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MAY 1966

Prepared by A. W. RIHACZEK
Electronics Research Laboratory
Laboratories Division
Laboratory Operations
AEROSPACE CORPORATION

Prepared for BALLISTIC SYSTEMS AND SPACE SYSTEMS DIVISIONS
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

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FOREWORD

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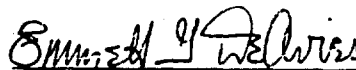


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ABSTRACT

When Doppler distortions of radar signals can be neglected, correlation or matched-filter processing is relatively simple. In those applications where high resolution requirements and high target speeds combine, the distortions in the waveform lead to severe processing problems. One way around these difficulties is the so-called Doppler-invariant waveform, which stays matched to the filter in the presence of an arbitrarily large Doppler effect. However, in many situations this waveform cannot be used. This paper extends the idea of Doppler invariance to only parts of the waveform, the complex modulation function, and the real envelope. We then obtain waveforms which simplify the Doppler search rather than eliminate it entirely, and hence are referred to as Doppler-tolerant. The addition of a constant-carrier term to the Doppler-invariant signal leads to the signal of which only the modulation function is Doppler-invariant. It permits independent measurement of range and range rate at the expense of having to search for the Doppler shift of the carrier. For applications of the principle to the envelope of a signal, the type of signal which is of particular interest is the pulse train. It is shown that a Doppler-tolerant pulse train can be designed such that it can be processed by a delay line with fixed taps even if the pulse spacing is significantly changed by the Doppler effect. This approach is useful for both coherent and incoherent pulse trains.

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I. INTRODUCTION

Correlation or matched-filter reception essentially amounts to a comparison of the noisy return signal with all waveforms that could have been received. This is a relatively simple task when the target is stationary, as the set of comparison waveforms then consists of delayed versions of the transmitted signal. The receiver performs a search in range by determining which of the delayed replicas correlates best with the received signal, and with a passive filter this search is automatic. The situation is much more complicated when the target is moving. Depending on the type of target motion and transmitted waveform, the received signal may vary considerably in shape; consequently, the receiver must search for the characteristics of the received signal as well as its delay. The practical problem of correlation reception consists in devising ways of performing this complicated search economically.

In order to simplify signal processing, it would be desirable to choose the radar signal so short that the target may be considered stationary over its duration. However, with modern requirements on target detectability this is seldom possible; and the same is true, of course, if targets are to be resolved on the basis of differences in their motion relative to the radar. Thus it is frequently necessary to make the signal so long that it becomes sensitive to target motion, even though it usually stays short enough to allow the assumption that the target moves with constant range rate over the duration of the signal. This is the domain of the radar theory dealing

with resolution in range and range rate. Conventional theory, however, makes the further assumption of a signal bandwidth so small that the effect of target range rate on the radar signal is a mere translation in the carrier frequency. This results in a relatively simple theory as well as a simple Doppler search procedure in the receiver. For larger bandwidths, on the other hand, the Doppler-distortions in the waveform become significant, leading to a receiver complexity which in some applications may be unmanageable, in particular with real-time processing. This paper deals with ways of overcoming the processing difficulties due to the Doppler-caused distortions in the radar waveform.

The assumptions of constant range rates yet large signal bandwidths are not made here merely in order to define a convenient problem; rather, they pertain to situations of great practical importance. Concerning more complicated types of target motion, since the measurability of the n th range derivative is roughly in the order of λ/T^n (where λ is the wavelength, T the coherent signal duration), effects of range acceleration will become noticeable only when T approaches the order of 100 milliseconds, even at relatively short waves. As for the second assumption, a "large" bandwidth in the present context means that $B \gtrsim 0.1c/R\dot{R}$, where c is the velocity of light and $\dot{R}$ the target range rate. Using typical numbers in this relation, it is easy to see that such bandwidths are indeed practical. Thus the problem of waveform distortion in the case of constant range rate is of considerable interest.

The problem of simplifying the Doppler search in the receiver has been studied before. In fact, in the narrow-band case, the linear FM or Chirp signal has the property of staying matched to a given filter even in the presence of a Doppler shift, and this may well be the most important reason for the popularity of this type of signal. The waveform that retains this property in the general case of large bandwidths is the so-called "Doppler-invariant" waveform, as described in Refs. 1 and 2. However, the Doppler-invariant signal does not solve all problems arising from distortions of the radar signal. First, many applications exist where the signal cannot be used; for example, those requiring coherent pulse trains. Second, the complete elimination of the Doppler search is paid for by a loss in the capability for independent measurements of range and range rate, with detrimental consequences on the resolution performance of the radar. In the following, we extend the idea of Doppler-invariance by applying it only to parts of the signal, the modulation function and the real envelope, rather than to the entire signal. This yields what might be called "Doppler-tolerant" waveforms, which eliminate the problem of waveform distortions by the Doppler effect, yet retain the capability for independent measurements of range and range rate.

II. DEGREES OF DOPPLER-TOLERANCE

As an introduction to the principle of Doppler-tolerance, it is best to start with a discussion of the waveform described in Refs. 1 and 2. It is a

segment of a constant-amplitude FM signal whose phase is changing logarithmically with time, written as

$$s(t) = \text{rect}\left(\frac{t}{T}\right) \cos \left[2\pi F \frac{1}{k} \ln(1 - kt) \right], \quad (1)$$

where F and k are constants. Since instantaneous frequency is defined as the time-derivative of the phase, the instantaneous frequency of the signal of Eq. (1) varies as

$$f_i = \frac{F}{1 - kt}. \quad (2)$$

The effect of range rate on the rf phase of the signal is an expansion or compression of the time scale, $t \rightarrow at$, where the factor a can be approximated as $a \approx 1 - 2\dot{R}/c$. Substituting in Eq. (1) the quantity at instead of t , the instantaneous frequency of the Doppler-transformed signal is seen to be

$$f'_i = \frac{aF}{1 - akt}. \quad (3)$$

As shown in Fig. 1, the plots of f_i and f'_i are congruent and can be brought to match by a simple translation in time. Hence, aside from an insignificant change in the amplitude, the effect of the Doppler on the output signal from a matched filter will be a translation in time. The filter stays matched in the presence of a Doppler shift, and in this sense it is justified to call the waveform Doppler-invariant.

To see more clearly the penalty paid for this property, we use Eqs. (2) and (3) and find the Doppler-caused time shift t_o from the relation $f_i(t) = f_i'(t - t_o)$ as

$$t_o = \frac{1}{k} \frac{a - 1}{a} \approx - \frac{1}{k} \frac{2\dot{R}}{c}. \quad (4)$$

Now the time shift t_o of the filter output signal simulates a change in target range by the amount $\Delta R = ct_o/2$. Since the time it takes the target actually to move through the distance ΔR is $\Delta R/\dot{R}$, that is,

$$\Delta t = - 1/k, \quad (5)$$

the time at which the output signal appears indicates the true target range Δt seconds after the signal impinges on the target rather than at the instant of signal impact. It follows that targets whose ranges and range rates at the time of signal impact are such that after (or before, depending on whether k is positive or negative) Δt seconds they would have the same range, within the resolution capability of the signal, cannot be resolved by the radar. This well-known result is a direct consequence of the Doppler-invariance property of the waveform. What happens is that the Doppler effect still causes a distortion of the waveform, but this "distortion" now is a simple time shift. Although it does not affect reception by the matched-filter, it may result in the coincidence of returns from targets that could be resolved with a different type of signal.

For these reasons and because of a variety of practical difficulties that arise in many applications, the Doppler-invariant waveform is not a

universal solution to the problem of processing signals returned from a moving target. However, instead of trying to design into a waveform an invariance to Doppler effects and eliminate the Doppler-search entirely, we may use the approach in order to solve the more limited problem of Doppler-caused waveform distortions. In other words, since the Doppler-shift of the carrier for narrow-band signals is manageable and the real difficulties in correlation processing appear when the signal bandwidth is so large that Doppler-distortions of the waveform become significant, it may be useful to design the signal such that only the distortions can be ignored. The receiver still would have to perform the search for the Doppler-shift of the carrier, but this relatively simple type of search would be adequate even if the signal bandwidth is very large. In the following, we discuss two types of signals falling into this class: the signal whose (complex) modulation function is Doppler-invariant, and the pulse train with Doppler-invariant real envelope. Since these signals themselves are not invariant in their form to the effects of Doppler, they will be referred to as Doppler-tolerant, indicating the fact that the Doppler effect can be ignored only to some degree.

III. WAVEFORM WITH DOPPLER-INVARIANT MODULATION FUNCTION

In accordance with its definition, the complex modulation function of a signal is obtained by translating the frequency spectrum down by an amount corresponding to the carrier frequency. Hence, in order to obtain a waveform

with Doppler-invariant modulation function, we simply take the Doppler-invariant function of Eq. (1) and translate it to some higher frequency. The desired signal takes the form

$$s(t) = \text{rect}\left(\frac{t}{T}\right) \cos \left[2\pi f_1 t - 2\pi F \frac{1}{K} \ln(1 - kt) \right], \quad (6)$$

where f_1 plays the part of the carrier frequency. Curves giving the instantaneous frequency of this signal and that of its Doppler-transformed version are shown in Fig. 2. The curves are still congruent and will match if translated in both time and frequency. Evidently, the necessary translation in frequency, that is, the value of the Doppler "mismatch", corresponds to the value of the Doppler shift of the carrier frequency f_1 .

Reception of the waveform of Eq. (6) is seen to require the usual search for the Doppler shift of the carrier, which may be performed, for example, by means of a filter bank whose filters have different center frequencies. Alternatively, the Doppler search can be implemented by adjusting the local oscillator frequency used in heterodyning the received signal. However, for the same reason that makes necessary the search for the carrier shift, the waveform also has an independent resolution capability in range rate. Targets with different range rates will be resolved regardless of what their ranges may be, with range rate resolution determined by signal duration and the wavelength corresponding to f_1 . We note from Eq. (4) that the Doppler-caused delay of the output signal depends only on waveform and range rate. Hence, all returns in a particular Doppler channel will be translated in time by the same amount, so that the true differential ranges for multiple returns

are correctly indicated. Since the range rate is also being measured, the error in absolute range can be corrected. This may be done permanently by inserting proper delays into the various Doppler channels, in which case the output signals from the receiver will give the true target distribution in range and range rate. The waveform thus eliminates the problem of Doppler-distortions in the large bandwidth case without sacrificing the capability for independent measurements of range and range rate.

Quantitatively, the decoupling of range and range rate caused by the translation of the Doppler-invariant signal in frequency can be seen by considering the shape of the central spike of the matched-filter response, or ambiguity function. The round-trip delay of the signal reflected from a target moving at constant range rate is

$$\Delta(t) = \tau_0 - \frac{v}{f_0} (t - \tau_0), \quad (7)$$

where τ_0 is the delay of the point at $t = 0$ on the waveform, and the relation between Doppler coefficient and range rate $\dot{R}$ is $v \approx -f_0 (2\dot{R}/c)$. Expressing in Eq. (7) v by $\dot{R}$ shows that the "carrier" frequency f_0 is immaterial. It is introduced here merely to conform with conventional notation. The received signal is $s_R(t) = s[t - \Delta(t)]$ or, in the complex notation,

$$v_R(t) = \text{rect} \frac{(t - \tau_R) \left(1 + \frac{v_R}{f_0}\right)}{T} e^{j2\pi \left\{ f_1(t - \tau_R) \left(1 + \frac{v_R}{f_0}\right) - \frac{F}{K} \ln \left[1 - k(t - \tau_R) \left(1 + \frac{v_R}{f_0}\right) \right] \right\}} \quad (8)$$

The receiver is assumed matched to this waveform, except for the Doppler coefficient which is taken as zero. From $\psi(\tau) = \int_{-\infty}^{\infty} \psi_R(t) \psi_F^*(t - \tau) dt$, the filter response becomes

$$\psi(\tau) = e^{j2\pi f_1 \tau} \int_{-\infty}^{\infty} \text{rect} \frac{t \left(1 + \frac{v}{f_0}\right)}{T} \text{rect} \frac{t - \tau}{T} e^{j2\pi f_1 t \frac{v}{f_0}} \\ \times e^{-j2\pi \frac{F}{k} \left\{ \ln \left[1 - kt \left(1 + \frac{v}{f_0}\right) \right] - \ln [1 - k(t - \tau)] \right\}} dt, \quad (9)$$

with v written instead of v_R and the time scale translated so that the peak appears at $\tau = 0$.

To simplify the calculation, we assume that the filter bandwidth is sufficiently large to accommodate the return signal even in the presence of the Doppler effect. This is the same as assuming that the reference signal with which the return is correlated has a larger duration than the transmitted signal, so that there is no energy loss due to the Doppler effect. In the well-known case of a Chirp signal, neglecting this energy loss would result in an ambiguity function whose ridge has constant height. For the purpose of analyzing the coupling between range and range rate, the drop in the height of the ridge is of no interest, both because it is not germane to the coupling effect and affects the results insignificantly. In Eq. (9), the rectangle functions then can be simply omitted and the limits of the integral taken as $(-T/2, T/2)$, corresponding to the duration of the transmitted signal. Since we are interested only in the envelope of the filter output signal, the factor in front of the integral can be neglected and the ambiguity function defined as

$$\chi(\tau, \nu) = \int_{-T/2}^{T/2} e^{-j2\pi \frac{F}{k} \ln \frac{1-kt(1+\frac{\nu}{f_0})}{1-k(t-\tau)}} e^{j2\pi f_1 \frac{\nu}{f_0} t} dt. \quad (10)$$

In studying the shape of the central spike of the ambiguity function, it is customary to expand $|\chi(\tau, \nu)|^2$ into a Taylor's series about the origin and drop terms of higher order than quadratic. The first derivatives of $|\chi(\tau, \nu)|^2$ are zero, so that

$$|\chi(\tau, \nu)|^2 = 1 + \frac{1}{2} \frac{\partial^2 |\chi_0|^2}{\partial \tau^2} \tau^2 + \frac{\partial^2 |\chi_0|^2}{\partial \tau \partial \nu} \tau \nu + \frac{1}{2} \frac{\partial^2 |\chi_0|^2}{\partial \nu^2} \nu^2 \quad (11)$$

in the parabolical approximation, where the index zero indicates that the derivatives are evaluated at $\tau = \nu = 0$. If a cut is taken through the response peak at a given level, for example, the half-power level, the contour for the cut is the ellipse

$$\frac{\partial^2 |\chi_0|^2}{\partial \tau^2} \tau^2 + 2 \frac{\partial^2 |\chi_0|^2}{\partial \tau \partial \nu} \tau \nu + \frac{\partial^2 |\chi_0|^2}{\partial \nu^2} \nu^2 = -1. \quad (12)$$

It is an easy matter to show that the ratio of overall width of the ellipse in either τ or ν domain to the width of the ellipse along the axis in the chosen domain is given by $1/\sqrt{1-Q}$, where the "correlation coefficient" Q is given as

$$Q = \frac{\left\{ \frac{\partial^2 |\chi_0|^2}{\partial \tau \partial \nu} \right\}^2}{\left\{ \frac{\partial^2 |\chi_0|^2}{\partial \tau^2} \right\} \left\{ \frac{\partial^2 |\chi_0|^2}{\partial \nu^2} \right\}}. \quad (13)$$

For the Doppler-invariant waveform, Q equals unity, corresponding to perfect coupling between range and range rate. The overall width of the "ellipse" obtained from the cut through the central spike is infinite, as the ridge has constant height under the earlier assumption. By studying the behavior of Q with different amounts of frequency translation, f_1 , we thus can determine how the coupling is changed in the Doppler-tolerant waveform. Noticing, from Fig. 1, that the instantaneous frequency of either waveform approaches infinity as $T/2$ approaches $1/k$, it is convenient to express the signal duration T in terms of unit $1/|k|$. The parameter $|k|T$ then is a measure for the curvature of the frequency function, with $(|k|T)_{\max} = 2$. We also introduce the signal bandwidth B , defined as the difference between maximum and minimum value of the instantaneous frequency,

$$B = \left| \frac{F}{1 - kT/2} - \frac{F}{1 + kT/2} \right| = F \frac{|k|T}{1 - (kT/2)^2}. \quad (14)$$

Calculating the second derivatives of $|\chi(\tau, \nu)|^2$ from Eq. (10), we obtain

$$Q = \frac{\left| \left[\ln \frac{1 - kT/2}{1 + kT/2} \right]^2 - \frac{(kT)^2}{1 - (kT/2)^2} \left[1 - \frac{|k|}{k} \frac{f_1}{B} \left(kT + \ln \frac{1 - kT/2}{1 + kT/2} \right) \right] \right|^2}{\left| \left[\ln \frac{1 - kT/2}{1 + kT/2} \right]^2 - \frac{(kT)^2}{1 - (kT/2)^2} \right|} \times \left[\left[\ln \frac{1 - kT/2}{1 + kT/2} \right]^2 - \frac{(kT)^2}{1 - (kT/2)^2} \left[1 - 2 \frac{|k|}{k} \frac{f_1}{B} \left(kT + \ln \frac{1 - kT/2}{1 + kT/2} \right) + \frac{1}{T^2} \left(\frac{f_1}{B} \right)^2 \frac{(kT)^4}{1 - (kT/2)^2} \right] \right]^{-1} \quad (15)$$

In Fig. 3, the coupling factor $1/\sqrt{1-Q}$ is shown as a function of the ratio f_1/B for several values of the parameter $|k|T$. When $|k|T$ is small, the signal approaches linear FM, so that large values of f_1/B are needed to produce a decoupling. Moreover, the degree of decoupling achievable for small values of $|k|T$ decreases, in agreement with the fact that the more linear the frequency sweep, the less the effect of any frequency translation. When the value of $|k|T$ is large, on the other hand, so that the instantaneous frequency function is strongly curved, a relatively small shift f_1 will result in almost complete decoupling of range and range rate.

IV. DOPPLER-TOLERANT PULSE TRAINS

Of perhaps greater practical importance is the application of the principle to pulse trains, both coherent and incoherent. For these types of signal, the time-bandwidth product becomes easily so large that the Doppler distortions are significant. Since such distortions primarily appear as a change in the pulse spacing of the return signal, they lead to the problem of pulse train processing when the pulse repetition period varies. We shall now describe the design principle by which these difficulties can be circumvented.

As a first step, consider the effect of range rate on a pulse train. Assume a radar transmitting a train of N identical pulses, each with envelope function $a(t)$ and phase modulation function $\phi(t)$. Introducing the complex modulation function $\mu(t) = a(t) e^{j\phi(t)}$, the pulse train can be written in the complex form

$$\psi(t) = e^{j(2\pi f_o t + \theta)} \sum_{n=1}^N \mu(t - \tau_n), \quad (16)$$

where f_o is the carrier frequency, θ the initial phase of the carrier, and the real signal is $s(t) = \text{Re}\{\psi(t)\}$. The pulse train thus is coherent and the discussion will be given for this type of pulse train; however, it will be evident that the property of concern applies equally to the incoherent pulse train. When the pulse train is reflected from a target moving at range rate $\dot{R}$, the round-trip travel time is given by Eq. (7).

Neglecting the changes in the amplitude, which are of no interest here, the form of the received pulse train is obtained by introducing into the transmitted signal the round-trip delay $\Delta(t)$. This will in general produce two types of effect, changes in the pulse spacing and distortions in the waveforms of the individual pulses. However, if the pulses are narrow-band, that is, if they meet the requirement $B_p \lesssim 0.1c/T_p \dot{R}$, the Doppler-distortions of the pulses themselves are negligible and the effect of range rate on the entire pulse train may be taken simply as a change in the pulse spacing. In what follows, we

assume that pulse distortions are insignificant. As an estimate for the seriousness of this restriction, even for range rates as high as 25,000 ft/sec, each pulse may have a time-bandwidth product up to about 4000 before the Doppler distortions must be taken into account. The restriction thus is not very serious. Note, however, that the principle of Doppler tolerance can also be used in those rare cases where each pulse of the pulse train has an extremely high time-bandwidth product. In order to prevent problems due to distortions of the individual pulses, we could use the waveform of the preceding section for the pulses. With the assumption of negligible pulse distortions, we are free to choose the pulse waveform arbitrarily.

In accordance with the above simplification, if the modulation function of the n th pulse is $\mu(t - \tau_n)$ upon transmission, its argument will change upon reception into

$$t - \tau_n - \Delta(t) = t - \tau_o + \frac{v}{c}(t - \tau_o) - \tau_n, \quad (17)$$

from Eq. (7). With the assumption of negligible pulse distortion, it is immaterial which point on the pulse is chosen to represent the delay for the entire pulse, so that we may choose the point $\mu(t=0)$ for convenience. For the received signal, the corresponding point is where the expression of Eq. (17) is zero, and this occurs at time

$$t \equiv \Delta_n = \tau_o + \frac{\tau_n}{1 + \frac{v}{c}} \approx \tau_o + \tau_n(1 - \frac{v}{c}). \quad (18)$$

If the origin of the time axis is put at the transmission of the first pulse, $\tau_1 = 0$, the quantity Δ_n simply gives the position in time of the n th received pulse.

All of what follows will be based on the approximation used in Eq. (18). The scope of the validity of the design principle developed below can thus be established by determining the error in the approximation. The largest error evidently occurs for the last pulse, for which τ_n is about equal to the duration T of the pulse train. This error is in the order of the largest term dropped from the series expansion of $\tau_n / (1 + v/f_0)$ and thus is about $T(v/f_0)^2$. In order for the error to be negligible, it must be small compared to the inverse bandwidth of the pulse train, or $T(v/f_0)^2 \ll 1/B$. Allowing an error of $0.1/B$ and substituting range rate for the Doppler coefficient, the limiting condition for the validity of this study can be expressed as

$$(TB)_{\text{pulse train}} \lesssim 0.03 (c/\dot{R})^2. \quad (19)$$

If the high range rate of 25,000 ft/sec is chosen as an example, the approximation holds for time-bandwidth products up to about 50×10^6 . For a bandwidth of 50 Mc, such a time-bandwidth product requires a pulse train duration of 1 sec. Since this is a signal duration where the effects of range acceleration may become significant, we see that still higher time-bandwidth products are of little interest for a theory concerned with range and range rate only.

Within the restriction of Eq. (19), the variable round-trip delay $\Delta(t)$ of the modulation function can be replaced by the approximate delay Δ_n of the

nth pulse, while the exact form of $\Delta(t)$ must be retained for the carrier term.

The received pulse train can then be written

$$\psi_R(t) = e^{j2\pi(f_o + \nu)(t - \tau_o)} + j\theta_R \sum_{n=1}^N \mu \left[t - \tau_o - \tau_n \left(1 - \frac{\nu}{f_o} \right) \right]. \quad (20)$$

The form of $\psi_R(t)$ used in the conventional ambiguity function differs from that of Eq. (20) only through omission of the term $\tau_n (1 - \nu/f_o)$ in the argument of the modulation function.

In implementing the idea of Doppler-tolerance, we now try to choose the pulse spacing in such a manner that transmitted and received pulse trains correlate in their envelopes even if the pulse train is returned from a moving target. In terms of a tapped delay line processor, a delay line with fixed taps should accommodate the received pulse train despite the fact that the spacing of the pulses depends on the unknown range rate. First, we design the pulse train such that transmitted and received versions match for a particular value of the range rate $\dot{R}_o$ or Doppler coefficient ν_o . Clearly, if the pulse spacing is chosen such that the Doppler effect changes each pulse interval into the preceding one, the envelopes of transmitted and received pulse trains will match if translated with respect to each other by the first pulse interval. Although for a train of N pulses only $(N-1)$ pulses will then correlate, the corresponding decrease in the signal-to-noise ratio will be of no consequence if N is large compared to unity. We now impose the additional requirement that when the target range rate is $2\dot{R}_o$, each pulse interval should be transformed by the Doppler into the second preceding one.

for a range rate $3\dot{R}_0$ into the third preceding one, and so forth. Correlation between received and transmitted pulse trains will then take place when both are translated with respect to each other (from the position for zero Doppler) by the first two pulse intervals, three pulse intervals, and so forth. By choosing the incremental range rate $\dot{R}_0$ sufficiently small, it must be possible to achieve practically continuous coverage over the entire range rate interval of interest.

Starting with the range rate $\dot{R}_0$ and Doppler coefficient v_0 , it is seen from the modulation function of Eq. (20) that the compression factor for the pulse intervals is $(1 - v_0/f_0)$. Hence, the above transformation requirement for the pulse interval $(\tau_n - \tau_{n-1})$ can be expressed as

$$(\tau_n - \tau_{n-1}) \left(1 - \frac{v_0}{f_0}\right) = \tau_{n-1} - \tau_{n-2}, \quad (21)$$

where $(\tau_{n-1} - \tau_{n-2})$ is the preceding pulse interval. With the same approximation as used in Eq. (18), and replacing n by $(n + 1)$ in Eq. (21), we also have

$$(\tau_n - \tau_{n-1}) \left(1 + \frac{v_0}{f_0}\right) = \tau_{n+1} - \tau_n. \quad (22)$$

This relation shows that when the target is receding rather than approaching, so that the sign of v_0 is reversed, each pulse interval is transformed into the succeeding (rather than preceding) interval. Hence, when a pulse train has the desired Doppler-invariance of the envelope, it will have it for both positive

and negative Doppler. As an illustration, Fig. 4 shows a pulse train with increasing repetition intervals and, below, the result of transforming each interval into the succeeding one. Both pulse trains will evidently correlate in their envelopes if the lower pulse train is translated in time as indicated, except for the loss of one pulse on the fringe.

It is easy to verify the Doppler-matching property for Doppler coefficients that are a multiple of ν_o . With $k\nu_o$ instead of ν_o in Eq. (21), successive application of the formula yields

$$(\tau_n - \tau_{n-1}) \left(1 - k \frac{\nu_o}{f_o}\right) \approx (\tau_n - \tau_{n-1}) \left(1 - \frac{\nu_o}{f_o}\right)^k = \tau_{n-k} - \tau_{n-k-1}. \quad (23)$$

Accepting the above approximation for the moment, the relation shows that a Doppler shift $k\nu_o$ will indeed transform each interval into one that is removed from it by k intervals, in agreement with the design goal. Thus we still obtain proper correlation, except that the signal-to-noise ratio degrades by the factor $N/(N-k)$, since k pulses will be lost for the correlation process. In principle, N can always be chosen large enough to make this loss acceptable. As for the approximation in Eq. (23), it is of the same type as introduced earlier and hence subject to the condition of Eq. (19). This can be shown by taking the error in Eq. (23), which is in the order of the third term in the series expansion of $(1 - \nu_o/f_o)^k$, i. e.,

$$\epsilon \approx (\tau_n - \tau_{n-1}) \frac{k-1}{2k} \left(\frac{k\nu_o}{f_o}\right)^2. \quad (24)$$

and summing it over all n . Since the maximum value of the Doppler shift, kv_0 , takes the place of the maximum value of v in Eq. (18), the summation leads to the requirement of (19) if the allowable error again is set equal to $0.1/B$.

In determining how small v_0 should be taken so that the coverage in Doppler may be considered continuous for practical purposes, it appears reasonable to select the value of v_0 such that the output signal from the correlator would drop by 3 db for this Doppler shift, if it were not for the Doppler-tolerance of the pulse train. This will ensure that the drop for Doppler shifts that are not multiples of v_0 will be negligible. Now, the range rate mismatch for which the correlator output drops by 3 db is in the order of $\dot{R} = 0.1c/TB$. When $\dot{R}$ is expressed in terms of the Doppler coefficient, the minimum incremental Doppler coefficient becomes

$$|v_0|_{\min} = \frac{0.2f_0}{TB} \quad (25)$$

The number of pulses required in order to keep the degradation in signal-to-noise ratio tolerable even for large Doppler shifts can be found by substituting in the loss factor $N/(N-k)$ for k from $kv_0 = v_{\max}$. This gives

$$L = N/(N - v_{\max}/v_0) = N/(N - \dot{R}_{\max}/\dot{R}_0) \quad (26)$$

For a maximum allowable loss of 3 db we find the required number of pulses to be

$$N = 2\dot{R}_{\max}/\dot{R}_0 = 20TB\dot{R}_{\max}/c \quad (27)$$

In deriving this relation, we have used Eq. (25) after expressing the Doppler coefficient through the range rate. As an example, for a time-bandwidth product of 10^6 and a maximum range rate (mismatch) of 5,000 ft/sec, the pulse train would have to use 100 pulses.

The law that governs the increase in the pulse interval of the Doppler-tolerant pulse train can be found by successive application of Eq. (21), which gives

$$\tau_n - \tau_{n-1} = \left(1 - \frac{v_o}{f_o}\right)^{-(n-2)} (\tau_2 - \tau_1) \quad (28)$$

$$\approx \left[1 + (n-2) \frac{v_o}{f_o}\right] T_1, \quad (28a)$$

where $T_1 \equiv \tau_2 - \tau_1$ is the first pulse interval. The pulse interval is seen to increase linearly from pulse to pulse, as would be expected from the analogy to the Doppler-invariant waveform, where the period of the rf also increases linearly with time. Instead of requiring that the Doppler effect transform each interval into the preceding one when the target is approaching, we could also require that this transformation take place for a receding target. This would merely change the sign of v_o in Eq. (28a) and thus would lead to a pulse train with linearly decreasing, rather than increasing, pulse spacing. Note that the total increase in the pulse interval over the duration of the pulse train is quite small, so that the Doppler-tolerant pulse train almost has the appearance of the uniform pulse train.

V. MEASUREMENT PROPERTIES OF THE DOPPLER-TOLERANT PULSE TRAIN

It appears desirable to substantiate the preceding reasoning and verify the fact that the Doppler-tolerant pulse train has the intended properties by considering the behavior of its ambiguity function. (This concerns, of course, the measurement properties of the coherent pulse train with Doppler-tolerance, whereas for incoherent pulse trains the measurement properties depend only on the individual pulses, and the Doppler tolerance is merely a means of simplifying the processor). From Eq. (20), we write the received pulse train

$$\psi_R(t) = e^{j2\pi(f_0 + \nu_R)(t - \tau_R) + j\theta_R} \sum_{n=1}^N \mu \left[t - \tau_R - \tau_n \left(1 - \frac{\nu_R}{f_0} \right) \right], \quad (29)$$

using the subscript R for denoting the particular values of delay and Doppler of the received signal. The receiver, on the other hand, is matched to different values of delay and Doppler, and entirely ignores the Doppler term in the modulation function. Thus it may be assumed matched to the signal

$$\psi_F(t) = e^{j2\pi(f_0 + \nu_F)(t - \tau_F) + j\theta_F} \sum_{n=1}^N \mu(t - \tau_F - \tau_n). \quad (30)$$

With these expressions, the receiver response $\psi(\tau) = \int_{-\infty}^{\infty} \psi_F(t) \psi_R^*(t - \tau) dt$

becomes

$$\psi(\tau) = e^{j2\pi(f_0 + \nu_R)(\tau + \tau_R - \tau_F)} e^{j(\theta_F - \theta_R)} \times \int_{-\infty}^{\infty} e^{j2\pi(\nu_F - \nu_R)t} \sum_{n=1}^N \sum_{m=1}^N \mu(t - \tau_n) \mu^* \left[t - \tau - \tau_R + \tau_F - \tau_m \left(1 - \frac{\nu_R}{f_0} \right) \right] dt. \quad (31)$$

The interesting quantity is the integral, that is, the complex modulation function of $\psi(\tau)$. As usual, we introduce the Doppler mismatch of the carrier, $\nu \equiv \nu_F - \nu_R$, and translate the time scale by the "mismatch" in delay, $(\tau_F - \tau_R)$. The complex modulation function, or ambiguity function, then can be written as

$$\chi(\tau, \nu, \nu_R) \equiv \sum_{n=1}^N \sum_{m=1}^N \int_{-\infty}^{\infty} e^{j2\pi \nu t} \mu(t - \tau_n) \mu^* \left[t - \tau - \tau_m \left(1 - \frac{\nu_R}{f_0} \right) \right] dt. \quad (32)$$

If it were not for the Doppler-mismatch of the envelope, ν_R , the above would be the conventional ambiguity function of a pulse train under the assumption that the Doppler effect merely causes a translation of the carrier frequency. The Doppler-tolerant pulse train is designed to eliminate the dependence of the ambiguity function on the Doppler mismatch of the envelope, as will be verified subsequently.

Since the pulse spacing is almost that of the uniform pulse train, the terms of the double sum having constant difference $(m-n)$ will be nearly coincident in time. Thus we collect these terms in the expansion

$$\sum_{n=1}^N \sum_{m=1}^N = \sum_{n=1}^N \left| \begin{array}{c} N \\ m=n \end{array} \right| + \sum_{r=1}^{N-1} \sum_{n=1}^{N-r} \left| \begin{array}{c} N-r \\ m=n+r \end{array} \right| + \sum_{r=1}^{N-1} \sum_{m=1}^{N-r} \left| \begin{array}{c} N-r \\ n=m+r \end{array} \right|, \quad (33)$$

which follows immediately from the correlation process of the uniform pulse train. The ambiguity function then becomes

$$\begin{aligned} \chi(\tau, \nu, \nu_R) = & \sum_{n=1}^N e^{j2\pi\nu\tau_n} \chi_{n,n} + \sum_{r=1}^{N-1} \sum_{n=1}^{N-r} e^{j2\pi\nu\tau_n} \chi_{n,n+r} \\ & + \sum_{r=1}^{N-1} \sum_{n=1}^{N-r} e^{j2\pi\nu\tau_{n+r}} \chi_{n+r,n}, \end{aligned} \quad (34)$$

$$\chi_{n,n} \equiv \int_{-\infty}^{\infty} e^{j2\pi\nu t} u(t) u^* \left[t - \tau + \tau_n - \tau_n \left(1 - \frac{\nu_R}{f_0} \right) \right] dt, \quad (35a)$$

$$\chi_{n,n+r} \equiv \int_{-\infty}^{\infty} e^{j2\pi\nu t} u(t) u^* \left[t - \tau + \tau_n - \tau_{n+r} \left(1 - \frac{\nu_R}{f_0} \right) \right] dt, \quad (35b)$$

$$\chi_{n+r,n} \equiv \int_{-\infty}^{\infty} e^{j2\pi\nu t} u(t) u^* \left[t - \tau + \tau_{n+r} - \tau_n \left(1 - \frac{\nu_R}{f_0} \right) \right] dt. \quad (35c)$$

As in the case of the uniform pulse train, we find a central peak that consists of the superposition of the N terms $\chi_{n,n}$, and subsidiary peaks for constant τ , made up of the $(N-r)$ terms $\chi_{n,n+r}$ or $\chi_{n+r,n}$. The difference to the uniform

pulse train lies in the translation along the delay axis of each such term, as implied by the quantity v_R in the argument of the modulation function.

It is these translations along the delay axis that are of interest. By repeated application of Eq. (28a), the position τ_n of the n th pulse in the train is

$$\tau_n = (n-1)T_1 + \frac{1}{2}(n-1)(n-2) \frac{v_0}{f_0} T_1. \quad (36)$$

We now consider the r th subsidiary spike of the ambiguity function, which is displaced along the delay axis from the central spike by about r pulse intervals. As seen from Eq. (35b), the displacement along the delay axis of a particular component $\chi_{n,n+r}$ is

$$\Delta\tau_n = \tau_{n+r} \left(1 - \frac{v_R}{f_0}\right) - \tau_n. \quad (37)$$

When the Doppler coefficient is a multiple of v_0 , $v_R = kv_0$, use of Eq. (36) gives this displacement as

$$\Delta\tau_n = T_1 \left[n(r-k) \frac{v_0}{f_0} + r + \left(k - kr + \frac{1}{2}r^2 - \frac{3}{2}r \right) \frac{v_0}{f_0} \right], \quad (38)$$

where the second order terms were dropped because of Eq. (19). As shown by Eq. (38), the displacement depends on n , so that the components $\chi_{n,n+r}$ which make up the r th spike are translated with respect to each other and hence add up to a smaller peak than would be the case without these translations. However, when $r = k$ the dependence of $\Delta\tau_n$ on n disappears and all

components are centered on the same point of the delay axis. This is the result of the design for Doppler tolerance. For a Doppler coefficient kv_0 , all components contributing to the k th subsidiary spike behave just as if no Doppler mismatch of the envelope existed.

To see the smearing of the spikes for which $r \neq k$, we take $k = r - p$ in Eq. (38) and obtain

$$\Delta\tau_n = T_1 \left\{ r + \frac{v_0}{f_0} \left[p(r-1) - \frac{r}{2}(r+1) \right] \right\} + T_1 \frac{v_0}{f_0} np. \quad (39)$$

The first term in the above expression describes the displacement common to all n components of the spike, whereas the second term describes the relative displacement of the n components and, hence, the spread of the spike. Since the number of contributing components is $(N-r)$, their over-all spread is

$$\Delta T = (N-r) T_1 \frac{v_0}{f_0} p = p \frac{(N-r)T_1}{2T} \frac{1}{B}, \quad (40)$$

with the use of Eq. (25). A significant drop in the amplitude of the spike occurs when p is large enough to make the factor in front of $1/B$ large compared to unity. This behavior is most easily seen on the delay axis, that is, when we assume that the carrier is Doppler-matched even though the envelope is not. Figure 5 shows, as an illustration, the ambiguity function of the uniform pulse train on the delay axis. The dashed line crudely indicates how these spikes might change for the Doppler-tolerant pulse train if the Doppler-mismatch of the envelope is $4v_0$.

The degradation of the signal-to-noise ratio by the factor $N/(N-k)$ for a Doppler mismatch $k\nu_0$ is evident from the fact that only $(N-k)$ components superpose to form the k th spike of the ambiguity function. In some situations, it may be preferable to accept a uniform loss throughout the entire Doppler interval. To this end, we may design the receiver for a Doppler-tolerant pulse train containing $N+M$ pulses, yet transmit only the center part of the pulse train containing N pulses.

Having considered the over-all behavior of the ambiguity function, we turn our attention to the fine structure in the Doppler domain for the particular spike for which the Doppler-match in the envelope occurs. As was found earlier, the relative shift in time disappears for the components making up this spike, and all the components have the same form. Hence, from Eq. (34), this spike may be written

$$\chi_k(\tau, \nu, \nu_R) = \chi_{n, n+k} \sum_{n=1}^{N-k} e^{j2\pi\nu\tau_n} \quad (41)$$

The ambiguity function for the signal pulse is seen to act as an envelope, with the fine structure generated by the sum in Eq. (41). Since the dependence of the fine structure on k is merely through the number of components, $(N-k)$, that contribute to the k th spike, we might as well study the behavior of the central spike, $k=0$. From Eq. (36), we then have

$$\sum_{n=1}^N e^{j2\pi\nu T_1 n} = e^{-j2\pi\nu \left(1 - \frac{\nu_0}{f_0}\right) T_1} \sum_{n=1}^N e^{j2\pi\nu T_1 n \left(1 - \frac{3}{2} \frac{\nu_0}{f_0}\right)} e^{j2\pi\nu T_1 n^2 \frac{1}{2} \frac{\nu_0}{f_0}}. \quad (42)$$

Since the interesting quantity is the envelope of the ambiguity function, the term in front of the summation signal may be ignored. The first term of the sum is the same as would be obtained for a uniform pulse train of repetition period $T_1 \left(1 - \frac{3}{2} \frac{\nu_0}{f_0}\right)$ and produces the well-known fine structure of the $\sin Nx / \sin x$ type (see, for example, Ref. 3). When N is relatively small, the effect of the term quadratic in n is negligible since $\nu_0 / f_0 \ll 1$, and the fine structure is that of the uniform pulse train. For large N , however, the term with n^2 destroys the linear phase progression in the sum, which results in a smearing of the ambiguous spikes in the Doppler fine structure.

It is interesting to estimate how large N must be before the smearing becomes pronounced. If the maximum phase shift due to the quadratic term is smaller than about $\pi/2$, its effect will evidently be negligible. This maximum phase shift occurs for $n = N$ and, from Eq. (42), we obtain the condition $\nu T_1 N^2 \frac{1}{2} \frac{\nu_0}{f_0} \lesssim \frac{1}{4}$. With Eq. (25), this becomes

$$\nu \lesssim \frac{TB}{N^2} \frac{1}{T_1}. \quad (43)$$

Since the m th ambiguous spike of the fine structure appears at $\nu \approx m/T_1$, the Doppler ambiguities up to the m th spike will remain unaffected by the non-uniform spacing of the Doppler-tolerant pulse train if

$$N \lesssim \sqrt{\frac{TB}{m}} \quad (44)$$

This means that a smearing of even the first ambiguous spike, for which $m = 1$, will take place only when the number of pulses in the pulse train is relatively large. In summary, then, we find that the Doppler-tolerant pulse train has essentially the same measurement and resolution properties as the uniform pulse train. This is intuitively clear from the fact that the Doppler-tolerant pulse train may be considered as a result of a slight staggering of the uniform pulse train. As is well known (Ref. 3), such a small amount of staggering does not change the properties of the pulse train to any significant degree. The advantage offered by the Doppler-tolerant pulse train is that it can be processed with a delay line having fixed taps, even if the pulse spacing is changed by the Doppler effect.

VI. CONCLUSIONS

It was shown that the principle of Doppler-invariance can be extended from the entire signal waveform to some of its characteristics only, such as the modulation function or the real envelope. These signals do not completely eliminate the Doppler search but merely simplify it, and for this reason are referred to as Doppler-tolerant rather than Doppler-invariant. The waveform whose (complex) modulation function is Doppler-invariant, and which thus is tolerant to distortions of the modulation function, is obtained from the known Doppler-invariant waveform by simply adding a constant-carrier term. Such a modification removes the coupling between range and range rate and allows

independent measurement of the two parameters, but at the expense of requiring the usual search for the Doppler shift of the carrier. The property of Doppler-invariance in the envelope of a signal appears to be most interesting for the pulse train, where it can be utilized primarily to circumvent the problems due to Doppler-caused changes in the pulse spacing. We have found that the corresponding Doppler-tolerant pulse train essentially has the measurement properties of the uniform pulse train. Waveforms of this type are of considerable practical interest for high-resolution radar and rapidly moving targets, where Doppler distortions in the modulation function would otherwise lead to severe processing problems.

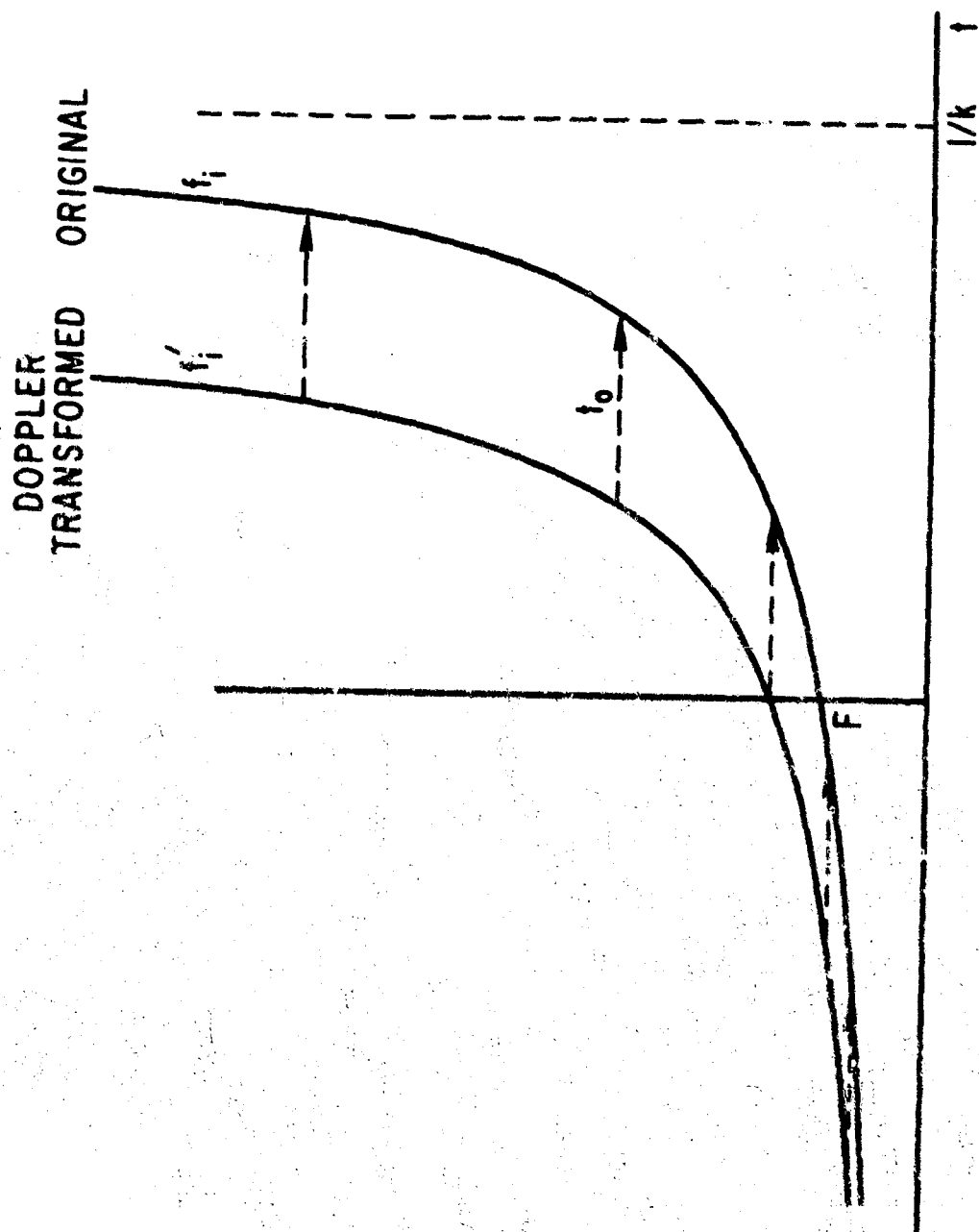


Fig. 1. Instantaneous Frequency for the Doppler-Invariant Waveform

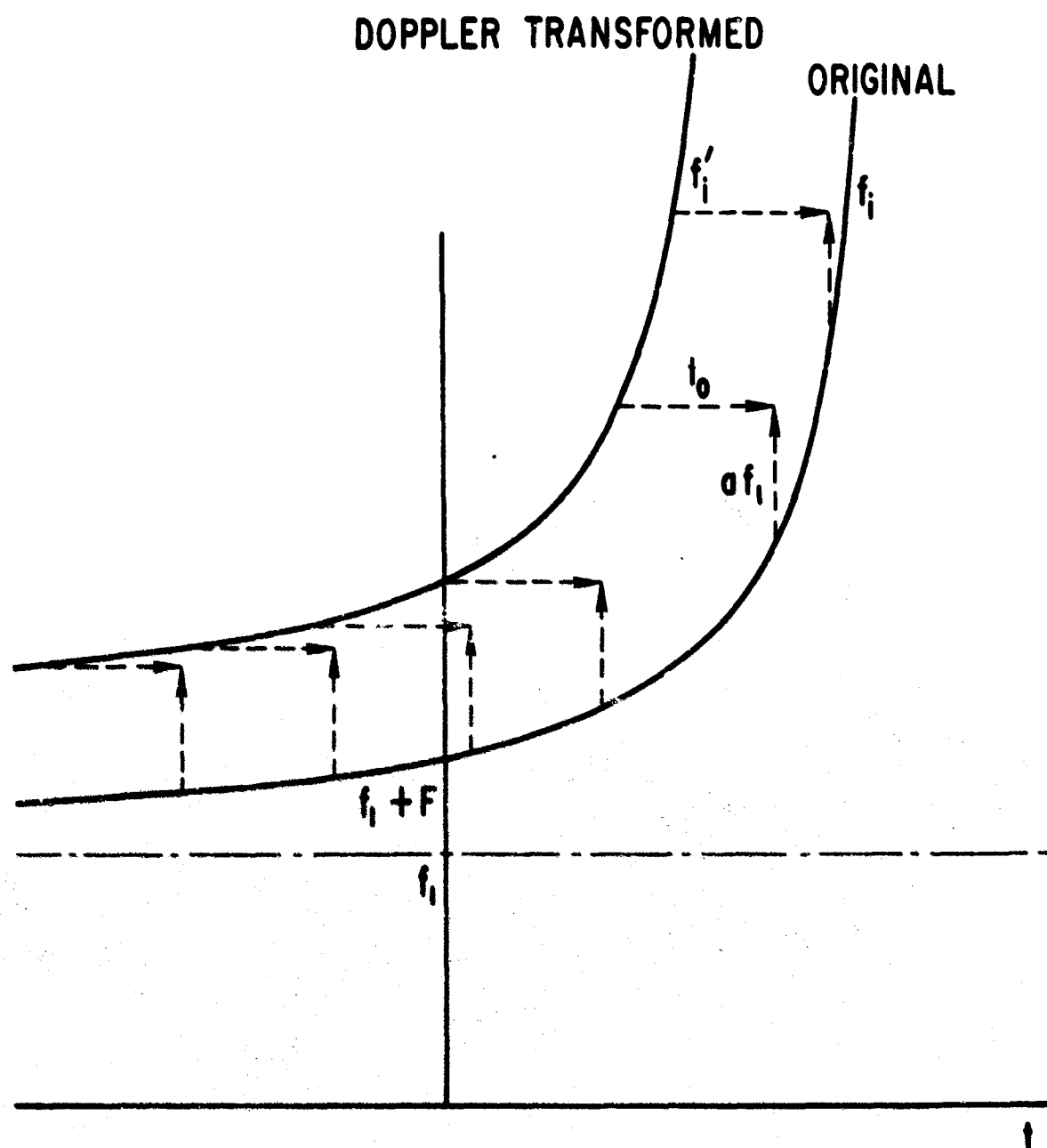


Fig. 2. Instantaneous Frequency for the Waveform with Doppler-Invariant Modulation Function

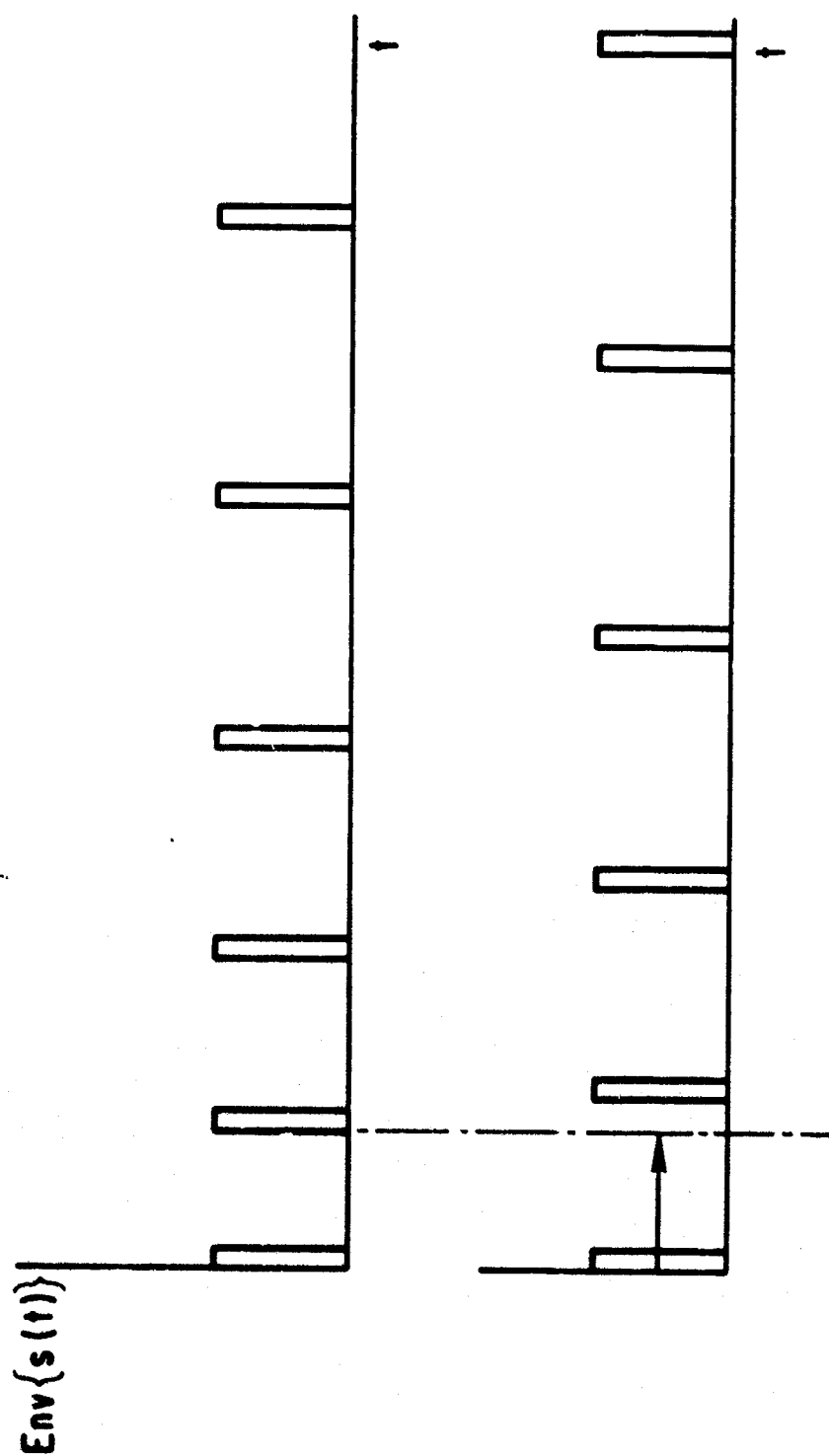


Fig. 3. Range and Range Rate Coupling of the Waveform of Fig. 2 as a Function of the Frequency Translation for Various Curvatures of the FM Sweep

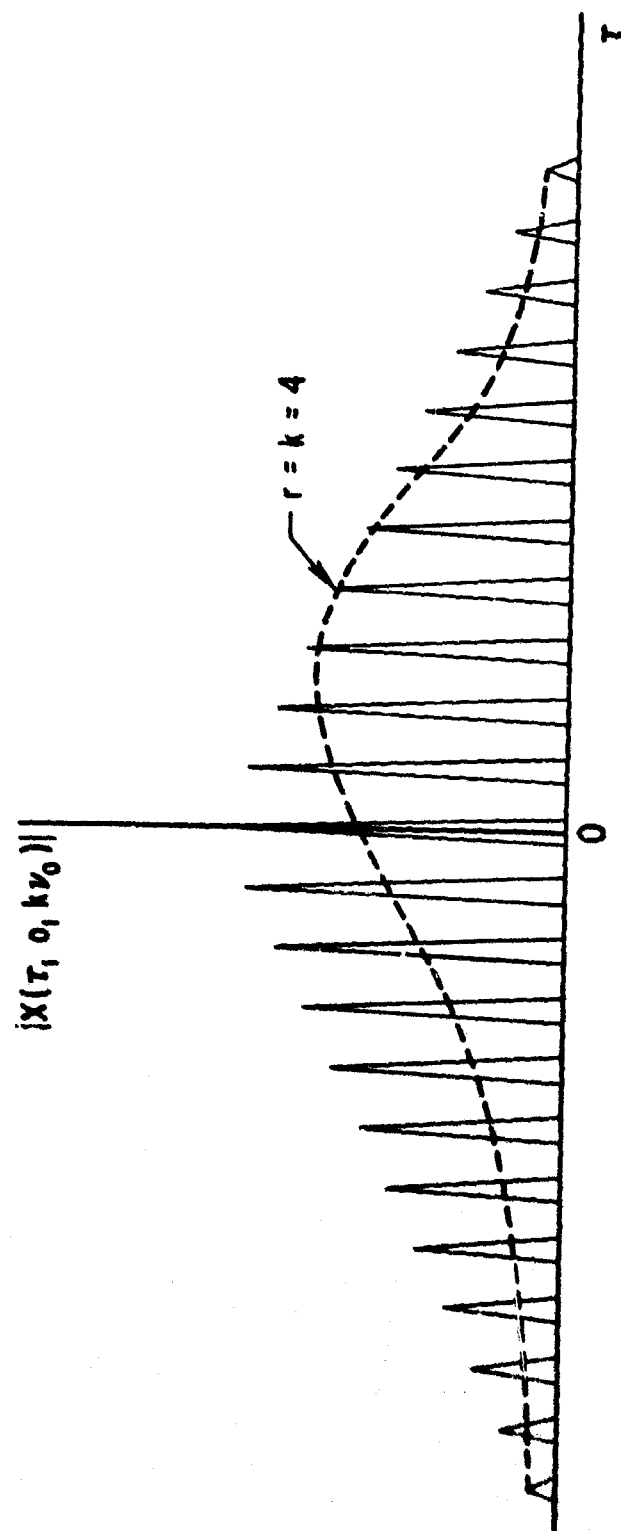


Fig. 4. Envelope of the Doppler-Tolerant Pulse Train

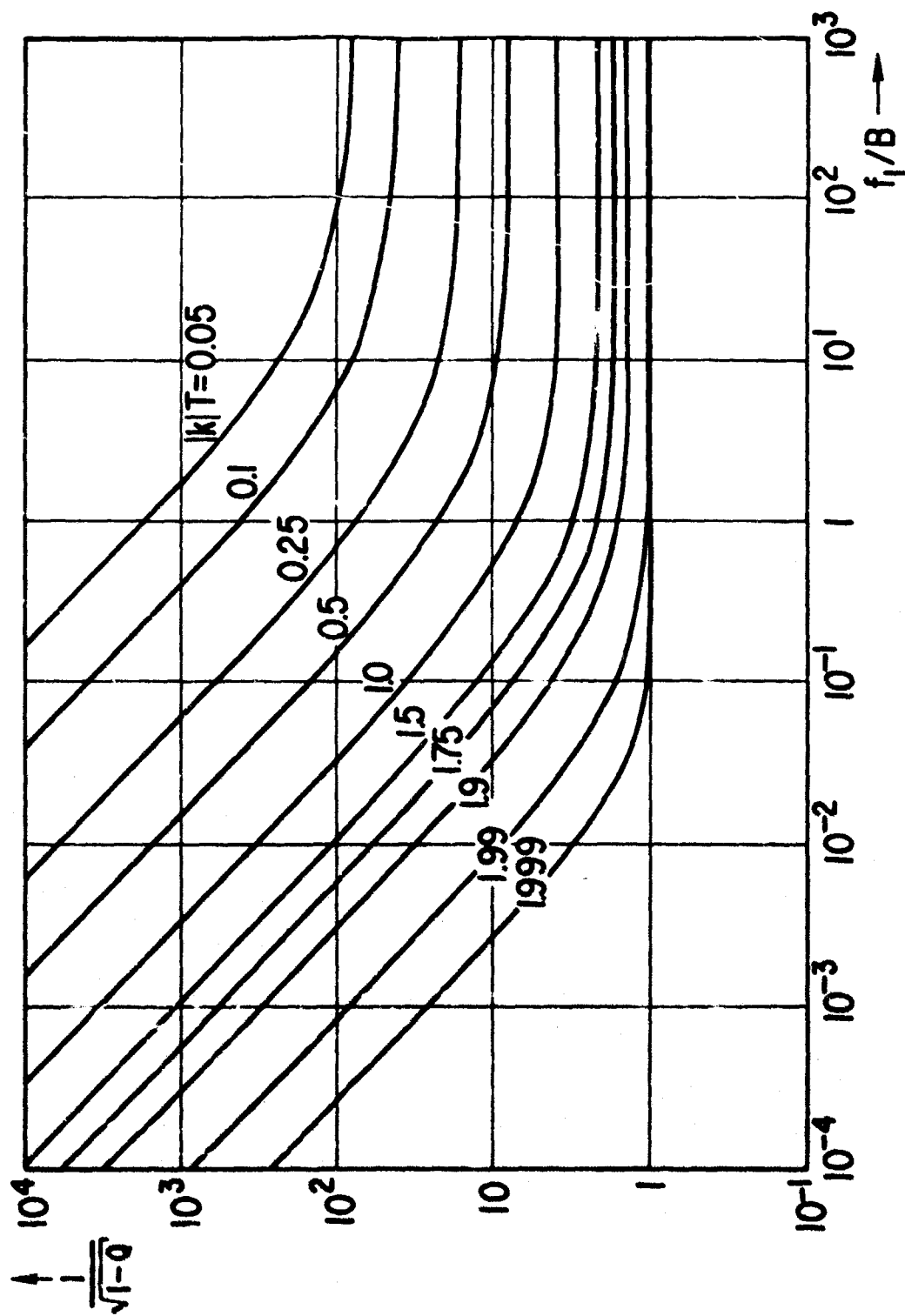


Fig. 5. Comparison of the Amplitude Behavior of the Ambiguity Functions for the Uniform Pulse Train and for the Doppler-Tolerant Pulse Train

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13. ABSTRACT When Doppler distortions of radar signals can be neglected, correlation or matched-filter processing is relatively simple. In those applications where high resolution requirements and high target speeds combine, the distortions in the waveform lead to severe processing problems. One way around these difficulties is the so-called Doppler-invariant waveform, which stays matched to the filter in the presence of an arbitrarily large Doppler effect. However, in many situations this waveform cannot be used. This paper extends the idea of Doppler invariance to only parts of the waveform, the complex modulation function, and the real envelope. We then obtain waveforms which simplify the Doppler search rather than eliminate it entirely, and hence are referred to as Doppler-tolerant. The addition of a constant-carrier term to the Doppler invariant signal leads to the signal of which only the modulation function is Doppler-invariant. It permits independent measurement of range and range rate at the expense of having to search for the Doppler shift of the carrier. For applications of the principle to the envelope of a signal, the type of signal which is of particular interest is the pulse train. It is shown that a Doppler-tolerant pulse train can be designed such that it can be processed by a delay line with fixed taps even if the pulse spacing is significantly changed by the Doppler effect. This approach is useful for both coherent and incoherent pulse trains.		

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