DAVIDSON LABORATORY

LETTER REPORT 1142

AN ANALYSIS OF THE FORCES AND MOMENTS ON RE-ENTRANT VEE-STEP PLANING SURFACES

by

P. Ward Brown

May 1966

STEVENS INSTITUTE
OF TECHNOLOGY

CASTLE POINT STATION
HOBOKEN, NEW JERSEY

DAVIDSON LABORATORY

LETTER REPORT 1142

May 1966

AN ANALYSIS OF THE FORCES AND MOMENTS ON RE-ENTRANT VEE-STEP PLANING SURFACES

by

P. Ward Brown

Sponsored by the Bureau of Ships
General Hydrodynamics Research Program
Administered by the David Taylor Model Basin
Contract Nonr 263(66)
(DL Project 3060 Code 077)

ABSTRACT

The effect of re-entrant vee-steps on the performance of planing surfaces is analyzed. Formulae are developed for the lift, drag, center of pressure and wetted area at high aspect ratio. It is found that the lift-drag ratio of the planing surface decreases as the re-entrant step angle becomes more acute.

INTRODUCTION

Conventional planing craft usually operate at a disadvantage compared to other craft which use dynamic support, such as hydrofoil boats and aircraft, because the planing craft's lifting surface is of low aspect ratio. Since low aspect ratio surfaces are relatively inefficient producers of lift more area has to be provided for a given lift with a resulting penalty in lift-drag ratio compared to a high aspect ratio surface.

The aspect ratio of a conventional planing boat is limited by the low trim at which the boat operates and by the geometry of the planing surface. In order to overcome the geometric limitation Clement[12] has suggested that planing surfaces with re-entrant vee steps in plan could be operated at low trim and high aspect ratio and hence at higher lift-drag ratios.

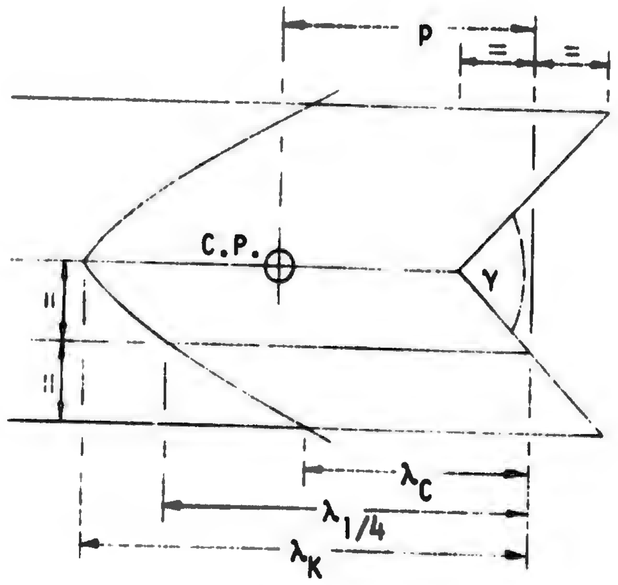
In order to explore this possibility a series of tests were run on re-entrant step planing surfaces at the Davidson Laboratory and the results published in a data report.[1] The prismatic models were of one foot beam and were made of 1/2 inch thick transparent plexi-glass. Three 20° deadrise surfaces having included re-entrant vee step angles of 60°, 120° and 180° were tested together with a fourth 10° deadrise surface having a 60° re-entrant step. The tests were made at constant trim and draft and at a speed of 34.05 fps ($C_v = 6.0$). A trim range of 2° to 12° was covered and the mean wetted lengths ranged from 0.13 to 1.31 beams (aspect ratios 7.7 to 0.77) and at each condition the lift, drag, pitching moment and wetted area was measured. The dynamic wetted areas were obtained from overwater photographs of the wetted area taken through the transparent model by a camera mounted above the model and travelling with it.

The anticipated improvement in performance of re-entrant step planing surfaces depends in part on the assumption that the planing formulae developed for low aspect ratio transverse step planing surfaces are applicable to high aspect ratio re-entrant step surfaces. In order to check this assumption and to obtain a rationalization of the experimental data the following analysis of re-entrant step planing data was supported by the Bureau of Ships.

NOMENCLATURE

A	aspect ratio, b^2/S $(1/\lambda_m)$
a_e	effective lift-curve slope, per radian
b	beam, ft
C_{D_b}	drag coefficient normalized on beam, $D/\frac{1}{2}\rho V^2b^2$
C_D	drag coefficient normalized on area, $D/\frac{1}{2}\rho V^2S$
C_{L_b}	lift coefficient normalized on beam, $L/\frac{1}{2}\rho V^2b^2$, $(\lambda_m C_L)$
C_L	lift coefficient normalized on area, $L/\frac{1}{2}\rho V^2S$
C_L'	sectional lift coefficient, Eq. (13)
C_f	skin friction coefficient
D	drag, lb
K	induced drag factor
L	lift, lb
p	position of the center of pressure forward of the mean step measured along the keel, beams
S	projected wetted area bounded by stagnation line, chines and step, measured on a plane normal to the centerline and containing the keel, $(b\lambda_m)$ sq.ft.
s	maximum wetted beam in chines dry planing, beams
V	horizontal velocity, fps
w	wave rise term, Eq. (7), beams
β	deadrise angle, deg
γ	included angle of re-entrant step projected on a plane containing the keel, deg

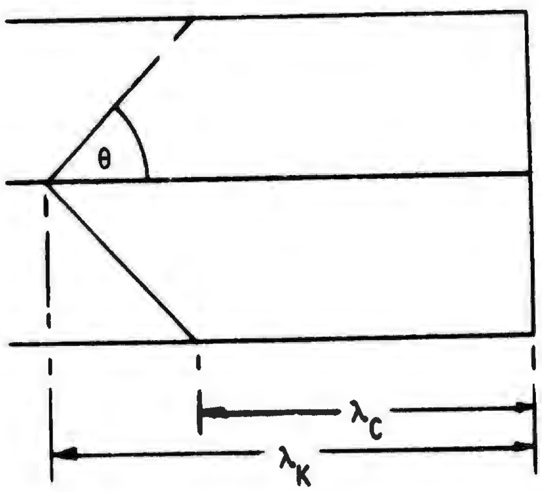
θ	half the included angle of the stagnation line projected on a plane containing the keel, deg
λ_C	chine wetted length, distance from the mean step position to the intersection of stagnation line and keel, measured along the chine, beams
λ_K	keel wetted length, distance from the mean step position to the stagnation line at keel, measured along the keel, beams
$\lambda_{1/4}$	distance from the mean step position to the stagnation line measured along the 1/4 beam buttock line, beams
λ_m	mean wetted length, S/b^2 , beams
λ_R	reference length, $\frac{1}{2}\cot\tau\tan\beta$, beams
ρ	mass density of water, slugs per cu.ft.
τ	trim angle, angle between keel and horizontal, deg

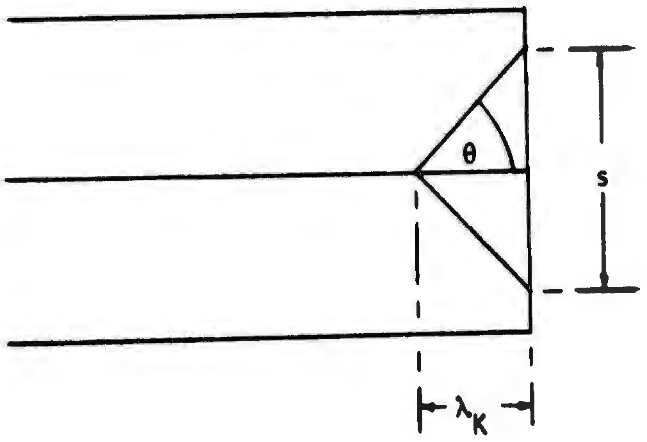


WETTED AREA

Since the aspect ratio of a planing surface is determined by the shape of its wetted area it seems appropriate to begin this analysis with a discussion of the wetted area data found in the experimental phase[1] of this study. A knowledge of the wetted area is in any event necessary for performance prediction.

The wetted areas were determined by a new method[1] believed to be more accurate than that used previously, particularly at high aspect ratio (short wetted lengths) and is described in the earlier report. The shape of the wetted area was found to vary in an unexpected manner as the aspect ratio increased.





Consider the above sketch of a transverse step planing surface at fixed trim, looking normal to the keel, in the chines wet and chines dry condition. It has generally been supposed that the angle θ of the stagnation line is constant for all wetted lengths and on this assumption it follows that:

Chines wet: $\lambda_C = \lambda_K - \frac{1}{2}\cot\theta \qquad \lambda_K \geq \frac{1}{2}\cot\theta$ (1)

Chines dry: $S = 2\lambda_K \tan\theta \qquad \lambda_K \leq \frac{1}{2}\cot\theta$ (2)

when $\lambda_K = \frac{1}{2}\cot\theta$ then $\lambda_C = 0$ and $S = 1$ (all linear dimensions are normalized with respect to the beam). The experimental values of λ_C and S as a function of λ_K are shown for the 20° deadrise transverse step surface at various trims on Fig. 1. The chines dry data has not previously been published and is given in Table 1.

Table 1

Wetted beam in chine dry planing - $\beta = 20^{\circ}$, $\gamma = 180^{\circ}$

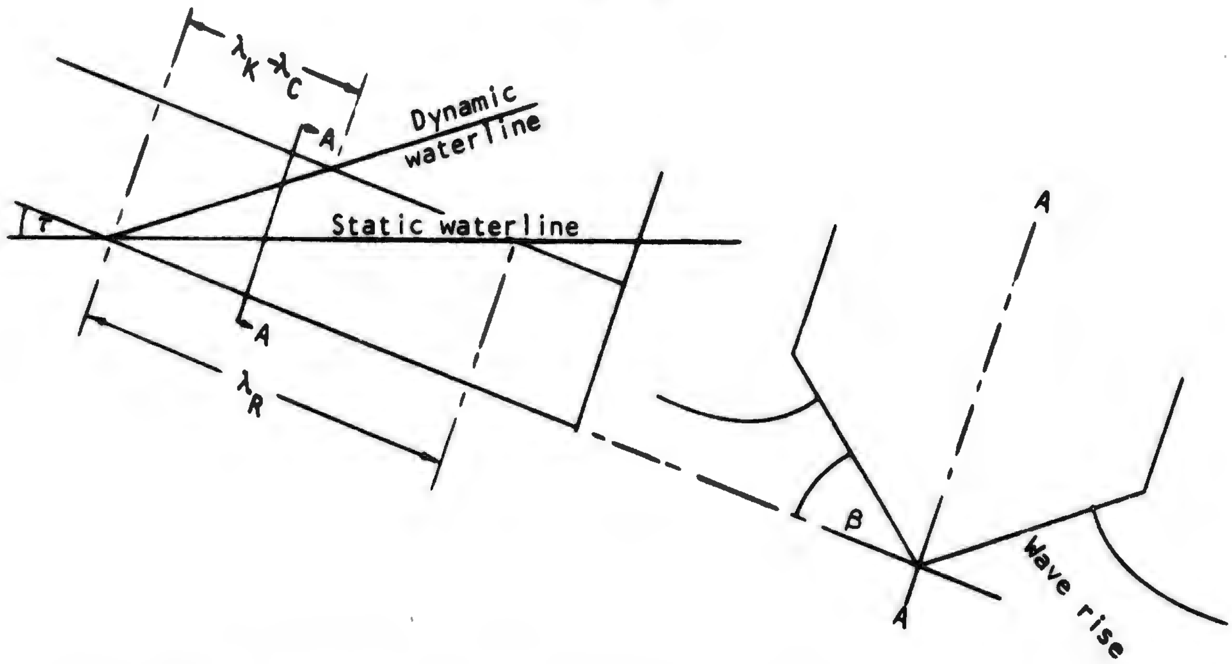
4° Trim		8° Trim		12° Trim	
λ_K	S	λ_K	S	λ_K	S
0.45	0.28	0.40	0.54	0.30	0.50
0.90	0.54	0.50	0.66	0.40	0.66
1.00	0.64	0.60	0.80	0.44	0.76
1.21	0.80	0.71	0.90	0.48	0.84
1.30	0.84				
1.46	0.94				

The straight lines implied by Eq. (1) are shown and it is evident that for chine lengths less than one beam the angle θ decreases as the wetted length decreases. This aft rotation of the stagnation line ceases after the chines become dry -- as is necessary from considerations of similarity -- and then the straight line relationship of Eq. (2) is followed.

Thus it appears that two constant values of θ, or of $\lambda_K - \lambda_C$, must be sought: one applicable for long wetted lengths and the other applying to the chines dry condition. Moreover the transitional behavior at short wetted lengths has to be considered.

The Value of $\lambda_K - \lambda_C$ at Long Wetted Lengths

It is well known that due to the phenomenon of wave rise about a planing surface the difference between keel and chine wetted lengths at speed is less than its static value.



The static value of this difference may be used as a reference length and is given by

$$\lambda_R = \frac{1}{2} \cot \tau \tan \beta \tag{3}$$

All the data presented in Ref 2 to 5 shows that difference between the keel and chine wetted lengths is constant for a given trim over a range of chine wetted lengths from one to seven. This data is summarized in Table 2 where the average value of $\lambda_K - \lambda_C$ and the number of data points, N , is shown for each trim at deadrise angles of 20°, 40°, 50° and 70°. (Shuford's[3] data for $\beta = 40^\circ$ has been omitted from Table 2 since it is inconsistent with all the other data, see his Fig. 18b).

Table 2

Average Values of $\lambda_K - \lambda_C$ $(\lambda_C > 1.0)$ Ref 2-5

Trim	$\beta = 20^\circ$		$\beta = 40^\circ$		$\beta = 50^\circ$		$\beta = 70^\circ$	
deg	N	$\lambda_K - \lambda_C$	N	$\lambda_K - \lambda_C$	N	$\lambda_K - \lambda_C$	N	$\lambda_K - \lambda_C$
2	8	2.94	-	-	-	-	-	-
4	30	1.42	16	3.45	3	5.06	-	-
6	31	1.01	17	2.29	10	3.44	-	-
9	-	-	-	-	16	2.18	2	5.31
12	33	0.45	31	1.02	20	1.60	3	3.84
18	20	0.26	34	0.71	18	0.92	6	2.50
24	20	0.19	26	0.41	30	0.60	13	1.65
30	14	0.12	26	0.30	16	0.35	21	1.27
34	7	0.09						

A simple equation fitting all the data is:

$$\lambda_K - \lambda_C = (0.57 + 0.001\,\beta)(\lambda_R - 0.006\beta) \qquad (4)$$

The average values of $\lambda_K - \lambda_C$ are shown plotted on Fig. 2 and 3 as a function of λ_R and the lines given by Eq. (4) are also shown.

It has generally been assumed that the difference $\lambda_K - \lambda_C$ can be deduced from the two-dimensional wave-rise given by Wagner's expanding plate analogy (see Savitsky[6] for instance) and this leads to:

$$\lambda_K - \lambda_C = 0.64\,\lambda_R \qquad (5)$$

This line is shown on Fig. 2 and 3 and as may be seen does not agree with the data. It should be noted that the use of Eq. (5) can lead to errors of the order of 15% in wetted area at practical values of trim and deadrise.

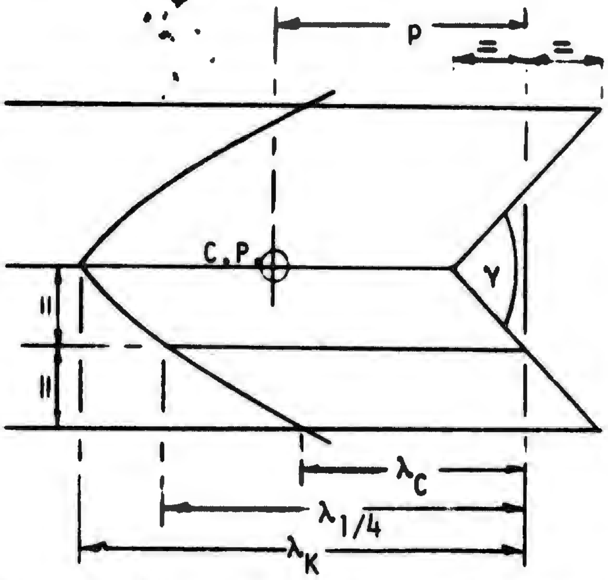
From Eq. (4) it follows that a suitable expression for the chine wetted length as a function of keel length for long wetted lengths is:

$$\lambda_C = \lambda_K - w \qquad (6)$$

where $w = (0.57 + 0.001\beta)(\lambda_R - 0.006\beta)$ (7)

The Variation of Chine Wetted Length with Keel Length at Large Aspect Ratio

The aft rotation of the stagnation line implied by Fig. 1 increases as the wetted length becomes smaller and hence, since shorter wetted lengths can be achieved by re-entrant step surfaces, is more marked for the re-entrant step surfaces. For these surfaces the various wetted lengths are defined as shown in the following sketch:



The variation of chine length with keel length is shown on Fig. 4 for the 20° deadrise surfaces. This data is fitted by the following empirical formula which is shown on the figure:

$$\lambda_C = (\lambda_K - w) - 0.2 \exp\left[- (\lambda_K - w)/0.3\right] \qquad (8)$$

where w is given by Eq. (7). Equation (6) applies for wetted chine lengths greater than one beam and Eq. (8) is a generalization which applies for all wetted lengths.

From Eq. (8) it is possible to calculate the two limiting values for the included angle of the stagnation line; one applying to long wetted lengths, being independent of step angle, and the other more acute angle applying to chine dry planing; as is done in the following table for a 20° deadrise surface.

Table 3

Included Angle of Stagnation Line

Trim	$\gamma = 180^\circ$		$\gamma = 120^\circ$		$\gamma = 60^\circ$	
4	37.8	34.9	37.8	33.9	37.8	31.5
6	55.5	49.7	55.5	47.7	55.5	43.3
8	71.9	62.7	71.9	59.8	71.9	51.1
10	85.6	73.5	85.6	69.6	85.6	60.9
12	98.6	83.5	98.6	78.7	98.6	68.1

Mean Wetted Length

As has been observed in previous planing studies the stagnation line is slightly curved. From the data in Ref 1 it is found that, regardless of wetted length, trim, deadrise and step angle,

$$\lambda_{1/4} = 0.5(\lambda_K + \lambda_C) + 0.045 \tag{9}$$

If it is assumed that the shape of the stagnation line is given by a second degree curve it follows that in general the mean wetted length is given by:

$$\lambda_m = \frac{1}{2}(\lambda_K + \lambda_C) + \frac{2}{3}\left[\lambda_{1/4} - (\lambda_K + \lambda_C)/2\right] \tag{10}$$

Thus using Eq. (9) a general expression for the mean wetted length is

$$\lambda_m = 0.5(\lambda_K + \lambda_C) + 0.03 \tag{11}$$

ANALYSIS OF LIFT

An attempt was made to analyze the lift along the lines used by Shuford.[3] While Shuford's formulation seems to be satisfactory at high trim and long wetted length it is not too accurate at small trims. Since the effect of the re-entrant step on the lift is small it seemed necessary to obtain a more representative rationalization of the high aspect ratio data in Ref 1. It should be noted that the following formulation applies for aspect ratios of a half and greater (wetted lengths less than two beams).

It is assumed, by analogy with wing theory, that the relationship between lift, trim and aspect ratio for "large" aspect ratios should be of the form:

$$C_L = a_e(\tau - K\,C_L/\pi A)$$

and hence
$$C_L = \frac{a_e \tau}{1 + a_e K/\pi A} \tag{12}$$

where a_e = effective lift curve slope
K = induced drag factor

It is believed, for reasons discussed by Brown[7,8] that the planing lift should properly be normalized with respect to the horizontal projection of the wetted area, whereas C_L is defined with respect to the wetted area projected on the keel plane. It is further believed that the lift varies linearly with the sine of the angle of attack. Incorporating these two modifications in Eq. (12) gives:

$$C_L = \frac{a_e \sin\tau \cos\tau}{1 + a_e K/\pi A} = \frac{C_L'}{1 + a_e K/\pi A} \tag{13}$$

In order to make a graphical presentation of the data Eq. (13) is rearranged, noting that $A\,C_{L_b} = C_L$, to give:

$$\frac{1}{C_{L_b}} = \frac{1}{C_L'}(A + a_e K/\pi) \tag{14}$$

Thus a linear relationship between $1/C_{L_b}$ and aspect ratio should hold for given trim at "large" aspect ratio.

The available data[1] is shown plotted in the form suggested by Eq. (14) on Fig. 5, 6 and 7 for the 20° deadrise surface with re-entrant step angles of $\gamma = 180^\circ$, 120° and 60° respectively. The expected linear relationship is obtained and C_L' is calculated from the slopes of the lines giving the following values:

Table 4

Experimental Values of C_L'

Trim, deg.	$\gamma = 180^\circ$	$\gamma = 120^\circ$	$\gamma = 60^\circ$
4	0.054	0.052	0.044
6	0.100	0.091	0.075
7.6	-	-	0.103
8	0.149	0.136	-
10	0.191	0.181	0.143
12	0.219	0.227	0.182
12*	0.241		
18*	0.367		
24*	0.458		
30*	0.521		

*Data from Chambliss and Boyd[2]

From Eq. (13) it is expected that:

$$C_L' \sec \tau = a_e \sin \tau$$

and this is shown plotted on Fig. 8 for 20° deadrise. A similar presentation for 10° deadrise and 60° re-entrant step angle is shown on Fig. 9.

The lines drawn on Fig. 8 and 9 are of the form

$$C_L' = a_e \cos\tau(\sin\tau - \sin\tau_o) \qquad (15)$$

where, from the analysis of this and other planing data

$$a_e = 1.68\ (1 - 0.4 \tan\beta)\cos\beta\ \frac{1 + \sin(\gamma/2)}{2} \qquad (16)$$

$$\sin \tau_o = 0.11\,(\tan\beta - 0.09) \tag{17}$$

Moreover it was found that the induced drag factor is given by:

$$K = \cos\beta\,\frac{2\sin(\gamma/2)}{1+\sin(\gamma/2)} \tag{18}$$

Since Eq. (13) can be put in the form:

$$C_{L_b} = \frac{C_L'\,\lambda_m}{1 + a_e K \lambda_m/\pi} \tag{19}$$

it follows that the effect of making the re-entrant step angle more acute, which diminishes C_L', is to reduce the lift at a given wetted area.

While the re-entrant step has an adverse effect on the lift, this effect is not too marked at small trims. The experimental lift values for a trim of 6° are shown plotted on Fig. 10 as a function of mean wetted length for step angles of $\gamma = 180^\circ$, 120° and 60°. The curves shown on the plot are calculated from Eq. (19) with Eqs. (16), (17) and (18).

DRAG ANALYSIS

The drag of a planing surface is generally taken to be of the form:

$$C_D = C_L \tan \tau + C_f(\tau, \beta, \lambda_m, C_L) \tag{20}$$

The frictional term is complicated by a number of considerations. The pressure under the planing surface is not uniform and there exists a complicated velocity distribution. Moreover there is a steep favorable pressure gradient immediately behind the stagnation line and consequently a considerable region of laminar flow may persist to quite high Reynolds Numbers.[9] The flow lines do not stream directly aft but diverge due to pressure field so that the selection of a characteristic length is obscure. These considerations make it difficult to establish representative values for Reynolds Number and skin friction coefficient.

Compounding this uncertainty is the fact that the planing surface generates a lateral sheet of spray ahead of the stagnation line which moves over the surface and whose velocity vector rotates in a horizontal plane from an aft direction at low trim to a forward direction at high trim.[10] This spray may increase the drag at low trim and decrease it at high trim.

Unfortunately it is not easy to obtain experimental clarification of the behavior of the frictional component. Equation (20) can be restated in the form

$$C_{D_b} - C_{L_b} \tan \tau = \lambda_m C_f(\tau, \beta, \lambda_m, C_L) \tag{21}$$

and experimental data[2] for a 20° deadrise surface at trims of 6° and 12° is shown plotted in this form on Fig. 11 and 12. While the anticipated increase in effective wetted area at low trim is evident on Fig. 11 it is clear that there is some latitude available in fitting a line to this data and hence in determining the skin friction coefficient. It should be remarked that it is difficult to determine the skin friction coefficient experimentally even when a large range of wetted length is covered and it should be possible to establish a trend line. When only small wetted lengths are investigated, as in the present study, it becomes even more difficult to find the skin friction coefficient.

In the case of the re-entrant step planing surface it is implied by Lippisch[11] that the effective removal of the low pressure wetted area at the aft end of the surface should result in a reduction in drag for a given lift. Analysis of the data does not bear out this expectation. One case at 6° trim is illustrated by Fig. 13. In general all the data shows the same trend -- an increase in the total drag at a given lift coefficient as the re-entrant step angle becomes more acute.

In view of the preceding arguments and the fact that the force measurements were made at small wetted lengths, it is thought that the experimental evidence for the increase in drag as the re-entrant step angle becomes more acute is not conclusive and should be subject to further investigation. On the other hand there now seems to be little likelihood of a drag reduction.

CENTER OF PRESSURE

The position of the center of pressure as a function of mean wetted length is given by

$$p = 0.7\,\lambda_m \tag{22}$$

and applies at all the conditions tested.

DISCUSSION

Lift/Drag Ratio

Interest in the re-entrant step planing surface stems from the fact that, as pointed out by Clement[12], such surfaces can be operated at higher aspect ratios than conventional transverse step surfaces. While an increase in aspect ratio will usually result in increased efficiency, in the case of the planing surface making the step re-entrant entails a reduction in lift curve slope that more than offsets the improvement anticipated from the increase in aspect ratio.

A measure of the efficiency of a lifting surface is the lift/drag ratio. Assuming for the purposes of illustration that the drag is given by

$$C_D = C_L \tan \tau + 0.003 \qquad (23)$$

and the lift by Eq. (13), (15), (16), (17) and (18), the variation of L/D with aspect ratio is shown for a 10° deadrise surface with a transverse step, and with re-entrant step angles of $\gamma = 120^\circ$ and 60° at a trim of 4° on Fig. 14. The lift/drag ratio increases with aspect ratio up to the point where the stagnation line crosses the chine at the step. This point represents the maximum aspect ratio attainable for the surface and hence the maximum lift/drag ratio.

It is evident that the efficiency of the planing surface progressively decreases for all aspect ratios as the re-entrant step angle becomes more acute and in particular the maximum lift/drag ratio attainable similarly decreases.

Wetted Area

While the bulk of this study is concerned with surfaces planing at wetted lengths less than one beam a wider range of wetted length was studied in order to define the wetted area.

The deficiency of the Wagner wave rise formula as a means of predicting chine wetted length has been pointed out in a number of reports, Ref 2 to 5. It is believed that Eq. (4) is a significant improvement over earlier formulas for chine wetted length, covering as it does a range of trim from 2° to 34° and of deadrise from 10° to 70°, moreover the equation is simple enough to be useful in routine computations.

Lift

The lift formula was specifically designed to fit the data obtained from the experimental study of re-entrant step planing surfaces.[1] While the trends indicated by other data reports, Ref 2-5, were considered it did not seem practical or necessary to analyze all this data in order to determine the effect of the re-entrant step. The data in the cited reports was obtained from planing surfaces having a beam of 4 inches. It is believed that the lift data obtained with a 12 inch beam using over-water photographs[1] of wetted area is more reliable, particularly at short wetted length. In any event there appear to be anomalies in the existing body of data, (for instance see Shuford's[3] Fig. 35 which shows that a 50° deadrise surface has a constant lift coefficient at given trim regardless of wetted length) which make it difficult to obtain a unified planing lift formula which is also accurate and make it desirable to obtain more experimental evidence.

The decision to limit the analysis to large aspect ratios conditions the form of the lifting equation. While it is thought that the resulting equation is sufficient for the purpose in hand, namely the evaluation of the re-entrant step, some features of the final formula merit discussion. The lift formula is, from Eq. (13), (15), (16), (17) and (18):

$$C_L = \frac{a_e\,(\sin\tau - \sin\tau_o)\,\cos\tau}{1 + a_e K/\pi A} \qquad (24)$$

where

$$a_e = 1.68\,(1 - 0.4\,\tan\beta)\cos\beta\,\frac{1 + \sin(\gamma/2)}{2} \qquad (25)$$

$$\sin\tau_o = 0.11(\tan\beta - 0.09) \qquad (26)$$

$$K = \cos\beta\,\frac{2\sin(\gamma/2)}{1+\sin(\gamma/2)} \qquad (27)$$

The basic formula, Eq. (24), is conventional in structure but the suggestion of a positive no-lift angle on a flat planing surface is unusual.

However the evidence for the existence of a finite no-lift angle presented on Fig. 8 and 9 seems quite conclusive. This implies that at some small positive trim the 'high aspect ratio' lift is zero; the magnitude of the no-lift angle being given by the empirical formula Eq. (26). For a 20° deadrise surface the no-lift angle is 1.72° and if the chines are wet the mean wetted length cannot be less than 1.85 beams, thus the aspect ratio is 0.54 and practically outside the range of high aspect ratio planing. It should also be remarked that at this low trim the included angle of the stagnation line is less than 16°: it seems plausible that when the sweep-back of the stagnation line becomes so large, i.e. 82°, the linear lifting term should cease to contribute effectively. Although the linear term is zero there could still be a contribution at very large wetted length due to cross-flow, however since this latter term varies as the square of the sine of the trim angle its contribution at a trim of 1.72° is unlikely to be significant.

It has long been known that the theoretical lift-curve slope of a two-dimensional planing surface is π, for instance see Pierson's[13] development of Wagner's theory. In practice there seems to be little similarity between the theoretical predictions of planing performance and the experimental realizations. The expression for lift-curve slope given by Eq. (25) is an empirical formula which seems to fit the data. The decrease in lift-curve slope as the re-entrant step angle becomes more acute represents the experimental findings. Lippisch[11] also forecast a decrease as the step was swept back, however he finds that the lift falls off like $\sin(\gamma/2)$ rather than the factor $\left[1 + \sin(\gamma/2)\right]/2$ of Eq. (25), thus Lippisch expects sweepback to have an even more deleterious effect on the lift.

Drag

The drag of a re-entrant step planing surface is no less than that of a transverse step surface.

A brief review of the data on planing surface drag suggests that there is little experimental support for the more complicated forms of the drag equation that have been put forward. For engineering purposes the drag may be calculated from:

$$C_D = C_L \tan \tau + C_f \sec\beta \tag{28}$$

where the skin friction coefficient is taken from the Schoenherr curve and the Reynolds Number is based on the forward speed and mean wetted length. Whatever the theoretical arguments there exists little experimental support for refinements of the above equation, such as allowing for the difference between the forward speed and the mean velocity under the surface.

On the other hand it seems desirable to make more precise measurements of the drag of planing surfaces. While the data on the re-entrant step surfaces shows no beneficial effect on the drag, being if anything somewhat adverse, the data from other sources is not only scattered but at times surprising. For instance Shuford's[3] Fig. 16 indicates skin friction coefficients of the order of 0.01 at high trims and wetted lengths at a Reynolds Number of 7 million.

Since a more sophisticated formulation of planing drag than that given by Eq. (28) is often used it would seem advisable to subject such formulae to experimental verification.

CONCLUDING REMARKS

From an analysis of the $C_v = 6$ data obtained by Brown and Van Dyck[1] on deadrise planing surfaces with re-entrant vee steps, extended by existing data in the literature, it was found that:

1. For aspect ratios greater than 0.5 (mean wetted lengths up to 2 beams) deadrise angles up to 40°, trims up to 12° and re-entrant step angles of 60° or greater, the lift is given by:

$$C_L = \frac{a_e (\sin\tau - \sin\tau_o) \cos\tau}{1 + a_e K/\pi A}$$

where $a_e = 1.68\ (1 - 0.4 \tan\beta) \cos\beta\ \frac{1 + \sin(\gamma/2)}{2}$

$$\sin \tau_o = 0.11\ (\tan\beta - 0.09)$$

$$K = \cos\beta\ \frac{2 \sin(\gamma/2)}{1 + \sin(\gamma/2)}$$

2. The drag is given by:

$$C_D = C_L \tan\tau + C_f \sec\beta$$

and is invariant with step angle

3. The distance of the center of pressure forward of the mean step position is given by:

$$p = 0.7\ \lambda_m$$

at all conditions

4. The mean wetted length may be calculated from the keel and chine wetted lengths:

$$\lambda_m = 0.5(\lambda_K + \lambda_C) + 0.03$$

5. The relationship between keel and chine wetted length is given by

$$\lambda_C = (\lambda_K - w) - 0.2 \exp\left[- (\lambda_K - w)/0.3\right]$$

where $w = (0.57 + 0.001\beta)(\lambda_R - 0.006\beta)$

$\lambda_R = 0.5 \cot \tau \tan\beta$

the exponential term being negligible for chine lengths greater than one beam.

6. The lift/drag ratio of re-entrant step planing surfaces is significantly lower than those with transverse steps.

RECOMMENDATIONS

In view of the trend obtained with the re-entrant steps and the evidence presented by Savitsky[14] it is recommended that a study be made of the performance of pointed vee-step planing surfaces. It is considered that such surfaces might have a higher lift-curve slope than transverse step surfaces.

The program of research on planing characteristics initiated by the National Advisory Committee for Aeronautics has resulted in a body of planing data which is particularly suited to be of use in the design of water-based aircraft. Planing performance at the low trims and low deadrises commonly met with in planing boat design is not thoroughly documented in the existing literature.

A brief review of existing planing drag data leads to the conclusion that this data is lacking in precision. There is therefore little experimental support for any but the simplest formulation of planing drag.

The above observations indicate a need for the acquisition of further planing data covering a range of variables more appropriate to planing boat design. Such data should be obtained using models having a beam larger than the 4 inch beam of the NACA series and emphasizing precision of drag measurement.

REFERENCES

1. Brown, P. Ward and Van Dyck, R.L.: An Experimental Investigation of Deadrise Planing Surfaces with Re-entrant Vee-Step.
   DL Letter Report 664, December 1964.

2. Chambliss, Derrill B. and Boyd, George M., Jr.: The Planing Characteristics of Two V-Shaped Prismatic Surfaces Having Angles of Dead Rise of 20° and 40°.
   NACA TN 2876, January 1953.

3. Shuford, Charles L., Jr.: A Theoretical and Experimental Study of Planing Surfaces Including Effects of Cross Section and Plan Form.
   NACA TN 3939, November 1956.

4. Springston, George B., Jr. and Sayre, Clifford L., Jr.: The Planing Characteristics of a V-Shaped Prismatic Surface with 50 degree Dead Rise.
   DTMB Report 920, February 1955.

5. Pope, James D.: The Planing Characteristics of a V-Shaped Prismatic Surface with 70 degree Dead Rise.
   DTMB Report 1285, December 1958.

6. Savitsky, Daniel: Hydrodynamic Design of Planing Hulls
   MARINE TECHNOLOGY, Vol. 1, No. 1, p. 75 et seq., October 1964.

7. Brown, P. Ward: The Force Characteristics of Surface-Piercing Fully Ventilated Dihedral Hydrofoils.
   DL Report 698, October 1958.

8. Brown, P. Ward: Improved First-Order Supercavitating Theory.
   DL Report 971, August 1963.

9. Korvin-Kroukowsky, V.V., Ross, W. and Savitsky, Daniel: Turbulence Stimulation in the Boundary Layer of Planing Surfaces: Part I Review of Analytical and Experimental Aspects of the Subject.
   DL Report 443, August 1952.

10. Savitsky, Daniel and Ross, Edward W.: Turbulence Stimulation in the Boundary Layer of Planing Surfaces: Part II Preliminary Experimental Investigation.
    DL Report 444, August 1952.

11. Lippisch, A.M., Colton, R.F. and Bonney, S.E.: Research in the Design and Testing of Planing Boats.
    Collins Engineering Report 1117-7, August 1962.

12. Clement, Eugene P.: A Lifting Surface Approach to Planing Boat Design
    DTMB Report 1902, September 1964.

13. Pierson, John D. and Leshnover, Samuel: An Analysis of the Fluid Flow in the Spray Root and Wake of Planing Surfaces.
DL Report 335, October 1948.

14. Savitsky, Daniel: Wetted Length and Center of Pressure of Vee-Step Planing Surfaces.
DL Report 378, September 1951.

VARIATION OF CHINE LENGTH WITH KEEL LENGTH
TRANSVERSE STEP β = 20°

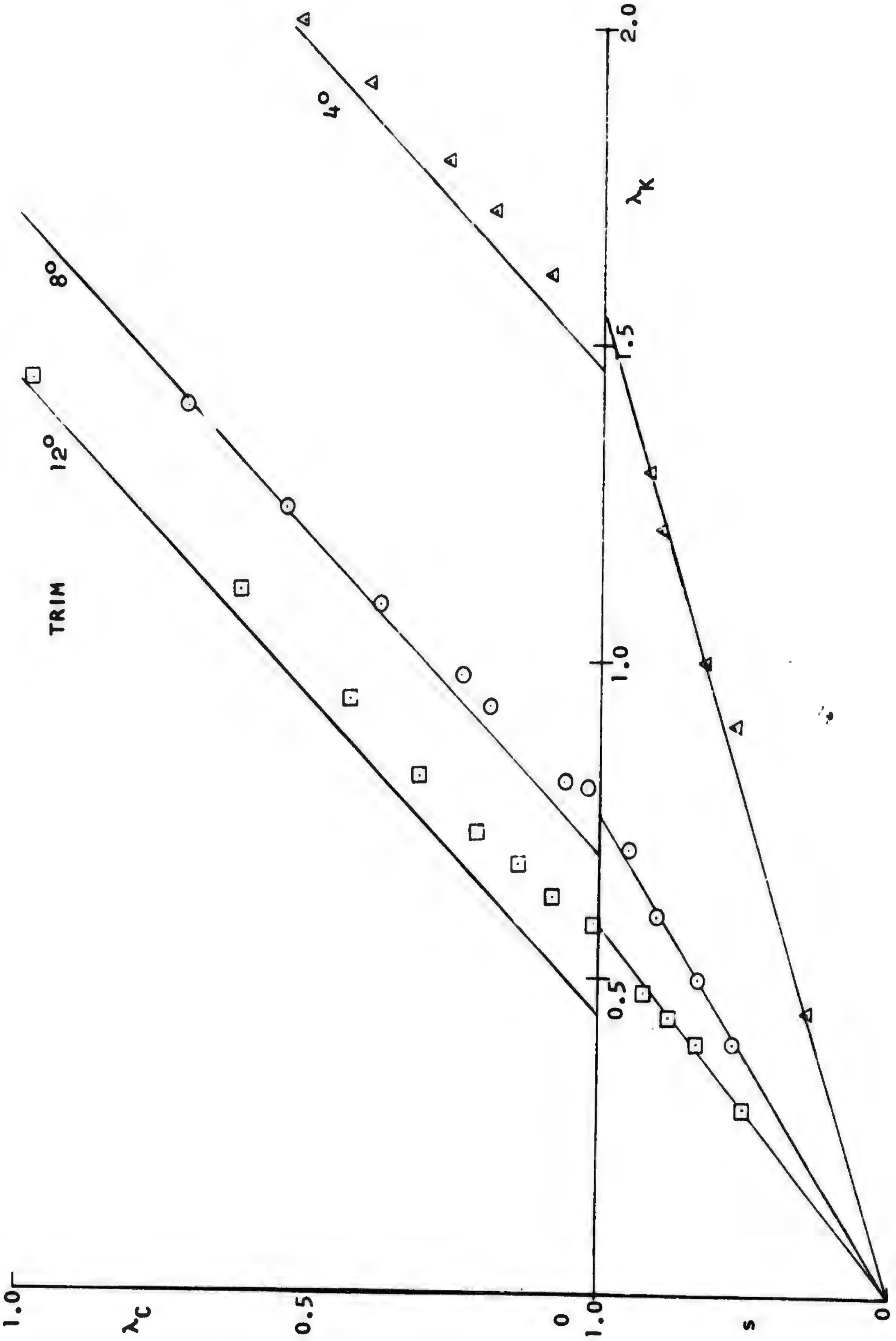
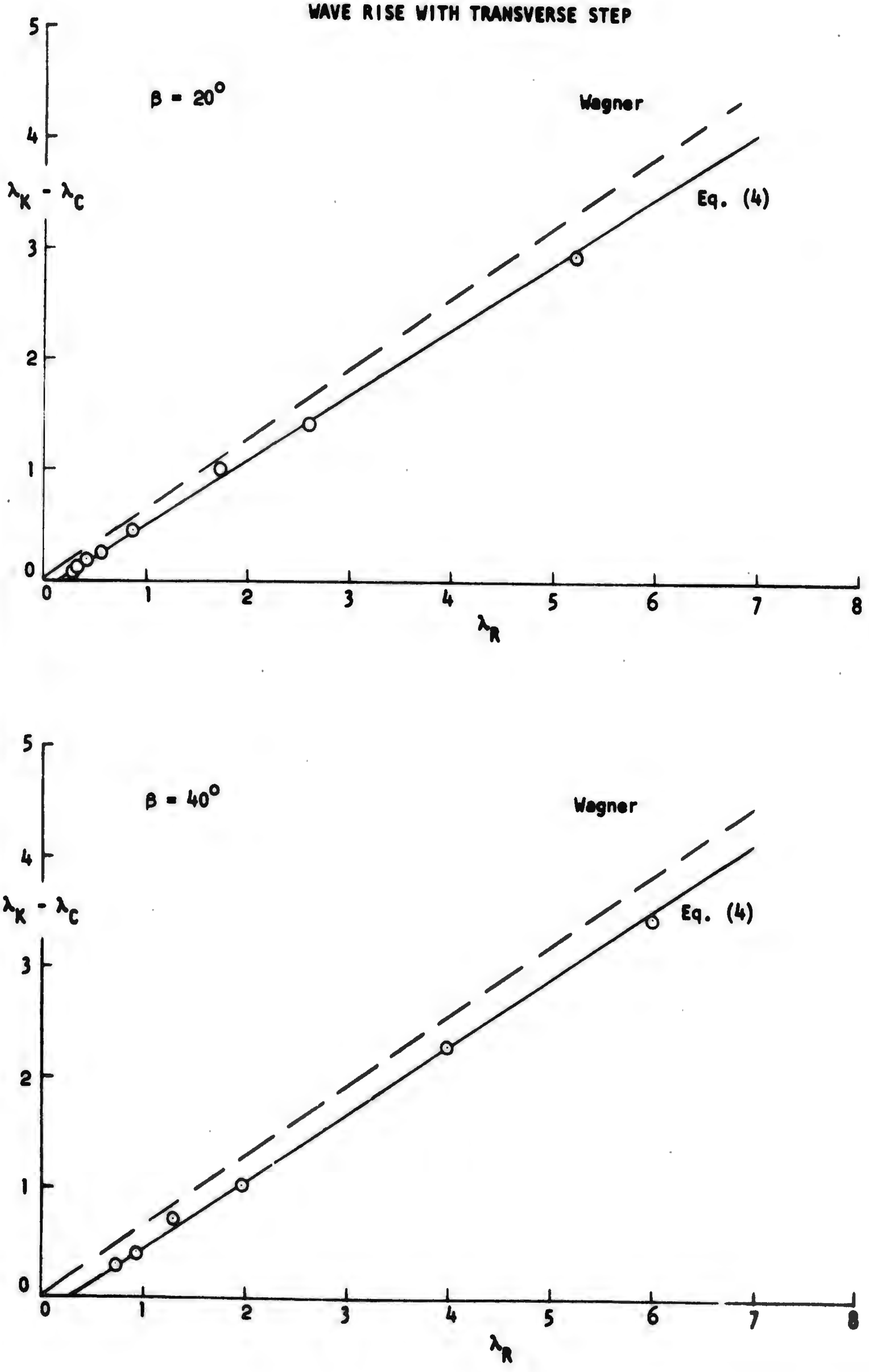
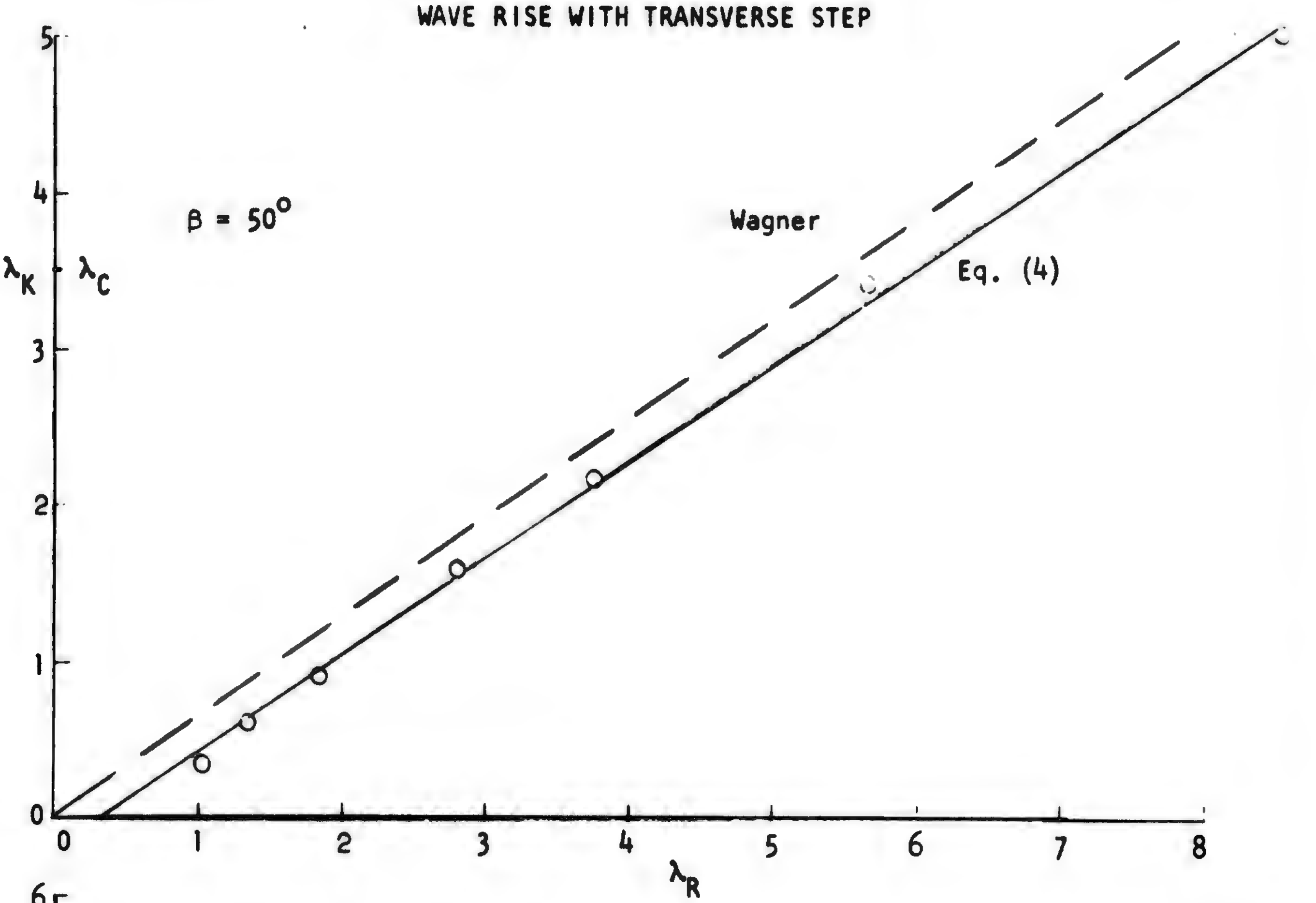


Fig 2

WAVE RISE WITH TRANSVERSE STEP



WAVE RISE WITH TRANSVERSE STEP



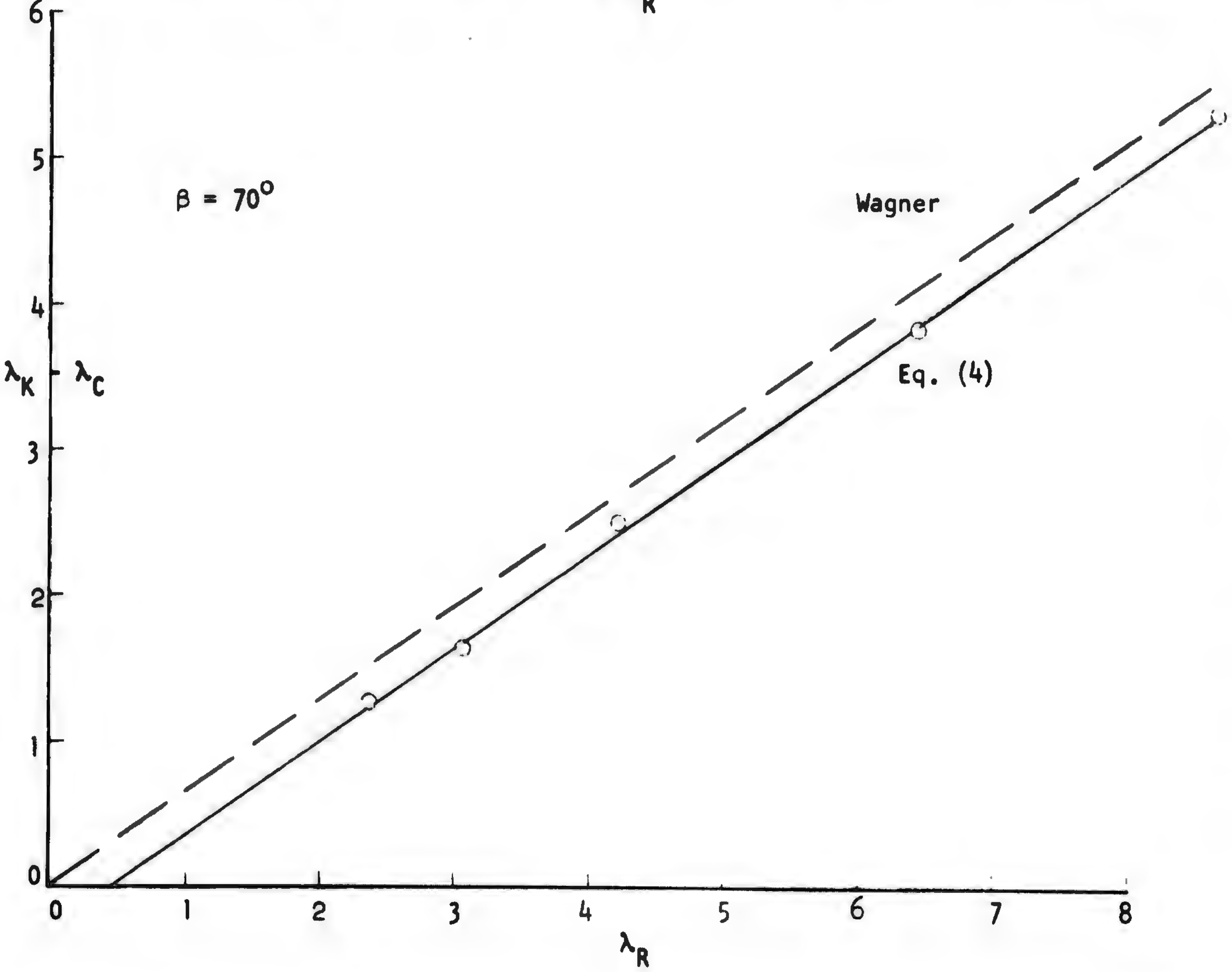


Fig 4

VARIATION OF KEEL AND CHINE WETTED LENGTH

$\beta = 20^\circ$

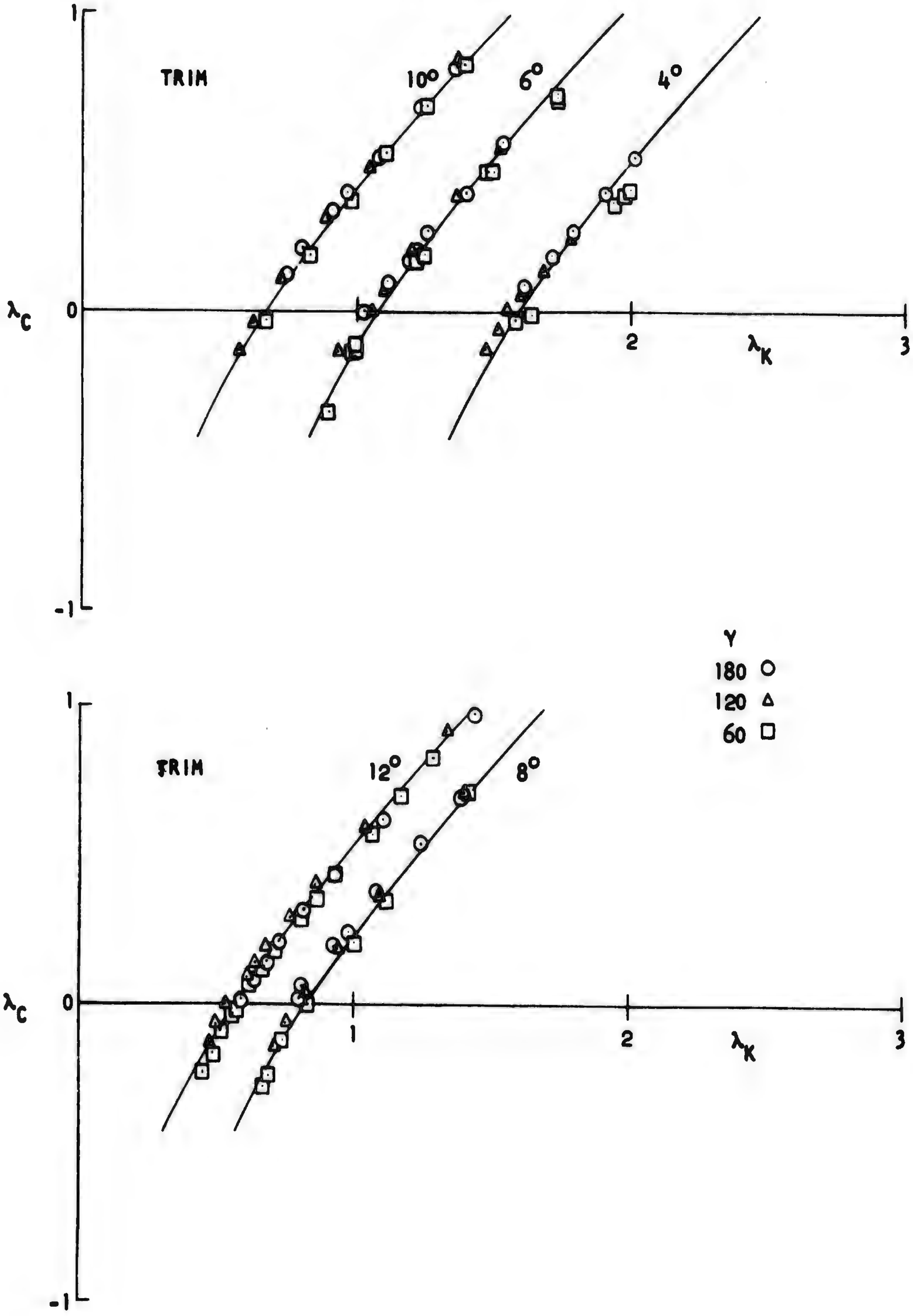


Fig 5

VARIATION OF LIFT WITH ASPECT RATIO
$\beta = 20^\circ$, $\gamma = 180^\circ$

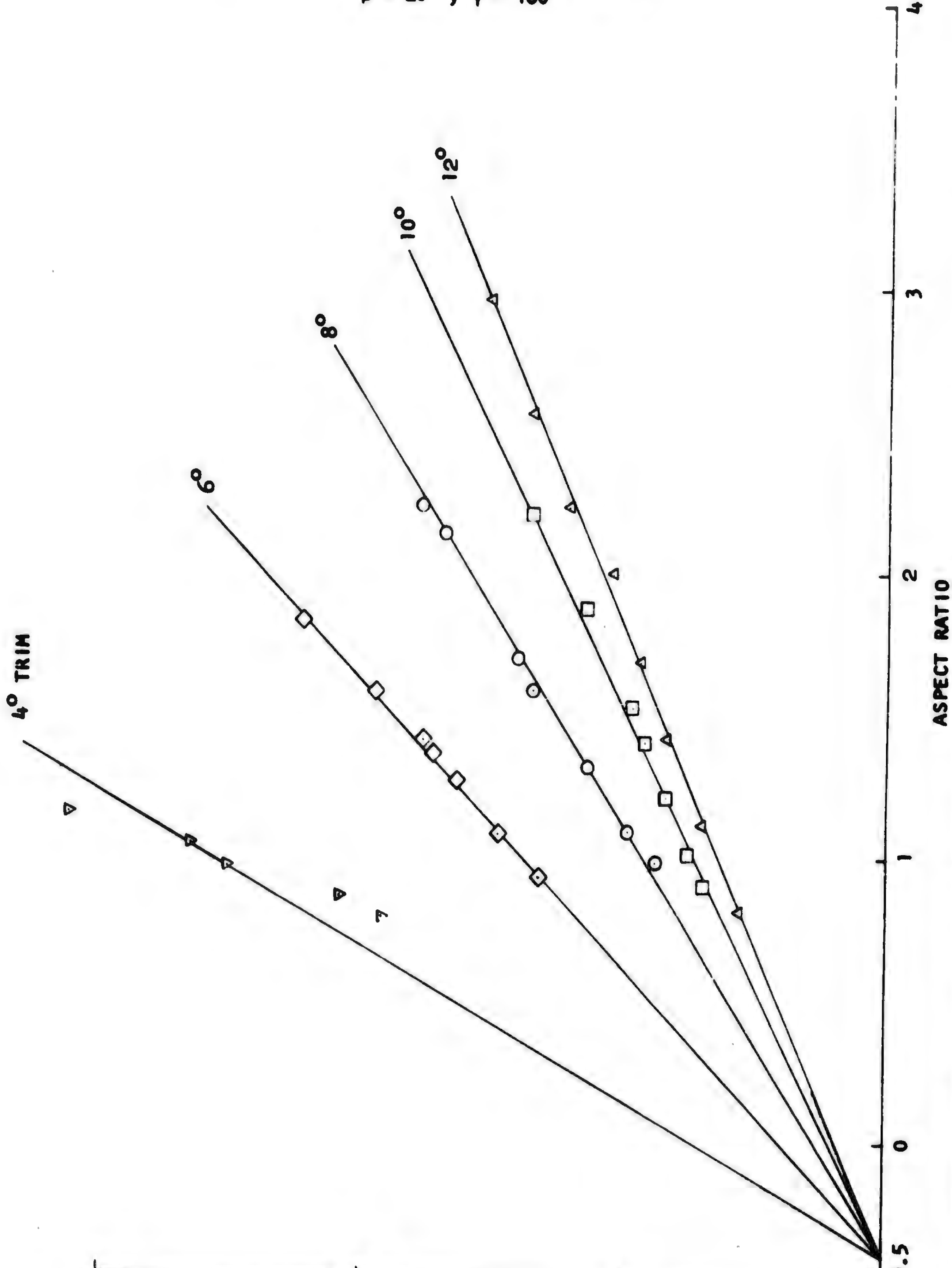


Fig 6

VARIATION OF LIFT WITH ASPECT RATIO

$\beta = 20^\circ$, $\gamma = 120^\circ$

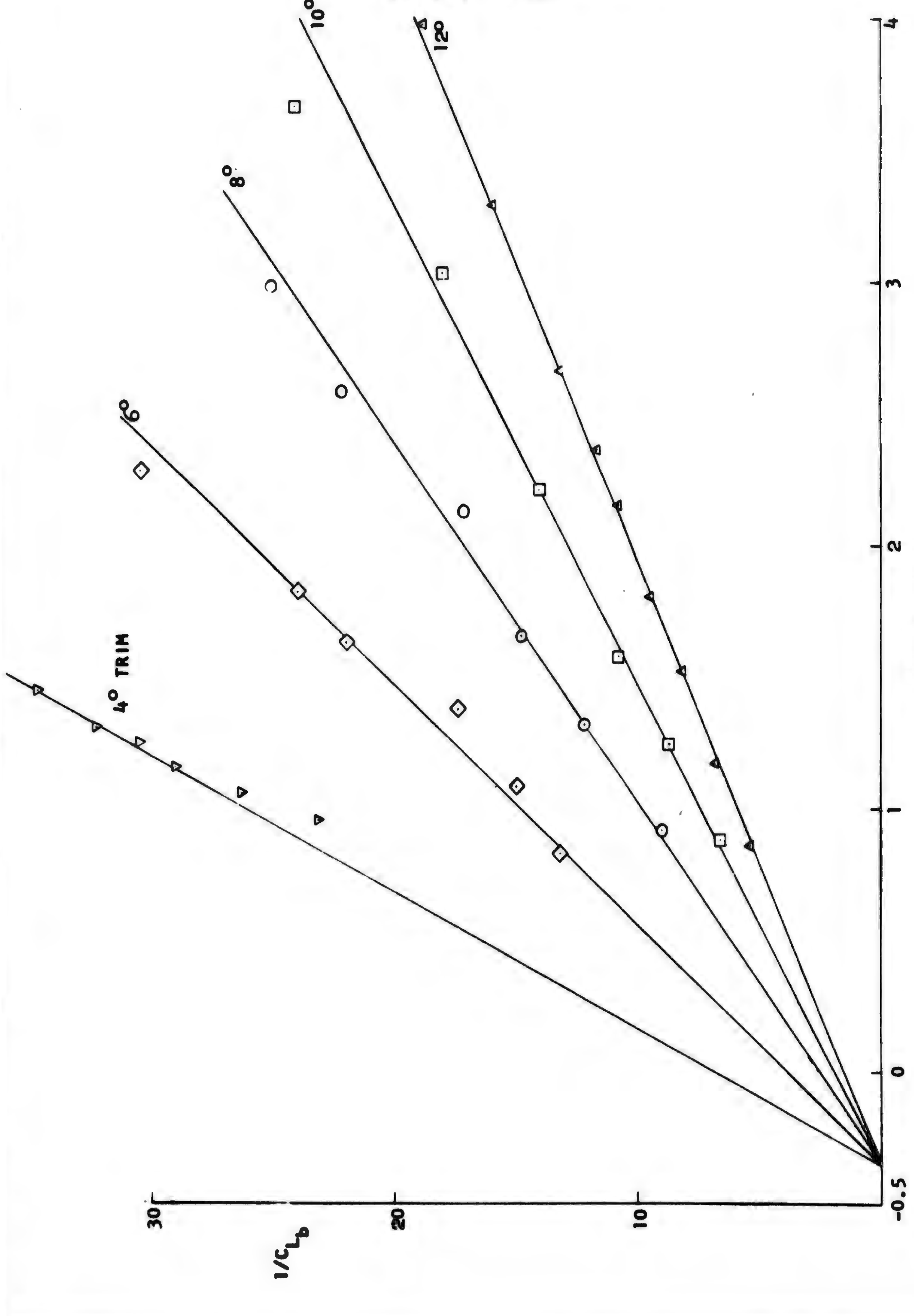
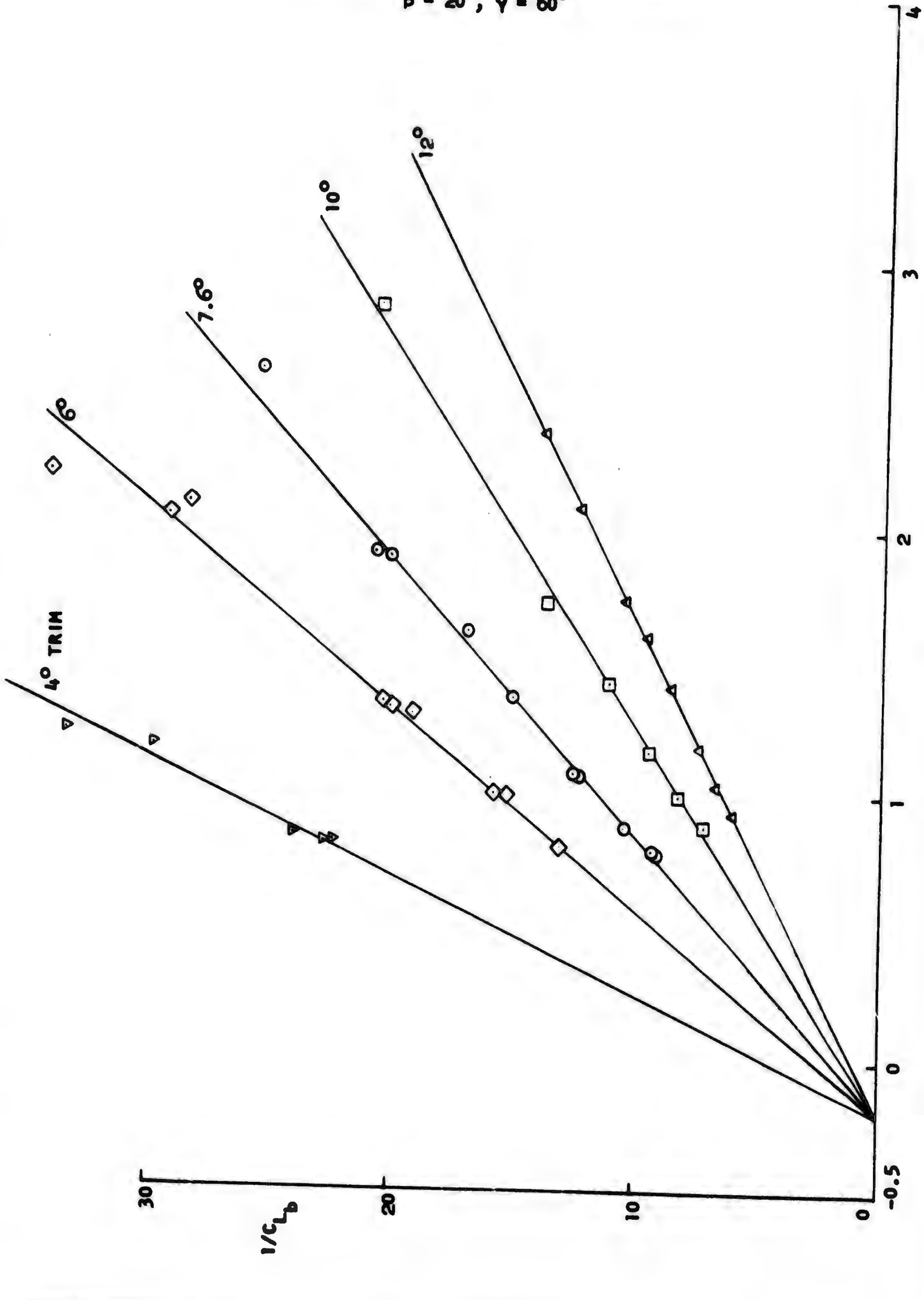


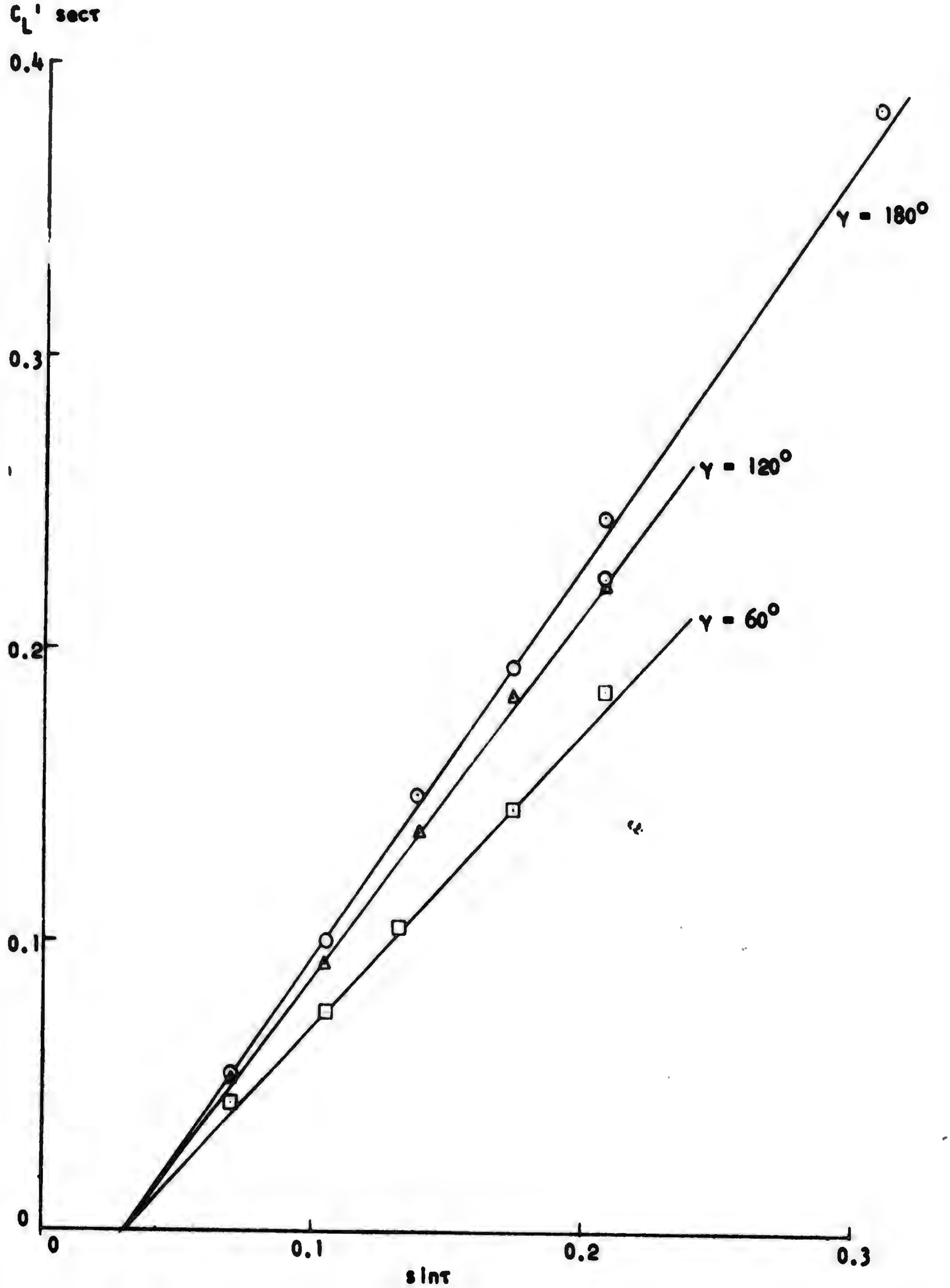
Fig 7

VARIATION OF LIFT WITH ASPECT RATIO

$\beta = 20^\circ$, $\gamma = 60^\circ$

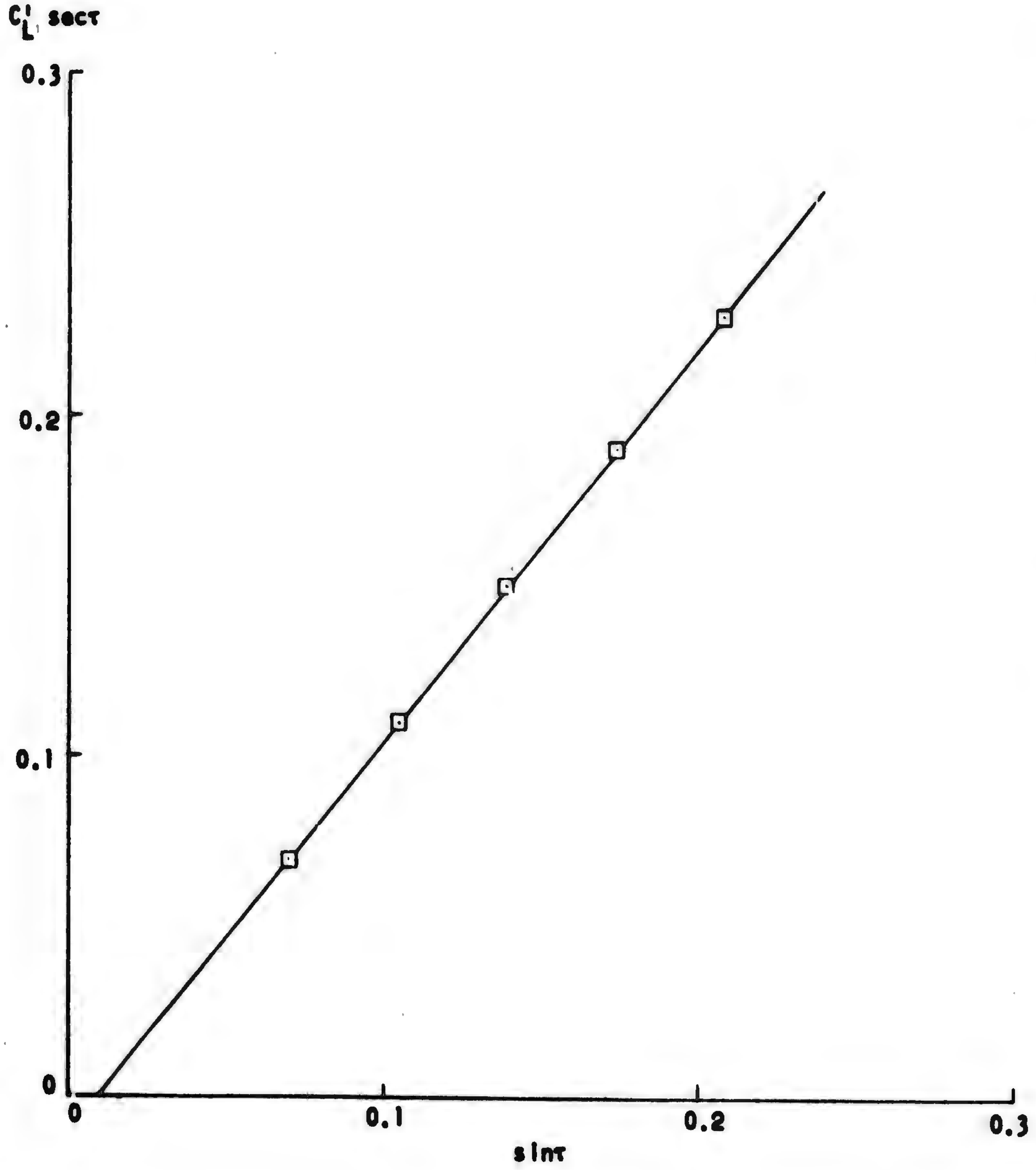


VARIATION OF LIFT WITH TRIM
$\beta = 20^\circ$



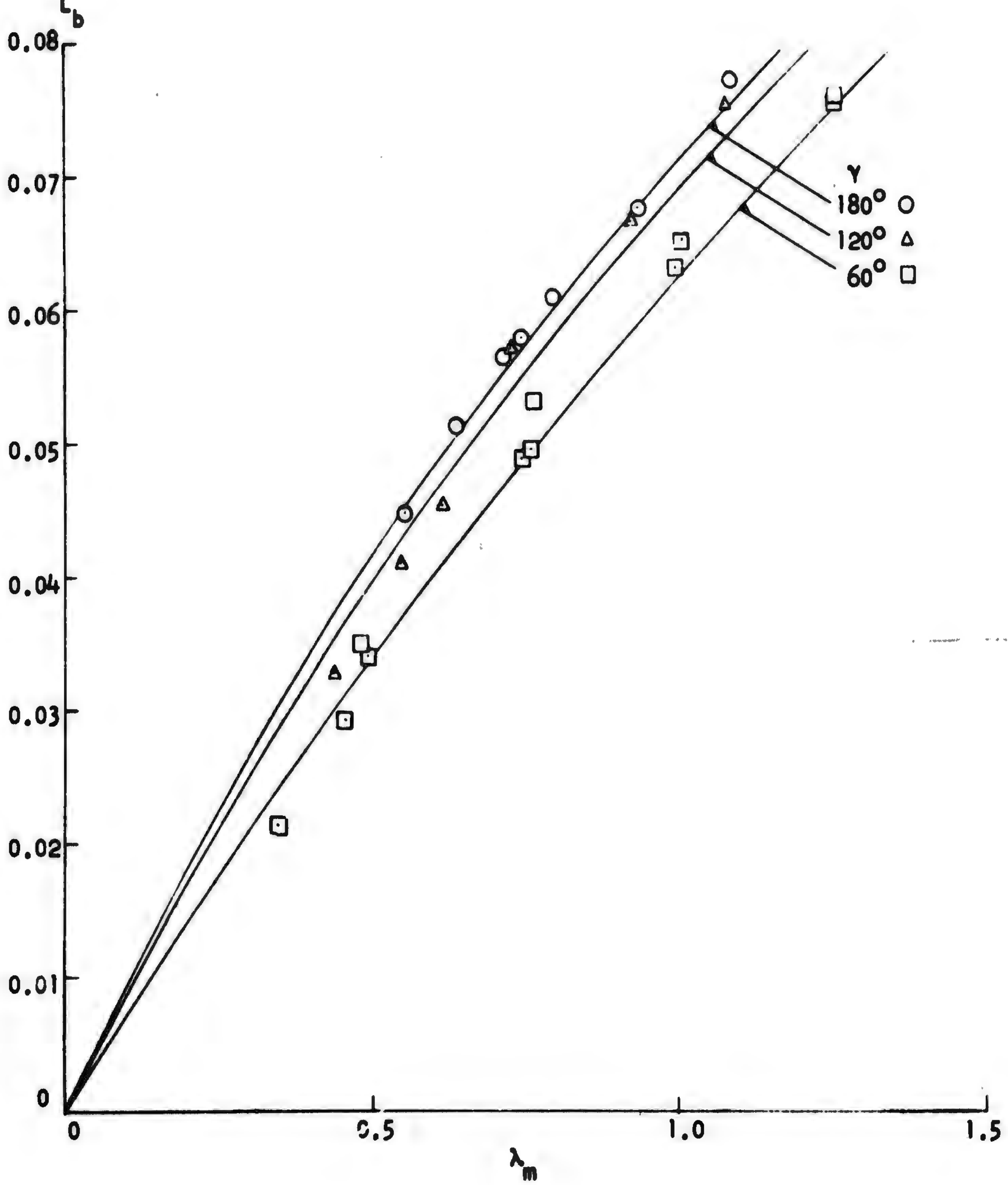
VARIATION OF LIFT WITH TRIM

$\beta = 10^\circ$, $\gamma = 60^\circ$



VARIATION OF LIFT WITH WETTED LENGTH

$\beta = 20^\circ$, $\tau = 6^\circ$



VARIATION OF FRICTION DRAG WITH WETTED LENGTH

$\beta = 20^\circ$, $\tau = 6^\circ$, $\gamma = 180^\circ$ FROM NACA TN 2876

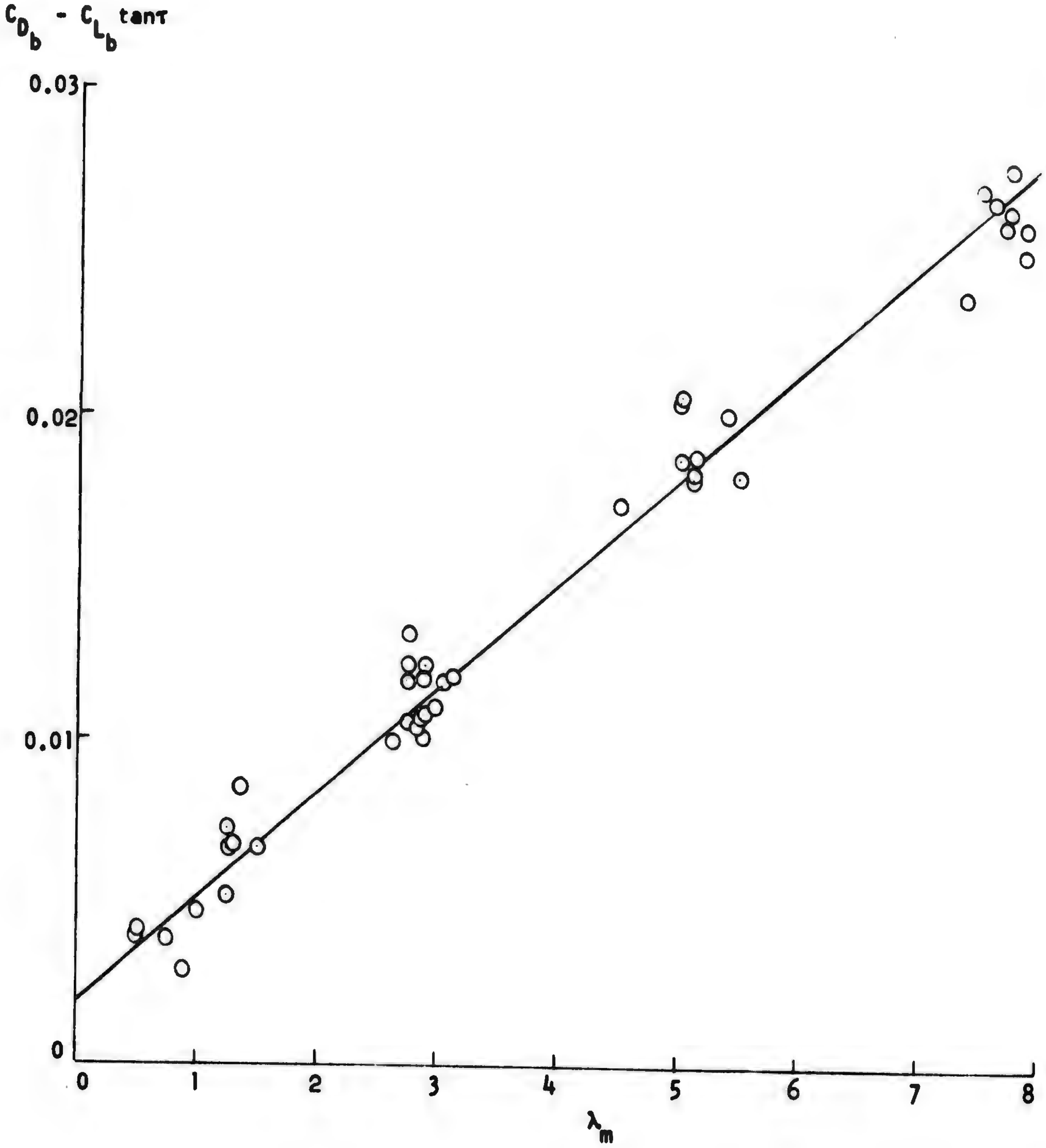
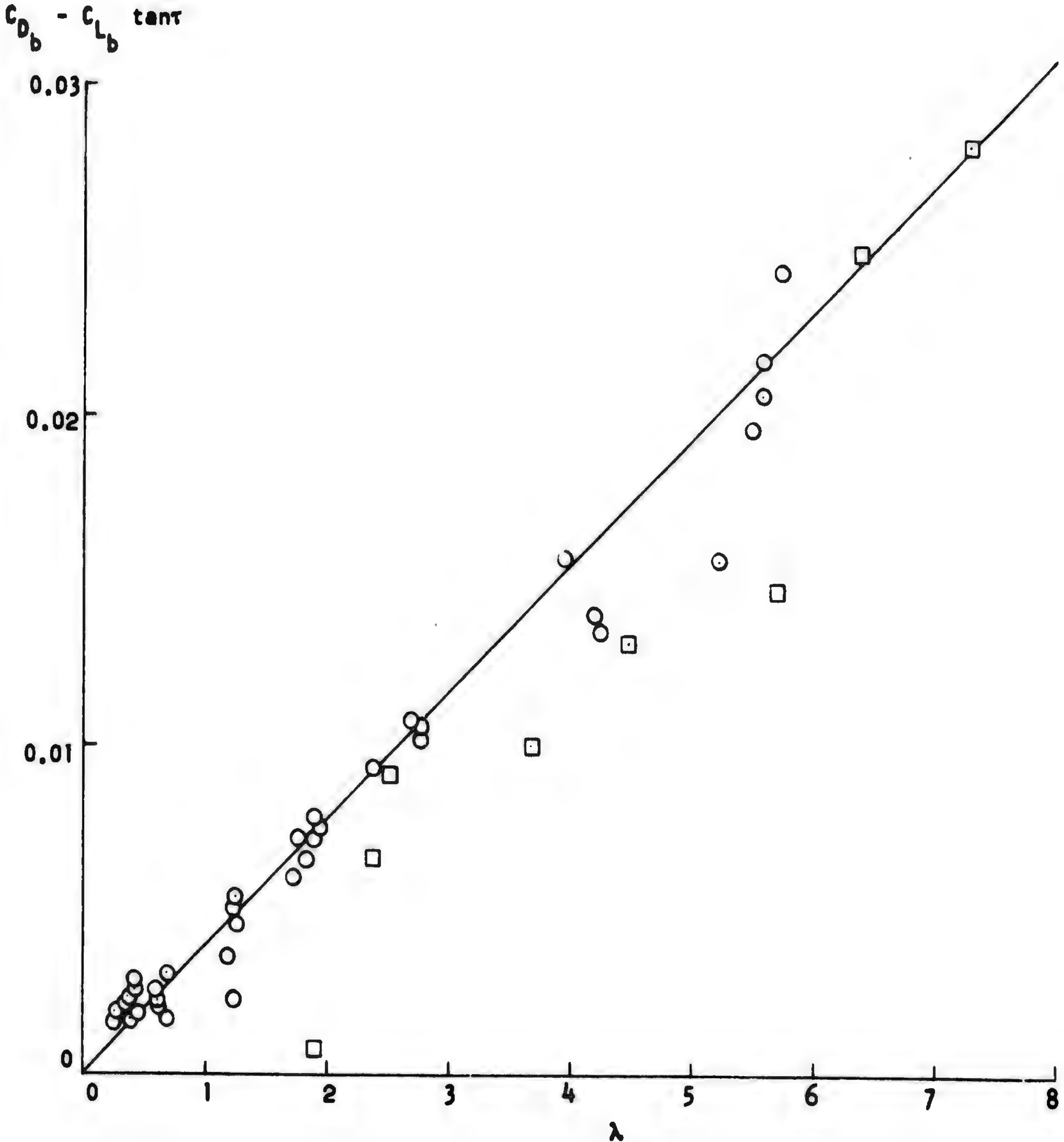


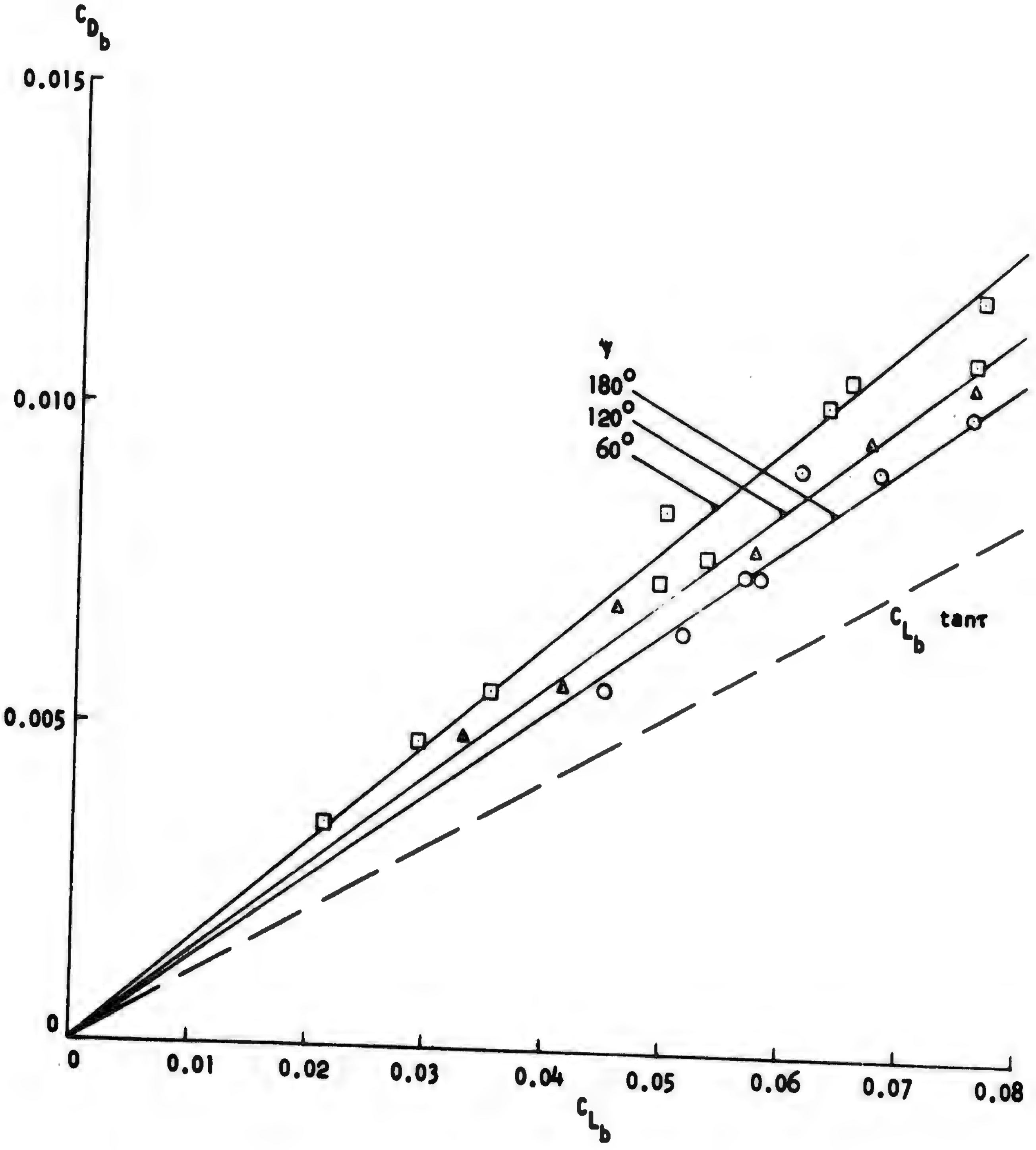
Fig 12

VARIATION OF FRICTION DRAG WITH WETTED LENGTH

$\beta = 20^\circ$, $\tau = 12^\circ$, $\gamma = 180^\circ$, FROM NACA TN 2876 ○ & TN 3939 □

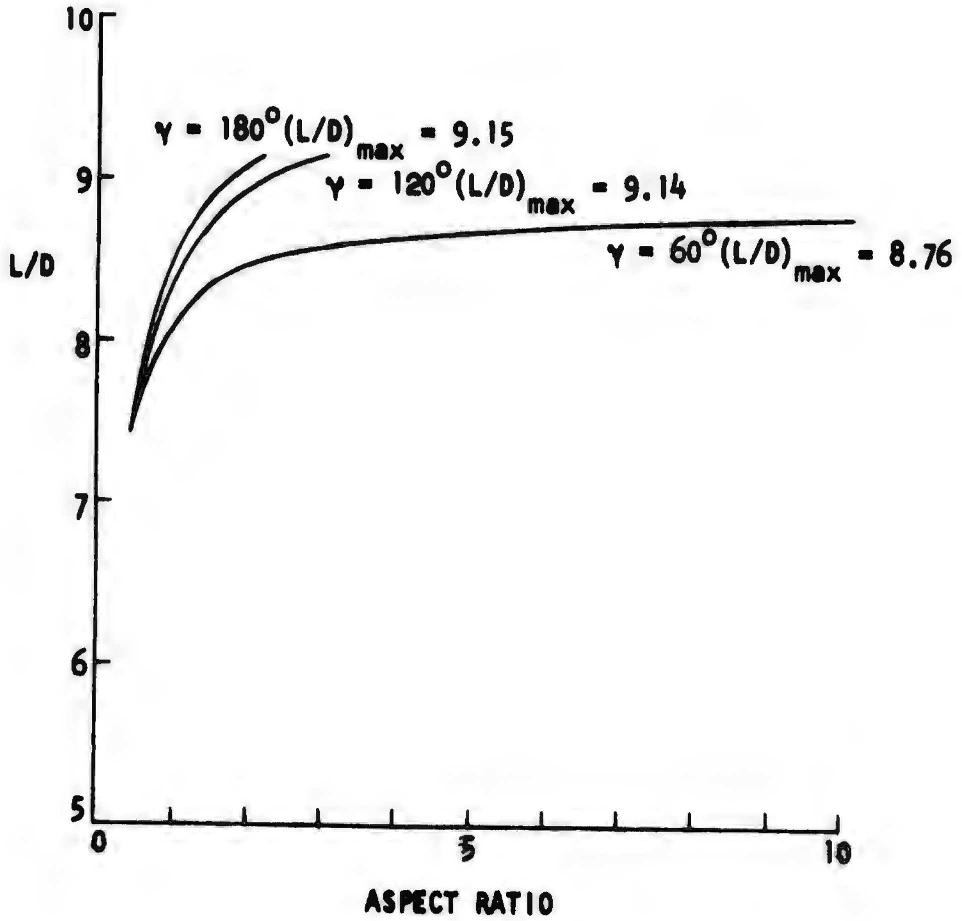


VARIATION OF DRAG WITH LIFT
$\beta = 20^\circ$, $\tau = 6^\circ$



VARIATION OF LIFT-DRAG RATIO WITH ASPECT RATIO

$\beta = 10^\circ$, $\tau = 4^\circ$, $C_f = 0.003$



UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*
DAVIDSON LABORATORY, STEVENS INSTITUTE OF TECHNOLOGY

2a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

2b. GROUP

3. REPORT TITLE
AN ANALYSIS OF THE FORCES AND MOMENTS ON RE-ENTRANT VEE-STEP PLANING SURFACES

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
FINAL REPORT (June 1965 - May 1966)

5. AUTHOR(S) *(Last name, first name, initial)*
BROWN, PETER W.

6. REPORT DATE
MAY 1966

7a. TOTAL NO. OF PAGES
36

7b. NO. OF REFS
14

8a. CONTRACT OR GRANT NO.
Nonr 263

b. PROJECT NO.

c. TASK 66

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
LR 1142

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. AVAILABILITY/LIMITATION NOTICES
QUALIFIED REQUESTERS MAY OBTAIN COPIES OF THIS REPORT FROM DDC

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
BuShips General Hydrodynamics Research Program Administered by David Taylor Model Basin

13. ABSTRACT

The effect of re-entrant vee-steps on the performance of planing surfaces is analyzed. Formulae are developed for the lift, drag, center of pressure and wetted area at high aspect ratio. It is found that the lift-drag ratio of the planing surface decreases as the re-entrant step angle becomes more acute.

DD FORM 1 JAN 64 1473

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
planing surface						
step						
aspect ratio						
lift						
drag						
center of pressure						
wetted area						
performance						
theory						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8*a*. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
____."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
____."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

DD FORM 1 JAN 64 1473 (BACK)