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SURFACE WAVES ON A UNIAXIAL PLASMA SLAB;
THEIR GROUP VELOCITY AND POWER FLOW

by

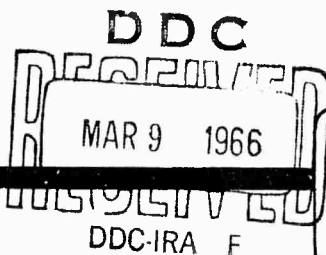
H. L. Bertoni and A. Hessel

Research Report No. PIBMRI-1294-65
Contract No. NONR 839(38)

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October 12, 1965



POLYTECHNIC INSTITUTE OF BROOKLYN
MICROWAVE RESEARCH INSTITUTE

Electrophysics Department

ARPA Order No. 529

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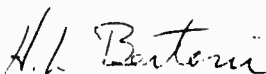
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SUMMARY

The relation between group velocity and the velocity of energy transport for surface waves in plane-stratified, anisotropic, dispersive media, which was derived in (1), is verified by direct calculation for the case of surface waves on a uniaxial, cold plasma slab located in free space. A superimposed D. C. magnetic field of infinite strength and parallel to the interfaces generates the uniaxial anisotropy in the slab. Surface waves having an arbitrary direction of propagation with respect to the D. C. magnetic field are considered. A useful graphical presentation of the dispersion relation is given, from which the direction of propagation of "surface wave rays" is directly obtained.

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INTRODUCTION

The group velocity of a surface wave propagating in a plane-stratified, anisotropic, dispersive medium that is also linear and lossless has been shown to be equal to the velocity of energy transport of the surface wave as a whole.⁽¹⁾ This velocity is defined as the integral of the real part of the Poynting vector over the coordinate in the direction of stratification divided by the integral of the stored energy density over this coordinate. In this paper the above relation is verified by direct calculation for the case of surface waves supported by a uniaxial, cold-electron plasma slab. The plasma slab is assumed to be of infinite extent and to be located in free space. A static magnetic field of infinite strength and parallel to the interfaces between the plasma and free space generates the anisotropy.

The characteristics of trapped surface waves propagating on anisotropic plasma slabs have been discussed in the literature for various specific directions of propagation relative to the static magnetic field. Wait⁽²⁾ has considered the surface waves propagating on a thin plasma slab with an arbitrary static magnetic field. Requiring the plasma slab to be thin reduces the effect of the static magnetic field to that which would be produced by the component normal to the slab alone. Meltz and Shore⁽³⁾ discuss the excitation of surface waves on a slab of arbitrary thickness when the static magnetic field is perpendicular to the slab and of infinite strength. In both of these cases, the anisotropy is such that the slab configurations have rotational symmetry about the coordinate normal to the slab and hence the characteristics of the surface waves will be independent of the direction of propagation. Furthermore, as in isotropic slab configurations, the velocity of energy transport of surface waves on these slab configurations will be parallel to the transverse wave vector.

When the static magnetic field is parallel to the air-plasma interfaces, the effect of the resultant anisotropy is more striking since then the characteristics of the surface waves on the slab depend on their directions of propagation with respect to the static magnetic field. Also, the velocity of energy transport will not be parallel, in general, to the transverse wave vector. Examples found in the literature, of surface waves on slab configurations with axis of anisotropy parallel to the interfaces, do not illustrate these anisotropic effects as they are restricted either to propagation along^(3, 4, 5) or normal⁽⁶⁾ to the static magnetic field. In either case, the velocity of energy transport is parallel to the transverse wave vector. In this paper, however, the surface wave fields are considered for arbitrary directions of propagation with respect to the static magnetic field of infinite strength. It will be shown that for this configuration, the velocity of energy transport of each surface wave is not parallel, in general, to the transverse wave vector, and that the direction, as well as the magnitude, of the real part of the complex Poynting vector varies

with the coordinate normal to the slab. Thus the slab configuration provides a non-trivial example of the equality of the surface wave's group velocity and its energy transport velocity. The excitation of the surface waves is not considered here.

In the first section of this paper the fields and dispersion relation of the E-type surface waves, which have no component of R. F. magnetic field along the static magnetic field, are found. A graphical procedure for solving the dispersion relation and the properties of the dispersion curves are discussed in the second section. The last section is devoted to an analytical verification of the equality of group velocity and energy transport velocity for the surface waves. The Appendix contains a proof that the uniaxial slab configuration can support only the E-type surface waves described in the body of the paper.

FIELDS AND DISPERSION RELATION

In this section, the fields and dispersion relation for surface waves on a uniaxial electron plasma slab are found. The plasma within the slab is homogeneous and the superimposed D.C. magnetic field, which is assumed to be of infinite strength, is parallel to the y axis (see Fig. 1). In the linear or small signal approximation, the interaction of the uniaxial plasma with a monochromatic electromagnetic field may be described by a relative dielectric tensor $\underline{\epsilon}'$. Neglecting collision loss, when the D.C. magnetic field is in the y direction, $\underline{\epsilon}'$ takes the form^(3, 4)

$$[\underline{\epsilon}'] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-X & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where $X = (\omega_p/\omega)^2$ and ω_p is the electron plasma frequency. Thus in the plasma slab $\underline{\epsilon} = \epsilon_0 \underline{\epsilon}'$ while in the air regions $\underline{\epsilon} = \epsilon_0 \underline{1}$, where $\underline{1}$ is the unit dyadic. The permeability tensor $\underline{\mu}$ is given everywhere by $\underline{\mu} = \mu_0 \underline{1}$.

The surface wave fields, which decay exponentially in the air regions, have transverse dependence $e^{-j(k_x x + k_y y)}$, k_x and k_y being real transverse wave numbers. These fields will be constructed from those plane wave solutions appropriate to the plasma region and those appropriate to the free space regions. The plane wave solutions appropriate to the plasma slab are those waves of the form

$$\begin{Bmatrix} \underline{E}(\underline{r}, \underline{k}) \\ \underline{H}(\underline{r}, \underline{k}) \end{Bmatrix} = \begin{Bmatrix} \underline{e}'(\underline{k}) \\ \underline{h}'(\underline{k}) \end{Bmatrix} e^{-j\underline{k} \cdot \underline{r}} \quad (2)$$

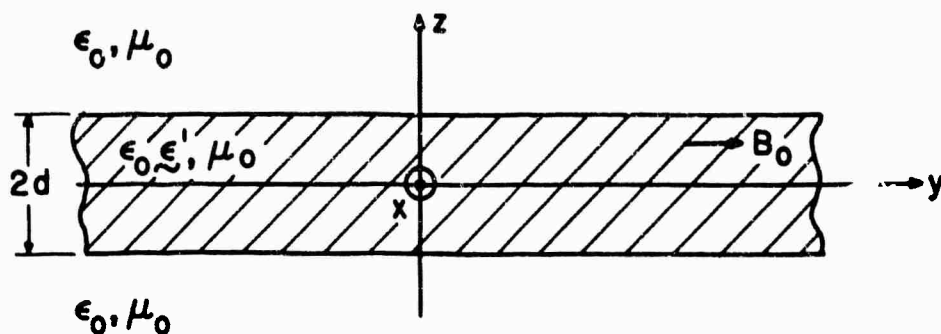


Fig. 1 Physical Configuration of the Anisotropic Plasma Slab.

which can propagate in an infinite homogeneous plasma described by the relative dielectric tensor $\underline{\epsilon}'$ given in (1). Similarly, the plane wave solutions appropriate to the free space are those having the form given in (2) which can exist when the plasma slab is absent.

The plasma plane waves are found by substituting $\underline{E}$ and $\underline{H}$ from (2) into Maxwell's equations. The resultant equations are, when the time dependence $e^{j\omega t}$ is suppressed,

$$\left. \begin{aligned} \underline{k} \times \underline{e}' &= \omega \mu_0 \underline{h}' \\ \underline{k} \times \underline{h}' &= -\omega \epsilon_0 \underline{\epsilon}' \cdot \underline{e}' \end{aligned} \right\} \quad (3)$$

Multiplying the first equation by $\underline{k} \times$ and substituting the second gives $\underline{k} \times (\underline{k} \times \underline{e}') = -k_0^2 \underline{\epsilon}' \cdot \underline{e}'$ with $k_0^2 = \omega^2 \epsilon_0 \mu_0$. Expanding the triple cross-product, this equation may be written in dyadic form as

$$(k_0^2 \underline{\epsilon}' + \underline{k} \underline{k} - k^2 \underline{1}) \cdot \underline{e}' = 0, \quad (4)$$

which is equivalent to three homogeneous equations in three unknowns. For there to be non-trivial solutions of (4), the determinant of the matrix representation of the dyadic operator $(k_0^2 \underline{\epsilon}' + \underline{k} \underline{k} - k^2 \underline{1})$ must vanish. Letting $\underline{k} = \underline{k}_t + \underline{z}_0 k_z$, with $\underline{k}_t = \underline{x}_0 k_x + \underline{y}_0 k_y$, the

vanishing of the determinant results in the plane wave dispersion relation $D_p(\underline{k}, \omega) = 0$ for an infinitely extended, homogeneous, uniaxial plasma. This plane wave dispersion relation may be solved for κ . Four solutions result, which are

$$\kappa = \left\{ \begin{array}{l} \pm \sqrt{k_o^2 - k_t^2} \\ \pm \sqrt{(1-X)(k_o^2 - k_y^2) - k_x^2} \end{array} \right\}. \quad (5)$$

The sign choice before each root refers to waves carrying power or decaying in the positive or negative z direction. Substituting each of the four solutions given in (5) into (4), the corresponding field vector $\underline{e}'$ can be found. Finally, the pertinent $\underline{h}'$ can be calculated from (3).

The above method may be repeated to find the plane wave fields $\underline{e}'_a$ and $\underline{h}'_a$ for free space. Since $\underline{k}_t$ must be the same for the entire surface wave if the transverse fields are to be continuous everywhere across the planes $z = \pm d$, it follows that the free space wave vector $\underline{k}_a = \underline{k}_t + \underline{z}_o \kappa_a$. Substituting the form of $\underline{E}$ and $\underline{H}$ given in (2) into Maxwell's equations for free space gives

$$\left. \begin{array}{l} \underline{k}_a \times \underline{e}'_a = \omega \mu_o \underline{h}'_a \\ \underline{k}_a \times \underline{h}'_a = -\omega \epsilon_o \underline{e}'_a \end{array} \right\}. \quad (6)$$

From (6), the homogeneous equations that determine $\underline{e}'_a$ are found, in dyadic form, to be

$$[(k_o^2 - k_a^2) \underline{1} + \underline{k}_a \underline{k}_a] \cdot \underline{e}'_a = 0. \quad (7)$$

From the requirement that the determinant of the matrix representation of $[(k_o^2 - k_a^2) \underline{1} + \underline{k}_a \underline{k}_a]$ vanish for non-trivial solutions of $\underline{e}'_a$ to exist, one can solve for κ_a as

$$\kappa_a = \pm \sqrt{k_o^2 - k_t^2}. \quad (8)$$

When these values of κ_a are used, (7) reduces to $\underline{k}_a \cdot \underline{e}'_a = 0$, i. e., the plane wave electric field is orthogonal to the wave vector $\underline{k}_a$, a condition that does not uniquely determine $\underline{e}'_a$. Commonly chosen solutions for $\underline{e}'_a$ are those corresponding to TM and TE modes with respect to the z direction. Other possible choices for $\underline{e}'_a$, which will

prove more useful in this analysis, are those of the so-called E-type and H-type modes, which are appropriate linear combinations of the TM and TE modes. The E-type modes with respect to y are characterized by the vanishing of the y component of the magnetic field, while the H-type modes are characterized by the vanishing of the y component of the electric field.

In each region, the surface wave fields will be a combination of the plane wave solutions appropriate to that region, the relative amplitudes of which can be found from the radiation condition and the continuity conditions at $z = \pm d$. Since the free space outside the slab is homogeneous, the surface waves are characterized by an exponential decay of their fields away from the slab. Such decay requires that κ_a be imaginary and that for Region 1 the sign choice in (8) be taken to give $\kappa_a = -j|\kappa_a|$, so that the fields will decay in the positive z direction. For the plane waves in Region 3, the sign must be taken so as to give $\kappa_a = j|\kappa_a|$, which will result in fields that decay in the negative z direction.

It will be shown later that a surface wave, whose fields in the plasma slab are a combination of those plane wave fields corresponding to $\kappa = \pm \sqrt{(1-X)(k_o^2 - k_y^2) - k_x^2}$, exists only for $X > 1$. In the Appendix it is shown that H-type surface wave modes, characterized by the vanishing of the y component of the electric field, cannot propagate on the uniaxial plasma slab. The plane wave fields in the plasma region corresponding to $\kappa = \pm \sqrt{(1-X)(k_o^2 - k_y^2) - k_x^2}$ are E-type modes and have the form

$$\left. \begin{aligned} \underline{e}' &= A[\underline{x}_o k_x k_y - \underline{y}_o (k_o^2 - k_y^2) + \underline{z}_o k_y \kappa] \\ \underline{h}' &= A \omega \epsilon_o [\underline{x}_o \kappa - \underline{z}_o k_x] \end{aligned} \right\} \quad (9)$$

with A an arbitrary constant.

As mentioned earlier, the only requirement on $\underline{e}'_a$ is that $\underline{k}_a \cdot \underline{e}'_a = 0$. Hence, we may arbitrarily select the transverse part of $\underline{e}'_a$ and then use the requirement $\underline{k}_a \cdot \underline{e}'_a = 0$ to find the corresponding z component of $\underline{e}'_a$. A particularly useful form of the transverse part of $\underline{e}'_a$ is obtained by choosing it to be identical with the transverse part of $\underline{e}'$ as given in (9). This choice will be seen to simplify the application of the continuity requirement on the transverse fields at $z = \pm d$. Following this procedure one finds

$$\left. \begin{aligned} \underline{e}'_a &= B[\underline{x}_o k_x k_y - \underline{y}_o (k_o^2 - k_y^2) + \underline{z}_o k_y \kappa_a] \\ \underline{h}'_a &= B \omega \epsilon_o [\underline{x}_o \kappa_a - \underline{z}_o k_x] \end{aligned} \right\} \quad (10)$$

with B an arbitrary constant. It is seen that the transverse part of $\underline{h}'_a$ has the same vector direction as the transverse part of $\underline{h}'$. It will thus be possible to satisfy the continuity conditions at $z = \pm d$ using only the plane waves exhibited in (9) and (10).

Since the wave number κ_a must be imaginary, let

$$\alpha = \sqrt{k_t^2 - k_o^2} \quad (11)$$

so that $\kappa_a = \pm j\alpha$ where α is real and positive. Then in Region 1, $\kappa_a = -j\alpha$, for decay in the positive z direction, and if $\underline{\rho} = \underline{x}_o x + \underline{y}_o y$, the fields are

$$\left. \begin{aligned} \underline{E} &= B_1 [\underline{x}_o k_x k_y - \underline{y}_o (k_o^2 - k_y^2) - \underline{z}_o j k_y \alpha] e^{-\alpha z} e^{-j \underline{k}_t \cdot \underline{\rho}} \\ \underline{H} &= B_1 \epsilon_o [-\underline{x}_o j \alpha - \underline{z}_o k_x] e^{-\alpha z} e^{-j \underline{k}_t \cdot \underline{\rho}} \end{aligned} \right\} \quad (12)$$

while in Region 3, $\kappa_a = j\alpha$, and the corresponding fields are

$$\left. \begin{aligned} \underline{E} &= B_3 [\underline{x}_o k_x k_y - \underline{y}_o (k_o^2 - k_y^2) + \underline{z}_o j k_y \alpha] e^{\alpha z} e^{-j \underline{k}_t \cdot \underline{\rho}} \\ \underline{H} &= B_3 \epsilon_o [\underline{x}_o j \alpha - \underline{z}_o k_x] e^{\alpha z} e^{-j \underline{k}_t \cdot \underline{\rho}} \end{aligned} \right\} \quad (13)$$

The constants B_1 and B_3 have yet to be determined.

For simplicity in what follows, we define β as

$$\beta = \sqrt{(1-X)(k_o^2 - k_y^2) - k_x^2} \quad (14)$$

so that in (5) $\kappa = \pm \beta$. It will be shown that for a surface wave to propagate on the slab β must be real. The fields in Region 2 will be the sum of the fields of the two plane waves having the vector form displayed in (9) and traveling in opposite directions along z . The most general form of such a sum is

$$\underline{E} = \left\{ [\underline{x}_o k_x k_y - \underline{y}_o (k_o^2 - k_y^2)] (A_1 e^{-j\beta z} + A_2 e^{j\beta z}) + \underline{z}_o k_y \beta (A_1 e^{-j\beta z} - A_2 e^{j\beta z}) \right\} e^{-j \underline{k}_t \cdot \underline{\rho}} \quad (15-a)$$

and

$$\underline{H} = \epsilon_o \left\{ \underline{x}_o \beta (A_1 e^{-j\beta z} - A_2 e^{j\beta z}) - \underline{z}_o k_x (A_1 e^{-j\beta z} + A_2 e^{j\beta z}) \right\} e^{-j \underline{k}_t \cdot \underline{\rho}} \quad (15-b)$$

with A_1 and A_2 to be determined from the boundary conditions at $z = \pm d$.

Requiring $\underline{E}_t$ and $\underline{H}_t$ to be continuous at $z = \pm d$ results in four homogeneous equations in four unknowns from which the relative amplitudes as well as the surface wave dispersion relation can be found. The continuity conditions at $z = d$ given the equations

$$\left. \begin{aligned} B_1 e^{-\alpha d} &= A_1 e^{-j\beta d} + A_2 e^{j\beta d} \\ -j\alpha B_1 e^{-\alpha d} &= \beta (A_1 e^{-j\beta d} - A_2 e^{j\beta d}) \end{aligned} \right\} \quad (16-a)$$

while those at $z = -d$ result in

$$\left. \begin{aligned} B_3 e^{-\alpha d} &= A_1 e^{j\beta d} + A_2 e^{-j\beta d} \\ j\alpha B_3 e^{-\alpha d} &= \beta (A_1 e^{j\beta d} - A_2 e^{-j\beta d}) \end{aligned} \right\} \quad (16-b)$$

Elimination of B_1 from the first two equations and B_3 from the second two gives the set

$$\left. \begin{aligned} 0 &= A_1 e^{-j\beta d} \left(1 + \frac{\beta}{j\alpha}\right) + A_2 e^{j\beta d} \left(1 - \frac{\beta}{j\alpha}\right) \\ 0 &= A_1 e^{j\beta d} \left(1 - \frac{\beta}{j\alpha}\right) + A_2 e^{-j\beta d} \left(1 + \frac{\beta}{j\alpha}\right) \end{aligned} \right\}, \quad (17)$$

which has a non-trivial solution for A_1 and A_2 only if the determinant of the coefficients is zero. The vanishing of the determinant yields the surface wave dispersion relation

$$e^{j4\beta d} - \left(\frac{j\alpha + \beta}{j\alpha - \beta}\right)^2 = 0 \quad (18)$$

If expressions (11) and (14) for α and β in terms of k_x , k_y and $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ are substituted into (18), the dispersion relation is seen to be of the form $D_s(\underline{k}_t, \omega) = 0$.

PROPERTIES OF THE DISPERSION RELATION

As given in (14), β is either real or imaginary for all real k_x and k_y . Let us first verify that no solutions of (18) exist for which β is imaginary. If β is imaginary, i.e., $\beta = \pm j|\beta|$, then (18) becomes

$$e^{\mp 4|\beta|d} = \left(\frac{\alpha \pm |\beta|}{\alpha \mp |\beta|} \right)^2. \quad (19)$$

The left-hand side is less (greater) than unity while the right-hand side is greater (less) than unity. This contradiction verifies the assertion. When β is real, however, both terms in (18) have magnitude unity so that a solution is possible. In order to find the range of frequencies for which (18) has solutions, based on the restriction that β be real, we plot for all $X > 0$ those regions in the $k_x - k_y$ plane where β is real and the region where α is real (see Fig. 2). From Fig. 2 it is seen that the regions where β is real and the region where α is real overlap only when $X > 1$. Hence the possibility that surface waves can propagate exists only for $X > 1$. In passing, observe that (18) remains invariant under the substitution of $-\beta$ for β . Thus it is sufficient to consider only positive values of β . Since the slab configuration has mirror symmetry in the plane $z = 0$, the surface wave fields will correspond to either an open-circuit or a short-circuit bisection of the slab (even and odd solutions in z). The dispersion relation given in (18) can be split into two independent dispersion relations, one giving the open-circuit bisection solutions and the other the short-circuit bisection solutions. These are

$$e^{j2\beta d} = \pm \frac{j\alpha + \beta}{j\alpha - \beta} \quad (20)$$

where the plus and minus signs correspond to short-circuit and open-circuit bisections, respectively. Using the plus sign for the short-circuit bisection case, the dispersion relation may be put in the form

$$\alpha = -\beta \cot \beta d, \quad (21)$$

whereas if the minus sign is used, the dispersion relation for the open-circuit bisection case can be written

$$\alpha = \beta \tan \beta d \quad (22)$$

with α and β as given in (11) and (14).

A graphical method for solving equations (21) and (22) is described below. In order to show that equations (21) and (22) are satisfied for real values of k_x, k_y and $\omega < \omega_p$, i. e., $X > 1$, the plots of (21) and (22) in the $\beta - \alpha$ plane are considered. Since α and β have been taken to be positive, only the first quadrant is of interest.

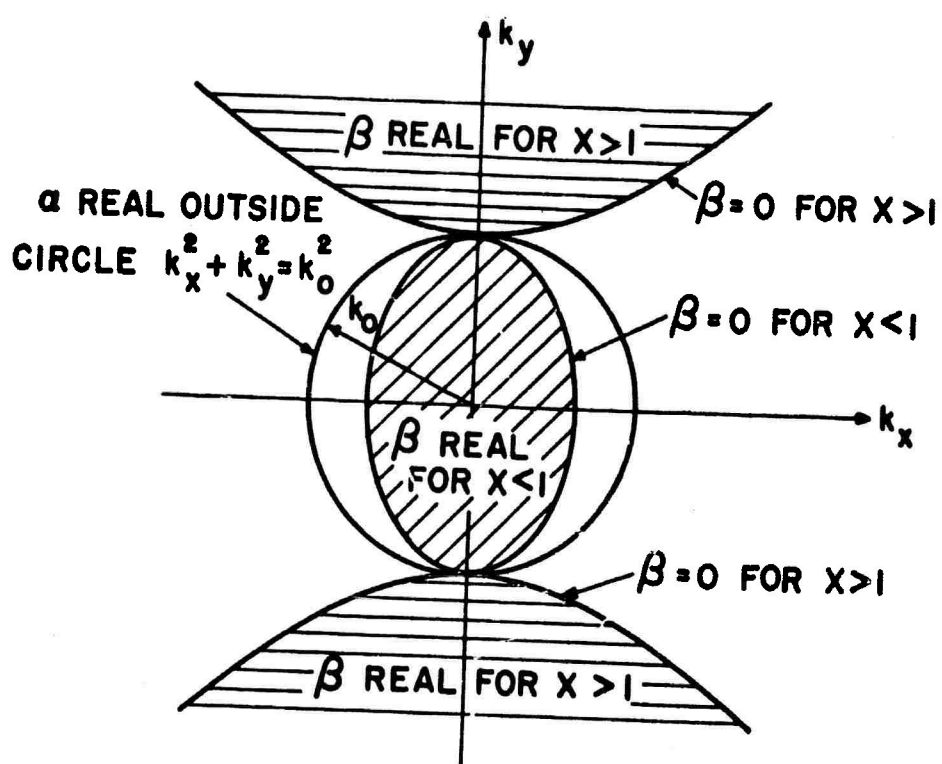


Fig. 2 Regions of Real β and α in the k_x - k_y Plane.

Adding α^2 to β^2 gives $\alpha^2 + \beta^2 = X(k_y^2 - k_0^2)$ when (11) and (14) are used. Because $k_y^2 > k_0^2$ for surface waves, as can be seen from Fig. 2, the plot of this relation in the β - α plane is a circle whose radius is $\sqrt{X(k_y^2 - k_0^2)}$. The intersection of this circle with the plot of (21) or (22) gives α and β from which k_x can be found using

$$k_x = \pm \sqrt{\frac{1}{X} [(X-1)\alpha^2 - \beta^2]} \quad (23)$$

But k_x must be real so that in the first quadrant only those intersections for which $\alpha \geq \frac{1}{\sqrt{X-1}} \beta$ give values of α and β which correspond to an actual surface wave. Since α and β depend only on k_y^2 and k_0^2 and not merely on k_y or k_x , constant ω surface wave dispersion curves will have mirror symmetry about the k_x and k_y axes in the k_x - k_y plane. Thus, knowing the relation between k_x and k_y for $k_x, k_y \geq 0$ is sufficient to determine the entire dispersion curve.

Figure 3 has been sketched to show the method outlined above for finding that k_x which satisfies (21) when k_y and k_0 are given. Each branch of $-\beta \cot \beta d$ depicted in Fig. 3 corresponds to a particular short-circuit bisection surface wave mode. Since there are an infinite number of such branches, there will be an infinite number of short-circuit bisection surface wave modes when $X(k_y^2 - k_0^2) \rightarrow \infty$. For a finite value of $X(k_y^2 - k_0^2)$ only a finite number of surface wave modes can propagate. For values of α and β in the shaded region of Fig. 3, k_x , as found from $k_x = \pm \sqrt{\frac{1}{X} [(X-1)\alpha^2 - \beta^2]}$, is imaginary. Thus it is seen that for fixed k_0 each mode has a minimum value of $k_y^2 > k_0^2$ at which $k_x = 0$ and below which no real solutions for k_x exist. The minimum value of k_y^2 for which a particular surface wave mode can exist is found from the condition that the circle $\alpha^2 + \beta^2 = X(k_y^2 - k_0^2)$, the line $\alpha = \frac{1}{\sqrt{X-1}} \beta$ and that branch of $\alpha = -\beta \cot \beta d$ corresponding to the mode in question all intersect at a common point. As k_y^2 increases from its minimum value, k_y and the corresponding solutions for k_x for each branch of $-\beta \cot \beta d$ trace out the surface wave dispersion curves in the k_x - k_y plane of the short-circuit bisection modes.

In a similar fashion, Fig. 4 depicts the method for finding that k_x which satisfies (22) when k_y and k_0 are given. From this figure and Fig. 3, it is seen that the lowest surface wave mode on the slab, i. e., the one with the smallest value of β , is that open-circuit bisection mode corresponding to the branch of $\beta \tan \beta d$ starting at $\beta = 0$. As in the case of the short-circuit bisection modes, k_x corresponding to values of β and α

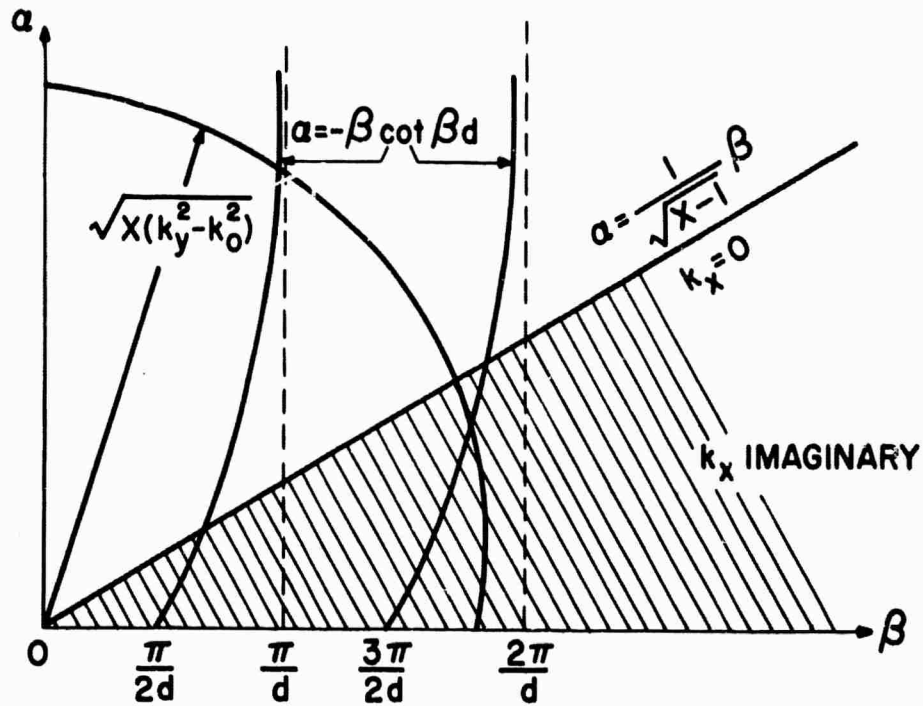


Fig. 3 Construction for Finding Solutions of the Surface Wave Dispersion Relation for the Short-Circuit Bisection Case.

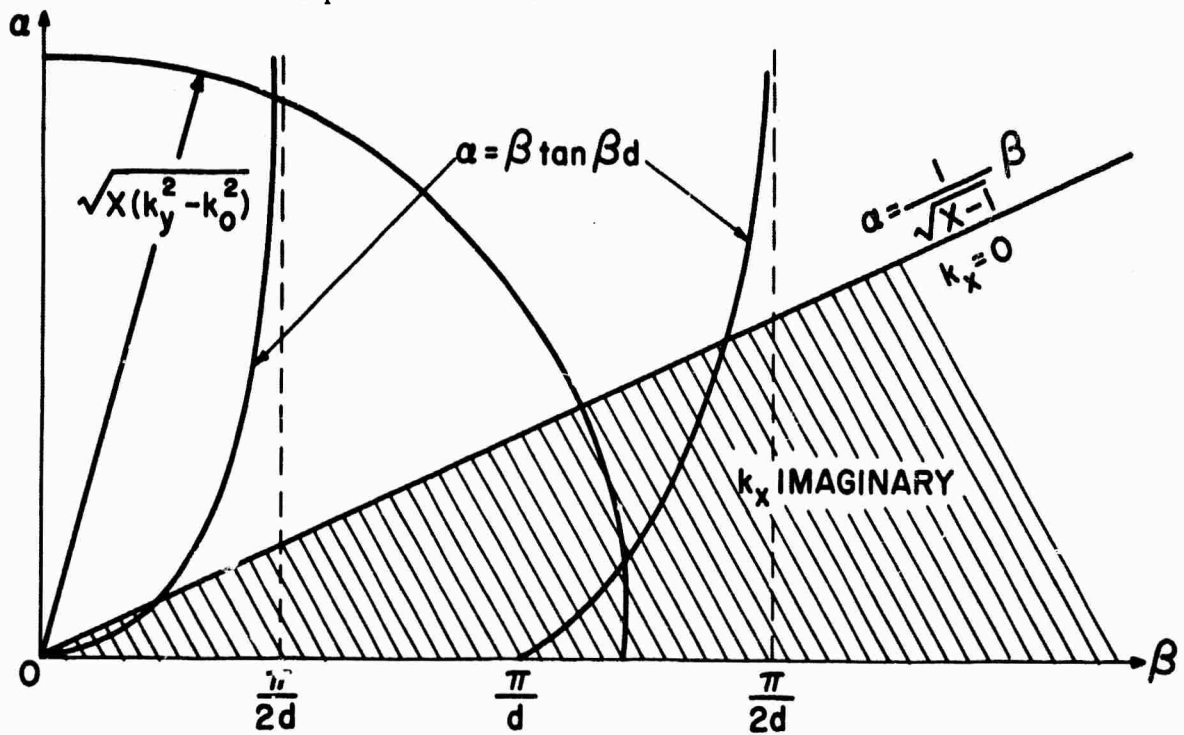


Fig. 4 Construction for Finding Solutions of the Surface Wave Dispersion Relation for the Open-Circuit Bisection Case.

in the shaded region of Fig. 4 is imaginary. Thus for each of the higher open-circuit bisection modes there will be a minimum value of $k_y^2 > k_o^2$ at which $k_x = 0$ and below which no real solution for k_x exists. For the lowest open-circuit bisection mode $[\frac{d}{d\beta}(\beta \tan \beta d)]_{\beta=0} = 0$ so that a part of the branch of $\beta \tan \beta d$ starting at $\beta=0$ lies in the shaded region of Fig. 4. Hence there will also be a minimum value of $k_y^2 > k_o^2$ for the lowest surface wave mode below which no real solution for k_x exists. As in the short-circuit bisection case, when k_y^2 increases from its minimum value for a particular mode and for a fixed k_o , k_y and the corresponding value of k_x trace out the dispersion curve of that open-circuit bisection mode.

In what follows, the basic properties of the surface wave dispersion curves will be derived. For any one mode, these properties lead to the form of the dispersion curves shown in Fig. 5, which has been drawn for two different frequencies $\omega_2 > \omega_1$. In order to find the shape of the dispersion curves of any one mode and for fixed ω , we consider the corresponding branch of $-\beta \cot \beta d$ in Fig. 3 or of $\beta \tan \beta d$ in Fig. 4. As pointed out previously, the dispersion curves are symmetric about the k_x and k_y axes so that we need find only that portion of the curves in the first quadrant of Fig. 5. Also, as was previously discussed, in the first quadrant of Fig. 5, k_y takes on its minimum value, which is greater than k_o , at $k_x = 0$, i. e., where the dispersion curve crosses the k_y axis. It will first be shown that in the first quadrant, k_x is a single-valued, monotonically increasing function of k_y . These two facts indicate that the inverse function, $k_y = k_y(k_x)$, is single-valued and monotonically increasing in the first quadrant as is depicted in Fig. 5. Other fundamental properties of that portion of the dispersion curve in the first quadrant of Fig. 5 that will be established are: 1) $dk_y/dk_x = 0$ at $k_x = 0$; 2) asymptotically as $k_y \rightarrow \infty$, $k_x \sim k_y/\sqrt{X-1}$ and the dispersion curve everywhere lies above the asymptote $k_y = k_x/\sqrt{X-1}$; 3) the value of k_y at $k_x = 0$, as well as the slope of the asymptote, increase with ω . One question that has not yet been answered analytically is whether the surface wave dispersion curves have inflection points.

To see that in the first quadrant of Fig. 5, k_x is a single-valued function of k_y , observe that for $\beta > 0, \alpha > 0$ each branch of $-\beta \cot \beta d$ in Fig. 3 and each branch of $\beta \tan \beta d$ in Fig. 4 intersects the circle $\alpha^2 + \beta^2 = X(k_y^2 - k_o^2)$ only once. Thus for a given ω_p and for each value of k_y and ω there will be only one set of values (β, α) for each mode and hence from (23) only one value of $k_x > 0$ for each mode. Therefore, in the first quadrant of Fig. 5, k_x is a single-valued function of k_y . That k_x is a monotonically increasing function of k_y can be inferred from the sign of dk_x/dk_y . Since k_x and k_y satisfy the

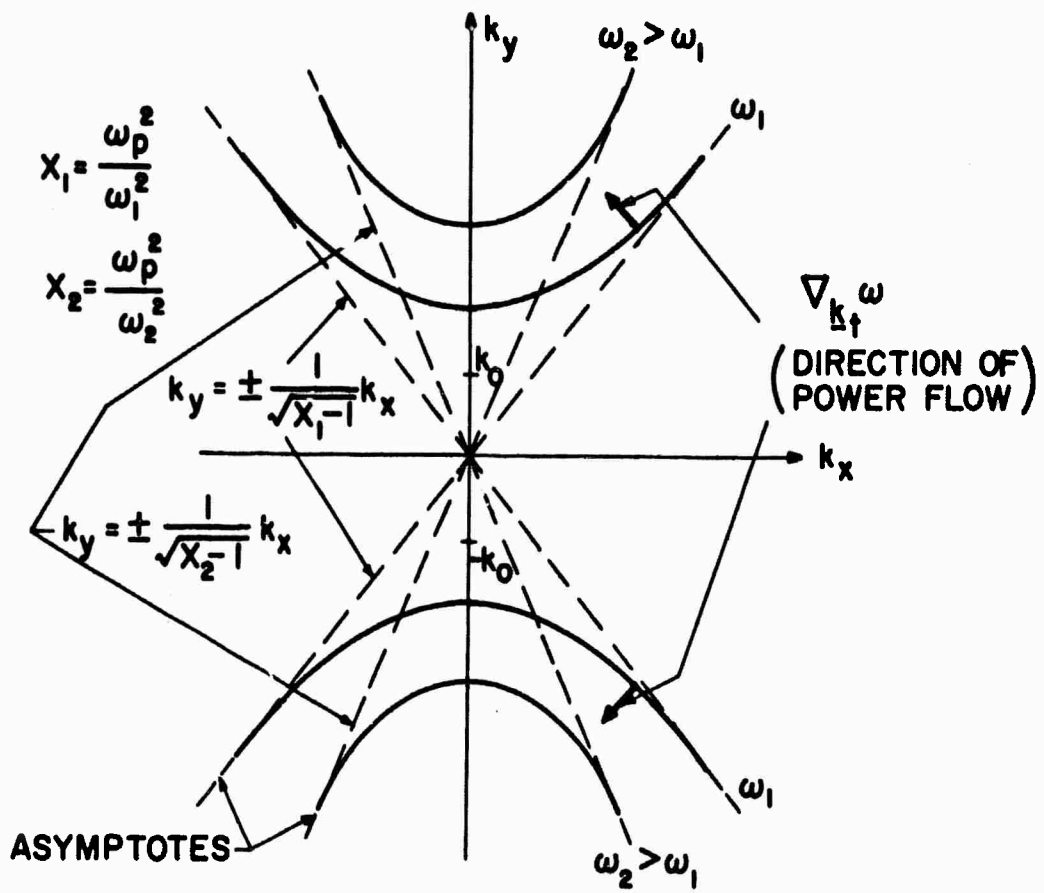


Fig. 5 Dispersion Curves for a Typical Surface Wave Mode With ω as a Parameter.

surface wave dispersion relation $D_s(k_x, \omega) = 0$, dk_x/dk_y for fixed ω is given by $dk_x/dk_y = - \frac{\partial D_s / \partial k_y}{\partial D_s / \partial k_x}$. Using D_s as given in the left-hand side of (18), with α and β defined in (11) and (14), it is found that

$$\frac{dk_x}{dk_y} = \frac{k_y}{k_x} - \frac{\alpha d(X-1) + k_x^2 / (k_y^2 - k_0^2)}{\alpha d + 1} \quad (24)$$

From (24) we see that in the first quadrant of Fig. 5, $dk_x/dk_y > 0$ and hence, $k_x(k_y)$ is a monotonically increasing function. Furthermore, (24) shows that $dk_y/dk_x = 0$ at $k_x = 0$ as is depicted in Fig. 5.

As $k_y \rightarrow \infty$, the value of β at the intersection of the circle $\alpha^2 + \beta^2 = X(k_y^2 - k_0^2)$ and any one branch of $-\beta \cot \beta d$ in Fig. 3 or any one branch of $\beta \tan \beta d$ in Fig. 4 approaches a constant. Thus, since k_0 has been assumed constant, $\alpha^2 \sim Xk_y^2$ as $k_y \rightarrow \infty$ and hence, from (23), k_x in the first quadrant of Fig. 5 is asymptotically given by $k_x \sim k_y \sqrt{X-1}$ or conversely $k_y \sim k_x / \sqrt{X-1}$. That the dispersion curve lies above the asymptote line $k_y = k_x / \sqrt{X-1}$, as shown in the first quadrant of Fig. 5, can be deduced from the definition of β given in (14). Since β is real for the surface waves and $X > 1$, $(X-1)k_y^2 - k_x^2 = \beta^2 + k_0^2(X-1) > 0$ and therefore in the first quadrant $k_y > k_x / \sqrt{X-1}$, which proves that the dispersion curve lies above the asymptote line. When ω increases but remains below ω_p , X decreases to unity and hence the slope of the asymptote, $1/\sqrt{X-1}$, increases as is depicted in Fig. 5. Furthermore, as ω increases the slope of the line $\alpha = \beta/\sqrt{X-1}$ in Fig. 3 and Fig. 4 increases. Hence the values of β and α at $k_x = 0$, as determined from the intersection of the line $\alpha = \beta/\sqrt{X-1}$ with any branch of $-\beta \cot \beta d$ in Fig. 3 or of $\beta \tan \beta d$ in Fig. 4, must increase. Because k_0 increases with ω and X decreases, the quantity $\frac{1}{X}(\alpha^2 + \beta^2) + k_0^2 = k_y^2$ must increase and thus the magnitude of k_y at $k_x = 0$ increases with ω . The above-described variation with ω of k_y at $k_x = 0$ is depicted in Fig. 5.

Thus the fundamental properties previously stated for the surface wave dispersion curves of any one mode are seen to hold. These properties indicate that the dispersion curves will have the form depicted in Fig. 5, with the possible exception of inflection points, for two different frequencies. From Fig. 3 and Fig. 4 it can also be seen that surface waves exist for all ω in the range $0 < \omega < \omega_p$. Lastly, since in the first quadrant of Fig. 5, $\frac{dk_y}{dk_x} \geq 0$, which follows from (24), and since the dispersion curve for $\omega = \omega_2$ lies

above that for $\omega = \omega_1 < \omega_2$, the x component of $\nabla_{\underline{k}} \omega$ must everywhere be negative. That the dispersion curve for $\omega = \omega_2$ lies above that for $\omega = \omega_1$ follows from the fact that at $k_x = 0$ the $\omega = \omega_1$ curve lies above the $\omega = \omega_2$ curve and the two curves never cross since $\nabla_{\underline{k}_t} \omega$, which is given in (42), is never infinite. The observation that $\underline{x}_0 \cdot \nabla_{\underline{k}_t} \omega > 0$ is confirmed by the analytic expression for $\nabla_{\underline{k}_t} \omega$ given in (42) and indicates that the surface waves are of the backward wave type with respect to the x direction.

GROUP VELOCITY AND ENERGY TRANSPORT VELOCITY

Having established the basic properties of the dispersion relation of the surface waves on a uniaxial plasma slab, the equality of group velocity and energy transport velocity for these surface waves will be verified by direct calculation. This relation is given in Reference (1) as

$$\nabla_{\underline{k}_t} \omega = \frac{\int_{-\infty}^{\infty} \underline{s} dz}{\int_{-\infty}^{\infty} w dz} \quad (25)$$

where $\underline{s}$ represents the real part of the complex Poynting vector $\underline{E} \times \underline{H}^*$ and w the time average stored energy density. To this end, the relative field amplitudes are first calculated. Since one of the coefficients A_1, A_2, B_1 and B_3 is arbitrary, for simplicity let

$$A_1 = -A_0(\beta - j\alpha)e^{j\beta d} \quad (26)$$

where A_0 is arbitrary. Then from (17) it is found that

$$A_2 = -A_0(\beta + j\alpha)e^{-j\beta d} \quad (27)$$

while from (16-a)

$$B_1 = -2\beta A_0 e^{\alpha d} \quad (28)$$

Using the dispersion relation in the form given in (18), which is valid for both open-circuit and short-circuit bisection modes, it follows that

$$B_3 = -2\beta A_0 \frac{\beta + j\alpha}{\beta - j\alpha} e^{-j2\beta d} e^{\alpha d} . \quad (29)$$

With these expressions in equations (12), (13) and (15) for the fields in the three regions, $\underline{s}$ in Region 1 is found to be

$$\underline{s} = 4|A_0|^2 \beta^2 \omega \epsilon_0 e^{-2\alpha(z-d)} [\underline{x}_0 k_x (k_o^2 - k_y^2) + \underline{y}_0 k_y (\alpha^2 + k_x^2)] \quad (30)$$

while in Region 3 it is

$$\underline{s} = 4|A_0|^2 \beta^2 \omega \epsilon_0 e^{2\alpha(z+d)} [\underline{x}_0 k_x (k_o^2 - k_y^2) + \underline{y}_0 k_y (\alpha^2 + k_x^2)] \quad (31)$$

and finally in Region 2 it is

$$\begin{aligned} \underline{s} = & |A_0|^2 \omega \epsilon_0 (\alpha^2 + \beta^2) \left\{ 2 [\underline{x}_0 k_x (k_o^2 - k_y^2) + \underline{y}_0 k_y (\beta^2 + k_x^2)] + \right. \\ & \left. + \left[\frac{\beta + j\alpha}{\beta - j\alpha} e^{j2\beta(z-d)} + \frac{\beta - j\alpha}{\beta + j\alpha} e^{-j2\beta(z-d)} \right] \cdot [\underline{x}_0 k_x (k_o^2 - k_y^2) + \underline{y}_0 k_y (k_y^2 - \beta^2)] \right\} . \end{aligned} \quad (32)$$

The quantity $\underline{S} = \int_{-\infty}^{\infty} \underline{s} dz$ is now calculated to be

$$\begin{aligned} \underline{S} = & 4|A_0|^2 \omega \epsilon_0 (\alpha^2 + \beta^2) \left\{ -\underline{x}_0 k_x (k_y^2 - k_o^2) \left[\frac{1}{\alpha} + d \right] \right. \\ & \left. + \underline{y}_0 k_y \left[\frac{1}{\alpha} k_x^2 + d(X-1)(k_y^2 - k_o^2) \right] \right\} . \end{aligned} \quad (33)$$

In order to determine the stored energy, observe that in the plasma slab

$$\left[\frac{\partial \omega \epsilon}{\partial \omega} \right] = \epsilon_0 \left[\frac{\partial \omega \epsilon'}{\partial \omega} \right] = \epsilon_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1+X & 0 \\ 0 & 0 & 1 \end{bmatrix} . \quad (34)$$

Thus in Region 2, the time averaged stored energy density, which is given by^(7, 8),

$$w = \frac{1}{2} \left[\underline{E}^* \cdot \frac{\partial \omega \epsilon}{\partial \omega} \cdot \underline{E} + \mu_0 |\underline{H}|^2 \right] , \quad (35)$$

is found to be

$$w = \frac{1}{2} |A_o|^2 \epsilon_o (\alpha^2 + \beta^2) \left\{ 4(k_y^2 - k_o^2)(Xk_y^2 - k_o^2) - \left[\frac{\beta + j\alpha}{\beta - j\alpha} e^{j2\beta(z-d)} + \frac{\beta - j\alpha}{\beta + j\alpha} e^{-j2\beta(z-d)} \right] \cdot [(\beta^2 - k_x^2)(k_o^2 + k_y^2) - (1+X)(k_y^2 - k_o^2)] \right\} \quad (36)$$

Outside the slab, the time averaged stored energy density has the form

$$w = \frac{1}{2} [\epsilon_o |\underline{E}|^2 + \mu_o |\underline{H}|^2] \quad (37)$$

so that in Region 1

$$w = 2 |A_o|^2 \epsilon_o \beta^2 [(k_o^2 + k_y^2)(k_x^2 + \alpha^2) + (k_y^2 - k_o^2)^2] e^{-2\alpha(z-d)} \quad (38)$$

while in Region 3

$$w = 2 |A_o|^2 \epsilon_o \beta^2 [(k_o^2 + k_y^2)(k_x^2 + \alpha^2) + (k_y^2 - k_o^2)^2] e^{2\alpha(z+d)} \quad (39)$$

Calculating $W = \int_{-\infty}^{\infty} w dz$, it is found that

$$W = 4 |A_o|^2 \epsilon_o (\alpha^2 + \beta^2) \left\{ d(k_y^2 - k_o^2)(Xk_y^2 - k_o^2) + \frac{1}{\alpha} [k_x^2 k_y^2 + (k_y^2 - k_o^2)^2] \right\} \quad (40)$$

In deriving the above power and energy formulas, extensive use has been made of the dispersion relation given in (18) and the formulas (11) and (14) for α and β . Using the above expressions for $\underline{S}$ and W , the energy transport velocity is seen to be

$$\frac{\underline{S}}{W} = w \frac{-\underline{x}_o k_x (k_y^2 - k_o^2)(\frac{1}{\alpha} + d) + \underline{y}_o k_y [\frac{1}{\alpha} k_x^2 + d(X-1)(k_y^2 - k_o^2)]}{d(k_y^2 - k_o^2)(Xk_y^2 - k_o^2) + \frac{1}{\alpha} [k_x^2 k_y^2 + (k_y^2 - k_o^2)^2]} \quad (41)$$

In order to compute the group velocity $\nabla_{\underline{k}_t} w$, the formula $\nabla_{\underline{k}_t} w = -\nabla_{\underline{k}_t} D_s / \frac{\partial D_s}{\partial w}$ from implicit function theory will be used where the function $D_s(\underline{k}_t, w)$ is the left-hand

side of (18). It is found that

$$\nabla_{\underline{k}} \omega = \omega \frac{\underline{x}_0 k_x \left(\frac{1}{\alpha} + d \right) + \underline{y}_0 k_y \left[d(X-1) + \frac{k_x^2 X}{\alpha(\alpha^2 + \beta^2)} \right]}{d(Xk_y^2 - k_o^2) + \frac{1}{\alpha}(k_y^2 - k_o^2) + \frac{k_x^2 k_y^2}{\alpha(k_y^2 - k_o^2)}} \quad (42)$$

If both the numerator and denominator of the above expression are multiplied by $(k_y^2 - k_o^2)$ and it is recognized that $X(k_y^2 - k_o^2) = \alpha^2 + \beta^2$, $\nabla_{\underline{k}} \omega$ will be seen to be identical with $\underline{S}/W$, as predicted in Reference 1.

The example worked out above also illustrates the fact that, in general, the direction of $\underline{s}$ as well as its magnitude can vary with z . This can be seen from equation (32) for $\underline{s}$ in Region 2 if it is recognized that the vectors $[\underline{x}_0 k_x (k_o^2 - k_y^2) + \underline{y}_0 k_y (\beta^2 + k_x^2)]$ and $[\underline{x}_0 k_x (k_o^2 - k_y^2) + \underline{y}_0 k_y (k_x^2 - \beta^2)]$ are parallel only for $k_x = 0$. Since the coefficient of the first vector is independent of z while the coefficient of the second vector depends on z , the direction of the vector sum, which gives $\underline{s}$, will depend on z for all surface waves for which $k_x \neq 0$.

APPENDIX

The purpose of this Appendix is to investigate the possible contribution to a surface wave field from the plane wave fields in the plasma that are associated with the solutions $\kappa = \pm \sqrt{k_o^2 - k_t^2}$ of the plasma plane wave dispersion relation. The vector character of these plasma plane waves is that of H-type modes and has the form

$$\left. \begin{aligned} \underline{e}' &= C[\underline{x}_o \kappa - \underline{z}_o k_x] \\ \underline{h}' &= \frac{C}{\omega \mu_o} [-\underline{x}_o k_y k_x + \underline{y}_o (k_o^2 - k_y^2) - \underline{z}_o \kappa k_y] \end{aligned} \right\} \quad (43)$$

with C an arbitrary constant. The vector character of those plane waves in the air regions that have the H-type mode form, and will thus allow a simple application of the continuity conditions at $z = \pm d$, is

$$\left. \begin{aligned} \underline{e}'_a &= D[\underline{x}_o \kappa_a - \underline{z}_o k_x] \\ \underline{h}'_a &= \frac{D}{\omega \mu_o} [-\underline{x}_o k_y k_x + \underline{y}_o (k_o^2 - k_y^2) - \underline{z}_o \kappa_a k_y] \end{aligned} \right\} \quad (44)$$

with $\kappa_a = \pm j\alpha$ and α as defined in (11). Note that $\kappa = \pm j\alpha$ also.

In Region 1, κ_a must be taken as $-j\alpha$ to ensure that $\underline{E}$ and $\underline{H}$ are zero at $z = \infty$. Similarly, in Region 3, κ_a must be taken as $j\alpha$. Denoting the amplitudes in Regions 1 and 3 as D_1 and D_3 , respectively, and letting C_1 and C_2 be the amplitudes of the plasma plane waves corresponding to $\kappa = -j\alpha$ and $\kappa = j\alpha$, respectively, the continuity conditions at $z = d$ result in the equations

$$\left. \begin{aligned} -D_1 e^{-\alpha d} &= -C_1 e^{-\alpha d} + C_2 e^{\alpha d} \\ D_1 e^{-\alpha d} &= C_1 e^{-\alpha d} + C_2 e^{\alpha d} \end{aligned} \right\} \quad (45)$$

when the fields in Region 2 are assumed to be the sum of the two H-type plane waves. The continuity conditions at $z = -d$ can be written as

$$\left. \begin{aligned} D_3 e^{-\alpha d} &= -C_1 e^{\alpha d} + C_2 e^{-\alpha d} \\ D_3 e^{-\alpha d} &= C_1 e^{\alpha d} + C_2 e^{-\alpha d} \end{aligned} \right\} \quad (46)$$

These equations have only the trivial solutions $C_1 = C_2 = D_1 = D_3 = 0$ and hence no surface wave can exist whose fields in the plasma are a sum of the two H-type plasma plane waves, which propagate as $\kappa = \pm j\alpha$.

The physical reason why no surface wave exists that contains the above-mentioned plane waves is that the waves of this polarization do not "see" the plasma, since the infinite D. C. magnetic field along y prevents the electrons from moving in response to an R. F. electric field that, as in this case, is purely transverse to y . In effect, for waves of this polarization, no slab on which to have surface waves is present.

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