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DIMENSIONAL ANALYSIS
OF
LOAD SINKAGE RELATIONSHIPS IN SOILS AND SNOW

December, 1964



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by

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DIMENSIONAL ANALYSIS
OF
LOAD SINKAGE RELATIONSHIPS IN SOILS AND SNOW

By
R. A. Liston and E. Hegedus

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LAND LOCOMOTION LABORATORY

ABSTRACT

Principles of dimensional analysis have been applied to load sinkage relationships in various types of soils and snow, in order to evaluate the possibility of predicting the behavior of prototype plates on the basis of load sinkage tests performed with model footings. Encouraging results were obtained in cohesionless soils and in soils possessing little cohesion.

A new dimensionally attractive load-sinkage equation has been developed whose accuracy was found to be superior to the Bekker method of load-sinkage evaluation when the work required to produce a certain amount of sinkage served as a basis of comparison. However, the Bekker method produced more accurate results if the measured and computed pressure-sinkage relationships were taken as the basis of comparison.

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1. INTRODUCTION:

a. Background:

A fundamental proposition to land locomotion mechanics is that the behavior of either a track or wheel can be predicted on the basis of the behavior of a plate. In order to describe the relationship between pressure and sinkage beneath a wheel*, a pressure-sinkage curve is obtained by means of a plate test. The resulting pressure-sinkage curve can be described by the familiar equation of Bekker (1):

$$p = kz^n \quad \dots \dots \dots (1).$$

This relationship is then applied to the wheel to determine, for example, the sinkage required to produce equilibrium in the soil.

There have been many objections raised concerning Equation (1) for a variety of reasons. The equation has constants whose dimensions vary; the equation is not compatible with bearing capacity theory; the equation does not include a form effect; and so on. Several investigators (2) (3) have proposed modifications to the equation which eliminated some of the objections. However, the suggested modifications have not shown a significant increase in accuracy over Equation (1).

*The reader is requested to add "or track" each time the word wheel is used in this section.

When examining the accuracy of a proposed pressure-sinkage equation based on plate sinkage, it seems reasonable to determine whether the plate sinkage data can be used to predict plate sinkage. That is, can the behavior of a small plate allow one to predict the behavior of a large plate? If a model-prototype relationship cannot be shown to be valid for constant load geometry, it is doubtful that any result can apply to a situation in which load geometry is widely divergent, either in form or size. When considering model-prototype relationships, an obvious approach is to treat the problem by dimensional analysis.

b. Dimensional Studies in Land Locomotion:

There has been ample precedent set for the treatment of land locomotion problems by means of dimensional analysis or dimensional similitude. Nuttall (4) was the forerunner in this approach followed by Vincent and Hicks (5). Dickson (6) treated both static and dynamic problems dimensionally. Each of the treatments were slightly different in the selection of parameters and the soil test instrumentation used to introduce soil properties.

Nuttall (4) conducted model studies of wheeled vehicles operating in sand and snow with considerable success. The unique development of Nuttall, beyond the considerable achievement of demonstrating that scaling was practical in sand or snow, was the delineation of a correlation factor. The correlation factor

was established to account for the fact that a prototype vehicle would operate in a different soil condition than the model if sinkage was properly scaled. The technique used by Nuttall has been described fully in several of his publications so will be treated very briefly here. Cohesion, qualify it as apparent, effective, actual, or what you will, is fundamental to shear strength. (The characteristic identified as cohesion changes with moisture content and in natural non-homogeneous soils the cohesion varies with depth). The shear strength is also associated with soil density which changes in some unpredictable way with depth. To account for the resulting changes in shear strength with sinkage, or soil depth, Nuttall proposed a correlation factor derived from a penetration test using plates of several sizes one of which could be considered as an arbitrary prototype and the remaining plates as models. Because the plates affected soil at depths proportional to the plate size, an index of the change in shear strength could be extracted from the plate test. By preparing a non-dimensional pressure-sinkage plot and obtaining constants that would cause all of the curves to collapse on the arbitrarily selected prototype curve, a series of correlation factors could be obtained for several scale factors.

Nuttall's results were highly encouraging to other investigators who attempted to follow his lead but assuming different

soil "constants" to avoid the use of a correlation factor. Vincent and Hicks (5) approached the problem of scaling wheel performance by considering the Bekker parameters. They were successful in sand but concluded that a soil having a "significant" cohesion and a non-zero friction angle made scaling inaccurate. The degree of cohesion considered significant will be discussed later in this report. As a result of the Vincent-Hicks studies, a proposal to devise scale soils was made. Attempts to develop such a material were not successful because of conflicting demands of a constant coefficient of friction between the soil and test wheel and a variable angle of internal friction. The obvious solution was to vary the material making up the wheel surface so that the coefficient could be maintained constant. However, the obvious was not observed because attempts to vary one property, cohesion for example, also changed density and friction angle. It is undoubtedly possible to devise a scale soil but the effort does not appear to justify the result because each natural soil of interest would demand a model soil which would be dependent both on the prototype soil and the scale factor.

Dickson (6) was doing very nicely in his study of the scaling of tracked vehicle performance. He used a "tilting plate dynamometer" to establish scaling relationships that produced

accurate predictions in purely cohesive or purely frictional material. Unfortunately, when he was well into his study, he completed his tour at the Canadian Army Mobility Laboratory and was posted to Western Canada.

Before abandoning this brief background discussion, the proposal of Spanski (7) to scale performance by establishing model weight empirically will be examined. He proposed to measure the Bekker soil values and predict prototype drawbar-pull to weight (DP/W) ratio. The soil is fixed, the model dimensions are fixed, then the only obvious parameter to vary is the model weight. To establish model weight, the DP/W for the model is taken as equal to that predicted for the prototype. A trial and error solution yields the proper model weight. It should be pointed out, however, that this approach does not produce a correct scale situation in that the sinkage is properly scaled only by accident.

c. Scope of Study:

The study with which this report is concerned was confined to frictional materials with the exception of one series of tests in Detroit Clay. The emphasis on frictional soils resulted from two factors: the analysis in Appendix A indicated that successful scaling of plates was quite likely for frictional materials and quite unlikely for cohesive materials; the experimental error involved in the study of frictional materials tends to be

much less than for cohesive materials. The latter reason is recognized as a rather weak justification.

The single test of a cohesive material was conducted to indicate the problems that can be expected once the study is extended to include cohesion.

Because of equipment limitations, the plate size did not exceed 72 square inches. Additional equipment has been subsequently constructed to permit the study of plates in excess of 100 square inches.

The objective of the study was to determine whether it is possible to treat simple plates as models and prototypes. A secondary objective was to derive an equation relating pressure and sinkage on the basis of dimensional analysis. If such an equation was forthcoming, the equation would be evaluated on the basis of its ability to predict the sinkage of a large plate from data established by means of a test using a small, or model, plate. Two criteria were proposed: the accuracy of predicting sinkage and the accuracy of predicting the work involved in sinking to a given depth.

2. DISCUSSION:

- a. The behavior of plates acting on soil has been studied in considerable depth. The problem is attractive since it is relatively simple from an experimental viewpoint and has application to a wide range of situations. In the field of land locomotion mechanics,

plates have been taken as a representation of either a wheel or a track. The pressure-sinkage relationship established by means of a plate test was applied directly to equations describing the sinkage of a wheel or track. The effect of horizontal deformation on sinkage was originally assumed to be slight - an assumption not supported by subsequent experiments. Depending on the application of the plate tests, a considerable variation in pressure-sinkage relationships and in the variable selected to describe the relationship has resulted. The civil engineers have been interested in bearing capacity and have, therefore, approached plate tests from the viewpoint of cohesion, angle of internal friction, and soil density. Their success has been possible because they were, in general, dealing with small deformation which did not produce a large deviation from the assumption of a direct relationship between shear and bearing capacity. Extension of the approach to large deformation did not produce satisfactory results.

The automotive engineer approached the plate-sinkage problem with a viewpoint biased by the fact that he was dealing with large deformations on the order of twice the width of the loading plate. Following the lead of the civil engineer, attempts were made to maintain a semblance of physical meaning to empirical soil parameters derived from plate-sinkage tests. For example, Bekker originally named his sinkage parameters as cohesive

and frictional moduli. Nuttall developed his correlation factor to describe the change in cohesive properties. Time has shown that the relationship between shear strength and the sinkage of a plate is not straightforward. Time has also indicated that rejection of the Bekker approach has been based on, or excused by, the pressure-sinkage relationship taken by Bekker. The relationship has a fundamental drawback in that the parameters k_c and k_ϕ have dimensions which vary. That is, if

$$p = \left(\frac{k_c}{b} + k_\phi \right) z^n \dots \dots \dots (2)$$

where k_c , k_ϕ , and n are soil parameters that vary with soil type, then the dimensions of k_c and k_ϕ depend upon the power of n . The parameters were originally taken to represent contributions of cohesion, k_c ; of friction, k_ϕ ; and density n . The lack of any demonstratable relationship between the parameters enumerated and associated properties detracted somewhat from the logic of equation. The fact that the equation worked quite well was generally overlooked.

There seems little doubt that a pressure-sinkage equation can be developed using cohesion, friction angle, density, and load variables. Whether the resulting expression will be useful is considerably more questionable.

b. Selection of variables:

In selecting the variables for the dimensional analysis shown in Appendix A, attempts were made to describe both plate

and soil characteristics. The original assumption concerning plate characteristics was that the width of the plate was the controlling variable. However, studies by Hanamoto (8) had indicated that the plate-sinkage relationship was sensitive to both width and length. This conclusion made sense when one considered the failure mechanisms involved: for plates having a large length to width ratio, the soil failure will approach a two-dimensional system; for plates having a small ratio, the failure will be three-dimensional; the relationship between local and general failure will change with plate size. It was concluded, therefore, that the plate dimension of interest was the perimeter since this dimension depends on width, length, and form. A serious doubt can be raised concerning the validity of the perimeter as a variable since the same value for the perimeter can be obtained from a number of different sized and shaped plates. This objection is recognized and data were examined on the basis of fixed shapes. Furthermore, only geometrically similar footings have been tested, that is, rectangular plates with constant aspect ratios, circular plates and square plates. Arguments for the square root of area or of the ratio of the area to the perimeter as being a better variable would be difficult to refute by the writers.

The following is a listing of the variables proposed and the variables selected:

- S = Circumference of plate (L)
 $\sqrt{A}$ = Area of plate (L)
 b = Characteristic width of plate (L)
 ℓ = Characteristic length of plate (L)
 z = Plate sinkage (L)
 h = Depth of soil layer (L)
 v = Velocity (LT^{-1})
 p = Average pressure beneath plate $(ML^{-1}T^{-2})$
 c = Cohesion $(ML^{-1}T^{-2})$
 ϕ = Angle of internal friction (1)
 μ = Coefficient of friction between plates and soil (1)
 γ = Density of soil $(ML^{-2}T^{-2})$

The soil type and plate material were to be kept constant and the soil was taken as having a depth producing semi-infinite conditions so that μ and h were eliminated at the outset. Schuring and Emori (9) found that the critical velocity where the inertia effects become important is equal to $v \geq \sqrt{5 g \ell}$. Yet experiments have been performed at a constant speed of 1 in./sec. which is smaller than the critical speed for all plate sizes tested. Therefore, the speed of penetration have also been neglected from the analysis. It was reasoned that the perimeter and length were better descriptors of plate size and geometry allowing the elimination of b and A . The dimensional analysis considered: S, ℓ, z, p, c, ϕ and γ . The selection of variables is better supported by an examination of test results than by discussion.

c. Cohesive and Frictional Materials:

The results of the study by Vincent and Hicks (5) indicated that scaling of wheel performance was possible if the soil was purely frictional. If cohesion were added to the soil, the scaling became distorted. It was mentioned earlier in this report that this study was confined to frictional materials. In order to investigate the range of cohesion which could be accepted before scaling became distorted, an analysis of the relative contribution of cohesion and friction to bearing capacity was made using Terzachi's equation (11). The results are shown in Figure 1 in which the contribution of cohesion is shown on the ordinate scale and is identified as q_c . The scale is the percentage of bearing capacity attributed to the cohesive component q_c . That is, bearing capacity q , was taken as equal to the sum of the contribution of cohesion, q_c , and the contribution of friction, q_ϕ . As can be seen in the figure, if q_c is to be less than 50% of q , the value of ϕ must be in excess of 35° if cohesion is 0.5 psi. If a soil has a cohesion in excess of 1 psi, it is quite unlikely that the friction angle could be large enough to affect the bearing capacity. The implication of the plot is that in moderately cohesive soils, the primary contribution to strength can be attributed to cohesion. Thus, the scaling laws used should correspond to the requirements of cohesion. However, if weight is computed to maintain dimensional similarity as shown in Appendix A, the result required that weight

be scaled by both the square and the cube. The dimensionless product c/p requires the square, and the product $\gamma l/p$ requires the cube. To establish the error involved in applying one or the other of the laws demands a thorough understanding of the failure mechanism. If we had that understanding, we would not be required to resort to similitude techniques at the outset.

The functional relationships that were developed in Appendix A were written for a semi-infinite soil mass, constant soil and plate materials, and constant sinkage rate. The relationships are:

$$\frac{Z}{S} = f_1 \left[\left(\frac{\gamma l}{p} \right), (\phi), \left(\frac{l}{S} \right) \right] \dots \dots \dots (3)$$

$$\frac{Z}{S} = f_2 \left[\left(\frac{\gamma l}{p} \right), \left(\frac{c}{p} \right), \left(\frac{c}{S \gamma} \right), \phi \right] \dots \dots \dots (4)$$

Equation 3 is for frictional soil and Equation 4 is for a soil having both cohesion and friction.

TEST FACILITIES AND PROCEDURES

Load sinkage curves used to check the theory based on dimensional analysis were obtained by means of the standard Land Locomotion Laboratory Bevameter which is capable of inducing a present constant rate of penetration between a speed range of 0 - 1750 in./min. For purposes of this investigation the penetration speed was maintained at 60 in./min. Operational principles of the test apparatus are shown in Figure 2.

Three different geometrical shapes of footings were used in the performance of load sinkage tests: circular, rectangular and square. The plate sizes selected from each group are listed in Table 1.

Test materials consisted of:

- a. Dry Ottawa Sand
- b. Wet Ottawa Sand
- c. Dry Iowa Sand
- d. Detroit Clay
- e. Snow

Classification characteristics and pertinent engineering properties of the materials tested are tabulated in Table II.

For the wet Ottawa sand, soil preparation consisted of flooding the entire soil bin by water through a drain pipe with lateral manifolds embedded on the bottom of the soil bin and then compacting the sand by quick drainage. Using such preparation the density readings reproduced within $\pm 1 \text{ lb./ft.}^3$, and the moisture content readings reproduced within $\pm 0.25\%$ in consecutive test runs.

T A B L E I.

Circular			Rectangular			Square	
Dia. (in.)	Area (in. ²)	Length (in.)	Width (in.)	Area (in. ²)	Width (in.)	Area (in. ²)	Area (in. ²)
2.0	3.14	4.5	1.0	4.5	2.0	4.0	4.0
3.0	7.06						
4.0	12.56	9.0	2.0	18.0	3.0	9.0	9.0
5.0	19.72						
6.0	28.26	13.5	3.0	40.5	4.0	16.0	16.0
8.0	50.24	18.0	4.0	72.0	5.0	25.0	25.0

T A B L E I I .

Soil Type	Mechanical Analysis			Water Content %	Density lb./ft. ³
	Sand %	Silt %	Clay %		
Dry Ottawa Sand	100	—	—	Air Dry	105.0
Wet Ottawa Sand	100	—	—	2.2	110.0
Dry Iowa Sand	100	—	—	Air Dry	100.0
Detroit Clay	20	35	45	26.0	120.0
Snow	—	—	—	—	17.0

PROPERTIES OF MATERIALS TESTED

No special preparation was required for dry Ottawa and Iowa sands to obtain uniform soil conditions. Two passes with a rake having 18-inch tines were made before each series of test runs produced uniform and consistent soil conditions as determined from the reproducibility of results.

The Detroit clay was remolded and compacted by a pneumatic hammer before each test run. Due to the difficulties involved in the preparation of large masses of clay the load sinkage results reported here cannot be considered as entirely reliable. The deviations involved in the density measurement were found to be approximately twice as high as for sands and the load sinkage curves obtained in repeated trial runs were considerably more scattered.

At least two trial runs were obtained with each penetration plate in each type of soil tested. Load sinkage curves reported here are therefore the averages of several measurements. The soil bins used in this study were sufficiently large to eliminate both side and bottom interference on the basis of a rule of thumb which states that a soil bin should have the minimum dimensions of: width equal to ten times the width of the widest plate and a depth equal to the sinkage, plus $2\frac{1}{2}$ times the plate width.

The snow tests were performed in the field at Houghton, Michigan during January and February, 1964. The test instrument used for the snow tests was a Bevameter mounted on a M-29C tracked "Weasel". The average snow density and snow depth were found to be 17 lb./ft.³ and three feet respectively. Approximately ten load sinkage curves were

obtained with each footing in snow in order to minimize errors inherent in the non-uniformity of snow cover. Again, the averages of these measurements are reported here.

The depth of penetrations in all sinkage measurements was at least six inches.

RESULTS

The results of the load-sinkage experiments and dimensional analysis are presented in graphs.

Figures 3 through 13 show load sinkage relationships obtained by using circular, rectangular and square plates in all the soils and snow tested. These load sinkage curves had been utilized to investigate the possibility of predicting the behavior of a larger (prototype) sinkage plate on the basis of a small (model) plate without utilizing model soils. Figures 14, 15, 16, and 17 show plots of load sinkage data in terms of dimensionless parameters $(\frac{Z}{S})$ vs. $(\frac{\gamma L}{p})$ for sands and snow. The above plots show remarkable collapse of data into a single curve for a given plate geometry which implies that the size effects were properly taken into account in the dimensionless parameters derived. This conclusion appears to be true for the entire length of the curves. For sinkages higher than three or four inches, however, the $(\frac{Z}{S})$ vs. $(\frac{\gamma L}{p})$ plots can be characterized with one curve for all plate geometries tested in both dry and wet Ottawa Sands. Therefore, it appears that beyond the realm of bearing capacity considerations, only plate size affects the

load sinkage relationships for a purely frictional soil.

Unfavorable results were obtained with clay. The $(\frac{Z}{S})$ vs. $(\frac{L}{p})$ plot of dimensionless parameters resulted in a distinct well-defined curve for each plate size tested. The clay tested had a zero friction angle and a cohesion of 1.3 psi. The cohesion (c) was the controlling parameter as shown in Figure 1, therefore, the data was plotted using the $(\frac{Z}{b})$ vs. $(\frac{C}{p})$ dimensionless parameters. Although these results looked somewhat encouraging, no satisfactory collapse of load sinkage data was achieved, particularly for larger sinkages. No doubt this inability is partially due to the non-uniformity of Detroit clay in which the load sinkage data was obtained. Therefore, it was decided that the experiments and the analysis of data pertaining to cohesive soils will be repeated and reported at a later date.

It is quite likely that a considerable objection will be raised concerning the selection of the dimensionless parameter $(\frac{ZL}{p})$. This form of the parameter produces very small values as compared to $(\frac{p}{LZ})$ and would not have the disadvantage of a discontinuity at zero pressure. The form was chosen because the result of the dimensional analysis indicated that we could accept a discontinuity either for the parameter $(\frac{LZ}{p})$ or $(\frac{p}{C})$. Because pressures near zero are of little or no interest in sinkage studies, the $(\frac{ZL}{p})$ form was selected.

ANALYSIS OF RESULTS

Figures 14, 15, 16, and 17 indicate that in a plot of the load sinkage data obtained with different sizes of plates in terms of dimensionless parameters, nearly all points are situated along an experimental curve for a given geometrical configuration of plates. Therefore, it seems feasible to assume that prediction of load sinkage relationships under large contact areas can be made on the basis of model load sinkage tests if the equations of the experimental curves are known.

It was found that by plotting $(\frac{Z}{S})$ against $(\frac{\sigma L}{p})$ on log-log paper a well defined straight-line function resulted for the sands and snow tested. Log-log plots of dimensionless parameters are shown in Figures 18, 19, 20 and 21. The equation of any of these straight lines may be described as follows:

$$\log \left(\frac{Z}{S} \right) - \log A = m \left[\log \left(\frac{\sigma L}{p} \right) - \log B \right] \dots \dots \dots 5$$

where "A" and "B" are the ordinate and abscissa of any point along the straight line respectively and "m" is the slope of the straight line. They are dimensionless constants associated with soil properties and plate geometry.

Eliminating the logs on both sides and solving equation 5 for p one gets:

$$p = \frac{\sigma L}{A^{\frac{1}{m}}} \left(\frac{Z}{S} \right)^{\frac{1}{m}} \dots \dots \dots 6$$

Equation 6 represents a "new" load sinkage relationship based on experiment within the framework of a dimensional analysis.

Observing that $\frac{1}{m} = n_o$ and that $\frac{\sigma}{A^{n_o} B S^{n_o}}$ is a constant value K for a given soil and plate size, Equation 6 can be reduced to the familiar simple form:

$$p = K z^{n_o}$$

Theoretically only one load-sinkage curve with a model footing is necessary for the evaluation of constants associated with Equation 6 and to predict the sinkage behavior of a large loading area from a small plate test. However, the accuracy of such predictions can obviously be improved if data from several plate sinkage tests are used in determining the plate sinkage equation.

In order to evaluate the usefulness of the proposed equation, two criteria were proposed: the accuracy of predicting the shape of the pressure-sinkage curve of a prototype plate using a small plate as a model and the accuracy of predicting the work involved in sinking a prototype plate to a given depth. The latter criterion is considered to be the more important measure because the application of the pressure-sinkage relationship to the vehicle problem involves the computation of work expended in sinking to a given depth. The resulting work computation is then related to motion resistance.

Two sets of data were analyzed: data for rectangular plates in dry Ottawa sand and data for circular plates in saturated Ottawa sand. In the case of the dry sand, the pressure-sinkage curve for a 4x18" plate

was predicted by means of the Bekker equation and by the similitude relationship. The Bekker parameters were obtained from a $1 \times 4\frac{1}{2}$ inch plate and a 2×9 inch plate. The dimensionless parameters were established from the data for the $1 \times 4\frac{1}{2}$ inch plate. In each case, the $p - z$ curve was predicted and the work required to force the 4×18 inch plate to a depth of 7 inches was computed. The results are shown by Figure 22 on where the work is tabulated in the upper righthand corner. The measured curve is shown in the figure to provide a visual comparison. The figure indicates that the curve predicted by the dimensional approach would be equally accurate as the Bekker equation for predicting work up to a sinkage of approximately three inches. At sinkages in excess of three inches and up to seven inches, the dimensional approach is more accurate. It is apparent that the sinkage prediction by means of the Bekker equation is much more accurate than the prediction by similitude.

In the examination of the circular plates, the Bekker parameters were obtained from two-inch and four-inch diameter plates and the dimensional parameters from the two inch plate. The pressure-sinkage curve for a six-inch plate was predicted from both sets of data and the work expended in sinking to seven inches was computed. The comparison between predicted and measured results is shown in Figure 23. The dimensional data produce a more accurate prediction of both sinkage and work for the circular plate. This result is consistent with the results of Hanamoto (8) who found that the circular plates produced much more uniform results than did rectangular plates.

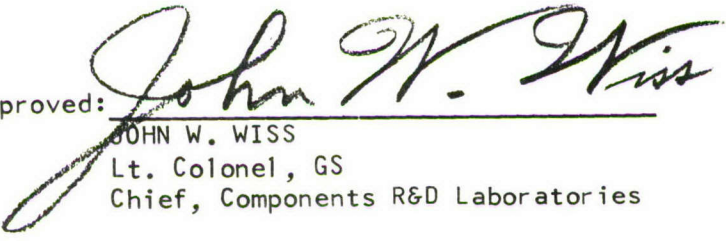
CONCLUSIONS:

The results of the experiments reported provide the following conclusions:

1. A pressure-sinkage relationship based on dimensional similitude techniques is possible for frictional soils.
2. A pressure-sinkage relationship based on dimensional similitude may not be possible for cohesive soils or soils having a "significant" amount of cohesion as defined by Figure 1 where "significant" cohesion is taken as producing a q_c in excess of 50%.
3. In frictional materials, the equation based on dimensional similitude predicts work more accurately than the Bekker equation.
4. The prediction technique based on dimensional similitude only requires a single plate test.

RECOMMENDATIONS:

The study of the scaling of the behavior of plates should be extended to include purely cohesive soils, soils having both cohesion and friction, and wet snow.

Approved: 

JOHN W. WISS

Lt. Colonel, GS

Chief, Components R&D Laboratories

APPENDIX A.

DIMENSIONAL ANALYSIS

By: Z. Janosi

It is assumed that the pressure-sinkage relationship is governed by the following variables*:

s:	circumference of the plate	(L)
γ :	bulk density of the soil	(M L ⁻² T ⁻²)
L:	characteristic length of the plate	(L)
p:	average pressure under the plate	(M L ⁻¹ T ⁻²)
c:	cohesion	(M L ⁻¹ T ⁻²)
h:	depth of soil layer	(L)
ϕ :	angle of internal friction	(1)

Accordingly, the following function must exist:

$$f[s^{\alpha_1} \times \gamma^{\alpha_2} \times L^{\alpha_3} \times p^{\alpha_4} \times c^{\alpha_5} \times h^{\alpha_6} \times \phi^{\alpha_7}] = 0$$

Since the dimension of the terms in the brackets is required to be one, one may write the following:

$$L^{\alpha_1} \times L^{\alpha_2} \times (M L^{-2} T^{-2})^{\alpha_3} \times L^{\alpha_4} \times (M L^{-1} T^{-2})^{\alpha_5} \times (M L^{-1} T^{-2})^{\alpha_6} \times L^{\alpha_7} \times 1^{\alpha_8} = 1$$

$$\text{or } L^{\alpha_1 + \alpha_2 - 2\alpha_3 + \alpha_4 - \alpha_5 - \alpha_6 + \alpha_7} \times M^{\alpha_3 + \alpha_5 + \alpha_6} \times T^{-2(\alpha_3 + \alpha_5 + \alpha_6)} = 1$$

*For a complete description of dimensional analysis techniques, see Reference 10.

If the last equation is to hold, in general, then it must be true that:

$$L \quad \alpha_1 + \alpha_2 - 2\alpha_3 + \alpha_4 - \alpha_5 - \alpha_6 + \alpha_7 = 1$$

$$M \quad \alpha_3 + \alpha_5 + \alpha_6 = 1$$

$$T \quad -2(\alpha_3 + \alpha_5 + \alpha_6) = 1$$

$$I \quad \alpha_8 = 1$$

which implies that:

$$\alpha_1 + \alpha_2 - 2\alpha_3 + \alpha_4 - \alpha_5 - \alpha_6 + \alpha_7 = 0 \dots (1)$$

$$\alpha_3 + \alpha_5 + \alpha_6 = 0 \dots \dots \dots (2)$$

$$\alpha_8 = \text{any number}$$

Thus, two of the seven unknowns may be eliminated.

It can be seen that:

$$\alpha_2 = -\alpha_1 + \alpha_3 - \alpha_4 - \alpha_7 \dots \dots \dots (3)$$

and $\alpha_5 = -\alpha_3 - \alpha_6 \dots \dots \dots (4)$

Using Equations (3) and (4) the original functional relationship may be rewritten as follows:

$$f \left[z^{\alpha_1} x_s^{-\alpha_1} x_s^{\alpha_3} x_s^{-\alpha_4} x_s^{-\alpha_7} x_s^{\alpha_3} x_s^{\alpha_4} x_p^{-\alpha_3} x_p^{-\alpha_6} x_c^{\alpha_6} x_h^{\alpha_7} x_\rho^{\alpha_8} \right] = 0$$

so that:

$$f \left[\left(\frac{z}{s}\right)^{\alpha_1} x \left(\frac{s}{p}\right)^{\alpha_3} x \left(\frac{p}{s}\right)^{\alpha_4} x \left(\frac{c}{p}\right)^{\alpha_6} x \left(\frac{h}{s}\right)^{\alpha_7} x (\rho)^{\alpha_8} \right] = 0$$

This can be rewritten as:

$$\left(\frac{z}{s}\right)^{\alpha_1} = f_1 \left[\left(\frac{s}{p}\right)^{\alpha_3} \left(\frac{p}{s}\right)^{\alpha_4} \left(\frac{c}{p}\right)^{\alpha_6} \left(\frac{h}{s}\right)^{\alpha_7} (\rho)^{\alpha_8} \right]$$

where f_1 is not the same function as f .

But equations (1) and (2) are satisfied no matter what arbitrary value is assigned to $\alpha_1, \alpha_3, \alpha_4, \alpha_6, \alpha_7$ and α_8

Thus, it is convenient to use

$$\begin{aligned} \alpha_3 &= \alpha_4 = \alpha_7 = \\ \alpha_8 &= 1 \\ \alpha_1 &= \alpha_6 = -1 \end{aligned}$$

so that

$$\frac{z}{s} = f_1 \left[\left(\frac{s}{p}\right) \left(\frac{p}{s}\right) \left(\frac{c}{p}\right) \left(\frac{h}{s}\right) (\rho) \right] \dots \dots \dots (5)$$

Since f_1 is an unknown function one is justified to write the following:

$$\frac{z}{s} = f_2 \left[\left(\frac{p}{h}\right) \left(\frac{c}{p}\right) \left(\frac{p}{h}\right) \left(\frac{c}{s}\right) (\rho) \right] \dots \dots \dots (6)$$

where f_2 is a new function.

The dimensionless products in the brackets of Equation 6 are obtained by linear combinations of those in Equation 5, as follows:

$$\frac{p}{s \sigma} \times \frac{s}{l} = \frac{p}{l \sigma}$$

$$\frac{p}{s \sigma} \times \frac{s}{h} = \frac{p}{h \sigma}$$

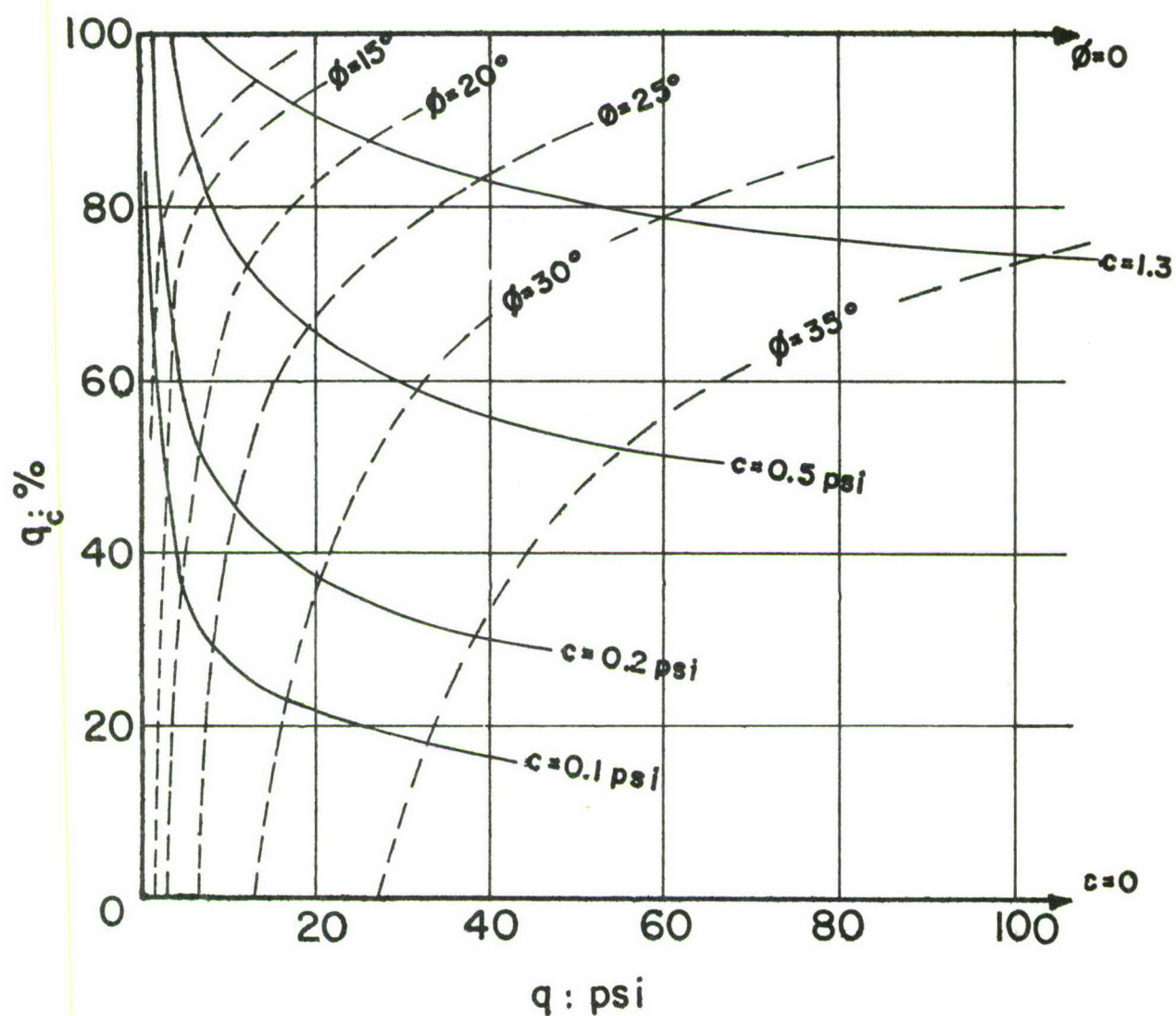
$$\left(\frac{p}{s \sigma}\right) \times \left(\frac{c}{p}\right) = \frac{c}{s \sigma}$$

When a transformation of dimensionless products is performed it is necessary to ascertain that there are as many new products as original products. Note that both equations, 5 and 6, contain five independent dimensionless products or master variables.

In dry sand $c = 0$ and assuming a deep homogeneous soil layer, $h = \infty$, and provided that geometrically similar ($\frac{l}{s} = \text{constant}$) plates are tested. Equation 6 reduces to

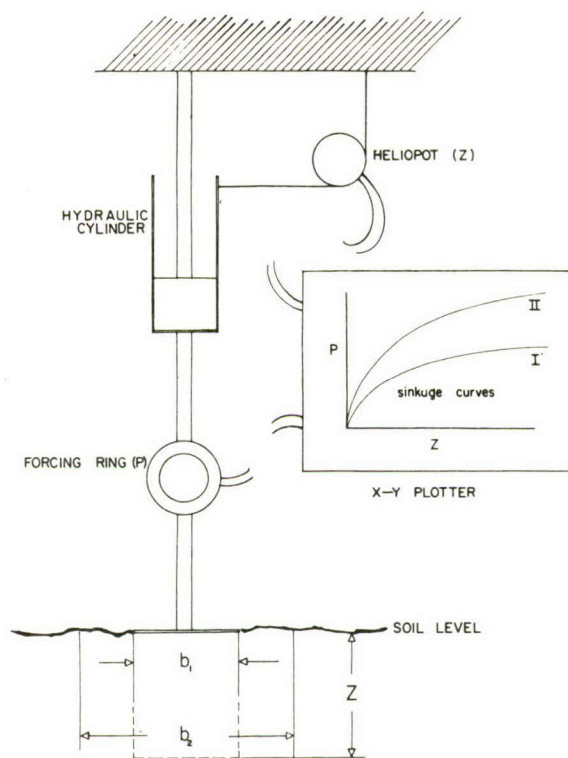
$$\frac{z}{s} = f \left[\left(\frac{l \sigma}{p} \right) (\phi) \right] \dots \dots \dots (7)$$

According to Equation 7, if one uses plates of several different lengths and circumferences in the same dry sand ($\sigma = \text{const}$, $\phi = \text{const}$), then for a given $\frac{l}{p}$ ration the $\frac{z}{s}$ fatios must be constant. In other words the $\frac{z}{s}$ versus $\frac{\sigma l}{p}$ durves must "collapse". If actual test results support this conclusion then the analysis may be judged successful.



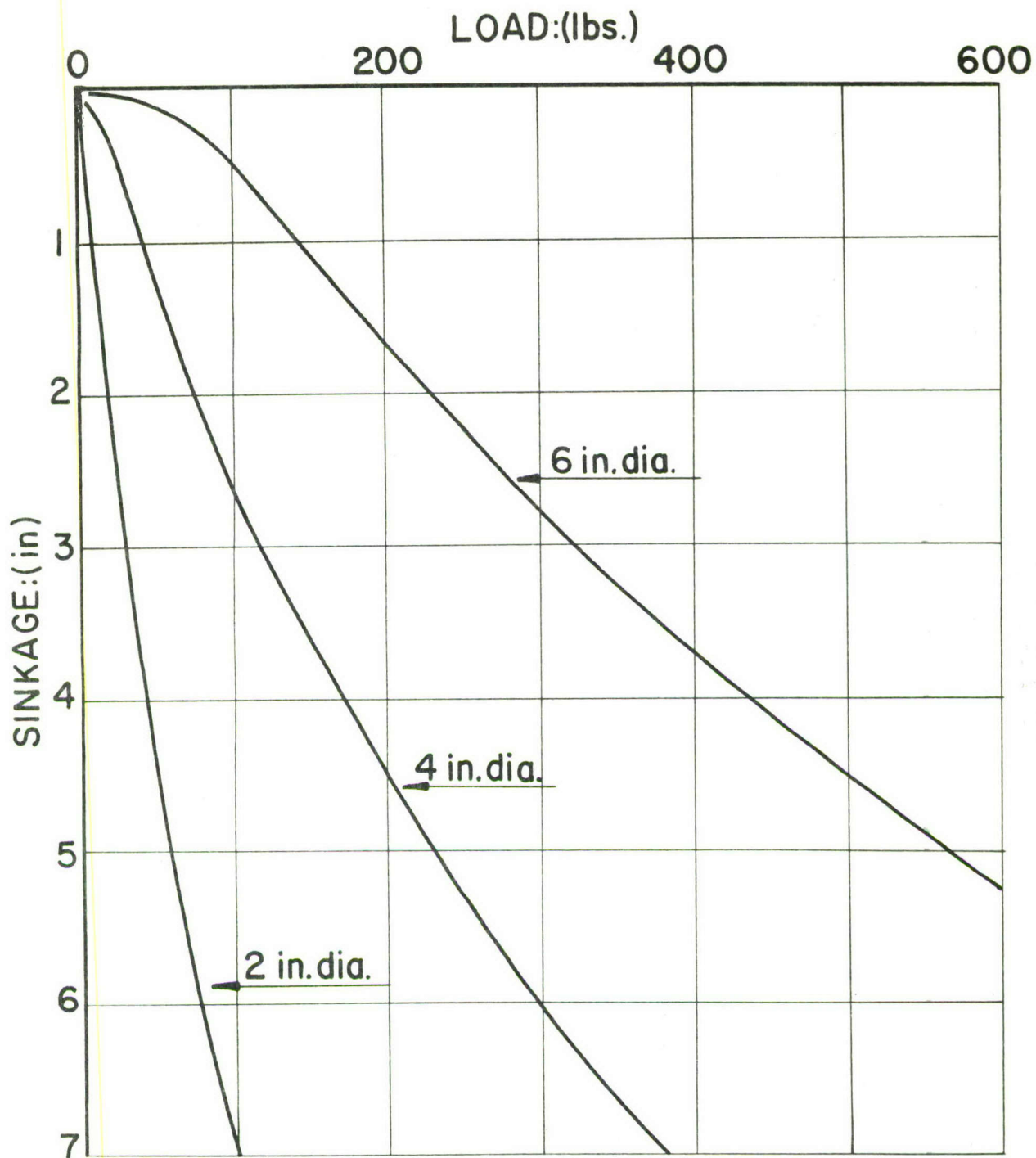
CONTRIBUTION OF COHESION TO
SURFACE BEARING CAPACITY.

Figure 1.



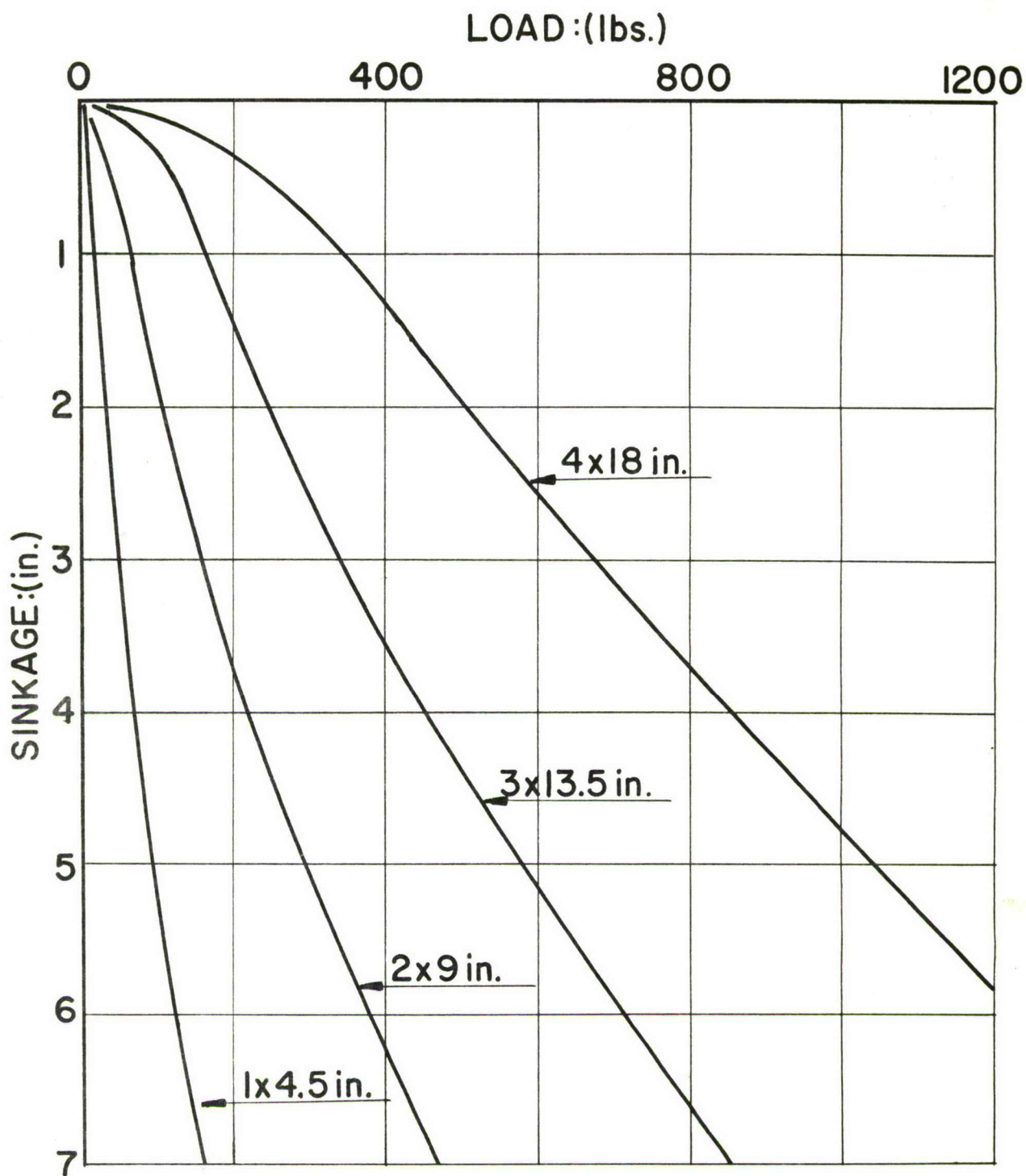
SCHEMATIC OF PLATE PENETRATION DEVICE

Figure 2.



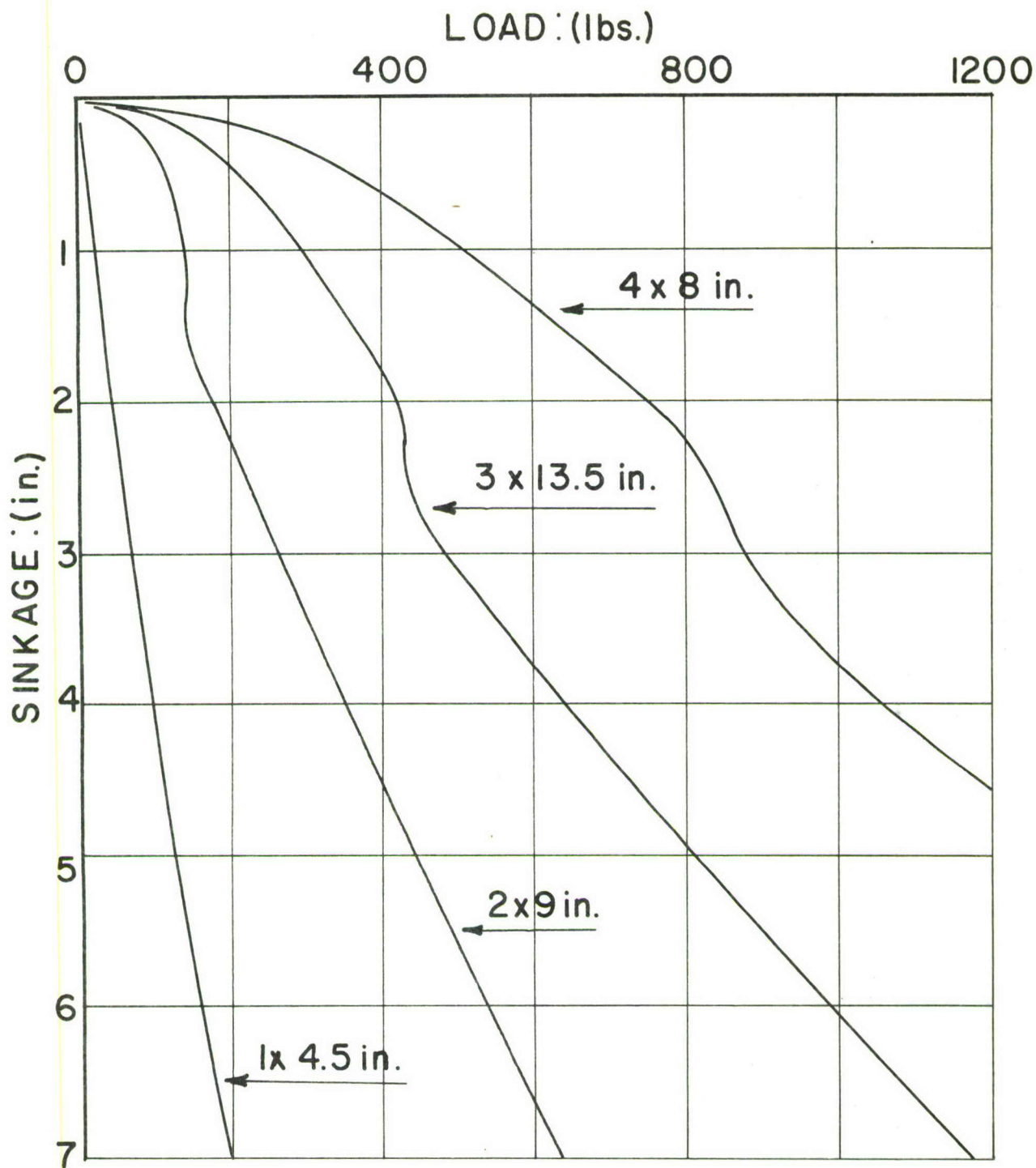
CIRCULAR PLATE SINKAGE TESTS
IN DRY OTTAWA SAND

Figure 3.



RECTANGULAR PLATE SINKAGE TESTS
IN DRY OTTAWA SAND

Figure 4.



RECTANGULAR PLATE SINKAGE TESTS
IN WET OTTAWA SAND

Figure 5.

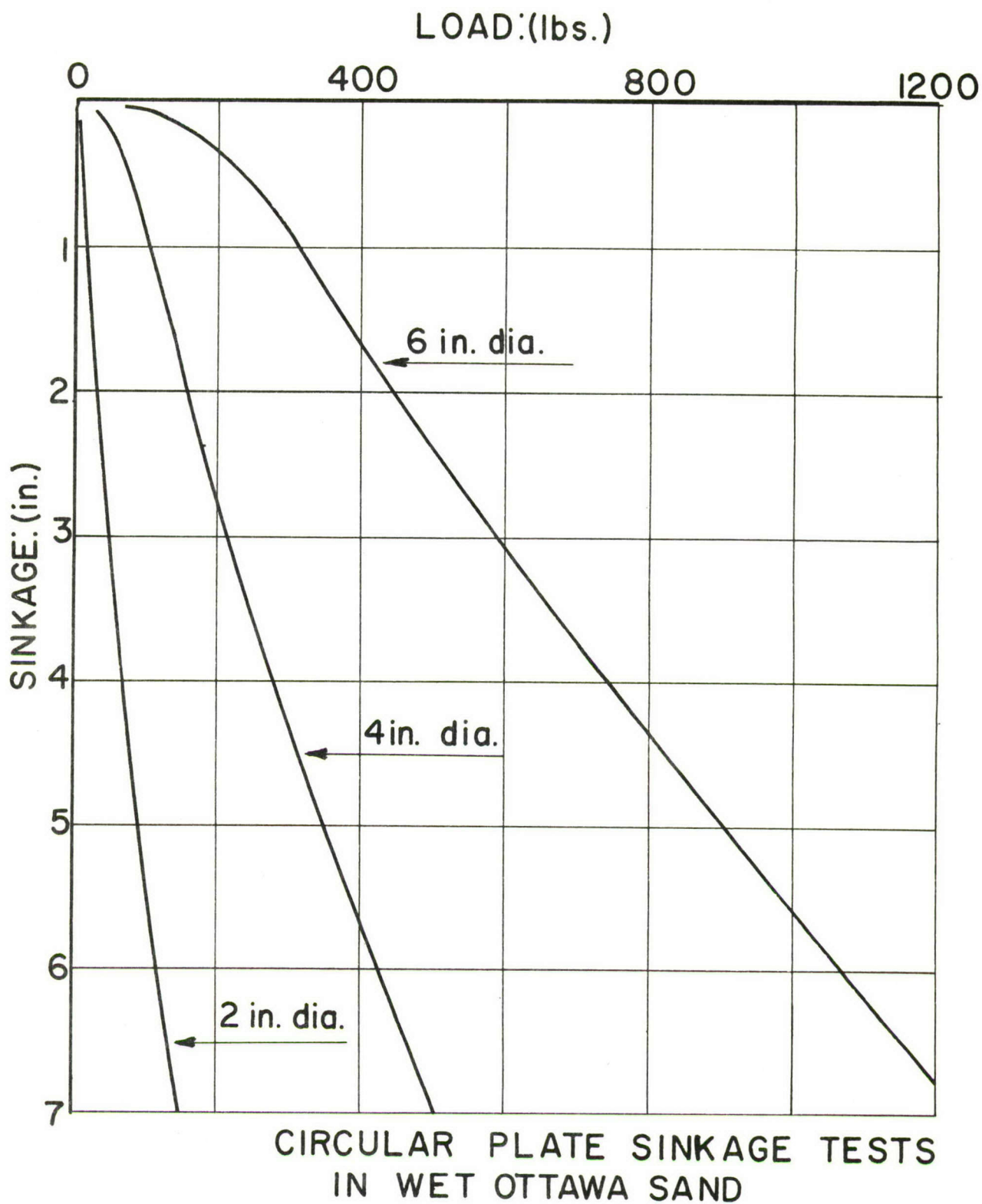
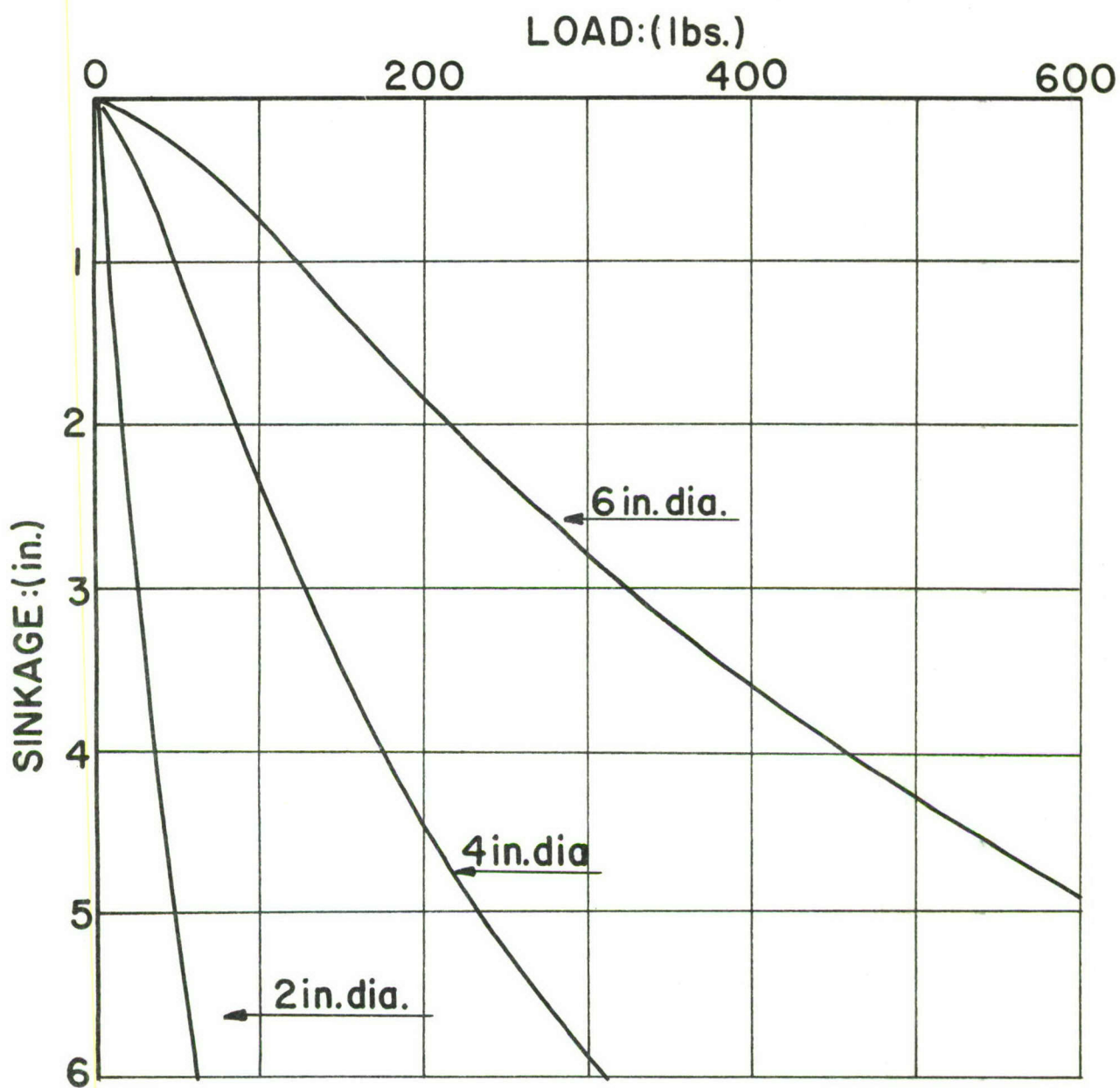
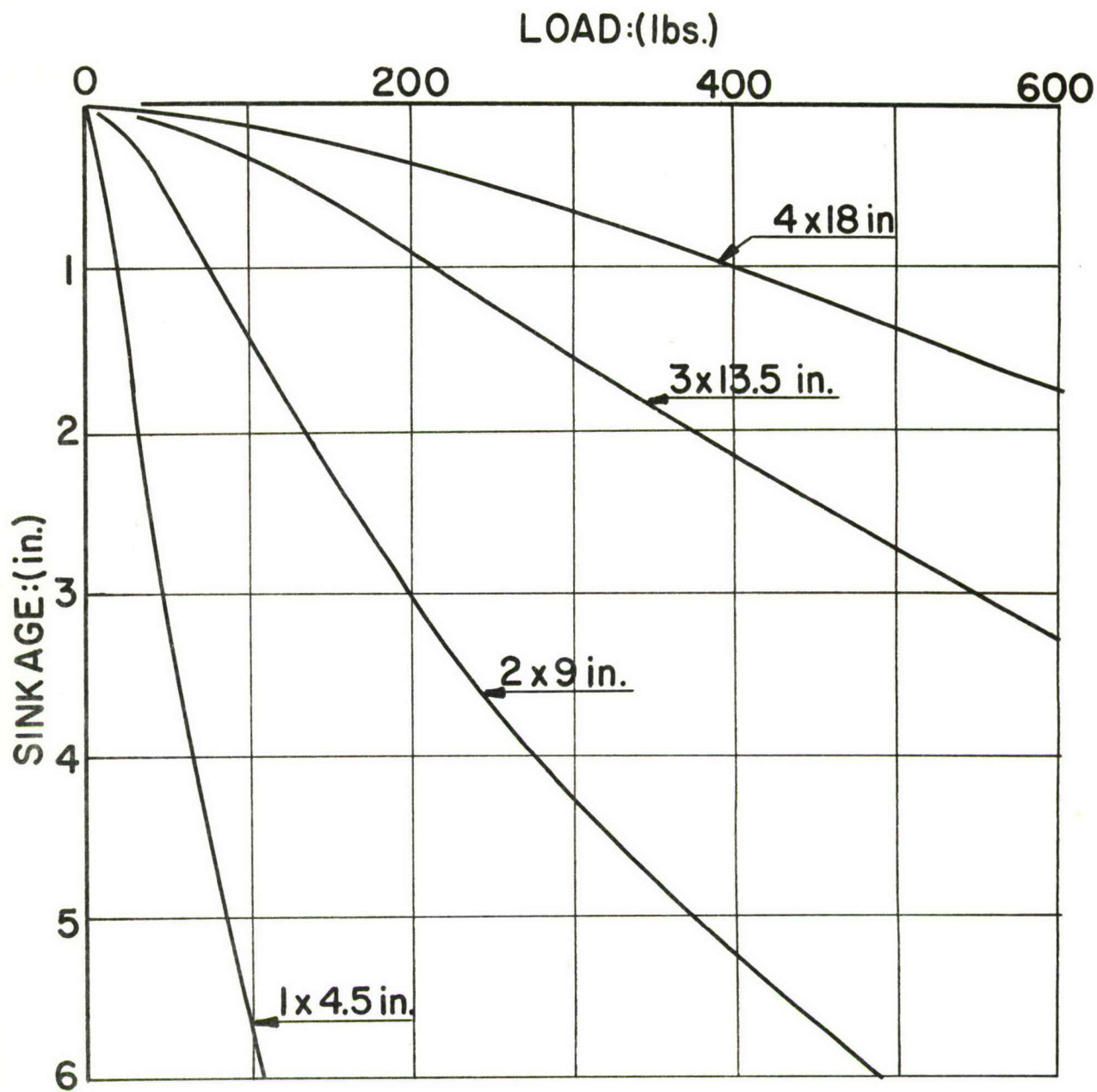


Figure 6.



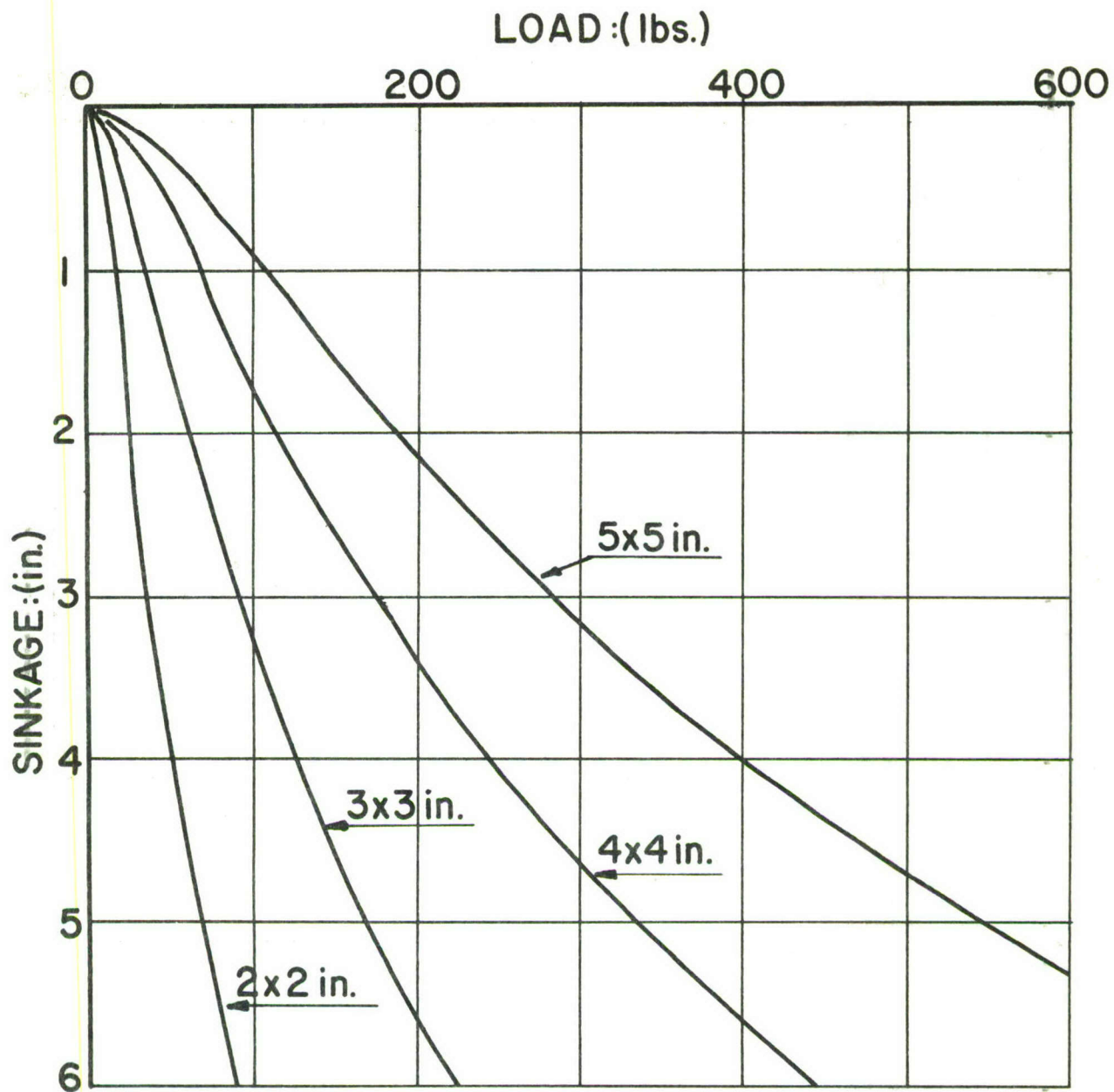
CIRCULAR PLATE SINKAGE TESTS
IN IOWA SAND

Figure 7.



RECTANGULAR PLATE SINKAGE TESTS
IN IOWA SAND

Figure 8.



SQUARE PLATE SINKAGE TESTS IN IOWA SAND

Figure 9.

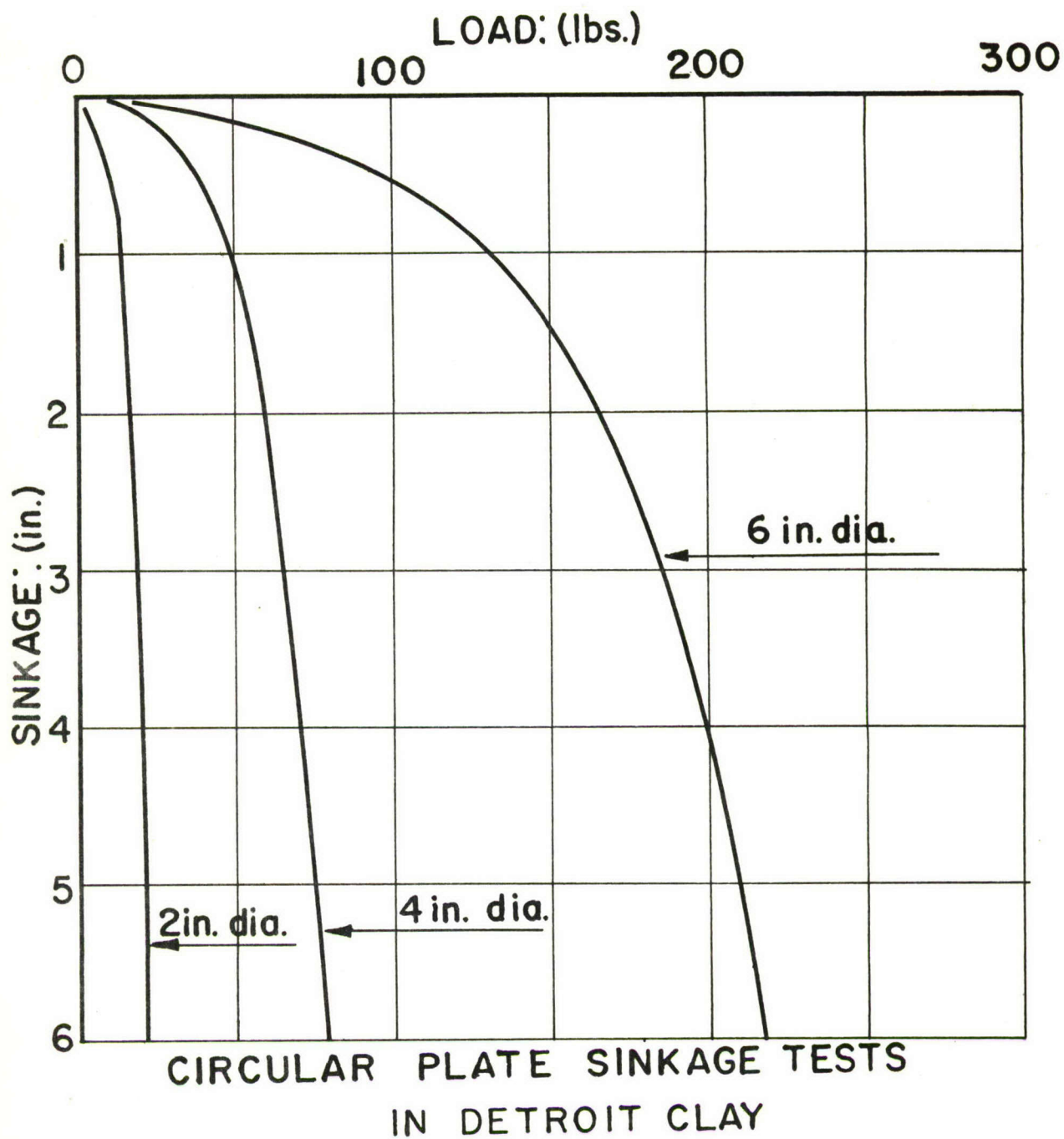
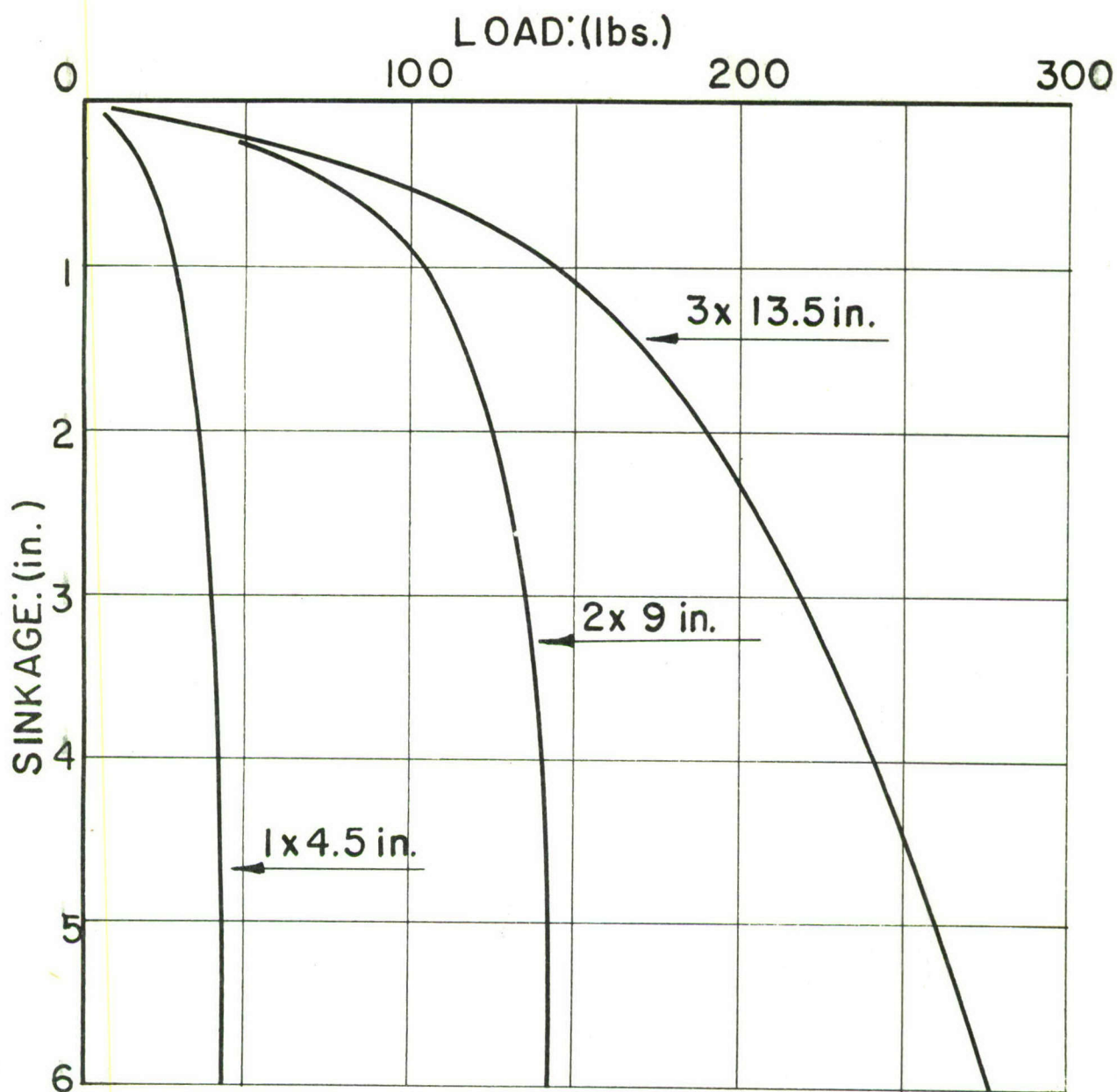


Figure 10.



RECTANGULAR PLATE SINKAGE TESTS
IN DETROIT CLAY

Figure 11.

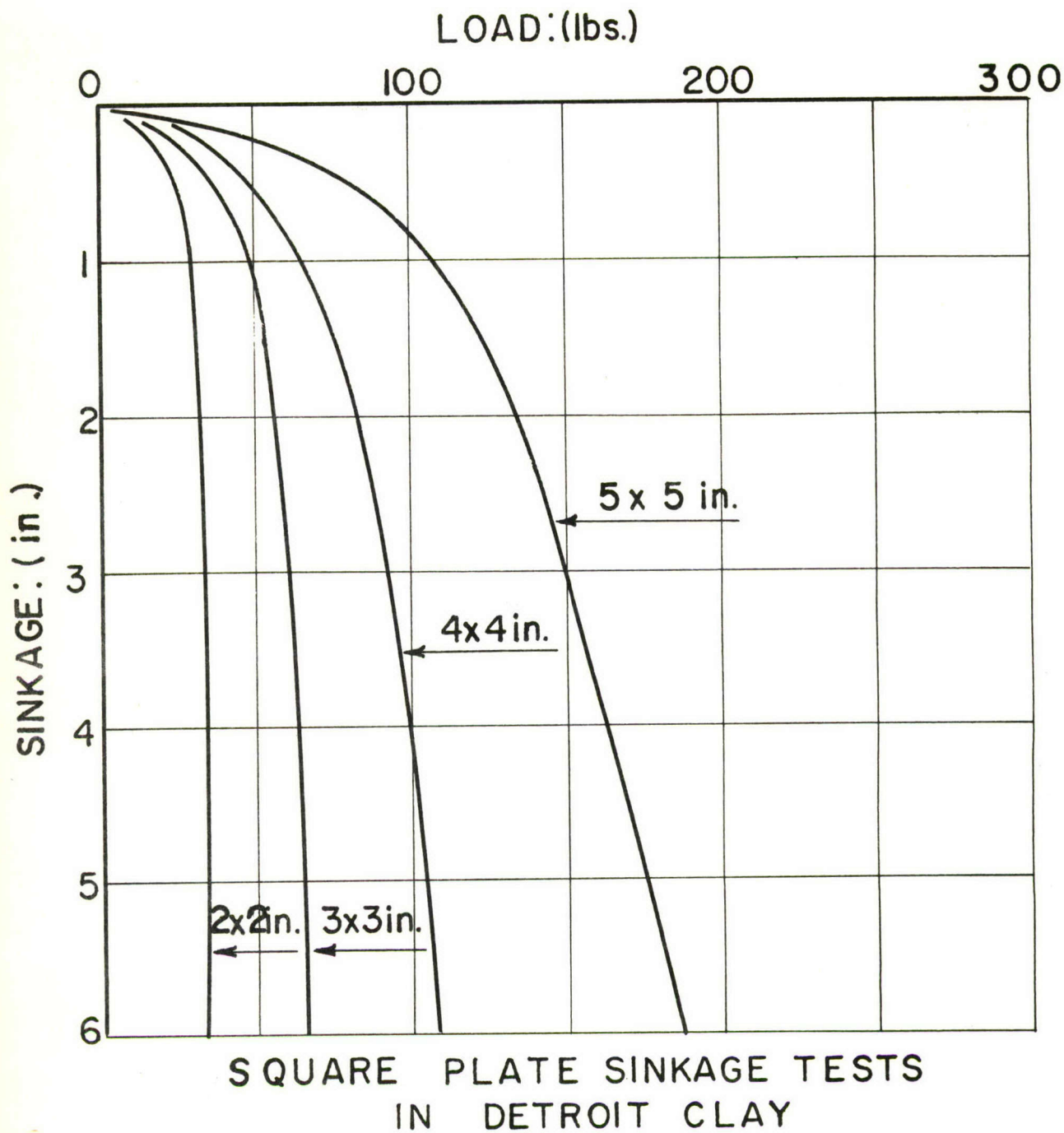
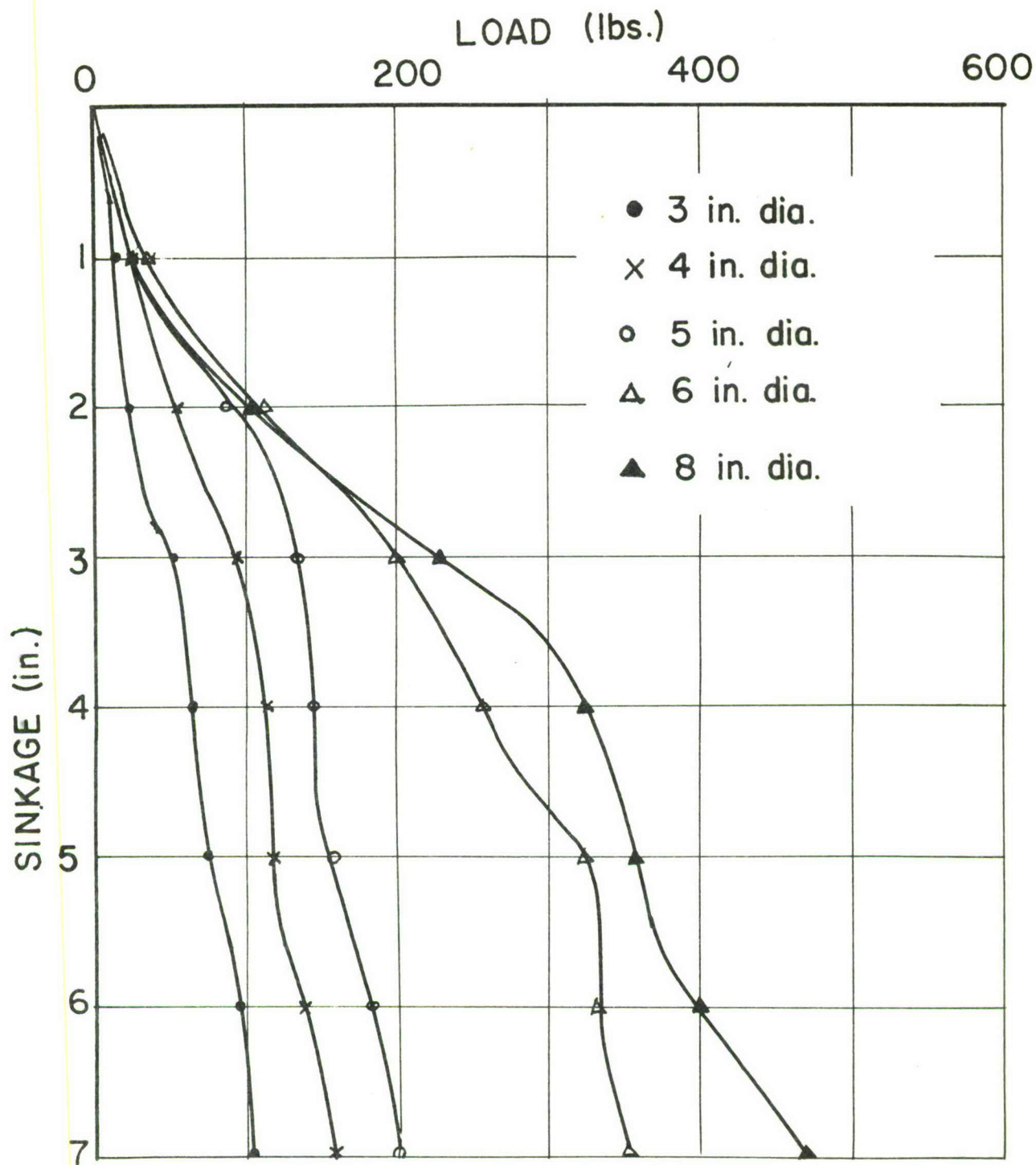
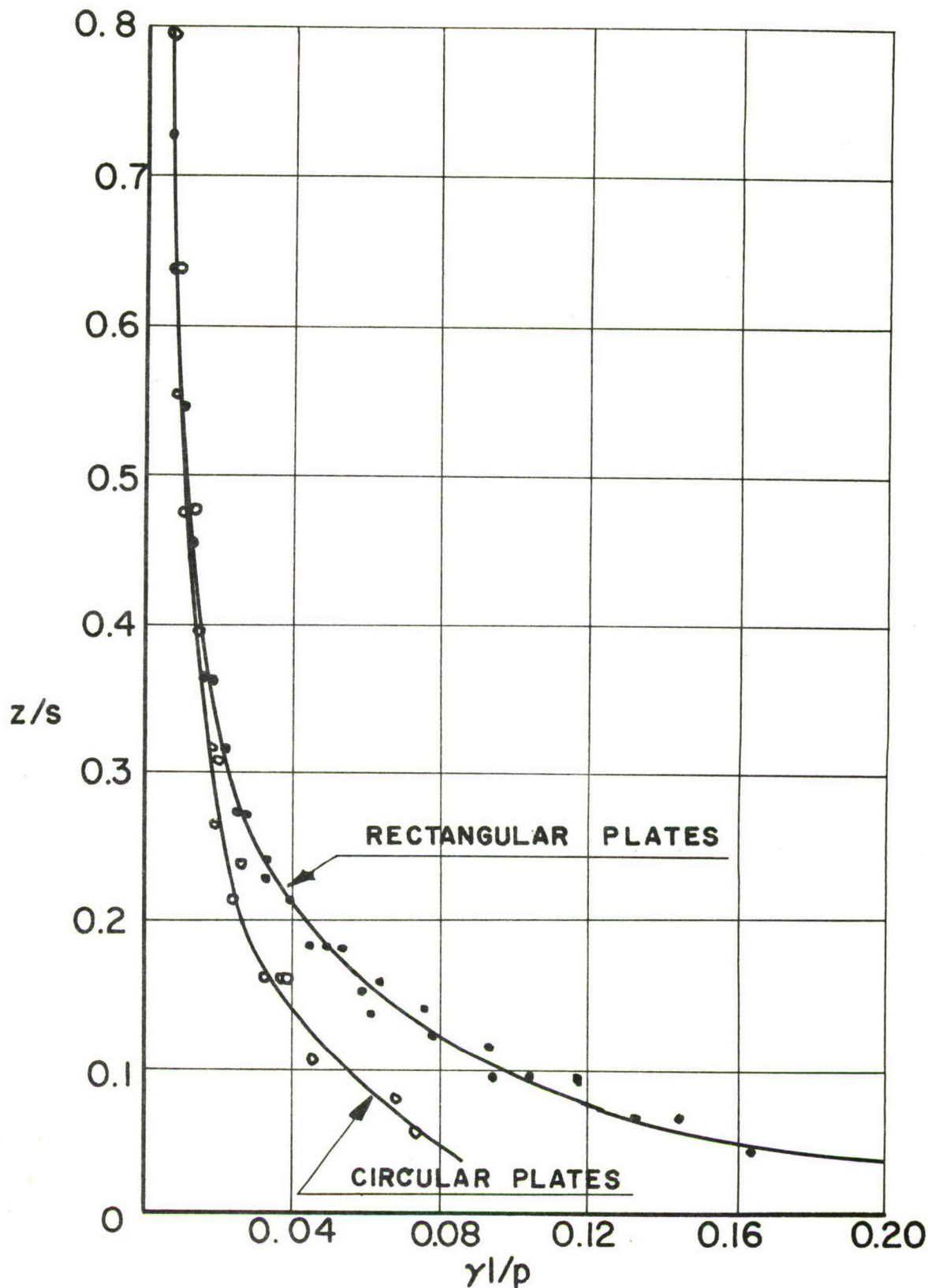


Figure 12.



CIRCULAR PLATE SINKAGE TESTS IN SNOW

Figure 13.



DIMENSIONLESS PLOT OF p vs. z
FOR DRY OTTAWA SAND

Figure 14.

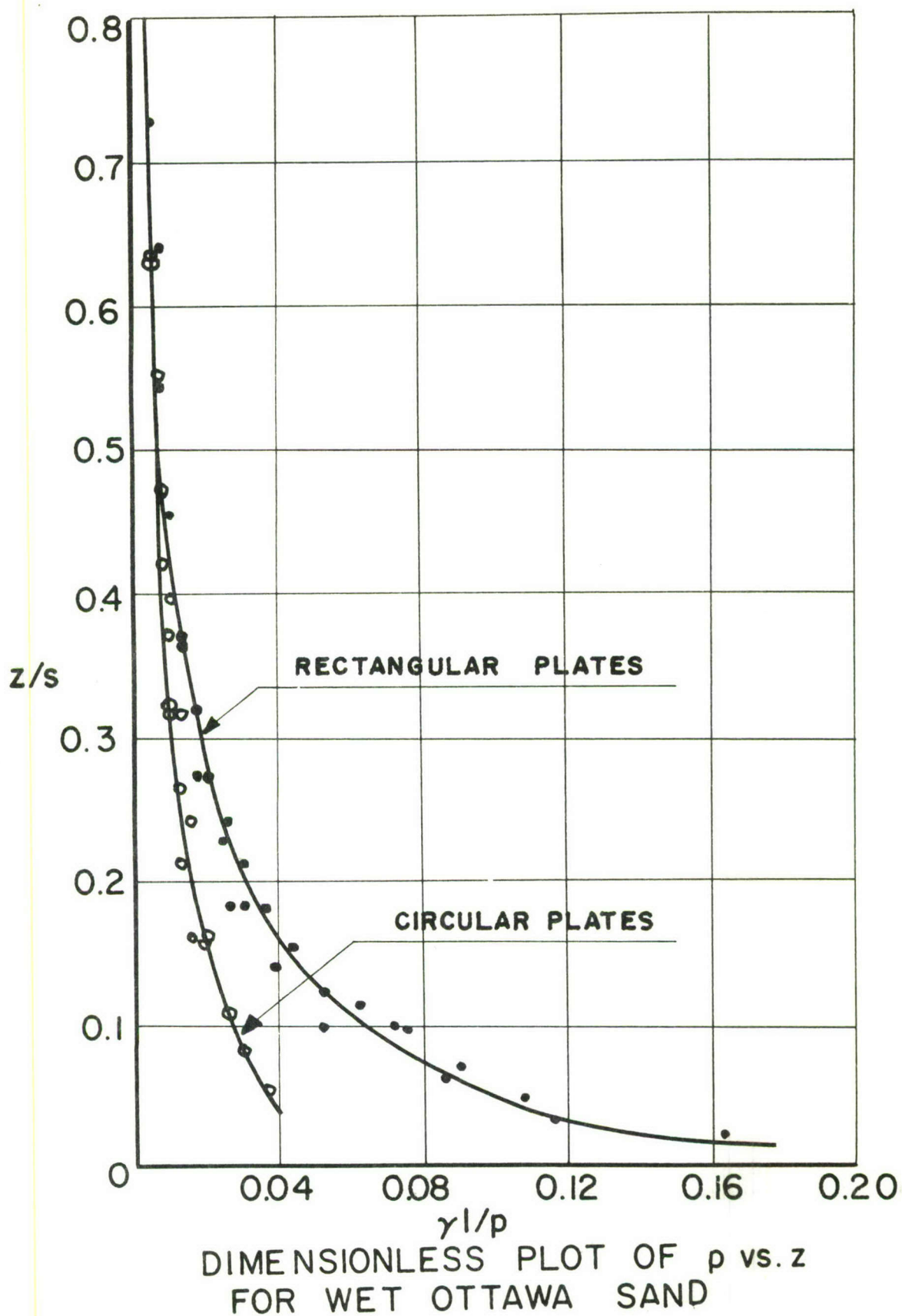


Figure 15.

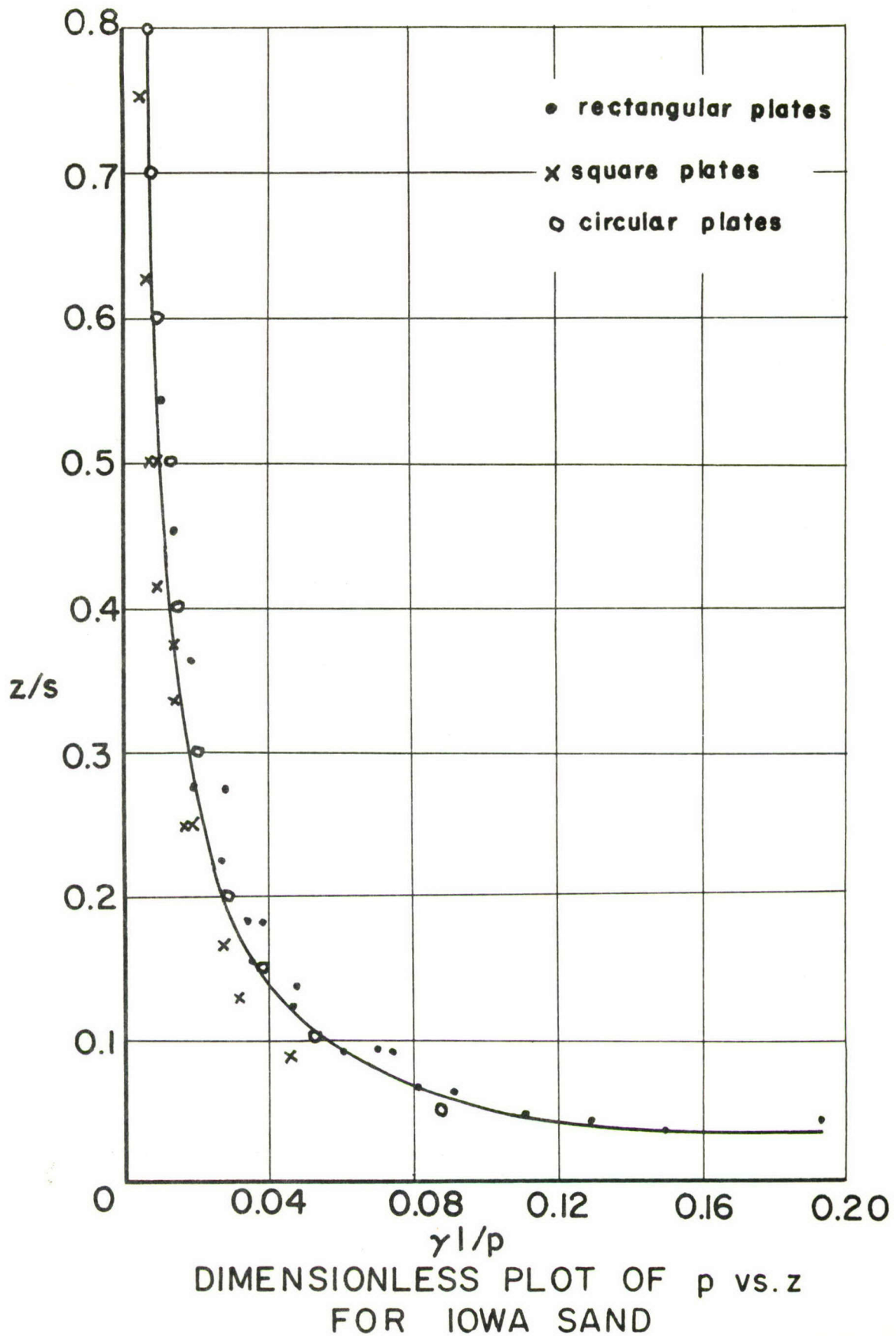


Figure 16.

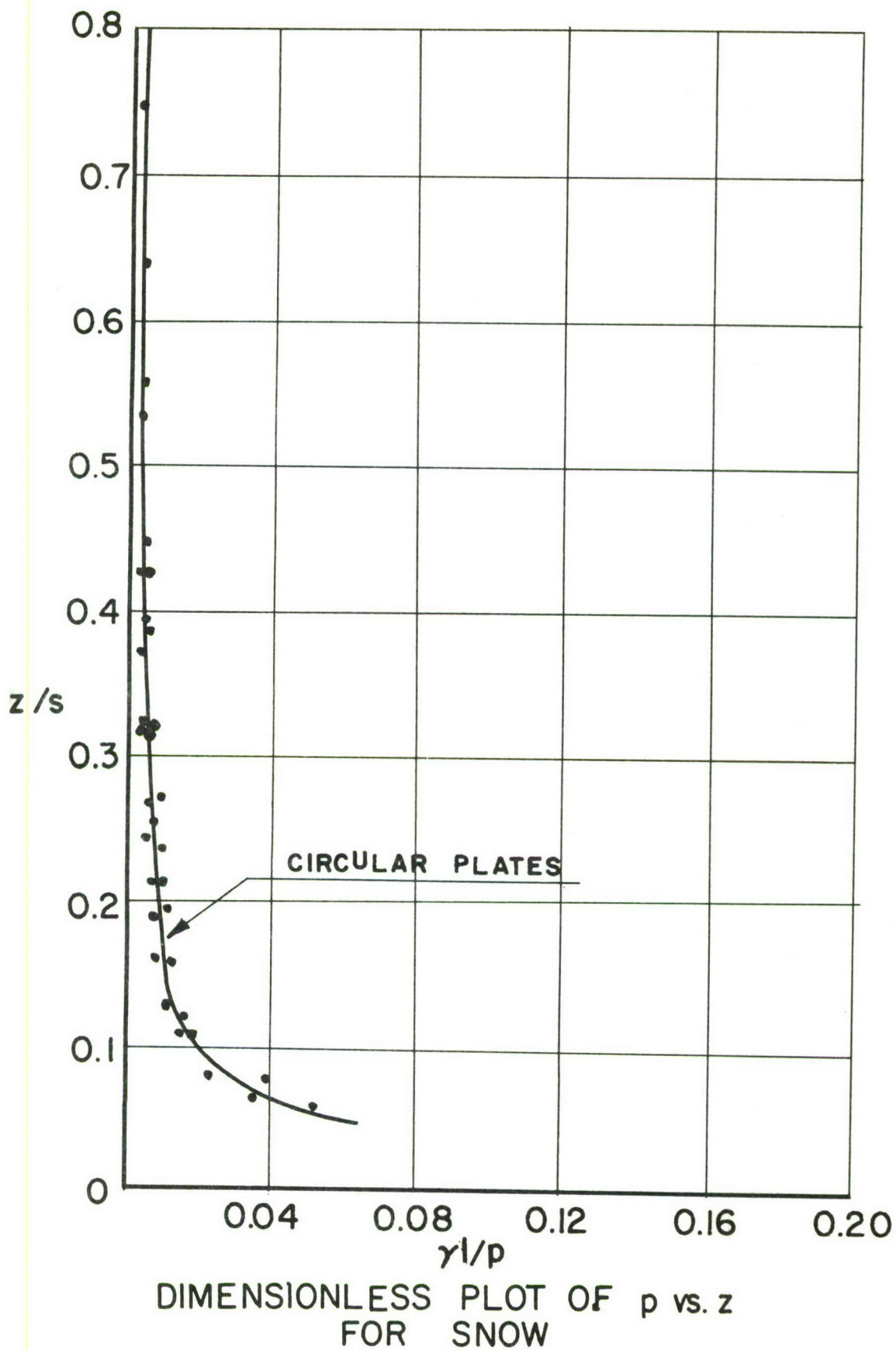
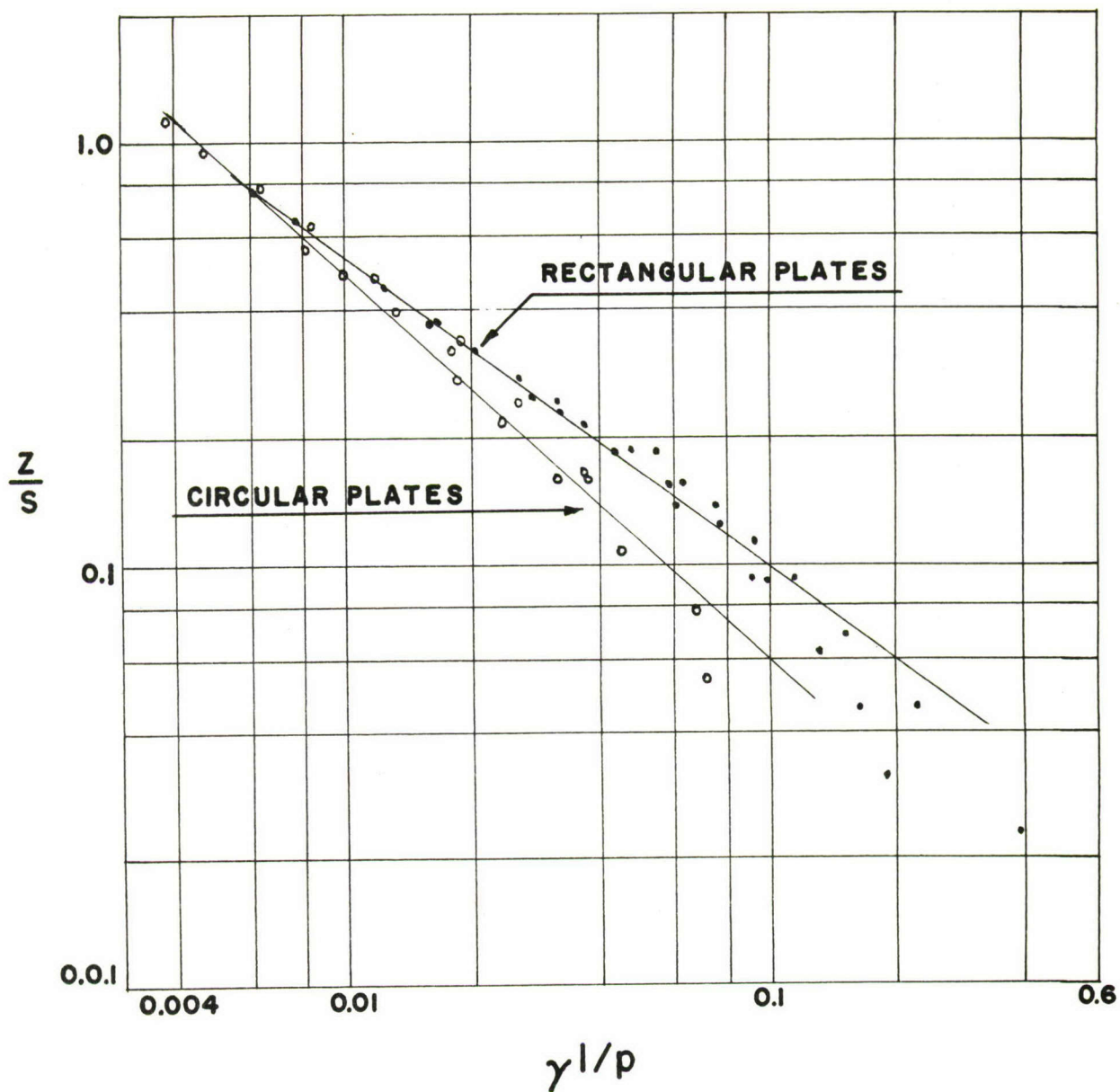
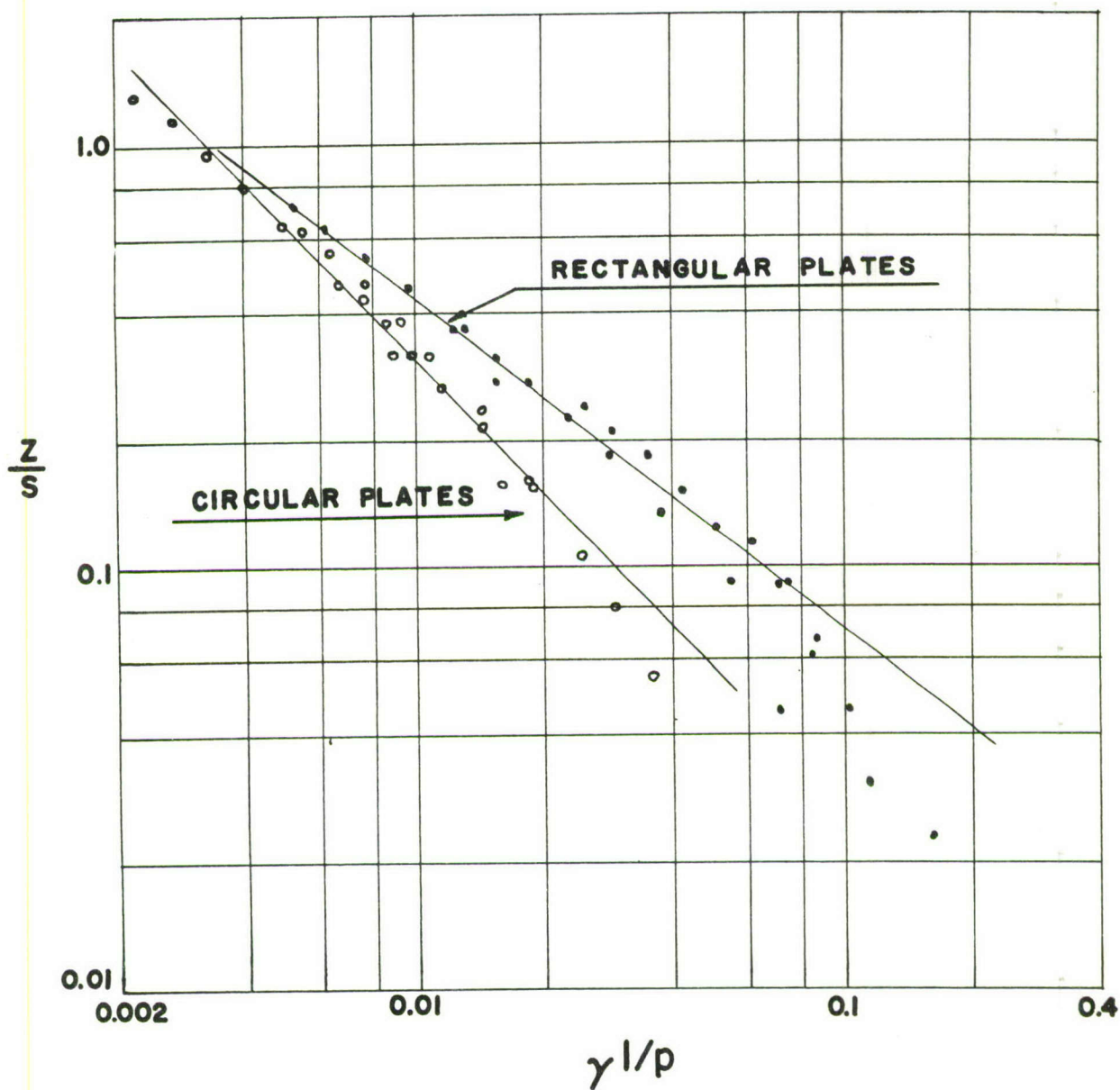


Figure 17.



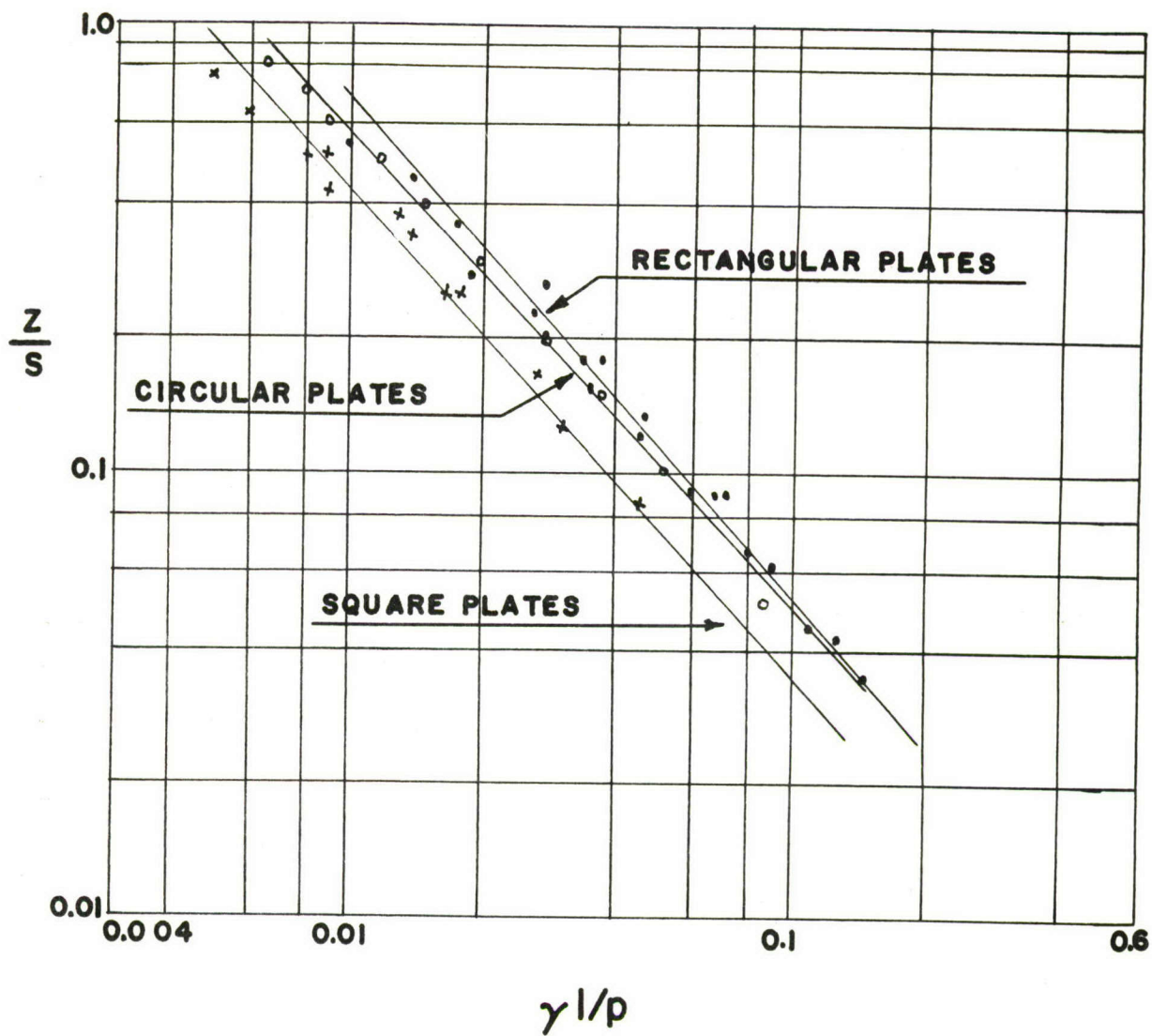
LOG(z/s) vs. LOG($\gamma l/p$) FOR DRY OTTAWA SAND.

Figure 18.



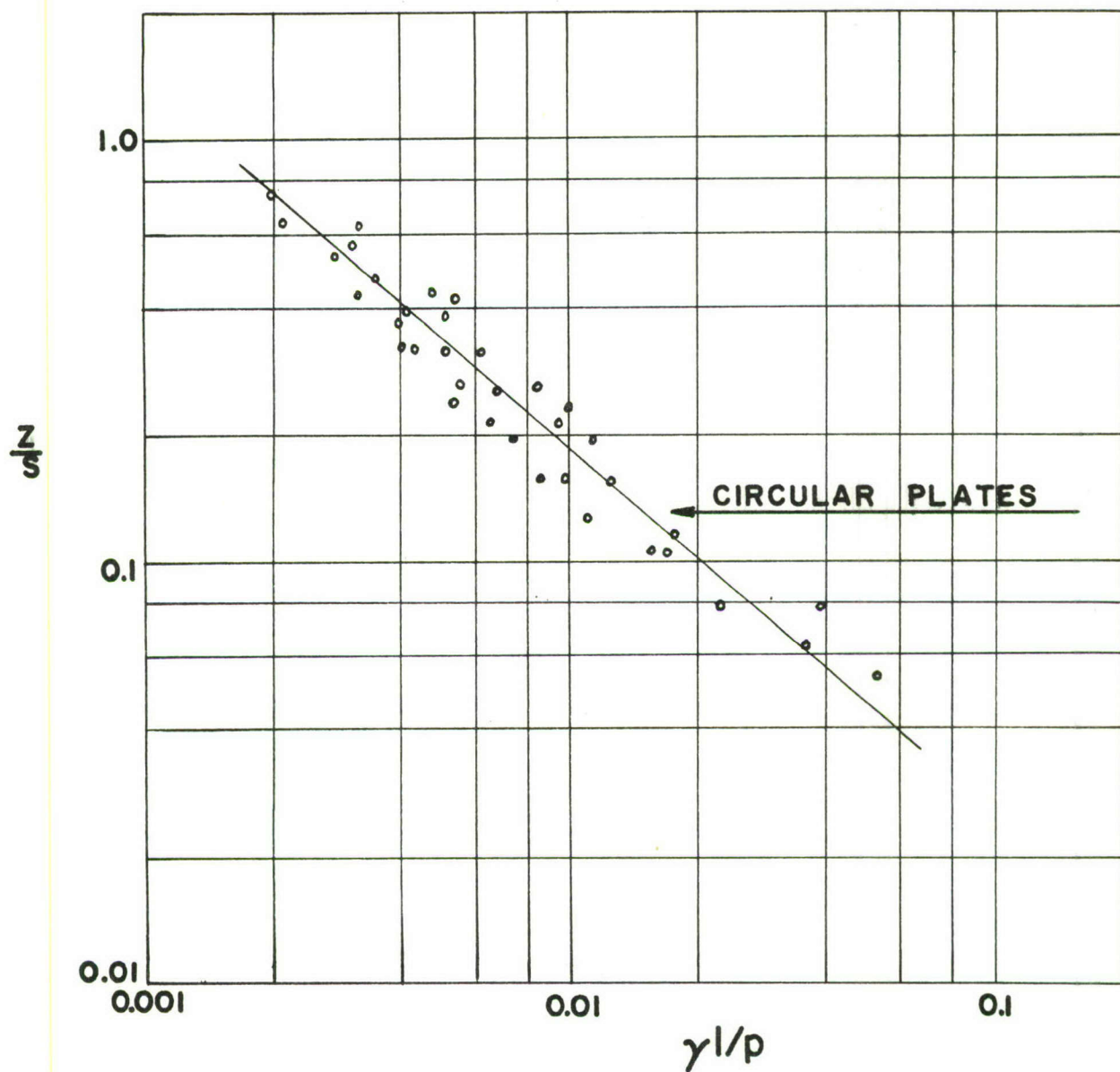
LOG(z/s) vs. LOG($\gamma l/p$) FOR WET OTTAWA SAND.

Figure 19.



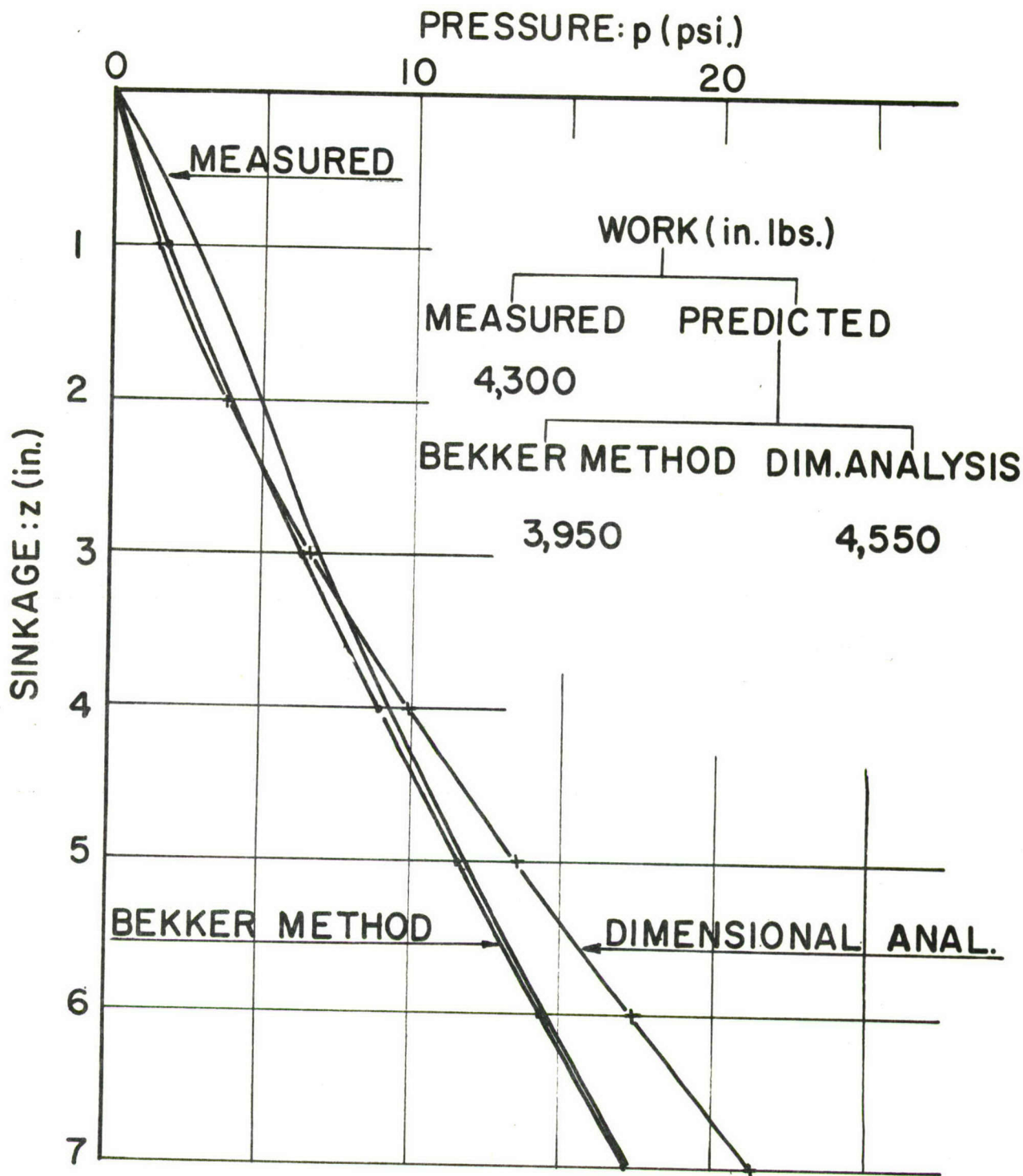
LOG (z/s) vs. LOG ($\gamma l/p$) FOR IOWA SAND.

Figure 20.



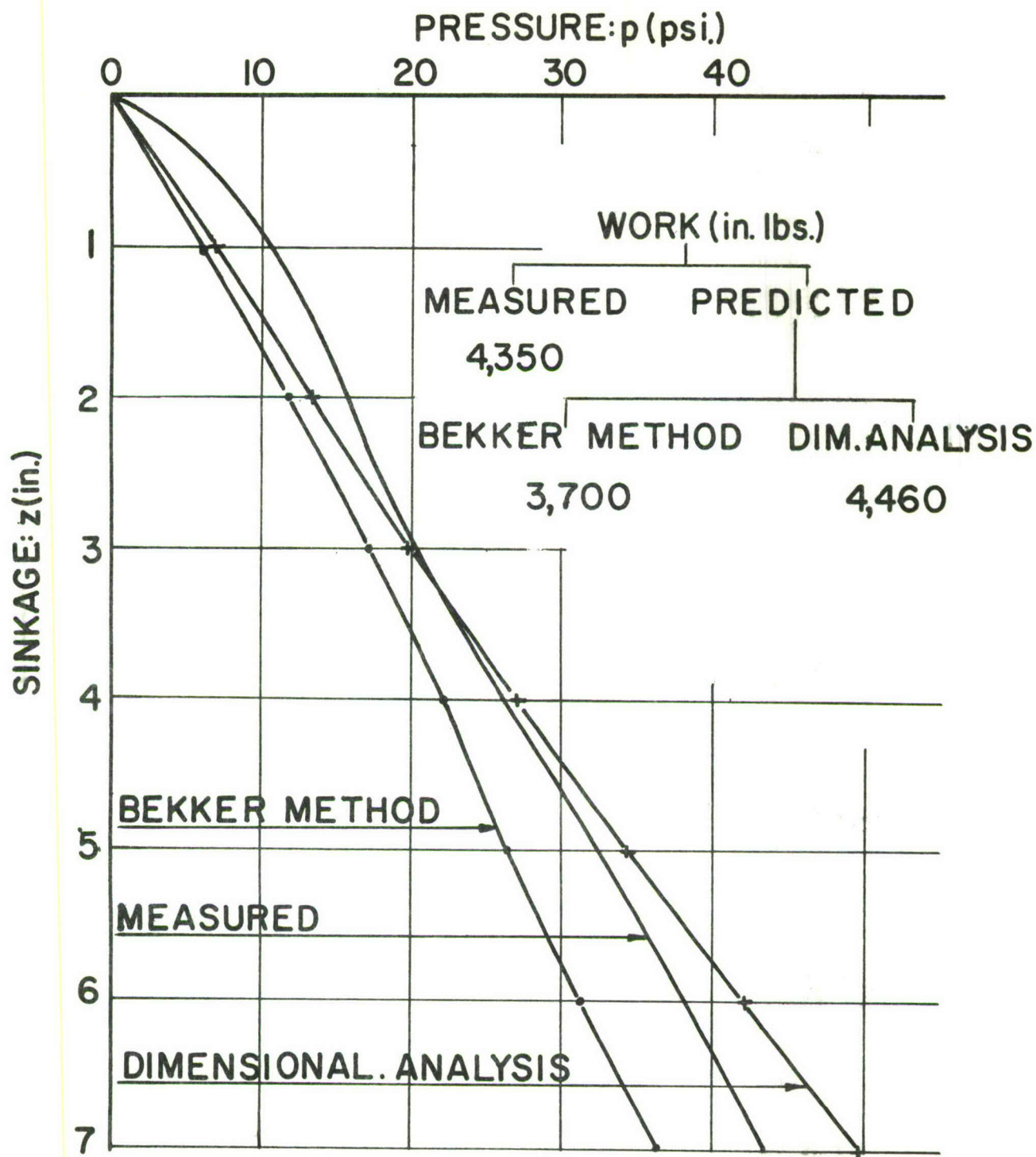
LOG(z/s) vs. LOG($\gamma l/p$) FOR SNOW

Figure 21.



COMPARISON OF RELATIONSHIPS FOR THE
4x18 in. FOOTING IN DRY OTTAWA SAND.

Figure 22.



COMPARISON OF RELATIONSHIPS FOR THE
6 in. Dia. FOOTING IN SATURATED OTTAWA
SAND.

Figure 23.

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