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TECHNICAL REPORT 231-6

SURFACE CURRENTS DUE TO A SUBMERGED DISTURBANCE IN A STRATIFIED OCEAN

By

K. K. Wong

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NOTATION

A	Source strength
b	Exponent of density stratification
f	Depth of submergence of disturbances
g	Acceleration due to gravity
H	Depth of thermocline
M	Quadrupole strength
N	Vaisala frequency
t	Time
u,v	Velocity components in x,y direction respectively
U,V	Transformed velocity components
x,y	Cartesian coordinates
α	Horizontal component of wave number
ω	Frequency
$\bar{\rho}$	Equilibrium density
τ	Thickness of thermocline
$\delta(x)$	Dirac delta function
φ	Fourier transform of V

INTRODUCTION

A submerged disturbance in a stratified ocean will, in general, generate motions that are quite different from those generated in a homogeneous ocean. By stratification we mean a density stratification due to either the presence of a salinity or temperature gradient or both, the fluid being assumed to be incompressible. The difference is not simply a modification of the motion by the density stratification, which is slight in a real ocean, but rather the appearance of other forms of motion whose existence depends entirely on the density stratification. The difference, then, is one of kind and not simply one of degree.

In this report, the surface currents, i.e., currents at the air-sea interface due to a two-dimensional submerged disturbance, are considered. The equilibrium density profile of the ocean is assumed to be of the form shown in Figure 1a. This profile is chosen for its simplicity and also for its close approximation to reality. The disturbance is assumed to be always below the thermocline: It is a quadrupole of constant strength amplitude and with a simple harmonic time dependency. In spite of its simplicity it provides a time scale (inverse frequency) and length scale (proportional to its depth below the thermocline) characterizing the disturbance.

In the Appendix similar calculations are carried out for another class of density profiles shown schematically in Figure 1b.

FORMULATION OF THE PROBLEM

The equilibrium density profile of the ocean is taken to be of the following form:

$$\bar{\rho} = \begin{cases} \rho_1 & \text{in } R_o: \ (f + H > y > f + \tau) \\ \rho_2 e^{-2b_1(y-f)} & \text{in } R_1: \ (f + \tau > y > f) \\ \rho \ e^{-2b_2 y} & \text{in } R_2: \ (f > y > -\infty) \end{cases}$$

The fluid is assumed to be inviscid and incompressible. In the above, ρ, ρ_1, ρ_2, f, τ and H are all positive real constants. A schematic diagram of the profile is shown in Figure 1a.

We are interested in the currents at the free surface $y = f + H$ generated by a two-dimensional quadrupole of constant strength amplitude M and with a simple harmonic time dependency of frequency ω located at the origin of the coordinate system $(x = 0, y = 0)$. The solution for a periodic source of strength A is found first. From this solution, the one for a quadrupole is obtained by differentiation.

It is convenient to work with the transformed velocity components U and V defined by

$$U = \bar{\rho}^{\frac{1}{2}} u$$

$$V = \bar{\rho}^{\frac{1}{2}} v$$

where

u, v are the physical velocity components.

Since the forcing function is simple harmonic, so, after a sufficient time has elapsed for the transients to die away, the whole system would acquire a simple harmonic motion of the same frequency. In this final quasi-steady state, the time dependency of the dependent variables may be factored out so that

$$U(x,y,t) = \overline{U}(x,y)e^{-i\omega t}$$

$$V(x,y,t) = \overline{V}(x,y)e^{-i\omega t}$$

If the disturbance is weak, so that the resulting motion may be considered as small perturbations about the state of hydrostatic equilibrium, then the differential equations governing $\overline{V}$ in the different regions are (see Wong, 1965)

$$\text{in } R_o \qquad \frac{\partial^2 \overline{V}_o}{\partial y^2} + \frac{\partial^2 \overline{V}_o}{\partial x^2} = 0 \qquad [1]$$

$$\text{in } R_1 \qquad \frac{\partial^2 \overline{V}_1}{\partial y^2} + \left(1 - \frac{2gb_1}{\omega^2}\right) \frac{\partial^2 \overline{V}_1}{\partial x^2} - b_1{}^2 \overline{V}_1 = 0 \qquad [2]$$

$$\text{in } R_2 \qquad \frac{\partial^2 \overline{V}_2}{\partial y^2} + \left(1 - \frac{2gb_2}{\omega^2}\right) \frac{\partial^2 \overline{V}_2}{\partial x^2} - b_2{}^2 \overline{V}_2$$

$$= 2\pi \sqrt{\rho}\, A\, \delta(x) \, [\dot{\delta}(y) - b_2 \delta(y)] \qquad [3]$$

where δ denotes the Dirac delta function and a dot denotes a differentiation. Subscripts have been added to $\overline{V}$ to denote the function in the various regions. The right hand side of Equation [3] represents the source.

The surface current is essentially due to the formation and propagation of internal waves. Because of the much larger density discontinuity at the air-sea interface the vertical movements there are negligible for all practical purposes. Therefore, the boundary condition there can be taken to be

$$\overline{V}_o = 0 \quad \text{at } y = f + H \tag{4}$$

At $y = f$ and $y = f + \tau$ both $\overline{U}$ and $\overline{V}$ have to be continuous. This gives rise to the following boundary conditions:

$$\overline{V}_o = \overline{V}_1 \qquad \text{at } y = f + \tau \tag{5}$$

$$\overline{V}_1 = \overline{V}_2 \qquad \text{at } y = f \tag{6}$$

$$\frac{\partial \overline{V}_o}{\partial y} = b_1 \overline{V}_1 + \frac{\partial \overline{V}_1}{\partial y} \qquad \text{at } y = f + \tau \tag{7}$$

$$b_1 \overline{V}_1 + \frac{\partial \overline{V}_1}{\partial y} = b_2 \overline{V}_2 + \frac{\partial \overline{V}_2}{\partial y} \qquad \text{at } y = f \tag{8}$$

The last two equations were obtained from the equation of continuity, viz.,

$$\frac{\partial \overline{U}}{\partial x} + \frac{\partial \overline{V}}{\partial y} + b\,\overline{V} = 2\pi\sqrt{\rho}\;A\;\delta(x)\;\delta(y)$$

which becomes

$$\frac{\partial \overline{U}_o}{\partial x} + \frac{\partial \overline{V}_o}{\partial y} = 0 \qquad \text{in } R_o \qquad [9]$$

$$\frac{\partial \overline{U}_1}{\partial x} + \frac{\partial \overline{V}_1}{\partial y} + b_1\overline{V}_1 = 0 \qquad \text{in } R_1 \qquad [10]$$

$$\frac{\partial \overline{U}_2}{\partial x} + \frac{\partial \overline{V}_2}{\partial y} + b_2\overline{V}_2 = 2\pi\sqrt{\rho}\;A\;\delta(x)\;\delta(y) \qquad \text{in } R_2 \qquad [11]$$

In addition to the above conditions other conditions have to be imposed at infinity to insure a unique solution. These additional conditions will be discussed as they arise.

SURFACE CURRENTS DUE TO A SUBMERGED SOURCE

Let φ_i denote the Fourier transform of $\overline{V}_i$ with respect to x so that

$$\varphi_i = \int_{-\alpha}^{+\infty} \overline{V}_i\, e^{-i\alpha x} dx \qquad , \qquad i = 0,\ 1,\ 2.$$

Then, from the equations governing $\overline{V}_1$ the following set of equations for φ_1 is obtained:

$$\ddot{\varphi}_o - \alpha^2 \varphi_o = 0 \qquad \text{for } f + H > y > f + \tau \qquad [12]$$

$$\ddot{\varphi}_1 - \left[b_1{}^2 + \left(1 - \frac{2gb_1}{\omega^2}\right) \alpha^2 \right] \varphi_1 = 0 \qquad \text{for } f + \tau > y > f \qquad [13]$$

$$\ddot{\varphi}_2 - \left[b_2{}^2 + \left(1 - \frac{2gb_2}{\omega^2}\right) \alpha^2 \right] \varphi_2 = 2\pi A \sqrt{\rho} \left[\mathring{\delta}(y) - b\delta(y) \right] \text{ for } y < f \qquad [14]$$

$$\varphi_o = 0 \qquad \text{at } y = f + H \qquad [15]$$

$$\varphi_o = \varphi_1 \qquad \text{at } y = f + \tau \qquad [16]$$

$$\varphi_1 = \varphi_2 \qquad \text{at } y = f \qquad [17]$$

$$\dot{\varphi}_o = b_1 \varphi_1 + \dot{\varphi}_1 \qquad \text{at } y = f + \tau \qquad [18]$$

$$b_1 \varphi_1 + \dot{\varphi}_1 = b_2 \varphi_2 + \ddot{\varphi}_2 \text{ at } y = f \qquad [19]$$

The most general solutions of Equations [12] to [14] are respectively,

$$\varphi_o = A_o e^{-|\alpha| y} + B_o e^{+|\alpha| y} \qquad [20]$$

$$\varphi_1 = A_1 e^{-\beta_1 y} + B_1 e^{+\beta_1 y} \qquad [21]$$

$$\varphi_2 = A_2 e^{-\beta_2 y} + B_2 e^{+\beta_2 y} + H(y)\, \pi A \sqrt{\rho} \left(\frac{b_2}{\beta_2} + 1 \right) e^{-\beta_2 y}$$

$$+ [1 - H(y)] \pi A \sqrt{\rho} \left(\frac{b_2}{\beta_2} - 1 \right) e^{+\beta_2 y} \qquad [22]$$

where A_i, B_i, are arbitrary functions of the parameter α, $H(y)$ denotes the Heaviside function and β_1, β_2 are given by

$$\beta_1 = + \sqrt{b_1{}^2 + \left(1 - \frac{2gb_1}{\omega^2} \right) \alpha^2}$$

$$\beta_2 = + \sqrt{b_2{}^2 + \left(1 - \frac{2gb_2}{\omega^2} \right) \alpha^2}$$

Because of Equation [15], φ_o must be of the form

$$\varphi_o = A_o \sinh |\alpha| \, [(f + H) - y] \qquad [23]$$

A_o now is some other arbitrary function of α. Because of conditions at $y \to -\infty$, $A_2 \equiv 0$ and Equation [22] becomes

$$\varphi_2 = B_2 e^{+\beta_2 y} + H(y)\ \pi A \sqrt{\rho} \left(\frac{b_2}{\beta_2} + 1 \right) e^{-\beta_2 y}$$

$$+ [1 - H(y)]\ \pi A \sqrt{\rho} \left(\frac{b_2}{\beta_2} - 1 \right) e^{+\beta_2 y} \qquad [24]$$

Substituting known expressions for φ_1 into Equations [16] to [19] yield the following set of four equations for the determination of A_o, A_1, B_1 and B_2:

$$A_o \sinh|\alpha|\ (H - \tau) = A_1 e^{-\beta_1 (f+\tau)} + B_1 e^{+\beta_1 (f+\tau)}$$

$$-|\alpha| A_o \cosh|\alpha|\ (H - \tau) = A_1 (b_1 - \beta_1) e^{-\beta_1 (f+\tau)} + B_1 (b_1 + \beta_1) e^{\beta_1 (f+\tau)}$$

$$B_2 e^{\beta_2 f} + \pi A \sqrt{\rho} \left(\frac{b_2}{\beta_2} + 1 \right) e^{-\beta_2 f} = A_1 e^{-\beta_1 f} + B_1 e^{+\beta_1 f}$$

$$B_2 (b_2 + \beta_2) e^{\beta_2 f} + (b_2 - \beta_2)\ \pi A \sqrt{\rho} \left(\frac{b_2}{\beta_2} + 1 \right) e^{-\beta_2 f} = A_1 (b_1 - \beta_1) e^{-\beta_1 f}$$

$$+ B_1 (b_1 + \beta_1) e^{+\beta_1 f}$$

Solving the above equations simultaneously, A_o is found to be:

$$A_o = - \frac{4\pi A \sqrt{\rho}\ (b_2 + \beta_2) e^{-\beta_2 f}}{(\xi+\eta)(b_1 - b_2 - \beta_1 - \beta_2) e^{+\beta_1 \tau} + (\xi - \eta)(b_1 - b_2 + \beta_1 - \beta_2) e^{-\beta_1 \tau}}$$

[25]

where

$$\xi = \sinh |\alpha| \; (H - \tau)$$

$$\eta = \frac{1}{\beta_1} \left[\; |\alpha| \; \cosh |\alpha| \; (H - \tau) + b_1 \; \sinh |\alpha| \; (H - \tau) \right]$$

Taking the inverse transform, V_o is expressed as a Fourier integral

$$\bar{V}_o = \frac{1}{2\pi} \int_{-\infty}^{+\infty} A_o \sinh|\alpha| \; [(f + H) - y] e^{i\alpha x} \, d\alpha \qquad [26]$$

$\bar{U}_o$ can be found from Equation [9] as follows:

$$\frac{\partial \bar{U}_o}{\partial x} = - \frac{\partial \bar{V}_o}{\partial y}$$

$$= + \frac{1}{2\pi} \int_{-\infty}^{+\infty} A_o |\alpha| \cosh|\alpha| \; [(f + H) - y] e^{i\alpha x} \, d\alpha$$

$$\bar{U}_o = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} A_o \frac{|\alpha|}{\alpha} \cosh|\alpha| \; [(f + H) - y] e^{i\alpha x} \, d\alpha \quad [27]$$

In arriving at the last expression for $\bar{U}_o$ the fact that $\bar{U}_o$ is antisymmetric in x was used.

The surface current induced by the source is, then, given by

$$\tilde{u}_o = \frac{e^{-i\omega t}}{2\pi i\sqrt{\rho_1}} \int_{-\infty}^{+\infty} A_o \frac{|\alpha|}{\alpha} e^{i\alpha x} d\alpha \qquad [28]$$

In what follows we shall always assume that

$$\epsilon = \frac{\rho_2 - \rho_1}{\rho_2} << 1$$

First consider the simplest special case where the thermocline is a discontinuity ($\tau = 0$) and the layer below the thermocline is homogeneous ($b_2 = 0$, $\rho = \rho_2$). For this special case

$$A_o = \frac{2\pi A \sqrt{\rho_2}\ \alpha\ e^{-|\alpha| f}}{|\alpha|\ e^{+|\alpha| H} - \frac{g\epsilon\alpha^2}{\omega^2} \sinh|\alpha| H}$$

Therefore

$$\tilde{u}_o = - i A e^{-i\omega t} \int_{-\infty}^{+\infty} \frac{\frac{\omega^2 |\alpha|}{\alpha} e^{-|\alpha|(f+H)} e^{i\alpha x} d\alpha}{\omega^2 - g\epsilon|\alpha| e^{-|\alpha| H} \sinh|\alpha| H} \qquad [29]$$

Since the integrand has poles on the real axis, the integral is undefined. Under such circumstances it is usual to deform the path of integration so that it goes around instead of through the poles. How the path is to be deformed depends on additional physical information. Here, the surface current is a consequence of the formation and propagation of interfacial waves at the thermocline. These waves are all outgoing waves from the central region of disturbances. Therefore, the current also possesses this outgoing wave property. For such a situation the path has to be deformed slightly above the real axis when $x < 0$ and slightly below when $x > 0$. The path of integration can be closed by a large semicircle in the upper half plane when $x > 0$ and the lower half plane when $x < 0$. It can easily be shown that the contribution from the integration along the semicircle vanishes as the radius tends to infinity. Since the integrand is analytic except for poles, the integral may be evaluated by the method of residues. The contributions from the poles having a non-vanishing imaginary part will vanish exponentially with x because of the factor $e^{i\alpha x}$. Therefore, for x large and positive only the positive real roots of the equation

$$\omega^2 - g\varepsilon\alpha\, e^{-\alpha H} \sinh \alpha H = 0 \qquad [30]$$

are important. This equation is plotted in Figure 2. It is seen that for a given ω there is only one real positive α, say, α_o which satisfies the above equation.

Hence, for large and positive x

$$\tilde{u}_o \sim -\pi A \frac{\omega e^{-\alpha_o(f+H)}}{\left(\frac{d\sigma}{d\alpha}\right)_{\alpha_o}} e^{i(\alpha_o x - \omega t)} \qquad [31]$$

where

$$\sigma^2 = g\epsilon |\alpha| e^{-|\alpha|H} \sinh|\alpha|H$$

and α_o is the positive real solution of Equation [30].

Next consider the effects of a thin thermocline on the surface currents generated by the source, i.e., τ is small compared with H but finite. As in the previous case b_2 is set equal to zero. Now

$$b_1 = \frac{1}{2\tau} \ln (1 + \epsilon) \approx \frac{\epsilon}{2\tau}$$

therefore

$$\beta_1 \tau = \tau \sqrt{b_1^2 + \left(1 - \frac{2gb_1}{\omega^2}\right) \alpha^2}$$

$$= \sqrt{\frac{\epsilon^2}{4} + \alpha^2 \tau^2 - g \epsilon \tau \frac{\alpha^2}{\omega^2}}$$

If τ is small, its effects on the dispersion relation (Equation [30]) may be expected to be small. The third term under the square root can then be approximated by

$$\frac{\tau\alpha}{e^{-\alpha H}\sinh\alpha H} = \frac{2\tau\alpha}{1 - e^{-2\alpha H}}$$

$$= \begin{cases} 2\tau\alpha & \text{for } \alpha H \text{ large} \\ \dfrac{\tau}{H} & \text{for } \alpha H \text{ small} \end{cases}$$

From Equation [31] it is seen that only waves with small wave numbers produce significant currents because of the factor $e^{-|\alpha|(f+H)}$. Therefore, $\alpha\tau$ is small since τ/H is assumed to be small. Hence, $\beta_1\tau$ is small and we have

$$e^{\beta_1\tau} \approx 1 + \beta_1\tau$$

$$e^{-\beta_1\tau} \approx 1 - \beta_1\tau$$

Under these circumstances and with $b_2 = 0$, A_o simplifies to

$$A_o = \frac{2\pi A\sqrt{\rho_2}\,|\alpha| e^{-|\alpha| f}}{|\alpha|(1+|\alpha|\tau)e^{-|\alpha|H} - g\epsilon\dfrac{\alpha^2}{\omega^2}\sinh|\alpha|H}$$

and the surface current to

$$\tilde{u}_o = -i\,A\,e^{-i\omega t}\int_{-\infty}^{+\infty} \frac{\dfrac{\omega^2|\alpha|}{\alpha(1+|\alpha|\tau)}\, e^{-|\alpha|(f+H-\tau)}\, e^{i\alpha x}\, d\alpha}{\left[\omega^2 - \dfrac{g\epsilon|\alpha|}{1+|\alpha|\tau}\, e^{-|\alpha|(H-\tau)} \sinh|\alpha|(H-\tau)\right]} \qquad [32]$$

The integral in Equation [32] can be evaluated as in the previous case where the thermocline is a discontinuity. For large positive x the surface current is given by

$$\tilde{u}_o \sim -\pi A \frac{\omega\, e^{-\alpha_o(f+H-\tau)}\, e^{i(\alpha_o x-\omega t)}}{\left(1+\alpha_o\tau\right)\left(\dfrac{d\gamma}{d\alpha}\right)_{\alpha_o}} \qquad [33]$$

where $\gamma^2 = \dfrac{g\epsilon|\alpha|}{1+|\alpha|\tau}\, e^{-|\alpha|(H-\tau)} \sinh|\alpha|(H-\tau)$

and α_o is the positive real root of the following equation:

$$\omega^2 - \frac{g\epsilon\alpha}{1+\alpha\tau}\, e^{-\alpha(H-\tau)} \sinh\alpha(H-\tau) = 0 \qquad [34]$$

Finally, let us consider the effects of a stratification below the thermocline; the thermocline is here taken to be a discontinuity, i.e., $\tau = 0$, b_2 is small so that

$$\beta_2 = \sqrt{b_2{}^2 + \left(1 - \frac{2gb_2}{\omega^2}\right)\alpha^2} \approx |\alpha| \sqrt{1 - \frac{2gb_2}{\omega^2}}$$

Under such circumstances A_o simplifies to

$$A_o \approx \frac{2\pi A\sqrt{\rho}\sqrt{1 - \frac{2gb_2}{\omega^2}}\, e^{-\sqrt{1 - \frac{2gb_2}{\omega^2}}|\alpha| f}}{\sqrt{1 - \frac{2gb_2}{\omega^2}} \sinh|\alpha| H + \cosh|\alpha| H - \frac{g\epsilon|\alpha|}{\omega^2}\sinh|\alpha| H}$$

and the surface current to

$$\tilde{u}_o \approx -iA\, e^{-i\omega t}\sqrt{1 - \frac{2gb}{\omega^2}} \int_{-\infty}^{+\infty} \frac{\frac{|\alpha|}{\alpha} e^{-\sqrt{1 - \frac{2gb_2}{\omega^2}}|\alpha| f}\, e^{i\alpha x}\, d\alpha}{\sqrt{1 - \frac{2gb_2}{\omega^2}} \sinh|\alpha|H + \cosh|\alpha|H - \frac{g\epsilon|\alpha|}{\omega^2}\sinh|\alpha|H}$$

[35]

When $\omega^2 < 2gb_2$, the integrand has no pole on the real axis. This can be shown as follows: Let α_o be a real root of the equation

$$\sqrt{1 - \frac{2gb_2}{\omega^2}} \sinh|\alpha| H + \cosh|\alpha| H - \frac{g\epsilon|\alpha|}{\omega^2}\sinh|\alpha| H = 0$$

[36]

If $\omega^2 < 2gb_2$ then α_o has to satisfy the following pair of equations:

$$\sqrt{\frac{2gb_2}{\omega^2} - 1}\ \sinh |\alpha_o|\ H = 0$$

$$\cosh |\alpha_o|\ H - \frac{g\epsilon|\alpha|}{\omega^2} \sinh |\alpha_o|\ H = 0$$

but this is impossible. Therefore, Equation [36] has no real solution for $\omega^2 < 2gb_2$. However, for every $\omega^2 > 2gb_2$ there is a unique positive solution (see Figure 2). Let this be denoted by α_o. Then for large positive x the surface current is given by

$$\tilde{u}_o \sim 2\pi A \frac{\sqrt{1 - \frac{2gb_2}{\omega^2}}\ e^{-\sqrt{1 - \frac{2gb_2}{\omega^2}}\ \alpha_o f}\ e^{i(\alpha_o x - \omega t)}}{\left(\frac{d\zeta}{d\alpha}\right)_{\alpha_o}} \qquad [37]$$

where

$$\zeta = \sqrt{1 - \frac{2gb_2}{\omega^2}} \sinh |\alpha|\ H + \cosh |\alpha|\ H - \frac{g\epsilon|\alpha|}{\omega^2} \sinh |\alpha|\ H$$

SURFACE CURRENTS DUE TO A SUBMERGED QUADRUPOLE

The quadrupole under consideration has a configuration shown in Figure 3. The strength M of the quadrupole is given by

$$M = A^2 \Delta^3$$

where A is the strength of the individual sources and sinks and Δ is the distance separating them.

The surface current w due to a quadrupole of strength amplitude M and time dependence $e^{-i\omega t}$ located at the origin is obtained by differentiating the source solution, i.e., $\tilde{u}_o$ with respect to x and replacing A by M.

Let us choose H as the reference length and $\sqrt{H/g\epsilon}$ as the reference time and introduce the following dimensionless variables

$$t' = t/\sqrt{H/g\epsilon}$$

$$x' = x/H$$

$$f' = f/H$$

$$\tau' = \tau/H$$

$$\alpha_o' = \alpha_o H$$

$$\omega' = \omega\sqrt{\frac{H}{g\epsilon}}$$

The surface currents generated by the quadrupole corresponding to the three cases analyzed in the previous section for the source are found to be

Case I $\tau' = 0, \quad b_2 = 0$

For large positive x the surface current due to the quadrupole is given by

$$w \sim \left(\frac{\pi M}{H^3}\right) \frac{\omega'(\alpha_o')^2 \; e^{-\alpha_o'(f'+1)} \; e^{i(\alpha_o' x' - \omega' t')}}{\left(\frac{d\sigma'}{d\alpha'}\right)_{\alpha_o'}}$$

The magnitude of the current is

$$|w| \sim \left(\frac{\pi M}{H^3}\right) \frac{\omega'(\alpha_o')^2 \; e^{-\alpha_o'(f' + 1)}}{\left(\frac{d\sigma'}{d\alpha'}\right)_{\alpha_o'}}$$

$$= \left(\frac{\pi M}{H^3}\right) U^* \qquad [38]$$

where

$$(\sigma')^2 = |\alpha'| \; e^{-|\alpha'|} \sinh |\alpha'|$$

and α_o', which is the asymptotic wave number of the current, is the positive real root of the equation

$$(\omega')^2 - \alpha' \; e^{-\alpha'} \sinh \alpha' = 0$$

Case II $\tau' \neq 0, \quad b_2 = 0$

For large positive x the surface current due to the quadrupole is

$$w \sim \left(\frac{\pi M}{H^3}\right) \frac{\omega'(\alpha_o')^2 \, e^{-\alpha_o'(f'+1-\tau')} \, e^{i(\alpha_o' x' - \omega' t')}}{(1 + \alpha_o'\tau')\left(\frac{d\gamma'}{d\alpha'}\right)_{\alpha_o'}}$$

The magnitude of the current is

$$|w| \sim \left(\frac{\pi M}{H^3}\right) \frac{\omega'(\alpha_o')^2 \, e^{-\alpha_o'(f'+1-\tau')}}{(1 + \alpha_o'\tau')\left(\frac{d\gamma'}{d\alpha'}\right)_{\alpha_o'}}$$

$$= \left(\frac{\pi M}{H^3}\right) U^* \qquad [39]$$

where

$$(\gamma')^2 = \frac{|\alpha'|}{(1 + |\alpha'|\tau')} e^{-|\alpha'|(1-\tau')} \sinh |\alpha'|(1 - \tau')$$

and α_o' is the positive root of

$$\omega'^2 - \frac{\alpha'}{1 + \alpha'\tau'} e^{-\alpha'(1-\tau')} \sinh \alpha'(1 - \tau') = 0$$

When $\tau' = 0$, this case reduces to Case I.

Case III $\tau' = 0, \quad b_2 \neq 0$

For large positive x the surface current due to the quadrupole is

$$w \sim \left(\frac{\pi M}{H^3}\right) \frac{2\sqrt{1 - \left(\frac{N'}{\omega'}\right)^2}\, e^{-\sqrt{1-\left(\frac{N'}{\omega'}\right)^2}\,\alpha_o' f'}\, e^{i(\alpha_o' x' - \omega' t')}}{\left(\frac{d\zeta'}{d\alpha}\right)_{\alpha_o}}$$

$$|w| \sim \left(\frac{\pi M}{H^3}\right) \frac{2\sqrt{1 - \left(\frac{N'}{\omega'}\right)^2}\, e^{-\sqrt{1-\left(\frac{N'}{\omega'}\right)^2}\,\alpha_o' f'}}{\left(\frac{d\zeta'}{d\alpha'}\right)_{\alpha_o'}}$$

$$= \left(\frac{\pi M}{H^3}\right) U^* \qquad [40]$$

where

$$\zeta' = \sqrt{1 - \left(\frac{N'}{\omega'}\right)^2}\ \sinh|\alpha'| + \cosh|\alpha'| - \frac{\alpha'}{(\omega')^2} \sinh|\alpha'|$$

and α_o' is the positive real root of the equation

$$\zeta' = 0$$

as N' tends to zero this case tends to Case I.

DISCUSSION OF RESULTS

The surface current due to a two-dimensional quadrupole of constant strength amplitude and simple harmonic time dependency was determined analytically. It was found that the current is a consequence of the formation and propagation of interfacial waves at the thermocline. For large distances from the source of disturbances, the interfacial waves form a regular train of outgoing waves having a definite amplitude and wave number. The surface current also possesses these characteristics.

The asymptotic wave number of the surface current is the wave number of free waves at the thermocline. It depends solely on the density profile and the frequency of the disturbance but not at all on its nature or depth of submergence. In Figure 2 the asymptotic wave number, denoted by α_o, is plotted against the frequency ω for various density profiles. For a given frequency ω and the same H, a thicker thermocline retards the phase velocity, hence increases the wave number. A stratified layer below the thermocline, however, increases the phase velocity, hence reduces the wave number. When the stratification in the lower layer is so large that its Vaisala frequency $N = \sqrt{2gb_3}$ is greater than the frequency of the disturbance, interfacial waves at the thermocline will attenuate as they propagate because of energy transfer in the form of true internal waves to the lower layer.

The asymptotic amplitude of the surface current depends on how effective the submerged disturbance is transmitting energy to that component of the interfacial wave which has a wave number equal to α_o (the wave number of free interfacial waves corresponding to the frequency ω of the submerged disturbance). In Figures 4 and 5 are shown the asymptotic amplitude of the surface current as a function of ω for various density profiles.

REFERENCE

1. Wong, K. K., "On Internal Gravity Waves Generated by Local Disturbances," HYDRONAUTICS, Incorporated Technical Report 231-5, February 1965.

APPENDIX

First consider the surface currents due to an oscillating source of amplitude A and simple harmonic time dependence $e^{-i\omega t}$ located at (x = 0, y = 0) in an ocean with the density profile shown in Figure 1b.

Express the physical vertical velocity component v(x,y,t) as

$$v = \bar{\rho}^{\frac{1}{2}}V = \rho^{\frac{1}{2}}\,\overline{V}\,e^{-i\omega t} \qquad [A-1]$$

The differential equations and boundary conditions governing φ, the Fourier transform of $\overline{V}$ with respect to x, are:

$$\frac{d^2\varphi_1}{dy^2} - \left[b_1{}^2 + \left(1 - \frac{2gb_1}{\omega^2}\right)\alpha^2\right]\varphi_1 = 0, \qquad f < y < H \qquad [A-2]$$

$$\frac{d^2\varphi_2}{dy^2} - \left[b_2{}^2 + \left(1 - \frac{2gb_2}{\omega^2}\right)\alpha^2\right]\varphi_2 = 2\pi A\rho^{\frac{1}{2}}\left[\dot{\delta}(y) - b_2\,\delta(y)\right]$$

$$\text{for } -\infty < y < f \qquad [A-3]$$

$$\varphi_1 = 0 \qquad \text{at } y = f + H \qquad [A-4]$$

$$\varphi_1 = \varphi_2 \qquad \text{at } y = f \qquad [A-5]$$

$$b_1\varphi_1 + \frac{d\varphi_1}{dy} = b_2\varphi_2 + \frac{d\varphi_2}{dy} \qquad \text{at } y = f \qquad [A-6]$$

$$\varphi_2 \to C \qquad \text{as } y \to -\infty \qquad [A-7]$$

Solving these equations simultaneously it is found that

$$\varphi_1 = C \sinh \beta_1[(f + H) - y] \qquad [A-8]$$

where

$$C = - \frac{2\pi A\rho^{\frac{1}{2}} (b_2 + \beta_2)\, e^{-\beta_2 f}}{(b_1 - b_2 - \beta_2) \sinh \beta_1 H - \beta_1 \cosh \beta_1 H} \qquad [A-9]$$

$$\beta_1 = \sqrt{b_1{}^2 + \left(1 - \frac{2gb_1}{\omega^2}\right)\alpha^2} \qquad [A-10]$$

$$\beta_2 = \sqrt{b_2{}^2 + \left(1 - \frac{2gb_2}{\omega^2}\right)\alpha^2} \qquad [A-11]$$

Therefore

$$V_1 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} C \sinh \beta_1[(f + H) - y]\, e^{i\alpha x}\, d\alpha \qquad [A-12]$$

from which the surface current u_o due to the oscillating source is found to be

$$u_o = \left[\bar{\rho}(y = f + H)\right]^{\frac{1}{2}} \frac{e^{-i\omega t}}{2\pi} \int_{-\infty}^{+\infty} C\beta_1 \frac{e^{i\alpha x}}{i\alpha} \, d\alpha \qquad [A-13]$$

The surface current due to an oscillating quadrupole of amplitude M can easily be found from that due to the source. It is

$$\tilde{u}_o = + \frac{e^{-i\omega t} M}{i} \int_{-\infty}^{+\infty} \frac{(b_2 + \beta_2)\, e^{-\beta_2 f} \beta_1 \alpha \, e^{i\alpha x} \, d\alpha}{(b_1 - b_2 - \beta_2) \sinh \beta_1 H - \beta_1 \cosh \beta_1 H} \qquad [A-14]$$

It has been assumed that

$$\epsilon = \frac{\rho - \bar{\rho}(y = f + H)}{\rho} << 1 \qquad [A-15]$$

Let it be further assumed that

$$\alpha >> b_1 \text{ or } b_2 \qquad [A-16]$$

so that

$$\beta_1 \approx |\alpha| \sqrt{1 - \frac{2gb_1}{\omega^2}}$$

$$\beta_2 \approx |\alpha| \sqrt{1 - \frac{2gb_2}{\omega^2}}$$

and expression [A-14] simplifies to

$$\tilde{u}_o \approx - e^{-i\omega t} \frac{M}{i} \sqrt{1 - \frac{2gb_2}{\omega^2}} \sqrt{1 - \frac{2gb_1}{\omega^2}} \;\times$$

$$\int_{-\infty}^{+\infty} \frac{|\alpha|\alpha \, e^{-\sqrt{1 - \frac{2gb_2}{\omega^2}}\,|\alpha| f} \, e^{i\alpha x} \, d\alpha}{\sqrt{1 - \frac{2gb_2}{\omega^2}} \sinh\sqrt{1 - \frac{2gb_1}{\omega^2}}\,|\alpha|H + \sqrt{1 - \frac{2gb_1}{\omega^2}} \cosh\sqrt{1 - \frac{2gb_1}{\omega^2}}\,|\alpha|H} \qquad \text{[A-17]}$$

For large x, contributions to the integral in Equation [A-17] come from the real poles of the integrand only. Therefore, let us examine the roots of the equation

$$\sqrt{1 - \frac{2gb_2}{\omega^2}} \sinh\sqrt{1 - \frac{2gb_1}{\omega^2}}\,|\alpha|H + \sqrt{1 - \frac{2gb_2}{\omega^2}} \cosh\sqrt{1 - \frac{2gb_1}{\omega^2}}\,|\alpha|H = 0 \qquad \text{[A-18]}$$

Assume $2gb_1 > 2gb_2$.

Case I: $\omega^2 < 2gb_2 < 2gb_1$

Equation [A-18] becomes

$$i \tan\sqrt{\frac{2gb_1}{\omega^2} - 1}\ |\alpha|H = -\sqrt{\frac{\frac{2gb_1}{\omega^2} - 1}{\frac{2gb_2}{\omega^2} - 1}}$$

which has no real roots. Therefore

$$\tilde{u}_o = 0 \qquad \text{for } \omega^2 < 2gb_2 < 2gb_1$$

Case II: $\omega^2 > 2gb_1 > 2gb_2$

Equation [A-18] becomes

$$\tanh\sqrt{1 - \frac{2gb_1}{\omega^2}}\ |\alpha|H = -\sqrt{\frac{1 - \frac{2gb_1}{\omega^2}}{1 - \frac{2gb_2}{\omega^2}}}$$

which also has no real roots. Therefore

$$\tilde{u}_o = 0 \qquad \text{for } \omega^2 > 2gb_1 > 2gb_2$$

<u>Case III</u>: $2gb_2 < \omega^2 < 2gb_1$

Equation [A-18] becomes

$$\tan\sqrt{\frac{2gb_1}{\omega^2} - 1}\ |\alpha| H = -\sqrt{\frac{\frac{2gb_1}{\omega^2} - 1}{1 - \frac{2gb_2}{\omega^2}}}$$

which has an infinite number of roots

$$\alpha_n = \pm \frac{n\pi - \tan^{-1}\sqrt{\frac{\frac{2gb_1}{\omega^2} - 1}{1 - \frac{2gb_2}{\omega^2}}}}{H\sqrt{\frac{2gb_1}{\omega^2} - 1}} \qquad n = 1, 2 \ldots$$

For x > 0 and large

$$\tilde{u}_o = -\frac{2\pi M}{H}\sqrt{1 - \frac{2gb_2}{\omega^2}}\ \times$$

$$\sum_{n=1}^{\infty} \frac{\alpha_n^2\, e^{-\sqrt{1 - \frac{2gb_2}{\omega^2}}\,\alpha_n f}\ e^{i(\alpha_n x - \omega t)}}{\sqrt{1 - \frac{2gb_2}{\omega^2}}\cos\sqrt{\frac{2gb_1}{\omega^2} - 1}\ |\alpha| H - \sqrt{\frac{2gb_1}{\omega^2} - 1}\ \sin\sqrt{\frac{2gb_1}{\omega^2} - 1}\ |\alpha| H}$$

[A-19]

Introduce dimensionless variables

$$f' = f/H \quad , \quad x' = x/H$$

$$\omega' = \omega/\sqrt{2gb_1}$$

$$N_2' = \sqrt{2gb_2}/\sqrt{2gb_1}$$

$$t' = t\sqrt{2gb_1}$$

Then

$$\tilde{u}_o = +\left(\frac{2\pi M}{H^3}\right)\sum_{n=1}^{\infty} \frac{\alpha_n'^2 \, e^{-\alpha_n' \sqrt{1 - \frac{N_2'^2}{\omega'^2}}\, f'} \, e^{i(\alpha_n' x' - \omega' t')}}{\sqrt{\frac{\frac{1}{\omega'^2} - 1}{1 - \frac{N_2'^2}{\omega'^2}}}\, \sin\sqrt{\frac{1}{\omega'^2} - 1}\,\alpha_n' - \cos\sqrt{\frac{1}{\omega'^2} - 1}\,\alpha_n'} \qquad [A-20]$$

$$\alpha_n' = \frac{n\pi - \tan^{-1}\sqrt{\frac{\frac{1}{\omega'^2} - 1}{1 - \frac{N_2'^2}{\omega'^2}}}}{\sqrt{\frac{1}{\omega'^2} - 1}} \qquad [A-21]$$

Write $\tilde{u}_o$ as

$$\tilde{u}_o = \frac{2\pi M}{H^3} (\tilde{u}_1 + \tilde{u}_2 + \ldots.)$$

Let

$$|\tilde{u}_1| = u_1{}^*$$

$$|\tilde{u}_2| = u_2{}^* \quad \text{etc.}$$

In Figure 6, $u_1{}^*$ and $u_2{}^*$ are plotted as a function of ω' for the special case $b_2 = 0$ and two different depths of submergence.

From the calculation, it is found that the asymptotic surface current is non-zero only when $2gb_2 < \omega^2 < 2gb_1$ (assuming that $2gb_1 > 2gb_2$).

When ω^2 lies within the above range, it is found that the surface current is in the form of a Fourier Series. This is in contrast to calculations for profiles which possess a well defined thermocline. In the latter case, there exists only one Fourier component.

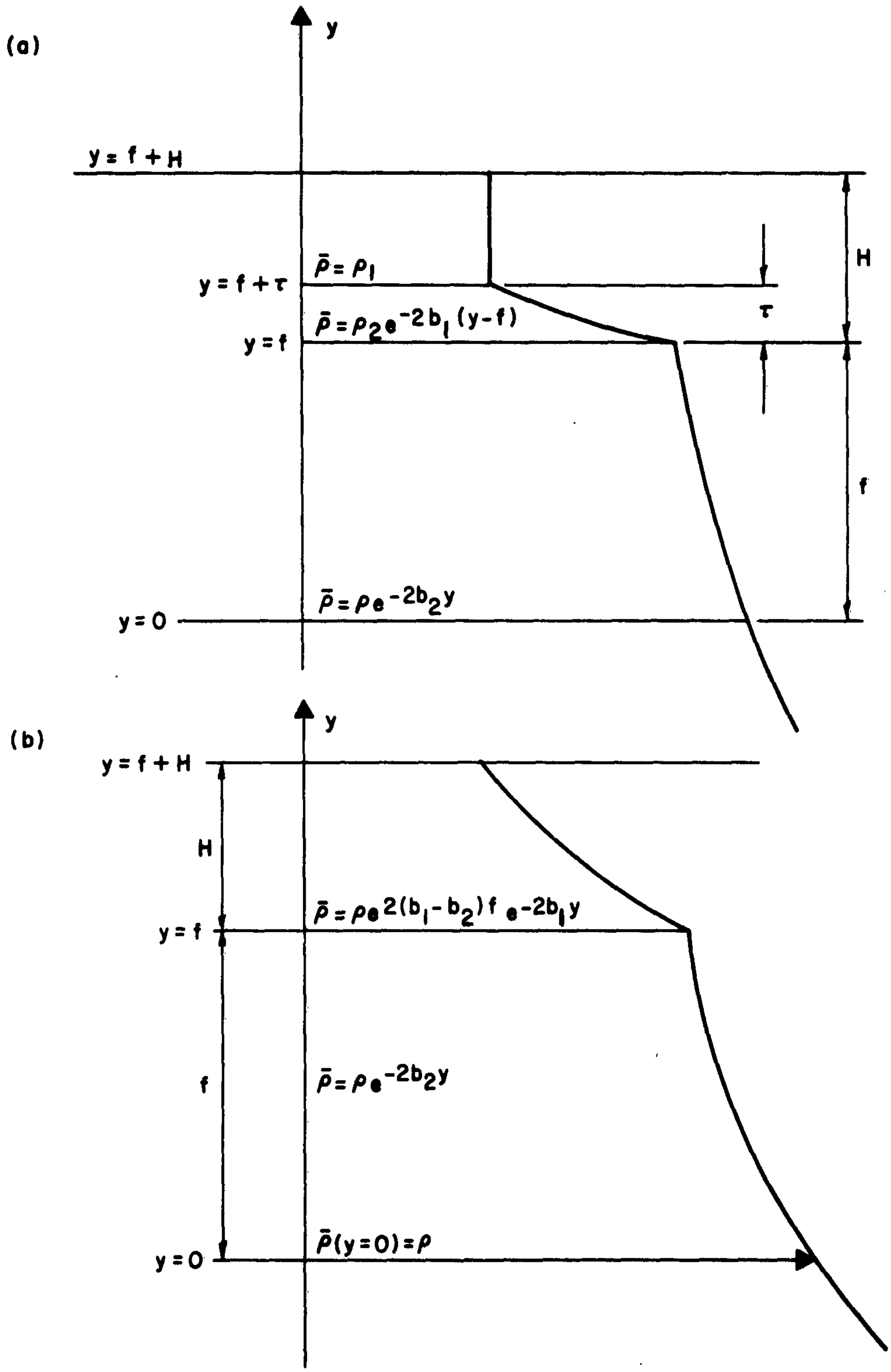

FIGURE 1 — SCHEMATIC DIAGRAM OF EQUILIBRIUM DENSITY PROFILES

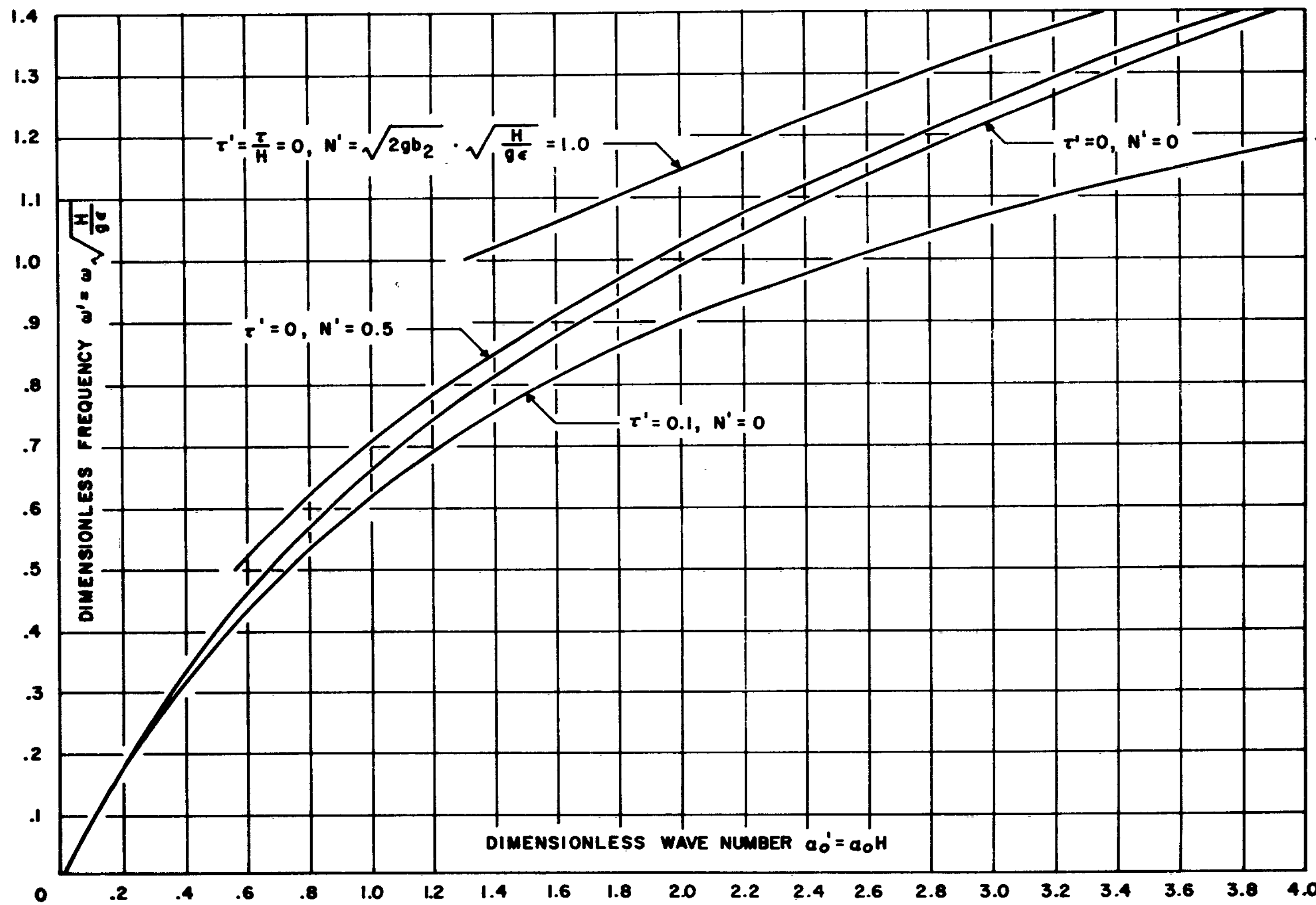

FIGURE 2 – ASYMPTOTIC WAVE NUMBER FOR DIFFERENT EQUILIBRIUM DENSITY PROFILES

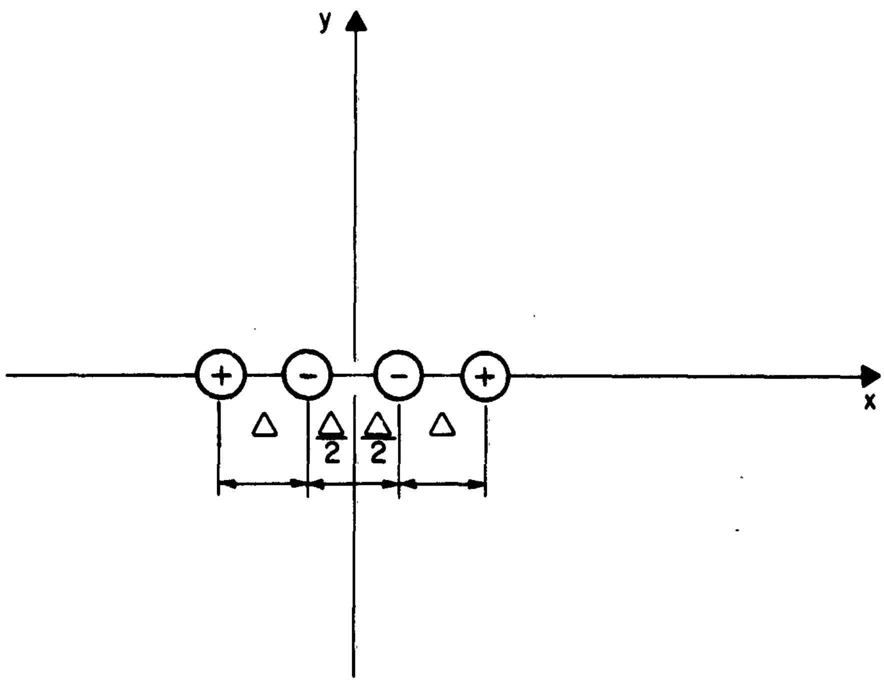

FIGURE 3 – CONFIGURATION OF QUADRUPOLE

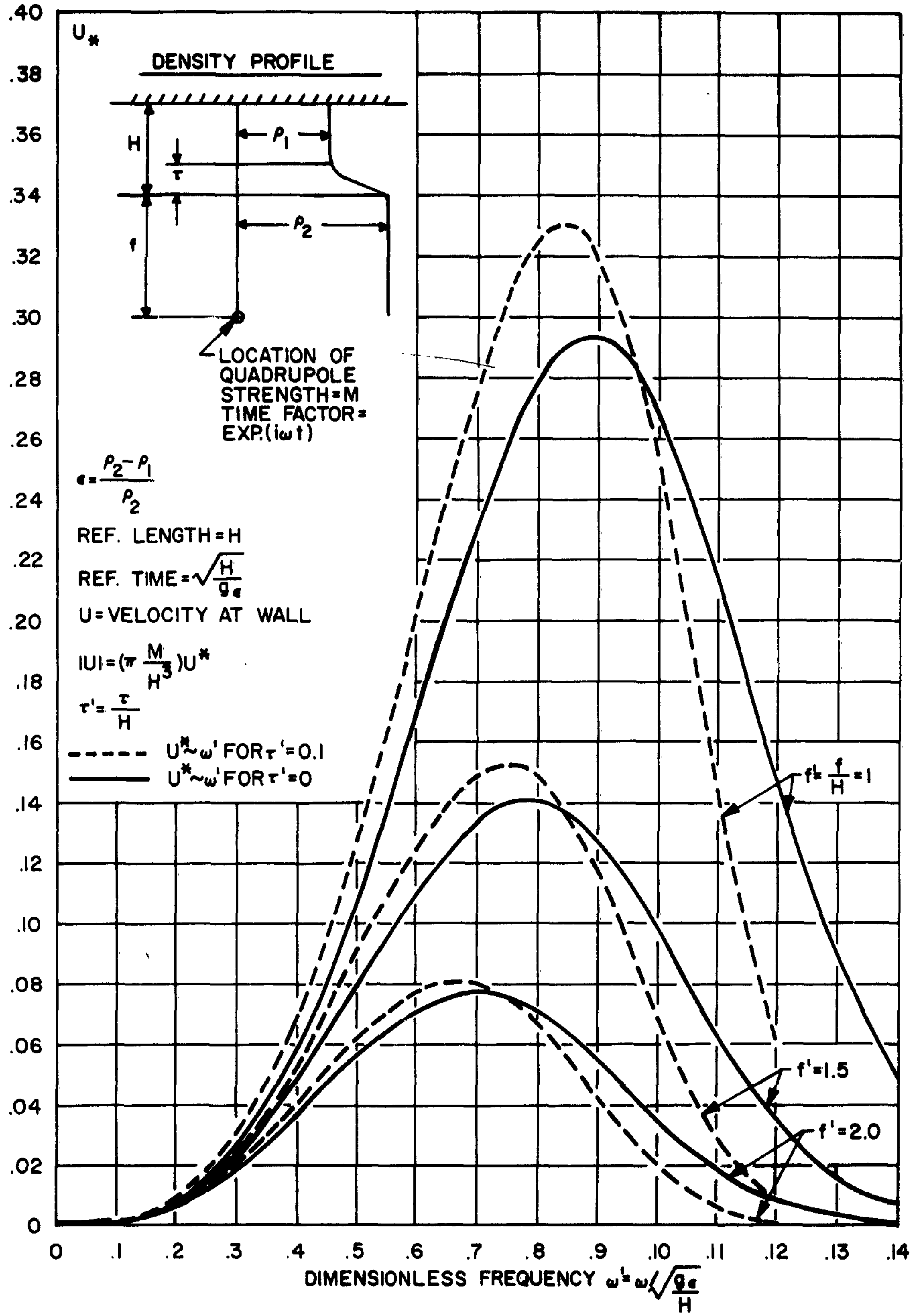

FIGURE 4 — EFFECT OF THERMOCLINE THICKNESS

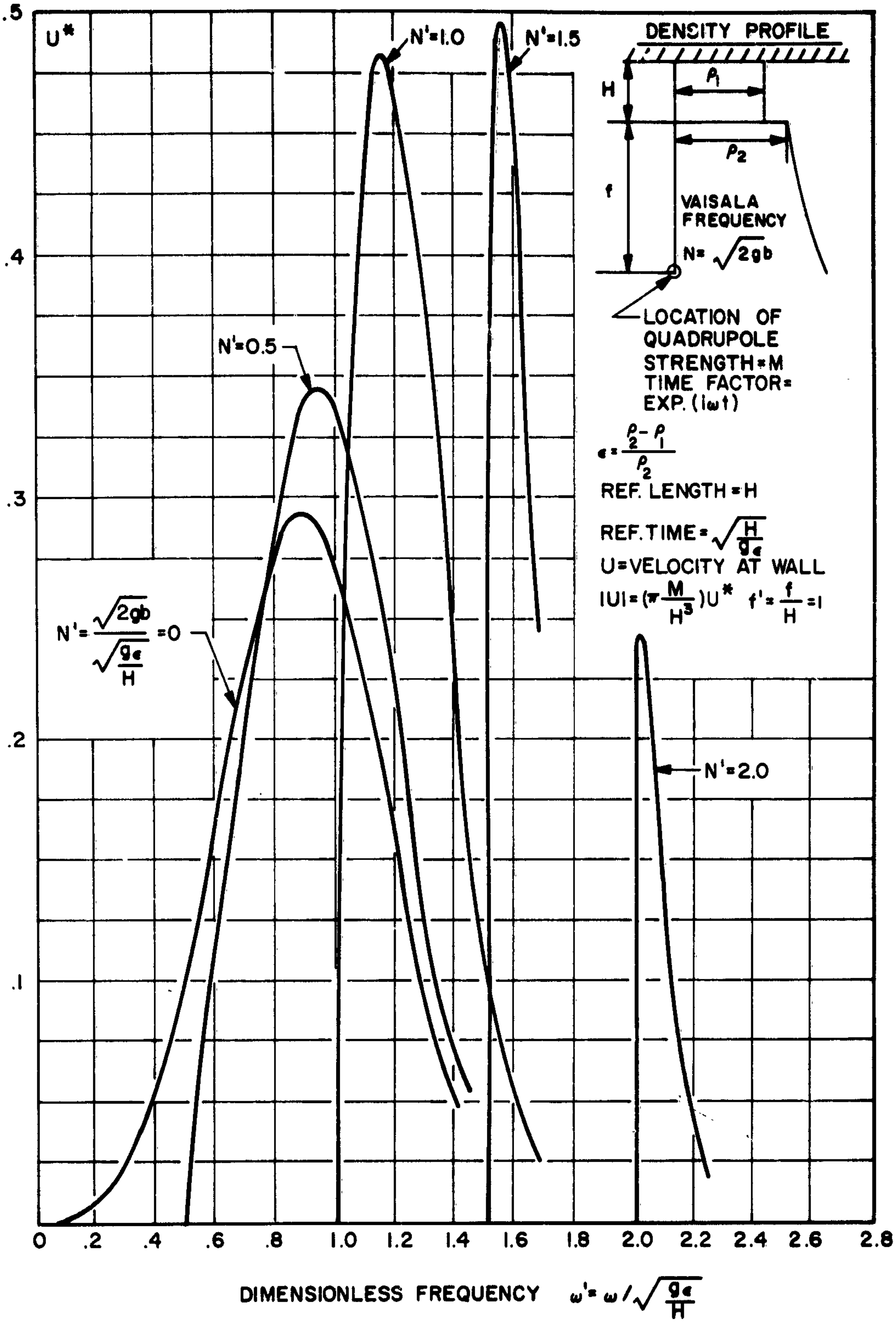

FIGURE 5-EFFECT OF DENSITY GRADIENT IN LOWER LAYER

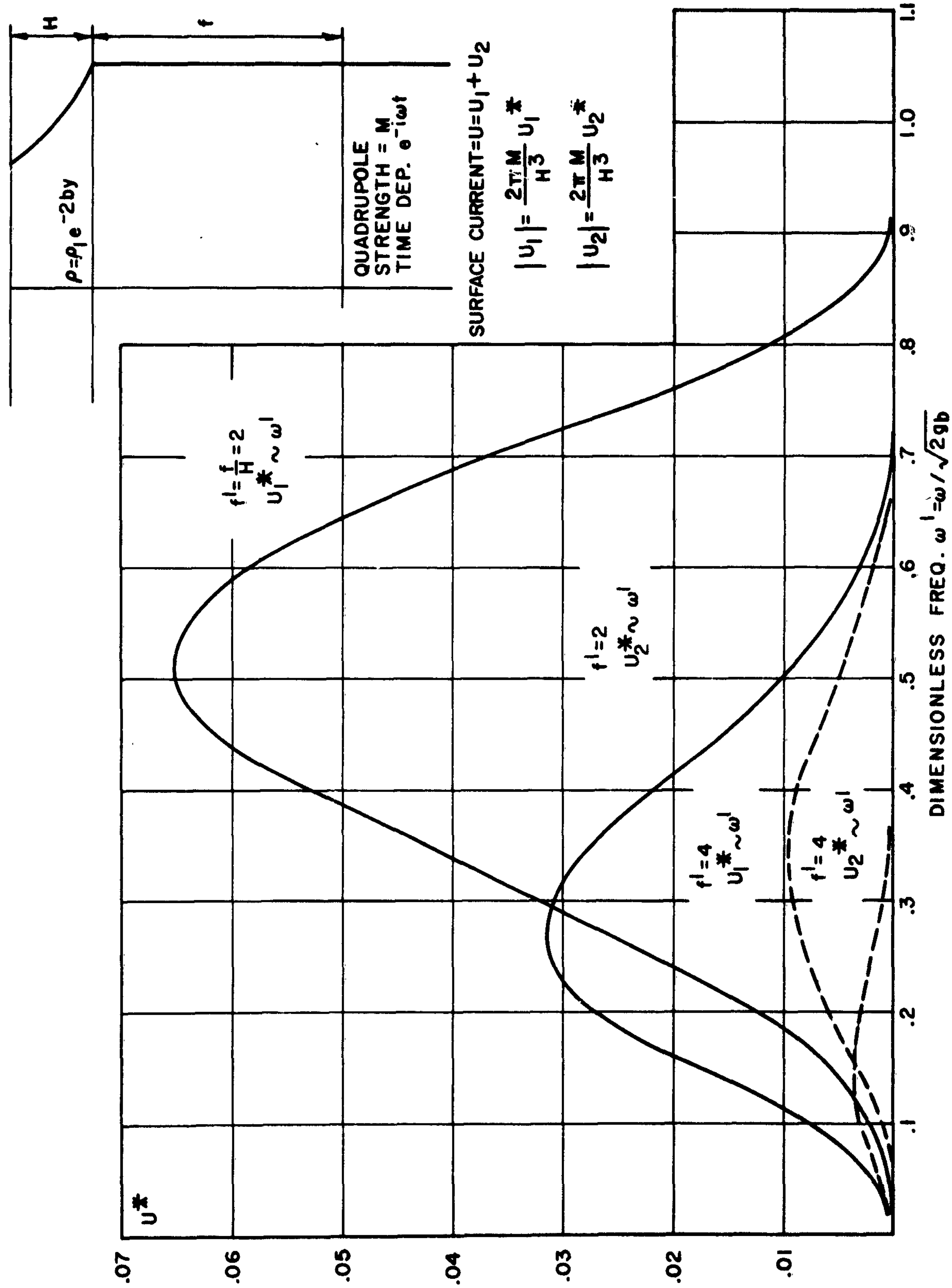

FIGURE 6 – EFFECT OF DENSITY GRADIENT IN UPPER LAYER

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Surface currents due to a submerged two-dimensional disturbance of simple harmonic time dependency in a stratified ocean were determined analytically. It was found that the current is a consequence of the formation and propagation of interfacial waves at the thermocline. For large distances from the source of disturbance, the interfacial waves form a regular train of outgoing waves having a definite amplitude and wave number. The surface current also possesses these characteristics. The asymptotic wave number of the surface current is the wave number of free waves at the thermocline.

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