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Appendix B

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APPENDIX B

THE INTERACTION OF BURIED STRUCTURES WITH GROUND SHOCK

of

CONCEPTS OF PRELIMINARY DESIGN OF

STRUCTURE PROJECTS FOR UNDERGROUND NUCLEAR DETONATIONS

by

A. H. Wiedermann

Armour Research Foundation

of

Illinois Institute of Technology

January 1960

HEADQUARTERS
AIR FORCE SPECIAL WEAPONS CENTER
Air Research and Development Command
Kirtland Air Force Base, New Mexico

Major General Charles M. McCorkle
Commander

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Director, Research Directorate

AFSWC-TR-60-3 - App-B

~~APPENDIX B~~

THE INTERACTION OF BURIED STRUCTURES WITH GROUND SHOCK

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CONCEPTS OF PRELIMINARY DESIGN OF STRUCTURE PROJECTS FOR UNDERGROUND NUCLEAR DETONATIONS, APPENDIX B.

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Research Directorate
AIR FORCE SPECIAL WEAPONS CENTER
Air Research and Development Command
Kirtland Air Force Base, New Mexico

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ABSTRACT

This appendix presents a theory or concept which can be used to estimate the forces acting on a buried structure when the structure is subjected to a ground shock wave. This work is intended to be applicable primarily for the type of rock found at the Nevada Test Site, such as white or reddish tuff.

A discussion of the free field variables is given together with an idealization of the stress wave forms of interest. The basis of the soil-structure interaction concept lies in the assumption of the nature of the forces acting on the buried structure. The force acting on the structure is assumed to be composed of two parts. One, the wave force, is due to the sudden motion of the surrounding media and the subsequent state of stress in the ground shock wave. The other, the arching force is due to the local deformation or displacement of the structure relative to the surrounding media. The motion of the structure is governed by the usual equation of motion, however, in this treatment the forcing term has been split into two uncoupled terms. The wave force term is a function of time and the arching force term is a function of the relative displacement between the structure surface and its corresponding soil position.

The response of the buried structure is given for a range of ground shock wave parameters, structure configurations and structure parameters, and includes some analysis of shock isolation systems.

PUBLICATION REVIEW

This report has been reviewed and is approved.

CAREY L. O'BRYAN, JR.
Colonel USAF
Deputy Commander

LIST OF SYMBOLS

Dimensional Symbol		Dimensionless Symbol
	Amplitude of Oscillation	A
	Minimum Amplitude	A_{1m}
	Coefficients	B_1, B_2, B_3, B_4, B_5
c_o	Seismic Velocity	
c'_o	Seismic Velocity	
	Coefficients	c_1, c_2
C_a	Arching Coefficient	
C_f	Friction Coefficient	
	Functions	C_1, C_2
D	Diameter of Structure	
E	Modulus of Elasticity of Soil	
E_b	Modulus of Elasticity of Buffer	$\mathcal{E}$
	Coefficients	E_1, E_2, E_3, E_4, E_5
F_a	Arching Force	
F_b	Force on Back Face of Structure	$\mathcal{F}_b$
F_f	Force on Front Face of Structure	$\mathcal{F}_f$
F_{ba}	Arching Force on Back Face of Structure	$\mathcal{F}_{ba}$
F_{bw}	Wave Force on Back Face of Structure	$\mathcal{F}_{bw}$
F_{fa}	Arching Force on Front Face of Structure	$\mathcal{F}_{fa}$
F_{fw}	Wave Force on Front Face of Structure	$\mathcal{F}_{fw}$
k	A Constant	$\mathcal{I}$
K	Flexibility Parameter	λ
ℓ	Buffer Thickness	$\mathcal{L}$
$\Delta \ell$	Change in Buffer Thickness	

LIST OF SYMBOLS
(Continued)

Dimensional Symbol		Dimensionless Symbol
L	Length of Structure	τ
L'	Compressed Length of Structure	
L_o	Critical Length of Structure	τ_o
m	Mass of a Structure Element	$\bar{\beta}^2$, $\bar{\mu}^2$
M_o	Mass of the Structure	β^2, μ^2
n	An Exponent	
t	Time	T
t_c	Time Difference	
t_d, t_o	Critical Times of Free-Field Waves	δ, γ
u	Particle Velocity of Soil	U_s
u_s	Particle Velocity of Soil at Front of Wave	
u_1, u'_1	Particle Velocities in Soil and Crushable Material	
U'	Compaction Front Velocity	
	Velocity of Structure	U, $\dot{Y}$
	Velocity of Structure at $T = \tau$	U^*
w	Displacement of Base Structure	W
x, x'	Displacement of Soil	X, X'
x_o	A Constant	
y, y'	Displacement of the Structure	Y
	Displacement of the Structure at $T = \tau$	Y^*
z, z'	Relative Displacement	Z, Z'
	Stress Relief Parameter	α, α'
Δ	Increment of Time	

LIST OF SYMBOLS
(Continued)

Dimensional Symbol		Dimensionless Symbol
ϵ_{xx}, ϵ	Strain in Soil	
ϵ_c	Crushing Strain	
	Buffer Parameter	ν
ρ	Density of Soil	
ρ_o	Ambient Density of Soil	
σ	Stress in Soil	
σ_o	A Constant	
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_1, \sigma'_1$	Stress	
σ_c	Crushing Stress	

o, o', r, r', r_1, r'_1, s, s'	Refer to States of the Material
{ }	Denotes Functional Relationship
$(\dot{\ })$	Denotes Derivative with Respect to Time

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APPENDIX B

THE INTERACTION OF BURIED STRUCTURES WITH GROUND SHOCK

I. INTRODUCTION

The purpose of this appendix is to present a theory which can be used to estimate the forces acting on a buried structure when the structure is subject to a ground shock wave.

A discussion of the free-field variables is given together with an idealization of the stress wave forms of interest. This is followed by a discussion of the nature of the forces acting on a buried structure. The last section deals with the response of the buried structure and includes a number of calculations to illustrate the effect of certain free-field variables, as well as structure parameters on the resulting motion of the structure. An analysis of isolation systems is also included in this section.

II. FREE-FIELD VARIABLES

The free-field variables are necessary inputs for the study of the interaction of ground shocks with buried structures. It is the purpose of this section to delineate these variables and to elaborate, somewhat, on the assumptions which must be made to make the problem tractable.

In general, interest lies in ground shock disturbances whose stress levels are in the range of hundreds of pounds per square inch. The range of earth media of interest is essentially unlimited; thus, at the level of stress of interest, we are dealing with media which behave physically in vastly different ways; that is, the behavior of the various media can be described as elastic, plastic, fluid, elastic-plastic, visco-elastic, etc. Any attempt to formulate a simple description of the behavior of the free-field variables must be tempered by the physical behavior of the media of interest. In general, this specific report will deal with a medium which is essentially elastic in its gross behavior.

The type of ground shock of interest in this report is restricted to plane waves which do not attenuate significantly in distances of the order of the size of structure of interest. Also, no boundary effects of the medium are considered (i.e., the medium is infinite in extent). The disturbances considered have fronts which are discontinuous (step pulse) or have finite rises; the stress fields behind the front are either uniform or decaying with time. Figure B-1 illustrates the stress-time variations and the following equations are idealizations of these variations:

$$\text{(a)} \quad \sigma = \sigma_o \qquad 0 \le t \le \infty \qquad \text{(B-1)}$$

$$\text{(b)} \quad \sigma = \sigma_o e^{-t/t_d} \qquad 0 \le t \le \infty \qquad \text{(B-2)}$$

$$\text{(c)} \quad \sigma = \sigma_o (1 - e^{-t/t_o}) \qquad 0 \le t \le \infty \qquad \text{(B-3)}$$

$$\text{(d)} \quad \sigma = \sigma_o (1 - e^{-t/t_o}) e^{-t/t_d} \qquad 0 \le t \le \infty \qquad \text{(B-4)}$$

where σ is stress, t is time, and σ_o, t_D and t_o are constants. Equation (B-4) is the general case. Thus, precursor type waves are excluded from the following analysis. The propagation velocity of the disturbance, c_o, is assumed to be constant and a function only of the media and is associated with a dilatational wave.

Since the disturbance is plane and in an infinite medium, only one non-zero component of strain exists in the free field, $\epsilon_{xx} = \epsilon$ (where x is the direction of propagation of the disturbance). The stress components are related to the strain component, but this relationship is unknown for the general case. For an elastic medium the most important stress is the normal stress in the direction of the propagation of the disturbance (σ_{xx}) and of considerably smaller magnitude and interest are the stresses normal to σ_{xx} (i.e., σ_{yy} and σ_{zz}). For the purposes of the following analysis the normal stress $\sigma_{xx} = \sigma$ will be the only stress considered. For a fluid-like behavior (which is the case treated in Appendix A), the hydrostatic pressure will correspond to this stress level.

Across a discontinuous front or quasi-stationary non-discontinuous front, the following equations are valid.

Continuity of Mass: $\rho_o c_o = \rho (c_o - u)$ (B-5)

Continuity of Momentum: $\sigma = \rho_o c_o u$ (B-6)

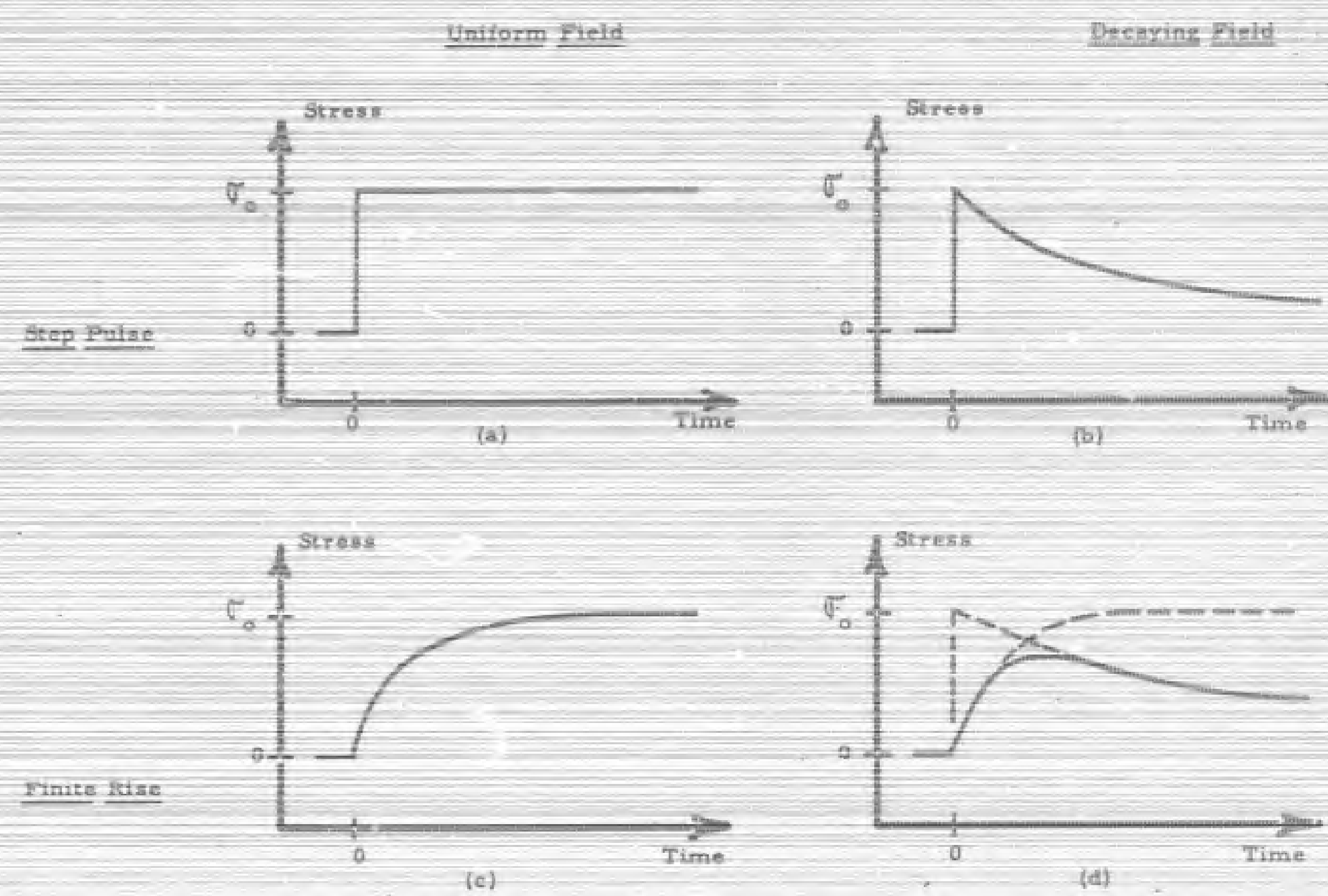


Fig. B-1 CLASSES OF GROUND SHOCK WAVES

where:

ρ_o = ambient density of the earth media

ρ = density of the media at the stress level σ

c_o = sound velocity

u = particle velocity of the media at the stress level σ

It should be pointed out that the disturbance is advancing into a medium which is at rest and at a zero state of stress. Thus, the stress due to gravity (overburden) is neglected when compared to the σ_{xx}.

Equation (B-6) yields a relationship for u, directly,

$$u = \frac{\sigma}{\rho_o c_o} \tag{B-7}$$

By using the equations for the propagation of a disturbance in an elastic medium,

$$c_o^2 = \frac{E}{\rho_o} , \tag{B-8}$$

we obtain

$$u = \frac{\sigma c_o}{E} , \tag{B-9}$$

where E is the modulus of elasticity for the medium. In any event, the particle velocity is proportional to the stress where the constant of proportionality is a function of the properties of the medium. The ambient density of the medium, ρ_o, and the sound velocity, c_o, are readily obtainable and should be known to a reasonable degree of accuracy.

The velocity-time variation cannot be derived; therefore, it is assumed that the instantaneous particle velocity is related to the instantaneous stress by Eq (B-7) or (B-9).

The absolute displacement of a particle of the medium is given by

$$x - x_o = \int_0^t u_t dt \tag{B-10}$$

where x_o is the original position. For the case corresponding to Eq (B-1)

$$x - x_o = u_p t \qquad \text{(B-11)}$$

or, using Eq (B-9) and letting $x_o = 0$,

$$x = \frac{\sigma_o c_o t}{E} \qquad \text{(B-12)}$$

III. NATURE OF THE FORCES ACTING ON A BURIED STRUCTURE

The purpose of this section is to present and discuss the types of forces which are assumed to act on a buried structure when this structure is struck by ground shock. The state of the art is quite meager at the present time and a considerable amount of work, both experimental and theoretical, must be done before the interaction phenomenon will be satisfactorily understood.

For the purpose of this program, the geometry of the buried structure is taken as a cylinder of diameter D and length L. Sections other than a circular section can also be treated in a similar manner with minor modifications. The direction of propagation of the plane disturbance is in the axial direction such that the interaction first occurs on the end of the cylinder.

Whatever forces are applied to the buried structure are applied to the three surfaces (front, back, and side) of the structure. These surfaces will form the basis for categorizing the forces in the following treatment.

For the purposes of making estimates of the response of a buried structure when subjected to ground shock, the following types of forces are assumed to act on the structure:

1. Wave Forces
2. Arching Forces
3. Shear Forces

These forces, which are explained in detail in the following paragraphs, do not have a firm basis. However, it was thought at this time, that the problem could best be approached by making assumptions as to the nature of the forces involved rather than making assumptions about any other physical parameters. In addition to the above forces there may exist normal compressive forces on the sides of the structure. These normal forces are symmetrically distributed and do not give rise to any net response in the direction of wave propagation. The effects of these forces are neglected in this report.

1. Wave Forces

It is well known that, when an acoustic wave reflects normally from an infinitely rigid boundary of infinite extent, the intensity of the disturbance is increased by a factor of two. If the boundary is another media of identical physical properties, the disturbance will be transmitted without any reflection and the intensity of the disturbance at this fictitious boundary will be equal in magnitude to the intensity of the original disturbance. If the acoustic impedance of the boundary media lies somewhere between the above two cases, the intensity of the reflected disturbance (at the boundary) will vary by a factor of one to two.

For the purposes of this program, it is assumed that the rigidity of the front face of the structure will be considerably greater than that of the surrounding media so that the reflection factor will be equal to two. Under certain circumstances, which will not be discussed here, the reflection factor could be greater than two or less than one.

We are dealing here with structures of finite size, such that any local wave reflections will be relieved in a period of time which is a function of the properties of the media and the size of the structure. To a first approximation, this time will be proportional to the size, in this case the diameter, D, and inversely proportional to the propagation velocity, c_o.

When any forces are applied to the structure, the structure will begin to move as a whole or to deflect locally. This motion will not, in general, correspond to the motion of the surrounding media. That is, there will be a relative displacement of the structure and, in the neighborhood of the structure, the displacement of the media will be distorted from the displacement which would have occurred if the structure had not been present. In attempting to separate the forces which act on a single face of the structure, we are assuming that the stresses in the media which give rise to these forces are also separable. This is only the case for a medium which is governed by linear differential equations. Thus, to proceed further using this approach, it is necessary to assume that the principle of superposition applies to this problem area, at least in an approximate manner.

By making use of the principle of superposition, it is possible to determine the magnitude of the wave force on the front face of the structure after local reflections have been relieved. The term "wave force" here refers to those forces which are due to the sudden motion of the surrounding media and the subsequent state of stress in the ground shock wave as differentiated from those forces arising from the local deformation or displacement of the structure relative to the surrounding media. Thus, the magnitude of the wave force on the front face of the structure after the local reflections have been relieved is equal to that force which would exist if the structure-soil interface were displaced so that the surrounding medium was not disturbed by the presence of the structure. The magnitude of the wave force will then be equal to the product of the front face area and the instantaneous stress (σ_{xx}) in the soil due to the ground shock wave.

For the case of a step pulse disturbance as defined by Eq (B-1), the wave force on the front face, F_{fw}, has an initial value of $2\,\sigma_o\,\pi D^2/4$ and then decays down to a value equal to $\sigma_o\,\pi D^2/4$. To formulate the problem analytically, it is necessary to make some specific assumptions as to the relief time of the reflected wave. We, therefore, assume that the relief time will be characterized by a time equal to $\alpha D/2\,c_o$ where α is a constant which should be of the order of one. We will use a value of $\alpha = 2$ for numerical purposes. Thus, the wave force on front face corresponding to the step disturbance can be written as

$$F_{fw} = \sigma_o\,\pi D^2/4\,\left[1 + e^{-t\,2\,c_o/\alpha D}\right] \qquad \text{(B-13)}$$

where $t = 0$ when the disturbance reaches the front of the structure (see Fig. B-4a). It should be emphasized here that the wave force is only a function of time, that is, the response of the structure itself does not affect this force.

The reflection of an acoustic disturbance of finite rise-time (Eq. (B-3)) on a rigid boundary of infinite extent can be approximated quite well by multiplying the instantaneous undisturbed stress at the boundary by a factor of two (the reflection factor). Thus, the wave force on the front face corresponding to this case can be expressed as

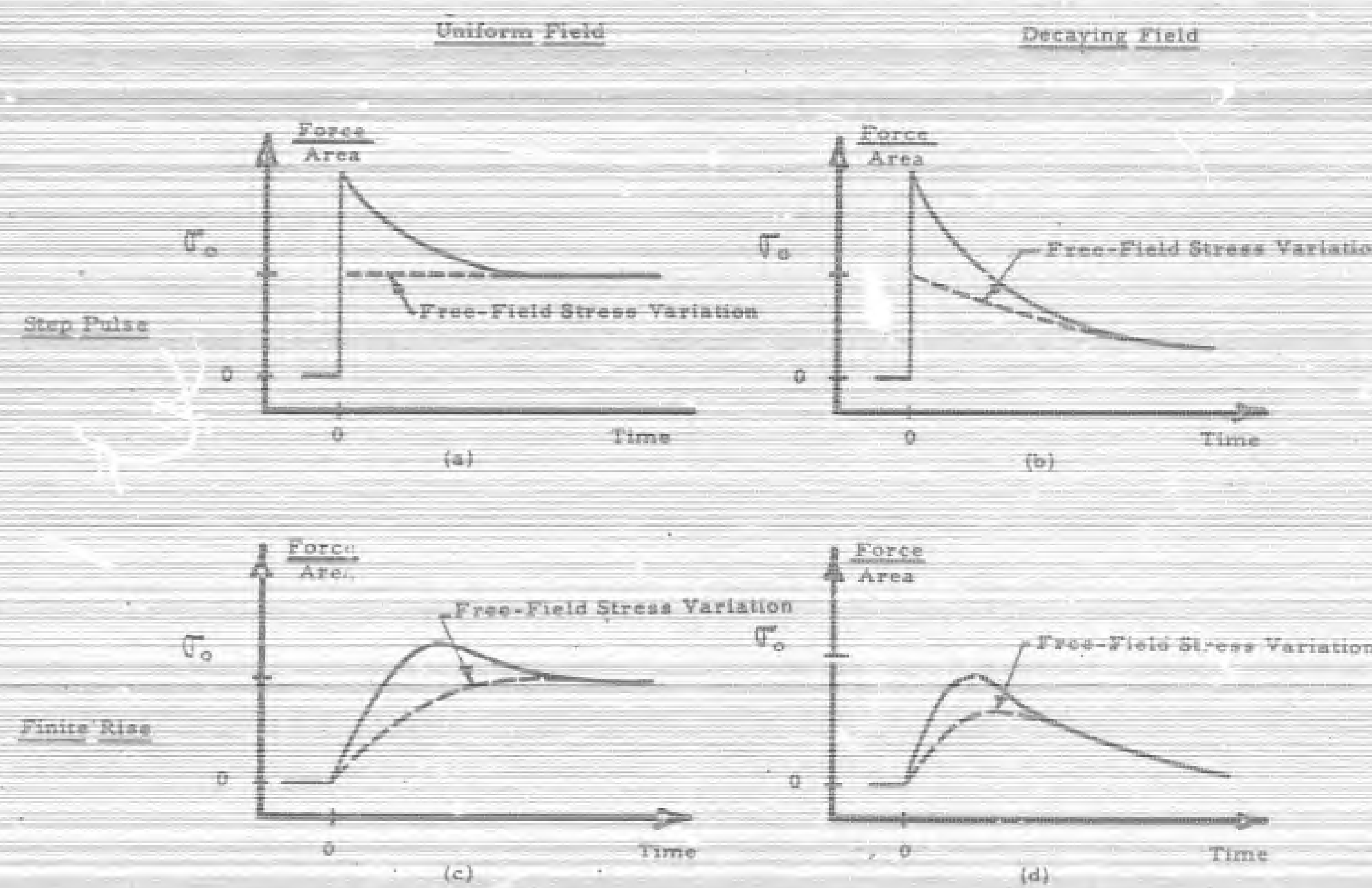


Fig. B-2 WAVE FORCES ON FRONT FACE OF STRUCTURE

$$F_{fw} = \sigma_o \pi D^2/4 \left[1 + e^{-t\,2\,c_o/\alpha D}\right]\left[1 - e^{-t/t_o}\right] \qquad \text{(B-14)}$$

Correspondingly, we can write equations for the free-field disturbances associated with the ground shock waves described by Eq (B-2) and (B-4), respectively, as

$$F_{fw} = \sigma_o \pi D^2/4 \left[1 + e^{-t\,2\,c_o/\alpha D}\right] e^{-t/t_d} \qquad \text{(B-15)}$$

and

$$F_{fw} = \sigma_o \pi D^2/4 \left[1 + e^{-t\,2\,c_o/\alpha D}\right]\left[1 - e^{-t/t_o}\right] e^{-t/t_d} \qquad \text{(B-16)}$$

Eq (B-16) being the general case. See Fig. B-2 for an illustration of the wave force on the front face of the structure.

A wave type of force also exists on the rear face of the structure. This force will not come into effect until the disturbance reaches the back face, that is, until $t = L/c_o$. The time required for the disturbance to propagate the slight additional distance that the rear face of the structure has moved during this period has been neglected. When the disturbance wave reaches the rear of the structure, it will propagate around the back corner and begin to increase the wave force, F_{bw}, from zero to a value corresponding to the free-field value of the disturbance. The build-up time is similar in character to the relief time; hence, the same assumptions and constants which were used in developing the wave force on the front face of the structure will be used. The wave force on the back face can then be written as

$$F_{bw} = \sigma_o \pi D^2/4 \left\{1 - e^{-\left[t\,2\,c_o/\alpha D - L2/D\alpha\right]}\right\} \qquad \text{(B-17)}$$

corresponding to the disturbance wave defined by Eq (B-1)

$$F_{bw} = \sigma_o \pi D^2/4 \left\{1 - e^{-\left\{t\,2\,c_o/\alpha D - L2/D\alpha\right\}}\right\} e^{-\left[t/t_d - L/c_o t_d\right]} \qquad \text{(B-18)}$$

corresponding to the disturbance wave defined by Eq (B-2), and

$$F_{bw} = \sigma_o \pi D^2/4 \left\{1 - e^{-\left[t\,2\,c_o/\alpha D - L2/\alpha D\right]}\right\}\left\{1 - e^{-\left[t/t_o - L/c_o t_o\right]}\right\} \tag{B-19}$$

corresponding to the disturbance wave defined by Eq (B-3), and the general case

$$F_{bw} = \sigma_o \pi D^2/4 \left\{1 - e^{-\left[t\,2\,c_o/\alpha D - L2/\alpha D\right]}\right\}\left\{1 - e^{-\left[t/t_o - L/t_o c_o\right]}\right\} e^{-\left[t/t_d - L/t_d c_o\right]} \tag{B-20}$$

corresponding to the disturbance wave defined by Eq (B-4). Figure B-3 illustrates this set of equations. These equations, Eq (B-17) through (B-20), are valid for $t \leq L/c_o$. During the interval of time $0 \leq t \leq L/c_o$, the wave forces on the back face of the structure vanish.

The effects of the disturbance wave on the side face(s) of the structure will be treated in the section on shearing forces.

2. Arching Forces

Those forces acting on the front and rear faces of the structure which arise from the relative displacement (or deformation) between the structure and the surrounding medium are called the arching forces. This term is perhaps not the best term; however, it is used here for the lack of a more descriptive one.

Consider the static force which exists on the end of a rigid cylinder when the cylinder is forced axially into an earth medium. The deflection of the cylinder face from the position of zero force is given by Z and the frictional forces on the side of the cylinder are neglected. Then this force, called the arching force, F_a is approximated by the simple power law

$$F_a = \pi D^2/4 \; C_a (Z)^n \tag{B-21}$$

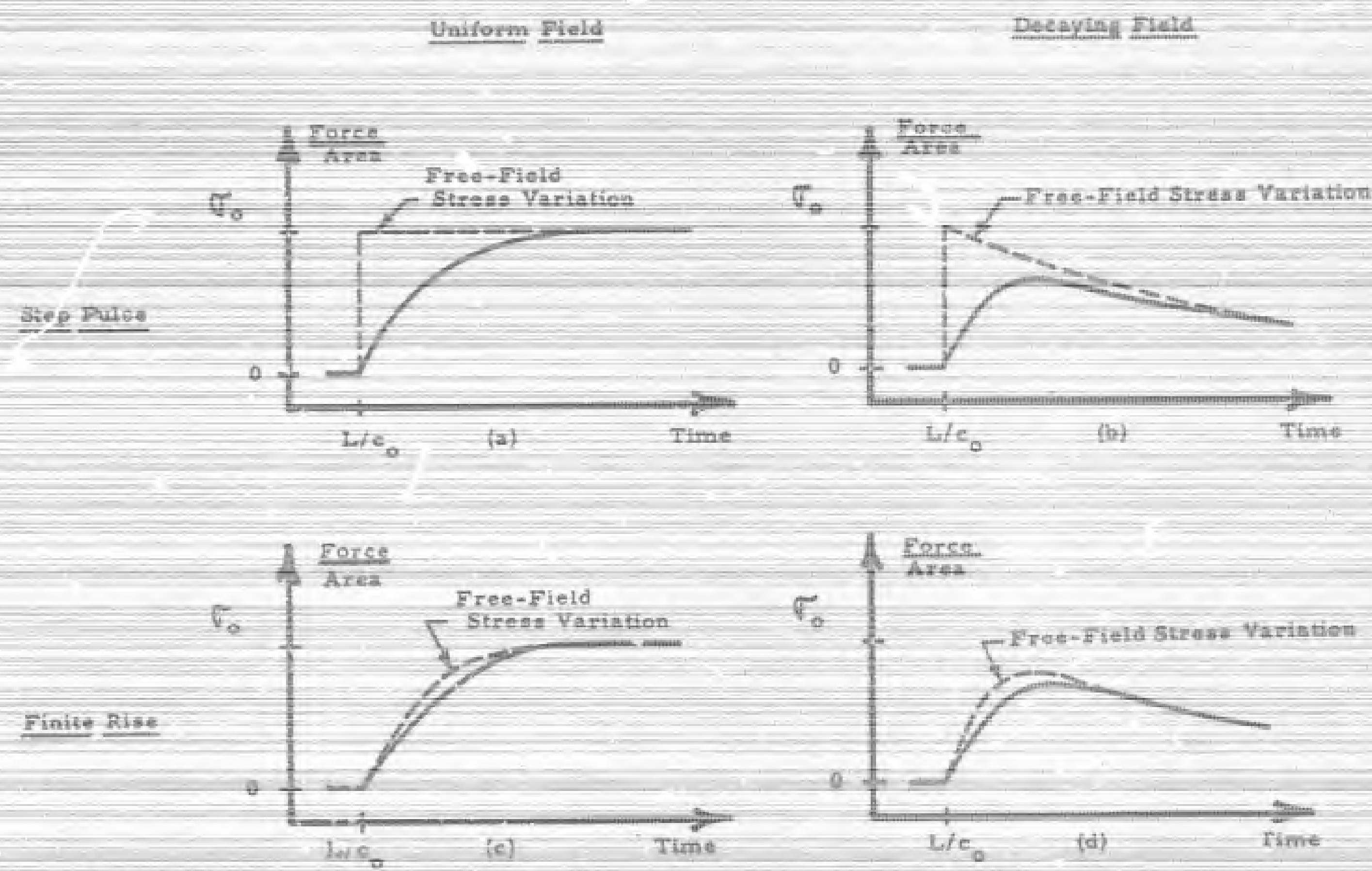


Fig. B-3 WAVE FORCES ON BACK FACE OF STRUCTURE where C_a and n are constants. This type of law does not take into consideration the many other factors which exist such as loading rates, hysteresis effects, ambient stress level, etc., but at the present time Eq (B-21) is more general than we can actually treat with the present state of the art. For the elastic case, the arching coefficient, C_a, is known for the cylinder[1] to be:

$$C_a = \frac{\pi E}{2D} \quad \text{(B-22)}$$

and n is, of course, equal to one.

Through the application of the principle of superposition, it is possible for the arching force to become negative; that is, it is only necessary to require that the total force acting on the front or back face of the structure be zero or greater. Since the total force acting on the front or back face of the structure is equal to the sum of the arching and wave forms, the arching force can become negative to the extent that the wave force is positive.

In defining the arching force as we have done, we are actually assuming that the end of the cylinder does not deflect locally, or if it does, that the relative displacement is a mean value. For this assumption to apply, the relative displacement must be large compared to any local deflections.

It is now necessary to write the more specialized equations for the arching forces acting on the front and back faces respectively. Let x be the absolute displacement of the soil particle at the position of the front face when the structure is not present. Let y be the absolute displacement of the front face of the structure. Then

$$z = x - y \quad \text{(B-23)}$$

is the relative displacement and Eq (B-21) becomes for the front face

$$F_{fa} = \pi D^2/4 \; C_a (x - y)^n \quad \text{(B-24)}$$

[1] Love, A. F. H., "The Stress Produced in a Semi-Infinite Solid by Pressure on Part of the Boundary", Transaction Royal Society (London), Series A, pp. 377-402, 1928.

If x', y', and z' are the corresponding variables for the back face of the structure, then for the back face

$$F_{ba} = \pi D^2/4 \; C_a \; (y')^n \qquad 0 \leq t \leq L/c_o \qquad (B-25)$$

and

$$F_{ba} = \pi D^2/4 \; C_h \; (y' - x')^n \qquad L/c_o \leq t \qquad (B-26)$$

It should be noted that x' = 0 during the time interval $0 \leq t \leq L/c_o$.

3. Shearing Force

A shear force between the side of the structure and the surrounding medium will exist. This force will resist any relative motion between the soil and the structure. If there is no sliding between the structure and the soil, one would be inclined to assume that the shearing force was approximately proportional to the relative displacement. However, it appears that there will be some relative motion at the soil structure interface; therefore, we have assumed, for the purpose of this program, that the shear force, F_s, acting on the side of the structure is proportional to the relative velocity of the soil (in the absence of the structure) and the structure. We can express this force analytically as

$$F_s = C_f \left\{ \pi D \; t \; c_o \; (u - \dot{y}) + \pi D \; (L - t \; c_o) \; \dot{y} \right\} \qquad 0 \leq t \leq L/c_o \qquad (B-27)$$

or

$$F_s = C_f \left\{ \pi DL \; (u - \dot{y}) \right\} \qquad L/c_o \leq t \qquad (B-28)$$

where:

$\dot{y}$ is the absolute velocity of the structure

C_f is the coefficient of friction

The value of the friction coefficient is not well known. However, by properly treating the surface on the side of the structure, one should be able to minimize the friction effect such that both C_f and F_s tend to zero. This force is of secondary importance in its influence on the arching phenomenon but does influence to some degree the response of buried structures. In conducting simple experiments one should attempt, at least initially, to minimize the effect of the shearing force. For the purpose of the subsequent analysis, we will asume that $C_f = 0$ and the shear force will be omitted.

IV. RESPONSE OF BURIED STRUCTURES

The previous sections have delineated the free-field variables and the subsequent forces which are assumed to act on the buried structure. The response of the buried structure will be treated in this section for the special cases of a rigid structure buried in an elastic medium.

The term "rigid structure" is used here to designate the treatment of the response of the structure as a single degree of freedom system. Thus, the absolute deflection of the structure as a whole (and the absolute deflection of both the front and back surfaces) can be characterized by a single variable, y. The treatment of a structure consisting of two rigid masses connected by an elastic (linear) spring will also be formulated and discussed.

The equation of motion of the rigid structure buried in the earth media can be written in differential form as

$$M_o\ddot{y} = F_{fw} + F_{fa} - F_{bw} - F_{ba} \qquad (B-29)$$

where M_o is the mass of the structure and $\ddot{y}$ is the acceleration of the structure.

We are limited here by a lack of knowledge of the properties of earth media and therefore must restrict the solution of the above differential equation to the case of an elastic soil medium. Solutions have been obtained to determine the effects of the various free-field and structural or geometric parameters and to evaluate the magnitude of the forces and deflection which will exist under given conditions.

The general differential equation of motion thus reduces, for the elastic case, to

$$M_o \ddot{y} = \frac{\sigma_o \pi D^2}{4} \left[1 + e^{-t\,2\,c_o/\alpha D}\right]\left[1 - e^{-t/t_o}\right] e^{-t/t_d}$$

$$+ \pi D^2/4 \cdot \pi E/2D \cdot (x - y) - \pi D^2/4 \;\; \pi E/2D\, y, \quad 0 \le t \le L/t_o$$

$$\text{(B-30)}$$

and

$$M_o \ddot{y}' = \frac{\sigma_o \pi D^2}{4} \left[1 + e^{-t\,2\,c_o/\alpha D}\right]\left[1 - e^{-t/t_o}\right] e^{-t/t_d}$$

$$- \frac{\sigma_o \pi D^2}{4} \left[1 - e^{-t\,2\,c_o/\alpha D + L2/\alpha D}\right]\left[1 - e^{-t/t_o + L/c_o t_o}\right]$$

$$e^{-t/t_d + L/c_o t_d} + \pi D^2/4 \cdot \pi E/2D\,(x - y) - \pi D^2/4 \cdot \pi E/2D$$

$$(y - x'), \quad L/c_o \cdot \le t \qquad \text{(B-31)}$$

noting that $y' = y$, $x = x\left\{t\right\}$, and $x' = x'\left\{t\right\}$. The initial conditions for the solution of Eq (B-30) are

$$y\left\{0\right\} = 0$$
$$\dot{y}\left\{0\right\} = 0 .$$

Also, the initial conditions of the solution of Eq (B-31) are such that the deflection and the velocity match those of the solution of Eq (B-30) at the time L/c_o.

1. <u>Response to a Step Pulse Wave</u>

For the purposes of illustration the solution for the step-pulse uniform field (Case a) will be treated in detail. Thus, Eq (B-30) and (B-31) reduce to the following by applying Eq (B-12):

$$M_o \ddot{y} = \frac{\sigma_o \pi D^2}{4} \left[1 + e^{-t c_o/D}\right] + \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \left[\frac{\sigma_o c_o t}{E} - 2y\right],$$

$$0 \leq t \leq L/c_o \qquad (B-32)$$

and

$$M_o \ddot{y} = \frac{\sigma_o \pi D^2}{4} \left[e^{-t c_o/D} + e^{-t c_o/D + L/D}\right] + \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D}$$

$$\cdot \left[\frac{2 \sigma_o c_o t}{E} - 2y - \frac{L \sigma_o}{E}\right], \quad L/c_o \leq t \qquad (B-33)$$

Note that $\alpha = 2$ was used and that $x' = x - L\,\sigma_o/E$. Figure B-4 presents a deflection time diagram for this case.

It will be convenient at this time to introduce a set of dimensionless variables and to obtain the solution in terms of these new variables.

Let

$$Y = \pi \frac{yE}{D\sigma_o},$$

$$X = \pi \frac{xE}{D\sigma_o},$$

$$Z = \pi \frac{zE}{D\sigma_o},$$

$$T = c_o t/D,$$

$$\tau = L/D, \qquad (B-34)$$

$$\mu^2 = \frac{M_o}{D^3 \rho_o},$$

$$\beta^2 = \frac{\pi^2}{4\mu^2},$$

and

$$\mathcal{F}_{fw} = \frac{F_{fw}}{\frac{\pi D^2}{4}\sigma_o}, \quad \text{etc.}$$

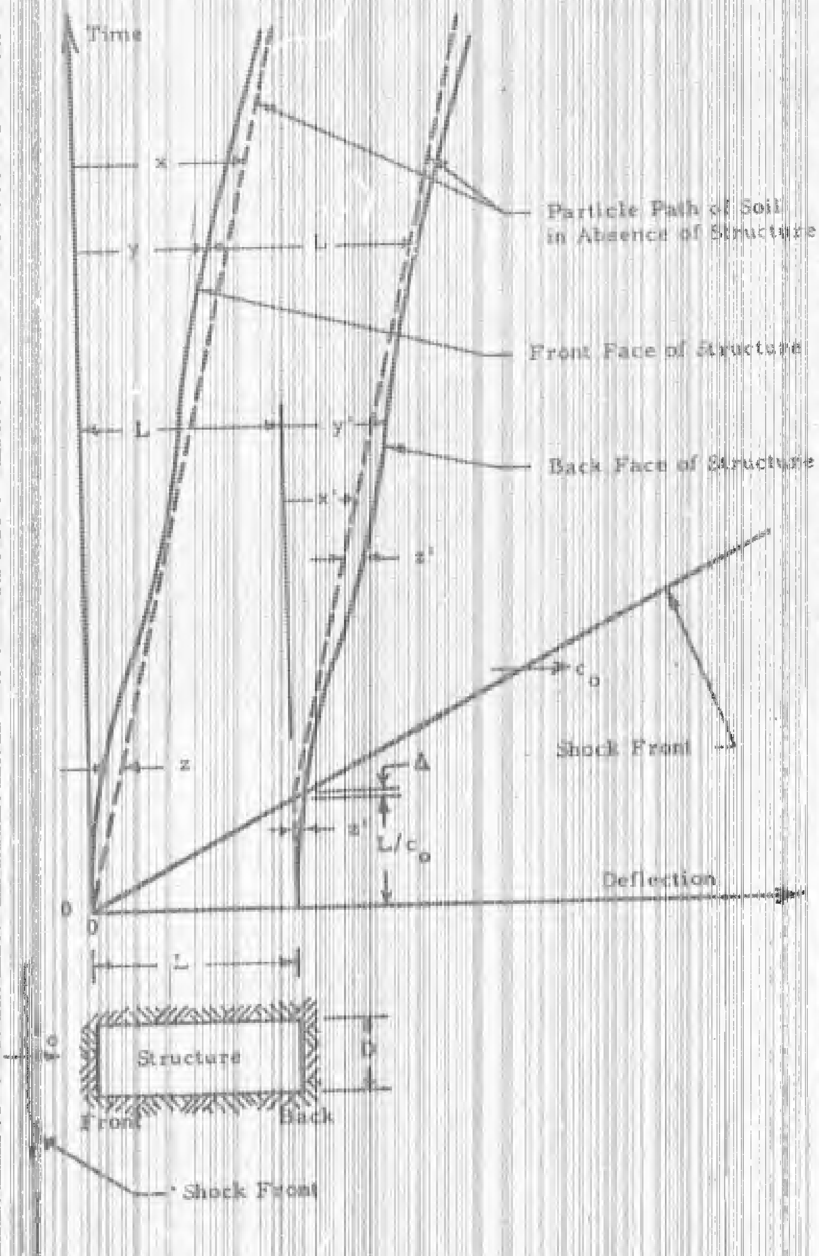


Fig. B-4 DEFLECTION-TIME DIAGRAM

It follows directly that

$$
\begin{aligned}
Z &= X - Y \\
X &= \pi T \text{ (using Eq (B-12))} \\
\dot{Y} &= \dot{y}\left[\frac{\pi E}{\sigma_o c_o}\right] \\
\ddot{Y} &= \ddot{y}\left[\frac{\pi E D}{\sigma_o c_o^2}\right] \\
\mathcal{F}_{fw} &= 1 + e^{-T} \\
\mathcal{F}_{fa} &= 1/2\, Z \\
\mathcal{F}_{ba} &= 1/2\, Y, \; 0 \leq T \leq \tau \\
\mathcal{F}_{ba} &= 1/2\, \left[\pi\tau - Z\right], \; \tau \leq T \\
\mathcal{F}_{bw} &= 1 - e^{-(T-\tau)}, \; \tau \leq T
\end{aligned}
\qquad \text{(B-35)}
$$

Note that the expressions for the dimensionless wave forces and for X apply only to the free-field condition of Case a.

Equations (B-32) and (B-33) can now be rewritten in terms of the dimensionless variables:

$$\ddot{Y} + \beta^2 Y = \beta^2 (1 + e^{-T}) + \pi/2\, \beta^2 T, \; 0 \leq T \leq \tau \qquad \text{(B-36)}$$

and

$$\ddot{Y} + \beta^2 Y = \beta^2 (1 + e^{+\tau})\, e^{-T} - \pi/2\, \beta^2 \tau + \beta^2 \pi T, \; \tau \leq T \qquad \text{(B-37)}$$

The solution to Eq (B-36) for the initial conditions $Y(0) = \dot{Y}(0) = 0$ is

$$Y = 1 + \pi/2\, T + \frac{e^{-T}}{\left[1 + \frac{1}{\beta^2}\right]} - \left[1 + \frac{1}{1 + \frac{1}{\beta^2}}\right] \cos(\beta T) + 1/\beta \left[\frac{1}{1 + \frac{1}{\beta^2}} - \pi/2\right] \sin(\beta T), \; 0 \leq T \leq \tau \qquad \text{(B-38)}$$

also

$$U = \dot{Y} = \pi/2 - \frac{e^{-T}}{\left[1 + \frac{1}{\beta^2}\right]} + \beta\left[1 + \frac{1}{1 + \frac{1}{\beta^2}}\right] \sin(\beta T) + \left[\frac{1}{1 + \frac{1}{\beta^2}} - \pi/2\right] \cos(\beta T), \; 0 \leq T \leq \tau \qquad \text{(B-39)}$$

We define

$$Y^* = Y\left\{\tau\right\}$$

$$U^* = U\left\{\tau\right\} \qquad \text{(B-40)}$$

hence,

$$Y^* = 1 + \pi/2\,\tau + \frac{e^{-\tau}}{\left[1 + \frac{1}{\beta^2}\right]} + \beta\left[1 + \frac{1}{1 + \beta^{-2}}\right] \cos(\beta\tau) + \left[\frac{1}{1 + \frac{1}{\beta^2}} - \pi/2\right]$$

$$\sin(\beta\tau) \qquad \text{(B-41)}$$

and

$$U^* = \pi/2 - \frac{e^{-\tau}}{\left[1 + \frac{1}{\beta^2}\right]} + \beta\left[1 + \frac{1}{1 + \frac{1}{\beta^2}}\right] \sin(\beta\tau) + \left[\frac{1}{1 + \frac{1}{\beta^2}} - \pi/2\right] \cos(\beta\tau) \qquad \text{(B-42)}$$

The solution of Eq (B-37) is

$$Y = \left\{1/\beta\left[\pi - \frac{1 + e^{-\tau}}{1 + \frac{1}{\beta^2}} - U^*\right] \sin(\beta\tau) - \left[\frac{\pi\tau}{2} + \frac{1 + e^{-\tau}}{1 + \frac{1}{\beta^2}} - Y^*\right]\right.$$

$$\left.\cos(\beta\tau)\right\} \cos(\beta T) + \left\{-\left[\frac{\pi\tau}{2} + \frac{1 + e^{-\tau}}{1 + \frac{1}{\beta^2}} - Y^*\right] \sin(\beta\tau) - 1/\beta\right.$$

$$\left.\left[\pi - \frac{1 + e^{-\tau}}{1 + \frac{1}{\beta^2}} - U^*\right] \cos(\beta\tau)\right\} \sin(\beta T) - \pi/2\,\tau + \pi T + \frac{\left[1 + e^{+\tau}\right] e^{-T}}{\left[1 + \frac{1}{\beta^2}\right]}$$

$$\tau \leq T \qquad \text{(B-43)}$$

Equations (B-38) and (B-43) were evaluated for the range of time $0 \leq T \leq 10$ and for $\mu^2 = 2$ and 5, and $\tau = 2$ and 4. Figures B-5 through B-18 present these results.

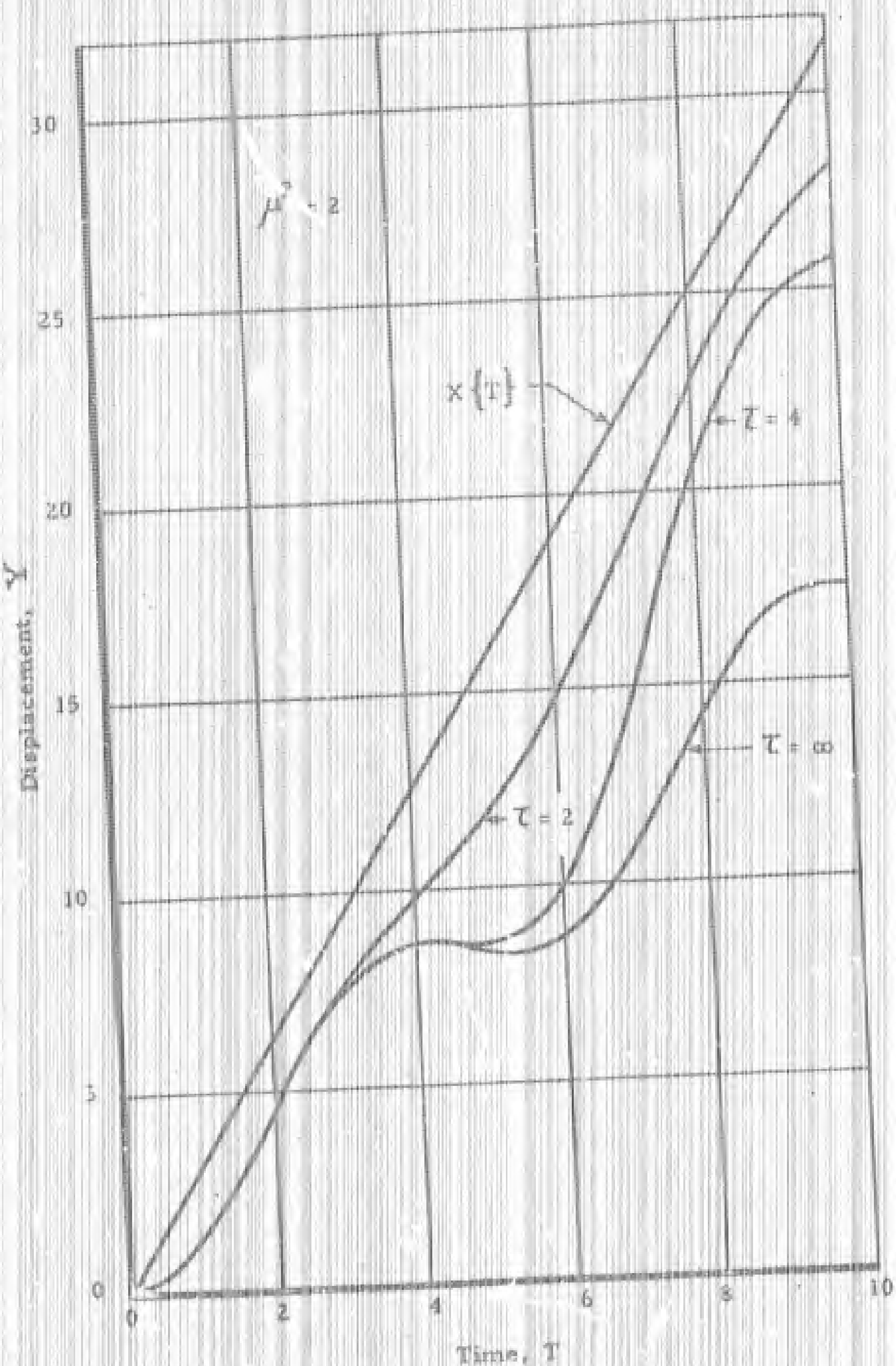


Fig. B-5 DISPLACEMENT VARIATION, $\mu^2 = 2$

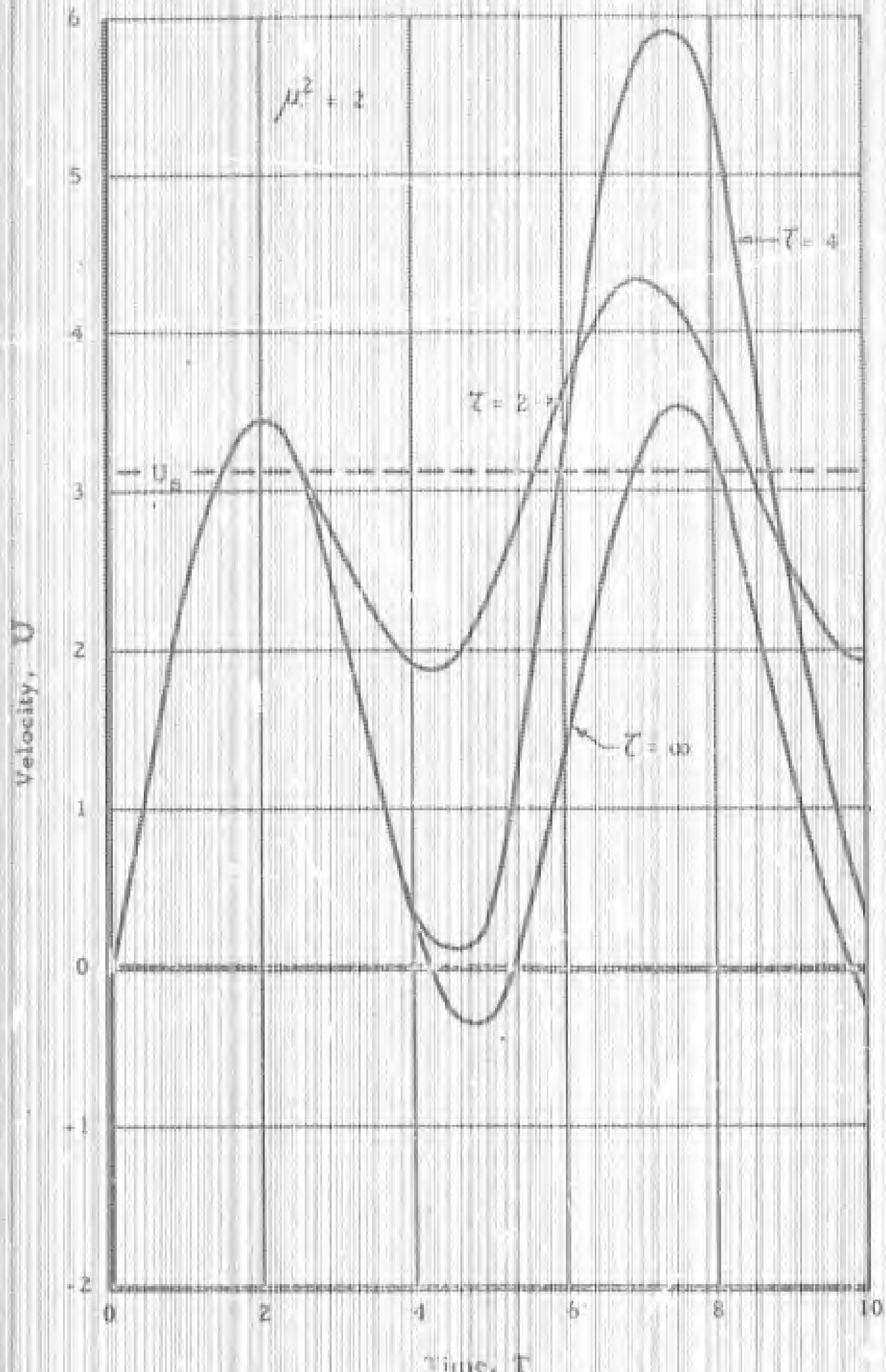


Fig. B-6 VELOCITY VARIATION, $\mu^2 = 2$

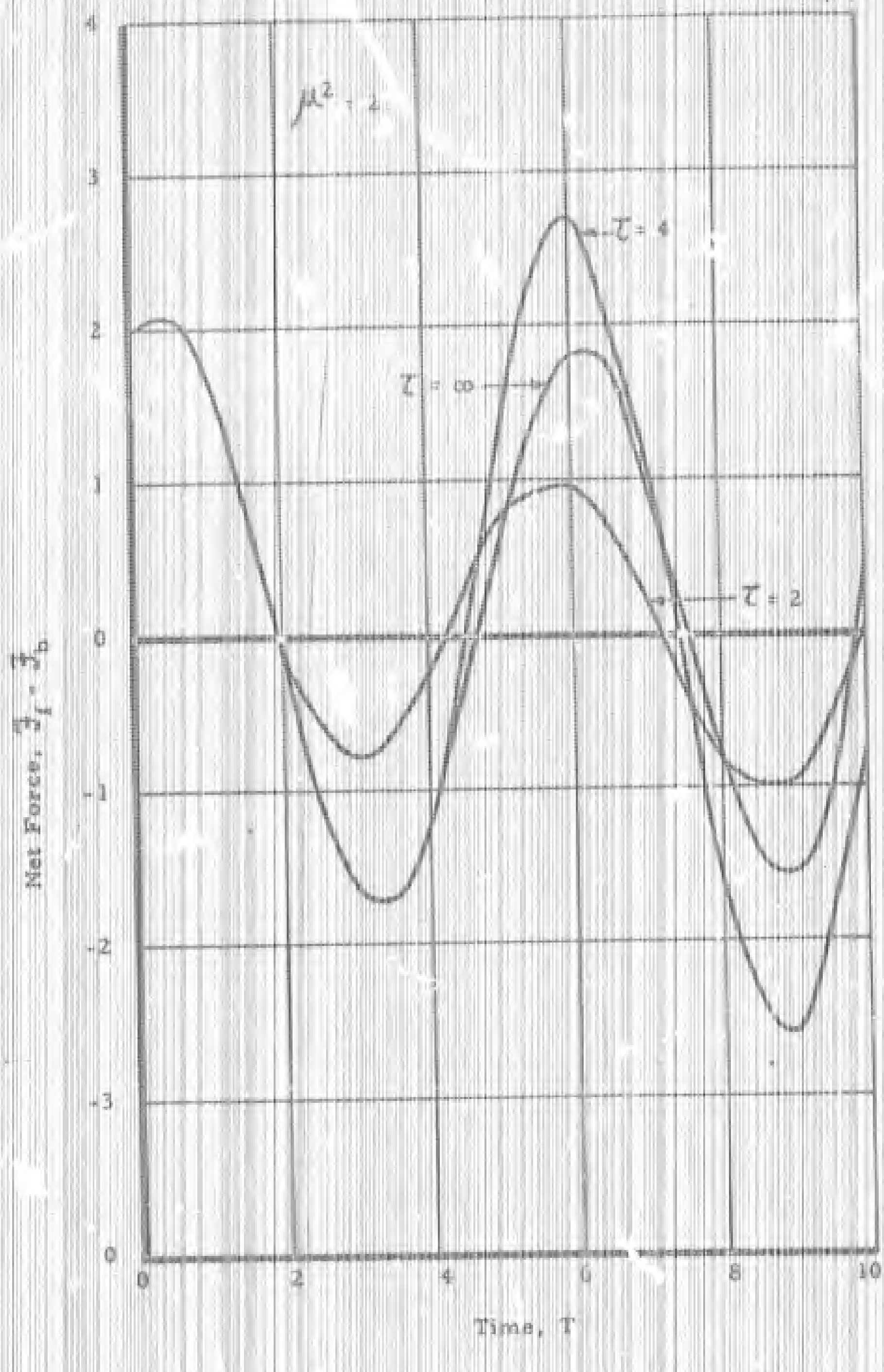


Fig. B-7 NET FORCE VARIATION, $\mu^2 = 2$

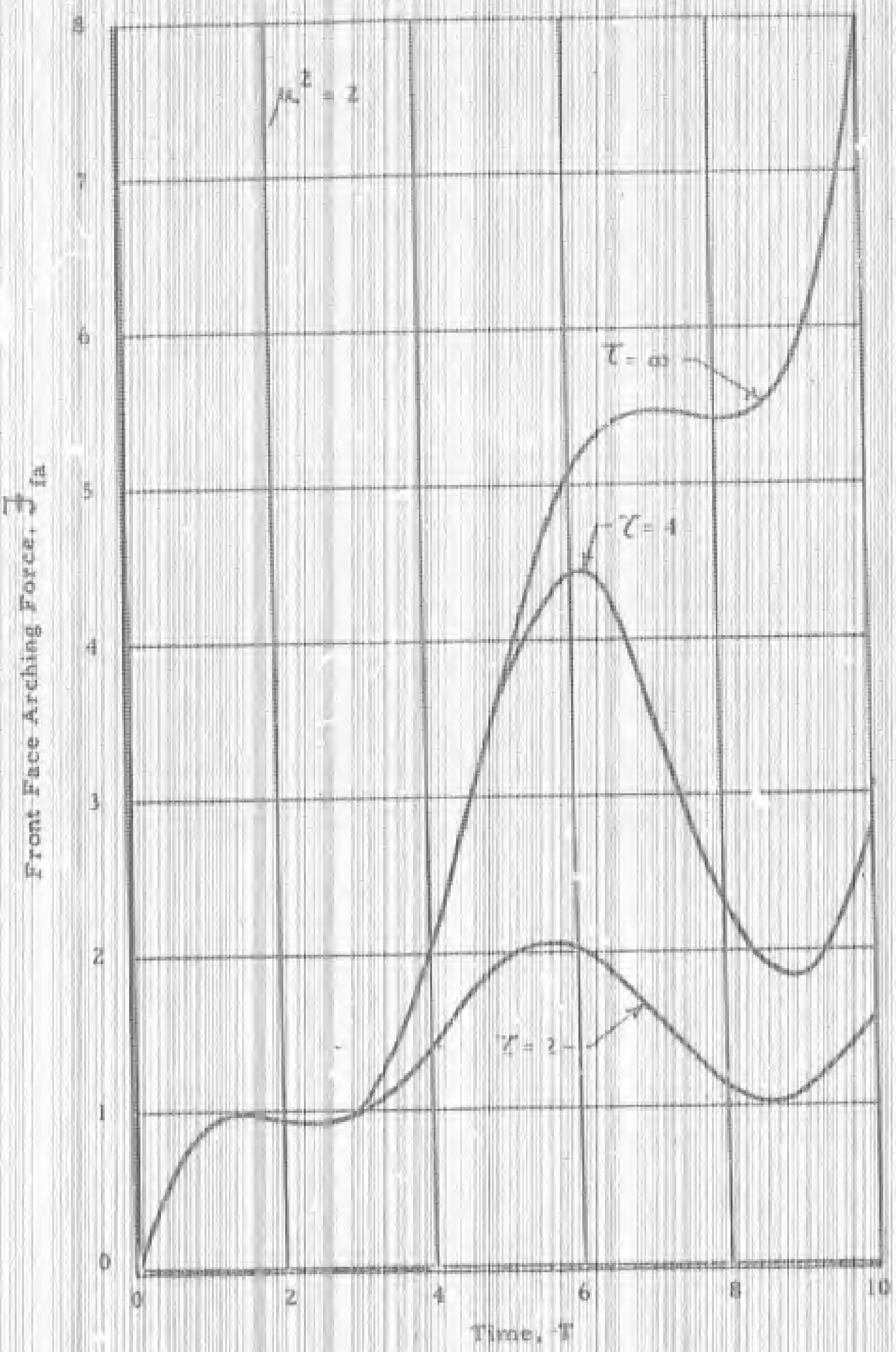


Fig. B-8 VARIATION OF ARCHING FORCE ON FRONT FACE, $\mu^2 = 2$

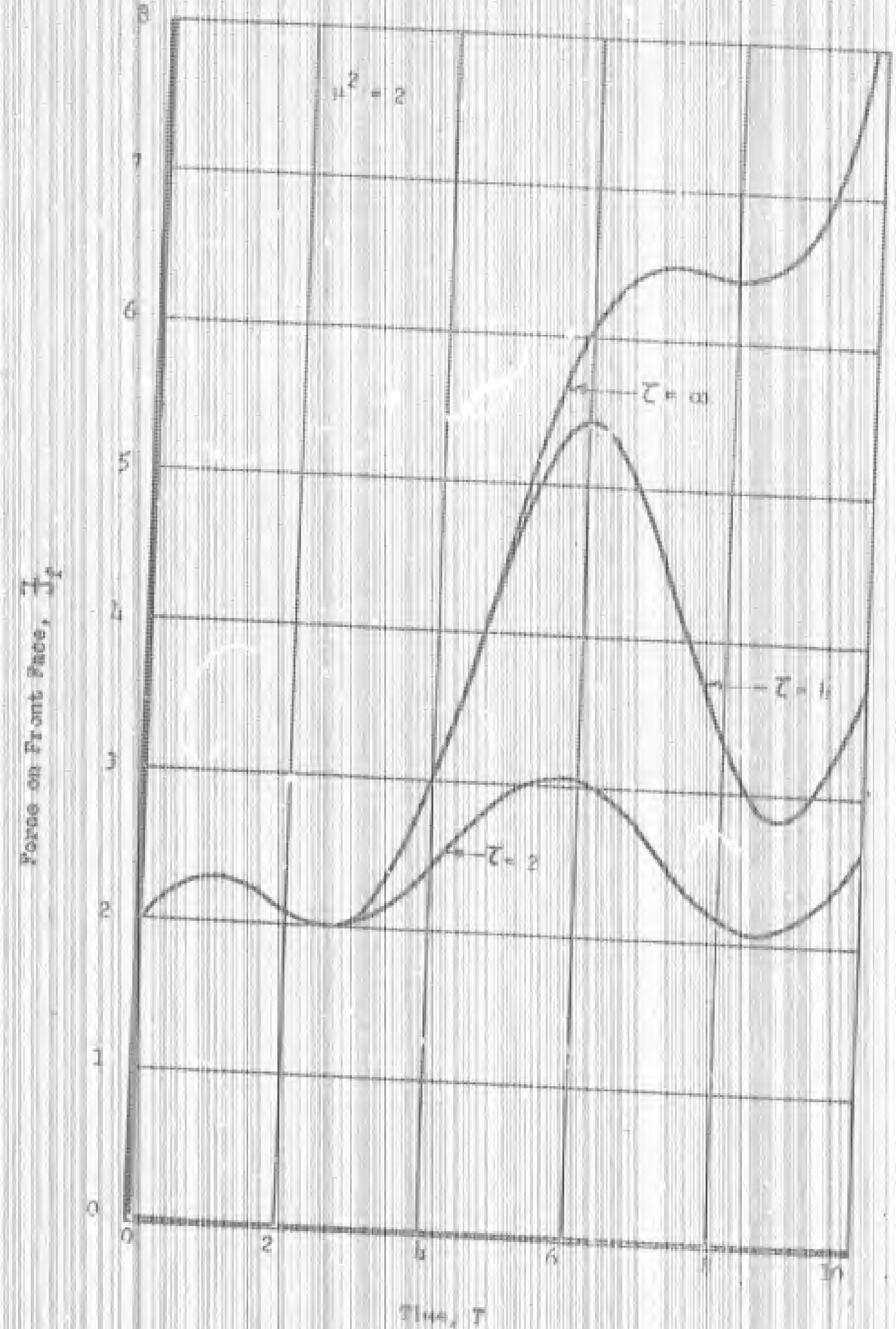


Fig. B-9 VARIATION OF FORCE ON FRONT FACE, $\mu^2 = 2$

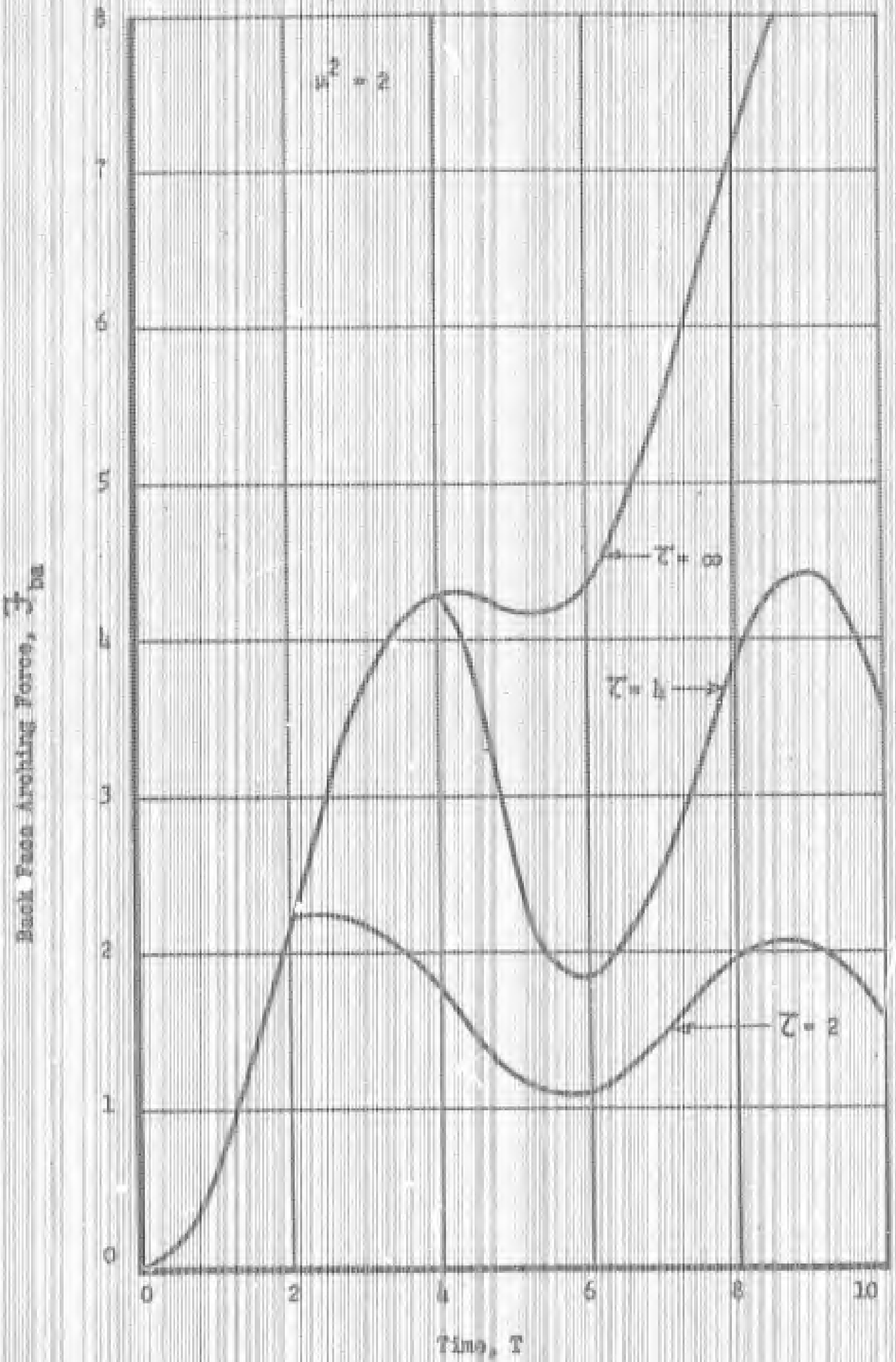
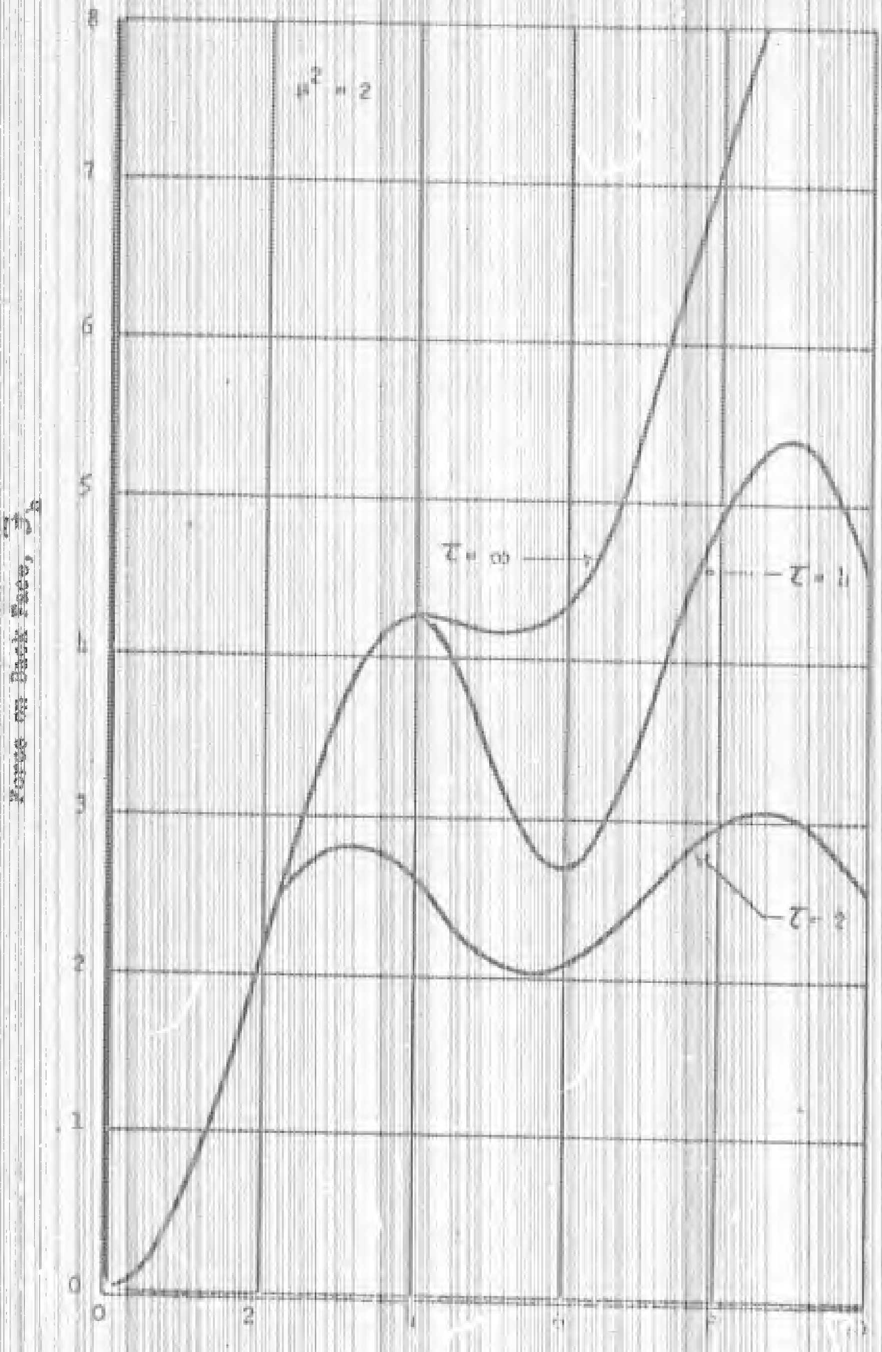


Fig. B-10 VARIATION OF ARCHING FORCE ON BACK FACE, $\mu^2 = 2$



Time, τ

Fig. B-11 [illegible] $\mu^2 = 2$

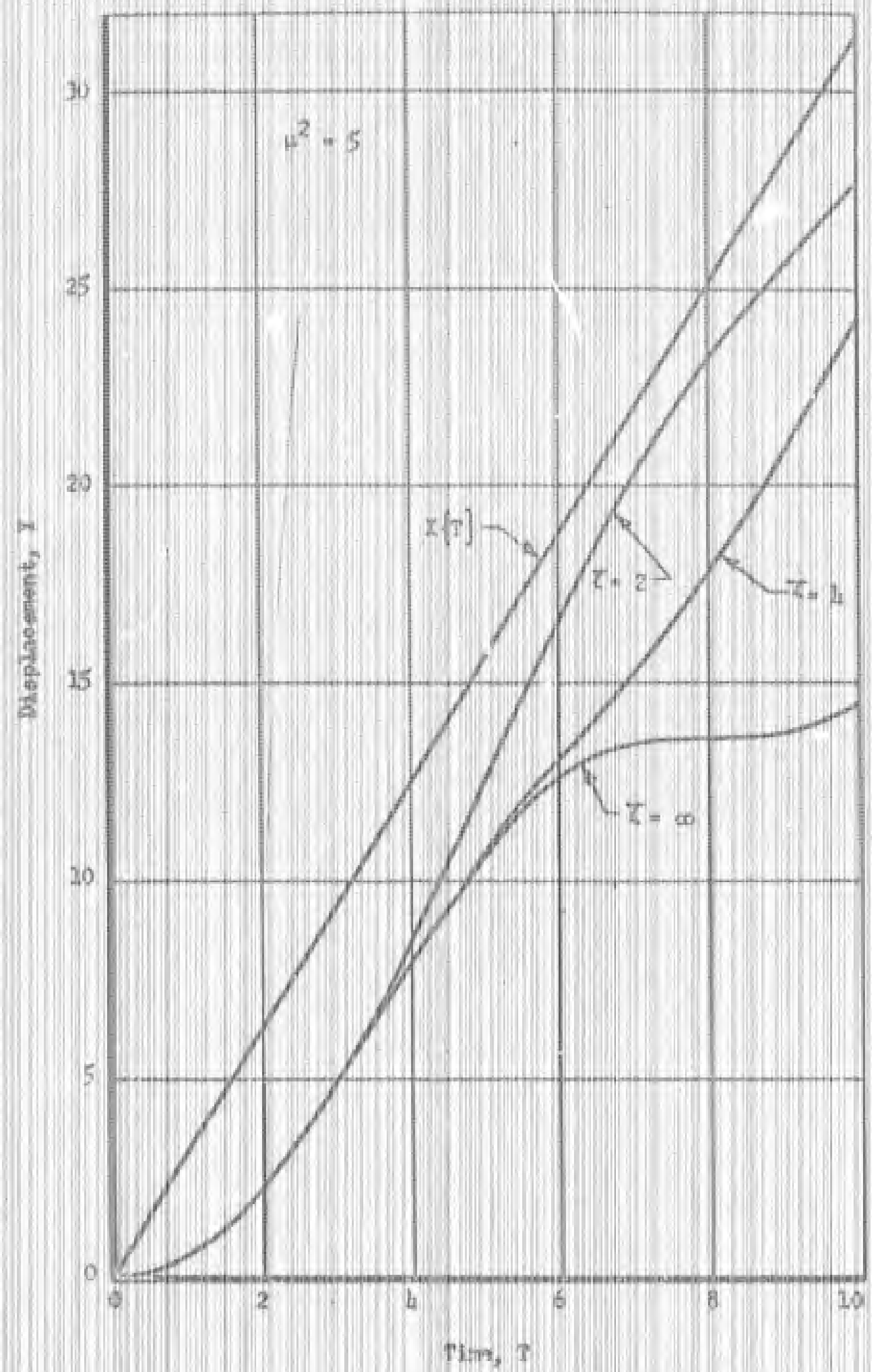


Fig. B-12 DISPLACEMENT-VARIATION, $\mu^2 = 5$

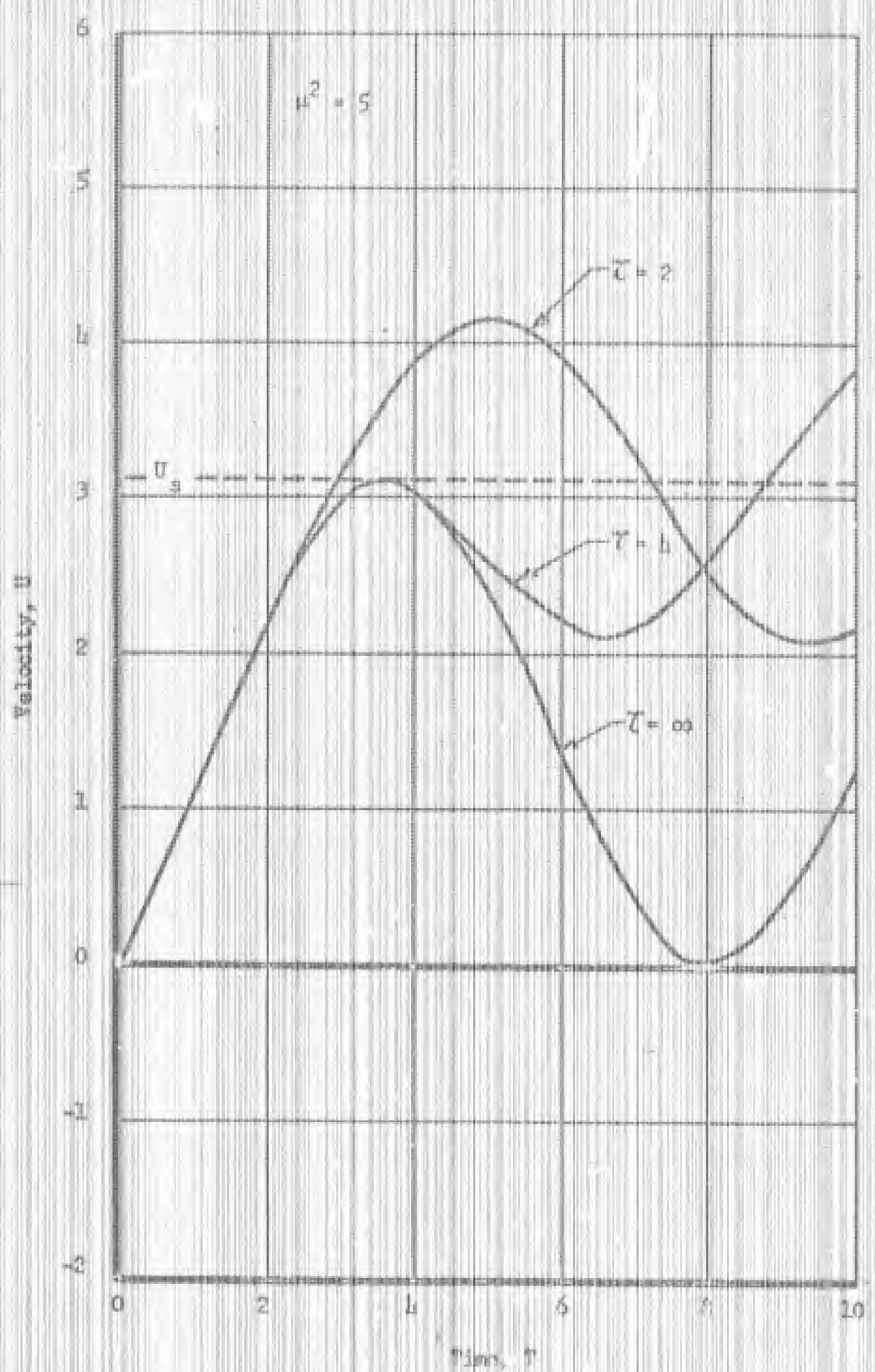


Fig. B-23 VELOCITY VARIATION, $\mu^2 = 5$

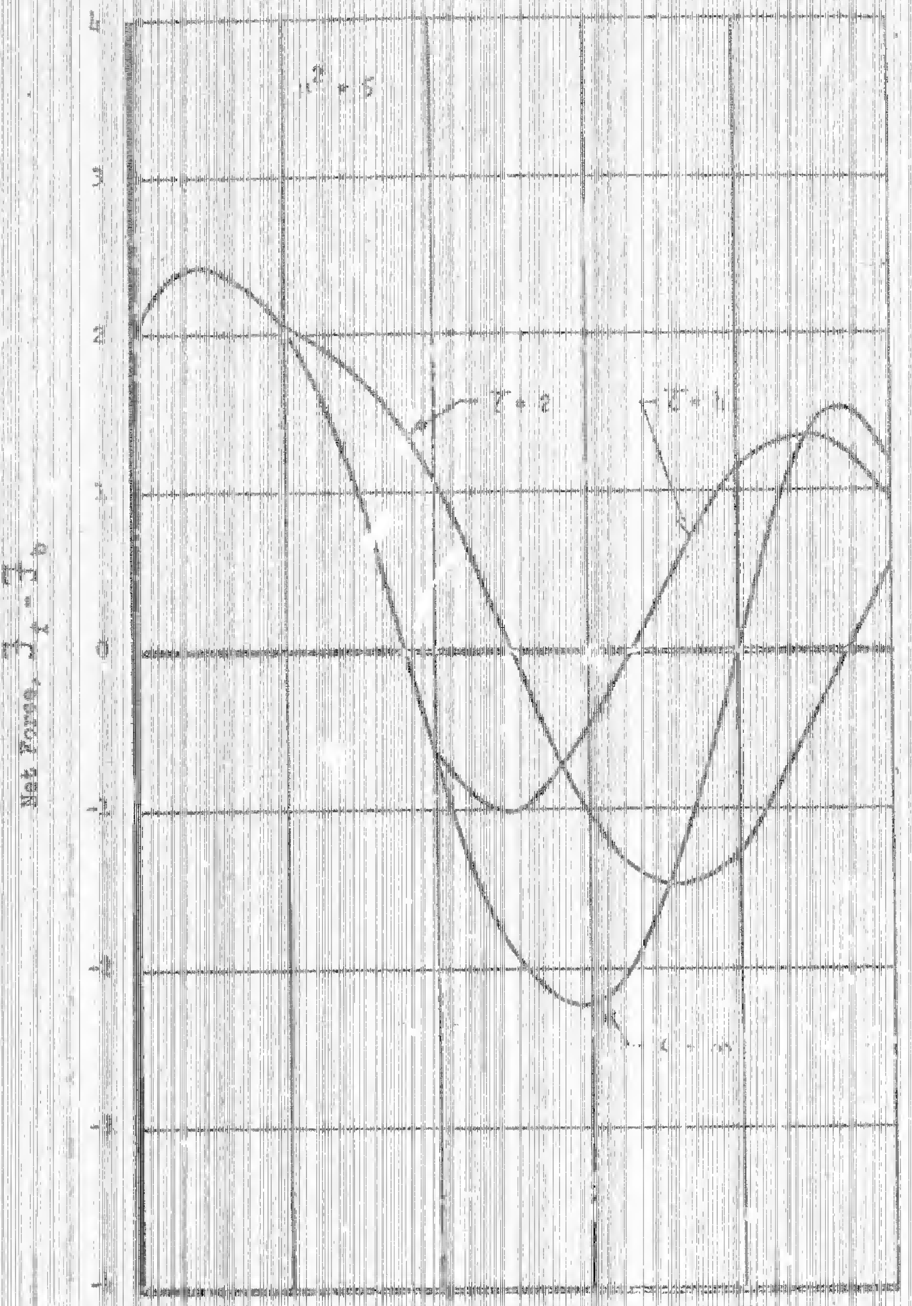


Fig. B-15 Net Force Variation, $\mu^2 = 5$

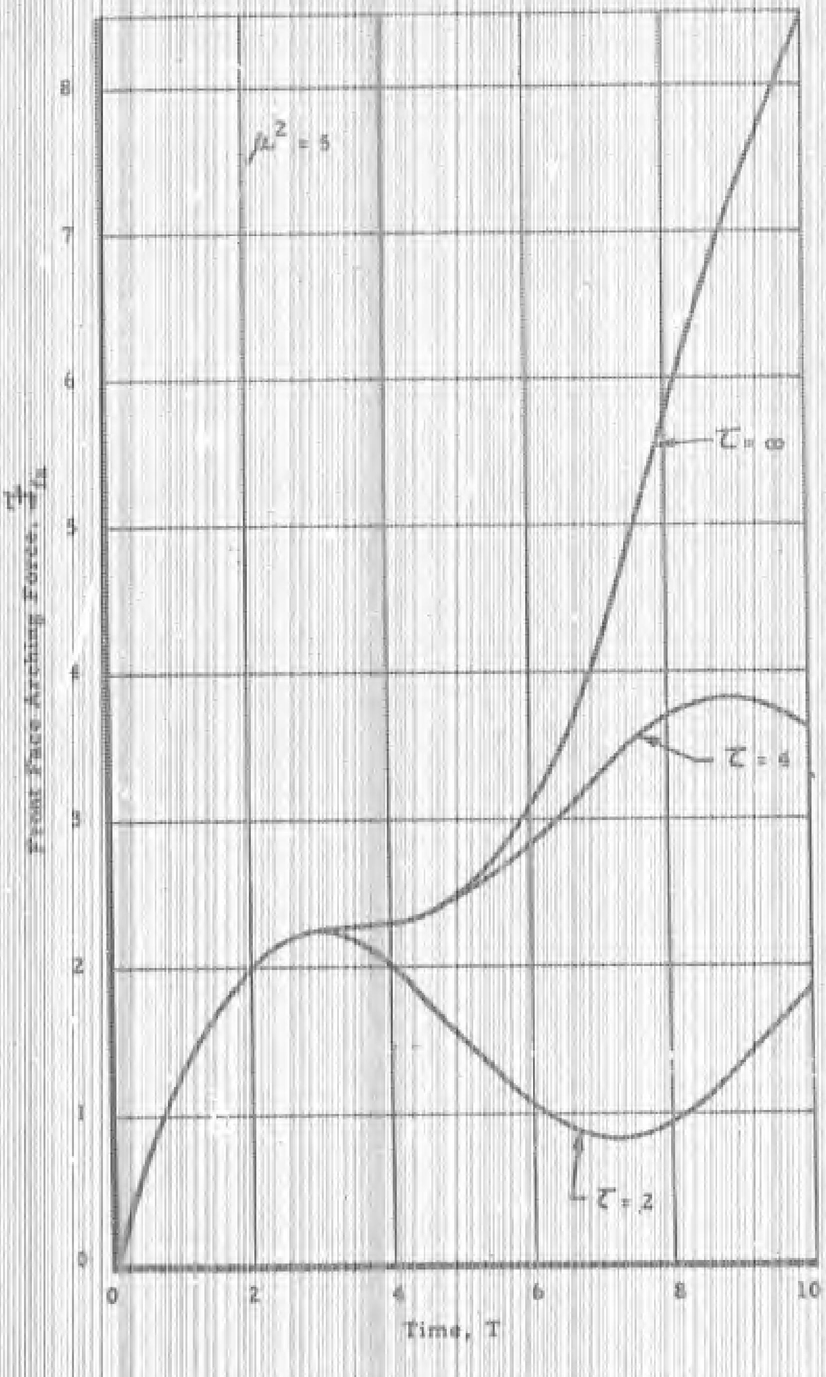


Fig. B-15 VARIATION OF ARCHING FORCE ON FRONT FACE, $\mu^2 = 5$

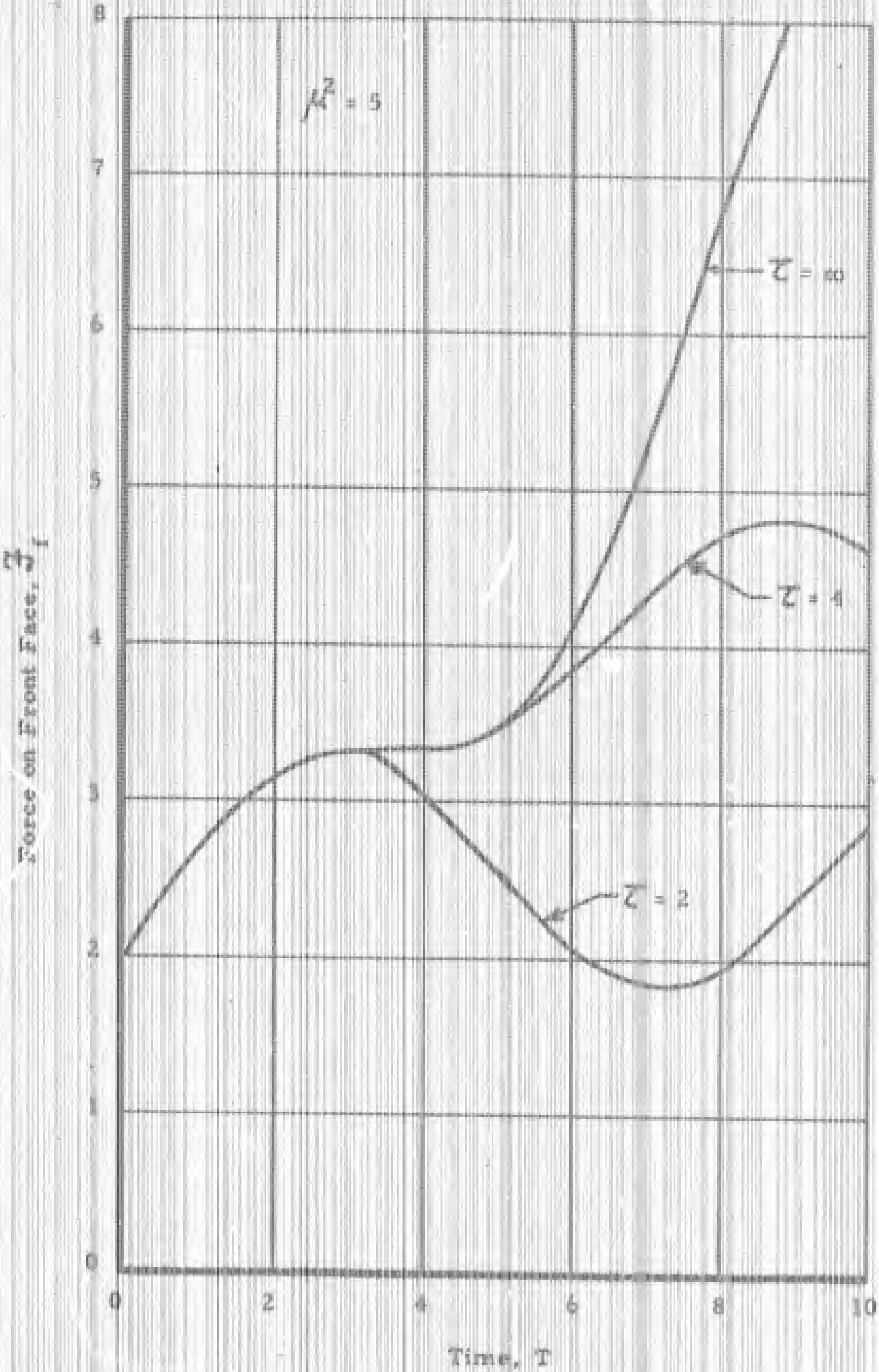


Fig. B-16 VARIATION OF FORCE ON FRONT FACE, $\mu^2 = 5$

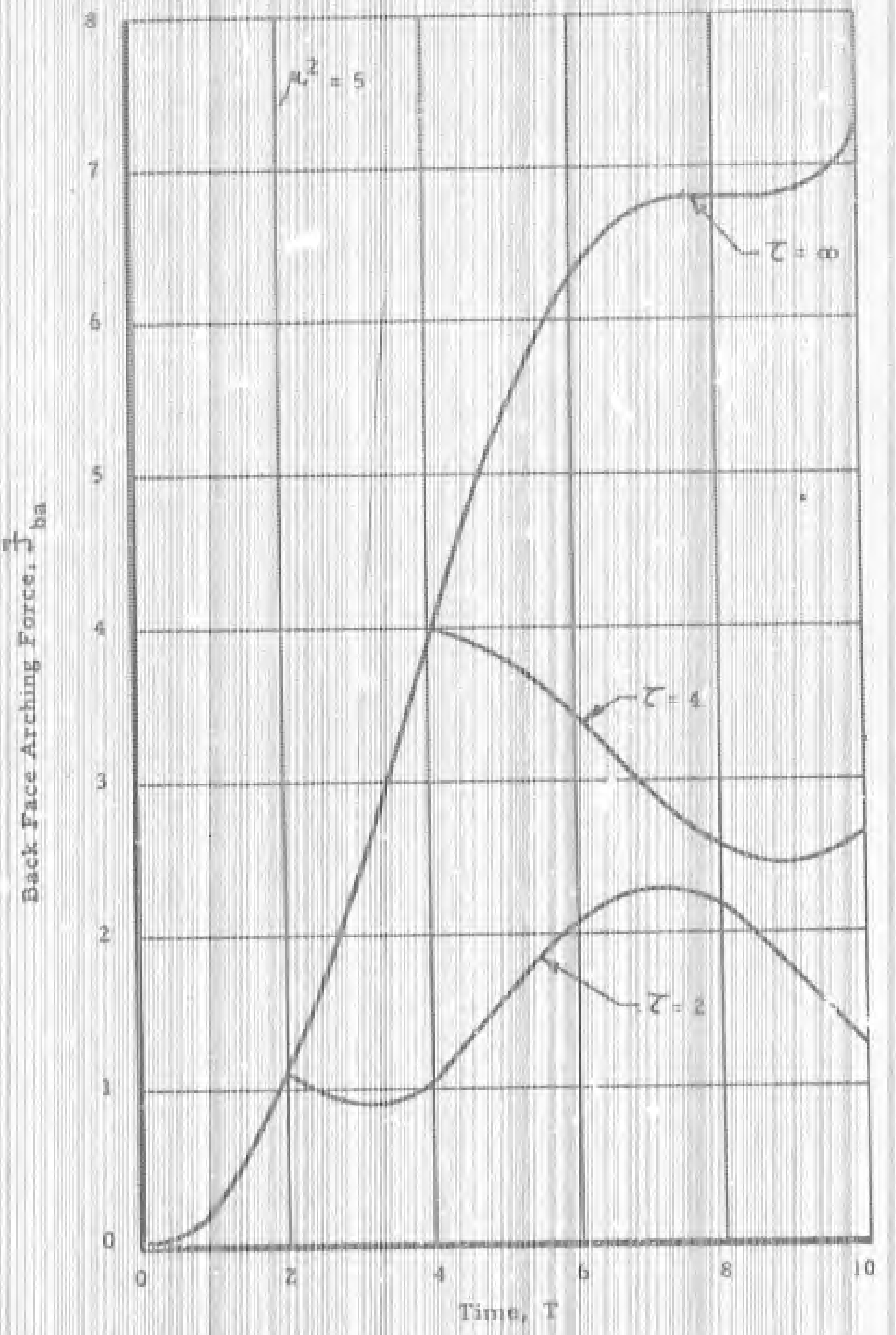


Fig. B-17 VARIATION OF ARCHING FORCE ON BACK FACE, $\mu^2 = 5$

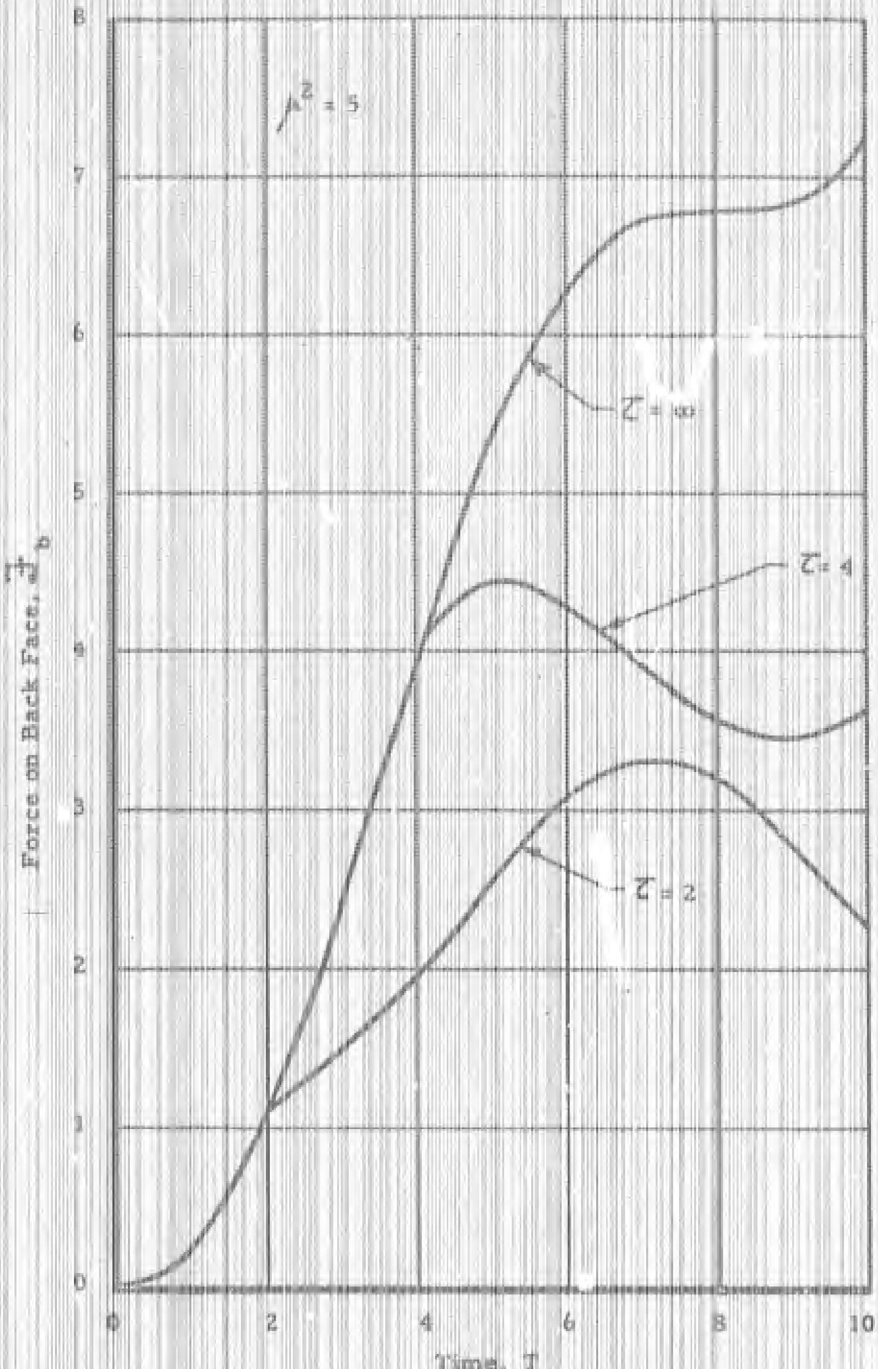


Fig. B-18 VARIATION OF FORCE ON BACK FACE, $\mu^2 = 5$

The displacements given by Eq (B-38) exhibit an oscillatory nature, superimposed upon a mean velocity of one-half the free-field velocity. The exponential term, due to the wave forces, decays quite rapidly and in general does not have any appreciable effect on the results except during the very early portion of the interaction. The velocity varies approximately between zero and the free-field velocity for the range of the values of the mass parameter $\mu^2 = 2$ to $\mu^2 = 10$.

The effect of the mass parameters on the structure velocity appears primarily in the period of oscillation; the larger the mass, the greater the period. The variation of the net force $(\mathcal{F}_f - \mathcal{F}_b)$ is also oscillatory in nature and is, of course, affected by the mass parameter in the same manner as is the velocity. The normalized net force varies approximately between the limits +2 to -2 for the above range of mass parameter values. Hence, the normalized peak acceleration decreases approximately linearly with increasing structure mass. The arching forces acting on the front and on the back faces of the structure and the total force on the front of the structure increases quite rapidly with time. The reason for this is that the relative displacement increases with time since the mean velocity of the structure is only one-half of the soil velocity.

Since the value of the relief time parameter α was selected intuitively, a comparison was made of the total force acting on the front face of the structure for the selected value of α (=2), for α =4 and for the limiting case of α =0. The results are presented in Fig. B-19 and do not show a great deviation between the three cases. It should be pointed out, however, that the limiting case, inherently, possesses a reflection coefficient of unity as opposed to the factor two which is being used. The above variation in α represents a variation of $\pm$ 100 per cent in the impulse (for α= 2) due to the reflection and relief of the shock wave; that is, the impulse due to the force in excess of $\sigma_0 \ \pi D^2/4$.

The more general form of the differential equation (B-38) which contains the relief time variable is

$$\ddot{Y} + \beta^2 Y = \beta^2 (1 + e^{-\alpha T}) + \frac{\pi}{2} \beta^2 \Gamma \quad 0 \leq T \leq \tau \qquad \text{(B-44)}$$

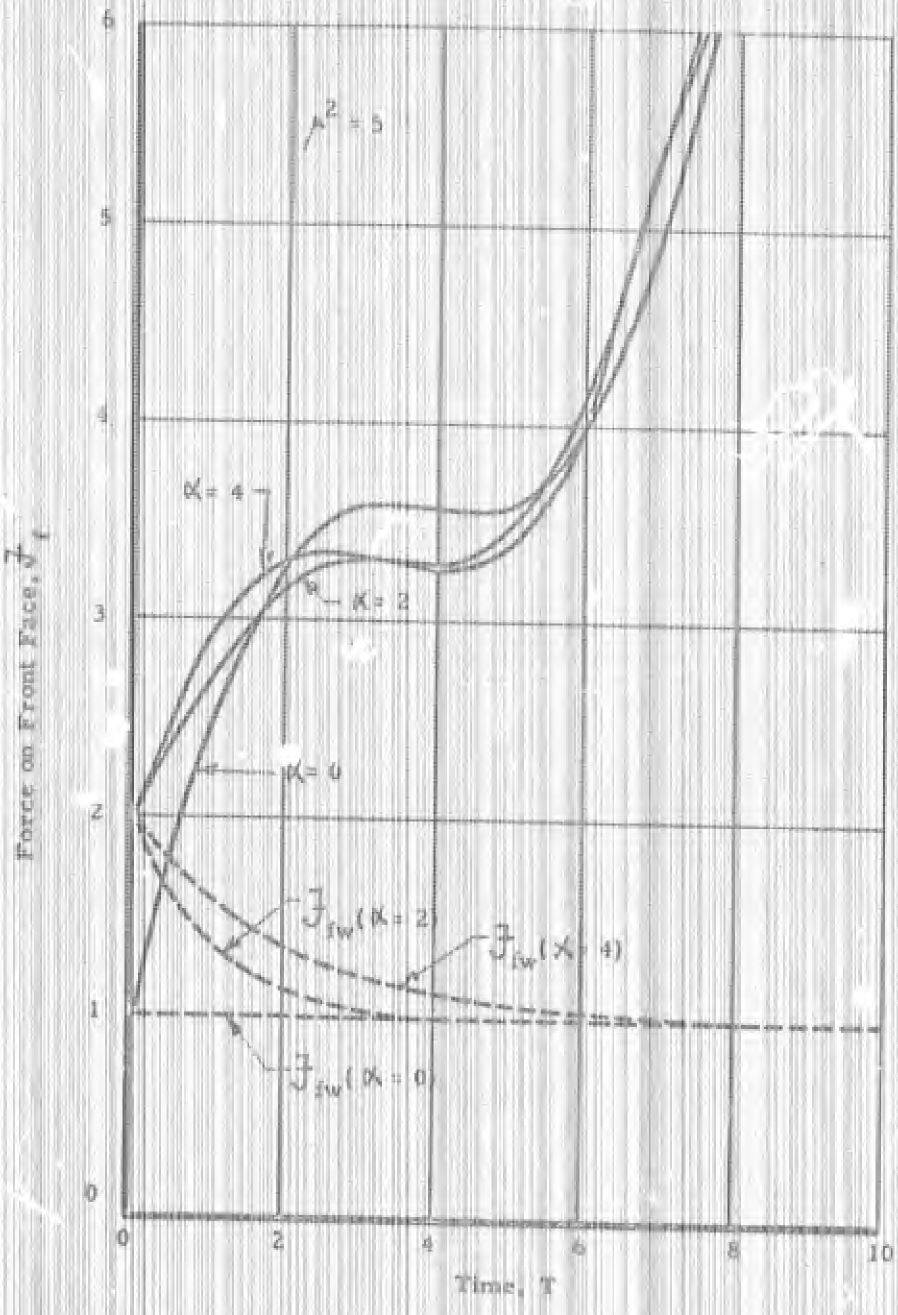


Fig. B-19 EFFECT OF THE RELIEF TIME PARAMETER, α, ON $\mathcal{F}_f$

where $\alpha' = 2/\alpha$.

The general solution for the displacement is

$$Y = 1 + \frac{\pi}{2} T + \frac{e^{-\alpha' T}}{\left[1 + \frac{1}{\beta^2}\right]} + 1/\beta \left[\frac{\alpha'}{1 + \frac{\alpha'^2}{\beta^2}} - \frac{\pi}{2}\right] \sin(\beta T)$$

$$- \left[1 + \frac{1}{1 + \frac{\alpha'^2}{\beta^2}}\right] \cos(\beta T), \qquad 0 \le T \le \tau \qquad \text{(B-45)}$$

For the limiting solution $\alpha = 0$ ($\alpha' = \infty$) we obtain

$$Y = 1 + \frac{\pi}{2} T - \frac{\pi}{2\beta} \sin(\beta T) - \cos(\beta T), \quad 0 \le T \le \tau \qquad \text{(B-46)}$$

Equation (B-38) yields the solution of the interaction problem during the period the disturbance is engulfing the structure ($0 \le T \le \tau$). For most structures of interest the length diameter ratio ($L/D = \tau$) will be in the range of from one to ten. This parameter, which appears in the solution (Eq (B-43)) for the interaction problem after the structure is completely engulfed, is quite significant.

The mean value of the total force on both the front and back faces of the structure is equal to $1 + \frac{\pi}{4}\tau$; the term $\frac{\pi}{4}\tau$ is due to the relative displacement of the structure face and the soil particles and the term "unity" is due to the free-field stress. The length parameter, τ, is also influential in another manner. Since there is no damping factor involved in the above equations, it is expected that oscillations, if set up, will persist.

The magnitude of these oscillations will be a function of the structural parameters, μ^2 and τ. The amplitude, A, of the oscillation of the displacement about the mean is given by

$$A = \Lambda\left\{\mu^2, \tau\right\} = \sqrt{\left[\frac{\pi\tau}{2} + \frac{1 + e^{-\tau}}{1 + \frac{1}{\beta^2}} - Y^*\right]^2 + 1/\beta^2 \left[\pi - \frac{1 + e^{-\tau}}{1 + \frac{1}{\beta^2}} - U^*\right]^2}$$

(B-47)

This function is plotted in Fig. B-20 for the range of interest and indicates that there exist several minimum values of the amplitude of oscillation. Since the magnitude of the forces acting on the structure is related to the relative displacement of the soil and structure, the magnitude of the force can be minimized by minimizing the function $A\left\{\mu^2, \tau\right\}$. Figure B-21 presents the value of the first minimum amplitude, A_{1m}, and the corresponding optimum length parameter, τ_o.

2. <u>Effect of the Wave Form on Structure Response</u>

The following paragraphs present the solutions for Cases b, c, and d (see Fig. B-1) together with computations illustrating the effect of the free-field wave form parameters on the response of a buried structure.

Case d, the general case, will be treated first. The free-field parameter, t_d, is a measure of the decay of the stress field with time and t_o is a measure of the rise time of the front of the wave. We define

$$\delta = \frac{D}{c_o t_d}$$

$$\gamma = \frac{D}{c_o t_o} \qquad \text{(B-48)}$$

thus

$$\sigma = \sigma_o \left(1 - e^{-\gamma T}\right) e^{-\delta T} \qquad \text{(B-49)}$$

and

$$X = \pi \left\{ 1/\delta \left[1 - e^{-\delta T} \right] - \frac{1}{(\delta + \gamma)} \left[1 - e^{-(\delta + \gamma) T} \right] \right\} \qquad \text{(B-50)}$$

$$X' = \pi \left\{ 1/\delta \left[1 - e^{-\delta (T - \tau)} \right] - \frac{1}{(\delta + \gamma)} \left[1 - e^{-(T - \tau)(\delta + \gamma)} \right] \right\} \qquad \text{(B-51)}$$

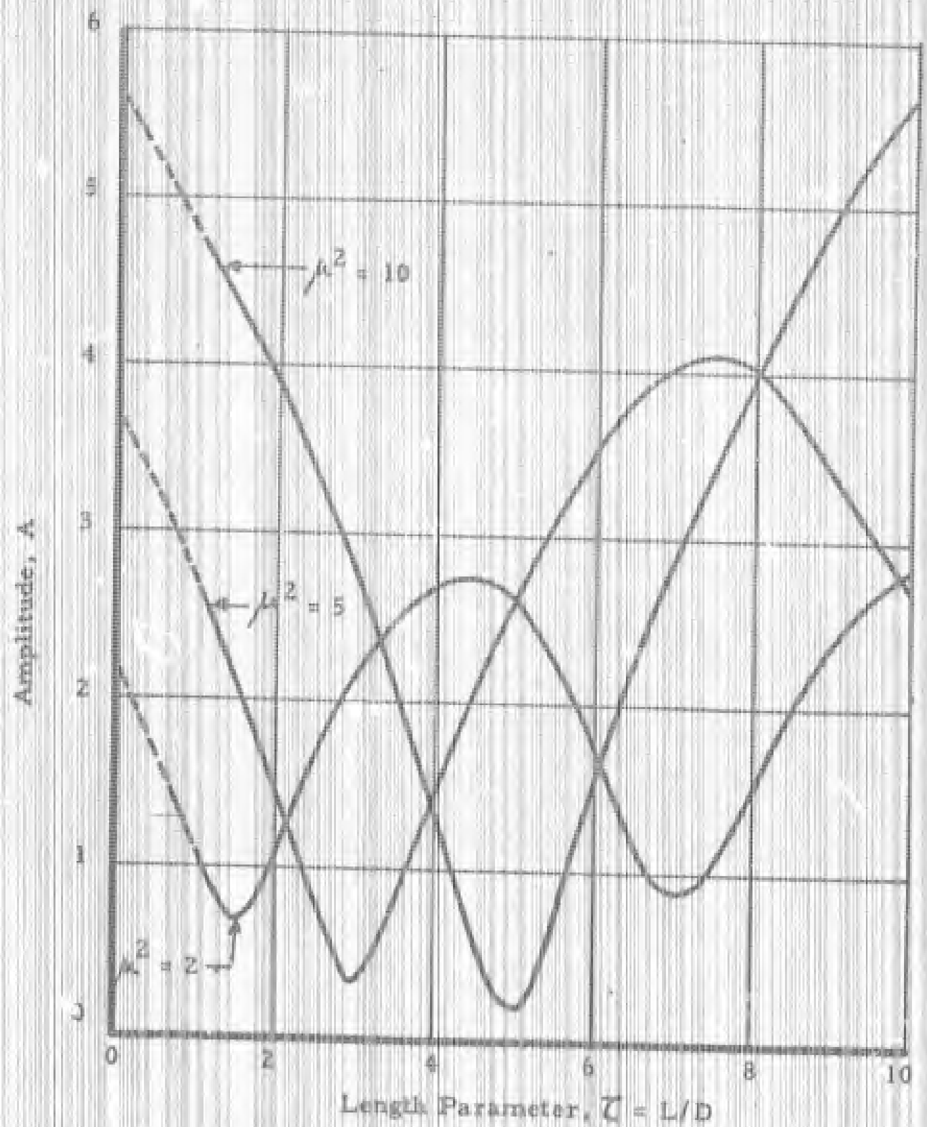


Fig. B-20 AMPLITUDE OF OSCILLATION

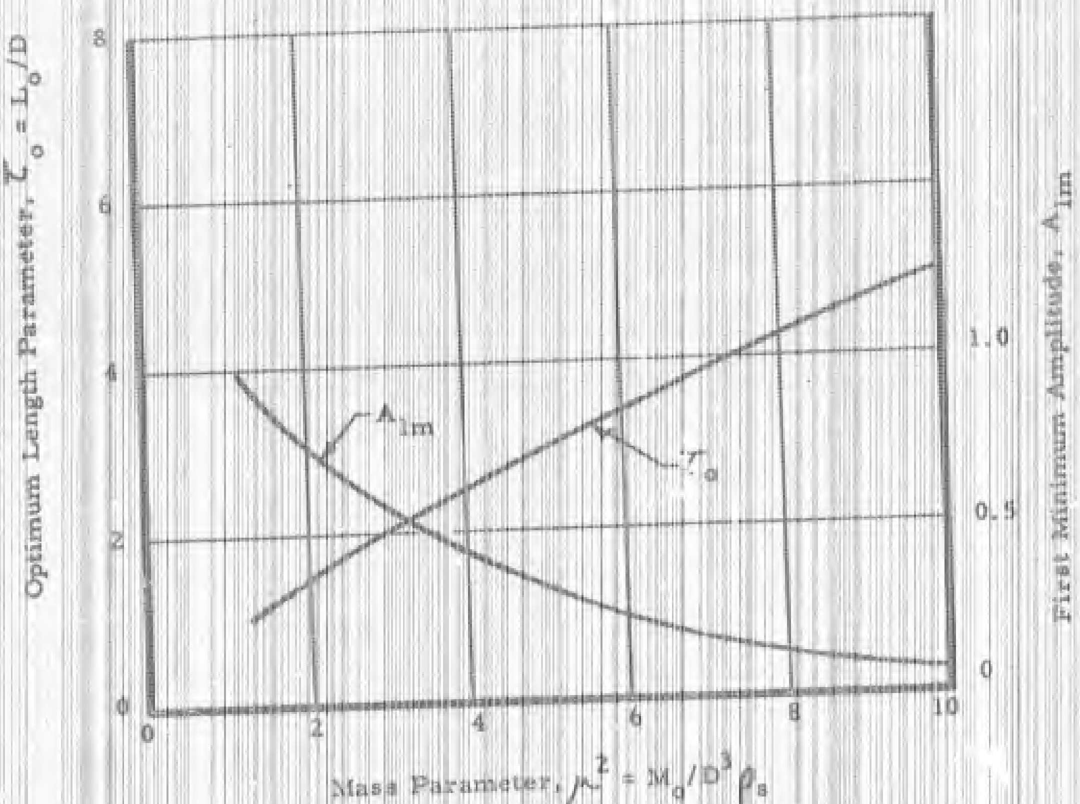


Fig. B-21 OPTIMUM LENGTH OF STRUCTURE

The general equation for the dimensionless forces is

$$\mathcal{F}_{fw} = \left[1 + e^{-\alpha' T}\right]\left[1 - e^{-\gamma T}\right] e^{-\delta T}$$

$$\mathcal{F}_{fa} = 1/2\, Z$$

$$\mathcal{F}_{bw} = 0 \qquad , \; 0 \le T \le \tau$$

$$\mathcal{F}_{bw} = \left[1 + e^{-\alpha'(T - \tau)}\right]\left[1 - e^{-\gamma(T - \tau)}\right] e^{-\gamma(T - \tau)} \qquad , \; \tau \le T$$

$$\mathcal{F}_{ba} = 1/2\, Y \qquad , \; 0 \le T \le \tau$$

$$\mathcal{F}_{ba} = 1/2\left[\pi \tau - Z\right] \qquad , \; \tau \le T$$

where

$$Z = Y - X' \tag{B-52}$$

and

$$X' = \pi \frac{\alpha'}{D} \frac{E}{\mathcal{J}_o}$$

The dimensionless form of the general equations of motion, Eq (B-30) and (B-31) is

$$\ddot{Y} + \beta^2 Y = \beta^2 \pi / 2 \left[1/\delta - \frac{1}{(\delta + \gamma)}\right] + \beta^2 e^{-\delta T}\left[1 - \frac{\pi}{2}\frac{1}{\delta}\right]$$

$$+ \beta^2 e^{-(\alpha' + \delta) T} - \beta^2 e^{-(\alpha' + \delta + \gamma) T}$$

$$- \beta^2 e^{-(\gamma + \delta) T}\left[1 - \frac{1}{(\gamma + \delta)} \pi/2\right], \; 0 \le T \le \tau$$

(B-53)

and

$$\ddot{Y} + \beta^2 Y = \beta^2 e^{-\delta T}\left[\left(1 - \frac{\pi}{2}\,\frac{1}{\delta}\right) - \left(1 + \frac{\pi}{2}\,\frac{1}{\delta}\right) e^{+\delta\tau}\right]$$

$$+ \beta^2 e^{-(\alpha' + \delta) T}\left[1 + e^{+(\alpha' + \delta)\tau}\right]$$

$$- \beta^2 e^{-(\gamma + \delta)T}\left[\left(1 - \frac{\pi}{2}\,\frac{1}{(\delta + \gamma)}\right) - \left(1 + \frac{\pi}{2}\,\frac{1}{(\gamma + \delta)}\right) e^{+(\gamma + \delta)\tau}\right]$$

$$- \beta^2 e^{-(\alpha' + \delta + \gamma)T}\left[1 + e^{+(\alpha' + \delta + \gamma)\tau}\right] + \beta^2\left[\frac{\pi}{\delta} + \frac{\pi}{(\delta + \gamma)}\right], \; \tau \le T$$

(B-54)

The solution to Eq (B-53) corresponding to the initial conditions

$Y(0) = 0$

$U(0) = 0$

is

$$Y = \frac{\pi}{2}\left[\frac{1}{\delta} - \frac{1}{(\delta + \gamma)}\right] + \frac{\left[1 - \frac{\pi}{2}\,\frac{1}{\delta}\right] e^{-\delta T}}{\left[1 + \frac{\delta^2}{\beta^2}\right]} + \frac{e^{-(\alpha' + \delta)T}}{\left[1 + \frac{(\alpha' + \delta)^2}{\beta^2}\right]}$$

$$- \frac{e^{-(\alpha' + \delta + \gamma)T}}{\left[1 + \frac{(\alpha' + \delta + \gamma)^2}{\beta^2}\right]} - \frac{\left[1 - \frac{\pi}{2}\,\frac{1}{(\gamma + \delta)}\right] e^{-(\gamma + \delta)T}}{\left[1 + \frac{(\gamma + \delta)^2}{\beta^2}\right]}$$

$$- \left\{\frac{\pi}{2}\left[\frac{1}{\delta} - \frac{1}{(\delta + \gamma)}\right] + \frac{\left[1 - \frac{\pi}{2}\,\frac{1}{\delta}\right]}{\left[1 + \frac{\delta}{\beta^2}\right]} + \frac{1}{\left[1 + \frac{(\alpha' + \delta)^2}{\beta^2}\right]} - \frac{1}{\left[1 + \frac{(\alpha' + \gamma + \delta)^2}{\beta^2}\right]}\right.$$

$$\left. - \frac{\left[1 - \frac{\pi}{2}\,\frac{1}{(\gamma + \delta)}\right]}{\left[1 + \frac{(\gamma + \delta)^2}{\beta^2}\right]}\right\} \cos(\beta T) + 1/\beta \left\{\frac{\left[\delta - \frac{\pi}{2}\right]}{\left[1 + \frac{\delta^2}{\beta^2}\right]} + \frac{(\alpha' + \delta)}{\left[1 + \frac{(\alpha' + \delta)^2}{\beta^2}\right]}\right.$$

$$
+ \frac{(\alpha' + \delta + \gamma)}{\left[1 + \frac{(\alpha' + \delta + \gamma)^2}{\beta^2}\right]} - \frac{\left[\gamma + \delta - \frac{\pi}{2}\right]}{\left[1 + \frac{(\gamma + \delta)^2}{\beta^2}\right]}\Bigg\} \sin(\beta T), \quad 0 \leq T \leq \tau \qquad \text{(B-55)}
$$

and the solution to Eq (B-54) corresponding to the initial conditions

$$
Y(\tau) = Y^*
$$

$$
U(\tau) = U^*
$$

is

$$
\begin{aligned}
Y = & \left\{C_1 \cos(\beta\tau) - C_2 \sin(\beta\tau)\right\} \cos(\beta T) + \left[\frac{\pi}{\delta} - \frac{\pi}{(\delta + \gamma)}\right] \\
& + \left\{C_1 \sin(\beta\tau) + C_2 \cos(\beta\tau)\right\} \sin(\beta T) + \frac{\left[\left(1 - \frac{\pi}{2}\frac{1}{\delta}\right) - \left(1 + \frac{\pi}{2}\frac{1}{\delta}\right) e^{+\delta\tau}\right] e^{-\delta T}}{\left[1 + \frac{\delta^2}{\beta^2}\right]} \\
& - \frac{\left[1 + e^{+(\alpha' + \delta)\tau}\right] e^{-(\alpha' + \delta)T}}{\left[1 + \frac{(\alpha' + \delta)^2}{\beta^2}\right]} - \frac{\left[1 + e^{+(\alpha' + \delta + \gamma)\tau}\right] e^{-(\alpha' + \delta + \gamma)T}}{\left[1 + \frac{(\alpha' + \delta + \gamma)^2}{\beta^2}\right]} \\
& - \frac{\left[\left(1 - \frac{\pi}{2}\frac{1}{(\gamma + \delta)}\right) - \left(1 + \frac{\pi}{2}\frac{1}{(\gamma + \delta)}\right) e^{+(\gamma + \delta)\tau}\right] e^{-(\gamma + \delta)T}}{\left[1 + \frac{(\gamma + \delta)^2}{\beta^2}\right]}, \quad \tau \leq T
\end{aligned}
\qquad \text{(B-56)}
$$

where

$$
\begin{aligned}
C_1 = & \; Y^* - \left[\frac{\pi}{\delta} - \frac{\pi}{(\delta + \gamma)}\right] - \frac{\left[\left(1 - \frac{\pi}{2}\frac{1}{\delta}\right) e^{-\delta\tau} - \left(1 + \frac{\pi}{2}\frac{1}{\delta}\right)\right]}{\left[1 + \frac{\delta^2}{\beta^2}\right]} \\
& - \frac{\left[1 + e^{-(\alpha' + \delta)\tau}\right]}{\left[1 + \frac{(\alpha' + \delta)^2}{\beta^2}\right]} + \frac{\left[\left(1 + \frac{\pi}{2}\frac{1}{(\gamma + \delta)}\right) e^{-(\gamma + \delta)\tau} - \left(1 + \frac{\pi}{2}\frac{1}{(\gamma + \delta)}\right)\right]}{\left[1 + \frac{(\gamma + \delta)^2}{\beta^2}\right]} \\
& + \frac{\left[1 + e^{-(\alpha' + \delta + \gamma)\tau}\right]}{\left[1 + \frac{(\alpha' + \delta + \gamma)^2}{\beta^2}\right]}
\end{aligned}
\qquad \text{(B-57)}
$$

and

$$C_2 = 1/\beta \left\{ U^* + \frac{\left[(\delta - \frac{\pi}{2}) e^{-\delta\tau} - (\delta + \frac{\pi}{2})\right]}{\left[1 + \frac{\delta^2}{\beta^2}\right]} \right.$$

$$- \frac{(\alpha' + \delta + \gamma)\left[1 + e^{-(\alpha' + \delta + \gamma)\tau}\right]}{\left[1 + \frac{(\alpha' + \delta + \gamma)^2}{\beta^2}\right]} + \frac{(\alpha' + \delta)\left[1 + e^{-(\alpha' + \delta)\tau}\right]}{\left[1 + \frac{(\alpha' + \delta)^2}{\beta^2}\right]}$$

$$\left. + \frac{\left[(\gamma + \delta - \frac{\pi}{2}) e^{-(\gamma + \delta)\tau} - (\gamma + \delta + \frac{\pi}{2})\right]}{\left[1 + \frac{(\gamma + \delta)^2}{\beta^2}\right]} \right\} \qquad \text{(B-58)}$$

where Y^* and $U^* = \dot{Y}^*$ are obtained from Eq (B-55) when $T = \tau$.

The above equations reduce to the following for Case c, which is the case of a finite rise to a uniform stress and velocity field. This limiting solution is obtained by letting $\delta \longrightarrow 0$: Thus

$$Y = 1 - \frac{\pi}{2}\frac{1}{\delta} + \frac{\pi}{2} T + \frac{e^{-T}}{\left[1 + \frac{1}{\beta^2}\right]} - \frac{e^{-(1+\gamma)T}}{\left[1 + \frac{(1+\gamma)^2}{\beta^2}\right]} - \frac{\left[1 - \frac{\pi}{2}\frac{1}{\gamma}\right] e^{-\gamma T}}{\left[1 + \frac{\gamma^2}{\beta^2}\right]}$$

$$- \left\{ 1 - \frac{\pi}{2}\frac{1}{\gamma} + \frac{1}{\left[1 + \frac{1}{\beta^2}\right]} - \frac{1}{\left[1 + \frac{(1+\gamma)^2}{\beta^2}\right]} - \frac{\left[1 - \frac{\pi}{2}\frac{1}{\gamma}\right]}{\left[1 + \frac{\gamma^2}{\beta^2}\right]} \right\} \cos(\beta T)$$

$$- 1/\beta \left\{ \frac{\pi}{2} - \frac{1}{\left[1 + \frac{1}{\beta^2}\right]} + \frac{(1+\gamma)}{\left[1 + \frac{(1+\gamma)^2}{\beta^2}\right]} + \frac{(\gamma - \frac{\pi}{2})}{\left[1 + \frac{\gamma^2}{\beta^2}\right]} \right\} \sin(\beta T), \quad 0 \le T \le \tau$$

(B-59)

and

$$Y = \left\{C_1 \cos(\beta\tau) - C_2 \sin(\beta\tau)\right\} \cos(\beta T) - \frac{\pi}{\gamma} - \frac{\pi}{2} T + \frac{(1 + e^{\tau}) e^{-T}}{\left[1 + \frac{1}{\beta^2}\right]}$$

$$+ \left\{C_1 \sin(\beta\tau) + C_2 \cos(\beta\tau)\right\} \sin(\beta T) - \frac{\left[1 + e^{+(1+\gamma)\tau}\right] e^{-(1+\gamma)T}}{\left[1 + \frac{(1+\gamma)^2}{\beta^2}\right]}$$

$$- \frac{\left[(1 - \frac{\pi}{2}\frac{1}{\gamma}) - (1 + \frac{\pi}{2}\frac{1}{\gamma}) e^{+\gamma\tau}\right] e^{-\gamma T}}{\left[1 + \frac{\gamma^2}{\beta^2}\right]} \quad , \tau \leq T \qquad \text{(B-60)}$$

where

$$C_1 = Y^* + \frac{\pi}{\gamma} - \frac{\pi\tau}{2} - \frac{(1 + e^{-\tau})}{\left[1 + \frac{1}{\beta^2}\right]} + \frac{\left[1 + e^{-(1+\gamma)\tau}\right]}{\left[1 + \frac{(1+\gamma)^2}{\beta^2}\right]}$$

$$+ \frac{\left[(1 - \frac{\pi}{2}\frac{1}{\gamma}) e^{-\gamma\tau} - (1 + \frac{\pi}{2}\frac{1}{\gamma})\right]}{\left[1 + \frac{\gamma^2}{\beta^2}\right]}$$

and

$$C_2 = 1/\beta \left\{U^* - \pi - \frac{(1+\gamma)\left[1 + e^{-(1+\gamma)\tau}\right]}{\left[1 + \frac{(1+\gamma)^2}{\beta^2}\right]} + \frac{1 + e^{-\tau}}{\left[1 + \frac{1}{\beta^2}\right]}\right.$$

$$\left. - \frac{\left[(\gamma - \frac{\pi}{2}) e^{-\gamma\tau} - (\gamma + \frac{\pi}{2})\right]}{\left[1 + \frac{\gamma^2}{\beta^2}\right]}\right.$$

In the preceding special equations for Case c and in the following equations for Case b, a value of $\alpha = 1$ was used.

Case b is the case of a step pulse followed by a decaying stress and velocity field. It is characterized by $\gamma \longrightarrow \infty$ and is given by the following equations:

$$Y = \frac{\pi}{2}\frac{1}{\delta} + \frac{\left[1 - \frac{\pi}{2}\frac{1}{\delta}\right] e^{-\delta T}}{\left[1 + \frac{\delta^2}{\beta^2}\right]} + \frac{e^{-(1+\delta)T}}{\left[1 + \frac{(1+\delta)^2}{\beta^2}\right]} + 1/\beta \left\{ \frac{(\delta - \frac{\pi}{2})}{\left[1 + \frac{\delta^2}{\beta^2}\right]} + \frac{(1+\delta)}{\left[1 + \frac{(1-\delta)^2}{\beta^2}\right]} \right\} \sin(\beta T) - \left\{ \frac{\pi}{2}\frac{1}{\delta} + \frac{\left[1 - \frac{\pi}{2}\frac{1}{\delta}\right]}{\left[1 + \frac{\delta^2}{\beta^2}\right]} + \frac{1}{\left[1 + \frac{(1+\delta)^2}{\beta^2}\right]} \right\} \cos(\beta T)$$

$$, \quad 0 \leq T \leq \tau \qquad \text{(B-61)}$$

and

$$Y = \left\{ C_1 \cos(\beta\tau) - C_2 \sin(\beta\tau) \right\} \cos(\beta T) + \frac{\pi}{\delta} + \frac{\left[1 + e^{+(1+\delta)\tau}\right] e^{-(1+\delta)T}}{\left[1 + \frac{(1+\delta)^2}{\beta^2}\right]} + \left\{ C_1 \sin(\beta\tau) + C_2 \cos(\beta\tau) \right\} \sin(\beta T) - \frac{\left[(1 - \frac{\pi}{2}\frac{1}{\delta}) - (1 + \frac{\pi}{2}\frac{1}{\delta}) e^{+\delta\tau}\right] e^{-\delta T}}{\left[1 + \frac{\delta^2}{\beta^2}\right]}$$

$$\tau \leq T \qquad \text{(B-62)}$$

where

$$C_1 = Y^* - \frac{\pi}{\delta} - \frac{\left[1 + e^{-(1+\delta)\tau}\right]}{\left[1 + \frac{(1+\delta)^2}{\beta^2}\right]} - \frac{\left[(1 - \frac{\pi}{2}\frac{1}{\delta}) e^{-\delta\tau} - (1 + \frac{\pi}{2}\frac{1}{\delta})\right]}{\left[1 + \frac{\delta^2}{\beta^2}\right]}$$

and

$$C_2 = 1/\beta \left\{ U^* + (1+\delta) \frac{\left[1 + e^{-(1+\delta)\tau}\right]}{\left[1 + \frac{(1+\delta)^2}{\beta^2}\right]} + \frac{\left[(\delta - \frac{\pi}{2}) e^{-\delta\tau} - (\delta + \frac{\pi}{2})\right]}{\left[1 + \frac{\delta^2}{\beta^2}\right]} \right\}$$

The equations for Case b were evaluated for $\delta = 0.1$ (and $\gamma = \infty$) and the equations for Case c were evaluated for $\gamma = 1$ (and $\delta = 0$). These values were considered as being reasonable and of interest to this program. The response of the buried structure for these two cases is compared with Case a ($\delta = 0$, $\gamma = \infty$, $\mu^2 = 5$, and $\tau = 4$) and is presented in Fig. B-22 through B-26.

The amplitude, A, of the oscillation of the deflection, Y, about its mean is only meaningful if the stress is constant for a period of time which is long compared to the period of oscillation of the buried structure. The amplitude is obtained from the coefficients of the sin (βT) and cos (βT) terms and is given in a general form as

$$A = A\left\{\mu^2, \tau, \gamma, \delta, \alpha'\right\} = \sqrt{C_1^{\,2} + C_2^{\,2}} \qquad \text{(B-63)}$$

3. Structures with Free Back Faces

The previous analysis treats the case of a completely buried structure and does not make any allowance for connecting tunnel or access-ways. This section deals with the special case where the back face of the structure is shielded from the ground shock disturbance; that is, the soil or rock in the neighborhood of the back face has been removed. The structure could be located at the end of a tunnel such that the front surface of the structure is in contact with the soil or rock and the structure would be free to move back away from the earth medium. There is, of course, a complete spectrum of conditions which could be imposed upon the back face. This analysis treats a limiting case and will be restricted to the free-field case of the step pulse followed by a uniform stress and velocity field.

The differential equation for this case is obtained by letting $\mathcal{F}_{bw} = \mathcal{F}_{ba} = 0$ and is given as

$$\ddot{Y} + \frac{1}{2}\beta^2 Y = \beta^2 \frac{\mathcal{F}}{2} T + \beta^2 + \beta^2 e^{-T} \qquad \text{(B-64)}$$

The initial conditions are

$Y(0) = 0$

$\dot{Y}(0) = 0$

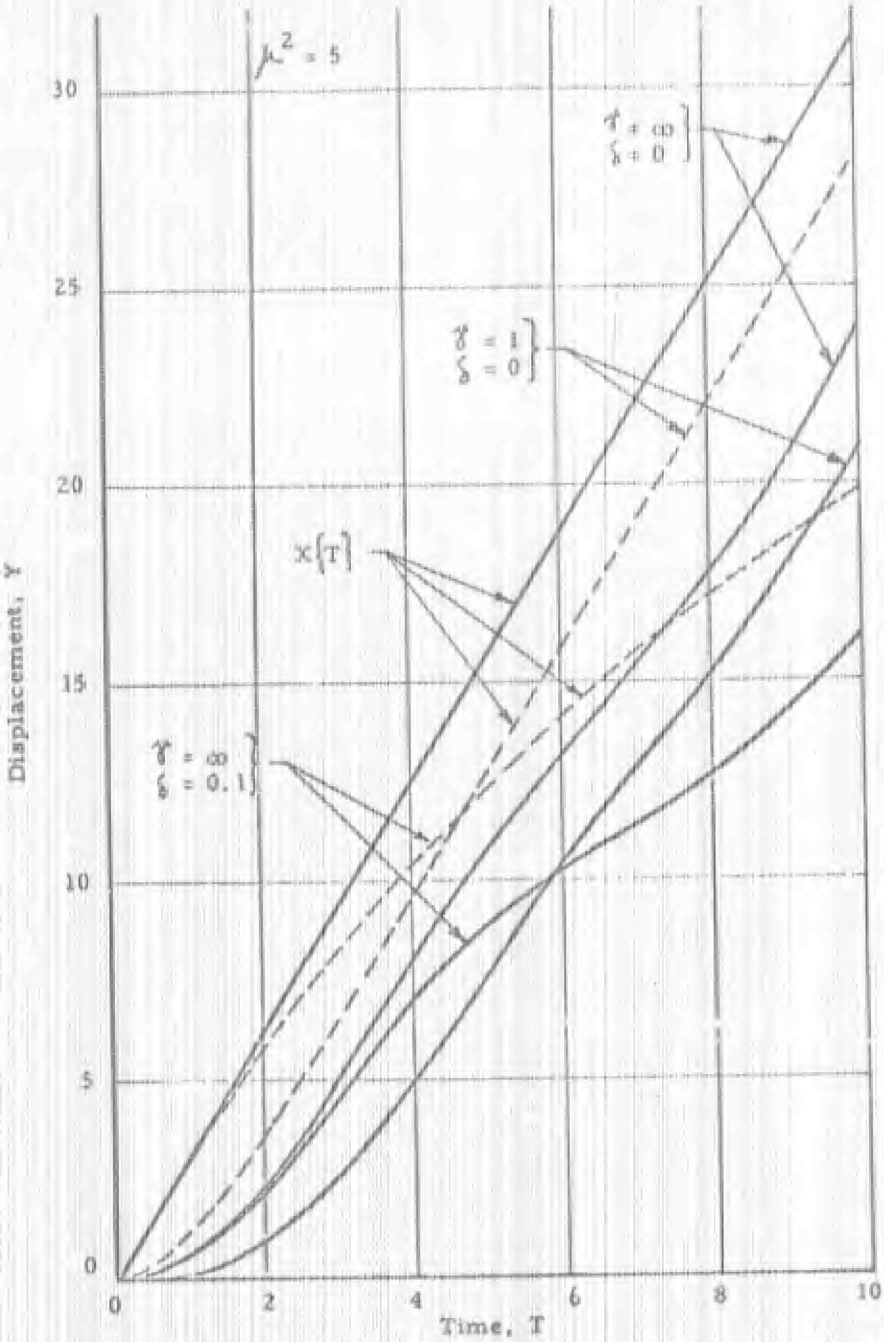


Fig. B-22 EFFECT OF FREE-FIELD WAVE FORM ON STRUCTURE DISPLACEMENT

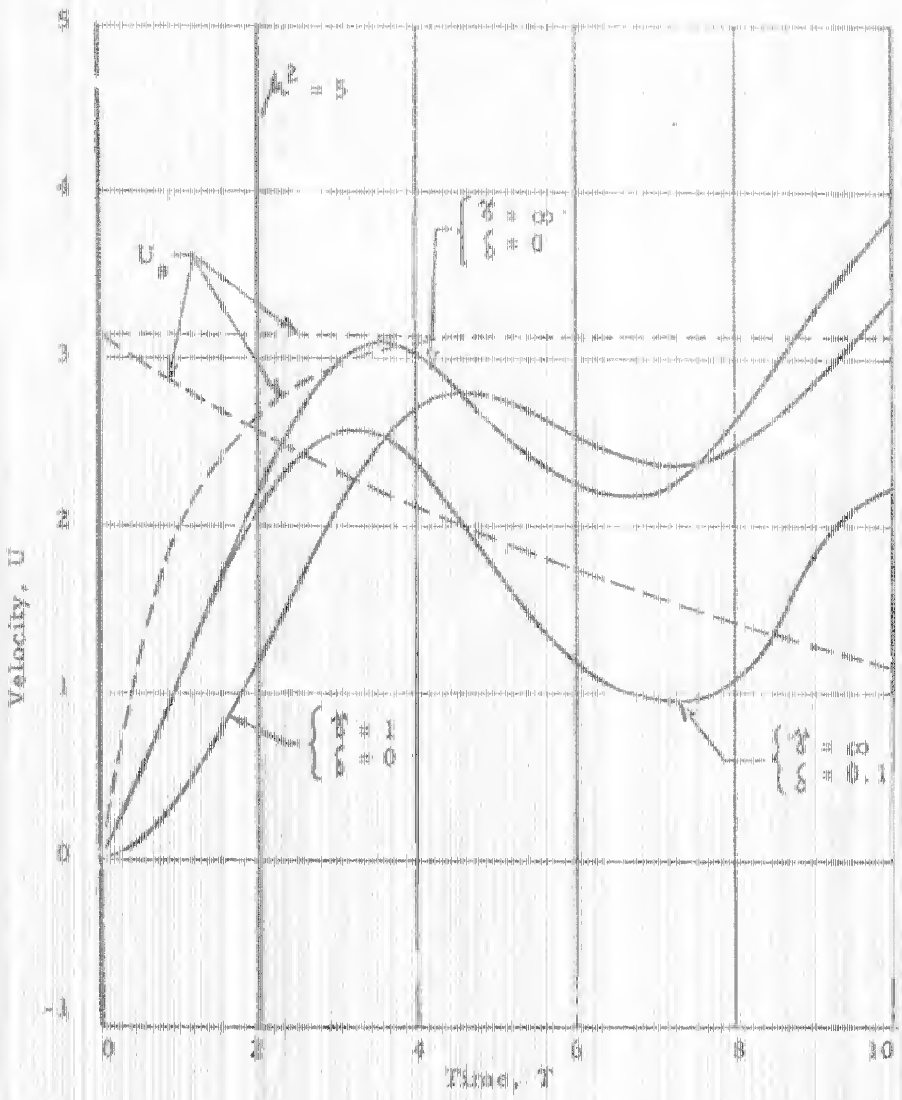


Fig. B-23 EFFECT OF FREE-FIELD WAVE FORM ON STRUCTURE VELOCITY

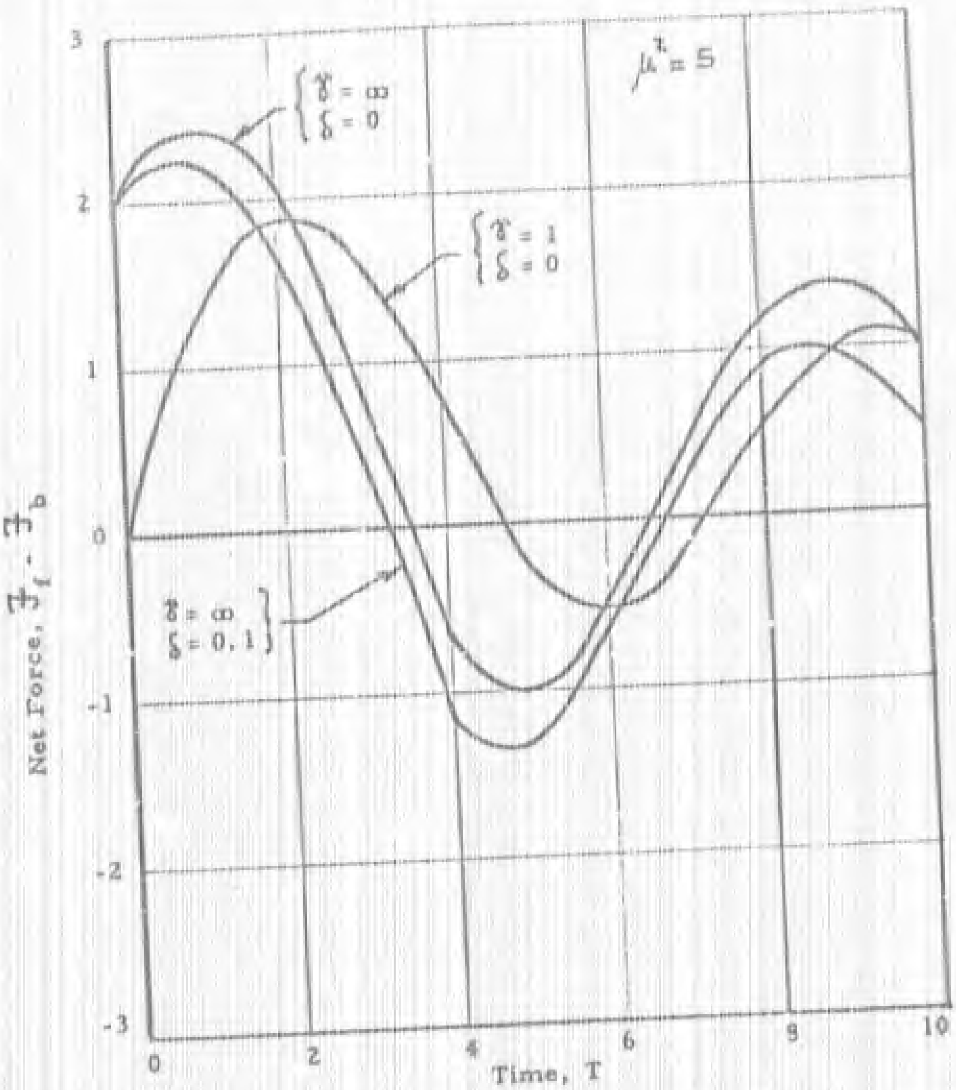


Fig. B-24 EFFECT OF FREE-FIELD WAVE FORM ON NET FORCE

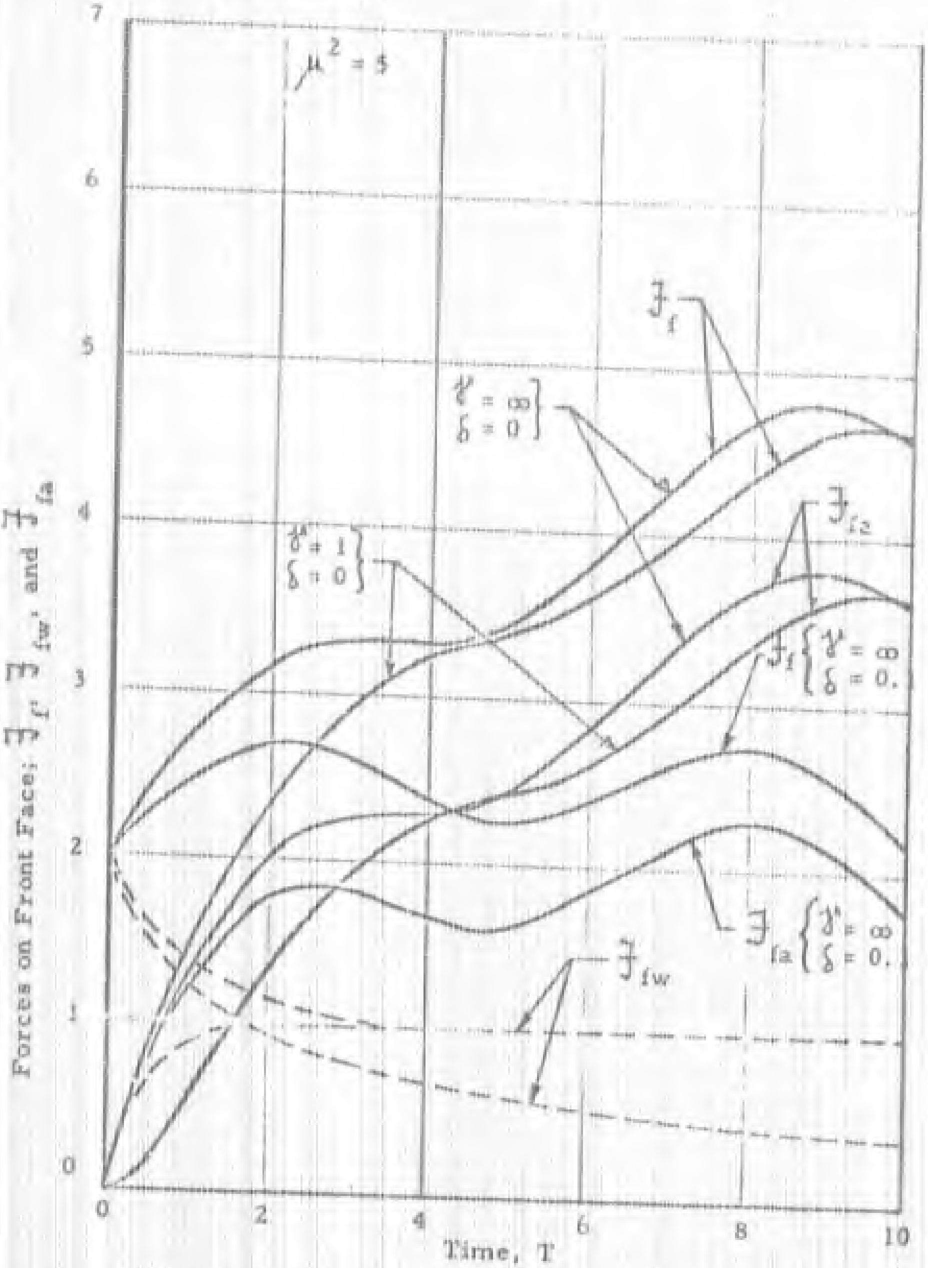


Fig. B-25 EFFECT OF FREE-FIELD WAVE FORM ON FORCES ACTING ON THE FRONT FACE

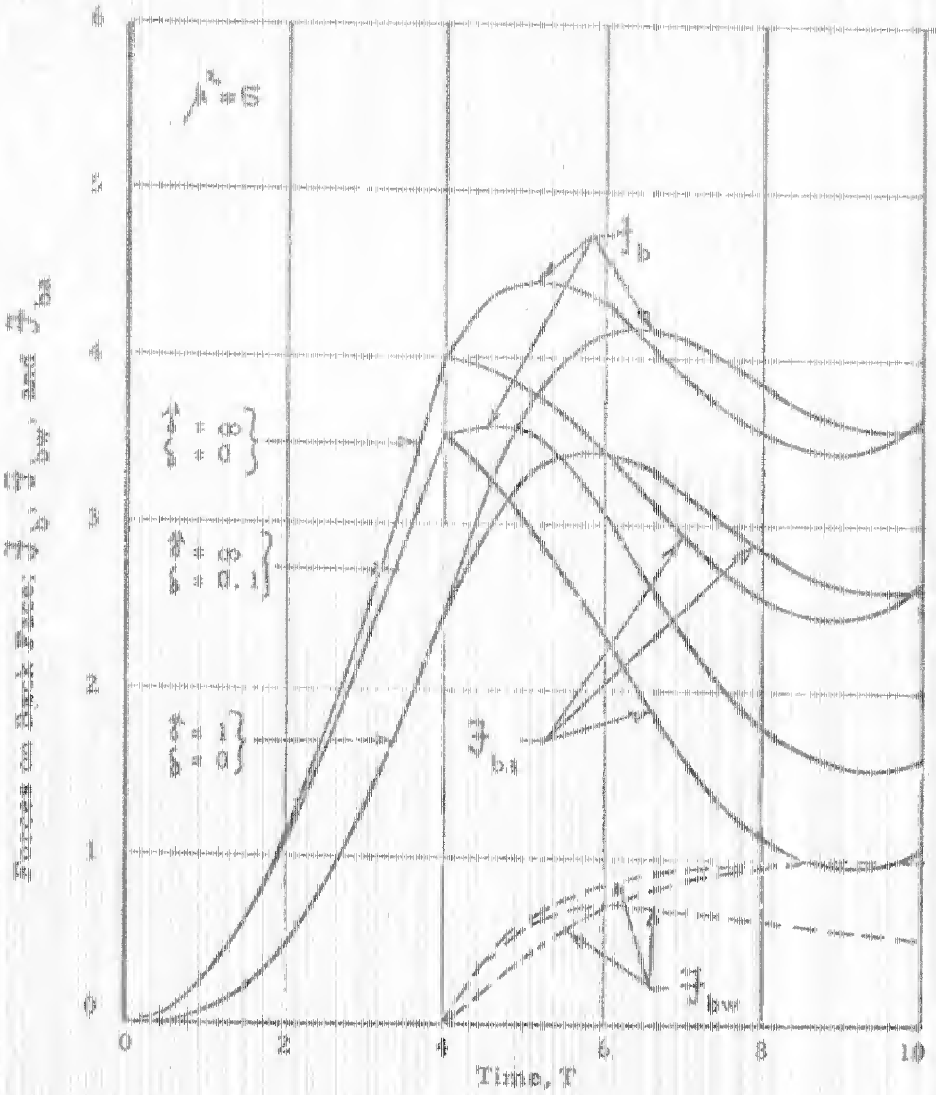


Fig. B-26 EFFECT OF FREE-FIELD WAVE FORM ON FORCES ACTING ON BACK FACE

The solution to Eq (B-64) is

$$Y = Z\left\{1 + \frac{\pi}{2}T + \frac{e^{-T}}{\left[1 + \frac{2}{\beta^2}\right]} - \left[1 + \frac{1}{1 + \frac{2}{\beta^2}}\right]\cos\left(\frac{\beta}{\sqrt{2}}T\right)\right.$$

$$\left. + \frac{\sqrt{2}}{\beta}\left[\frac{1}{1 + \frac{2}{\beta^2}} - \frac{\pi}{2}\right]\sin\left(\frac{\beta}{\sqrt{2}}T\right)\right\} \qquad \text{(B-65)}$$

Equation (B-35) for $\mathcal{F}_{fw}$ and $\mathcal{F}_{fa}$ is valid for the above equation (Note: $\alpha' = 1$ was used). Figure B-27 and B-28 present $\mathcal{F}_{fa}$ and $\mathcal{F}_{t}$ respectively for $\mu^2 = 2$, 5, 10, and ∞.

The following paragraph presents a minor variation of the above analysis. Consider the structure which possesses a resistance to motion which is proportional to the absolute displacement, y.

The equation of motion is

$$M_o \ddot{y} = F_{fw} + F_{fa} - ky \qquad \text{(B-66)}$$

or if we define

$$\zeta = \frac{8k}{\pi^2 DE} \qquad \text{(B-67)}$$

We can rewrite (B-66) in a dimensionless form as

$$\ddot{Y} + \left(\frac{1+\zeta}{2}\right)\beta^2 Y = \beta^2 \frac{\pi}{2} T + \beta^2 + \beta^2 e^{-T} \qquad \text{(B-68)}$$

The solution to Eq (B-68) corresponding to the same initial condition imposed on the solution of Eq (B-64) is

$$Y = \left[\frac{2}{1+\zeta}\right]\left\{1 + \frac{\pi}{2}T + \frac{e^{-T}}{\left[1 + \frac{2}{(1+\zeta)\beta^2}\right]} - \left[1 + \frac{1}{1 + \frac{2}{(1+\zeta)\beta^2}}\right]\cos\left(\beta\sqrt{\frac{1+\zeta}{2}}\,T\right)\right.$$

$$\left. + 1/\beta\sqrt{\frac{2}{1+\zeta}}\left[\frac{1}{1 + \frac{2}{(1+\zeta)\beta^2}} - \frac{\pi}{2}\right]\sin\left(\beta\sqrt{\frac{1+\zeta}{2}}\,T\right)\right\} \qquad \text{(B-69)}$$

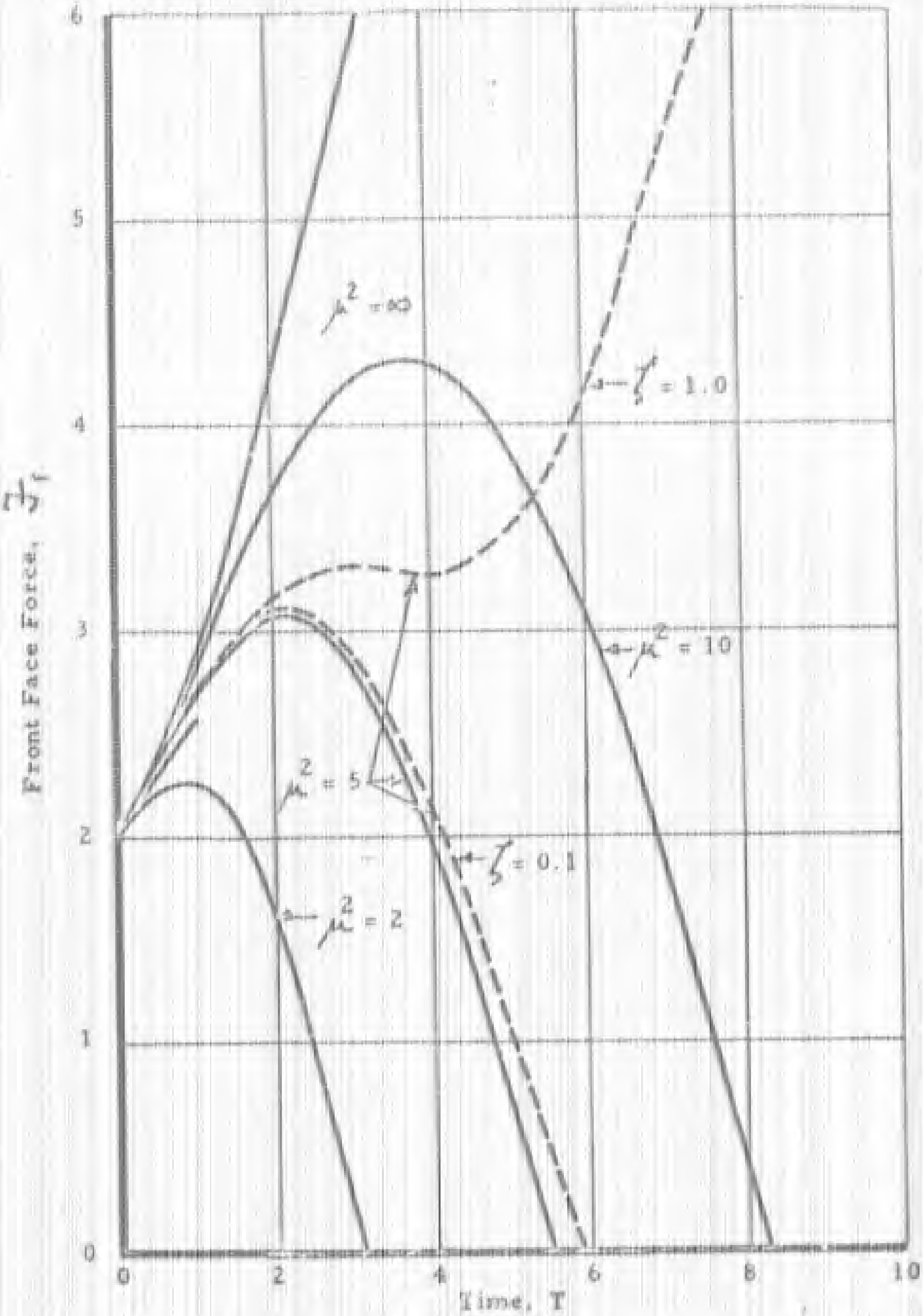


Fig. B-27 VARIATION OF FORCE ON FRONT FACE FOR STRUCTURE WITH FREE BACK FACE

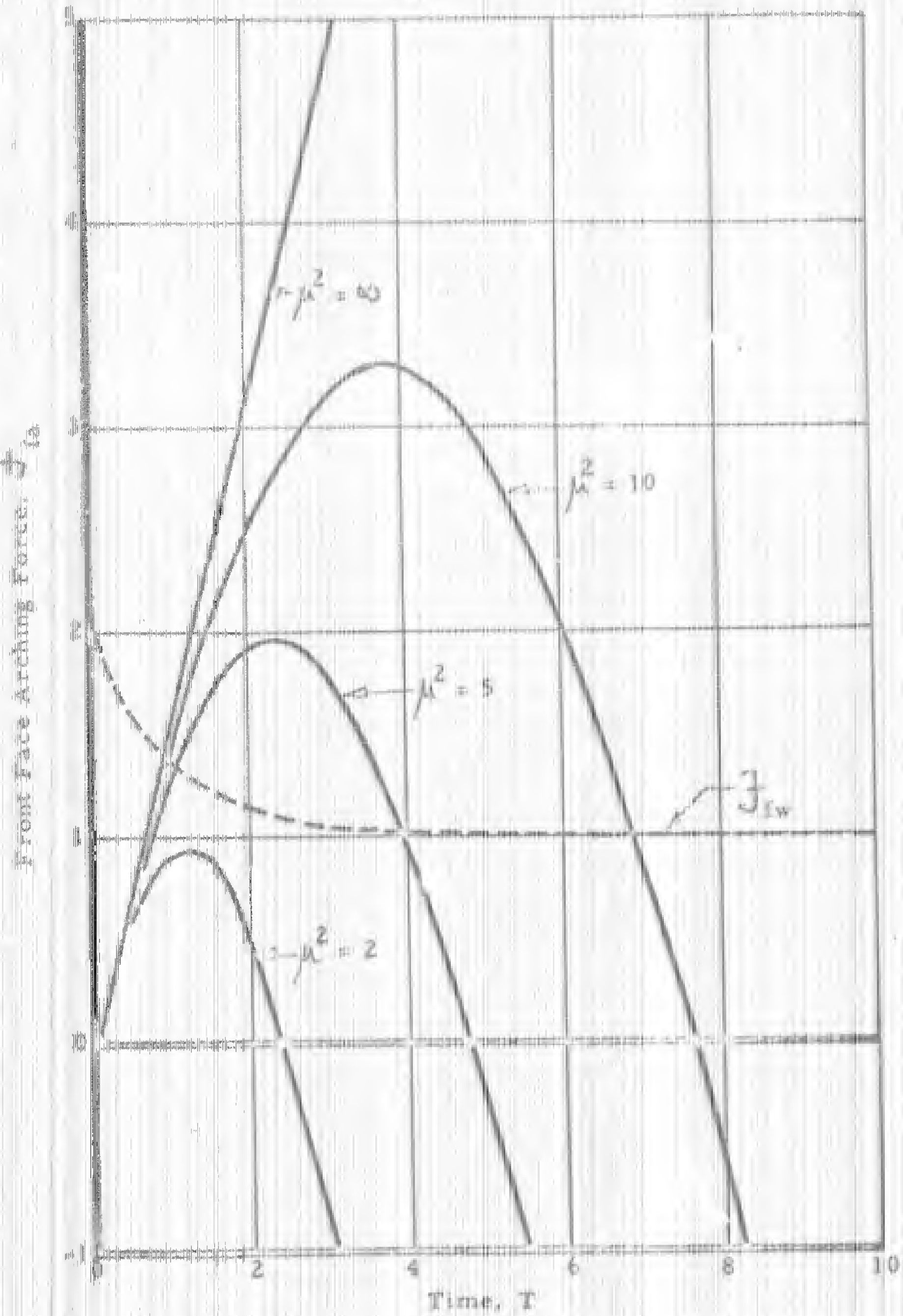


Fig. B-28 VARIATION OF ARCHING FORCE ON FRONT FACE FOR STRUCTURE WITH FREE BACK FACE

Thus, ζ corresponds to the solution given by Eq (B-64). If $\zeta = 1$, the solution corresponds to Eq (B-38). A more practical value of ζ would be 0.1. Eq (B-69) was evaluated for this value of ζ and the results are presented in Fig. B-28 for $\mu^2 = 5$.

4. Isolation and Shock Absorbing Systems

It is desirable to reduce both the loads acting on a buried structure and the accelerations of components housed within the structure when the structure is struck by a ground shock wave. These reductions may be accomplished by isolating the structure from the surrounding soil (or rock). To be consistent with the analysis carried out thus far in this appendix, we will limit our treatment to considerations of the front and back faces of the structure. This section discusses three classes of isolation or shock absorbing systems.

The first system consists of surrounding the structure (front and back face) with a crushable material. This system could have two principal characteristics. First, it could reduce the initial load (impact) and, hence, the acceleration acting on the structure; and secondly, it could, by effectively changing the length of the structure, reduce the subsequent or final compressive load. The specific properties of the crushable material will, of course, influence the effectiveness of this type of system.

The second system consists of providing a void around the buried structure. Thus, the load on the front surface of the structure will be zero until the free surface of the soil displaces the depth of the void. This system has the disadvantage that the soil (or rock) may spall and cause local impact loads. Also, the soil must be able to maintain the free surface under static conditions.

The third system consists of surrounding the buried structure with an elastic buffer. The material should have a modulus of elasticity which is smaller than the modulus of the earth medium which surrounds the structure. This system has the same general characteristics as above. However, the possibility of impacts, which could occur with the other two systems, does not exist.

There are some materials which have the property of being able to dissipate a great deal of energy when a shock wave traverses the material, thus causing the pressure or stress level to drop sharply. This property is usually restricted to a specific stress range for a given material. Thus, it is possible to lower the stress level of an advancing wave, at least locally, and for a short period of time. However, the volume which must be filled with this special material to accomplish any significant results must be at least the order of magnitude of the volume of the structure. The isolation systems discussed herein are restricted to the case where the depth of the isolating material (or void) is smaller than the characteristic dimension of the structure (in the present case the diameter D).

The initial interaction for the three systems will be discussed here prior to treating the elastic buffer case in detail. We will limit the discussion of the initial interaction to the case of a step pulse wave.

For the purpose of this discussion, the properties of the crushable material can be idealized by the stress-strain curve given in Fig. B-29 (c). When the stress reaches the crushing level, σ_c, the material changes volume corresponding to the strain ϵ_c, and thereafter is considered a rigid material. The crushing strength of the material must be sufficiently great to support any static loads which are imposed upon it by the structure or the surrounding soil. The wave diagram (a) of Fig. B-29 illustrates the interaction of the incident shock (step stress pulse) with the crushable material and the structure. When the shock strikes the crushable material, the material crushes and a stress wave propagates forward at the velocity U' given by:

$$U = u'_1 / \epsilon_c \qquad \text{(B-70)}$$

where u'_1 is the particle velocity in the crushed material. The stress in the wave traveling through the crushable material is σ_c. The condition at the interface between the soil and the crushable material requires that the particle velocity and the stress be continuous across the interface; that is,

$$\sigma_1 = \sigma'_1 = \sigma_c$$

$$u_1 = u'_1 \qquad \text{(B-71)}$$

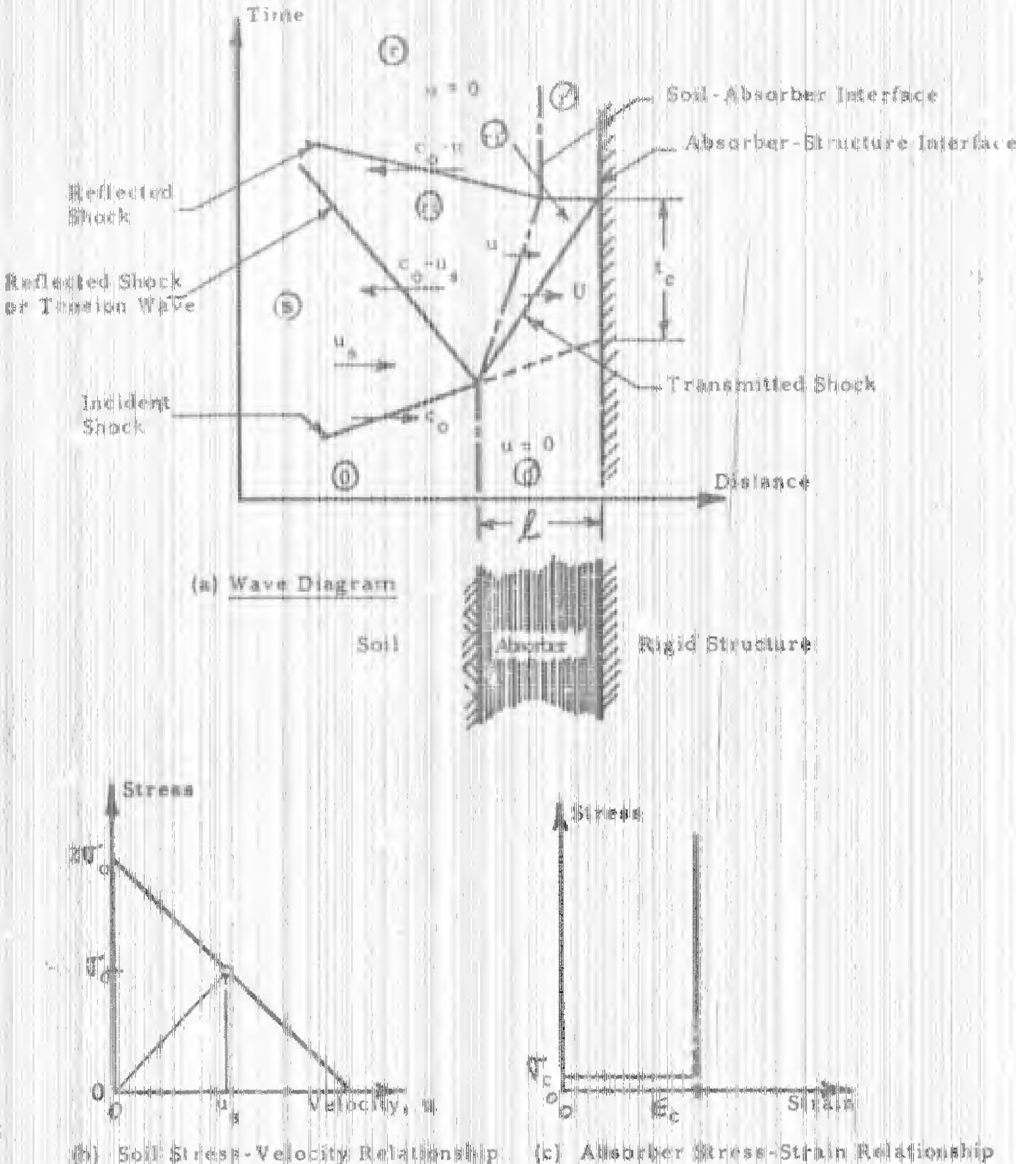


Fig. B-29 SCHEMATIC DIAGRAMS FOR ABSORBER APPLICATION where σ_1 and σ'_1 are the stresses in the soil and crushable material, respectively, and u_1 is the particle velocity in the soil. Since the soil is being treated as an elastic material, the stress is related to the velocity by the following equation:

$$\sigma = \sigma_o \left[2 - \frac{u}{u_s} \right] \qquad \text{(B-72)}$$

This relation is illustrated in Fig. B-29 (b). Using Eq (B-71) and (B-72) we obtain

$$u'_1 = u_s \left[2 - \frac{\sigma_c}{\sigma_o} \right] \qquad \text{(B-73)}$$

and therefore Eq (B-70) can be rewritten as

$$U = \frac{u_s}{\epsilon_c} \left[2 - \frac{\sigma_c}{\sigma_o} \right] \qquad \text{(B-74)}$$

It should be noted that $\sigma_c \leq 2\sigma_o$ in order for this analysis to be valid.

When the stress wave which is propagating through the crushable material strikes the rigid structure it reflects from the structure. For the idealized material assumed here the reflected wave will propagate back at infinite velocity. Thus, the soil is brought to rest at the soil-crushable material interface, resulting in the propagation of a reflected wave back into the soil. The stress in soil behind this wave and in the crushable material is increased to $2\sigma_o$. Thus the initial load per unit area acting on the structure is $2\sigma_o$, which is the same initial load per unit area which would act on the structure if the crushable material were replaced by soil. Actually, an arching effect does exist but since this treatment is rather cursory its influence has been neglected.

There will be a slight time difference between the arrival of the initial stress wave at the structure for the system discussed here and the system without the crushable material. This time difference is given as

$$t_c = \ell \left[\frac{1}{U} - \frac{1}{c_o} \right] \qquad \text{(B-75)}$$

The crushable material located at the back of the structure may or may not crush depending upon the values of σ_o and σ_c. In any event, it is possible for this type of system to impose large impact loads whenever the structure breaks loose from the soil (or vice versa) and then, due to the motion of both the soil and the structure, regains contact.

For the isolation system which utilizes a void space around the structure, the incident shock or stress wave will be reflected from the free surface of the soil as a tension wave (Fig. B-30 (b)). The free surface moves forward with a velocity equal to $2u_s$. If the soil does not fail locally due to the tension wave, so that spallation will not occur, nothing will happen to the structure until the free surface of the soil has traversed the void space. (End effects have been neglected). At the time that the free surface impacts with the rigid structure, the soil is brought to rest. The stress increases instantaneously from zero to $2\,\sigma_o$ and a stress wave moves back into the soil. The impact load per unit area on the structure is again equal to $2\,\sigma_o$; hence, no real benefits are derived from this system. Also, impact loads are quite probable. The impact loads to which this system and the system which utilizes a crushable material are susceptible tend to disqualify these systems as suitable isolation systems.

The elastic buffer system is illustrated in Fig. B-30 (a). In this system the incident stress wave is transmitted through the buffer and propagates at the seismic (sound) velocity of the buffer material. Depending upon the acoustic impedance of the soil and buffer material, a wave will be reflected from the soil-buffer interface. Generally, both the density and the seismic velocity of the buffer material will be less than the corresponding property of the soil so that the signal (not shown in the figure) reflected from the interface will be a tension wave, although perhaps quite weak. The stress wave propagating through the buffer will reflect from the rigid structure increasing the stress to nearly $2\,\sigma_o$. The stress wave reflected from the structure will move out into the soil and increase the stress in the soil. In order to simplify the treatment somewhat, we will assume that the acoustic impedance of both materials is the same; hence, the structure will experience an initial stress per unit area of $2\,\sigma_o$. The benefits of this isolation system will be demonstrated in the following analysis.

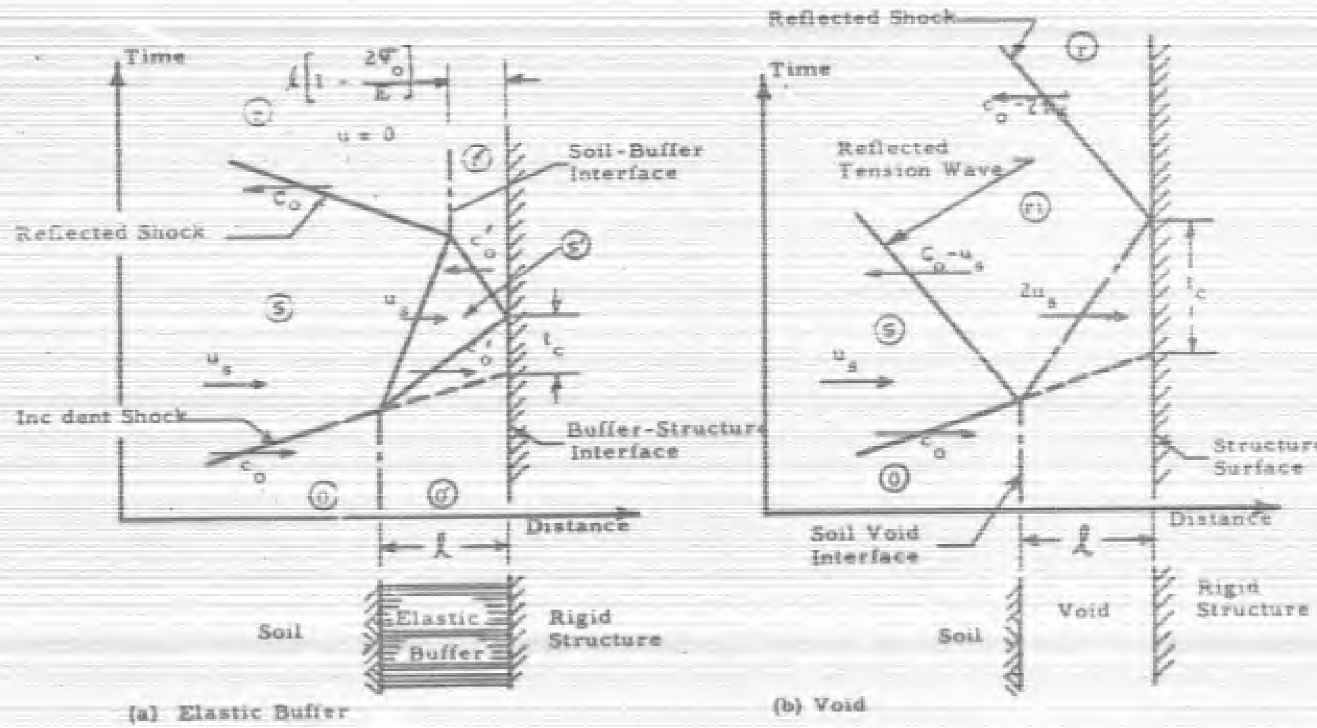


Fig. B-30 SCHEMATIC WAVE DIAGRAMS FOR ISOLATION TECHNIQUES

For the case of an elastic buffer, the arching force is modified in that smaller forces are required for a given relative deflection. A layer of material, characterized by an elastic modulus, E_b, and a thickness, ℓ, is placed at both ends of the structure. In general, $\ell \ll D$, so that the edge effects are not predominant. Corresponding to the load deflection relationship assumed previously, we assumed that for the soil

$$\sigma = \frac{F_a}{\pi D^2/4} = \frac{\pi E}{2D} z' \qquad \text{(B-76)}$$

where z' is the relative deflection of the soil-buffer interface, and for the buffer

$$\sigma = \frac{\Delta \ell}{\ell} E_b \qquad \text{(B-77)}$$

where $\Delta \ell$ is the change in thickness of the buffer. If z is the relative deflection of the structure (face) and the soil, then

$$z = z' + \Delta \ell \qquad \text{(B-78)}$$

or using Eq (B-76) and (B-77) yields

$$F_a = \frac{\pi D^2}{4} \cdot \frac{z}{\left[\frac{2D}{\pi E} + \frac{\ell}{E_b}\right]} = \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \cdot \frac{z}{\left[1 + \frac{\pi E \ell}{2 E_b D}\right]} \qquad \text{(B-79)}$$

or, in a dimensionless form,

$$\mathcal{J}_a = \frac{Z}{z} \frac{1}{\left[1 + \frac{\pi}{2} \frac{\mathcal{L}}{\mathcal{E}}\right]} = \nu \frac{Z}{z} \qquad \text{(B-80)}$$

where

$\mathcal{L} = \ell / D$

$\mathcal{E} = E_b / E \ll 1$

For the case of a step pulse (Case a) the equations of motion Eq (B-36) and (B-37) become

$$\ddot{Y} + \nu\beta^2 Y = \beta^2\left[1 + e^{-T}\right] + \beta^2 \frac{\pi}{2}\nu T, \quad 0 \leq T \leq \tau \qquad \text{(B-81)}$$

and

$$\ddot{Y} + \nu\beta^2 Y = \beta^2\,(1 + e^{+\tau})\,e^{-T} + \nu\beta^2\pi T - \beta^2\nu\frac{\pi}{2}, \quad \tau \leq T \qquad \text{(B-82)}$$

The solution of these differential equations, subject to the same initial conditions which were imposed upon Eq (B-36) and (B-37) is

$$Y = 1/\nu + \pi/2\ T + \frac{e^{-T}}{\nu + \frac{1}{\beta^2}} + \frac{1}{\sqrt{\nu}\beta}\left[\frac{1}{\nu + \frac{1}{\beta^2}} - \frac{\pi}{2}\right]\sin(\sqrt{\nu}\beta T)$$

$$- \left[1/\nu + \frac{1}{\nu + \frac{1}{\beta^2}}\right]\cos(\sqrt{\nu}\beta T), \quad 0 \leq T \leq \tau \qquad \text{(B-83)}$$

and

$$Y = \left\{C_1\cos(\beta\sqrt{\nu}\tau) - C_2\sin\beta\sqrt{\nu}\tau)\right\}\cos(\beta\sqrt{\nu}\ T) + \left\{C_1\sin(\beta\sqrt{\nu}\tau)\right.$$

$$\left. + C_2\cos(\beta\sqrt{\nu}\tau)\right\}\sin(\beta\sqrt{\nu}\ T) + \pi T - \frac{\pi}{2}\tau + \frac{\left[1 + e^{+\tau}\right]e^{-T}}{\left[\nu + \frac{1}{\beta^2}\right]}, \quad \tau \leq T \qquad \text{(B-84)}$$

where

$$C_1 = Y^* - \frac{\pi}{2}\tau - \frac{\left[1 + e^{-\tau}\right]}{\left[\nu + \frac{1}{\beta^2}\right]}$$

$$C_2 = \frac{1}{\beta\sqrt{\nu}}\left\{U^* - \pi + \frac{\left[1 + e^{-\tau}\right]}{\left[\nu + \frac{1}{\beta^2}\right]}\right\}$$

A comparison of the response of a rigid structure was made for $\nu = 0.5$ and $\nu = 1.0$. These results are presented in Fig. B-31 through B-34. The amplitude of the oscillation of the displacement for this case is given by the general equation, Eq (B-63). The velocity does not differ greatly during the first portion of the interaction. For the example given here (after $T = \tau$) the magnitude of the oscillation of the velocity for the buffer case is considerably smaller; however, this is for the most part fortuitous. With the exception of the initial value the net force or acceleration is somewhat smaller for the elastic buffer case. However, as was mentioned previously, this type of isolation system will not be very effective in reducing the initial impact of a steep fronted (step pulse) wave. As is evident from the argument of the harmonic terms, the period of oscillation is somewhat larger for the buffer case. The arching and net forces on both the front and back faces are reduced considerably for this example. The arching force on both the front and back faces is reduced by the factor, ν. Thus,

$$F_{fa} = \nu \frac{Z}{2}$$

$$F_{ba} = \left[\frac{\nu}{2} \pi \tau - Z\right], \ \tau \leq T \qquad \text{(B-85)}$$

Hence, the mean value of the total force on both the front and back faces of the structure is equal to $1 + \nu \frac{\pi}{4} \tau$.

5. Other Structural Parameters

The previous sections have treated a variety of special cases covering the effects of both structural and free-field parameters. The basic loading or interaction theory presented at the beginning of this appendix could be applied to a variety of complex structural arrangements. It is the purpose of this last section to explore briefly two rather important structural parameters, both dealing with the flexibility of the buried structure.

The first problem concerns the shortening of the structure due to the compressive loads which exist during the interaction. The second problem deals with a special case of a structure which consists of two masses connected together by a linear spring.

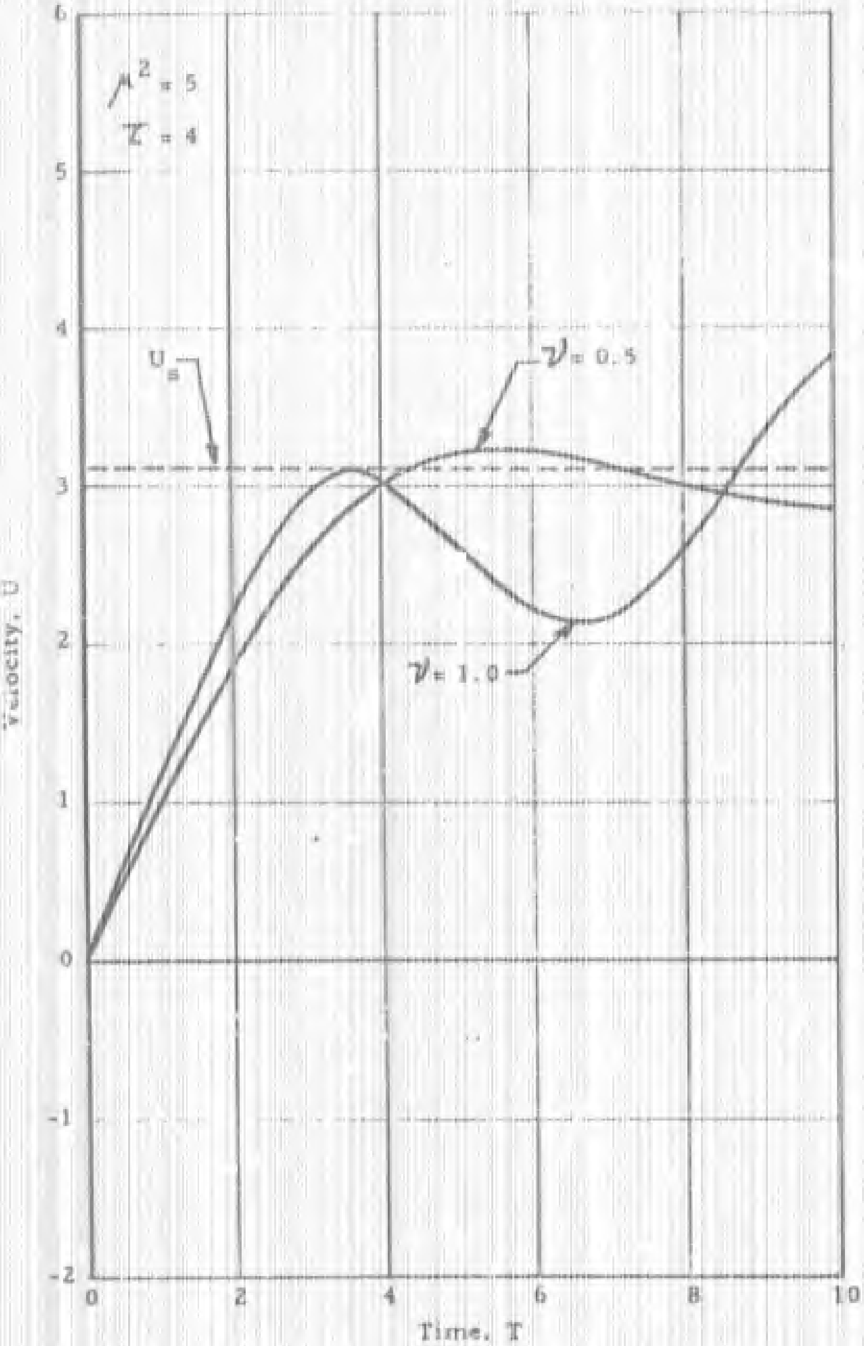


Fig. B-31. EFFECT OF ELASTIC BUFFER ON VELOCITY

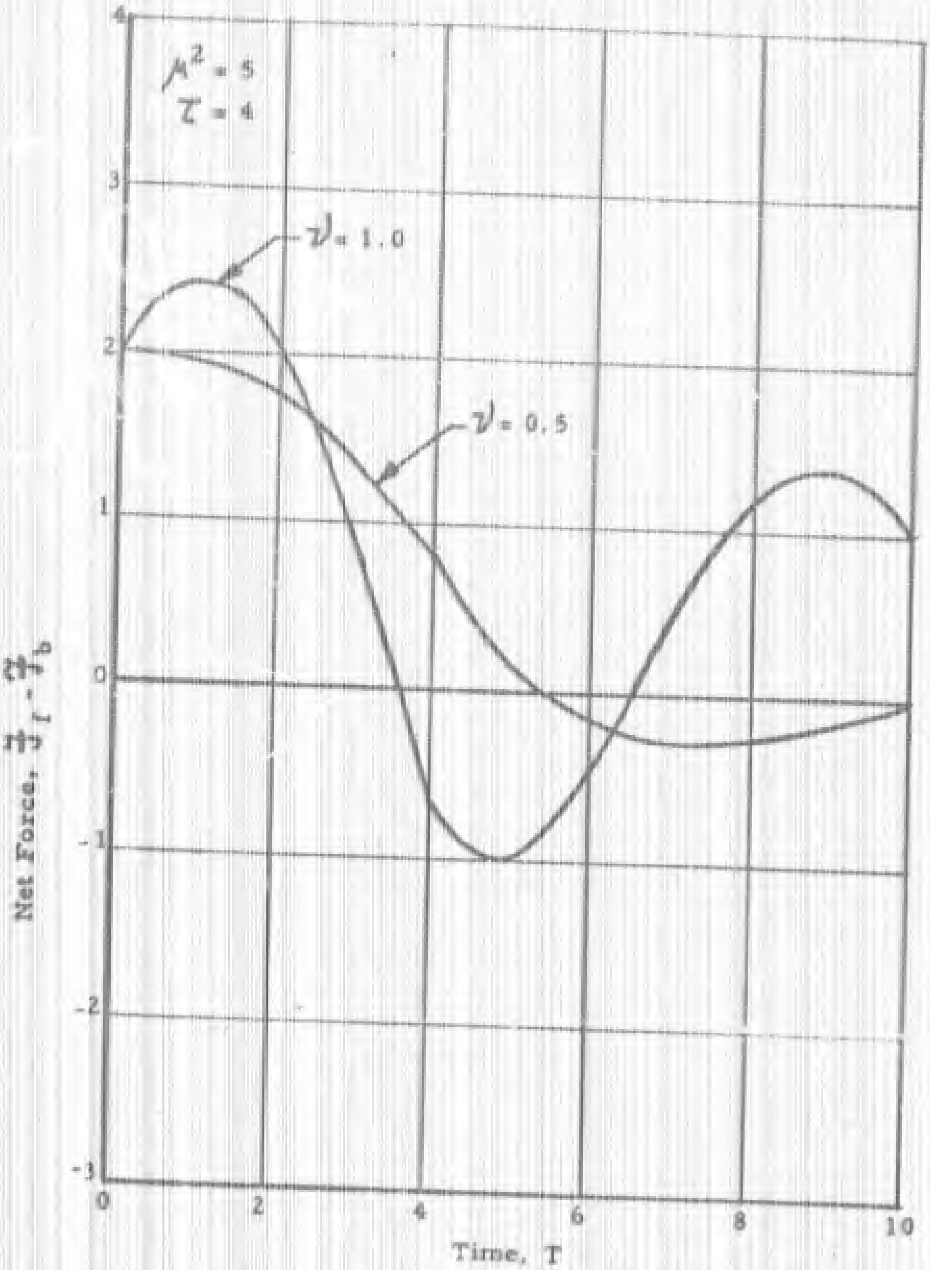


Fig. B-32. EFFECT OF ELASTIC BUFFER ON NET FORCE

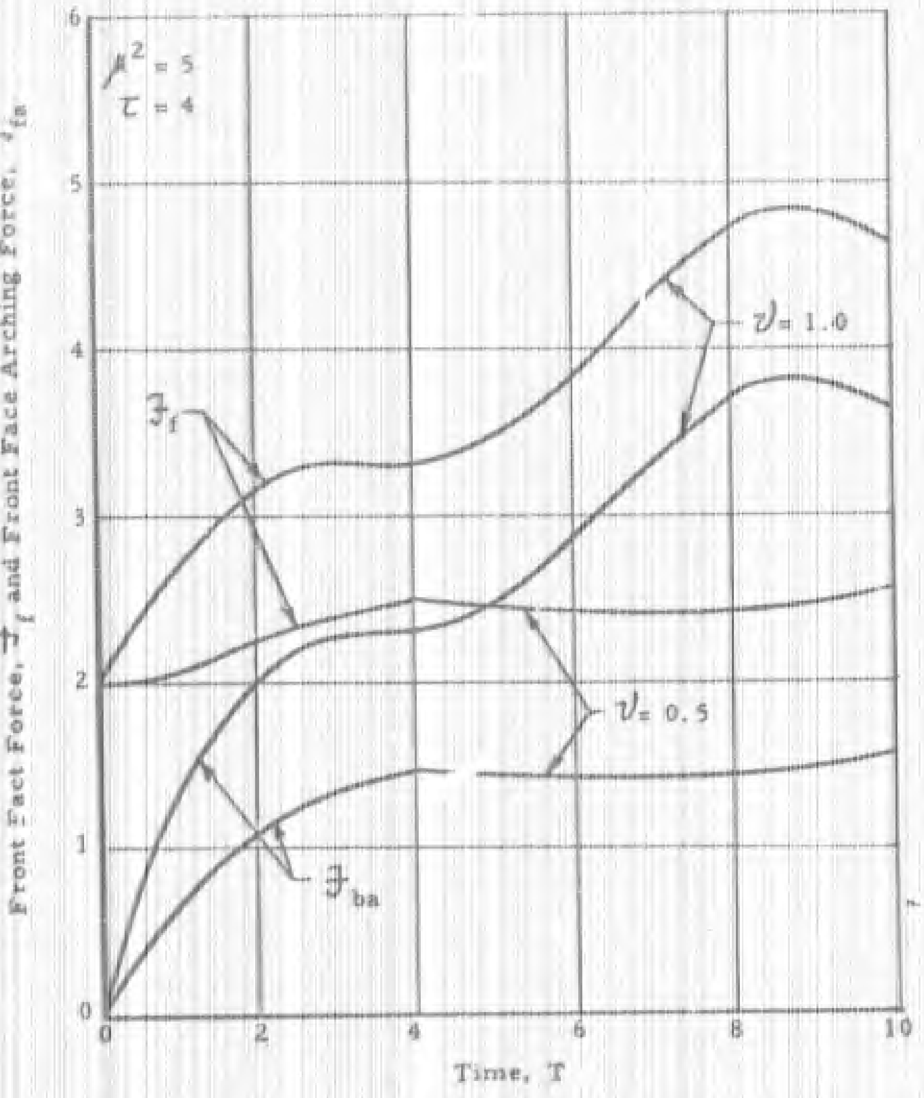


Fig. B-33. EFFECT OF ELASTIC BUFFER ON THE FORCES ACTING ON THE FRONT FACE

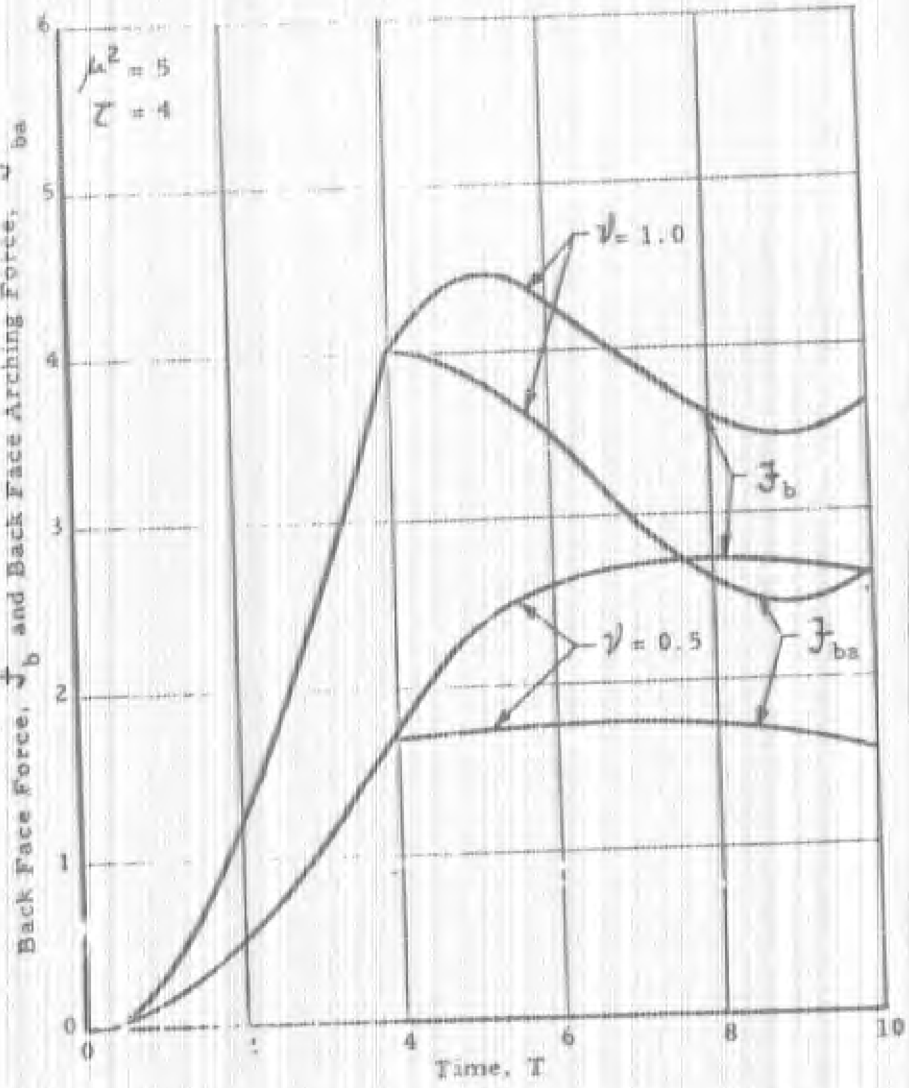


Fig. B-34. EFFECT OF ELASTIC BUFFER ON FORCES ACTING ON BACK FACE OF STRUCTURE

In treating the first problem, it should be pointed out that it is desirable to use a single variable to determine the position of the structure. In this analysis, the coordinate, y, is used to specify the absolute displacement of the center (the mass center) of the structure. It will be further assumed that the structure will be shortened uniformly and proportional to the force applied to the structure; that is,

$$L' = L - K(F_f + F_b) \qquad \text{(B-86)}$$

where L' is the length of the structure corresponding to the compressive load $(F_f + F_b)$ and K is a flexibility parameter of the structure. It is possible to use a more equitable form for Eq (B-86), such as one which allows the two halves of the structure to be compressed to different lengths. The change in length being proportional to the load acting on the respective half would complicate the analysis considerably and may not improve the results appreciably. It is thought that the use of Eq (B-86) will yield a good first order solution.

The absolute deflection or displacement of the front and back faces of the structure are given respectively as

$$y + 1/2\,(L - L')$$

and

$$y - 1/2\,(L - L')$$

so that the arching forces are given as

$$F_{fa} = \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D}\left[x - y - 1/2\,(L - L')\right] = \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D}\left[x - y - K/2\,(F_{fw} + F_{fa}\right.$$

$$\left. + F_{bw} + F_{ba})\right],\ 0 \leq t \qquad \text{(B-87)}$$

and

$$F_{ba} = \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \left[y - 1/2\,(L - L') \right]$$

$$= \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \left[y - K/2\,(F_{fw} + F_{fa} + F_{bw} + F_{ba}) \right], \quad 0 \leq t \leq L/c_o$$

(B-88)

$$F_{ba} = \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \left[y - 1/2\,(L - L') - x + \frac{\sigma}{E} L \right]$$

$$= \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \left[y - x + \frac{\sigma}{E} L - K/2\,(F_{fw} + F_{fa} + F_{bw} + F_{ba}) \right], \quad L/c_o \leq t$$

(B-89)

Expanding both Eq (B-87) and(B-88), and solving for F_{fa} and F_{ba}, yields

$$F_{fa} = \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \left\{ x \left[\frac{1 + \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \cdot \frac{K}{2}}{1 + \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D}\, K} \right] - y - \left[F_{fw} + F_{bw} \right] \frac{\frac{\pi D^2}{4} \cdot \frac{\pi E}{2D}\, \frac{K}{2}}{\left[1 + \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D}\, K \right]} \right\}$$

$$F_{ba} = \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \left\{ y - \left[\frac{\frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \cdot \frac{K}{2}}{1 + \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \cdot K} \right] x - \left[F_{fw} + F_{bw} \right] \frac{\frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \cdot \frac{K}{2}}{\left[1 + \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \cdot K \right]} \right\}, \quad 0 \leq t \leq L/c_o \qquad \text{(B-90)}$$

or, in a dimensionless form,

$$\mathcal{F}_{fa} = \frac{(1 + \lambda)}{(1 + 2\lambda)}\, X/2 - Y/2 - \frac{\lambda(\mathcal{F}_{fw} + \mathcal{F}_{bw})}{(1 + 2\lambda)}$$

$$\mathcal{F}_{ba} = Y/2 - \frac{\lambda}{(1 + 2\lambda)} \; X/2 - \frac{\lambda(\mathcal{F}_{fw} + \mathcal{F}_{bw})}{(1 + 2\lambda)} \; , \; 0 \le T \le \tau \quad \text{(B-91)}$$

where $\lambda = \frac{\pi D^2}{4} \cdot \frac{\pi E}{2D} \cdot \frac{K}{2}$.

Similarly, by expanding Eq (B-87) and (B-89) we obtain

$$\mathcal{F}_{fa} = X/2 - Y/2 - \tau/2 \; \frac{\tau\lambda}{(1 + 2\lambda)} - \frac{\lambda(\mathcal{F}_{fw} + \mathcal{F}_{bw})}{(1 + 2\lambda)}$$

and

$$\mathcal{F}_{ba} = Y/2 - X/2 + \frac{(1 + \lambda)}{(1 + 2\lambda)} \frac{\pi\tau}{2} - \frac{\lambda(\mathcal{F}_{fw} + \mathcal{F}_{bw})}{(1 + 2\lambda)} \; , \tau \le T \quad \text{(B-92)}$$

It should be noted that the wave forces, $\mathcal{F}_{fw}$ and $\mathcal{F}_{bw}$ are unaffected by the flexibility of the structure. It will be of interest to compute the net arching force $\mathcal{F}_{fa} - \mathcal{F}_{ba}$:

$$\mathcal{F}_{fa} - \mathcal{F}_{ba} = X/2 - y, \; 0 \le T \le \tau \quad \text{(B-93)}$$

and

$$\mathcal{F}_{fa} - \mathcal{F}_{ba} = X + Y - \pi\tau/2 \; , \; \tau \le T$$

The equation of motion of the structure, Eq (B-29), in dimensionless form is

$$\ddot{Y} = \beta^2 \left[\mathcal{F}_{fa} - \mathcal{F}_{ba} + \mathcal{F}_{fw} - \mathcal{F}_{bw} \right] . \quad \text{(B-94)}$$

As is evident from Eq (B-92), this equation is independent of the parameter λ. Therefore, the motion of the mass center $y\{t\}$ is identical to that presented previously in this appendix; however, the arching forces acting on the structure are reduced and may even be negative up to the value of the wave force. For the step pulse case the solution for the displacement, Y, is given by Eq (B-39) and (B-43). The mean force acting on each face of the flexible structure for the step pulse case is

$$1 + \frac{\pi\xi}{4}\left[\frac{1}{1 + 2\lambda}\right] - \frac{2\lambda}{(1 + 2\lambda)}$$

where the term unity is due to the free-field stress and last two terms are due to the relative displacement between the soil and structure.

The second problem, which is treated below, concerns the motion of a structure which consists of two masses connected together by a linear spring. The two masses are collinear in the direction of the motion of the ground shock wave. The mass located in the front is to be considerably smaller than the second mass, which could be called the base structure. For simplicity, the soil will be removed from the back of the base structure so that the force acting on the back of this structure complex will be zero. The two mass systems could be applied to an actual structure for the purpose of shock isolation.

The independent variables y and w denote the absolute displacement of the front mass and base structure, respectively. Then the equation of motion for the two masses can be written as

$$m\ddot{y} = F_{fw} + F_{fa} - k(y - w) \tag{B-95}$$

$$M_o\ddot{w} = k(y - w)$$

where m and M_o are the masses of the front mass and base structure, respectively, and k is the spring constant. Using the same notation which was employed previously, we can rewrite Eq (B-95) in a dimensionless form as

$$\ddot{Y} = \beta^2\left[\mathcal{F}_{fw} + \mathcal{F}_{fa} - \xi/2\,(Y - W)\right]$$

and

$$\ddot{W} = \beta^2\ \xi/2\left[Y - W\right], \tag{B-96}$$

where

$$W = \frac{\pi w}{D} \cdot \frac{E}{\sigma_o}$$

and

$$\beta^2 = \frac{\pi^2}{4\bar{\lambda}^2} = \frac{\pi^2 D^3 \rho_0}{4m}$$

We further limit this analysis to the step pulse case so that the arching and wave forces are specified. The equations of motion then become

$$\ddot{Y} + \frac{\beta^2}{2}\left[1 + \zeta\right]Y - \frac{\beta^2 \zeta}{2} W = \beta^2 + \pi/2\, \beta^2 T + \beta^2 e^{-T}$$

and

$$\ddot{W} + \zeta/2\, \beta^2 W - \frac{\zeta}{2}\beta^2 Y = 0 \qquad \text{(B-97)}$$

The initial conditions are

$$Y\{0\} = \dot{Y}\{0\} = W\{0\} = \dot{W}\{0\} = 0 \qquad \text{(B-98)}$$

The simultaneous solution of the above set of second order linear differential equations, obtained by using the method of Laplace Transforms, is

$$Y\{T\} = 2 + \pi T + B_1 e^{-T} + B_2 \cos\left[c_1 T\right] + B_3 \cos\left[c_2 T\right]$$

$$+ B_4 \sin\left[c_1 T\right] + B_5 \sin\left[c_2 T\right] \qquad \text{(B-99)}$$

where

$$B_1 = \frac{\beta^2\left(1 + \frac{\zeta}{2}\beta^2\right)}{\left[1 + c_1^2\right]\left[1 + c_2^2\right]}$$

$$B_2 = -\frac{\beta^2\left[\frac{\zeta}{2}\beta^2 - c_1^2\right]\left[1 + 2c_1^2\right]}{c_1^2\left[c_2^2 - c_1^2\right]\left[1 + c_1^2\right]}$$

$$B_3 = - \frac{\bar{\beta}^2 \left[\frac{\zeta}{2}\beta^2 - c_2^2\right]\left[1 + 2c_2^2\right]}{c_2^2\left[c_1^2 - c_2^2\right]\left[1 + c_2^2\right]}$$

$$B_4 = - \frac{\bar{\beta}^2 \left[\frac{\zeta}{2}\beta^2 - c_1^2\right]\left[\frac{\pi}{2}(1 + c_1^2) - c_1^2\right]}{c_1^3\left[c_2^2 - c_1^2\right]\left[1 + c_1^2\right]} \qquad \text{(B-100)}$$

$$B_5 = - \frac{\bar{\beta}^2 \left[\frac{\zeta}{2}\beta^2 - c_2^2\right]\left[\frac{\pi}{2}(1 + c_2^2) - c_2^3\right]}{c_2^3\left[c_1^2 - c_2^2\right]\left[1 + c_2^2\right]}$$

$$c_1 = \sqrt{\left[\frac{\zeta}{4}\beta^2 + \frac{\bar{\beta}^2}{4}(1+\zeta)\right] + \sqrt{\left[\frac{\beta^2}{4}\zeta + \frac{\bar{\beta}^2}{4}(1+\zeta)\right]^2 - \frac{\zeta}{4}\bar{\beta}^2\beta^2}}$$

$$c_2 = \sqrt{\left[\frac{\zeta}{4}\beta^2 + \frac{\bar{\beta}^2}{4}(1+\zeta)\right] - \sqrt{\left[\frac{\beta^2}{4}\zeta + \frac{\bar{\beta}^2}{4}(1+\zeta)\right]^2 - \frac{\zeta}{4}\bar{\beta}^2\beta^2}}$$

and

$$W\{T\} = 2 + \pi T + E_1 R^{-T} + E_2 \cos\left[c_1 T\right] + E_3 \cos\left[c_2 T\right]$$

$$+ E_4 \sin\left[c_1 T\right] + E_5 \sin\left[c_2 T\right] \qquad \text{(B-101}$$

where

$$E_1 = \frac{\beta^2 \bar{\beta}^2 \zeta/2}{(1 + c_1^2)(1 + c_2^2)}$$

$$E_2 = + B_2 \left[\frac{\frac{\zeta}{2}\beta^2}{\frac{\zeta}{2}\beta^2 - c_1^2}\right]$$

$$E_3 = B_3 \quad \frac{\frac{5}{2}\beta^2}{\frac{5}{2}\beta^2 - c_2^2}$$

$$E_4 = B_4 \left[\frac{\frac{5}{2}\beta^2}{\frac{5}{2}\beta^2 - c_1^2} \right] \qquad \text{(B-102)}$$

$$E_5 = B_5 \left[\frac{\frac{5}{2}\beta^2}{\frac{5}{2}\beta^2 - c_2^2} \right]$$

The above equations are used and a numerical example is presented in the text of this report.