8  | 3.0  | 3.14 | 100  | S9   | 0.4 | 12.6 | 128 | S10                                                                                                                                                                                                                                                       | 1.10                 | 2.82 | 114 | <table><tr><th>FILTER</th><th>RANGE (rad/sec)</th><th><math>\rho</math></th></tr><tr><td>B1</td><td>0.75 TO 6</td><td>0.97</td></tr><tr><td>B2</td><td>0.75 TO 3</td><td>0.91</td></tr><tr><td>B3</td><td>3 TO 9</td><td>0.92</td></tr><tr><td>B4</td><td>6 TO 27</td><td>0.90</td></tr><tr><td>B5</td><td>27 TO 81</td><td>0.92</td></tr><tr><td>B6</td><td>81 TO 243</td><td>0.90</td></tr><tr><td>B7</td><td>6 TO 3</td><td>0.99</td></tr><tr><td>B8</td><td>3 TO 6</td><td>0.97</td></tr><tr><td>B9</td><td>6 TO 9</td><td>0.91</td></tr><tr><td>B10</td><td>9 TO 12</td><td>0.76</td></tr></table> | FILTER | RANGE (rad/sec) | $\rho$ | B1 | 0.75 TO 6 | 0.97 | B2 | 0.75 TO 3 | 0.91 | B3 | 3 TO 9 | 0.92 | B4 | 6 TO 27 | 0.90 | B5 | 27 TO 81 | 0.92 | B6 | 81 TO 243 | 0.90 | B7 | 6 TO 3 | 0.99 | B8 | 3 TO 6 | 0.97 | B9 | 6 TO 9 | 0.91 | B10 | 9 TO 12 | 0.76 | <p>ELKIND [1956]</p> |
| FORCING FUNCTION                                                                                                                                                                                                                                                                                                                     | K (sec)                                                   | $1/\tau$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | $\tau$                    |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S1                                                                                                                                                                                                                                                                                                                                   | 3.0                                                       | 4.76                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 123                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S2                                                                                                                                                                                                                                                                                                                                   | 1.8                                                       | 5.03                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 107                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S3                                                                                                                                                                                                                                                                                                                                   | 1.0                                                       | 12.6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 90                        |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S4                                                                                                                                                                                                                                                                                                                                   | 0.3                                                       | 12.6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 128                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S5                                                                                                                                                                                                                                                                                                                                   | 1.1                                                       | 1.89                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 100                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S6                                                                                                                                                                                                                                                                                                                                   | 1.27                                                      | 1.00                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 140                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S7                                                                                                                                                                                                                                                                                                                                   | 2.82                                                      | 1.18                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 178                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S8                                                                                                                                                                                                                                                                                                                                   | 3.0                                                       | 3.14                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 100                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S9                                                                                                                                                                                                                                                                                                                                   | 0.4                                                       | 12.6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 128                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| S10                                                                                                                                                                                                                                                                                                                                  | 1.10                                                      | 2.82                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 114                       |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| FILTER                                                                                                                                                                                                                                                                                                                               | RANGE (rad/sec)                                           | $\rho$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B1                                                                                                                                                                                                                                                                                                                                   | 0.75 TO 6                                                 | 0.97                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B2                                                                                                                                                                                                                                                                                                                                   | 0.75 TO 3                                                 | 0.91                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B3                                                                                                                                                                                                                                                                                                                                   | 3 TO 9                                                    | 0.92                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B4                                                                                                                                                                                                                                                                                                                                   | 6 TO 27                                                   | 0.90                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B5                                                                                                                                                                                                                                                                                                                                   | 27 TO 81                                                  | 0.92                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B6                                                                                                                                                                                                                                                                                                                                   | 81 TO 243                                                 | 0.90                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B7                                                                                                                                                                                                                                                                                                                                   | 6 TO 3                                                    | 0.99                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B8                                                                                                                                                                                                                                                                                                                                   | 3 TO 6                                                    | 0.97                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B9                                                                                                                                                                                                                                                                                                                                   | 6 TO 9                                                    | 0.91                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |
| B10                                                                                                                                                                                                                                                                                                                                  | 9 TO 12                                                   | 0.76                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                           |          |          |          |               |      |      |     |               |      |       |      |               |      |      |      |                                                                                                                                                                                                                                                                    |        |                 |        |           |          |      |           |          |      |           |         |      |                      |      |      |     |      |      |      |      |     |      |     |                                                                                                                                                                                                                                                           |                      |      |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |        |                 |        |    |           |      |    |           |      |    |        |      |    |         |      |    |          |      |    |           |      |    |        |      |    |        |      |    |        |      |     |         |      |                      |

Table 2 Summary of Operator Describing Functions in Compensatory Tasks with Various Controlled Elements.

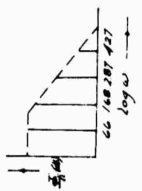
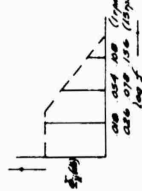
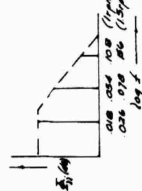
| TYPE OF FORCING FUNCTION                                                                                                                 | GENERAL CONTROL TASK                                                                 | CONTROLLED ELEMENT TRANSFER FUNCTION (GAIN NOT SHOWN)                                                       | "BEST FIT" HUMAN OPERATOR TRANSFER FUNCTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | FREQUENCY RANGE OF HUMAN OPERATOR MEASUREMENTS | AVERAGE LINEAR CORRELATION | INVESTIGATORS AND REMARKS                                                                                                                          |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
|------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------|----------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|---------------|------|------|-----|------|----|-----|------|-----|-----|----|-----|------|-----|-----|----|------|------|-----|-----|----|-----------------------------------------------|------|--------------------------------------------------------------------------------------------|
| ① RANDOM ADDING OF SUPERPOSITION OF 4 SINUSOIDS<br>   | HANDWHEEL TYPE CONTROL WITH NO RESTRAINTS                                            | $\frac{1}{(s+1)}$<br>$\frac{1}{(\frac{s}{0.5}+1)}$<br>$\frac{1}{(\frac{s}{0.5}+1)}$                         | $\frac{K e^{-T_d} (\frac{s}{T_d} + 1)}{(\frac{s}{T_d} + 1)(\frac{s}{T_d} + 1)}$ <table><tr><th>CONTROLLED ELEMENT</th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th></tr><tr><td>0.05</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>0.1</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>1.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>10.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr></table> <p>where: <math>T_d = 1/5</math></p> | CONTROLLED ELEMENT                             | $\frac{1}{T_d}$            | $\frac{1}{K}$                                                                                                                                      | $\frac{1}{T_d}$ | $\frac{1}{K}$ | 0.05 | 0.01 | 100 | 0.1  | 10 | 0.1 | 0.01 | 100 | 0.1 | 10 | 1.0 | 0.01 | 100 | 0.1 | 10 | 10.0 | 0.01 | 100 | 0.1 | 10 | SAME FREQUENCIES AS SHOWN IN FORCING FUNCTION |      | RUSSELL [1951]                                                                             |
| CONTROLLED ELEMENT                                                                                                                       | $\frac{1}{T_d}$                                                                      | $\frac{1}{K}$                                                                                               | $\frac{1}{T_d}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\frac{1}{K}$                                  |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.05                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.1                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 1.0                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 10.0                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| ② RANDOM ADDING OF SUPERPOSITION OF 4 SINUSOIDS<br>   | HANDWHEEL TYPE CONTROL WITH NO RESTRAINTS                                            | $\frac{1}{s}$                                                                                               | $\frac{K e^{-T_d} (\frac{s}{T_d} + 1)}{(\frac{s}{T_d} + 1)}$ <table><tr><th>CONTROLLED ELEMENT</th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th></tr><tr><td>0.05</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>0.1</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>1.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>10.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr></table>                                                         | CONTROLLED ELEMENT                             | $\frac{1}{T_d}$            | $\frac{1}{K}$                                                                                                                                      | $\frac{1}{T_d}$ | $\frac{1}{K}$ | 0.05 | 0.01 | 100 | 0.1  | 10 | 0.1 | 0.01 | 100 | 0.1 | 10 | 1.0 | 0.01 | 100 | 0.1 | 10 | 10.0 | 0.01 | 100 | 0.1 | 10 | SAME FREQUENCIES AS SHOWN IN FORCING FUNCTION |      | RUSSELL [1951]                                                                             |
| CONTROLLED ELEMENT                                                                                                                       | $\frac{1}{T_d}$                                                                      | $\frac{1}{K}$                                                                                               | $\frac{1}{T_d}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\frac{1}{K}$                                  |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.05                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.1                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 1.0                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 10.0                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| ③ RANDOM ADDING OF SUPERPOSITION OF 3 SINUSOIDS<br>  | SIMULATED TANK TARGET TRACKING WITH SLOW GRID HANDWHEEL "DISPLACEMENT" SPEED CONTROL | $\frac{1}{s^2} \left( \frac{s}{0.5} + 1 \right)$ <p>THIS IS 1/2 OVER THE FREQUENCY RANGE OF MEASUREMENT</p> | $\frac{K e^{-T_d} (\frac{s}{T_d} + 1)}{(\frac{s}{T_d} + 1)(\frac{s}{T_d} + 1)}$ <table><tr><th>CONTROLLED ELEMENT</th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th></tr><tr><td>0.05</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>0.1</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>1.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>10.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr></table>                                      | CONTROLLED ELEMENT                             | $\frac{1}{T_d}$            | $\frac{1}{K}$                                                                                                                                      | $\frac{1}{T_d}$ | $\frac{1}{K}$ | 0.05 | 0.01 | 100 | 0.1  | 10 | 0.1 | 0.01 | 100 | 0.1 | 10 | 1.0 | 0.01 | 100 | 0.1 | 10 | 10.0 | 0.01 | 100 | 0.1 | 10 | SAME FREQUENCIES AS SHOWN IN FORCING FUNCTION | 0.94 | TUSTIN [1947] INDICATED TRANSFER FUNCTION DOES NOT AGREE EXACTLY WITH THAT GIVEN BY TUSTIN |
| CONTROLLED ELEMENT                                                                                                                       | $\frac{1}{T_d}$                                                                      | $\frac{1}{K}$                                                                                               | $\frac{1}{T_d}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\frac{1}{K}$                                  |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.05                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.1                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 1.0                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 10.0                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| ④ RANDOM ADDING OF SUPERPOSITION OF 3 SINUSOIDS<br> | SIMULATED TANK TARGET TRACKING WITH SLOW GRID HANDWHEEL "GUIDED-LIVING"              | $\frac{1}{s^2} \left( \frac{s}{0.5} + 1 \right)$ <p>THIS IS 1/2 OVER THE FREQUENCY RANGE OF MEASUREMENT</p> | $\frac{K e^{-T_d} (\frac{s}{T_d} + 1)}{(\frac{s}{T_d} + 1)(\frac{s}{T_d} + 1)}$ <table><tr><th>CONTROLLED ELEMENT</th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th></tr><tr><td>0.05</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>0.1</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>1.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>10.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr></table>                                      | CONTROLLED ELEMENT                             | $\frac{1}{T_d}$            | $\frac{1}{K}$                                                                                                                                      | $\frac{1}{T_d}$ | $\frac{1}{K}$ | 0.05 | 0.01 | 100 | 0.1  | 10 | 0.1 | 0.01 | 100 | 0.1 | 10 | 1.0 | 0.01 | 100 | 0.1 | 10 | 10.0 | 0.01 | 100 | 0.1 | 10 | SAME FREQUENCIES AS SHOWN IN FORCING FUNCTION |      | TUSTIN [1947] INDICATED TRANSFER FUNCTION DOES NOT AGREE EXACTLY WITH THAT GIVEN BY TUSTIN |
| CONTROLLED ELEMENT                                                                                                                       | $\frac{1}{T_d}$                                                                      | $\frac{1}{K}$                                                                                               | $\frac{1}{T_d}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\frac{1}{K}$                                  |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.05                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.1                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 1.0                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 10.0                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| ⑤ RANDOM ADDING OF SUPERPOSITION OF 4 SINUSOIDS<br> | HANDWHEEL TYPE CONTROL WITH NO RESTRAINTS                                            | $\left( \frac{s+1}{s} \right)$<br>$\left( \frac{s+1}{s} \right)$                                            | $\frac{K e^{-T_d} (\frac{s}{T_d} + 1)}{(\frac{s}{T_d} + 1)}$ <table><tr><th>CONTROLLED ELEMENT</th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th></tr><tr><td>0.05</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>0.1</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>1.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>10.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr></table>                                                         | CONTROLLED ELEMENT                             | $\frac{1}{T_d}$            | $\frac{1}{K}$                                                                                                                                      | $\frac{1}{T_d}$ | $\frac{1}{K}$ | 0.05 | 0.01 | 100 | 0.1  | 10 | 0.1 | 0.01 | 100 | 0.1 | 10 | 1.0 | 0.01 | 100 | 0.1 | 10 | 10.0 | 0.01 | 100 | 0.1 | 10 | SAME FREQUENCIES AS SHOWN IN FORCING FUNCTION |      | RUSSELL [1951]                                                                             |
| CONTROLLED ELEMENT                                                                                                                       | $\frac{1}{T_d}$                                                                      | $\frac{1}{K}$                                                                                               | $\frac{1}{T_d}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\frac{1}{K}$                                  |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.05                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.1                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 1.0                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 10.0                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| ⑥ RANDOM ADDING OF SUPERPOSITION OF 4 SINUSOIDS<br> | HANDWHEEL TYPE CONTROL WITH NO RESTRAINTS                                            | $\frac{10T_d + 10}{10T_d + 1} \left( \frac{s}{0.5} + 1 \right)$                                             | $\frac{K e^{-T_d} (\frac{s}{T_d} + 1)}{(\frac{s}{T_d} + 1)}$ <table><tr><th>CONTROLLED ELEMENT</th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th><th><math>\frac{1}{T_d}</math></th><th><math>\frac{1}{K}</math></th></tr><tr><td>0.05</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>0.1</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>1.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr><tr><td>10.0</td><td>0.01</td><td>100</td><td>0.1</td><td>10</td></tr></table>                                                         | CONTROLLED ELEMENT                             | $\frac{1}{T_d}$            | $\frac{1}{K}$                                                                                                                                      | $\frac{1}{T_d}$ | $\frac{1}{K}$ | 0.05 | 0.01 | 100 | 0.1  | 10 | 0.1 | 0.01 | 100 | 0.1 | 10 | 1.0 | 0.01 | 100 | 0.1 | 10 | 10.0 | 0.01 | 100 | 0.1 | 10 | SAME FREQUENCIES AS SHOWN IN FORCING FUNCTION |      | RUSSELL [1951]                                                                             |
| CONTROLLED ELEMENT                                                                                                                       | $\frac{1}{T_d}$                                                                      | $\frac{1}{K}$                                                                                               | $\frac{1}{T_d}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\frac{1}{K}$                                  |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.05                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.1                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 1.0                                                                                                                                      | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 10.0                                                                                                                                     | 0.01                                                                                 | 100                                                                                                         | 0.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 10                                             |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
|                                                                                                                                          |                                                                                      |                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                |                            | <table><tr><td><math>T_d</math></td><td><math>\beta_0</math></td></tr><tr><td>0.5</td><td>0.60</td></tr><tr><td>2.0</td><td>0.97</td></tr></table> | $T_d$           | $\beta_0$     | 0.5  | 0.60 | 2.0 | 0.97 |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| $T_d$                                                                                                                                    | $\beta_0$                                                                            |                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 0.5                                                                                                                                      | 0.60                                                                                 |                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |
| 2.0                                                                                                                                      | 0.97                                                                                 |                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                |                            |                                                                                                                                                    |                 |               |      |      |     |      |    |     |      |     |     |    |     |      |     |     |    |      |      |     |     |    |                                               |      |                                                                                            |





Table 3 Summary of Operator Remnant Character

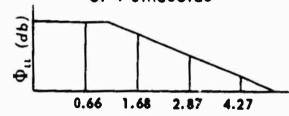
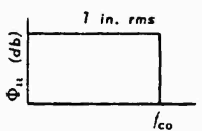
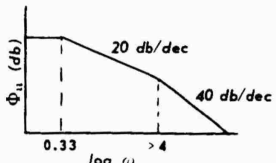
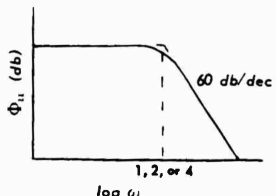
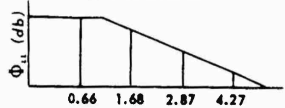
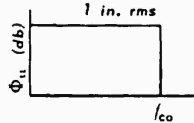
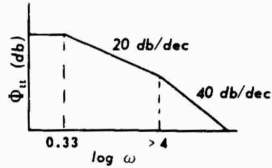
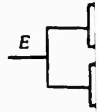
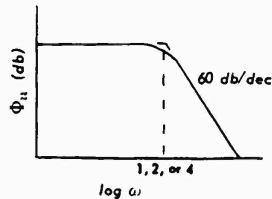
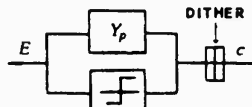
| INVESTIGATOR | GENERAL CONTROL TASK INCLUDING<br>CONTROLLED ELEMENT DYNAMICS<br>(GAIN NOT SHOWN)                                                                                                                                                                                       | FORCING FUNCTION                                                                                                                                                                                                                                                                                                  | AVERAGE<br>LINEAR<br>CORRELATION                                                                                              | ALL REMNANT<br>DUE TO THE OP              |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------|
|              |                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                   |                                                                                                                               | $\Phi_{nnf} =$                            |
| RUSSELL      | Simple tracker, handwheel type control<br>with no restraints<br>$Y_c = 1$                                                                                                                                                                                               | Random appearing superposition<br>of 4 sinusoids<br>                                                                                                                                                                           | 0.9                                                                                                                           |                                           |
| ELKIND       | Simple following with pip trapper<br>$Y_c = 1$                                                                                                                                                                                                                          | Random appearing function made up of<br>40-120 sinusoids, giving any<br>desirable rectangular spectra<br>$f_{co} \div .16, .24, .4, .64, .96, 1.6, 2.4$ cps<br>or<br>$\omega_{co} \div 1, 1.5, 2.5, 4, 6, 10, 15$ rad/sec<br> | R.16 - 0.995<br>R.24 - .99<br>R.40 - .995<br>R.64 - .98<br>R.96 - .92<br>R.1.6 - .75<br>R.2.4 - .58                           | White<br>$\Phi_{nnf}(0)$<br>var<br>R.16 - |
| GOODYEAR     | Simulated longitudinal aircraft control in pitching mockup;<br>stick with inertial, spring, and damping restraints<br>$Y_c = \frac{T_c s + 1}{s(T_1 s + 1)[(s/\omega_n)^2 + 2\zeta s/\omega_n + 1]}$<br>$1/T_c = 1.37, 1/T_1 = 2.4$<br>$\omega_n = 4.17, \zeta = 0.52$  | Random appearing<br>                                                                                                                                                                                                         |                                                                                                                               |                                           |
| FRANKLIN     | Simulated F-80A in tail chase; aileron and elevator controlled<br>$Y_{cA} = \frac{(s/0.242)(s/-2.92+1)(s/4.11+1)(s/8.32+1)}{s^2(s/0.0017+1)(s/5.65+1)[(s/3.88)^2 + 2(0.084)s/3.88 + 1]}$<br>$Y_{cF} = \frac{(s/0.28+1)(s/1.58+1)}{s^2[(s/3.98)^2 + 2(0.56)s/3.98 + 1]}$ | Random appearing white noise through<br>third order binomial filter giving<br>available corner frequencies of<br>1, 2, and 4 rad/sec<br>                                                                                     | Elevator,<br>$\rho_a \div 0.6$<br>Aileron,<br>$\rho_a \div 0.5$<br>( $\rho$ was a<br>strong<br>function<br>of $\omega_{co}$ ) |                                           |

Table 3 Summary of Operator Remnant Characteristics in Compensatory Tasks.

| FORCING FUNCTION                                                                                                                                                                                                                                                                                          | AVERAGE LINEAR CORRELATION                                                                                           | REMNANT SOURCES and BEST FIT DATA                                                                                                                       |                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                          |                                                                                                                                                                               |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                                                                                                                                                                                                                                                                           |                                                                                                                      | ALL REMNANT ASSUMED TO BE DUE TO NOISE INJECTED AT THE OPERATOR'S INPUT<br>$\Phi_{nn_e} = \frac{1}{ H ^2} \frac{\Phi_{nn}}{\int_0^\infty \Phi_{ii} df}$ | ALL REMNANT ASSUMED TO BE DUE TO NOISE INJECTED AT THE OPERATOR'S OUTPUT<br>$\Phi_{nn_e} = \frac{ Y_p ^2}{ H ^2} \frac{\Phi_{nn}}{\int_0^\infty \Phi_{ii} df}$                                                                                                                                        | ALL REMNANT ASSUMED TO BE DUE TO NONSTEADY OPERATOR BEHAVIOR                                                                                                                                                                                                                             | NONLINEAR                                                                                                                                                                     |
| Random appearing superposition of 4 sinusoids<br>                                                                                                                                                                         | 0.9                                                                                                                  |                                                                                                                                                         |                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                          | $\Phi_{nn_e} = A_d^2$                                                                                                                                                         |
| Random appearing function made up of 40-120 sinusoids, giving any desirable rectangular spectra<br>$f_{co} \div .16, .24, .4, .64, .96, 1.6, 2.4$ cps<br>or<br>$\omega_{co} \div 1, 1.5, 2.5, 4, 6, 10, 15$ rad/sec<br> | R.16 - 0.995<br>R.24 - .99<br>R.40 - .995<br>R.64 - .98<br>R.96 - .92<br>R.16 - .75<br>R.2.4 - .58                   | White noise,<br>$\Phi_{nn_e}(0) \div \frac{1}{\pi} \int_0^\infty \Phi_{ii} df$<br>valid for<br>R.16 - R.64 only                                         | $\Phi_{nn_e}(\omega) \div 2T\sigma_{n_e}^2 \{(\sin \frac{1}{2}\omega T)/(\frac{1}{2}\omega T)\}^2$<br>or<br>$R_{nn_e}(r) = \sigma_{n_e}^2(1 -  r /T)$<br>$T \div 0.75/f_{co}$ (All cases)<br>$\frac{\sigma_{n_e}^2}{\int_0^\infty \Phi_{ii} df} \div \frac{1.75}{\sqrt{f_{co}}} \quad (R.40 - R.2.4)$ | $R_{\Delta H \Delta H}(r) \div \sigma_{\Delta H}^2(1 -  r /T)$<br>where: $T_1 \div \frac{0.5}{\omega_{co}} \div 0.25$ sec<br>$\sigma_{\Delta H}^2 = 0.7/f_{co}$<br>and<br>$R_{\Delta H \Delta H}(r) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \Delta H(t) \Delta H(t+r) dt$ | No di<br>Small thre<br>is pos                                                                                                                                                 |
| Random appearing<br>                                                                                                                                                                                                    |                                                                                                                      |                                                                                                                                                         |                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                          | $\Phi_{nn_e} \div 2T_A(E_A/\delta + \pi A_d^2[\delta$<br>$T_A = 2.7$ sec<br><br>COMPUTER |
| Random appearing white noise through third order binomial filter giving available corner frequencies of 1, 2, and 4 rad/sec<br>                                                                                         | Elevator,<br>$\rho_a \div 0.6$<br>Aileron,<br>$\rho_a \div 0.5$<br>( $\rho$ was a strong function of $\omega_{co}$ ) | ALTHOUGH THE FRANKLIN F-80 DATA REMNANT POWER IS CONSISTENT WITH ALL OF THESE MODELS IT IS NOT UNEQUIVOCALLY ASSIGNABLE TO ANY ONE SOURCE               |                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                          |                                                                                                                                                                               |

| AVERAGE<br>LINEAR<br>CORRELATION                                                                                                          | REMNANT SOURCES and BEST FIT DATA                                                                                                             |                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                          |
|-------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                                                                                                           | ALL REMNANT ASSUMED TO BE<br>DUE TO NOISE INJECTED AT<br>THE OPERATOR'S INPUT<br>$\Phi_{nn_e} = \frac{1}{ H ^2} \int_0^\infty \Phi_{nn} d f$  | ALL REMNANT ASSUMED TO BE<br>DUE TO NOISE INJECTED AT<br>THE OPERATOR'S OUTPUT<br>$\Phi_{nn_c} = \frac{ Y_p ^2}{ H ^2} \int_0^\infty \Phi_{nn} d f$                                                                                                                                                                                  | ALL REMNANT ASSUMED TO BE DUE TO<br>NONSTEADY OPERATOR BEHAVIOR                                                                                                                                                                                                                                           | NONLINEAR OPERATION AND DITHER                                                                                                                                                                                                                                                                                                                           |
| 0.9                                                                                                                                       |                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                           | $\Phi_{nn_c} = A_d^2 [\delta(\omega - \omega_d) + \delta(\omega + \omega_d)]$<br>$\omega_d \doteq 7.73 \text{ rps}$                                                                                                                                                                                                                                      |
| R.16 - 0.995<br>R.24 - .99<br>R.40 - .995<br>R.64 - .98<br>R.96 - .92<br>R.1.6 - .75<br>R.2.4 - .58                                       | White noise,<br>$\Phi_{nn_e}(0) \doteq \frac{1}{8} \int_0^\infty \Phi_{nn} d f$<br>valid for<br>R.16 - R.64 only                              | $\Phi_{nn_c}(\omega) \doteq 2T\sigma_{n_c}^2 [(\sin \frac{1}{2}\omega T)/(\frac{1}{2}\omega T)]^2$<br>or<br>$R_{nn_c}(r) = \sigma_{n_c}^2 (1 -  r /T)$<br>$T \doteq 0.75/\omega_{co} \quad (\text{All cases})$<br>$\frac{\sigma_{n_c}^2}{\int_0^\infty \Phi_{nn} d f} \doteq \frac{1.75}{\sqrt{f_{co}}} \quad (\text{R.40 - R.2.4})$ | $R_{\Delta H \Delta H}(r) \doteq \sigma_{\Delta H}^2 (1 -  r /T)$<br>where: $T_1 \doteq \frac{0.5}{\omega_{co}^2} \doteq 0.25 \text{ sec}$<br>$\sigma_{\Delta H}^2 = 0.7/f_{co}$<br>and<br>$R_{\Delta H \Delta H}(r) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \Delta H(t) \Delta H(t+r) dt$ | No dither observed;<br>Small threshold nonlinearity<br>is possibly present                                                                                                                                                                                                                                                                               |
|                                                                                                                                           |                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                           | $\Phi_{nn_c} \doteq 2T_A(E_A/2)^2 [(\sin \frac{1}{2}\omega T_A)/(\frac{1}{2}\omega T_A)]^2$<br>$+ \pi A_d^2 [\delta(\omega_d + \omega) + \delta(\omega_d - \omega)]$<br>$T_A = 2.7 \text{ sec}, \quad \omega_d = 8.8 \text{ rad/sec}$<br><br>COMPUTER MECHANIZATION |
| Elevator,<br>$\rho_a \doteq 0.6$<br><br>Aileron,<br>$\rho_a \doteq 0.5$<br><br>( $\rho$ was a<br>strong<br>function<br>of $\omega_{co}$ ) | ALTHOUGH THE FRANKLIN F-80 DATA REMNANT POWER IS CONSISTENT WITH ALL OF THESE MODELS,<br>IT IS NOT UNEQUIVOCALLY ASSIGNABLE TO ANY ONE SOURCE |                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                          |



function. Thus, the most frequently accepted generalized human operator transfer function is the composite of these three terms (see Figure 7):

$$H(s) = \frac{Ke^{-\tau s}(T_L s + 1)}{(T_I s + 1)(T_N s + 1)} \quad (1)$$

### 3. Symbol Values

Ham (1958) points out that  $K$ ,  $T_L$  and  $T_I$  "are highly variable from one set of conditions to another." (p. 16) Assuming a simple compensatory tracking task in one coordinate, Ham reports that in the reaction time term,  $e^{-\tau s}$ ,  $\tau \cong 0.2$  seconds with an approximate maximum variation of  $\pm 20\%$ .

The reaction time delay contributes a phase lag that increases linearly with frequency. This intrinsic phase lag of the human operator is his dominant phase characteristic. Reaction time delay establishes a definite upper limit to the rapidity with which an operator can act on error signal stimuli. In practice it means that a (simple) closed loop system ... having an operator in the loop cannot reproduce input signals ... having frequency spectra extending above about 3 cycles per second. (p. 17)

Similarly,  $T_N$  is a limiting factor, since motor activity requires time once the motor nerves have sparked the appropriate muscles. Ham states that  $T_N \cong 0.1$  second, with a variation of approximately  $\pm 20\%$ . He further suggests that it is often possible to replace  $e^{-\tau s}/(1 + T_N s)$  by  $e^{-(\tau + T_N)s}$  for modelling purposes. (p. 17)

The gain,  $K$ , has been observed to have values of from 1 to 100 when the controlled element is designed to respond instantly to control movements of the operator.  $K$  depends upon the bandwidth of the stimulus. (The higher the uppermost frequency in the forcing function spectrum, the broader the bandwidth.) Values of  $K$  have been observed to decrease rapidly as bandwidth was increased. Thus,  $K$  is a function of signal spectrum and control element characteristics. A human operator tends to adjust his  $K$  as high as possible and still maintain system stability (Ham, p. 18).

$T_I$  has been observed to have values ranging from 0 to 20 seconds. It is usually only a few seconds, but tends to increase as input bandwidth decreases. "By introducing a  $T_I$  the operator may be able to raise his  $K$  and hence improve low frequency tracking without making the system unstable." (p. 18)

The lead time constant,  $T_L$  generally ranges between 0 and 2.5 seconds, "although its upper limit is not known." This lead, which acts like anticipation, may tend to improve feedback loop stability depending on "the particular situation, upon the spectral character of (the signal) and upon the character of the system" being controlled (p. 19).

#### 4. Another Form of the Linear Model

From McRuer and Krendel's tables, it is clear that several studies have yielded data which fit a simpler form of equation (1) resulting when one or more of the time constants (except for  $\tau$ ) approach zero. For example, Elkind's derived transfer functions (McRuer and Krendel, 1957, p. 214) all have no  $T_L$  term. And in one of his equations,  $T_N$  was another time constant found not to be significant.

A different appearing human operator linear transfer function has also been found to fit experimental data (see Table 1, items 1 and 2):

$$H(s) = \frac{e^{-\tau s}}{\frac{T^2 s^3}{K_{(2)}} + \frac{2\zeta T s^2}{K_{(2)}} + \frac{s}{K_{(2)}} + 1} \quad (2)$$

where  $\zeta$  equals the damping ratio,  $K_{(2)}$  = gain for this form of the equation, and  $T$  = an overall time constant. When the input forcing function is a step function, the resulting human response data fits equation (2), but also is very closely approximated by  $\frac{e^{-\tau s}}{T_N s + 1}$  where  $T_N \cong 1/K_{(2)}$  (see Table 1 item 1).

Equation (2) also describes the results obtained when the input forcing function is a regular sine or square wave. For these particular periodic inputs, however,  $\tau \cong 0$ , which seems to mean that the regularity of the stimulus, resulting in learning ease and prediction accuracy, makes it possible for a man to perform as if he required no reaction time.

#### 5. The Remnant

Investigators who have considered the remnant term in their quasi-linear approach have ascribed the remnant to one or more of four sources: (1) noise at the operator's input, (2) noise at the operator's output, (3) non-steady behavior of the operator, or (4) "nonlinear operation and dither." (See Table 3.) Ham (1958) states that "in compensatory tracking the remnant is largely noiselike or random in character ... [but] its origin

and quantitative character is not yet fully understood." It has been found that the mean square value of  $n(t)$  "increases as the bandwidth of the system stimulus signal ... increases." (p. 20) In other words, as the input signal varies faster and faster, the operator tracks more and more erratically.

The most descriptive and comprehensible property of a remnant function due to noise or dither is its power spectrum or spectral density,  $\Phi_{nn}$ .<sup>1</sup> (McRuer and Krendel, 1957, p. 35)

$$\Phi_{nn} = \Phi_{hh} - |C(s)|^2 \Phi_{ii} \quad (3)$$

where  $C(s)$  is the Laplace transform transfer function of the entire closed loop system,  $\Phi_{hh}$  is the human operator output power spectrum, and  $\Phi_{ii}$  is the power spectrum of the input forcing function. The useful property of the remnant function due to nonsteady human operator behavior is the autocorrelation of the time-dependent portion in the system transfer function  $C(s)$ . Letting  $\varphi_{\Delta c \Delta c}$  represent that autocorrelation function,

$$\varphi_{\Delta c \Delta c} = \frac{\varphi_{nn}}{\varphi_{ii}} \quad (4)$$

where  $\varphi_{nn}$  and  $\varphi_{ii}$  are the autocorrelation functions of the remnant and input forcing function, respectively. (See McRuer and Krendel, 1957, p. 106.)

## 6. Approaches by Individual Researchers

Earlier it was mentioned that Bekey (1962) is one of the supporters of the Sampled-Data Model concept. The model he proposes is one containing (a) reaction time and neuromuscular system functions,

$$H_1(s) = \frac{K e^{-\tau s}}{T_N s + 1}$$

plus (b) hold circuit characteristics,

$$H_2(s) = \left( \frac{T_\sigma s + 1}{T} \right) \left( \frac{1 - e^{-T_\sigma s}}{s} \right)$$

---

<sup>1</sup> See the discussion of the general concept of the power spectral density in the introduction.

Thus, his model (which is diagrammed in Figure 9) turns out to be

$$H(s) = H_1(s)H_2(s) = \left( \frac{Ke^{-\tau s}}{T_N s + 1} \right) \left( \frac{T_\sigma s + 1}{T_\sigma} \right) \left( \frac{1 - e^{-T_\sigma s}}{s} \right) \quad (5)$$

In Bekey's experiment subjects were required to perform compensatory tracking in which the forcing function was a sum of 10 random-appearing nonharmonic sinusoids. He compared the results of applying his model and a linear continuous model with the actual experimental data, and he found that "the discrete models do indeed result in input-output behavior which more closely approximates results than that which results from linear continuous models." He goes on to say, "the continuous models were considered adequate representations of tracking behavior when the input function bandwidth did not exceed approximately 3/4 cps. In the present study the band extended to 1.6 cps; spectral peaks were noted in the range of 1-1.6 cps (which are consistent with previous data), and these peaks were shown to be consistent with linear sampling models as well," (p. 49). Previously the peaks were unexplainable when it was attempted to fit experimental data to linear continuous models.

Diamantides (1958) chose to study an analog computer, nonlinear model of the human operator using a model adjustment technique. While his model resembles the generalized one shown in Figure 9, it differs in that he omits dither and saturation limits on rate response. (See Figure 11.) He includes some difficult nonlinearities, however, such as is illustrated in Figure 12, which gives an example of the type of discontinuous function that cannot readily be included in a linear mathematical model, particularly when more than one function has discontinuities.

Diamantides' experiment required that pilots correct for flight simulated changes in pitch angle due to "a wind disturbance." By simultaneously observing computer and pilot performances resulting from identical inputs to each, he could manually adjust analog computer potentiometers representing simulated human parameters so that the analog performance matched a given subject's performance.

The computer technique used by Ornstein (1961) was a little more sophisticated than that of Diamantides, but the former's model was much simpler. Ornstein began with the assumption of an equation (1) model, but put the model into the equivalent form

$$H(s) = \frac{(a s + 1) e^{-\tau s}}{c s^2 + b s + d} \quad (6)$$

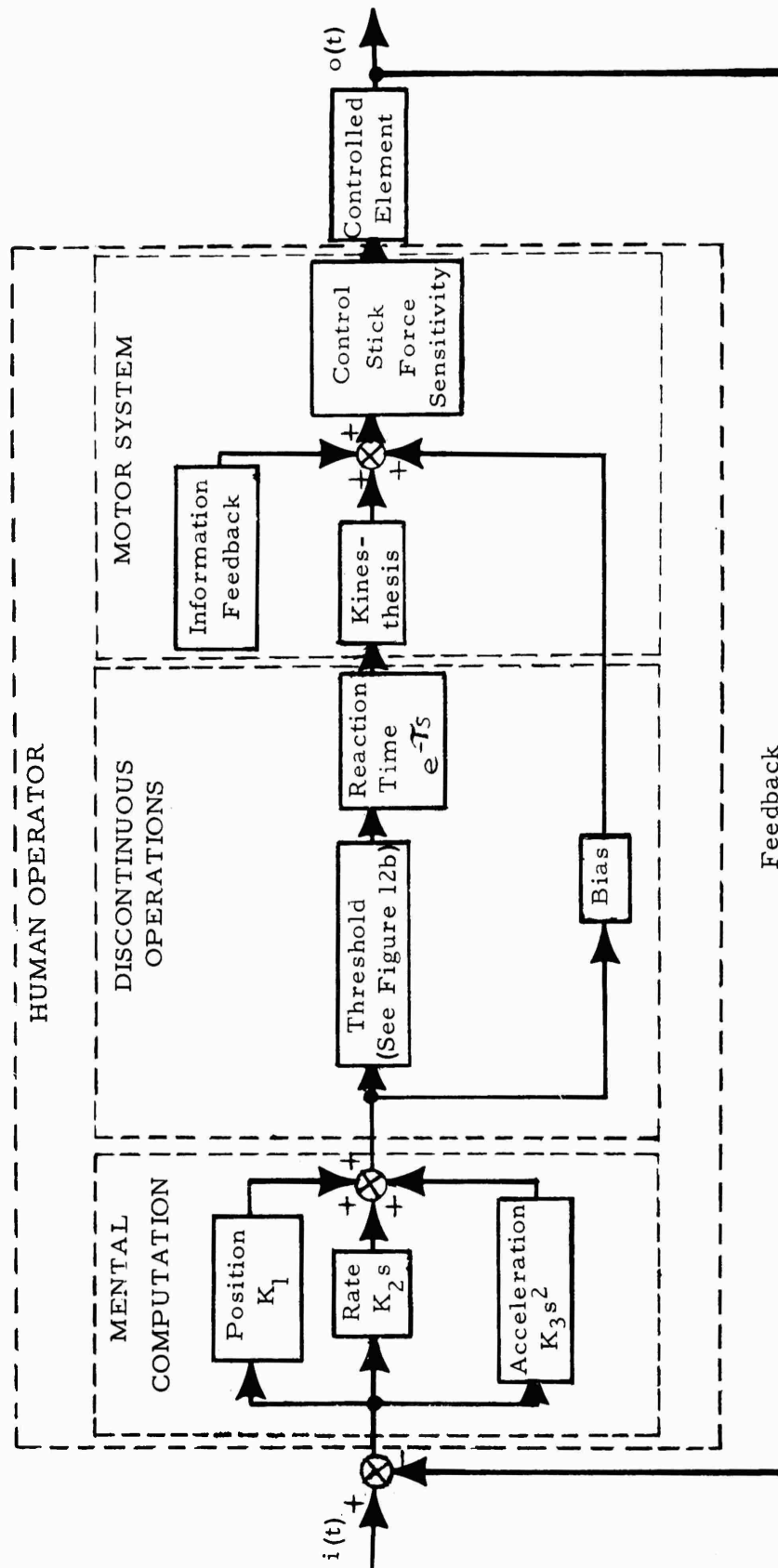


Figure 12a. Block diagram of Diamantides' pilot analog system (1958, p. 365)

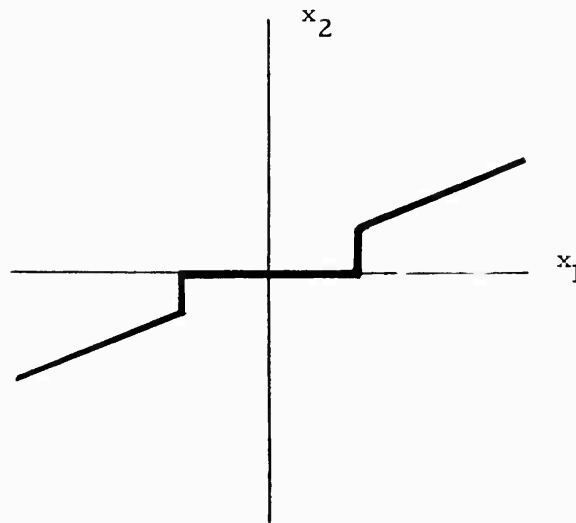


Figure 12b. Form of describing function for the threshold characteristic in Figure 12a.

By means of some complex mathematics, he then programmed the computer to derive automatically, repeatedly and continually, the values of the parameters  $a$ ,  $b$ ,  $c$ , and  $d$  based on inputs to and performance measures of human operators using a control stick for compensatory tracking; because of equipment limitations,  $\tau$  was assigned the value 0.2 seconds. He refers to his setup as a "manalog." This automatic method for computing the coefficients was found to be sensitive as well as "effective from the points of view of reliability, validity and efficiency. The sensitivity of the technique is such that its further application may reveal nuances and relationships which permit the amalgamation of modern and classical efforts in psycho-motor research." (Ornstein, p. 45)

Elkind, et al, who also used a model adjustment technique, comment on the difficulties inherent in Ornstein's techniques. They claim that:

the coefficients of this approximation, the coefficients of the differential equation, have to be determined by "cut-and-try" or "hill-climbing" procedures. There does not seem to be an analytic procedure for finding the coefficients. The technique also requires an

assumption of the form of pilot transfer function. Since there may be a strong interaction among the coefficients being adjusted (the value of one coefficient influences the values of the others), inaccurate results may be obtained if the operator's actual characteristics are not of the assumed form. (1963, p. 5)

Therefore, although Elkind, et al, take a model adjustment approach, they employ an operator-matching concept of mathematical functions used by Huggins. (Elkind, et al, 1963, p. 6) It is noteworthy, by the way, that these investigators used equation (1) as the transfer function which the filter model was adjusted to fit.

Sender also used a model-adjustment technique. However, he starts off by assuming the simplest possible model he can,

$$H(s) = \frac{K e^{-\tau s}}{T_N s + 1}$$

Then, Senders forces this model into his system and adjusts other system parameters to make the human operator (pilot) model fit. "We want the pilot to behave as a simple amplifier, except that he is constrained by his reaction time delay and his neuromuscular lag." In other words, this simplified model may result in system errors, unless "all dynamics that are needed to reduce the error [are] put in the controlled system, rather than expecting the pilot to learn them." (p. 9)

Senders' problem was to find the "optimum control system dynamics for the barest human operator model in a simple tracking system." He gathered results of previous studies, and by programming a computer to search for coefficient values leading to minimum error he derived the most effective-appearing control system functions. In diagrammatic form, the total tracking system he concluded to be optimum appears as Figure 13.

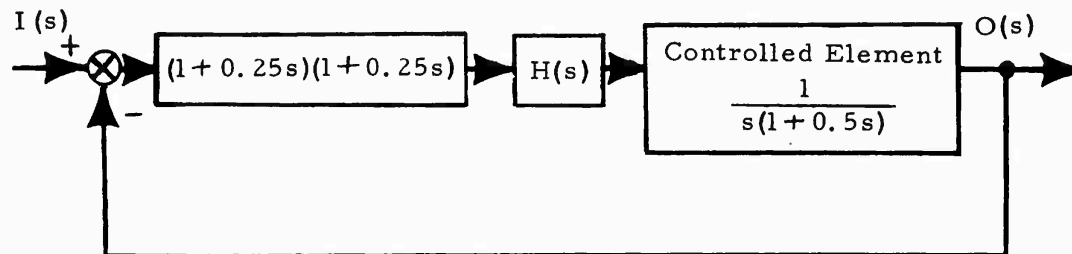


Figure 13. Senders' optimum control system dynamics for simplifying H(s) (after Senders, 1959, p. 10)

## 7. Two Recent Applications of the Model

Rathert, et al, (1961) used an analog computer simulation of the human operator to evaluate characteristics of the total flight system simulator and to evaluate subjective pilot opinions. These investigations adopted equation (1) as their model and utilized results of previous studies to assign all but two parameter values, K and  $T_L$

$$H(s) = \frac{K e^{-0.2 s} (T_L s + 1)}{(0.1 s + 1) (0.1 s + 1)}$$

The gain, K, was varied at two constant values of  $T_L$ , zero and 0.1 in order to see what changes in these terms were needed to best "match the performance of the real pilot." (p. 6) The results of using this model provided numerical values of the range of acceptable gain values for two categories of control dynamics. "With good control dynamics, the human pilot could have used broad range of gain ... before the appearance of any instability ... With poor dynamics, the human pilot must have had to adjust his gain closely on a very narrow band to avoid either poor performance or ... instability."

Frost (1961) used an even simpler model for his evaluative study. He wished to determine the range of K values, possible for good performance and stability of a tracking system in which the quickening<sup>1</sup> of the display signal was varied. His model was

$$H(s) = \frac{K e^{-\tau s}}{T_N s + 1} = \frac{K e^{-0.2 s}}{0.1 s + 1}$$

The schematic model he describes for his study appears in Figure 14.

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<sup>1</sup> A quickened system continuously displays to the human operator where he must position his control to meet a given system output criterion. (Morgan, et al, Eds., 1963, p. 241) This is done by feeding derivative information back to the display, as indicated in the simple example of Figure 14, which represents the writer's conception of the Frost system. Frost varied quickening by means of a variable gain,  $K_Q$ . Using his simple operator model, "Even this very poor pilot was able to maintain acceptable control when the display signal was suitably quickened...." (Frost, p. 2)



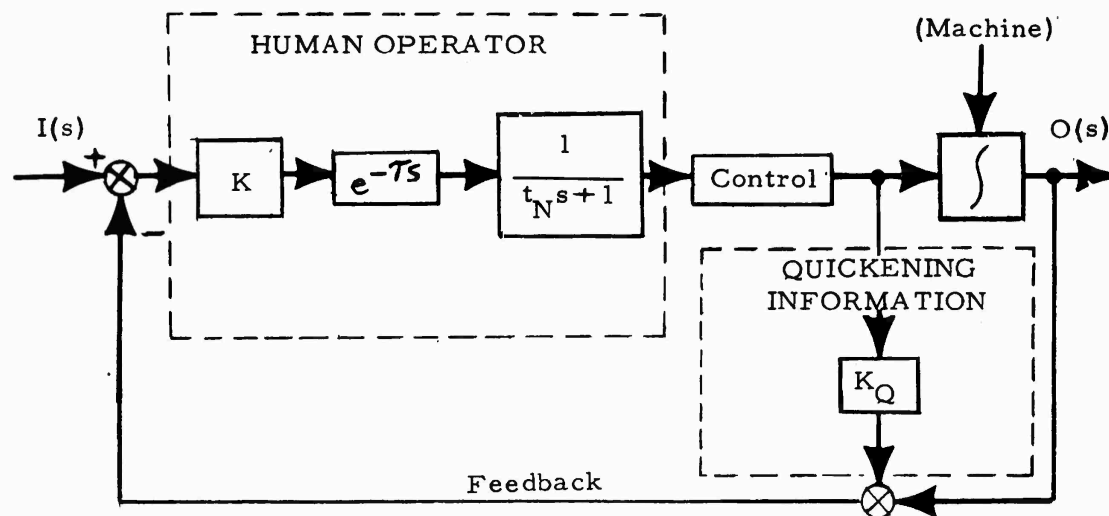


Figure 14. A simple quickened man-machine system.

#### 8. Concluding Comments

In his excellent article, Licklider (1960) summarizes so well the apparent current difficulties in the human operator modelling area that it seems appropriate here to reiterate his conclusions. Licklider says that there appears to be

.... a disproportion between the mathematical apparatus ... ready to be brought to bear upon the results of tracking experiments and the results upon which the apparatus can be set. The reason for the existence of this disproportion is two-fold: First, the problem of processing data for use in testing models has proven difficult. Only very recently has there been more than a trickle of experimentally determined transfer functions. Second, many of the experiments on tracking have not been formulated in relation to models at all. There are many bits and pieces, and few real sets of modellable data.

The main need, therefore, is for reliable and practical data processing equipment. The whole field could be revolutionized by a few comprehensive experiments, but at the present rate they would take years and hundreds of thousands of dollars. (p. 273)

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