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(6) PERT FOR SMALL PROJECTS.

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SUMMARY

The various graphical and mathematical techniques available for controlling research and development projects are considered in relation to the needs of the manager of a small project. The PERT/TIME model is recommended.)

The implementation of PERT/TIME in the R & D field is discussed, and a brief account is given of experience gained from its use in managing the engineering development and manufacture of the data handling sub-systems of the U.K.3 satellite.)

Appendices are included which describe the basic PERT/TIME system and postulate operating instructions for its use. Mention is made of derivatives of the original techniques which may be of use in the future.

Departmental Reference: Space 120

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1 INTRODUCTION

Much of the time of members of the scientific and professional classes in the Civil Service is spent in management. Almost every recruit must expect that some part of his career will be devoted to managerial duties, however distasteful this may appear to him at the time.

Nearly all scientific Managers perform their managerial duties without recourse to techniques and facilities which they would use in their technical work as a matter of routine. Indeed, many of them regard scientific techniques for management with a healthy but irrational scepticism. It is a little surprising that a man who has employed an electronic computer to evaluate a mathematical model of a control system should scorn to use the same technique to assist in organising the development of the resulting hardware.

This attitude is gradually changing, at least at the highest levels of management. In Britain, a Committee was set up in May 1958; "To enquire into the techniques employed by Government Departments and other bodies wholly financed by the Exchequer for the management and control of research and development carried out by them or on their behalf, and to make recommendations". In the Committee's report¹ fifty-two recommendations were made covering a very wide field of scientific policy, and some of them have been adopted. In particular, the Committee made detailed recommendations about the procedure to be adopted in the letting and control of development contracts. It laid great stress on careful planning both in terms of cost and time-scale.

In the U.S.A. the priority allocated to the development of a nuclear weapons system immune to interdiction (Polaris) led to the development of network analysis as a tool of management².

In the Ministry of Aviation network analysis is already being used in the control of major projects, of which Sea Dart is probably the best known. On the 1st January, 1965 a section was set up within the Controllorate of Aircraft for project time and cost analysis.

However, the use of network techniques for R. and D. Projects of small size, say of less than £500 000 total cost, is unusual. When they have been used, it has often been at the instigation of the contractor.

It has frequently been said that network techniques do not make a valid contribution to small projects. This is not so. Any programme of work which involves the co-ordination of inter-dependent activities can use them. Valuable savings of time and cost have been achieved in plant maintenance, when activity durations have been measured in hours rather than weeks.

Early in 1964 it was decided, in a Section of Space Department, to overhaul the techniques that it was using in managing the small projects with which it was concerned. This paper is largely based on the experience so gained. It is concerned with the examination of the needs of the Manager, the choice of a mathematical model technique to meet these needs, the experimental introduction of the technique into a small project (the data handling sub-system of the U.K.3 satellite), and the possibilities of future extension. It assumes a knowledge of the basic principles of network analysis, but includes in Appendix A a brief summary of the Programme Evaluation and Review Technique (PERT), which was the system selected. Readers who are unfamiliar with the management type of network analysis should read it, at least up to Section A.3, before reading beyond Chapter 3 of the main text.

2 THE NEEDS OF THE MANAGER

The primary function of the Manager is the making of decisions. He may possess powers of persuasion and leadership, and a large fist for banging the table, but he must still decide when and in what way to use these powers and how hard to bang. He must allocate his resources, manpower and equipment, to achieve his objective within his specified time-scale and overall cost; if possible sooner and cheaper. He must ensure that the inputs and outputs of the device he is producing are compatible, in every respect including time, with the other equipments with which it must work, or as the jargon is, he must control his interfaces. He must ensure that all the people working on the project are aware of their precise function and their relation with each other. Above all, he must monitor the progress of a project sufficiently closely to modify his original directives and allocations to meet changing circumstances, before things get out of hand.

The recommended procedure for decision making is usually a variant of the following:-

- (i) Define the problem.
- (ii) Analyse it.
- (iii) Postulate possible solutions.
- (iv) Select one.

The Manager of a development project normally has his problem defined for him in general terms by means of an operational requirement, a design aim or a performance specification. He may also, but not necessarily, be told when the equipment is required and how much he can spend. Often, he will be required to make estimates of time-scale and cost. Occasionally, the problem may not be clearly defined and the Manager will have to do this for himself.

In order to make the problem amenable to analysis, it must be broken down in some degree of detail. This division may be done chronologically, in terms of technical difficulty, in terms of the various technologies to be employed, or in terms of the man power and facilities to be used. The list is not exhaustive, nor are the criteria for division mutually exclusive. If the analysis is to be useful, it must be disciplined. The Manager needs to determine the areas of greatest difficulty, and to relate each activity to those dependent upon it, and to determine its bearing on the overall time scale and cost. All this can be achieved by constructing a mathematical model of the problem.

During the next phase of the analysis he may wish to measure alternative technical solutions against the project requirements and against internal and external interfaces. He will need to decide what parts of his development programme can be carried out in series, and what must be done in parallel to avoid over-running the time-scale.

Having isolated his problems and planned their solution the Manager now needs to communicate his plan to his subordinates or his contractors. It is obviously advantageous if the perfected mathematical model is in such a form that it can readily be understood by them. It is even better if those concerned with the most detailed aspects of the work can analyse the problems delegated to them by means of the same technique. In this way the difficulties of information feedback are minimised, but the choice of model is thereby limited.

Feedback of information is of vital importance because, as the project proceeds, it will become apparent that the overall plan must be changed. This may be because it was wrong, or because the requirements of the project may have changed. The feedback must be sufficiently rapid for the Manager to recognise the new difficulties and resolve them before the project as a whole is adversely affected. For this to be achieved all concerned must "speak the same language".

Most important of all, the manager needs a status reporting system that will enable him to "manage by exception". The technique of management by exception is based on the premise that no Manager can effectively control every detail in a project. He must direct his endeavours only towards those parts of the project which, exceptionally he hopes, need his attention. He must, therefore be able to discover which activities determine the completion date and which have the greatest effect on overall cost (they may not be the same). In this way he will not waste time in speeding up activities which do not directly affect the overall time scale, and in devising false or negligible economies. This method is particularly valuable in the Civil Service, where more or less rigid staff complements tend to limit the availability of management expertise.

3 TECHNIQUES CONSIDERED

The source of most of the more fashionable management techniques is the aerospace industry of the United States. A list of these³, published in March 1964, contains some fifty techniques from ABLE (Activity Balance Line Evaluation) to WHISIT (Where In the Hell Is It?). It was only the lack of published information on the latter which prevented its immediate adoption.

In fact there is an interesting dichotomy between the practise of industry in the U.S. and the U.K. In America many firms employ network systems dignified with a unique acronym which may include their own name, but which are basically PERT. In the United Kingdom many firms claim to use PERT, whereas in fact some use unique systems based on the memory of one of their executives of a half-day session at a management training course held some time ago, and which are not PERT at all.

The proliferation of techniques is more apparent than real. They may be divided without significant loss of precision into two types: bar charts and networks.

3.1 Bar chart techniques

A bar chart is a graphical model of a project in which each aspect to be represented is shown separately as a bar or line. The bars are normally displayed horizontally against a time scale, and can thus be made to indicate the beginning and end of work on each aspect. This information can be extended by width modulation and/or colour coding to show the magnitude and type of effort and facilities involved. An example is shown in Fig.1.

This type of chart can be of assistance in allocating effort and resources from a limited pool. It can also be used to develop or check the development cost plan required by M.O.A. of its tenderers and contractors⁴. It does not show the inter-dependence of the various activities. Another limitation is that the only events which can be allocated a point in the time scale are the beginning and end of each of the activities displayed. If the number of bars is extended to include all the significant events, even in quite a small project, it becomes a very long and confusing strip of paper. It makes no provision for the evaluation of work in progress, i.e. progress along each bar, and thus is of no assistance to the manager in measuring achievements in terms of time and cost against his targets.

The bar chart is most useful in resource allocation for production programmes, and for the presentation of a very broad (though possibly misleading) picture of a project to higher management. It has, however, been developed in various ways to overcome its limitations.

3.1.1 Gantt charts⁵

It is common practice to refer to all types of bar charts by this title, but strictly it refers to production scheduling charts of the type shown in Fig.2.

This is a shop loading chart which shows the scheduled time for the completion of each item as an open bar and the performance actually achieved as a solid bar. The system was devised by H. L. Gantt to assist with munitions production in America in the 1914-1918 war. It will be noticed that the system does not allow for uncertainty in the time of completion of each activity.

Although it is limited in scope, many variants of the Gantt Chart have been widely applied in Industry and it has not been confined to the production situation for which it was originally designed. The present writer has used the technique in progressing the activities of a number of contractors making similar items of instrumentation. Perhaps the most valuable results were achieved by displaying the Gantt Chart on the office wall, so that the lagging contractors could be spurred on by the visibly superior efforts of their competitors.

3.1.2 Line of balance

This technique has been applied almost exclusively to production planning, and is thus of limited interest. For this reason it is not described here. There is a brief but clear explanation in Polaris Management⁶. Its purpose is to highlight the areas in which production is falling behind, and can show where the remedial attentions of management should be directed. In other words, the technique is directed towards achievement of the "management by exception" principle in that only those components which fall significantly below the line of balance need be hastened.

3.1.3 Milestone charts

Another technique for increasing the usefulness of bar or Gantt charts in the control of R. & D. projects is the milestone method. A milestone is a carefully defined situation in the lifetime of a project; for example, the delivery of the first prototype or the completion of manufacturing drawings. These milestones are usually shown by a numbered asterisk or arrow against the particular bar of which they form part. The position of the milestone is established by determining the point in time when it ought to occur if the project is to be completed on schedule.

This technique has been used quite extensively in the control of Ministry projects. The "development plan" which is a mandatory part of the documentation specified by G.W./S.A. Project Costs in their contractors development costs plans⁴, is to be a milestone bar chart.

Like other forms of bar charts, the milestone method does not take account of the interdependence of activities or milestones, neither does it allow for any uncertainty in estimated times for activities. Reference 4 does envisage that the milestone chart may be derived from a network, in which case these limitations need not apply.

3.2 Network techniques

The next step in the evolution of graphical models of R. & D. projects was the network. Fig.3 shows how it was derived from the milestone bar chart by introducing lines linking the milestones to indicate the constraints which they impose on each other.

A good account of the background to this development is given by Miller⁷. Several studies carried out in the United States showed that military and commercial development and production programmes had vastly exceeded their estimated time and cost. This led to the development of the Critical Path Method (C.P.M.) by M. R. Walker of DuPont and J. E. Kelly of Remington-Rand⁸. Independently, and at the same time, a research team in the Navy Special Projects Office developed the Programme Evaluation and Review Technique (PERT) for the fleet ballistic missile programme, otherwise Polaris. The first published paper on PERT appeared in September 1959⁹.

Although the two techniques have the same basis and many similarities, their vocabularies are different. To avoid confusion, this paper will use the PERT vocabulary exclusively. In the original form, C.P.M. included cost as a parameter, but PERT did not. An extension of the PERT technique to include cost has since been made and this is known as PERT/COST. This has led to the original version of PERT being known as PERT/TIME. The distinction also will be made in this paper.

The basic network technique used for both C.P.M. and PERT involves the construction of a flow diagram or network of "events", shown as circles or rectangles, and "activities" shown as arrows. The event is defined as a distinguishable, unambiguous point in time that coincides with the beginning and/or end of a specific task or activity. The arrow shows either a specific activity which must necessarily be completed in order to achieve the event to which it points, or a constraint which, while not involving time-consuming activity,

nevertheless governs the achievement of the event. This latter is known as a "dummy activity". The achievement of any event will be governed by one or more activities, and it will initiate one or more further activities, the first and last events being excepted. Thus, all the activities in the project are shown on the network in their proper order and in their correct relationship to each other.

Estimates are made for the time to be taken by each activity, and it is then possible to compute the total time to reach any event in the network from the start. From this can be deduced the activities which determine the completion date of the project - the "critical path" through the network. It is in this analysis that C.P.M. and PERT differ. A fuller treatment of network construction is given at Appendix A.

3.2.1 The critical path method - C.P.M.

Up to the point of the completion of the network the techniques used in C.P.M. and PERT are identical, although as mentioned above the terminology is different. The C.P.M. model is deterministic, that is a single time estimate is made for each activity, and no allowance is made for uncertainty. Further, in the original C.P.M. system it was assumed that the estimator would be on sufficiently familiar ground to make a cost estimate as well. From these estimates it is possible to deduce the total time and cost for the project. If the total time is too great to meet the scheduled requirements, a second set of estimates, known as "crash" is made. The crash estimates for each activity are the minimum possible time for its achievement, and the minimum, but inflated, cost involved. Thus the normal estimates represent the most economical way of carrying out the project, and the crash estimates the quickest.

From the normal and crash schedules a third schedule is developed to determine the minimum cost to meet the crash time schedule-by reverting from crash to normal those activities which have sufficient slack, i.e. those activities which have time in hand on the crash schedule. Further schedules may then be produced at arbitrary times between normal and crash in order to generate a "time/cost curve" for the project. From this and from knowledge of the customers requirements, a final "optimum" schedule for the project is prepared.

It will be apparent that to carry out a meaningful analysis of this kind, the manager must have available accurate estimates, both in time and cost, for each activity. It is assumed in C.P.M. that these estimates will be made on the basis of past experience. The method has been applied with considerable success in production and civil engineering projects.

3.2.2 The programme evaluation and review technique - PERT/TIME

The major difference between PERT/TIME and C.P.M. is that the former is a stochastic model. That is, it takes account of uncertainties in the times estimated for individual activities by expressing them as distributed variables. It assumes a beta distribution with a standard deviation (σ) of 1/6th of the range. Of course, engineers are not accustomed to expressing their estimated times for activities in these terms. Furthermore, rigorous network analysis would present formidable problems, including the solution of a cubic equation, to find the mean and variance of each distributed variable.

The PERT technique uses a simplified approach to obtain an "expected time" and variance for each activity. Three time estimates are made for each: optimistic - t_o , most likely - t_m , and pessimistic - t_p . These are carefully defined - see Appendix A. A weighted arithmetic mean of the three then gives the expected time - t_e .

$$t_e = \frac{t_o + 4t_m + t_p}{6} .$$

Since the range is defined by the optimistic and pessimistic estimates, the variance, is given by:

$$\sigma^2 (t_e) = \left(\frac{t_p - t_o}{6} \right)^2 .$$

The mathematical basis of this simplification is not rigorous, and it is known that there is a slight bias towards an optimistic result. However, the equations can be regarded as empirical rules which have been applied successfully. Appendix C describes the PERT model of an activity in more detail.

Thus the stochastic model is converted into a deterministic one in which the elapsed time to reach any event can be calculated by adding the expected times for the activities in the longest path to it from the initial event, and a measure of the uncertainty can be arrived at by adding the variances. This measure can be used to arrive at the probability of the event meeting a scheduled date.

To this end, an assumption is made that, where there are a substantial number of activities on the critical path, the distribution of the possible total elapsed times for the path will approach the normal (symmetrical) distribution. The theoretical justification for this assumption is the central limit theorem, which holds when there are a number of activities (e.g. more than ten) and their individual distributions are random. The probability of meeting a scheduled date

is found by expressing the difference between the scheduled time (T_s) and the total expected time (T_E) in terms of the standard deviation:

$$\frac{T_s - T_E}{\sigma}$$

The result is compared with a set of probability distribution tables to give a figure for the probability of achieving T_s .

This approximation has been the subject of criticism. It depends, inter alia, on the assumption that the critical path is "enough longer" than all other paths through the network for the latter to be negligible. In practice, this situation does not often obtain. If parallel paths of duration approaching that of the critical path exist, the probability calculation will produce an optimistic answer. The point is dealt with fully in Para. 5.6(g).

It can certainly be argued that most managers are not statisticians, and are liable to make inaccurate deductions from statistical information. One of the foremost practitioners of PERT in the U.K. has privately informed the author that he does not estimate probabilities, but merely uses the variance of the objective event as a rough guide.

However this may be, the PERT algorithm does provide the manager with a substantial amount of information about his project which he is unlikely to obtain in any other way, to use as he thinks fit. In considering a mathematical model of this kind, it should not be forgotten that he can introduce a powerful compensating mechanism into the computation - himself.

The size of PERT and C.P.M. networks may range from perhaps thirty events to many thousands. In general, there will be about fifty per cent more activities than events. In all but the smallest projects, there will be more than one level of management, and each will wish to operate to an appropriate degree of detail. In PERT, this fact is acknowledged by breaking down the total work content of the project into tasks or "end-items" at different degrees of detail or "levels of indenture". The result is displayed as a family tree which, it is emphasized, is not necessarily related to the organisational tree. An example of part of the work breakdown structure is shown in Fig.A1. Each "block" on the tree will normally generate one or more networks. For networks of up to one hundred events, manual computation is easiest and quickest, at least initially. For networks of three hundred events or more, electronic data processing is almost essential. It seems probable that the majority of the small projects with which this paper is concerned would generate networks, at the lowest level of

indenture, of between one hundred and three hundred activities. In this borderline region, the computation, though arithmetically simple, is tedious and time-consuming. Means of tackling this problem are discussed in Section 5.

Thus, the basic PERT/TIME system does offer, in exchange for effort spent in the construction and analysis of networks, a solution to the majority of the needs of the manager as postulated in Section 2. It allows the project to be broken down into fine detail, without obscuring the major objectives. It provides a mathematical model of the project, which allows for uncertainties in time, but which does not involve complex computation. It allows alternative solutions to the manager's problems to be tried out in the model, and the effect on the time-scale determined. By having the network and its analysis brought up to date at regular intervals, he can ensure that he is adequately informed on progress, and take account of any necessary changes to the programme. Most important of all, the PERT system directs the manager's attention to parts of the project that are likely to lag, before they actually do so. It is the management by exception technique par excellence.

The disadvantages of PERT are that it makes a significant demand on the manager's own time and that of his subordinates or contractors; the statistical part of its data output is open to misinterpretation; its performance is absolutely determined by the validity of the data which are fed into it. It does not include a formal process for evaluating the accuracy of the providers of this data, which could be done by comparing past estimates with the time actually taken. The conscientious manager does no doubt generate "calibration factors" for his staff, but there is no reason why this process should not be brought into the open so that accuracy may improve. Its implementation has also been known to cut across organisational hierarchies, but this may not always be a disadvantage.

4 CHOICE OF TECHNIQUE

The reader has known from the beginning that PERT/TIME was the chosen technique. For those who are prepared to take the job of management seriously, the advantages would certainly appear to outweigh the disadvantages. There were, however, other reasons for the choice.

In its activities, Space Department is involved with a number of research organisations and establishments in the U.K. and abroad. Among them are The National Aeronautics and Space Administration (N.A.S.A.) in the United States, The European Launcher Development Organisation (E.L.D.O.) and the Weapons Research Establishment of the Australian Department of Supply. All these organisations use PERT. A number of firms in the British aerospace industry use

it or are dabbling with it. Of the one hundred and twenty one firms and divisions of firms in the aerospace industry of the United States listed by Frambes³, one hundred and seven were using PERT or a close derivative.

PERT thus has at least the makings of a universal "language" for communication of information about projects and their management. In particular, satellite networks could be integrated with those for the launching rocket, and both with those for the range facilities involved. In the strictly limited context of the Section's own responsibilities, a common language was desirable to co-ordinate the different sub-systems for which it would be responsible.

The choice of PERT/TIME was thus made on two grounds. It appeared to be the best model, fully developed, for the special requirements of the manager of a small development, or research and development project. It had found wide acceptance among organisations with which the section was likely to have dealings.

5 IMPLEMENTATION OF PERT/TIME

5.1 Size of project

In Section 1, a "small project" was arbitrarily defined as one costing less than half a million pounds. There is no doubt that the application of PERT/TIME to projects approaching this figure in total cost will yield substantial benefits. If we are to take cost as our criterion, the question of a lower limit arises.

Battersby¹⁰ quotes as an example a small network modelling the preparation of sweet and sour pork. This was formulated by a member of the staff of the College of Aeronautics and his wife - presumably for fun. It is at least doubtful whether the increase in culinary efficiency in their family kitchen justified the effort involved. If, however, he had been involved in the management of a Chinese Restaurant, increased efficiency would yield financial dividends.

This is the crux of the situation; in deciding whether or not to use the PERT technique in the management of a particular project the manager has to decide whether the benefit derived will justify the expenditure involved. Thus, in plant maintenance, where the efficiency of comparatively small and inexpensive operations govern the availability of facilities of high capital value, PERT and C.P.M. have both been used successfully. In the R. & D. field, the same principle applies. On the time scale of a very small project, a much larger programme may depend. It is suggested that, in any case, a project which seems likely to cost £10000 or more should be considered as a possible candidate for the PERT/TIME treatment.

It is implicit in the foregoing that the cost of implementing PERT/TIME, as a proportion of project cost, rises as the project gets smaller. This is borne out in practice. A number of figures have been quoted by various authorities, and these may be summarised by saying that the cost of PERT ranges from 0.1 per cent of project cost for very large projects to 1 or perhaps 2 per cent for small ones.

5.2 Extent of implementation

Having decided to employ PERT on a particular project one has to decide how far to go. Some people merely draw the network and use it as a qualitative guide to project planning. Others go the whole hog and establish computer programmes for generating many different arrangements of the basic PERT data for different purposes. Both these approaches have their disadvantages.

The difficulty with the qualitative method lies mainly in the fact that it is only in the analysis that the first indications of the networks validity or otherwise are obtained. An unanalysed network must be scrutinised extremely carefully to discover such logical inconsistencies as "loops" (a situation where an event is its own predecessor, shown in Fig.4) and "dangles" (an event other than the initial event which has no predecessor, or an event other than the objective event which has no successor). These faults are quite readily found in manual analysis, and computer programmes are arranged so that network validation is the first process performed by the machine. Once again, one must consider the value of the information obtained. From a qualitative network it may be determined that Task 'B' should commence before Task 'A'. With some extra effort devoted to analysis, optimum starting dates for both tasks can be established.

At the other extreme, there is a danger of generating so much data in the form of computer print-outs that the manager has insufficient time to absorb them, let alone base his decisions on them. Computer time is expensive, and the writing of PERT computer programmes is a difficult and lengthy process. However, new computers almost always have suitable PERT programmes in their associated "software", so this difficulty may be expected to disappear in time.

Perhaps the best approach is to begin with manual analysis and, when the work load becomes too great for the manager or his assistants, to introduce electronic data processing later. Even then, the reports called for should be limited to those which are strictly necessary.

The amount of effort involved is greatly affected by the degree of detail or "level of indenture" of the networks. A useful rule of thumb is that the longest of the expected times t_e of the activities at the lowest level of indenture should approximate to the technical reporting interval - or the

interval between progress meetings. Another useful technique is to place an arbitrary restriction on the number of events in any one network and, if necessary, to integrate a number of these small networks by means of milestones and interface events into a manageable summary network at a higher level of indenture.

This technique is taken to its logical conclusion by the Australian Weapons Research Establishment. Their system is to produce a number of networks at the Principal Officer level of their organisation chart, this could be said to be somewhere between P.S.O. and S.P.S.O. level. The Principal Officer's contribution to any one trials programme is a small network consisting of no more than twenty or thirty events. Time estimates are then made for all the activities on the network and these data are transferred to punched cards by the PERT Analysis Section. By means of an I.B.M. 1401 Computer, the data on the punched cards are transferred to a magnetic input tape for the I.B.M. 7090 Computer which is to do the analysis. Since the interface events between the networks are carefully coded, the project network is, in effect, built up on the computer master file, and the analysis is done from this. It is quite unnecessary for the overall network to be drawn at all and this is not done (except occasionally for publicity purposes). This technique does, of course, depend on the availability of a powerful computer with large storage capacity.

5.3 Intra-mural implementation

It is unlikely that anyone would be able to implement PERT in his own work without the co-operation of his colleagues. He may have to justify the consumption of effort to his superiors, and he will need the assistance of other people in preparing his input data. This introduces problems of persuasion and education. The origin of the present paper lay in requests for colleagues and superiors for the writer to undertake these functions. It is hoped that its arguments will be enough to convince a hypothetical juror that PERT/TIME is a valuable tool of management for small R. and D. projects "beyond all reasonable doubt".

In the past, professional consultants in operations research have often been employed by organisations introducing PERT procedures for the first time. In the writer's opinion, this is only justified where a complex project of great importance is involved. There is now a great deal of published information on PERT and its derivatives. It would be quite easy, though very tedious for all concerned, to provide this paper with a bibliography of 500 references. Instead of this, a brief guide to further reading appears at the end of Appendix A.

If the manager is prepared to make a reasonably careful study of published information, and if he is prepared for his first attempt at PERT implementation to be experimental in character, the need for employing consultants should not arise. Another useful source of information is the training courses in PERT techniques run by schools of management and some computer companies.

Once the people concerned have absorbed the basis of the technique, and the objective event of the project has been defined, implementation can begin. The first task is to establish a work breakdown structure. This, together with other PERT/TIME terms used in this section, is fully defined in Appendix A. Essentially, it is a chart in the form of a family tree which defines the levels of indenture (or detail) at which networks will be prepared. For small projects, there are unlikely to be more than three levels of indenture, at least at the beginning. The temptation to draw an overall network at the highest, or management, level should be resisted if possible. Instead, objective events for the lowest level networks should be established by management, and the individual officers responsible should start to draw their networks. It is immaterial whether they use the "backward method" (starting with the objective event and working backwards), which was favoured by some early PERT users¹¹, or the more obvious forward method. It will soon become apparent that network construction is an iterative process. As soon as the earliest drafts are ready, all concerned should confer for constructive criticism and establishment of interfaces. The process of adjustment and redrawing of networks at the lowest level of indenture will take some time, and no attempt should be made at this stage to produce tidy networks for reproduction. When some measure of agreement is achieved, the manager can select milestones and interface events for his network at the next higher level of indenture.

If this shows that the basic relationships between the detailed networks are sensible, time estimates can be prepared.

The choice of estimator for each activity is one of balance between responsibility and experience. Obviously, the individual who is responsible to management for the completion of a particular activity must be convinced of the accuracy of the estimates. On the other hand, he may not be sufficiently experienced in the work involved in the activity to make the estimates himself. In this case, it would perhaps be best for the preparation of the three time estimates to be undertaken by a small group which includes the most experienced as well as the responsible people. If the activity is completely new, it would be wise to seek for some analagous situation for which information is available. Guesses should only be used in the last resort.

As soon as all the estimates are available, a manual analysis should be performed, preferably by those responsible for the networks. The results of the analyses are likely to instigate another loop of network refinement and replanning. There are some basic PERT/TIME rules for this operation, and they will be found useful. Alternative techniques and plans can be set up in the model, and their effect on time scales determined. During this process, it will be found that the critical path is continually changing, and it is essential to proceed slowly; because, for example, enthusiasm for shortening time scales can lead to overloading of resources and facilities.

The result of these labours will be a model (not a plan) of the project, from which the probability of meeting a scheduled target date can be determined. If this is acceptable, detailed planning can begin. This will include the assignment of personnel to the activities, and requests for Drawing Office and Workshop facilities and the like. A programme for revising the networks at regular intervals should be laid down. If manual analysis has been found very onerous, arrangements should be made for the revisions to be processed on a computer. The computer programme should be tested by running the analysis of the initial network and checking against the results of manual analysis.

It may have been noticed that no mention has been made of a "PERT specialist" or even of a PERT analysis staff. It is fatal to the whole concept to set up an independent organisation to run PERT. There is then a tendency for Project Engineers to regard it as incidental to the real work of running the project. If this attitude becomes established, PERT is no longer a tool of management, it becomes an unnecessarily sophisticated reporting system.

5.4 Extra-mural implementation

Within the Department, this may be taken to include the use of PERT by Design and R. and D. Authorities to manage the efforts of their contractors, and its use by the contractors themselves within their own organisation. The principles of implementation set out above are equally applicable, but the situation is complicated by the contractual relationship and, perhaps, by the fact that the contractor may already be using the technique.

Because PERT costs money, the requirements for its use must be made known to tenderers at the outset, i.e. they must be stated in the tender documents. If it is decided to introduce it during the lifetime of an existing project, it should be incorporated in a contract amendment. Even if the contractor is using PERT internally, it is unsatisfactory for a design authority to have to ask for the substantial amount of work involved in rescheduling the job, when he has no contractual right to do so.

When PERT is already being used by the firm, a conflict may arise between the intentions of the Project Manager, and the procedures already in use. While the possibility of a reasonable compromise is not excluded, there is much to be said for specifying precisely what is required of prime and subsidiary contractors. This point is pursued in the next section.

There is a strong argument for an approved procedure to be laid down on an Establishment - or Department - wide basis, but this raises larger issues.

When an otherwise suitable firm has no experience whatever of network techniques, the question of training arises. It is not unreasonable to ask that the contractor should undertake the training of his own personnel at his own expense, but a simple manual of basic techniques and operating procedure could well be provided by the Department. An example of such a manual might be Appendices A and B of this paper.

5.5 Application of the PERT/TIME rules

It is the author's contention that any manager setting out to use PERT for the first time should be prepared to adhere to the basic rules set out in Appendix A. The man who adds a few cross-linking activity lines to a Milestone Bar Chart and thereby imagines that he has constructed a PERT network is not only deceiving himself about PERT, he is deceiving himself about his project. His network is unlikely to be a valid model, and any analysis will yield misleading answers.

The critics of the PERT algorithm have generally taken the view that it is an over-simplification, and there is some truth in this. Any further simplification is, therefore, likely to be dangerous.

There is converse tendency; to attempt to overcome the limitations of the PERT technique by extensions or variations of the computation. It is suggested, however, that the best basis for work of this kind is experience of the existing system.

Another reason for sticking to the rules is the desire to preserve the communication capability of the technique. Indeed, the basic rules allow sufficient alternatives to endanger this. For this reason it is recommended that, before the introduction of PERT on a new project, a set of operating instructions should be drawn up. These would be supplementary to the basic rules and would ensure the coherence of networks and analyses prepared by different contributors to the project. Such instructions have been drawn up by the writer for use by his own Section, and these are reproduced at Appendix B. It is

considered essential that, particularly in the case of contractors, such requirements are made known at outset. It is, of course, essential that they be included in the documentation accompanying competitive tenders, so that the firms can include the cost of PERT in their quotations. They can also be required to include preliminary networks and analyses with their technical proposals, and the operating instructions will ensure that the efforts of competing firms are directly comparable.

It is intended that the operating instructions will be appended to the design aim for the project, together with a list of milestones. These are events which are considered by the Design Authority to be of particular importance, and which the contractor is to include in his network. They may be derived from the Design Authority's own network, but this is deliberately excluded from the documentation. The reason for this is that PERT/TIME is essentially a "bottom up" system. Imposing the Design Authority's management level network on the contractor would place artificial constraints upon on his detailed networks, and might tend to invalidate them. Indeed, it is argued by Boverie¹² that a list of milestones tends to have this effect. In the present writer's opinion, this is going too far. The manager ought to indicate stages during a project at which, for instance, design reviews are required. Nevertheless, the list of milestones should be confined to the necessary minimum and no attempt should be made to provide it with scheduled dates. A target may, of course, be set for the objective event.

When the contractor has completed his networks, the manager must be prepared to co-operate in resolving conflicts between them and his own. His realisation of the problems may not be the best.

It may be of interest to examine some of the other rules in Appendix 'B' and the reasons for their inclusion:

(a) Procedure

The procedure laid down is intended to fulfil two main objects. Firstly, to ensure that the PERT network and analysis really are prepared as a genuine attempt to schedule the project. The best way is to insist on their inclusion in the contract proposal. Their merit should be given full weight in considering acceptance of tenders. Secondly, the procedural requirements are intended to ensure that the contractor keeps his networks and analyses up to date. Not only must he "update" them at each reporting interval, but the updating must be carried out within the week immediately preceeding the reporting (or progress) meeting.

(b) Events

Networks are to be event oriented. There are two schools of thought about this. It is often suggested that because activities represent what actually has to be done, the networks should be formed from labelled activities and that events should merely be numbered conjunctions of activities to facilitate analysis. This contention has some appeal at the working level, but it does not meet the requirements of the manager.

PERT/TIME is not very good at evaluating work in progress, so the manager, who is concerned with what has actually been achieved, must look to completed events as his main source of information on progress. If events are not defined, as in an activity oriented network, this information is hard to come by. Furthermore, a large activity oriented network is very difficult to read. Either the descriptions of the activities are written along the activity line, which makes it necessary to spread out the network, or they are listed separately, which makes interpretation tedious and difficult.

In an event oriented network, all the qualitative information is included within the event symbols. Thus the network can be self-contained, without growing to inordinate size. Furthermore, it is much easier to produce an activity oriented analysis from an event oriented network than the converse. Examples of event and activity oriented networks are given in Figs. A2 and A3.

Fulkerson's rule¹³, is specified for the numbering of events. This gives effect to the proposition that, for any activity, the number j of the end event must always be greater than the number i of its beginning event. It has the advantage that a manual analysis in approximate chronological order can immediately be produced from the network. Its use will also automatically detect logical inconsistencies such as "loops" in the network. Some computer routines will automatically rank activities in chronological order from networks numbered in random fashion, but this can consume a lot of expensive computer time.

The number of events in the contractors network is limited for the reasons given above.

(c) Activities

Under this heading in the operating instructions appear reiterations of two basic PERT rules. It is essential that each activity shall be the responsibility of one individual in the contractors organisation. This is based on the time-honoured maxim that any manager must have only one bottom to kick in respect of any one job.

The other insists that the three time estimates are to be made throughout the project, except for dummy activities and for those where a precise time is known in advance. Experience has shown that there is a reluctance on the part of some people to acknowledge that there is any marked uncertainty about the time of any activity whatsoever.

(d) Analysis

Analysis is deputed to the contractors themselves. This has the obvious advantage of relieving the Design Authority of the business of doing it, but there are other reasons. Perhaps the most important is that the contractor will be more concerned to ensure the accuracy of the information which is to be analysed if he has to do it himself. Where areas of difficulty arise he will have advance notice, and be able to evaluate possible remedies before he is called upon to give verbal account at a meeting. Experience in the United States has shown that many of the failures of PERT implementation have stemmed from the requirement for PERT control being regarded as something additional to existing systems of management and control. This is fatal; one way of inducing the contractor to implement PERT properly is to involve him as much as possible in its operation.

The operations called for are the standard activity-oriented network analysis showing expected times and variances, and a slack path analysis. As previously explained, these may readily be derived from the event-oriented network, if the events are suitably labelled. Within the system as defined, these analyses may be performed manually without difficulty. Mention is made of an inexpensive (5 U.S. dollars) computing aid of the circular slide rule type, which may be used. The contractor is not discouraged from using sophisticated analysis techniques but, to preserve coherence, the results of applying the basic PERT formulae are to be shown in any case.

(e) Reports

The intention in this section is to minimise paperwork and maximise information. For this reason, the reports are limited to the network analysis and the slack path analysis, but the form of the network analysis is so drawn as to give all the information that could reasonably be expected from a network. The only manipulation of this data which is called for is the slack path analysis. This directs attention to the critical and sub-critical paths, and assists in finding ones way about the network. To avoid repeated redrawing of the network, the concept of "NO" is introduced; this is an artificial event which defines the situation at a reporting date. If a list of events actually

achieved is provided, the "NOW" event can be transferred to an existing network by drawing a line which intersects all activities which are actually in progress. The analysis is then performed using "NOW" as the initial event. The only eventuality which would involve the redrawing of a network is a change in its logic.

5.6 Some pitfalls

It is not unknown for PERT/TIME to be adopted by an organisation, only to be rejected after brief experience. However, since it has been successfully implemented elsewhere, it is arguable that the fault lies with the users rather than the technique. From experience, and from the literature, it is possible to suggest some common pitfalls in implementation which may account for these failures. Some of them have already been mentioned, e.g.:

- (a) Not observing the basic rules of the technique.
- (b) Creating a specialised PERT staff divorced from project management.
- (c) Generating unnecessarily voluminous analyses.
- (d) Producing a network which is either too large to be readily understood, or which can only be understood by reference to separate lists of events or activities.
- (e) Imposing a management level network on the more detailed networks, i.e. using a "top-down" approach instead of the correct "bottom-up" system.
- (f) Neglecting to provide an adequate set of operating instructions (work statement) for subordinates or contractors.

There are a few additional pitfalls that ought to be mentioned:

- (g) It is dangerous to concentrate attention on the critical path in a network to the exclusion of the sub-critical ones. The probability calculations for achievement of objective events is based on the assumption that the critical path is "sufficiently longer" than the others for the latter to be neglected. This assumption is not often true. Experience shows that when a network is rescheduled to reduce the overall estimated time, the reduction of the length of the critical path almost always results in the creation of a new one. Furthermore, it is possible for a highly critical activity to be on a non-critical path if, for instance, there are two activities of equal importance occurring in parallel, and one is estimated to take marginally longer than the other. The slack path analysis can be of great assistance in probing these difficult areas.

MacCrimmon and Ryavec¹⁴ have shown theoretically that the error between the PERT-calculated mean duration and the actual mean duration for a critical path increases with an increasing number of parallel paths and with their criticality. It therefore behoves the PERT user to scrutinise parallel paths carefully. It should be noted that cross-linking between paths reduces the error. In general, the PERT calculation of mean duration gives an optimistic result, because the uncertainties in the sub-critical paths are not included. On the other hand, the PERT calculation tends to overstate the standard deviation.

(h) When the estimated time for completion of a project is substantially later than the required time, it is normally rescheduled by applying certain rules, which are set out in Appendix A. When this is done, it is possible to achieve a highly ingenious arrangement of the network which gives an acceptable estimated duration, but which is completely impossible to put into effect. This is because the manager has neglected to consider his resource limitations. Of course, in theory, new men can be hired and new equipment purchased; in practice this is often not feasible or even justified. Even if it can be done, there will be a substantial delay before the new resource is operational. When parallel paths are created either initially, or when rescheduling, resource limitation must be borne very much in mind.

(i) It has already been pointed out that the success of PERT is absolutely dependent upon the validity of the data fed into it. It follows that all concerned in the management and operation of a project will be involved in the implementation of PERT. They cannot contribute effectively unless they are adequately informed of the basis of the system. It follows that the manager will have to train his people, or to arrange for them to be trained by others. Boverie¹¹ puts the point very neatly: "Training of both the Programme Management and the Project Engineers must be accomplished with authority and in detail, for without an adequate understanding of PERT by these potential users PERT can only hope to be an ancillary, bureaucratic function consuming time, money and energy. A common mistake is to provide only a surface presentation to top management. A surface presentation can inculcate only a surface desire to see the system work, and if top management is not enthusiastically supporting PERT, the odds are against PERT being properly employed."

Successful users of PERT emphasise the importance of proper training. For example, in 1963, 670 people attended the NASA PERT system "workshop", the highest attendance of the NASA management training programme¹⁵.

(j) PERT/TIME has been presented in this paper as a complete and all-embracing tool of management up to Project Manager level. If it is implemented, there is no need for it to be supplemented by less effective techniques. Indeed, it is positively undesirable that it should. Human inertia is such that the older techniques will be relied upon because people are used to them, although experience has shown that they are inadequate. In these circumstances, PERT will never "get off the ground".

An effort should thus be made to substitute PERT information for the older systems which are specified in existing procedures, e.g. the milestone bar chart of the development cost plan. If this cannot be done, it must be made absolutely clear that such charts must be compiled from PERT analyses.

6 EXPERIENCE WITH U.K.3 DATA HANDLING PROJECT

The writer's section is responsible for the design and manufacture of data handling and data storage equipment for the U.K.3 international satellite. Since it was of the utmost importance that the various models of this equipment should be delivered to the satellite prime contractor on the scheduled dates laid down by the overall Project Manager, it was decided to implement PERT/TIME for the management of the development and manufacture of these sub-systems. The overall management of the U.K.3 project employs the milestone bar chart technique, with interdependence links, to generate the scheduled dates for the sub-systems. The estimated cost for the data handling equipment is of the order of £110K, and for the data storage system between £20K - £30K, so both come within our arbitrary definition of a small project.

The writer has previously permitted himself one or two generalisations about the attitude of British firms to PERT. It must be said at this point that these comments do not in any way apply to either of the contractors employed, who have given their fullest co-operation.

It was decided to introduce PERT in February 1964, at which time the electrical design of both sub-systems had been virtually completed within R.A.E. At the time, contract proposals for the engineering development and manufacture of the electronic sub-systems had been made, but financial sanction had not yet been given. A satellite technology contract on the agency factory of the General Electric Company at Portsmouth was in being, and was used, in part, to support the U.K.3 work on data handling equipment.

An American PERT practitioner¹⁶ has written "after more than two years of effort, there are still very few Managers who are willing to admit that a single

significant decision has been based on PERT". In the U.K.3 Programme, most of the significant decisions affecting the data handling and data storage sub-systems have been based on PERT, so far without disastrous results.

6.1 Data handling sub-system

This consists of three units; a high and a low speed encoder, and a programmer. Some idea of the complexity of the equipment may be obtained from the fact that they contain a total of over a twelve hundred active devices, with an overall component count of 2729. "Worst-case" design was used throughout.

In February 1964, the electrical design of the entire sub-system was virtually completed by R.A.E., except for adjustments at circuit interfaces to meet requirements which were still being formulated. An engineered model of an early version was undergoing mechanical design at G.E.C., as part of the satellite technology development. Thus, the basic circuit techniques and methods of assembly would be proven, it was hoped, before work on the U.K.3 system started. In the absence of contract action, it was decided to construct a detailed PERT model of the project to determine the feasibility of compliance with the overall U.K.3 Programme. This network consisted of 127 events. It was found that there was a high probability of producing the flight models to schedule but that the short interval between the delivery of the electrical compatibility model (D.2), and the prototype (P.1) might give rise to difficulties. The area of greatest uncertainty was financial approval and contractual negotiation. Before these processes were completed, up-dating the analysis showed that it would no longer be possible to produce the D.2 model by the scheduled date. This led to the first PERT-based decision: to change the design of the technology model, which was on the drawing board and for which components had been ordered, to the latest U.K.3 standard. Thus it could be used as the D.2 model, at the cost of some loss of information on basic technology. The decision was justified in that the D.2 model was finished on schedule.

By this time, some information on manufacturing times was forthcoming and, when the job was rescheduled to take account of the decision on D.2, it was shown that it would not be possible to perform all the manufacture of the various models in series; i.e. to allow for design changes after each series of tests on the previous models. This led to the second major PERT-based decision; to freeze the design of the basic functional assemblies - or brick units - at the earliest possible stage in the project, and to accommodate all subsequent design stages at the sub-assembly level. There was of course a risk of wastage, if any of the brick designs were proved to be inadequate. The PERT model showed that this risk had to be accepted in the light of the possible alternatives.

As soon as financial sanction was granted, and tenders issued, a party representing the various design authorities within R.A.E. visited all the tenderers. It was made clear to them all that the Design Authority for the data handling sub-system would require the use of PERT in project management. Four of the seven firms visited said that they were already using the technique, but two of these did not seem to know much about it. The remaining two supported their proposals with PERT-based information, and one presented the results of the full treatment - eight networks together with computer print-out of the analysis.

In the event, the contract was let to one of the two latter firms - the Applied Electronics Laboratory of the General Electric Company, which is a Government Agency Factory, and the responsibility for detailed networking was at once handed over. Their first model was rejected on the ground that, although the critical path contained eight weeks of slack, the variance was very high and the model did not allow sufficient time for the insertion of design changes, stemming from compatibility and environmental tests, into the later models. A second model was then prepared, and this proved to be satisfactory. It also showed, as had the R.A.E. network, that there was insufficient time between D.2 and P.1 delivery dates. This situation was aggravated by a requirement for a model of the sub-system to be provided for an early series of heat balance tests. For various reasons, it had been decided to manufacture an extra equipment for this purpose. The G.E.C. network and analysis showed clearly that the strain on resources imposed by the introduction of this equipment would prevent P.1 equipment being delivered on schedule. It was therefore decided to delete this model from the programme and to refurbish the D.2 model for the heat balance tests.

A further difficulty arose at this time. Two of the sub-systems of the satellite, the data handling and the power supply electronics were to be constructed by means of welded interconnections. The welding was performed using specialised equipment in a controlled atmosphere. This made it impossible to increase the capacity of the welding facility without capital expenditure. The construction of PERT models for both these sub-systems provided a clear case for the approval of expenditure to increase the capacity of the welding facility by 75 per cent, and for the introduction of more efficient utilisation of the existing operators and equipment. They also provided information on which the U.K.3 Project Manager could base decision on the priorities to be observed in allocating resources up to the P.1 date.

A useful practical point which has emerged from this work is that, where there is a series of scheduled dates to be met in the course of a project, these dates should be used as the objective events of a series of networks. This has been shown to be easier and more useful than attempting to draw a complete detailed network for the project as a whole. The detailed networks are, of course, linked by management level network to reflect their interdependence. Figs.5 and 6 show the management networks for the data handling equipment on 1st January and 15th July 1965 respectively.

6.2 Date storage

The data storage sub-system for U.K.3 consists of an endless loop tape recorder. The principal design constraints of this equipment are high reliability, low power consumption, survival in the launch environment, and low weight.

In February 1964, a basic tape recorder design had been established and two prototypes had been manufactured in R.A.E. A contract proposal for further engineering development and manufacture of prototype and flight models had been put forward, but it was in the same state of stagnation as were the data handling contract proposals. Consequently, an 80 event detailed PERT network for the further work was drawn up in R.A.E. This established that compliance with the U.K.3 overall programme was feasible, if certain risks were taken. These involved, for example, the commencement of assembly of model P.1 before compatibility tests of D.2 were complete.

The contract was let to the Atomic Weapons Research Establishment, and work commenced in July 1964. The contractor's personnel concerned had no knowledge of PERT whatever, but were willing and ready to co-operate. An 86 event network with delivery of the D.2 model as the objective event was jointly established. A.W.R.E. suggested that, as the work had a manufacturing rather than a development bias, single time estimates should be used. Because of the experimental nature of the PERT implementation at the time, this was agreed. It must be said that the single time estimates were not noticeably more accurate, in the event, than estimates for other parts of the programme. For a small network such as this, the saving in estimating effort was negligible and, subjectively, it is felt that the use of three time estimates would have marginally improved the accuracy. It must be said, however, that this exercise is by no means conclusive.

PERT has contributed to the making of several decisions during the development and manufacture of the D.2 model of the tape recorder, which was delivered on schedule. Perhaps the most important of these were decisions to

obtain alternative clean room facilities at A.W.R.E. because the room which was to be constructed for the assembly of tape recorders would not be ready in time, and to reduce the amount of testing to be applied to the D.2 model.

A further detailed network has been constructed by A.W.R.E. for the P.1 and R.1 recorders. A management level network for the whole project was constructed at R.A.E. It is significant that, in this example, PERT/TIME has been implemented successfully by a contractor who had no previous knowledge of the technique.

7 DERIVATIVES OF PERT/TIME AND C.P.M.

Mention has been made of certain limitations of the PERT/TIME technique. Both PERT and C.P.M. are products of Operational Research and, since their introduction, other practitioners of the art have been busily engaged in exposing these limitations and inaccuracies. Others have been seeking to refine and extend the basic network techniques to overcome them. A comprehensive review of these activities is outside the scope of this paper, but it has been thought worthwhile to make brief mention of those most likely to interest the reader, and to refer him to the source literature for further information.

7.1 Computational variants

7.1.1 Monte Carlo methods

The basic PERT algorithm turns the stochastic PERT network into a deterministic one by making an assumption about the nature of the distribution of the activity durations, and by making an approximation to determine the mean and variance of the distribution. This approach has the advantage of arithmetical simplicity, but it is not without error. In this paper, attention has been drawn to known sources of error and it has been suggested that the manager's own judgement can supply the necessary correction.

Among others, Van Slyke¹⁷ has suggested that the errors could be avoided by use of Monte Carlo simulation. By taking the mean of some thousands of samples of possible project durations (the distribution being determined by the probability functions) it is possible to achieve a much closer approximation to the true mean of the model. The accuracy achieved can be increased by increasing the number of samples.

If Van Slyke's methods are used, an exceedingly useful by-product of the computation is available. This is the probability of an activity being on the critical path. The disadvantage of Monte Carlo methods is that they depend on the availability of a powerful and fast computer to perform the many thousands of

calculations involved. The manager in the engineering field may also feel that he needs an Operational Research scientist to drive the computer.

7.1.2 Decomposition of large networks

If the reader ignores the advice given in this paper and generates a very large network, he will undoubtedly find it difficult to analyse. It may even be beyond the storage capacity of most computers. The technique recommended by the pundits for dealing with the situation is to decompose the large network into a number of small ones which can be individually analysed, and then to regenerate the original network from the resulting data. This can be achieved quite readily by "cut and try" methods, but formal procedures for doing so have been proposed by Parikh and Jewell¹⁸, among others. They deal not only with time networks, but with cost-time networks such as C.P.M. and PERT/COST.

7.1.3 An electronic analogue

R. H. Barker¹⁹ has suggested that an electronic analogue of a network might usefully bridge the gap between manual analysis which is suitable for small networks and electronic computation which is really only justified by quite large ones. In his proposal, the activity would be represented by a monostable trigger circuit with adjustable time constant. Conjunctive events could be simulated by switches driven by bistables which would only operate when pulses had been received from all the delay circuits.

This intriguing suggestion has great attraction for the electronic engineer. It should be possible to construct an analogue for a 100 event network for the same cost as that of writing and proving a computer programme. Like the computer, it could be used for any network (if suitable patching facilities were provided); but, unlike the computer, running costs would be negligible.

7.2 PERT/COST

In that PERT/TIME facilitates the continuous employment of resources and the elimination of duplication of effort at interfaces and elsewhere, it contributes to the reduction of project cost as well as project time. However, when Malcolm et al²⁰ developed PERT, they deliberately excluded cost (or resources), and system performance from their consideration.

Thus, when PERT/TIME was first introduced, cost control was carried out using earlier techniques, notably the cumulative cost curve derived from milestone charts. In this technique, the estimated cumulative cost of the project is plotted against time, and milestones are marked on the curve. The actual cumulative cost is plotted on the same piece of paper and, providing that actual

cost does not exceed projected cost at the moment of achievement of each milestone, all is assumed to be well. It is not difficult to envisage a situation in which, because of slow progress on work contributing to later milestones, a model of this kind would give a completely misleading picture. Similar curves are used in the PERT/COST system, but a third curve is drawn to show "value of work performed" (see Fig.7).

PERT/COST imposes the disciplines of PERT on the planning and control of expenditure. It uses a network in which the activities are "work packages" - which are individually estimated and costed. These packages are frequently formed by grouping together activities in the PERT/TIME network for the project which are related in the nature of their work content and in their direction to a defined subsidiary objective. The PERT/COST network can however be completely independent of the PERT/TIME network, so long as the subsidiary objectives can be clearly defined. In either case, the PERT/COST network should include work packages for such activities as direct supervision and project management, unless these are covered by overheads.

The size of the work packages is largely determined by the requirement that the activities in each package should be of a similar work content to enable them to be separately costed. It might be logical to include laboratory assembly in a package together with machine shop time, but such logic is unlikely to appeal to the accountants. The other factors are arbitrary; the approximate value and the time for completion of each package. These will tend to be related to total project cost and to cost reporting interval in the same sort of way as activities in a PERT/TIME network.

Unlike PERT/TIME, a mechanism exists for evaluating work in progress on a particular package. This parameter is "value of work performed to date". This is given by the formula:

$$\text{Value of work performed} = \frac{\text{Actual cost to date} \times \text{Original estimate for complete package}}{\text{Latest revised estimate}}$$

This information can be displayed on a "cost of work report" which is analogous to the cumulative expenditure reports of the milestone system, but contains more information (see Fig.7). This report may be prepared for each work package or, more usually, for groups of packages - sometimes known as end items. If the size of the project justifies it, a whole family of reports can be generated for different levels of management, i.e. in different degrees of detail. This is analogous to the levels of indenture in the PERT/TIME system. The information supplied to each level of management includes the following:

- (a) Original estimated cost of work performed to date.
- (b) Actual cost of work performed to date.
- (c) Cost over-run, or under-run.
- (d) Original estimate for total cost on completion.
- (e) Latest revised estimate for total cost on completion.
- (f) Projected over-run, or under-run of total cost.
- (g) PERT/TIME information.

As with PERT/TIME, the value of PERT/COST is absolutely determined by the validity of the information fed into it. It may be necessary to alter estimating and accounting procedures in order to achieve this.

The foregoing is a necessarily inadequate summary of PERT/COST, which is by no means so well documented as PERT/TIME. The most useful reference is probably the D.O.D. and N.A.S.A. PERT/COST Guide²¹. This was intended by its originators as an experimental system, but it has become an accepted standard.

An attempt was made at the request of, and in co-operation with, G.W./S.A. Project Costs to apply PERT/COST to the U.K.3 Data Handling Sub-System. The management level PERT/TIME network was sub-divided into PERT/COST packages of median value £10000. Unfortunately, these work packages were too small for the contractors' accounting system to handle. Since the system was dictated by the Agency agreement between the firm and the department, it was not possible to modify it. This is mentioned to stress the point that the introduction of sophisticated systems of project control may stand or fall on the willingness or ability of those concerned to alter their existing procedures.

7.3 LESS - Least cost estimating and scheduling

This is a network based technique derived from C.P.M. Its purpose is obvious.

7.4 FRISM - Programme reliability information system for management

This is an attempt to evaluate the third of the variables considered originally by the PERT/TIME team - technical performance. Like PERT, it was developed for the U.S. Navy Polaris programme, and its methods are derived from PERT. It uses two independent measures of reliability:

- (a) RMI - reliability maturity index

This is described as "a measure of compliance with the planned reliability activities in the development programme".

(b) REM - reliability performance measure

This is a prediction of the eventual reliability of the equipment concerned, which is continuously revised throughout the development process.

The author has been able to find practically no published information about PRISM, but the objective is so important that further efforts will be made.

7.5 RAMP - Resource allocation and multi-project scheduling

This technique is based on PERT and C.P.M. Its object is to allocate resources - i.e. people, facilities, materials and finance, among several projects according to a defined management policy. Such a policy might involve completion of one or more projects in the shortest time, completion of all projects at minimum cost, continuous employment of labour or facilities, or some combination of these. The system was devised by CEIR Inc., and is quite fully described in reference 22.

7.6 The decision box network

All the project management techniques hitherto described assume successful completion, not only of the project, but of each activity within it. In a research project this situation is unlikely to obtain. Even in a development project a line of attack in a difficult area may prove unfruitful, and an alternative must be tried.

The application of network techniques to this situation was first considered by Eisner²³. He proposed the db (Decision Box) network. This allows the use of alternative paths to achieve the same object. These may be conjunctive - if they are both to be attempted, or disjunctive if only one is to be tried. The alternative paths may merge to arrive at a known and fixed objective, or there may be several possible outcomes. If the latter is the case, Eisner's model allows the probability of each outcome occurring to be calculated.

In a recent article²⁴ Scott and Hopkins stress the danger of applying PERT and C.P.M. techniques, as they stand, to projects where there is a substantial element of risk that particular activities will not be completed successfully. They, too, suggest the use of alternative paths in networks assigning probabilities to each. They also allow the use of "recursive loops" to accommodate the situation when an activity has proved to be unsuccessful, and the job must be tried again a different way.

Some such extension of the basic PERT model would undoubtedly be of value in the management of research and might also be applicable to more advanced development projects.

8 CONCLUSIONS

- 8.1 The basic PERT/TIME technique is an effective model for management of small projects. Its deficiencies can be allowed for by the use of human judgement because the scale is small.
- 8.2 Since PERT/TIME and its variants are by far the most commonly used technique of their kind, it is a good communication "language".
- 8.3 For successful implementation, something approaching an act of faith is required - existing techniques must be abandoned, and project management must operate the tool itself; it should not be delegated to a specialist group.
- 8.4 It is highly desirable that the procedure to be adopted should be clearly established. An example of such a procedure is shown in Appendix B.
- 8.5 It is too soon to attempt to evaluate the effectiveness of PERT/TIME as applied to the U.K.3 Data Handling and Storage Sub-Systems. It is not suggested that the decisions made with the benefit of PERT would not have been made without it. However, they were made earlier and with more confidence in the outcome than would otherwise have been the case, and consequently were more effective.
- 8.6 While it is suggested that, for initial implementation, the basic rules of PERT/TIME should be adhered to, there is no doubt that further developments of the technique are desirable. In particular, techniques for cost control such as PERT/COST, and performance and reliability predictions such as PRISM are needed. Another area where further work is required is the research project, in which the outcome is in doubt. A technique along the lines of the decision-box network is needed here.
- 8.7 There is no doubt that the use of mathematical and cybernetic models in management is growing. Already models of large business enterprises and of stock markets are being constructed. Serious consideration is being given to models of a nation's entire economy to make it possible to damp out the alternate cycles of boom and recession. There may be a place in management in the future for the man who declines to consider the use of these techniques, but it is likely to be a lonely one.

9 ACKNOWLEDGEMENT

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Appendix A

AN OUTLINE OF PERT/TIME

A.1 The basic concept

PERT is an acronym for Programme Evaluation and Review Technique. The mechanism of its evaluation and review processes is a model of the programme. This model can be described as a logic diagram, flow chart or, more usually, a network. The technique includes procedures for analysing the model to establish time relationships between different parts of the programme and, in particular, to establish the sequence of activities which determines the total time taken for its completion. This is the "critical path". The technique takes account of uncertainties in time-scale, such as may be encountered in research and development projects.

If this evaluation leads to a viable plan for the project, the progress of the plan may be reviewed by the technique. This is achieved by "up-dating" the model, i.e. deleting past happenings and revising estimates, at regular intervals. From analysis of the up-dated model, it is possible both to review progress, and to predict areas of difficulty which will jeopardise the project objectives.

This Appendix describes the model and its procedures in chronological order, i.e. the order in which each would be met in implementing the technique.

A.2 Work breakdown structure

In all but the smallest projects, there will be several levels of management and supervision, and each will wish to operate to its own particular degree of detail. In the PERT model, this fact is acknowledged by breaking down the total work content of the project into tasks or "end-items" at different degrees of detail or "levels of indenture". The result is displayed as a family tree which, it is emphasised, is not necessarily related to the organisational family tree. An example of part of a work breakdown structure is given in Fig.A1.

It is essential that, before the work breakdown structure is established, the objectives of the project are defined, clearly and in detail. It will then be possible for the objectives of each task to be defined with equal clarity. The next operation is to construct a model - or network - for each part of the project at the lowest level of indenture.

A.3 The network

The network is a chart composed of two basic types of symbol; activities and events. These are defined as follows:

(a) Real activities

These are the time consuming events of the project. They represent real work and involve the employment of labour or resources, or both.

(b) Dummy activities

These are constraints which, while not themselves consuming time or resources, nevertheless govern the logic of the network and the achievement of objectives. An example of such dummy activity would be the undesirability of commencing the mechanical design of an electronic assembly until the circuit design was completed. In addition to such naturally-occurring dummies, artificial ones are sometimes introduced to make a network easier to decipher.

(c) An event is defined as a distinguishable, unambiguous point in time that coincides with the beginning and/or end of a specific task or activity. The achievement of any event will be governed by one or more activities, and it will initiate one or more further activities. The initial and objective (last) events in any network are obvious exceptions to this rule.

(d) Interface events are those which simultaneously form part of more than one network. An example of this would be decisions on interchangeability between parts of a large system.

It will be noticed that no mention is made of interface activities. The reasons for this will become clear as the description proceeds.

The universal symbol for an activity in both PERT and C.P.M. is the arrow; dummy activities are sometimes distinguished by dotted arrows. Various symbols are used for events, but they all involve an outlined area in which information can be written. It is usual to distinguish between interface and ordinary events. The symbols used in this paper are shown, with a key, in Fig.A2.

It will be found in practice that a valid model of any project, of which the eventual outcome is not in doubt, can be constructed by connecting activities and events together to form a network. This statement is subject to the proviso that the following rules are observed:

(i) Each activity must have, and may be defined by, predecessor and successor events. In the same way, each event must have one or more predecessor activities and one or more successor activities, except for the initial and objective events.

(ii) No activity may commence until its predecessor event has been achieved. Similarly, no event is achieved until all its predecessor activities have been completed.

These two rules govern the logic, or as some writers will have it, the topology of the network. Their importance cannot be too greatly stressed. Arising from them there are further procedural rules:

(iii) No event may be followed by a series of activities which lead back to the same event. This logical fault in a network is known as a "loop" and it is illustrated in Fig.4. Although the example shown may seem obvious, the formation of loops is by no means uncommon in complex networks.

(iv) It is not permissible to define two concurrent activities by reference to the same predecessor and successor events, even if this would be an accurate representation. The correct way of handling this situation is to introduce a third event and a dummy activity as shown in Fig.A4.

(v) A single event may not be used to initiate successor activities which are not dependent on all its predecessor activities. This rule is illustrated in Fig.A5; in this we assume that activity 4 is dependent on both activity 1 and activity 2, but activity 3 is dependent on activity 1 and independent of activity 2. In this situation Fig.A5(a) contravenes this independent activity rule, but A5(b) does not.

(vi) No real activity may be shown as more than one activity line in the network. One solution to the problem which arises from the situation shown in Fig.A6(a) is an optional rule which requires each real activity to have unique predecessor and successor events. The application of this is shown in Fig.A6(b), in which two dummy activities have been introduced. If the rule is applied throughout a network however, many unnecessary dummy activities are created. It is usually considered best, therefore, to confine the use of unique predecessor and/or successor events to those situations where rule (v) requires it, or where increased clarity of the network is obtained by its use.

Once the initial event is achieved, i.e. after the programme has started, rule 1 may still be observed by using an artificial event "NOW" as the single initial event. As time goes on, NOW succeeds each event as it is reached.

Networks may be event-oriented or activity-oriented. The difference lies in the way in which the activities are defined. In an activity-oriented network, the activities are defined by giving them a plain language description, and events are merely numbered conjunctions of activities to facilitate analysis, as in Fig.A3.

In an event-oriented network, the events are defined in plain language and numbered, and the activities are defined by quoting their predecessor and successor

events, as in Fig.A2. For management purposes, the event-oriented network is usually preferred, although at the working level, there is something to be said for using the activity-oriented network.

Both types of network may, with advantage, have the numbers for the events determined by Fulkerson's rule. The procedure is as follows:-

- (a) Number the initial or starting event "1".
- (b) Delete all the activity arrows emerging from the numbered events (i.e. "1" in the first iteration). This will generate at least one new initial event.
- (c) Number all the new initial events "2", "3" and so on, in any order.
- (d) Reiterate (b) and (c) until the objective event is reached.

The use of this rule will automatically detect logical inconsistencies in the network. It also makes manual analysis much easier than if the numbers were allocated in random fashion.

A.4 Time estimates

When the network is complete, a time estimate is made for each activity. When the time is known precisely in advance, this is stated as a number, usually in units of weeks, to one place of decimals. In a research and development programme such estimates are usually confined to dummy activities which are, by definition, time zero and, perhaps, procedural activities.

For all other activities, it is likely that some uncertainty about their duration will exist. This may be expressed statistically by considering the activity as a distributed variable. The range of each distribution is determined by optimistic and pessimistic estimates, and the mode (or peak of the probability distribution curve) by an estimate of the "most likely" duration. These estimates are given the symbols t_o , t_p and t_m respectively. An illustration of three possible probability curves is given in Fig.A7. It was decided by the originators of the technique that, of the known probability distributions, that which most nearly fitted the practical situation was the beta distribution. The mean or expected value of this distribution is that which divides the area under the probability curve into two equal portions. This is called the expected time - t_e . The probabilities of completing the activity earlier or later than t_e are equal - at 50 per cent.

The rigorous calculation of the value of t_e , and its standard deviation, would be tedious. However, a simplified arithmetical approach is used for PERT calculations. The expected time is defined by the formula:

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

and the standard deviation is defined as one sixth of the range. In practice, the variance (square of the standard deviation) is a more useful quantity since it may be added arithmetically along a path of many activities to give the variance of the expected time for the whole path. Thus, the variance is given by the formula:

$$\sigma^2 (t_e) = \left\{ \frac{t_p - t_o}{6} \right\}^2 .$$

It may be said that the proof of these formulae has been the subject of some controversy, but they may be regarded as empirical rules which have been applied successfully. Appendix C describes the PERT model of an activity in more detail.

It is assumed that the estimates will be made by the person who has most experience of the type of activity concerned. It is therefore necessary that each of these persons should be given a clear definition of each of the three estimates. Statistically speaking, the optimistic and pessimistic estimate can be defined as each having a 1 per cent probability of occurring on any one occasion and the most likely time is the mode of the distribution. However, this information is of little use to those unfamiliar with statistical methods. The following definitions are therefore suggested:

(i) Optimistic estimate t_o

The time that will be taken if "everything goes right" - i.e. if exceptionally good luck is experienced.

(ii) Most likely time t_m

The time which, in the opinion of the estimator, is the one which would occur most often in normal circumstances.

(iii) Pessimistic time t_p

The maximum time that the activity will take if "everything goes wrong". "Everything" in this case excludes "acts of God" - fire, flood, tempest, etc.

When time estimates have been made for each activity, analysis of the network may begin.

A.5 Analysis

The first step is to determine the expected time and variance for each activity. When this has been done, it is possible to calculate the cumulative

expected time T_E and the cumulative event variance $\sigma^2(T_E)$ for any event on the network by adding the individual expected times and variances of all the activities along the longest path to it from the initial event. The next process in the analysis is to perform this calculation for each event.

The cumulative expected time for the objective event will be the best estimate of the time for completing the entire project. This may, of course, be earlier or later than the scheduled, or required, time. The cumulative variance will give an indication of the probability of achieving this date.

The next step is to take the scheduled time, if there is one, or otherwise the expected time as a starting point, and to work backwards through the network to calculate the latest allowable time T_L for each event. This calculation is made in exactly the same way as the forward pass through the network; by adding together the expected times of the activities on the longest path from the objective event to the event concerned. A cumulative variance of T_L can also be computed if required.

The next step is to deduct the cumulative expected time from the latest allowable time, this gives the "slack" for each event, that is to say the amount of time by which the event may be delayed without affecting the scheduled date for the entire project. Slack may be positive, zero, or negative. If it is negative, the project may be expected to over-run by this amount. The path through the network with the smallest amount of slack is called the "critical path", it is this path that determines the overall timescale for the project, and it is only by shortening this path that the end date can be brought forward.

It will be seen that the arithmetic of this analysis is very simple. It can however become extremely tedious for large networks, and it is usual to use an electronic computer for these. For networks of up to one or two hundred events however, manual analysis is quite feasible and an example is given in Table A1, in which the network of Figure A2 is analysed. Table A1 is an activity oriented analysis, and it will be noticed that it has been produced from an event-oriented network. It is possible to produce event-oriented analyses, in greater or less detail for different levels of management. It is usual, however, to base these on the activity oriented analysis at the lowest level of indenture. Another useful permutation of these data is the slack path analysis shown in Table A2. This merely ranks the various paths through the network in order of increasing amounts of slack. It directs the manager's attention to those parts of the network which may need his attention.

It may assist the reader to assimilate the analysis technique if we consider the example (Fig. A2, Tables A1 and A2) in more detail. The network has been numbered in accordance with Fulkerson's rule, so we first list the activities in ascending order of their predecessor event numbers. It will be noted that the event descriptions have been chosen so that deduction of the correct activity description is easy. To ensure this, the "unique predecessor and successor event" rule has been observed where necessary.

When the activities are listed, we calculate the expected time t_e and variance $\sigma^2(t_e)$ for each in accordance with the formulae given in A4. Thus for activity 1 - 2:-

$$t_e = \frac{1.5 + 3 + 4 \times 2}{6} = 2.1 \text{ (to first place of decimals)}$$

and

$$\sigma^2(t_e) = \left\{ \frac{3.0 - 1.5}{6} \right\}^2 = 0.0625 .$$

for this purpose, 0.06 is near enough.

These simple calculations are repeated for each activity in turn.

The cumulative expected time T_E and cumulative event variance $\sigma^2(T_E)$ are then calculated for each (successor) event. Thus for event 2:-

$$T_E = 0 + 2.1 = 2.1$$

$$\sigma^2(T_E) = 0 + 0.06 = 0.06$$

and for event 3:-

$$T_E = 2.1 + 7.5 = 9.6$$

$$\sigma^2(T_E) = 0.06 + 1.36 = 1.42 .$$

The calculation is always made for the longest path through the network to the event concerned. Where alternative paths exist, e.g. event 12, (sometimes called a backward node), they are each examined in turn. In practice, this merely means that the expected time of each activity, for which it is the successor, are added to the cumulative expected time of the respective predecessor event:-

$$T_E (\text{Event } 10) + t_e (\text{Activity } 10 - 12) = 21.1 + 0 = 21.1$$

and

$$T_E (\text{Event } 8) + t_e (\text{Activity } 8 - 12) = 18.8 + 0 = 18.8 .$$

Thus the largest path to event 12 is through activity 10 - 12 and this is indicated by circling the appropriate T_L , thus 21.1. It is this value, and its associated variance, that determines the T_E and variance for event 12.

When this forward pass through the network is complete, a backward pass is made, using precisely the same technique, to establish the latest allowable time T_L for the completion of each activity. In practice, most people find this pass more difficult, presumably because they are working from right to left and from the bottom of the paper upwards. This difficulty tends to ease with experience. In our example, the scheduled time T_S is 42 weeks and this sets T_L for the objective event 20. Then:

$$T_L (\text{Event } 19) = T_L (\text{Event } 20) - t_e (\text{Activity } 19 - 20) = 42.0 - 1.6 = 40.4.$$

Where an event initiates more than one activity (forward node or burst point) the situation is analogous to that of a backward node on the forward pass, in that it is the longest path from the objective event which determines the T_L . Thus for event 2:-

$$T_L = T_L (\text{Event } 3) - t_e (\text{Activity } 2 - 3) = 11.9 - 7.5 = 4.4$$

$$T_L = T_L (\text{Event } 4) - t_e (\text{Activity } 2 - 4) = 28.2 - 5.2 = 23.0$$

so it is the path through Event 3 that determines the T_L (Event 2) and T_L (Event 3) is ringed to record this fact.

It is possible also to calculate $\sigma^2 T_L$, by adding the variances along the largest path from the objective event, if this parameter is thought by the manager to be useful.

The final stage of the analysis is the calculation of slack ($T_L - T_E$), thus:-

$$\text{Slack (Activity } 19 - 20) = 42.0 - 39.7 = 2.3 .$$

Obviously, if T_E is greater than T_L , the slack will be negative and this indicates that the project, if carried out according to the network and time estimates, has 50% probability of overrunning by this amount of time.

The slack path analysis, given in Table A2, is simply a matter of listing the events which relate to each successive value of slack in the analysis and checking to see that the list corresponds to an actual path through the network. The path with the least amount of slack (critical path) is listed first. If any subcritical path, taking into account its cumulative variance, approaches the critical path in length, this fact is noted.

It will be observed that the individual or department head responsible for each real activity is listed on the analysis. The value of this is obvious.

The amount of slack in the critical path and the variance of the cumulative expected time for the objective event will give an indication of the probability of achieving the scheduled date. A quantitative value for the probability of doing so can, however, be established. It is assumed that, where there are a substantial number of activities on the critical path, the distribution of the possible total elapsed times for the path will approach the normal (symmetrical) distribution. This assumption is based on the central limit theorem, which holds when there are a number of activities (e.g. more than ten) and their individual distributions are random. The probability of meeting any scheduled date is found by expressing the difference between the scheduled time T_S and the overall expected time T_E in terms of the standard deviation:

$$\frac{T_S - T_E}{\sigma} .$$

The result is compared with a set of probability distribution tables to give a figure for the probability of achieving T_S . As an example, we may take the figures from our sample analysis in Table A1:

$$\frac{42.0 - 39.7}{2.23} = + 1.03$$

$$\text{Probability of achieving } T_S = 84.8\% .$$

It should be borne in mind that, both the total expected time and the variance and probability figures are based on the assumption that the critical path is "enough longer" than the parallel paths for the latter not to affect the result.

If there are parallel paths with slack which is only slightly smaller than that in the critical path then these calculations will give optimistic answers. This optimism is in addition to the slight optimistic bias which has been found to be inherent in the technique. Methods have been suggested for avoiding these errors, but these are outside the scope of this brief summary. The manager can, of course, make his own subjective corrections.

A.6 Re-scheduling

The conclusion reached from the results of the analysis of the network may well be that it does not represent an acceptable plan for carrying out the project. In all probability a shortening of the overall timescale will be required. To achieve this, it is necessary to attack the activities on the critical path. There are three basic rules for this replanning:

(i) Review all activities to determine whether each is absolutely essential to the programme. Activities which are desirable rather than essential may be deleted.

(ii) Employ additional resources to reduce activity times on the critical path. It may be possible to transfer resources from paths with large positive slack, otherwise total resources may have to be increased. This may be done by overtime working, the use of better people, or the recruitment of additional personnel. Another possible alternative is to subcontract some activities.

(iii) Two or more activities which, in the original network, were planned to occur in series, may be re-planned to take place in parallel. This course may involve some technical risk, and will certainly involve an increase in resources in the same way as rule 2.

In a project of any size, there may be a number of alternative methods of achieving the desired result. Each of these alternatives may be tried in the PERT model, and their effects determined to enable the best solution to be found.

When the critical path has been reduced to the required length, and the analysis has been run through again, it is highly likely that a new critical path will have been created. This must be treated in the same way, until all the areas of criticality have been dealt with. In considering the final analysis, two points must be borne in mind. The creation of a number of paths with nearly the same criticality leads to an optimistic prediction, as stated above. Secondly, the peak demand for resources should not exceed those likely to be available. In any case, excessive peaks in demand for labour lead to inefficient working.

The perfected PERT model thus becomes the basis for planning, i.e. allocating materials, labour and facilities to the project. It may also be used as a channel of communication to convey planning information backwards and forwards between management and operations. This leads in to the second function of PERT - review.

A.7 Progress reviews

Once the project has started, the manager will wish to review progress at regular intervals. When PERT is used, immediately before each progress report is due, the network is revised or "up-dated" to bring it into line with the current situation. All events that have been achieved are deleted and replaced with the artificial event NOW. Any changes in the logic of a network that have been brought about by the changing situation are made, and revised estimates made for activities if necessary. The analysis is run through again, and a slack path analysis prepared. For a large project, less detailed reports for higher levels of management may be made. The result of this work may indicate that re-planning of parts of the project is needed to ensure that the objectives are reached. The value of PERT is that it directs management's attention to these areas, and enables them to try possible solutions in the model as in the initial re-scheduling phase. The predictive capability of the technique enables corrective action to be taken before it is too late.

A.8 A guide to further reading

This outline is a compression of a very large amount of published information about PERT, in which much has been intentionally omitted. In addition to omissions intentional and unintentional, it is probable that data compression on this scale has introduced some distortion. It does however include what, from the writer's own experience, appears to be essential.

Anyone who wishes to study the subject further may find this list of recommended reading useful:

- (a) The work of developing PERT, and the foundations upon which it was based, are described by its originators in reference 10. This has since been supplemented by an Addendum²⁵.
- (b) Perhaps the most practical of the manuals of PERT is that of Miller⁸.
- (c) More general treatments of network techniques are references 3 and 22.
- (d) Reference 11 contains material for training courses on PERT and C.P.M.
- (e) Most of the statistical arguments about PERT have been ventilated in the columns of Operations Research; the Journal of the Operations Research Society

of America, from 1959 onwards. Further material of this kind appears in Management Science; the Journal of the Institute of Management Sciences, Baltimore, Maryland.

(f) More empirical arguments have been conducted in the columns of the now defunct journal: Aerospace Management.

(g) A "programmed teaching" book has recently been published.

Paradoxically, only a person untrained in the subject can express an objective view of a book of this kind. However, good reports have been received of:- "A programmed introduction to PERT", issued by the I.T. & T. Federal Electric Corporation, and published by Wiley.

Table A1 - ACTIVITY ANALYSIS

Predecessor event	Successor event	Activity	t_o	t_m	t_p	t_e	T_E	
1	2	Write design speco.	1.5	2	3	2.1	2.1	(1)
2	3	Electrical design	5	7	12	7.5	9.6	24
2	4	Write test spec.	3	5	8	5.2	7.3	1
3	5	Mechanical design	2	3	5	3.2	12.8	(13)
3	6	Order components	2	2	2	2	11.6	28.
4	11	Dummy	0	0	0	0	7.3	33.
4	17	Dummy	0	0	0	0	7.3	(21)
5	7	Order material	2	2	2	2	14.8	27.
5	8	Prepare drawings	5	6	7	6	18.8	28.
6	9	Obtain components	10	14	20	14.3	25.9	27.7
7	10	Obtain materials	4	6	10	6.3	21.1	(25.9)
8	12	Dummy	0	0	0	0	18.8	(21.1)
9	11	Dummy	0	0	0	0	(25.9)	27.7
9	15	Dummy	0	0	0	0	(25.9)	(28.2)
10	12	Dummy	0	0	0	0	(21.1)	30.7
11	14	Reliability assessment	10	12	15	12.2	38.1	27.7
12	13	Make chassis	2	3	4	3	24.1	40.4
13	15	Dummy	0	0	0	0	24.1	30.7
14	19	Dummy	0	0	0	0	(38.1)	30.
15	16	Prototype assembly	1.5	2.5	3	2.4	28.3	46
16	17	Dummy	0	0	0	0	(28.3)	33.
17	18	Prototype tests	5	7	11	7.3	35.6	33.
18	19	Dummy	0	0	0	0	35.6	40.
19	20	Design review	1	1.5	2.5	1.6	39.7	42.

Table A2 - SLACK PATH

Weeks of slack	2.3	4.8	6
Event numbers	1 2 3 6 9 11 14 19 20	9 15 16 17 18 19	
Remarks	Critical path $\sigma(T_E) = 2.23$	Sub-critical but $\sigma(T_E) = 1.96$	Sub.

ELECTRONIC DECUSTICATOR

T_E	T_L	Slack	activity variance $\sigma^2 (t_E)$	event variance $\sigma^2 (T_E)$	Remarks	Activity responsibility
2.1	4.4	2.3	0.06	0.06	For an explanation of the circled numbers, see text-page 41	Manager
9.6	(11.9)	2.3	1.36	1.42		Engineering
7.3	28.2	20.9	0.69	0.75		Eng.
12.8	19.4	6.6	0.25	1.67		D.O.
11.6	(13.9)	2.3	0	1.42		Stores
7.3	28.2	20.9	0	0.75		
7.3	33.1	25.8	0	0.75		
14.8	(21.4)	6.6	0	1.67		Stores
18.8	27.7	8.9	0.11	1.88		D.O.
25.9	28.2	2.3	2.79	4.21		Stores
21.1	27.7	6.6	1.00	2.67		Stores
18.8	27.7	8.9	0	1.88		
(25.9)	(28.2)	2.3	0	4.21		
(25.9)	30.7	4.8	0	4.21		
(21.1)	27.7	6.6	0	2.67		
38.1	40.4	2.3	0.69	4.90		Eng.
24.1	30.7	6.6	0.11	2.78		Shops
24.1	30.7	6.6	0	2.78		
(38.1)	40.4	2.3	0	4.90		
(28.3)	33.1	4.8	0.06	2.84		Model shop
(28.3)	33.1	4.8	0	2.84		
35.6	40.4	4.8	1.00	3.84		Eng.
35.6	40.4	4.8	0	3.84		
39.7	42.0	2.3	0.06	4.96		Manager
					$\sigma (T_E) = 2.23$ Scheduled time: 42 weeks Objective event: Prototype approval	

- SLACK PATH ANALYSIS

4.8	6.6	8.9	20.9
9	3	5	2
5	5	8	4
6	7	12	11
7	10		
3	12		
	13		
	15		
critical	Sub-critical	Sub-critical	Sub-critical
1.96			

I

Appendix B

PROGRAMME EVALUATION AND REVIEW TECHNIQUE -
CONTRACTUAL REQUIREMENTS

B.1 General

All development and manufacturing work to meet the attached design aim will be planned and progressed by means of the Programme Evaluation and Review Technique (PERT/TIME). The following clauses specify the procedure to which all tenderers and contractors are expected to adhere.

B.2 Procedure

All tenders and contractual proposals are to be accompanied by PERT/TIME network(s) and a preliminary analysis report. On receipt of contract or an I.T.P., the contractor is required to review his preliminary networks in conjunction with the Design Authority to arrive at an agreed programme (the "initial" report). He may also be required to enter into liaison with other contractors and to adjust his interface events to conform to an overall plan.

The contractor is also to bring his networks up to date and to produce a Progress Analysis Report at intervals to coincide with the technical reporting interval (normally one calendar month). The effective date of the Progress Analysis Report shall be not more than one working week before the Progress Meeting at which it will be discussed, and it must be in the hands of the Design Authority not less than two working days before the meeting.

The contractor may use any version or variant of PERT/TIME which is compatible with the detailed requirements set out below. The Design Authority will assist contractors who are not familiar with the technique, but cannot undertake training of personnel.

B.3 Networks

B.3.1 Events

Networks are to be event oriented. That is, each event shall be shown by a symbol containing a number and an abbreviated description of the event. Activities will not be numbered or labelled. Two different event symbols are required, one for events which have an interface with, or form part of, other networks, and one for all other events. Preferred symbols are shown at Fig.B1, but others may be used providing they are clear and unambiguous.

The milestones in the attached list are to be incorporated by the contractor, unchanged, as events in the appropriate networks.

No network, at any level of indenture, is to contain more than one hundred and fifty events. The preferred maximum is one hundred events. Where, because of this rule, or for any other reason, the contractor uses more than one network to cover the work under the contract, he is required to produce an integrated network meeting the same rules at a higher level of indenture which does cover the entire project.

As specified in the PERT literature, each network will have one (only) initial event and one (only) objective event. These will normally be the specified milestones. As the network is amended to conform with progress through the project, the initial event will be the artificial event "NOW"; that is, the last reporting date.

Event numbers will be allocated in accordance with Fulkerson's rule. This rule is stated as follows:

- (a) Number the initial or starting event "1".
- (b) Delete all the activity arrows emerging from the numbered events (i.e. "1" in the first iteration). This will generate at least one new initial event.
- (c) Number all the new initial events "2", "3" and so on, in any order.
- (d) Reiterate (b) and (c) until the objective event is reached.

It is permissible to use partial numbering, i.e. to omit one or two integers in each decade in the initial numbering process, so that they may be used for later additions. If an allocation of event numbers is made for the contractor's use, it will be given after the list of milestones.

B.3.2 Activities

Two kinds of activities will be shown on the network: real or time-consuming activities, and dummy activities. They are to be shown by solid and dotted lines respectively. In each case the line will be terminated by an arrow pointing to the successor event.

The rule which specifies unique predecessor and successor events for each activity need not be followed, because this produces an excessive number of dummy activities. Its use should be confined to situations where it is necessary in the interests of clarity.

The degree of detail required in the network is difficult to define. It is, in part, governed by the limitation on the number of events. It is also required that the content of activities shall be chosen such that the longest of the expected times t_e shall approximate to the reporting period - four weeks. Each activity must fall within the responsibility of one individual supervisor within the contractor's organisation. It is important that activities shall not be defined in such a way that they involve the work of two or more departments.

Three time estimates will be made for each activity. They are to be the optimistic t_o , most likely t_m and pessimistic t_p times of the basic PERT. The contractor will be required to calculate the expected time t_e and variance for each activity. Where activities on the contractor's network are the responsibility of the Design Authority or another contractor, he must give adequate notice to the party concerned that three time estimates are required. This system of estimating is to be used throughout the project, except for dummy activities and for activities where a precise time is known in advance.

B.4 Analysis

These requirements have been drawn up in such a way that manual analysis is feasible. There is, of course, no objection to the use of an electronic computer, but the information specified must always be provided. If the analysis is to be done manually, a calculating aid known as the PERT-O-GRAPH II Critical Path Computer will be found useful. This inexpensive device is obtainable from James Halcombe Associates at Beverley Hills, California. The accuracy obtainable by its careful use is acceptable.

Two Analysis Reports are required, and their preferred forms are shown in Table 1 and Table 2. Manual analysers will have no difficulty in adhering to the precise form of these tables. A computer print-out in different form is acceptable, providing that it yields the same information, omissions can be added manually if necessary. Contractors who wish to use an existing computer programme should consult the Design Authority to arrive at an acceptable compromise. It will be observed that the basic analysis is activity oriented, this may readily be derived from the event oriented network if the events are suitably labelled.

The time estimates and the results of the analysis should not be written on to the master copy of the network, because this will make later amendments more difficult. "Squared up" networks, which are arranged for events to be aligned with a horizontal timescale, are not required.

The latest allowable time T_L for the objective event is to be the scheduled time, if one is stated in the contractual requirement, or the total estimated time T_E for that event from the contractor's own analysis.

The basis of the analysis is to be the original PERT formulae given in Table 1. It is appreciated that the use of these formulae can lead to minor errors. If the contractor wishes to use Monte Carlo or other techniques to minimise these errors, he may do so, but the basic analysis should still be performed and shown in the reports.

B.5 Reports

The preliminary and initial reports will consist of a print of the network(s), the network analysis and the slack path analysis. Progress reports will contain the network analysis and the slack path analysis referred to the reporting date. A list of events achieved since the previous report is also required. These can then be transferred to the network to give the position of "NOW". If any modifications have been made to the network in addition to the change in position of "NOW", a copy of the modified network will also be required.

Table 1

NETWORK ANALYSIS

Predecessor event	Successor event	Activity	Optimistic estimate t_o	Most likely estimate t_m	Pessimistic estimate t_p	Expected time t_e	Cumulative expected time T_E	Latest allowable time T_L	Slack $T_L - T_E$	Activity variance $\sigma^2(t_e)$	Event variance (cumulative) $\sum^2(t_e)$	Activity responsibility
Number	Number	Abbreviated description	Weeks	Weeks	Weeks	$t_o + 4t_m + t_p$ 6 Weeks	Weeks from start	Weeks from start	$T_L - T_E$	$\sigma^2(t_e) = \left\{ \frac{t_p - t_o}{6} \right\}^2$		Contractors department etc.

Table 2

SLACK PATH ANALYSIS

0	Weeks of slack								
Events (Critical path)	Events	Events	Events	Etc.					

Appendix C

THE MODEL OF AN ACTIVITY

The model of the duration of a real activity in a network can be deterministic (as in the single time estimate of C.P.M.) or stochastic (as in PERT) in that it can be expressed as a random variable with a specific distribution.

MacCrimmon and Ryavec¹⁴ have suggested that three properties might be postulated for such distribution:

(a) Unimodality

The curve of probability versus time should have a single mode (or peak) which is in PERT terminology, the "most likely" time for completion. One can imagine some special cases in research and development where this might not be true, but the vast majority of activity distributions may be expected to be unimodal.

(b) Continuity

The distribution and hence its curve should be continuous. Again, it is possible to postulate special cases with discrete distributions, but a continuous distribution will be a good approximation.

(c) Two non-negative abscissa intercepts

This property merely reflects the certainty that an activity cannot be completed in negative time.

To these, the present writer would add a fourth:

(d) The possibility of asymmetry

The majority of distribution of time estimates are skewed, and one feels intuitively that this reflects the practical situation. In any case, the use of a symmetrical distribution, e.g. the normal distribution, would be to impose a restriction which it would be difficult to justify.

A distribution which possesses these four properties is the beta, and this was chosen by the originators of PERT^{10,25} to be the model for the duration of an activity. The probability density function of the beta distribution is:

$$f(t) = K(t - a)^{\alpha} \cdot (b - t)^{\beta} \quad (C1)$$

Some examples are given in Fig.A7. It will be noted that the first is symmetrical, the second is skewed to the left and the third is skewed to the right.

In this expression, the two end points of the distribution A and B are specified by the optimistic and pessimistic time estimates t_o and t_p . The exponents α and β are determined by the assumption that the standard deviation of the distribution is one-sixth of its range ($b - a$) and the estimate of the value of the mode (m), or most likely time t_m .

In the PERT algorithm, the two parameters used are the mean (or expected time t_e) and variance (square of the standard deviation) although these do not define a unique distribution. To obtain the mean precisely it is necessary to solve a cubic equation but, to simplify the application of the model a linear approximation was made:

$$\text{Mean} = \frac{(a + 4m + b)}{6} \quad (C2)$$

or in PERT terminology

$$t_e = \frac{(t_o + 4t_m + t_p)}{6} \quad (C3)$$

and the variance

$$\sigma^2(t_e) = \left[\frac{t_p - t_o}{6} \right]^2 \quad (C4)$$

This approximation and assumption are not rigorously valid for all beta distributions. Battersby²⁶, who deals with the PERT statistics at some length, shows that they are true when the parameters of the beta distribution have certain specific values, and that they are not grossly in error over quite a wide range of values. There is, however, a tendency to underestimate the variance when using equation C4.

Various alternative methods of computing activity and network parameters have been suggested by Fulkerson¹³, MacCrimmon and Ryavec¹⁴, Van Slyke¹⁷ and others. Notwithstanding the mathematical merit of these alternatives, managers have, in practice, continued to use the original PERT method almost exclusively. It is simple, it has been shown to work in practice and its acknowledged deficiencies can be minimised by intelligent implementation.

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TASK

SPA/P/1843

Fig.1&2

A

B

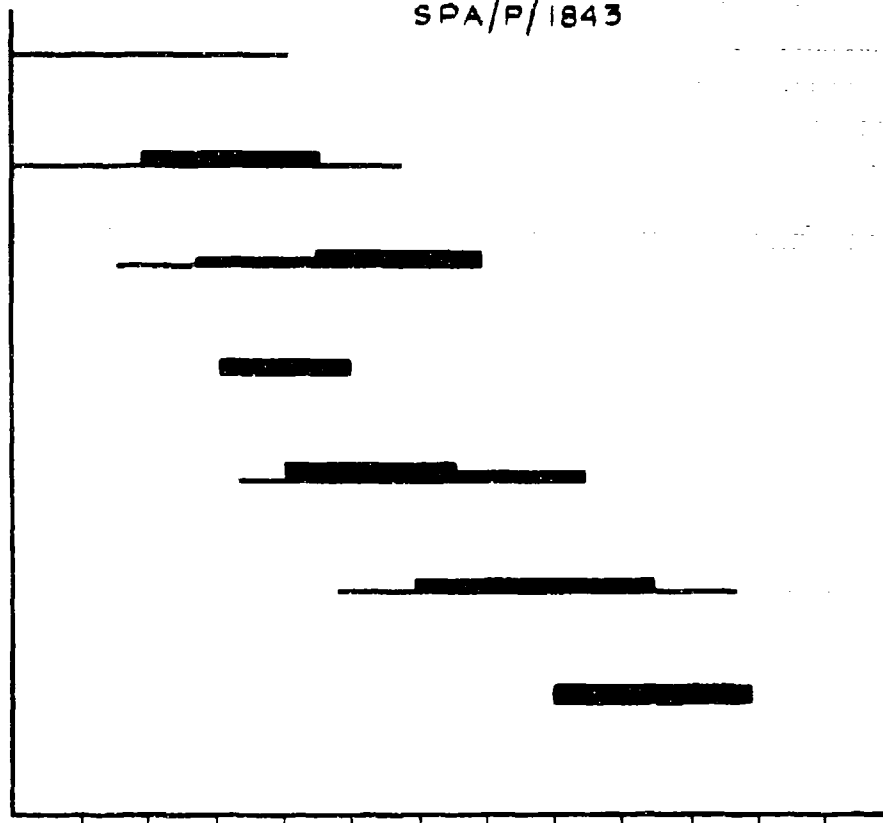
C

D

E

F

G



TIME

FIG.1 SIMPLE BAR CHART.

PART NUMBER	QUANTITY	WEEKS			
		1	2	3	4
4006-1	150	[Bar spanning weeks 1 to 2]			
4006-2	50	[Bar spanning weeks 1 to 1.5]			
4006-7	100	[Bar spanning weeks 1 to 2.5]			
4006-11	250	[Bar spanning weeks 2 to 2.5]			
4006-12	150	[Bar spanning weeks 2.5 to 4]			

TIME
NOW

FIG. 2 GANTT CHART.

Fig.3&4

SPA/P/1844

TASK

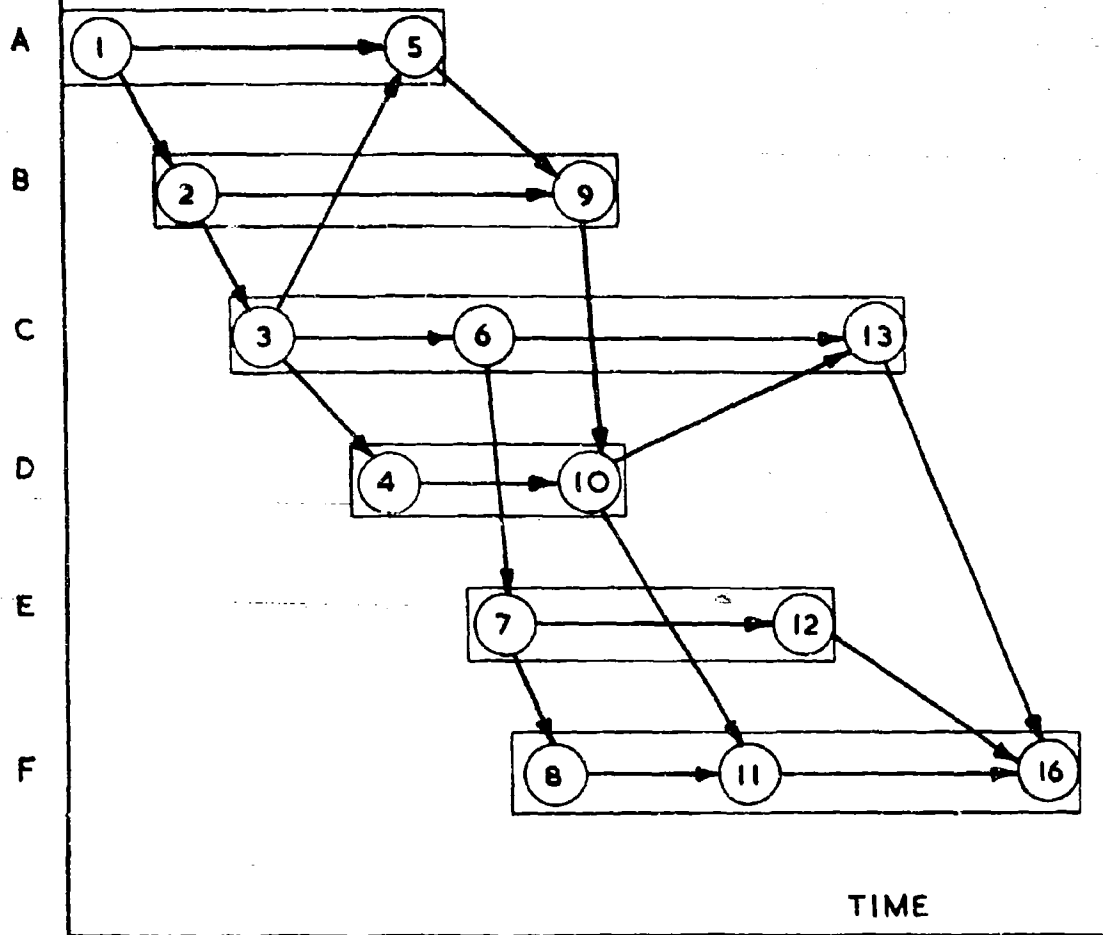


FIG. 3 DEVELOPMENT OF THE NETWORK FROM THE MILESTONE BAR CHART

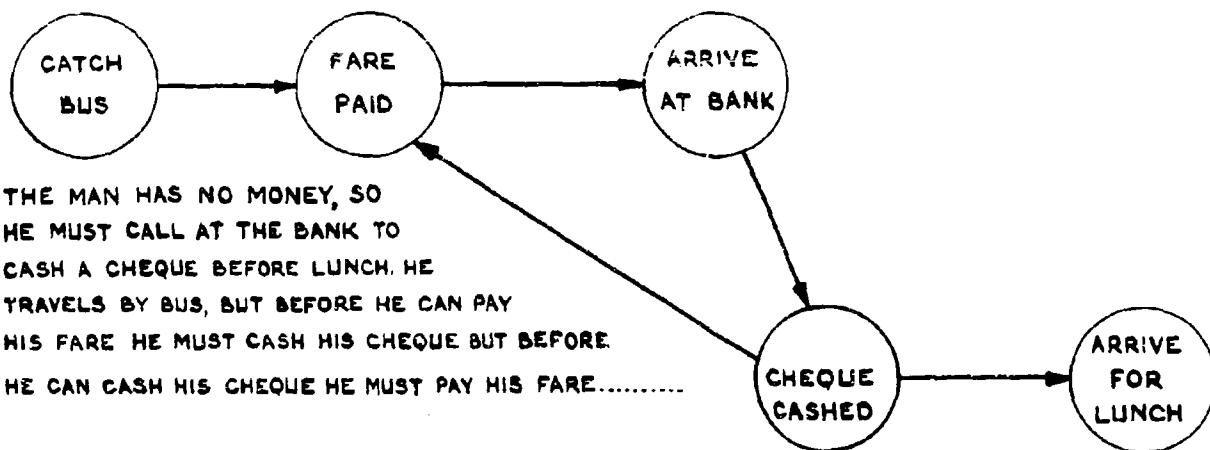


FIG. 4 A NETWORK "LOOP"—THE "GOOD LORD I HAVE NO MONEY" PROJECT

UK3 PERT/TIME

NOW TO FLIGHT HARDWARE
DATA HANDLING

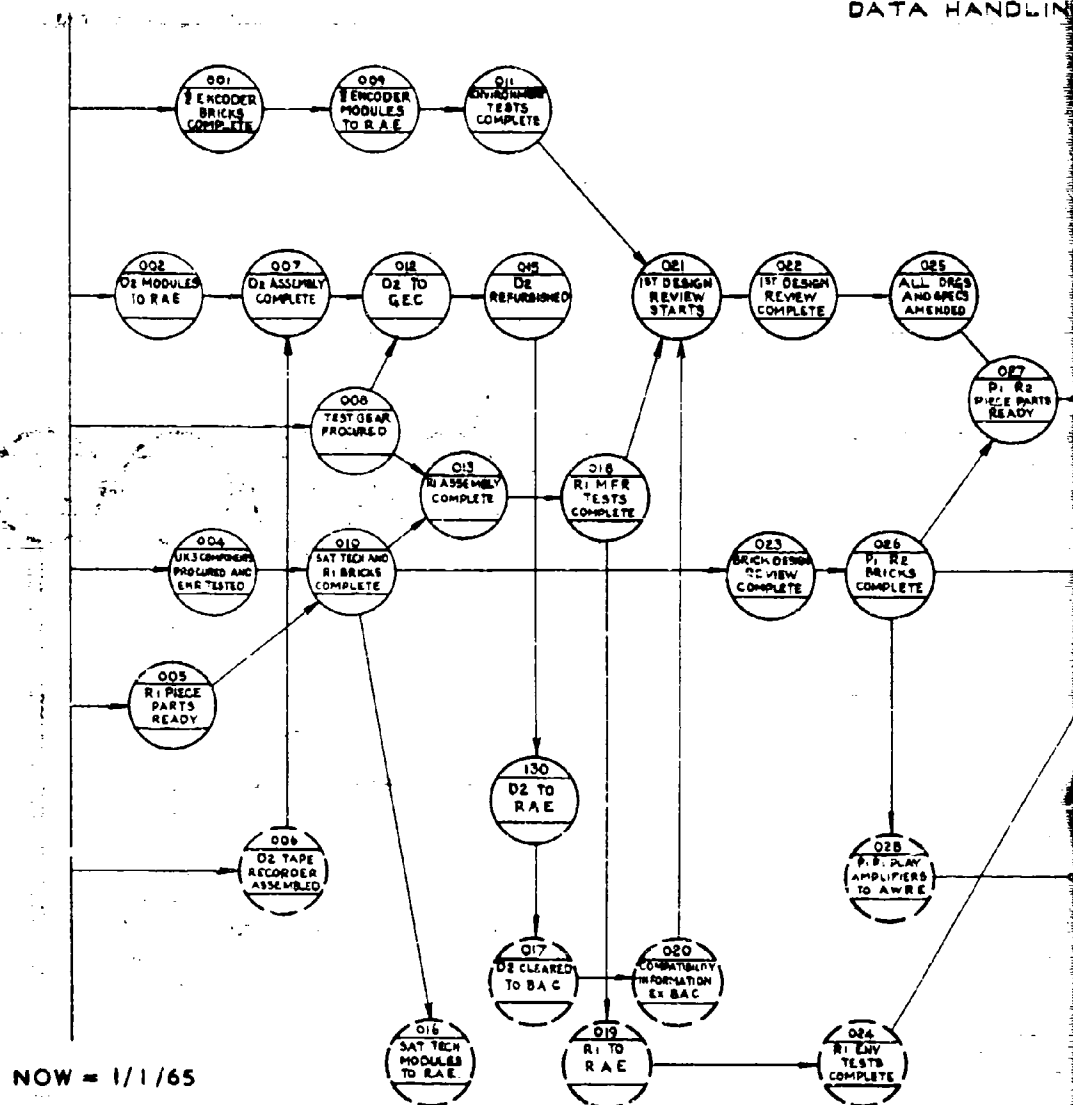
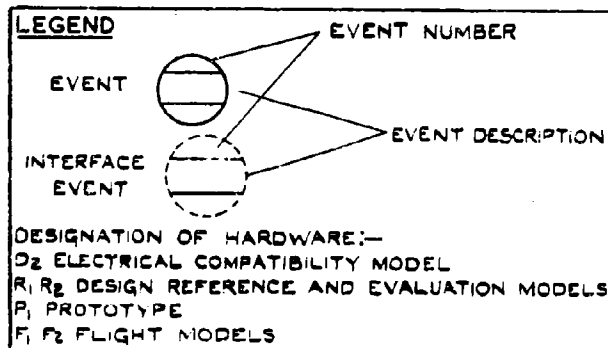
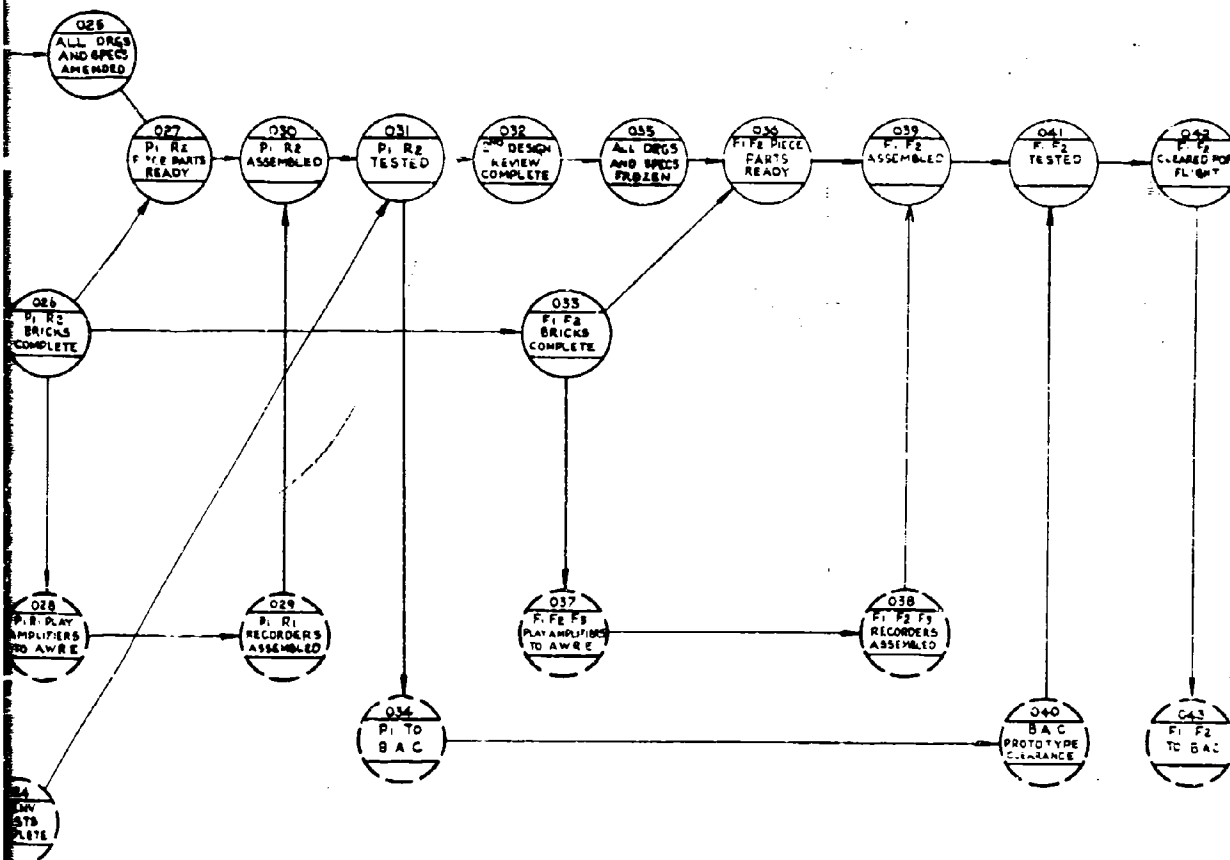


FIG. 5 UK3 DATA HANDLING MANAGEMENT

3 PERT/TIME NETWORK

FLIGHT HARDWARE DELIVERED
DATA HANDLING SYSTEM



MANAGEMENT LEVEL NETWORK AS AT 1/1/65

2

UK3 PERT:TIME

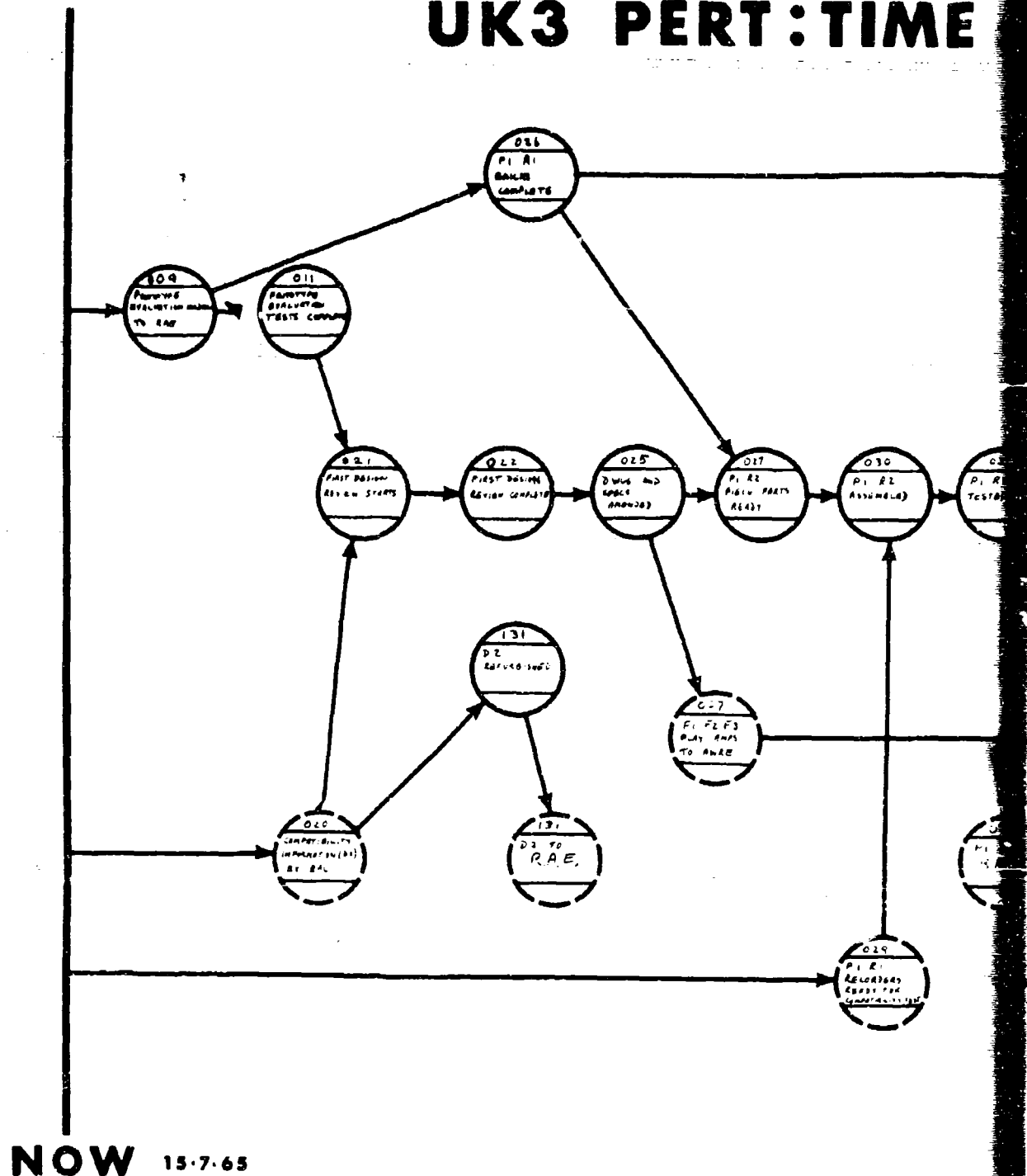


FIG. 6 UK3 DATA HANDLING MANAGEMENT LEVEL

[illegible]

2

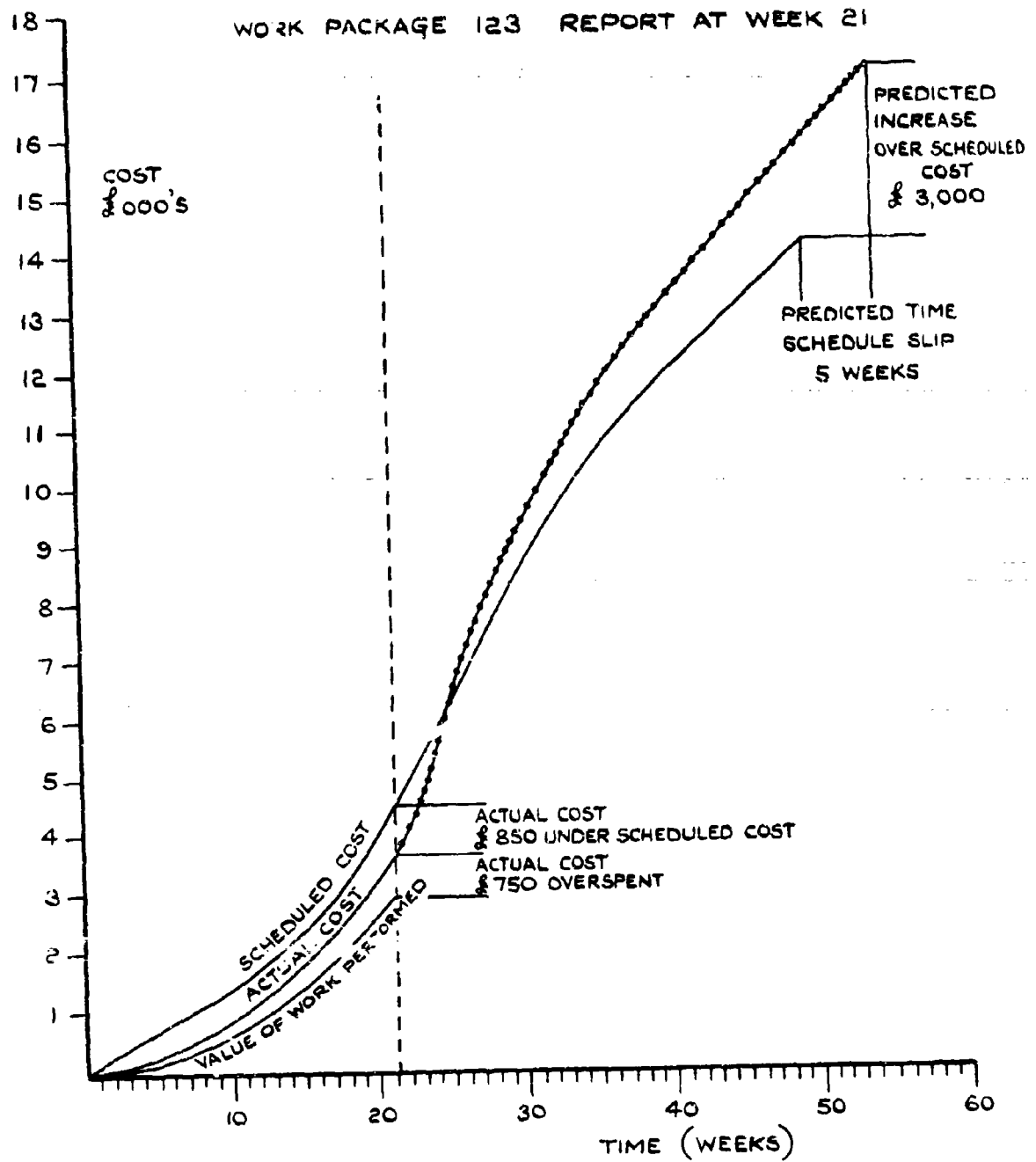


FIG.7 PERT/COST COST OF WORK REPORT

LEVELS OF INDENTURE

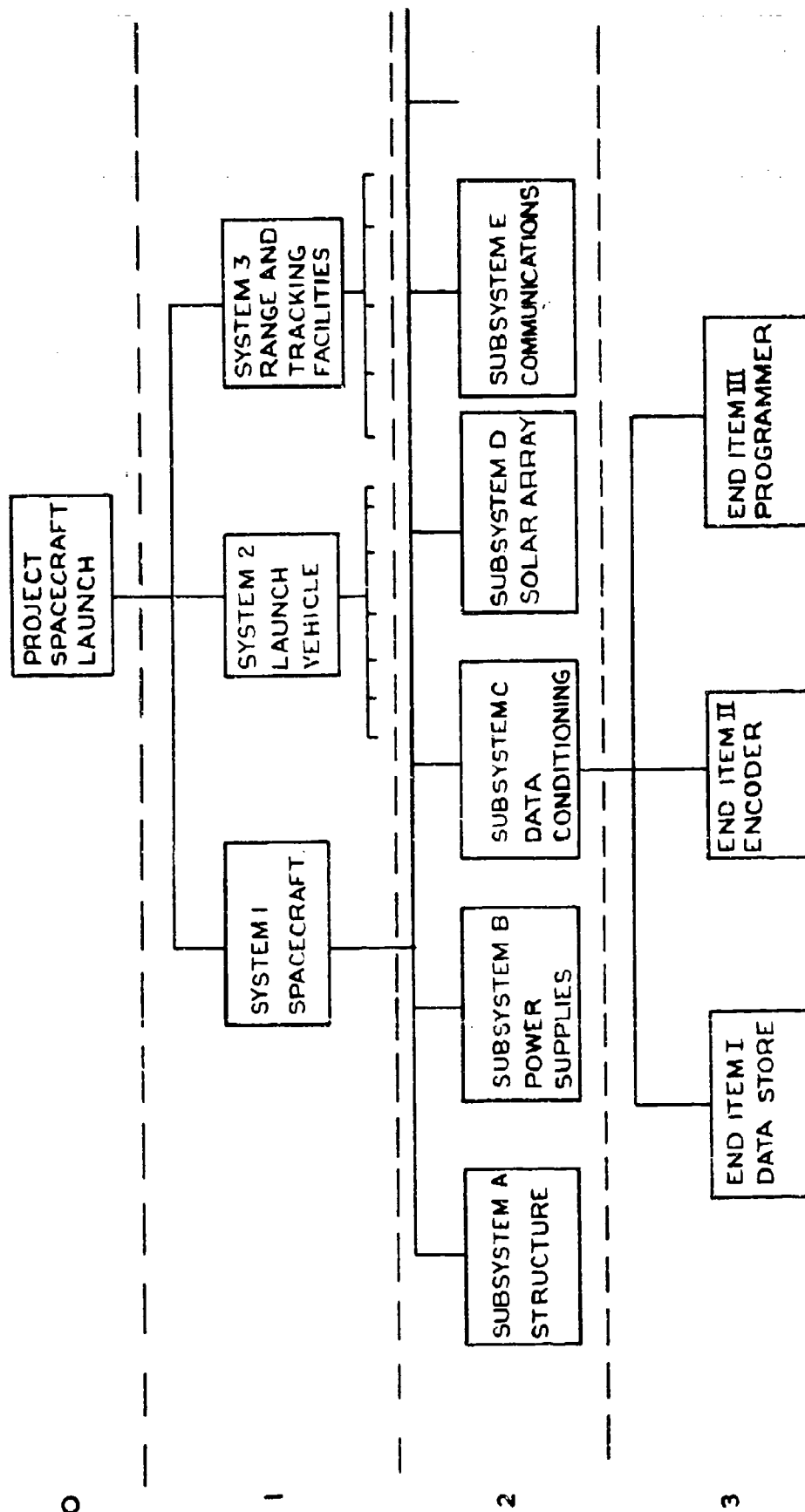


FIG. A1 PERT WORK BREAKDOWN STRUCTURE

Fig. A2

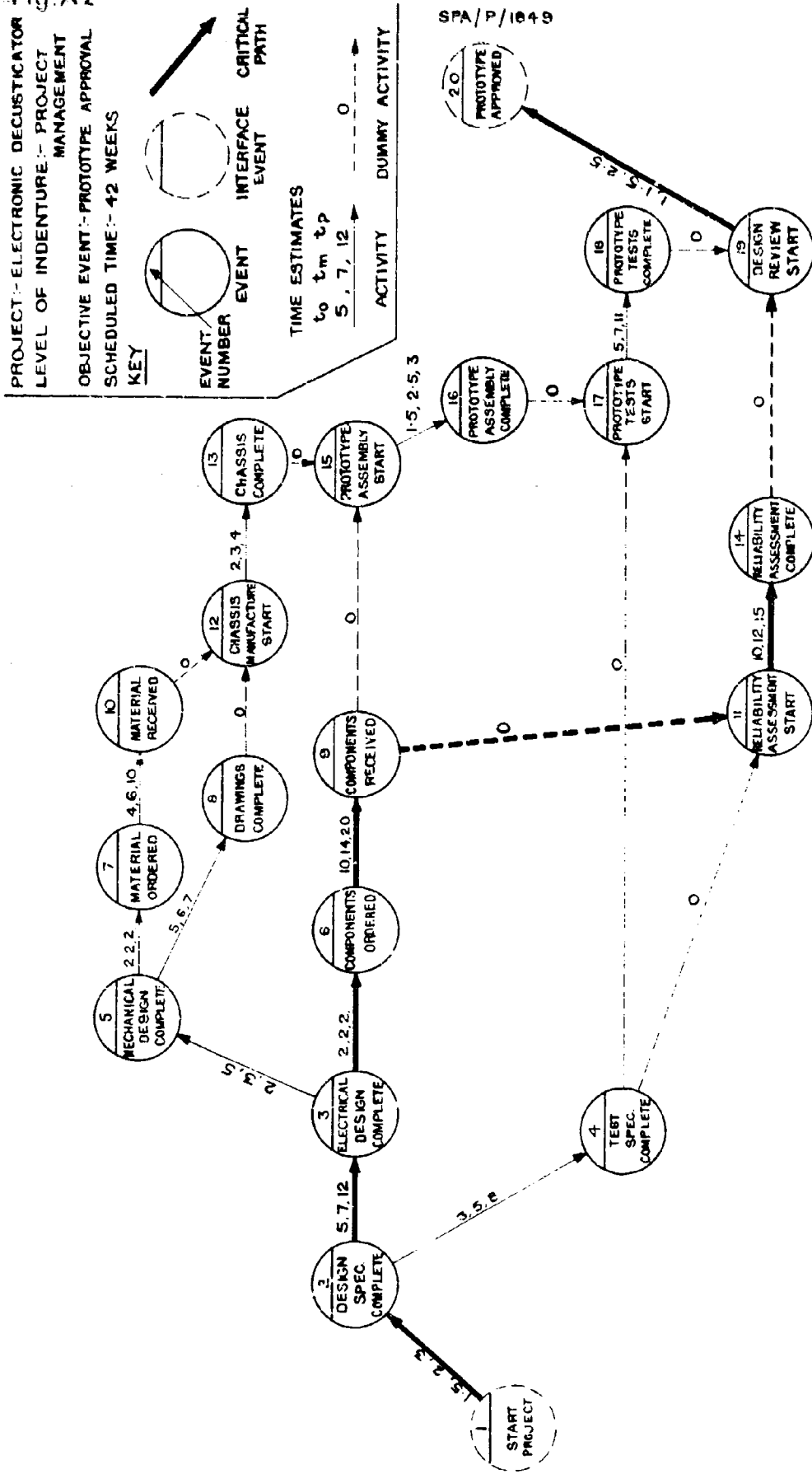
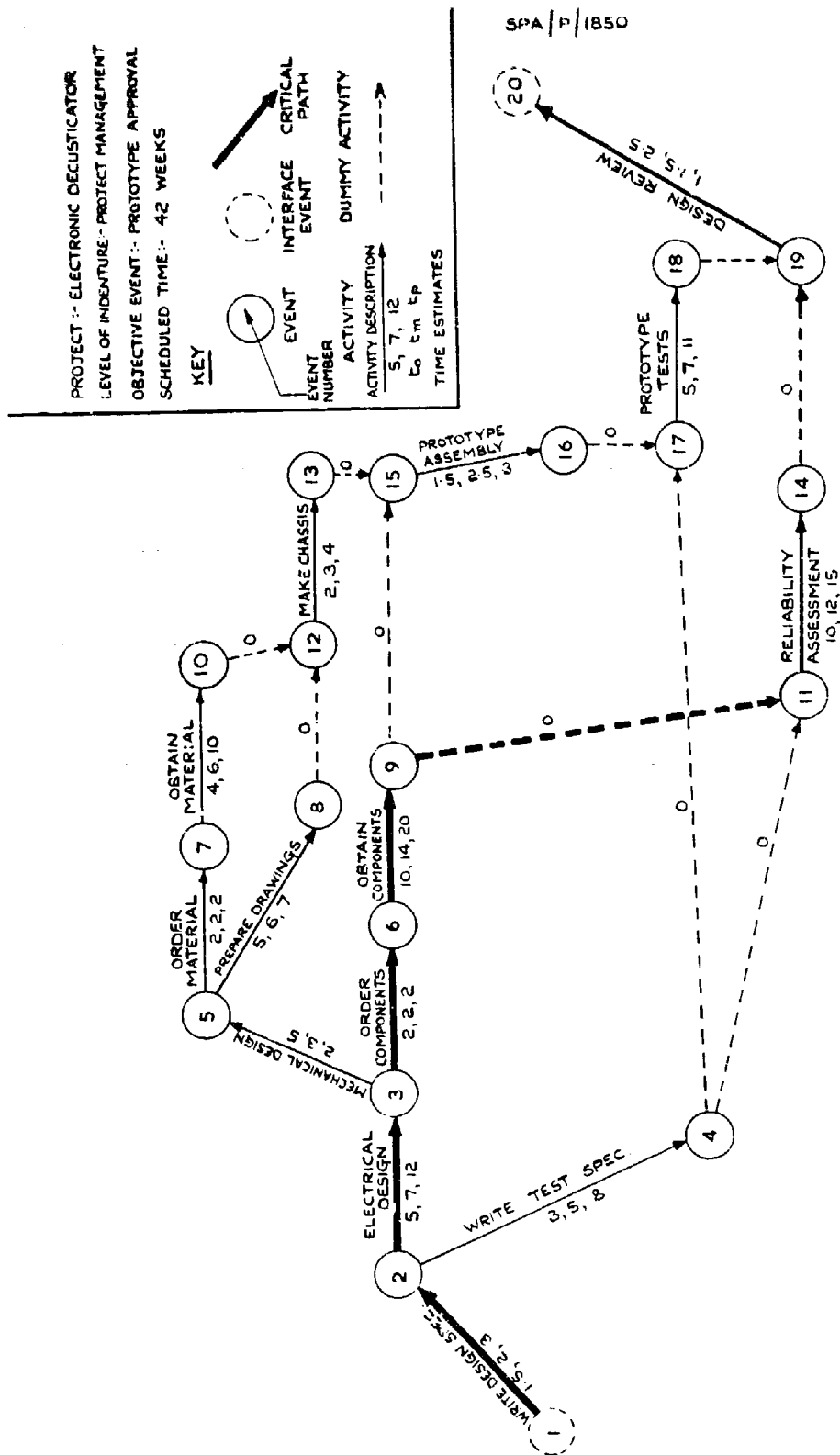


FIG. A2 EXAMPLE OF PERT NETWORK - EVENT ORIENTED



SPA / P / 1850

Fig A3

FIG. A3 EXAMPLE OF PERT NETWORK - ACTIVITY ORIENTED

Fig.A 4&5

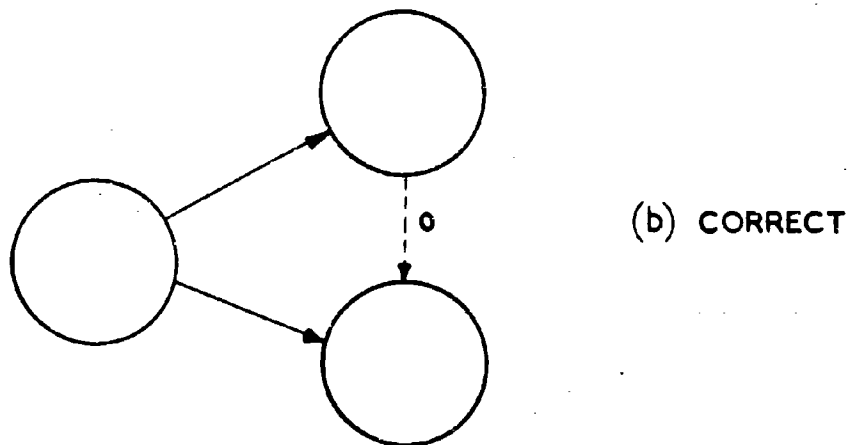
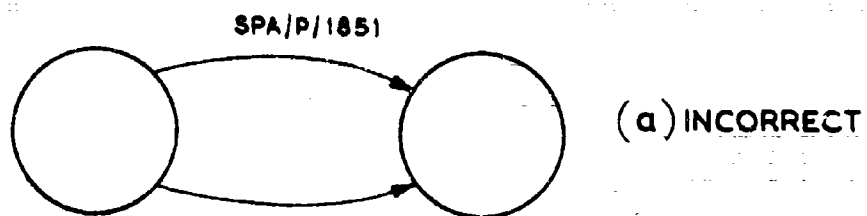


FIG. A4 CONCURRENT ACTIVITIES RULE

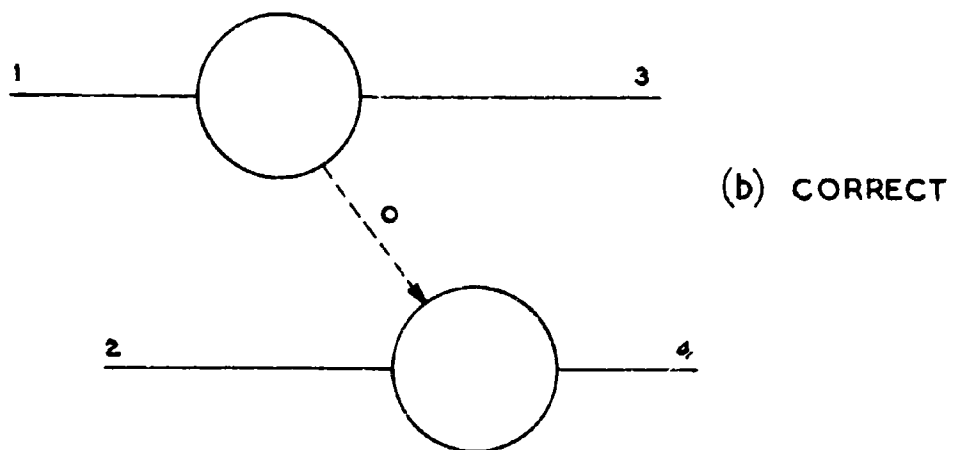
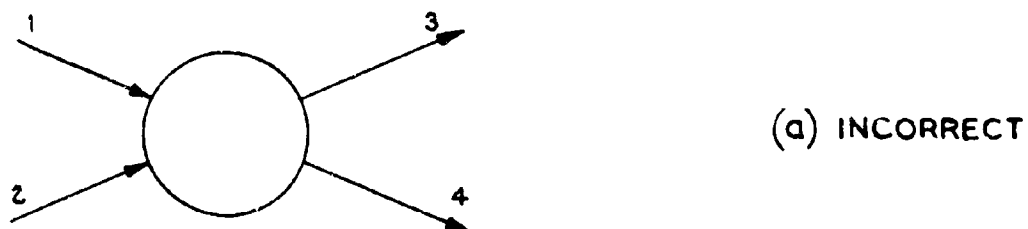
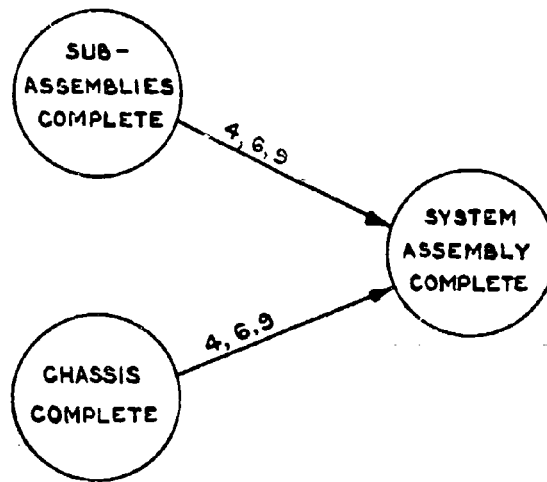
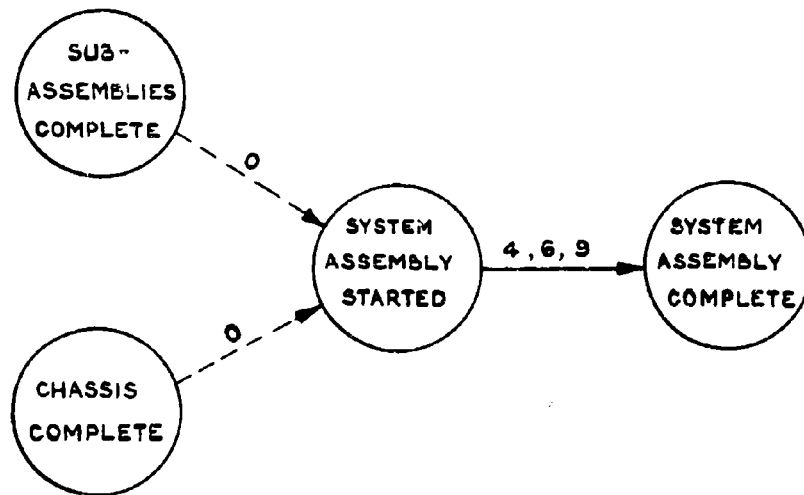


FIG. A5 INDEPENDENT ACTIVITY RULE



(a) INCORRECT



(b) CORRECT

FIG. A6 UNIQUE PREDECESSOR
AND SUCCESSOR EVENT RULE