AD NUMBER

AD464816

NEW LIMITATION CHANGE

TO

Approved for public release, distribution
unlimited

FROM

Distribution authorized to U.S. Gov't.
agencies and their contractors;
Administrative/Operational Use; MAR 1965.
Other requests shall be referred to Air
Force Materials Laboratory, Attn; Research
and Technology Division, Wright-Patterson,
AFB, OH 45433.

AUTHORITY

AFML ltr, 7 Apr 1975

AFML-TR-64-415

464816

A THEORY OF STAGE II FATIGUE CRACK PROPAGATION
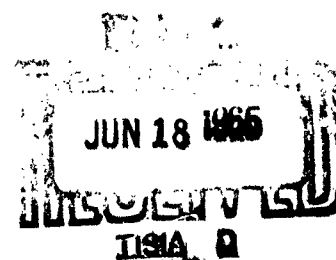
.

J. C. GROSSKREUTZ

MIDWEST RESEARCH INSTITUTE

TECHNICAL REPORT No. AFML-TR-64-415

MARCH 1965

AF MATERIALS LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

JUN 18 1965

NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

DDC release to CFSTI not authorized. The distribution of this report is limited because it contains technology identifiable with items on the strategic embargo lists excluded from export or re-export under U. S. Export Control Act of 1949 (63 STAT.7), as amended (50 U.S.C. App. 2020.2031), as implemented by AFR 400-10.

Copies of this report should not be returned to the Research and Technology Division unless return is required by security considerations, contractual obligations, or notice on a specific document.

FOREWORD

This report was prepared by Midwest Research Institute under USAF Contract No. AF 33(657)-10883. This contract was initiated under Project No. 7353, "Characterization of Solid Phase and Interphase Phenomena in Crystalline Substances," Task No. 735301, "Mechanical Metallurgy." The work was monitored by the AF Materials Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, with Mr. K. D. Shimmin acting as Project Engineer.

This report covers work done during the period 1 September 1963 to 31 October 1964.

The author served as project leader during this research and is grateful for the assistance of several co-workers: Mr. Jet Wimp was responsible for computer programming; Mr. C. Q. Bowles and Mrs. Wanda Chinnery performed some of the numerical computations. The helpful comments of Dr. Sheldon L. Levy toward the formulation of the theory are gratefully acknowledged.

This technical report has been reviewed and is approved.

W. J. TRAPP
Chief, Strength and Dynamics Branch
Metals and Ceramics Division
Air Force Materials Laboratory

ABSTRACT

A continuum model of crack extension is used to derive a crack propagation law for the case of constant plastic strain amplitude fatigue. The rate of crack growth is found to be proportional to the square root of the crack length. Integration over the total number of cycles to failure, N_f , yields an expression of the form $N_f(\Delta\bar{\epsilon}_p)^{n+1} = f(\Delta\bar{\epsilon}_p)$, where $\Delta\bar{\epsilon}_p$ is the applied plastic strain range, and f is a function which varies rapidly in the region of large strains, but approaches a constant as $\Delta\bar{\epsilon}_p$ becomes small. The strain hardening coefficient, n , and the fracture strain enter as material constants. Comparison with experimental data gives good agreement for $\Delta\bar{\epsilon}_p \geq 0.01$, which is consistent with the assumptions used in the theory. A discussion is given which interprets the well-known power law $N_f^{1/2}\Delta\bar{\epsilon}_p = \text{const.}$ in terms of crack propagation.

iii

TABLE OF CONTENTS

I. INTRODUCTION

The process of fatigue failure is normally divided into crack nucleation and crack propagation stages. Forsythe[1] has shown that the latter process is itself composed of at least two sequential stages which he has termed Stage I, slip (shear) plane cracking, and Stage II, tensile mode cracking. Stage II propagation takes place on a plane oriented 90° to the tensile stress and the crack tip deformation leaves characteristic striations on the fracture surface. Each striation has been shown to correspond to a single stress (or strain) cycle. The study of fracture surfaces has revealed the existence of these striations (or ripples) on a wide variety of materials and demonstrates the generality of the Stage II process.

The Stage II mechanism has been studied by Laird and Smith[2] in aluminum fatigued at high stress levels. By sectioning specimens strained to various stages of the stress cycle, it was possible to demonstrate directly the crack tip deformation which leads to the formation of striations. The crack is extended and blunted on the tensile half-cycle, then resharpened on the compressive half-cycle. The latter process restores the stress-concentration factor necessary for repetition of the cycle. It is this mechanism which has suggested the model used in this report.

The fraction of total life spent in Stage II propagation increases as the applied stress (or strain) amplitude increases.[3] Laird and Smith[4] have correlated the number of ripples (observed optically) on the fracture surfaces of Al and Ni with total cycles to failure and find that for lives less than 100 cycles, at least 70 per cent of life is occupied by Stage II. For lives of $\sim 10^3$ cycles, at least half the life consisted of Stage II propagation. Recent studies[5] of a variety of materials have confirmed the fact that crack growth by ripple formation occupies greater than 75 per cent of life in high strain - low cycle fatigue.

Nucleation of a propagating Stage II crack therefore requires something less than 25 per cent of total life in low cycle fatigue. At the crack origin the narrow spacing ($\sim 0.1~\mu$) of the ripples and possible obliteration due to "banging" of the crack surfaces make it difficult to place a definite lower limit on the nucleation time. Nevertheless, in light of the work just cited, it appears logical to approach the problem of low-cycle fatigue in terms of Stage II crack propagation and a detailed analysis of the crack-tip deformation during any given cycle.

1

The treatment to be given in this report, while based on continuum theory, makes no pretense for elegance. Rather, it seeks to develop a simple model, based on experimental evidence and plausible assumptions, such that the mathematics remains tractable and the results capable of experimental verification. The case of fully reversed, constant plastic strain amplitude was chosen both for its simplicity and the fact that a well-documented empirical law exists which relates the number of cycles to failure to the applied strain amplitude.[6,7]

II. CRACK GROWTH MODEL

The experimental results of Laird and Smith[2] on aluminum and of McEvily et al.[8] on polyethylene (which also produces Stage II striations) have been used to construct the continuum model of crack extension illustrated in Figure 1. The crack is a two-dimensional ellipse and considered to be located somewhere in a thick specimen such that conditions of plane strain exist at the tip. In the fully compressed state, the crack has semi-major axis equal to c and semi-minor axis equal to b_o . As the applied strain increases to the maximum tensile state, plastic deformation at the crack tip results in the creation of new surface and the crack assumes the shape with semi-major axis equal to $c + \delta c$ ($\delta c = RR'$) and semi-minor axis equal to b_m . In the derivation to follow, it is assumed that $\delta c \ll c$. During the compression half-cycle, the crack-tip strain reached during tension is considered irreversible such that the arc $P'R'$ becomes $P''R''$ in the fully compressed state. In this manner an extension equal to Δx_2 is achieved during one complete cycle. The folding which leads to ripple formation has been neglected in the calculation of Δx_2 .

The assumption of strain irreversibility at the crack tip is an example of the instability of ductile materials under reversed loading which has been discussed by McClintock.[9] In fact, the model presented here can be considered a special case of McClintock's "growth by progressive deformation" case, for which he has shown the instability to arise from either the Bauschinger effect or low strain hardening rates.

Referring to Figure 1, the root strain, ϵ_{rl} , is defined as $(OP'-OP)/OP$, where O is chosen so that OR is the root radius, r_o , of the compressed crack. To relate Δx_2 to this strain, consider the triangles $OP'R'$ and $OP''R''$. These right triangles are approximately equal since the arcs $P'R'$ and $P''R''$ are taken to be equal and $OR' \cong OP''$. Thus $RR'' \cong PP'$; and since PP' can also be expressed as $OP'-OP = r_o\epsilon_{rl}$, the following relation holds,
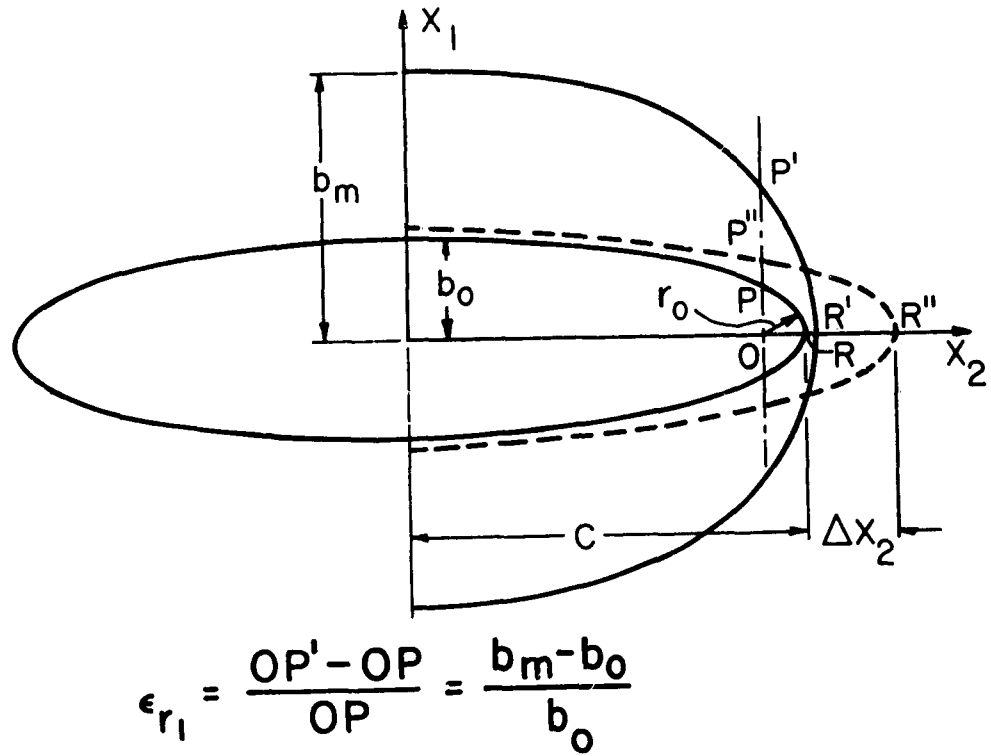
CRACK GROWTH MODEL

$$\epsilon_{r_1} = \frac{OP' - OP}{OP} \cong \frac{b_m - b_0}{b_0}$$

Figure 1 - Continuum Model for Stage II Fatigue Crack Growth

3

$$\Delta x_2 = \beta r_0 \epsilon_{rl} \tag{1}$$

where β is a constant of the order 1 which allows Equation 1 to be written as an exact equality.

III. DERIVATION OF CRACK GROWTH LAW

The framework of the derivation is as follows: The root strain is first expressed in terms of the applied strain through a strain concentration factor. The strain hardening coefficient of the material, n , is introduced at this point. Next, the crack growth rate, $dc/dN = \Delta x_2$, is integrated between the limits N_0 , the number of cycles to initiate a Stage II crack, and N_f , the number of cycles to fracture. The limiting values of the crack length are c_0 , the initial Stage II crack length, and c_f , the final crack length, for which the next quarter-cycle of strain will bring ductile fracture. The fracture strain of the material, $\bar{\epsilon}_f$, is introduced at this point. Thus, two material constants, n and $\bar{\epsilon}_f$, appear in the final result.

The root strain ϵ_{rl} can be expressed in terms of the changes in the semi-minor axis, $\epsilon_{rl} = (b_m - b_0)/b_0$, provided $\delta c \ll c$. The calculation relating this strain to the macroscopic applied strain is more accurately carried out using logarithmic strains since the magnitudes are finite. Thus, we define the incremental, logarithmic strain as $d\bar{\epsilon}_{rl} = db/b = 1/2(dr/r)$, where r is the root radius. The total logarithmic root strain is therefore $\bar{\epsilon}_{rl} = 1/2 \ln(r_m/r_0)$ at the conclusion of the tensile half-cycle.

To evaluate the strain concentration factor, we recognize, as Drucker[10] has suggested, that this factor will vary as the crack-tip geometry changes during the cycle. An incremental strain concentration factor, k_ϵ , is therefore defined by $d\bar{\epsilon}_{rl} = k_\epsilon d\bar{\epsilon}$, where $\bar{\epsilon}$ is the applied logarithmic strain. Neuber[11] has given an expression, valid in shear, which relates the Hookian stress concentration factor, k_H , to the stress and strain concentration factors, k_σ and k_ϵ , for any arbitrary nonlinear stress-strain law,

$$(k_\sigma k_\epsilon)^{1/2} = k_H \tag{2}$$

Neuber has further stated that the relation may be extended to arbitrary states of stress without modification. Rhee and McClintock[12] have questioned the validity of this procedure in the tensile case for nonhardening, ideally

plastic materials. In the present calculation, for want of a better relation, we shall use the Neuber expression as valid for materials in tension with finite strain hardening. Assuming that the power law, $\sigma = \sigma_0 \epsilon^n$, holds for the materials in question, we find

$$k_\epsilon = \left(4\,\frac{c}{r}\right)^{\frac{1}{n+1}} \tag{3}$$

where k_H has been specified for an elliptical crack of length $2c$ and root radius r . The strain hardening coefficient, n , should strictly be interpreted as characteristic of the material in the vicinity of the crack tip as the tensile half-cycle is applied to it. In the sequel, because such values are unknown, we shall take n to be the usual value determined from a tensile stress-strain curve.

Equation 3 may now be used in the incremental expression for $d\bar{\epsilon}_{rl}$ to find the relation between the root strain and the applied strain. We have

$$\frac{1}{2}\,\frac{dr}{r} = \left(4\,\frac{c}{r}\right)^{\frac{1}{n+1}} d\bar{\epsilon}$$

Integrating between r_0 and r_m on the left, and 0 and $\Delta\bar{\epsilon}_p$, the total plastic strain range, on the right, we obtain

$$\left(\frac{r_m}{r_0}\right)^{\frac{1}{n+1}} = \left\{ \frac{2\Delta\bar{\epsilon}_p}{n+1}\left(4\,\frac{c}{r_0}\right)^{\frac{1}{n+1}} + 1 \right\} = \alpha \;,$$

or,

$$\epsilon_{rl} = \frac{n+1}{2}\,\ln\,\alpha \tag{4}$$

Here it has been assumed that the applied strains are large enough to neglect the elastic component, and that the total plastic strain range corresponds to opening the crack from its fully compressed state to its maximum opening.

5

We next assume that at the conclusion of each strain cycle, the crack is returned in its fully compressed state to a root radius, r_0 . Therefore, Equation 4 represents the maximum root strain for any crack length c . Converting Equation 4 to conventional strains gives,

$$\epsilon_{r1} = \alpha^{\frac{n+1}{2}} - 1 \; ,$$

and thus the crack growth rate is

$$dc/dN = \beta r_0 \left(\alpha^{\frac{n+1}{2}} - 1 \right) \; . \tag{5}$$

Equation 5 predicts that the rate of crack growth in Stage II is approximately proportional to the square root of the crack length under the conditions of constant plastic strain amplitude. This dependence on crack length is quite sensitive both to the way in which the incremental root strain $d\bar{\epsilon}_{r1}$ is defined and to the assumption about r_0 . If, for example, the root strain is taken simply as dr/r , then dc/dN will depend linearly on c . Moreover, if r_0 actually increases as the crack lengthens, then Equation 5 is altered so that dc/dN increases more rapidly with c . For the balance of this paper the simple model which led to Equation 5 will be retained.

Equation 5 can be integrated between suitable limits to find the number of cycles spent in Stage II crack growth. The integration can be expressed in terms of α to give,

$$\int_{\alpha_0}^{\alpha_f} \frac{(\alpha-1)^n d\alpha}{\left(\alpha^{\frac{n+1}{2}} - 1 \right)} = \beta \; \frac{2^{n+3}}{(n+1)^{n+2}} \, (\Delta\bar{\epsilon}_p)^{n+1} \int_{N_0}^{(N_f-1/4)} dN \tag{6}$$

Here N_0 is the number of cycles to initiate a Stage II crack of length c_0 , and N_f the total number of cycles to failure. α_0 and α_f are obtained by substituting the values for c_0 and c_f into the expression for α . The final crack length c_f is achieved one quarter-cycle before complete failure by ductile fracture. The integral on the left in Equation 6, designated by I in the sequel, cannot be integrated in closed form except for the limiting cases, $n = 0, \; 1$.

To choose a reasonable value for c_f , we define a critical root fracture strain such that when $c = c_f$, $\bar{\epsilon}_{r1} = \bar{\epsilon}_{rf} = \frac{n+1}{2} \ln \alpha_f$. A limiting

condition is given when the applied strain range $\Delta\overline{\epsilon}_p$ is equal to the fracture strain $\overline{\epsilon}_f$, and c_f is essentially the void size. For convenience, we take the void size to be of the order of c_o . One then obtains

$$c_f = c_o\left(\frac{\overline{\epsilon}_f}{\Delta\overline{\epsilon}_p}\right)^{n+1} \tag{7}$$

The limits α_o and α_f are given by

$$\alpha_f = \left\{\frac{2\overline{\epsilon}_f}{n+1}\left(4\,\frac{c_o}{r_o}\right)^{\frac{1}{n+1}} + 1\right\}$$

$$\tag{8}$$

$$\alpha_o = \left\{\frac{2\Delta\overline{\epsilon}_p}{n+1}\left(4\,\frac{c_o}{r_o}\right)^{\frac{1}{n+1}} + 1\right\}$$

The only parameter to enter the evaluation of the integral, I , is the ratio c_o/r_o ; the values of n and $\overline{\epsilon}_f$ depend on the material under test, and $\Delta\overline{\epsilon}_p$ is given by the experimental conditions.

Because we have taken the macroscopic fracture strain $\overline{\epsilon}_f$ as the criterion for determining c_f , it should be noted that the larger the parameter c_o/r_o , the larger the required root strain to achieve fracture, $\overline{\epsilon}_{rf}$. If $\overline{\epsilon}_{rf}$ were known, the best value for c_o/r_o could then be chosen without difficulty. As it is, a choice for c_o/r_o must be made from comparison of Equation 6 with experimental life data in the low-cycle range. Even so, the derivation of Equation 6 with only one adjustable parameter (β enters only as a constant multiplier) can be considered fortunate.

Equation 6 can be rewritten in the following form,

$$(N_f - 1/4 - N_o)(\Delta\overline{\epsilon}_p)^{n+1} = \frac{(n+1)^{n+2}}{2^{n+3}\beta}\,I \tag{9}$$

If we take N_o to be small compared with N_f in the high strain region, an assumption consistent with the trend of recent experimental results, we have

$$N_f(\Delta\overline{\epsilon}_p)^{n+1} = \frac{(n+1)^{n+2}}{2^{n+3}\beta}\,I \tag{10}$$

7

an equation which bears some resemblance to the Manson-Coffin power law relation[6,7] for low-cycle fatigue. Weiss[13] has derived a similar relation starting from somewhat different assumptions, but with a constant term on the right-hand side. On the other hand, the right-hand side of Equation 10 is a function of $\Delta\bar{\epsilon}_p$ through the integral I, which approaches a limiting value as $\Delta\bar{\epsilon}_p \longrightarrow 0$.

In the following section, Equation 10 is compared with the experimental results obtained by a number of workers for constant plastic strain, low-cycle fatigue tests. In all cases the factor β was taken equal to 1, and the integral, I, was evaluated by Simpson's Rule using an IBM 1620 automatic computer.

IV. COMPARISON WITH EXPERIMENT

In Figure 2, $\Delta\bar{\epsilon}_p$ versus N_f curves for nickel* and three different values of the parameter c_0/r_0 are compared with total life data for Ni taken from the work of Coffin and Tavernelli.[14] In these and subsequent curves, one-quarter cycle has been added to the value of N_f calculated from Equation 10 so that the theoretical and experimental data are effectively normalized at the fracture strain. It is clear that the value $c_0/r_0 = 10$ provides the best agreement with experiment in the region $\Delta\bar{\epsilon}_p \gtrsim 0.1$ where the assumptions leading to Equation 10 should be most applicable. This value for c_0/r_0 is also more satisfying from a physical standpoint than either of the other values. Further consequences of this choice for c_0/r_0 will be discussed at the end of this section.

The sensitivity of the computed life values to various choices for n and $\bar{\epsilon}_f$, is shown in Figures 3 and 4 for nickel. The fact that $n = 0.4$ is the accepted value is quite clear. The curve for $\bar{\epsilon}_f = 2.0$ in Figure 4 would be the result if $\Delta\bar{\epsilon}_p$ at one-fourth cycle were chosen as $2\bar{\epsilon}_f$ instead of $\bar{\epsilon}_f$, as has been suggested by Willner and McClintock.[15] Clearly this choice would not produce better agreement with the experimental data in the present case.

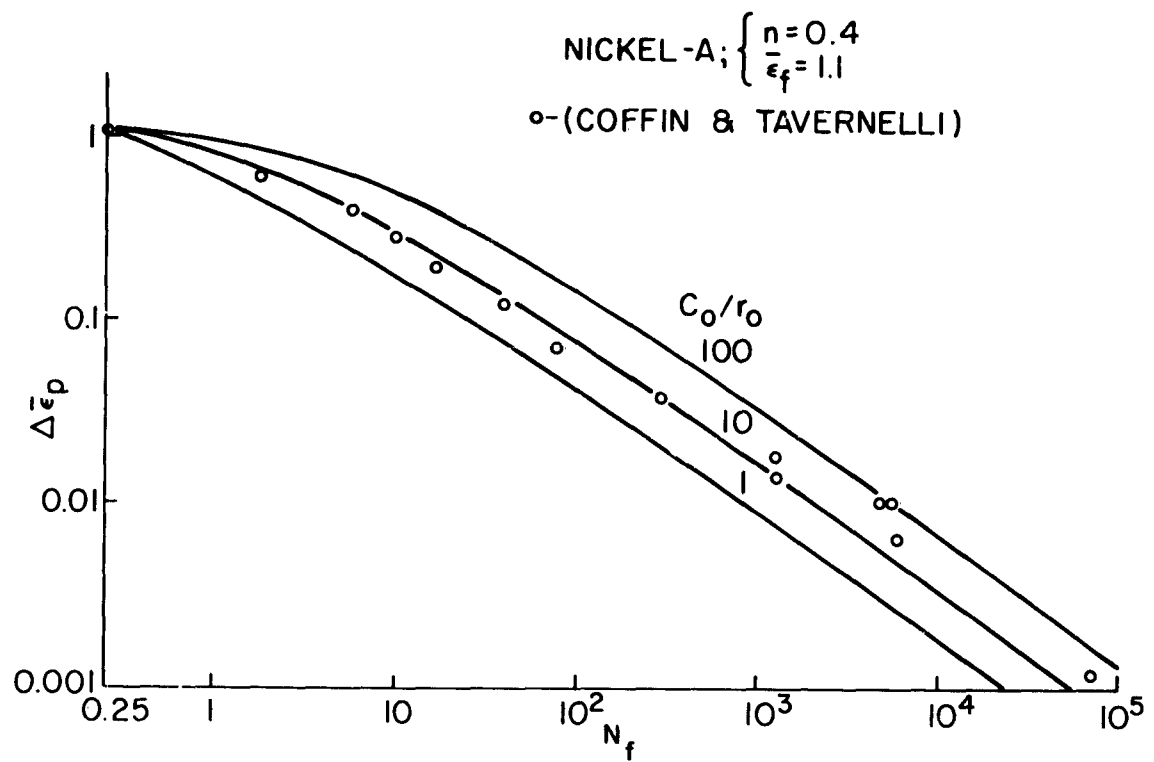* Sources of the material constants n and $\bar{\epsilon}_f$ are given in Table 1.

Figure 2 - $\Delta\bar{\epsilon}_p$ versus N_f Calculated from Equation 10 for Nickel and Three Different Values of c_o/r_o . Experimental Points Taken from Reference 14.

9

TABLE 1

MATERIAL CONSTANTS

Metal	n	$\bar{\epsilon}_f$[e]
1100 Al	0.5[a]	1.71
Ni	0.4[b]	1.10
2024 Al	0.2[c]	0.4
OFHC Cu	0.54[d]	1.4

---

[a] A. M. Willner and F. A. McClintock, Paper No. 61-WA-273 Annual Meeting of ASME (1961).

[b] G. W. Geil and N. L. Carwile, NBS Circular 520, 67 (1952).

[c] J. R. Low and F. Garfalo, Proc. Soc. Exptl. Stress Anal. 4, 16 (1947).

[d] J. R. Low, "Properties of Metals in Materials Engineering," ASTM (1949).

[e] J. F. Tavernelli and L. F. Coffin, Jr., Trans. of ASME 84 D, 533 (1962).
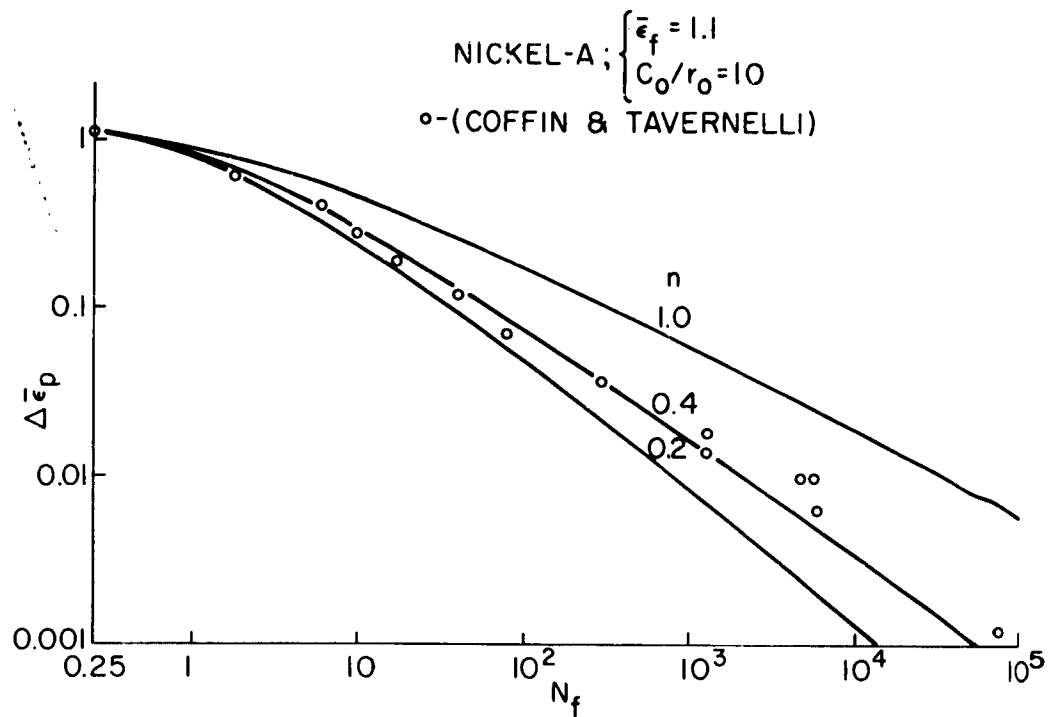
Figure 3 - $\Delta\bar{\epsilon}_p$ versus N_f Curves for Nickel with Different Values
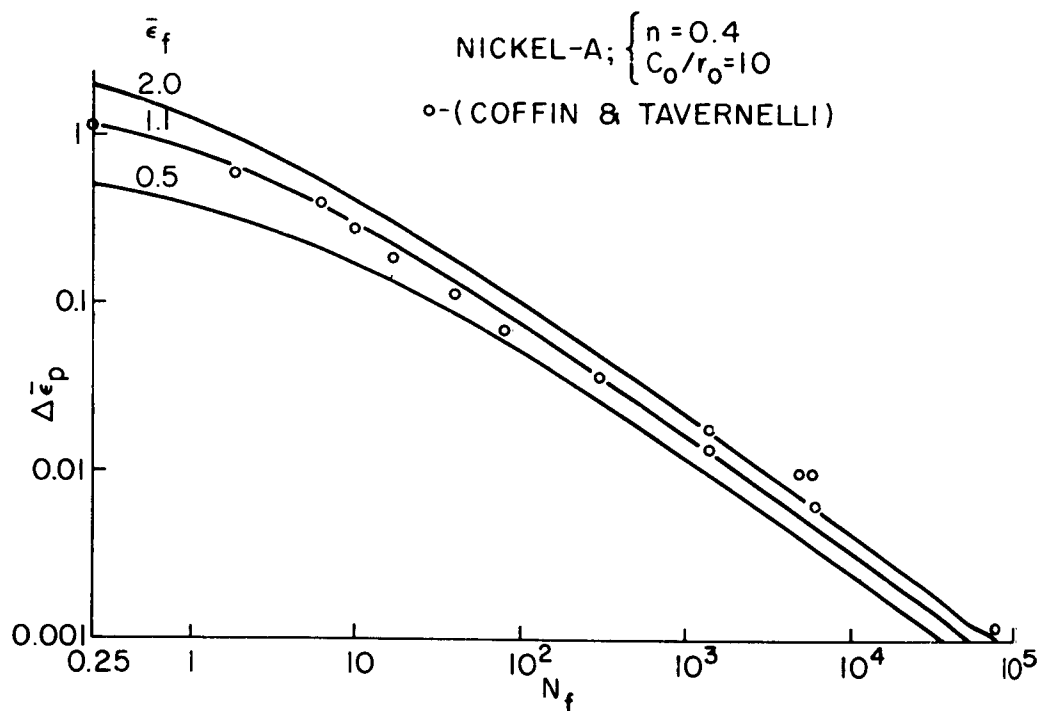of the Strain Hardening Coefficient, n .
(Accepted Value n = 0.4 .)

Figure 4 - $\Delta\bar{\epsilon}_p$ versus N_f Curves for Nickel with Different Values
of the Fracture Strain, $\bar{\epsilon}_f$.

(Accepted Value, $\bar{\epsilon}_f$ = 1.1 .)

11

In Figure 5, the results of Equation 10 are compared with the experimental data for 1100 Al from two sources.[5,14] Again, the choice of $c_o/r_o = 10$ is consistent with the data. Figure 6 compares the theory with the data for OFHC copper, and again shows the sensitivity of the theory to the choice of n (accepted value, $n = 0.5$). Figure 7 compares the theory with data for 2024 Al. This was the only case in which a choice of c_o/r_o higher than 10 was indicated. The low ductility of this alloy compared to the other three materials may account for this fact.

An estimate of the order of magnitude of c_o and r_o can be obtained from experimental observations on crack growth rates in Stage II. If Equation 5 is rewritten in nondimensional form, $(dc/dN)/r_o$ may be plotted against the ratio c/c_o for various choices of c_o/r_o . Such a plot for $n = 0.4$ and $\Delta \bar{\epsilon}_p = 0.02$ is shown in Figure 8. If $\bar{\epsilon}_f$ is chosen as 1.5, the ratio c_f/c_o is fixed, and the upper limit to $(dc/dN)/r_o$ can be found for the three cases shown. McEvily et al.[5] have found for OFHC $Cu(n = 0.5, \bar{\epsilon}_f = 1.4)$ that the ripple spacing just prior to fracture is of the order of 10^{-3} cm. when $\Delta \bar{\epsilon}_p = 0.02$. This information, coupled with the upper limit for $(dc/dN)/r_o$ can be used to determine c_c and r_o as shown in Figure 8. The value $c_o = 10^{-3}$ cm. when $c_o/r_o = 10$ is reasonable in the light of observations by Forsythe[3] and Grosskreutz[16] at much lower strain amplitudes where the transition from Stage I to Stage II cracking in aluminum occurs at crack lengths of the order of 10 μ. The value for r_o when $c_o/r_o = 10$ seems a bit large on first glance, but if one recalls that the original model is an idealized elliptical crack, then $r_o = 10^{-4}$ cm. is not unreasonable.

Finally, the ratio of final to initial crack growth rate in Figure 8 is 38, which compares with 17 for the ratio of largest to smallest ripple spacing found by McEvily et al.[5] in Cu. The theory thus predicts that somewhat smaller striations should exist than those actually resolved by electron microscopy of fracture surface replicas.
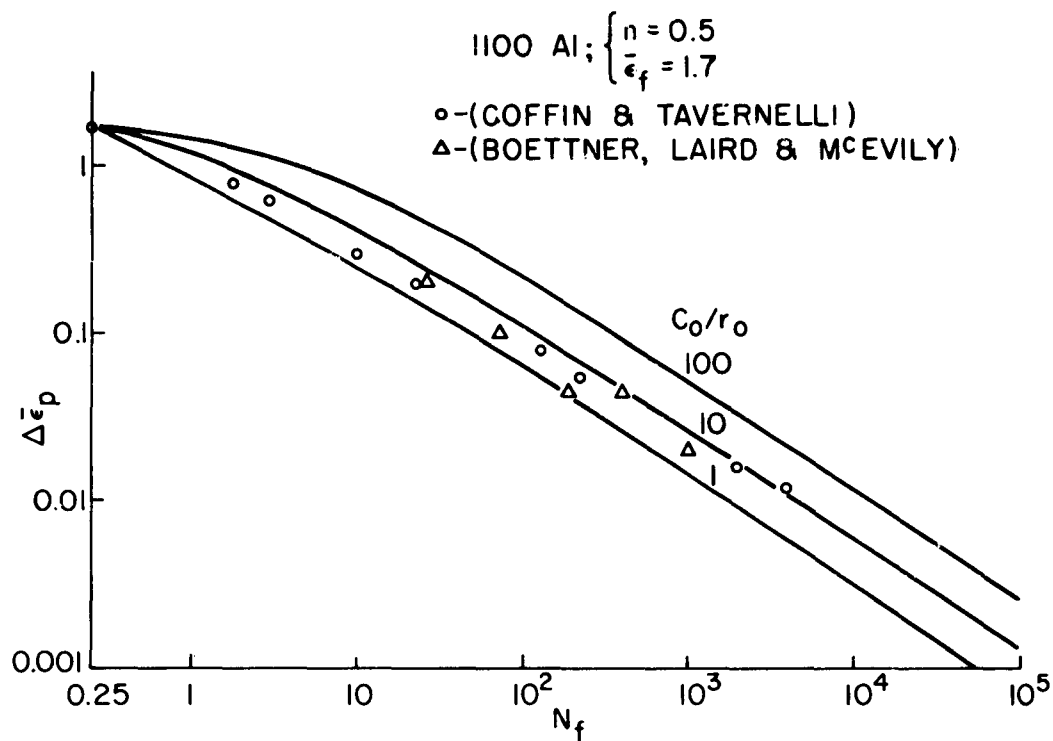
12

Figure 5 - $\Delta\bar{\epsilon}_p$ versus N_f Curves for 1100-Al with Different Values
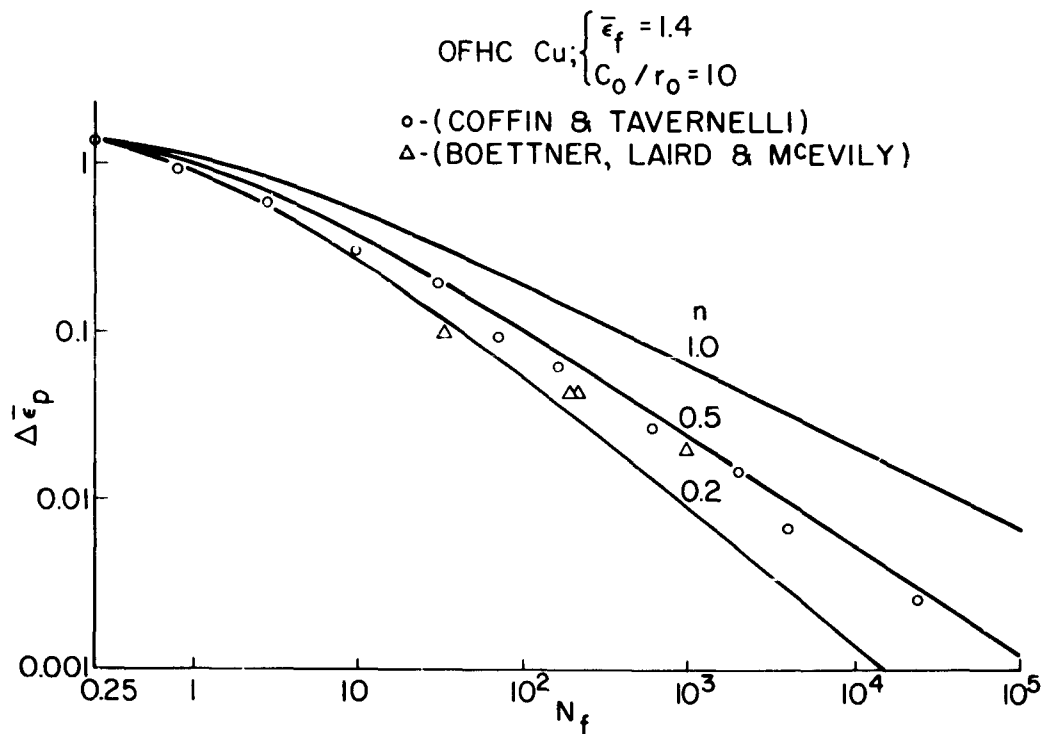of c_o/r_o. Experimental Points Taken from
References 5 and 14.

Figure 6 - $\Delta\bar{\epsilon}_p$ versus N_f Curves for OFHC-Cu with Different Values
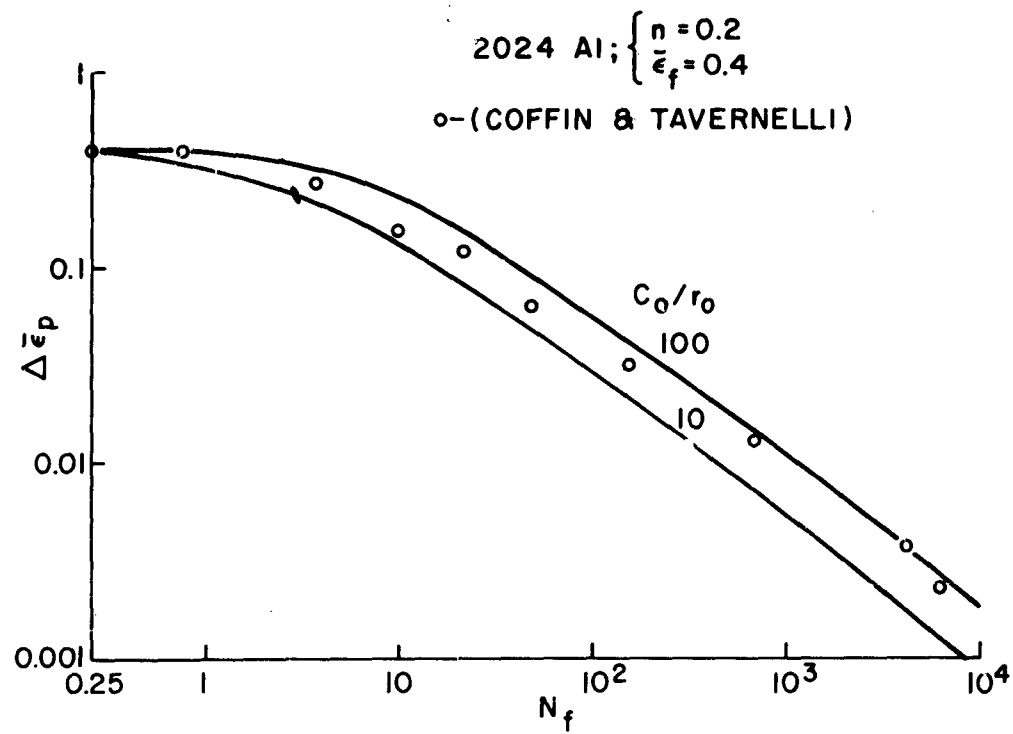for n. (Accepted Value $n = 0.5$.) Experimental Data,
References 5 and 14.

13

Figure 7 - $\Delta\bar{\epsilon}_p$ versus N_f Curves for 2024-Al with Different Values of c_o/r_o . Experimental Data, Reference 14.
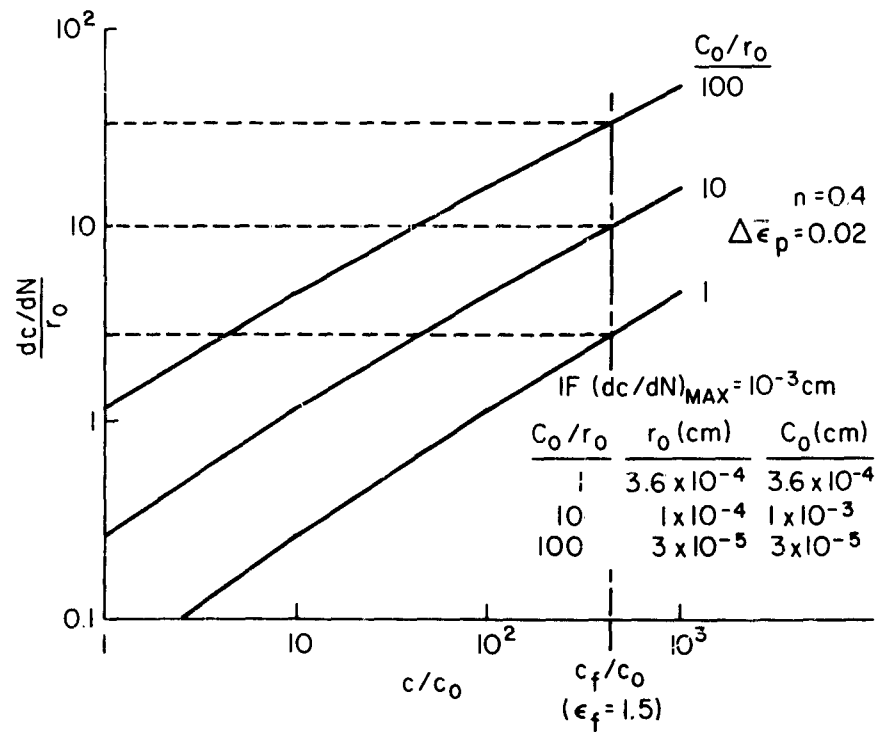

Figure 8 - Dimensionless Plot of Crack Growth Rate versus Crack Length (Reference 5) for Various Values of c_o/r_o . Other Parameters as Shown

14

V. DISCUSSION

The good agreement between Equation 10 and the experimental data over several decades of life lends good support to the existing experimental data which show that low-cycle, high-strain fatigue can be viewed mainly as Stage II crack propagation. The expression first proposed by Manson and Coffin[6,7] is therefore a crack propagation law. The cracks involved are never very large, ranging from about 10 μ at the outset to the order of millimeters near the end of life.

The theory which has been described suggests that the number of cycles to initiate a Stage II crack, N_o, is indeed quite small compared to N_f until the strain range, $\Delta\bar{\epsilon}_p$, falls below ~ 0.01, or $N_f \gtrsim 5 \times 10^3$ cycles. It is below this value of $\Delta\bar{\epsilon}_p$ that the theoretical curves in Figures 2 - 7 begin to fall away from the experimental points. It is surprising that the agreement should hold out to these relatively long lives. Laird and Smith[4] have observed that only 30 per cent of the fatigue life is taken up with ripple-propagation for $N_f \sim 5 \times 10^3$ cycles. However, their observations were limited to optical magnifications, and it is quite possible, in view of later work,[5] that a much larger percentage would have been observed at electron-optical levels. Another possibility is that the simple model which leads to Equation 5 is, in fact, not accurate enough, and that the crack growth rate takes an altered form. It has already been suggested above that some elementary changes in the definitions of $d\bar{\epsilon}_{rl}$ and r_o lead to a linear, or higher power, dependence of dc/dN on c. In this case, computed values of (N_f-N_o) would fall away from the actual life curves sooner than in the present case. There is some recent evidence for a linear relation between dc/dN and c,[5] which suggests that N_o becomes appreciable at larger values of $\Delta\bar{\epsilon}_p$ than those predicted here.

The sensitivity of the theory to variations in n and $\bar{\epsilon}_f$ (Figures 3, 4 and 6) argue that the continuum model on which the calculation is based is realistic. To be sure, the model has a "built-in" inverse dependence between N_f and $\Delta\bar{\epsilon}_p$. However, with only one adjustable parameter, c_o/r_o, it is reasonable to conclude that the agreement between Equation 10 and the experimental data is more than fortuitous.

It is also clear why many materials behave so nearly alike under the same constant plastic strain conditions. Values for n are nearly all in the range 0.2 - 0.5, and most values for $\bar{\epsilon}_f$ are of the order of unity. Thus, the rate of crack propagation for a given $\Delta\bar{\epsilon}_p$ and c will not vary much over 20 per cent over the range of n.

15

The theory here developed is restricted to axial loading tests in which a single crack dominates most of the test life. It has been shown, however, that materials obey a power law relation when fatigued in high-strain torsion.[15] Microscopic observation at somewhat lower strains[17] has shown that torsional fatigue produces a multitude of Stage I cracks at the surface before Stage II propagation begins. A dilemma is posed, therefore, in the interpretation of high-strain torsion tests. It would be extremely interesting to study the fracture surfaces of such specimens to determine the fraction of life spent in Stage II propagation.

Finally, it should be pointed out that the theory predicts (Equation 15) that near the end of life, the ripple spacing for a given material will be independent of the value of $\Delta \bar{\epsilon}_p$. It would be interesting to explore this possibility experimentally.

REFERENCES

1. P. J. E. Forsythe, <u>Proceedings of the Crack Propagation Symposium</u>, Cranfield, The College of Aeronautics, 76 (1962).

2. C. Laird and G. C. Smith, <u>Phil. Mag.</u> 7, 847 (1962).

3. P. J. E. Forsythe, <u>Acta Metallurgica</u> 11, 703 (1963).

4. C. Laird and G. C. Smith, <u>Phil. Mag.</u> 8, 1945 (1963).

5. R. C. Boettner, C. Laird, and A. J. McEvily, Jr., <u>Trans. AIME Met. Soc.</u> (in press) (1964).

6. S. C. Manson, NACA Report 1170 (1954).

7. L. F. Coffin, Jr., <u>Trans. AIME Met. Soc.</u> 76, 923 (1954).

8. A. J. McEvily, Jr., R. C. Boettner, and T. L. Johnston, <u>Fatigue - An Interdisciplinary Approach</u>, Syracuse Univ. Press, 95 (1964).

9. F. A. McClintock, Fracture of Solids, <u>Interscience</u>, 65 (1963).

10. D. C. Drucker, Fracture of Solids, <u>Interscience</u>, 3 (1963).

11. H. Neuber, <u>Journal of Appl. Mech.</u> 28, 544 (1961).

12. S. Rhee and F. A. McClintock, <u>Proceedings of the Fourth Congress of Applied Mechanics</u>, ASME, 1007 (1962).

13. V. Weiss, <u>Fatigue - An Interdisciplinary Approach</u>, Syracuse University Press, 179 (1964).

14. L. F. Coffin and J. F. Tavernelli, <u>Trans. AIME Met. Soc.</u> 215, 794 (1959).

15. A. M. Willner and F. A. McClintock, Paper No. 61-WA-273, Annual Meeting of ASME (1961).

16. J. C. Grosskreutz, <u>J. Appl. Phys.</u> 33, 1787 (1962).

17. W. A. Wood, S. Cousland, and K. R. Sargent, <u>Acta Metallurgica</u> 11, 643 (1963).