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SATELLITE DRAG COEFFICIENTS

by

G. E. Cook

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MINISTRY OF AVIATION
PARLOR HIGH STREET

ROYAL AIRCRAFT ESTABLISHMENT

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SATELLITE DRAG COEFFICIENTS

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SUMMARY

The drag of artificial earth satellites is reconsidered in the light of recent studies of gas-surface interactions and atmospheric composition. Between heights of 140 and 400 km at times of low solar activity, or heights of 140 and 600 km at times of high solar activity, the drag coefficient is almost independent of height, and at present, because of the lack of decisive evidence, there is perhaps not sufficient reason to abandon the value of 2.2 which has been widely used in recent years for the drag coefficient. It must be recognized, however, that this value is subject to some uncertainty and may be too low, perhaps by as much as 10 per cent. At heights above 400 km (low solar activity) or 600 km (high solar activity) the drag coefficient increases with height, since both the degree of energy transfer and the molecular speed ratio decrease as the molecular weight of the atmosphere decreases.

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1 INTRODUCTION

Since a satellite normally experiences free-molecule flow, knowledge of the mechanism of molecular reflexion at its surface is necessary if the aerodynamic drag is to be determined. When the early satellites were launched only meagre information on gas-surface interactions was available; drag was estimated¹, assuming the atmospheric molecules were diffusely re-emitted with velocities corresponding to the surface temperature of the vehicle. Even today there is still no experimental evidence on surface phenomena in the appropriate energy range, although recent theoretical studies have indicated the important factors involved. In the light of these investigations it is now possible to indicate how the drag coefficients may differ from the values currently in use.

2 FREE-MOLECULE FLOW

The conventional requirement for free-molecule flow is that the mean free path in the free stream, λ_0 , is large compared with a typical linear dimension L of the body. The ratio λ_0/L is known as the Knudsen number K , so that free-molecule flow occurs if $K \gg 1$. For a body moving at very high speed², however, this condition is not sufficient to ensure free-molecule flow if the surface temperature is low and appreciable energy accommodation takes place.

If the number density in the free stream is n_0 and the molecules impinge on the body with a speed v_i , the number of molecules arriving at unit projected area in unit time is $n_0 v_i$. The molecules will be re-emitted at a speed v_r which generally differs from v_i . The number density of re-emitted molecules near the surface will then be of the order $n_0 v_i / v_r$. Since mean free path is inversely proportional to density, the mean free path of an incident molecule passing through the re-emitted molecules near the surface is given by

$$\lambda \sim \lambda_0 \frac{v_r}{v_i}.$$

If free-molecule flow is to exist, the Knudsen number based on λ must be greater than unity and the true requirement is $\lambda/L \gg 1$, i.e.

$$\frac{\lambda_0}{L} \gg \frac{v_i}{v_r}. \quad (1)$$

In practice the largest value of v_i/v_r for a satellite is about 13, but this will only occur when the molecules are re-emitted with a speed corresponding to the surface temperature.

Values of mean free path given by³ the U.S. Standard Atmosphere (1962) are sufficiently accurate for order-of-magnitude studies and are shown in Fig.1 for heights between 100 and 250 km. At a height of 200 km the mean free path is over 200 m and free-molecule flow will probably be experienced by all satellites so far launched. Even at 175 km, where the mean free path is over 100 m, only a few satellites, e.g. the large carrier rockets, are likely to have entered the transition region, where collisions between incident and re-emitted particles lead to a reduction in the drag coefficient. A satellite whose greatest dimension is about 1 m will continue in free-molecule flow down to about 140 km.

An important parameter governing the drag coefficient of a body in free-molecule flow is the molecular speed ratio s defined by

$$s = \frac{\text{speed of satellite}}{\text{most probable molecular speed}} \quad (2)$$

The most probable molecular speed v_m at any altitude is given by

$$v_m = \sqrt{\frac{2RT}{M}} \quad (3)$$

where T is the atmospheric temperature, M the molecular weight and R the gas constant. v_m is related to the scale height H by the equation

$$v_m = \sqrt{2Hg} \quad (4)$$

g being the local value of the acceleration due to gravity. Using values of H from⁴ the COSPAR International Reference Atmosphere (1964), the most probable molecular speed is found to increase from about 0.7 km/sec at a height of 150 km to around 1.5 km/sec at 700 km, whereas the satellite's speed is always greater than 7.5 km/sec at these altitudes. Consequently the molecular speed ratio at perigee always exceeds 5 for perigee heights below 700 km and the random thermal motion of the atmospheric molecules can be neglected, i.e. the flow is hyper-thermal⁵. For a satellite with perigee at 1500 km, however, the molecular speed ratio is less than 4 and the thermal motion cannot be ignored.

3 MOLECULAR REFLEXION AT A SOLID SURFACE

3.1 The accommodation coefficient

The degree of energy transfer between a gas molecule and a surface is usually expressed in terms of the accommodation coefficient α , which is defined as the ratio of the energy change experienced by the impinging molecules to the

maximum energy change that could take place. If the average kinetic energies of the incident molecules and the re-emitted molecules are denoted by E_i and E_r respectively, and the average kinetic energy of a molecule re-emitted with a velocity corresponding to the surface temperature is E_s , then

$$\alpha = \frac{E_i - E_r}{E_i - E_s} \quad (5)$$

At satellite speeds the incident molecules are almost uni-directional and they impinge on the surface with energies in the range 0.2 eV (hydrogen atom at 6 km/sec) to 12 eV (oxygen atom at 8.5 km/sec). Consequently we are interested in incident energies which are high compared with the energy at which molecules are likely to be trapped at the surface by physisorption, but still well below the level at which appreciable sputtering will occur. So far no experimental measurements of accommodation coefficients have been made in this energy range⁶. Over the past few years, however, a number of theoretical studies have been performed and these have indicated the main factors affecting the energy accommodation.

3.2 Theoretical values of α

The theoretical investigation of accommodation coefficients is extremely difficult because of the large number of fundamental assumptions required to make progress. These assumptions concern the motion of the gas atom and the configuration and motion of the surface atoms. It is also usual to assume that the surface is clean.

In terms of classical theory the simplest possible model is obtained by considering the incident and surface atoms to be smooth and perfectly elastic spheres. The energy exchange is then independent of gas temperature and is greatest in a head-on collision for which we have the elementary result

$$\alpha = \frac{4\mu}{(1 + \mu)^2} \quad (6)$$

where μ is the ratio of the mass of the incident gas atom to the mass of the surface atom. The value given by (6) is valid for $\mu < 1$, when only a single collision takes place. For $\mu > 1$ the incident gas atom is still travelling towards the surface after a single collision and the accommodation coefficient cannot be estimated without considering the motion of the surface atoms. For an oblique collision, when the velocity of the incident atom is inclined at an angle ϕ to the line of centres at impact, a factor $\cos^2 \phi$ is introduced on the right-hand side of (6).

Using classical mechanics Goodman⁷⁻⁹ has recently made an extensive study of accommodation coefficients assuming the surface to be a simple cubic lattice. The incident atom is assumed to impinge directly onto one of the surface atoms and the latter are considered to be initially at rest. In his latest paper⁹ a Morse potential is used to represent the interaction between the gas atom and the surface atom. For a single atom the general form of the variation of the accommodation coefficient with incident energy E_i is independent of the gas-solid system and is as shown in Fig. 2. At low temperatures (i.e. low incident energies) the gas atom is trapped and the energy accommodation is complete. Above a certain temperature the gas atom has sufficient energy to escape and the energy accommodation is incomplete. After reaching a minimum the accommodation coefficient increases towards the value given by the elastic-spheres model as the incident energy becomes large. Using the calculated values of accommodation coefficient for single atoms, Goodman proceeds to find the 'true' accommodation coefficient on the assumption that the incident beam has a one-dimensional Maxwellian velocity distribution. If the parameters of the Morse potential are suitably chosen, the resulting values of accommodation coefficient show adequate quantitative agreement with the experimental data of Thomas and Schofield (mentioned in Ref. 9) over a limited range of gas temperature.

The high energy limit given by equation (6) will only apply when a single collision takes place. It appears for the elastic-spheres model and also for other one-dimensional lattice models that a second collision does not occur until μ exceeds 1. With a three-dimensional lattice model, however, Goodman has found⁷ that there is always a second collision when μ exceeds 0.84. Consequently the accommodation coefficient is reduced below the value given by (6) for $0.84 < \mu < 1$ and it becomes small as μ tends to infinity. Poor accommodation for large values of μ has also been noted by Gilbey¹⁰ for a continuum model of the surface. This phenomenon is due to the fact that a three-dimensional lattice possesses rigidity and always completely restores itself after a series of collisions, whereas a one-dimensional lattice does not.

It is informative to consider the high-energy limit obtained when a quantum theoretical approach is used. Again an incident gas atom is assumed to impinge directly onto a lattice atom. It was originally believed that first-order quantum-mechanical perturbation theories were inadequate for the treatment of high energy collisions. Gilbey¹⁰ has recently re-examined Devonshire's theory and shown that it can successfully predict multi-phonon transitions and should, therefore, predict correct values of accommodation

coefficients for both small and large energy transfers, provided the accommodation coefficient is itself small. When the incident energy of the gas atom is sufficiently high, he found that Devonshire's result reduces to $\alpha = 4\mu$. This is an approximation to (6) valid for small values of μ .

So far we have only mentioned theories which assume the incident gas atom to impinge directly on a lattice atom. Although this assumption is useful in initial investigations it is difficult to tell how the predicted values will differ from the values which occur in practice. It seems that only three attempts have been made to consider collisions in which the gas atom does not approach and leave the solid along a normal to its surface. In Baule's original treatment¹¹ the surface was represented by an oscillating cubic lattice. Collisions were based on the elastic-spheres model and the energy transfer was determined by averaging over all angles of incidence ϕ . The result, obtained assuming only a single collision with the surface, gave

$$\alpha = \frac{2\mu}{(1 + \mu)^2}, \quad (7)$$

which is half the value predicted by equation (6).

In practice, neither equation (6) nor equation (7) will give the correct result, and the true value of α will probably lie somewhere between these two extremes. Öpik¹² has also attempted to calculate average values of accommodation coefficients using the elastic-spheres model. He made some allowance for the fact that all impacts are not head-on collisions and determined an average accommodation coefficient. His computed values of α , which are slightly less than those predicted by equation (1), are shown in Fig. 3, together with those from other theoretical models. More recently, Oman, Bogan, Weiser and Li¹³ have used a detailed classical model in which gas atom trajectories are not confined to head-on collisions. The results of sample calculations for helium, neon and argon on nickel¹⁴ are shown in Fig. 3 for atomic beams incident at 15 or 45° to the surface normal. These results suggest that a value between the two curves representing equations (6) and (7) is of the right order at energies of a few electron volts.

3.3 Distribution of reflected molecules

As with the accommodation coefficient, no measurements of the distribution of reflected molecules have been made in the appropriate energy range. Experimental evidence that does exist is to some extent conflicting, and, in contrast to the accommodation coefficient, there are no theories which can be used to assist in the reconciliation of the results.

The extreme mechanisms of re-emission are generally regarded as specular and diffuse. Specular reflexion occurs when the angle of reflexion of a particle is equal to the angle of incidence. Diffuse re-emission takes place according to the Knudsen cosine law, which states that the number of molecules emitted between the angles ϕ and $\phi + \delta\phi$ from the normal to the surface is proportional to $\cos \phi \delta\phi$.

At thermal energies both diffuse re-emission and lobate reflexion near the specular ray have been obtained in laboratory experiments with molecular beams. The early investigations using this technique were principally concerned with scattering by cleaved alkali halides and this work has recently been discussed by Hurlbut¹⁵. For these surfaces quasi-specular reflexion can occur at all angles of incidence. The scattering of deuterium and helium from polycrystalline platinum has been studied by Datz, Moore and Taylor¹⁶, who found that the angular distribution conformed to two patterns, depending on the temperature of the surface. On a surface at room temperature the distributions were diffuse, whereas on a high temperature surface (about 1900°C) they were remarkably specular. For both gases the beam reflected from the hot surface was found to lie more nearly along the specular ray as the beam temperature was increased, an effect that has been noted by several investigators. The reflexion of low energy beams of rare gases, nitrogen and oxygen from polycrystalline platinum and nickel has been studied recently by Hinchey and Foley¹⁷. Again diffuse re-emission was observed when the surfaces were at room temperature. For argon on platinum a change from diffuse re-emission to lobate reflexion was found as the surface was heated, the transition occurring in the region of 500°C. The lobe maxima were found to be displaced from the specular ray towards the normal, the displacement being greater for incident angles nearer the surface. The difference in the observed distributions between the hot and cold surfaces is most probably due to the presence of adsorbed gaseous contaminants at low temperatures.

3.4 Application to satellites

For light gases on clean surfaces both theory and the limited experimental evidence now available suggest that low values of the accommodation coefficient are appropriate. The success of Goodman's theory in predicting the correct dependence of the accommodation coefficient on various parameters cannot be ignored, and, for a clean surface, the accommodation coefficient will probably vary with incident energy in the manner shown qualitatively in Fig.2. The accommodation coefficient at the surface of a satellite can probably be computed on the basis of the hard-spheres model so that α is proportional to

$\mu/(1 + \mu)^2$ for $\mu < 1$. Since the factor of proportionality is likely to lie between 2 and 4, the accommodation coefficient will probably have a value between the continuous and dashed curves shown in Fig.3.

Before an accommodation coefficient can be chosen for a particular satellite at a specific altitude it is necessary to decide on an average value for μ . The main atmospheric constituents are N_2 , O_2 , O, He, A and H. Atomic oxygen predominates over most of the altitude range of chief interest (150 to 800 km) while atomic hydrogen is only present in significant quantities above 500 km. Strictly the drag due to each constituent should be computed separately, but because of other uncertainties it is adequate to take the mass of the incident molecules equal to the ambient molecular weight if μ is less than one. The variation of molecular weight with altitude, as given by the COSPAR International Reference Atmosphere, 1964, is shown in Fig.4 for extreme levels of solar activity and local times corresponding to maximum and minimum values of the diurnal density variation.

When the incident energy of the impinging molecules is sufficiently large for the 'hard-spheres' model to be valid, the mass of the surface atoms used in calculating μ will depend on the nature of the outermost layer of atoms on the surface, and not on the actual material used in constructing the vehicle. For only a few satellites have the surface materials been publicized. Typical materials used on the outside of satellites are gold, stainless steel, aluminium, plastic, quartz, synthetic sapphire and various paints. Quartz or synthetic sapphire is used to protect solar cells from radiation damage, while paints are used to vary the surface characteristics and hence to control the temperature of the vehicle.

It is extremely difficult to define the exact state of a solid surface owing to the ease with which gases are adsorbed. The type of adsorption depends on the nature of the attractive force holding the adsorbed gas to the surface. Physical adsorption depends on van der Waals' forces and the heat of adsorption is relatively low, i.e. a few kcal/mole. Chemisorption displays a large heat of adsorption, implying the existence of a chemical bond. In contrast to physisorption, chemisorption is not readily reversible, high temperatures and low pressures being required to produce desorption.

Chemisorption takes place under conditions in which incipient chemical reaction between gas and surface might be expected. For example, most metals are capable of reacting with oxygen to form oxides, so that it is not surprising that oxygen is chemisorbed by all metals except gold. Although an energy of activation is sometimes necessary before chemisorption occurs, it can

occur extremely rapidly, especially with oxygen, and a complete monolayer can be formed on a metal surface in a few seconds, even at a pressure of 10^{-6} mm of mercury (which corresponds to a height of about 200 km).

Since oxygen is fairly abundant at ground level and atomic oxygen is usually the main atmospheric constituent at the orbital heights of interest, we would expect, on the basis of the foregoing discussion, that the surfaces of all metals except gold would be covered by a monolayer of adsorbed oxygen. Many metals would possess more than a monolayer since oxide films can be formed by a diffusion mechanism. Aluminium is known to form an oxide film in this manner, while stainless steel owes its 'stainless' property to the formation of a film of chromic oxide. It is reasonable to suppose that in the final atomic arrangement the metallic surfaces of a satellite would most probably present oxygen atoms to the ambient atmospheric particles; only gold-plated surfaces are likely to remain uncontaminated. Although an atom will occasionally be desorbed, adsorption sites are unlikely to remain vacant for long in the presence of energetic atoms of oxygen.

For non-metals the nature of the outermost layer of atoms is not so easily defined. Physisorbed atoms will be readily desorbed by the incident atoms while the chemisorption of energetic atoms on non-metals is virtually unexplored. Sapphire is Al_2O_3 and, since the ionic radius of aluminium is small compared with that of oxygen, a gas atom is unlikely to hit aluminium. With quartz (SiO_2), a free space near a silicon atom would constitute a site for chemisorption of an oxygen atom and would be unlikely to remain unfilled for long. Since quartz possesses a somewhat open structure, an occasional hit on a silicon atom cannot be ruled out. The atomic weights of aluminium and silicon are, however, not so very different from oxygen, and the small uncertainty in μ will not significantly contribute to the already large uncertainty in the accommodation coefficient.

The outermost atoms of a painted surface are even more difficult to specify. Paints consist of inorganic pigments and binders of silicone, acrylic or epoxy resins. These resins are synthetic polymers with organic groups on the polymer chain. Since the organic groups contain hydrogen, the surface atoms may sometimes be lighter than the incident atoms, which would then undergo a second collision. At present it is impossible to specify even an approximate value of α under these circumstances. Fortunately it is not usually necessary to paint more than 20 to 30% of a satellite's surface for thermal control purposes.

A topic which should be mentioned at this stage is sputtering. A few years ago it was common practice to discuss threshold energies below which sputtering did not occur, values of 50-60 eV being quoted for most materials. With the more accurate measuring techniques now available, very small yields have been measured at lower energies. Extrapolation of these results¹⁸ suggests that sputtering is insignificant at energies below 20-25 eV. It is therefore reasonable to conclude that sputtering will have a negligible effect on drag, both directly through the removal of momentum by sputtered material, and indirectly through the modification of μ by the removal of adsorbed atoms. McKeown and his collaborators¹⁹ claim to have measured sputtering on two Discoverer satellites at a height of about 200 km. The erosion rates were found to be $0.1 \pm 0.05 \text{ \AA/day}$ for gold and $0.15 \pm 0.05 \text{ \AA/day}$ for silver, corresponding to sputtering yields of 10^{-6} atoms/molecule and 2×10^{-6} atoms/molecule respectively. These values have recently been disputed²⁰, however.

4 DRAG OF SIMPLE SHAPES IN HYPERTHERMAL FREE-MOLECULE FLOW

From equation (5) the ratio of the speed of a re-emitted molecule v_r to the speed of an incident molecule v_i is related to the thermal accommodation coefficient by the equations

$$\left. \begin{aligned} \frac{v_r}{v_i} &= \left(\frac{E_r}{E_i} \right)^{\frac{1}{2}} = \left\{ 1 + \alpha \left(\frac{E_s}{E_i} - 1 \right) \right\}^{\frac{1}{2}} \\ \text{and} \quad \frac{E_s}{E_i} &= \frac{T_w}{T_i} \end{aligned} \right\} \quad (8)$$

where T_i is the kinetic temperature of incidence and T_w is the surface temperature. Since we are unable to make any statement about the variation of α with the angle of incidence of a surface element, all reflected molecules are assumed to have the same value of v_r . After making this assumption, there is little point in taking into account the random thermal motion for molecular speed ratios above 5.

The drag of satellites in hypersonic free-molecule flow has been studied by Schamberg⁵ and his results for certain simple shapes are quoted in Table 1. The drag coefficient C_D is defined by

$$C_D = \frac{D}{\frac{1}{2} \rho V^2 S} \quad (9)$$

where D is the drag acting in the opposite direction to the satellite's velocity vector $\underline{V}$, ρ is the air density and S is a representative cross-sectional area. The ratio v_r/v_1 is denoted by r . Since T_w/T_1 is very small, r is approximately equal to $\sqrt{1-\alpha}$.

Table 1

DRAG IN HYPERTHERMAL FREE-MOLECULE FLOW

Shape	Drag coefficient (based on projected area perpendicular to direction of motion)	
	Diffuse re-emission	Accommodated specular reflexion
Flat plate (normal to flow)	$2(1 + \frac{2}{3} r)$	$2(1 + r)$
Flat plate at incidence θ	$2(1 + \frac{2}{3} r \sin \theta)$	$2(1 - r \cos 2 \theta)$
Sphere	$2(1 + \frac{4}{9} r)$	2
Cylinder perpendicular to flow	$2(1 + \frac{\pi}{6} r)$	$2(1 + \frac{1}{3} r)$
Cone of semi vertex angle ψ with vertex forwards and axis parallel to flow	$2(1 + \frac{2}{3} r \sin \psi)$	$2(1 - r \cos 2 \psi)$

The variation of the drag coefficient with α is shown in Figs. 5 to 7 for a flat plate, a sphere and a circular cylinder of infinite length respectively. C_D was evaluated assuming $T_w/T_1 = 0.006$. Fig. 5, which gives the drag coefficient of a flat plate based on the projected area perpendicular to the direction of motion for various angles of incidence, serves as a reminder that C_D would not necessarily exceed 2 if there were appreciable reflexion of molecules along the specular ray. A flat plate is not a practical shape, however, and Figs. 6 and 7 give a fairer impression of the likely value of C_D .

5 DRAG OF A SPINNING CIRCULAR CYLINDER IN HYPERTHERMAL FREE-MOLECULE FLOW

Most satellites used for density determination are unstabilized, so that their attitudes are variable. It is known that a spinning satellite will eventually rotate about its axis of maximum moment of inertia if it is either acted upon by small external torques²¹ or experiences energy dissipation internally²². Hence, to evaluate drag, a non-spherical satellite is normally assumed to rotate about its axis of maximum moment of inertia after it has been in orbit for a few

days. Unfortunately the position of the axis of rotation is, in general, unknown. The extreme modes of rotation for a long thin satellite spinning about its axis of maximum moment of inertia are (a) spinning propellerwise and (b) tumbling end-over-end. Since the axis of rotation may be inclined at any angle to the direction of motion, it is best to take for the drag the mean of its values under modes of rotation (a) and (b).

Many spinning satellites are approximately circular cylinders and this is the only shape which need be considered in detail. The drag under mode (a) has already been given in the previous section; it remains to consider the drag under mode (b).

Using the equations given by Schamberg⁵ for the forces acting on an element of area, the drag coefficient of a circular cylinder tumbling end-over-end is readily shown to be given by

$$C_D = 2 \left\{ 1 + \frac{\pi^2 (\ell + d)}{6 (4\ell + \pi d)} r \right\} \quad (10)$$

for diffuse re-emission. Here ℓ is the length of the cylinder and d is its diameter. The reference area, which is taken as the mean of the projected area perpendicular to the direction of motion, is given by

$$S = \frac{2}{\pi} \left(\ell d + \frac{\pi d^2}{4} \right) . \quad (11)$$

The drag coefficient given by equation (10) is almost independent of the length/diameter ratio as can be seen from Fig.8. It is worth noting that the drag coefficient of a rotating body is close to the value for a sphere if the mean projected area is taken as the reference area.

6 DRAG AT LOW MOLECULAR SPEED RATIOS

As the altitude increases above about 800 km the most probable molecular speed becomes so large that the random thermal motion of the atmospheric molecules must be considered. The drag produced by impinging molecules is readily calculated, but, unless specular reflexion occurs, the drag due to the re-emitted molecules is not so readily obtained.

Here we consider only spheres, for which the drag due to the incident molecules only is the same as the total drag with specular reflexion. If i refers to incident molecules and S to specular reflexion, the corresponding drag coefficients are given by

$$C_{D_i} = C_{D_s} = 2 \left(1 + \frac{1}{s^2} - \frac{1}{4s^4} \right) \operatorname{erf}(s) + \frac{2s^2 + 1}{\sqrt{\pi} s^3} \exp(-s^2) \quad (12)$$

When considering drag with diffuse re-emission most authors^{1,23,24} assume that the most probable re-emission speed v_r is constant over the entire surface. The contribution of the re-emitted molecules to the drag coefficient is then given by $2\sqrt{\pi}/3s_r$ so that the total drag is

$$C_D = C_{D_i} + \frac{2\sqrt{\pi}}{3s_r} \quad (13)$$

Although the assumption of constant re-emission speed is valid for complete accommodation, it does not apply for poor accommodation; so, for molecular speed ratios above one, the molecular speed ratio of re-emission will be much larger at the rear than at the front.

With present knowledge of gas-surface interactions, detailed calculations allowing for variations in the re-emission speed over the surface are not justified. For accommodation coefficients near zero and molecular speed ratios greater than one, it is at present preferable to assume one re-emission speed ratio s_r for all surface elements at the front and another re-emission speed ratio s'_r for all surface elements at the rear. It can then be shown that the drag coefficient due to diffuse re-emission C_{D_d} is given by

$$C_{D_d} = \frac{\sqrt{\pi}}{3} \left(\frac{1}{s_r} + \frac{1}{s'_r} \right) + \frac{1}{3} \left\{ \sqrt{\pi} \operatorname{erf}(s) + \frac{1}{2s^3} \left[1 + (2s^2 - 1) \exp(-s^2) \right] \left(\frac{1}{s_r} - \frac{1}{s'_r} \right) \right\}, \quad (14)$$

so that the total drag coefficient is

$$C_D = C_{D_i} + C_{D_d} \quad (15)$$

We can calculate what is probably the largest possible value of C_D by taking $s_r = 1$ and $s'_r = s$. This means that we are assuming the molecules incident on the front are re-emitted with a speed equal to the satellite's speed, and the molecules incident on the rear are re-emitted at a speed corresponding to the ambient atmospheric temperature. Fig.9 indicates that a practical upper

limit to the drag coefficient is probably best estimated by drawing a curve from the hyperthermal limit at $s = 6$ to the upper limit for the drag at $s = 1$.

The lowest curve in Fig.9 gives the drag coefficient of a sphere for either specular reflexion or diffuse re-emission with $T_r/T = 0$, T_r being the temperature of re-emission. The drag with diffuse re-emission and complete accommodation can be obtained for any value of $\sqrt{T_r/T}$ by linear interpolation between the three lowest curves. The drag coefficient for diffuse re-emission and incomplete accommodation can be obtained by interpolating between the practical upper limit discussed above and the appropriate value for complete accommodation, bearing in mind the variation of C_D with α for hyperthermal free-molecule flow given in Fig.6.

For values of s near and below unity the most probable molecular speed is close to the local escape speed and the assumption of a Maxwellian velocity distribution in the free stream is no longer justified. For a circular orbit the drag coefficient should not be taken from Fig.9 for molecular speed ratios less than about 1.5.

7 CHARGE DRAG

A satellite moving in the upper atmosphere can acquire an electric charge and there will then be a contribution to the drag from the interaction of the vehicle with the ambient ions. Several analyses of charge drag have been made and many conflicting results obtained. On the basis of the present evidence^{25,26}, charge drag appears to be negligible up to at least the altitude where helium becomes the main constituent. This occurs near 550 km at night when solar activity is low and at much higher altitudes at other times. Charge drag may still be negligible at much greater altitudes, however.

An experimental investigation²⁷ of the drag of six tin dipoles, 34 cm long and 0.043 cm in diameter, at an altitude of about 3000 km has shown that the effect of charge drag on small satellites at this altitude is not catastrophic, as has sometimes been predicted, although it may exceed the neutral particle drag. The smallness of the electric drag has been confirmed by observation of the West Ford test belt²⁸.

8 DISCUSSION

In the light of the discussion on surface interaction in section 3, it is evident that there is still considerable uncertainty in the estimation of satellite drag coefficients. There is now ample evidence, both theoretical and experimental, that low values of accommodation coefficient are appropriate

for light gases incident on heavy surface atoms and α can be taken to increase from zero for a mass ratio μ of zero to a value near unity for a mass ratio of 1. We have seen that the mass ratio depends on the outermost layer of atoms on the surface and on the ambient molecular weight. Most satellite surfaces, unless they are either gold plated or painted, will present mainly atoms of oxygen to the incident gas so that μ will be less than 1 when the ambient molecular weight is less than about 16. In the important region where the molecular weight exceeds 16 it is probably best, at present, to take the value of α appropriate to $\mu = 1$ for the majority of surfaces. This is because high energy collisions take place practically atom with atom. Also, since the dissociation energies of O_2 and N_2 molecules are 5.08 and 9.76 eV respectively, there will usually be some dissociation at low altitudes and this will have the effect of increasing the accommodation coefficient.

The exact distribution of the reflected molecules is unknown and it is probably best to assume diffuse re-emission until further experimental evidence is available. Fortunately, for accommodation coefficients near unity, the drag coefficient is not particularly sensitive to the distribution of the reflected molecules and little error will ensue from assuming an incorrect distribution. As the accommodation coefficient decreases, however, the drag becomes more sensitive to the distribution of the reflected molecules.

In the absence of a proper understanding of gas-surface interactions, it is only appropriate to use an elementary model to calculate drag. For satellites below 700 km the molecular speed ratio exceeds 5 and the random thermal motion of the atmospheric molecules can be ignored. The drag of various simple shapes can be obtained from the formulae in Table 1 or from Figs. 5 to 8. For purposes of illustration the variation of the drag coefficient of a sphere with altitude is shown in Fig. 10 for three variations of α with μ . The four curves for each variation of α correspond to the values of molecular weight shown in Fig. 4.

The accommodation coefficient at the surface of most satellites probably exceeds 0.8 at heights up to about 400 km, at all times of day and for all levels of solar activity. For a sphere the smallest value of the drag coefficient is then 2.07 and occurs when α is unity. A drag coefficient of 2.2, which is the value normally used in density determinations, corresponds to $\alpha \approx 0.95$: it must be emphasized that this value may well be too low, perhaps by as much as 10 per cent, especially if α decreases appreciably as the angle between the incident beam and the surface normal increases. Nevertheless the possible range of values of C_D given by the theory discussed above is insufficient to explain²⁹ the large discrepancy (a factor of 2) between the density values

measured³⁰ by ionization gauges aboard Explorer 17 (1963-09A) at a height of 280 km and those obtained from orbital studies.

At altitudes above 400 km the mass ratio μ can be quite small, particularly at times of low solar activity, and a somewhat higher value of C_D is then appropriate. The drag coefficient of any particular satellite can only be obtained after making a judicious choice of the various parameters involved. The recent discussion of satellite drag by Izakov³¹ is oversimplified, since the degree of energy transfer is assumed to be the same at all heights.

For satellites above 700 to 800 km it is necessary to take the thermal motion into account and the remarks made in section 6 must be borne in mind. The drag coefficients of spheres in high-altitude circular orbits are given in Table 2 for various values of exospheric temperature and two values of molecular weight. The values of C_D were obtained from Fig. 9 by interpolation, assuming $\alpha = 3.6\mu/(1 + \mu)^2$. C_D does not vary appreciably if the numerical factor in this equation is changed to 3.2 or 4. A temperature of 700°K corresponds to

Table 2

DRAG COEFFICIENTS OF SPHERES IN HIGH-ALTITUDE CIRCULAR ORBITS

Molecular weight	1		4		
Temperature (°K)	700	1000	1000	1500	2000
Height (km)					
1000	3.4	3.6	2.8		
2000	3.5	3.7	2.8	2.9	3.1
3000	3.5	3.8		3.0	3.1

very low solar activity³², when atomic hydrogen is likely to be the dominant constituent above about 800 km. A temperature of 2000°K corresponds to very high solar activity when there is a strong possibility that helium will be the dominant constituent above about 1500 km. The drag coefficients given in Table 2 apply if the surface atoms have a molecular weight of 16. When hydrogen is a major constituent these values of C_D serve as an upper limit, since for some surfaces the outmost atoms may not then be oxygen.

9 CONCLUSIONS

Between heights of 140 and 400 km at times of low solar activity, or heights of 140 and 600 km at times of high solar activity, the drag coefficient is almost independent of height, and at present for most satellites, because of

the lack of decisive evidence, there is perhaps not sufficient reason to abandon the value of 2.2 which has been widely used in recent years for the drag coefficient. It must be recognized, however, that this value is subject to some uncertainty and may be too low, perhaps by as much as 10 per cent.

At heights above 400 km (low solar activity) or 600 km (high solar activity) the drag coefficient increases with height, since both the energy transfer and the molecular speed ratio decrease as the molecular weight of the atmosphere decreases. It should be noted that the uncertainty in the accommodation coefficient is less when the molecular weight of the atmosphere is low. This leads to only a small spread in the possible values of drag coefficient in the 700-800 km region at times of low solar activity, as can be seen from Fig.10; under these conditions the drag coefficient can be predicted to within a few per cent.

Acknowledgment

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SYMBOLS

C_D	drag coefficient of satellite
C_{D_d}	contribution of diffusely re-emitted molecules to drag coefficient
C_{D_i}	contribution of incident molecules to drag coefficient
C_{D_s}	drag coefficient with specular reflexion
d	diameter
D	drag of satellite
E_i	average kinetic energy of incident molecules
E_r	average kinetic energy of re-emitted molecules
E_s	kinetic energy of a molecule re-emitted with a velocity corresponding to surface temperature
g	acceleration due to gravity
H	scale height
K	Knudsen number
l	length
L	typical linear dimension
M	molecular weight of atmosphere
n_0	number density in free stream
r	$= \frac{v_r}{v_i}$
R	gas constant
s	molecular speed ratio
s_r	molecular speed ratio of re-emission
S	representative cross-sectional area of satellite
T	atmospheric temperature
T_i	kinetic temperature of incident molecules
T_r	temperature of re-emission
T_w	temperature of surface
v_i	speed of incident molecule
v_m	most probable molecular speed
v_r	speed of re-emission
V	velocity of satellite
α	accommodation coefficient
θ	angle of incidence for a flat plate

SYMBOLS (Contd)

λ	mean free path near front surface
λ_0	mean free path in free stream
μ	ratio of mass of gas atom to mass of surface atom
ρ	air density
ϕ	see section 3.2
ψ	semi vertex angle of cone

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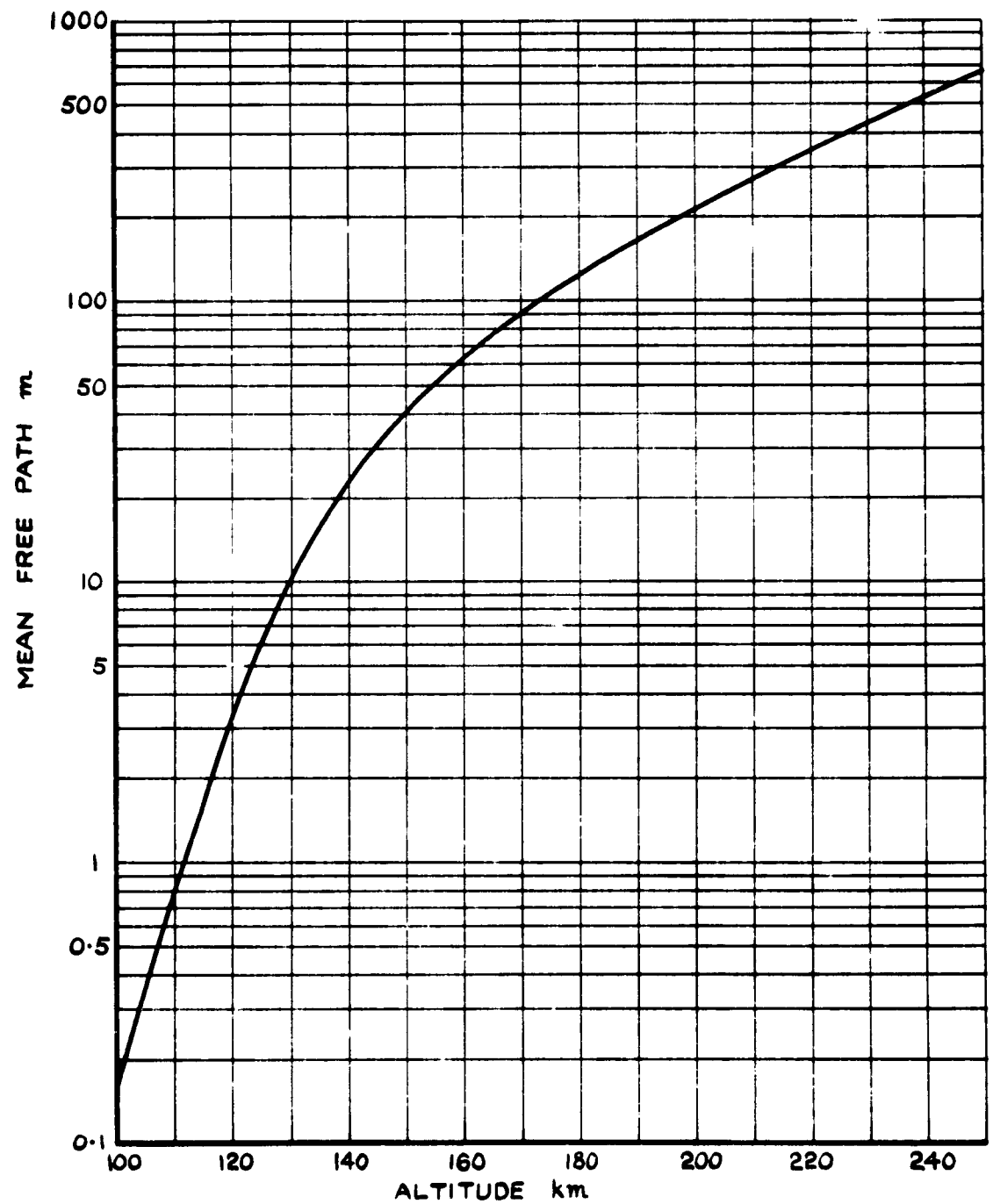


FIG. 1 MEAN FREE PATH FOR HEIGHTS BETWEEN 100 AND 250 km, AS GIVEN BY THE U.S. STANDARD ATMOSPHERE (1962)

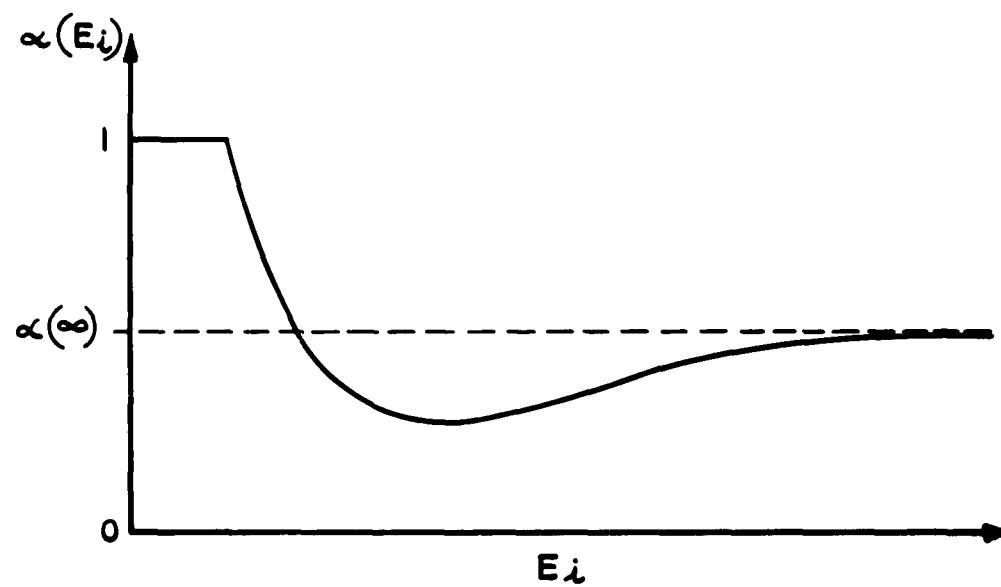


FIG.2 FORM OF THE VARIATION OF ACCOMMODATION COEFFICIENT WITH INCIDENT ENERGY (AFTER GOODMAN⁹)

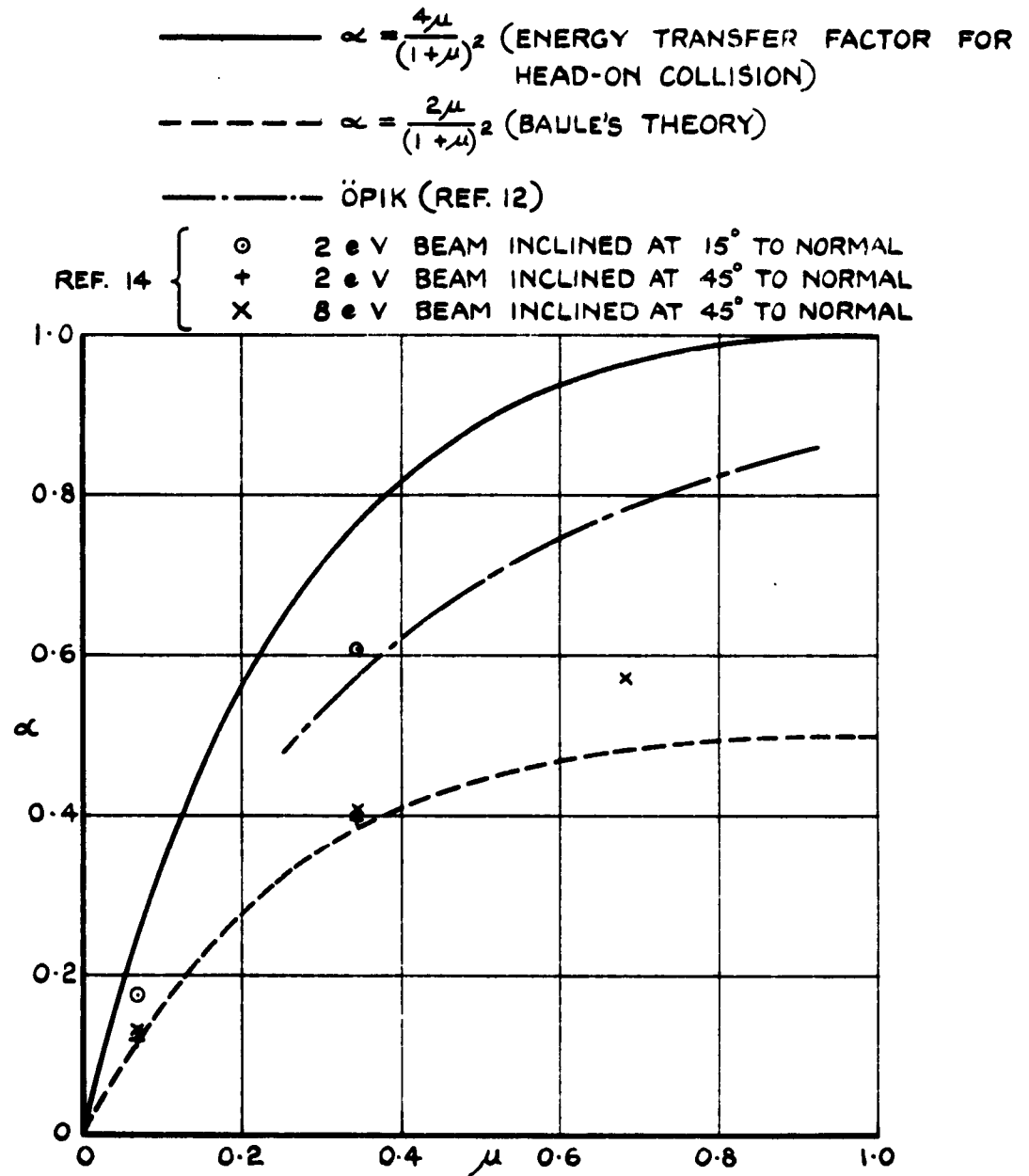


FIG.3 VARIATION OF ACCOMMODATION COEFFICIENT α WITH MASS RATIO μ , AS GIVEN BY VARIOUS THEORIES

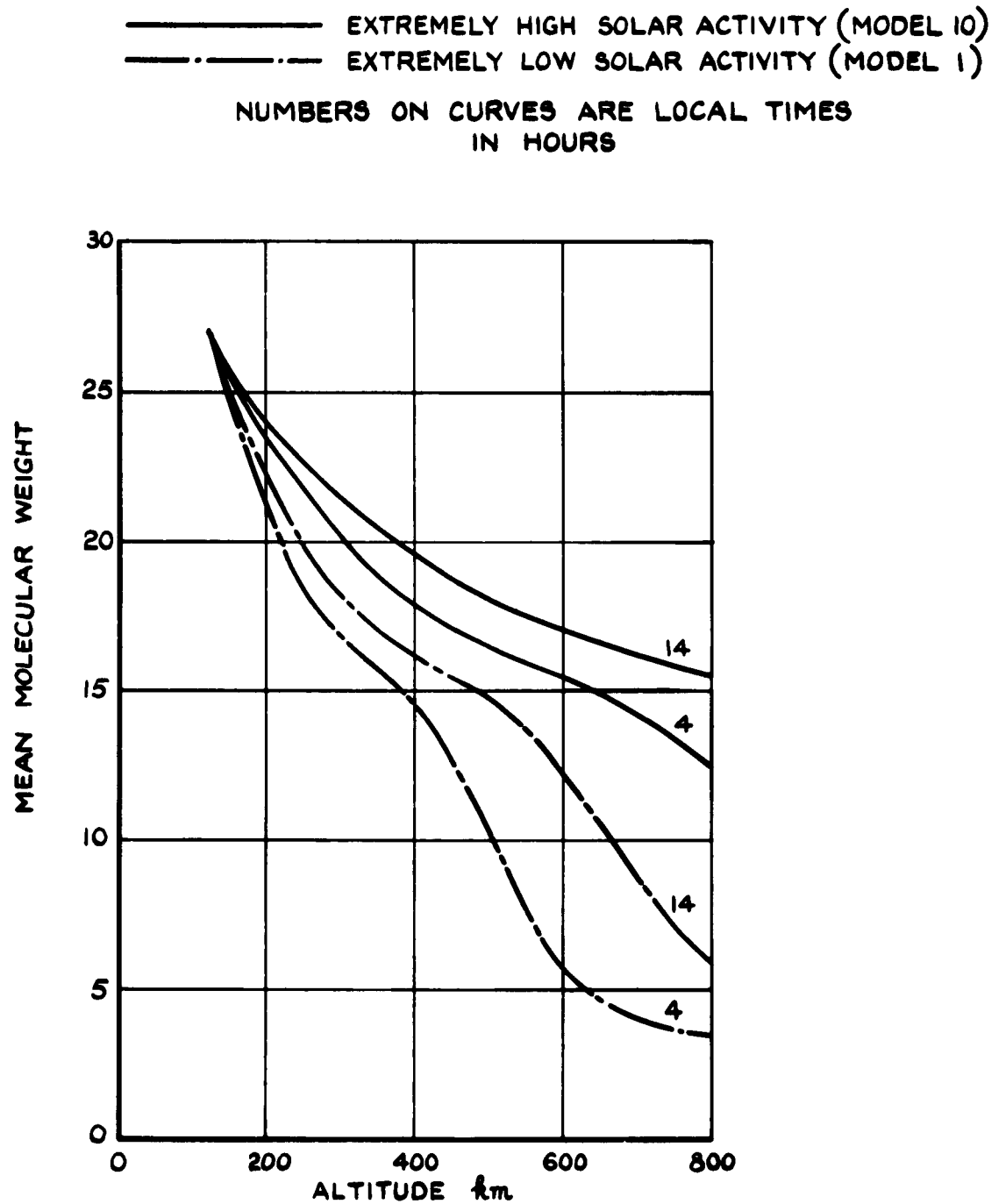


FIG. 4 EXTREME VALUES OF THE MOLECULAR WEIGHT OF
THE ATMOSPHERE AS GIVEN BY CIRA (1964)

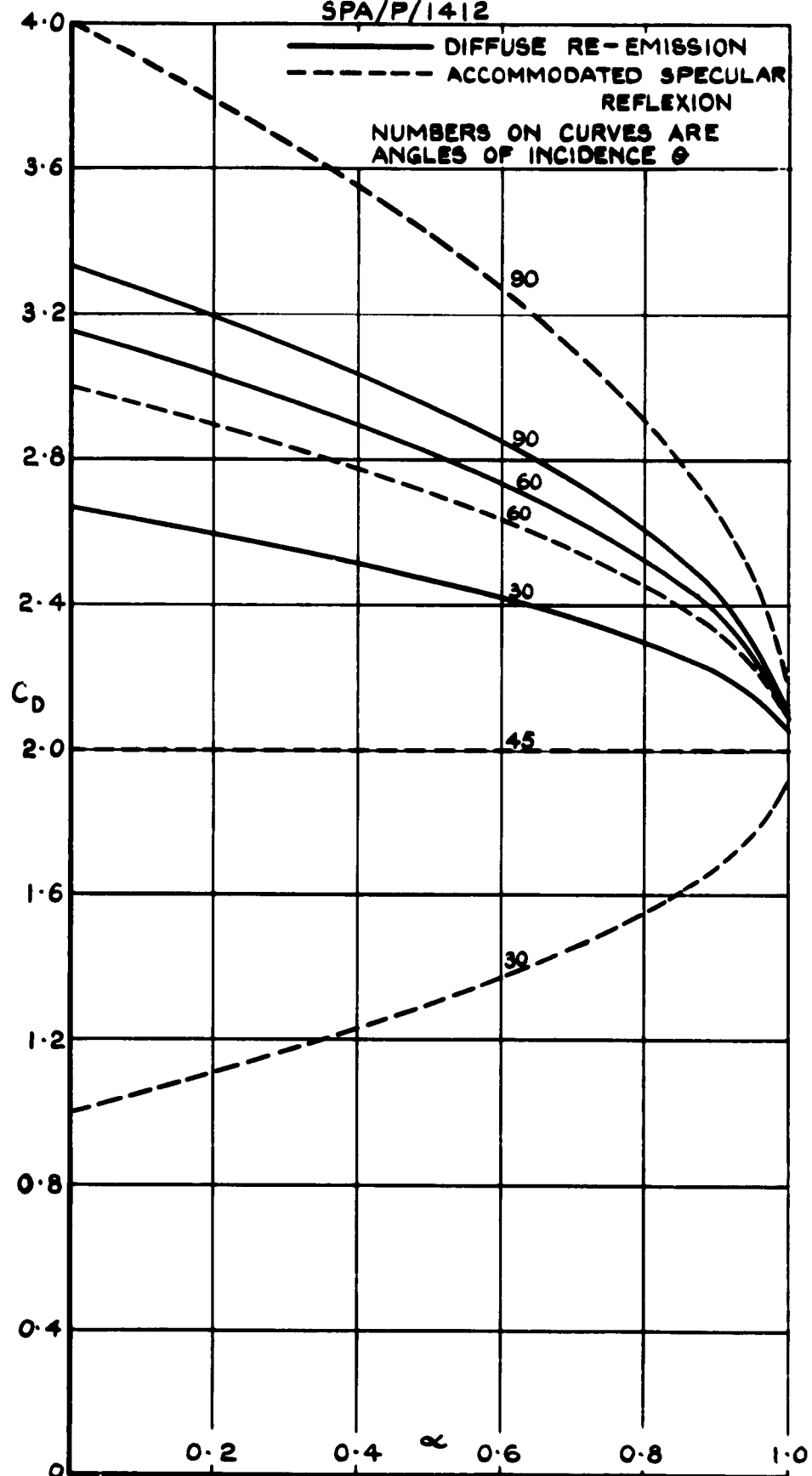


FIG.5 DRAG COEFFICIENT OF A FLAT PLATE IN HYPERThermal FREE-MOLECULE FLOW. THE DRAG COEFFICIENT IS BASED ON THE PROJECTED AREA PERPENDICULAR TO THE DIRECTION OF MOTION. ($\frac{T_w}{T_i} = 0.006$)

Fig.6

SPA/P/1413

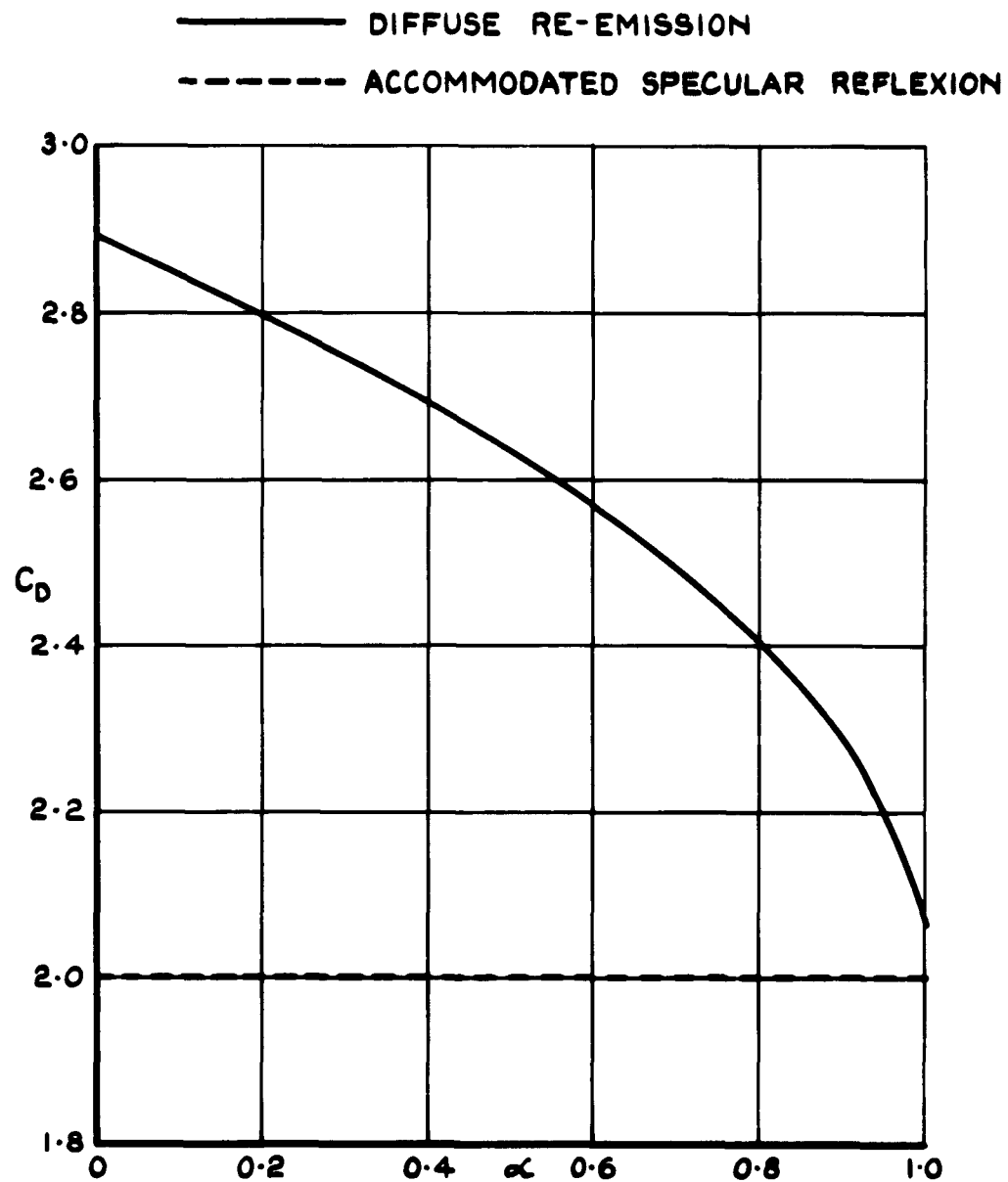


FIG. 6 DRAG COEFFICIENT OF A SPHERE IN HYPERTHERMAL
FREE-MOLECULE FLOW. $\left(\frac{T_w}{T_i} = 0.006\right)$

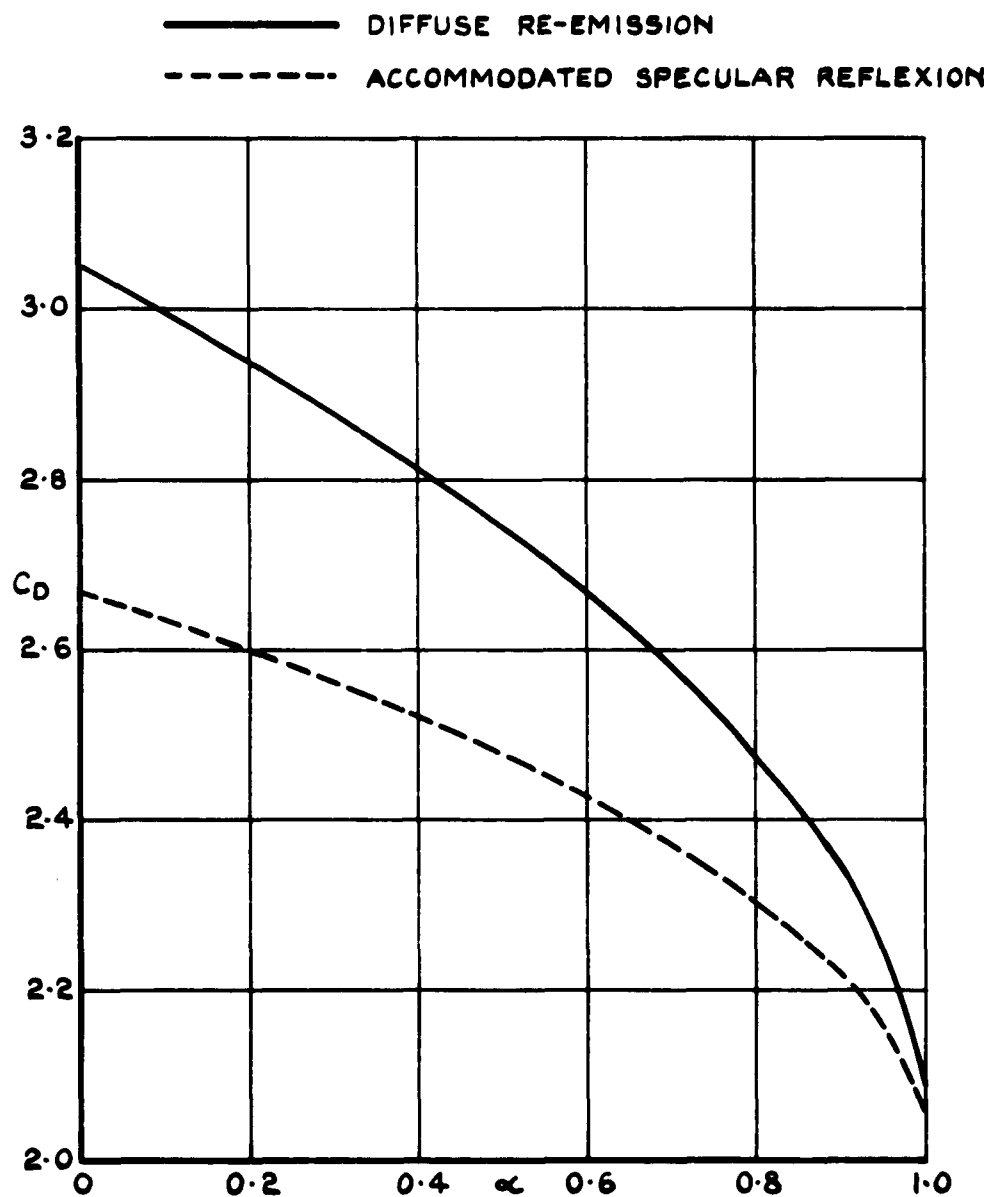


FIG.7 DRAG COEFFICIENT OF AN INFINITE CIRCULAR CYLINDER WITH AXIS PERPENDICULAR TO DIRECTION OF MOTION, IN HYPERTHERMAL FREE-MOLECULE FLOW. $\left(\frac{T_w}{T_i} = 0.006\right)$

NUMBERS ON CURVES ARE VALUES
OF LENGTH/DIAMETER.

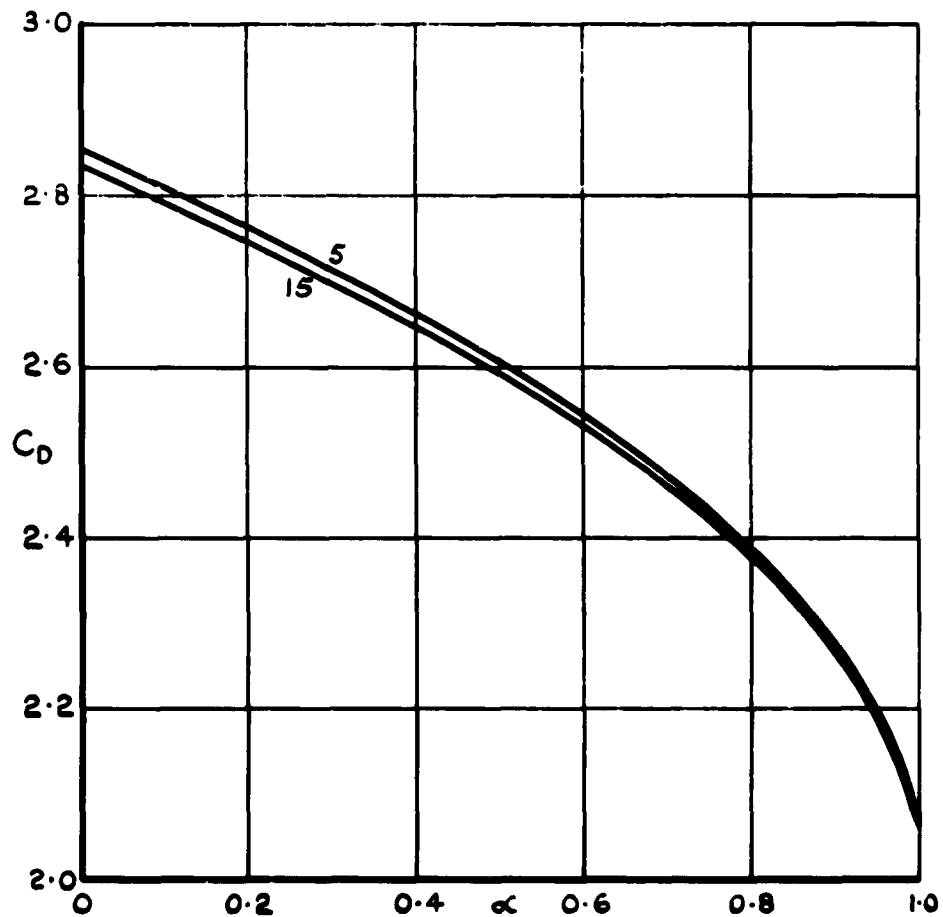


FIG.8 DRAG COEFFICIENT OF A CIRCULAR CYLINDER TUMBLING
END-OVER-END, WITH DIFFUSE RE-EMISSION. THE DRAG
COEFFICIENT IS BASED ON THE AVERAGE PROJECTED
AREA PERPENDICULAR TO THE DIRECTION OF MOTION.

$$\left(\frac{T_w}{T_i} = 0.006\right)$$

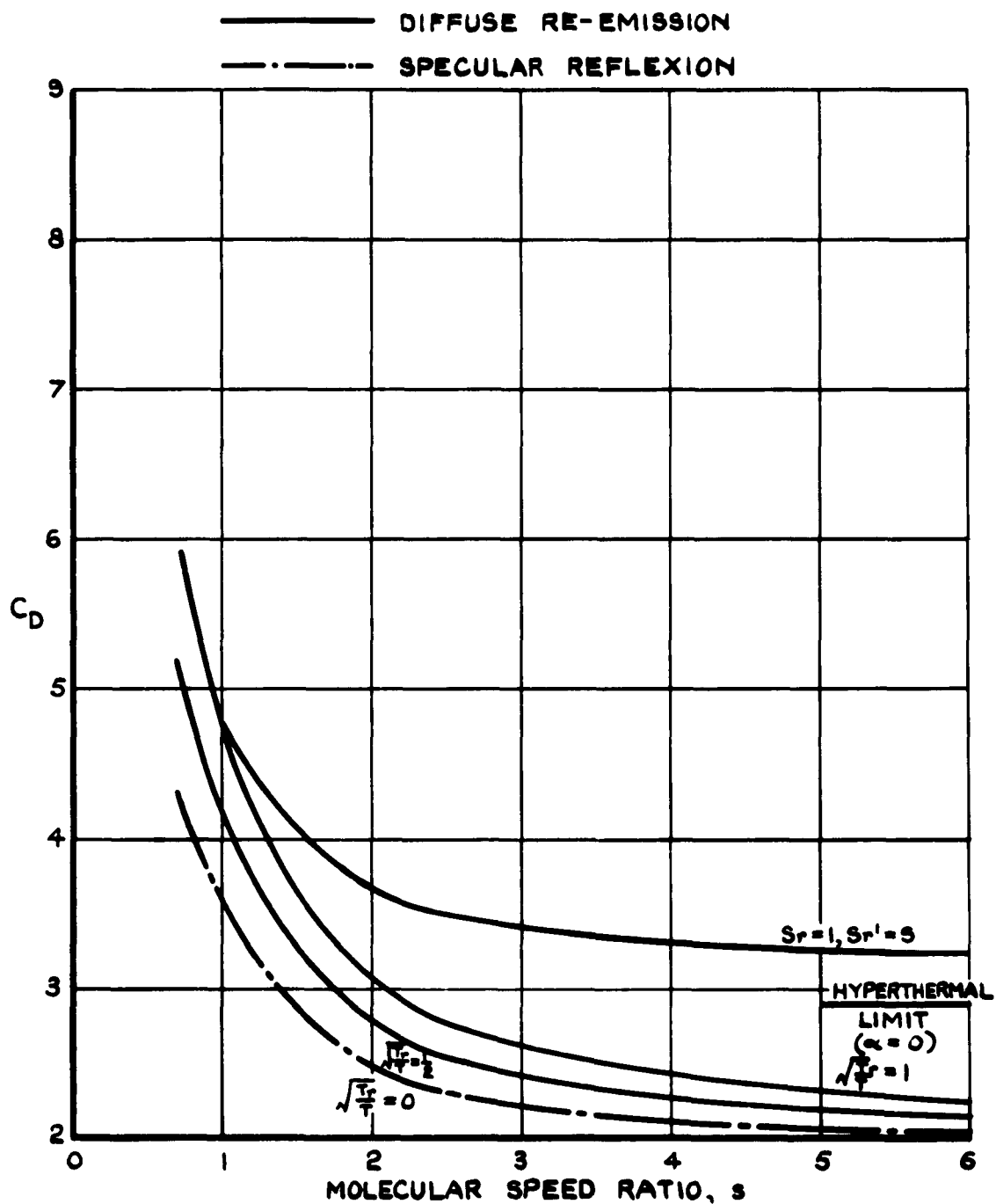


FIG. 9 VARIATION OF THE DRAG COEFFICIENT OF A SPHERE WITH THE MOLECULAR SPEED RATIO

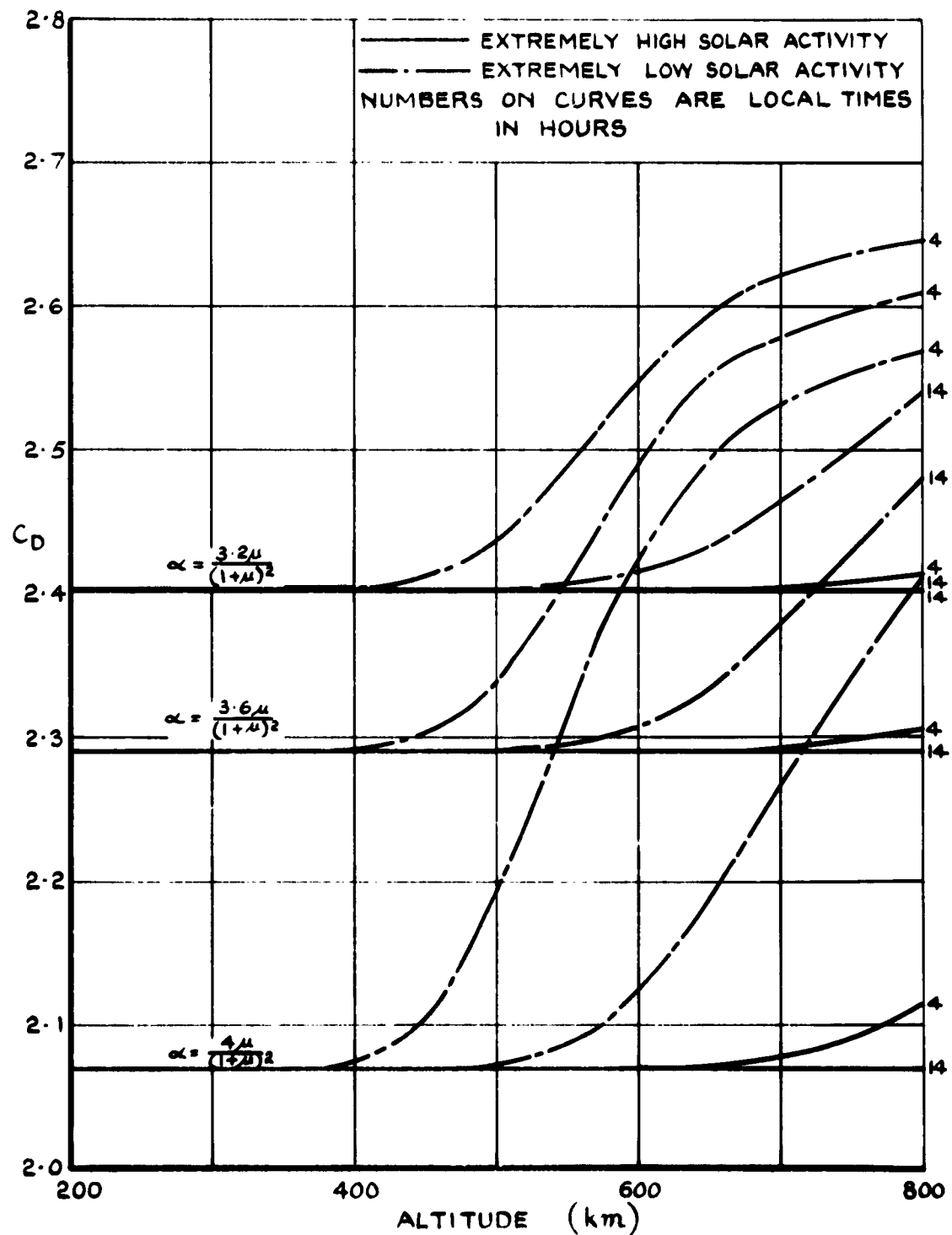


FIG.10 VARIATION OF THE DRAG COEFFICIENT OF A SPHERE WITH ALTITUDE ASSUMING HYPERTHERMAL FREE-MOLECULE FLOW

Cook, G.E. 533.6.013.12 : 629.195 SATELLITE DRAG COEFFICIENTS Royal Aircraft Establishment Technical Report 65005 January 1965 The drag of artificial earth satellites is reconsidered in the light of recent studies of gas-surface interactions and atmospheric composition. Between heights of 140 and 400 km at times of low solar activity, or heights of 140 and 600 km at times of high solar activity, the drag coefficient is almost independent of height, and at present, because of the lack of decisive evidence, there is perhaps not sufficient reason to abandon the value of 2.2 which has been widely used in recent years for the drag coefficient. It must be recognized, however, that this value is subject to some uncertainty and may be too low, perhaps by as much as 10 per cent. At heights above 400 km (low solar activity) or 600 km (high solar activity) the drag coefficient increases with height, since both the degree of energy transfer and the molecular speed ratio decrease as the molecular weight of the atmosphere decreases.	Cook, G.E. 533.6.013.12 : 629.195 SATELLITE DRAG COEFFICIENTS Royal Aircraft Establishment Technical Report 65005 January 1965 The drag of artificial earth satellites is reconsidered in the light of recent studies of gas-surface interactions and atmospheric composition. Between heights of 140 and 400 km at times of low solar activity, or heights of 140 and 600 km at times of high solar activity, the drag coefficient is almost independent of height, and at present, because of the lack of decisive evidence, there is perhaps not sufficient reason to abandon the value of 2.2 which has been widely used in recent years for the drag coefficient. It must be recognized, however, that this value is subject to some uncertainty and may be too low, perhaps by as much as 10 per cent. At heights above 400 km (low solar activity) or 600 km (high solar activity) the drag coefficient increases with height, since both the degree of energy transfer and the molecular speed ratio decrease as the molecular weight of the atmosphere decreases.
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