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Submitted in partial fulfillment of
the requirements for the degree of

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ABSTRACT

A method is developed to analyze and compare the effectiveness of ASW vehicles. The measure of effectiveness is the probability that the vehicle, after detecting a submarine with passive sensors, can transit to the contact area and re-establish contact with the submarine. A computer simulation is developed and an example using three hypothetical ASW vehicles is illustrated.

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1. Introduction.

As a result of increased emphasis currently being placed on ASW weapon systems, it is advantageous to possess a computer simulation capable of analyzing and comparing proposed ASW vehicles and operational ASW vehicles. The computer simulation for the comparison of ASW vehicles developed in this paper is intended to be used for this purpose.

The problem at hand is to develop a method for comparing ASW vehicles under operational conditions using vehicle, submarine, and tactical situation parameters. The simulation allows one to compute a probability which is used as a measure of effectiveness. This probability, hereafter referred to as the "probability of success," is defined in this paper as the probability that an ASW vehicle successfully relocates a submarine at an estimated position given that a detection has occurred.

A description of the simulation and its limitations is made in Section 2. It is very important to understand the simulation and the background assumptions before the results of the simulation are analyzed. After the problem is developed mathematically in Section 2, a discussion and an illustration is given in Section 3 to demonstrate how to use the computer program. In Section 4, several applications are discussed. Finally a comparison is made of three hypothetical ASW vehicles--a High Speed Surface Vehicle, a VERTOL Vehicle, and a Seaplane-Type Vehicle--using the results generated from the computer simulation. The output of one computer run is illustrated in Figures 5 and 6. Curves for the probability of success versus range to the submarine for each vehicle are displayed in Figures 7, 8, and 9. These curves are used to analyze and compare the systems.

The results of these comparisons illustrate the usefulness of the simulation in comparing the performance of different vehicles and for studying the effect of various parameters such as expected sensor range on the probability of success.

2. Simulation Description and Mathematical Development.

Effort has been directed toward the development of a computer simulation that represents a real world conflict between an ASW vehicle and a submarine. The setting is one of an ASW vehicle assigned to a patrol, surveillance, or barrier mission. Initial detection is assumed to have been made using passive sensors. The vehicle then proceeds as rapidly as possible to an estimated position (EP) in order to relocate the submarine. The EP is the predicted position of the submarine when the actual position is not known, but the vehicle's sensors indicate the presence of a submarine. The objective of the vehicle is to compute an EP of the submarine, close the position at maximum speed, and attempt to relocate the submarine using active sensors. No information about the actions of the submarine is available while the vehicle closes to the EP.

Certain other restrictions are assumed in the model. Ahead thrown weapons and nuclear weapons are not available for use by the vehicle. Multiple targets are excluded. A passive detection range of at least five nautical miles is necessary; otherwise, an active sensor would be used and the problem of transiting to an estimated position is irrelevant. Also, the vehicle is assumed to lie motionless in the water until detection has been achieved to insure passive sensor capability.

The simulation is developed for two different tactical situations. The first case is that of a submarine which is unaware of the activity of the vehicle. The second case is that of a submarine which is alerted by the activity of the vehicle. Common to both of these cases is a maneuvering board problem which will be discussed first.

The maneuvering board solution consists of describing geometrically the positions and motions of a vehicle and a submarine with time. The passive sensor of the vehicle provides the last known position, course and speed of the submarine, and the bearing and range to the submarine from the vehicle. Knowing these values and the vehicle speed, an EP, transit time of the vehicle, relative closing speed, and the distance traveled to EP are computed. For overall ease in reading the same notation is used in the mathematical development that has been used in the computer simulation (Appendix C).

The solution is computed by solving a velocity vector problem. The vehicle is located at the origin (0,0) of a rectangular coordinate system. One side of the velocity triangle (Figure 1) represents the estimated velocity vector of the submarine (U1) which is re-established at the origin. The velocity vector for the vehicle (Z) is unknown in direction, but known in magnitude. The remaining relative velocity vector (RELSPD) represents the relative course and speed of the vehicle closing the submarine. Only the direction of RELSPD is known which is parallel to the true bearing line from the vehicle at (0,0) to the submarine at (X2, Y2). Then the intersection (X3, Y3) between the relative velocity vector and the vehicle velocity circle is computed.

In order to solve the maneuvering board problem various points and vectors in the rectangular coordinate system must be computed. The values for the components of U1 are:

$$X1 = U1 \times SIN\ (THETA) \tag{1}$$

$$Y1 = U1 \times COS\ (THETA) \tag{2}$$

The last known position (X2,Y2) of the submarine is given by:

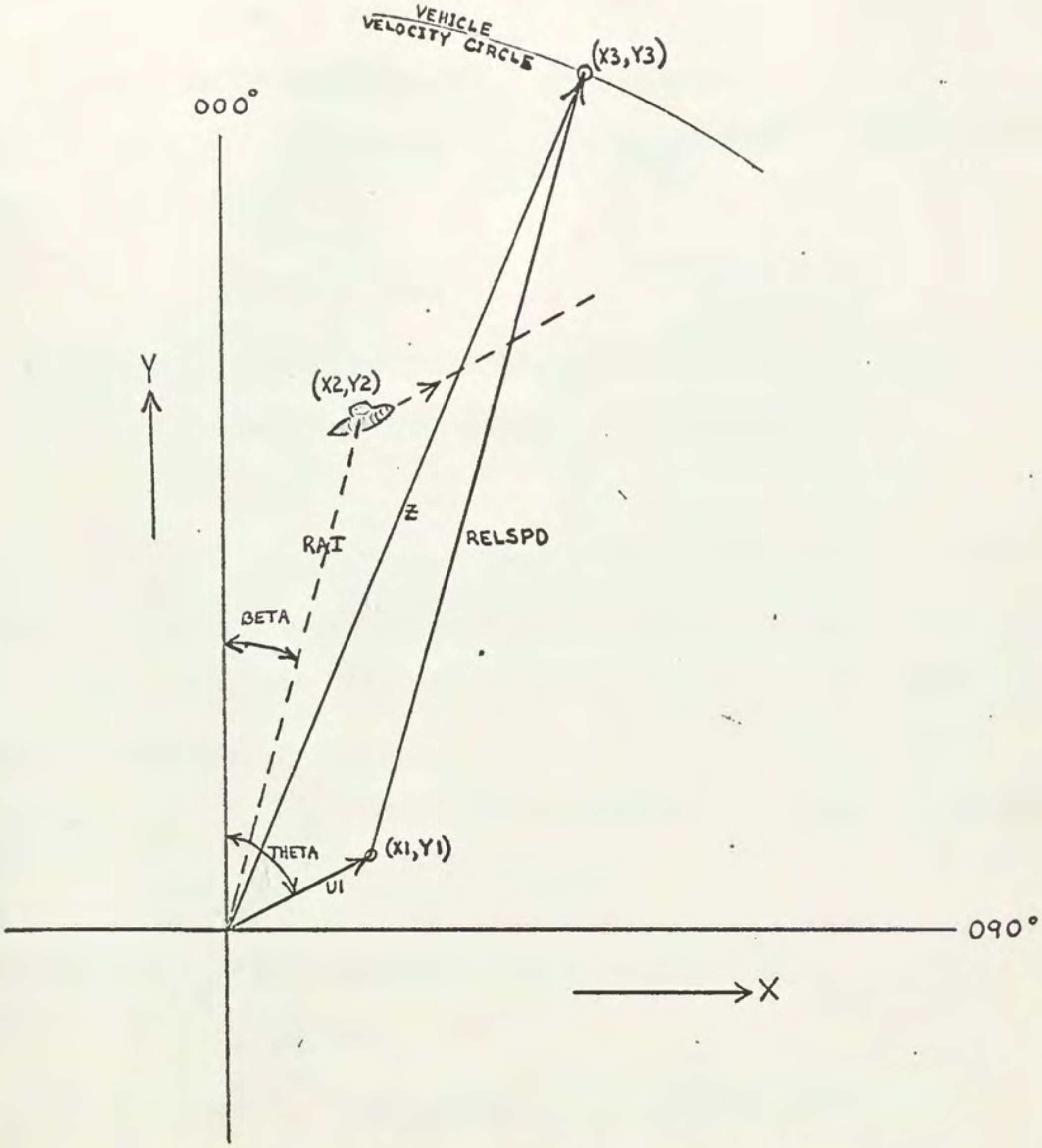

Figure 1. The Maneuvering Board Solution

$$X2 = RAI \times SIN\ (BETA) \tag{3}$$

$$Y2 = RAI \times COS\ (BETA)\ , \tag{4}$$

where BETA is the bearing in degrees of the submarine from the vehicle and RAI is the range in nautical miles of the submarine from the vehicle. The RELSPD vector is represented by the line through the point (X1, Y1) and parallel to the line connecting the origin and (X2, Y2). The equation of this line is:

$$Y3 = \frac{Y2}{X2}\,(X3-X1) + Y1\ . \tag{5}$$

The RELSPD vector intersects the vehicle velocity circle at the point (X3, Y3). The equation of the vehicle velocity circle is:

$$Y3^2 + X3^2 = V^2, \tag{6}$$

where V is the average vehicle transit speed. The point (X3, Y3) is determined by solving equations (5) and (6) simultaneously. The direction of the vector from the origin to (X3, Y3) represents the course of the vehicle when traveling inbound to EP. The relative speed vector is represented by the vector between (X1, Y1) and (X3, Y3) and its magnitude is:

$$RELSPD = \sqrt{(X3-X1)^2 + (Y3-Y1)^2}\ . \tag{7}$$

The vehicle's transit time (TD) to EP is defined as:

$$TD = RAI/RELSPD\ . \tag{8}$$

Let the vehicle reaction time (RT) be the following sum:

$$RT = PTOT + TOT + CLOT + RELT\ , \tag{9}$$

where, PTOT - pretake-off time which commences when the vehicle can no longer observe the submarine and includes decision time, warm-up time, and sensor retrieval time,

TOT - take-off time which includes take-off roll and time to take-off into the wind, CLOT - land time which includes time to land and taxi time needed to arrive at EP in order to deploy active sensors,

RELT - target relocation time which includes the time necessary to deploy sensors and ends when the submarine is relocated.

Any of these times can have a zero value. For example, a helicopter could have TOT equal to zero if it needs no take-off time. Also, the times are mutually exclusive so that no time interval is added twice. Obviously the blind time (TB) is:

$$TB = RT + TD . \tag{10}$$

Evasion for the alerted submarine begins when blind time commences. At that time the submarine is assumed to be at (X2, Y2). If RT is equal to zero, the submarine is at (X2, Y2) when evasion begins; but if RT is not equal to zero, the submarine is not actually at (X2, Y2) when evasion begins because it takes RT for the vehicle to commence moving towards EP. During the reaction time the vehicle is motionless, but the submarine is moving away from (X2, Y2). Because of the small displacement of the submarine from (X2, Y2) due to RT, the assumption that the submarine is at (X2, Y2) when evasion begins does not significantly affect the results of the simulation.

It is necessary to determine the estimated position (EP) of the submarine after a time interval of TB and a submarine speed U3. The equation of the estimated distance (DIST) traveled by the submarine is:

$$DIST = U3 \times TB . \tag{11}$$

The EP is also the position at which the vehicle estimates that an intercept will be made with the submarine. The components of EP relative to the origin are:

$$X4 = DIST \times SIN\ (THETA) + X2 \tag{12}$$

$$\mathrm{Y4} = \mathrm{DIST} \times \mathrm{COS}\ (\mathrm{THETA}) + \mathrm{Y2}\ . \qquad (13)$$

The equation for the distance DISDAT) traveled by the vehicle to EP is

$$\mathrm{DISDAT} = \sqrt{\mathrm{X4}^2 + \mathrm{Y4}^2}\ . \qquad (14)$$

The values for TB and DIST are very important in determining the probabilities of success for the two cases developed below.

Case I. The unalerted submarine:

It is conceivable that the vehicle could transit to EP without the submarine being cognizant of the impending danger. In this case the unalerted submarine does not evade and proceeds at cruise course and velocity. Prediction errors for the submarine course and velocity are made which are characteristic of the vehicle's equipment. The errors are assumed to be distributed normally about the predicted course and velocity. Because of long blind times expected, the predicted positions of the submarine are not accurately represented by a circular-normal distribution, and a numerical integration must be performed. Probabilities of success are computed for each of a representative distribution of predicted positions consisting of 25 points or cells (see Figure 2). This concept has been adopted from [3].

The last known submarine position (X2, Y2) is established at the origin of a rectangular coordinate system. Since DIST is the distance traveled by the submarine during TB, then EP is (DIST,0) in this coordinate system.

Using the prediction errors, a submarine position is computed for each of the 25 cell midpoints. The distance (VV) and angle (THETER) for each cell are computed using the submarine cruise speed (U1), the submarine

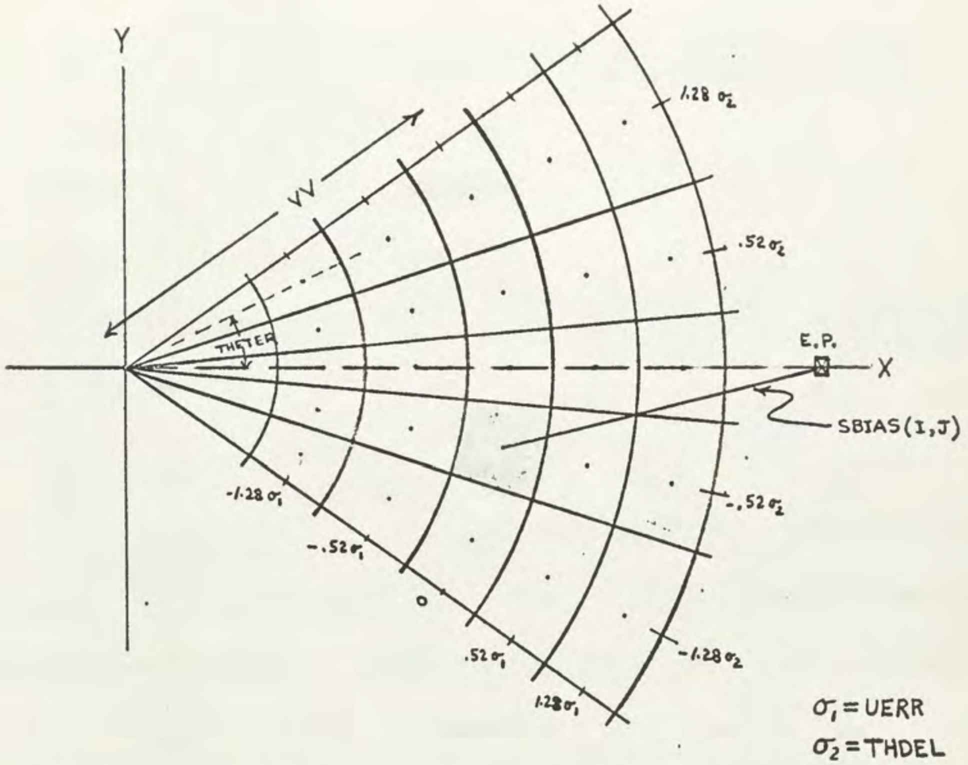

Figure 2 Numerical Integration For The Unalerted Case

speed error (UERR), the submarine course error (THDEL), and the blind time (TB). The equations for VV and THETER are:

$$VV = (U1-AA(I) \times UERR)\ TB \tag{15}$$

$$THETER = AA\ (J) \times THDEL\ , \tag{16}$$

with AA(I) and AA(J) = 0, ± .52, ± 1.28 depending on which cell is being considered. These numbers can be obtained from any normal density tables, [2].

The equations for the submarine position (SUBX (I,J), SUBY(I,J)) at the center of each of the 25 cells are:

$$SUBX\ (I,J) = VV \times COS\ (THETER) \tag{17}$$

$$SUBY\ (I,J) = VV \times SIN\ (THETER)\ . \tag{18}$$

A bias is computed between EP and the center of each cell. The equation for the bias of the $(i,j)^{th}$ cell is:

$$SBIAS\ (I,J) = \sqrt{(SUBX(I,J)-DIST)^2 + (SUBY(I,J))^2}\ . \tag{19}$$

Various errors have to be considered to give the simulation realism. It is assumed that the navigation error and sensor locating error are additive about the point EP. The distribution of the vehicle location about EP due to these errors is assumed to be circular normal.

The standard deviation of navigation error (SIGNAV) is:

$$SIGNAV = PRCNTN \times RAI \tag{20}$$

where PRCNTN is a percentage of the last known range (RAI). The standard deviation of sensor locating error consists of the standard deviation of bearing (SIGSBR) and the standard deviation of range (SIGSR) which are defined by:

$$SIGSR = PRCNTS \times RAI \tag{21}$$

$$SIGSBR = RAI \times SIN\ (BRGER) \tag{22}$$

where BRGER is the bearing error and PRCNTS is the percentage error of the last known range. Since (21) is greater than (22), because of the passive sensor, an elliptical normal distribution exists; but it is approximated by a circular normal distribution for ease of computation. This is accomplished by equating the area of an ellipse to the area of a circle. The result is an approximate standard deviation (SIGAPR).

$$\pi \sigma_{XE} \sigma_{YE} = \pi \sigma^2_{CIR} \text{, with } \begin{array}{l} \sigma_{XE} = \text{SIGSBR} \\ \sigma_{YE} = \text{SIGSR} \end{array} \tag{23}$$

$$\therefore \text{SIGAPR} = \sigma_{CIR} = \sqrt{\sigma_{XE} \sigma_{YE}} \tag{24}$$

Therefore, the total standard deviation of error (SIGJT) is the sum of the squares of the approximate standard deviation of error and the standard deviation of navigation error according to the equation:

$$\text{SIGJT} = \sqrt{(\text{SIGNAV})^2 + (\text{SIGAPR})^2} \;. \tag{25}$$

Other error models were considered in order to more accurately represent the errors involved. Due to lack of time the error model described above, although not the best, was adopted.

To compute the probability of success for the unalerted submarine the circular coverage function is used, [1]. In order to use this function, the parameters SBIAS(I,J), SIGJT, and EXPRNG are required. The first two have been defined above. The term EXPRNG, which is defined as the expected range of the active sensor used by the vehicle, is a characteristic of the sensor employed. The quantities $\frac{\text{SBIAS(I,J)}}{\text{SIGJT}}$ and $\frac{\text{EXPRNG}}{\text{SIGJT}}$ are used in the circular coverage function to compute the probability of success (PK(I,J)) that a circular disk of radius EXPRNG will cover a point (SUBX(I,J), SUBY(I,J) from the EP since the probable position of the disk is described by a Gaussian distribution. This probability PK(I,J) is the probability of success for the ijth cell. Since it is equally likely that the submarine is in each of the 25 cells, the probability that it is within each cell is .04. The probabilities of success for each cell are summed and multiplied by .04 to give the "probability of success (PROB) against an unalerted submarine." The probability of success is:

$$\text{PROB} = \sum_{I} \sum_{J} (\text{PK}(I,J) \times .04) \tag{26}$$

Case II. The alerted submarine:

The submarine is aware of the presence of an ASW vehicle and evades by changing course and speed. The speed changes are characteristics of the submarine but the course changes are determined in number and degree by the situation being simulated. A probability of success is computed for each evasion turn and an arithmetic average taken over the number of evasion tactics used.

This case is designed to give practically full evasion to the submarine (see Figure 3). The submarine evasion turns are limited to a maximum turn of 90 degrees to either side of the estimated submarine course. No additional turns are granted after the initial turn is executed. For turns greater than 90 degrees the submarine would deviate from its initial track to such an extent that it would not be able to make up the lost time necessary to make good a mission speed of advance.

The range of values for the set of evasion turns is determined by the situation being simulated. For example, a slow submarine would

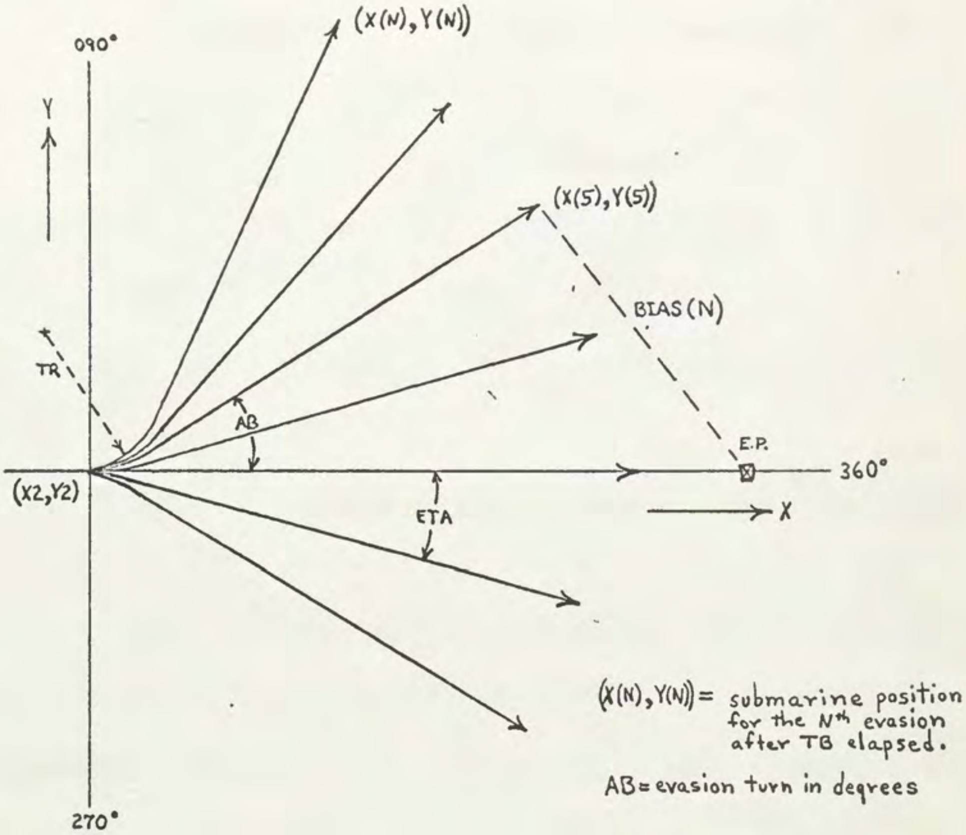

Figure 3 Alerted Case

almost always turn away from the vehicle when the vehicle's presence is known. There are two limiting turns which bound the set of evasion turns to the right and the left. The bounds must be no greater than 90 degrees to either side of the estimated submarine course. PSI and DELTA are the left bound and right bound, respectively. The range of values for the set of evasion turns is determined by (PSI-DELTA). The interval (ETA) between evasion turns is $\mathrm{ETA} = \frac{\mathrm{PSI-DELTA}}{\mathrm{NU}}$, NU being the number of evasion turns considered. If the set of turns encompasses turns to the right of the submarine course then $\mathrm{ETA} = \frac{\mathrm{PSI} \quad (360\text{-}\mathrm{DELTA})}{\mathrm{NU}}$. In Figure 3, PSI = 060 degrees, DELTA = 330°, and $\mathrm{ETA} = \frac{060 \quad (360\text{-}330)}{6} = 15^{\circ}$. Therefore, the evasion turns considered for this example are 330°, 345°, 360°, 015°, 030°, 045°, and 060°.

In the alerted submarine case the submarine course and velocity prediction errors that were present in the unalerted case are ignored. Since in this case a set of evasion turns are selected which are only subjective predictions, it is valid to exclude prediction errors because they are comparatively insignificant to the computations. However, vehicle sensor and navigation errors that were assumed in the unalerted case still prevail.

Initially, the submarine is at cruise speed. When evasion commences, the assumed submarine speed is increased to fast speed. The characteristics of the submarine determine how long the submarine remains at fast speed (length of time is T1) after which it slows to silent speed. The time intervals of acceleration and deceleration are considered negligible when compared with the long blind times encountered.

The alerted case uses a rectangular coordinate system with the last known position of the submarine (X2,Y2) at the point of origin when evasion begins. From this point positions (X(N), Y(N)), which are the positions at the end of each of the N evasion turns, are calculated. (See Figure 3.) The time to turn (TTT), evasion turn in degrees (AB), and the turning radius (TR) are needed to compute X(N) and Y(N). The equations for TTT, X(N), and Y(N) are computed as follows:

$$\mathrm{TTT} = \frac{\text{arc length}}{\text{sub speed}} = \frac{\mathrm{TR} \times \mathrm{AB}}{\mathrm{U3}}$$

If $\mathrm{TI} \geqslant \mathrm{TB}$, then

$$\mathrm{X(N)} = \mathrm{TR} \times \mathrm{SIN(AB)} + \mathrm{U3} \times (\mathrm{TB}-\mathrm{TTT})\ \mathrm{COS(AB)} \tag{27}$$

$$\mathrm{Y(N)} = \mathrm{TR} - \mathrm{TR} \times \mathrm{COS(AB)} + \mathrm{U3} \times (\mathrm{TB}-\mathrm{TTT})\ \mathrm{SIN(AB)}\ . \tag{28}$$

If $\mathrm{TI} < \mathrm{TB}$, then

$$\mathrm{X(N)} = \mathrm{TR} \times \mathrm{SIN(AB)} + \mathrm{U3} \times (\mathrm{TB}-\mathrm{TTT})\ \mathrm{COS(AB)} + (\mathrm{TB}-\mathrm{T1})\mathrm{U2} \times \mathrm{COS(AB)} \tag{29}$$

$$\mathrm{Y(N)} = \mathrm{TR}-\mathrm{TR} \times \mathrm{COS(AB)} + \mathrm{U3} \times (\mathrm{TB}-\mathrm{TTT})\mathrm{SIN(AB)} + (\mathrm{TB}-\mathrm{T1})\ \mathrm{U2} \times \mathrm{SIN(AB)}. \tag{30}$$

The time T1 is the length of time the submarine remains at fast speed (U3); after which it changes to silent speed (U2). Equations (29) and (30) account for the speed change occurring before TB has terminated. The point (X(N),Y(N)) can be considered the submarine's actual position after the N^{th} evasion turn is taken.

The estimated position EP is the predicted position of the submarine after the interval of blind time has elapsed. Therefore, EP is (DIST,0). The distance between each (X(N),Y(N)) and EP constitutes a bias (BIAS(N)). This is an important factor in computing the probability of success. The bias for the N^{th} evasion turn is:

$$\mathrm{BIAS(N)} = \sqrt{(\mathrm{X(N)}-\mathrm{DIST})^2 + (\mathrm{Y(N)})^2}\ . \tag{31}$$

As in the unalerted case, the parameters BIAS, EXPRNG, and SIGJT must be computed (EXPRNG and SIGJT are the same as in the unalerted case) in order to use the circular coverage function. The circular coverage function is used to compute the probability (PPK(N)), for each N, that the vehicle can regain contact at EP given the total standard deviation of error (SIGJT), the bias (BIAS(N)) between the terminal point of the N^{th} evasion and EP, and the expected active sonar range (EXPRNG).

The two ratios of the parameters that are needed for the circular coverage function are $\frac{BIAS(N)}{SIGJT}$ and $\frac{EXPRNG}{SIGJT}$. With these ratios the probabilities, PPK(N), are computed for each N, and then all PPK(N), N = 1, ..., NU+1, are summed and averaged over all NU+1. The average probability is:

$$PAVG = \frac{\sum_{N=1}^{NU+1} PPK(N)}{NU+1} \quad . \tag{32}$$

It might be appropriate to use the option to weight each of the evasion turns. If it is believed that the turns within the set of evasion turns are not equally likely, then the probability vector UNU(N), N = 1, ..., NU+1 can be used which gives probabilities that each of the N evasions is actually taken. If they are not equally likely, then the "weighted probability of success (WTPAVG) against an alerted submarine" is computed. The equation for WTPAVG is

$$WTPAVG = \frac{\sum_{N=1}^{NU+1} (PPK(N) \times UNU(N))}{(NU+1)} \tag{33}$$

The results of the mathematical development have produced methods of computing the probability of success (PAVG) for an alerted submarine with equally likely evasion turns, the probability of success (WTPAVG) for an alerted submarine with evasion turns that are not equally likely, and finally the probability of success (PROB) for an unalerted submarine. These probabilities are used to analyze and compare ASW vehicles.

3. Input Data Rules.

A complete run for the computer program (Appendix C) consists of 25 input cards. Each input data card represents the value of one variable except for the last two cards, one of which contains the values for the AA(i), i=1, ..., 5 and the other which contains the UNU(N) values, N=1, ..., NU+1 .

One of the first functions of the program is to set the indices NO=1 and M=1. (See Appendix B--Flow Charts.) The index NO is used to number the pages of the program output. The index M is compared with NEXT (initially set equal to one). The comparison dictates whether the program will continue or terminate. The value for NEXT determines the number of runs to be executed and is an input to the program. The computer then reads the input data cards from SUBROUTINE INPUT after which the 25 values are printed on page 1 of the output. The index NO is increased by one. The programmed computations are performed and the results are printed on page 2 of the output (Figure 6). The indices NO and M are then increased by one. The index M is compared to NEXT and if NEXT is greater than M, SUBROUTINE INPUT searches for more input data cards. Since the program is used to compare and evaluate systems, only a few variables will be changed for each successive run. However, more or all the variables can be changed as long as the input procedure is followed. For example, to compare the three vehicles in Table 1, first place the parameters for the High Speed Surface Vehicle, RAI = 5.0, and a value for NEXT in the data input for Run 1. (See Figure 4.) The next run consists of a Locator Card which initiates the reading of Variable Change Card, which in turn changes the value of RAI to 10. The subsequent

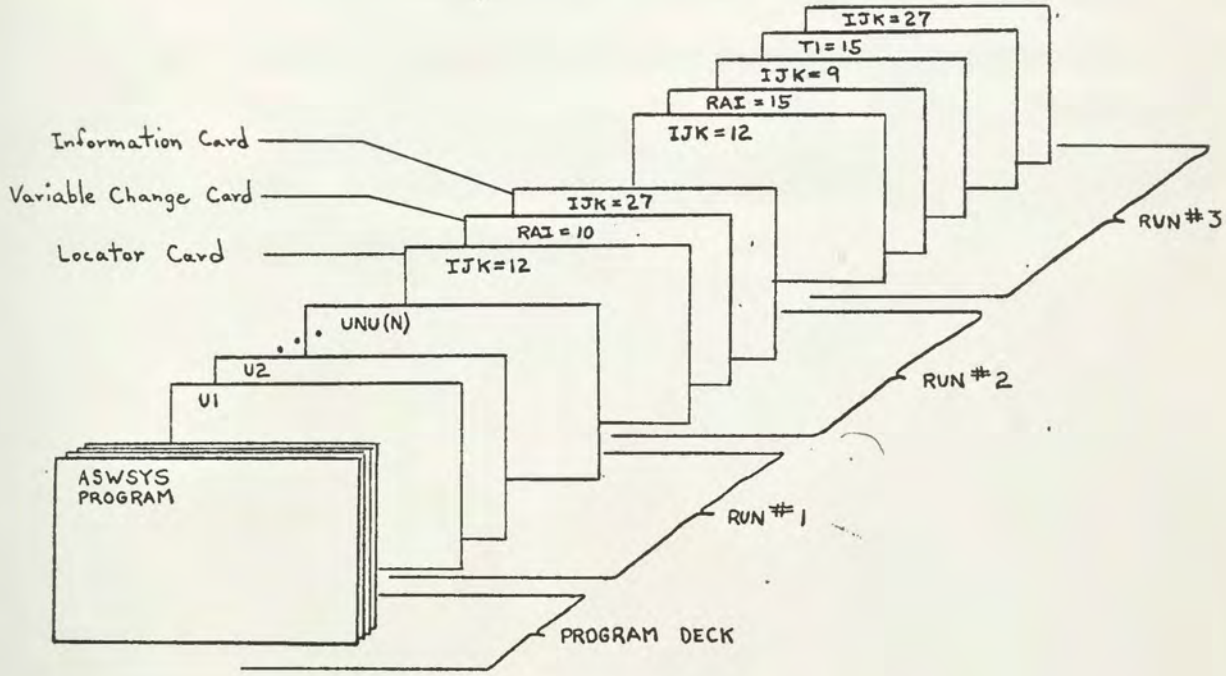

Figure 4 Input Data Procedure

card would be the Information Card, which indicates no further changes for the second run -- start computations. For this run the variable RAI has changed and all other parameter values remain the same.

If another variable change were desired, another Locator Card, Variable Change Card, and Information Card would be placed after the RAI Variable Change Card as indicated in Run 3 in Figure 4.

A run following the initial run takes a minimum of three cards. If only one variable is being changed (e.g. RAI = 10), then three cards are necessary. This procedure is followed for RAI = 5, 10, ..., 60 for each system in Table 1 in order to compute probabilities of success for each vehicle over the entire RAI range.

TABLE 1

PARAMETER INPUTS FOR SAMPLE ASW SYSTEMS

	HIGH SPEED SURFACE VEHICLE	VERTOL VEHICLE	SEAPLANE TYPE VEHICLE
U1	8.0 SLO SUB 18.0 FAST SUB		
U2	3.0 8.0	(SAME SUB SPEEDS FOR THE 3 SYSTEMS)	
U3	15.0 30.0		
Z	45.00	120.00	200.00
PTOT	3.00	4.00	5.00
TOT	.00	1.00	5.00
CLDT	.00	2.00	5.00
RELT	3.00	5.00	5.00
T1	10.00	10.00	10.00
THETA	45.00	45.00	45.00
BETA	10.00	10.00	10.00
TR	4000.00	4000.00	4000.00
PSI	60.00	60.00	60.00
DELTAL	330.00	330.00	330.00
BRGER	1.00	2.00	.50
EXPRNG	3.00	5.00	6.00
PRCNTN	4.00	2.00	4.00
PRCNTS	5.00	5.00	5.00
THDEL	5.00	5.00	5.00
UERR	5.00	2.00	5.00
NU	6	6	6
AA(I)	-1.28 , -.52 , .00 , .52 , 1.28		(SAME FOR 3 SYSTEMS)
UNU(N)	.10 , .05 , .00 , .10 , .40 , .30 , .05		(SAME FOR 3 SYSTEMS)

4. An Example and Some Applications.

The simulation possesses the capability of being used as a tool to analyze and compare ASW vehicles as explained in the previous sections. In order to illustrate this capability, the parameters listed for each system in Table 1, and ranges RAI=5, 10, 15, ..., 60 nautical miles were used as input data in the computer program (Appendix B) and processed in the CDC-1604 computer. The program is written in FORTRAN-60. The flow chart is illustrated in Appendix C.

A sample output for the Seaplane Type Vehicle using the parameters of Table 1 and RAI=45 is shown in Figures 5 and 6. The former is a list of all the parameter values for the indicated run. This list enables one to insure that the correct parameters for that run are being properly read into the computer. The latter page contains the probabilities of success for one run. The probabilities for each RAI are plotted on graphs. Each vehicle is depicted on an individual graph. Each graph has six curves -- three drawn from values calculated from using a slow submarine as an adversary and three drawn from using a fast submarine as an adversary (see Figures 7, 8, 9).

The parameter values in this sample are strictly hypothetical and demonstrate only the use of the simulation. However, with these chosen parameter values it can be seen in Figures 7, 8, and 9 that throughout the range of RAI the High Speed Surface Vehicle has probabilities of success considerably lower than the other systems. The slower the speed of the vehicle, the lower the probability because of a low closing rate and long blind time. The probabilities of success are higher at shorter RAI for the VERTOL Vehicle as compared with the Seaplane-Type Vehicle,

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OUTPUT RUN 9 OF 22                                                            PAGE 17

                              COMPLETE INPUT PARAMETER SET

PARAMETER  VALUE       MEANING

U1           8.00      SUBMARINE CRUISE SPEED (KNOTS)
U2           3.00      SUBMARINE SILENT SPEED (KNOTS)
U3          15.00      SUBMARINE FAST SPEED (KNOTS)
Z          200.00      VEHICLE TRANSIT SPEED TO DATUM (NEGLECT ACCEL. OR DECEL.)

PTOT         5.00      PRE  TAKE OFF TIME (MIN)
TOT          5.00      TAKE OFF TIME (MIN)
CLOT         5.00      LAND TIME (MIN)
RELT         5.00      TARGET RELOCATION TIME (MIN)
T1          10.00      ELLAPSED TIME OF SUB AT SPEED U3 (MIN)

THETA       45.00      COURSE OF SUB AT LAST KNOWN POSITION (DEGREES)
BETA        10.00      TRUE BEARING OF SUB FROM VEHICLE (DEGREES)

RAI         45.00      RANGE TO SUB AT LAST KNOWN POSITION FROM VEHICLE (MILES)

TR        4000.00      TURNING RADIUS OF SUB (YDS)
PRCNTN       4.00      NAV. ERROR IN PERCENT DISTANCE TO SUB
PRCNTS       5.00      SENSOR RANGE ERROR IN PERCENT SENSOR RANGE
BRGER         .50      SENSOR BEARING ERROR IN DEGREES

EXPRNG       6.00      RANGE EXPECTED FROM SENSOR FOR LOCALIZATION AT DATUM,MILES

PSI         60.00      LEFT BOUND FOR SUB EVASIVE TURN IN DEGREES
DELTAL     330.00      RIGHT BOUND FOR SUB EVASIVE TURN IN DEGREES
NU          6          NUMBER OF INCREMENTS CONSIDERED IN PSI-DELTA RANGE
NEXT       24          NUMBER OF SETS OF INPUT DATA
UERR         5.00      SUBMARINE SPEED ERROR(KNOTS)
THDEL        5.00      SUBMARINE COURSE ERROR (DEGREES)
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FIGURE 5

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OUTPUT RUN 9 OF 22                                                                      PAGE 18


                                        INPUT PARAMETERS

                                                TARGET        SUBMARINE   SUBMARINE   SUBMARINE
VEHICLE   PRE TAKE   TAKE OFF      LAND    RELOCATION       CRUISE      SILENT        FAST
 SPEED    OFF TIME     TIME        TIME       TIME           SPEED       SPEED        SPEED
(KNOTS)     (MIN)      (MIN)      (MIN)      (MIN)          (KNOTS)     (KNOTS)      (KNOTS)

200.00      5.00       5.00        5.00       5.00           8.00        3.00         15.00

                                                              SENSOR                 EXPECTED
SUBMARINE                    TRUE BEARING   NAVIGATION       RANGE       SENSOR     RANGE OF
TO VEHICLE    SUBMARINE        SUB FROM        ERROR          ERROR      BEARING   LOCALIZATION
 RANGE          COURSE          VEHICLE      (PERCENT)      (PERCENT)     ERROR       SENSOR
 (MILES)      (DEGREES)       (DEGREES)     DISTANCE)        RANGE)     (DEGREES)    (MILES)

  45.00         45.00           10.00          4.00           5.00         .50         6.00


          AVERAGE SUCCESS PROBABILITY FOR 7 EVASION TACTICS IS   .539
 AVERAGE WEIGHTED SUCCESS PROBABILITY FOR 7 EVASION TACTICS IS   .073
           AVERAGE SUCCESS PROBABILITY FOR UNALERTED SUBMARINE IS   .679


                           COMPUTED VALUES FOR SUBMARINE EVASION

                                             BLIND   DISTANCE TRANSIT   RELATIVE
EVASIVE   EVASION TURN     SUCCESS           TIME       TO DATUM          SPEED       SIG TOTAL
NUMBER     (DEGREES)     PROBABILITY         (MIN)      (MILES)          (KNOTS)      (MILES)

   1        -30.00          .539             33.96       52.18           193.39        2.03
   2        -15.00          .621             33.96       52.18           193.39        2.03
   3           .00          .656             33.96       52.18           193.39        2.03
   4         15.00          .621             33.96       52.18           193.39        2.03
   5         30.00          .539             33.96       52.18           193.39        2.03
   6         45.00          .441             33.96       52.18           193.39        2.03
   7         60.00          .355             33.96       52.18           193.39        2.03
```

FIGURE 6

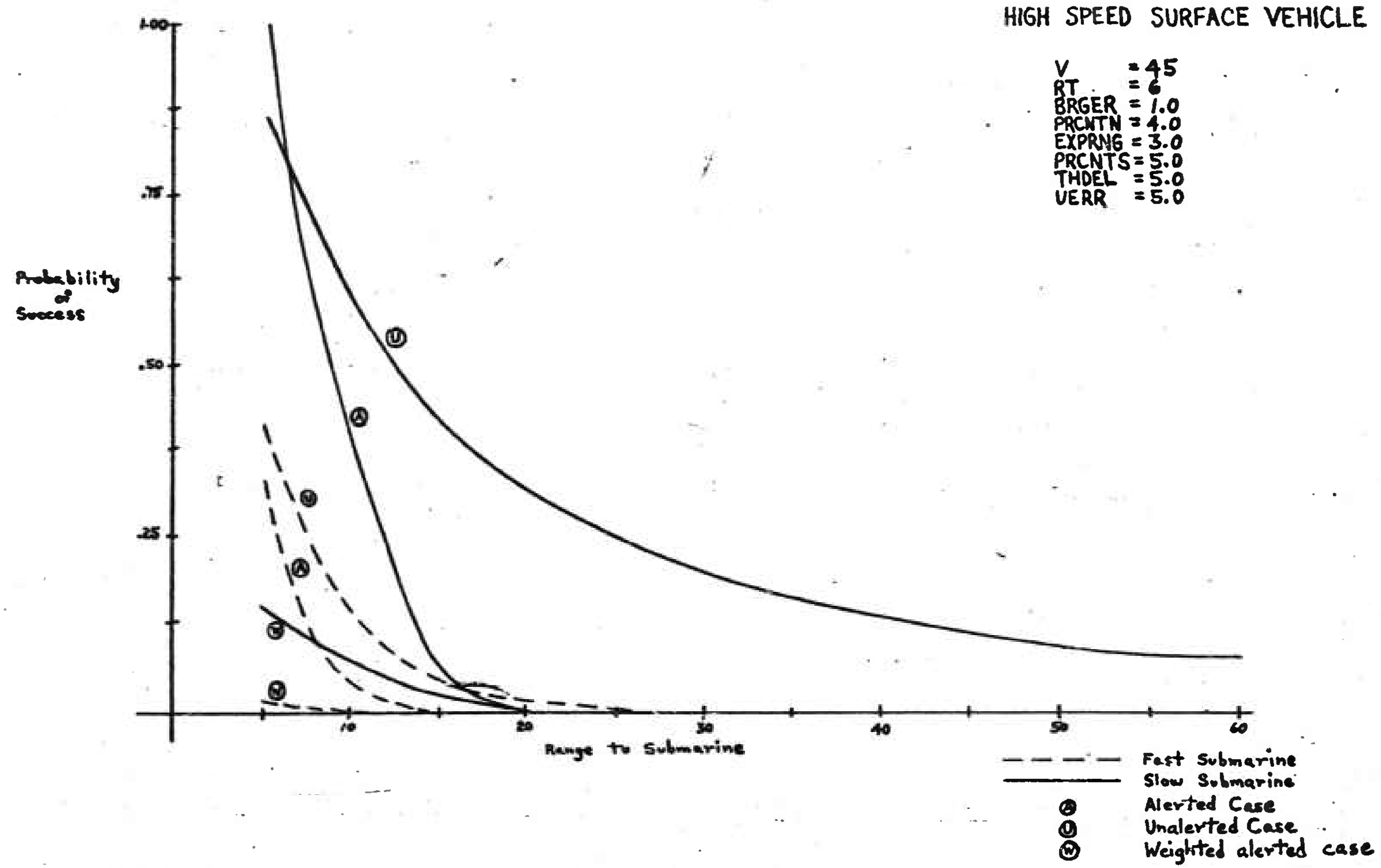

HIGH SPEED SURFACE VEHICLE
V = 45
RT = 6
BRGER = 1.0
PRCNTN = 4.0
EXPRNG = 3.0
PRCNTS = 5.0
THDEL = 5.0
VERR = 5.0
Probability of Success
1.00
.75
.50
.25
10
20
30
40
50
60
Range to Submarine
Ⓤ
Ⓐ
Ⓦ
Ⓐ
Ⓤ
Ⓦ
Fast Submarine
Slow Submarine
Ⓐ Alerted Case
Ⓤ Unalerted Case
Ⓦ Weighted alerted case

Figure 7

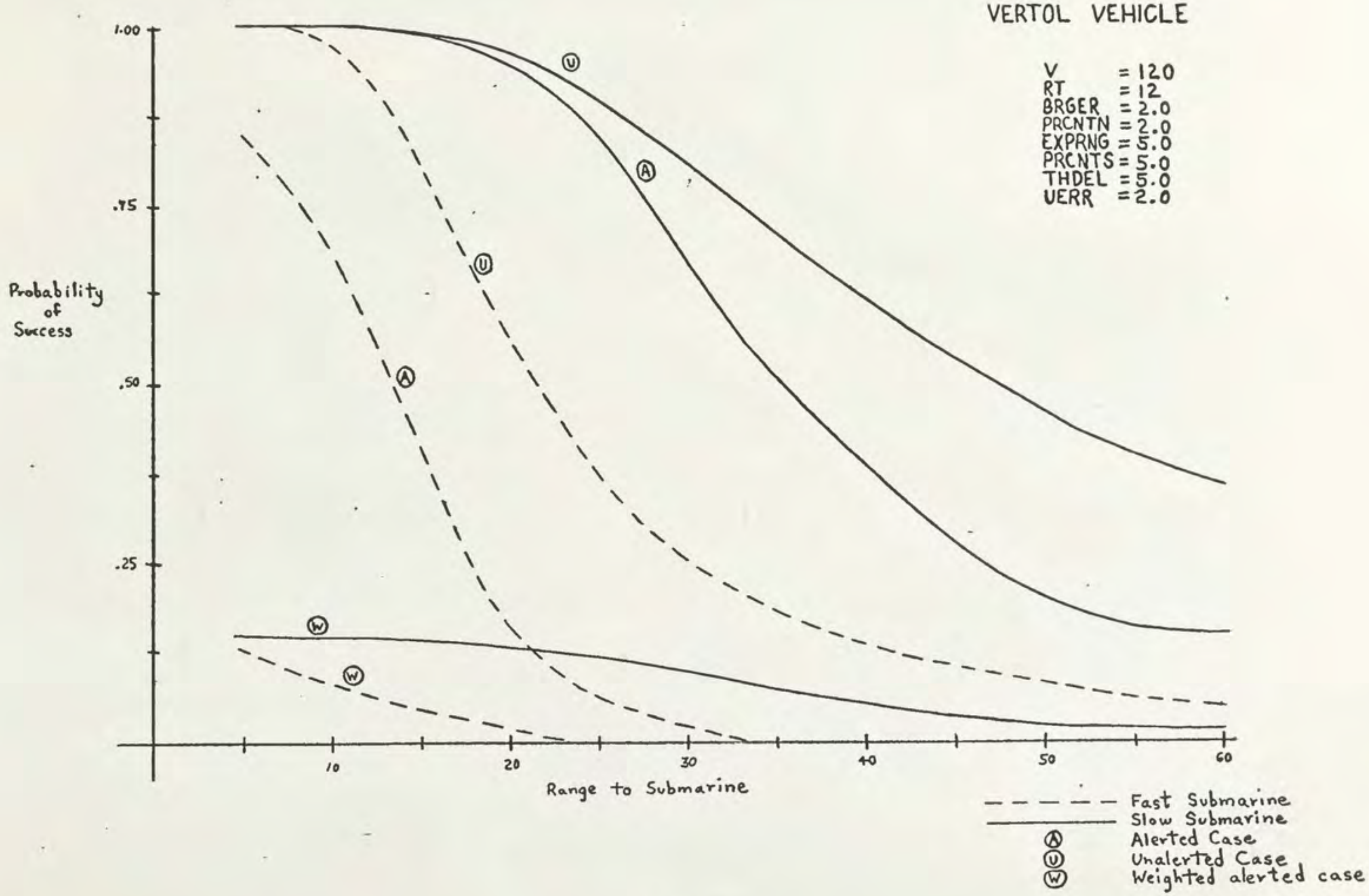

VERTOL VEHICLE
V = 120
RT = 12
BRGER = 2.0
PRCNTN = 2.0
EXPRNG = 5.0
PRCNTS = 5.0
THDEL = 5.0
UERR = 2.0
Probability of Success
1.00
.75
.50
.25
10
20
30
40
50
60
Range to Submarine
Ⓤ
Ⓐ
Ⓤ
Ⓐ
Ⓦ
Ⓦ
Fast Submarine
Slow Submarine
Ⓐ Alerted Case
Ⓤ Unalerted Case
Ⓦ Weighted alerted case

Figure 8

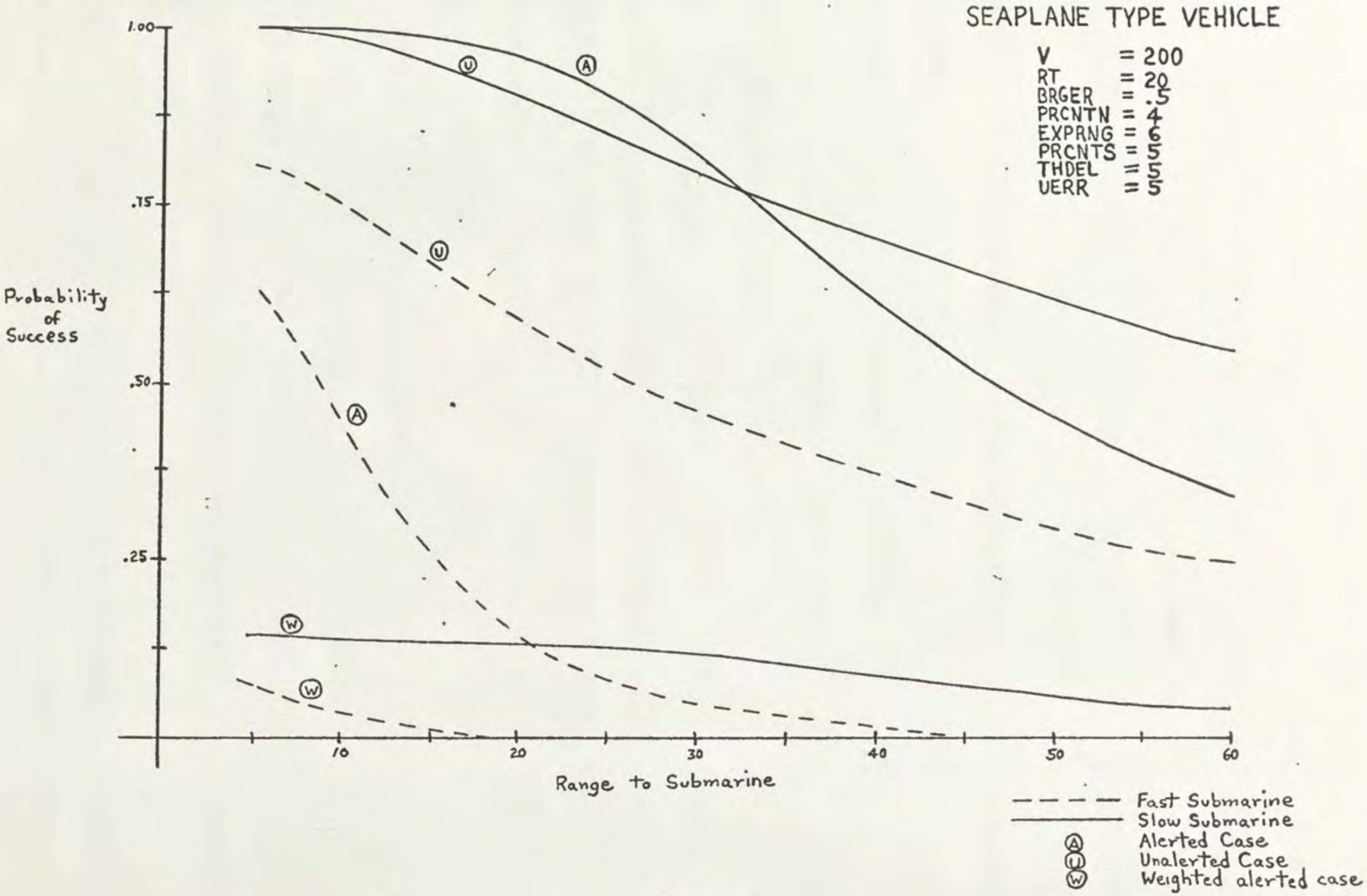

SEAPLANE TYPE VEHICLE
V = 200
RT = 20
BRGER = .5
PRCNTN = 4
EXPRNG = 6
PRCNTS = 5
THDEL = 5
UERR = 5
Probability of Success
1.00
.75
.50
.25
10
20
30
40
50
60
Range to Submarine
Ⓤ
Ⓐ
Ⓦ
Fast Submarine
Slow Submarine
Ⓐ Alerted Case
Ⓤ Unalerted Case
Ⓦ Weighted alerted case

Figure 9

but a crossover point occurs at approximately RAI=25, where the Seaplane-Type Vehicle has higher probabilities of success. The crossover is a result of the values selected in the sample for the vehicle speed, reaction time, and expected sonar range. From these results the best vehicle can be chosen and the other vehicles investigated for improvements.

In addition to the example stated above several applications are suggested for the simulation. One application is to use the simulation to determine what parameters or characteristics the vehicle and the sensors must have to obtain a given probability of success. For example, if the state-of-the-art for active sensors dictates that five nautical miles is the maximum range expected for active sensors for the next ten years, then it would be advantageous to find values of vehicle speed, reaction time, navigation errors, and prediction errors that can be tolerated for the vehicle and still maintain a given probability of success. The development of a new ASW vehicle could be directed towards achieving these parameter values.

Another application can be made if a new sensor is being proposed for a given vehicle. The values for the expected active sensor range and the sensor errors can be computed for a given probability of success. These parameter values can be used as design limits in the development of the proposed sensor.

Finally the simulation can be used if intelligence indicated that a new enemy submarine is in existence. The simulation can be used to indicate how well the present vehicles will perform against the new threat relative to the old threat.

5. Conclusions.

The example in Section 4 is the type of comparison that can be performed for various kinds of ASW vehicles. There are several decisions--evasion tactics used, submarine type used, and so on--that have to be made prior to using the program; but once the decisions are made for a given situation, they have to remain the same for all systems being compared. Several modifications and suggestions for the model are discussed below.

It might be argued that an elliptical coverage function should be used instead of a circular coverage function because of the elliptical sensor locating error. This change can be made by rewriting SUBROUTINE OCIP of the computer program (Appendix C).

Instead of evasion turns limited to 90 degrees either side of the course line, they could be programmed for 360 degrees evasion; i.e., 180 degrees either side of the course line.

It must be realized that the depth of the submarine has no direct influence on this simulation. Therefore, this has to be kept in mind when parameter values such as EXPRNG are put into the simulation. It is possible that EXPRNG could be less in an evasion problem where the submarine is more likely to increase its depth, and therefore, EXPRNG should be reduced.

It would be possible to give the vehicle freedom of movement before transit commences, rather than being motionless. But the advantages of the passive tactic for the vehicle are then relinquished.

The restriction on T1 in the alerted submarine case could be abolished by programming for the possibility of having a speed change executed before a turn is completed.

Possibly a passive sensor detached from the vehicle could transmit current information about the submarine's actions while the vehicle is inbound to intercept the submarine. This would increase the probability of success.

In the foregoing sections, the simulation has been described and the mathematical development shown. In addition, the use of the computer program has been illustrated and an example and its results briefly analyzed. The use of the simulation is simple and the results easily understood. Therefore, it is recommended that the use of this simulation be employed to compare and analyze proposed ASW vehicles and operational ASW vehicles.

BIBLIOGRAPHY

1. Operations Research Incorporated, Silver Spring, Maryland. Use of the Circular Coverage Function in Calculation of A/S Kill Probabilities, by G. B. Yntema, H. D. Kushner, and R. A. Gibbons. August 26, 1958. Prepared under Navy Bureau of Ordnance, Contract NOrd 17976.

2. Parzen, E., Modern Probability Theory and Its Applications. John Wiley & Sons, Inc., 1960.

3. Operations Evaluation Group, Washington, D. C., Model and Computer Program for an Attack on an Evading Submarine, by S. H. Howe, J. F. Hammerle, and R. D. Mason, Jr., June 8, 1962. IRM-18.

4. RAND. Circular Coverage Function, by H. H. Germand. January 26, 1950. RM-330.

APPENDIX A

GLOSSARY

Notation	Meaning
A(1),...,A(5)	Number of standard deviations from the mean of the normal density function. Each A(i) is the midpoint of a segment of area which represents 20% of the total area under the normal density function. The A(i)'s are used in the prediction error of the unalerted submarine case.
AB	Evasion turn in degrees from the estimated submarine course.
BETA	True bearing of the submarine from the vehicle (degrees).
BIAS(N)	Distance from EP to the terminal point of the N^{th} evasion.
BRGER	Sensor bearing error in degrees.
CLDT	Land time (min.) which includes time to land into the wind and taxi time to get into position to employ active sensors.
DELTA	Bound to the right of the known submarine course. This bounds to the right the set of possible evasion turns (degrees).
DIST	Distance the submarine travels during blind time, assuming a constant course and speed.
EP	Estimate position of the submarine after blind time has expired.
EXPRNG	Expected range of the active sensor used by the vehicle in re-establishing contact at the EP (in nautical miles).
NEXT	Number of data runs for the computer program.
NU	Number of increments considered with the (PSI-DELTA) range of the set of possible turns.
PAVG	Average probability of success for N evasions considered in the alerted submarine case.
PRCNTN	Navigation error in percent of distance traveled.

Notation	Meaning
PRCNTS	- Sensor range error in percent of sensor range.
PROB	- Probability of success for the unalerted submarine case.
PSI	- Bound to the left of the last known submarine course. This bounds to the left the set of possible evasion turns (degrees).
PTOT	- Pre take-off time (min.) which commences when the vehicle can no longer observe the submarine and includes decision time, warm-up time, and sensor retrieval time.
RAI	- Range from the vehicle to the submarine at the last known position (nautical miles).
RELT	- Target relocation time (min) which includes the time to deploy sensors and ends when the submarine is relocated.
RT	- Reaction time which is the sum of PTOT, TOT, CLDT, and RELT.
SBIAS(I,J)	- Distance between the $(i,j)^{th}$ cell and EP.
SIGJT	- Total standard deviation of errors.
TB	- Blind time which is the sum of the reaction time (RT) and the transit time (TD).
TD	- Time it takes the vehicle to travel from its initial position to EP.
T1	- Tactical interval of time (min.) after evasion commences during which the submarine travels at fast speed. After T1 has elapsed, the submarine slows to silent speed.
THDEL	- Submarine course error (degrees).
THETA	- Course of the submarine (degrees) at the last known position.
TOT	- Take-off time (min.) which includes take-off roll and the time to maneuver into the wind.
TR	- Turning radius of the submarine.

Notation Meaning

UERR - Submarine speed error (knots).

UNU(1),...,UNU(NU+1) - The set of probabilities for $N = 1, \ldots, NU+1$ evasion turns. These are used when evasion turns considered are not equally likely.

U1 - Submarine cruise speed (knots).

U2 - Submarine silent speed (knots).

U3 - Submarine fast speed (knots).

V - Average vehicle transit speed (knots)

WTPAVG - Weighted probability of success for the alerted submarine case.

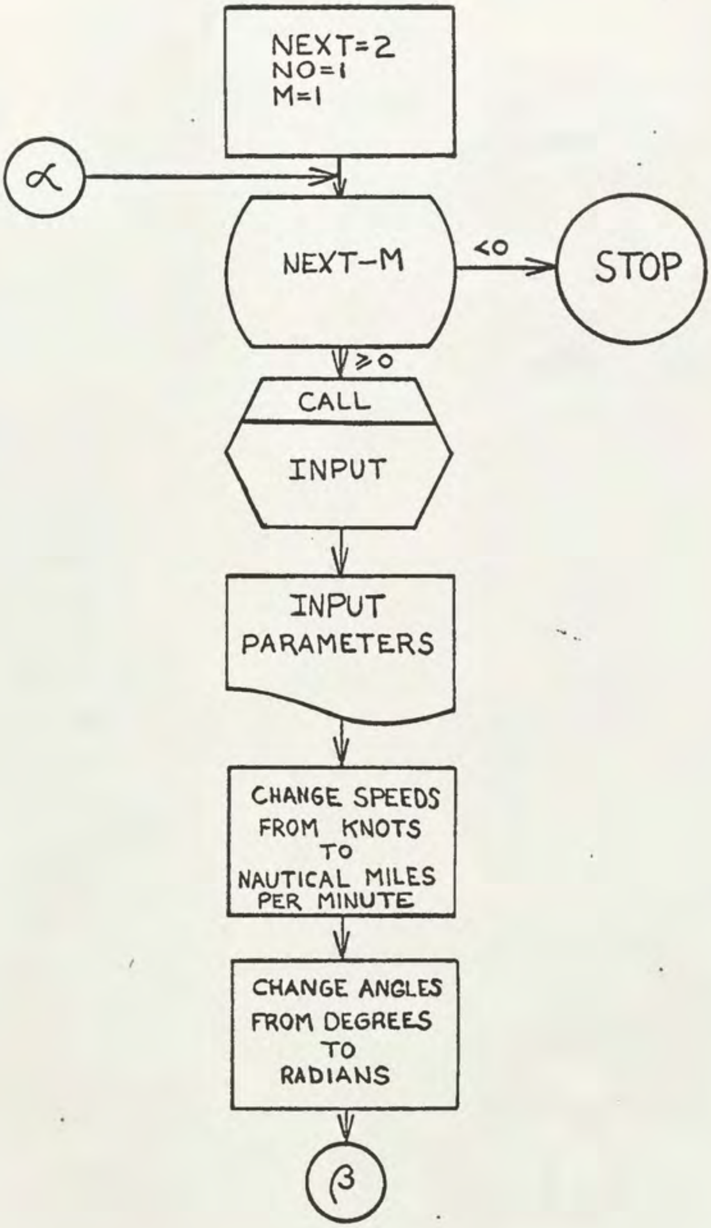
(X2,Y2) - Last known position of the submarine.

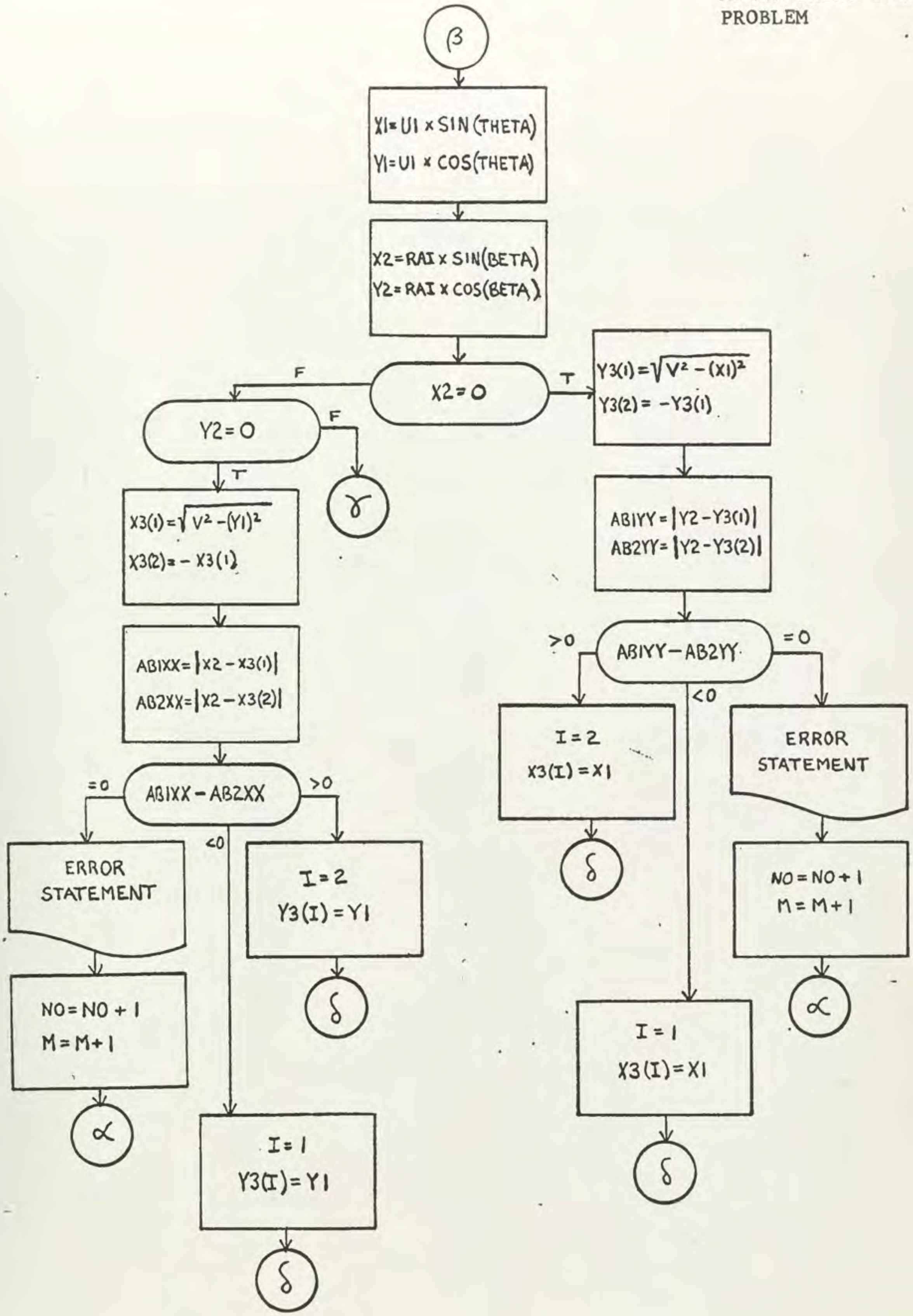
(X(N),Y(N)) - Terminal point of the N^{th} evasion after blind time has expired.

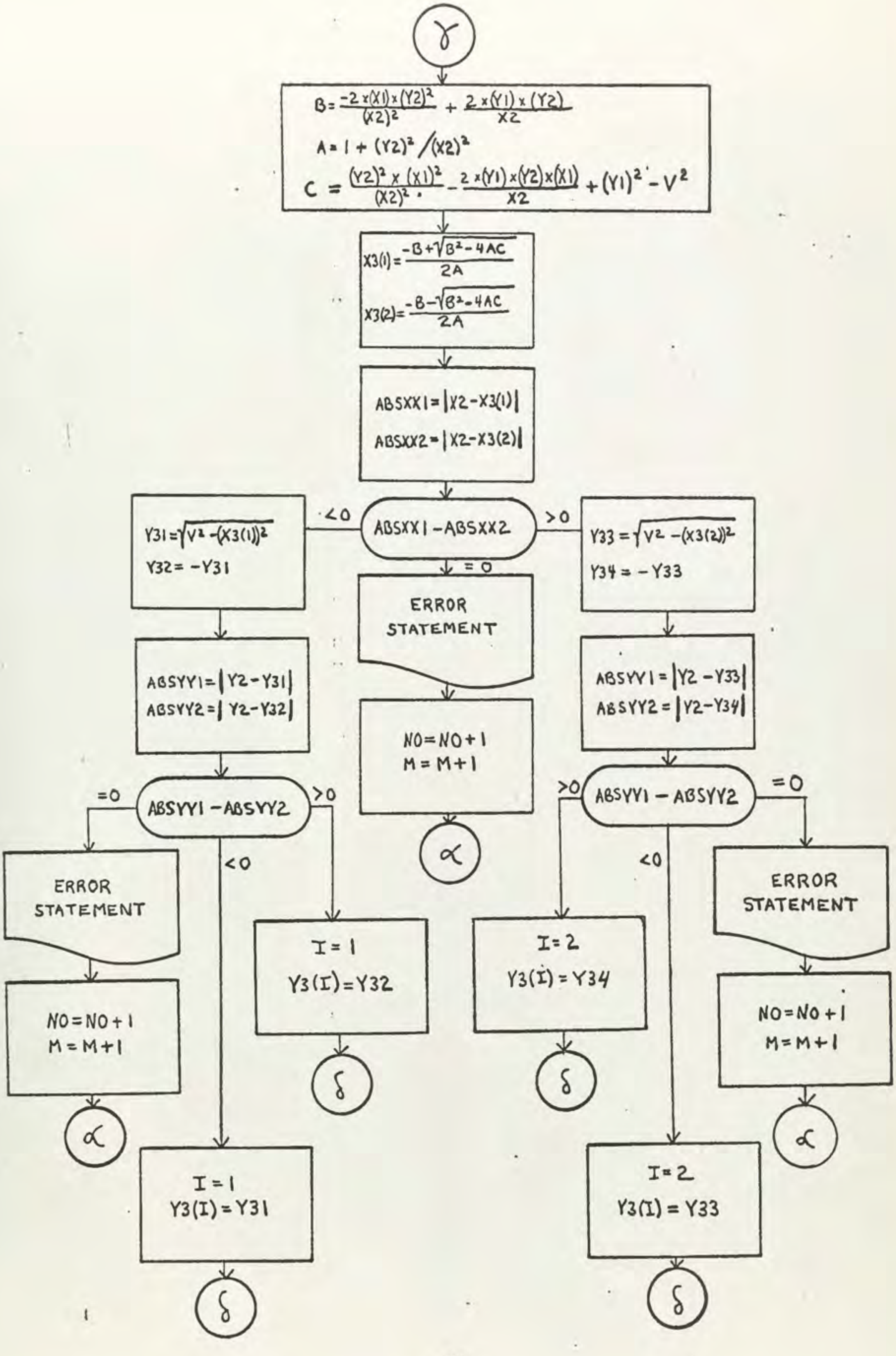
Z - Average vehicle transit speed (knots). Neglect acceleration and deceleration time.

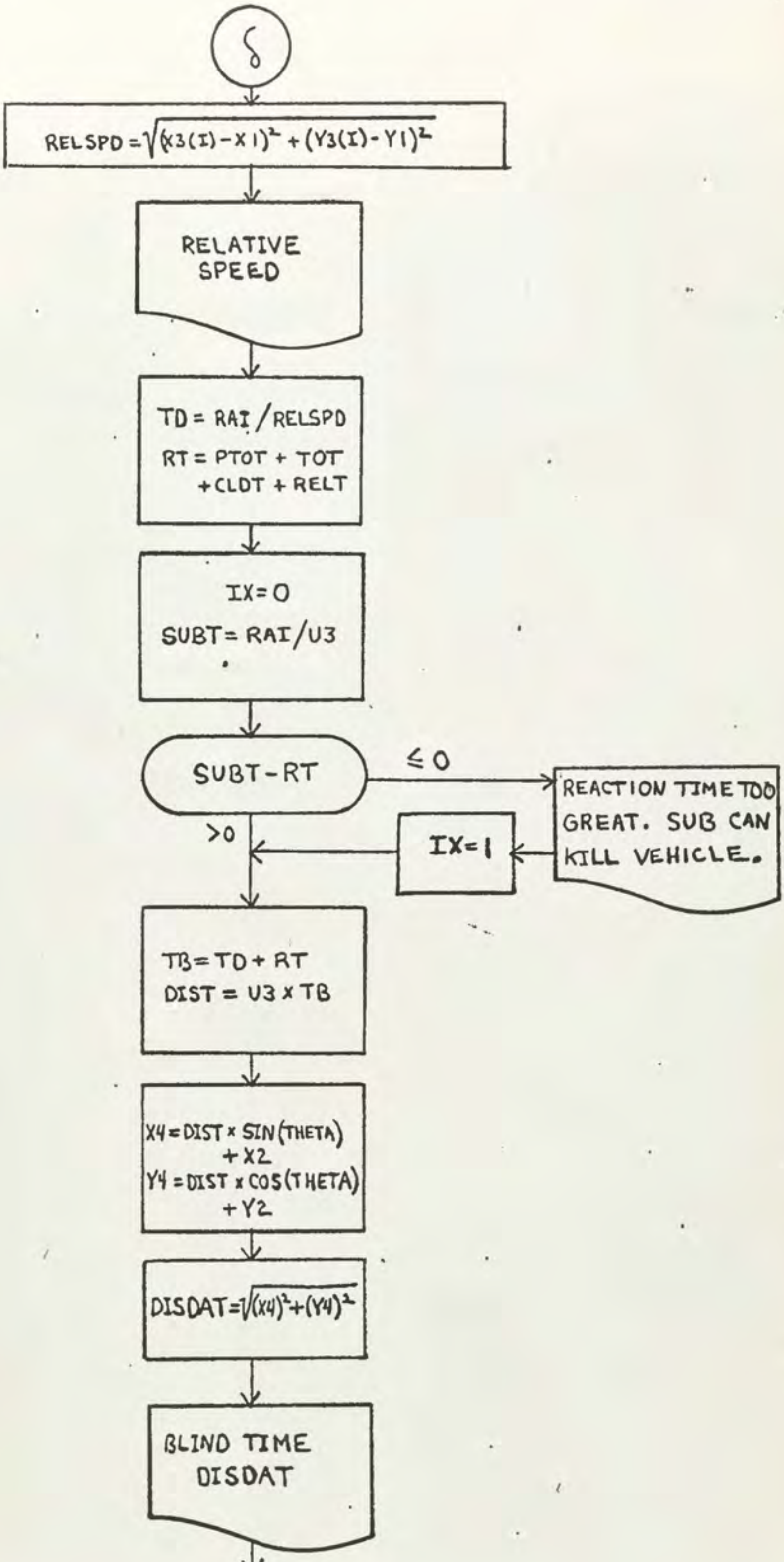
APPENDIX B

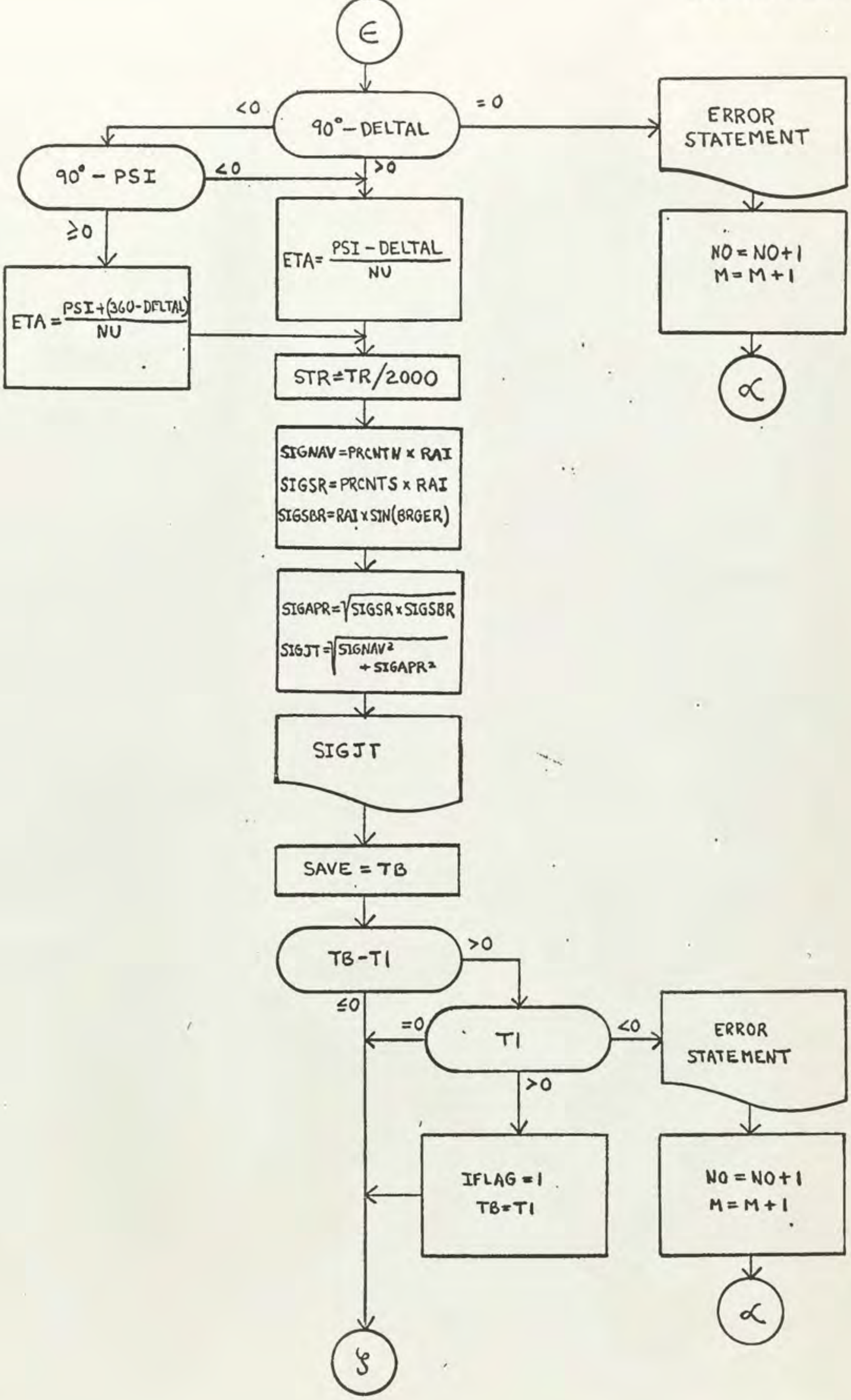
INPUT AND
UNIT CHANGES

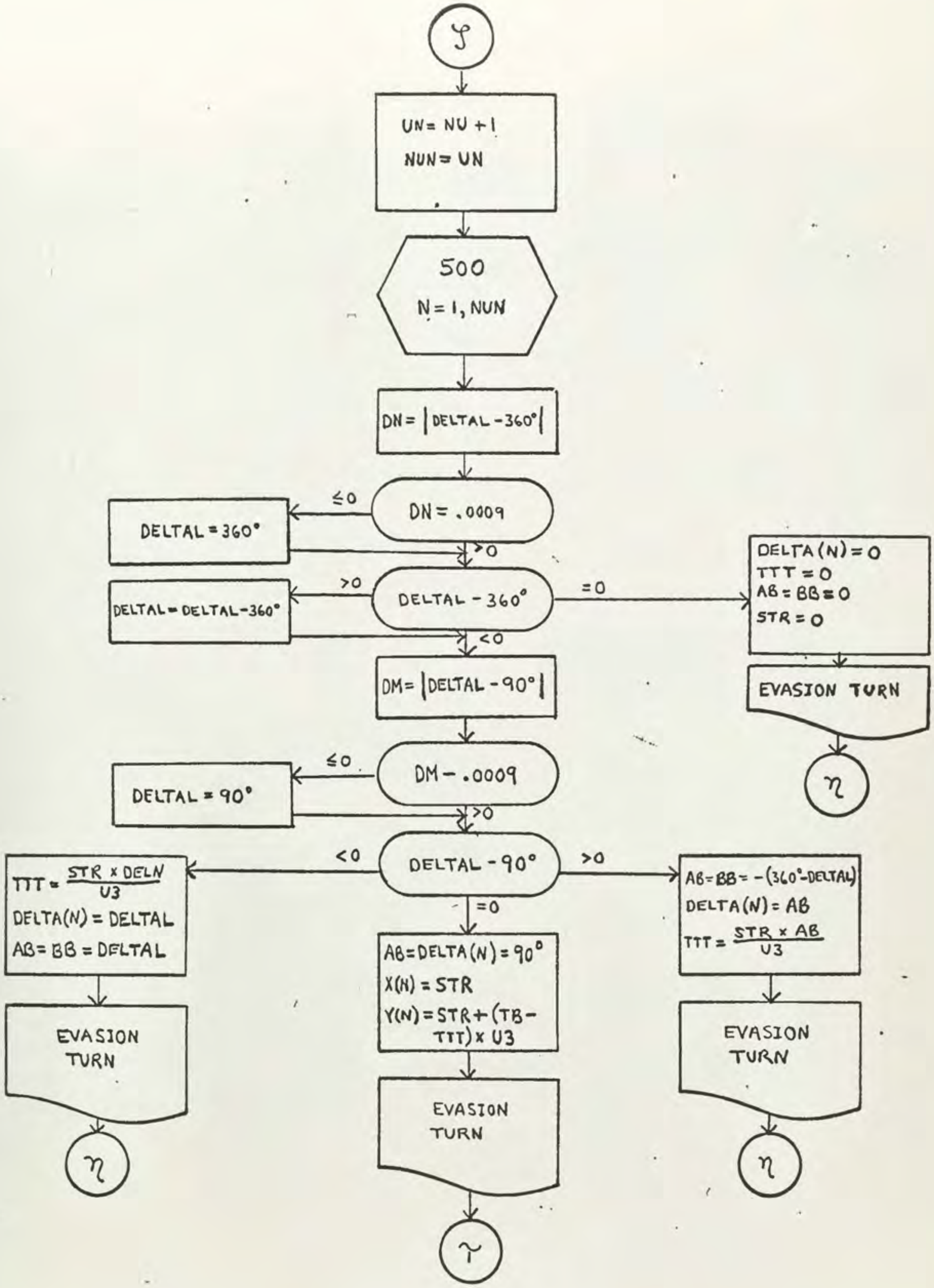



β
X1 = U1 × SIN(THETA)
Y1 = U1 × COS(THETA)
X2 = RAI × SIN(BETA)
Y2 = RAI × COS(BETA)
X2 = 0
F
T
Y3(1) = √(V² − (X1)²)
Y3(2) = −Y3(1)
Y2 = 0
F
T
γ
X3(1) = √(V² − (Y1)²)
X3(2) = − X3(1)
AB1YY = |Y2 − Y3(1)|
AB2YY = |Y2 − Y3(2)|
AB1XX = |X2 − X3(1)|
AB2XX = |X2 − X3(2)|
>0
AB1YY − AB2YY
=0
<0
I = 2
X3(I) = X1
ERROR STATEMENT
δ
NO = NO + 1
M = M + 1
α
I = 1
X3(I) = X1
δ
=0
AB1XX − AB2XX
>0
<0
ERROR STATEMENT
I = 2
Y3(I) = Y1
δ
NO = NO + 1
M = M + 1
α
I = 1
Y3(I) = Y1
δ

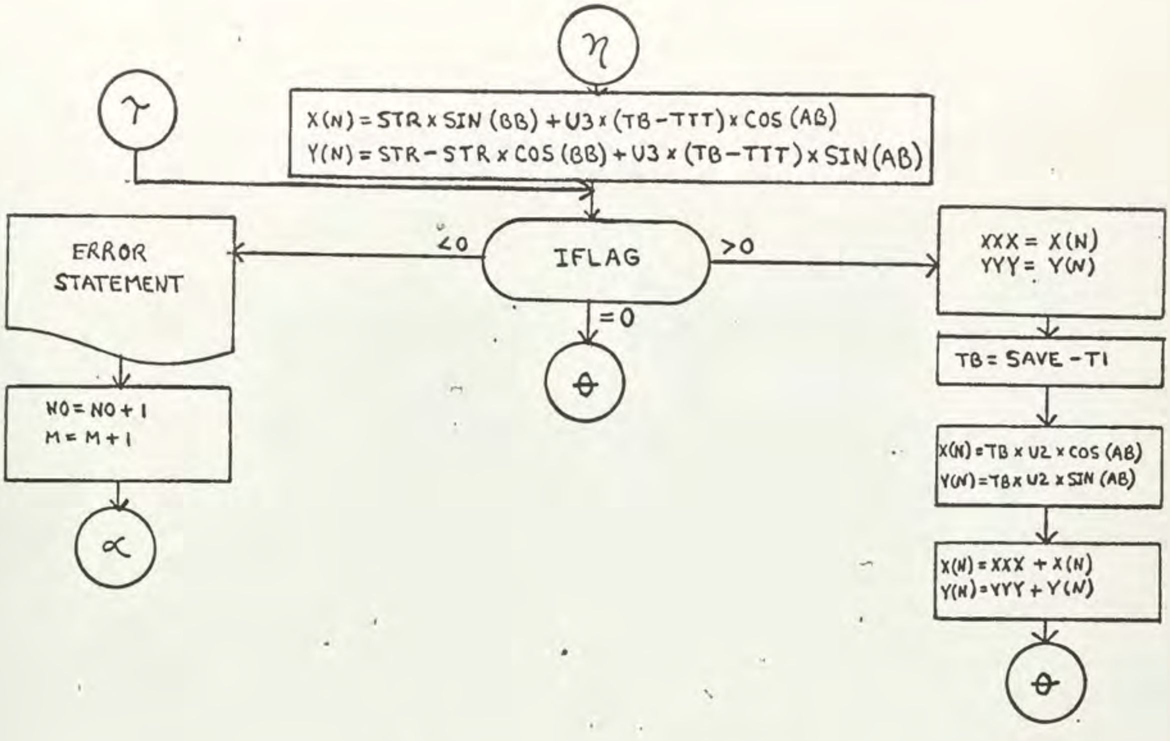

γ
$B = \frac{-2 \times (X1) \times (Y2)^2}{(X2)^2} + \frac{2 \times (Y1) \times (Y2)}{X2}$
$A = 1 + (Y2)^2 / (X2)^2$
$C = \frac{(Y2)^2 \times (X1)^2}{(X2)^2} - \frac{2 \times (Y1) \times (Y2) \times (X1)}{X2} + (Y1)^2 - V^2$
$X3(1) = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$
$X3(2) = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$
$ABSXX1 = |X2 - X3(1)|$
$ABSXX2 = |X2 - X3(2)|$
<0
ABSXX1 − ABSXX2
>0
=0
$Y31 = \sqrt{V^2 - (X3(1))^2}$
$Y32 = -Y31$
ERROR STATEMENT
$Y33 = \sqrt{V^2 - (X3(2))^2}$
$Y34 = -Y33$
$ABSYY1 = |Y2 - Y31|$
$ABSYY2 = |Y2 - Y32|$
NO = NO + 1
M = M + 1
$ABSYY1 = |Y2 - Y33|$
$ABSYY2 = |Y2 - Y34|$
=0
ABSYY1 − ABSYY2
>0
>0
ABSYY1 − ABSYY2
=0
α
<0
<0
ERROR STATEMENT
ERROR STATEMENT
I = 1
Y3(I) = Y32
I = 2
Y3(I) = Y34
NO = NO + 1
M = M + 1
NO = NO + 1
M = M + 1
δ
δ
α
α
I = 1
Y3(I) = Y31
I = 2
Y3(I) = Y33
δ
δ


δ
RELSPD = √(X3(I) − X1)² + (Y3(I) − Y1)²
RELATIVE SPEED
TD = RAI / RELSPD
RT = PTOT + TOT + CLDT + RELT
IX = 0
SUBT = RAI/U3
SUBT − RT
≤ 0
REACTION TIME TOO GREAT. SUB CAN KILL VEHICLE.
IX = 1
>0
TB = TD + RT
DIST = U3 x TB
X4 = DIST x SIN(THETA) + X2
Y4 = DIST x COS(THETA) + Y2
DISDAT = √(X4)² + (Y4)²
BLIND TIME DISDAT
ε

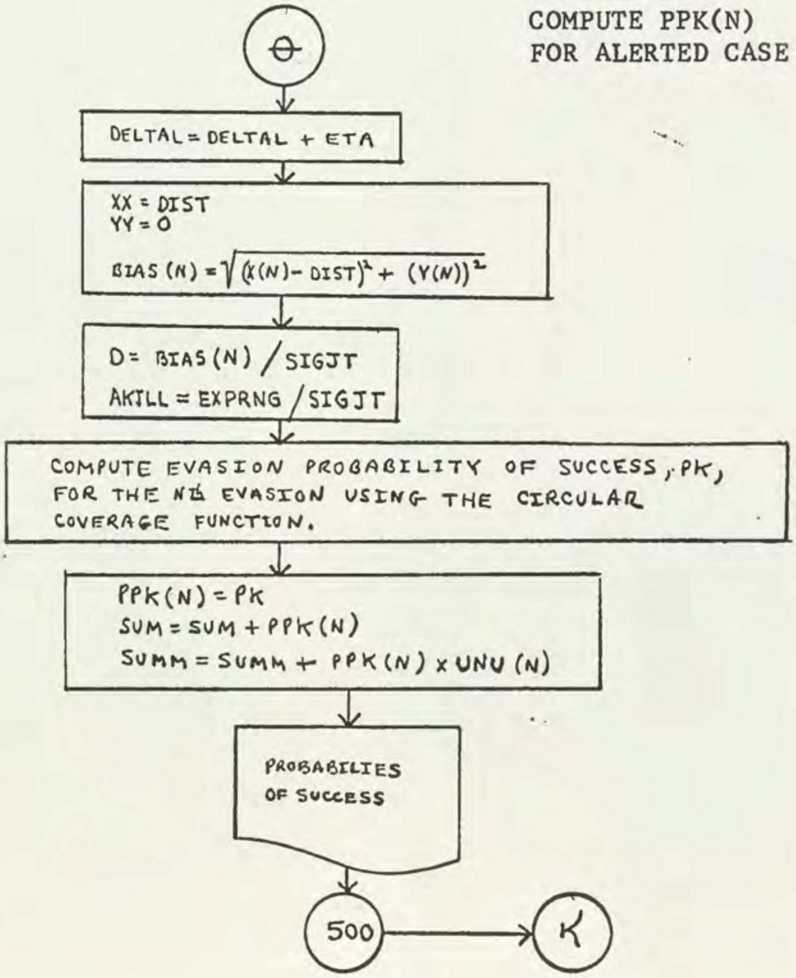

∈
90° − DELTAL
<0
=0
>0
ERROR STATEMENT
90° − PSI
<0
≥0
ETA = PSI − DELTAL / NU
ETA = PSI + (360 − DELTAL) / NU
NO = NO + 1
M = M + 1
α
STR = TR/2000
SIGNAV = PRCNTN × RAI
SIGSR = PRCNTS × RAI
SIGSBR = RAI × SIN(BRGER)
SIGAPR = √SIGSR × SIGSBR
SIGJT = √SIGNAV² + SIGAPR²
SIGJT
SAVE = TB
TB − T1
>0
≤0
T1
=0
<0
>0
ERROR STATEMENT
IFLAG = 1
TB = T1
NO = NO + 1
M = M + 1
α
γ

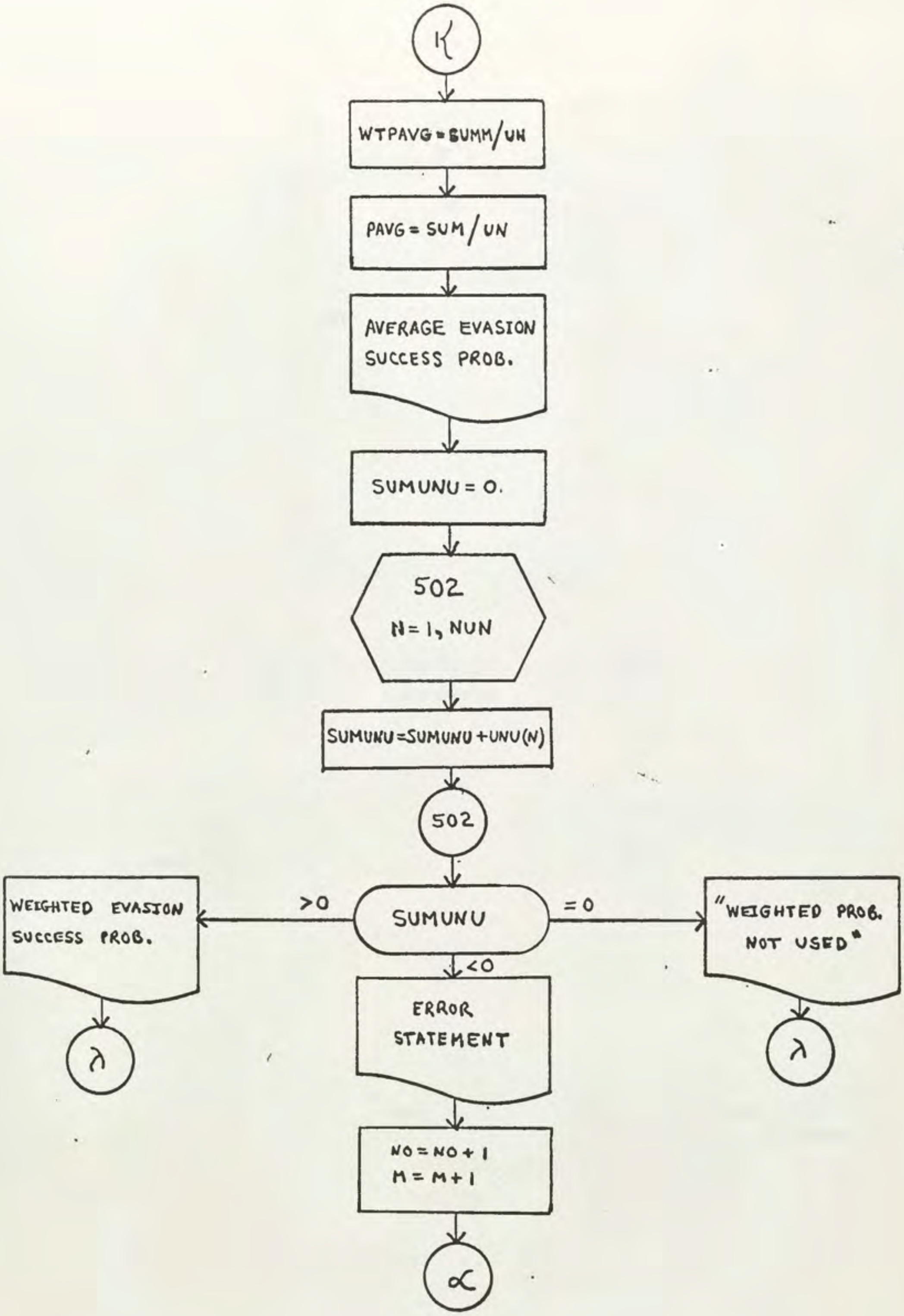


$\mathcal{S}$
UN = NU + 1
NUN = UN
500
N = 1, NUN
DN = |DELTAL − 360°|
DN = .0009
≤0
DELTAL = 360°
>0
DELTAL − 360°
>0
DELTAL = DELTAL − 360°
=0
DELTA(N) = 0
TTT = 0
AB = BB = 0
STR = 0
EVASION TURN
η
<0
DM = |DELTAL − 90°|
DM − .0009
≤0
DELTAL = 90°
>0
DELTAL − 90°
<0
TTT = STR × DELN / U3
DELTA(N) = DELTAL
AB = BB = DELTAL
EVASION TURN
η
=0
AB = DELTA(N) = 90°
X(N) = STR
Y(N) = STR + (TB − TTT) × U3
EVASION TURN
γ
>0
AB = BB = −(360° − DELTAL)
DELTA(N) = AB
TTT = STR × AB / U3
EVASION TURN
η



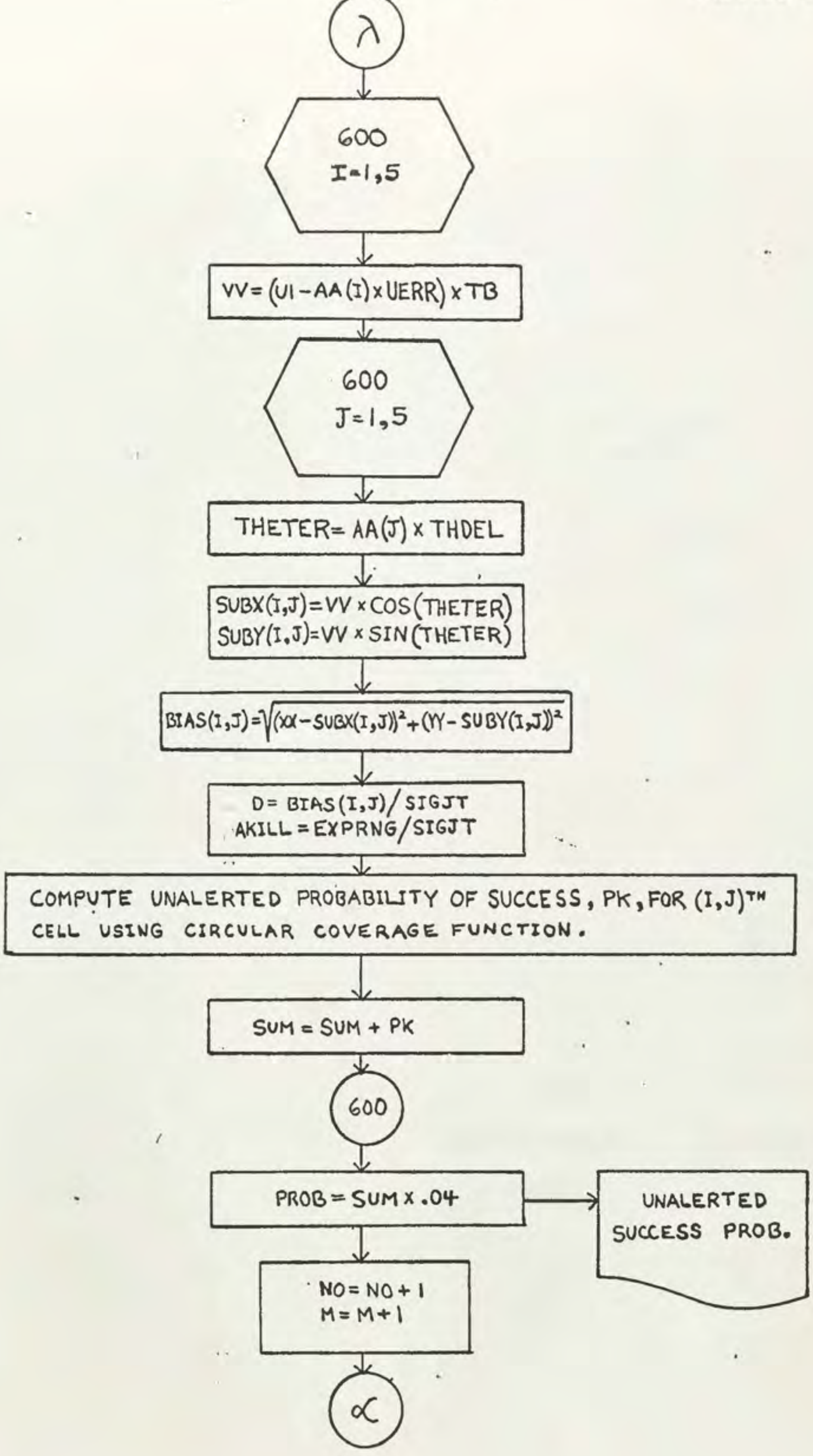
COMPUTE PPK(N)
FOR ALERTED CASE

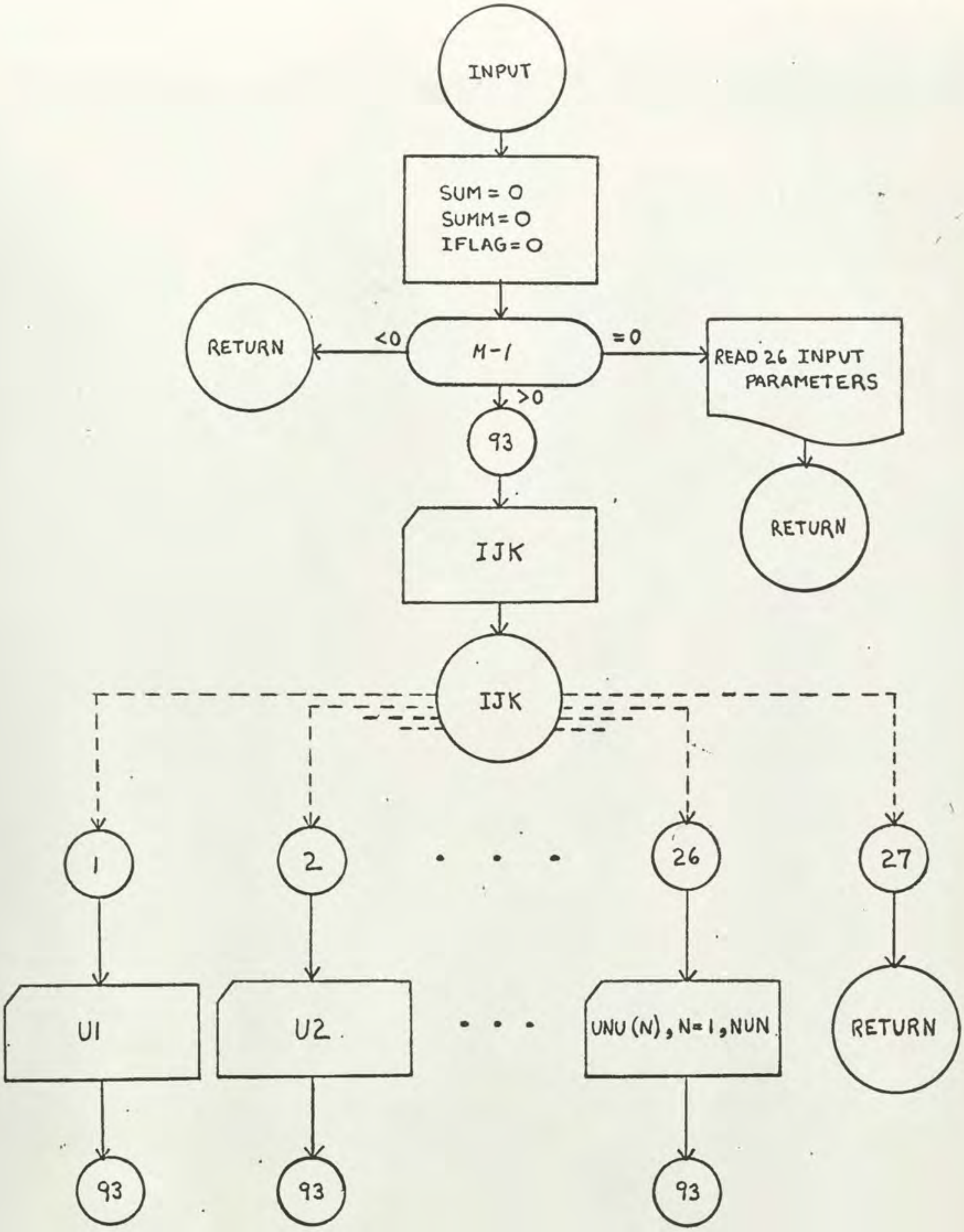




K
WTPAVG = SUMM/UN
PAVG = SUM/UN
AVERAGE EVASION SUCCESS PROB.
SUMUNU = 0.
502
N = 1, NUN
SUMUNU = SUMUNU + UNU(N)
502
SUMUNU
>0
=0
<0
WEIGHTED EVASION SUCCESS PROB.
"WEIGHTED PROB. NOT USED"
ERROR STATEMENT
NO = NO + 1
M = M + 1
λ
λ
∝

UNALERTED CASE




INPUT
SUM = 0
SUMM = 0
IFLAG = 0
RETURN
<0
M-1
=0
READ 26 INPUT PARAMETERS
>0
93
RETURN
IJK
IJK
1
2
. . .
26
27
U1
U2
. . .
UNU (N), N = 1, NUN
RETURN
93
93
93

```
      PROGRAM ASWSYS
C     THE PROGRAM COMPUTES PROBABILITIES OF AN ASW VEHICLE VS. AN
C     EVASIVE SUBMARINE AS A FUNCTION OF REACTION OF THE VEHICLE.
      DIMENSION Y3(2),X3(2),DELTA(30),X(30),Y(30),BIAS(30),AA(5),
     1SUBX(5,5),SUBY(5,5),SBIAS(5,5),PPK(15),UNU(15),ZZ(4)
      NEXT=2
      NO=1
      M=1
   20 IF(NEXT-M)9000,30,30
   30 CALL INPUT(U1,U2,U3,Z,PTOT,TOT,CLDT,RELT,T1,THETA,BETA,RAI,     TR,
     1PRCNTN,PRCNTS,BRGER,EXPRNG,PSI,UERR,THDEL,NU,NEXT,DELTAL,SUM,AA,IF
     2LAG,UNU,DELN,SUMM,M)
C     OUTPUT ALL INPUTS.
      GO TO 1999
C     CONVERT KNOTS TO MILES PER MINUTE, AND DEGREES TO RADIANS
   40 SV=Z*.01667
      SU1=U1*.01667
      SU2=U2*.01667
      SU3=U3*.01667
      SUERR=UERR*.01667
      STHETA=THETA/57.29578
      SBETA=BETA/57.29578
      SPSI=PSI/57.29578
      SDELTA=DELTAL/57.29578
      SBRGER=BRGER/57.29578
      STHDEL=THDEL/57.29578
C     SET UP MANEUVERING BOARD SOLUTION TO SOLVE FOR THE POSITON THE
C     SUBMARINE WILL BE   WHEN THE VEHICLE INTERCEPTS IT, ASSUMING A
C     CONSTANT COURSE/SPEED FOR THE SUBMARINE.
      X1=SU1*SINF(STHETA)
      Y1=SU1*COSF(STHETA)
C     (X1,Y1) AND (0,0) ARE TWO POINTS THAT INDICATE THE CUS/SPD OF THE
C     SUBMARINE.
      X2=RAI*SINF(SBETA)
      Y2=RAI*COSF(SBETA)
C     (X2,Y2) IS THE LAST KNOWN POSITION OF THE SUBMARINE RELATIVE TO
C     VEHICLE AT (0,0). NOW CHECK SPECIAL CASES.
C     THE FOLLOWING 13 STATEMENTS ARE USED TO ROUND OFF X1,X2,Y1,Y2 TO
C     ZERO IF THEY ARE WITHIN + OR - .001 OF ZERO.
      ZZ(1)=X1
      ZZ(2)=Y1
      ZZ(3)=X2
      ZZ(4)=Y2
      DO 52 I=1,4
      ABZZ=ABSF(ZZ(I))
      IF(ABZZ-.001)51,51,52
   51 ZZ(I)=0.
   52 CONTINUE
```

```
      X1=ZZ(1)
      Y1=ZZ(2)
      X2=ZZ(3)
      Y2=ZZ(4)
      IF(X2)105,100,105
 100  Y3(1)=SQRTF(SV**2-X1**2)
      Y3(2)=-SQRTF(SV** 2-X1**2)
      Y3A=Y3(1)
      Y3B=Y3(2)
      AB1YY=ABSF(Y2-Y3A)
      AB2YY=ABSF(Y2-Y3B)
      IF(AB1YY-AB2YY)102,9050,103
 102  I=1
      GO TO 104
 103  I=2
 104  X3(I)=X1
      GO TO 200
 105  IF(Y2)110,106,110
 106  X3(1)=SQRTF(SV**2-Y1**2)
      X3(2)=-X3(1)
      X3A=X3(1)
      X3B=X3(2)
      AB1XX=ABSF(X2-X3A)
      AB2XX=ABSF(X2-X3B)
      IF(AB1XX-AB2XX)107,9050,108
 107  I=1
      GO TO 109
 108  I=2
 109  Y3(I)=Y1
      GO TO 200
 110  B=((-2.*X1*Y2*Y2)/(X2**2))+((2.*Y1*Y2)/X2)
      A=1.+(Y2**2/X2**2)
      C=((Y2**2*X1**2)/X2**2)-(2.*Y1*Y2*X1)/X2+Y1**2-SV**2
      X3(1)=(-B+SQRTF(B**2-4.*A*C))/(2.*A)
      X3(2)=(-B-SQRTF(B**2-4.*A*C))/(2.*A)
      X3A=X3(1)
      X3B=X3(2)
      ABSXX1=ABSF(X2-X3A)
      ABSXX2=ABSF(X2-X3B)
      IF(ABSXX1-ABSXX2) 125,9050,155
 125  I=1
      Y31=SQRTF(SV**2-X3A**2)
      Y32=-Y31
      ABSYY1=ABSF(Y2-Y31)
      ABSYY2=ABSF(Y2-Y32)
      IF(ABSYY1-ABSYY2)130,9050,135
 130  I=1
      Y3(I)=Y31
```

```
      GO TO 200
  135 I=1
      Y3(I)=Y32
      GO TO 200
  155 I=2
      Y33=SQRTF(SV**2-X3B**2)
      Y34=-Y33
      ABSYY1=ABSF(Y2-Y33)
      ABSYY2=ABSF(Y2-Y34)
      IF(ABSYY1-ABSYY2)140,9050,150
  140 I=2
      Y3(I)=Y33
      GO TO 200
  150 I=2
      Y3(I)=Y34
  200 X3II=X3(I)
      Y3II=Y3(I)
      RELSPD=SQRTF((X3II-X1)**2+(Y3II-Y1)**2)
      RSPD=RELSPD/.01667
C     COMPUTE FLYING TIME TO DATUM
      TD=RAI/RELSPD
C     COMPUTE REACTION TIME
      RT=PTOT+TOT+CLDT+RELT
      IX=0
      SUBT=RAI/SU3
      IF(SUBT-RT)202,202,201
  202 IX=1
  201 CONTINUE
C     COMPUTE OVER ALL REACTION TIME (CALLED BLIND TIME).
      TB=TD+RT
C     COMPUTE DISTANCE TRAVELED BY SUBMARINE DURING BLIND TIME.
      DIST=SU3*TB
C     (X4,Y4)=PREDICTED DATUM POSITION OF SUBMARINE RELATIVE TO VEHICLE
C     AT(0,0).
      X4=DIST*SINF(STHETA)+X2
      Y4=DIST*COSF(STHETA)+Y2
C     DISTANCE FLOWN TO DATUM.
      DISDAT=SQRTF(X4**2+Y4**2)
C     COMPUTE PROBABILITIES FOR ALERTED SUBMARINE EVADING. PLACE SUB IN
C     (0,0) POSITION HEADING 000. RANGE OF SUB EVASION IS FROM DELTA
C     DEGREES TO RIGHT OF 000 TO PSI DEGREES TO THE LEFT WITH INCREMENTS
C     OF ETA DEGREES. FOR COMPUTING PURPOSES 000 IS LAST KNOWN SUB CUS.
      RNU=NU
      IF(1.5708-SDELTA)204,9050,210
  204 IF(1.5708-SPSI)210,205,205
  205 ETA=(SPSI+(6.28319-SDELTA))/RNU
      GO TO 212
  210 ETA=(SPSI-SDELTA)/RNU
```

```
  212 STR=TR/2000.
      SAVE1=STR
C     COMPUTE ERRORS
      SIGNAV=.01*PRCNTN*RAI
      SIGSR= .01*PRCNTS*RAI
      SIGSBR=RAI*SINF(SBRGER)
      SIGAPR=SQRTF(SIGSR*SIGSBR)
      SIGJT= SQRTF(SIGNAV**2+SIGAPR**2)
      SAVE=TB
      IF(TB-T1)299,299,405
  405 IF(T1)9050,299,407
  407 IFLAG=1
      TB=T1
  299 UN=NU+1
      NUN=UN
      DO 500 N=1,NUN
      IF(N-1)9050,301,310
  301 DELN=SDELTA
  310 DN=ABSF(DELN-6.2832)
      IF(DN-.0009)312,312,314
  312 DELN=6.2832
  314 IF(DELN-6.2832)300,315,666
  315 CONTINUE
      DELTA(N)=0.
      TTT=0.
      AB=0.
      STR=0.
      BB=0.
      GO TO 350
  666 DELN=DELN-6.2832
  300 DM=ABSF(DELN-1.5708)
      IF(DM-.0009)313,313,316
  313 DELN=1.5708
  316 IF(DELN-1.5708)320,330,340
  320 TTT=STR*DELN/SU3
      AB=DELN
      DELTA(N)=DELN*57.29578
      BB=AB
      GO TO 350
  330 AB=1.5708
      DELTA(N)=90.
      X(N)=STR
      Y(N)=STR+(TB-TTT)*SU3
      GO TO 409
  340 AB=-(6.2832-DELN)
      DELTA(N)=-(6.2832-DELN)*57.29578
      TTT=STR*AB/SU3
      BB=AB
```

```
      STR=-STR
      TTT=-TTT
  350 X(N)=STR*SINF(BB)+SU3*(TB-TTT)*COSF(AB)
      Y(N)=STR-STR*COSF(BB)+SU3*(TB-TTT)*SINF(AB)
  409 IF(IFLAG)9050,400,410
  410 XXX=X(N)
      YYY=Y(N)
      TB=SAVE-T1
      X(N)=TB*SU2*COSF(AB)
      Y(N)=TB*SU2*SINF(AB)
      X(N)=XXX+X(N)
      Y(N)=YYY+Y(N)
  400 DELN=DELN+ETA
      XX=DIST
      YY=0.
C     COMPUTE BIAS FOR EACH (X(N),Y(N)) RELATIVE TO (XX,YY).
      XNX=X(N)
      YNY=Y(N)
      BIAS(N)=SQRTF((XNX-XX)**2+(YNY-YY)**2)
      D=BIAS(N)/SIGJT
      AKILL=EXPRNG/SIGJT
      STR=SAVE1
      TB=T1
C     COMPUTE PROBABILITY FOR EACH EVASION BY SOLVING CIRCULAR COVERAGE
C     FUNCTION.
      CALL GCIP(D,AKILL,PK)
      PPK(N)=PK
      SUMM=SUMM+PPK(N)*UNU(N)
  500 SUM=SUM+PPK(N)
C     COMPUTE PROBABILITY FOR EVASIVE SUBMARINE USING WEIGHTED PROBABILITIES
      WTPAVG=SUMM/UN
      PAVG=SUM/UN
C     COMPUTE PROBABILITY ASSUMING SUBMARINE NOT ALERTED OF VEHICLES
C     PRESENCE.
      SUMUNU=0.0
      DO 502 N=1,NUN
  502 SUMUNU=SUMUNU+UNU(N)
C     LAST KNOWN POSITION OF SUBMARINE IS THE ORIGIN. PREDICTED SUBMARI
C     NE POSITION AFTER BLIND TIME (WITH NO EVASION) IS (XX,YY). COMPUTE
C     POSITIONS OF CENTERS OF CELLS.
      SUM=0.
      DO 600 I=1,5
      TB=SAVE
      VV=(SU1-AA(I)*SUERR)*TB
      DO 600 J=1,5
      THETER=AA(J)*STHDEL
      SUBX(I,J)=VV*COSF(THETER)
      SUBY(I,J)=VV*SINF(THETER)
```

```
      XSUB=SUBX(I,J)
      YSUB=SUBY(I,J)
      SBIAS(I,J)=SQRTF((XX-XSUB)**2+(YY-YSUB)**2)
      D=SBIAS(I,J)/SIGJT
      AKILL=EXPRNG/SIGJT
      CALL CCIP (D,AKILL,PK)
  600 SUM=SUM+PK
      PROB=SUM*.04
      GO TO 3010
C     PROB IS PROBABILITY THAT VEHICLE RELOCATES SUB AT DATUM IF SUB
C     DOES NOT EVADE
C     PRINT OUT INPUT PARAMETER SET
 1999 WRITE OUTPUT TAPE 3,2001,M,NEXT,NO
 2001 FORMAT(1H1,//,5X,10HOUTPUT RUN ,I3,1X,2HOF,I3,79X,5HPAGE ,I3)
      WRITE OUTPUT TAPE 3,2005
 2005 FORMAT(1H0,46X,28HCOMPLETE INPUT PARAMETER SET)
      WRITE OUTPUT TAPE 3,2010
 2010 FORMAT(///,20X,9HPARAMETER,11X,5HVALUE,15X,7HMEANING)
      WRITE OUTPUT TAPE 3,2015,U1
 2015 FORMAT(///,20X,2HU1,18X,F10.4,10X,30HSUBMARINE CRUISE SPEED (KNOTS
     1))
      WRITE OUTPUT TAPE 3,2020,U2
 2020 FORMAT(/ ,20X,2HU2,18X,F10.4,10X,30HSUBMARINE SILENT SPEED (KNOTS)
     1)
      WRITE OUTPUT TAPE 3,2025,U3
 2025 FORMAT(/ ,20X,2HU3,18X,F10.4,10X,28HSUBMARINE FAST SPEED (KNOTS))
      WRITE OUTPUT TAPE 3,2030,Z
 2030 FORMAT(/ ,20X,1HZ,19X,F10.4,10X,57HVEHICLE TRANSIT SPEED TO DATUM
     1(NEGLECT ACCEL. OR DECEL.))
      WRITE OUTPUT TAPE 3,2035,PTOT
 2035 FORMAT(/ ,20X,4HPTOT,16X,F10.4,10X,23HPRE TAKE OFF TIME (MIN))
      WRITE OUTPUT TAPE 3,2040,TOT
 2040 FORMAT(/ ,20X,3HTOT,17X,F10.4,10X,19HTAKE OFF TIME (MIN))
      WRITE OUTPUT TAPE 3,2045,CLDT
 2045 FORMAT(/ ,20X,4HCLDT,16X,F10.4,10X,15HLAND TIME (MIN))
      WRITE OUTPUT TAPE 3,2050,RELT
 2050 FORMAT(/ ,20X,4HRELT,16X,F10.4,10X,28HTARGET RELOCATION TIME (MIN)
     1)
      WRITE OUTPUT TAPE 3,2055,T1
 2055 FORMAT(/ ,20X,2HT1,18X,F10.4,10X,38HELLAPSED TIME OF SUB AT SPEED
     1U3 (MIN))
      WRITE OUTPUT TAPE 3,2060,THETA
 2060 FORMAT(/ ,20X,5HTHETA,15X,F10.4,10X,46HCOURSE OF SUB AT LAST KNOWN
     1 POSITION (DEGREES))
      WRITE OUTPUT TAPE 3,2065,BETA
 2065 FORMAT(/ ,20X,4HBETA,16X,F10.4,10X,42HTRUE BEARING OF SUB FROM VEH
     1ICLE (DEGREES))
      WRITE OUTPUT TAPE 3,2070,RAI
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```
2070 FORMAT(/ ,20X,3HRAI,17X,F10.4,10X,56HRANGE TO SUB AT LAST KNOWN PO
    1SITION FROM VEHICLE (MILES))
     WRITE OUTPUT TAPE 3,2080,TR
2080 FORMAT(/ ,20X,2HTR,18X,F10.4,10X,27HTURNING RADIUS OF SUB (YDS))
     WRITE OUTPUT TAPE 3,2085,PRCNTN
2085 FORMAT(/ ,20X,6HPRCNTN,14X,F10.4,10X,37HNAV. ERROR IN PERCENT DIST
    1ANCE TO SUB)
     WRITE OUTPUT TAPE 3,2090,PRCNTS
2090 FORMAT(/ ,20X,6HPRCNTS,14X,F10.4,10X,42HSENSOR RANGE ERROR IN PERC
    1ENT SENSOR RANGE)
     WRITE OUTPUT TAPE 3,2095,BRGER
2095 FORMAT(/ ,20X,5HBRGER,15X,F10.4,10X,31HSENSOR BEARING ERROR IN DEG
    1REES)
     WRITE OUTPUT TAPE 3,2100,EXPRNG
2100 FORMAT(/  ,20X,6HEXPRNG,14X,F10.4,10X,58HRANGE EXPECTED FROM SENSO
    1R FOR LOCALIZATION AT DATUM,MILES)
     WRITE OUTPUT TAPE 3,2105,PSI
2105 FORMAT(/ ,20X,3HPSI,17X,F10.4,10X,42HLEFT BOUND FOR SUB EVASIVE TU
    1RN IN DEGREES)
     WRITE OUTPUT TAPE 3,2110,DELTAL
2110 FORMAT(/ ,20X,6HDELTAL,14X,F10.4,10X,43HRIGHT BOUND FOR SUB EVASIV
    1E TURN IN DEGREES)
     WRITE OUTPUT TAPE 3,2115,NU
2115 FORMAT(/ ,20X,2HNU,18X,I5,15X,47HNUMBER INCREMENTS CONSIDERED IN P
    1SI-DELTA RANGE)
     WRITE OUTPUT TAPE 3,2120,NEXT
2120 FORMAT(/ ,20X,4HNEXT,16X,I5,15X,28HNUMBER OF SETS OF INPUT DATA)
     WRITE OUTPUT TAPE 3,2125,UERR
2125 FORMAT(/ ,20X,4HUERR,16X,F10.4,10X,29HSUBMARINE SPEED ERROR (KNOTS
    1))
     WRITE OUTPUT TAPE 3,2130,THDEL
2130 FORMAT(/ ,20X,5HTHDEL,15X,F10.4,10X,32HSUBMARINE COURSE ERROR (DEG
    1REES))
    1))
     NO=NO+1
     GO TO 40
9050 WRITE OUTPUT TAPE 3,9055,M,NEXT,NO
9055 FORMAT(1H1,///,5X,11HOUTPUT RUN ,I3,1X,2HOF,I3,79X,5HPAGE ,I3,////
    1,5X,67HERROR. INCORRECT SOLUTION BEING CALCULATED. CONTINUE WITH N
    2EXT RUN.)
     NO=NO+1
     M=M+1
     GO TO 20
3010 WRITE OUTPUT TAPE 3,3015,M,NEXT,NO
3015 FORMAT(1H1,//,5X,10HOUTPUT RUN ,I3,1X,2HOF,I3,79X,5HPAGE ,I3)
     WRITE OUTPUT TAPE 3,3020
3020 FORMAT(///,50X,16HINPUT PARAMETERS)
     WRITE OUTPUT TAPE 3,3025,Z,PTOT,TOT,CLOT,RELT,U1,U2,U3,RAI,THETA
```

```
3025 FORMAT(///,46X,56HTARGET      SUBMARINE   SUBMARINE   SUBMARINE   S
    1UBMARINE/1H ,5X,110HVEHICLE    PRE TAKE    TAKE OFF    LAND   RELOCAT
    2ION    CRUISE     SILENT        FAST       TO VEHICLE   SUBMARINE/1
    3H ,6X,106HSPEED   OFF TIME      TIME       TIME       TIME        SPEE
    4D      SPEED      SPEED         RANGE             COURSE/1H ,5X,110H(KNO
    5TS)    (MIN)      (MIN)      (MIN)       (MIN)        (KNOTS)     (KNOT
    6S)     (KNOTS)     (MILES)       (DEGREES)//1H ,6X,F6.2,2X,F6.2,6X,F
    76.2,3X,F6.2,3X,F6.2,6X,F6.2,7X,F6.2,6X,F6.2,6X,F6.2,7X,F6.2)
     WRITE OUTPUT TAPE 3,3030,BETA,PRCNTN,PRCNTS,BRGER,EXPRNG
3030 FORMAT(///,49X,32HSENSOR                             EXPECTED/1H ,20X,61HTRUE
    1 BEARING   NAVIGATION    RANGE       SENSOR          RANGE OF/1H ,21X,
    263HSUB FROM           ERROR          ERROR        BEARING      LOCALIZATION
    3/1H ,23X,59HVEHICLE          (PERCENT          (PERCENT     ERROR          S
    4ENSOR/1H ,21X,59H(DEGREES)        DISTANCE)      RANGE)     (DEGREES)
    5    (MILES)//1H ,22X,F6.2,9X,F6.2,6X,F5.2,7X,F6.2,6X,F6.2)
     IF(IX)9050,3034,3333
3333 WRITE OUTPUT TAPE 3,3032
3032 FORMAT(///,10X,105H** NOTE-RANGE TO SUB IS CLOSE ENOUGH FOR SUB TO
    1 CLOSE VEHICLE AND FIRE WEAPONS BEFORE VEHICLE CAN ESCAPE.)
3034 CONTINUE
     WRITE OUTPUT TAPE 3,3035,NUN,PAVG
3035 FORMAT(///,30X,32HAVERAGE SUCCESS PROBABILITY FOR ,I2,20H EVASION
    1TACTICS IS ,F5.3)
     IF(SUMUNU)9050,3036,3031
3036 WRITE OUTPUT TAPE 3,3037
3037 FORMAT(//,30X,45HAVERAGE WEIGHTED SUCCESS PROBABILITY NOT USED)
     GO TO 3039
3031 WRITE OUTPUT TAPE 3,3038,NUN,WTPAVG
3038 FORMAT(//,21X,41HAVERAGE WEIGHTED SUCCESS PROBABILITY FOR ,I2,20H
    1EVASION TACTICS IS ,F5.3)
3039 WRITE OUTPUT TAPE 3,3041,PROB
3041 FORMAT(////,42X,39HCOMPUTED VALUES FOR UNALERTED SUBMARINE//1H ,30-
    1X,28HAVERAGE SUCCESS PROBABILITY ,F5.3)
     WRITE OUTPUT TAPE 3,3040
3040 FORMAT(///,42X,  37HCOMPUTED VALUES FOR SUBMARINE EVASION///1H ,62
    1X,37HBLIND       DISTANCE TRANSIT   RELATIVE/1H ,17X,94HEVASION
    2EVASION TURN        SUCCESS         TIME           TO DATUM          SPE
    3ED        SIG TOTAL/1H ,17X,94HNUMBER              (DEGREES)        PROBABILI
    4TY      (MIN)            (MILES)            (KNOTS)       (MILES)/)
     DO 3055 N=1,NUN
     WRITE OUTPUT TAPE 3,3050,N,DELTA(N),PPK(N),TB,DISDAT,RSPD,SIGJT
3050 FORMAT(/1H ,16X,I5,8X,F8.2,10X,F5.3,8X,F6.2,8X,F6.2,11X,F6.2,6X,F5
    1.2)
3055 CONTINUE
     NO=NO+1
     M=M+1
     GO TO 20
9000 END
```

```
      SUBROUTINE INPUT(U1,U2,U3,Z,PTOT,TOT,CLDT,RELT,T1,THETA,BETA,RAI,
     1   TR,PRCNTN,PRCNTS,BRGER,EXPRNG,PSI,UERR,THDEL,NU,NEXT,DELTAL,SUM
     2,AA,IFLAG,UNU,DELN,SUMM,M)
      DIMENSION AA(5),UNU(15)
      SUM=0.
      SUMM=0.
      IFLAG=0
      DELN=0.
      IF(M-1)111,91,93
   91 READ 110,U1
      READ 110,U2
      READ 110,U3
      READ 110,Z
      READ 110,PTOT
      READ 110,TOT
      READ 110,CLDT
      READ 110,RELT
      READ 110,T1
      READ 110,THETA
      READ 110,BETA
      READ 110,RAI
      READ 110,TR
      READ 110,PSI
      READ 110,DELTAL
      READ 110,BRGER
      READ 110,EXPRNG
      READ 110,PRCNTN
      READ 110,PRCNTS
      READ 110,THDEL
      READ 110,UERR
  110 FORMAT(F12.4)
      READ 112,NU
      READ 112,NEXT
  112 FORMAT(I4)
      READ 113,(AA(I),I=1,5)
  113 FORMAT(5F12.4)
      NUN=NU+1
      READ 114,(UNU(N),N=1,NUN)
  114 FORMAT(12F6.4)
      GO TO 111
   93 READ 94,IJK
   94 FORMAT(I4)
      GO TO(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,
     124,25,26,27),IJK
    1 READ 110,U1
      GO TO 93
    2 READ 110,U2
      GO TO 93
```

```
    3 READ 110,U3
      GO TO 93
    4 READ 110,Z
      GO TO 93
    5 READ 110,PTOT
      GO TO 93
    6 READ 110,TOT
      GO TO 93
    7 READ 110,CLDT
      GO TO 93
    8 READ 110,RELT
      GO TO 93
    9 READ 110,T1
      GO TO 93
   10 READ 110,THETA
      GO TO 93
   11 READ 110,BETA
      GO TO 93
   12 READ 110,RAI
      GO TO 93
C     THIS SPACE FREE FOR AN INPUT VARIABLE
   13 GO TO 93
   14 READ 110,TR
      GO TO 93
   15 READ 110,PSI
      GO TO 93
   16 READ 110,DELTAL
      GO TO 93
   17 READ 110,BRGER
      GO TO 93
   18 READ 110,EXPRNG
      GO TO 93
   19 READ 110,PRCNTN
      GO TO 93
   20 READ 110,PRCNTS
      GO TO 93
   21 READ 110,THDEL
      GO TO 93
   22 READ 110,UERR
      GO TO 93
   23 READ 112,NU
      NUN=NU+1
      GO TO 93
   24 READ 112,NEXT
      GO TO 93
   25 READ 113,(AA(I),I=1,5)
      GO TO 93
   26 READ 114,(UNU(N),N=1,NUN)
```

```
      GO TO 93
   27 GO TO 111
  111 RETURN
      END
      SUBROUTINE OCIP(A,V,PAV)
      IF(V-A-5.0)1,1,2
    2 PAV=1.0
      RETURN
    1 IF(V-A+5.0)3,4,4
    3 PAV=0.0
      RETURN
    4 PI=3.14159265
      HALPI=PI*0.5
      TWPI=2.0*PI
      RTWPI=SQRTF(TWPI)
      RHAPI=SQRTF(HALPI)
      IF(A-4.)5,6,6
C     A LESS THAN FOUR
    5 ASQ=A*A
      AFOUR=ASQ*ASQ
      ASIX=AFOUR*ASQ
      Z=-ASQ*0.25
      EXP=EXPF(Z)
      CALL  IZERO(-Z,BESSZ)
      CALL  IONE(-Z,BESSO)
      EM1P=RHAPI*EXP*(BESSZ+ASQ*(BESSZ+BESSO)*0.5)
      EM3P=RHAPI*EXP*((ASQ+3.0)*BESSZ+(AFOUR+4.0*ASQ)*(BESSZ+BESSO)*0.5)
      EM5P=RHAPI*EXP*((AFOUR+11.0*ASQ+15.0)*BESSZ+(ASIX+12.0*AFOUR+23.0*
     1ASQ)*(BESSZ+BESSO)*0.5)
      EM2P=ASQ+2.0
      EM4P=AFOUR+8.0*ASQ+8.0
      EM6P=ASIX+18.0*AFOUR+72.0*ASQ+48.0
      SIGSQ=EM2P-(EM1P**2)
      SIGMA=SQRTF(SIGSQ)
      EM1P2=EM1P**2
      EM1P3=EM1P*EM1P2
      EM1P4=EM1P2**2
      EM1P5=EM1P4*EM1P
      EM1P6=EM1P3**2
      EM3=EM3P-3.0*EM1P*EM2P+2.0*EM1P3
      EM4=EM4P-4.0*EM1P*EM3P+6.0*EM1P2*EM2P-3.0*EM1P4
      EM5=EM5P-5.0*EM1P*EM4P+10.0*EM1P2*EM3P-10.0*EM1P3*EM2P+4.0*EM1P5
      EM6=EM6P-6.0*EM1P*EM5P+15.0*EM1P2*EM4P-20.0*EM1P3*EM3P+15.0*EM1P4*
     1EM2P-5.0*EM1P6
      TERM1=EM3/(SIGMA*SIGSQ)
      TERM2=EM4/(SIGSQ**2)-3.0
      TERM3=EM5/(SIGSQ**2*SIGMA)-10.0*EM3/(SIGMA*SIGSQ)
      TERM4=EM6/(SIGSQ**3)-15.0*EM4/(SIGSQ**2)+30.0
```

```
      GO TO 7
6     ASQ=A*A
      ACUBE=A*ASQ
      AFOUR=ASQ**2
      AFIVE=AFOUR*A
      ASIX=ACUBE**2
      ASVEN=ASIX*A
      EM1P=A+0.5/A+1.0/(8.0*ACUBE)+3.0/(16.0*AFIVE)+75.0/(128.0*ASVEN)
      SIGMA=1.0-1.0/(4.0*ASQ)-9.0/(32.0*AFOUR)-97.0/(128.0*ASIX)
      TERM1=1.0/ACUBE+3.0/AFIVE
      TERM2=-3.0/AFOUR
      TERM3=0.0
      TERM4=0.0
7     WHY=(V-EM1P)/SIGMA
      WHY2=WHY**2
      WHY3=WHY*WHY2
      WHY4=WHY2**2
      WHY5=WHY*WHY4
      CALL PHI(WHY,W,ERF,CPHI)
      PAV=CPHI-(EXPF(-WHY2*0.5)/RTWPI)*((WHY2-1.0)/6.0*TERM1+(WHY3-3.0*
     1WHY)/24.0*TERM2+(WHY4-6.0*WHY2+3.0)/120.0*TERM3+(WHY5-10.0*WHY3
     2+15.0*WHY)/720.0*TERM4)
      RETURN
      END
      SUBROUTINE PHI (WHY,W,ERF,CPHI)
      W=ABSF(WHY)/SQRTF(2.0)
      CALL ERRORR(W,ERF)
      IF(WHY-0.000001)600,601,600
600   CPHI=0.5*(1.0+(WHY/ABSF(WHY)*ERF))
      RETURN
601   CPHI=0.5
      END
      SUBROUTINE ERRORR (W,ERF)
      P= ((((0.00328975*W-0.00039446)*W+0.02743349)*W+0.08864027)*W +
     1  0.14112821)*W+1.0
      P=P*P
      P=P*P
      P=P*P
      ERF=1.0-1.0/P
      RETURN
      END
      SUBROUTINE IZERO(EX,BESSZ)
      BESSZ =1.0
      EXSQ = EX*EX
      DENOM=1.0
      XNUM=EXSQ/4.0
      FNUM=1.0
      DO 100 I=1,10
```

```
      D=I
      FNUM=FNUM*XNUM
      DENOM=DENOM*D
100   BESSZ=BESSZ+(FNUM/(DENOM**2))
      END
      SUBROUTINE IONE(EX,BESSO)
      BESSO =1.0
      EXSQ=EX*EX
      DEN1=1.0
      DEN2=1.0
      XNUM=EXSQ/4.0
      FNUM=1.0
      DO 101 I=1,10
      D=I
      FNUM =FNUM*XNUM
      DEN1=DEN1*D
      DEN2=DEN2*(D+1.0)
101   BESSO=BESSO +FNUM/(DEN1*DEN2)
      BESSO =BESSO*EX*0.5
      END
      END
```