MANAGING COMPLEX INVENTORY CONTROL PROBLEMS USING MATHEMATICAL DECISION RULES

GEORGE K. ARMSTRONG

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By

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The complexity of the inventory manager's problem requires that he seek out new and better ways of examining his alternatives and solving his day to day operating problems. To do so and still remain effectively in control, he must understand the limitation and capabilities of any mathematically conceived scheme that he utilizes and continuously exercise his management prerogative of setting, adjusting and insuring compliance with the parameters within which the system operates. This paper recognizes the methods that are available to management which allow computation of optimal order quantities, the time to order and associated safety level and forecasting of requirements. In addition to these three functional activities, management can use the three in combination to successfully plan and subsequently control the activities of an agency charged with the efficient and economical management of a large and diversified inventory.

May 1962
Master of Science in Management
Navy Management School

MANAGING COMPLEX INVENTORY CONTROL

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A Research Paper

Presented To

the Faculty of the Navy Management School

U. S. Naval Postgraduate School

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science of Management

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May 1962

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PART I

INTRODUCTION

"Inventory management is that phase of military logistics which controls the input, availability and disposal of material in the Naval Establishment."[1] Ten years ago, and only to a slightly lesser extent today, the assertion, within an organizational unit, that scientific or mathematical techniques offer a reliable and efficient tool to management created consternation and concern both to management and the rank and file worker. Fears are manifest within the former group due to the feeling of inadequacy to cope with the language of the mathematician or with the resulting mathematical model for rapid simulation of everyday business transactions and in the latter for job security or disparagement of personal or professional ability. The reaction is analagous to the results experienced during and for a hundred years after the industrial revolution when one man management was slowly forced to give way to corporate leadership and the skills of the artisan yielded to the machine. The pay off in any new endeavor offering change and promises of betterment is accelerated on its course by building a foundation of confidence and trust at inception. In retrospect, this hindsight gained from observing the growing pains of sound and experiementally tested inventory control systems

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[1]Office of Naval Material, Navy Policy and Standards for Inventory Management, NAVEXOS P-1500 (Washington: Department of the Navy, 1960), p. 1-1

developed for Navy inventories, the prospect of radical change from human decisions based on loosely guided human judgments and forecasts to a system readable only in a strange language and based on the outputs of a formula comprised of foreign characters or "dumped" from a machine was not in itself acceptable. Neither the inventory manager nor the stock analyst could place much confidence in the results of such a system allowing for little if any judgment based on years of personal experience with specific stock units and the customers. It has been noted recently that a new, possibly more palatable term has entered the literature on the subject of application of quantitative techniques for the making of management decisions, information technology.[2] Narrowing the field of interest specifically to the subject matter of this paper, the one and only objective of inventory management within the Navy is the decision made relative to stockage policy for items having support significance to the fleet and shore establishment. A decision, no matter how it was reached, is not good or bad in itself but is relative to the consequences realized during the time in which the influence of the decision is being felt. The job of the Navy inventory manager is complex enough due to the sheer volume of numbers and conflicting demand patterns for his product to deny the necessity for aids capable of providing current and consistent raw and computational data on scheduled demand or when index criteria

[2]George P. Shultz and Thomas L. Whisler, Management Organization and the Computer (Glencoe: The Free Press of Glencoe, Illinois, 1960), p. 3.

indicate action is required. Within the field of informational technology, three areas of activity can be involved: 1) the use of mathematical and statistical methods with or without the aid of electronic computers; 2) the use of computers for mass integrated data processing and 3) the direct application of computers to simulation techniques which provide the data on which decisions are based.[3] Although mathematical and statistical processes are vital to any system which removes individual, repeated judgments concerning the item control, these processes can be routinized while retaining the elements of sophistication which act for consistency and relative reliability of forecasts and decisions. The theories involved in a program such as "step wise regression", the fitting of a formula to a set of data, are with very little effort understandable as only a compilation of cost data applied to the performance function of the activity and the predictability of incurring such costs under varied possible courses of action.

The management authority for the setting of policy and establishing of parameters of effectiveness can never be delegated to, appropriated by nor abrogated by a control system or computer, manual or electronic, bounded by such limits. By no means is such a system infallible in all respects and management when assigning the objective functions to be optimized must do so with the full realization that:

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[3]The application of each area will be treated insofar as applicable to total inventories or segments of inventory in Part III.

1) give and take between multiple objectives is sometimes mandatory (parameters may be illogical or unobtainable in the light of other restraints or changing conditions.)

2) without some vigilance, the decision process may become too rigidified by the over dependence on quantitative data with proper qualitative judgments obscured by the volume and rapidity of solution and data flow from a scientifically formulated system. The overweighing, advantageous considerations of the use of an informational technology system for the control of inventories are:

1) Quantification of all pertinent elements of interest.

2) Broadening the scope of each decision and allowing for the automatic investigation of alternatives under current conditions.

3) Shortening the planning or review period.

4) Reducing the incidence of poor decisions caused by internal and external time lags in transaction reporting and data flow.

5) Consolidating item records and insuring availability in various combinations of item history or forecast for the use of managers in the various functional areas of inventory control responsibility; i.e., stock status review, stratification, budgeting, production planning, disposal, etc.

6) De-personalizing decisions and insuring only the properly constituted "safety level" of stock over the administrative and production lead time.

After the basic decisions of management have been made, a systematic, scientific inventory control system can operate to effect a time sequence of decisions not manifestly inherent in any other decision making process. Any one decision will be relative to the preceding one and the following ones and to preceding demands and forecasted demands (as differentiated from past average demand). In essence, through simulation of results to be anticipated by planned manipulation and computation of variables acting on an inventory, management can evaluate with more assurance of accuracy inventory wide the effect of alternative action decisions on:

1) Total customer service; material effectiveness

2) Total inventory holding cost; financial effectiveness

3) Budget requirements; effective planning prior to commitment of funds and other resources rather than after the bins are full and funds depleted.

Since methods are available to allow optimization of inventory decision rules, it now becomes the duty of the systems personnel and mathematicians to delineate clearly the capabilities, limitations and organizational relationships essential to optimal implementation and utilization of the rules. In a word, the inventory manager must learn to select and use the system best adapted to his specific problem.

C. Northcote Parkinson, in his latest article, "The Art of Being No. 2", provides lucid insight into the problems that can occur if management is hoodwinked into a situation of subservience to a scientifically conceived program of operations, the language and mechanics of which are completely foreign to top management personnel. Mr. Parkinson points out vividly that the president or the "No. 1" must retain control and to do so must require of himself sufficient understanding of any situation to make a sound, well grounded decision. To quote from the article the words of a mythical "No. 1" at a board meeting following a presentation by a consulting mathematician as to the mathematically formulated approach to the management of his organization;

> All this sounds like gobbledegook. I haven't the least idea what you are talking about and no reason to think that it matters. If you have any constructive comments to make on our organization, make them in plain language, stating what you think should be done. But don't talk to me as you might to a digital computer. I don't like it, don't grasp it, and won't have it.

The language common to the board room must be used within the board room just as the language of the computer and the computer technician must be used in the computer room. There is a common ground between when terms are properly defined and at this level of conversation the decisions will be made and the system feed back evaluated.

The research undertaken to produce this paper has had as its chief objective the compilation of the basic methods and techniques of scientific inventory management compatible with the problems of the Navy. The essence of the paper is to explore the total possibilities of each significant method and primarily to communicate the net effect of each so that management can reach well founded decisions and realize when and to what extent it should participate.

PART II

PRECEDENTS

"The objective of Navy Inventory Managers is the efficient and economical adjustment of supply to demand."[4] The elimination of the word "Navy" in the above quotation from the basic policy guidance offered to all Navy Inventory Managers would leave the phrase applicable to any segment of industry or government. Neither economists, businessmen nor Congressmen the world over could find quarrel with this tenet. The only issue for argument is the "how" of accomplishment. In order to insure compliance with the objective as stated, a system of measurement of effectiveness must be employed and comparative possible processes must be evaluated in the search for the most efficient and economical method.

It is immediately and obviously clear that the dollar profit or plant expansion should not be the effectiveness criteria utilized by the Navy. It must also be realized that a statement to a manager of a segment of a large and diversified inventory that "100% support" or "100% effectiveness" is required of him has absolutely no valid meaning in terms of his real ability to perform; rather, timeliness of support or the fulfillment of material demands is the important factor. To meet anticipated demands for items having application within the Navy whether they be beans, boots,

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[4]Office of Naval Material, op. cit.

bolts or battleships, the cognizant inventory manager may establish a stocking policy for the item requiring its availability from:

1) current Navy (or single manager) owned stocks

2) industry through purchase at time of need

3) repair or substitution of like item

4) fabrication by a Naval activity.

These basic alternatives have not changed since the first U. S. Navy ship was provisioned, loaded with the necessary voyage supplies, despite all of the changes to the organizations charged with the responsibilities of outfitting and maintaining the fleet. For years the determination of gross requirements for a particular cruise was tantamount to inventory management with an occasional inventory by the purser or paymaster and a "top off" of store rooms if and when the required goods could be located in a port of call. The advent of the steam engine required more attention be given the problem of fuel replacement and machinery maintenance. As technology advanced in the field of engineering and as the Navy grew to thousands of ships in service, the art of inventory management lagged far behind, just a few steps ahead of the Dark Ages. Although data accumulation was recognized as a necessity for management, it was stored and used cumulatively in raw form and eventually either haphazardly discarded or never purged. As a result, decisions relative to inventories, particularly of technical repair parts as differentiated from provisions, clothing or general housekeeping articles, were based on faulty population data and poorly conceived forecasts based on inaccurate records of past demand. These forecasts were best guesses of future demand based on long run average past demand in some cases or extremely short run past demand. Very little conscious effort was exerted to determine cause of demand but all were averaged and expected to reoccur. Prior to the appearance of the integrated Navy Supply System in 1947, each Navy bureau and field command had important responsibilities for management of required items of supply. Vertical control was vested in the bureaus for classes of items while shore commands exercised horizontal controls across the entire field of inventory. In total, there was not and could never be efficient and economical management of the inventory problem. Reconsolidation of the management function for specified segments of the Navy-wide inventory into the bureaus and supply demand control points, collectively termed inventory points (ICP), was the initial step leading toward the development and implementation of scientific techniques that could truly put control of vast inventories within the grasp of management if management would take the reins. Not only could the facilities be made available at one central location to perform the day to day functions of item management, but the data affecting that item could be systematically collected, collated and maintained current with respect to total Navy application and usage. Before anything approaching a scientific method of manipulation of item factors could be formalized and implemented, the previously used systems in the decentralized operation were expanded and applied to the world-wide inventories. This was the best and in fact all that was available at the time.

To say that there were no innovations would be an injustice. The introduction of 1) formalized stock control records recognizing and requiring the recording of salient features of the item and the demand for it; 2) improved data storage and sorting facilities such as electric accounting machines; and 3) maintenance procedures to allow for if not to insure the updating and purging of records to meet the rapidly changing configuration of the fleet and its requirements were but several indications of the recognition of the need for an improved inventory management system. Even with these aids, the job of computing requirements and deciding when and how many to buy for stock was a perfunctory duty of a stock analyst. Depending upon the time of review of the status of the item, the recommendation would probably correspond closely to the number of items issued since the last review or when stock on hand reached some figure long before determined to be a reorder level, an annual average usage, or increment or multiple thereof, would be recommended for order. It is true that the analyst did make use of certain index points and recorded information about the items to be controlled but there was no consistency or reliability of conclusions. Forecasts were based entirely on overall past performance and individual intuition. The clearest and most comprehensible example in graphic form of the fallacy of logical or intuitive reasoning in determining stockage quantities for technical support items is obtained from the demand frequency curve of sample items. Figure 1 is a simple plot of the number of demands occurring in each successive quarter over a period of four years. Figure 2 is a histogram resulting from the demand pattern of

Figure 1

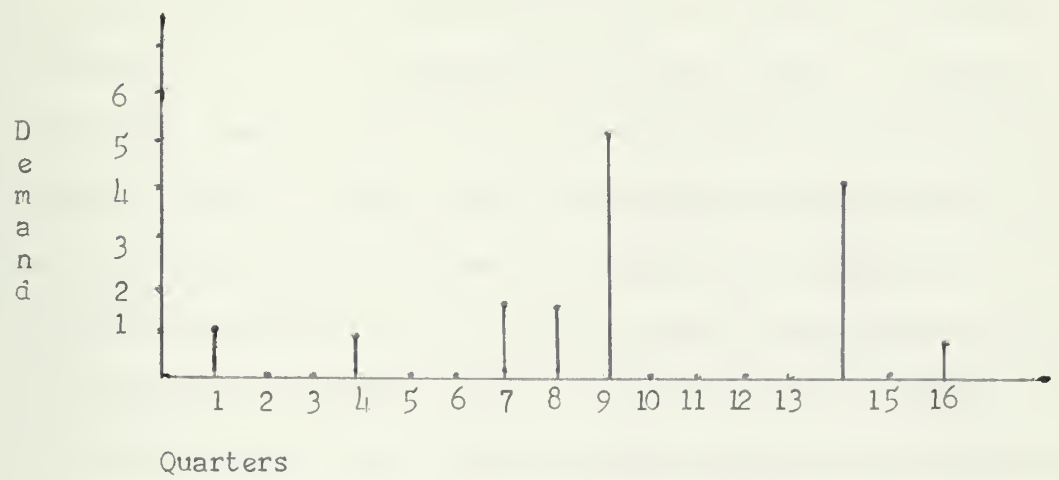


Figure 2

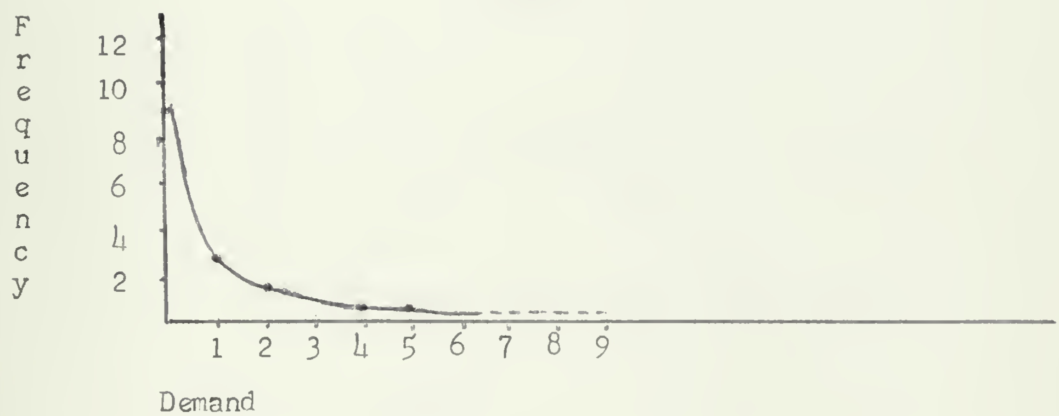
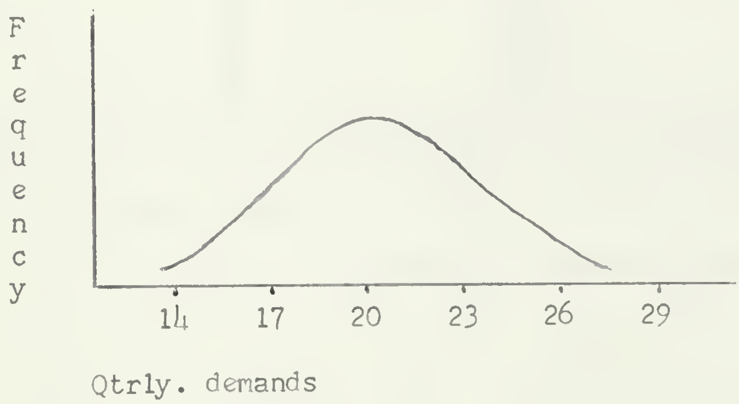


Figure 1. The histogram merely sums the quarters in which identical demands occur and by the ratio of this figure to the total number of quarters from which demand data was taken, a frequency of any expected demand quantity can be computed from the graph. For example, a demand of 1 per quarter occurred in 3 of the 16 quarters (3/16 = 18.7). This data will be further expanded in Part III. Clearly the Navy supply system would have been paying high holding

costs for an excessively large inventory over a long period of time had decisions been based intuitively on short run increases in demand of the item represented in Figures 1 and 2. Although statistical laws of distribution and probability have existed and been well proven in theory, only in the past decade have they been seriously applied to the inventory problem. The Poisson or Bernoulli distribution illustrated above most nearly characterizes the technical or insurance type item with slow moving tendencies. The item in constant and repetitive demand more nearly conforms to the normal distribution with similar periodic fluctuations on either side of the mean.

Figure 3



For example consider the sample item exemplified by Figure 3 and evidencing the following characteristics:

Mean or average demand (μ) = 20

Standard deviation (σ) = 2; a measure of dispersion of demands and calculated as a function of the mean of a sample distribution, sample size and values of the variant (x_1, x_2,...x_i,...x_n);

$$\sigma = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - x)^2\right]^{\frac{1}{2}}$$

n - sample size
x - arithmetic average of n quarterly demand samples
x_i - ith demand in sample

From the laws of normal distribution, 99.74% of the quarterly demands will fall in the range of 14 - 26. The probability of exceeding the following demands are indicated in Table 1.

Table 1

Qtrly. demand	Prob. of exceeding
20	.5
21	.31
22	.16
23	.07
24	.0228
25	.0062
26	.0013

An inventory manager would have 84% confidence of filling all demands with a quarterly average of 22 items in stock. If all of the items under his cognizance indicated the same demand pattern then he could be confident of filling all demands for all items over many quarters approximately 84% of the time by maintaining a quarterly average stock of each item of 22.

Although of paramount importance to the inventory manager in setting effectiveness standards and evaluating performance, the rigorous examination of the laws of probability and statistics is not the major concern of this paper and assumptions will be made regarding characteristics of particular distributions with no attempt at proof. The manager must fractionate or segregate his inventory by movement history and apply to the items within each category the proper consideration of probabilistic movement.

In addition, while totally suceptible to the methods of scientific management discussed herein, the functional areas of inventory control dealing with provisioning, stratification and mobilization requirements, disposal and others are not specifically treated. Any assumption made herein generally regards these factors as non imperative to the issue of the repetitive demand item and the day by day management of stocked items, unless otherwise stated.

Although the elements of cost or risk associated with item management are discussed at more length in Part III, recognition of the more important factors is deemed necessary prior to further analytical discussion. Even the most conservative list of factors that should be considered in making item decisions would number at least a dozen. In practical application, precise development of decision rules and formulae allow for consolidation of many factors into a constant value for a class of items or the approximation of others due to insignificant error rate or impractibility of obtaining precise values. Other factors are assigned or modified by policy decisions or by experimentation of a range of values for samples to determine which best fits the empirical results of operations. To name a few, some obvious and others not due primarily to non-recognition in the past, with no particular attention to order of significance, the list would include:

1) funding restraints
2) usage including nature of (replenishable or nonreplenishable) demands.

3) order costs including but not limited to

a) purchasing cost, usually a consolidation of administrative costs involved in creating and administering a contract

b) receiving costs

c) requirements review and other stock control and financial control portion of effort expended.

4) holding cost including but not limited to

a) interest on dollar inventory investment[5]

b) obsolescence risk

c) storage and maintenance including inventory, preservation, etc.

d) deterioration and shrinkage (as applicable)

5) shortage cost based upon essentiality analysis of the item.

6) reparability of the item.

7) trend of population growth.

This list must not be assumed to be exhaustive. The assignment of weights to each element and the accuracy in compiling all relevant costs will determine the validity of the recommendations resulting from the application of decision making techniques or speaking broadly, from informational technology.

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[5] Office of Naval Material, op. cit., p. 6-7. The general rate of interest is now prescribed at 4%.

PART III

INVENTORY CONTROL:

TECHNIQUES, METHODS, and RAMIFICATIONS

> All levels of the naval establishment must constantly seek new and better ways of obtaining, analyzing and translating the facts into meaningful forecasts and predictions.[6]

Economic Order Principle. The concept of an economic purchase quantity was formulated as a result of the recognized need to dampen inventory fluctuations and the search for optimum inventory levels. Although the resulting formula, developed as early as the middle 1920's, has been modified under many circumstances to recognize prime criteria, the basis remains the same. The essence of the principle prescribes that the size of stock acquisitions will be established at the point where total variable costs of operation are at a minimum. Since the use of this principle is mandatory within the Naval Establishment (though not completely complied with) except where other overriding considerations dictate otherwise, only minimum treatment will be given the subject. This algorithm has been fully investigated and rigorous proofs are available in the literature.[7] Figure 4 offers graphic evidence

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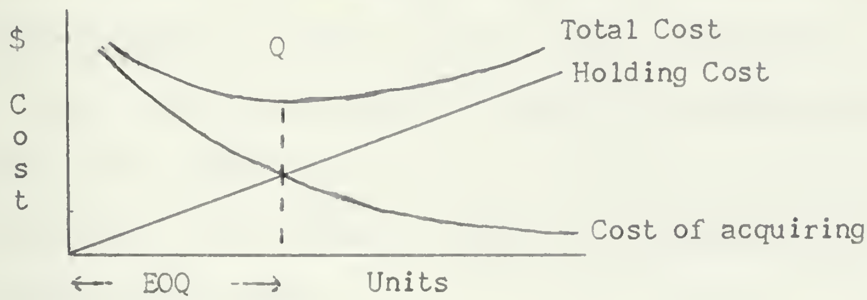
[6]Office of Naval Material, op. cit., p. 5-1

[7]Thomson M. Whitin, The Theory of Inventory Management (Princeton: Princeton University Press, 1957), pp. 32-38; John F. Magee, Production Planning and Inventory Control (New York: McGraw-Hill Book Co., Inc., 1958), p. 48 and Appendix A, pp. 305-315.

of the nature of the basic costs affecting the order quantity costs.

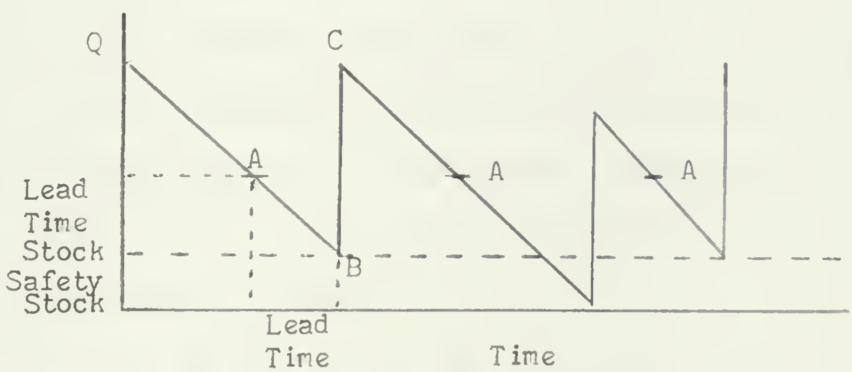
Figure 4



Holding costs tend to increase linearly with an increase in the number of units ordered while the costs of acquisition or purchase cost decreases with an increase in quantity of the buy.

In the simplest form, Figure 5 shows the application of the order quantity in an inventory cycle of an item having a stable demand pattern and one extended lead time.

Figure 5



A Order Point

B Receipt of Stock

CB Order Quantity

The concept of order quantity and order point have now been established. In addition to the demand patterns illustrated in Figures 1, 2 and 3 it must be remembered that change in trend is a factor that must be detected and evaluated to provide continuing optimum response. Figure 5 does not exist in actuality.

The economic order quantity represents the functional relationships of the annual value of sales, the cost to order and the holding cost. The following symbolism of costs will be used throughout this paper:

- Q economic order quantity
- A annual value of sales or demand
- C order cost: c_i elements comprising C
- H holding cost; h_i elements comprising H
- n number of significant elements in any cost

By summation of the costs involved, $C = \sum_{i=1}^{n} c_i$ and $H = \sum_{i=1}^{n} h_i$, and setting up the equation to express total annual variable costs,

$TVC = \frac{QH}{2} + \frac{A}{Q}C$; ($\frac{QH}{2}$ = annual carrying charge and $\frac{A}{Q}C$ = annual procurement expense disregarding safety levels of stock), the calculus allows minimization of TVC very simply. Differentiation of TVC with respect to Q and setting the result equal to zero minimizes Q in solution (point Q in Figure 4):

$$\frac{H}{2} - \frac{AC}{Q^2} = 0; \quad Q = \frac{2AC}{H}$$

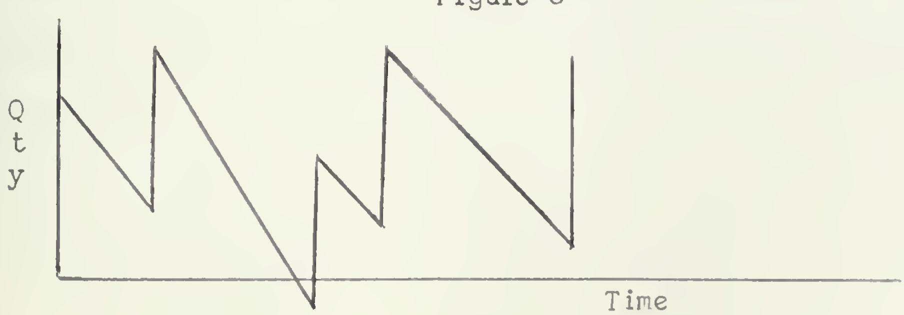
Also from Figure 4, Q occurs at the point of intersection of the two cost curves; hence $\frac{QH}{2} = \frac{AC}{Q}$; $\quad Q = \frac{2AC}{H}$

The reference earlier made to the works of Whitin and Magee expand the cost equation adequately to allow for variations necessary in any of the cost functions and quantity discounts.

Thus far, it has been presupposed that there is no safety level of supply and that receipts of material occur at the point in time when inventory reaches zero; an assumption of certainty.

Reorder Point. Since certainty is not assured in demand patterns, it then becomes a problem of management of uncertainty to provide optimal stock levels. Utilization of the economic order principle assures optimal order quantities bounded by the cost constraints established, so, the time to order relative to stock status must be determined. Figure 6 has expanded the over-simplified cycle chart of Figure 5 to express what can and does happen in practice. In effect, both demand and lead time are uncertain and should be considered in this way rather than in any other combination with one important exception based on a selective management approach to inventory control. This exception may well apply to low value items where minimum management is desirable and a fixed safety level of stock, Figure 5, is established because of its simplicity and ease of calculation based on a moving average of past demand. This concept necessarily implies an inventory fractionation in terms of item value as well as demand.

Figure 6



Consider three items, A, B, and C each having an average demand of 6 units per quarter over a period of eight quarters:

Table 2

Qtr	A	B	C
1	6	5	0
2	6	7	10
3	6	6	0
4	6	5	0
5	6	9	24
6	6	4	0
7	6	7	0
8	6	5	14
	8 | 48	8 | 48	8 | 48
average	6	6	6

Each item shows clearly a different pattern of demand yet the arithmetic average is the same. The difference is clearly in the variance between the demand quantities. For a reliable estimate of lead time, no safety level of stock would be required for item A; a small safety level for B and a large one for C. The variation in demand for the item would determine its safety level; hence a variable safety level. From statistics, the variation is the square of the standard deviation earlier discussed. In the example of Table 2, the differences for item B of quarterly demand and average are:

Table 3

Qtr	Diff	$(\text{Diff})^2$
1	-1	1
2	1	1
3	0	0
4	-1	1
5	3	9
6	-2	4
7	1	1
8	-1	1
	0	8 ⟌ 18
		2.25 variance

Standard deviation $(\sigma) = \sqrt{\text{var}} = 1.5$

Had the average quantity been on hand at the beginning of each quarter for item B, the results would have been as indicated in the first part of Table 4. Part 2 of Table 4 assumes a quarterly start of the average plus one standard deviation (rounded to one unit) and Part 3 the effect of one standard deviation rounded to two units or a total of eight units.

The addition of one standard deviation, one unit, allows the filling of all demands but two and all quarterly total demands in seven of the eight quarters sampled, but also importantly creates more excesses. The only way to avoid excess material is to know with certainty what demands will be. From Part II of this paper, one standard deviation above the mean in a distribution approximating the normal will create an effectiveness of 84% and an assumed shortage risk of 16%. Had two standard deviations been added or 3 units above the mean, effectiveness could be approximated at 97.7% or a shortage risk assumed for any quarter of 2.3%.

Table 4

	On hand	Demand	Results
Part 1	6	5	1 excess
	6	7	* 1 short
	6	6	
	6	5	1 excess
	6	9	* 3 short
	6	4	2 excess
	6	7	* 1 short
	6	5	1 excess
Part 2	7	5	2 excess
	7	7	
	7	6	1 excess
	7	5	2 excess
	7	9	* 2 short
	7	4	3 excess
	7	7	
	7	5	2 excess
Part 3	8	5	3 excess
	8	7	1 excess
	8	6	2 excess
	8	5	3 excess
	8	9	* 1 short
	8	4	4 excess
	8	7	1 excess
	8	5	3 excess

The setting of shortage risk appropriate to the item is the heart of the entire inventory management function and control system.

Referring back to Figures 1 and 2 the class of items which evidence slow moving characteristics or a low average demand pose a quite different one than that just discussed. The probabilistic vagaries of these items can be approached in one of two ways. Collection and analysis of past demand data will allow for determining actual empirical distributions and the resulting frequencies of observations used for calculation. The other method is to attempt to match the demand pattern for the item to a known distribution function, the properties of which will be applied to the slow moving item. The Poisson distribution in which the variance equals the mean ($\mu = \sigma^2$) is a useful approximation if the yearly average demand is low and erratic. Table 5 contains data extracted from Figures 1 and 2 and will be used to illustrate the setting of safety levels for variance for these types of items.

Table 5

Yearly average: 4 Quarterly average: 1 each

Possible quarterly demand	No. of qtrs of demand	Frequency	Cum
0	9	56	56
1	3	18.7	74.7
2	2	12.5	87.2
3	0		
4	1	6.25	
5	1	6.25	
6 or more			

Recognizing that this is data for only one item, it still allows consideration of three important phenomena:

1) Zero demand occurs most frequently (56%)

2) The demand went as high as five in a quarter and can conceivably go higher

3) The average quarterly demand of one occurred only 18.7% of the time

To facilitate further discussion and comparison, a shortage risk of 15% is assumed acceptable; hence an effectiveness standard of 85% is the long run goal for this item and its class of low average demand items. Again it is emphasized that the setting of effectiveness standards is a management decision and should vary because of considerations of unit cost, part essentiality and funding limitations. From Table 5 it is seen that the expected value of demand being 2 or less is 87%. A confidence interval has been established, not a guarantee of any finite value of demand. With reference to the concept of informational technology, it is obvious that the collection of, analysis and transfer to useable empirical form of such data for a wide range of inventory is a monumental undertaking. In the absence of high speed data processors, or time for complete analysis of the items of the inventory for individual density functions, the possibility exists for fitting a known function to the problem at hand. Examination of the Poisson for $\mu = 1$ reveals the data in Table 6. Using the Poisson function, again it is found that for $\mu = 1$ a stock level of 2 affords a confidence interval of over 85%.

Considering an inventory of one hundred stockkeeping units with an average demand of one per quarter, a decision to stock none would result in an expected shortage of 100 each quarter. This does not mean an expected shortage of one for each item. Assuming Poisson applicability, a decision to stock one of each unit would allow expected results of 62 items used (probable demand of 1 or more), 38 items surplus (zero demand) and 38 unsatisfied item requests (expected demand less expected use). A decision to stock 2 of each allows an expectation of; 88 items used, 112 items not used and a shortage of 10 items or 10% of expected demands for the quarter not satisfied. The decision to stock 2 results in a 56% non usage as compared to 38% for stockage of 1 each.

Table 6[8]

Poisson Distribution

Variable demand equal to or less than:	Prob
Quarterly average: 1 each (μ)	
0	.38
1	.74
2	.92
3	.98
4	.997
Quarterly average: 2 each (μ)	
0	.14
1	.42
2	.67
3	.85
4	.94
5	.98
Quarterly average: 3 each (μ)	
4	.82
5	.92

$P\ (\text{demand} = x) = P\ (d \leq x) - P\ (d \leq [x-1])$

Two methods have now been discussed briefly indicating means to analyze items and set safety levels covering a quarter or any other useful time period. This discussion is preliminary to and actually the first phase of calculations necessary to determine the reorder point. It is the lead time for resupply that must be protected by the safety level and safety level is determined by

[8]Edward H. Bowman, Robert B. Fetter, Analysis for Production Management, (Homewood, Ill.: Richard D. Irwin, Inc., 1961), p. 200. Poisson distribution curves.

the variance in demand. By the nature of the variance in demand, it has been shown that demands may exceed the average in any given quarter considerably but the probability of this happening in several succeeding quarters is very low. If high demands did succeed themselves, this trend must be recognized and the parameters (μ and σ) changed accordingly depending on the cause of the demands. The method of change or recognition will be discussed in more detail later. The intuitive result of straight line multiplication of one quarter's safety level by the lead time expressed in quarters would be too high, too much insurance. Assuming the variance to remain the same over each quarter of the lead time, the statistical solution is the safety level, which is an expression of standard deviation times the square root of the lead time in quarters. If the lead time is three quarters, σ for three quarters equals $\sqrt{var_1 + var_2 + var_3}$ (where var = σ^2). This equation reduces to: SD 3 qtrs = SD 1 qtr $\sqrt{3}$. Having made provision for the aspect of uncertainty in demand by expecting average demand to occur and adding a safety level for variance, a general formula can be set down for re-order point determination:

Re-order point = Expected (average) demand over the lead time period + Variable safety level over lead time.

R = Re-order point

Q = Quarterly average demand

LT_Q = Lead time expressed in quarters

a = Number of standard deviations to insure effectiveness

$R = (Q \times LT_Q) + (a \times \sigma \sqrt{LT_Q})$

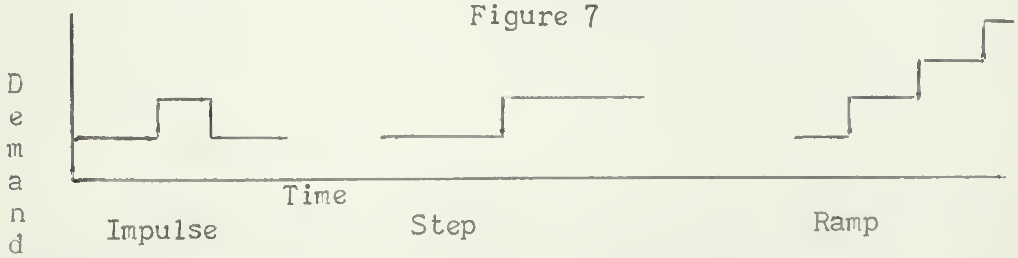
From the earlier example of $\mu = 6$, $\sigma = 1.5$, to assure 84% effectiveness over a nine month lead time (a = 1):

$$R = (6 \times 3) + (1 \times 1.5\sqrt{3}) = 18 + 2.6 = 21$$

When in the periodic stock status review the assets on hand and due in reach this level, order action should be triggered.

Forecasting. The simplest method of forecasting in the light of uncertainty is the cumulative average but this system is relatively slow in response as many periods of data are accumulated. This can be rectified by adoption of a more satisfactory and meaningful system by limiting the number of time periods and moving the average successively forward. Should the effective span be chosen as four quarters of demand data then the moving average would always be computed using the immediate preceding four quarters. The criteria used to select the effective time period for carrying data is dependent on the characteristics of the segment of inventory under consideration. There are three standard kinds of change in demand: an impulse, a step and a ramp.[9] These are easily illustrated and variations of any and all should be recognized.

Figure 7



[9] Robert G. Brown, Statistical Forecasting for Inventory Control. (New York: McGraw-Hill Book Co., 1959), p. 30.

The time period selected must necessarily be long enough to differentiate changes in trend from periodic impulses or steps but not so long that the response becomes too slow for effective action.

> If the average is computed over a short period of time, it is subject to a large "sampling error," but if it is computed over a long period of time, we may be averaging together several different markets. The moving average is an attempt at compromise.[10]

The moving average method is simplicity in extreme but still an effective tool. A slight refinement can produce marked improvements in the forecast results as the most current information is recognized as most significant. This refinement has been given the name exponential smoothing and acts primarily as a catalyst in that the system can be made more or less sensitive to new data and the response more acute or damped. Furthermore, errors of accumulation tend to be washed out and there is not the necessity to carry a large volume of prior periodic demand data in the model, only prior averages. Three key equations are cited which serve to show the process clearly.[11]

New Average = a (new demand) + (1-a) (old average)

New Trend = a (current trend) + (1-a) (old trend)

$$\text{Expected demand} = (\text{new average}) + \frac{(1-a)}{a}(\text{new trend})$$

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[10] Ibid., pp. 29

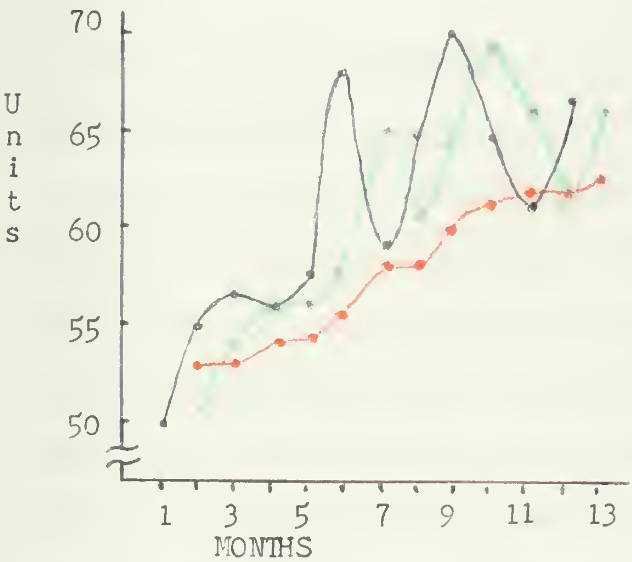
[11] Ibid., pp. 45-51; Robert G. Brown and Richard F. Meyer, The Fundamental Theorem of Exponential Smoothing, Gen Memo #4, (Cambridge: Arthur D. Little Inc., 1961), pp. 1-25.

a is a constant expressed as a fraction and represents a fractional portion of the difference between the demand experienced in a period just ended and the forecast for that period which will be put in the formula to update the forecasting model; in effect, assigning weights to the periods of time constituting the forecast. There is, of course, no optimal value of the smoothing constant, a, for it will depend entirely on demand pattern. It is a simple action of experimentation to determine that value suitable for a fractionated segment of the inventory. Management must never abdicate its responsibility to recognize the dynamics of any inventory problem and must periodically test the validity of the smoothing constant in use.[12] If a small value of a is chosen, such as on the order of .01, the response will be slow and gradual. If a high value is chosen, say .5, estimates will respond very quickly but not only to real changes in trend or a step but to random high magnitude fluctuations as well: clear evidence again that the manager must know his inventory overall but not hand massage every item. New program data concerning individual items must be pumped into the model and when such data indicates, specific items should be extracted for detailed analysis by the stock analyst. Forecasts of this type easily and readily lend themselves to accuracy checks by the variance method associated with the laws of probability

[12]Charles C. Holt, Franco Modigliani, John F. Muth, Herbert A. Simon, Planning Production, Inventories and Work Force, (Englewood Cliffs: Prentice Hall Inc., 1960), pp. 268-271.

previously discussed.[13] Although this paper has not attempted even the briefest formalization of or derivation of exponential smoothing, it is postulated that it is an effective management forecasting tool and the necessary proofs of formulation are clear in the references cited. Figure 8 has been prepared from a 12 month sample of demand for an item showing estimated response for a = .1 and a = .5 and starting average demand of 54 per month.

Figure 8



No.	Demand	Expected Demand .1	Expected Demand .5
1	50		
2	55	53	51
3	57	53	54
4	56	54	56
5	58	54	56
6	68	55	58
7	59	58	65
8	65	58	61
9	70	60	64
10	65	62	69
11	62	63	66
12	67	63	63
13		64	66

Brown extends this forecasting system carefully and completely through variations such as will be encountered in any complex

[13]Brown, Statistical Forecasting for Inventory Control, op, cit., p. 54-57.

inventory problem. His individual papers for Arthur D. Little, Inc., give wide scope to the statistical forecasting methods.[14]

Methods. Reference was earlier made to the phrase informational technology encompassing the following approaches to decision making:

1) the use of mathematical and statistical models with or without the aid of electronic computers or other high speed data processors.

2) the use of EAM or EDPM for mass, integrated data processing and

3) the direct application of computers to decision making through simulation techniques.

The first of these methods has been in use to some degree from man's first attempt to quantify the functional elements constituting a problem to and including the present day. No computers presently contemplated offer the slightest hope of eliminating this method of data manipulation to reach an inventory decision. Within the scientific approach to inventory management, this plus purely judgmental decisions is the essence of management by exception. Critical items or high cost items requiring this method of management need not be excluded from the volume production effort of the activity but they should evidence clearly in their characteristics that when and if certain parameters are exceeded they should receive the personal attention of the stock analyst or higher authority.

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[14]Robert G. Brown, Transient Responses of Exponential Smoothing, General Memo #21 (Cambridge: Arthur D. Little, Inc., 1961), pp. 1-53.

The second method of data handling and decision making is in wide-spread use and is truly a composite of one and three of which there will seldom be pure application anymore in a large scale, complex inventory system. Electric Accounting Machines, EAM, and Electronic Data Processing Machines, EDPM, are recognized by all segments of industry and government to offer benefits in volume, integrated data processing. The idea in this phase is to accumulate in proper form the statistics of the business and report them out in required form for analysis preparatory to decision making by the analyst. The final decision may be reached by relatively simple mental calculations, the use of graphs or charts or through the use of such devices as depicted in Appendix A.[15] This device is only one of many developed to manipulate consistently and conveniently the variety of constants and variants influencing optimal decisions; optimal within the management established criteria. Without substantial reason of judgment, personalized influences should not be encouraged or allowed to modify either the item or system parameters established for the model or the result obtained therefrom. The item restraining values should be those obtained from provisioning and future updating by technical or stock control personnel based on actual usage and failure data. The system parameters are those of the inventory control activity affecting all items in identical or percentile ways.

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[15]Inventory Control Calculator developed for the Bureau of Supplies and Accounts by the Planning Research Corporation of Los Angeles, California.

Since data is processed in the clerical or routine sense by electric accounting machines and electronic computers at most activities engaged in volume business, it is clearly logical that utilization of EDPM to its utmost capability is highly desirable and economical. The third phase of information technology requires the electronic computer to compute, not just massage and manipulate data. This requires the loading of the machine with a program to compute reorder quantity, reorder point and forecast. There is no limit to the number of mathematical formulae that could be provided or the inputs but the number can be drastically reduced by the fitting of a good model to the inventory characteristics. To do this, systems personnel and mathematicians must work together to achieve the proportions of each of the costs and other parameters so generally discussed herein. Again it is essential to realize that careful model construction and collection of input data will result in good item decision rules. The tendency, unfortunately, is to check item by item the outputs of the model and to change to more intuitive decisions based on past ordering or stocking philosophies or crystal ball forecasts lacking adequate justifications. The Systems Development Department of the International Business Machine Company has recently published the final report, Task No. 0453, on a simulation system for a military inventory problem.[16] This report is a composite of the Navy problem and

[16] IBM, Use of the IBM Inventory Management Control Simulator in a Military Supply System, 15 Sept. 1961, pp 1-70.

with variations could be made applicable to practically all classes and cognizances of inventory. Of paramount importance, this model and others tested at specific Inventory Control Points do provide results that test out superior to past experience using less refined or non-scientific approaches. The third phase does not rule out selective management for certain items but rather allows for more immediate flagging of items in such a category as transaction data is rapidly analyzed within the computer. A prime function of management is to stick to its conviction if input data and model are carefully conceived and not allow for other arbitrary judgments when they are not warranted because of non-quantitative influences. Dr. John M. Pfiffner, Professor of Public Administration, University of Southern California stated in a recent lecture that computer use would actually require greater intellectualization of the management functions. A Navy Department official states,

> EDP systems give to the defense departments what is truly the most limited resource with which to plan and program logistic effort. It gives time, the most perishable of all resources.[17]

Mr. Pehrson has in a word exemplified the true benefits to be gained from computerized inventory programs; timeliness of actions allowing for consistency of rules and attention to all significant restraints and requirements. Managers of large, diversified

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[17]Gordon O. Pehrson, Management Control in the Military Departments reproduced in Management Control Systems, (New York: John Wiley and Sons, Inc., 1960), p. 67.

inventories must manage and allocate time of personnel and machines to insure proper item management consistent with relative item worth to the system.

Ramifications. The management tool recognized as most effective to the manager is the budget of his organization. The four basic budget phases, formulation, justification, execution and review or audit are truly the scribe marks on the yardstick by which his net effectiveness is measured. The budget is a price out of plans or forecasts for a coming period. Mathematical decision rules, such as those discussed, which form the nucleus of the inventory manager's repertoire of decision making aides, have as their chief value the ability to predict what is going to happen (demand or requirement) before it happens and hence allow management to justify forecast or budget and most importantly, allow adjustments to be made in an organized way.[18] A fallacy of thinking allows managers to speak in terms of "the requirement" for the coming year; that is a single quantity of overall inventory that must be available based on initial forecasts. Within a given set of criteria of course, the requirement may be singularized but that is no assurance that this is the only requirement that will allow satisfactory operation of the system. In effect, "satisfactory" must now be defined and within the definition is the operating space that the manager must have to adjust with the dynamics of his

[18]ALRAND Working Memorandum No. 6 of 7 June 1961 and No. 7 of 13 June 1961, Ships Parts Control Center, Mechanicsburg, Pa., contains budgeting philosophy and an initial program for that inventory.

problem. For example, it has been more the rule than the exception that even when Navy field management has seen the commitment rate running too high to get through the year within the budgeted total, the commitment rate does not change for there is "a requirement" to be met. Stock answers to over commitments include as a whipping boy the mathematical formulae by which the decisions of individual item requirements were reached. "The formula says we need this much." The problem is not static nor are the formulae. The results obtained from the operating parameters established by management and funding limitations seem to have historically been accepted as non-restrictive. The budget concept for Navy Stock Fund (NSF) inventories is published annually by the Bureau of Supplies and Accounts as guidance to inventory managers in determining budget requirements for the coming budget year. The following quotation is typical of NSF budgeting guidance:

> Requirements based on net sales. The NSF budgetary requirements are basically tied directly to 'issue of material with charge,' i.e., net sales. Since a large percentage of the funds apportioned to the NSF are used to replace sales, sales must be correctly estimated. The base year for projection of sales for the apportionment (FY 1962) and budget (FY 1963) will be the current year (FY 1961). Net sales are the sum of the last four expenditure entries on the Financial Inventory Control Report . . .[19]

The Formula then appears as:

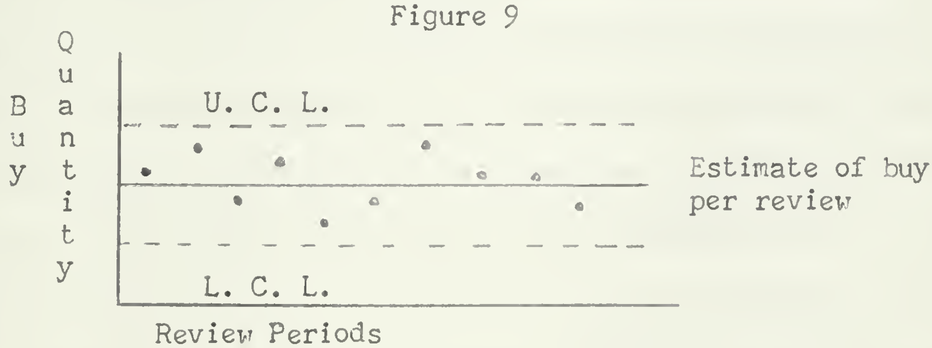
Funds required = Sales + ending inventory (on hand and on order) - beginning inventory.

[19] Bureau of Supplies and Accounts Instruction 7113.1A of 4 April 1961, para 4.i.

This expression is too simple and must still recognize special, identifiable programs scheduled for the budget year and transfers of items between managers as well as planned "cut down" of stocks in calculated long supply. Although the foregoing expression satisfies the quoted guidance for budgeting in that projected sales estimates, or forecasts, are computed, these estimates rely too heavily on the raw pastdemand data. It becomes too easy to visualize future funding as the means to replace what has been sold. The philosophy, theoretical and in practice, should surely be that the budget provide for what is going to be sold in the future. Also, the budget should provide a statement of consequences (risks or costs) which would result from greater or smaller amounts than requested.

To estimate the dollar value of next year's routine stock buys, as differentiated from provisioning new items, one readily available method is to price out next years expected buys. There may in time be other predictors. By having the means to forecast demand, compute the reorder point and obtain current asset data (on hand plus due in) for each item or sample of items, a prediction can be made as to whether or not the item will hit its reorder point and how many times if it does. With the economic order quantity calculation, the frequency of buys and unit price during the year, the expected budget requirement for the item is known. The cumulative total of these expectations is the expected fund requirement for the inventory. If the total inventory has similar demand characteristics, simple sampling techniques will suffice for budgeting. If the demand patterns are diverse then some other selective device is necessary. If a large part of dollar demand comes from a small percentage of items, then this segment should be forecast individually while broader estimates should be made for remaining large item volume of inventory. Item budgets should be retained in records for later analysis if buys occur in large variance to the forecasts. With such an analysis, budget adjustment justification will be facilitated. It is of importance to note here the waste resulting from failure to retain computational results of economic order quantity, reorder point, forecasts, etc., for these pieces of data are called for time and time again in interactive type processes of comparison and tests. Consider a control system for the commitment rate of an inventory activity such as Figure 9.

Figure 9



Over the course of the year there will be an established period of time between inventory reviews at which time certain items will have reached their reorder point and buys will occur. In other words, small portions of the budget will be commited periodically throughout the budget year and this portion can be expressed as an average commitment per review period (budget divided by number of review periods). The actual buys per review are then fed back into the control system and variances from the average determined. From past experience the upper and lower variance control limits can be established. So long as actual buys stay within limits without bias either way, the commitment rate is in control and on schedule. Bias to one side of the estimate indicates something has changed significantly and further analysis and adjustments are required. Management must be alert to the possibility of the extreme case of no buys at all being made for items reaching reorder point toward end of budget year or very large buys at this time which may throw the system into a state of imbalance.

The actual adjustments to the buying rate can be achieved through the manipulation of the internal interest rate which normally is set at 4%. Remembering the effect of interest on lot sizes or purchase quantities, an increase in the interest rate tends to decrease the buying rate while a decrease will increase the buying rate. By comparing the actual buy or commitment rate to the planned rate, the percent increase or decrease necessary to get back to the budgeted rate can be computed. Initial budget estimates should in fact be made for several different interest rates (4%, 8%, 36%, etc.) to provide the consequences that could be anticipated. The so called "required" budget, or better that one submitted for basic consideration within specified criteria, will have been computed at the 4% rate. Now, in order to manage, management will of necessity be required to trade off or play give and take between budget, shortages of stock and operating costs. Also it is necessary to know just how stringent the funding limitations are from the bureau level or if bargaining for supplements is still in vogue. Assuming that both parties, inventory manager and management bureau, are playing by thesame rules and recognize the interrelationships of constraints fostered by the mathematical approach (actually inherent in all approaches but recognizable and identifiable in the mathematical approach) the decisions on routine stock replenishments can be made so as to commit or buy at a rate consistent with the budget. No matter what the budget size, an interest rate is calculable to correspond.

The net beneficial effect of this method of budgeting and subsequently managing the inventory is its flexibility to meet restraints while still allowing for optimization within these restraints. Budgeting is based on forecasts of new demands rather than on past sales and explicitly considers buying policies of the activity or system. The yearly buy quantities when priced out and summed provide the budget, its justification, and an effective control tool for the execution phase.

As was stated in Part II, no mention has been made of provisioning, stratification allowances or disposal as other functional elements of the inventory problem. It appears axiomatic that in "cradle to grave" item management, the same type of considerations which cause demand are determinative of stocking policy. The inherent beauty of mathematics is its ability to accept modifiers of various kinds and the same formula or model will operate in the same way within any set of parameters. Thus, what has so far been written is not foreign matter to any functional area and indeed they should be treated all as one problem theoretically (not physically), the inventory management problem.

PART IV

CONCLUSION

Before writing the conclusion, the inventory problem will be restated and the problem under investigation will be restated. The inventory problem consists of three basic elements affecting the answer to the question "how much, if any, to stock, when and where in a multi-outlet system?"; 1) customer satisfaction, 2) inventory maintenance costs and 3) inventory investment. The problem under investigation is the net effect the use of mathematical approaches will have on the manager's job of decision making and to what extent the manager should participate in establishing and using a mathematically oriented system. The management function would then seem to include the assignment of weights to the three elements listed above in order to keep the system in balance within the mission of the inventory control activity. Here there seems to be a service versus cost consideration; element one versus elements two and three.

The first hurdle to be overcome in setting down initial inventory policy decisions is for the systems personnel and the mathematicians to develop a common language at least in terms of the objective function. It is conceivable that in the course of arriving at the definition of the common goal, the factors supposedly comprising the statement of the objective are inadequate or superfluous. This information is vital in itself for it serves to indicate that the actual problem is not the supposed problem. The preliminary analysis of the situation has pointed up to top management considerations not previously contemplated in their proper perspective. Attention to data accumulation should provide the correct empirical material that will serve as the inputs to the system developed to fit the pattern of operations of the particular inventory control point. This heuristic approach has led management to the point that basic inventory policy decisions must be made to control, both internally and externally, the operating system; these decisions are facilitated by system simulations of real life which can show management in advance the results of its decisions based on forecasts of several contingent possibilities. Alternative courses of action are presented for analysis with documented justification which can be fully evaluated as a part of the informational technology system. The best mix of system and item parameters to achieve the stated objective function can be pulled from the results and the service versus costs decision made quantitatively; shortage costs, associated with shortage risk and essentiality, used for service measure. Once established, the mathematical or scientific system will operate only within the bounds specified until and unless changed by the operator with or without the approval of managerial authority. Management's job then becomes three fold in the realm of participation in the inventory control system. First, it is management's responsibility to determine finally the objective function of the activity. Second, management must establish the restricting parameters within which the objective must be optimally realized and third, insure that the system does operate to optimum efficiency in accordance with the prescribed criteria until such time as management changes that criteria. After the basic decisions are made establishing a systemitized, mathematical system, the job of management is as Barnard puts it,

> not that *of* the organization, but the specialized work of *maintaining* the organization in operation.[20]

The system, after check out and acceptance, has been proved useful for the day to day processes, "the work volume of the organization." If the system has built in the necessary warning signals for selective management or selective attention to changes, the management has the aid that allows it to remain on top of the ever changing situation and to make the adjustments in time to realize beneficial effects.

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[20]Chester I. Barnard, *The Functions of the Executive*, (Cambridge: Harvard University Press, 1960), p. 215.

BIBLIOGRAPHY

Abramovitz, Moses. Inventories and Business Cycles. Washington: National Bureau of Economic Research Inc., 1959. 499 pp.

Arrow, Kenneth J., Samuel Karlin, Herbert Scarf. Studies in the Mathematical Theory of Inventory and Production. Stanford: Stanford University Press, 1958. 340 pp.

Barnard, Chester I., The Functions of the Executive. Cambridge: Harvard University Press, 1960. 322 pp.

Bowman, Edward H., Robert B. Fetter. Analysis for Production Management. Homewood, Ill: Richard D. Irwin, Inc., 1961. 554 pp.

Brown, Robert G., Statistical Forecasting for Inventory Control. New York: McGraw-Hill Book Co., Inc. 1959. 227 pp.

Brown, Robert G., Richard F. Meyer. The Fundamental Theorem of Exponential Smoothing. Cambridge: Arthur D. Little, Inc. 1961. 25 pp.

Holt, Charles C., Franco Modigliani, John F. Muth, Herbert A. Simon. Planning Production, Inventories, and Work Force. Englewood Cliffs: Prentice Hall Inc. 1960. 404 pp.

Magee, John F., Production Planning and Inventory Control. New York: McGraw-Hill Book Co., Inc. 1958. 330 pp.

Malcolm, Donald G., Alan J. Rowe. Management Control Systems. New York: John Wiley and Sons Inc., 1960. 360 pp.

Morse, Phillip M., Queues, Inventories and Maintenance. New York: John Wiley and Sons Inc., 1958. 197 pp.

Optner, Stanford L., Systems Analysis for Business Management. Englewood Cliffs; Prentice-Hall Inc., 1960. 268 pp.

Shultz, George P., Thomas L. Whisler. Management Organization and the Computer. Glencoe: The Free Press of Glencoe, 1960. 244 pp.

Whitin, Thomas M. The Theory of Inventory Management. Princeton: Princeton University Press, 1957. 342 pp.

Wilson, E. Bright Jr., An Introduction to Scientific Research. New York: McGraw-Hill Book Co., Inc. 1952. 365 pp.

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A

APPENDIX 1