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NONLINEAR EQUATIONS FOR CIRCUITS CONTAINING EXPLODING WIRES

by

F. D. Bennett

September 1965

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ABERDEEN PROVING GROUND, MARYLAND

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F. D. Bennett

Exterior Ballistics Laboratory

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NONLINEAR EQUATIONS FOR CIRCUITS CONTAINING EXPLODING WIRES

ABSTRACT

A set of nonlinear, ordinary differential equations applicable to R-L-C circuits with exploding wires is developed. From these an exact first integral, or conservation law, is obtained and approximate solutions of the set are found by the Picard method. The ranges of validity of these approximate solutions are examined and criteria developed for scaling exploding wire experiments so that the theoretical expressions will be applicable. Calculations of time-to-melt are made and compared with experimental values. Agreement is good enough to justify confidence in the approximations used. The major regimes of circuit behavior are delineated and discussed.

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INTRODUCTION

The variable resistance introduced by an exploding wire into the R-L-C circuit of which it is a part produces a nonlinearity which prevents solution of the circuit equations in analytical form. In a previous note[1*] I have given an approximate solution which ought to hold for early times, i.e., near switch-on. The same regime has been studied by Nash and Olsen[2] under the assumption that resistance varies linearly with time. They are able to solve the circuit equations exactly, in terms of confluent hypergeometric functions. The difficulty with their solution lies in the fact that it cannot apply, even in the very early regime, to a wire whose resistance varies linearly with temperature: for, as our approximate solution[1] shows, the first time dependent term in the resistance function is <u>cubic</u>. General agreement with this deduction is shown by Hobson and Manka[3] who give a perturbation treatment of the circuit equations using the classical result for constant R-L-C as the unperturbed solution. The limitations on their approach are such that, despite the apparent perturbation of the oscillatory solution, the only regime in which their solution is applicable is during the first quarter cycle where the unperturbed solution is nearly linear. Their experimental results tend to confirm the cubic law just cited and a fifth power term in current introduced by the variable resistance.

A similar problem has been considered by R. C. Good, Jr.[4] who attempts to solve the circuit equations with variable resistance by a modified WKB method. In order to obtain a solution he assumes R to vary inversely with time as a Laurent series truncated at the fifth term. Because of this decrease of resistance with time, his approximation is clearly of use only in the late stages of the discharge. This is the regime after the wire has expanded to the point of producing both peripheral and axial arcs[5] and the resistance in question is that of the plasmas produced by the explosion.

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* *Superscript numbers denote references which may be found on page 20.*

The central regime, during which the wire undergoes its major expansion, has been relatively little studied because of its very nonlinear nature. The author and his collaborators[6] have been able to treat this regime by assuming the existence of a vaporization wave which reduces the conducting cross section of the wire and thereby increases its resistance. An approximate solution is given which reproduces quite well, the current, voltage and resistance functions of the exploding wire circuit during the expansion up to arc formation.

There is little likelihood that a comprehensive treatment of the circuit equations, valid for the entire explosion, will be given in the near future. There are two main reasons for this state of affairs, viz., (1) the resistance function of the metal applicable to the entire phenomenon cannot be given as a function of specific energy, and (2) even where resistance is linear with specific energy (or absolute temperature), as it is before melting and for a short interval thereafter, the circuit equations are nonlinear and an exact solution appears impossible except by numerical techniques. For the present we are forced to be content with approximate solutions, valid over short ranges of the independent variables and capable of representing only parts of the exploding wire history.

In this paper, I consider again the early regime up to and through the melting point, and present approximate solutions to the circuit equations. The procedure used indicates a way of classifying the existing approximations. The usefulness of some of these equations will be indicated both for calculating the time of melt and for bridging the gap between melt and onset of the vaporization wave.

CIRCUIT EQUATIONS

We consider an R-L-C circuit in which resistance is a function of time and consists of the constant circuit resistance R_c plus the variable exploding wire resistance R_w. The inductance of the wire is assumed to be much smaller than that of the circuit. In exploding wire experiments switching is usually achieved by a triggered spark gap whose resistance falls, at switch-on, to values assumed small when compared to R_c and R_w. There is some controversy as to the amounts of energy expended in the spark-switch, and these appear to depend upon the configuration of the switch and the amount of over-voltage. Moses and Korneff[7] report considerably larger switch losses than is the experience at the author's laboratory. For this treatment, switch resistance is assumed negligible. Then, with the previous notation[1], we write the circuit equations as,

$$\frac{di}{dt} = - \frac{(R_c + R_w)i}{L} - \frac{q}{LC} , \tag{1}$$

$$\frac{dq}{dt} = i , \tag{2}$$

and

$$\frac{d \ln R_w}{dt} = \Lambda i^2 , \tag{3}$$

where q is the charge on the condenser, in general $\Lambda = (1/m)(\partial R_w/\partial e)$ and in particular $\Lambda = \beta R_{wo}/m$, which may be seen as follows.

The third equation arises from the resistance variation with temperature. For considerable ranges, both above and below the melting transition, R_w is linear in either specific energy e or absolute temperature T. Accordingly we may take

$$R_w = R_{wo}[1 + \beta(e - e_o)] , \tag{4}$$

or

$$R_w = R_{wo}[1 + \alpha(T - T_o)] , \tag{4a}$$

and find

$$\frac{dR_w}{dt} = \beta R_{wo} \frac{de}{dt} . \tag{5}$$

The specific energy e may be calculated from

$$e = e_o + \frac{1}{m} \int_{t_o}^{t} R_v i^2 d\tau \quad , \tag{6}$$

which yields

$$\frac{de}{dt} = \frac{R_v i^2}{m} \quad . \tag{6a}$$

Equations (5) and (6a) then give Equation (3). Equations (1), (2) and (3) constitute the third-order, nonlinear set of ordinary differential equations in which we are interested. Had some other law governing resistance variation with energy or temperature been chosen than Equation (4), the difference would have appeared in Equation (3) which governs both the coupling with the linear circuit equations and the degree of nonlinearity. The present set of equations is based on the experimental observation of approximately linear variation of resistance with temperature and is therefore a phenomenological, mathematical model.

The initial conditions at switch-on are $t = t_o = 0$, $i = i_o = 0$, $q = q_o = - CV_o$, $R_v = R_{vo}$, $T = T_o$, $e = e_o$. If the initial conditions on q and t are suitably modified, these conditions and differential equations apply at any time during the interval from switch-on until onset of the vaporization wave in the liquid, provided only that the interval of application is one for which R_v obeys Equation (4) or Equation (4a). Equation (4) has the advantage that precise values of specific heat are not needed so long as resistance variation with specific energy is measured. Currently available experimental techniques provide good measurements, during the explosion, of current and voltage from which resistivity can be calculated. Time-resolved, thermal measurements of specific heat with temperature are less well developed at present.

Equation (1) shows that a rapid variation in resistance, such as occurs at the melting point, will produce a corresponding change in the slope of current. This effect is especially visible in the current-time curves if melting occurs while current is increasing almost linearly.

Under such conditions an abrupt slope change in current is observed. In metals such as nickel, other transition points, e.g., the magnetic Curie point, can be seen in the voltage and current curves of the wire explosion. Study of these transitions by the exploding wire technique is still in the early stages of development.

FIRST INTEGRAL

Multiply Equation (1) by i and use Equation (2). The result, clearly in energy conservation form, is

$$d\left[\frac{Li^2}{2} + \frac{q^2}{2C}\right] + (R_c + R_w)i^2\,dt = 0 \quad . \tag{7}$$

With Equation (3) we find upon integrating

$$\frac{Li^2}{2} + \frac{q^2}{2C} + \int \frac{(R_c + R_w)}{\Lambda}\,d\ln R_w = K \quad , \tag{7a}$$

with $q = CV$, the constant form for Λ and the initial conditions for switch-on,

$$1/2\left[Li^2 + C(V^2 - V_o^{\,2})\right] + \frac{m}{\beta}\left[\frac{R_c}{R_{wo}}\ln\frac{R_w}{R_{wo}} + \left(\frac{R_w}{R_{wo}} - 1\right)\right] = 0 \ . \tag{8}$$

Equation (8) provides an exact, first integral, or conservation law, for the nonlinear system. Two uses may be anticipated. Given the circuit parameters and two measured variables, e.g., i and condenser voltage V, the third (R_w) can be calculated. Secondly, the exact relation may be useful in evaluating the accuracy of various approximations. It may also turn out that an improved approximation can be obtained from the first integral.

Qualitative information on the nature of the solutions can be seen from Equation (8) because the second bracket as a function of R_w, is non-decreasing. This indicates that in the (i, V) or (i, q) planes the trajectory through an initial V_o or q_o will be a spiral winding around the origin to zero; or, alternatively, until R_w exceeds the critical damping resistance, after which an exponential type of decrease to zero will occur.

APPROXIMATE SOLUTIONS

Approximate solutions to the differential equations, Equations (1), (2), (3), can be obtained by the method of successive approximations (Picard method)[8]. Substitution of $\phi = \ln R_w$ shows immediately that the necessary conditions are satisfied, in particular the Lipschitz condition, since the right hand sides are differentiable. Rather than use this substitution we leave the equations as stated above because the resistance approximations are somewhat more convenient to handle.

The first approximation is found by substituting the initial conditions in place of the variables on the right hand sides of Equations (1), (2), (3) and integrating. Thus,

I

$$i_1 = (V_o/L)t, \qquad q_1 = q_o = -CV_o, \qquad R_{w1} = R_{wo}\,. \tag{9}$$

The second approximation is found by substituting the first approximation for the variables in Equations (1), (2), (3) and integrating a second time. We obtain

II

$$i_2 = \frac{V_o t}{L}\left(1 - \frac{(R_c + R_{wo})}{2L}\,t\right), \tag{10a}$$

$$q_2 = -CV_o\left(1 - \frac{t^2}{2LC}\right), \tag{10b}$$

$$R_{w2} = R_{wo}e^{A^*t^3}, \tag{10c}$$

where

$$A^* = (\Lambda V_o^2/3L^2)\,. \tag{10d}$$

From this point on, further approximations become both increasingly complicated and more difficult to obtain. Proceeding one more cycle in the same fashion, we find for the third approximation,

III

$$i_3 = \frac{V_o t}{L}\left[1 - \frac{R_c t}{2L} - \left(\frac{1}{LC} - \frac{R_c(R_c + R_{wo})}{L^2}\right)\frac{t^2}{6}\right]$$

$$+ \frac{R_{wo}(R_c + R_{wo})}{6L^2A^*}\left(e^{A^*t^3} - 1\right) - \frac{R_{wo}}{L}t^2e^{A^*t^3}S \quad , \text{(11a)}$$

$$q_3 = -CV_o\left[1 - \frac{t^2}{2LC} + \frac{(R_c + R_{wo})}{6L^2C}t^3\right] \quad , \qquad \text{(11b)}$$

$$R_{w3} = R_{wo}\exp\left[A^*t^3\left(1 - \frac{3(R_c + R_{wo})t}{4L} + \frac{3(R_c + R_{wo})^2\,t^2}{20L^2}\right)\right], \text{(11c)}$$

where

$$S = \sum_0^\infty \frac{(-1)^n\,(3A^*t^3)^n}{2.5.8\,..\,(3n + 2)} \quad . \qquad \text{(11d)}$$

While the last two terms in (11a) arise naturally from the integrations performed, they clearly contribute higher order terms that would be meaningless in this approximation if we choose to regard it as giving the first few terms of a series expansion being developed by the process. Accordingly, we save only the leading terms in Equation (11a) and write it as

$$i_3 = \frac{V_o}{L}\left\{t - \frac{(R_c + R_{wo})t^2}{2L} - \left[\frac{1}{LC} - \frac{(R_c + R_{wo})^2}{L^2}\right]\frac{t^3}{6} - \frac{R_{wo}A^*t^5}{5L}\right.$$

$$\left. + \frac{R_{wo}(R_c + R_{wo})A^*t^6}{12\,L^2} - \cdots\right\} \quad . \qquad \text{(11aa)}$$

For the circuit with R-L-C constant the oscillatory solution for current is $i = (V/\omega L)e^{-\alpha t}\sin \omega t$, with $\alpha = R/2L$ and $\omega = (1/LC - R^2/4L^2)^{1/2}$. If this solution is expanded in series, comparison with Equation (11aa) shows that the terms up to cubic can be regarded as originating in the oscillatory properties of the circuit and that the temperature change of resistance affects the current no earlier than the terms of fifth and sixth degree as found also by Hobson and Manka[3]. The fifth degree term, however, is likely to be comparable in magnitude to the earlier one in t^3, as can be seen by substitution of representative values. Values of A* are readily obtained from Equation (10d) if β is available from direct experiment[6]. Where this is not the case, tabulated A values[1] may be used instead because $A = (\alpha/\beta s)A^*$, where α is the temperature coefficient of resistivity and s is the specific heat.

All the terms of i_3 are decreasing except the first and last. The cubic term has a coefficient resembling ω^2 but smaller in magnitude since a factor of 1/4 is missing. The total resistance, $R_{wo} + R_c$, is small enough so that both ω and this coefficient are positive in most practical circuits, i.e., one usually does not arrange an overdamped circuit even by chance. Thus, current falls below the linear ramp, $i = (V_o/L)t$, during the entire interval for which the higher order terms are suitably small.

Of course with R_w increasing, a condition will occur for which the circuit is overdamped. Such an overdamped regime usually begins sometime after the wire melts, during the rapid rise of resistance caused by the vaporization wave[6], and persists for a short interval of time until either the expansion is complete and current shuts off, or until restrike occurs via the peripheral and/or axial arcs.

Further interesting properties of the third approximation may be seen in Equations (11b) and (11c). To this order the charge, q_3, is still unaffected by wire heating, as no terms in A* appear. On the other hand, resistance R_{w3} now has three terms of a series in the exponent, and sample calculations show that it increases somewhat less rapidly at early times than does the second approximation. Comparison of the second and third terms in the exponent shows that the negative term predominates in the interval $0 \leq t \leq 5L/(R_c + R_{wo})$; and, for representative values[5], the upper limit is about 5 μsec indicating that throughout the entire region of interest before melt, which in the cited cases is $0 \leq t \leq 0.5$ μsec, the wire resistance increases at a variable, slower rate than shown by the second approximation.

LIMITATIONS

An interesting fact revealed by the present hierarchy of approximations is that the rather convenient equations presented in our earlier note[1] are i_1, q_1 and R_{w2} in the present notation; thus, the assumptions used there achieve a hybrid combination of the first and second approximations found here. The major assumptions under which (i_1, q_1, R_{w2}) appear in the former note as an approximation are: (1) conservation of energy, (2) Equation (4a) for variation of resistance, (3) q and consequently $V \doteq V_o$ nearly constant and, (4) $0 < (Ri/V_o) \leq k \ll 1$.

The first two of these requirements are common to both treatments. Numbers (3) and (4) are strictly required only of the first approximation, I. In terms of the hybrid solution the last condition, (4), can be formulated as

$$1 + \frac{R_{wo}}{R_c} e^{At^3} \leq \frac{k\,L}{t\,R_c} \quad . \tag{12}$$

The upper bound on time, which satisfies the inequality for the given value of k, is then found as the solution of the transcendental Equation (12). In essence, one wishes to locate the intersection of the exponentially increasing function on the left, with the equilateral hyperbola on the right side of the inequality. This intersection will always lie to the left (i.e. at smaller times) of the intersection with the hyperbola of the horizontal line $y = 1 + (R_{wo}/R_c)$, which intersection occurs at $t_1 = kL/(R_c + R_{wo})$. If $k = 1/10$ is considered satisfactory for the approximation and, as frequently happens, $L/(R_c + R_{wo}) \sim 10^{-6}$ sec, then $t_1 \sim 10^{-7}$ sec. Such a bound is insufficient to allow use of the approximation to calculate times to melt for our data on copper wires[5]. We see clearly, then, the desirability of increasing the ratio $L/(R_c + R_{wo})$ to extend the interval of validity of the theory. Recalling that $A = (R_{wo}/3)(\alpha/ms)(V_o/L)^2$ we see also that R_{wo} occurs both in A and as a coefficient of the exponential.

The dependence on R_{wo} is especially interesting because it indicates that the interval of validity of the theory can be increased by proper dimensioning of the wire. In brief, decreasing R_{wo} will move the intersection toward larger times by decreasing A. Decreasing R_c will move the hyperbola away from the axes, with the same effect. Since $R_{wo} = 4k_o \ell/\pi d^2$ and $m = \rho \ell \pi d^2/4$, where k_o, ρ, ℓ, d are resistivity, density, length and diameter respectively, a variety of scaling operations is possible, e.g. one can hold mass constant and vary ℓ/d so as to decrease R_{wo} or one can hold number of atoms, N_a, and ℓ/d constant and vary the diameter and material of the wire.

All of the above considerations become easier, and the effectiveness of the scaling is greater if the circuit resistance R_c can be made much smaller than R_{wo}. The inequality (12) simplifies to $R_{wo} e^{At^3} = kL/t$ and proper scaling will increase maximum time by as much as a factor of ten. To minimize R_c one must use large conductors in transmission line configurations and very low-loss capacitors. A possibly important contribution to R_c may come from the method of switching. Use of a coaxial, triggered, over-volted spark gap confines losses from this source to a small fraction of the current rise at switch-on.

The inequality of condition (4) can be written in terms of the second approximation, II, as

$$\left(1 + \frac{R_{wo}}{R_c} e^{At^3}\right)\left(1 - \frac{t^2}{2LC}\right)^{-1}\left[1 - \frac{(R_c + R_{wo})t}{2L}\right] \le \frac{kL}{R_c t} \qquad (13)$$

and a discussion similar to that given above holds. To increase the range of applicability of the theory and the goodness of the approximation one wishes to increase L/R_c and decrease R_{wo}. The additional terms will tend to decrease the rate at which the left hand side increases until the term quadratic in time exceeds the first order term. This is the tendency that one would expect from the experimental fact that current decreases and falls below the linear ramp in the first quarter cycle. For this reason the rate of heating of the wire will fall below that implied by the first approximation, i_1, and the rate of increase of resistance will be smaller than that implied by R_{2w}. Thus it would appear that scaling criteria developed from inequality (12) ought to provide a reliable guide to the use of the third approximation, but that actual criteria for its validity should rest on the relative sizes of the successive terms in i_3, q_3, R_{w3}.

If one takes ratios of successive terms in the third approximation, the quantity $a(R_c + R_{wo})t/L$ occurs almost everywhere with $a < 1/2$. With $(R_c + R_{wo})/L \sim 10^6$, one sees that $t \leq 0.2$ μsec will assure rapid convergence via descending powers of ten. If $t \sim 0.5$ μsec convergence will not be so good, but may be adequate to allow calculation of the various quantities up to the time of melt. Since R_{wo} is usually small, or can be made small, efforts to reduce R_c without greatly changing $1/LC$ will have the beneficial effect of extending the range of validity of the approximation.

TIME TO MELT

Because of the convenient form in which the approximations for R_w appear, an expression for the melting time, i.e. the time interval from switch-on until beginning of melt, can easily be derived. Setting $T = T_m$, $R_w = R_{wm}$ in Equation (4a) and using Equation (11c) in the form $R_w = R_{wo} \exp[At^3 f(t)]$ we find that melting time, t_m, is given by

$$t_m\left[f(t_m)\right]^{1/3} = \left\{\frac{\ln\left[1 + \alpha(T_m - T_o)\right]}{A}\right\}^{1/3} \qquad (14)$$

where f(t) = 1, if the second approximation is used, or

$$f(t) = 1 - \frac{3(R_c + R_{wo})t}{4L} + \frac{3(R_c + R_{wo})^2 t^2}{20 L^2} \qquad (14a)$$

for the third approximation. A first value for t_m can be found by setting f(t) = 1. Slightly increasing this value of t_m and substituting into Equation (14) will quickly give values which bracket the root and allow an interpolation. The effect of f(t) is not large because its magnitude is close to unity and it appears only to the 1/3 power; thus, the size of the time-dependent correction terms in Equation (14a) is diminished by a factor of about three.

As a test of Equation (14) we have calculated times until melting begins for the series of copper wires already reported[5] and have compared the results with experimental values in Table I. The agreement is good to better than 8 percent although neither experimental nor calculated results are known to better precision than two significant figures. The value of inductance used is an effective value obtained

TABLE I

d(in)	V_o(kv)	t_m(μsec)	
		exp.	calc.
.004	9.2	0.38	0.36
.004	10.8	0.31	0.32
.004	12.4	0.29	0.27
.005	10.8	0.44	0.41
.005	12.4	0.37	0.36

Table I. Intervals, t_m, until beginning of melt for exploding wires of Cu (Reference 5). d - wire diameter, V_o - condenser voltage.

from the initial current slope and the applied voltage on the condenser. The effective inductance is nearly twice as large as the value obtained from the circuit damping curve with the exploding wire replaced by a shorting bar; yet, the exploding wire inductance is estimated as less than 10 percent that of the circuit. The inability to obtain agreement between inductance measured by these different methods, or equivalently between current slopes found experimentally and from theory, led us to an elaborate investigation[9] of transient skin effects in the current measuring shunt, the condenser and the other conductors in the circuit. These were exonerated, but no satisfactory explanation has yet been found for the discrepancy. The attempt to attribute it to losses in the trigger gap[7] has not seemed satisfactory since our measurements of gap resistance indicate at most 5 to 10 percent losses due to this cause and are far too small to account for the low values of current slope.

Calculated values of t_m are, with one exception, smaller than the measured values, which fact suggests some systematic error in either the experimental situation or in the evaluation of the dimensions and constants of the wire. We will not undertake in this paper any detailed examination of sources of error because at this stage of development and with the precisions possible the gains would hardly repay the effort. The good agreement obtained between theoretical and experimental values of t_m suggests that the third approximations are reliable for situations where melting occurs before the first quarter cycle has elapsed.

MELTING AND BEYOND

The phenomenology of the wires[5] suggests that for many purposes the current and resistance curves can be broken into four regions, denoted here by Roman numerals.

In the first of these, region I, current is nearly linear and wire resistance exponential. The approximation of our note[1] or i_3, R_{W3} can be used up to beginning of melt.

In region II current is roughly constant at a maximum i_m. This value can be read directly from measured current curves or estimated by finding the time of the first horizontal tangent from $(di_3/dt) = 0$ and substituting back into i_3. As a check, conservation of energy gives $(Li_m^2/2) \le (CV_o^2/2)$ from which $i_m \le (V_o/\sqrt{L/C})$. One must use an effective inductance in this inequality; otherwise the bound will be too high. From another point of view, if the total resistance $R_c + R_{wo}$ were constant at the optimum value[10] of $1.10 \sqrt{L/C}$, then $i_m = (V_o/1.92 \sqrt{L/C})$ would be true. Inspection of experimental current curves shows this to be a lower bound, which fact is reasonable considering that all circuit-plus-wire resistances start from the low side of the optimum value and do not attain it until the peak of current has passed.

With i_m determined, the variation of resistance can be obtained directly by integrating Equation (3); and the resulting exponential variation of resistance will apply for an interval above melting where Equation (4a) holds and before the vaporization wave starts.

The voltage curves suggest that melting occurs over a finite time interval. Values of resistivity for the solid and melted metal are known and some information can be gleaned from equating electrical power consumed to that used in providing the latent heat of melting. Some assumptions are necessary in order to prescribe the resistance of the wire.

Because of the finite time interval involved, one must infer an ordered process of melting, i.e. a melting wave or similar mechanism proceeding in the radial direction. Pinch pressures would tend to raise the melting point and, since they increase toward the center, the inner parts will melt at later times; therefore, the wave should proceed from outer to inner radii. If such a wave is assumed, its velocity, obtained from the voltage time curves, is ~ 1.5 - 2.0 mm/μsec. Assuming further that the wire resistance is formed by parallel conduction through a solid core with a molten jacket and that the interface proceeds inward with the above velocity, one can calculate the time of melting and the intermediate resistance values.

Beyond melting, as has been noted above, approximately linear increase of resistance with temperature occurs over an interval which extends up to time of onset of the vaporization wave. While this law holds for such metals as Cu, Au, Ag, Al, it is not true for others like Mo, Fe, Ni, W[11] where an actual decrease with specific energy appears to occur above melting. For these, perhaps a linear approximation with negative slope would be adequate approximation, although, whether the slope changes from negative to positive before the vaporization wave begins is not certain at present.

To return to the classification of the regimes of current, region III corresponds to that covered by the vaporization expansion wave. We have already found[6] approximate expressions for resistance, voltage and current in this region which covers the steep rise in both resistance and voltage. When arc formation is avoided, this theory accounts for the electrical behavior of the circuit up to current shut-off, i.e. commencement of "dwell". In circuits with the well-matched wires there is no further electrical oscillation to speak of.

When arc breakdown occurs either at the voltage peak or after sufficient expansion (dwell) has taken place, circuit resistance steadily decreases to low values characteristic of arc plasma conduction. For this region IV, perhaps the approximation of Good[4] will be useful although to date no attempt has been made to compare his assumed resistance variation with experiment.

The qualitative nature of this discussion indicates that much work remains to be done to provide a complete and accurate theory of exploding wire circuit behavior. Nevertheless, major divisions of the phenomenon can be recognized and for several of these useful approximate theories exist. In particular, circuit behavior up to beginning of melt should be reasonably well represented by the equations developed here.

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F. D. BENNETT

REFERENCES

1. F. D. Bennett, Phys. Fluids 7, 147 (1964).
2. C. P. Nash and C. W. Olsen, Phys. Fluids 7, 209 (1964).
3. A. Hobson and C. K. Manka, Journal of Applied Physics, February 1966.
4. Exploding Wires, Vol III, edited by W. G. Chace and H. K. Moore Plenum Press, New York, 1964, p. 23.
5. F. D. Bennett, H. S. Burden and D.D. Shear, Phys. Fluids 5, 102 (1962).
6. F. D. Bennett, G. D. Kahl and E. H. Wedemeyer in Reference 4, p. 65.
7. K. G. Moses and T. Korneff in Reference 4, p. 391.
8. E. L. Ince. Ordinary Differential Equations. Dover Publications, New York, 1944, Chapter III, p.63.
9. F. D. Bennett and J. W. Marvin, Rev. Sci. Instr. 33, 1218 (1962).
10. F. D. Bennett, Phys. Fluids 1, 515 (1958). The details of the derivation are contained in BRL Report 1056, October 1958, Appendix I.
11. F. H. Webb, Jr., H. H. Hilton, P. H. Levine and A. V. Tollestrup in Exploding Wires, Vol. II, edited by W. G. Chace and H. K. Moore Plenum Press, New York, 1962, p. 37.