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Inertial Compensation of Energy Losses
in Timing Mechanisms

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INERTIAL COMPENSATION OF
ENERGY LOSSES IN
TIMING MECHANISMS

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Approved

A handwritten signature in dark ink, appearing to read "R. C. Geldmacher", is written over a horizontal line.

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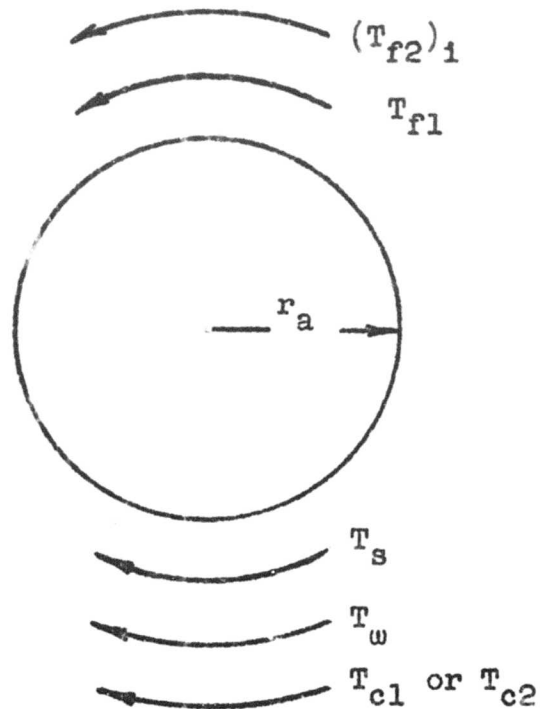
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ABSTRACT

The behavior of timing mechanisms under spin conditions is affected by: 1) changes in gear train friction, 2) changes in friction between main spring leaves, 3) changes in mainspring torque. The purpose of this study has been to examine these effects in the light of the possibility of synthesizing an inertial (spin energized) compensator which will provide enough torque to balance out both spin induced and run-down torque losses.

The problem has been approached by analyzing various sources of torque loss or gain and then combining these analyses to show the pertinent design parameters. In the work that follows, all torques will be referred to the spring arbor as shown in Fig. 1.



$(T_{f2})_i$	Torque due to friction between i th and $i-1$ spring leaves
T_{f1}	Torque due to spin induced friction in gear train
T_s	Torque due to windup of main spring
T_w	Torque due to spin of main spring
T_{c1}	Torque due to segmental gear compensator
T_{c2}	Torque due to attached mass compensator

Fig. 1

ANALYSIS

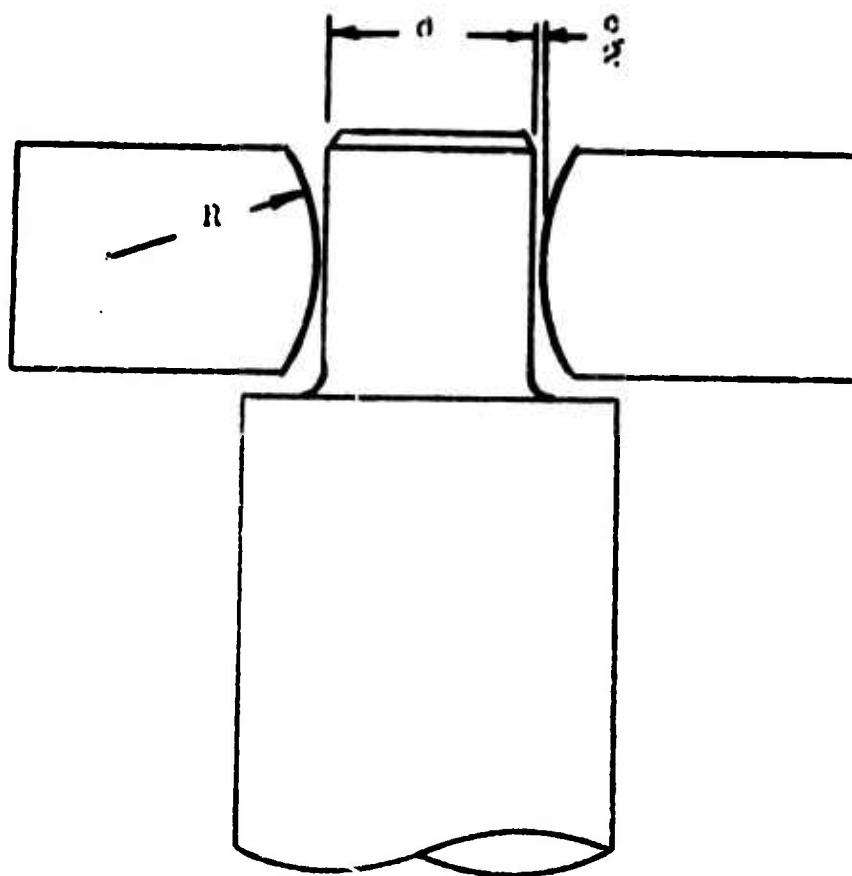
Gear Train Friction The loads, stress levels, and friction torques of timer-mechanism bearings have been studied in detail in [1] and [2]. For the case of the conventional timer (shafts parallel to the axis of spin) the major spin induced friction torques are generated in journal bearings. A cross-section view of such a bearing is shown in Fig. 2. A cylindrical shaft of diameter d turns in part of a toroidal journal bearing which has an inside diameter $d + c$ and meridian radius R . As a limiting case the bearing may be cylindrical, i.e., $R = \infty$ and the axial clearance shown may not exist or may be replaced by some provision for withstanding a small thrust load.

If a cosine distribution of normal force in the circumferential direction is assumed, the following relations can be derived for maximum pressure and friction torque [2] for the particular shaft-gear assembly

$$1) \quad (p_m)_k = \frac{4(P_{tr})_k}{\pi d_k}$$

$$2) \quad (T_f)_k = \frac{2\mu_1 d_k (P_{tr})_k}{\pi}$$

* Bracketed numbers designate references listed on page 44



SCHEMATIC OF JOURNAL BEARING

Fig. 2

where $(P_{tr})_k$ is the transverse load applied to the k th bearing, $(P_m)_k$ is the maximum force per unit length of circumference, d_k is the diameter of the k th shaft, μ_1 is the coefficient of friction, and $(T_f)_k$ is the friction torque at the k th shaft. If it is assumed that transverse forces produced by transmitted torque are negligible compared with those produced by angular acceleration, relation 2) may be written [2]

$$3) \quad (T_f)_k = \frac{2\mu_1 d_k m_k e_k \omega^2}{\pi}$$

where m_k is the mass of the shaft-gear assembly, e_k is the eccentricity of the shaft, and ω is the angular velocity of the spinning spring.

The friction torque $(T_{f1})_k$ reflected to the main-spring for each shaft-gear assembly is

$$4) \quad (T_{f1})_k = (T_f)_k g_k$$

where g_k is the gear ratio to the main spring. If relation 3) is used, 4) becomes

$$5) \quad (T_{f1})_k = \frac{2\mu_1 d_k m_k e_k g_k \omega^2}{\pi}$$

and the total friction torque reflected to the main spring is

$$6) \quad T_{f1} = \frac{2u_1}{\pi} \left[\sum_{k=1}^u d_k m_k e_k g_k \right] \omega^2$$

where u is the number of gear-shaft assemblies in the gear train.

Mainspring Friction The rotating spring is simulated by concentric sections offset as shown in Fig. 3.

A general expression for the length l_1 of the i th turn of the spring is

$$7) \quad l_1 = \pi \left[2r_a + (2i - 1)t + d_1 + d_2 + \dots + d_i \right]$$

where t is the thickness of the spring and r_a is the radius of the arbor.

The radius of the i th turn is $l_1 / 2\pi$ and the distance q_1 from the center of rotation of the arbor to the center of gravity of the i th turn is

$$8) \quad q_1 = \frac{l_1}{2\pi} - (r_a + \frac{t}{2}) - it$$

Assuming a density of 500 pounds per cubic foot, the mass of the i th turn is

$$9) \quad m_1 = l_1 (t) (w) (120) 10^{-4} \quad (\text{oz. sec.}^2/\text{in.})$$

where w is the width of the spring.

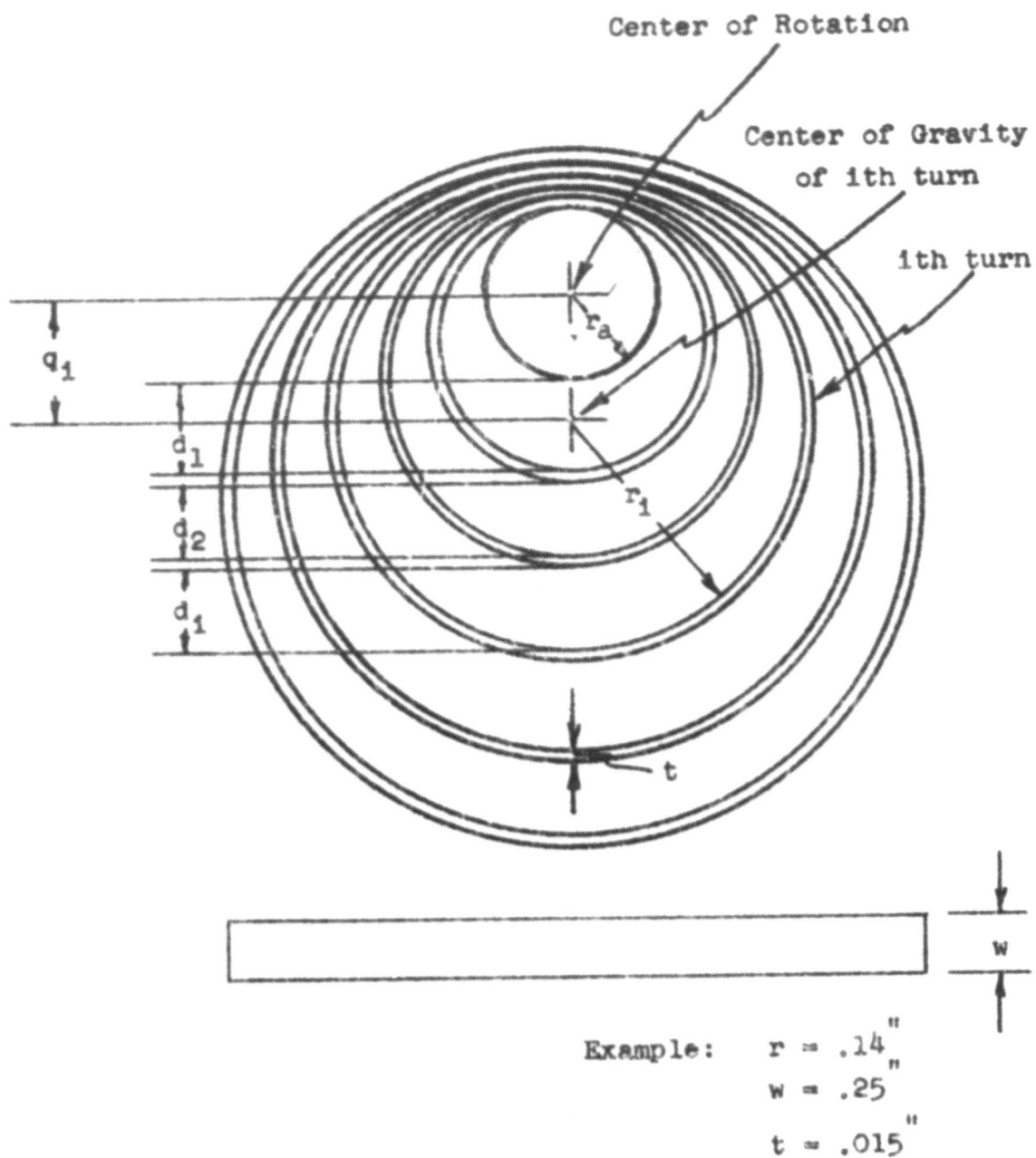


Fig. 3

The normal force exerted on the i-l turn by the ith turn then is

$$10) \quad F_1 = \sum_{j=1}^n m_j q_j \omega^2$$

Substituting in 10) from 9), 8), and 7) gives

$$11) \quad F_1 = \frac{120tw}{10^4} \left[\frac{1}{2\pi} \sum_{j=1}^n (\ell_j)^2 - \left(r_a + \frac{t}{2}\right) \sum_{j=1}^n \ell_j - t \sum_{j=1}^n j \ell_j \right] \omega^2$$

where

$$12) \quad \ell_j = \pi [2r_a + (2j - 1)t + \sum_{i=1}^j d_i]$$

The friction torque $(T_{f2})_1$ is that occurring between the ith and i-l turn and may be written

$$13) \quad (T_{f2})_1 = [r_a + (i - 1)t] \mu_2 F_1$$

where μ_2 is the coefficient of friction.

Static run-down photographs indicate that to a good approximation the distances $d_1, d_2, \dots, d_i$ may be

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assumed to remain equal. These distances will, however, depend upon the number of turns of the spring. Thus from

7) when $d_1 = d_2 = \dots = d_1 = d_n$

$$14) \quad l = \pi \sum_{i=1}^n \left[2r_a + (2i - 1)t + (d_n)_1 + (d_n)_2 + \dots + (d_n)_i \right]$$

and

$$15) \quad l = \pi \left[n(2r - t) + tn(n + 1) + d_n \frac{n(n + 1)}{2} \right]$$

and

$$16) \quad d_n = \frac{2 \left[l/\pi - tn^2 - 2r_a n \right]}{n(n + 1)}$$

Equation 11 may then be reduced to

$$17) \quad P_1 = \frac{120tw}{10^4} \left[A \sum_{j=1}^n 1 + B \sum_{j=1}^n j + C \sum_{j=1}^n j^2 \right] w^2$$

where the coefficients A, B, and C are

$$A = - \pi(2r_a t - t^2)$$

$$18) \quad B = \frac{\pi}{2} (2r_a d_n - 3t d_n - 4t^2)$$

$$C = \frac{\pi}{2} (d_n^2 + 2t d_n)$$

Summing 17) gives

$$F_1 = \frac{120tw}{10^4} \left\{ A(n+1-1) + \frac{B}{2} [n(n+1) - 1(1-1)] \right.$$

19)

$$\left. + \frac{C}{6} [n(n+1)(2n+1) - 1(1-1)(21-1)] \right\} \omega^2$$

and finally

$$(T_{f2})_1 = \frac{120 \mu_2 tw}{10^4} [r_a + (1-1)t] \left\{ A(n+1-1) \right.$$

$$20) \quad \left. + \frac{B}{2} [n(n+1) - 1(1-1)] + \frac{C}{6} [n(n+1)(2n+1) \right.$$

$$\left. - 1(1-1)(21-1) \right] \right\} \omega^2 \quad (\text{in. lb.})$$

Mainspring Torque Changes in spring torque occur as a result of inertial forces acting on spring leaves during spin. These inertial forces are proportional to the square of the spin speed ω and to the distance of the spring leaves from the arbor. For a given spin speed

inertial forces become larger as the spring unwinds.

In contrast to the inertial effects which ideally increase the spring torque, static spring torque decreases as the spring unwinds. This decrease, when the spring is not spinning, is for all practical purposes proportional to angle of unwind. For the effective range of the spring, a theoretical result that agrees well with experiment is

$$21) \quad T_s = \frac{EI}{l} (\theta_0 - \theta) + D$$

where T_s is the spring torque, E is Young's modulus, I is cross-section moment of inertia of the spring, l is the length, θ is the angle of unwind of the arbor of the spring, and θ_0 is the fully wound angle of the arbor. The constant D must be determined experimentally and is the torque intercept of the static run-down curve as shown in Fig. 4.

For the case of a spring having the following spiral shape [3]

$$r = R_2 - \frac{R_2 - R_1}{\phi} \beta$$

where ϕ is the total angle of the spiral, β is the an-

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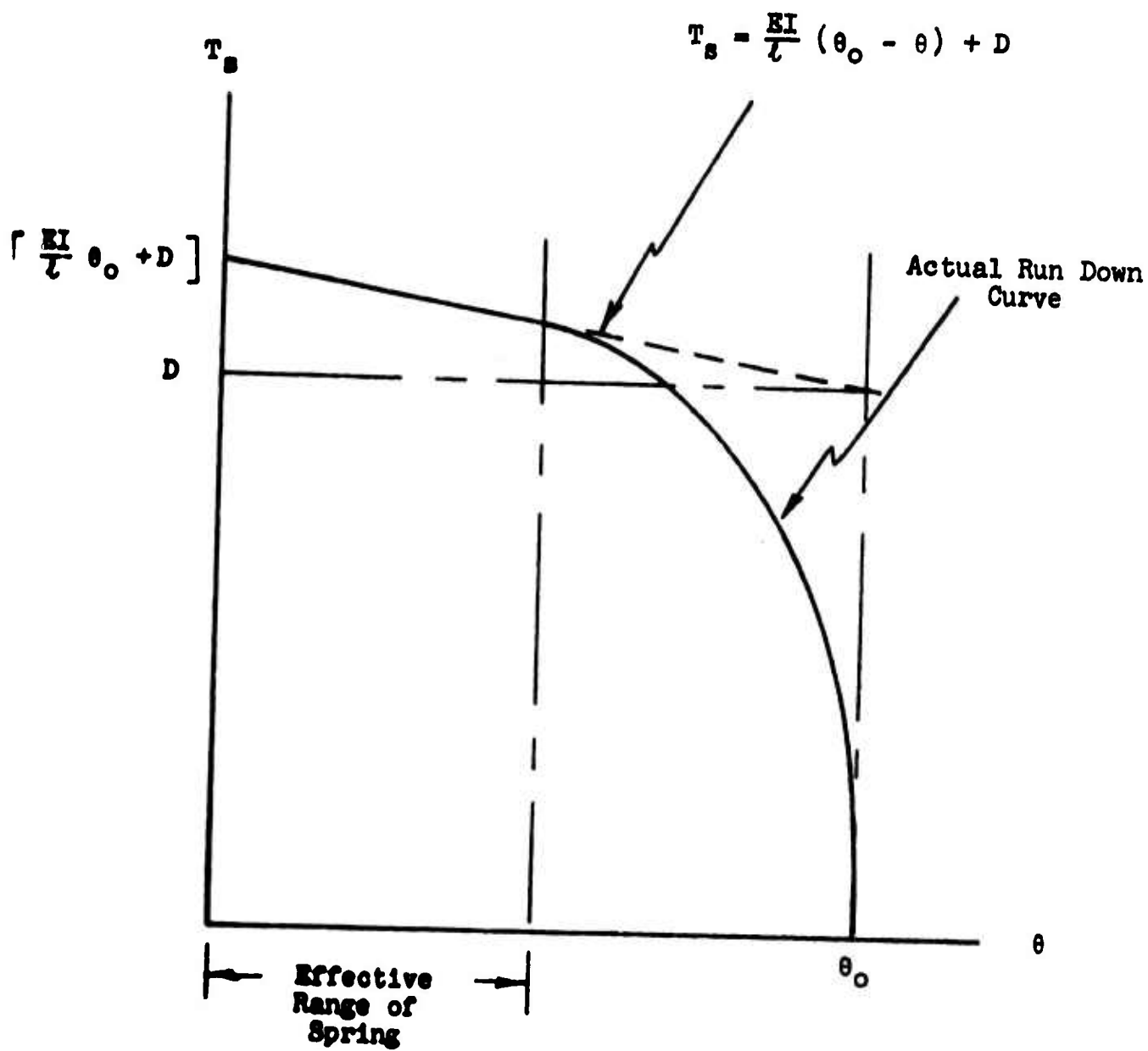


Fig. 4

gular coordinate of r , and R_1 and R_2 are the inside and outside radii of the spring respectively, the relation between the spin induced angle of unwind θ_w , the torque T_w and the angular velocity of spin ω is [4]

$$22) \quad \theta_w = \frac{1}{EI} \left\{ T_w l - \rho R_2^4 \left[1 - 2 \frac{R_2 - R_1}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2} + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^3} \right] (2n + 1) \pi \omega^2 \right\}$$

where ρ is mass per unit length of spring and θ is angle of unwind. Setting $\theta_w = 0$ gives the spin induced torque as a function of the spring geometry

$$23) \quad T_w = \frac{\rho R_2^4}{l} \left[1 - 2 \frac{R_2 - R_1}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2} + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^3} \right] (2n + 1) \pi \omega^2$$

Spring Turns as a Function of Angle of Unwind The number of spring turns n as a function of θ may be written

$$24) \quad n = n_0 - \frac{\theta}{2\pi}$$

where n_0 is the initial number of turns.

Spin Decay To a good approximation, spin decay may be taken to be a linear function of θ [5], ie.,

$$25) \quad \omega = \omega_0 (1 - q\theta)$$

where ω_0 is the angular velocity at the muzzle of the gun and q is a constant determined by experiment. Assuming ω decays p percent per turn of the spring arbor, then

$$26) \quad q = \frac{p}{200\pi}$$

and

$$27) \quad \omega = \omega_0 \left(1 - \frac{p\theta}{200\pi}\right)$$

or

$$28) \quad \omega/\omega_0 = 1 - \frac{p\theta}{200\pi}$$

Segmental Gear Compensator One type of compensator [5] consists of two weighted segmental gears engaging a pinion. Such an arrangement (with both gears represented by a single gear) is shown schematically in Fig. 5. Referring to the figure, the force accelerating the compensator mass toward the center of rotation is

$$29) \quad F = M \omega^2 v$$

where M may be considered to be the mass associated with two segmental gears. The moment of F about the center of rotation of the gear is

$$30) \quad T' = M \omega^2 v (r_p + R_g) \sin \alpha$$

and the reaction tangential force at the rim of the gear is

$$31) \quad F' = \frac{M \omega^2 v}{R_g} (r_p + R_g) \sin \alpha$$

The opposite of this force acts on the pinion and gives rise to the pinion torque

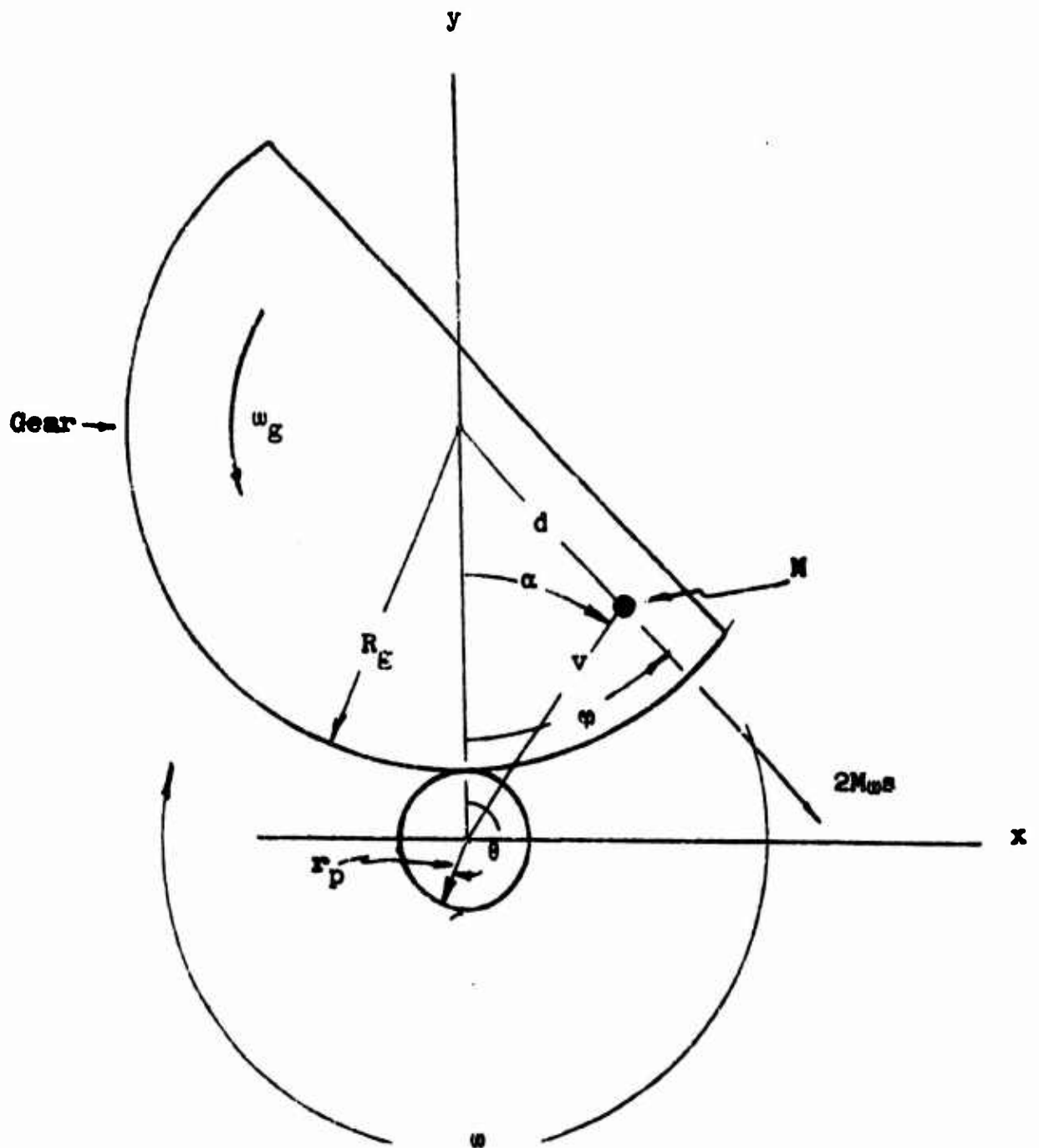


Fig. 5

$$32) \quad T_{c1} = \frac{r_p}{R_g} M \omega^2 v (r_p + R_g) \sin \alpha$$

however,

$$33) \quad v \sin \alpha = d \sin \phi$$

therefore 32) may be written

$$34) \quad T_{c1} = Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] \omega^2 \sin \phi$$

In addition

$$35) \quad r_p \theta = R_g \phi$$

and

$$36) \quad T_{c1} = Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] \omega^2 \sin \frac{r_p}{R_g} \theta$$

There will be a Coriolis force $2M\omega s$ acting through the center of rotation of the gear. The speed s in the rotating frame of reference is

$$37) \quad s = \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{1/2}$$

where

$$\frac{dx}{dt} = (\omega_g \cos \omega_g t) d$$

$$\frac{dy}{dt} = (\omega_g \sin \omega_g t) d$$

and ω_g is the angular velocity of the segmental gear. Assuming ω to be 1000π radians per second and ω_g to be $\pi/50$ radians per second the Coriolis force is

$$38) \quad F_c = \frac{40 \pi^2 \omega_g d}{386}$$

$$\approx \omega_g d$$

where ω_g is the weight of the gear and mass and d is the distance from the center of rotation of the gear to the center of gravity of the gear and mass. Setting $d = 1.21$ inches and $\omega_g = .165$ pounds, the Coriolis force is .2 pounds.

Connected Mass Compensator Another type of compensator that could be employed is shown schematically in Fig. 6. The force accelerating the compensator mass toward the center of rotation is

$$39) \quad F = Mw^2h$$

From equilibrium, the force transmitted to the arbor is

$$40) \quad P = Mw^2h \cos \gamma$$

$$= Mw^2L$$

The torque about the center of the arbor due to P is

$$41) \quad T_{c2} = Mw^2Lr_a$$

which may be written

$$42) \quad T_{c2} = Mr_a (L_o + r_a \theta) \omega^2$$

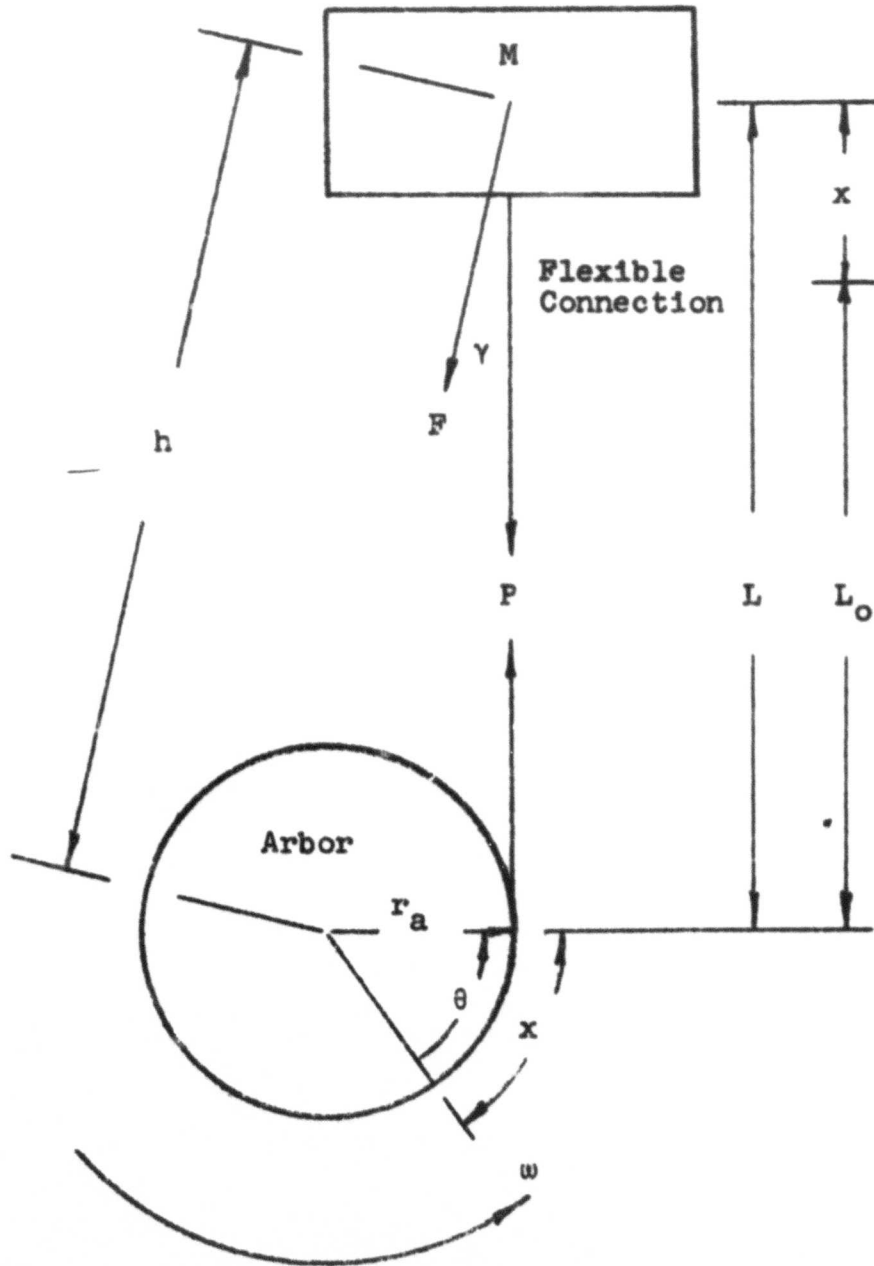


Fig. 6

SYNTHESIS

Symbols

$(T_{f2})_1$	Friction torque between i th and $i - 1$ leaves
T_{f1}	Torque due to spin induced friction in gear train
T_s	Torque due to windup of mainspring
T_w	Torque due to spin of mainspring
T_{c1}	Torque due to segmental gear compensator
T_{c2}	Torque due to attached mass compensator
r_a	Radius of mainspring arbor
m_k	Mass of a particular gear-shaft assembly
d_k	Diameter of a particular shaft
e_k	Eccentricity of a particular shaft
g_k	Gear ratio from shaft to main spring
u	Number of gear-shaft assemblies
u_1	Coefficient of friction between shaft and bearing
u_2	Coefficient of friction between spring leaves
l_i	Length of i th spring turn
t	Thickness of spring
w	Width of spring

l	Length of spring
E	Young's modulus
I	Moment of inertia of cross section of spring
ρ	Mass of spring leave per unit length
R_1	Inside radius of spring
R_2	Outside radius of spring
θ	Angle of unwind of spring arbor
θ_0	Angle of windup of spring arbor
ω	Angular velocity of projectile spin
D	Spring constant (see Fig. 4)
d_n	Distance between spring leaves
n	Number of leaves (turns) of spring
i	Spring leave number counting from arbor
M	Compensator mass
L_0	Initial length of compensator link
r_p	Radius of segmental-gear pinion
R_g	Radius of segmental gear
d	Distance from c.g. of segmental gear to center of rotation of gear

For ease of reference, the relations pertinent to a compensated design are repeated.

a) Gear Train Friction (Relation 6)

$$T_{f1} = \frac{2\mu_1}{\pi} \left[\sum_{j=1}^u d_k m_k e_k g_k \right] \omega^2$$

b) Friction Between Spring Leaves (Relation 20)

$$\begin{aligned} (T_{f2})_1 &= \frac{120 \mu_2 t w}{10^4} \left[r_a + (1 - 1)t \right] \{ A(n + 1 - 1) \\ &+ \frac{B}{2} \left[n(n + 1) - 1(1 - 1) \right] + \frac{C}{6} \left[n(n + 1)(2n + 1) \right. \\ &\left. - 1(1 - 1)(21 - 1) \right] \} \omega^2 \end{aligned}$$

c) Windup Torque of Spring (Relation 21)

$$T_s = \frac{EI}{l} (\theta_o - \theta) + D$$

d) Torque Due to Spin of Spring (Relation 23)

$$\begin{aligned} T_w &= \frac{\rho R_2^4}{l} \left[1 - 2 \frac{(R_2 - R_1)}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2} \right. \\ &\quad \left. + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^3} \right] (2n + 1) \pi \omega^2 \end{aligned}$$

- e) Number of Spring Turns as a Function of Angle of Unwind (Relation 24)

$$n = n_0 - \frac{\theta}{2\pi}$$

- f) Spin Decay (Relation 27)

$$\omega = \omega_0 \left(1 - \frac{p\theta}{200\pi}\right)$$

- g) Segmental Gear Compensator (Relation 36)

$$T_{c1} = Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] \omega^2 \sin \frac{r_p}{R_g} \theta$$

- h) Connected Mass Compensator (Relation 42)

$$T_{c2} = Mr_a (L_0 + r_a \theta) \omega^2$$

To synthesize a fully compensated system requires that the above relations be combined to give an expression for torque as a function of θ and that the parameters of the system then be adjusted until the coefficients of θ become zero.

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Segmental Gear Compensator Combining a), b), c),

d), and g) gives

$$\begin{aligned}
 T = & \left[- \frac{2\mu_1 u}{\pi} \sum_{k=1}^n d_k m_k e_k g_k \right. \\
 & - \frac{120 \mu_2 t w}{10^4} [r_a + 1 - 1)t] \{A(n + 1 - 1) \\
 & + \frac{B}{2} [n(n + 1) - 1(1 - 1)] + \frac{C}{6} [n(n + 1)(2n + 1) \\
 & - 1(1 - 1)(21 - 1)] \} + \frac{\rho R_2^4}{t} [1 - 2 \frac{(R_2 - R_1)}{R_2} \\
 & + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2} + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^3}] (2n + 1)\pi \\
 & + M d \frac{r_p}{R_g} (r_p + R_g) \sin \frac{r_p}{R_g} \theta \Big] w^2 \\
 & + \left[\frac{EI}{t} (\theta_0 - \theta) + D \right]
 \end{aligned}$$

43)

where

$$A = - \pi(2r_a t - t^2)$$

$$18) \quad B = \frac{\pi}{2} (2r_a d_n - 3td_n - 4t^2)$$

$$C = \frac{\pi}{2} (d_n^2 + 2td_n)$$

and

$$15) \quad d_n = \frac{2 [t/\pi - tn^2 - 2r_a n]}{n(n+1)}$$

Relation f) may be introduced in 43) in order to bring in spin decay effects. Thus using f) and setting

$$44) \quad A' = \frac{2\mu_1 u}{\pi} \sum_{j=1} d_k m_k e_k g_k$$

$$45) \quad B' = \frac{120\mu_2 tw}{10^4} [r_a + (1-1)t] \{A(n+1-1) + \frac{B}{2} [n(n+1) - 1(1-1)] + \frac{C}{6} [n(n+1)(2n+1) - 1(1-1)(21-1)]\}$$

$$46) \quad C' = \frac{\rho R_2^4}{t} \left[1 - 2 \frac{(R_2 - R_1)}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2} \right]$$

$$+ \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^3} \Big] (2n + 1)\pi$$

$$47) \quad D' = Md \frac{r_p}{R_g} (r_p + R_g) \sin \frac{r_p}{R_g} \theta$$

and re-grouping gives

$$\begin{aligned} & [(C' + D') - (A' + B')] \omega_o^2 + \frac{EI}{l} \theta_o + D \\ 48) \quad & - \left\{ [(C' + D') - (A' + B')] \omega_o^2 \frac{2p}{200\pi} + \frac{EI}{l} \right\} \theta \\ & + \left\{ [(C' + D') - (A' + B')] \omega_o^2 \left(\frac{p}{200\pi} \right)^2 \right\} \theta^2 = T \end{aligned}$$

Use of relation e) in order to bring in rundown effects introduces very great complexity because of B' , consequently the synthesis procedure that follows is based upon torque conditions at various specific states of rundown.

Compensation under the conditions implied in 48) can be obtained if the following relations are satisfied

$$49) \quad [(C' + D') - (A' + B')] \omega_o^2 + \frac{EI}{l} \theta_o + D = T$$

$$50) \quad [(C' + D') - (A' + B')] = \theta_o^2 \frac{2p}{200\pi} + \frac{EI}{l} = 0$$

$$51) \quad [(C' + D') - (A' + B')] = \theta_o^2 \left(\frac{p}{200\pi} \right)^2 = 0$$

Thus if

$$52) \quad C' + D' = A' + B'$$

and

$$53) \quad \frac{EI}{l} \ll T$$

then

$$54) \quad T = \frac{EI}{l} \theta_o + D$$

which is to say that output torque will remain constant with a magnitude T.

Relation 53) appears to be usually satisfied in standard designs, i.e., $EI/l \approx .1$ inch pounds.

It may be seen that A' and B' introduce friction torque losses due to bearing friction and spring friction respectively while C' and D' introduce torque gain

due to inertial effects on spring leaves and the compensating mass respectively. It should be pointed out that the physical configuration from which B' and C' are derived are inconsistent inasmuch as B' requires asymmetry whereas C' requires symmetry. It must therefore be assumed that C' is on the order of magnitude of the torque gain for an asymmetrical spring.

It should also be pointed out that 47) is a function of θ and that effective compensation requires that r_p/R_g be limited to the interval in the neighborhood of $\pi/2$. This limitation implies that $\sin \frac{r_p}{R_g} \theta$ has been expanded in a Taylor series about the point $\frac{r_p}{R_g} \theta = \frac{\pi}{2}$ and that all but the first term of the series has been discarded. Other equations similar to 52) can, for example, be developed by expanding $\sin \frac{r_p}{R_g} \theta$ about a point in the interval $0 \leq \frac{r_p}{R_g} \theta \leq \pi/2$ and discarding all but the first or the first two terms. However, the investigation of optimum points about which to expand $\sin \frac{r_p}{R_g} \theta$ should be held in abeyance until more insight is gained into the basic characteristics of the overall device.

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Connected Mass Compensator For the connected mass compensator relations a), b), c), d), and h) give

$$\begin{aligned}
 T = & \left[-2\mu_1 \sum_{k=1}^u d_k m_k e_k g_k - \frac{120 \mu_2 t w}{10^4} \right. \\
 & [r + (1 - 1)t] \{ A(n + 1 - 1) \\
 & + \frac{B}{2} [n(n + 1) - 1(1 - 1)] + \frac{C}{6} [n(n + 1)(2n + 1) \\
 54) & \left. - 1(1 - 1)(21 - 1)] \right\} + \frac{\rho R_2^4}{t} \left[1 - 2 \frac{(R_2 - R_1)}{R_2} \right. \\
 & + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2} + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^3} \left. \right] (2n + 1) \pi \\
 & + Mr_a (L_o + r_a \theta) \omega^2 \\
 & + \left[\frac{EI}{t} (\theta_o - \theta) + D \right]
 \end{aligned}$$

From f), 44), 45), 46), and 54)

$$T = \{ [C' - (A' + B')] + Mr_a L_o \} \omega_o^2 + \frac{EI}{t} \theta_o + D \}$$

$$\begin{aligned}
 & - \left\{ [C' - (A' + B') + Mr_a L_o] \frac{2p}{200\pi} \omega_o^2 \right. \\
 55) & \quad \left. - Mr_a^2 \omega_o^2 - \frac{EI}{l} \right\} \theta \\
 & + \left\{ [C' - (A' + B') + Mr_a L_o] \left(\frac{p}{200\pi} \right)^2 \right. \\
 & \quad \left. - Mr_a^2 \frac{2p}{200\pi} \right\} \omega_o^2 \theta^2 \\
 & + \left\{ Mr_a^2 \left(\frac{p}{200\pi} \right)^2 \omega_o^2 \right\} \theta^3
 \end{aligned}$$

Compensation under the conditions implied in 55) can be obtained if the following relations are satisfied

$$56) \quad [C' - (A' + B') + Mr_a L_o] \omega_o^2 + \frac{EI}{l} \theta_o + D = T$$

$$\begin{aligned}
 57) \quad & [C' - (A' + B') + Mr_a L_o] \frac{2p}{200\pi} \omega_o^2 \\
 & - Mr_a^2 \omega_o^2 + \frac{EI}{l} = 0
 \end{aligned}$$

$$58) \quad [C' - (A' + B') + Mr_a L_o] \left(\frac{p}{200\pi} \right)^2$$

$$- Mr_a^2 \frac{2p}{200\pi} = 0$$

$$59) \quad Mr_a^2 \left(\frac{p}{200\pi} \right)^2 w_o^2 = 0$$

If

$$60) \quad C' + Mr_a L_o = A_1 + B'$$

then from 56)

$$61) \quad \frac{EI}{l} \theta_o + D = T$$

and from 57)

$$62) \quad - Mr_a^2 w_o^2 + \frac{EI}{l} = 0$$

and from 58)

$$63) \quad Mr_a^2 \frac{2p}{200\pi} = 0$$

From 61) and 62)

$$64) \quad T = Mr_a^2 \omega^2 \theta_0 + D$$

but 59) and 63) cannot be satisfied unless spin decay is zero. Thus the design equation 60) will give good results only if

$$65) \quad Mr_a^2 \left(\frac{p}{200\pi}\right)^2 \omega_0^2 \ll T$$

and

$$66) \quad Mr_a^2 \left(\frac{2p}{200\pi}\right) \ll T$$

Dividing 63) by 59) gives

$$\frac{Mr_a^2 \left(\frac{2p}{200\pi}\right)}{Mr_a^2 \left(\frac{p}{200\pi}\right) \omega_0^2} = \frac{400\pi}{p\omega_0^2}$$

Assuming $\omega_0 \geq 10$ and $p \geq 1$, 63) will be larger than 59) and it is only necessary to satisfy 66).

EXAMPLES

Gear Train Friction The gear train friction torque reflected to the main spring may be calculated from relation 6)

$$6) \quad T_{f1} = \left[\frac{2\mu_1}{\pi} \sum_{k=1}^u d_k m_k e_k g_k \right] \omega^2$$

where μ_1 is coefficient of friction, u is the number of gear-shaft assemblies, d_k is the diameter of a particular shaft, m_k is the mass of a particular shaft-gear assembly, e_k is the eccentricity (distance from center of shell rotation) of a particular shaft, and g_k is the gear ratio from a particular shaft to the mainspring. The bracketed term in 6) has been calculated using the values shown in Table 1 and gives

$$67) \quad T_{f1} = 3.95 (10^{-6}) \omega^2 \quad \text{in.oz.}$$

$$= A' \omega^2$$

Mainspring Friction The torque due to friction between the i 'th and $i - 1$ spring leaves may be calculated from 20)

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$$\begin{aligned}
 (T_{f2})_1 &= 120\mu_2 \tan(10^{-4}) [r_a + (1 - 1)t] \{A(n + 1 - 1) \\
 20) &+ \frac{B}{2} [n(n + 1) - 1(1 - 1)] + \frac{C}{6} [n(n + 1)(2n + 1) \\
 &- 1(1 - 1)(21 - 1)] \} \omega^2
 \end{aligned}$$

where

$$A = -\pi(2r_a t - t^2)$$

$$B = \frac{\pi}{2} (2r_a d_n - 3td_n - 4t^2)$$

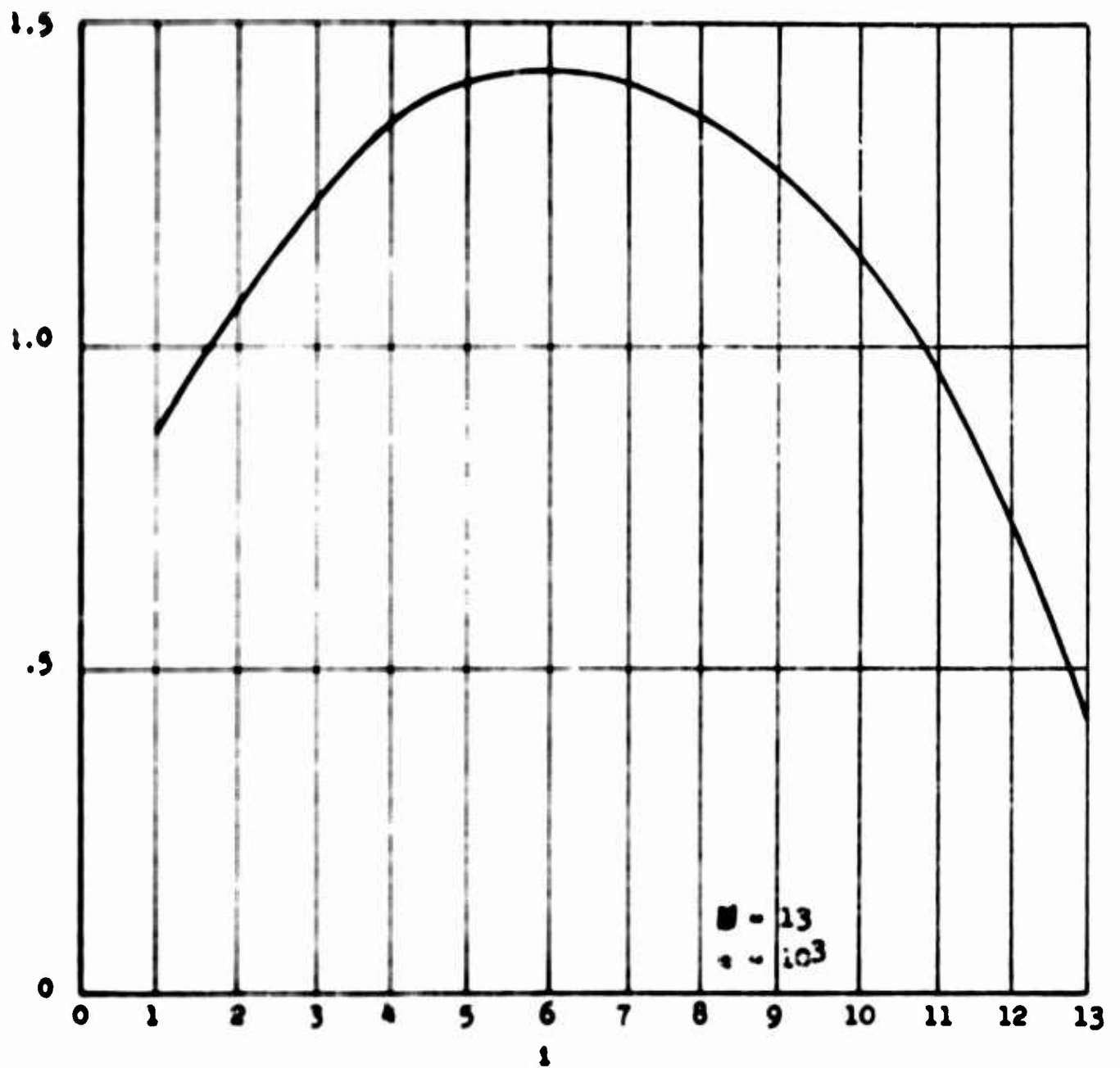
$$C = \frac{\pi}{2} (d_n^2 + 2td_n)$$

$$d_n = \frac{2[1/\pi - tn^2 - 2r_a n]}{n(n + 1)}$$

The variation of $(T_{f2})_1$ with respect to i is shown on page 36 for the following quantities

$$\mu_2 = .17 \quad \text{coefficient of friction}$$

$$r_a = .14'' \quad \text{radius of arbor}$$



SPIN INDUCED FRICTION TORQUE
BETWEEN 1TH AND 1-1 LEAVES OF
SPRINGS

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TABLE 1

Gear Train Friction

Mechanism in Center

$$\mu_1 = .17$$

k (Shaft) no.	d_k (inches)	m_k (oz. sec ² /in.)	e_k (inches)	g_k	$\frac{2\mu_1}{\pi} d_k m_k e_k g_k$
us	$97.8(10^{-3})$	$228(10^{-6})$	0	1	0
4	$49.2(10^{-3})$	$134(10^{-6})$	.314	2.33	$521(10^{-9})$
3	$30.0(10^{-3})$	$40.3(10^{-6})$	.284	7.0	$270(10^{-9})$
2	$22.1(10^{-3})$	$25.6(10^{-6})$	.312	21.10	$401(10^{-9})$
1	$18.3(10^{-3})$	$20.9(10^{-6})$	.210	78.8	$684(10^{-9})$
0	$18.6(10^{-3})$	$13.8(10^{-6})$	.211	354	$2070(10^{-9})$
BL	$18.0(10^{-3})$	$7.69(10^{-6})$	0	-	-

$$\frac{2\mu_1}{\pi} \sum_{k=1}^u d_k m_k e_k g_k = 3946(10^{-9})$$

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$w = .25''$ width of spring

$t = .015''$ thickness of spring

$l = 26.5''$ length of spring

$n = 13$ number of turns of spring
($n = 16.2$ when $d_n = 0$)

$\omega = 1000$ angular velocity of spin

The shape of the torque versus spring-leave curve suggests that relative motion between spring leaves begins at the outside of the spring where the friction torque is the least and that gross readjustments may occur as the spring unwinds.

For $n = 13$, the maximum friction torque between spring leaves is, from the above theory

$$(68) \quad (T_{f2})_6 = 1.425(10^{-6}) \omega^2$$

$$= B' \omega^2 \quad \text{in.oz.}$$

Mainspring Torque If the length, width, and thickness of the mainspring are taken as above and if E is $48(10^{-7})$ ounces per square inch, then EI/l is 1.28 inch ounces and [relation 21)]

$$69) \quad T_s = .0797 (\theta_0 - \theta) + D$$

The torque gain due to the spin of the spring may be calculated from 23)

$$23) \quad T_s = \frac{2R_2^4}{l} \left[1 - 2 \frac{(R_2 - R_1)}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2} + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^3} \right] (2n + 1) \pi \omega^2$$

If R_1 and R_2 , the inside and outside radii of the spring, are taken to be .14" and .6" respectively, and if n and l are taken as above then 23) becomes

$$70) \quad T_s = 9(10^{-6}) \omega^2 \quad \text{in.oz.} \\ = C' \omega^2$$

Segmental Gear Compensator The torque gain due to the segmental compensator is [relation 36]

$$36) \quad T_{cl} = Md \left[\frac{r_p}{R_g} (r_p + R_g) \omega^2 \sin \frac{r_p}{R_g} \theta \right]$$

If $\sin \frac{r_p}{R_g} \theta$ is approximated by the first term of a Taylor's expansion about $\theta = \frac{\pi R_g}{r_p}$ then 36) becomes

$$\begin{aligned} 71) \quad T_{c1} &= Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] \omega^2 \\ &= D' \omega^2 \end{aligned}$$

If the appropriate parts of 67), 68), 70), and 71) are substituted in 52), then for $n = 13$ the design criterion for a segmental gear compensator becomes

$$72) \quad 9(10^{-6}) + Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] = 3.95(10^{-6}) + 1.425(10^{-6})$$

where the terms in 72) may be identified from

$$52) \quad C' + D' = A' + B'$$

It thus appears that C' , the term contributed by the torque gain due to the spinning spring, is by itself sufficiently large to compensate for A' and B' which are the terms contributed by torque losses due to gear train and spring friction respectively. However, it should be remembered that C' strongly depends upon n , R_2 , and the analytical function chosen to represent the spring. It

would seem to be highly desirable to investigate this term more thoroughly.

If C' is neglected then 72) becomes

$$73) \quad Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] = 5.375(10^{-6})$$

and if [5]

$$r_p + R_g = .452$$

$$r_p = \frac{1}{3} R_g$$

$$d = .19$$

then 73) gives $M = 1.875(10^{-4})$ which corresponds to a weight of .0725 oz., a feasible order of magnitude.

Connected Mass Compensator If the appropriate parts of 67), 68) and 70) are substituted in 60) then for $n = 13$ the design criterion for a connected mass compensator becomes

$$74) \quad 9(10^{-6}) + Mr_a L_o = 3.95(10^{-6}) + 1.425(10^{-6})$$

where the terms in 74) may be identified from

$$60) \quad C' + Mr_a L_o = A' + B'$$

Inasmuch as the remarks made earlier about C' also apply to 74) it will be neglected. Relation 74) then becomes

$$75) \quad Mr_a L_o = 5.375(10^{-6})$$

If r_a and L_o are taken to be .14" and .2" respectively then $M = 1.92(10^{-4})$ which corresponds to a weight of .0741 ounces, a feasible order of magnitude.

Relation 66) gives

$$76) \quad \frac{4.62}{10^6} p \ll T$$

which is satisfied for any conceivable value of p . Thus 74) is valid.

CONCLUSIONS

It would seem that inertially generated torques can be used to compensate for friction, run down, and spin decay energy losses in timing mechanisms. However, it appears that two elements of the timing mechanism should be investigated further before dependable design proce-

dures can be developed. These areas are the torque gain due to spring spin, and torque loss due to friction between spring leaves.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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