A SIMULATOR INVESTIGATION OF A SELF-ADAPTIVE PITCH DAMPER FOR A HIGH PERFORMANCE FIGHTER AIRCRAFT

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Submitted in partial fulfillment of
the requirements for the degree of

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IN
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ABSTRACT

The extreme speed and altitude ranges of which modern, high performance, supersonic airplanes are capable have become manifested in wide variations in airframe transfer function parameters as flight conditions change. Airframe control systems and stability augmentation devices utilized in these aircraft require some form of gain changing to provide satisfactory stability and control characteristisc over the complete flight envelope. In present day operational aircraft, system gains are scheduled as functions of airspeed and altitude, however these mechanizations have some shortcomings which are pointed out in this paper, and also, the desirability of self-adaptive aircraft control systems is discussed. A survey of adaptive techniques which have been applied to the airplane control problem is presented, and a self-adaptive pitch-stability augmentor which utilizes a digital computer in the adaptation loop is proposed. The proposed self-adaptive system is tested in conjunction with a digital three degree of freedom flight simulation of the pitch dynamics of a representative supersonic fighter-attack aircraft, and is found to be capable of performing the adaptation process in two cycles of airplane short-period oscillation. Requirements for practical mechanization in a flight vehicle and recommendations for future research are discussed.

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SYMBOLS

A	dynamic pressure multiplied by wing area
b	wing span, ft
C_D	drag coefficient, $\frac{\text{Drag}}{1/2\rho V^2 S}$
C_L	lift coefficient, $\frac{\text{Lift}}{1/2\rho V^2 S}$
C_l	rolling-moment coefficient, $\frac{\text{Rolling moment}}{1/2\rho V^2 Sb}$
C_m	pitching-moment coefficient, $\frac{\text{Pitching moment}}{1/2\rho V^2 Sc}$
C_n	Yawing-moment coefficient, $\frac{\text{Yawing moment}}{1/2\rho V^2 Sb}$
C_Y	latera;-force coefficient, $\frac{\text{Lateral Force}}{1/2\rho V^2 S}$
$\bar{c}$	wing mean aerodynamic chord, ft
g	acceleration due to gravity, ft/sec^2
h_p	pressure altitude, ft
i_t	stabilizer deflection, deg
I_X	moment of inertia of airplane about X-axis, slug-ft^2
I_{X_e}	moment of inertia of rotating engine parts about X-axis, slug ft^2
I_{XZ}	product of inertia referred to X- and Z-axes, slug-ft^2
I_Y	moment of inertia of airplane about Y-axis, slug-ft^2
I_Z	moment of inertia of airplane about Z-axis, slug-ft^2
K	system gain with respect to indicated subscript
M	mach number
M	pitching moment, ft-lb
m	airplane mass, $\frac{W}{g}$, slugs
N	yawing moment, ft-lb
p	rolling velocity, radians/sec
T	engine thrust, lb

C_{D_0}	profile drag coefficient
AR	wing aspect ratio, b^2/S
q	pitching velocity, radians/sec
r	yawing velocity, radians/sec
S	wing area, sq ft
t	time, sec
V	true airspeed, ft/sec
W	airplane weight, lb
X,Y,Z	body axes of airplane
	angle of attack of airplane body axis, radians or deg
α_{L_0}	angle of attack at zero lift, radians or deg
α_0	initial angle of attack of airplane body axis, deg
β	angle of sideslip, radians or deg
	increments from initial conditions, radians or deg
δ_{a_t}	total aileron deflection (positive for right rolls), radians or deg
δ_r	rudder deflection, radians or deg
ζ_θ	fraction of critical damping in pitch of non-rolling aircraft
ζ_ψ	fraction of critical damping in yaw of non-rolling aircraft
ρ	mass density of air, slugs/cu ft
$\Delta\varphi$	increment in angle of bank, deg
ω_e	rotational velocity of engine rotor, radians/sec
ω_θ	undamped natural frequency in pitch of non-rolling aircraft
ω_ψ	undamped natural frequency in yaw of non-rolling aircraft

C_{L_α}, $C_{L_{i_t}}$, C_{l_β}, $C_{l_{\delta_{a_t}}}$, C_{m_α}, C_{m_β}, $C_{m_{\delta_e}}$, $C_{m_{i_t}}$, C_{n_β}, $C_{n_{\delta_{a_t}}}$, $C_{n_{\delta_r}}$, C_{Y_β}, M_α, N_β — indicates derivative with respect to subscript

C_{l_p}, C_{l_r}, C_{n_p}, C_{n_r}, C_{Y_p}, C_{Y_r}, N_r — indicates derivative with respect to to $\frac{b}{2V}$ x subscript

$C_{m_{\dot{\alpha}}}$, C_{m_q}, M_q — indicates derivative with respect to $\frac{\bar{c}}{2V}$ x subscript

Dot over a symbol indicates derivative with respect to time.

I - INTRODUCTION

As the speed and altitude capabilities of modern high performance supersonic aircraft have increased in recent years, the range of variation of the dynamic characteristics of these aircraft have increased proportionately. A ten to one variation in damping ratio and natural frequency of some of the dominant modes of aircraft motion is not uncommon for modern airplanes with Mach 2 and 70,000 ft. altitude capabilities. As an example, for the airplane considered in this study, the damping ratio of the dominant longitudinal mode varies from $\zeta = 0.445$ at M = 0.9 @ 10,000 ft. to $\zeta = 0.064$ at M = 1.2 @ 45,000 ft.

Various stability augmentation devices have been used to decrease the range of variation of airplane parameters and to hold them as near as possible to optimum values. These devices, in their historical order of application, include fixed gain dampers, pilot-adjusted-gain dampers, speed-and-altitude-programmed-gain dampers, and self-adaptive-gain-adjusting dampers. A number of methods have been investigated for mechanizing self-adaptive stability augmentation systems for aircraft, utilizing various techniques which will be discussed later. And, the purpose of this study is to investigate the use of an airborne stored program digital computer for generation of the adaptation commands in a self-adaptive pitch damping system to be used in a high performance fighter-interceptor type aircraft. The dynamics of the airplane, including control system components, and the adaptation computer were simulated on a Control Data Corporation 1604 general purpose digital computer.

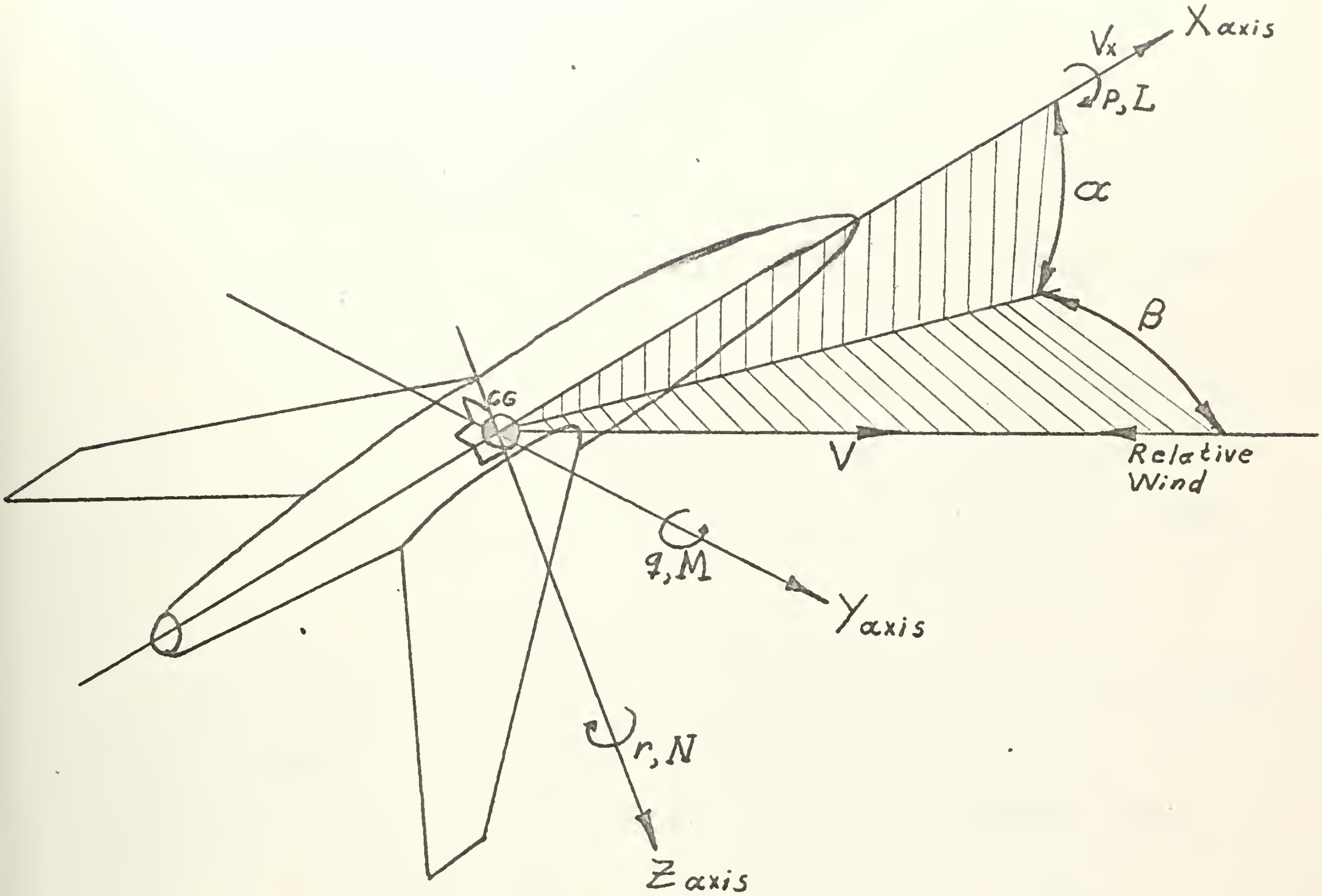
Before beginning the discussion of adaptive control systems as applied to aircraft, it is felt that a better understanding of the system requirements may be gained by a short review of the characteristic dynamic modes of a supersonic aircraft and the theory underlying the analysis and synthesis of these motions.

II DYNAMICS OF THE AIRCRAFT

The airplane body axis system which is generally used in airplane stability and control studies is shown in figure 1, as three mutually perpendicular axes intersecting at the airplane center of gravity. The longitudinal, or X, axis lies in the airplane's vertical plane of symmetry and is arbitrarily oriented by the airplane designer so as to define a horizontal plane of symmetry of the fuselage. In almost all aircraft the X body axis does not coincide with the longitudinal principal axis (longitudinal axis about which the moment of inertia is least) due to the high location of the mass of the tail surfaces and due to the assymetric top-to-bottom configuration of the fuselage. Consequently, the longitudinal principal axis normally slopes downward from tail to nose and coincides with the X body axis only at the center of gravity. The dynamic unbalance of mass about the X axis thus causes the airplane to have an I_{XZ} product of inertia which will be important when cross coupling of airplane motions is discussed.

The Z body axis, or normal axis, is perpendicular to the X axis and to the wing plane and has positive direction downward. And, the Y, or lateral axis, is simply defined as being normal to the X and Z axes. Fortunately, due to the lateral symmetry, an airplane has no dynamic unbalance about the Y axis and thus only one product of inertia, I_{XZ}, exists.

A second set of references, the earth reference system, must also be considered due to action of components of gravity along the three axes. To determine the gravity components, the airplane's attitudes in pitch and roll must be determined and the acceleration of gravity resolved along the three axes by utilizing the Euler equations. The pitch and roll angles are illustrated in figure 2, where pitch angle is shown with wings level and



AXIS	x	y	z	Units
1. Forces acting on the airplane along the axes:	X	Y	Z	lb.
2. Moments acting on the airplane about the axes:	L	M	N	ft-lb.
3. Moments of inertia of the airplane about the axes:	Ix	Iy	Iz	slug-ft^2
4. Linear velocities of the airplane's CG in space:	u	v	w	ft/sec
5. Angular displacements of the airplane about the axes:	φ	θ	ψ	rad.
6. Angular velocities of the airplane about the axes:	p	q	r	rad/sec

Figure 1. Body axis system and parameters used in the digital simulation

roll angle is shown with body level. In these two conditions the earth reference angles are equivalent to integrations of airplane angular velocities about their respective body axes. But, if the craft is in an attitude with non-zero values of both pitch and roll, these angles must be obtained from the projection of the X and Y axes onto two vertical planes containing these axes.

A third earth reference angle which has not been illustrated and which plays no part in the dynamics of motion is the yaw, or heading, angle which is the projection of the longitudinal body axis onto the horizontal plane.

Characteristic Airplane Dynamics

The six degrees of freedom of motion of an aircraft can now be defined as three angular freedoms in roll pitch and yaw about the X, Y, and Z axes respectively; and longitudinal, lateral, and normal translations along the X, Y, and Z axes respectively. Thus, the dynamics can be represented in terms of six simultaneous differential equations of motion. Before discussing the makeup of the six equations of motion, it is felt that a better understanding of the following material may be obtained by a review of the various modes of aircraft motion. The motions can generally be divided into two classes, longitudinal modes and lateral-directional modes. The first class involves pitch rotation combined with translations along the X and Z axes, whereas the lateral-directional modes involve rolling, yawing, and translation along the Y axis. This division of modes occurs because of the low level of aerodynamic and inertial cross coupling between longitudinal and lateral motions. For small amplitude oscillations (excursions in pitch, roll, and yaw less than 15-20 degrees) the two classes can be assumed dynamically independent.

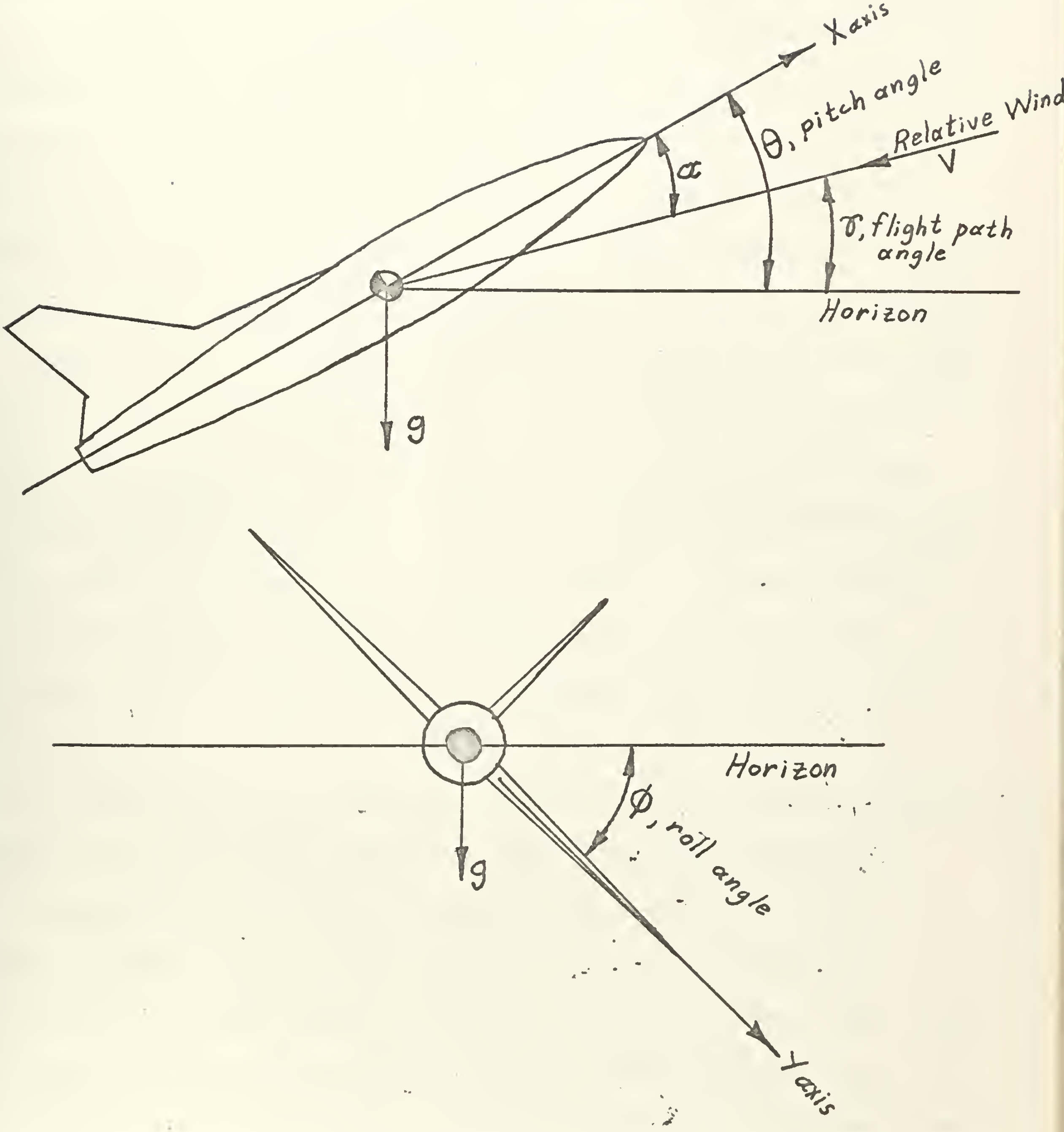


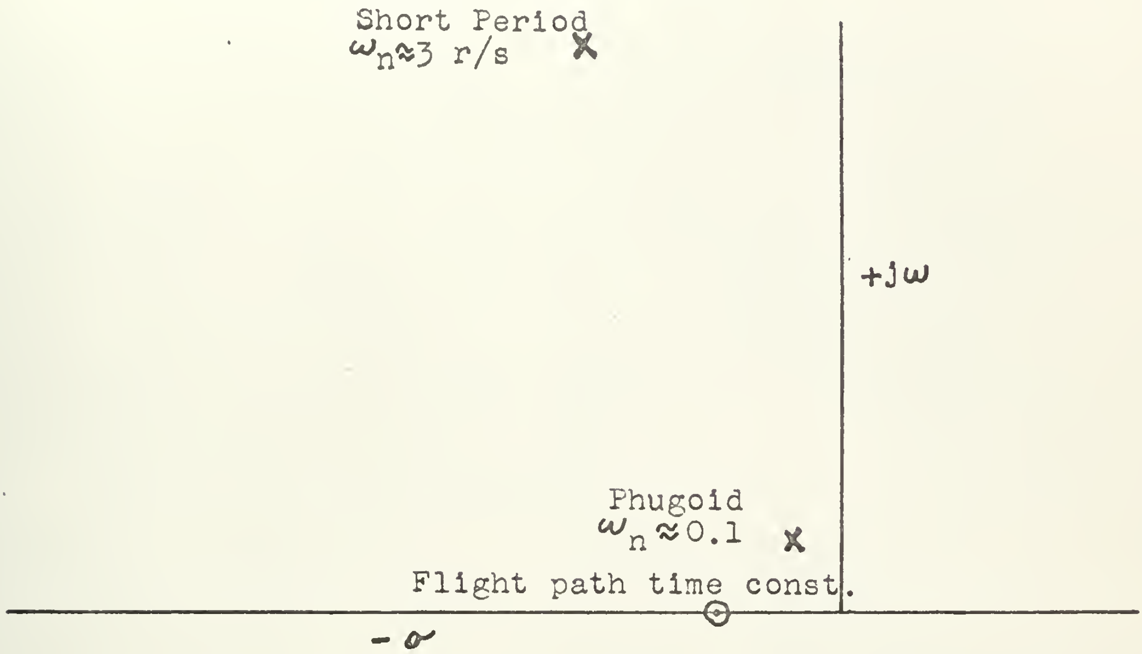
Figure 2. Earth reference system used in the digital simulation

Figure [illegible]. shows pole-zero plots of the longitudinal and lateral directional modes of a representative supersonic airplane. The top half of the S plane is shown for each case. It is seen that the longitudinal motions occur in two oscillatory modes, one with a relatively long period, the phugoid mode, and one with a much shorter period, the short-period mode. The damping of the phugoid is usually low and in some cases it is a neutral or even divergent oscillation. However, due to its long period, the motion is easily kept under pilot control with little or no conscious attempt at correction, and no artificial stability augmentation is required for this mode. The phugoid occurs at constant angle of attack and manifests itself as a continuous trade-off between airspeed and alt tude.

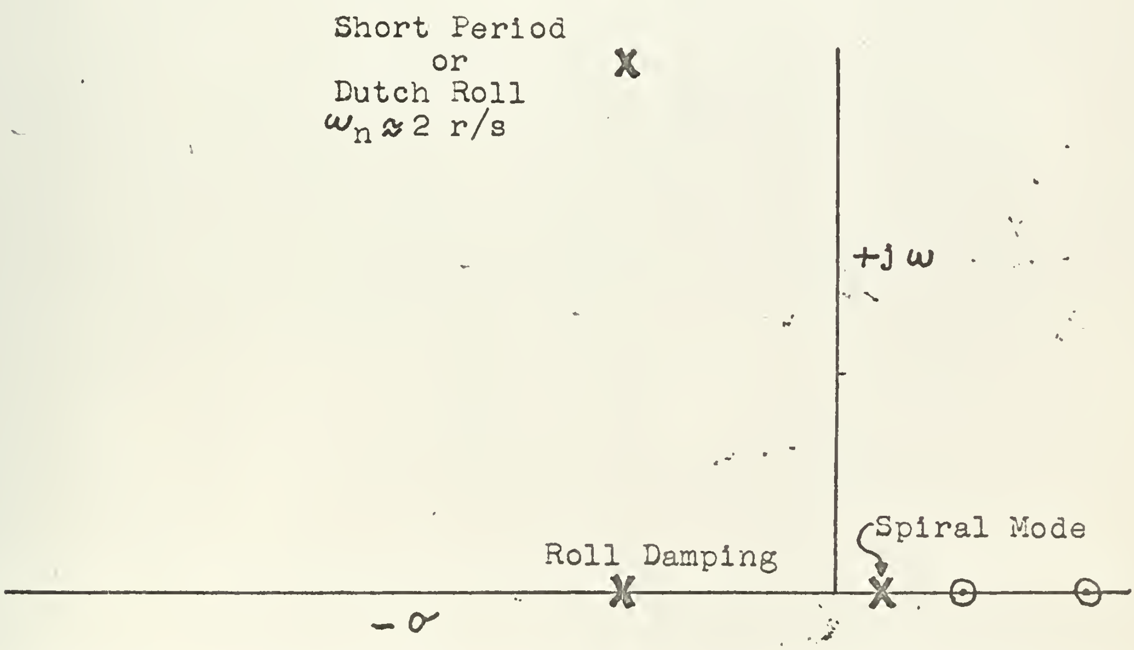
Figures 4. and 5. show that the period and damping ratio of the short period mode of a supersonic aircraft can vary over wide limits, and the inclusion of the time lag of the human pilot in the control loop can sometimes cause the short-period poles to move into the right hand side of the S plane. This necessitates the use of pitch stability augmentation not only for improvement of handling qualities but for reasons of safety. This mode occurs at constant airspeed and appears as a pitching oscillation accompanied by small amplitude translations up and down the Z axis. It is extremely important in maneuvering flight and is of prime interest to designers of airborne fire control and stability augmentation systems.

The lateral-directional pole-zero plot for a supersonic aircraft normally exhibits only one pair of complex poles and two real poles as shown in fig. 3. The frequency and damping ratio of the lateral-directional short period, or Dutch roll, mode vary as flight conditions change in a manner similar to the variation of longitudinal short-period characteristics. This mode is a combined rolling-yawing oscillation with small amplitude lateral translations, and its name is derived from the swaying motions of

Figure 3 Pole-Zero Plot of the Characteristic Modes of Motion of a Supersonic Aircraft.



Longitudinal modes arising from translation along the X and Z axes, and pitching motion about the Y axis.



Lateral-directional modes arising from lateral translation along the Y axis and rolling & yawing motions about the X and Z axes respectively.

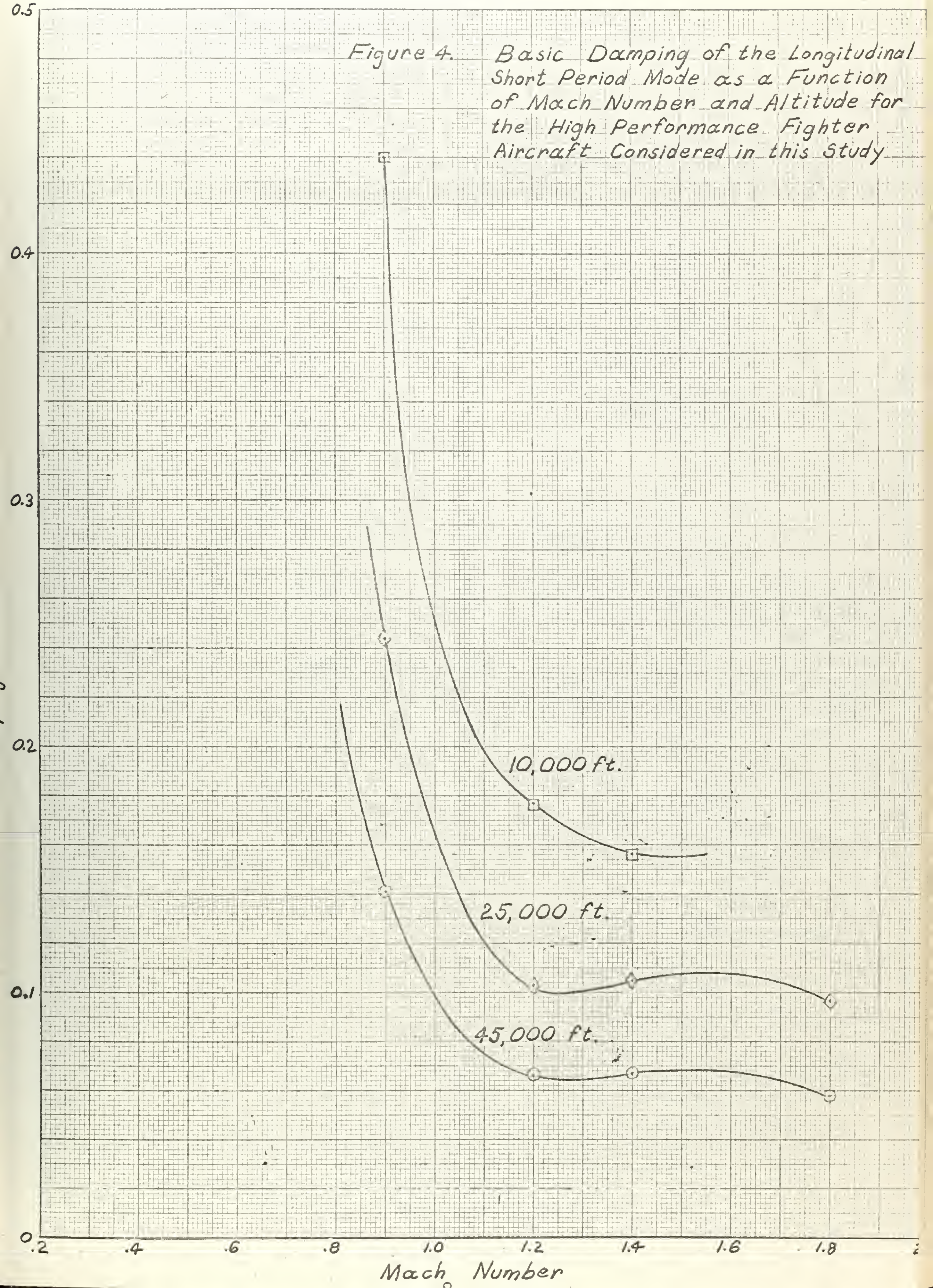


Figure 4. Basic Damping of the Longitudinal Short Period Mode as a Function of Mach Number and Altitude for the High Performance Fighter Aircraft Considered in this Study
0.5
0.4
0.3
0.2
0.1
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
10,000 ft.
25,000 ft.
45,000 ft.
Mach Number

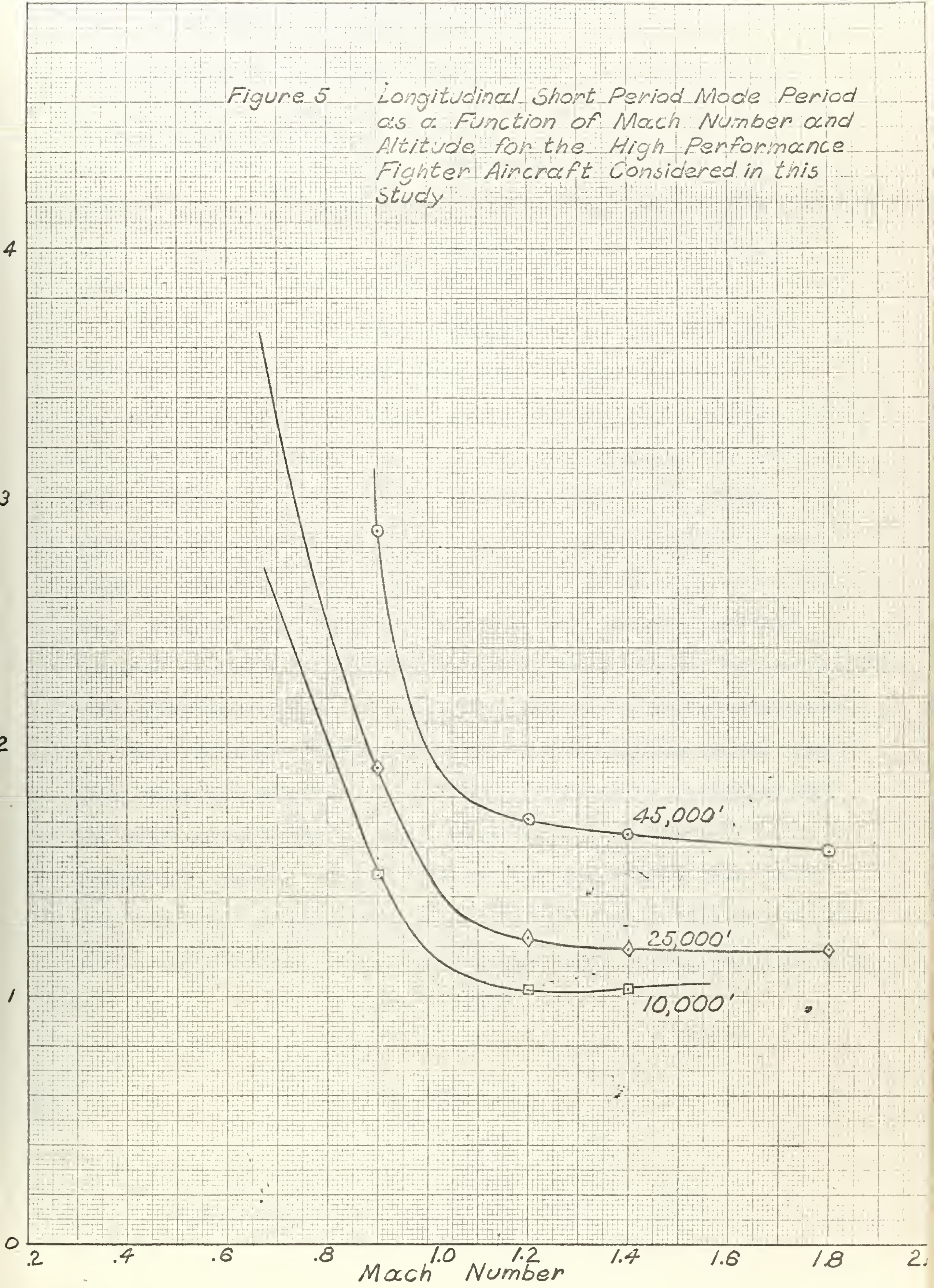


Figure 5 Longitudinal Short Period Mode Period as a Function of Mach Number and Altitude for the High Performance Fighter Aircraft Considered in this Study

a Dutch skater. The Dutch roll oscillations of a supersonic aircraft usually require artificial damping in the same manner as required by the longitudinal short-period mode. Therefore, even though this study is concerned with an adaptive longitudinal stability augmentation system, the results and conclusions are applicable to the lateral-directional case as well.

The roll damping, or roll subsidence, mode is shown as a negative real pole and it represents the viscous roll damping of the aircraft. And, the spiral mode is usually a steady divergence which manifests itself as a gradually steepening turn with accompanying sideslip. The divergence is fortunately slow and, as in the case of the longitudinal phugoid mode, is easily controlled by the pilot. The roll subsidence and spiral mode poles can occasionally form a complex pole pair in the case of some subsonic airplanes with large dihedral effect. This lateral-directional "phugoid" mode will exhibit small amplitude, long period rolling oscillations with considerable lateral translation and very little yawing oscillation.

This concludes the survey of normal modes of airplane motion. However, there are other "pathalogical" modes which can be experienced when the rates and amplitudes of motion become large. Some of these are the phenomena of inertial cross coupling in roll, autorotation (Ref. 125.), and longitudinal pitch-up (Ref. 86); all of which can be eliminated or alleviated by the use of well designed airplane stability augmentation systems.

Aerodynamic Stability Derivatives

One final topic, the concept of aerodynamic stability derivatives, should be mentioned before proceeding to the equations of motion. The forces and moments which act on an aircraft are generally expressed in terms of non-dimensional coefficients multiplied by appropriate aircraft and flight condition parameters. As an example, the equation for airplane lift is shown below.

$$\text{Lift} = C_L \frac{\rho}{2} V^2 S$$

Where:

Lift is in pounds

C_L is the airplane lift coefficient

ρ = atomspheric density in slugs/ft^3

V = airplane forward velocity in ft/sec.

S = airplane wing area in ft^2.

The expression $\frac{\rho}{2} V^2$ is a measure of the kinetic energy of the air-stream and is termed the dynamic pressure because it also represents the force which would theoretically be exerted against a unit square surface held normal to the stream. Thus, the product of dynamic pressure and wing area represents a hypothetical aerodynamic force from which the actual airplane lift can be obtained through multiplication by the lift coefficient. Moments acting about the aircraft can be obtained in the same manner with the inclusion of a moment arm, for dimension considerations, as is done in the following pitching moment equation.

$$\text{Pitching Moment} = C_m \frac{\rho}{2} V^2 S\bar{c}$$

Where the moment arm $\bar{c}$ is the wing mean aerodynamic chord. Corresponding expressions for drag force, side force, rolling moment, and yawing moment coefficients are included in the table of symbols. It is seen that there exists an aerodynamic coefficient for each of the six degrees of freedom of motion. These coefficients and their derivatives are usually determined empirically in wind tunnel and flight studies because of the inaccuracy of analytical determinations.

Each coefficient can be expressed in a Taylor's series such as:

$$C_L = \left(\frac{\partial C_L}{\partial \alpha}\right)\alpha + \left(\frac{\partial^2 C_L}{\partial \alpha^2}\right)\frac{\alpha^2}{2!} + \cdots\cdots + \left(\frac{\partial C_L}{\partial \dot{\alpha}}\right)\dot{\alpha} + \left(\frac{\partial^2 C_L}{\partial \dot{\alpha}^2}\right)\frac{\dot{\alpha}^2}{2!} + \cdots$$

$$\cdots + \left(\frac{\partial C_L}{\partial q}\right)q + \left(\frac{\partial^2 C_L}{\partial q^2}\right)\frac{q^2}{2!} + \cdots\cdots$$

The effects of the second order and higher terms are insignificant in aerodynamics and are dropped. So, for practical purposes the coefficient can be expressed:

$$C_L = \left(\frac{\partial C_L}{\partial \alpha}\right)\alpha + \left(\frac{\partial C_L}{\partial \dot{\alpha}}\right)\dot{\alpha} + \left(\frac{\partial C_L}{\partial q}\right)q + \cdots\cdots$$

The partial derivative terms such as $\left(\frac{\partial C_L}{\partial \alpha}\right)$ are called aerodynamic stability derivatives and are usually written for brevity in the form $C_{L\alpha}$

$$\therefore C_L = C_{L\alpha}\alpha + C_{L\dot{\alpha}}\dot{\alpha} + C_{Lq}q$$

Figure 6, a curve of lift coefficient vs. angle of attack, can be used to illustrate the physical meaning of stability derivatives. Here, $\left(\frac{\partial C_L}{\partial \alpha}\right)$ is the slope, or derivative, of the airplane lift curve. It is seen that the curve is linear up to $\alpha \approx 12°$. This indicates that $\Delta C_L = C_{L\alpha}\alpha$ is physically valid for small variations in α. The fact that almost all aerodynamic stability derivatives remain nearly linear over a similar range of their corresponding variables enables the small amplitude motions of the aircraft to be expressed as a set of simultaneous linear differential equations.

How many stability derivatives must be considered in the equations of motion? Given the six force and moment coefficients C_X*, C_Y, C_Z*, C_l, C_m,

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*The lift coefficient C_L and the drag coefficient C_D act perpendicular and parallel, respectively, to the relative wind, but it can be assumed with little error that they are also normal and parallel to the X body axis. Then $C_Z = C_L$ and $C_X = C_D$.

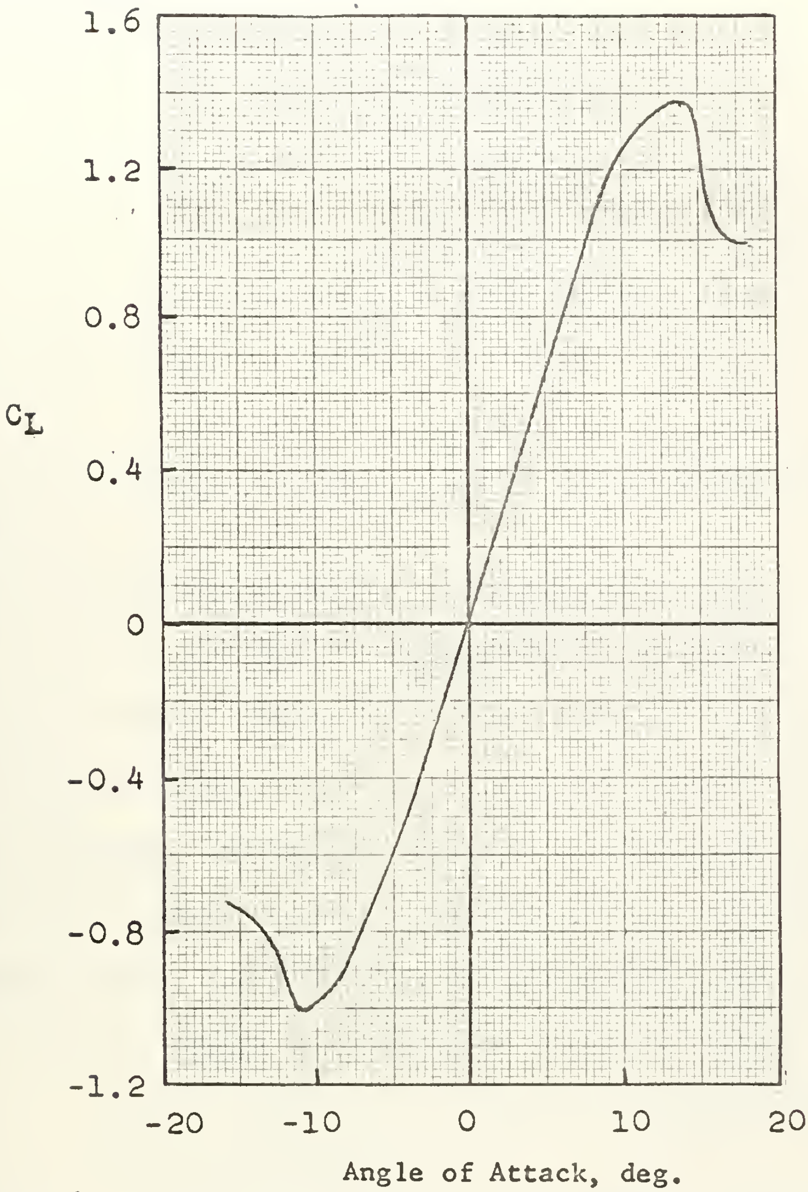


Figure 6. Plot of Airplane Lift Coefficient vs. Angle of Attack.

and C_n and the fact that each coefficient can be influenced by:

1. angular displacement about each axis,
2. angular rate about each axis,
3. velocity along each axis, and
4. acceleration along each axis

there are 12 possible derivatives for each coefficient or 72 in all. Also, control surface deflections can add 36 more terms. Fortunately, airplane symmetry, which causes a lack of cross coupling between longitudinal and lateral - directional modes, greatly reduces the number. The magnitudes of some of the remaining derivatives are so small as to make them insignificant. As a result there are approximately 25 derivatives considered important enough to be included in a simulation program. These are summarized in Appendix III. A list of stability derivatives for a representative supersonic aircraft is tabulated in Appendix I. The aircraft is the same one for which the longitudinal short period mode characteristics are illustrated in figures 4. and 5.

The Airplane Equations of Motion and the Longitudinal Short Period Transfer Function.

The six equations of motion corresponding to the six degrees of freedom of an aircraft are derived in appendix IV and the final results are presented in table I. It should be noted that the equations represent the balance of inertial and aerodynamic forces and moments along and about the airplane body axes, and that the terms are composed of airplane physical constants, aerodynamic stability derivatives, and aerodynamic variables. Assumptions used in the derivation of the equations are:

1. The change in aerodynamic forces and moments due to deviations from an equilibrium condition are assumed directly proportional to the deviation.
2. The deviations of the airplane from equilibrium are assumed small.
3. The airplane is a rigid body.
4. Constant thrust is assumed during an airplane distrubance, and secondary power effects are neglected.

The equations are used directly in the digital computer simulation of the airplane dynamics, which is outlined in appendix VI, and the airplane longitudinal short-period transfer function is derived from the pitching and normal translation equations of motion in appendix V. This transfer function is of prime importance in the study of the airplane pitch damper because the short-period mode is the dominant longitudinal mode. The short-period transfer function is written below both in its general form and also in terms of the airplane physical characteristics, airplane stability derivatives, and aerodynamic parameters. The second form of the transfer function is shown to illustrate the dependence of the dynamics of the short period mode on changes in stability derivatives and aerodynamic parameters, and it is possible from inspection of the function to determine which derivatives and parameters exert influences on a given characteristic, such as damping ratio, of the short-period mode.

Longitudinal Short Period Transfer Function

A. General Form

$$\frac{\theta}{i_t} = \frac{K_\theta (S + T_\theta)}{S(S^2 + 2\xi \omega_n S + \omega_n^2)}$$

B. Detailed Form

$$\frac{\theta}{i_t} = \frac{\frac{A\bar{C}}{I_Y} C_{m_{i_t}} \left(S + \frac{A}{mV} C_{L\alpha}\right)}{S\left(S^2 + S\left(\frac{A}{mV} C_{L\alpha} - \frac{AC^2}{2VI_Y} C_{m_{q+\dot{\alpha}}}\right) - \left(\frac{AC^2}{2VI_Y} C_{m_q} \frac{A}{mV} C_{L_\alpha} - \frac{AC}{I_Y} C_{m\alpha}\right)\right)}$$

$$\omega_n = \sqrt{-\frac{A\bar{C}^2}{2VI_Y} C_{m_q} \frac{A}{mV} C_{L\alpha} - \frac{A\bar{C}}{I_Y} C_{m\alpha}}$$

$$\xi = \frac{\frac{A}{mV} C_{L\alpha} - \frac{A\bar{C}^2}{2VI_Y} C_{m_{q+\dot{\alpha}}}}{2\sqrt{\frac{A\bar{C}^2}{2VI_Y} C_{m_q} \frac{A}{mV} C_{L\alpha} - \frac{A\bar{C}}{I_Y} C_{m\alpha}}}$$

(referenced to body axis)

$$\dot{p} = \left(\frac{I_Y - I_Z}{I_X}\right)qr + \frac{I_{XZ}}{I_X}\dot{r} + \frac{I_{XZ}}{I_X}pq + \frac{Ab}{I_X}C_{l_{\delta_{a_t}}}\delta_{a_t} + \frac{Ab}{I_X}C_{l_{\delta_r}}\delta_r + \frac{Ab^2}{2VI_X}C_{l_p}p + \frac{Ab^2}{2VI_X}C_{l_r}r + \frac{Ab}{I_X}C_{l_\beta}\beta + \frac{Ab^2}{2VI_X}C_{l_{\dot{\beta}}}\dot{\beta}$$

$$\dot{q} = \left(\frac{I_Z - I_X}{I_Y}\right)pr + \frac{I_{XZ}}{I_Y}r^2 - \frac{I_{XZ}}{I_Y}p^2 - \frac{I_{X_e}\omega_e}{I_Y}r + \frac{A\bar{c}}{I_Y}C_{m_{i_t}}i_t + \frac{A\bar{c}^2}{2VI_Y}C_{m_q}q + \frac{A\bar{c}^2}{2VI_Y}C_{m_{\dot{\alpha}}}\dot{\alpha} + \frac{A\bar{c}}{I_Y}C_{m_\alpha}\Delta\alpha + \frac{A\bar{c}}{I_Y}C_{m_\beta}\beta$$

$$\dot{r} = \left(\frac{I_X - I_Y}{I_Z}\right)pq + \frac{I_{XZ}}{I_Z}\dot{p} - \frac{I_{XZ}}{I_Z}qr + \frac{I_{X_e}\omega_e}{I_Z}q + \frac{Ab}{I_Z}C_{n_{\delta_r}}\delta_r + \frac{Ab^2}{2VI_Z}C_{n_r}r + \frac{Ab^2}{2VI_Z}C_{n_p}p + \frac{Ab}{I_Z}C_{n_\beta}\beta + \frac{Ab^2}{2VI_Z}C_{n_{\dot{\beta}}}\dot{\beta} + \frac{Ab}{I_Z}C_{n_{\delta_{a_t}}}\delta_{a_t}$$

$$\dot{\beta} = \frac{g}{V}m_3 - r + \alpha p + \frac{A}{mV}C_{Y_\beta}\beta + \frac{Ab}{2mV^2}C_{Y_p}p + \frac{Ab}{2mV^2}C_{Y_r}r + \frac{Ab}{2mV^2}C_{Y_{\dot{\beta}}}\dot{\beta}$$

$$\dot{\alpha} = q + \frac{g}{V}n_3 - p\beta - \frac{A}{mV}C_{L_\alpha}\alpha + \frac{A}{mV}C_{L_\alpha}\alpha_{L_0} - \frac{A}{mV}C_{L_{i_t}}i_t.$$

$$\dot{V}_X = \frac{T}{m} - \frac{A}{m}C_{D_0}\cos\alpha - \frac{A}{m}\frac{(C_{L_\alpha}\alpha)^2}{\pi AR}\cos\alpha - \frac{A}{m}C_{D_\beta}|\beta|\cos\beta + \frac{A}{m}C_{L_\alpha}\alpha\sin\alpha - g\sin\theta$$

$$V = \frac{V_X}{\cos\alpha\ \cos\beta}$$

where $A = \frac{\rho V^2 S}{2}$

$m_3 = \sin\varphi\ \cos\theta$

$n_3 = \cos\varphi\ \cos\theta$

θ pitch angle

φ roll angle

(Angles between the body axis and earth gravity axis)

TABLE II

Short Period Pitch-Rate-per-Stabilizer-Deflection Transfer Functions Used in this Study

Flight Condition			$\dot{\theta}/i_t$
1.	M = 0.9	Alt. = 10,000'	$\frac{20.875(S+1.277)}{(S^2+3.695S+17.60)}$
2.	1.2	10,000'	$\frac{24.128(S+1.687)}{(S^2+2.170S+37.41)}$
3.	1.4	10,000'	$\frac{25.171(S+1.363)}{(S^2+1.958S+39.33)}$
4.	0.9	25,000'	$\frac{13.791(S+0.777)}{(S^2+1.600S+10.730)}$
5.	1.2	25,000'	$\frac{16.728(S+0.8771)}{(S^2+1.042S+25.751)}$
6.	1.4	25,000'	$\frac{16.786(S+0.8334)}{(S^2+1.100S+27.468)}$
7.	1.8	25,000'	$\frac{17.857(S+0.7928)}{(S^2+1.011S+27.656)}$
8.	0.9	45,000'	$\frac{6.465(S+0.337)}{(S^2+0.616S+4.808)}$
9.	1.2	45,000'	$\frac{8.641(S+0.397)}{(S^2+0.484S+13.511)}$
10.	1.4	45,000'	$\frac{8.777(S+0.379)}{(S^2+0.515S+14.518)}$
11.	1.8	45,000'	$\frac{9.577(S+0.352)}{(S^2+0.640S+15.774)}$

III - THE USE OF A PITCH DAMPER FOR LONGITUDINAL STABILITY AUGMENTATION

The purpose of airplane stability augmentation is to reduce the range of variation of a dynamic characteristic, or characteristics, and to hold the variables of interest as close as possible to predetermined optimum values. In the case of an aircraft these optimum values are defined in terms of "airplane handling qualities" which describe an airplane's response to manual control by a pilot. Even though the determination of desirable handling qualities is subjective, and is dependent on pilot opinion, a number of studies have been conducted to specify these qualities in quantitative engineering terms. One of the better known studies, conducted at the Cornell Aeronautical Laboratory, (Ref. 34), concludes that the desirable natural frequency and damping ratio of the short-period mode of a fighter airplane lie in the range ω_n = .4 to .5 cycles per second and damping ratio = .65 to .70. The same study also concludes that even though the longitudinal natural frequency is of importance in maneuvering flight, it can vary over wide limits and the handling qualities will still be considered tolerable, as long as the short period damping ratio is held in the .65 to .70 range. Thus it is considered more desirable to equip a high performance airplane with artificial damping than to provide static stability augmentation. It should be noted that for the craft considered in this study, as well as for almost all other airplanes of similar type and mission, the longitudinal damping ratio of the un-augmented airplane is always below the desired $\zeta = .7$, as is shown in figure 4. This means that the pitch damping system will work to supplement the basic aerodynamic damping in all cases.

Mechanization of the Pitch Damper

In the discussion of the airplane longitudinal short-period transfer function it was shown that the short-period motion can be represented as a second order system with characteristic damping ratio and natural frequency. It was also shown that the natural frequency and damping ratio can be expressed as functions of the aerodynamic stability derivatives. The expression for short period damping ratio being:

$$\zeta = \frac{\frac{A}{mV} C_{L_\alpha} - \frac{A\bar{c}^2}{2VI_Y} C_{m_{q+\dot{\alpha}}}}{2\sqrt{\frac{A\bar{c}^2}{2VI_Y} C_{m_{q+\dot{\alpha}}} \frac{A}{mV} C_{L_\alpha} - \frac{A\bar{c}}{I_Y} C_{m_\alpha}}}$$

From the above relationship it can be seen that the most effective means of varying the damping ratio is to artificially vary the pitch damping derivative, $C_{m_{q+\dot{\alpha}}}$, which represents the airplane's aerodynamic viscous resistance to pitching velocity.

The pitch damper must then provide an aerodynamic pitching moment which is proportional to airplane pitch rate, and which has a direction in opposition to the angular rate. Thus, the moment produced is in phase with the viscous aerodynamic damping in pitch and the two moments combine to form the total pitch damping moment. The artificial damping moment is usually generated by deflecting the airplane's horizontal stabilizer in direct proportion to measured pitch rate.

Figure 7. is a block diagram of the pitch damper used in this study. The airplane transfer function shown in the diagram represents the longitudinal short-period mode only because, as will be demonstrated later, the phugoid mode and the lateral-directional dynamics are of negligible importance in longitudinal stability augmentation considerations.
The stabilizer actuator in the diagram is an irreversible hydraulic position servo, and the parameters in the transfer function represent the state-of-

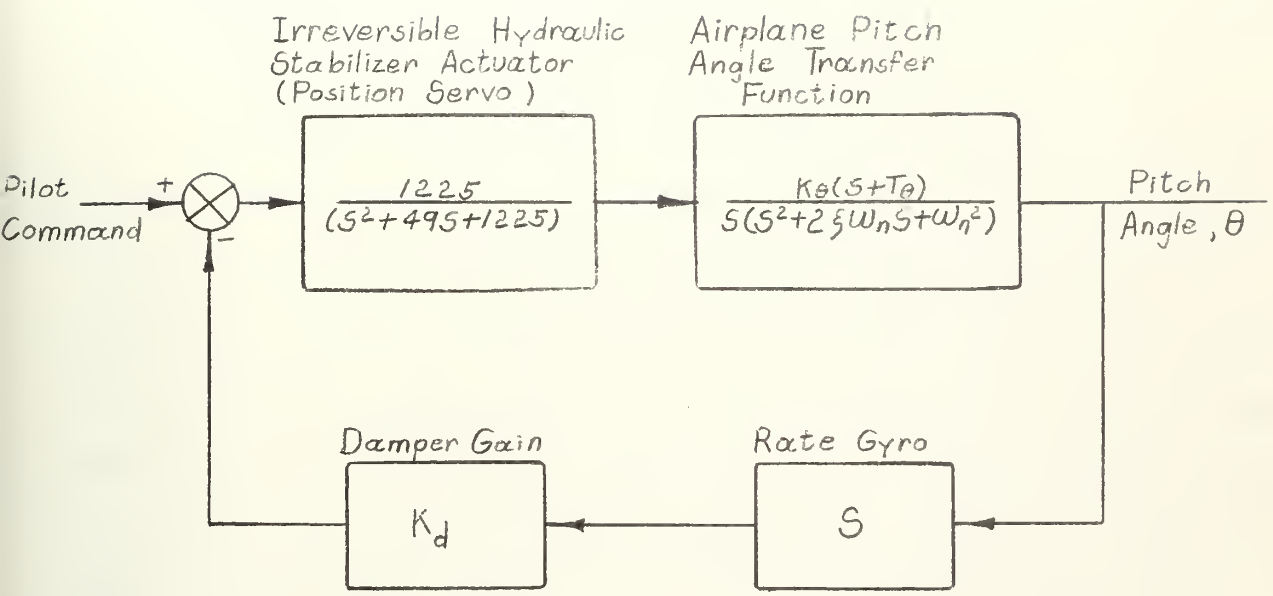


Figure 7. Block Diagram of the Pitch Damper

the-art as used in modern aircraft. It is generally assumed that the dynamics of the rate gyro have negligible effect on system operation because the ratio of the characteristic frequency of state-of-the-art rate gyros to the short-period mode frequency is of the order of 20 to 30 (Ref. 43), and thus the gyro transfer function is assumed to be unity. Table II lists the 11 airplane transfer functions used in the study along with corresponding flight conditions. And, figures 8 thru 18 are root locus plots showing the behavior of the system with varying damper gain at each condition. The square symbols on the plots indicate the gain values at which the airplane short-period damping ratio has been increased to 0.7. It is seen that the use of the damper has very little influence on the short period frequency when operated over the range of gains required to bring ξ up to 0.7 at each flight condition. And, it is also seen that the actuator pole moves in such a direction, with increasing damper gain, that the actuator damping ratio remains close to 0.7 over the damper operating range. The actuator mode will become unstable at extremely high gains, but operation at such gains is of no interest in this study.

It is known that the closing of a feedback loop around a plant having varying characteristics will cause the plant output to vary over a smaller range, as plant characteristics vary, than the range of variation of output with no feedback. From this it would be expected that the pitch damper should not only increase the average value of damping ratio over all flight conditions, but also that the range of variation of damping ratio, as conditions vary, should decrease. This is borne out in Figure 19. which is a plot of short-period damping ratio vs. speed and altitude with constant damper gain = 0.3. The chosen value of damper gain is approximately the median of the gains required for 0.7 damping ratio over all 11 flight conditions. In the case of the basic airplane, ξ varies from 0.445 at M = 0.9 @ 10,000 ft. to ξ = 0.064 at M = 1.2@ 45,000 ft. whereas the variation of ξ with damper gain = 0.3 is from 1.1 to 0.436 for the same two flight conditions. Thus the range of variation about the mean has been reduced from approximately 190% to approximately 95%.

Even though the use of the constant gain damper has caused a considerable improvement in the dynamics of the airplane, a further restriction in the range of variation of damping ratio is usually desired for fighter-interceptor type aircraft. One method of solving this problem, which is in general use today, is the programming of stability augmentation system gains, as functions of measured flight parameters, in such a manner that the programmed gains will cause near optimum system response over all flight conditions. The techniques of gain scheduling methods range from

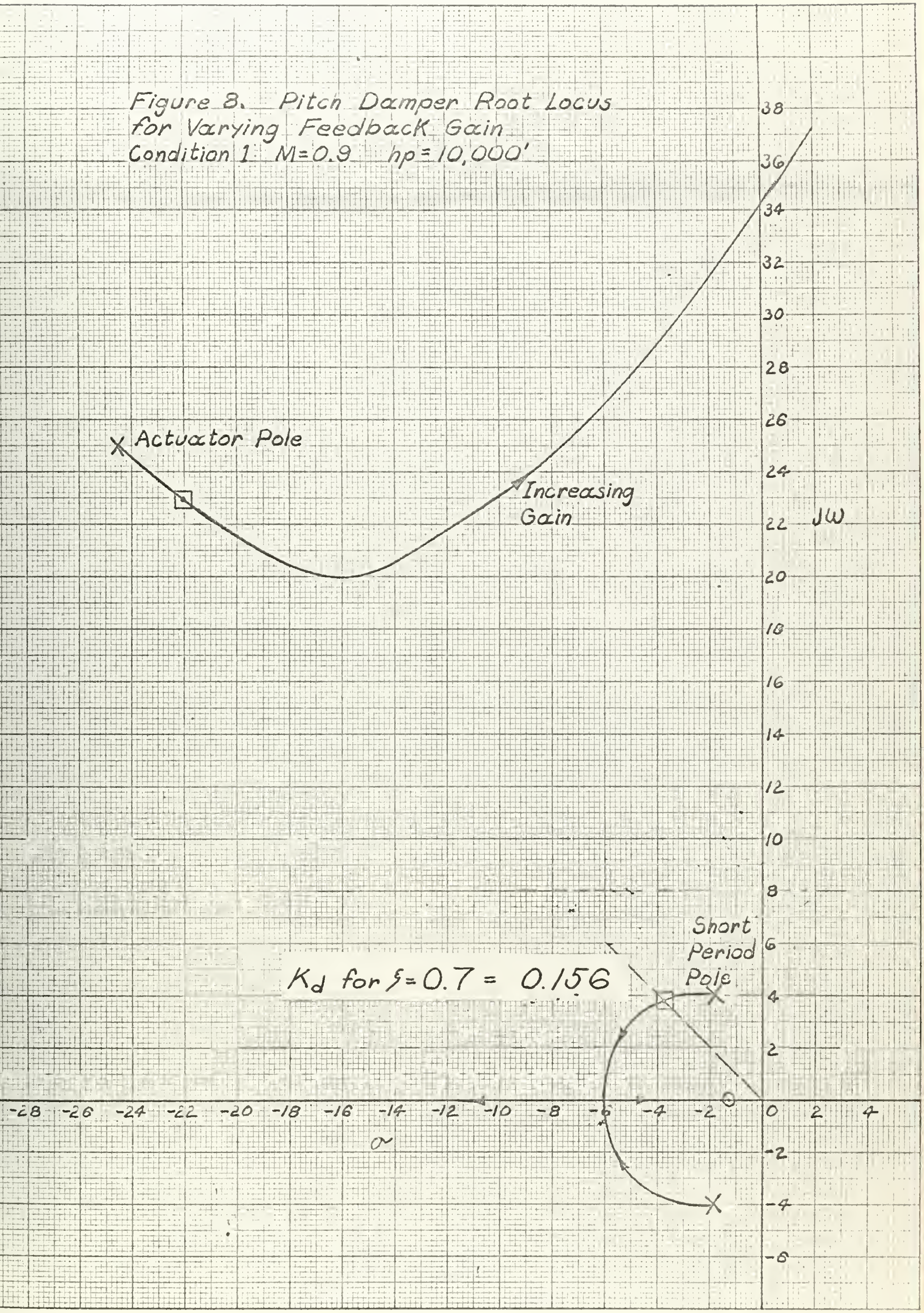


Figure 8. Pitch Damper Root Locus for Varying Feedback Gain
Condition 1 M=0.9 hp=10,000'

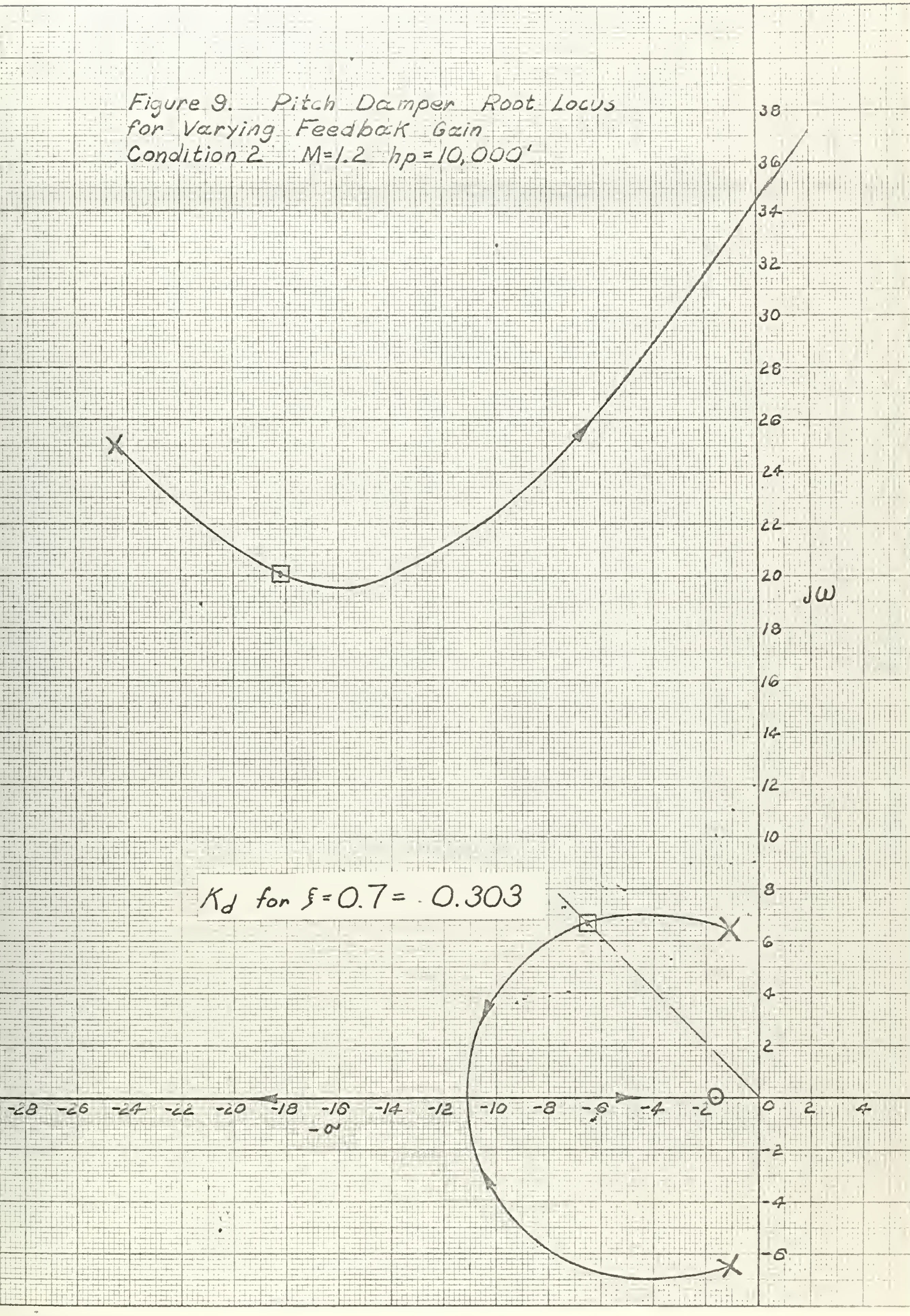


Figure 9. Pitch Damper Root Locus
for Varying Feedback Gain
Condition 2 M=1.2 hp=10,000'
K_d for ζ = 0.7 = 0.303
jω
-σ
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
-2
-4
-6
-28
-26
-24
-22
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4

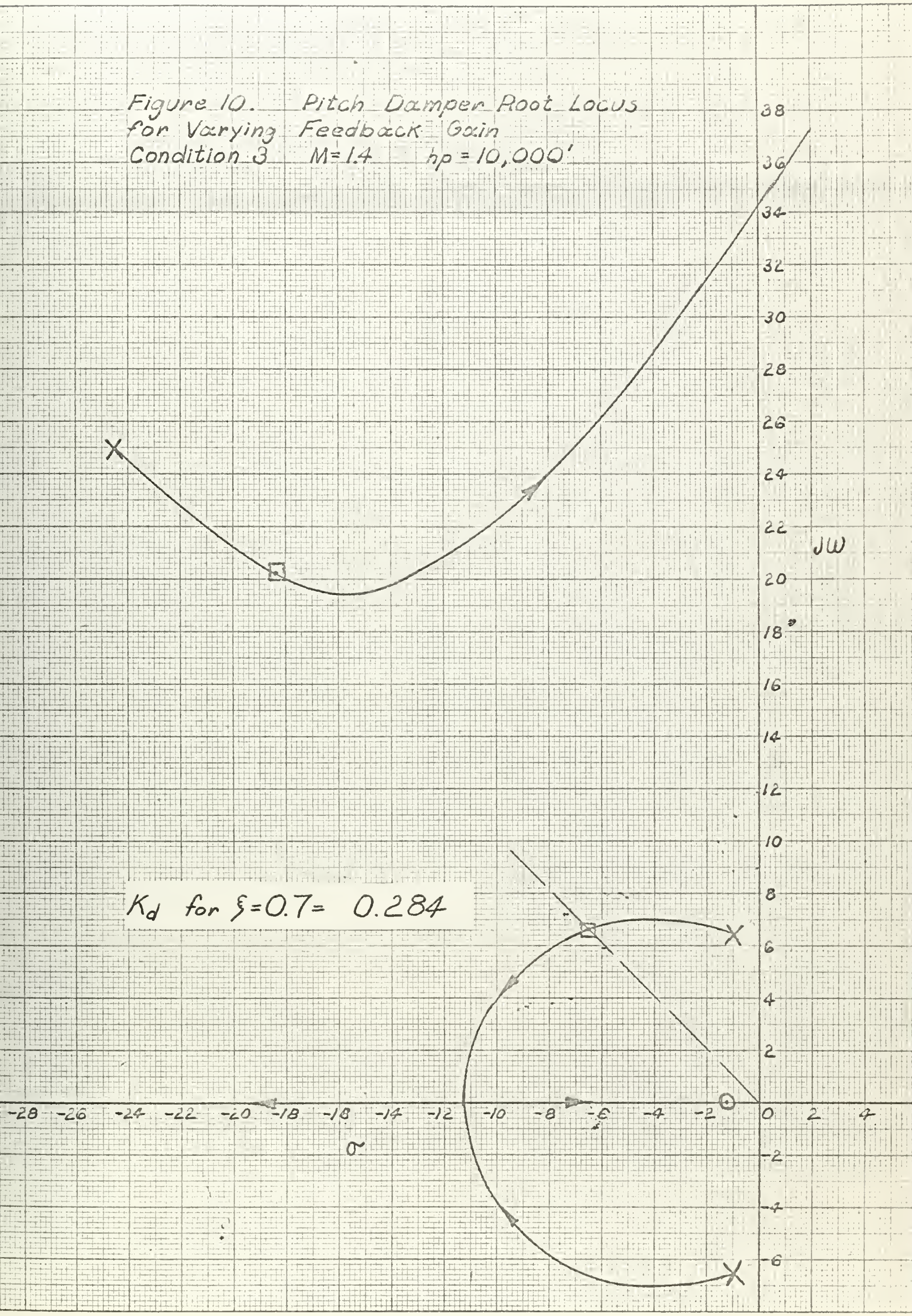


Figure 10. Pitch Damper Root Locus
for Varying Feedback Gain
Condition 3 M=1.4 hp = 10,000'
K_d for ζ=0.7= 0.284
jω
σ
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
-2
-4
-6
-28 -26 -24 -22 -20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4

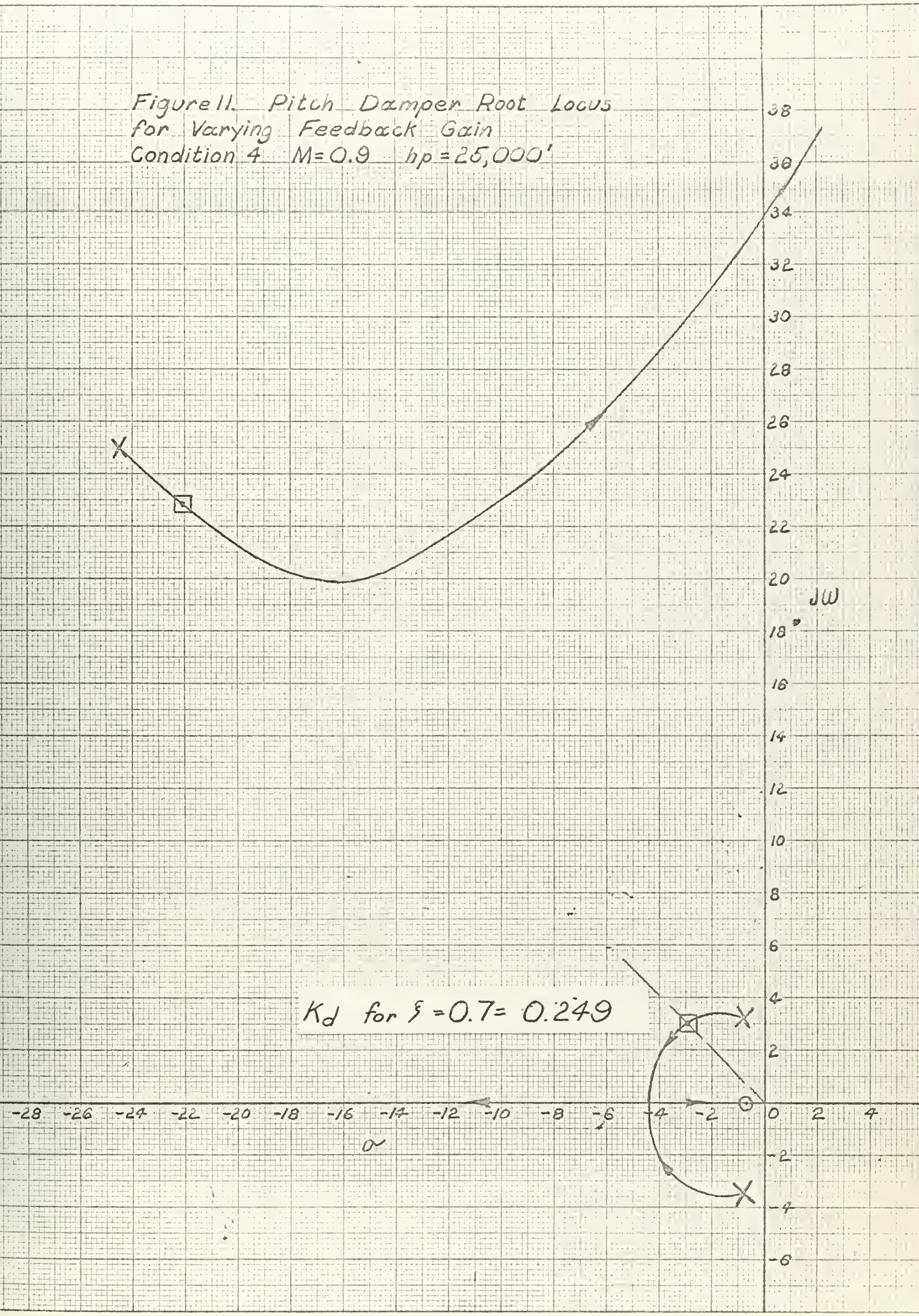


Figure 11. Pitch Damper Root Locus
for Varying Feedback Gain
Condition 4 M=0.9 hp=25,000'
K_d for ζ = 0.7 = 0.249
jω
σ

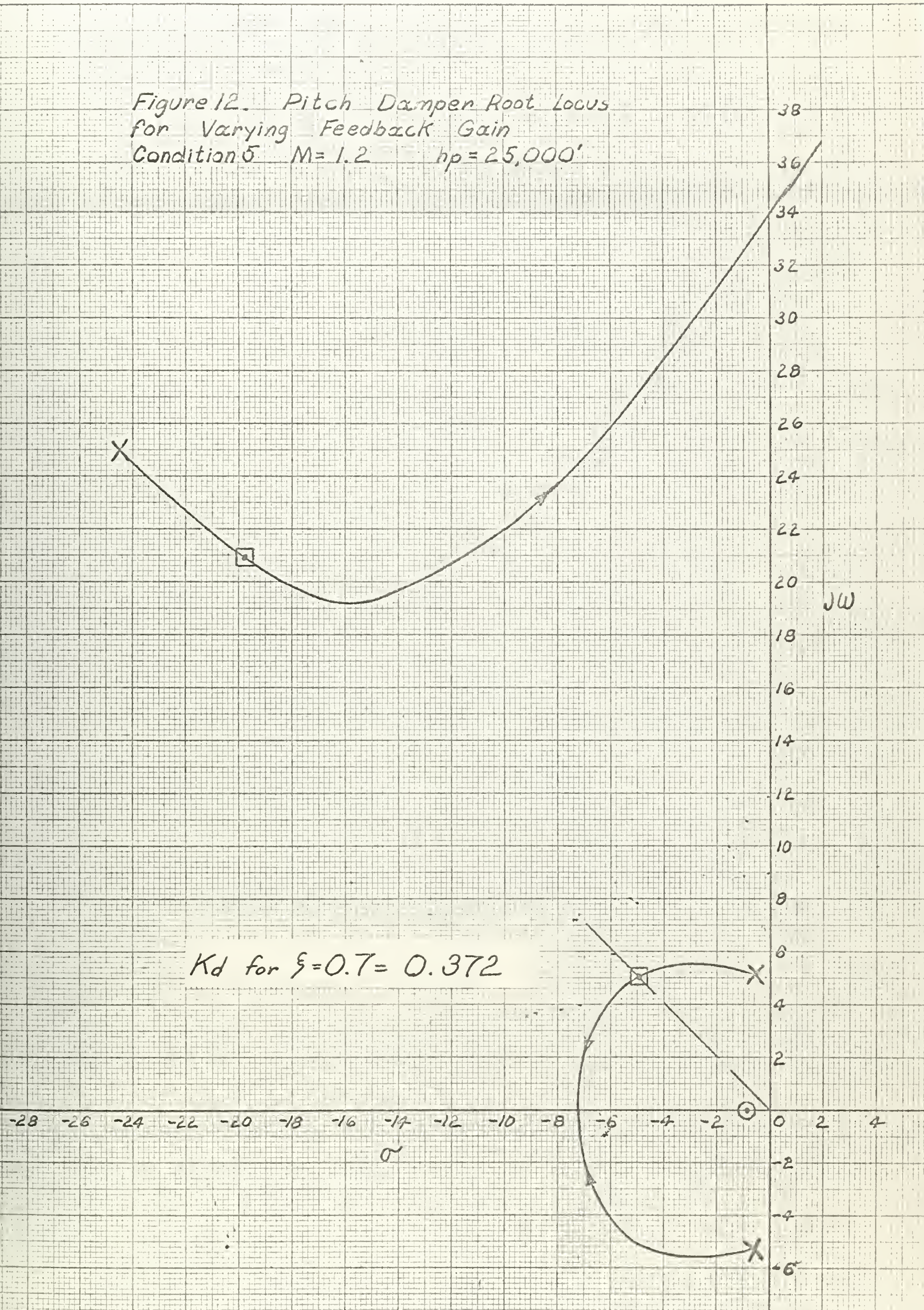


Figure 12. Pitch Damper Root Locus
for Varying Feedback Gain
Condition 5 M= 1.2 hp = 25,000'
Kd for ζ=0.7= 0.372
jω
σ

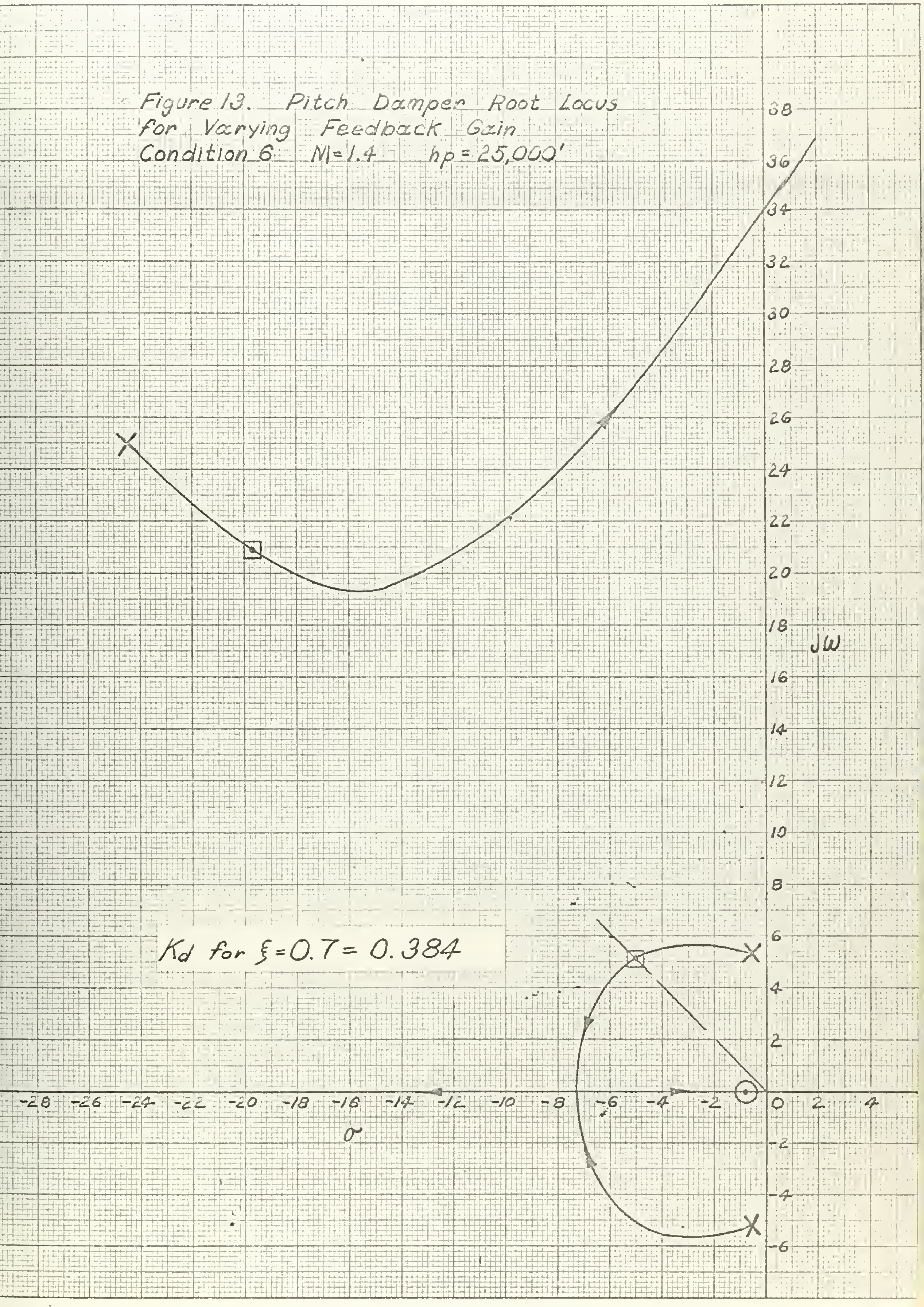


Figure 13. Pitch Damper Root Locus
for Varying Feedback Gain
Condition 6 M=1.4 hp=25,000'
Kd for ξ=0.7= 0.384
jω
σ
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
-2
-4
-6
-28
-26
-24
-22
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4

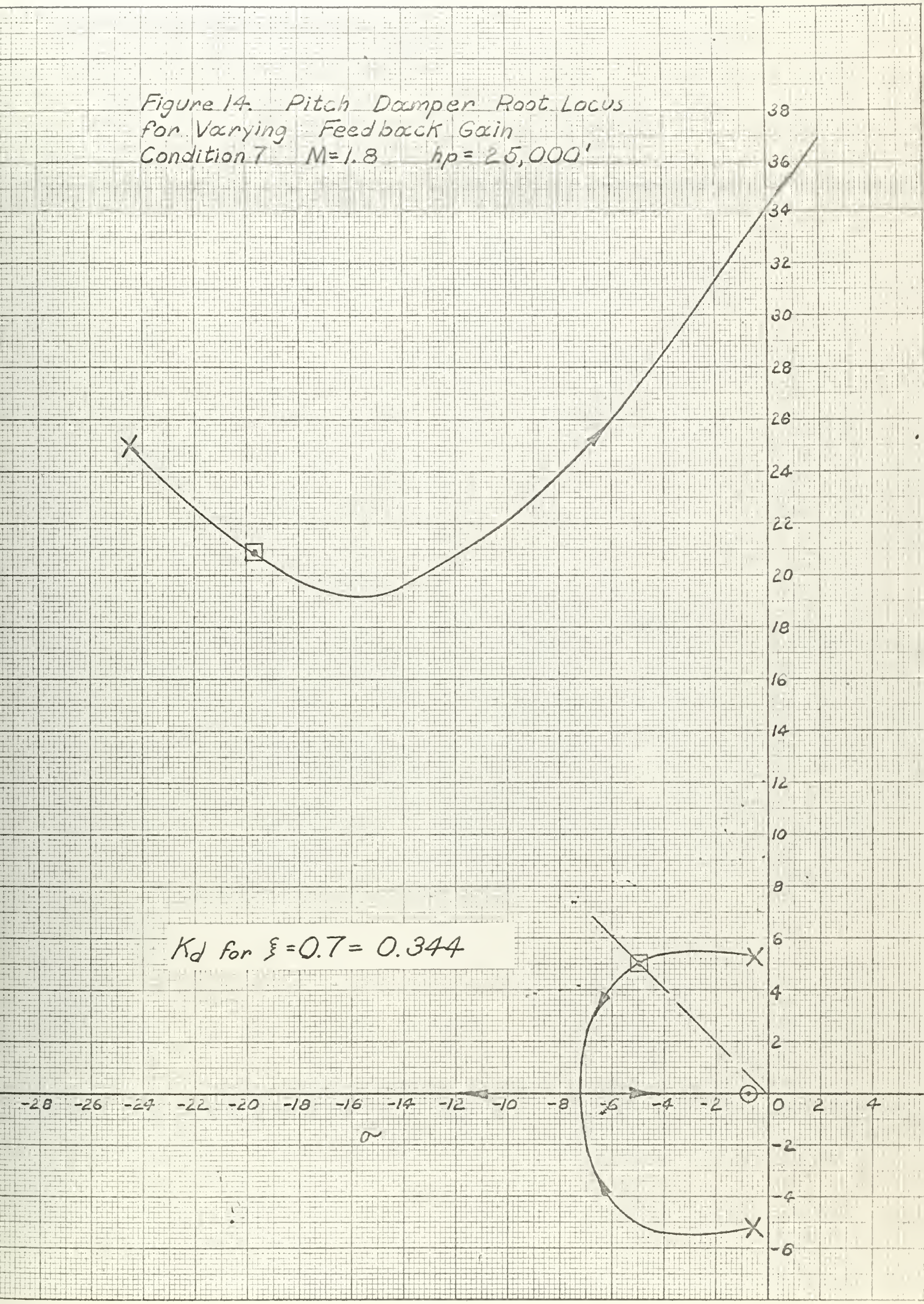


Figure 14. Pitch Damper Root Locus for Varying Feedback Gain
Condition 7 M=1.8 hp = 25,000'

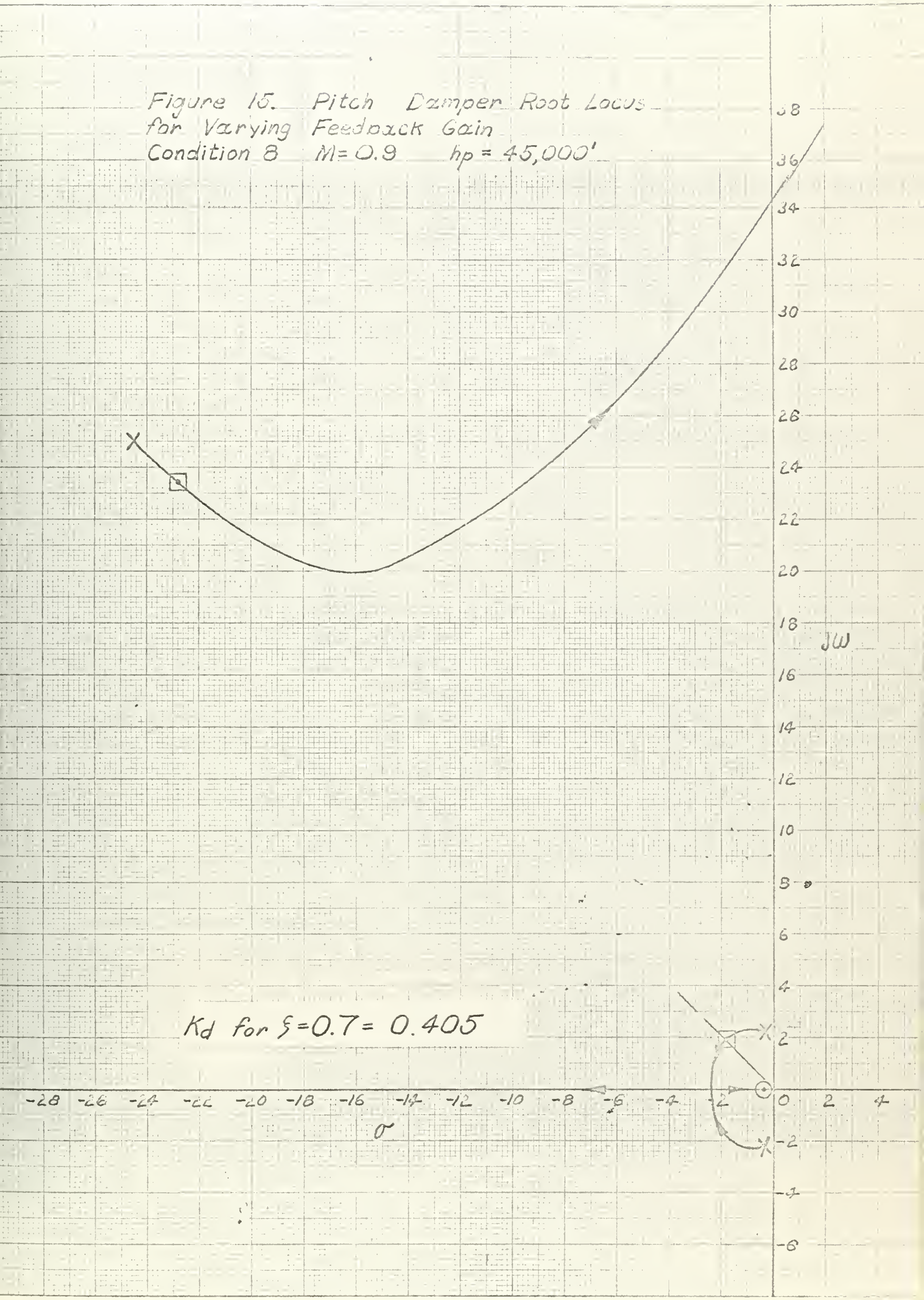


Figure 15. Pitch Damper Root Locus for Varying Feedback Gain
Condition 8 M= 0.9 hp = 45,000'

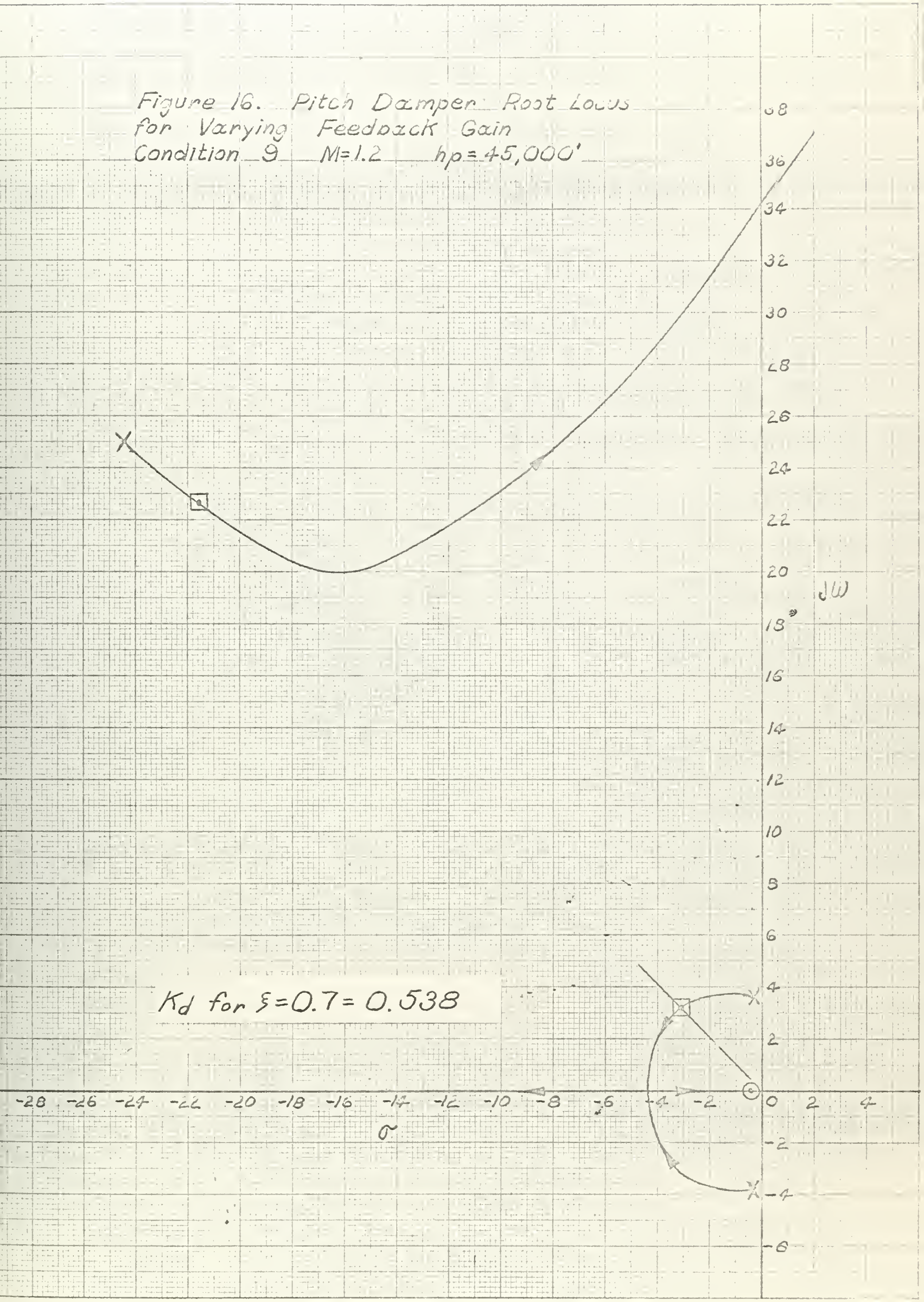


Figure 16. Pitch Damper Root Locus
for Varying Feedback Gain
Condition 9 M=1.2 hp=45,000'
Kd for ξ=0.7= 0.538
jω
σ
-28 -26 -24 -22 -20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4
38 36 34 32 30 28 26 24 22 20 18 16 14 12 10 8 6 4 2 -2 -4 -6

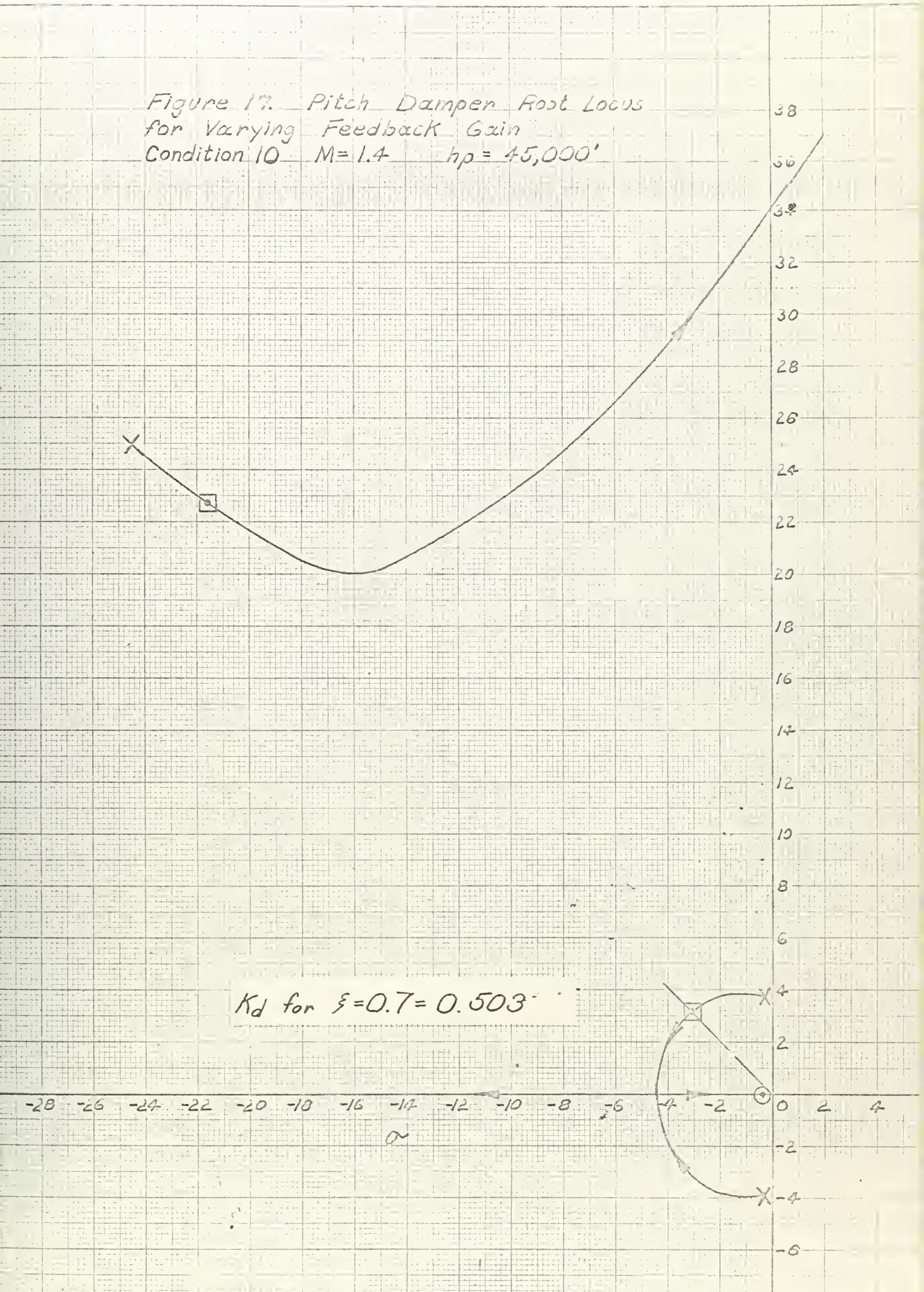


Figure 17. Pitch Damper Root Locus
for Varying Feedback Gain
Condition 10 M= 1.4 hp = 45,000'
Kd for ξ=0.7= 0.503
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
-2
-4
-6
-28 -26 -24 -22 -20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4
σ

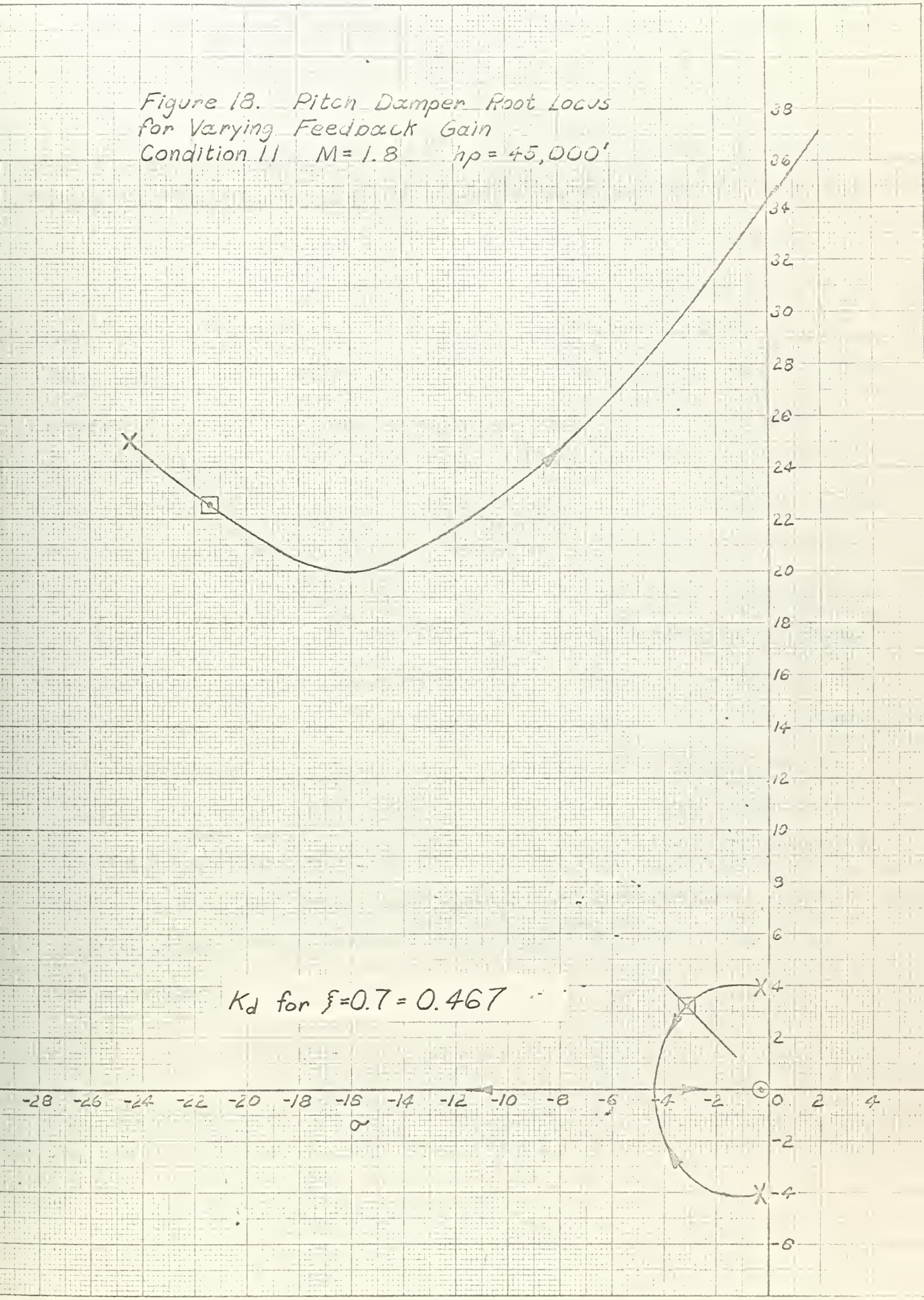


Figure 18. Pitch Damper Root Locus for Varying Feedback Gain
Condition II M = 1.8 hp = 45,000'

variation of damper gain as a function of measured airspeed to systems which program as functions of airspeed, altitude, fuel load, center of gravity position, wing flap position, landing gear position, and various combinations thereof. One fact becomes obvious from the preceeding discussion; the dynamic characteristics of the aircraft must be accurately known in advance at all flight conditions and for all airplane configurations if the programming system is to operate successfully. Such a detailed determination of airplane characteristics requires a large amount of wind tunnel and flight testing which is both time consuming and costly. And, even after the desired information has been determined it is usually necessary to conduct lengthy flight tests with the programming and stability augmentation systems installed in the airplane for final "tailoring" of the aircraft-system combination. In addition, accurate in-flight air data and airplane configuration information must be continusouly fed into the system during flight to provide the basis for gain scheduling. The instrumentation and transducers required are generally costly, and are usually complex enough that they create reliability and fail-safety problems.

A possible solution to these control problems is to design a system which is intelligent enough to determine when the airplane's dynamic performance has changed, and in what manner it has changed, and to use this information to adjust the stability augmentation system gains to obtain optimum performance over all flight conditions. This is the concept of the self-adaptive control system.

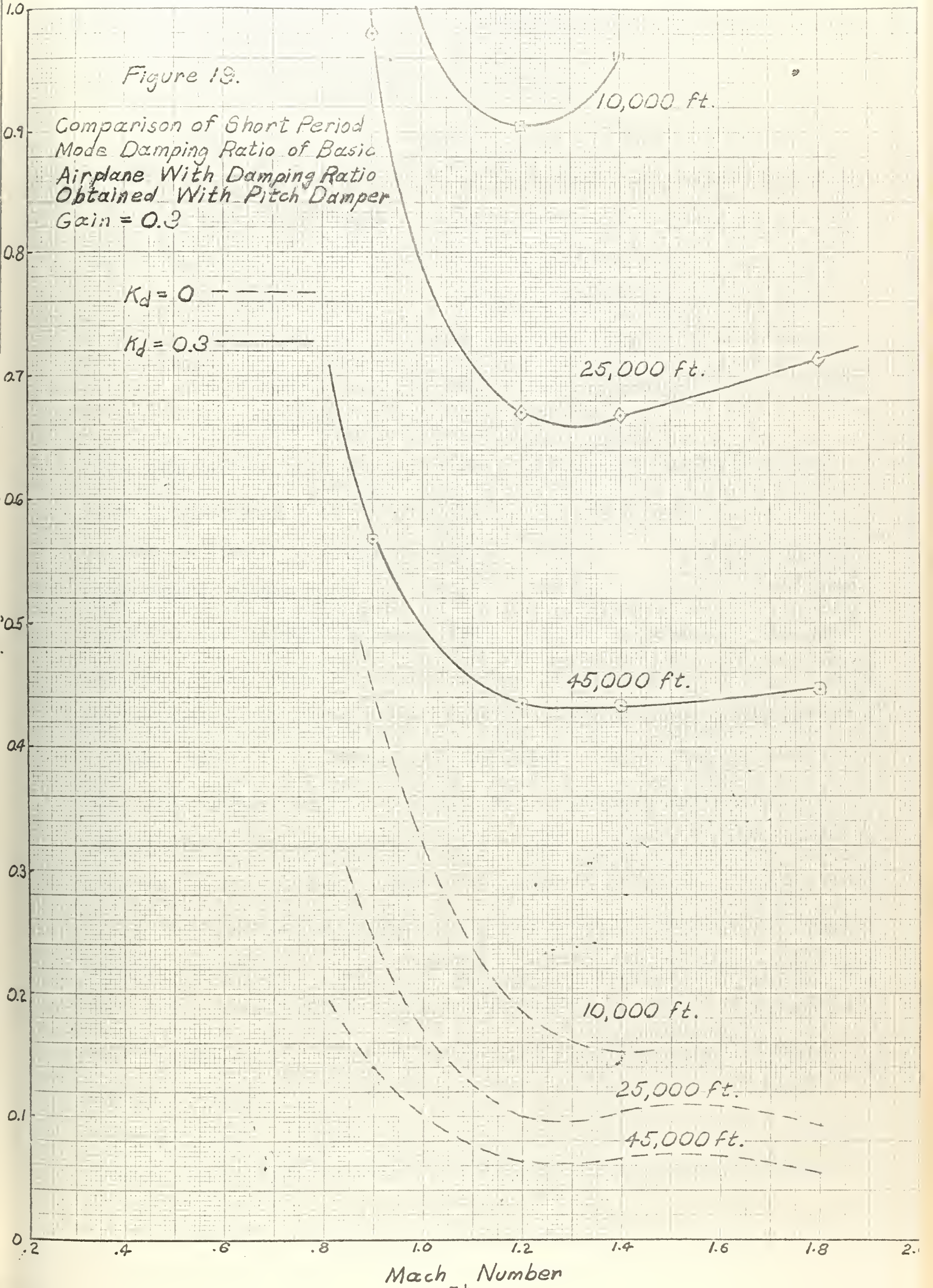


Figure 19.
Comparison of Short Period Mode Damping Ratio of Basic Airplane With Damping Ratio Obtained With Pitch Damper Gain = 0.3
K_d = 0
K_d = 0.3
10,000 ft.
25,000 ft.
45,000 ft.
10,000 ft.
25,000 ft.
45,000 ft.
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.
Mach Number

IV - SELF-ADAPTIVE CONTROL SYSTEMS

The concept of adaptive control has been applied to a wide range of control system types with the result that a number of definitions of adaptive control exist. However, the most often used definition, and also the one which appears to best fit the aircraft control problem is as follows:

> An adaptive system is a control system configuration in which the measurement of process dynamics or signal characteristics is utilized to automatically adjust the controller in an attempt to achieve optimum operation at all times.

From this definition it can be seen that the adaptive process must perform two functions, the first of which is the "identification" function, or the measurement of process dynamics or signal characteristics to determine present system state and compare it with the desired state. And, the second is the "command" function which is automatic changing of the system so that it will approach a desired state. In most adaptive control studies it is assumed that the variation of system characteristics takes place slowly with respect to the frequency of the system mode of interest, and thus the identification process can be allowed a relatively long measurement interval during which to determine system state. However, in the aircraft problem, it is desirable to perform the adaptation process relatively rapidly in comparison to the quantity being controlled, in this case the short period mode dynamics. This is because virtually instantaneous changes can take place in the airframe transfer function due to release of payload or certain Mach number effects on stability derivatives. For this reason, one of the objectives of this study is the investigation of an adaptive technique capable of rapid identification and command. Another consideration in mechanization of the adaptive process is the generally accepted requirement that, in order for the system to be considered truly adaptive, the identification process can not involve measurement of any variable which is external to the system. To illustrate the concepts and requirements discussed above, a block diagram of Gibson's ideal generalized adaptive system (Ref. 76) is shown in Figure 20.

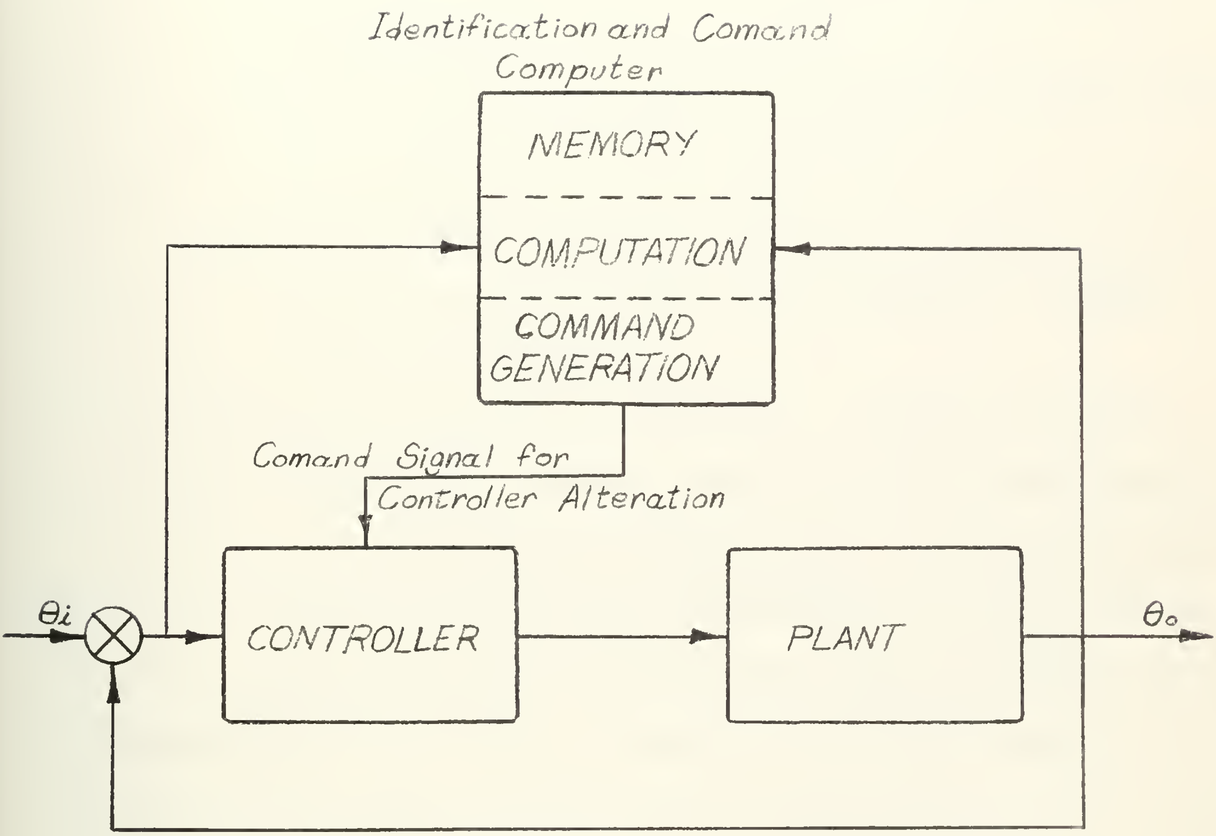


Figure 20. Gibson's Ideal Generalized Adaptive System

Classification of Adaptive Systems

In order to compare the approaches which have been taken to the design of adaptive control systems, Aseltine, in reference 4. has seperated them into 5 classes which, in some cases, have a certain amount of overlapping, but even so, they form a convenient framework for comparison. Aseltine's classes are:

1) Passive Adaptation: Systems which adapt without automatic changes in system characteristics, but rather through design for uniform operation over wide variations in environment.

2) Input Signal Adaptation: Systems which automatically adjust their parameters in accordance with input signal characteristics.

3) Extremum Adaptation: Systems which adjust to obtain a maximum or minimum of some system variable.

4) System Variable Adaptation: Systems in which the command process is actuated directly by measurement of a system variable or variables.

5) System Characteristic Adaptation: Systems in which the identification process measures system variables in order to determine system characteristics. The system characteristics thus determined are compared with desired characteristics and the differences used to generate command signals.

It has been noted from a survey of adaptive systems which have been investigated for possible use in aircraft control that, to date, all such systems fit into classes 1, 4, or 5. And, it has also been noted that an alternate classification system might be applied to aircraft adaptive control systems. The suggested classes are:

1) High Gain Adaptation: Systems in which the characteristics of the aircraft are submerged by the use of high feedback and/or forward loop gains. Some form of idealized model is normally used to provide either system input signals or output comparison signals, and the techniques usually involve either nonlinear processes, with attendant limit cycle, or

operation of a minor system mode near instability.

2) Impulse Response Adaptation: Systems in which the identification process computes a time history of the plant impulse response. The impulse response is then utilized by the command process to generate plant alteration signals.

3) Parameter Adaptation: Systems in which plant parameters are determined by direct computation. The error between the desired value of the parameter and its actual value is used as the command signal for plant alteration.

Survey of Adaptive Techniques which have Been Applied to the Airplane Stability Augmentation Problem

1) High Gain Adaptation Systems

The earliest studies of aircraft adaptive control involved systems which operated at high gains in order to force the variation of airframe parameters into a small range. One of the first of these studies was conducted by Rath of WADC (Ref. 134) on a method which was not truly adaptive, in that parameters were not automatically varied, however, it may be classed as passive adaptation. Rath's system used a parallel complementary optimum response model as shown in Figure 21. The goal of this study was to design a system which would produce optimum airframe dynamic characteristics, as airframe parameters varied, with constant system gain. A fourth order transfer function was selected as the optimum model for the pitch dynamics and the same input signal was applied to the airframe and to the model. The difference between model and plant output was obtained and various functions of the difference signal were utilized to improve the airframe response. Both the forward loop gain and the shaping loop gain were made high in order to "force" the system

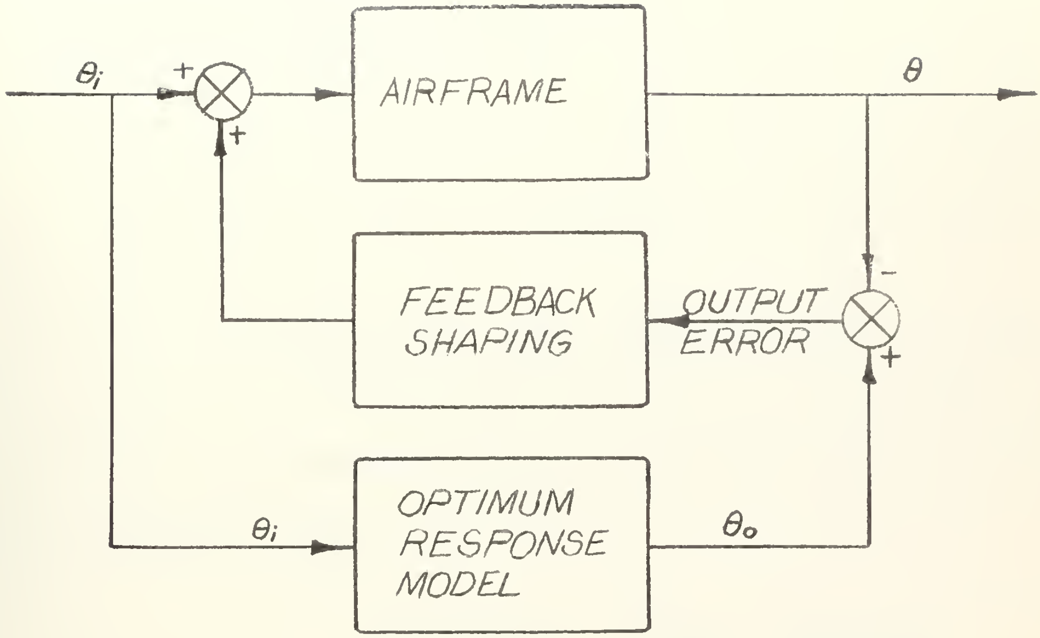


Figure 21. Rath's System for Passive Adaptation Using a Complementary Optimum Response Model

overall characteristics to resemble the model. Feedback functions studied included pitch angle, or position, error feedback, integrated position error feedback, rate error feedback, and rate-error-plus-position-error feedback. It should be noted that error, as referred to here, means the difference between plant output and model output rather than plant input-output error. The first two feedback functions produced overall instability at high gains, whereas rate error feedback resulted in a stable system with infinite steady state position error. Rate-error-plus-position-error feedback allowed very high gains to be used before the system became unstable, and while the system was stable, the airframe output matched the model output acceptably at most flight conditions while using a constant value of feedback gain. It was also found that, whatever feedback shaping was used, the basic airframe and hydraulic actuator must form a stable system with the shaped feedback loop alone. That is, the feedforward loop could not correct instabilities of the airframe and shaped feedback combination. The optimum transfer function feedforward in combination with the shaped feedback had the effect of changing the input to the airframe to a value which caused the airframe to have the desired output, assuming the airframe had the desired control rates and high gain control power.

An approach similar to Rath's passive adaptation method was taken by Dandois of Convair who utilized the inverse of the optimum response model in the feedback loop in combination with a high forward loop gain system as shown in Figure 22. (Ref. 43).

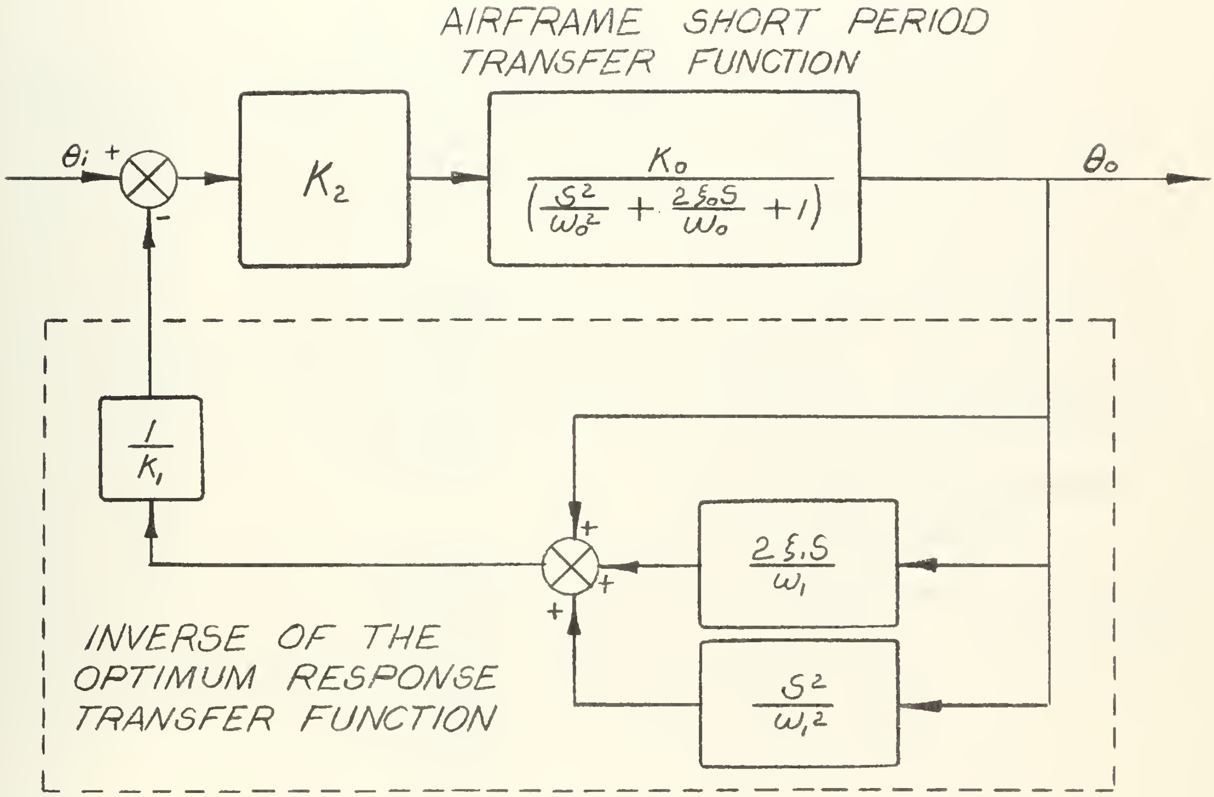


Figure 22. Dandois' System Using the Inverse of the Optimum Response Model in the Feedback Loop

He demonstrated that, with such a feedback arrangement, the overall system characteristics approached the characteristics of the optimum response model as forward loop gain was increased. In actual operation, the system held airframe parameters within acceptable limits over a wide range of flight conditions, and the higher the gain the better the performance. However, gain limits, dependent upon flight conditions, were reached at which the system became unstable. The instability was shown to be due to sensor nonlinearities rather than a basic, theoretical, system instability.

One of the first "high gain" adaptive systems to actually use an identification and command process was the system-variable-adaptation system of Flugge-Lotz and Taylor (Ref. 64), which is illustrated in Figure 23. As is shown in the block diagram, one of a number of alternate

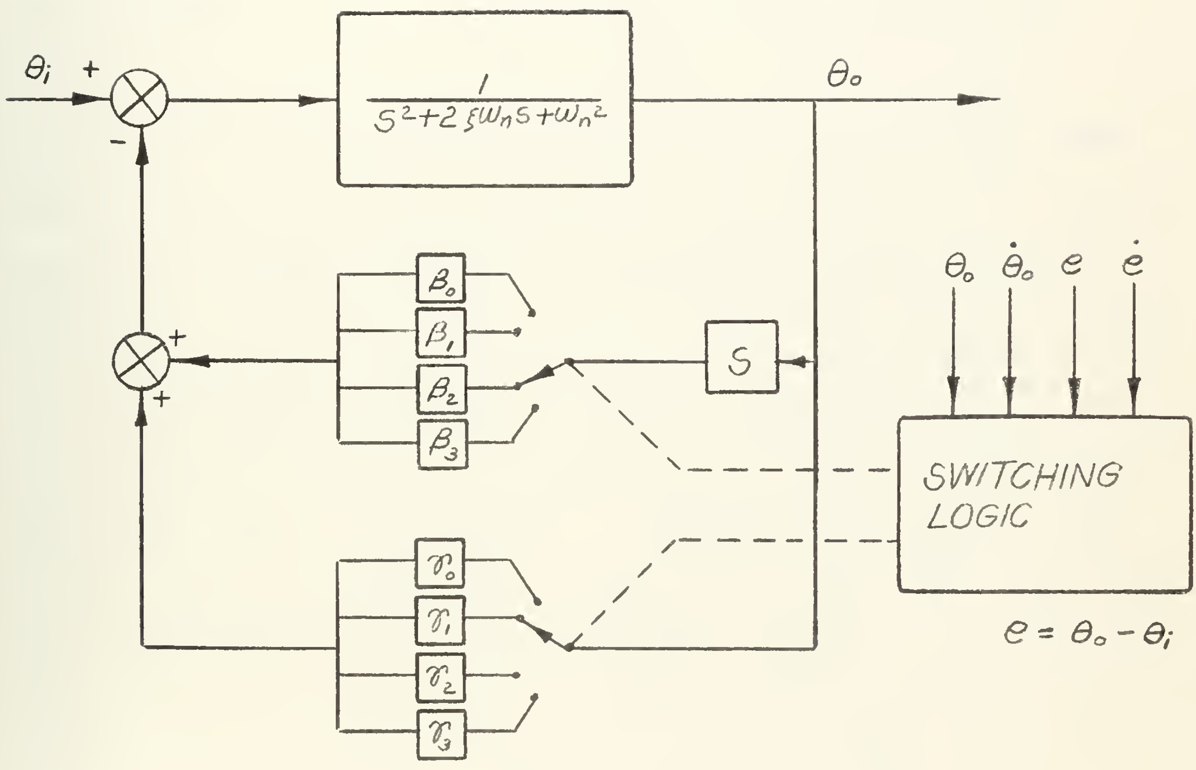


Figure 23. Nonlinear Adaptive System of Flugge-Lotz and Taylor

feedback paths was used according to a switching logic process based on simultaneous measurements of system output, error signal, and derivatives of these. The identification process thus consisted of the measurements and switching logic which determines the appropriate feedback path, and the command consisted of switching to the chosen path. Forward gain was kept high in order to "force" a small instantaneous error between input and output, and the system was nonlinear during any length of time comparable to the natural period of the plant because of the switching between feedback loops during one cycle of operation. The arrangement was shown to be insensitive to variations in plant parameters over wide limits, however a limit cycle persisted during operation due to the feedback nonlinearity.

The Minneapolis-Honeywell relay actuated adaptive system (Ref. 165), which is presently undergoing flight tests, is similar to the Flugge-Lotz mechanization but uses only one feedback path with switching logic in the forward loop. The switching logic derives its input from the error signal between optimum response model output and the process feedback signal as shown in Figure 24. The switching logic actuates a relay, or on-off controller, to operate the process at a high gain with the objective of maintaining a small error signal between model output and plant output. As in the Flugge-Lotz system, the airframe output approximately duplicates the model output when the forward gain is high, and the characteristic limit cycle is also present. A second adaptive feature of this system, aside from the switching logic, is the use of limit cycle amplitude measurement to vary forward loop gain. That is, forward gain is adjusted in such a manner that the limit cycle amplitude is held constant over all flight conditions. The prototype of this system was successfully tested in an F-94C fighter aircraft and an improved version is presently being flight tested in the X-15 high altitude research airplane.

Another application of the utilization of the inverse optimum response transfer function in the feedback loop, in conjunction with

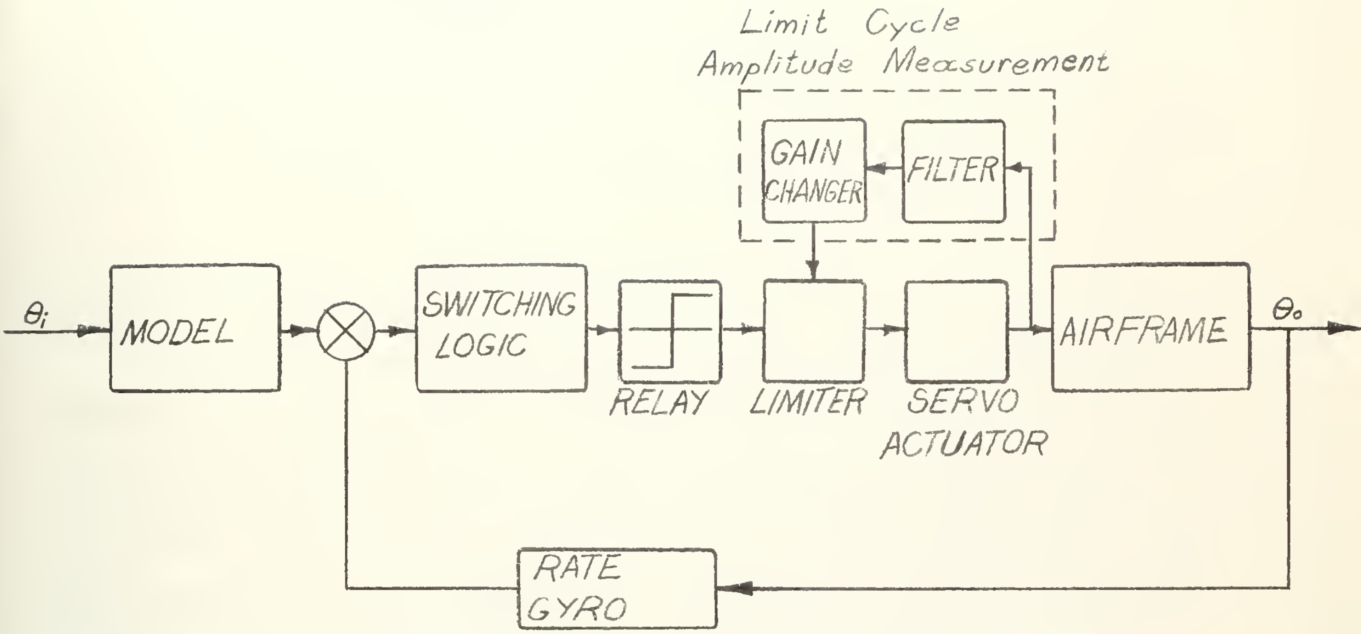


Figure 24. Minneapolis-Honeywell Relay Actuated Adaptive System

high forward loop gain, is the General Electric system (Ref. 32), which was recently tested in an F-102 interceptor aircraft. The mechanization is similar to that of Dandois shown in Figure 22, except that an active system-variable-adaptation feature has been added. This feature involves variation of the forward loop gain, as flight conditions change, to maintain the system just below the boundary of instability. As was previously mentioned, when considering systems of this type, the higher the gain, the closer will be the airframe characteristics to the optimum response characteristics and thus if the system is operated near the instability boundary, the maximum performance will be extracted. In operation, the adaptation system monitors the frequency of the control surface actuator because the actuator mode is the one which becomes unstable as forward gain is increased. The gain is then adjusted to maintain the frequency of this mode constant over all flight conditions. It should be noted that the actuator mode is a lightly damped oscillation and that normal control inputs are utilized to provide the required perturbations. The frequency measurement is accomplished with a narrow band pass filter and zero crossing counter arrangement.

2) Impulse Response Adaptation Systems

The function of the identification process in the impulse response adaptation class of systems is first, the determination of a time history of the plant impulse response through measurement of system input and output signals over a period of time, and second, the determination of plant characteristics through examination of the impulse response. An early method of impulse response self optimization which was investigated by Aseltine (Ref. 4) serves to illustrate the process, even though this particular mechanization would not be suitable for aeronautical use. The impulse response was obtained by periodic input pulses which were widely spaced with respect to the period of the plant mode of interest. After each perturbation, the system damping ratio was computed by separate integration of the positive and negative portions of the impulse response and obtaining the ratio of positive to negative area. Damping ratio was computed from this ratio and the system damping coefficient was varied in a closed loop manner to cause actual damping ratio to approach a desired value. This method was considered undesirable for aircraft use primarily because normal operating signals must be excluded from the input during the comparatively lengthy identification period.

A method of impulse response determination in which normal input signals could be included in the identification process was investigated by Anderson of Aeronutronics (Ref. 1). The mechanization made use of the convolution theorem whereby the system input convolved in time with the impulse response results in a time history of the system output. In operation, random noise with autocorrelation function $\phi_{ii}(\tau)$ was superimposed on the normal input signal as shown in Figure 25. The plant output was filtered to separate the noise component from the total response, and the input and output noise components were cross-correlated to obtain $\phi_{io}(\tau) = \int_{-\infty}^{\infty} g(X)\phi_{ii}(\tau - X)\,dX$. The noise input had a bandwidth 3 to 10 times the plant bandwidth so that $\phi_{ii}(\tau)$ was effectively an impulse and $\phi_{io}(\tau)$ was approximately the plant impulse response. Twelve channels of corss correlation were used and each channel provided one point on a time history of the impulse response. Damping ratio was determined and utilized as discussed in the preceeding paragraph.

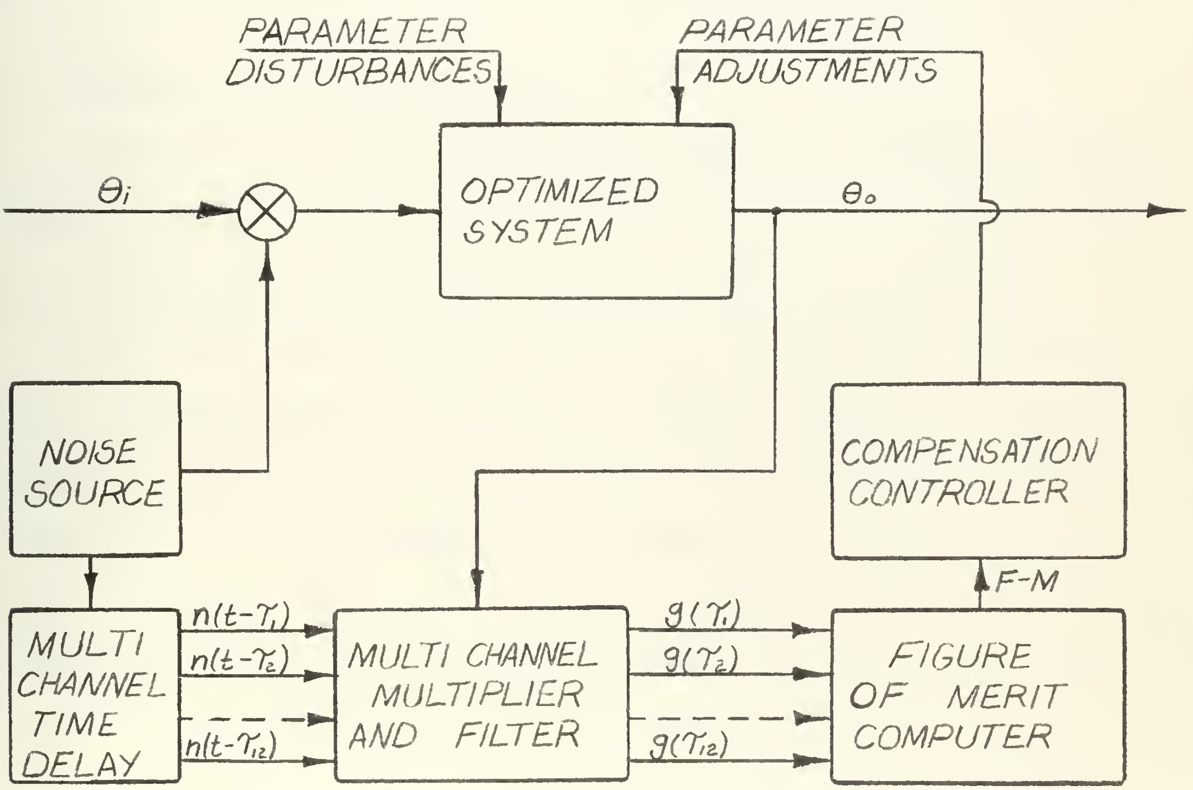


Figure 25. Block Diagram of Aeronutronic's Impulse Response Self Optimizing System

Another method of impulse response determination utilizing the convolution theorem in conjunction with simultaneous input and output measurements is currently under investigation at Hughes Aircraft. In this method, discrete samples of the input and output time histories are used as coefficients in two polynomials. The output polynomial is periodically divided by the input polynomial through the use of a high speed digital computer. The coefficients of the resultant polynomial can be shown to be amplitude samples of the system impulse response. One requirement for this method is that the amplitudes of the system output samples must be corrected for initial conditions existing at the beginning of the interval over which samples are taken. The correction process requires a knowledge of the impulse response and thus the impulse response must be determined in a convergent succession of polynomial divisions. At present, the requirements for convergence to the true impulse response are under investigation.

3) Parameter Adaptation Systems

The function of the identification process in a parameter adaptation system is the direct determination of plant parameters through measurements of, and operations upon, selected signals and their derivatives.

One of the first studies of parameter adaptation was conducted by Kalman, of Columbia University, who used a digital computer to determine plant parameters (Ref. 91). In Kalman's system, coefficients of the plant's pulse transfer function were computed by continuously measuring and storing sampled values of plant input and output. After a series of samples had been stored, values of the transfer function coefficients were assumed and a series of sample output points were digitally calculated using the known past input samples. The mean square error between calculated and known output points was determined and the assumed values of the coefficients were altered through the use of a technique of least squares filtering which changed coefficients in a manner that reduced the mean square error. This process was repeated at very high speed until the MSE was reduced below a desired limit. After the plant coefficients had been determined with sufficient accuracy, an optimal controller was synthesized in a second digital computer routine and the results were used to adjust the system controller.

A different approach to parameter adaptation was taken by Whitaker, of MIT, who used a model reference as shown in Figure 26. (Ref. 171).

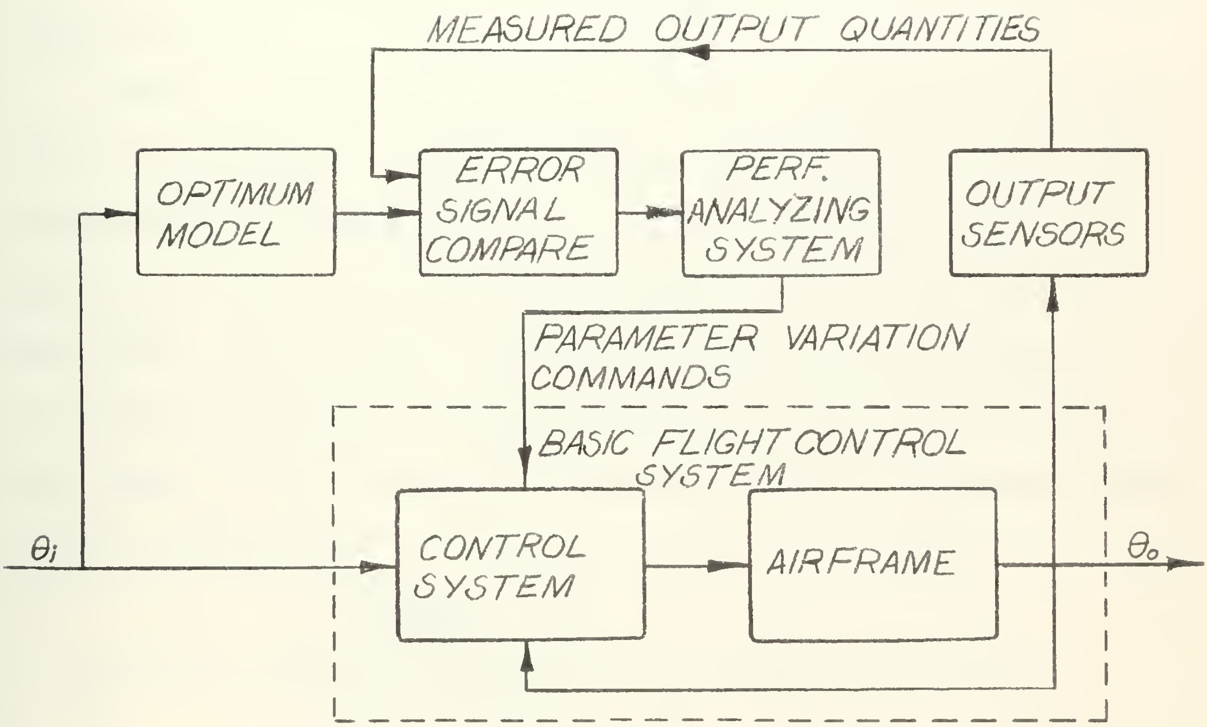


Figure 26. MIT Model Reference Parameter Adaptation System

The input signal was fed to both the plant and to the ideal model, and a difference function between plant output and model output was used to form an integral squared error. The ISE thus formed was required to be related to a single plant parameter which was then varied in a closed loop manner to cause the ISE to attain its minimum value. The primary objective of the study was the determination of what functions of the plant-output-to-model-output error were indicative of what parameter variations. After the desired relationships were determined, the system was mechanized and flight tested in an F-94A fighter aircraft and it was found that adpatation could occur within an interval corresponding to two or three time constants of the dominant system response modes. However, recent developments have made possible a significant reduction in convergence time and the elimination of much of the complexity of the previous mechanization (Ref. 127). These developments were derived from a recent doctoral research program at MIT which evolved an analytical design method for parameter adaptation model reference systems.

A method of adaptation employing an analog computer, which calculated values of instantaneous forward loop gain, was studied by Corbin of Brooklyn Polytechnic Institute (Ref. 38). The gain value calculated was compared with a desired level and the difference used as a signal to the gain controller. The differential equations of motion of the plant were set up in the analog computer as integral equations and were solved for the gain value, given continuous measurements of the plant input and output signals, as shown in Figure 27.

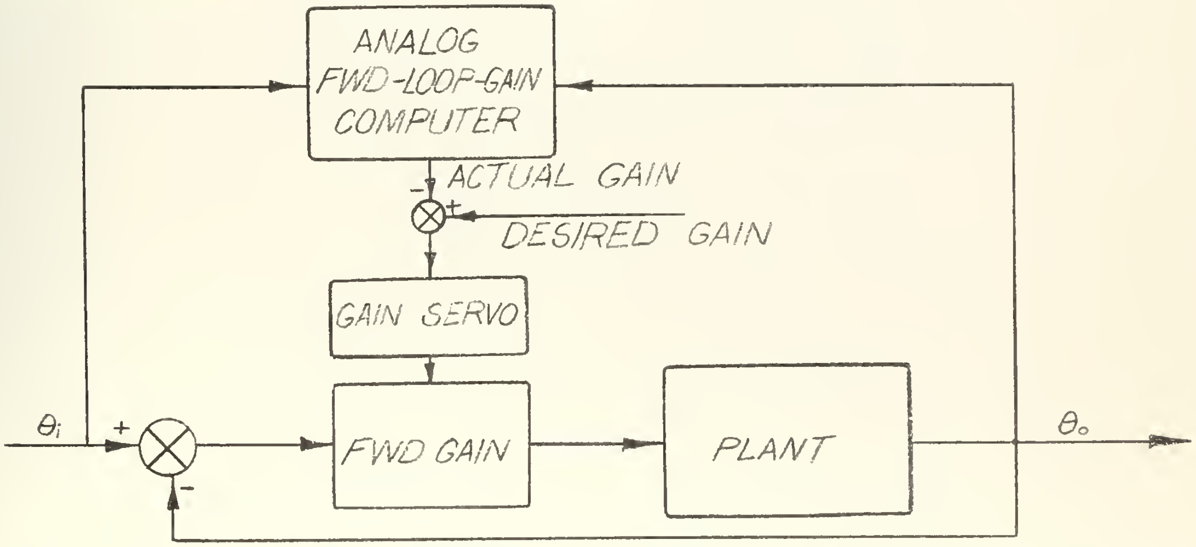


Figure 27. Corbin's Gain Adaptation Method Utilizing an Analog Computer in the Identification Process

A method similar to Corbin's approach, but which employed a digital computer to directly solve for the plant parameters, was investigated by Azgapetian of Servomechanisms Incorporated (Ref. 7.). In this study, the equations of motion of the second order system under investigation were converted to sample data notation, and relationships for damping ratio, natural frequency, and forward loop gain were derived in terms of past sample points of plant input and output. The equations were then mechanized in a digital computer routine which periodically sampled the input and output signals and computed plant parameters at a high rate. Parameters thus calculated were used in a closed loop manner to vary a controller which caused overall system dynamics to remain constant as plant characteristics were varied.

The adaptation method investigated in this study is similar to that discussed in the preceeding paragraph in that a digital computer is utilized to solve second order equations of motion for system damping ratio and natural frequency given system output samples. The primary differences are the method of solution, and the fact that the system under investigation is actually a sixth order system which includes a second order hydraulic servo and a fourth order airframe transfer function.

V - PROPOSED SYSTEM

As mentioned in the preceding section, it is proposed that an aircraft pitch damping system could be made to produce invariant short period damping ratio over all flight conditions by digital computation of the short-period damping ratio and closed loop adjustment of the pitch damper gain to cause the damping ratio to equal a desired value. A block diagram of the proposed system is shown in Figure 28 below. The proposed identification process thus consists of solving the airframe equations of motion

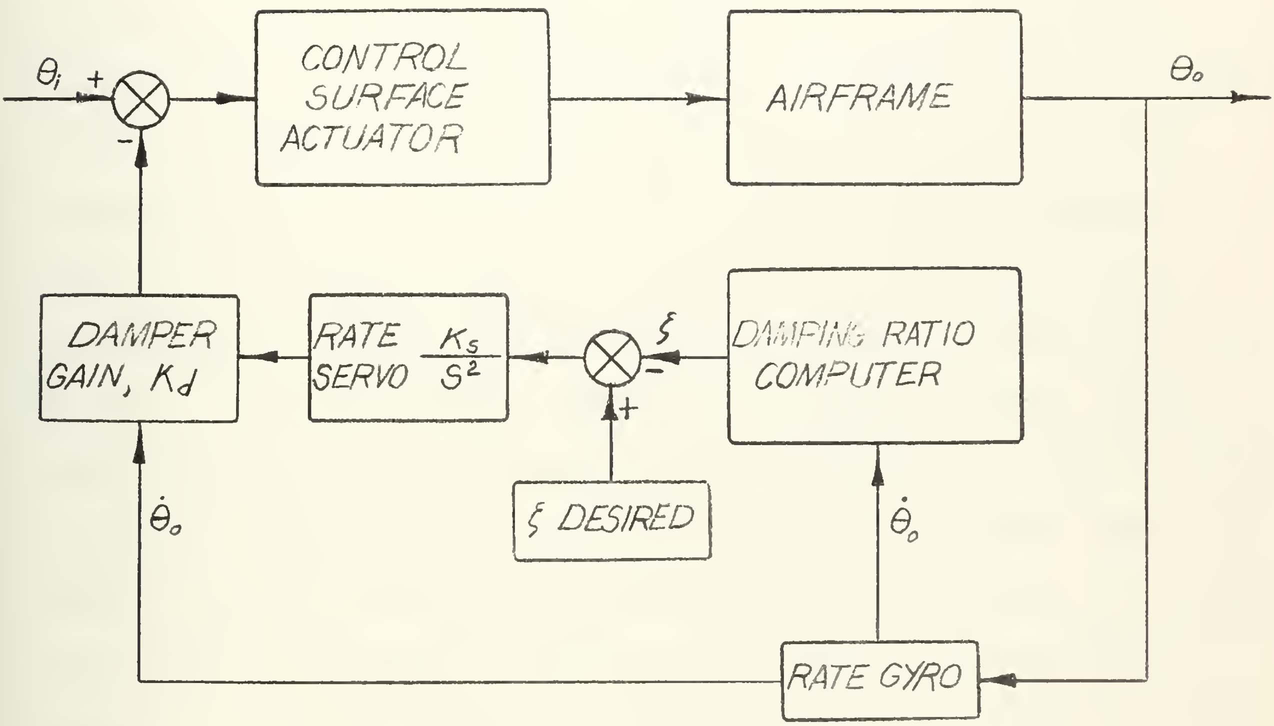


Figure 28. Proposed Self-Adaptive Pitch Damper

for values of damping ratio given samples of pitch angle and its derivatives, and the command process consists of varying damper gain at a rate proportional to the error between desired and computed ξ and in the direction indicated by the sign of the error. Basic assumptions and conditions for this study are:

1. The airframe and control system are linear. This has been shown to be true for small perturbations.
2. There is no control surface rate limiting or deflection limiting.
3. Airframe parameters are time variant but are assumed constant over the length of time required for one identification cycle.
4. System disturbances due to external noise (atmospheric turbulence) will not be considered.

It is also assumed that the short-period mode is dominant in the longitudinal dynamics and thus the airframe longitudinal response can be closely described by the short-period frequency and damping ratio. This assumption was verified by conducting two longitudinal flight simulation programs at each of the 11 conditions listed in Table II. The first program was run in two degrees of freedom, pitch and normal translation, and thus simulated the short-period mode dynamics, and the second program simulated the short-period and phugoid modes by inclusion of the longitudinal translation degree of freedom. Damping ratio and period were determined from pitch angle time histories obtained by pulsing the aircraft about the pitch axis and allowing it to return to equilibrium. Characteristics obtained from the simulation programs are listed in Table III. It is seen that the two degree and three degree of freedom characteristics are in close agreement, and thus the decision was made to investigate a computational method of determining the "pseudo" second order damping ratio of a sixth order, three degree of freedom, airframe-control system combination from sampled data.

Table III - Short-Period Mode Characteristics Compared with Damping Ratio and Period Obtained from a Longitudinal Three Degree of Freedom Flight Simulation.

Flight Condition		Damping Ratio		Period, Sec	
M	Altitude	2 DF	3DF	2DF	3DF
0.9	10,000[1]	0.444	0.440	1.650	1.700
1.2	10,000[1]	0.185	0.177	1.050	1.005
1.4	10,000[1]	0.155	0.156	1.025	1.020
0.9	25,000[1]	0.245	0.244	1.974	2.025
1.2	25,000[1]	0.100	0.1028	1.225	1.275
1.4	25,000[1]	0.104	1.105	1.199	1.175
1.8	25,000[1]	0.095	0.096	1.200	1.225
0.9	45,000[1]	0.140	0.141	2.900	2.975
1.2	45,000[1]	0.064	0.066	1.725	1.725
1.4	45,000[1]	0.066	0.067	1.650	1.675
1.8	45,000[1]	0.057	0.058	1.600	1.599

Proposed Computational Method

The generalized differential equation of motion for a second order system is:

$$\ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2\theta = K\delta_e$$

from which an expression for damping ratio can be derived as:

$$\xi = \frac{K\delta_e}{2\omega_n\dot{\theta}} - \frac{\ddot{\theta}}{2\omega_n\dot{\theta}} - \frac{\omega_n\theta}{2\dot{\theta}}$$

It is seen that, in this form, the damping ratio is a function of the instantaneous values of θ and its first and second derivatives, the input forcing function, and the natural frequency. Azgapetian has shown in Ref. 7. that an expression for, ξ which is independent of ω_n , can be derived as an integral equation. But it was decided, for this study, to avoid long term integrations and their attendant drifts, and rather to make use of the high speed iterating capabilities of digital computers by deriving a companion expression for natural frequency to be used in an iterative process for arriving at ξ and ω_n simultaneously. The expression for natural frequency being:

$$\omega_n = -\frac{\xi\dot{\theta}}{\theta} \pm \sqrt{\left(\frac{\xi\dot{\theta}}{\theta}\right)^2 - \frac{\ddot{\theta}}{\theta} + \frac{K\delta_e}{\theta}}$$

which is in terms of instantaneous values of θ and its first two derivatives, control input, and damping ratio. It is noted that the term, k, by which the control deflection is multiplied, corresponds to the airplane elevator effectiveness, or steady state airframe gain, which is also a function of flight conditions and which therefore must be determined in a spearate computation routine. Corbin, in Ref. 38. has devised a method of computing the instantaneous value of forward loop gain of a second order system by utilizing integral equations. Corbin's system utilized an analog computer which was supplied continuous values of plant input and output, but, the method could be converted to digital form with sampled inputs and used in this study if desired. It was decided not to introduce this additional complexity into the program but rather to concentrate solely on the plant parameter computation method and its application to an adaptive system because the computation method was, as yet, untried. This decision necessitated the exclusion of pilot control inputs to the system because, if pilot inputs were to be included, the value of the airframe gain would have to be known. It should be made clear that control deflections called for by the pitch damper are not included in the preceeding expressions for natural frequency and damping ratio because the airplane pitching moment created by the damping system is considered to be added to the natural aerodynamic damping moment to make up total damping. If pilot inputs were to be included, the control deflection created by the pitch damper would be subtracted from total deflection and the remaining control deflection would be used as the input term, δ_e , in the ξ and ω_n equations. With pilot control input excluded, the equations are reduced to:

$$\omega_n = -\frac{\xi\dot{\theta}}{\theta} \pm \sqrt{\left(\frac{\xi\dot{\theta}}{\theta}\right)^2 - \frac{\ddot{\theta}}{\dot{\theta}}}$$

$$\xi = -\frac{\ddot{\theta}}{2\omega_n\dot{\theta}} - \frac{\omega_n\theta}{2\dot{\theta}}$$

The proposed iterative computational routine is as follows:

1) Measure and store sampled values of θ, $\dot{\theta}$, and $\ddot{\theta}$ at two separate instants in time.

2) Assume a value of damping ratio and compute ω_n using the first sample set of θ, $\dot{\theta}$, & $\ddot{\theta}$ and assumed ξ . The ω_n equation is used in the initial calculation because the value of ω_n is dependent upon variations in ξ to a lesser extent than is the value of ξ dependent upon variations in ω_n .

3) Use the value of ω_n computed in 2) and the second sample set of θ, $\dot{\theta}$, & $\ddot{\theta}$ to compute ξ .

4) Use the new value of ξ and the first sample set to compute a new value of ω_n .

5) Repeat steps 3) and 4) in sequence until the successive values of damping ratio and natural frequency either converge or diverge.

Although it might seem that only one sample set of θ and its first two derivatives is required to define the parameters of a second order system, it is seen that the method described above requires two sets for convergence. If only one set were used in both equations, the ω_n obtained in the first calculation, when substituted into the second equation would produce the originally assumed ξ , and convergence would be impossible. This is because the relationships for ω_n and ξ are actually just two forms of the same equation.

An appropriate question to ask at this time is, will the computations converge, and if so, under what conditions will they converge? The answer is given in the following section.

VI APPLICATION OF PROPOSED ITERATIVE DIGITAL DAMPING RATIO COMPUTER TO A SECOND ORDER SYSTEM

It was felt that it would be better to test the proposed plant identification method first with a simple 2nd order system before adding the complexity of the phugoid and actuator modes of the three degree of freedom flight simulation. When done in this manner, the validity of the basic idea could either be proved or disproved with certainty because the method is based on the assumption that the airframe closely resembles a second order system. Objectives of the program were to determine:

1) Will the iterative computing method converge to correct values of ζ and ω_n ?
2) Conditions for convergence.
3) Should both forms of the ω_n equation be used, and if so when is a particular form applicable?
4) Effect of system damping ratio on convergence.
5) Effect of sample spacing on convergence.

The second order system used in the following programs was of the form:

$$\ddot{\theta} + (\text{DAMPING FACTOR})(12)\dot{\theta} + 36\theta = 0$$

where the natural frequency = 6 radians per second was approximately the median of the airframe short period ω_n of this study, and the damping factor could be varied to produce damping ratios over the range from 0.1 to 1.0 thus approximating the variation of the short-period mode damping ratio. Outputs were obtained by releasing the system from an initial displacement, θ_0 at time = 0 and allowing it to oscillate freely. Samples of θ, $\dot{\theta}$, & $\ddot{\theta}$ were periodically taken and stored for use in the ζ calculations. The dynamic simulation was used in four programs which were performed to test the damping ratio calculator and the influence of certain variables on its operation. The programs and results are discussed in the following pages and sample listings of each are included in appendix VI.

Program 1. Calculation of ω_n Using Known Values of ζ, and Calculation of ζ Using Known Value of ω_n.

As a check on the accuracy of the dynamic simulation, the samples of output were used in a series of non-iterative calculations in which ω_n was determined using the known value of ζ, and ζ was determined using the known value of ω_n.

It is recalled that the relationship for natural frequency,

$$\omega_n = -\frac{\zeta\dot{\theta}}{\theta} \pm \sqrt{\left(\frac{\zeta\dot{\theta}}{\theta}\right)^2 - \frac{\ddot{\theta}}{\theta}}$$

has two forms which will be referred to for identification, as the positive and negative forms. Both forms were used in the calculations in order to determine under what conditions each was applicable. A third purpose of the program was to obtain a basis for comparing the accuracy of ζ and ω_n computed by the iterative method. Nine runs were made in which damping ratio was varied from 0.1 to 0.9 in 0.1 steps, and partial listings of the runs made at $\zeta = 0.2$ and 0.7 are presented in table IV.

It is seen from table IV that the calculated values of ζ and ω_n have approximately 0.5% accuracy except in regions where the ratios $\frac{\dot{\theta}}{\theta}$ and $\frac{\ddot{\theta}}{\theta}$ become large. The errors in ζ and ω_n obtained with this series of runs are due to errors in the generated output values; θ, $\dot{\theta}$, & $\ddot{\theta}$; and thus are indicative of the solution accuracy of the simulated second order system, and they also represent the highest accuracy which can be expected from the iterative method. It is also observed that the calculated values of ζ have very little variation whereas the ω_n error increases in the regions where θ is small, causing the ratios of $\frac{\dot{\theta}}{\theta}$ and $\frac{\ddot{\theta}}{\theta}$ to be large, and it is in these regions, before θ changes sign, that the negative form of the ω_n equation is applicable. Increasing damping ratio had the effect of widening the regions in which ω_n errors appear and in which the negative ω_n

Table IV Calculation of Natural Frequency Given Correct Damping Ratio, and Calculation of Damping Ratio Given Correct Natural Frequency for a Second Order System With $\omega_n = 6.0$ Rad/Sec and $\zeta = 0.2$ and 0.7

Sample Spacing 0.0125 Cycle.

PLANT IDENTIFICATION ROUTINE - SECOND ORDER

CONDITION= 2 DAMPING RATIO= .2000

ACCELERATION	VELOCITY	DISPLACEMENT	OMEGAN1	OMEGAN2	DAMPRAT
-34.909819	-.434735	.997322	6.004203	-5.829842	.190494
-33.595382	-.863053	.989194	6.004834	-5.655841	.194619
-32.133230	-1.274004	.975818	6.005478	-5.483248	.195985
-30.535811	-1.665819	.957424	6.006144	-5.310185	.196672
-28.815956	-2.036888	.934259	6.006840	-5.134753	.197089
-26.986802	-2.385762	.906594	6.007577	-4.954950	.197372
-25.061701	-2.711158	.874713	6.008367	-4.768574	.197578
-23.054149	-3.011961	.838918	6.009225	-4.573107	.197736
-20.977702	-3.287225	.799521	6.010172	-4.365574	.197863
-18.845898	-3.536173	.756847	6.011234	-4.142336	.197968
-16.672187	-3.758197	.711229	6.012447	-3.898810	.198057
-14.469854	-3.952858	.663006	6.013863	-3.629053	.198134
-12.251956	-4.119879	.612522	6.015557	-3.325122	.198203
-10.031248	-4.259145	.560125	6.017644	-2.976075	.198264
-7.820131	-4.370700	.506159	6.020313	-2.566302	.198321
-5.630584	-4.454739	.450972	6.023889	-2.072655	.198373
-3.474119	-4.511603	.394904	6.028996	-1.459177	.198422
-1.361722	-4.541775	.338293	6.036992	-.666768	.198469
.696184	-4.545873	.281469	6.051495	.408725	.198514
2.689788	-4.524639	.224752	6.086341	1.966338	.198558
4.609921	-4.478935	.168455	6.272509	4.362838	.198602
6.448092	-4.409733	.112877	9.794177	5.832562	.198645
8.196513	-4.318107	.058305	23.690226	5.934103	.198688
11.396610	-4.072334	-.046742	5.972716	-40.821967	.198778
12.836410	-3.920762	-.096718	5.979735	-22.194992	.198825
14.162731	-3.751897	-.144689	5.984347	-16.356640	.198875
15.371551	-3.567184	-.190449	5.987638	-13.479787	.198929
16.459617	-3.368112	-.233809	5.990128	-11.752290	.198986
17.424440	-3.156207	-.274598	5.992097	-10.589657	.199049
18.264289	-2.933022	-.312667	5.993708	-9.745970	.199120
18.978178	-2.700125	-.347884	5.995064	-9.099695	.199200
19.565846	-2.459094	-.380136	5.996233	-8.583825	.199292
20.027742	-2.211504	-.409334	5.997261	-8.158339	.199402
20.364998	-1.958920	-.435403	5.998182	-7.797821	.199535
20.579404	-1.702891	-.458292	5.999021	-7.485314	.199704
20.673379	-1.444938	-.477967	5.999795	-7.209030	.199925
20.649939	-1.186546	-.494414	6.000520	-6.960482	.200234
20.512663	-.929164	-.507635	6.001207	-6.733358	.200700
20.265655	-.674188	-.517653	6.001866	-6.522824	.201496
19.913508	-.422961	-.524506	6.002506	-6.325066	.203192
19.461264	-.176767	-.528248	6.003134	-6.136986	.209474
18.914369	.063176	-.528951	6.003758	-5.955983	.168654
18.278638	.295721	-.526699	6.004383	-5.779798	.192338
17.560206	.519797	-.521593	6.005016	-5.606394	.195131
16.765485	.734408	-.513744	6.005666	-5.433857	.196223
15.901123	.938644	-.503276	6.006340	-5.260313	.196810

Table IV - Continued

ACCELERATION	VELOCITY	DISPLACEMENT	OMEGAN1	OMEGAN2	DAMPRAT
14.973961	1.131675	-.490325	6.007046	-5.083842	.197180
13.990984	1.312760	-.475034	6.007797	-4.902394	.197437
12.959284	1.481246	-.457558	6.008604	-4.713691	.197627
11.886012	1.636569	-.438058	6.009486	-4.515100	.197775
10.778344	1.778253	-.416701	6.010463	-4.303480	.197895
9.643431	1.905914	-.393660	6.011564	-4.074955	.197995
8.488369	2.019255	-.369113	6.012829	-3.824603	.198080
7.320155	2.118068	-.343239	6.014315	-3.545987	.198155
6.145654	2.202232	-.316222	6.016107	-3.230430	.198221
4.971565	2.271711	-.288245	6.018338	-2.865862	.198281
3.804387	2.326550	-.259490	6.021224	-2.434886	.198336
2.650391	2.366876	-.230141	6.025154	-1.911379	.198387
1.515589	2.392890	-.200378	6.030897	-1.254149	.198436
.405711	2.404869	-.170378	6.040203	-.394233	.198482
-.673819	2.403160	-.140314	6.058135	.792692	.198527
-1.717905	2.388172	-.110354	6.107544	2.548848	.198571
-2.721799	2.360380	-.080663	6.565521	5.139424	.198614
-3.681113	2.320313	-.051396	12.176150	5.882218	.198657
-4.591836	2.268554	-.022704	34.023796	5.944415	.198701
-5.450344	2.205735	.005272	5.964874	-173.323327	.198745
-6.253409	2.132527	.032396	5.975077	-32.305450	.198791
-6.998207	2.049643	.058545	5.981237	-19.985186	.198839
-7.682317	1.957825	.083600	5.985395	-15.352948	.198890
-8.303729	1.857846	.107456	5.988418	-12.904145	.198945
-8.860837	1.750500	.130016	5.990736	-11.376240	.199004
-9.352439	1.636598	.151191	5.992589	-10.322459	.199069
-9.777735	1.516966	.170907	5.994118	-9.544512	.199142
-10.136312	1.392433	.189095	5.995415	-8.940881	.199225
-10.428141	1.263836	.205701	5.996540	-8.454162	.199322
-10.653565	1.132007	.220678	5.997534	-8.049409	.199437
-10.813283	.997771	.233991	5.998430	-7.704088	.199579
-10.908339	.861944	.245615	5.999248	-7.402979	.199761
-10.940106	.725327	.255536	6.000007	-7.135387	.200003
-10.910265	.588699	.263748	6.000720	-6.893539	.200347
-10.820793	.452819	.270257	6.001398	-6.671603	.200881
-10.673937	.318418	.275075	6.002051	-6.465079	.201841
-10.472195	.186199	.278226	6.002687	-6.270382	.204106
-10.218301	.056831	.279742	6.003313	-6.084575	.216423
-9.915192	-.069053	.279662	6.003936	-5.905169	.184186
-9.565996	-.190857	.278033	6.004563	-5.729981	.193503
-9.174003	-.308025	.274909	6.005200	-5.557017	.195530
-8.742644	-.420043	.270353	6.005856	-5.384384	.196424
-8.275467	-.526441	.264432	6.006538	-5.210202	.196932
-7.776116	-.626795	.257218	6.007256	-5.032525	.197262
-7.248304	-.720726	.248789	6.008021	-4.849246	.197497
-6.695796	-.807900	.239228	6.008848	-4.658001	.197673
-6.122380	-.888033	.228621	6.009755	-4.456031	.197812
-5.531850	-.960888	.217057	6.010763	-4.240008	.197925
-4.927986	-1.026274	.204630	6.011907	-4.005796	.198020
-4.314526	-1.084048	.191432	6.013228	-3.748096	.198102
-3.695155	-1.134112	.177561	6.014792	-3.459917	.198174
-3.073483	-1.176417	.163112	6.016693	-3.131756	.198239
-2.453023	-1.210955	.148182	6.019084	-2.750264	.198297
-1.837183	-1.237762	.132870	6.022218	-2.295984	.198351
-1.229240	-1.256917	.117270	6.026563	-1.739318	.198402
-.632336	-1.268539	.101478	6.033073	-1.032846	.198449

Table IV - Continued

CONDITION= 7 DAMPING RATIO= .7000

ACCELERATION	VELOCITY	DISPLACEMENT	OMEGAN1	OMEGAN2	DAMPRAT
-32.518830	-.420371	.997379	6.012666	-5.422600	.671394
-29.005397	-.804694	.989677	6.012680	-4.874358	.685868
-25.687407	-1.146321	.977440	6.012694	-4.370803	.690646
-22.563123	-1.447687	.961187	6.012708	-3.904104	.693036
-19.629943	-1.711196	.941405	6.012722	-3.467937	.694478
-16.884500	-1.939218	.918555	6.012737	-3.057109	.695446
-14.322759	-2.134075	.893063	6.012753	-2.667295	.696145
-11.940102	-2.298034	.865332	6.012769	-2.294832	.696676
-9.731413	-2.433303	.835732	6.012786	-1.936570	.697096
-7.691155	-2.542022	.804610	6.012804	-1.589752	.697437
-5.813440	-2.626260	.772284	6.012823	-1.251924	.697721
-4.092098	-2.688010	.739047	6.012843	-.920861	.697963
-2.520738	-2.729188	.705169	6.012865	-.594501	.698173
-1.092801	-2.751627	.670895	6.012890	-.270896	.698358
.198384	-2.757078	.636449	6.012917	.051839	.698522
1.359554	-2.747209	.602032	6.012947	.375569	.698670
2.397471	-2.723603	.567826	6.012981	.702179	.698805
3.318882	-2.687758	.533993	6.013020	1.033626	.698929
4.130484	-2.641089	.500677	6.013065	1.371978	.699045
4.838895	-2.584926	.468006	6.013120	1.719973	.699153
5.450624	-2.520520	.436089	6.013186	2.070580	.699254
5.972048	-2.449037	.405022	6.013269	2.452076	.699351
6.409392	-2.371569	.374888	6.013377	2.843134	.699444
6.768709	-2.289128	.345754	6.013524	3.255442	.699533
7.055865	-2.202652	.317676	6.013737	3.693355	.699619
7.276529	-2.113008	.290701	6.014078	4.162070	.699703
7.436159	-2.020993	.264861	6.014711	4.667840	.699786
7.539795	-1.927337	.240183	6.016315	5.217939	.699867
7.593052	-1.832705	.216682	6.028037	5.813242	.699948
7.600117	-1.737703	.194366	6.510341	6.006137	.700029
7.565747	-1.642875	.173238	7.266745	6.009910	.700110
7.494262	-1.548714	.153292	8.133335	6.010925	.700192
7.389753	-1.455656	.134516	9.138630	6.011397	.700275
7.256079	-1.364091	.116894	10.325568	6.011672	.700361
7.096868	-1.274361	.100406	11.757073	6.011852	.700448
6.915523	-1.186762	.085026	13.528653	6.011979	.700538
6.715228	-1.101552	.070727	15.792533	6.012075	.700633
6.498949	-1.018948	.057477	18.807121	6.012150	.700731
6.269440	-.939133	.045242	23.049256	6.012211	.700836
6.029253	-.862256	.033986	29.507034	6.012261	.700947
5.780741	-.788436	.023672	40.616290	6.012304	.701066
5.526069	-.717763	.014262	64.445604	6.012341	.701194
5.267215	-.650302	.005715	153.293882	6.012373	.701335
5.005985	-.586093	-.002009	6.012401	-414.409029	.701489
4.744016	-.525155	-.008951	6.012427	-88.150044	.701662
4.482787	-.467489	-.015152	6.012450	-49.208001	.701856
4.223622	-.413077	-.020652	6.012472	-34.015191	.702079
3.967704	-.361885	-.025492	6.012492	-25.886909	.702338
3.716082	-.313867	-.029712	6.012510	-20.801486	.702645
3.469674	-.268962	-.033352	6.012527	-17.302693	.703017
3.229280	-.227100	-.036449	6.012544	-14.735417	.703479
2.995589	-.188202	-.039042	6.012560	-12.761324	.704074
2.769182	-.152180	-.041166	6.012575	-11.187999	.704872
2.550546	-.118940	-.042858	6.012590	-9.897905	.706010
2.340076	-.088382	-.044151	6.012604	-8.815159	.707773
2.138085	-.060402	-.045078	6.012618	-7.888553	.710899
1.944809	-.034893	-.045671	6.012632	-7.082255	.718024

Table IV - Continued

CCELERATION	VELOCITY	DISPLACEMENT	OMEGAN1	OMEGAN2	DAMPRAT
1.760412	-.011745	-.045960	6.012645	-6.370415	.751011
1.584997	.009154	-.045974	6.012659	-5.733891	.637835
1.418607	.027917	-.045740	6.012673	-5.158183	.680693
1.261231	.044657	-.045285	6.012687	-4.632083	.688602
1.112814	.059486	-.044632	6.012701	-4.146765	.691943
.973255	.072514	-.043805	6.012715	-3.695161	.693794
.842418	.083853	-.042826	6.012729	-3.271520	.694977
.720133	.093611	-.041715	6.012745	-2.871087	.695802
.606202	.101892	-.040492	6.012760	-2.489869	.696412
.500400	.108799	-.039174	6.012777	-2.124461	.696885
.402483	.114434	-.037777	6.012794	-1.771913	.697265
.312188	.118893	-.036318	6.012812	-1.429621	.697577
.229238	.122270	-.034809	6.012832	-1.095246	.697840
.153343	.124654	-.033265	6.012853	-.766647	.698066
.084206	.126132	-.031697	6.012877	-.441823	.698263
.021523	.126786	-.030115	6.012902	-.118860	.698437
-.035015	.126695	-.028530	6.012930	.204111	.698594
-.085719	.125935	-.026951	6.012962	.528959	.698735
-.130900	.124575	-.025384	6.012998	.857599	.698865
-.170867	.122684	-.023838	6.013040	1.192032	.698985
-.205927	.120324	-.022319	6.013090	1.534400	.699096
-.236382	.117555	-.020832	6.013149	1.887041	.699201
-.262529	.114432	-.019382	6.013222	2.252559	.699301
-.284657	.111009	-.017972	6.013316	2.633904	.699395
-.303050	.107332	-.016608	6.013440	3.034477	.699486
-.317981	.103447	-.015290	6.013614	3.458259	.699574
-.329718	.099395	-.014022	6.013876	3.909975	.699659
-.338515	.095216	-.012806	6.014320	4.395298	.699742
-.344620	.090944	-.011642	6.015248	4.921011	.699824
-.348268	.086611	-.010532	6.018406	5.494219	.699906
-.349685	.082246	-.009477	6.158836	5.991120	.699986
-.349086	.077877	-.008476	6.854105	6.008693	.700067
-.346677	.073527	-.007530	7.659854	6.010503	.700149
-.342650	.069217	-.006638	8.587396	6.011184	.700231
-.337189	.064966	-.005799	9.671900	6.011543	.700315
-.330467	.060792	-.005013	10.964613	6.011765	.700401
-.322644	.056709	-.004279	12.541722	6.011917	.700490
-.313874	.052730	-.003595	14.521378	6.012028	.700582
-.304298	.048866	-.002960	17.097171	6.012113	.700679
-.294047	.045126	-.002373	20.609855	6.012180	.700780
-.283244	.041517	-.001832	25.720114	6.012236	.700887
-.272002	.038046	-.001335	33.899489	6.012282	.701002
-.260425	.034718	-.000880	49.225428	6.012322	.701125
-.248609	.031537	-.000466	88.733917	6.012356	.701259
-.236640	.028504	-.000091	432.996579	6.012387	.701406
-.224599	.025621	.000247	6.012414	-151.100646	.701568
-.212556	.022889	.000550	6.012438	-64.247932	.701750
-.200577	.020307	.000820	6.012461	-40.679637	.701957
-.188719	.017874	.001059	6.012481	-29.651905	.702196
-.177035	.015588	.001268	6.012500	-23.229714	.702476
-.165568	.013447	.001449	6.012518	-19.006207	.702811
-.154359	.011448	.001604	6.012535	-16.002548	.703221
-.143443	.009587	.001736	6.012552	-13.745592	.703739
-.132848	.007860	.001845	6.012567	-11.978633	.704418
-.122599	.006264	.001933	6.012582	-10.550309	.705354
-.112717	.004794	.002002	6.012596	-9.365561	.706735

time

equation is applicable. It would be expected from the results of this series of runs that the iterative method of determining ζ and ω_n might experience difficulty in converging when θ is small and that the region of non convergence will become wider as system damping ratio is increased.

Program 2. Iterative Calculation of ζ and ω_n Using Both Forms of the ω_n Equation.

The purpose of this run was to test the iterative damping ratio computer described in section V. The previously described second order system, with $\omega_n = 6$ & $\zeta = 0.2$, was used to supply output samples of θ, $\dot{\theta}$, and $\ddot{\theta}$ which were stored, two sets at a time, and used as inputs to the computer. Sample spacing in this run was 0.05 cycle and each ζ-ω_n calculation was iterated 20 times using each of the two forms of the ω_n equation. At the termination of each iterating process, the results of each of the 20 iterations was printed out for study, the oldest set of samples of θ, $\dot{\theta}$, & $\ddot{\theta}$, discarded, a new set generated and stored, and the ζ-ω_n calculation cycle repeated using the two new sample sets. A sample series of the printouts from the ζ-ω_n computer is listed in Table V where the results of one convergent and one divergent solution are shown.

The primary result of this program was the demonstration that the iterative method would converge, and it did so for 51% of the sample sets (ζ = 0.2). Accuracy obtained was of the same order as that obtained in Program #1, approximately 0.05%, and in regions of small values of θ the accuracy of the iterative method was much greater. As an example, ω_n from the iterative method was 6.00359 as compared to ω_n = 6.086341 obtained from Program #1 when the identical sample set was used with each method in a region of small values of θ. It was found that a necessary but not sufficient condition for convergence of the iterative method was that θ and $\dot{\theta}$ be of opposite sign.

As is shown in Table V, another interesting property of the iterative method is that when convergence occurs, it occurs simultaneously with both forms of the ω_n equation, and the iterative series using the negative form of the ω_n equation converges to the negative values of ζ and ω_n arrived at using the positive ω_n equation. Also, it was found that the use of the negative ω_n equation would not be required in future programs because the iterations performed when using the positive form converged in areas where the results of Program #1 indicated that the negative ω_n equation was applicable. It had thus far been established that, at $\zeta = 0.2$, the iterative computations would converge for a length of time corresponding to approximately one quarter of the system period at a rate of two times per system cycle. It was felt that this would suffice for the proposed adaptive system, however, it was considered desirable to determine the effect of different values of damping ratio on convergence.

Program 3. Iterative Calculation of ζ & ω_n at 9 Values of System Damping Ratio.

Nine runs were made in this program, with system damping ratio varying from 0.1 to 0.9 in 0.1 steps, and a sampling interval = 0.05 period. Only the positive form of the ω_n equation was used in the iterative process and the solution was tested for divergence after each iteration to prevent unnecessary calculations. Table VI is a partial listing of the runs made with ζ = 0.2 and 0.7. It is seen that the region of divergence is appreciably wider with ζ = 0.7 than with 0.2. The percentage of convergent iterations to total iterations is tabulated below as a function of damping ratio for each of the nine conditions:

Table V Iterative Calculation of Natural Frequency and Damping Ratio, Using Both Forms of the ω_n Equation, of a Second Order System with $\omega_n = 6.0$ Rad/Sec and $\zeta = 0.2$.

Sample Spacing = 0.05 Cycle

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.200000	-5.630577	-4.454730	0.450977

	OMEGAN	DAMPR
	7.629993	0.303380
	6.878375	0.252439
	6.413184	0.226073
	6.210091	0.212571
	6.107890	0.205695
+ Form	6.056324	0.202204
	6.030267	0.200434
	6.017089	0.199537
	6.010423	0.199083
	6.007049	0.198854
	6.005342	0.198737
	6.004477	0.198678
	6.004040	0.198648
	6.003819	0.198633
	6.003707	0.198626
	6.003650	0.198622
	6.003621	0.198620
	6.003607	0.198619
	6.003599	0.198618
	6.003595	0.198618
	OMEGAN	**DAMPR**
	-2.860373	0.076158
	-4.239639	-0.065534
	-5.078711	-0.132633
	-5.528898	-0.165552
	-5.761842	-0.181965
	-5.880874	-0.190210
− Form	-5.941390	-0.194367
	-5.972086	-0.196468
	-5.987639	-0.197533
	-5.995516	-0.198067
	-5.999504	-0.198339
	-6.001522	-0.198477
	-6.002544	-0.198547
	-6.003062	-0.198582
	-6.003323	-0.198600
	-6.003456	-0.198609
	-6.003523	-0.198613
	-6.003557	-0.198616
	-6.003574	-0.198617
	-6.003583	-0.198617

Table V - Continued

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.399999	18.978094	-2.700115	-0.347882
	OMEGAN		DAMPR
	4.205901		0.564624
	3.300829		0.852037
	2.354981		1.340582
	1.558809		2.154067
	0.988132		3.492864
	0.613964		5.684419
	0.378345		9.264260
	0.232391		15.107418
	0.142551		24.643762
	0.087360		40.222131
	0.053459		65.734811
	0.032669		107.570982
	0.019852		177.027143
	0.011444		307.084564
	0.005859		599.775604
	0.002441		1439.462204
	-0.002930		-1199.551773
	44936.139648		-2894.778473
	108440.639648		-6985.727661
	261690.726562		-16858.072266
	OMEGAN		DAMPR
	-19.190211		1.053100
	-41.399521		2.582062
	-97.553423		6.248351
	-234.412361		15.085813
	-565.269005		36.408317
	-1363.942886		87.862394
	-3291.417694		212.031506
	-7942.874573		511.678204
	-19167.864990		1234.790436
	-46256.204590		2979.816833
	-111626.239258		7190.943481
	-269378.281250		17353.303223
	-650068.140625		41877.279297
	-1568755.156250		101058.941406
	-3785745.781250		243877.103516
	-9135824.375000		588528.257812
	-22046720.500000		1429246.000000
	-53203507.000000		3427360.906250
	-128391576.000000		8270963.625000
	-77459164.000000		19959625.000000

Table VI - Percent of Convergent Iterations as a Function of System Damping Ratio

Damping Ratio	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
% Convergence	52%	51%	51%	49%	48%	41%	34%	27%	19%

The percentage of convergent iterations drops off sharply for values of exceeding 0.5 but it is not desired to increase the airframe damping ratio above 0.7 and thus the minimum obtainable ratio of convergent to total iterations should never be less than approximately 1/3.

Program 4. Influence of Sample Spacing on the Percentage of Convergent Iterations.

The final program was run to determine what influence, if any, the width of sample spacing has on the ability of the iterative calculations to converge. All runs were made with ζ = 0.2, and six values of sample spacing ranging from 0.01 cycle to 0.5 cycle were investigated. Results of the program are tabulated below.

Table VIII - Percent of Convergent Iterative Calculations as a Function of Sample Spacing $\zeta = 0.2$

Spacing	0.01 cycle	0.02	0.05	0.1	0.2	0.5
% Convergence	56%	53%	51%	50%	48%	46%

It is seen that narrow sample spacing favors convergence, however it can be varied over wide limits with little effect on convergence. It was arbitrarily decided to set sample spacing at 0.025 seconds for the three degree of freedom airframe simulation; the primary consideration being conservation of computer time. This sample spacing corresponded to approximately 50 samples per short period cycle, depending on flight condition.

Table VII Iterative Calculation of Natural Frequency and Damping Ratio of a Second Order System With ω_n=6.0 Rad/Sec and ζ= 0.2 & 0.7
Sample Spacing = 0.05 Cycle

Condition 2 Damping Ratio = 0.2

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.200000	-5.630577	-4.454730	0.450971
OMEGA-N	DAMPRAT		
7.629993	0.303380		
6.818375	0.252439		
6.413184	0.226073		
6.210091	0.212571		
6.107890	0.205695		
6.056324	0.202204		
6.030267	0.200434		
6.017089	0.199537		
6.010423	0.199083		
6.007049	0.198854		
6.005342	0.198737		
6.004477	0.198678		
6.004040	0.198648		
6.003819	0.198633		
6.003707	0.198626		
6.003650	0.198622		
6.003621	0.198620		
6.003607	0.198619		
6.003599	0.198618		
6.003595	0.198618		

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.250000	2.689775	-4.524627	0.224752
OMEGA-N	DAMPRAT		
9.251958	0.261913		
6.966586	0.215692		
6.256753	0.202902		
6.066626	0.199669		
6.019025	0.198874		
6.007354	0.198680		
6.004508	0.198633		
6.003815	0.198622		
6.003646	0.198619		
6.003605	0.198618		
6.003595	0.198618		
6.003592	0.198618		
6.003592	0.198618		
6.003592	0.198618		
6.003592	0.198618		
6.003592	0.198618		
6.003592	0.198618		
6.003592	0.198618		

Table VII Continued   Damping Ratio = 0.2

6.003592	0.198618
6.003592	0.198618

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.299999	9.848088	-4.205210	0.005013
OMEGA-N	DAMPRAT		
15.324385	0.085544		
-7.279872	-0.165185		
-4.234596	-0.279041		
-1.191589	-0.983381		

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.349999	15.371489	-3.567171	-0.190448
OMEGA-N	DAMPRAT		

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.399999	18.978094	-2.700115	-0.347882
OMEGA-N	DAMPRAT		
4.205901	0.564624		
3.300829	0.852037		

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.449999	20.579304	-1.702884	-0.458290
OMEGA-N	DAMPRAT		
4.907358	0.570963		
4.181908	0.882182		

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.499998	20.265548	-0.674185	-0.517650
OMEGA-N	DAMPRAT		
5.377643	0.730322		

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.549998	18.278530	0.295719	-0.526696
OMEGA-N	DAMPRAT		
5.757618	-0.240352		

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.599997	14.973863	1.131666	-0.490321

OMEGA-N	DAMPRAT
6.119884	0.244751
6.030041	0.209181
6.009638	0.201036
6.004975	0.199172
6.003908	0.198745
6.003664	0.198647
6.003608	0.198625
6.003595	0.198619
6.003593	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618
6.003592	0.198618

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.649996	10.778271	1.778240	-0.416698

OMEGA-N	DAMPRAT
6.525987	0.300233
6.262415	0.249806
6.132749	0.224382
6.068288	0.211580
6.036061	0.205138
6.019904	0.201897
6.011790	0.200267
6.007714	0.199447
6.005664	0.199035
6.004634	0.198828
6.004116	0.198723
6.003855	0.198671
6.003724	0.198645
6.003658	0.198631
6.003625	0.198625
6.003608	0.198621
6.003600	0.198620
6.003596	0.198619
6.003594	0.198618
6.003593	0.198618

Table VII - Continued

Condition 7   Damping Ratio = 0.7

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.200000	1.359547	-2.747204	0.602031
OMEGA-N	DAMPRAT		
4.221364	0.521158		
4.917109	0.589098		
5.325055	0.629943		
5.575466	0.655293		
5.732646	0.671299		
5.832533	0.681505		
5.896477	0.688051		
5.937594	0.692266		
5.964107	0.694986		
5.981233	0.696743		
5.992308	0.697880		
5.999476	0.698616		
6.004117	0.699093		
6.007122	0.699402		
6.009069	0.699602		
6.010331	0.699731		
6.011148	0.699815		
6.011677	0.699870		
6.012021	0.699905		
6.012243	0.699928		

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.250000	4.838883	-2.584920	0.468005
OMEGA-N	DAMPRAT		
2.861347	0.586139		
4.887297	0.633940		
5.364674	0.660114		
5.622871	0.675476		
5.773553	0.684772		
5.864458	0.690488		
5.920257	0.694035		
5.954843	0.696248		
5.976406	0.697633		
5.989896	0.698501		
5.998354	0.699046		
6.003663	0.699389		
6.007000	0.699605		
6.009097	0.699740		
6.010416	0.699825		
6.011245	0.699879		
6.011767	0.699913		
6.012096	0.699934		

Table VII- Continued   Damping Ratio = 0.7

```
      6.012302      0.699947
      6.012432      0.699956

        TIME        ACCELERATION   VELOCITY     DISPLACEMENT
      0.299999      6.768687     -2.289121      0.345753
      OMEGA-N       DAMPRAT
     -3.249022     -0.700412
     -1.717651     -0.990456
   SOLUTION DIVERGING

        TIME        ACCELERATION   VELOCITY     DISPLACEMENT
      0.349999      7.539968     -1.927330      0.240182
      OMEGA-N       DAMPRAT
     -9.915033     -0.815083
     -2.306982     -0.991636
   SOLUTION DIVERGING

        TIME        ACCELERATION   VELOCITY     DISPLACEMENT
      0.399999      7.494233     -1.548708      0.153291
      OMEGA-N       DAMPRAT
   SOLUTION DIVERGING

        TIME        ACCELERATION   VELOCITY     DISPLACEMENT
      0.449999      6.915493     -1.186757      0.085026
      OMEGA-N       DAMPRAT
   SOLUTION DIVERGING

        TIME        ACCELERATION   VELOCITY     DISPLACEMENT
      0.499998      6.029223     -0.862252      0.033986
      OMEGA-N       DAMPRAT
    -44.580629     -0.957002
   SOLUTION DIVERGING

        TIME        ACCELERATION   VELOCITY     DISPLACEMENT
      0.549998      5.065958     -0.586090     -0.002009
      OMEGA-N       DAMPRAT
    -64.266227      0.043701
   SOLUTION DIVERGING

        TIME        ACCELERATION   VELOCITY     DISPLACEMENT
      0.599997      3.967681     -0.361883     -0.025492
      OMEGA-N       DAMPRAT
     10.228298      0.175711
     20.289338     -0.444423
   SOLUTION DIVERGING
```

Table VII- Continued Damping Ratio = 0.7

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.649996	2.995569	-0.188200	-0.039041

OMEGA-N	DAMPRAT
8.028860	0.158457
10.427491	-0.318348

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.699996	2.138070	-0.060402	-0.045078

OMEGA-N	DAMPRAT
7.040965	-0.113642

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.749995	1.418596	0.027917	-0.045740

OMEGA-N	DAMPRAT

SOLUTION DIVERGING

TIME	ACCELERATION	VELOCITY	DISPLACEMENT
0.799994	0.842411	0.083853	-0.042826

OMEGA-N	DAMPRAT
5.818552	0.622535
5.961974	0.679927
5.999497	0.694778
6.009243	0.698625
6.011769	0.699621
6.012424	0.699880
6.012594	0.699946
6.012637	0.699964
6.012649	0.699968
6.012652	0.699969
6.012653	0.699970
6.012653	0.699970
6.012653	0.699970
6.012653	0.699970
6.012653	0.699970
6.012653	0.699970
6.012653	0.699970
6.012653	0.699970
6.012653	0.699970
6.012653	0.699970

VII - APPLICATION OF THE DAMPING RATIO COMPUTER TO A SELF ADAPTIVE PITCH DAMPING SYSTEM

Investigation of the Ability of the ζ-ω_n Computer to Determine Short-Period Mode Characteristics

It has thus far been established that the damping ratio computer operates satisfactorily when supplied input samples generated by a second order system, and it has also been established that the short period mode, a second order oscillation, is the dominant mode of the airplane longitudinal dynamics. The next step in the program is then to investigate the operation of the computer when supplied samples generated by the three degree of freedom, fourth order airframe simulation. The simulation program is discussed in detail in Appendix VI.

Flight condition number seven, M = 1.8 @ 25,000 ft., having short-period mode damping ratio = 0.095 and period = 1.2 seconds, was chosen for the test. At the start of the computer run, the airframe was released from a non-trim pitch condition and allowed to oscillate. Periodic samples of pitch angle, θ, and its first two derivatives were taken and supplied to the ζ-ω_n computer as described in the preceeding section. At the beginning of each set of ζ-ω_n calculation an initial value of $\zeta = 0.4$ was introduced into the equation and the iterations were carried out 15 times. The iterations converged for 49 percent of the sample sets, which is in good agreement with the previous second order system investigation, however the computed values of damping ratio ranged from 0.1448 to 0.024565 during a 2.5 second (flight time) run. Such a wide variation of computed ζ was inconsistent with the results of the preceeding study in which it was demonstrated that the longitudinal aircraft dynamics very closely approximate the dynamics of a second order system. A review of foregoing investigations was conducted to determine if there were any significant differences between the second order study and the present application of the ζ-ω_n computer, and the answer was found in examination of the airplane short-period pitch-angle-to-elevator deflection transfer function

$$\frac{\theta}{i_t} = \frac{K_\theta\,(S+T_\theta)}{S(S^2+2\xi\omega_n S+\omega_n^2)}$$

It is seen that the 1/S term causes pitch angle output to exhibit a theoretically infinite steady state error or, if the airframe is perturbed from its trim position at a given flight condition, the ensuing short period oscillations will not decay to the original trim angle. Further, it can be shown that the airframe will return to trim condition as a function of the longitudinal phugoid mode. The primary difference between the ideal second order system and the short period dynamics is then the fact that the former system oscillates about θ equal zero whereas the airframe oscillates about an average pitch angle which is constantly varying as a function of the phugoid dynamics, and thus the pitch angle samples introduced into the computer were erroneous. The solution to this problem was seen in the pitch-rate-per-stabilizer-deflection transfer function,

$$\frac{\dot{\theta}}{i_t} = \frac{K_\theta(S+T_\theta)}{(S^2+2\xi\omega_n S+\omega_n^2)}$$

which exhibits zero steady state error, and which also shows that free airframe short period oscillations oscillate about, and decay to, zero <u>pitch rate</u>. It was realized that pitch rate is the true second order variable of the airframe longitudinal dynamics and in the study to be outlined next, pitch rate and its first two derivatives $\ddot{\theta}$ and $\dddot{\theta}$ were utilized as inputs to the ξ-ω_n computer.

A second test of the ξ-ω_n computer using the three degree of freedom flight simulation was performed, as stated above, and a partial listing of the results obtained when sampled values of $\dot{\theta}$, $\ddot{\theta}$ & $\dddot{\theta}$, were used as computer inputs is presented in Table IX. As before, the calculations converged for 49% of the sample sets but, as can be seen, the calculated values of damping ratio compare favorably with the known $\xi = 0.095$. It can also be seen from examination of Table IX that, in most cases, the calculations have not quite settled out to steady values of ξ and ω_n after 15 iterations, and as a result, it was decided to repeat the iterative calculation cycle 20 times in all future programs.

Application of the ξ-ω_n computer to the Adaptation Loop

It was felt, at this time, that the ξ-ω_n computer was generating sufficiently accurate values of damping ratio to allow its use in the adaptation loop, however one obvious discrepancy remained to be corrected before the loop could be closed. That is, what should be done regarding the calculations which do not converge and thereby generate highly inaccurate values of ξ ? Before discussing the technique which was used to detect and account for divergent calculations, possibly a better understanding may be obtained from a short review of the proposed mechanization of the adaptation loop.

It will be recalled that it was proposed to compare the output of the damping ratio computer with the desired value of short period damping ratio ; the error between ξ - desired and ξ - computer being used as the input signal to a rate servo which would vary pitch damper gain in the proper direction to null out the ξ- error, and at a rate proportional to the magnitude of the error. It is apparent that the erroneous values of ξ resulting from divergent calculations could not be allowed to reach the computer output, and it was found that this could be accomplished by simply including a routine which examined the resultant values of ω_n and ξ at the termination of the series of iterative calculations for each sample set. After examination of a large number of past output records from both, the second order, and the airframe studies, it was determined that, in every case, when the calculations diverged the resultant calculated values of either ξ or ω_n or both were completely out of the range of variation of the real values of ξ and ω_n . For examples of divergent calculations,

Table IX Iterative Calculation of Airframe Short Period Damping Ratio and Natural Frequency Using $\dot{\theta}$, $\ddot{\theta}$, & $\dddot{\theta}$, Generated by a Longitudinal Three Degree of Freedom Flight Simulation, as Computer Inputs

Natural Frequency = 5.24 Rad/Sec
Damping Ratio = 0.095

TIME-SEC	THETA	THETADOT	FWD VELOCITY
1.099999	0.244806	[illegible].776224	1805.982346
	OMEGAN		DAMPR
	7.98[illegible]181		[illegible].323749
	7.226988		[illegible].266[illegible]7[illegible]
	6.68514		[illegible].222587
	6.294[illegible]84		[illegible].189897
	6.[illegible]614		[illegible].165376
	5.8[illegible]4166		[illegible].147[illegible]16
	5.65315[illegible]		[illegible].133287
	5.542262		[illegible].123[illegible]32
	5.46[illegible]581		[illegible].115378
	5.4[illegible]26		[illegible].1[illegible]9669
	5.355627		[illegible].1[illegible]5412
	5.32255[illegible]		[illegible].1[illegible]2239
	5.298[illegible]		[illegible].[illegible]99874
	5.279785		[illegible].[illegible]98113
	5.266242		[illegible].[illegible]968[illegible]
1.124999	[illegible].262[illegible]78	[illegible].6[illegible]4[illegible]64	18[illegible]5.235[illegible]92
	OMEGAN		DAMPR
	9.[illegible]27217		[illegible].31[illegible]96[illegible]
	7.823235		[illegible].248395
	7.[illegible]3878		[illegible].2[illegible]2836
	6.447[illegible]		[illegible].17[illegible]358
	6.[illegible]67493		[illegible].147335
	5.8[illegible]7712		[illegible].13[illegible]72
	5.629[illegible]54		[illegible].119614
	5.5[illegible]5673		[illegible].111559
	5.42[illegible]174		[illegible].1[illegible]59[illegible]4
	5.36[illegible]772		[illegible].1[illegible]1937
	5.319418		[illegible].[illegible]99158
	5.29[illegible]589		[illegible].[illegible]97211
	5.27[illegible]469		-[illegible].[illegible]95847
	5.256418		[illegible].[illegible]94893
	5.246599		[illegible].[illegible]94224
TIME-SEC	THETA	THETADOT	FWD VELOCITY
1.149999	[illegible].274967	[illegible].4[illegible]6163	18[illegible]4.43[illegible]54
	OMEGAN		DAMPR
	1[illegible].784[illegible]76		[illegible].289516
	8.594914		[illegible].216917
	7.2528[illegible]4		[illegible].17[illegible]112
	6.44289[illegible]		[illegible].14[illegible]453
	5.9567[illegible]		[illegible].121912
	5.664582		[illegible].11[illegible]431
	5.488537		[illegible].1[illegible]3367

Table IX - Continued

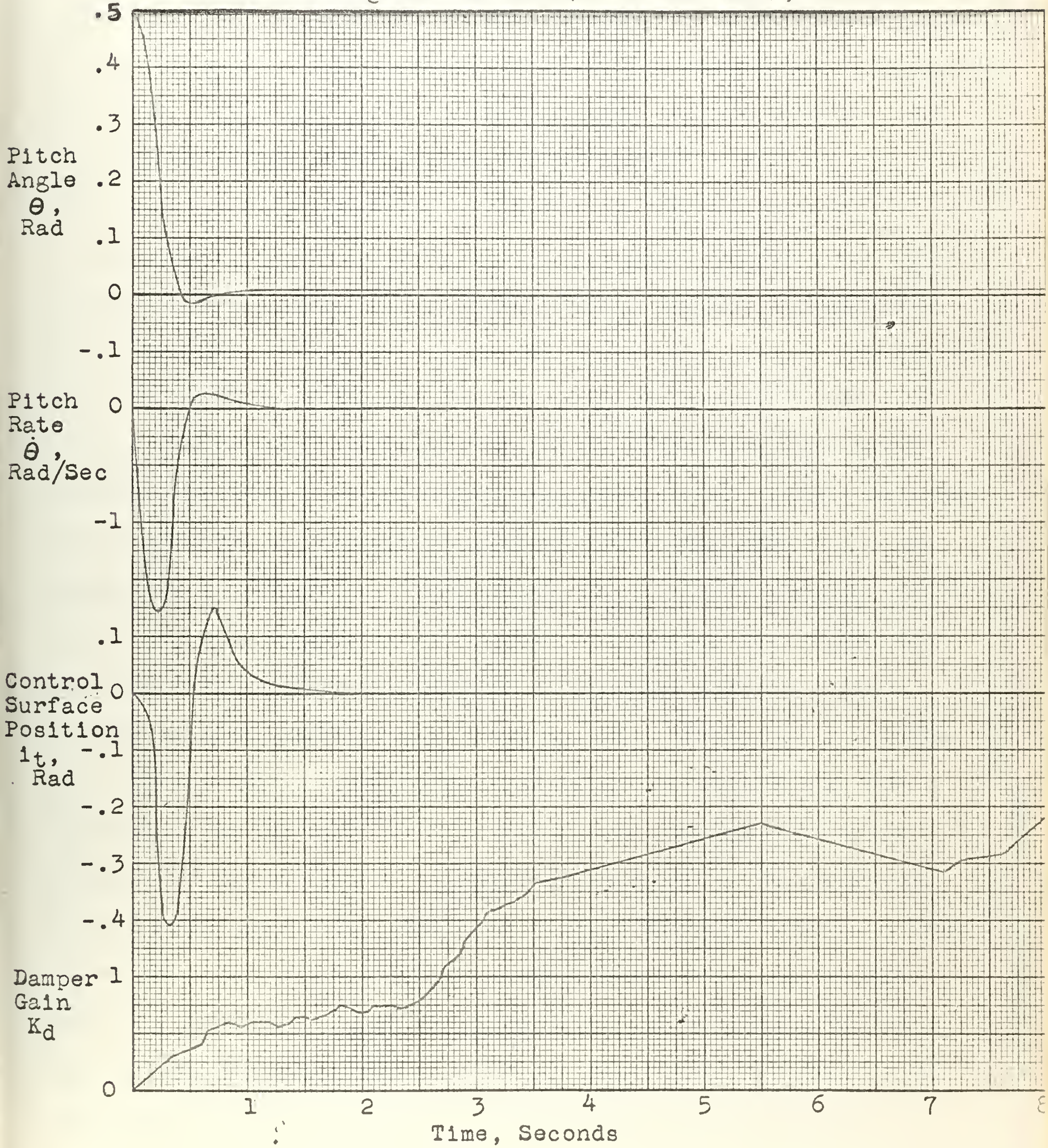
TIME-SEC	THETA	THETADOT	FWD VELOCITY
	5.21134[illegible]		0.[illegible]91963
	5.21[illegible]679		0.[illegible]91953
	5.21[illegible]419		0.[illegible]91948
	5.21[illegible]317		0.[illegible]91947
	5.21[illegible]277		0.[illegible]91946
TIME-SEC	THETA	THETADOT	FWD VELOCITY
1.199999	0.287253	0.045585	18[illegible]2.735855
	OMEGAN		DAMPR
	23.5[illegible]8[illegible]3		0.1228[illegible]6
	7.[illegible]8[illegible]167		0.[illegible]82383
	4.621625		0.[illegible]97[illegible]62
	5.559436		0.[illegible]89[illegible]65
	5.133489		0.[illegible]93294
	5.292757		0.[illegible]91[illegible]43
	5.154972		0.[illegible]92196
	5.225596		0.[illegible]91594
	5.188687		0.[illegible]91975
	5.2[illegible]778[illegible]		0.[illegible]91743
	5.197857		0.[illegible]91827
	5.2[illegible]3[illegible]5		0.[illegible]91784
	5.2[illegible]328		0.[illegible]918[illegible]6
	5.2[illegible]1718		0.[illegible]91794
	5.2[illegible]996		0.[illegible]918[illegible]1
1.224999	0.286672	-0.111[illegible]32	18[illegible]1.893[illegible]73
	OMEGAN		DAMPR
	86.334987		-[illegible].681[illegible]69
	-0.52[illegible]734		-1.3251[illegible]9
	-[illegible].266954		-2.59[illegible]797
	-[illegible].13647[illegible]		-5.[illegible]71[illegible]11
	-[illegible].[illegible]69763		-9.92149[illegible]
	-[illegible].[illegible]35843		-19.31159[illegible]
	-[illegible].[illegible]18646		-37.122271
	-[illegible].01[illegible]223		-67.7[illegible]6779
	-[illegible].[illegible]6775		-1[illegible]2.1732[illegible]9
	-[illegible].[illegible]5615		-123.27[illegible]599
	-[illegible].[illegible]5859		-118.134319
	-[illegible].[illegible]5493		-126.[illegible]9948
	-0.[illegible]5249		-131.87[illegible]88[illegible]
	-[illegible].[illegible]5737		-12[illegible].647818
	-[illegible].[illegible]5371		-128.873812

see Tables V or VII. Also, it was known with certainty, that for the aircraft considered in this study, damping ratio was restricted well within the range from 0. to 1.0 and natural frequency within the range from 0. to 8.0 radians/second. Thus a divergence detection routine was programmed to use a simple logical process for determining whether or not the calculated values of both ζ and ω_n fell within the allowable ranges. If both values were acceptable to the detection routine, the calculated value of ζ was set into the damping ratio comparison process, and if either ζ or ω_n or both were unacceptable to the detection routine, the last acceptable computed value of ζ was reset into the comparator. The values of ω_n and ζ resulting from a divergent series were always orders of magnitude different from the allowable ranges of ω_n and ζ, and thus the allowable ranges could be altered considerably to suit the characteristics of other systems with no deterioration in the effectiveness of the divergence detection routine. As an example, if it were expected that the airframe might have short-period poles in the right half plane over some flight conditions, the allowable ζ range could be increased from $\zeta = -1.0$ to $+1.0$.

A variable which has not yet been discussed, but which was quite important in the evaluation of the adaptation loop, is the gain of the rate servo (hereinafter called the adaptation servo) used to vary pitch damper gain. When ζ- error signals are supplied to the adaptation servo which in turn causes variations in airframe damping ratio, the entire adaptation loop appears to resemble a second order servomechanism and should be amenable to conventional systems analysis techniques. However analysis was complicated by the intermittent action of the damping ratio computer and thus an "optimum" value of adaptation-servo-gain, K_s, was determined experimentally by performing a series of three degree of freedom computer runs with varying values of K_s. Flight condition 7, M = 1.8 @ 25,000 ft., was again used

Figure 29. Time History of Airplane & Control Surface Motion and Pitch Damper Gain During Adaptation Process With No Adaptation Servo Lock

Initial $K_d = 0$, Adaptation servo gain = 1.
Flight Condition 7 - M = 1.8 @ 25,000 ft.



for all runs and adaptation-servo gain was varied from $K_s = 0.05$ to 2.0 in six steps.

Figure 29 is a typical time history of the run made with $K_s = 1.0$. Airplane pitch angle, pitch rate, control surface position, and pitch damper gain are shown as functions of time, and again the run was started by releasing the airframe from a non-trim pitch position and allowing it to oscillate. The damping ratio of the basic airplane at this condition was 0.096 and the steady state damper gain for correct adaptation, from the root locus plot, was 0.344. Initial damper gain was set at zero at time = zero and it is seen that $K_d = 0.344$ was achieved in 0.45 seconds, and that the airframe motion was heavily damped by the control action. However, it is also seen that damper gain did not level out at the desired value of $K_d = 0.344$ but instead continued increasing in a somewhat random manner until at time = 10 seconds (not shown) it reached the required value for instability of the actuator mode and a violent divergent oscillation ensued. In this condition, the actuator motions comprised the dominant longitudinal mode, and it was interesting to note that the ζ-ω_n computer was still performing its duty in that it was now computing natural frequency and damping ratio of the actuator mode. The crossover frequency for this mode was predicted to be 34 radians per second from the root locus plot, which was verified by the fact that computed damping ratio changed from positive to negative sign when computed ω_n was approximately 34.5 rad/sec. But, the operation of the adaptation loop was obviously unacceptable with $K_d = 1.0$. An improvement in performance was detected when lower values of adaptation-servo-gain were investigated, and with $K_d = 0.1$ the damper gain did settle out reasonably well, but approximately eight seconds were required to complete the adaptation process when using the K_d. Osburn has shown in Ref. 127 that the MIT self adaptive system is capable of completing the adaptation process on a similar airframe during two cycles of plant oscillation, which would correspond to adaptation time = 2.4 seconds for flight condition 7, presently under discussion. Thus, a review was made of the adaptation method to determine if the process could be speeded up without attendant drift in pitch damper gain.

The inability of the adaptation loop to set proper damper gain when operating at high adaptation-servo-gains was found to be caused by four interacting factors:

1) The reader will recall that when the ξ-ω_n process failed to converge, the computer set the most recently computed convergent value of ξ into the damping ratio comparator, and the adaptation servo continued to change damper gain at a rate proportional to the indicated ξ - error.

2) It was found that during the adaptation process, the constantly changing damper gain caused errors in computed ω_n and ξ due to the fact that system parameters were altered appreciably during one sampling interval. And, the computed values of ξ were caused to vary in a random manner and were generally 0.05 to 0.1 below the true value of damping ratio.

3) As pitch damper gain increased, the effect of the time rate of change of system parameters on the ξ-ω_n calculations was magnified due to the increased system loop gain.

4) When system motions damped to zero, the ξ-ω_n computer was not supplied with sufficient information to perform calculations.

It can be seen how these factors combined to cause erratic operation of the adaptation mechanism. As an example, if a value of ξ = 0.6 were computed and set into the ξ comparator, and no new convergent values of ξ were generated for two seconds, a steady ξ - error signal = 0.1 would be fed to the adaptation servo and it would increase damper gain at a constant rate during the two second period. This is illustrated in Figure 29. where K_d was increased linearly from time = 3.5 seconds to 5.5 seconds. At 5.5 seconds a convergent value of ξ, exceeding 0.7, was obtained, set into the ξ - comparator, and damper gain began a downward drift. However, the drift of Kd was generally upward because the erroneous damping ratio solutions were usually smaller than 0.7.

The problem of drifting damper gain was solved by an alteration of the $\xi - \omega_n$ computer output routine in which output logic was re-programmed to set $\xi = 0.7$ into the ξ comparator if no convergent solution were obtained. This action effectively "locked" the adaptation servo at its present setting, and damper gain was not allowed to vary until generation of a new convergent solution. The above process will be referred to as the "adaptation-servo lock" in the remainder of this report. An important benefit accruing from the use of the servo lock was the fact that the plant was allowed to settle to a linear system during each period of nonconvergent calculations and thus the accuracy of subsequent convergent calculations was improved. It is recalled that, when the plant is in motion, the calculations converge during one quarter period in each half system cycle, and from this, it would be expected that damper gain would be changed in quarter period steps during the adaptation process. This is borne out in Figure 30 which is a time history of the adaptation process with the servo lock in operation.

The run shown in Figure 30 is identical in all other respects, except the use of the servo lock, to the run illustrated in Figure 29 that is, flight condition 7, and adaptation-servo gain = 1.0. It is seen that pitch damper gain increased in steps, as predicted, and it settled out close to the predicted value of $K_d = 0.344$ in 2.3 seconds. After three seconds running time, the airplane motion damped out and no values of ζ were available until $t = 5$ sec. During this interval damper gain remained constant at $K_d = 0.3$. At $t = 5$ sec., the airframe was perturbed by an impulse and the computer began generating values causing damper gain to be moved into the $K_d = 0.34$ to 0.35 region again. The time history of motions resulting from the second impulse also serves to illustrate the behavior of the adapted airframe which displays a damping ratio = 0.7.

A series of runs was made with the servo lock in operation using eight different values of adaptation servo gain ranging from $K_s = 0.1$ to 5.0. It was found from this series that values of K_s exceeding 1.0 caused an excessive overshoot in damper gain during the adaptation process and that values of K_s less than 1.0 resulted in a smooth adaptation transition, but time required to fully adapt was approximately inversely proportional to servo gain. On the basis of these results, it was decided that $K_s = 1.0$ was optimum, and a second series of runs was conducted to investigate the operation of the adaptation loop, with this gain setting, at each of the 11 flight conditions. Table X is a sample listing of the computer output for the three degrees of freedom run performed for flight condition 7, from which the reader can see the stepwise manner in which adaptation is accomplished. Items of interest in this program were the time required to adapt the airframe from its basic un-augmented condition to damping ratio = 0.7, and the steady state value of damper gain upon completion of adaptation. These values are listed in Table XI for each flight condition. Other items listed are the theoretically predicted gain for adaptation and the basic airframe short period damping ratio. Fgiure 30 shows that the steady state damper gain exhibited small amplitude fluctuations about a mean value during operation, and it is

Figure 30. Time History of Airplane & Control Surface Motion and Pitch Damper Gain During Adaptation Process With Adaptation Servo Lock

Initial $K_d = 0$, Adaptation Servo Gain = 1.
Flight Condition 7 - M = 1.8 @ 25,000 ft.

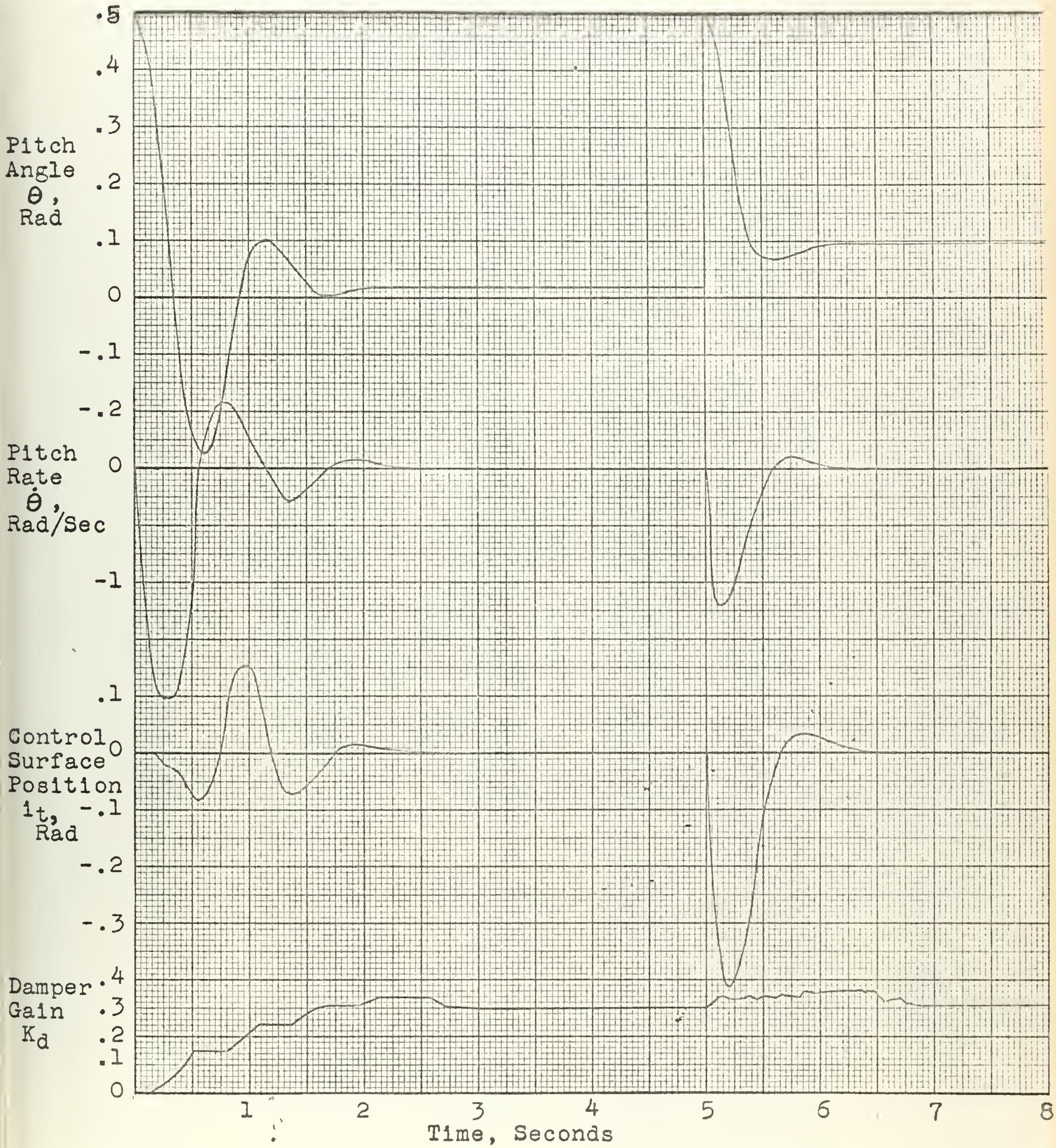


Table X Listing of Computer Output from a Three Degree of Freedom Flight Simulation With Adaptation Loop Using Servo Lock

Flight Condition 7.: M 1.8 @ 25,000 ft.
Adaptation Servo Gain 1.

TIME-SEC	THETA	THETADOT	DAMPR	DAMPER GAIN	ACTUATOR POS
.000000	.500000	.000000	.700000	.000000	.000000
.025000	.496086	-.324431	.700000	.000000	.000000
.050000	.484054	-.634788	.700000	.000000	.000000
.075000	.464490	-.926028	.700000	.000000	.000000
.100000	.437929	-1.193615	.700000	.000000	.000000
.125000	.405005	-1.434342	.700000	.000000	.000000
.150000	.366441	-1.644364	.700000	.000000	.000000
.175000	.323036	-1.821223	.700000	.000000	.000000
.200000	.275644	-1.962866	.700000	.000000	.000000
.225000	.225167	-2.067863	.700000	.000000	.000000
TIME-SEC	THETA	THETADOT	DAMPR	DAMPER GAIN	ACTUATOR POS
.250000	.172532	-2.135406	.700000	.000000	.000000
.275000	.118680	-2.165315	.700000	.000000	.000000
.300000	.064546	-2.158025	.089002	.000000	.000000
.325000	.011050	-2.114576	.700000	.015275	-.000452
.350000	-.040712	-2.034858	.700000	.015275	-.009470
.375000	-.090410	-1.917663	.127703	.015275	-.021140
.400000	-.136543	-1.766754	.181400	.029582	-.028282
.425000	-.178529	-1.596439	.700000	.042547	-.037272
.450000	-.215658	-1.378673	.700000	.042547	-.049352
.475000	-.247287	-1.147634	.203708	.042547	-.057443
TIME-SEC	THETA	THETADOT	DAMPR	DAMPER GAIN	ACTUATOR POS
.500000	-.272925	-.901144	.199693	.054955	-.056564
.525000	-.292284	-.646515	.126940	.057462	-.052560
.550000	-.305229	-.388959	.129633	.081789	-.047015
.575000	-.311750	-.133684	.137643	.096048	-.039044
.600000	-.311775	.113410	.700000	.110107	-.026311
.625000	-.306190	.345884	.700000	.110107	-.007546
.650000	-.294838	.557601	.700000	.110107	.0156[illegible]4
.675000	-.278500	.744048	.700000	.110107	.040677
.700000	-.257[illegible]46	.902573	.700000	.110107	.064190
.725000	-.233590	1.031756	.700000	.110107	.084929
TIME-SEC	THETA	THETADOT	DAMPR	DAMPER GAIN	ACTUATOR POS
.750000	-.206468	1.131944	.700000	.110107	.102290
.775000	-.177211	1.202945	.700000	.110107	.116157
.800000	-.146532	1.245857	.700000	.110107	.126621
.825000	-.115119	1.261995	.700000	.110107	.133832
.850000	-.083520	1.253045	.266386	.110107	.137956
.875000	-.052639	1.221023	.201998	.120947	.139554
.900000	-.022729	1.167525	.700000	.133397	.141761
.925000	.005579	1.092940	.700000	.133397	.146542
.950000	.031771	.998985	.348036	.133397	.147954
.975000	.055417	.890160	.394769	.142196	.142796
TIME-SEC	THETA	THETADOT	DAMPR	DAMPER GAIN	ACTUATOR POS
1.000000	.076203	.771047	.259382	.149827	.134114
1.025000	.093717	.644919	.244128	.160843	.123815
1.050000	.108419	.514754	.246414	.172239	.112307
1.075000	.119646	.383499	.273031	.183579	.099190
1.100000	.127510	.254355	.289741	.194253	.083715
1.125000	.132406	.130686	.700000	.204510	.065511
1.150000	.134210	.015725	.700000	.204510	.044834
1.175000	.133279	-.087718	.700000	.204510	.022451
1.200000	.129927	-.177608	.700000	.204510	.000005
1.225000	.124508	-.252939	.700000	.204510	-.020869
TIME-SEC	THETA	THETADOT	DAMPR	DAMPER GAIN	ACTUATOR POS
1.250000	.117390	-.313570	.700000	.204510	-.039126
1.275000	.108936	-.359950	.700000	.204510	-.054307
1.300000	.099493	-.392883	.700000	.204510	-.066342
1.325000	.089385	-.413380	.700000	.204510	-.075370
1.350000	.078909	-.422578	.700000	.204510	-.081615
1.375000	.068332	-.421691	.433248	.204510	-.085331
1.400000	.057891	-.411983	.396940	.211179	-.086813
1.425000	.047790	-.394597	.220489	.218755	-.087055
1.450000	.038212	-.370377	.185767	.230743	-.086603
1.475000	.029316	-.340152	.169195	.243599	-.085652

this mean value which has been tabulated.

Table XI - Comparison of Steady State Damper Gain Generated by the Adaptation System, with Theoretical Gain Required for = 0.7 at Each Flight Condition, time to adapt, & Basic Airframe Damping Ratio also Tabulated.

Adaptation Servo Gain 1.0

Flight Condition	Basic ξ	Time to Adapt	Theoretical Kd	Experimental Kd
1 M = 0.9 10,000 ft	0.440	1.4 Sec.	0.156	0.16
2 M = 1.2 10,000 ft	0.177	2.2	0.303	0.29
3 M = 1.4 10,000 ft	0.156	2.1	0.284	0.28
4 M = 0.9 25,000 ft	0.244	1.7	0.249	0.26
5 M = 1.2 25,000 ft	0.109	2.5	0.372	0.36
6 M = 1.4 25,000 ft	0.105	2.5	0.384	0.38
7 M = 1.8 25,000 ft	0.096	2.5	0.344	0.34
8 M = 0.9 45,000 ft	0.141	2.2	0.405	0.44
9 M = 1.2 45,000 ft	0.066	3.1	0.538	0.56
10 M = 1.4 45,000 ft	0.067	2.9	0.503	0.51
11 M = 1.8 45,000 ft	0.058	2.8	0.467	0.47

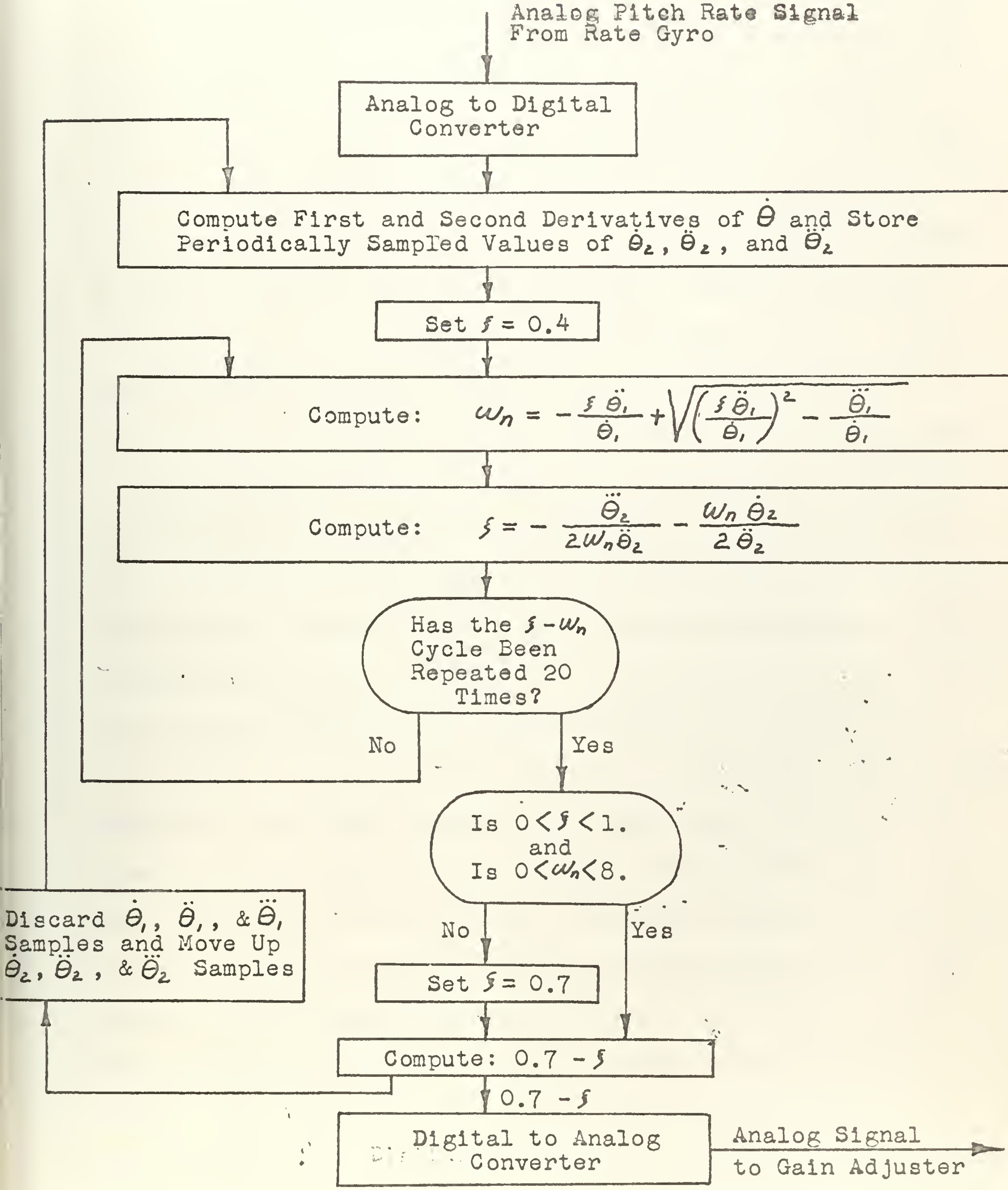
It is seen that the adaptation times compare favorably with those of the MIT system, and as would be expected, those flight conditions with lower values of basic damping ratio exhibited longer adaptation times. The experimental values of steady state damper gain could be determined at best with two place accuracy, but even so, reasonable agreement with the theoretical values was obtained.

Considerations in the Practical Mechanization of an Airborne Digital Damping Ratio Computer

Perhaps the best method of reviewing the finally "optimized" Adaptation process is to follow through the flow diagram of Figure 31. which illustrates the program sequence to be followed by the proposed airborne damping ratio computer. The primary difference between the airborne computer and the ξ-ω_n solution programmed into the three degree of freedom flight simulation is the requirement for an analog to digital converter for pitch rate input signals and a digital to analog converter for output signals to the adaptation servo. It was found from the simulation program that simple trapezoidal differentiation of $\dot{\theta}$ and $\ddot{\theta}$ yielded sufficiently accurate values of $\ddot{\theta}$ and $\dddot{\theta}$ for use in the ξ-ω_n calculations.

From a short preliminary survey of computer requirements it was determined that a special purpose digital computer having a 130 word storage capacity and a repertoire of 12 arithmetic and manipulating instructions would suffice for this application. And, if it is assumed that each instruction requires 5 microseconds for execution (a representative state-of-the-art-estimation), the sequence from raw data input to generation of a new adaptation-servo command will require 12,600 microseconds, or in other words, the total cycle will have a maximum possible repetition rate of 83 times per second. However, the cycle utilized in this study was repeated only 40 times/second, which leaves a comfortable execution-speed margin.

Figure 31. Flow Diagram of the Proposed Damping Ratio Computer for Use in the Adaptation Loop



VIII - CONCLUSIONS

It has been demonstrated that a simple, special-purpose digital computer can be utilized to determine the natural frequency and damping ratio of the short-period mode of the free longitudinal oscillations exhibited by a representative high performance fighter aircraft. And, it has also been shown that the calculated values of damping ratio can be used in a closed loop manner to vary the gain of a self adaptive pitch damper which is capable of maintaining constant airframe short-period damping ratio over the full speed and altitude range of the vehicle. The system investigated is capable of performing the adaptation process, from basic unaugmented airplane to $\zeta = 0.7$, in a period of time ranging from 1.4 to 3.1 seconds depending on flight condition; and this speed of operation was shown to be similar to that of the most recent MIT flight-vehicle self-adaptive system.

In conducting an evaluation program with the proposed mechanization, in conjunction with a longitudinal three degree of freedom flight simulation, it was found that measurements of pitch rate rather than pitch angle were necessary to extract the short period characteristics from the combined phugoid and short period motions. First and second derivatives of pitch rate are then determined in the adaptation computer and sampled values of $\dot{\theta}$, $\ddot{\theta}$, and $\dddot{\theta}$ set into the iterative routine for determination of short period damping ratio and natural frequency. The system requires no perturbation signals or limit cycling for operation of the identification process, however, the system will adapt only while the airframe exhibits pitching motion. It should be emphasized that the mechanization investigated in this study is not a complete longitudinal control system, in that there is no provision for external pilot-input signals. However, it was proposed in Chapter V that Corbin's forward-loop-gain computing system could be converted from analog to digital form, and used in conjunction with the present computer routine to enable the introduction of external control inputs.

Perhaps the most promising area for further investigation of this particular adaptation method lies in a study of possible means of forcing the convergence of each series of ξ-ω_n iterative calculations. It is felt that, if the ξ-ω_n calculations could be made 100 percent convergent, the speed and accuracy of the adaptation process could be greatly enhanced. Other factors which would require investigation before a practical mechanization could be accomplished are: 1. the effect of noise (atmospheric turbulence) introduced into the forward loop, 2. the effects of control surface rate and amplitude limiting, and 3. the influence of transducer inaccuracies and noise on the operation of the adaptation loop.

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Table of Stability Derivatives of a Representative High Performance Fighter-Attack Aircraft

Altitude 10,000 ft.

	M = 0.9	M = 1.2	M = 1.4	M = 1.8
$C_{m_{it}}$	-.678	-.441	-.338	
C_{m_q}	-10.93	-4.358	-3.244	
C_{m_α}	-.475	-.665	-.515	
$C_{m_{\dot{\alpha}}}$	-.891	+2.03	+.785	
C_{L_α}	4.22	4.18	2.895	
$C_{L_{it}}$	.503	.279	.214	
C_{n_β}	.205	.198	.112	
$C_{n_{\delta r}}$	-.344	-.293	-.224	
$C_{n_{\delta a}}$	.0134	.0031	.0029	
C_{n_r}	-.431	-.444	-.360	
C_{n_p}	.090	.064	.054	
C_{l_β}	-.130	-.129	-.117	
$C_{l_{\delta r}}$	.111	.098	.085	
$C_{l_{\delta a}}$	.009	.0065	.0037	
C_{l_r}	.161	.144	.119	
C_{l_p}	-.345	-.255	-.21	
C_{Y_β}	-1.15	-1.01	-.91	
C_{Y_r}				
$C_{Y_{\delta r}}$	.720	.573	.435	
C_{Y_p}				
C_{D_o}				

All derivatives per radian
or radian per second

Table of Stability Derivatives

Altitude 25,000 ft.

	M = 0.9	M = 1.2	M = 1.4	M = 1.8
$C_{m_{it}}$	−.831	−.567	−.418	−.269
C_{m_q}	−6.31	−3.95	−3.19	−2.36
C_{m_α}	−.607	−.865	−.676	−.4125
$C_{m_{\dot{\alpha}}}$	−1.11	+2.20	+.973	+.850
C_{L_α}	4.49	3.80	3.095	2.29
$C_{L_{it}}$	.608	.359	.264	.170
C_{n_β}	.234	.241	.158	.048
$C_{n_{\delta r}}$	−.370	−.328	−.256	−.168
$C_{n_{\delta a}}$	.0131	.0029	.0028	.0028
C_{n_r}	−.472	−.498	−.410	−.270
C_{n_p}	.100	.079	.070	.062
C_{l_β}	−.148	−.15	−.139	−.087
$C_{l_{\delta r}}$	.124	.113	.100	.075
$C_{l_{\delta a}}$	.0116	.00516	.00402	.00336
C_{l_r}	.183	.159	.138	.114
C_{l_p}	−.385	−.31	−.26	−.205
C_{Y_β}	−1.22	−1.1	−1.0	−.85
C_{Y_r}				
$C_{Y_{\delta r}}$	.808	.642	.500	.350
C_{Y_p}				
C_{D_o}				

Table of Stability Derivatives

Altitude 45,000 ft.

	M = 0.9	M = 1.2	M = 1.4	M = 1.8
$C_{m_{i_t}}$	−.991	−.745	−.556	−.367
C_{m_q}	−4.98	−4.71	−3.83	−2.90
C_{m_α}	−.722	−1.16	−.915	−.602
$C_{m_{\dot\alpha}}$	−1.36	+2.37	+1.07	+.90
C_{L_α}	4.74	4.19	3.43	2.47
$C_{L_{i_t}}$	.725	.471	.352	.232
C_{n_β}	.267	.287	.206	.095
$C_{n_{\delta r}}$	−.399	−.364	−.291	−.201
$C_{n_{\delta a}}$	.0131	.010	.0102	.010
C_{n_r}	−.540	−.551	−.470	−.323
C_{n_p}	.12	.102	.084	.082
C_{l_β}	−.158	−.171	−.160	−.095
$C_{l_{\delta r}}$	.134	.129	.118	.087
$C_{l_{\delta a}}$	.0139	.0127	.0098	.009
C_{l_r}	.242	.178	.139	.114
C_{l_p}	−.42	−.36	−.315	−.26
C_{Y_β}	−1.30	−1.19	−1.10	−.57
C_{Y_r}				
$C_{Y_{\delta r}}$	.870	.710	.568	.419
C_{Y_p}				
C_{D_0}				

APPENDIX II

Table of Airplane Physical Characteristics

Airplane mass	1,110 slugs
I_X	19,642 slug-ft.^{2}
I_Y	129, 533 slug ft.^{2}
I_Z	142,398 slug-ft.^{2}
I_{XZ}	8,219 slug-ft.^{2}
Wing area	394.84 ft.^{2}
Wing span	36.58 ft.
Wing mean aerodynamic chord	12.24 ft.
Wing aspect ratio	3.39
I_{Xe}	13.95 slug-ft.^{2}
ω_e	1,259 rad/sec. (Cruising)

APPENDIX III

A SUMMARY OF MAJOR AIRCRAFT STABILITY DERIVATIVES

DERIVATIVE	COMMENTS
$C_{l\delta a}$	Rolling moment/ aileron deflection - little importance in dynamic stability.
$C_{l\delta r}$	Rolling moment/ rudder deflection - cross coupling term of little importance in dynamic stability.
C_{l_p}	Rolling moment/ roll rate - viscous damping in roll, balances aileron moment to produce a constant rolling velocity for a given aileron deflection, has small damping effect on lateral-directional spiral mode.
C_{l_r}	Rolling moment/ yawing velocity - contributes slightly to Dutch roll, most important cause of spiral divergence mode.
$C_{l\beta}$	Rollimg moment/ sideslip angle - dehedral effect, large negative value will produce good spiral characteristics and poor Dutch roll characteristics.
$C_{l\dot{\beta}}$	Rolling moment/ sideslip angular rate, usually negligible in magnitude.
$C_{m_{i_t}}$	Pitching moment/ elevator deflection, control derivative, little importance in dynamic stability.
C_{m_q}	Pitching moment/ pitch rate, viscous damping in pitch, large effect on damping of longitudinal short period mode.
$C_{m\dot{\alpha}}$	Pitching moment/ rate of change of angle of attack, due to aerodynamic time lag and aeroelastic effects, contributes to damping of longitudinal short period mode.
$C_{m\alpha}$	Pitching moment/ angle of attack - produces restoring moment, establishes frequency of longitudinal short period mode.
$C_{m\beta}$	Pitching moment/ sideslip angle - aerodynamic cross coupling derivative, usually negligible in magnitude.
$C_{n\delta r}$	Yawing moment/ rudder deflection - control derivative, little importance in dynamic stability.
C_{n_r}	Yawing moment/ yaw rate - viscous damping in yaw, major contributor to damping in Dutch roll (lateral-directional short period mode).

C_{n_r} — Yawing moment/ roll rate - aerodynamic cross coupling derivative, contributes slightly to damping in Dutch roll.

C_{n_β} — Yawing moment/ sideslip angle - produces directional restoring moment, establishes frequency of Dutch roll mode.

$C_{n_{\dot\beta}}$ — Yawing moment/ sideslip angular rate - contributes slightly to damping of Dutch roll.

$C_{n_{\delta_a}}$ — Yawing moment/ aileron deflection - control cross coupling derivative, little importance in dynamic stability.

C_{Y_β} — Side force/ sideslip angle - contributes to damping of Dutch roll.

C_{Y_p} — Side force/ roll rate - aerodynamic cross coupling derivative, magnitude usually negligible.

C_{Y_r} — Side force/ yaw rate - magnitude usually negligible.

$C_{Y_{\dot\beta}}$ — Side force/ sideslip angular rate - magnitude usually negligible.

C_{L_α} — Lift/ angle of attack - effects frequency of longitudinal phugoid mode, has major effects on aircraft maneuverability.

$C_{L_{\dot\alpha}}$ — Lift/ rate of change of angle of attack - due to aerodynamic time lag and aeroelastic effects, has negligible effect on dynamics.

$C_{L_{i_t}}$ — Lift/ elevator deflection - magnitude negligible for conventional aircraft, has effect on tailless aircraft and those with short tail length.

C_D — Airplane drag coefficient - primary contributor to damping of longitudinal phugoid oscillation.

C_{D_α} — Drag/ angle of attack - negligible effect onlongitudinal phugoid mode because this mode takes place at constant angle of attack.

APPENDIX IV

EQUATIONS OF MOTION

The equations of motion are derived from the two familiar expressions: Force = Mass x Acceleration and

Moment = Moment of Inertia x Angular Acceleration

From classical mechanics, the equations of motion of a rigid body in free space with no aerodynamic or gravity forces acting are.

$$F_X = m(\dot{u} + qw - rv)$$

$$F_Y = m(\dot{v} + ru - pw)$$

$$F_Z = m(\dot{w} + pv - qu)$$

$$L = \dot{p} I_X + qr(I_Z - I_Y) - I_{XZ}(pq + \dot{r}) + I_{XY}(rp - \dot{q}) + I_{YZ}(r^2 - q^2)$$

$$M = \dot{q} I_Y + (pq - \dot{r}) I_{YZ} - (\dot{p} + qr) I_{XY} + rp(I_X - I_Z) + (p^2 - r^2) I_{XZ}$$

$$N = \dot{r} I_Z + (qr - \dot{p}) I_{XZ} - (\dot{q} + rp) I_{YZ} + pq(I_Y - I_X) + (q^2 - p^2) I_{XY}$$

Note that all symbols are defined in figure 1. It is seen that the last two terms in each of the first three equations are the product of an angular and a linear velocity and thus represent centripetal accelerations directed along the respective axes. The last four terms in each of the last three equations represent moments created by inertial cross coupling. These terms arise from a variety of sources including gyroscopic precessional torques and moments due to dynamic unbalance. A detailed derivation of the inertial coupling terms will not be given in this paper, but it should be pointed out that these terms can sometimes become dominant, with disastrous results, when the amplitudes and rates of aircraft motion become large (ref. 3). It is recalled that the aircraft possesses vertical symmetry which causes the products of inertia I_{XY} and I_{ZY} to be zero. This eliminates two inertial coupling terms from each equation. Next, the components of force due to gravity, and the aerodynamic forces and moments are included.

$$m\dot{u} = rv - qw - mg\sin\theta - C_D\frac{\rho}{2}V^2S + \text{Engine Thrust}$$

$$m\dot{v} = pw - ru + mg\cos\theta\sin\phi + C_Y\frac{\rho}{2}V^2S$$

$$m\dot{w} = qu - pv - mg\cos\theta\cos\phi - C_L\frac{\rho}{2}V^2S$$

$$\dot{p}I_X = qr(I_Y - I_Z) + \dot{r}I_{XZ} + pqI_{XZ} + C_l\frac{\rho}{2}V^2Sb$$

$$\dot{q}I_Y = pr(I_Z - I_X) + r^2I_{XZ} - p^2I_{XZ} - I_{Xe}\omega_e r + C_m\frac{\rho}{2}V^2S\bar{c}$$

$$\dot{r}I_Z = pq(I_X - I_Y) + \dot{p}I_{XZ} - qrI_{XZ} + I_{Xe}\omega_e q + C_n\frac{\rho}{2}V^2Sb$$

The $I_{Xe}\,\omega_e$ terms represent the angular momentum of rotating engine parts, which when multiplied by a yaw or pitch angular velocity, produces a gyroscopic precessional torque about the pitch or yaw axis respectively. These terms are significant in a turbojet powered aircraft. It should be noted that the wingspan, b, is used as moment arm for the aerodynamic moments in the roll and yaw equations whereas mean aerodynamic chord, $\bar{c}$, is used as moment arm in the pitch equation.

A further refinement of the equations consists of converting the components of linear velocity, u,v, and w, along the X,Y, and Z axes into more familiar aerodynamic quantities. The relationships used are:

$$u \approx V, \quad \text{airplane forward velocity}$$

$$v \approx V \sin\beta \approx V\beta$$

$$w \approx V \sin\alpha \approx V\alpha$$

$$\dot{v} \approx V\dot{\beta}$$

$$w \approx V\dot{\alpha}$$

Where α and β, the aerodynamic attack and sideslip angles, define the inclination of the forward velocity vector to the X body axis. The complete equations of motion with the aerodynamic coefficients in expanded form are shown in table I. The reader will see that the aerodynamic rotary derivative terms are multiplied by certain additional flight condition parameters in order that the value of the derivative can remain as constant as possible over a wide range of flight conditions. An example is the use of the term $\frac{b}{2V}$ with derivatives taken with respect to roll rate. Similar terms for other derivatives are listed in the table of symbols.

One final set of equations must be used to enable the conversion of angular rates about the body axes to angular rates with respect to the earth coordinate system. The necessary Euler equations are (ref. 5.).

$$\dot{\theta} = q \cos\phi - r \sin\phi$$

$$\dot{\phi} = p + q \sin\phi \tan\phi + r \cos\phi \tan\theta$$

$$\dot{\psi} = (q \sin\phi + r \cos\phi) \sec\theta$$

$\dot{\theta}$ and $\dot{\phi}$ can be integrated to obtain θ and ϕ which are used in the equations of motion.

APPENDIX V

DERIVATION OF THE LONGITUDINAL SHORT-PERIOD TRANSFER FUNCTION FROM THE EQUATIONS OF MOTION

Of the two longitudinal modes of airplane motion, the phugoid and the short-period, only the latter mode is of interest in stability augmentation considerations because the phugoid normally does not cause any deterioration of handling qualities. In Section III, a transfer function of the mode of interest in stability augmentation systems must be included in the block diagram of the proposed pitch damper to enable an analysis of the system. Thus, the longitudinal short-period transfer function must be derived from the airplane equations of motion. The degrees of freedom of interest being pitch and normal translation for which the corresponding equations are shown below.

$$\ddot{\theta} = \frac{A\bar{c}}{I_Y} C_{m_{i_t}} i_t + \frac{A\bar{c}^2}{2VI_Y} C_{m_{q+\dot{\alpha}}} \dot{\theta} + \frac{A\bar{c}}{I_Y} C_{m_\alpha} \Delta\alpha$$

$$\dot{\alpha} = \dot{\theta} + \frac{g}{V} \cos\theta - \frac{A}{mV} C_{L_\alpha} \alpha$$

The pitch damping derivatives, C_{m_q} and $C_{m_{\dot{\alpha}}}$, are usually added together to form an equivalent $C_{m_{q+\dot{\alpha}}}$, and engine torque and tail lift terms have been neglected because of their low magnitudes. Assumptions necessary for the derivation of the transfer function are: 1. an equilibrium condition exists along the Z axis when airplane weight = airplane lift, $W = \frac{\rho}{2} S V^2 C_{L_\alpha} \alpha\text{-trim}$, and 2. $\cos\theta \approx 1$.

The difference between α and α-trim will be denoted α, The equations of motion then become:

$$\ddot{\theta} = \frac{A\bar{c}}{I_Y} C_{m_{i_t}} i_t + \frac{A\bar{c}^2}{2VI_Y} C_{m_{q+\dot{\alpha}}} \dot{\theta} + \frac{A\bar{c}}{I_Y} C_{m_\alpha} \alpha_1$$

$$\dot{\alpha} = \dot{\theta} - \frac{A}{mV} C_{L_\alpha} \alpha_1$$

The next step is the introduction of the differential operator, S, and the elimination of angle of attack from the equations in order that the $\frac{\theta}{i_t}$ equation may be obtained.

$$S^2\theta = \frac{A\bar{c}}{I_Y} C_{m_{i_t}} i_t + \frac{A\bar{c}^2}{2VI_Y} C_{m_{q+\dot{\alpha}}} \theta S + \frac{A\bar{c}}{I_Y} C_{m_\alpha} \alpha_1$$

$$\alpha S = \theta S - \frac{A}{mV} C_{L_\alpha} \alpha_1$$

or: $$\alpha_1 = \frac{\theta S}{S + \frac{A}{mV} C_{L_\alpha}}$$

Substituting α_1

$$\theta S^2 - \frac{A\bar{c}^2}{2VI_Y} C_{m_{q+\dot{\alpha}}} S\theta - \frac{A\bar{c}}{I_Y} C_{m_\alpha} \frac{S\theta}{S + \frac{A}{mV} C_{L_\alpha}} = \frac{A\bar{c}}{I_Y} C_{m_{i_t}} i_t$$

The $\frac{\theta}{i_t}$ transfer function is:

$$\frac{\theta}{i_t} = \frac{\frac{A\bar{c}}{I_Y} C_{m_{i_t}} \left(S + \frac{A}{mV} C_{L_\alpha}\right)}{S\left(S^2 + S\frac{A}{mV} C_{L_\alpha} - \frac{A\bar{c}^2}{2VI_Y} C_{m_q} S - \frac{A\bar{c}^2}{2VI_Y} C_{m_q} \frac{A}{mV} C_{L_\alpha} - \frac{A\bar{c}}{I_Y} C_{m_\alpha}\right)}$$

Rearranging

$$\frac{\theta}{i_t} = \frac{\frac{A\bar{c}}{I_Y} C_{m_{i_t}} \left(S + \frac{A}{mV} C_{L_\alpha}\right)}{S\left(S^2 + S\left(\frac{A}{mV} C_{L_\alpha} - \frac{A\bar{c}^2}{2VI_Y} C_{m_q}\right) - \left(\frac{A\bar{c}^2}{2VI_Y} C_{m_q} \frac{A}{mV} C_{L_\alpha} - \frac{A\bar{c}}{I_Y} C_{m_\alpha}\right)\right)}$$

The same equation in more general notation is shown below.

$$\frac{\theta}{i_t} = \frac{K_\theta (S + T_\theta)}{S(S^2 + 2\zeta \omega_n S + \omega_n^2)}$$

Where:

K_θ is the airplane elevator effectiveness

T_θ is the flight path time constant

ζ is the short-period damping ratio

ω_n is the short-period natural frequency

It can be seen that the steady state transfer function is essentially a pitch rate command system for a given horizontal stabilizer deflection.

APPENDIX VI

NOTES ON THE DIGITAL COMPUTER SIMULATION OF A DYNAMIC SYSTEM

Although analog computers are generally used to simulate system dynamics, the systems investigated in this study were simulated on a stored program, general purpose digital computer using techniques similar to analog computer programs. The techniques are similar in that both methods use some form of integration combined with other operations such as summation, subtraction, and multiplication to solve a set of simultaneous differential equations of motion; each equation of motion representing a degree of freedom of the system. An analog computer program may be regarded as a continuous parallel solution of the equations of motion, whereas the digital method employs a step by step serial solution. A number of digital integration techniques are available, ranging from simple rectangular integration to highly accurate, but time consuming methods, such as the Runge-Kutta numerical integration. The technique selected for this study is a modified form of the trapezoidal rule and is shown below.

$$X_n = X_{n-1} + \frac{h}{2}[3\dot{X}_{n-1} - \dot{X}_{n-2}]$$

Where: n-k denotes values computed in previous cycles
h is the width, in time, of the computation interval.

A major difference between the digital solution and the concept of the analog flow diagram lies in the fact that the digital method performs integration of lower ordered derivatives first. As an example, the steps in the computation cycle of a second order system solution are shown below:

$$X_n = X_{n-1} + \frac{h}{2}[3\dot{X}_{n-1} - \dot{X}_{n-2}]$$
$$\dot{X}_n = \dot{X}_{n-1} + \frac{h}{2}[3\ddot{X}_{n-1} - \ddot{X}_{n-2}]$$
$$\ddot{X}_n = -2\zeta\omega_n\dot{X}_n - \omega_n^2 X_n$$

The computation cycle is repeated at a high rate and the values of $X, \dot{X}, \& \ddot{X}$ generated in each pass represent points on a time history of system motion. When a more complex system having a greater number of equations of motions is simulated, the integrations for each equation of motion are performed in a serial manner such that one pass is executed through all equations before recycling.

The accuracy of this technique is, of course, a function of the width of time increment between computing cycles, and it was noted that when using the method described above, integrator errors were manifested as an undamping of the system dynamics. In some lightly damped cases it was found that cumulative integrator errors actually caused the solution to diverge, the corrective action being the use of finer calculation intervals. It has also been noted that the required fineness of calculation intervals is a function of both the period of the system mode of highest frequency and the damping of this mode. A rough rule of thumb is the use of 200 divisions per period for $\zeta = 0.1$, down to 50 divisions per period for $\zeta = .5$, to obtain one percent solution accuracy.

The following five pages are sample listings of two of the FORTRAN-programming-language programs written for this study. The first is a second order system and the second a three degree of freedom flight simulation. The programs were run on both the CDC 1604 and the IBM 7090 general purpose digital computers.

SECOND ORDER PLANT IDENTIFICATION ROUTINE 3

```
      M=1
    1 READ INPUT TAPE 3,2, ACHM, ALT, DENSTY, VELX, DYNPRS, THETA, CTHET
     1A,STHETA, ALPHAT, THRUST
    2 FORMAT (F4.1, I6, F8.7, F7.2, F8.1 , 4F7.6, F8.2)
      READ INPUT TAPE 3,3, CMIT, CMQA, CMA, CLA, CLDELE, CDU, THEDD
    3 FORMAT (6F6.3, F8.4)
      WRITE OUTPUT TAPE 2,4, ACHM, ALT
    4 FORMAT (1H1//58H   THREE DEGREE OF FREEDOM FLIGHT SIMULATION  MACH
     1NUMBER= F4.1, 12H  ALTITUDE= I6)
      INITIAL CONDITIONS
      J=1
      J1=1
      K=1
      L=1
      L1=1
      TIME=0.
      SRVGN=.25
      DAMPR=.7
      DMPGN=0.
      DAMPR2=.4
      ACCLN=0.
      OMEGAN=0.
   50 ALPHA=.5
      ALPHD=0.
      ALPHD1=0.
      THETD=0.
      THETD1=0.
      THEDD1=THEDD
      THTTD=0.
      THTTD2=0.
      VELCTY=VELX
      VELXD=0.
      VELXD1=0.
      ACTPOS=0.
      ACTVL=0.
      ACTVL1=0.
      ACTCL=0.
      ACTCL1=0.
      SIGNAL=0.
      ALPHA2=.5
      ALPHD2=0.
      THETA2=.5
      THETD2=0.
      THEDD2=THEDD
    5 WRITE OUTPUT TAPE 2,6
    6 FORMAT (105H   TIME-SEC        THETA        THETADOT    FWD VELOCITY ACT
     1UATOR POS NORM ACCEL  DAMPER GAIN OMEGAN        DAMPR)
    7 WRITE OUTPUT TAPE 2,8,TIME, THETA, THETD,VELCTY, ACTPOS, ACCLN,
     1DMPGN, OMEGAN, DAMPR
    8 FORMAT (9F12.6)
      IF(L-1000)9,12,12
    9 L=L+1
      IF(L1-200)51,52,52
   51 L1=L1+1
      GO TO 14
```

```
 52 L1=1
    GO TO 50
 12 CALL EXIT
    ACTUATOR SIMULATION
 14 ACTPOS=ACTPOS+.0025*(3.*ACTVL-ACTVL1)
    ACTVL1=ACTVL
    ACTVL=ACTVL+.0025*(3.*ACTCL-ACTCL1)
    ACTCL1=ACTCL
    ACTCL=-ACTVL*49.-ACTPOS*1225.+SIGNAL*1225.
    INTEGRATION OF LONITUDINAL ACCELERATION TO OBTAIN VELOCITY
    VELX=VELX+.0025*(3.*VELXD-VELXD1)
    VELXD1=VELXD
    VELXD=(THRUST/1110.)-(AOVERM*CDO)-(AOVERM*((CLA*ALPHA)**2.)/(3.141
   159*3.39))-(32.18*SINF(THETA))
    VELCTY=VELX
    CALCULATION OF AERODYNAMIC PARAMETERS DEPENDENT ON VELOCITY
    DYNPRS=DENSTY*(VELCTY**2.)*394.84
    ACIYY=DYNPRS*12.24/129533.
    AC2VIY=(ACIYY*12.24)/(2.*VELCTY)
    AMV=DYNPRS/(VELCTY*1110.)
    AOVERM=DYNPRS/1110.
    SOLUTION OF EQUATIONS OF MOTION
    THETA=THETA+.0025*(3.*THETD-THETD1)
    THETD1=THETD
    THETD=THETD+.0025*(3.*THEDD-THEDD1)
    THEDD1=THEDD
    THEDD=(ACIYY*CMIT*ACTPOS)+(AC2VIY*CMQA*THETD)+(ACIYY*CMA*(ALPHA-AL
   1PHAT))
    ALPHA=ALPHA+.0025*(3.*ALPHD-ALPHD1)
    ALPHD1=ALPHD
    ALPHD=THETD+(32.18*COSF(THETA)/VELCTY)-(AMV*CLA*ALPHA)
    CALCULATE NORMAL ACCELERATION
    ACCLN=(CLA*ALPHA*DYNPRS)/(1110.*32.18)
    PITCH DAMPER SIMULATION
    SIGNAL=THETD*DMPGN
    TIME=TIME+.005
    DIFFERENTIATE PITCHING ACCELERATION
    THTTD=(THEDD-THEDD1)/.005
    ADAPTATION SERVO
    DMPGN=DMPGN+((.7-DAMPR)*SRVGN*.005)
    IF(J-5)15,16,16
 15 J=J+1
    GO TO 14
 16 J=1
    CALCULATE NATURAL FREQUENCY AND DAMPING RATIO- ASSUMED DAMPR=.4
    DAMPR1=DAMPR2
 21 GAMMA=DAMPR1*THEDD2/THETD2
    BETA=SQRTF(GAMMA**2.-THTTD2/THETD2)
    OMEGAN=-GAMMA+BETA
    DAMPR1=-(THTTD/(2.*OMEGAN*THEDD))-((OMEGAN*THETD)/(2.*THEDD))
 27 IF(J1-20)30,31,31
 30 J1=J1+1
    GO TO 21
 31 J1=1
    IF(OMEGAN-8.)25,26,26
 25 IF(OMEGAN)26,32,32
```

```
32 IF(DAMPR1-1.)33,26,26
33 IF(DAMPR1)26,34,34
34 DAMPR=DAMPR1
   DAMPR2=DAMPR1
   GO TO 29
26 DAMPR1=DAMPR2
35 GAMMA=DAMPR1*THEDD2/THETD2
   BETA=SQRTF(GAMMA**2.-THTTD2/THETD2)
   OMEGAN=-GAMMA-BETA
   DAMPR1=-(THTTD/(2.*OMEGAN*THEDD))-((OMEGAN*THETD)/(2.*THEDD))
36 IF(J1-20)37,38,38
37 J1=J1+1
   GO TO 35
38 J1=1
   IF(OMEGAN-8.)39,40,40
39 IF(OMEGAN)40,41,41
41 IF(DAMPR1-1.)42,40,40
42 IF(DAMPR1)40,34,34
40 DAMPR=.7
   WRITE OUTPUT TAPE 2,43
43 FORMAT (31H  NO SOLUTION FOR DAMPING RATIO)
29 THETD2=THETD
   THEDD2=THEDD
   THTTD2=THTTD
   IF(K-10)17,18,18
17 K=K+1
   GO TO 7
18 K=1
   GO TO 5
   END(1,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
```

```
      XEQ
      I=1
      J=1
      K=1
      XDO=0.
      XDDO=0.
      XO=1.
      XDN=0.
      XN=1.
      TIME=0.
      XDOO=0.
      XDDOO=0.
      WRITE OUTPUT TAPE 2,1
    1 FORMAT (1H1//44H PLANT IDENTIFICATION ROUTINE - SECOND ORDER)
    2 DAMPR=.4
      XDDN=-2.4*XDN-36.*XN
      XDINCR=(XDDN+XDDO)*.00025
      XDN=XDO+XDINCR
      XDDO=XDDN
      XINCR=(XDO+XDN)*.00025
      XN=XN+XINCR
      XDO=XDN
      TIME=TIME+.0005
      IF(I-1000)4,5,5
    4 I=I+1
      GO TO 2
    5 I=1
      WRITE OUTPUT TAPE 2,6
    6 FORMAT (1H //55H        TIME        ACCELERATION  VELOCITY     DISPLA
     1EMENT)
      WRITE OUTPUT TAPE 2,7, TIME, XDDN, XDN, XN
    7 FORMAT (4F13.6)
C     ITERATIVE CALCULATION OF NATURAL FREQUENCY AND DAMPING RATIO GIVE
C      A STARTING VALUE OF DAMPING RATIO = 0.4
      WRITE OUTPUT TAPE 2,8
    8 FORMAT (25H     OMEGA-N      DAMPRAT)
    9 ALPHA=DAMPR*XDOO/XO
      BETA=SQRTF(ALPHA**2.-XDDOO/XO)
      OMEGAN=-ALPHA+BETA
      DAMPR=-(XDDN/(2.*OMEGAN*XDN))-((OMEGAN*XN)/(2.*XDN))
      OMEG=ABSF(OMEGAN)
      IF(OMEG-100.)15,16,16
   15 DMP=ABSF(DAMPR)
      IF(DMP-1.)17,16,16
   16 WRITE OUTPUT TAPE 2,18
   18 FORMAT (19H SOLUTION DIVERGING)
      DAMPR=.4
      J=1
   20 ALPHA=DAMPR*XDOO/XO
      BETA=SQRTF(ALPHA**2.-XDDOO/XO)
      OMEGAN=-ALPHA-BETA
      DAMPR=-(XDDN/(2.*OMEGAN*XDN))-((OMEGAN*XN)/(2.*XDN))
      OMEG=ABSF(OMEGAN)
      IF(OMEG-100.)21,22,22
   21 DMP=ABSF(DAMPR)
      IF(DMP-1.)23,22,22
```

```
22 WRITE OUTPUT TAPE 2,24
24 FORMAT (19H SOLUTION DIVERGING)
   GO TO 12
23 WRITE OUTPUT TAPE 2,25, OMEGAN, DAMPR
25 FORMAT (2F13.6)
   IF(J-20)26,12,12
26 J=J+1
   GO TO 20
17 WRITE OUTPUT TAPE 2,10, OMEGAN, DAMPR
10 FORMAT (2F13.6)
   IF(J-20)11,12,12
11 J=J+1
   GO TO 9
12 J=1
   XO=XN
   XDOO=XDN
   XDDOO=XDDN
   IF(K-40)13,14,14
13 K=K+1
   GO TO 2
14 CALL EXIT
   END(1,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
```