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DIGITAL COMPUTER PROGRAM  
FOR  
FLEXURAL ANALYSIS OF BEAMS

\* \* \* \*

Gary B. Lowe

DIGITAL COMPUTER PROGRAM  
For  
FLEXURAL ANALYSIS OF BEAMS

by  
Gary B. Lowe  
//  
Lieutenant, United States Navy

Submitted in partial fulfillment of  
the requirements for the degree of

MASTER OF SCIENCE  
IN  
MECHANICAL ENGINEERING

United States Naval Postgraduate School  
Monterey, California

1963

DIGITAL COMPUTER PROGRAM  
FOR  
FLEXURAL ANALYSIS OF BEAMS

\* \* \* \*

Gary B. Lowe

This work is accepted as fulfilling  
the thesis requirements for the degree of

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

from the

United States Naval Postgraduate School

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## Abstract.

This paper describes a solution to the flexural analysis of a beam by digital computer. Subroutines were written which calculate shear, moment, slope and deflection at any specified point on a beam if the loading is known. Shear deflection may be included for many support conditions. Neither elastic supports nor column effects are considered. The subroutines solve problems of variable beam cross-section and loading by utilizing an extension of "McCaulay's Method", a generalized step function.

The main program provides input-output facilities and also solves for indeterminate reactions on the beam.

Problems and their solutions, showing the versatility of this program, are also included.

## Acknowledgment.

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## 1. Introduction.

The routine solution to engineering problems by digital computer appears to be limited only by the scope of the available programs. A flexible program permitting the solution of routine problems in flexural analysis of beams would seemingly fill a need in the field of digital computer applications to Mechanics. The present work is devoted to such an idea.

Five subroutines, "LOAD", "SHEAR", "MOMENT", "SLOPE", and "DEFLECT", were written which will calculate the load, shear, moment, slope, and deflection, respectively, at any indicated point on a beam if the indeterminate quantities are calculated elsewhere. The use of generalized step functions in this project allows for piecewise linear variation of distributed loading, bending compliance,  $\frac{1}{EI}$ , and shear compliance,  $\frac{K}{AG}$ .

The main program, "BEAM3", first generates and then solves a set of simultaneous linear equations, calculating the indeterminate quantities of the beam. The above mentioned subroutines are then utilized to calculate the remaining flexural quantities. This program also provides input-output facility.

## 2. Definition of the problem.

The problem of structural beam analysis is one in which the external loading applied to the beam is well defined, as is the geometric configuration and the nature of the beam itself. Where information about such a beam is required, this information will most likely fall into the following categories:

1. Internal shear forces,  $V$ , and moments,  $M$ .
2. Slope,  $\theta$ .
3. Deflection,  $y$ .
4. Redundant reactions, either forces,  $R_y$ , or moments,  $RM_z$ .

"Flexural analysis" is used here as meaning the process by which this information is generated from the known data.

In attacking this problem of flexural analysis, the intention was to limit the scope as little as possible. However, several customary simplifications and limitations were made. First, elastic supports and beam column effects were not considered. Limitations concerning the type and number of external loads and the geometry of the beam will be discussed later. Also the basic relations which follow, inherently restrict the deflection of the beam to small values and the internal stresses to values below the elastic limit of the structural material. Shear deflection, which is generally neglected in a problem of this type, has been included in some problems.

### a. Bending deflection ( $y_1$ ).

The Bernoulli-Euler-Navier equation governing the elastic curve of a beam due to bending, consistent with the sign convention shown in Appendix B is:

$$\frac{M}{EI} = \text{curvature} = (d^2y/dx^2) [1+(dy/dx)^2]^{-3/2}$$

and we use the approximation, curvature  $\approx d^2y/dx^2$ .

The following equations, derived from the above equation and shown in Timoshenko /5/,<sup>1</sup> with differences due to the chosen sign convention, describe the flexure of a beam (See Appendix B for notation).

$$\begin{aligned} W &= q(x) , \\ V &= \int W dx , \\ M &= \int V dx , \\ \theta_1 &= \frac{dy_1}{dx} = \int \frac{M}{EI} dx , \\ y_1 &= \int \theta_1 dx , \end{aligned}$$

b. Shear deflection ( $y_2$ ).

The equation governing the shear deflection of a beam, also given in Timoshenko /5/, compatible with the stated sign convention becomes:

$$\frac{dy_2}{dx} = - \frac{KV}{AG}$$

It is evident that:

$$y_2 = \int \left( - \frac{KV}{AG} \right) dx$$

c. Total deflection ( $y$ ).

The equations used in this thesis are the result of the sum of the above shear and bending deflections. They are:

$$W = q(x) ,$$

<sup>1</sup>Numbers so indicated refer to the bibliography.

$$V = \int_0^x W dx + \sum F_1^*$$

$$M = \int_0^x V dx + \sum M_1^*$$

$$\theta = \int_0^x \frac{M}{EI} dx - \frac{KV}{AG} + C_1^{**}$$

$$Y = \int_0^x \theta dx + C_2^{**}$$

Note that step-discontinuities may appear in all of the above equations, except in the equation for deflection. These discontinuities limit the methods that can be used in handling the equations.

Three methods that could be used to solve the equations are a pure analytical approach, a step-by-step numerical integration scheme, and the "curly bracket" method. Of these methods, the analytical approach is generally restricted to a specific problem and is unsuited for adaptation to digital computer programming. The numerical integration scheme can be used in the computer solution of flexural analysis and may have been programmed; however, our search of literature has not uncovered any such program. The "curly bracket" method was chosen for use in this thesis because it was felt that it would result in a more exact solution requiring less

\*In these equations,  $\sum F_1$  and  $\sum M_1$ , represent the sum of the point forces and moments, respectively, applied to the beam.

\*\* $C_1$  and  $C_2$  are constants of integration.

### 3. "Curly bracket" method

The "curly bracket" method is based upon the properties of a convenient notation defined as follows:

$$\begin{aligned} \{x-a\}^n &= 0 && \text{if } x < a \\ &= (x-a)^n && \text{if } x > a \end{aligned}$$

where  $n$  is an integer  $\geq 0$ .

Appendix A contains further properties of expressions containing curly brackets. The notation is generally credited to McCauley /1/, but similar, though less useful notations can be traced to A.Föppel and St. Venant. Also, the ideas involved are related to the use of the Laplace Transformation with the result that this method or approximations to it are reinvented frequently /8/ /9/.

Using this method allows one equation, valid over the entire length of the beam, to be expressed for each of the basic equations of flexure. The particular value of this method is that it permits the integration of these equations to be carried out easily and has the virtue of adjusting the constants of integration most conveniently.

In addition, for use with a digital computer, the "switching" property of the curly bracket can be easily handled in FORTRAN by use of an "IF" statement.

A simple example should convince most readers of the convenience of this method. Examine the following problem, which consists of a uniform beam, simply supported at each end with a concentrated load at the center of the span.

stant variation of bending compliance,  $\frac{1}{EI}$ . In this thesis the method has been further expanded to include beams with piecewise linear variation of both bending compliance and shear compliance,  $\frac{K}{AG}$ .

#### 4. Implementation.

Although the formulas shown in Appendix A could be expanded further, a piecewise linear variation of distributed loads, bending compliance, and shear compliance was considered capable of providing sufficient accuracy and flexibility for the resulting program.

Fig. 2(a) illustrates a distributed load on a beam whose origin is at  $x=0$ . Fig. 2(b) shows an element of the distributed load.

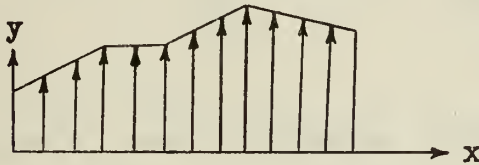


Fig. 2(a)

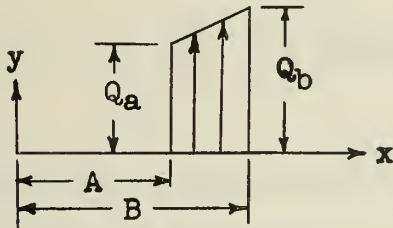


Fig. 2(b)

The analytical expression for this element is:

$$Q(x) = Q_a \{x-A\}^0 + \frac{Q_b-Q_a}{B-A} \{x-A\}^1 - Q_b \{x-B\}^0 - \frac{Q_b-Q_a}{B-A} \{x-B\}^1$$

The same relationship is used for an element,  $f(x)$ , of bending compliance and an element,  $g(x)$ , of shear compliance upon substituting  $f$  for  $Q$  and  $g$  for  $Q$  respectively.

The basic equations of flexure can now be expressed in the same manner as they were utilized in programming. They

are:<sup>2</sup>

$$W(x) = \sum_{i=1}^{NOL} Q_i(x) \quad (1)$$

$$V(x) = \int_0^x W(x)dx + \sum_{i=1}^{NOF} F_{y_i} \{x-x_{f_i}\}^0 + \sum_{i=1}^{NOR} R_{y_i} \{x-x_{r_i}\}^0 \quad (2)$$

where  $F_{y_i}$  are upward, known forces at  $x=x_{f_i}$  and  $R_{y_i}$  are upward, unknown reactions at  $x=x_{r_i}$

$$M(x) = \int_0^x V(x)dx + \sum_{i=1}^{NOM} M_{z_i} \{x-x_{m_i}\}^0 + \sum_{i=1}^{NORM} RM_{z_i} \{x-x_{rm_i}\}^0 \quad (3)$$

where  $M_{z_i}$  are clockwise, known moments at  $x=x_{m_i}$  and  $RM_{z_i}$  are clockwise, unknown reactive moments at  $x=x_{rm_i}$

$$\theta = \frac{dy}{dx} = \int_0^x \left[ M(x) \sum_{i=1}^{NOI} f_i(x) \right] dx - \left[ V(x) \sum_{i=1}^{NOK} g_i(x) \right] + C_1 \quad (4)$$

where  $C_1$  is the constant of integration.

Appendix A indicates how the product and the integration of the product of the curly bracket symbols are formed.

$$y = \int_0^x \int_0^x \left[ M(\xi) \sum_{i=1}^{NOI} f_i(\xi) \right] d\xi dx - \int_0^x \left[ V(x) \sum_{i=1}^{NOK} g_i(x) \right] dx + C_1 x + C_2 \quad (5)$$

where  $C_2$  is the constant of integration.

Equations (1) through (5) when properly coded in FORTRAN

<sup>2</sup>The numbers, NOL, NOF, NOR, NORM, NOI and NOK, appearing above the summation signs are program input parameters indicating the number of terms for each problem.

form the core of the five basic subroutines: LOAD, SHEAR  
MOMENT, SLOPE and DEFLECT. It must be noted before discuss-  
ing the subroutines that equations (2) through (5) involve  
the unknown values  $C_1$ ,  $C_2$ ,  $R_{y_1}$ , and  $RM_{z_1}$ , where there may be  
as many as ten values each of  $R_y$  and  $RM_z$ . The calculation of  
these values is done in the main program BEAM3 and will be  
discussed in section 6.

## 5. The basic subroutines.

### Subroutines LOAD, SHEAR and MOMENT.

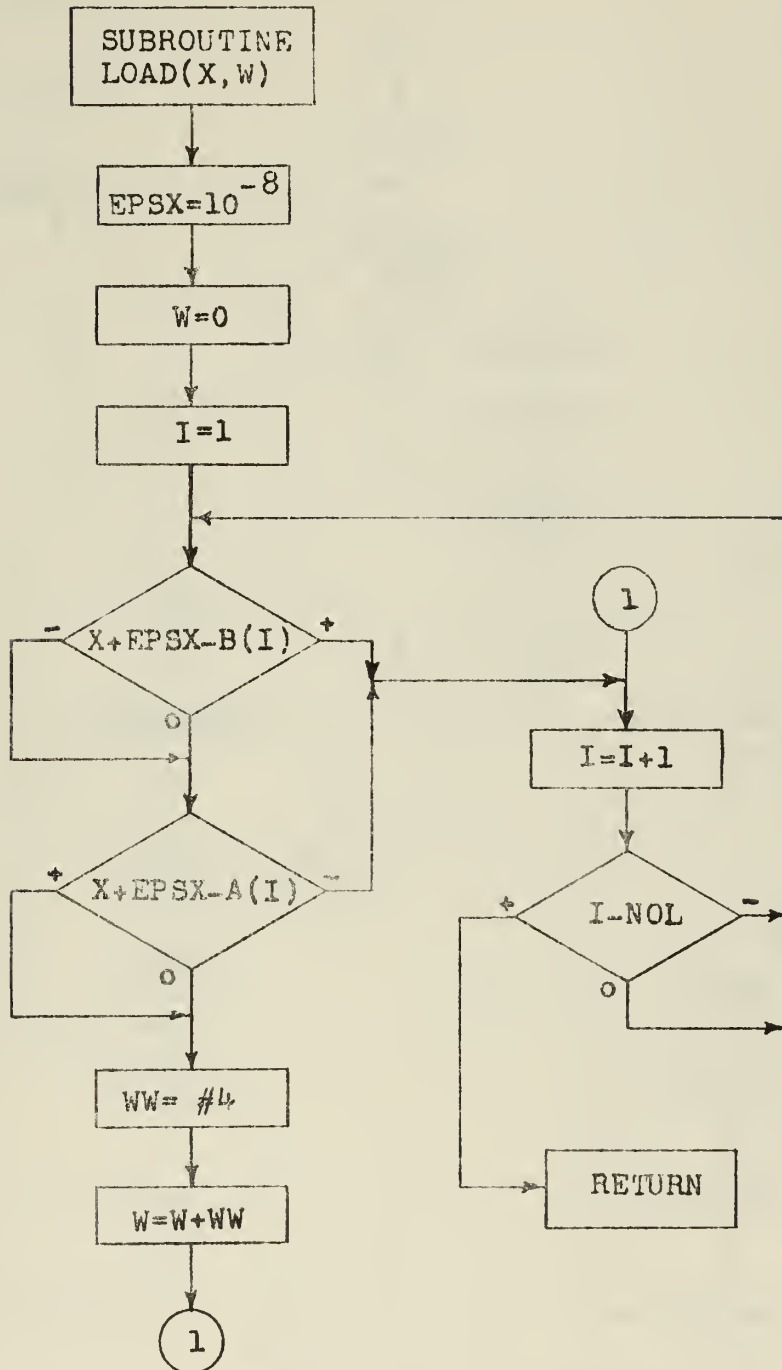
The above named subroutines, as their names imply, are employed to calculate the total distributed load, shear and moment, respectively, at any desired point on the beam. The expanded forms of equations (1), (2) and (3) were used in writing these subroutines. Utilizing the distance,  $x_1$ , as the entering argument, the subroutines select and sum the appropriate terms in the corresponding equation by inspecting all loading terms and discarding those in which the value within the curly bracket is negative. Actually the load, shear and moment are calculated a small increment,  $\epsilon = 10^{-8}$ , to the right of  $x_1$  in order to obtain the right side value of any step-discontinuities occurring at the point,  $x_1$  (See flow charts on pages 13, 14 and 15).

### Subroutine SLOPE.

This subroutine is equation (4), excluding the constant of integration,  $C_1$ , in FORTRAN language (See flow chart on page 17). Note that the equation can be expressed as a summation of terms, which are constants times the multiplication of two step-functions of the form  $C\{x-a\}^n\{x-b\}^m$ , or the summation of the integral of the same type of terms, where  $n$  equals either 0 or 1.

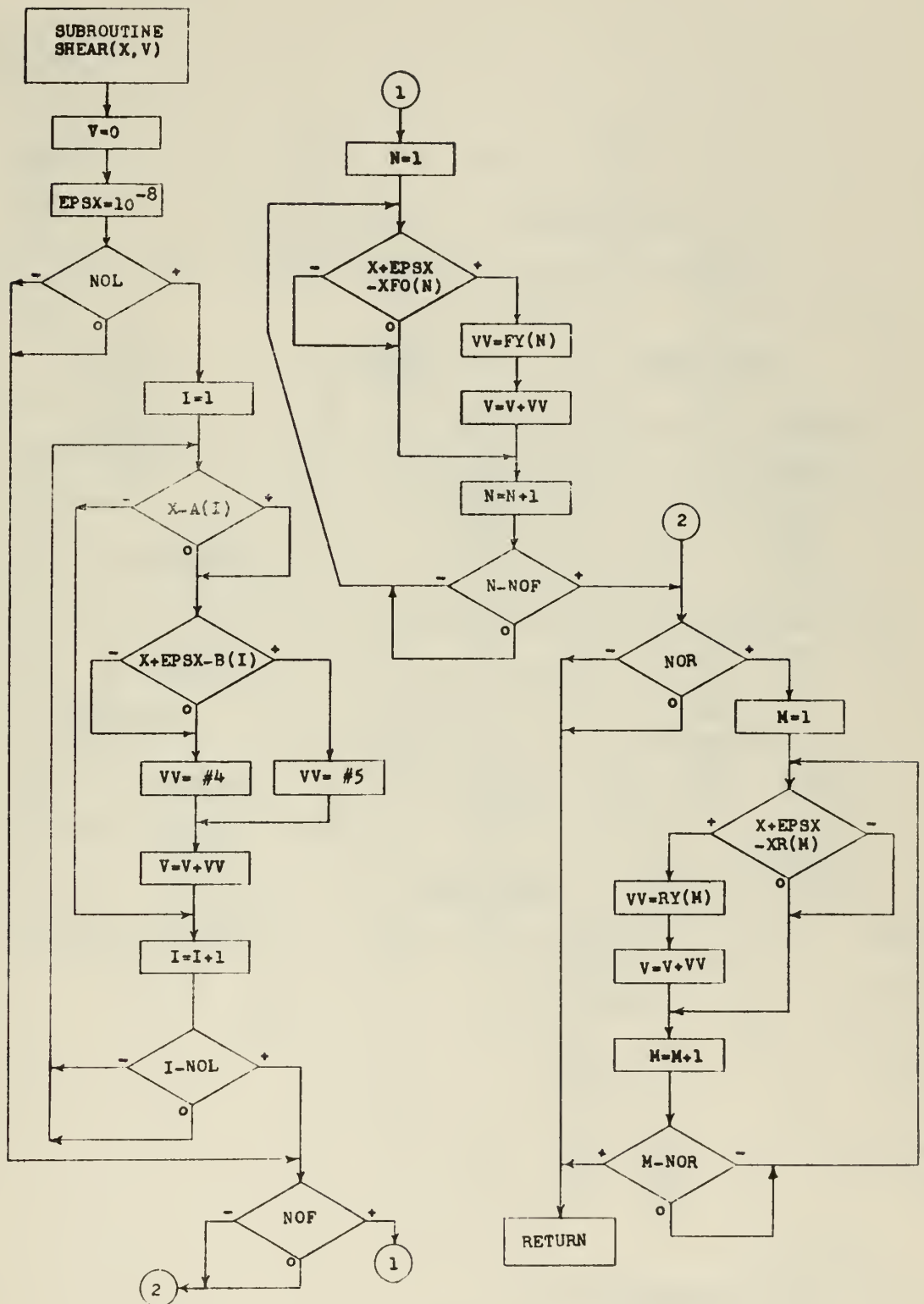
Function subroutines, FTON and FT1N (see Appendix A), will calculate the value of  $\{x-a\}^0\{x-b\}^m$  and  $\{x-a\}^1\{x-b\}^n$ , respectively. Function subroutines, ENTON and ENT1N (See

# Flow Chart of LOAD



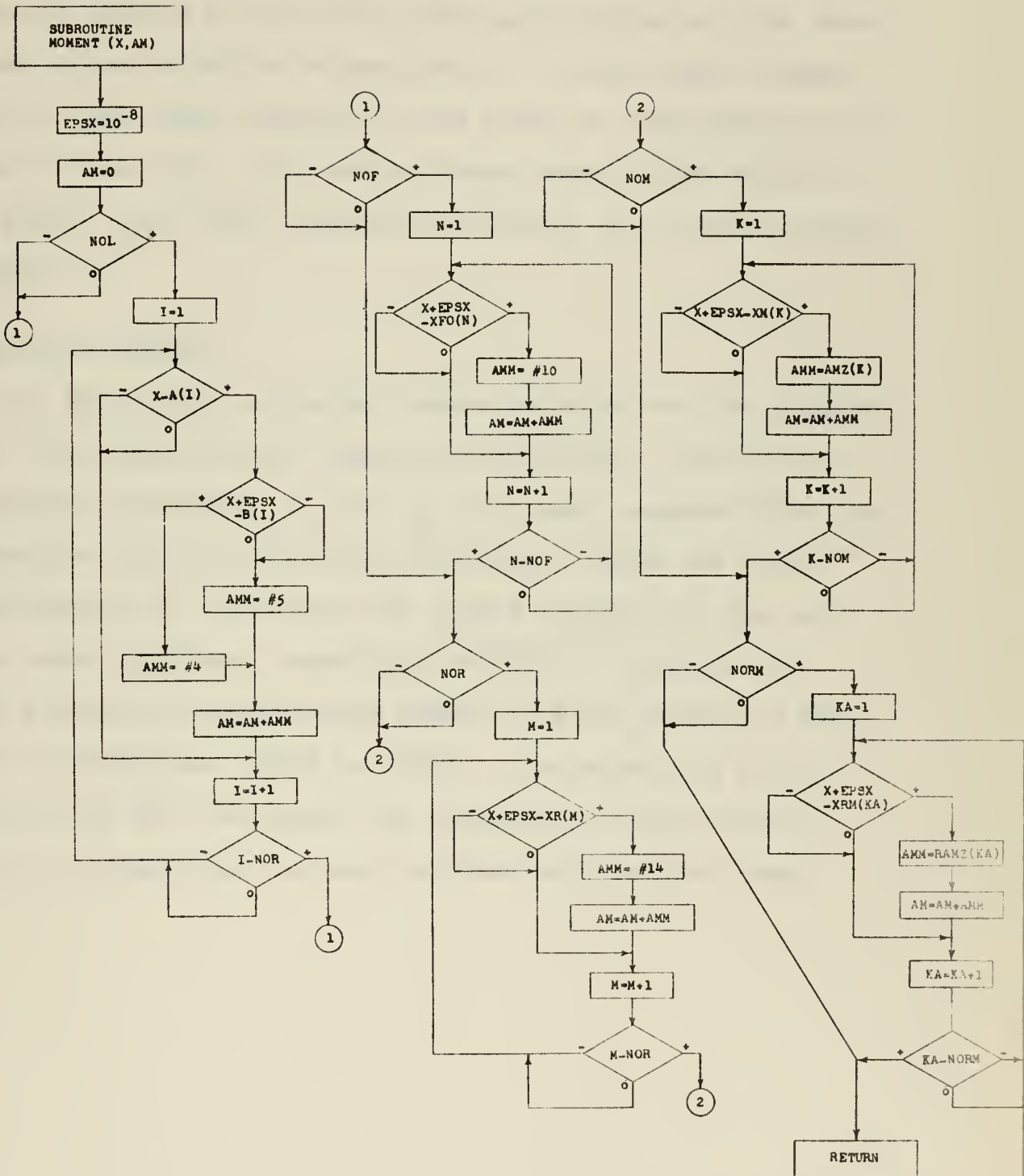
Note: Numbers preceded by a number sign, #, refer to the statement number in the listing of LOAD.

Flow Chart of SHEAR



Note: Numbers preceded by a number sign, #, refer to the statement number in the listing of SHEAR.

Flow Chart of MOMENT



Note: Numbers preceded by a number sign, #, refer to the statement number in the listing of MOMENT.

Appendix A), will calculate the first integral of these functions. Equation (4) can now be expressed as the summation of constants multiplied by the appropriate function subroutines.

SLOPE calculates all combinations of  $M(x)$  and  $f(x)$  by stepping through all the  $f(x)$  terms and varying all the terms in the moment equation at each step. It then steps through all the  $g(x)$  terms varying all the terms in the equation for shear at each step. The sum of these terms is the value of the slope, minus the integration constant,  $C_1$ , at the entered distance,  $x$ .

#### Subroutine DEFLECT.

At this point the author wishes to point out the similarity of the equations for slope and deflection. Since the integration constants,  $C_1$  and  $C_2$ , are again excluded from the subroutine, and the function subroutines, DITON and DITLN (See Appendix A), calculate the second integral of the multiple step-functions, subroutine DEFLECT is identical to SLOPE except for substituting ENTON for FTON, ENTLN for FTLN, DITON for ENTON and DITLN for ENTLN. The result is a subroutine that will calculate the deflection, minus the constants of integration, at any required point on the beam.



## 6. Program BEAM3.

The main program, BEAM3, can be divided into four distinct steps. These steps are: reading the input data, calculation of indeterminate quantities, calculation of the flexural quantities, and the output of the solution.

The program first reads the input data which includes the length of the beam, all external loading, the location of all reactions, the flexural properties of the beam, and a group of parameters giving the number of each type of load or reaction. It also provides for reading a predetermined value of deflection, PDEFT, at each reaction and a predetermined value of slope, PSLP, at each moment reaction.

The program then proceeds to solve for the indeterminate quantities of the beam by utilizing the two equations of static equilibrium plus equations obtained from the known boundary condition at each reaction or reactive moment. The first two equations are simply the summation of the forces on the beam equals zero and the summation of the moments about the right end of the beam equals zero. The remainder of the equations are obtained from the fact that the slope at each reactive moment or the deflection at each reaction is known. Using this fact in the appropriate one of either equation (4) or (5) results in a set of linear equations for the indeterminate quantities.

The equations thus obtained are generated in a matrix form that is compatible with subroutine GAUSS2 (See Appendix E). This subroutine is then called to solve the equations simulta-

neously. If the matrix is singular, GAUSS2 exits to the calling program which in turn prints "MATRIX SINGULAR" and then stops. If not, the subroutine solves the equations and returns the calculated values to the main program. The most serious limitations of this program are imposed by the generation or solution of the simultaneous equations, therefore it is appropriate to include and explain several of them at this point.

First, at least one reaction, even if it is later forced to equal zero, must be included with the input, or a singular matrix will be generated.

Including shear deflection in the solution must also be greatly limited at this point. Since in general the shear includes step discontinuities, they will also appear in the slope if shear deflection is included. The program generates an equation for slope at each moment reaction. If by chance, either a reaction or point force occurs at the same point as a moment reaction, the slope at this point is double valued. The program is unable to distinguish which of the two values is required except when it occurs at the extreme ends of the beam. For example a beam "built in" at both ends is acceptable. If the beam is "built in" at only one end, it must be solved in a manner such that the "built in" is at the origin of the beam.

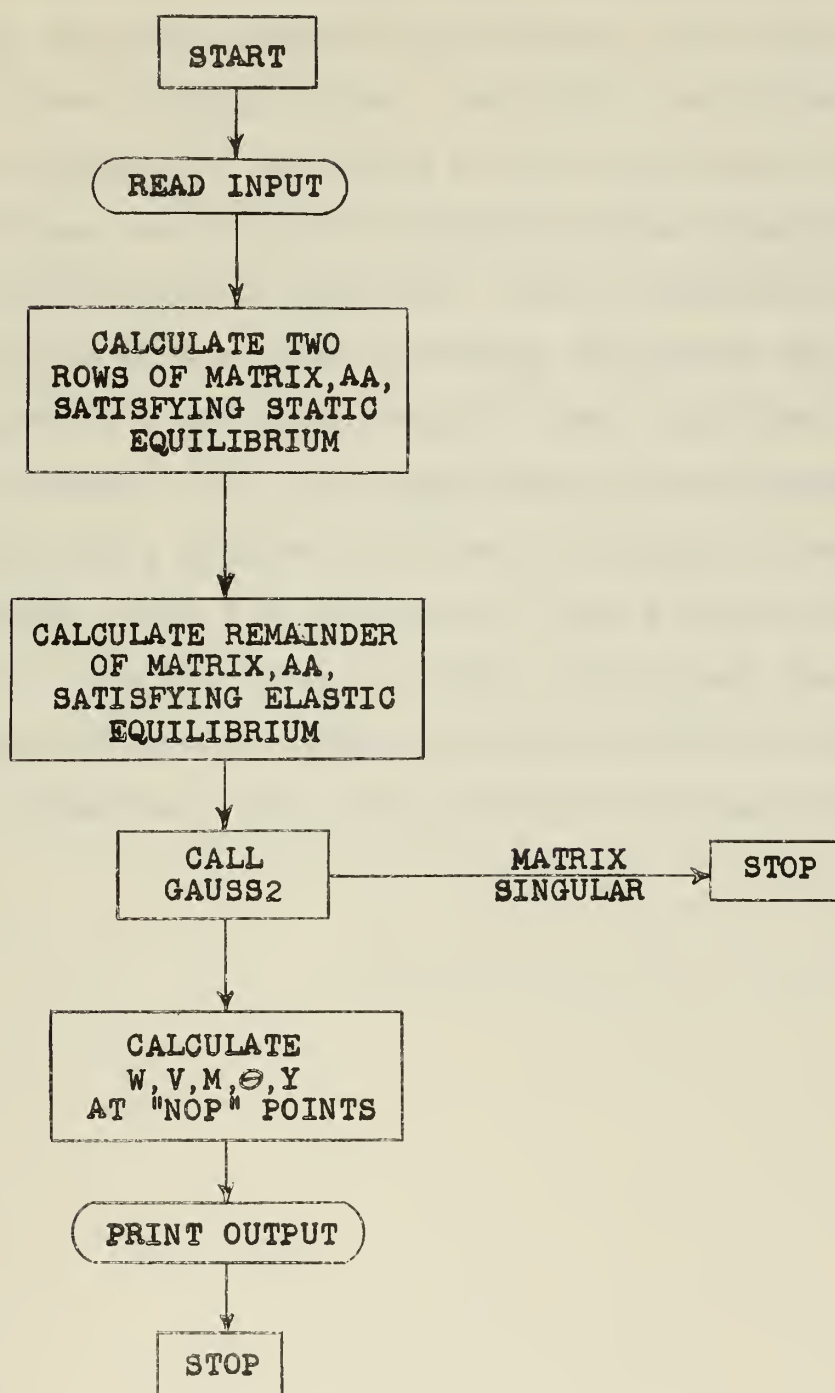
The program then divides the beam into a number of increments, NOP-1,\* and calls the subroutines, LOAD, SHEAR,

\*NOP is another parameter read in at the start of BEAM3.

MOMENT, SLOPE and DEFLECT to calculate the flexural quantities at each of these increments.

The output consists of printing the distance from the left end of the beam to the point in question and the values of the load, shear, moment, slope and deflection at that point. The values of all of the indeterminate quantities are also printed out.

Fig. 7 Simplified Flow Chart of BEAM3



## 7. Testing of Program.

The testing of the program and the subroutines could, in theory, continue indefinitely because of the unlimited number of beam configurations. The author has attempted to show (See Appendix D) solutions to test problems of simple configurations and constant bending and shear compliance to compare with classical solutions. Then a statically determinate problem with a varying bending compliance was solved and compared to a solution obtained from a numerical integration scheme. The most rigid test for the program was in constructing a problem for a beam of unlikely configuration (See Problems 6 and 7 of Appendix D), and solving the flexural analysis of this beam from both ends. Note that the only significant difference between the two solutions is the opposite signs for shear and slope, which would of course, be expected.

## 8. Conclusions and Recommendations.

The thesis shows a method of including piecewise linear variation of both bending compliance and shear compliance in the flexural analysis of beams by extending "McCaulay's Method". Adapting this idea to the digital computer resulted in a program capable of analyzing a very general type of beam. However, had time permitted, several changes would have been made to the program. First, internal monitoring of the input data would be an asset to any user. Also, using the same general method of attack, this program would have been expanded to include bending compliance and shear compliance of a piecewise parabolic nature.

Other additions to the program, much larger in scope, would be including elastic supports and/or beam column effect.

It would be interesting to compare the results of this program to those obtained from a program utilizing the numerical integration method. Also, a more exhaustive study of the program could be made to determine accuracy of, and time requirement for, solutions.

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## APPENDIX A

### "Curly bracket" method, definitions and formulas

We define:

$$\begin{aligned} 1. \quad \{x-a\}^n &= (x-a)^n, & x > a \\ &= 0, & x < a \end{aligned}$$

where  $n$  is an integer  $\geq 0$

From this definition, it is possible to establish the following formulas. Although it would be possible to express these results in terms of curly brackets, in most cases the resulting expressions would be rather complicated. The forms exhibited below have the advantage of being most readily conformable to the decision commands available in FORTRAN.

$$\begin{aligned} 2. \quad \int_0^x \{x-a\}^n dx &= \frac{(x-a)^{n+1}}{n+1}, & x > a \\ &= 0, & \text{Otherwise} \end{aligned}$$

$$\begin{aligned} 3. \quad \{x-a\}^0 \{x-b\}^n &= (x-b)^n, & x > a \text{ and } x > b \\ &= 0, & \text{Otherwise} \end{aligned}$$

$$\begin{aligned} 4. \quad \{x-a\}^1 \{x-b\}^n &= (x-b)^{n+1} + (b-a)(x-b)^n, & x > a \\ & & \text{and } x > b \\ &= 0, & \text{Otherwise} \end{aligned}$$

$$\begin{aligned} 5. \quad \int_0^x \{x-a\}^0 \{x-b\}^n dx &= \frac{(x-b)^{n+1}}{n+1} - \frac{(a-b)^{n+1}}{n+1}, & x > a > b \\ &= \frac{(x-b)^{n+1}}{n+1}, & x > b \geq a \\ &= 0, & \text{Otherwise} \end{aligned}$$

$$\begin{aligned}
6. \int_0^x \{x-a\}^1 \{x-b\}^n dx &= \frac{(x-b)^{n+2}}{n+2} + \frac{(b-a)(x-b)^{n+1}}{n+1} \\
&+ \frac{(a-b)^{n+2}}{(n+1)(n+2)}, \quad x > a > b \\
&= \frac{(x-b)^{n+2}}{n+2} + \frac{(b-a)(x-b)^{n+1}}{n+1}, \quad x > b > a \\
&= 0, \quad \text{Otherwise}
\end{aligned}$$

$$\begin{aligned}
7. \int_0^x \int_0^x \{\xi-a\}^0 \{\xi-b\}^n d\xi dx &= \frac{(x-b)^{n+2}}{(n+1)(n+2)} - \frac{(x-a)(a-b)^{n+1}}{(n+1)} \\
&- \frac{(a-b)^{n+2}}{(n+1)(n+2)}, \quad x > a > b \\
&= \frac{(x-b)^{n+2}}{(n+1)(n+2)}, \quad x > b \geq a \\
&= 0, \quad \text{Otherwise}
\end{aligned}$$

$$\begin{aligned}
8. \int_0^x \int_0^x \{\xi-a\}^1 \{\xi-b\}^n d\xi dx &= \frac{(x-b)^{n+3}}{(n+2)(n+3)} + \frac{(b-a)(x-b)^{n+2}}{(n+1)(n+2)} \\
&+ \frac{(a-a)(a-b)^{n+2}}{(n+1)(n+2)} + \frac{2(a-b)^{n+3}}{(n+1)(n+2)(n+3)}, \quad x > a > b \\
&= \frac{(x-b)^{n+3}}{(n+2)(n+3)} + \frac{(b-a)(x-b)^{n+2}}{(n+1)(n+2)}, \quad x > b > a \\
&= 0, \quad \text{Otherwise}
\end{aligned}$$

Function Subroutines FTON, FT1N, ENTON, ENT1N, DITON, DIT1N

The above named function subroutines were written and utilized to evaluate each of the preceeding formulas numbered 3 through 8. The values x, a, b and n were used as entering

arguments to these functions. The function then became the variable as follows:

$$\text{FTON} = \{x-a\}^0 \{x-b\}^n$$

$$\text{FT1N} = \{x-a\}^1 \{x-b\}^n$$

$$\text{ENTON} = \int_0^x \{x-a\}^0 \{x-b\}^n dx$$

$$\text{ENT1N} = \int_0^x \{x-a\}^1 \{x-b\}^n dx$$

$$\text{DITON} = \int_0^x \int_0^x \{\xi-a\}^0 \{\xi-b\}^n d\xi dx$$

$$\text{DIT1N} = \int_0^x \int_0^x \{\xi-a\}^1 \{\xi-b\}^n d\xi dx$$

As examples, flow charts for FTON and ENTON are shown in Figs. 8 and 9.

Fig. 8 Flow Chart of FTON

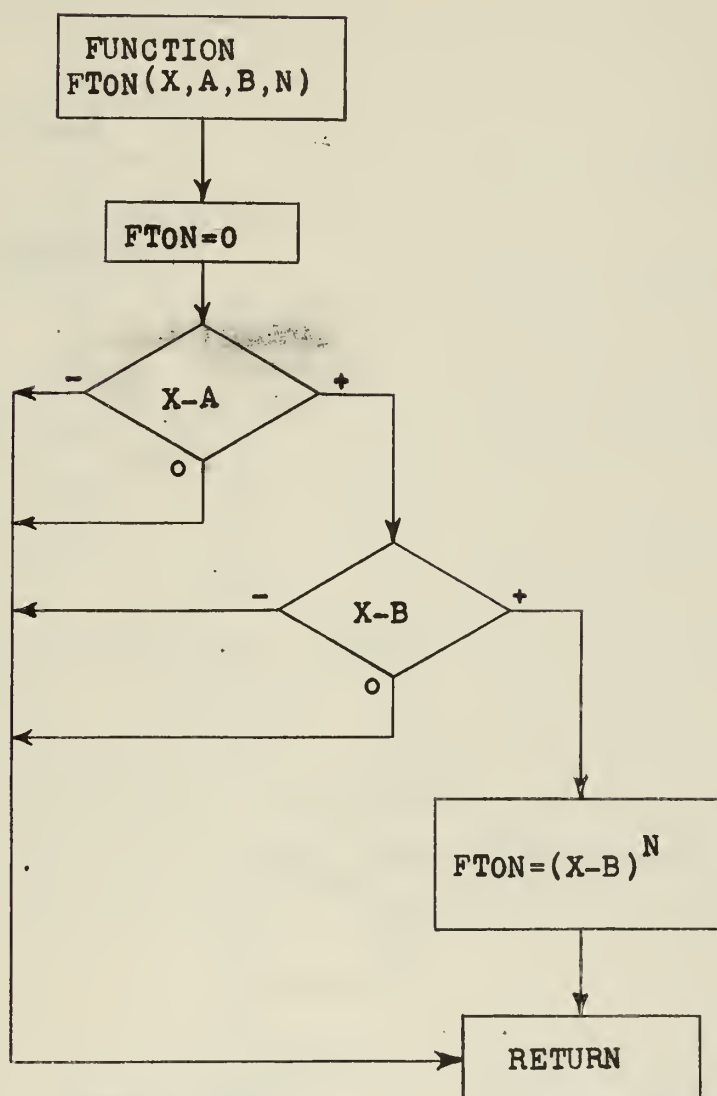
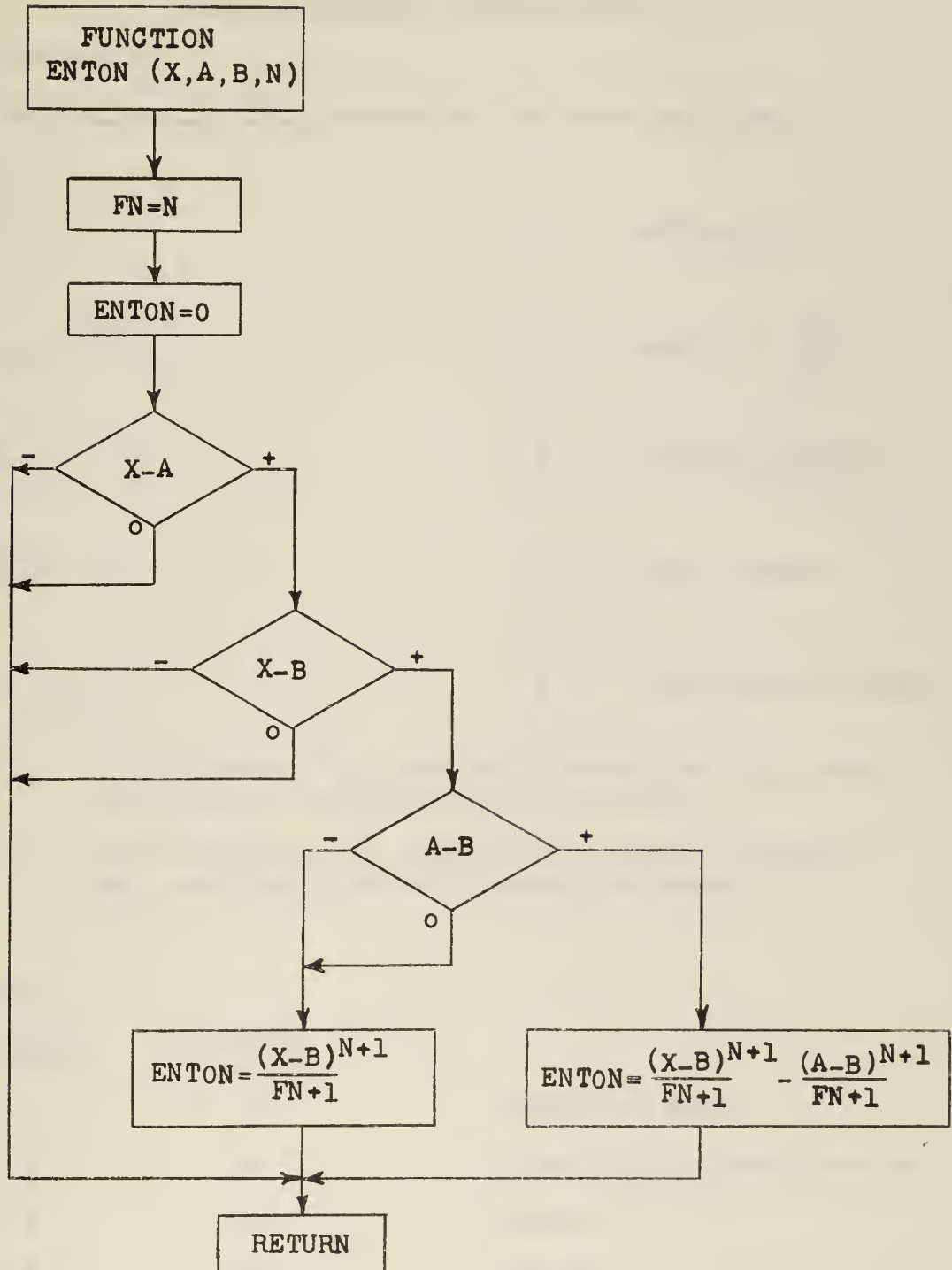


Fig. 9 Flow Chart of ENTON



## APPENDIX B

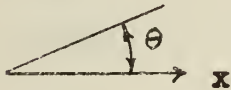
### Sign Convention and Notation

#### Sign Convention

The following sign convention has been employed:



$y$  = deflection



$\theta$  = slope =  $\frac{dy}{dx}$



$M$  = bending moment



$V$  = shear force



$W$  = distributed loads

- Note: 1. Point forces,  $F_y$ , and point reactions,  $R_y$ , are positive in the positive  $y$  direction.
2. Point moments,  $M_z$ , and point reactive moments,  $RM_2$ , are positive in a clockwise sense.

#### Notation:

| THESIS   | PROGRAM |                                   |
|----------|---------|-----------------------------------|
|          | C       | Length of beam                    |
| W        | W       | Total distributed load at a point |
| V        | V       | Shear                             |
| M        | AM      | Moment                            |
| $\theta$ | Y1      | Slope                             |
| y        | Y       | Deflection                        |

|        |      |                                               |
|--------|------|-----------------------------------------------|
| $F_y$  | FY   | Point load or force                           |
| $R_y$  | RY   | Point reaction                                |
| $M_z$  | AMZ  | Point moment                                  |
| $RM_z$ | RAMZ | Point reactive moment                         |
| $Q(x)$ |      | Element of distributed load                   |
| $Q_a$  | AQ   | Distributed load (left end)                   |
| $Q_b$  | QB   | Distributed load (right end)                  |
| $f(x)$ |      | Element of bending compliance                 |
|        | EA   | Bending compliance<br>(left end of element)   |
|        | EB   | Bending compliance<br>(right end of element)  |
| $g(x)$ |      | Element of shear compliance                   |
|        |      | $(\frac{1}{EI})$                              |
|        | GA   | Shear compliance<br>(left end of element)     |
|        | GB   | Shear compliance<br>(right end of element)    |
| $X_f$  | XFO  | Distance to $F_y$                             |
| $X_r$  | XR   | Distance to $R_y$                             |
| $X_m$  | XM   | Distance to $M_z$                             |
| $X_m$  | RXM  | Distance to $RM_z$                            |
| $A_g$  | AG   | Distance to $G_a$                             |
| $B_g$  | BG   | Distance to $G_b$                             |
| $A_e$  | AE   | Distance to $E_a$                             |
| $B_e$  | BE   | Distance to $E_b$                             |
| A      | A    | Distance to $Q_a$                             |
| B      | B    | Distance to $Q_b$                             |
| NOL    | NOL  | Number of distributed loads (25) <sup>3</sup> |

<sup>3</sup>Numbers following explanation of terms NOL--NOK indicate the arbitrarily chosen maximum value of each for this program.

|       |       |                                                                       |
|-------|-------|-----------------------------------------------------------------------|
| NOF   | NOF   | Number of point forces (100)                                          |
| NOR   | NOR   | Number of point reactions (10)                                        |
| NOM   | NOM   | Number of point moments (50)                                          |
| NORM  | NORM  | Number of point reactive moments (10)                                 |
| NOI   | NOI   | Number of bending compliance elements (25)                            |
| NOK   | NOK   | Number of shear compliance elements (25)                              |
| PDEFT | PDEFT | Predetermined deflection at a reaction                                |
| PSLP  | PSLP  | Predetermined slope at a reactive moment                              |
| NOP   | NOP   | Number of points at which the flexural quantities will be calculated. |

## APPENDIX C

### General Information Concerning Use of Program

The composite program, BEAM3 plus subroutines, was written in FORTRAN-60, compiled and tested on a Control Data Corporation 1604 digital computer. It consists of 412 FORTRAN statements requiring about 580 cards. The computer storage requirement is about 12,500 cells, however, this number can be reduced by changing the dimension statements which would of course, reduce either the maximum number of external loads, elements of shear compliance or bending compliance, or number of points at which flexural quantities could be calculated.

The FORTRAN statements used in this program are: GO TO, computed GO TO, IF, STOP, DO, CONTINUE, FORMAT, READ, PRINT, WRITE OUTPUT TAPE, FUNCTION, SUBROUTINE, RETURN, CALL, DIMENSION, COMMON, and END.

There are no conversion constants incorporated in this program. As such all input must be in a consistent set of units.

#### "FORMAT" for Input Data

The input data for a problem is read by the program with a FORMAT of I4 for all "fixed point" variables and F20.0 for all floating point variables. The arrangement of the data on the input cards is shown as follows;

| Group no. | No. of cards in group | Fields | Entries                                                           |
|-----------|-----------------------|--------|-------------------------------------------------------------------|
| 1         | 1                     | F20.0  | C                                                                 |
| 2         | 1                     | 8I4    | NOP, NOL, NOF, NOR                                                |
| 3         | NOL                   | 4F20.0 | NOM, NORM, NOI, NOK<br>A, B, A <sub>a</sub> , A <sub>b</sub>      |
| 4         | NOF                   | 2F20.0 | x <sub>f</sub> , F <sub>y</sub>                                   |
| 5         | NOR                   | 3F20.0 | x <sub>r</sub> , 0.0, PDEFT                                       |
| 6         | NOM                   | 2F20.0 | x <sub>m</sub> , M <sub>z</sub>                                   |
| 7         | NORM                  | 3F20.0 | x <sub>m</sub> , 0.0, PSLP                                        |
| 8         | NOI                   | 4F20.0 | A <sub>e</sub> , B <sub>e</sub> , E <sub>a</sub> , E <sub>b</sub> |
| 9         | NOK                   | 4F20.0 | A <sub>g</sub> , B <sub>g</sub> , G <sub>a</sub> , G <sub>b</sub> |

As an example the input data cards for test problem #1 in Appendix D was as follows:

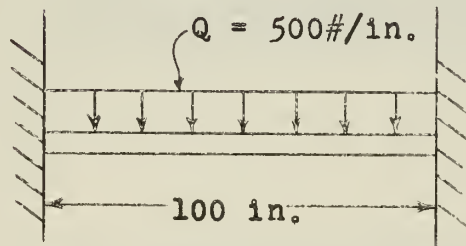
|         |                  |      |                       |
|---------|------------------|------|-----------------------|
| CARD 1: | 100              |      |                       |
| CARD 2: | 21 1 0 2 0 2 1 0 |      |                       |
| CARD 3: | 0.               | 100  | -500. -500.           |
| CARD 4: | 0.               | 0.   | .0                    |
| CARD 5: | 100              | 0.   | .0                    |
| CARD 6: | 0.               | 0.   | 0.                    |
| CARD 7: | 100              | 0.   | 0.                    |
| CARD 8: | 0.               | 100. | .000000001 .000000001 |

## APPENDIX D

### Test problems and solutions

#### Test problem 1

This problem consists of a beam of uniform cross-section, "built-in" at both ends, and loaded with a uniformly distributed weight of 500#/in. (See Fig. 10)



$$\frac{1}{EI} = 10^{-9} \frac{1}{\#in.^2}$$

$$\frac{K}{AG} = 0$$

Fig. 10

#### Test problem 2

This problem is identical to problem 1 except that the slope at either end of the beam is forced to values other than zero.

$$\theta_{\text{left end}} = -.002$$

$$\theta_{\text{right end}} = .002$$

# SOLUTION TO TEST PROBLEM 1

| DISTANCE | LOAD    | SHEAR    | MOMENT   | SLOPE    | DEFLECTION |
|----------|---------|----------|----------|----------|------------|
| 00       | 500.00  | 25000.0  | -416667. | 00000000 | 00000000   |
| 5.00     | -500.00 | 22500.0  | -297917. | 00000000 | 00000000   |
| 10.00    | -500.00 | 20000.0  | -191667. | 00000000 | 00000000   |
| 15.00    | -500.00 | 17500.0  | -97917.  | 00000000 | 00000000   |
| 20.00    | -500.00 | 15000.0  | -16667.  | 00000000 | 00000000   |
| 25.00    | -500.00 | 12500.0  | 520833.  | 00000000 | 00000000   |
| 30.00    | -500.00 | 10000.0  | 1083333. | 00000000 | 00000000   |
| 35.00    | -500.00 | 7500.0   | 1520833. | 00000000 | 00000000   |
| 40.00    | -500.00 | 5000.0   | 1833333. | 00000000 | 00000000   |
| 45.00    | -500.00 | 2500.0   | 2020833. | 00000000 | 00000000   |
| 50.00    | -500.00 | 0        | 2020833. | 00000000 | 00000000   |
| 55.00    | -500.00 | -2500.0  | 1833333. | 00000000 | 00000000   |
| 60.00    | -500.00 | -5000.0  | 1520833. | 00000000 | 00000000   |
| 65.00    | -500.00 | -7500.0  | 1083333. | 00000000 | 00000000   |
| 70.00    | -500.00 | -10000.0 | 520833.  | 00000000 | 00000000   |
| 75.00    | -500.00 | -12500.0 | -16667.  | 00000000 | 00000000   |
| 80.00    | -500.00 | -15000.0 | -97917.  | 00000000 | 00000000   |
| 85.00    | -500.00 | -17500.0 | -191667. | 00000000 | 00000000   |
| 90.00    | -500.00 | -20000.0 | -297917. | 00000000 | 00000000   |
| 95.00    | -500.00 | -22500.0 | -416667. | 00000000 | 00000000   |
| 100.00   | -500.00 | -25000.0 | -416667. | 00000000 | 00000000   |

```

RY( 1) = 25000.
RY( 2) = 25000.
RAMZ( 1) = -416667.
RAMZ( 2) = 416667.
C1 = .00000000
C2 = .00000000

```

# SOLUTION TO TEST PROBLEM 2

| DISTANCE | LOAD    | SHEAR    | MOMENT   | SLOPE     | DEFLECTION |
|----------|---------|----------|----------|-----------|------------|
| .00      | -500.00 | 25000.0  | -376667. | .00200000 | .00000000  |
| 5.00     | -500.00 | 22500.0  | -257917. | .00358125 | -.01420052 |
| 10.00    | -500.00 | 20000.0  | -151667. | .00460000 | -.03487500 |
| 15.00    | -500.00 | 17500.0  | -57917.  | .00511875 | -.05933333 |
| 20.00    | -500.00 | 15000.0  | 233333.  | .00520000 | -.08533333 |
| 25.00    | -500.00 | 12500.0  | 920833.  | .00490625 | -.11087421 |
| 30.00    | -500.00 | 10000.0  | 1483333. | .00430000 | -.13333333 |
| 35.00    | -500.00 | 7500.0   | 1920833. | .00340000 | -.15382500 |
| 40.00    | -500.00 | 5000.0   | 2233333. | .00240000 | -.16771719 |
| 45.00    | -500.00 | 2500.0   | 2483333. | .00123125 | -.17711719 |
| 50.00    | -500.00 | 0.0      | 2633333. | .00000000 | -.18020833 |
| 55.00    | -500.00 | -2500.0  | 2683333. | .00123125 | -.17800000 |
| 60.00    | -500.00 | -5000.0  | 2633333. | .00240000 | -.15332552 |
| 65.00    | -500.00 | -7500.0  | 2483333. | .00340000 | -.11074219 |
| 70.00    | -500.00 | -10000.0 | 2233333. | .00430000 | -.05933333 |
| 75.00    | -500.00 | -12500.0 | 1920833. | .00490625 | -.01420052 |
| 80.00    | -500.00 | -15000.0 | 1483333. | .00520000 | .00000000  |
| 85.00    | -500.00 | -17500.0 | 920833.  | .00511875 | .03487500  |
| 90.00    | -500.00 | -20000.0 | 57917.   | .00520000 | .05933333  |
| 95.00    | -500.00 | -22500.0 | -257917. | .00460000 | .08533333  |
| 100.00   | -500.00 | -25000.0 | -376667. | .00200000 | .00000000  |

```

RY( 1) = 25000.
RY( 2) = 25000.
RAMZ( 1) = -376667.
RAMZ( 2) = 376667.
C1 = -.00200000
C2 = .00000000

```

### Test problem 3

This problem consists of a beam of uniform cross-section, "built-in" at one end and simply supported at the other, and loaded with a uniformly distributed weight. (See Fig. 11)

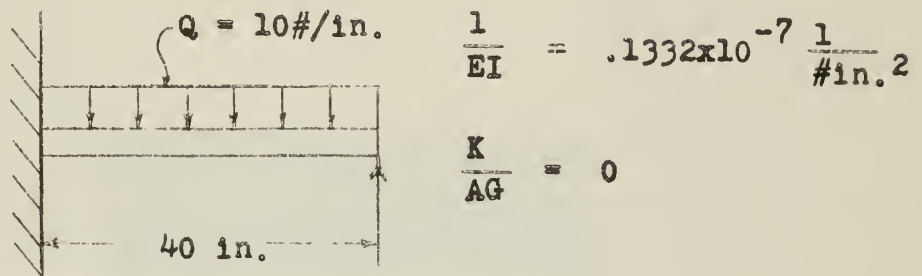


Fig. 11

### Test problem 4

This problem is identical to problem number 3 except shear deflection is included in the solution.

$$\frac{K}{AG} = .159 \times 10^{-6} \frac{1}{\#in.}$$

# SOLUTION TO TEST PROBLEM 3

| DISTANCE | LOAD   | SHEAR   | MOMENT   | SLOPE     | DEFLECTION |
|----------|--------|---------|----------|-----------|------------|
| .00      | -10.00 | 250.00  | -2000.00 | .00000000 | .00000000  |
| 2.00     | -10.00 | 230.00  | -1520.00 | .00004680 | -.00004893 |
| 4.00     | -10.00 | 210.00  | -1080.00 | .00008134 | -.00017902 |
| 6.00     | -10.00 | 190.00  | -680.00  | .00010470 | -.00036683 |
| 8.00     | -10.00 | 170.00  | -320.00  | .00011793 | -.00059105 |
| 10.00    | -10.00 | 150.00  | .00      | .00012210 | -.00083250 |
| 12.00    | -10.00 | 130.00  | 280.00   | .00011828 | -.00107412 |
| 14.00    | -10.00 | 110.00  | 520.00   | .00010754 | -.00130101 |
| 16.00    | -10.00 | 90.00   | 720.00   | .00009093 | -.00150036 |
| 18.00    | -10.00 | 70.00   | 880.00   | .00006953 | -.00166154 |
| 20.00    | -10.00 | 50.00   | 1000.00  | .00004440 | -.00177600 |
| 22.00    | -10.00 | 30.00   | 1080.00  | .00001661 | -.00183736 |
| 24.00    | -10.00 | 10.00   | 1120.00  | .00001279 | -.00184136 |
| 26.00    | -10.00 | -10.00  | 1120.00  | .00004271 | -.00178586 |
| 28.00    | -10.00 | -30.00  | 1080.00  | .00007211 | -.00167086 |
| 30.00    | -10.00 | -50.00  | 1000.00  | .00009990 | -.00149850 |
| 32.00    | -10.00 | -70.00  | 880.00   | .00012503 | -.00127304 |
| 34.00    | -10.00 | -90.00  | 720.00   | .00014643 | -.00100086 |
| 36.00    | -10.00 | -110.00 | 520.00   | .00016304 | -.00069051 |
| 38.00    | -10.00 | -130.00 | 280.00   | .00017378 | -.00035262 |
| 40.00    | -10.00 | -150.00 | .00      | .00017760 | -.00000000 |

```

RY( 1)= 250.00
RY( 2)= 150.00
RAMZ( 1)=-2000.00
C1=.00000000
C2=.00000000

```

# SOLUTION TO TEST PROBLEM 4

| DISTANCE | LOAD   | SHEAR   | MOMENT   | SLOPE    | DEFLECTION |
|----------|--------|---------|----------|----------|------------|
| 00       | -10.00 | 248.91  | -1956.22 | 00003958 | 000000000  |
| 2:00     | -10.00 | 228.91  | -1478.41 | 00008206 | -00012375  |
| 4:00     | -10.00 | 208.91  | -1040.59 | 00011234 | -00032009  |
| 6:00     | -10.00 | 188.91  | -642.78  | 00013149 | -00056570  |
| 8:00     | -10.00 | 168.91  | -284.97  | 00014058 | -00083936  |
| 10:00    | -10.00 | 148.91  | 32.84    | 00014067 | -00112203  |
| 12:00    | -10.00 | 128.91  | 310.65   | 00013283 | -00139677  |
| 14:00    | -10.00 | 108.91  | 548.46   | 00011812 | -00164877  |
| 16:00    | -10.00 | 88.91   | 746.27   | 00009760 | -00186536  |
| 18:00    | -10.00 | 68.91   | 904.08   | 00007235 | -00203602  |
| 20:00    | -10.00 | 48.91   | 1021.89  | 00004343 | -00215232  |
| 22:00    | -10.00 | 28.91   | 1099.70  | 00001190 | -00220799  |
| 24:00    | -10.00 | 8.91    | 1137.51  | 00002117 | -00219889  |
| 26:00    | -10.00 | -11.09  | 1135.32  | 00005471 | -00212300  |
| 28:00    | -10.00 | -31.09  | 1093.14  | 00008766 | -00198044  |
| 30:00    | -10.00 | -51.09  | 1010.95  | 00011896 | -00177345  |
| 32:00    | -10.00 | -71.09  | 888.76   | 00014753 | -00150642  |
| 34:00    | -10.00 | -91.09  | 726.57   | 00017225 | -00118585  |
| 36:00    | -10.00 | -111.09 | 524.38   | 00019225 | -00082039  |
| 38:00    | -10.00 | -131.09 | 282.19   | 00020626 | -00042080  |
| 40:00    | -10.00 | -151.09 | .00      | 00021329 | -00000000  |

RY( 1)= 248.91  
 RY( 2)= 151.09  
 RAMZ( 1)= -1956.22  
 C1=-.00000000  
 C2=.00000000

# Test problem 5

This problem consists of a homogeneous circular shaft, three inches in diameter at the center and tapered to one and a half inches in diameter at both ends. The total weight of the shaft is 100# and a uniformly distributed load totaling 200# is placed on the central section. The shaft is supported by a uniformly distributed reaction at each end. Points A and B are points of zero deflection. This problem was approximated for the program as shown in Fig. 12.

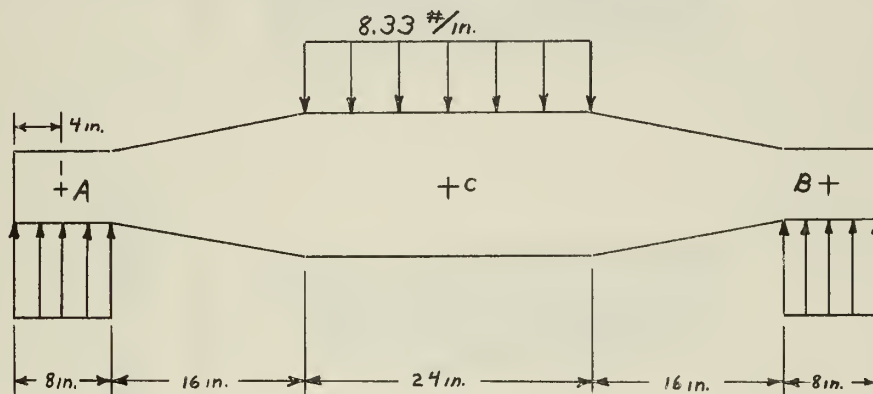


Fig. 12

Weight of shaft:

|           |                                 |
|-----------|---------------------------------|
| 0" - 8"   | -0.5315#/in.                    |
| 8" - 24"  | -0.5315 to -2.081#/in. (linear) |
| 24" - 36" | -2.081#/in.                     |

Bending compliance  $\left(\frac{1}{EI}\right)$ :

|           |                         |             |                                              |
|-----------|-------------------------|-------------|----------------------------------------------|
| 0" - 8"   | $0.1420 \times 10^{-6}$ | $1/\#-in^2$ |                                              |
| 8" - 12"  | $0.1420 \times 10^{-6}$ | to          | $0.0583 \times 10^{-6}$ $1/\#-in^2$ (linear) |
| 12" - 16" | $0.0583 \times 10^{-6}$ | to          | $0.0281 \times 10^{-6}$ $1/\#-in^2$ (linear) |
| 16" - 20" | $0.0281 \times 10^{-6}$ | to          | $0.0152 \times 10^{-6}$ $1/\#-in^2$ (linear) |
| 20" - 24" | $0.0152 \times 10^{-6}$ | to          | $0.0089 \times 10^{-6}$ $1/\#-in^2$ (linear) |
| 24" - 36" | $0.0089 \times 10^{-6}$ |             | $1/\#-in^2$                                  |

Shear compliance  $\left(\frac{K}{AG}\right) = 0.$

# SOLUTION TO TEST PROBLEM 5

| DISTANCE | LOAD  | SHEAR  | MOMENT  | SLOPE      | DEFLECTION |
|----------|-------|--------|---------|------------|------------|
| 00       | 18.22 | 00     | 00      | 00144040   | 00573400   |
| 4.00     | 18.22 | 72.96  | 145.75  | 001441280  | 00000000   |
| 8.00     | 18.22 | 142.93 | 583.37  | 001219554  | 00534754   |
| 12.00    | 18.22 | 142.93 | 1161.37 | 00088582   | 009533806  |
| 16.00    | 18.22 | 138.48 | 1724.71 | 00064188   | 01256778   |
| 20.00    | 18.22 | 132.49 | 2267.50 | 00047103   | 01477502   |
| 24.00    | 18.22 | 128.41 | 2782.94 | 00035015   | 01640427   |
| 28.00    | 18.22 | 124.00 | 3198.80 | 00024330   | 01759611   |
| 32.00    | 18.22 | 120.41 | 3532.09 | 00012461   | 01853491   |
| 36.00    | 18.22 | 117.64 | 3848.94 | 00000000   | 01853491   |
| 40.00    | 18.22 | 115.64 | 4198.50 | 000024330  | 01759611   |
| 44.00    | 18.22 | 113.28 | 4527.15 | 000035015  | 01640427   |
| 48.00    | 18.22 | 110.48 | 4827.71 | 000047103  | 01477502   |
| 52.00    | 18.22 | 107.64 | 5161.37 | 000064188  | 01256778   |
| 56.00    | 18.22 | 104.83 | 5483.75 | 000088582  | 009533806  |
| 60.00    | 18.22 | 102.00 | 5833.37 | 000088582  | 00534754   |
| 64.00    | 18.22 | 99.17  | 6145.75 | 0001219554 | 00000000   |
| 68.00    | 18.22 | 96.00  | 6483.75 | 000144040  | 00000000   |
| 72.00    | 18.22 | 92.93  | 6833.37 | 000144040  | 00573400   |

RY( 1) = .08  
 RY( 2) = .08  
 C1 = -.00144040  
 C2 = .00573400

### Test problem 6

This hyperstatic problem consists of a beam with a very unlikely configuration as shown in Fig. 13. The bending compliance varies as shown in Fig. 14. Shear deflection has been omitted, i.e.,  $\frac{K}{AG} = 0$ .

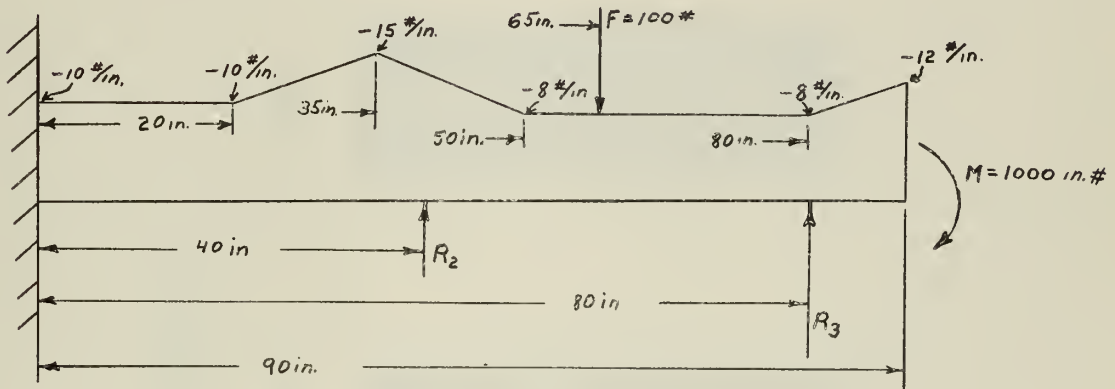


Fig. 13

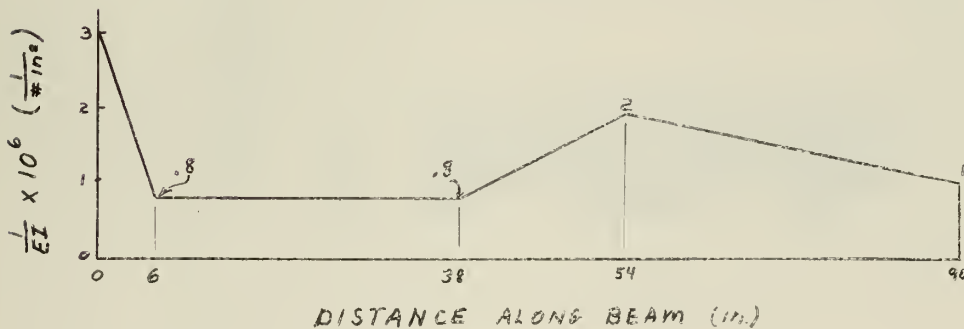


Fig. 14

### Test problem 7

This problem is identical to problem number 6 except the beam was turned end for end.

# SOLUTION TO TEST PROBLEM 6

| DISTANCE | LOAD   | SHEAR   | MOMENT   | SLOPE      | DEFLECTION |
|----------|--------|---------|----------|------------|------------|
| 00       | -10.00 | 218.61  | -1672.36 | -0.0000000 | 0.0000000  |
| 4.50     | -10.00 | 173.61  | -1789.89 | -0.0057642 | -0.0142942 |
| 9.50     | -10.00 | 128.61  | -109.89  | -0.0080462 | -0.0466376 |
| 13.50    | -10.00 | 83.61   | 367.59   | -0.0071632 | -0.0819457 |
| 18.50    | -10.00 | 38.61   | 642.57   | -0.0037739 | -0.1073300 |
| 22.50    | -10.83 | -7.56   | 714.18   | -0.0011200 | -0.1136361 |
| 27.50    | -12.83 | -59.43  | 565.01   | -0.0061222 | -0.0970579 |
| 33.50    | -13.83 | -118.66 | 168.94   | -0.0092768 | -0.0612746 |
| 40.50    | -14.43 | -183.18 | -509.21  | -0.0079686 | -0.0202241 |
| 45.50    | -12.43 | 208.95  | -1250.21 | -0.0001886 | -0.0036347 |
| 49.50    | -10.23 | 156.18  | -432.53  | -0.0054019 | -0.0139851 |
| 54.50    | -8.00  | 79.12   | 176.58   | -0.0061080 | -0.0412062 |
| 58.50    | -8.00  | 43.12   | 613.61   | -0.0045671 | -0.0657186 |
| 63.50    | -8.00  | 7.88    | 888.65   | -0.0018145 | -0.0804487 |
| 67.50    | -8.00  | -128.88 | 1001.68  | -0.0016365 | -0.0810019 |
| 72.50    | -8.00  | -164.88 | 702.72   | -0.0049528 | -0.0657555 |
| 76.50    | -8.40  | -200.88 | 41.25    | -0.0063414 | -0.0394510 |
| 81.50    | -10.20 | 91.95   | -781.42  | -0.0050588 | -0.0126894 |
| 85.50    | -10.20 | 49.00   | -1437.42 | -0.0065353 | -0.0012410 |
| 90.00    | -12.00 | 0.00    | -1000.00 | -0.0043562 | -0.0067038 |
|          |        |         |          | -0.0144534 | -0.0465377 |

RY( 1) = 218.61  
 RY( 2) = 452.51  
 RY( 3) = 328.88  
 RAMZ( 1) = -1672.36  
 C1 = .0000000  
 C2 = .0000000

# SOLUTION TO TEST PROBLEM 7

| DISTANCE | LOAD   | SHEAR   | MOMENT   | SLOPE     | DEFLECTION |
|----------|--------|---------|----------|-----------|------------|
| 00       | -12.00 | .00     | -1000.00 | .01445346 | -.04653768 |
| 4.50     | -10.20 | -49.95  | -1115.42 | .00435523 | -.00670377 |
| 9.00     | -8.40  | -91.80  | -1137.40 | .00065355 | -.00124103 |
| 13.50    | -8.00  | 200.88  | -781.25  | .00505885 | -.01268946 |
| 18.00    | -8.00  | 164.88  | -41.72   | .00634141 | -.03945104 |
| 22.50    | -8.00  | 128.88  | 702.65   | .00495289 | -.06575552 |
| 27.00    | -8.00  | -43.12  | 1001.61  | .00163650 | -.08100193 |
| 31.50    | -8.00  | -79.12  | 888.58   | .00181471 | -.08044870 |
| 36.00    | -8.00  | -115.18 | 613.53   | .00456710 | -.06571865 |
| 40.50    | -8.00  | -156.95 | 176.21   | .00610080 | -.04120624 |
| 45.00    | -10.20 | -208.66 | -432.21  | .00544019 | -.01398517 |
| 49.50    | -12.40 | -183.63 | -1250.94 | .00018866 | -.00036344 |
| 54.00    | -13.80 | 118.56  | -509.94  | .00796888 | -.02022419 |
| 58.50    | -12.80 | 159.43  | 168.01   | .00927688 | -.06127469 |
| 63.00    | -10.00 | -38.61  | 565.18   | .00612216 | -.09705796 |
| 67.50    | -10.00 | -38.61  | 714.57   | .00112008 | -.11363612 |
| 72.00    | -10.00 | -38.61  | 642.59   | .00377395 | -.10733003 |
| 76.50    | -10.00 | -128.61 | 367.59   | .00716320 | -.08194571 |
| 81.00    | -10.00 | -173.61 | -789.88  | .00804624 | -.04663761 |
| 85.50    | -10.00 | -218.61 | -1672.36 | .00576422 | -.01429420 |
| 90.00    | -10.00 | -218.61 | -1672.36 | .00000000 | -.00000000 |

RY( 1)= 328.88  
 RY( 2)= 452.51  
 RY( 3)= 218.61  
 RAMZ( 1)= 1672.36  
 C1=.01445346  
 C2=-.04653768

## APPENDIX E

### Subroutine GAUSS2

This routine is a modified form of the GAUSS2 used by C. B. Bailey /7/ in his program, LINEQN, for solving simultaneous linear equations by Gaussian elimination. Bailey was solving the matrix equation,  $Ax=B$ , with a possibility of 50 vectors B for each matrix A, but this program requires only one vector B with each matrix A. Also since the maximum number of equations in BEAM3 is 22, the size of matrix A was reduced from 100x101 to 22x23.

At each step of the elimination, the value of the diagonal element is compared to a defined "zero". If it is smaller than this quantity, the matrix is considered singular and an error output is returned to the calling program. This is the "matrix singular" test used by BEAM3.

## APPENDIX F

### Listing of program and subroutines

| Name            | Page |
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| LOAD            | 54   |
| SHEAR           | 55   |
| MOMENT          | 56   |
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| FTON and FT1N   | 64   |
| ENTON and ENT1N | 65   |
| DITON and DIT1N | 66   |
| GAUSS2          | 67   |

```

PROGRAM BEAM3
  DIMENSION W(1001),V(1001),AM(1001),Y1(1001),Y(1001),XX(1001),
1  A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10),
2  XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25),
3  AAA(22,23),XA(22),PDEFT(10),PSLP(10)
4  ,ITITLE(10),GA(25),GB(25),AG(25),BG(25)
  COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,AMZ,AE,BE,EA,EB,NOL,
1  NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
  READ 70,C
70 FORMAT (F20.0)
  READ 71,NOP,NOL,NOF,NOR,NOM,NORM,NOI,NOK
71 FORMAT(8I4)
  IF(NOL) 51,51,50
50 READ 72,(A(I),B(I),QA(I),QB(I),I=1,NOL)
72 FORMAT(4F20.0)
51 IF(NOF)53,53,52
52 READ 73,( XFO(N),FY(N),N=1,NOF)
73 FORMAT (2F20.0)
53 IF(NOR)55,55,54
54 READ 74,(XR(M),RY(M),PDEFT(M),M=1,NOR)
74 FORMAT (3F20.0)
55 IF(NOM)57,57,56
56 READ 73,(XM(K),AMZ(K),K=1,NOM)
57 IF(NORM)59,59,58
58 READ 74,(RXM(KA),RAMZ(KA),PSLP(KA),KA=1,NORM)
59 IF(NOI)61,61,60
60 READ 72,(AE(IA),BE(IA),EA(IA),EB(IA),IA=1,NOI)
61 IF(NOK)1,1,62
62 READ 72,(AG(IB),BG(IB),GA(IB),GB(IB),IB=1,NOK)
1 NOR1 = NOR+1
  C1=0.
  C2= 0.
  NORM2 =NORM + 2
  NORM3 =NORM + 3
  NOQ = NOR +NORM
  NOQ1 = NOQ + 1
  NOQ2 = NOQ + 2

```

```

NOQ3 = NOQ + 3
EPSX=10.**(-8)
DO 2 K=1,NCR
2  AA(1,K) = 1.
DO 3 K= NOR1,NOQ2
3  AA(1,K) = 0.
AAA=0.
IF(NOF)45,45,35
35 DO 4 N=1,NOF
4  AAA= AAA + FY(N)
45 IF(NOL)48,48,46
46 DO 5 I=1,NOL
5  AAA = AAA + ((QA(I)+QB(I))/2.)* (B(I)-A(I))
48 AA(1,NOQ3)=-AAA
DO 6 K =1,NCR
6  AA(2,K) = C-XR(K)
IF(NCRM)9,9,7
7  DO 8 K=NOR1,NOQ
8  AA(2,K)=1.
9  DO 10 K=NCQ1,NOQ2
10 AA(2,K) = 0.
CALL MOMENT (C,AAA)
AA(2,NOQ3) =-AAA
IF (NORM)17,17,11
11 DO 16 J=3,NORM2
DO 13 K=1,NCR
AA(J,K)=0.
DO 12 IA=1,NOI
AAA=EA(IA)*ENTON(RXM(J-2),AE(IA),XR(K),1) - EB(IA)*ENTON(RXM(J-2),
1BE(IA),XR(K),1) +((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*(ENT1N(RXM(J-2)
2,AE(IA),XR(K),1)-ENT1N(RXM(J-2),BE(IA),XR(K),1))
12 AA(J,K) = AA(J,K) + AAA
IF (NOK) 13,13,900
900 DO 910 IB=1,NOK
IF(NORM-1)902,901,902
902 IF(J-NORM2)901,904,901
901 X=RXM(J-2) -EPSX

```

```

AAA = GA(1B)* FT0N( X      ,AG(1B),XR(K),0) - GB(1B)* FT0N(X
1      ,BG(1B),XR(K),0) + ((GB(1B)-GA(1B))/(BG(1B)-AG(1B)))* ( FT1N(
2 X      ,AG(1B),XR(K),0)- FT1N( X      ,BG(1B),XR(K),0))
GO TO 910
904 X=RXM(J-2) +EPSX
AAA = GA(1B)* FT0N( X      ,AG(1B),XR(K),0) - GB(1B)* FT0N(X
1      ,BG(1B),XR(K),0) + ((GB(1B)-GA(1B))/(BG(1B)-AG(1B)))* ( FT1N(
2 X      ,AG(1B),XR(K),0)- FT1N( X      ,BG(1B),XR(K),0))
910 AA(J,K) =AA(J,K) -AAA
13 CONTINUE
IF(NOC-NOR1)155,135,135
135 DO 15 K=NOR1,NOC
AA(J,K)= 0.
DO 14 1A=1,NOI
AAA= EA(1A)*ENT0N(RXM(J-2),AE(1A),RXM(K-NOR),0) -EB(1A)* ENT0N
1 (RXM(J-2),BE(1A),RXM(K-NOR),0) + ((EB(1A)-EA(1A))/(BE(1A)-AE(1A))
2 ) *(ENT1N(RXM(J-2),AE(1A),RXM(K-NOR),0) - ENT1N(RXM(J-2),BE(1A),
3 RXM(K-NOR),0))
14 AA(J,K) = AA(J,K) + AAA
15 CONTINUE
155 AA(J,NOC1)=1.
AA(J,NOC2)= 0.
IF(J-NORM2)156,157,156
156 X=RXM(J-2) - EPSX
CALL SLOPE(X,AAA)
GO TO 16
157 X=RXM(J-2) + EPSX
CALL SLOPE(X,AAA)
16 AA(J,NOC3) =-AAA + PSLP(J-2)
17 DO 20 J=NORM3,NOC2
DO 18 K=1,NOR
AA(J,K) =0.
DO 181 1A=1,NOI
AAA = EA(1A)* D1TON(XR(J-NORM2),AE(1A),XR(K),1) -EB(1A)*D1TON
1 (XR(J-NORM2),BE(1A),XR(K),1) +((EB(1A)-EA(1A))/(BE(1A)-AE(1A)))
2 *(D1T1N(XR(J-NORM2),AE(1A),XR(K),1) - D1T1N(XR(J-NORM2),BE(1A),
3 XR(K),1))

```

```

181 AA(J,K) = AA(J,K) + AAA
    IF (NOK)18,18,920
920 DO 930 IB=1,NOK
    AAA =GA(IB)* ENT0N(XR(J-NORM2),AG(IB),XR(K),0) - GB(IB)*ENT0N
1 (XR(J-NORM2),BG(IB),XR(K),0) +((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
2 *(ENT1N(XR(J-NORM2),AG(IB),XR(K),0) -ENT1N(XR(J-NORM2),BG(IB),
3 XR(K),0))
930 AA(J,K) =AA(J,K) -AAA
18 CONTINUE
    IF(N0Q-N0R1)195,185,185
185 DO 19 K=N0R1,N0Q
    AA(J,K) = 0.
    DO 19 IA=1,N0I
    AAA = EA(IA)* DIT0N(XR(J-NORM2),AE(IA),RXM(K-N0R),0) - EB(IA)*
1 DIT0N(XR(J-NORM2),BE(IA),RXM(K-N0R),0) +((EB(IA)-EA(IA)) / (BE(IA
2 )- AE(IA))) * (DIT1N(XR(J-NORM2),AE(IA),RXM(K-N0R),0) - DIT1N
3 (XR(J-NORM2),BE(IA),RXM(K-N0R),0))
19 AA(J,K) = AA(J,K) + AAA
195 AA(J,N0Q1) =XR(J-NORM2)
    AA(J,N0Q2) = 1.
    CALL DEFLECT (XR(J-NORM2),AAA)
20 AA(J,N0Q3) =-AAA+ PDEFT(J-NORM2)
    CALL GAUSS2(N0Q2,.00000001,AA,XA,K1)
    GO TO (22,21),K1
21 WRITE OUTPUT TAPE 4,77
77 FORMAT (16H MATRIX SINGULAR)
STOP
22 DO 23 K=1,N0R
23 RY(K) = XA(K)
    IF(N0RM)26,26,24
24 DO 25 K=N0R1,N0Q
25 RAMZ(K-N0R) = XA(K)
26 C1 = XA(N0Q1)
    C2 = XA(N0Q2)
    X=0.
NOPE =NOP-1
FNOP = NOP-1

```

```

DO 27 J=1,NCPE
  CALL LOAD (X,W(J))
  CALL SHEAR (X,V(J))
  CALL MOMENT (X,AM(J))
  X1 =X+EPSX
  CALL SLOPE(X1,Y1(J))
  Y1(J)=Y1(J) +C1
  CALL DEFLECT (X,Y(J))
  Y(J)=Y(J)+C1*X+C2
  XX(J) = X
27  X = C/FNOP +X
  XX(NOP)=X
  X=X-3.*EPSX
  CALL LOAD(X,W(NOP))
  CALL SHEAR(X,V(NOP))
  CALL MOMENT(X,AM(NOP))
  CALL SLOPE(X,Y1(NOP))
  Y1(NOP)=Y1(NOP)+C1
  CALL DEFLECT(X,Y(NOP))
  Y(NOP)=Y(NOP)+C1*X +C2
  WRITE OUTPUT TAPE 4,76
76  FORMAT ( 3X,1HJ,7X,4HLOAD,10X,5HSHEAR,10X,6HMOMENT,8X,5HSLOPE,
  1 9X,1CHDEFLECTION,8X,1HX)
  WRITE OUTPUT TAPE 4,78,(J,W(J),V(J),AM(J),Y1(J),Y(J),XX(J),J=1,
  1NOP)
78  FORMAT(I4,6E15.8)
  IF (NCR)282,282,280
280 PRINT 281,(M,RY(M),M=1,NOR)
281 FORMAT ( 3HRY(,I2,2H)=,E20.8)
282 IF(NORM)288,288,283
283 PRINT 284,(KA,RAMZ(KA),KA=1,NORM)
284 FORMAT (5HRAMZ(,I2,2H)=,E20.8)
288 PRINT 286,C1
286 FORMAT(5X,3HC1=,E20.8)
  PRINT 287,C2
287 FORMAT(5X,3HC2=,E20.8)
285 CONTINUE

```

STOP

END

```

SUBROUTINE LOAD(X,W)
  DIMENSION A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10)
  1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
  2,GA(25),GB(25),AG(25),BG(25)
  COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
  1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
  EPSX=10.**(-8)
  W=0.
  1 DO 5 I=1,NOL
  2 IF(X+EPSX-B(I))3,3,5
  3 IF(X+EPSX-A(I))5,4,4
  4 WW=QA(I)+((QB(I)-QA(I))*(X-A(I)))/(B(I)-A(I))
  W=W+WW
  5 CONTINUE
  RETURN
  END

```

```

SUBROUTINE SHEAR(X,V)
  DIMENSION A(25),B(25),QA(25),QB(25),XFC(100),FY(100),XR(10),RY(10)
  1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
  2,GA(25),GB(25),AG(25),BG(25)
  COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
  1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
  EPSX=10.**(-8)
  V=0.
  IF(NOL)7,7,1
  1 DO 6 I=1,NOL
    IF(X-A(I))6,2,2
  2 IF(X+EPSX-B(I))4,4,3
  3 VV= QA(I)*(X-A(I))+((QB(I)-QA(I))*((X-A(I))**2))/(2.*(B(I)-A(I)))
    1-QB(I)*(X-B(I)) - ((QB(I)-QA(I))*((X-B(I))**2))/(2.*(B(I)-A(I)))
    GO TO 5
  4 VV=+QA(I)*(X-A(I))+((QB(I)-QA(I))*((X-A(I))**2))/(2.*(B(I)-A(I)))
  5 V=V+VV
  6 CONTINUE
  7 IF(NOF)10,10,75
  75 DO 9 N=1,NOF
    IF(X+EPSX-XFO(N))9,9,8
  8 VV=FY(N)
    V=V+VV
  9 CONTINUE
  10 IF(NOR)13,13,105
  105 DO 12 M=1,NOR
    IF(X+EPSX-XR(M))12,12,11
  11 VV= RY(M)
    V=V+VV
  12 CONTINUE
  13 CONTINUE
    RETURN
  END

```

```

SUBROUTINE MOMENT(X,AM)
  DIMENSION A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10)
  1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
  2,GA(25),GB(25),AG(25),BG(25)
  COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
  1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
  EPSX=10.**(-8)
  AM=0.
  IF(NOL)8,8,2
  2 DO 7 I=1,NOL
    IF(X-A(I))7,3,3
  3 IF(X+EPSX-B(I))5,5,4
  4 AMM=(+QA(I)/2.)*((X-A(I))**2)+((QB(I)-QA(I))*((X-A(I))**3))/(6.*(
  1B(I)-A(I)))-(QB(I)/2.)*((X-B(I))**2)-(QB(I)-QA(I))*((X-B(I))**3)
  2/(6.*(B(I)-A(I)))
  GO TO 6
  5 AMM=(+QA(I)/2.)*((X-A(I))**2)+((QB(I)-QA(I))*((X-A(I))**3))/(6.*(
  1(B(I)-A(I)))
  6 AM=AM+AMM
  7 CONTINUE
  8 IF(NOF)12,12,9
  9 DO 11 N=1,NOF
    IF(X+EPSX-XFO(N))11,11,10
  10 AMM= FY(N)*(X-XFO(N))
    AM=AM+AMM
  11 CONTINUE
  12 IF(NOR)16,16,13
  13 DO 15 M=1,NOR
    IF(X+EPSX-XR(M))15,15,14
  14 AMM= RY(M)*(X-XR(M))
    AM=AM+AMM
  15 CONTINUE
  16 IF(NOM)28,28,17
  17 DO 19 K=1,NOM
    IF(X+EPSX-XM(K))19,19,18
  18 AMM=AMZ(K)
    AM=AM+AMM

```

```
19 CONTINUE
28 IF(NORM) 32,32,29
29 DO 31 KA=1,NORM
    IF(X+EPSX-RXM(KA))31,31,30
30 AMM=RAMZ(KA)
    AM=AM+AMM
31 CONTINUE
32 CONTINUE
    RETURN
    END
```

```

SUBROUTINE SLOPE(X,Y1)
  DIMENSION A(25),B(25),QA(25),QB(25),XFC(100),FY(100),XR(10),RY(10)
  1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
  2,GA(25),GB(25),AG(25),BG(25)
  COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
  1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
  Y1=0.
  DO 17 IA=1,NOI
    IF(X-AE(IA))17,17,2
  2 IF(NOL)5,5,3
  3 DO 4 I=1,NOL
20 EYY = EA(IA)*((QA(I)/2.)*ENTON(X,AE(IA),A(I),2) +((QB(I)-QA(I))/
  1(6.*(B(I)-A(I))))*(ENTON(X,AE(IA),A(I),3)-ENTON(X,AE(IA),B(I),3))
  2 -(QB(I)/2.)*ENTON(X,AE(IA),B(I),2)) +((EB(IA)-EA(IA))/(BE(IA)-
  3 AE(IA)))*((QA(I)/2.)*ENTIN(X,AE(IA),A(I),2) +((QB(I)-QA(I)) /
  4(6.*(B(I)-A(I))))*(ENTIN(X,AE(IA),A(I),3)-ENTIN(X,AE(IA),B(I),3))
  5 -(QB(I)/2.)*ENTIN(X,AE(IA),B(I),2))
  Y1=Y1+EYY
21 EYY= -EB(IA)*((QA(I)/2.)*ENTON(X,BE(IA),A(I),2) +((QB(I)-QA(I))/
  1(6.*(B(I)-A(I))))*(ENTON(X,BE(IA),A(I),3)-ENTON(X,BE(IA),B(I),3))
  2 -(QB(I)/2.)*ENTON(X,BE(IA),B(I),2)) -((EB(IA)-EA(IA))/(BE(IA)-
  3 AE(IA)))*((QA(I)/2.)*ENTIN(X,BE(IA),A(I),2) +((QB(I)-QA(I)) /
  4(6.*(B(I)-A(I))))*(ENTIN(X,BE(IA),A(I),3)-ENTIN(X,BE(IA),B(I),3))
  5 -(QB(I)/2.)*ENTIN(X,BE(IA),B(I),2))
  Y1=Y1+EYY
  4 CONTINUE
  5 IF(NOF)8,8,6
  6 DO 7 N=1,NOF
22 EYY= EA(IA)*FY(N)*ENTON(X,AE(IA),XFO(N),1) -EB(IA)*FY(N)*ENTON(X,
  1BE(IA),XFO(N),1) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*FY(N)*
  2(ENTIN(X,AE(IA),XFO(N),1) - ENTIN(X,BE(IA),XFO(N),1))
  Y1=Y1+EYY
  7 CONTINUE
  8 IF(NOR)11,11,9
  9 DO 10 M=1,NOR
23 EYY= EA(IA)*RY(M)*ENTON(X,AE(IA),XR(M),1)-EB(IA)*RY(M)* ENTON(X,
  1BE(IA),XR(M),1) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*RY(M) *

```

```

2(ENT1N(X,AE(IA),XR(M),1)- ENT1N(X,BE(IA),XR(M),1))
Y1=Y1+EYY
10 CONTINUE
11 IF(NOM) 14,14,12
12 DO 13 K=1,NOM
24 EYY= EA(IA)*AMZ(K)*ENTON(X,AE(IA),XM(K),0) -EB(IA)*AMZ(K)*ENTON(X,
1BE(IA),XM(K),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))* AMZ(K) *
2 (ENT1N(X,AE(IA),XM(K),0)- ENT1N(X,BE(IA),XM(K),0))
Y1=Y1+EYY
13 CONTINUE
14 IF(NORM) 17,17,15
15 DO 16 KA= 1,NORM
25 EYY=EA(IA)*RAMZ(KA)*ENTON(X,AE(IA),RXM(KA),0)-EB(IA)*RAMZ(KA)*
1 ENTON(X,BE(IA),RXM(KA),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*
2RAMZ(KA)*(ENT1N(X,AE(IA),RXM(KA),0)- ENT1N(X,BE(IA),RXM(KA),0))
Y1=Y1+EYY
16 CONTINUE
17 CONTINUE
IF(NOK)595,595,500
500 DO 59C IB=1,NOK
IF (X -AG(IB))590,59C,509
509 IF (NCL)530,530,510
510 DO 52C I=1,NOL
26 EYY = GA(IB)*(QA(I)*FTON(X,AG(IB),A(I),1) + ((QB(I) -QA(I)) /
1 (2.*(B(I)-A(I))))*(FTON(X,AG(IB),A(I),2)-FTON(X,AG(IB),B(I),2))
2 -QB(I)*FTON(X,AG(IB),B(I),1)) + ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
3 *(QA(I)*FT1N(X,AG(IB),A(I),1) + ((QB(I)-QA(I))/(2.*(B(I)-A(I))))
4* (FT1N(X,AG(IB),A(I),2) -FT1N(X,AG(IB),B(I),2)) - QB(I)* FT1N
5 (X,AG(IB),B(I),2))
Y1 =Y1 -EYY
27 EYY =-GB(IB)*(QA(I)*FTON(X,BG(IB),A(I),1) + ((QB(I) -QA(I)) /
1 (2.*(B(I)-A(I))))*( FTON(X,BG(IB),A(I),2)-FTON(X,BG(IB),B(I),2))
2-QB(I)* FTON(X,BG(IB),B(I),1))- ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
3*(QA(I)* FT1N(X,BG(IB),A(I),1)+ ((QB(I)-QA(I))/(2.*(B(I)-A(I))))
4*( FT1N(X,BG(IB),A(I),2)- FT1N(X,BG(IB),B(I),2)) - QB(I)* FT1N
5 (X,BG(IB),B(I),2))
Y1 =Y1 -EYY

```

```

520 CONTINUE
530 IF(NOF)560,560,540
540 DO 55C N=1,NOF
  28 EYY=GA(IB)*FY(N)* FTON(X,AG(IB),XFO(N),0) -GB(IB)*FY(N)* FTON(X,
    1BG(IB),XFO(N),0) + ((GB(IB)-GA(IB)) / (BG(IB)-AG(IB))) * FY(N)*
    2( FT1N(X,AG(IB),XFO(N),0) - FT1N(X,BG(IB),XFO(N),0))
    Y1 =Y1 -EYY
550 CONTINUE
560 IF(NOR)590,590,570
570 DO 58C M=1,NOR
  29 EYY =GA(IB)*RY(M)* FTON(X,AG(IB),XR(M),0) -GB(IB)*RY(M)* FTON(X,
    1BG(IB),XR(M),0) + ((GB(IB)-GA(IB)) / (BG(IB)-AG(IB))) * RY(M) *
    2( FT1N(X,AG(IB),XR(M),0) - FT1N(X,BG(IB),XR(M),0))
    Y1 =Y1 -EYY
580 CONTINUE
590 CONTINUE
595 CONTINUE
  RETURN
  END

```

```

SUBROUTINE DEFLECT(X,Y)
  DIMENSION A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10)
  1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
  2,GA(25),GB(25),AG(25),BG(25)
  COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,AMZ,AE,BE,EA,EB,NOL,
  1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
  Y=0.
  DO 17 IA=1,NOI
    IF(X-AE(IA))17,17,2
  2 IF(NOL)5,5,3
  3 DO 4 I=1,NOL
20 EYY = EA(IA)*((QA(I)/2.)*DITON(X,AE(IA),A(I),2) + ((QB(I)-QA(I))/
  1(6.*(B(I)-A(I))))*(DITON(X,AE(IA),A(I),3)-DITON(X,AE(IA),B(I),3))
  2 -(QB(I)/2.)*DITON(X,AE(IA),B(I),2)) + ((EB(IA)-EA(IA))/(BE(IA)-
  3 AE(IA)))*((QA(I)/2.)*DITIN(X,AE(IA),A(I),2) + ((QB(I)-QA(I)) /
  4(6.*(B(I)-A(I))))*(DITIN(X,AE(IA),A(I),3)-DITIN(X,AE(IA),B(I),3))
  5 -(QB(I)/2.)*DITIN(X,AE(IA),B(I),2))
  Y=Y+EYY
21 EYY= -EB(IA)*((QA(I)/2.)*DITON(X,BE(IA),A(I),2) + ((QB(I)-QA(I))/
  1(6.*(B(I)-A(I))))*(DITON(X,BE(IA),A(I),3)-DITON(X,BE(IA),B(I),3))
  2 -(QB(I)/2.)*DITON(X,BE(IA),B(I),2)) - ((EB(IA)-EA(IA))/(BE(IA)-
  3 AE(IA)))*((QA(I)/2.)*DITIN(X,BE(IA),A(I),2) + ((QB(I)-QA(I)) /
  4(6.*(B(I)-A(I))))*(DITIN(X,BE(IA),A(I),3)-DITIN(X,BE(IA),B(I),3))
  5 -(QB(I)/2.)*DITIN(X,BE(IA),B(I),2))
  Y=Y+EYY
  4 CONTINUE
  5 IF(NOF)8,8,6
  6 DO 7 N=1,NOF
22 EYY= EA(IA)*FY(N)*DITON(X,AE(IA),XFO(N),1) -EB(IA)*FY(N)*DITON(X,
  1 BE(IA),XFO(N),1) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*FY(N)*
  2(DITIN(X,AE(IA),XFO(N),1) - DITIN(X,BE(IA),XFO(N),1))
  Y=Y+EYY
  7 CONTINUE
  8 IF(NOR)11,11,9
  9 DO 10 M=1,NOR
23 EYY= EA(IA)*RY(M)*DITON(X,AE(IA),XR(M),1)-EB(IA)*RY(M)* DITON(X,
  1BE(IA),XR(M),1) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*RY(M) *

```

```

2(DIT1N(X,AE(IA),XR(M),1)- DIT1N(X,BE(IA),XR(M),1))
Y=Y+EYY
10 CONTINUE
11 IF(NOM)14,14,12
12 DO 13 K=1,NOM
24 EYY=EA(IA)*AMZ(K)*DITON(X,AE(IA),XM(K),0) -EB(IA)*AMZ(K)*DITON(X,
1BE(IA),XM(K),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*AMZ(K)*
2(DIT1N(X,AE(IA),XM(K),0)- DIT1N(X,BE(IA),XM(K),0))
Y=Y+EYY
13 CONTINUE
14 IF(NORM) 17,17,15
15 DO 16 KA= 1,NORM
25 EYY=EA(IA)*RAMZ(KA)*DITON(X,AE(IA),RXM(KA),0)-EB(IA)*RAMZ(KA)*
1DITON(X,BE(IA),RXM(KA),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA))) *
2RAMZ(KA)*(DIT1N(X,AE(IA),RXM(KA),0)- DIT1N(X,BE(IA),RXM(KA),0))
Y=Y+EYY
16 CONTINUE
17 CONTINUE
IF(NOK)595,595,500
500 DO 59C IB=1,NOK
IF (X -AG(IB))590,590,509
509 IF (NCL)530,530,510
510 DO 52C I=1,NCL
26 EYY=GA(IB)*(QA(I)* ENTON(X,AG(IB),A(I),1) + ((QB(I) -QA(I) ) /
1(2.*(B(I)-A(I))))*(ENTON(X,AG(IB),A(I),2)-ENTON(X,AG(IB),B(I),2))
2-QB(I)*ENTON(X,AG(IB),B(I),1)) + ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
3*(QA(I)*ENT1N(X,AG(IB),A(I),1) + ((QB(I)-QA(I))/(2.*(B(I)-A(I))))
4*(ENT1N(X,AG(IB),A(I),2)-ENT1N(X,AG(IB),B(I),2)) - QB(I)*ENT1N
5(X,AG(IB),B(I),2))
Y=Y-EYY
27 EYY=-GB(IB)*(QA(I)*ENTON(X,BG(IB),A(I),1) +((QB(I)-QA(I))/
1(2.*(B(I)-A(I))))*(ENTON(X,BG(IB),A(I),2)-ENTON(X,BG(IB),B(I),2))
2-QB(I)*ENTON(X,BG(IB),B(I),1))- ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
3*(QA(I)*ENT1N(X,BG(IB),A(I),1)+ ((QB(I)-QA(I))/(2.*(B(I)-A(I))))
4*(ENT1N(X,BG(IB),A(I),2)-ENT1N(X,BG(IB),B(I),2)) - QB(I)* ENT1N
5(X,BG(IB),B(I),2))
Y=Y-EYY

```

```

520 CONTINUE
530 IF(NOF)560,560,540
540 DO 550 N=1,NOF
  28 EYY=GA(IB)*FY(N)*ENTON(X,AG(IB),XFO(N),0) -GB(IB)*FY(N)*ENTON(X,
    1BG(IB),XFO(N),0) + ((GB(IB)-GA(IB)) / (BG(IB)-AG(IB))) * FY(N)*
    2(ENT1N(X,AG(IB),XFO(N),0) - ENT1N(X,BG(IB),XFO(N),0))
    Y=Y-EYY
550 CONTINUE
560 IF(NOR)590,590,570
570 DO 580 M=1,NOR
  29 EYY =GA(IB)*RY(M)*ENTON(X,AG(IB),XR(M),0) -GB(IB)*RY(M)*ENTON(X,
    1BG(IB),XR(M),0) + ((GB(IB)-GA(IB)) / (BG(IB)-AG(IB))) * RY(M) *
    2(ENT1N(X,AG(IB),XR(M),0) - ENT1N(X,BG(IB),XR(M),0))
    Y=Y-EYY
580 CONTINUE
590 CONTINUE
595 CONTINUE
  RETURN
  END

```

```

FUNCTION FT0N(X,A,B,N)
  FT0N = 0.
  IF(X-A)3,3,1
1 IF(X-B)3,3,2
2 FT0N=(X-B)**N
3 RETURN
END

```

```

FUNCTION FT1N (X,A,B,N)
  FT1N =0.
  IF (X-A)3,3,1
1 IF (X-B)3,3,2
2 FT1N = (X-B)**(N+1) + (B-A)*((X-B)**N)
3 RETURN
END

```

```

FUNCTION ENT0N (X,A,B,N)
  FN=N
  ENT0N = 0.
  IF(X-A)5,5,1
1 IF(X-B)5,5,2
2 IF(A-B)3,3,4
3 ENT0N = (X-B)**(N+1)/(FN+1.)
  GO TO 5
4 ENT0N = (X-B)**(N+1)/(FN+1.) - (A-B)**(N+1)/(FN+1.)
5 RETURN
END

```

```

FUNCTION ENT1N(X,A,B,N)
  FN=N
  ENT1N = 0.
  IF(X-A)5,5,1
1 IF(X-B)5,5,2
2 IF(A-B)3,3,4
3 ENT1N = (X-B)**(N+2)/(FN+2.) + (B-A)*(X-B)**(N+1)/(FN+1.)
  GO TO 5
4 ENT1N = (X-B)**(N+2)/(FN+2.) + (B-A)*(X-B)**(N+1)/(FN+1.)
1 -(A-B)**(N+2)/(FN+2.) -((B-A)*(A-B)**(N+1))/(FN+1.)
5 RETURN
END

```

FUNCTION DITON(X,A,B,N)

FN=N

DITON =0.

IF(X-A)5,5,1

1 IF(X-B)5,5,2

2 IF(A-B)3,3,4

3 DITON = (X-B)\*\*(N+2)/((FN+1.)\*(FN+2.))

GO TO 5

4 DITON = (X-B)\*\*(N+2)/((FN+1.)\*(FN+2.)) - (X\*(A-B)\*\*(N+1))/(FN+1.)  
1-(A-B)\*\*(N+2)/((FN+1.)\*(FN+2.)) + (A\*(A-B)\*\*(N+1))/(FN+1.)

5 RETURN

END

FUNCTION DIT1N(X,A,B,N)

FN=N

DIT1N=0.

IF(X-A)5,5,1

1 IF(X-B)5,5,2

2 IF(A-B)3,3,4

3 DIT1N = (X-B)\*\*(N+3)/((FN+2.)\*(FN+3.)) + ((B-A)\*(X-B)\*\*(N+2))/  
1((FN+1.)\*(FN+2.))

GO TO 5

4 DIT1N = (X-B)\*\*(N+3)/((FN+2.)\*(FN+3.)) + ((B-A)\*(X-B)\*\*(N+2))/  
1((FN+1.)\*(FN+2.))- (X\*(A-B)\*\*(N+2))/(FN+2.)-(X\*(B-A)\*(A-B)\*\*(N+1))/  
2 (FN+1.) - (A-B)\*\*(N+3)/((FN+2.)\*(FN+3.)) - (B-A)\*(A-B)\*\*(N+2)/  
3 ((FN+1.)\*(FN+2.)) + (A\*(A-B)\*\*(N+2))/(FN+2.) + (A\*(B-A)\*(A-B)\*\*  
4 (N+1))/(FN+1.)

5 RETURN

END

```

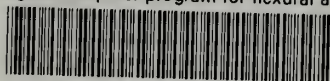
      SUBROUTINE GAUSS2(N,EP,A,X,KER)
      DIMENSION A(22,23) ,X(22)
      NPM=N+1
10   DO 34 L=1,N
      KP=0
      Z=0.0
      DO 12 K=L,N
      IF(Z-ABSF(A(K,L)))11,12,12
11   Z=ABSF(A(K,L))
      KP=K
12   CONTINUE
      IF(L-KP)13,20,20
13   DO 14 J=L,NPM
      Z=A(L,J)
      A(L,J)=A(KP,J)
14   A(KP,J)=Z
20   IF(ABSF(A(L,L))-EP)50,50,30
30   IF(L-N)31,40,40
31   LP1=L+1
      DO 34 K=LP1,N
      IF(A(K,L))32,34,32
32   RATIO=A(K,L)/A(L,L)
      DO 33 J=LP1,NPM
33   A(K,J)=A(K,J)-RATIO*A(L,J)
34   CONTINUE
40   DO 43 I=1,N
      II=N+1-I
      DO 43 J=1,1
      JPN=J+N
      S=0.0
      IF(II-N)41,43,43
41   IIP1=II+1
      DO 42 K=IIP1,N
42   S=S+A(II,K)*X(K)
43   X(II) =(A(II,JPN)-S)/A(II,II)
      KER=1
75  RETURN

```

50 KER=2  
RETURN.  
END

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