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HYPERSONIC WAKE SEMIANNUAL REPORT

1 July-31 December 1965

By

L. A. Hromas, TRW Systems

March 1966

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Ballistic Systems Division
Deputy for Ballistic Missile Reentry Systems
Air Force Systems Command
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FOREWORD

This report was prepared for the Hypersonic Wake Project, under Contract No. AF 04(694)-748, by TRW Systems, Redondo Beach, California. Inclusive dates of research were 1 July to 31 December 1965; the report was submitted in March 1966. TRW Systems Report No. 4456-6007-R0000 is assigned.

This technical report has been reviewed and is approved.

T. W. Swartz, LT, BSYDM

ABSTRACT

A summary is given of the work performed from 1 July 1965 to 31 December 1965 under the Hypersonic Wake Project. Primary emphasis is placed on the solution to the near wake viscous interaction problem by both the integral and finite difference methods. Results of the rotational characteristics program with the inclusion of shock waves are also presented.

CONTENTS

ILLUSTRATIONS

SECTICN I

INTRODUCTION

In the previous semiannual report the importance of the interaction parameter $\bar{\chi}$ was discussed in relation to the distance required for the completion of the initial boundary layer expansion at high Mach numbers. The subsequent examination of ballistic range photographs and experimental data indicated that outer streamlines of the boundary layer undergo only a very slight turn at the base of the body, while the inner streamlines (low Mach number) turn through very large angles. Simple hand calculations utilizing rotational characteristics indicated that this was a proper conclusion which could be obtained by analytical methods alone. A major portion of the fluid in the boundary layer appears to be only slightly influenced by viscous forces after the expansion. Only the fluid near the dividing streamline seems to be influenced by the action of shear force. The vorticity induced by the body boundary layer, however, introduces entropy gradients into the flow which play a very important role in determining the post-expansion flow field. In particular, the formation of the lip shock in two-dimensional flow is a direct result of vorticity in the field. In addition, the formation of the wake shock would certainly influence the fluid mechanical as well as chemical features of the near wake. The treatment of the near inviscid flow (which is the "wake" of the boundary layer) therefore becomes increasingly important as the Mach number increases.

The primary objective of the previous work of Webb, et al. (Reference 1) was to demonstrate that a solution to the viscous near wake problem could be obtained using the integral technique with the Crocco-Lees critical point concept. Since the main interest was with the viscous region, a simplified model of the outer inviscid region was employed. In particular, the outer inviscid flow was assumed to be isentropic with all streamlines parallel and at uniform Mach number upstream of the near wake region. The Prandtl-Meyer function was used to calculate the angle of the streamlines entering the viscous region locally, once the Mach number at the edge of that region was calculated. This procedure permitted integration of the governing equation to be performed in the upstream direction from the near stagnation point, but did not provide information on the location of the vehicle with respect to the stagnation point.

It is the object of the current analysis to begin the calculation at the base of the vehicle and to calculate the viscous near wake region in conjunction with a more realistic outer inviscid flow. This is achieved by employing the method of rotational characteristics starting with an initial line which extends radially from the base of the vehicle to the bow shock and which includes the supersonic portion of the boundary layer. This region is treated as an inviscid flow through the expansion process. The inner boundary of the inviscid flow is obtained from the viscous wake program; in particular, the Mach number and location are obtained and the purpose of the characteristics program is to calculate the remaining flow properties at this location, including the flow angle. It should be made clear that the inner boundary is not, in general, a streamline boundary as is typical in jet plume applications of the method of characteristics. In this case the boundary location and Mach number are known, whereas for the streamline boundary case only the Mach number and streamline are known. In an actual run it may be shown that the inner boundary is a streamline boundary immediately after the expansion, but then becomes a nonstreamline boundary at the beginning of the interaction region where "inviscid" fluid enters the viscous layer through the mixing process.

A principle feature of the inviscid flow field is the presence of the "lip" and wake shock waves. These are illustrated in Figure 1 along with the neighboring flow field and boundaries. (Figure 1 is a sketch, not an actual run, of the model upon which the current analysis is based.) The proper treatment of these flow discontinuities is believed to be an important consideration in the near wake viscous interaction analysis.

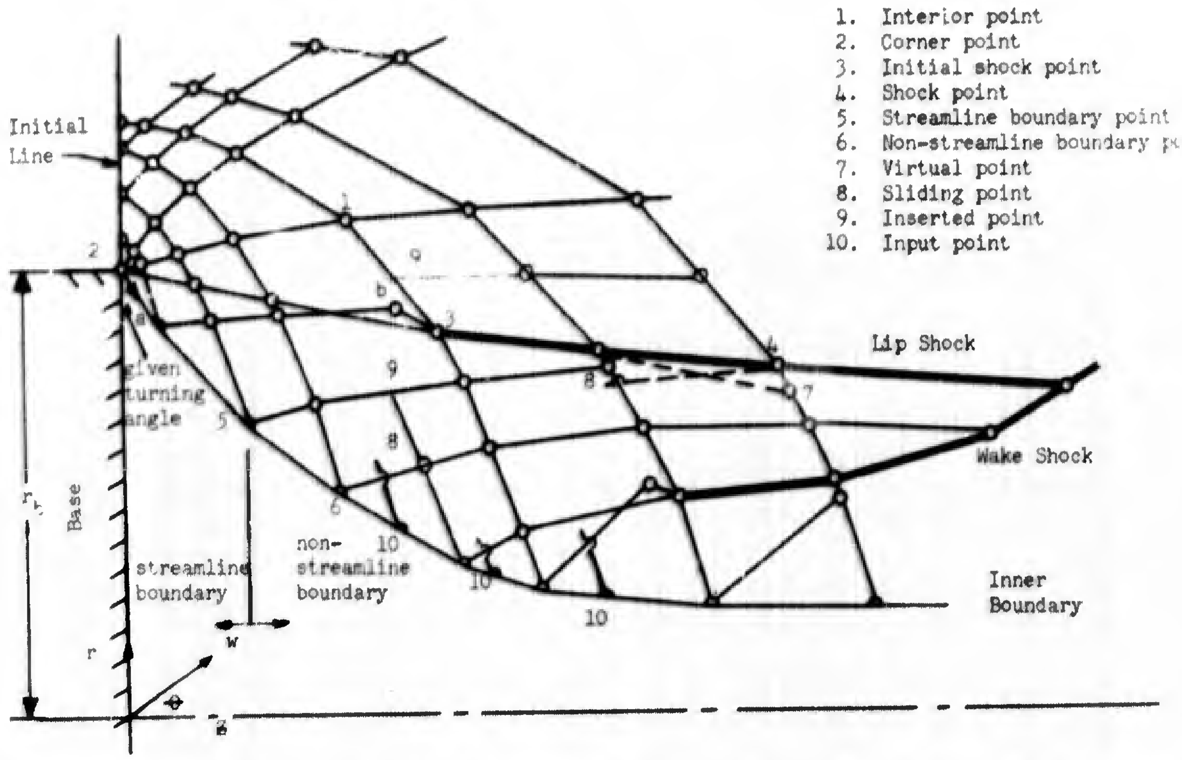


Figure 1. Characteristics Net for Determining Inviscid Near Wake Flow.

SECTION II

ROTATIONAL METHOD OF CHARACTERISTICS PROGRAM

The finite difference equations for rotational characteristics are presented in Reference 2. These are fairly standard in form and will not be repeated here. The principle variables selected are Mach number, Prandtl-Meyer function, flow angle, total or stagnation pressure, total or stagnation temperature, and the stream function. These are specified along an initial line which extends radially from the base. The initial line input data is arbitrary but will reflect the vorticity encountered in the boundary layer and that due to the bow shock. The entropy, i. e., total pressure and total temperature is constant on streamlines except across the lip and wake shock waves, where the total pressure changes in step fashion. Total temperature varies across streamlines. Referring to Figure 1, the analysis proceeds from mesh point to mesh point starting at the initial line along right running characteristics to the boundary. The flow undergoes an expansion about the corner which, due to vorticity, is more complicated than a simple centered corner expansion in Prandtl-Meyer flow. The right running characteristics, which are necessary in a vortical expansion, reflect from the boundary as compression waves and must be accounted for. In fact, it is the crossing of the first reflected wave with the last expansion wave which leads to the formation of the lip shock. This is shown in Figure 1 where the line between points 2 and 3, and a and b are the last expansion and first compression waves, respectively. In the computational procedure, the initial shock point is determined when a foldback in the right running characteristic currently being calculated occurs. The points 2, 3, and 4 in Figure 2 depict the foldback. It is seen from the figure that this condition results from the crossing of left running characteristics 1-3 and 5-4. The initial shock point is taken to be point 3, and the initial shock angle is assumed to be that of the characteristic between points 1 and 3. Point 4 is thereafter ignored. To locate the foldback in the program, a comparison of the axial location of the mesh point is made with the preceding point on the right running characteristic being calculated.

The preceding scheme for starting a shock interior to the flow was outlined by Moe and Troesh (Reference 3). Their problem was concerned with the boundary shock obtained in jet flows.

The wake shock is formed when two reflected characteristics cross in the manner shown in Figures 1 and 2. This is expected to occur in that region where the pressure is increasing significantly on the axis in the viscous wake, i. e., on the approach to the rear stagnation point.

The lip shock is typically very weak in comparison with the wake shock. The reason is apparent from Figure 1 where it is seen that the lip shock is "fed" on the upstream side by the expansion waves from the corner which tend to weaken the shock, but is reinforced on the downstream side by the reflected compression waves which maintain the shock. In contrast, the wake shock is reinforced on both sides by compression waves and so is much stronger than the lip shock.

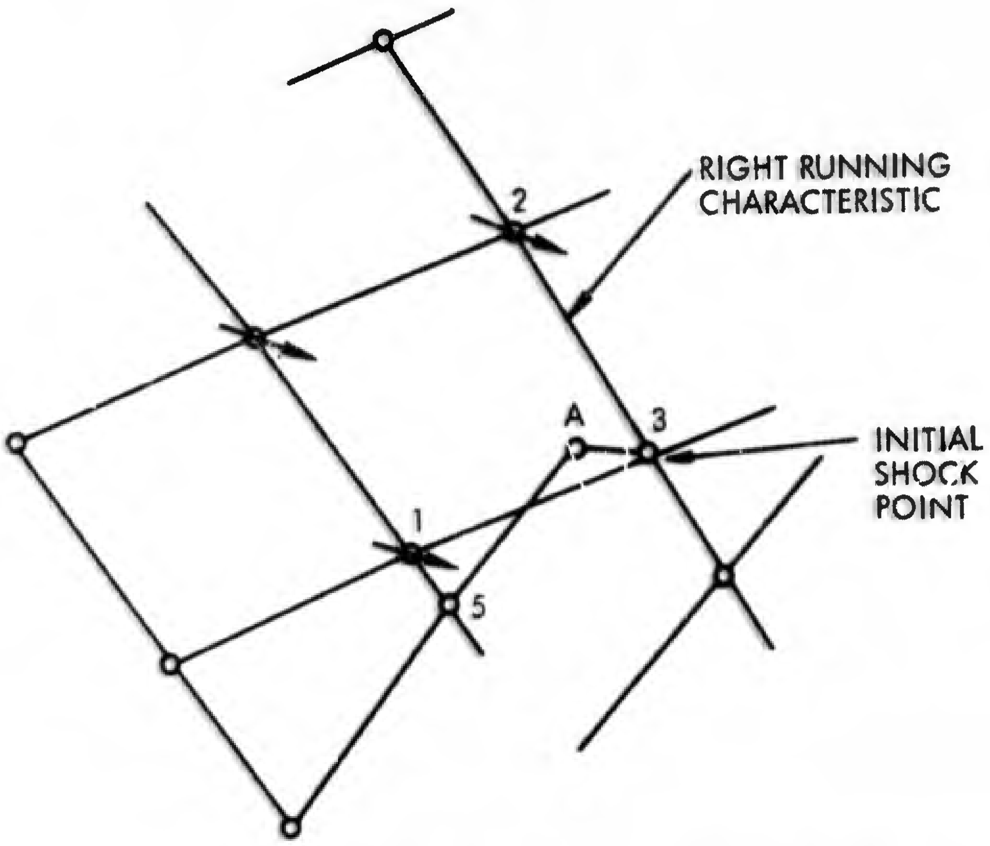


Figure 2. Foldback Scheme for Initial Shock Point

One of the questions of concern in the current study is the dependence that the initial line vorticity and the inner boundary conditions have on the formation and character of the lip shock. Of specific concern are the conditions or family of conditions that lead to the formation of the single lip shock so often observed. Calculations indicate that the conditions very near the corner strongly influence the lip shock. Hence, its formation is less directly influenced by the interaction region in the near wake than is the wake shock. The problem is really that the initial conditions before the expansion are unknown. It has been suggested by some that the expansion process really begins upstream of the corner, i. e., the actual boundary layer begins to adjust and by the time the corner is reached, is not a boundary layer at all, e. g., terms omitted from the full Navier-Stokes equations in boundary layer theory become important. For example, a significant normal pressure gradient may exist across the layer at the corner, and calculations reflecting this are planned. The sensitivity of these initial conditions on the lip shock formation is a question which has not yet been resolved.

The previous comments are applicable to both two-dimensional and axisymmetric flow; however, a lip shock can be formed in a nonvortical axisymmetric flow with a streamline boundary. This is exactly analogous to the boundary or "bottle" shock found in supersonic jet spreading flows (see Reference 3). The current program is designed to handle both two-dimensional and axisymmetric flow and so the axisymmetric effect on the generation of the lip shock can be studied.

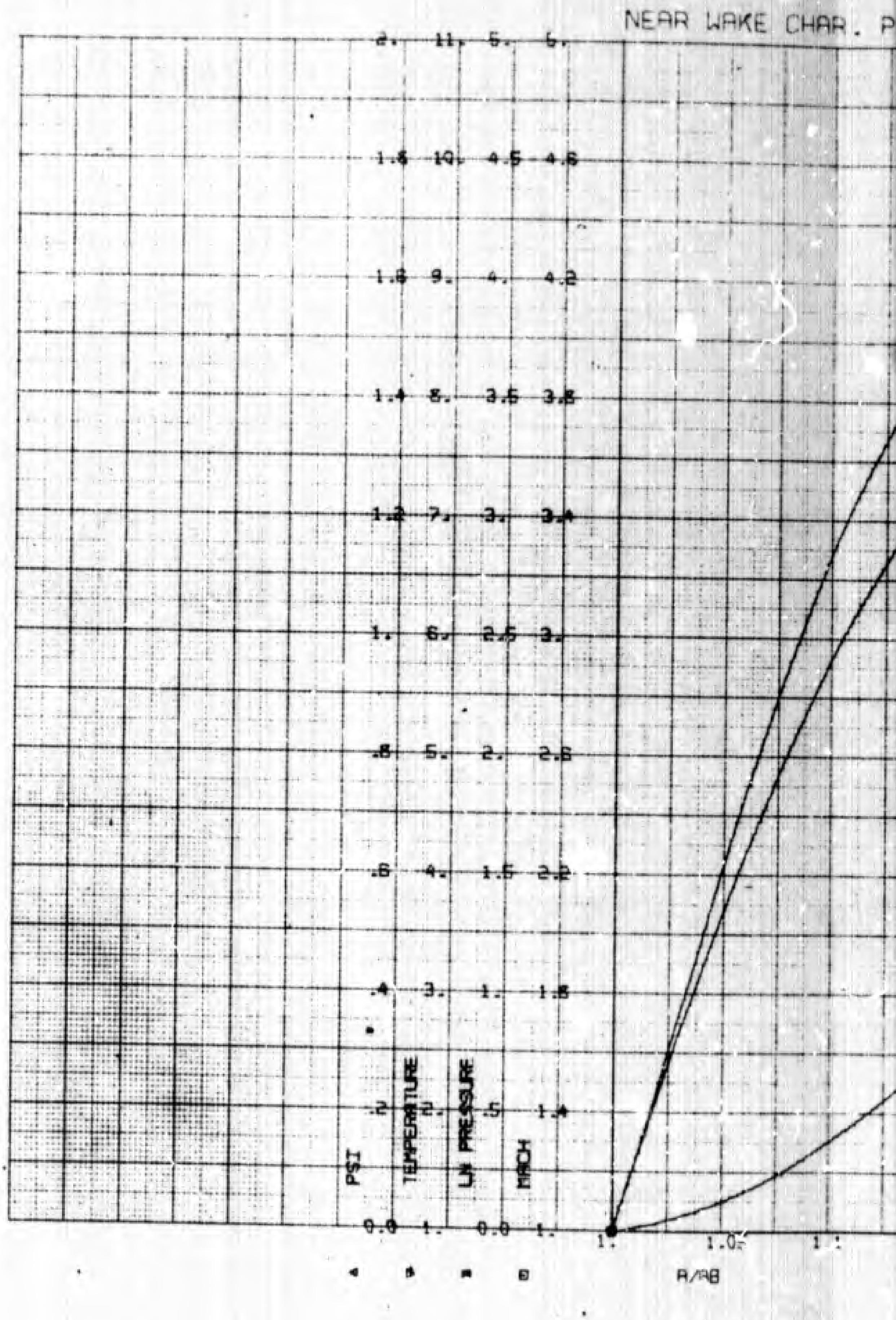
2.1 STATUS

The finite difference and related equations and the flow model are outlined in Reference 2. The computational procedure and equations are discussed in detail in Reference 4, which forms the basis for the machine program. The programming itself is about 90 percent complete, and approximately half of this has been checked out. Early emphasis was placed on a simplified version of the rotational characteristics program which permitted calculation of a vortical flow expanding about a corner to a free or streamline boundary. The reflected waves were accounted for and provision for detecting and calculating the lip shock was made. This version was successful and a result from it is shown in Figure 3. It is felt that the capability of predicting and calculating the lip shock is a significant milestone in the current study. A review of literature, particularly Weinbaum (Reference 5) and Moretti (Reference 6), indicated that these investigators illustrated the converging nature of the reflected wave system but did not actually calculate the lip shock wave and, therefore, its effect on the downstream flow.

Checkout of the program with both the lip and wake shock waves is now underway. When that is completed, emphasis will be placed on checkout of the nonstreamline boundary. This includes the logic necessary for coupling the characteristics program to the viscous wake program. Several refinements are also intended, such as inserting characteristics when the mesh size exceeds input limits.

2.2 EXAMPLE

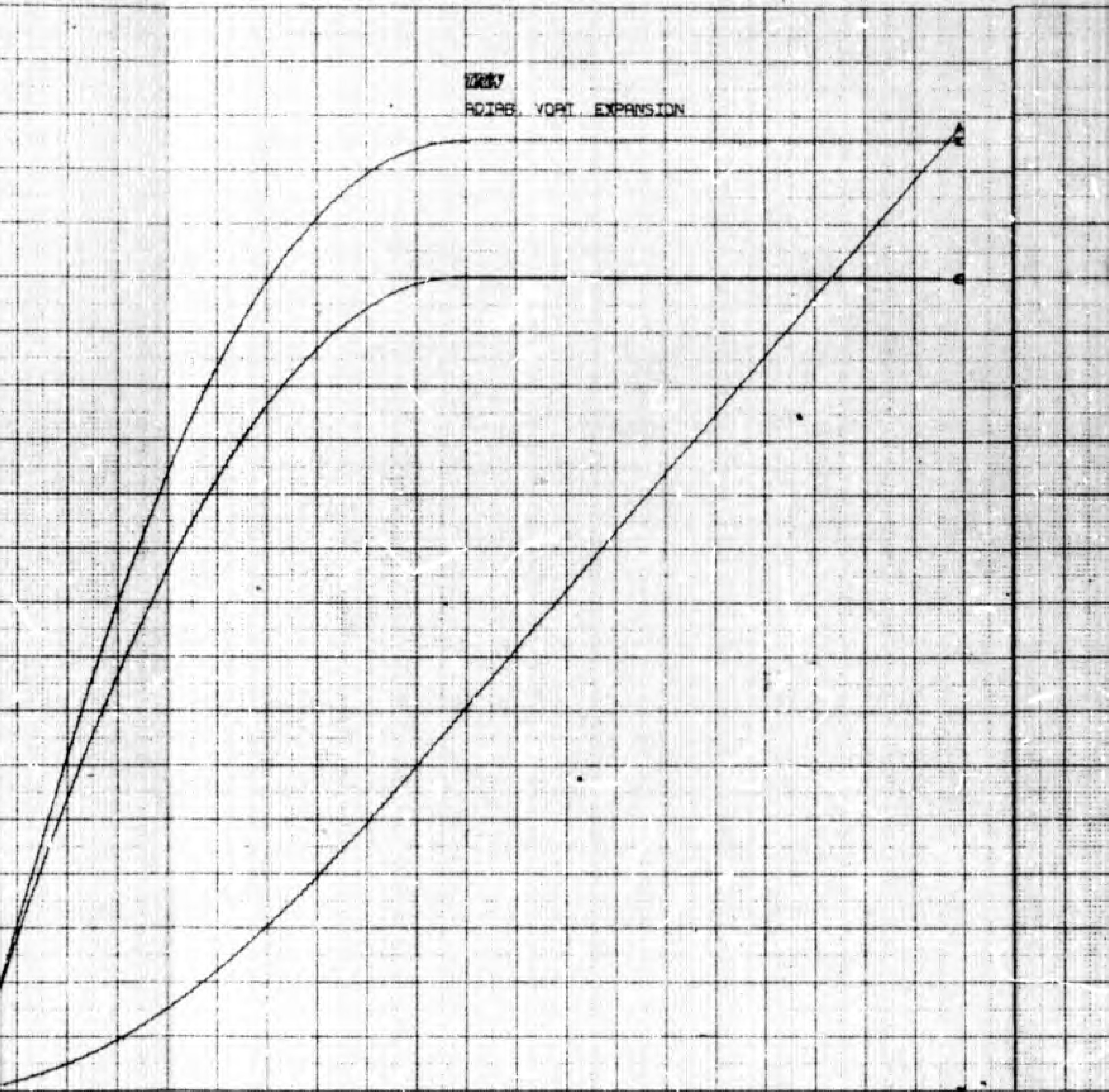
A result obtained from the current program is shown in Figure 3. Every third characteristic computed is plotted. The initial conditions are also shown and approximate a laminar boundary layer profile with constant total temperature and static pressure. On the plot of initial conditions "temperature' means "total temperature," "ln pressure" means "ln total pressure," and "psi" is the stream function. The inner boundary is a Mach 2 streamline boundary. The foldback scheme previously discussed predicted the initial point of the lip shock to be at r = 1.207 and z = 0.868, shock points were successively calculated. In this example, the initial lip shock point was formed by the intersection of the fifth and sixth reflected waves from the inner boundary. These originated on the inner boundary at locations z = 0.0478 and z = 0.0669, respectively. It was noted that the first few reflected waves from the boundary were expansion rather than compression waves. It may be for this reason that the initial lip shock point is so far downstream. If so, it appears that initial conditions in the immediate neighborhood of the corner will greatly effect the post-expansion flow field and particularly influence the location of the lip shock.


NEAR WAKE CHAR. P
2. 11. 5. 6.
1.8 10. 4.5 4.6
1.6 9. 4. 4.2
1.4 8. 3.5 3.8
1.2 7. 3. 3.4
1. 6. 2.5 3.
.8 5. 2. 2.6
.6 4. 1.5 2.2
.4 3. 1. 1.8
.2 2. .5 1.4
PSI
TEMPERATURE
LN PRESSURE
MACH
0.0 1. 0.0 1.
1.
R/RB

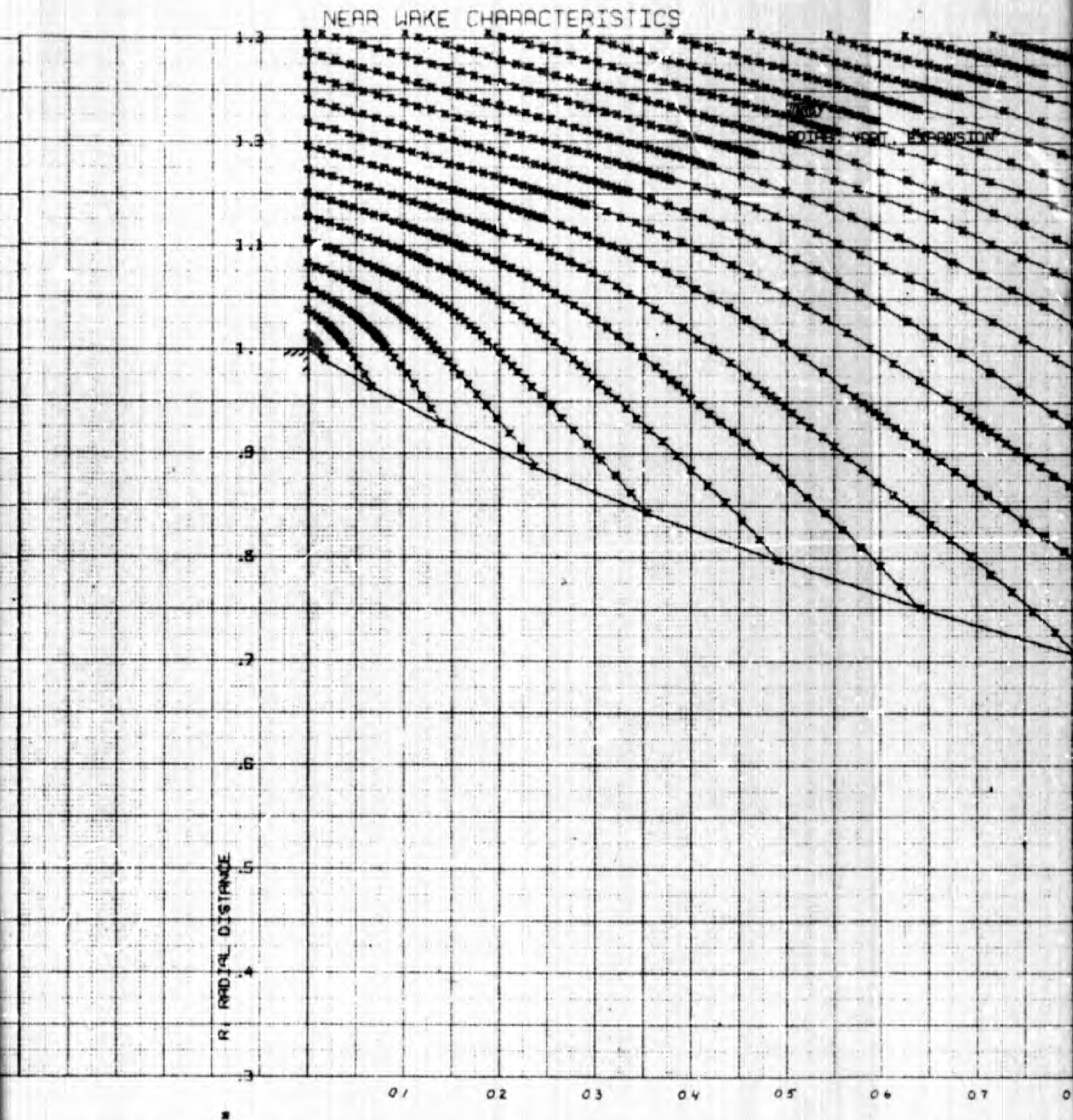
1

2

AR WAKE CHAR. PROGRAM, INITIAL LINE DATA



2


NEAR WAKE CHARACTERISTICS
RADIAL VERT. EXPANSION
R, RADIAL DISTANCE
1.3
1.2
1.1
1.
.9
.8
.7
.6
.5
.4
.3
01
02
03
04
05
06
07

3

Fig 3

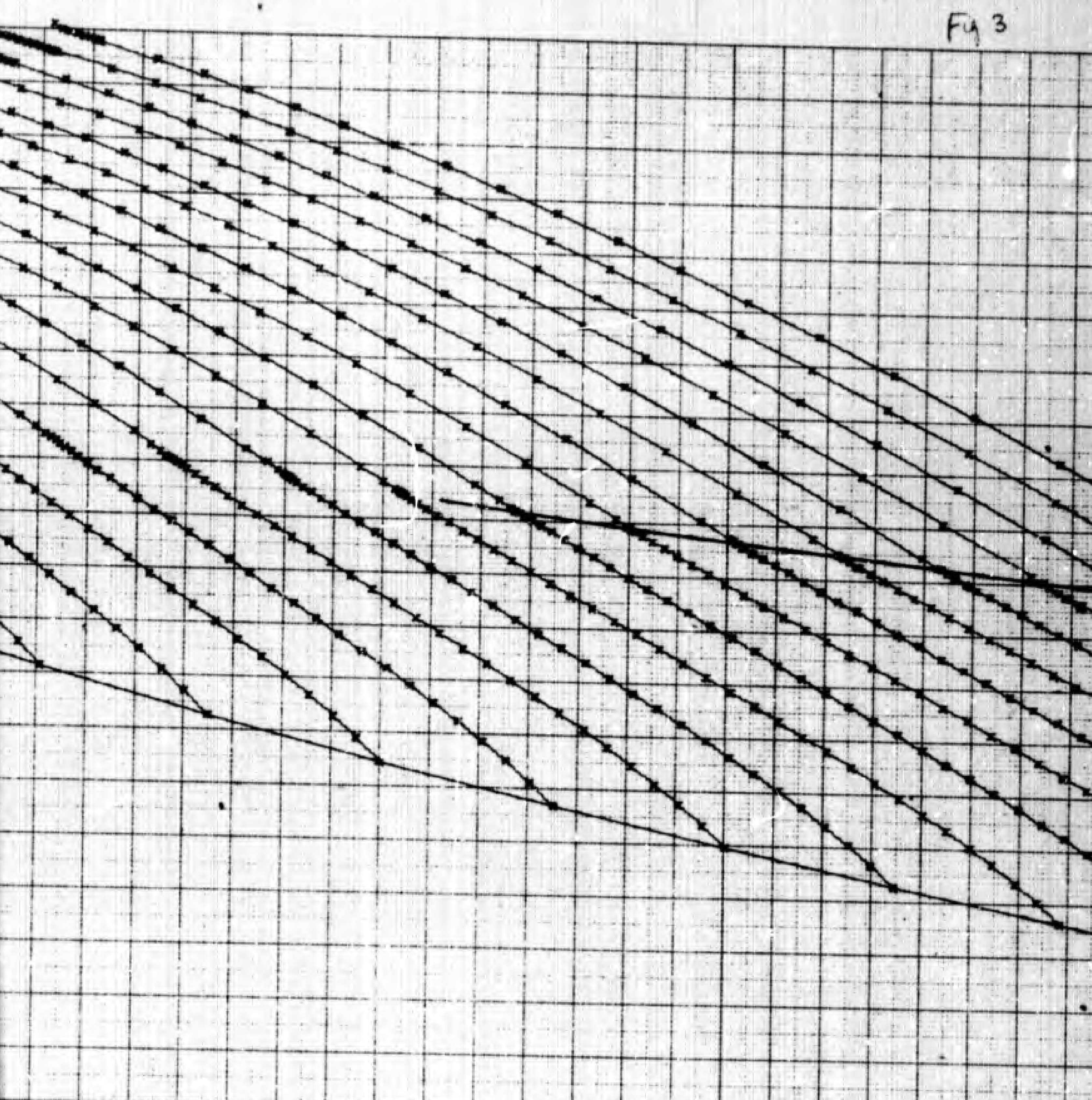


Figure 3. Two Dimensional Adiab
Vortical Expansion to Constant Pre
(Mach 2) Boundary

Fig 3

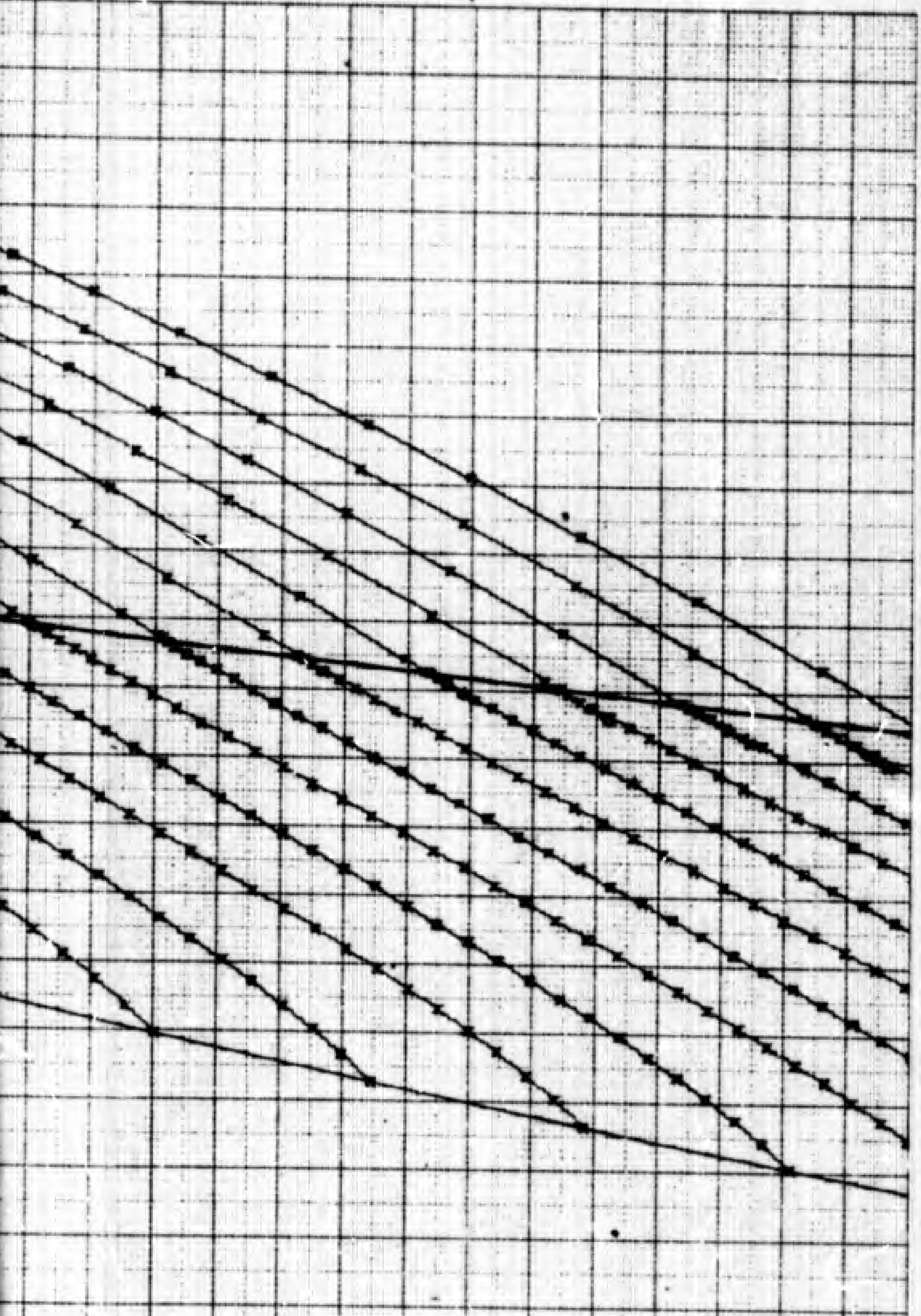


Figure 3. Two Dimensional Adiabatic Vortical Expansion to Constant Pressure (Mach 2) Boundary

SECTION III

VISCOUS LAYER ANALYSIS

In previous studies of wakes, the supersonic viscous interaction problem in the heat conducting laminar near wake was solved (Reference 1). The viscous region, described by the two-dimensional boundary layer equations, was treated by a multimoment integral method while the inviscid external flow was assumed to be governed by the Prandtl-Meyer relation. The principal feature of this study is to determine uniquely the inviscid flows which will permit the conservation relations in the viscous flow field to be satisfied. Mathematically, the problem reduces to a determination of the initial condition (inviscid pressure) which permits an integral curve through a saddle point singularly. That is, the local inviscid flow angle (hence the pressure) is determined by finding the solution which passes through the Crocco-Lees critical point for a given free stream Reynolds number and Mach number. In the actual calculations, velocity profiles of the Stewartson wake solution family and polynominals have both been used and the results show that between these two, the difference is only slight downstream of the wake stagnation point. Upstream, however, the Stewartson profiles exhibit a more favorable behavior since they could reach asymptotically a constant pressure value taken to be the base pressure. It has also been discovered numerically that for a given free stream Mach number, there exists a limiting free stream Reynolds number below which no solutions are obtainable. This means mathematically, the saddle point in the integration can no longer be found. A systematic exploration of these solution limits resulted in the curve shown in Figure 4. The solution limit line is given approximately by:

$$R_{e_\infty} \simeq 2M_\infty^{4.5}$$

The Stewartson profiles again are superior in this respect, but the difference is not appreciable.

The goal of the present study is to find methods that will extend the domain in which a saddle point is obtained in the integration. The main effort, at present, is to investigate in detail the effect of the normal pressure gradient in the near wake. It is believed that this quantity becomes increasingly important as the Mach number increases and the Reynolds number decreases.

3.1 NORMAL PRESSURE GRADIENT

The viscous shear layer in the previous studies was assumed to have a boundary layer behavior (governed by the boundary layer equations) and therefore the normal pressure gradient ($\partial p/\partial y$) was considered to be vanishingly small. This assumption appears to be valid for high Reynolds number and low Mach number flows. As the Mach number increases and the Reynolds number decreases, the shear layer thickens. The velocity

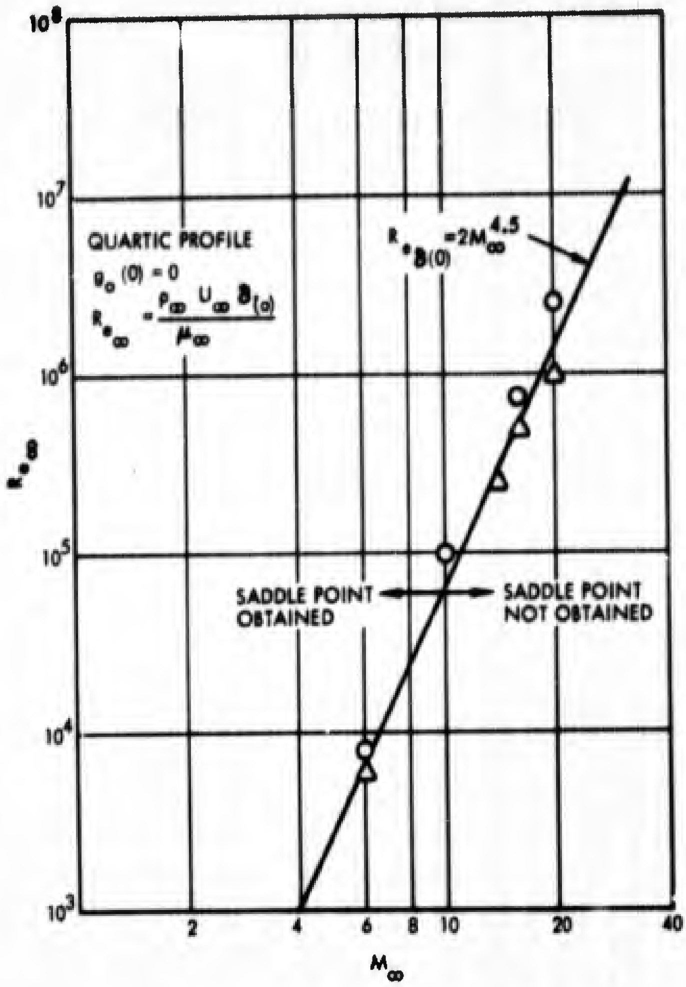


Figure 4. Solution Limit in Mach Number—Reynolds Number Domain

profile must adjust itself to match the low subsonic speed on the axis to a high supersonic one at the edge. Near the edge, a good portion of the local velocity does not differ very much from its edge value. Numerical results also indicate that in this region, the curvature of the streamlines change very little from its edge value, hence large normal pressure gradients are produced. It has been reported in previous monthly progress reports that the values of the normal pressure gradient at the edge are at least of the same order of magnitude as the transverse pressure gradients ($\partial p/\partial x$). The edge portion of the shear layer therefore may behave quite differently from the part near the axis where $\partial p/\partial y$ is identically zero due to the symmetry requirement.

It seems that in a range of interest (namely, high Mach number and low Reynolds numbers) the conventional boundary layer treatment is not quite adequate and some modifications must be made. Since any basic change would mean additional mathematical difficulties in the quantitative study, it would be desirable to introduce this effect in the most simple manner while still preserving the essential features of the problem. In the following analysis, the normal pressure gradient is introduced into the framework of the boundary layer equations by expressing the local static pressure in terms of the velocity profile. This is made possible by assuming that the streamline inclinations are small and in so doing, an extra term is added to the x-momentum equation. Multimoment integral methods again can be applied and the idea of a critical point would still exist. This, of course, introduces difficulties elsewhere. For instance, the density ratio ρ_δ/ρ no longer has a simple expression, and in its place the full equation of state must be used. As a consequence, the Crocco integral relation ceases to be valid in the non-adiabatic case. In general then, the main features of the problem are unchanged (i. e., unique solutions are obtained by the critical point), although the algebra becomes more complicated.

Since the present aim is to explore the nature of the integration, the present analysis is limited to the simplest possible case, namely, a two-dimensional adiabatic flow possessing a quartic velocity profile. The extension from this simple profile to others is only a matter of algebra, but the extension to the nonadiabatic case requires further explorations. Finally, all symbols and notations used hereafter are in complete agreement with that of Reference 1, with additional ones defined in the text.

3.2 ANALYSIS

The flow is assumed to be governed by the following equations:

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = o \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{d p_\delta}{dx} + \frac{\partial}{\partial y}\left(\nu \frac{\partial u}{\partial y}\right) - \frac{\partial}{\partial x}\left(P - P_\delta\right) \tag{2}$$

$$\rho u^2 K_\delta = - \frac{\partial p}{\partial y} \tag{3}$$

where K_δ is the streamline curvature evaluated at the edge of the shear layer.

Equation (1) is the usual continuity relation, and Equation (2) is the x-momentum in the boundary layer theory with an added term signifying the y dependence of the static pressure. Equation (3) is an approximate form of the y-momentum, where the inertial term is assumed to predominate over the viscous term and the local streamline curvature differs very little from the edge value. This approximation appears valid where the normal pressure gradient is most important (i. e., at the outer edge). Near the axis K goes to zero, however, u^2 is very small and the normal pressure gradient is reduced at this point. In the far downstream, K_δ approaches zero as the flow gradually becomes parallel, hence the normal pressure gradient vanishes, and the flow regains its boundary layer character. Equation (3), therefore, has the form which assures the right physical behavior without the introduction of mathematical difficulties inherent in a more sophisticated treatment.

The definition of the dependent and independent transformed variables is the same as that appearing in Reference 1.

In the transformed plane, the x-momentum equation becomes

$$f_\eta f_{\xi\eta} - f_\xi f_{\eta\eta} = \frac{M_\delta^1}{M_\delta} f f_{\eta\eta} + \left(\frac{\rho_\delta}{\rho} - f_\eta^{\,2}\right)\frac{u_\delta^1}{u_\delta} + \frac{f_{\eta\eta\eta}}{R_\infty}$$

$$\left\{-\frac{\rho_\delta}{\rho}\,\frac{1}{\rho_\delta u_\delta^{\,2}\xi_x}\left[\xi_x \frac{\partial}{\partial\xi}\left(P - P_\delta\right) + \eta_x \frac{\partial}{\partial\eta}\left(P - P_\delta\right)\right]\right\} \tag{4}$$

and the y-momentum equation after integrating with respect to η yields

$$P - P_\delta = R_\infty M_\delta u_\delta \int_\eta^{\eta_\delta} U d\eta^1 \tag{5}$$

Note that Equation (4) is merely the boundary layer equation with an additional term in the braces which accounts for the normal pressure gradient effects. Also the quantity ρ_δ/ρ may no longer be evaluated from the Crocco integral.

Utilizing Equations (5) and (9), the overall continuity relation, the zeroth and the first moment of the transformed x-momentum equation takes the forms

$$\phi = \frac{\epsilon^1}{\epsilon}\eta_\delta\left[\int_o^1 U dv - \left(a - \frac{1}{2\lambda}\right)\int_o^1 \frac{\rho_\delta}{\rho} dv\right] + \eta_\delta \int_o^1 \frac{\partial u}{\partial\xi} du$$

$$- \eta_\delta \int_o^1 \frac{\partial}{\partial\xi}\left(\frac{\rho_\delta}{\rho}\right) dv \tag{6}$$

$$-\theta^1 = \frac{\epsilon^1}{\epsilon}\left(1 + \frac{1}{2\lambda}\right)\theta + \frac{\epsilon^1}{\epsilon}\frac{1}{2\lambda}\eta_\delta \int_o^1 \left(\frac{\rho_\delta}{\rho} - U\right) dv - \frac{K_\delta}{\xi_x}\eta_\delta^{\,2}\frac{\epsilon^1}{\epsilon}\left(1 + \frac{1}{2\lambda}\right)\int_o^1 \frac{\rho_\delta}{\rho}\int_v^1 U^2 dv^1\, dv$$

$$-\frac{K_\delta \eta_\delta^{\,2}}{\xi_x}\int_o^1 \frac{\rho_\delta}{\rho}\int_o^1 \frac{\partial u^2}{\partial\xi} dv^1\, dv - \frac{K_\delta}{\xi_x}\eta_\delta^1 \eta_\delta \int_o^1 \frac{\rho_\delta}{\rho} dv \tag{7}$$

$$+\frac{K_\delta}{\xi_x}\left(\frac{d}{d\xi}\ln \xi_x\right)\eta_\delta^{\,2}\int_o^1 U^2 dv \int_o^v \frac{\rho_\delta}{\rho} dv^1 - \frac{K_\delta}{\xi_x}\eta_\delta^{\,2}\int_o^1 U^2 dv \int_o^v \frac{\partial}{\partial\xi}\left(\frac{\rho_\delta}{\rho}\right) dv^1$$

$$
-\frac{\theta_1{}^1}{2} = \frac{1}{2}\,\frac{\epsilon^1}{\epsilon}\left(1+\frac{1}{\lambda}\right)\theta_1 + \frac{\epsilon}{\epsilon}\,\frac{1}{2\lambda}\,\eta_\delta - \frac{\theta_2}{R_\infty} - \frac{K_\delta}{\xi_x}\,\eta_\delta{}^2\,\frac{\epsilon^1}{\epsilon}\left(1+\frac{1}{2\lambda}\right)\int_o^1 \frac{\rho_\delta}{\rho} U\,dv \int_v^1 U^2\,dv^1
$$

$$
-\frac{K_\delta}{\xi_x}\,\eta_\delta{}^2 \int_o^1 \frac{\rho_\delta}{\rho}\,U\,dv \int_v^1 \frac{\partial u^2}{\partial \xi}\,dv^1 - \frac{K_\delta}{\xi_x}\,\eta_\delta{}^1\,\eta_\delta \int_o^1 \frac{\rho_\delta}{\rho}\,U\,dv \qquad (8)
$$

$$
+\frac{K_\delta}{\xi_x}\left(\frac{d}{d\xi}\ln \xi_x\right)\eta_\delta{}^2 \int_o^1 U^3 dv \int_o^v \frac{\rho_\delta}{\rho}\,dv^1 - \frac{K_\delta}{\xi_x}\,\eta_\delta{}^2 \int_o^1 U^3 dv \int_o^v \frac{\partial}{\partial \xi}\left(\frac{\rho_\delta}{\rho}\right) dv^1
$$

where $\lambda = 1 - 1/\epsilon$. The added terms in the x-momentum equations are of the order of K_δ and should vanish in the far wake where the flow becomes parallel. Thus the far wake would approach the boundary layer behavior in this formulation. Equations (6), (7), and (8) allow a total of three unknown functions $P_i\ (\xi)$, $(i = 1, 3)$, to be determined. Upon the introduction of assumed velocity profiles into these equations, an autonomous system of three first-order ordinary differential equations results.

The singularities of this system are similar to those discussed in Reference 1, and a saddle point exists as the uniqueness condition. The method of integration of this set is also identical to that of Reference 1. In order to do the integration, one has chosen the quartic profile for U.

$$
U = 1 - (1 - u_o)\,(1 - v^2)^2 \qquad (9)
$$

where $U_o\ (\xi)$ is the velocity ratio on the axis and $v = \frac{\eta}{\eta_\delta}$.

3.3 PRELIMINARY RESULTS

The numerical program for handling the integration of these equations has just been completed, and the first case computed is at $M_\infty = 6$. Preliminary results indicate that the value of R_∞ is lowered to 8×10^3 (the original theory was not able to find the saddle point below 2×10^4) and the saddle point integration is still possible. There is also strong indication that the present theory is capable of producing solutions at lower values of R_∞ than those that have just been computated. This is planned as soon as the checking of the newly developed program is completed. The preliminary results obtained so far indicate that the normal pressure gradient influence is most pronounced in the neighborhood and immediate downstream of the rear stagnation point. The values of K_δ approach zero rapidly and at a distance of about ten initial shear layer thickness (the shear layer thickness at the rear stagnation point), the flow resumes a boundary layer behavior. The gross trends observed so far appear to coincide with physical reality; however, further checkout is necessary before any definite conclusions can be stated.

SECTION IV

NEAR WAKE FINITE DIFFERENCE METHOD

In order to investigate the basic nature of the near wake interaction problem in detail, a finite difference method has been developed for the calculation of the laminar wake with displacement interaction between the viscous and inviscid flows. The motion of the fluid is assumed to be governed by the following equations

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = o \tag{10}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(u \frac{\partial u}{\partial y}\right) \tag{11}$$

$$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} \tag{12}$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left(u \frac{\partial H}{\partial y}\right) \tag{13}$$

The steamwise momentum Equation (11) differs from the boundary layer equation only in the appearance of the pressure gradient term as a partial rather then ordinary derivative. The transverse momentum Equation (12) includes only the predominating inertia and pressure terms. The lower order transverse viscous terms may be included; however, an additional boundary condition on the v-component must be introduced if they are considered. Since the predominating effect is that of streamline curvature, simplicity dictates the exclusion of these terms.

The above set of equations is then transformed into the von Mises coordinate system where the integration proceeds along streamlines. At a reference streamline located in the nearly "inviscid" flow, the Prandtl-Meyer relation is employed to furnish a relation between pressure, flow angle and Mach number. The integration is started by assuming normalized shapes for the initial transverse distribution of pressure, velocity, and flow angle. Two families of solutions are obtained during the integration downstream depending on the initial value of the Mach number $M_\delta(o)$ at ψ_δ. In one family, the axis velocity increases rapidly at some point while the pressure decreases rapidly and the viscous layer collapses (sink type flow). In the other family, the edge pressure increases rapidly at some point while the axis velocity decreases and subsequently reverses (source type flow). A unique solution to the problem is thus obtained by finding the initial edge Mach number that leads to an integration which divides these two families (i. e., where sources and sinks are of zero strength).

SECTION V

WAKE TURBULENCE ANALYSIS

During the reporting period, an investigation was conducted on the feasibility of including advanced theoretical descriptions of turbulence structure in hypersonic wake calculations. In particular, the statistical mechanical theory formulated by R. H. Kraichnan in his report "Turbulent Mixing in the Chemically Reacting Wake" was investigated.

The initial effort consisted of a background acquisition and survey of the many papers in the Physics of Fluids and other open literature concerning this theory. In the face of moderately negative advice from Professor H. W. Liepmann of the California Institute of Technology, a consultant to TRW Systems, the effort was slackened and the manpower was diverted to an experimental study of wake turbulence, which has not been supported by this contract. Recently, the advice from Professor Liepmann concerning the Kraichnan theory has turned strongly negative.

The theory itself has many attractive features. It establishes an approximation to the Navier-Stokes equations that describe incompressible turbulence. Several conservation properties of the original equations are retained, together with a guarantee of a positive energy spectrum. A recent revision of the theory gave the Kolmogorov spectrum, one of the best-established properties of turbulence at high Reynolds number. In the revision, the guarantee of positive spectra was lost, however. No negative spectra have yet occurred in the numerical calculations.

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A summary is given of the work performed from July 1, 1965 to December 31, 1965 under the Hypersonic Wake Project. Primary emphasis is placed on the solution to the near wake viscous interaction problem by both the integral and finite difference methods. Results of the rotational characteristics program with the inclusion of shock waves are also presented.

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