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HYDROMECHANICS

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## ANALYSIS FOR DETERMINING STRESSES IN STIFFENED CYLINDRICAL SHELLS NEAR STRUCTURAL DISCONTINUITIES

by

Robert D. Short and Robert Bart



STRUCTURAL MECHANICS LABORATORY
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# NOTATION

| | |
|---|---|
| $A_F$ | Area of frame |
| $A_1, A_2, A_3, A_4$ | Integration constants |
| $\|B\|$ | Bay matrix; see Equation [8] |
| $b$ | Faying width of frame |
| $D$ | $\dfrac{Eh^3}{12(1-\nu^2)}$ |
| $E$ | Young's modulus |
| $\|F\|$ | Frame matrix |
| $f_1, f_2, f_3, f_4$ | Deflection functions |
| $g_1, g_2, g_3, g_4$ | Deflection functions |
| $H =$ | $(A_F + bh)\left[\sqrt{1+\alpha^2\beta^2}\sinh K_1 l + \sqrt{1-\alpha^2\beta^2}\sin K_2 l\right] +$ $\dfrac{2h}{\alpha}\sqrt{1-\alpha^4\beta^4}\,(\cosh K_1 l - \cos K_2 l)$ |
| $h$ | Shell thickness |
| $I$ | Moment of inertia of frame section about radial axis |
| $K_1$ | $\alpha\sqrt{1-\alpha^2\beta^2}$ |
| $K_2$ | $\alpha\sqrt{1+\alpha^2\beta^2}$ |
| $L$ | Center-to-center distance of frames |
| $l$ | Clear distance between frames |
| $M$ | Longitudinal bending moment in shell |
| $P$ | Pressure |
| $P_c$ | Collapse pressure |
| $R$ | Radius to middle surface of shell |
| $R_F$ | Radius to center of gravity of frame section |
| $\|S\|$ | Bay matrix; see Equation [11] |
| $T$ | $-\dfrac{2h\sigma_1}{P_c R}$ |
| $V$ | Longitudinal shear stress in shell |
| $x$ | Distance along shell generator |
| $y$ | Radial deflection of shell |

| | |
|---|---|
| $y_u$ | $\frac{PR^2}{2Eh}(2-\nu)$ |
| $\alpha^4$ | $\frac{3(1-\nu^2)}{R^2h^2}$ |
| $\beta^2$ | $\frac{PR^3}{2Eh}$ |
| $\delta$ | Differential shear function |
| $\nu$ | Poisson's ratio |
| $\sigma$ | Stress |
| $\phi_1, \phi_2, \phi_3, \phi_4$ | Functions defined by Equations [41], [44], [46], and [46], respectively |

## ABSTRACT

Recent developments in structural research on circular ring-stiffened cylinders subjected to hydrostatic pressure have indicated that refinements of the standard strength analysis are required to account for the effects of discontinuities such as a heavy frame or bulkhead or variations in shell thickness. A procedure has been developed for computating axisymmetric stresses near these discontinuities in a cylinder in which the thickness of the material does not vary between a pair of stiffeners but may change from one side of a stiffener to the other. The results of this analysis indicate that stresses higher than those predicted by a standard computation will usually exist near these discontinuities. A method of reducing this effect by modifying the geometry near these points is also presented.


## INTRODUCTION

The strength of thin-walled, stiffened, cylindrical shells against axisymmetric yielding is usually computed from an elastic analysis of Von Sanden and Gunther.[1] This analysis gives the stress distribution in the shell, and collapse is assumed to occur when the stresses at the middle of a bay exceed the yield point of the material. A more recent analysis developed by Salerno and Pulos[2] accounts for the longitudinal moments resulting from the end pressure together with the radial deflection of the shell and, therefore, gives more accurate stresses. Although neither of these analyses indicates any difference in strength between bays, it has been observed experimentally that failure by axisymmetric yielding almost invariably occurs in the full-length bay nearest a bulkhead or heavy frame. To determine the effect of end conditions on the stresses in the shell, a modified Salerno and Pulos analysis is derived which takes into account variable frame spacing and size and changes in shell thickness; it can be easily extended to variations in radius between adjacent bays.

To offset the weakening effect of the heavy member, it has become customary to reduce the spacing of the first frame from this member. This practice, however, merely forces the weakening effect of the heavy member into the next bay without appreciably increasing the collapse pressure. Since it is usually undesirable to reduce the length of more than one bay, another method to increase the strength of the end bays is proposed in this report.

## ELASTIC STRESSES IN FINITE CYLINDER

The theory of Salerno and Pulos assumes an infinite closed cylinder composed of a linearly elastic material of uniform thickness and radius reinforced by equally spaced, equal-sized frames and subjected to hydrostatic pressure. A typical bay of this cylinder illustrating

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[1] References are listed on page 24.

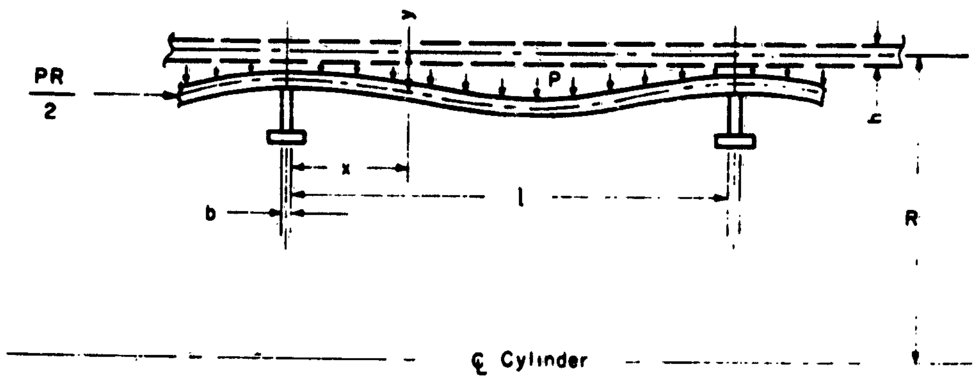


Figure 1 – Typical Bay of the Salerno and Pulos Cylinders

the initial and deformed shapes is shown in Figure 1.

The deformation defined by the radial deflection $y$ as a function of $x$ must satisfy the differential equation

$$\frac{d^4y}{dx^4} + \frac{12(1-\nu^2)\,PR}{2Eh^3}\,\frac{d^2y}{dx^2} + \frac{12(1-\nu^2)}{R^2h^2}\left[y - \frac{PR^2}{Eh}\,(1-\nu/2)\right] = 0 \qquad [1]$$

where $E$ is Young's modulus, $\nu$ is Poisson's ratio, and the other terms are defined in Figure 1. Setting

$$\alpha^4 = \frac{3(1-\nu^2)}{R^2h^2}, \quad \beta^2 = \frac{PR^3}{2Eh}, \quad \text{and} \quad y_u = \frac{PR^2}{2Eh}\,(2-\nu)$$

this simplifies to*

$$y^{IV} + 4\alpha^4\beta^2\,y'' + 4\alpha^4(y-y_u) = 0 \qquad [2]$$

Note that $y_u$ is the deflection which would occur if the stiffening frames were not present.

One form of the general solution of Equations [1] or [2] is

$$y = A_1 \cosh K_1x \cos K_2x + A_2 \sinh K_1x \sin K_2x + A_3 \cosh K_1x \sin K_2x$$
$$+ A_4 \sinh K_1x \cos K_2x + y_u \qquad [3a]$$

where $K_1 = \alpha\sqrt{1-\alpha^2\beta^2}$ and $K_2 = \alpha\sqrt{1+\alpha^2\beta^2}$. The derivatives of $y$ are readily obtained in the following form

$$y' = (A_1K_1 + A_2K_2)\sinh K_1x \cos K_2x + (A_2K_1 - A_1K_2)\cosh K_1x \sin K_2x$$
$$+ (A_3K_2 + A_4K_1)\cosh K_1x \cos K_2x + (A_3K_1 - A_4K_2)\sinh K_1x \sin K_2x \qquad [3b]$$

*The solution which follows is taken from Reference 3.

$$y'' = [A_1(K_1^2 - K_2^2) + 2A_2K_1K_2] \cosh K_1x \cos K_2x + [A_2(K_1^2 - K_2^2) - 2A_1 K_1 K_2] \sinh K_1x \sin K_2x + [A_3(K_1^2 - K_2^2) - 2A_4K_1K_2] \cosh K_1x \sin K_2x + [A_4 (K_1^2 - K_2^2) + 2A_3 K_1K_2] \sinh K_1x \cos K_2x \qquad [3c]$$

$$y''' = [A_1K_1(K_1^2 - 3K_2^2) + A_2K_2(3K_1^2 - K_2^2)] \sinh K_1x \cos K_2x + [A_1K_2 (K_2^2 - 3K_1^2) + A_2K_1(K_1^2 - 3K_2^2)] \cosh K_1x \sin K_2x + [A_3K_2(3K_1^2 - K_2^2) + A_4K_1 (K_1^2 - 3K_2^2)] \cosh K_1x \cos K_2x + [A_3K_1(K_1^2 - 3K_2^2) + A_4K_2(K_2^2 - 3K_1^2)] \sinh K_1x \sin K_2x \qquad [3d]$$

Let

$$H = (A_F + bh)(\sqrt{1+\alpha^2\beta^2} \sinh K_1l + \sqrt{1-\alpha^2\beta^2} \sin K_2l) + \frac{2h}{\alpha}\sqrt{1-\alpha^4\beta^4} (\cosh K_1l - \cos K_2l) \qquad [4]$$

where $A_F$ is the area of the frame cross section* and $b$ is the faying width of the frame. Applying conditions of coutinuity and equilibrum, Equations [3] for the Salerno and Pulos cylinders may be reduced to:

$$y = y_u\left[1 - \frac{A_F}{H} f_1(x)\right] \qquad [5a]$$

$$\frac{y'}{\alpha} = -y_u\frac{A_F}{H} f_2(x) \qquad [5b]$$

$$\frac{y''}{2\alpha^2} = -y_u\frac{A_F}{H} f_3(x) \qquad [5c]$$

$$\frac{y'''}{2\alpha^3} = -y_u\frac{A_F}{H} f_4(x) \qquad [5d]$$

where

$$f_1(x) = \sqrt{1-\alpha^2\beta^2}\,[\cosh K_1x \sin K_2(l-x) + \cosh K_1(l-x) \sin K_2x] + \sqrt{1+\alpha^2\beta^2}\,[\sinh K_1x \cos K_2(l-x) + \sinh K_1 (l-x) \cos K_2x] \qquad [6a]$$

$$f_2(x) = \frac{f_1'(x)}{\alpha} = 2\,[\sinh K_1x \sin K_2(l-x) - \sinh K_1(l-x) \sin K_2x] \qquad [6b]$$

$$f_3(x) = \frac{f_1''(x)}{2\alpha^2} = \sqrt{1-\alpha^2\beta^2}\,[\cosh K_1x \sin K_2(l-x) + \cosh K_1(l-x) \sin K_2x] - \sqrt{1+\alpha^2\beta^2}\,[\sinh K_1x \cos K_2(l-x) + \sinh K_1(l-x) \cos K_2x] \qquad [6c]$$

*Greater accuracy is obtained by using for $A_F$ the value obtained by multiplying the true area of the section by $R/R_F$ where $R_F$ is the radius of the center of gravity of the frame.

$$f_4(x) = \frac{f_1'''(x)}{2\alpha^3} = -2\left\{\alpha^2\beta^2 \left[\sinh K_1 x \sin K_2(l-x) - \sinh K_1(l-x) \sin K_2 x\right] + \sqrt{1-\alpha^4\beta^4} \right.$$
$$\left. \cdot \left[\cosh K_1(l-x) \cos K_2 x - \cosh K_1 x \cos K_2(l-x)\right]\right\} \qquad [6d]$$

In an actual structure, the presence of bulkheads or heavy frames precludes the assumption of one deformation pattern for all bays as assumed in the Salerno and Pulos analysis. However, an approximate relationship can be found by computing $y$ and its derivatives at a distance from the bulkhead in terms of the conditions at the bulkhead and by assuming that these will approach the values computed by a Salerno and Pulos analysis as the distance increases. Basically, symmetric deflection assumptions of Salerno and Pulos are replaced by the matrix equation

$$|B|_{0,S\&P} = |B|_{0,i}^{*} \qquad [7]$$

where

$$|B| = \begin{vmatrix} y \\ \dfrac{y'}{\alpha} \\ \dfrac{y''}{2\alpha^2} \\ \dfrac{y'''}{2\alpha^3} \\ y_u \end{vmatrix} \qquad [8]$$

The subscript $0, i$ denotes that $|B|$ is to be evaluated at the bulkhead or at a frame ($x = 0$) in the $i$th bay from the bulkhead, and the subscript 0, S&P denotes values at $x = 0$ in the infinite cylinder. Since Equation [7] is rarely exactly satisfied, $|B|_{0,i}$ will be taken such that it is most nearly satisfied by a "least-squares" approximation consistent with the bulkhead conditions.

The basic problem, then, is to determine $|B|_{0,i}$ in terms of the bulkhead condition $|B|_{0,1}$ so that Equation [7] and a least-square estimate can be used to determine $|B|_{0,1}$. If Equations [3] are evaluated at $x = 0$, they can be reduced to

$$A_1 = y(0) - y_u \qquad [9a]$$

---

*We could as easily have used $|B|_{l,S\&P} = |B|_{l,i}$.

$$A_2 = \left(\frac{y''(0)}{2\alpha^2}\right)\left(\frac{1}{\sqrt{1-\alpha^4\beta^4}}\right) + (y(0) - y_u)\,\frac{\alpha^2\beta^2}{\sqrt{1-\alpha^4\beta^4}} \qquad [9b]$$

$$A_3 = \frac{1}{2}\left[\left(\frac{y'(0)}{\alpha}\right)\left(\frac{1+2\alpha^2\beta^2}{\sqrt{1+\alpha^2\beta^2}}\right) + \left(\frac{y'''(0)}{2\alpha^3}\right)\left(\frac{1}{\sqrt{1+\alpha^2\beta^2}}\right)\right] \qquad [9c]$$

$$A_4 = \frac{1}{2}\left[\left(\frac{y'(0)}{\alpha}\right)\left(\frac{1-2\alpha^2\beta^2}{\sqrt{1-\alpha^2\beta^2}}\right) - \left(\frac{y'''(0)}{2\alpha^3}\right)\left(\frac{1}{\sqrt{1-\alpha^2\beta^2}}\right)\right] \qquad [9d]$$

Now, substitution of Equations [9] into Equations [3], which will yield conditions at any point of the bay, gives

$$|B|_{x,i} = |S|_{x,i} \times |B|_{0,i} \qquad \text{when } x \leq l_i \qquad [10]$$

where $|S|_{x,i}$ is defined by Equation [11] on page 6.

The identity equation $y_u \equiv y_u$, which holds throughout the bay, is included in Equation [10] for reasons of conformableness which will be apparent later. Replacing $x$ by $l_1$ and letting $i = 1$ in Equation [10] gives

$$|B|_{l,1} = |S|_{l,1} \times |B|_{0,1} \qquad [12]$$

$|B|_{l,1}$ now denotes the boundary conditions at the end of the bay away from the bulkhead. It is possible to assume that $|B|_{l,1} = |B|_{l,\,S\&P}$ and to solve immediately for $|B|_{0,1}$. This will not give a very good approximation because $l_1$ is usually too small. In order to obtain $|B|_{x,i}$ at a greater distance from the bulkhead, $|B|$ must be found on the other side of the frame. The conditions on both sides of a frame are shown in Figure 2.

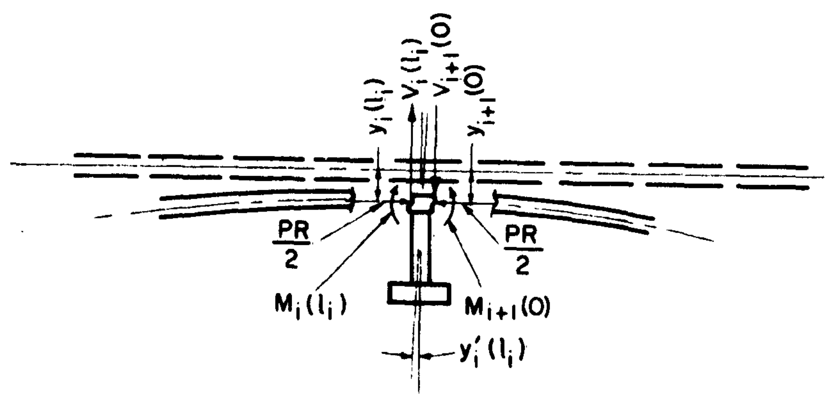


**Figure 2 – Loads and Deformations at a Frame**

$$
|S|_{z,i} = \begin{vmatrix}
\left\{\begin{matrix} \cosh K_1 z \cos K_2 z \\ + \dfrac{\alpha^2\beta^2}{\sqrt{1-\alpha^4\beta^4}} \sinh K_1 z \sin K_2 z \end{matrix}\right\} &
\dfrac{1}{2}\left\{\begin{matrix} \dfrac{1+2\alpha^2\beta^2}{\sqrt{1+\alpha^2\beta^2}} \cosh K_1 z \sin K_2 z \\ + \dfrac{1-2\alpha^2\beta^2}{\sqrt{1-\alpha^2\beta^2}} \sinh K_1 z \cos K_2 z \end{matrix}\right\} &
\left\{\dfrac{\sinh K_1 z \sin K_2 z}{\sqrt{1-\alpha^4\beta^4}}\right\} &
\dfrac{1}{2}\left\{\begin{matrix} \dfrac{\cosh K_1 z \sin K_2 z}{\sqrt{1+\alpha^2\beta^2}} \\ - \dfrac{\sinh K_1 z \cos K_2 z}{\sqrt{1-\alpha^2\beta^2}} \end{matrix}\right\} &
\left\{\begin{matrix} 1 - \cosh K_1 z \cos K_2 z \\ - \dfrac{\alpha^2\beta^2}{\sqrt{1-\alpha^4\beta^4}} \sinh K_1 z \sin K_2 z \end{matrix}\right\} \\
\left\{\begin{matrix} \dfrac{\sinh K_1 z \cos K_2 z}{\sqrt{1-\alpha^2\beta^2}} \\ - \dfrac{\cosh K_1 z \sin K_2 z}{\sqrt{1+\alpha^2\beta^2}} \end{matrix}\right\} &
\left\{\begin{matrix} \cosh K_1 z \cos K_2 z \\ + \dfrac{\alpha^2\beta^2}{\sqrt{1-\alpha^4\beta^4}} \sinh K_1 z \sin K_2 z \end{matrix}\right\} &
\left\{\begin{matrix} \dfrac{\cosh K_1 z \sin K_2 z}{\sqrt{1+\alpha^2\beta^2}} \\ + \dfrac{\sinh K_1 z \cos K_2 z}{\sqrt{1-\alpha^2\beta^2}} \end{matrix}\right\} &
\left\{\dfrac{\sinh K_1 z \sin K_2 z}{\sqrt{1-\alpha^4\beta^4}}\right\} &
\left\{\begin{matrix} \dfrac{\cosh K_1 z \sin K_2 z}{\sqrt{1+\alpha^2\beta^2}} \\ - \dfrac{\sinh K_1 z \cos K_2 z}{\sqrt{1-\alpha^2\beta^2}} \end{matrix}\right\} \\
\left\{- \dfrac{\sinh K_1 z \sin K_2 z}{\sqrt{1-\alpha^4\beta^4}}\right\} &
\dfrac{1}{2}\left\{\begin{matrix} \dfrac{\sinh K_1 z \cos K_2 z}{\sqrt{1-\alpha^2\beta^2}} \\ - \dfrac{\cosh K_1 z \sin K_2 z}{\sqrt{1+\alpha^2\beta^2}} \end{matrix}\right\} &
\left\{\begin{matrix} \cosh K_1 z \cos K_2 z \\ - \dfrac{\alpha^2\beta^2}{\sqrt{1-\alpha^4\beta^4}} \sinh K_1 z \sin K_2 z \end{matrix}\right\} &
\dfrac{1}{2}\left\{\begin{matrix} \dfrac{\cosh K_1 z \sin K_2 z}{\sqrt{1+\alpha^2\beta^2}} \\ + \dfrac{\sinh K_1 z \cos K_2 z}{\sqrt{1-\alpha^2\beta^2}} \end{matrix}\right\} &
\left\{\dfrac{\sinh K_1 z \sin K_2 z}{\sqrt{1-\alpha^4\beta^4}}\right\} \\
\left\{\begin{matrix} - \dfrac{\cosh K_1 z \sin K_2 z}{\sqrt{1+\alpha^2\beta^2}} \\ - \dfrac{\sinh K_1 z \cos K_2 z}{\sqrt{1-\alpha^2\beta^2}} \end{matrix}\right\} &
\left\{- \dfrac{\sinh K_1 z \sin K_2 z}{\sqrt{1-\alpha^4\beta^4}}\right\} &
\left\{\begin{matrix} \dfrac{1-2\alpha^2\beta^2}{\sqrt{1-\alpha^2\beta^2}} \sinh K_1 z \cos K_2 z \\ - \dfrac{1+2\alpha^2\beta^2}{\sqrt{1+\alpha^2\beta^2}} \cosh K_1 z \sin K_2 z \end{matrix}\right\} &
\left\{\begin{matrix} \cosh K_1 z \cos K_2 z \\ - \dfrac{\alpha^2\beta^2}{\sqrt{1-\alpha^4\beta^4}} \sinh K_1 z \sin K_2 z \end{matrix}\right\} &
\left\{\begin{matrix} \dfrac{\cosh K_1 z \sin K_2 z}{\sqrt{1+\alpha^2\beta^2}} \\ + \dfrac{\sinh K_1 z \cos K_2 z}{\sqrt{1-\alpha^2\beta^2}} \end{matrix}\right\} \\
0 & 0 & 0 & 0 & 1
\end{vmatrix} \quad [11]
$$

Assuming that the faying width $b$ remains straight, four equations can be obtained from considerations of continuity and equilibrium. Obviously

$$y_{i+1}(0) = y_i(l_i) + b\, y_i'(l_i) \qquad [13a]$$

$$y_{i+1}'(0) = y_i'(l_i) \qquad [13b]$$

Equilibrium of vertical forces yields

$$V_i(l_i) - V_{i+1}(0) - Pb + E\left[\frac{y_i(l_i) + y_{i+1}(0)}{2R}\right]\cdot\left(\frac{A_F + bh}{R}\right) + \left(\frac{\nu PR}{2}\right)\left(\frac{b}{R}\right) = 0 \qquad [13c]$$

Moment equilibrium yields

$$M_i(l_i) - M_{i+1}(0) + \frac{b}{2}\left[V_i(l_i) + V_{i+1}(0)\right] - y_i'(l_i)\,\frac{E\left(I_F + \frac{b^3h}{12}\right)}{R^2} = 0 \qquad [13d]$$

where $I_F$ * is the moment of inertia of the frame about the vertical (radial) axis. It should be noted that $h$ may be selected from either bay as long as $A_F + bh$ represents the total area of frame and faying width of shell and $I_F + (b^3h/12)$ represents the total moment of inertia about the radial centerline. The moments and shears are obtained in terms of $y''$ and $y'''$ by the relations

$$M = -Dy'' \qquad [14]$$

$$V = -Dy''' \qquad [15]$$

where $D = \dfrac{Eh^3}{12(1-\nu^2)}$ for a cylinder.

Substituting these expressions into Equations [13] and rearranging gives:

$$|B|_{0,\,i+1} = |F|_{i,\,i+1} \times |B|_{l,\,i} \qquad [16]$$

where

---

*Greater accuracy is obtained by using for $I_F$ the value obtained by multiplying the true $I$ of the section $\dfrac{R}{R_F}$.

$$|F|_{i,i+1} = \begin{vmatrix} 1 & b\alpha_i & 0 & 0 & 0 \\ 0 & \dfrac{\alpha_i}{\alpha_{i+1}} & 0 & 0 & 0 \\ -(b\alpha_{i+1})^2 \dfrac{(A_F + bh)}{bh_{i+1}} & (b\alpha_i)(b\alpha_{i+1})^2 \left[ \dfrac{\left(2I_F + \dfrac{b^3 h}{6}\right)}{b^3 h_{i+1}} - \dfrac{(A_F + bh)}{2bh_{i+1}} \right] & \left(\dfrac{\alpha_i}{\alpha_{i+1}}\right)^2 \left(\dfrac{Di}{D_{i+1}}\right) & \left(\dfrac{\alpha_i}{\alpha_{i+1}}\right)^2 \left(\dfrac{Di}{D_{i+1}}\right)(b\alpha_i) & (b\alpha_{i+1})^2 \left(\dfrac{y_{u,i+1}}{y_{u,i}}\right) \\ -2b\alpha_{i+1} \dfrac{(A_F + bh)}{bh_{i+1}} & -(b\alpha_i)(b\alpha_{i+1}) \dfrac{(A + bh)_i}{bh_{i+1}} & 0 & \left(\dfrac{\alpha_i}{\alpha_{i+1}}\right)^3 \left(\dfrac{D_i}{D_{i+1}}\right) & 2b\alpha_{i+1} \left(\dfrac{y_{u,i+1}}{y_{u,i}}\right) \\ 0 & 0 & 0 & 0 & \dfrac{y_{u,i+1}}{y_{u,i}} \end{vmatrix} \quad [17]$$

The double subscript $i$, $i+1$ indicates a frame between $i$ and $i+1$ bays from the bulkhead. Equation [16] includes the identity equation

$$(y_u)_{i+1} = (y_u)_i \frac{(y_u)_{i+1}}{(y_u)_i}$$

and allows the shell parameters (except radius) to change.

Substitution of Equation [11] into Equation [16] gives

$$|B|_{0,i+1} = |F|_{i,i+1} \times (|S|_{l,i} \times |B|_{0,i}) \qquad [18]$$

By repeated application of Equation [18] we may obtain

$$|B|_{0,i+m} = |F|_{i+m-1,\, i+m} \times |S|_{l,i+m-1} \times \ldots \times |F|_{i,i+1} \times |S|_{l,i} \times |B|_{0,i} {}^{*} \qquad [19]$$

For $i = 1$ Equation [19] may be shortened to

$$|B|_{0,m+1} = |F,S|_{m+1,1} \times |B|_{0,1} \qquad [20]$$

where $|F,S|_{m+1,1}$ is the product of the $|F|$ and $|S|$ matrices in the order of Equation [19] for all frames and bays between the $(m+1)$ bay and the bulkhead. Now if the product for matrix $|F,S|_{m+1,1}$ ends with at least two consecutive identical bays, Equation [7] will be very nearly satisfied by $i = m+1$. Almost all problems of practical interest can be carried far enough to satisfy this condition, and it will be assumed hereafter that $|F,S|_{m+1,1}$ does in fact terminate at such a point. Substituting Equation [20] into Equation [7] gives

$$|B|_{0,S\&P} = |F,S|_{m+1,1} \times |B|_{0,1} \qquad [21]$$

As previously noted this is not exactly true, but a least-squares process can be used to estimate two of the bulkhead conditions if the other two are given. Any pair may be used as the knowns, but $y$ and $y''$ will be used here because they are the most readily evaluated at a bulkhead at pressures sufficiently high to cause yielding at the bulkhead.** Setting

$$\left(\frac{y'}{\alpha}\right)_{0,\,1} = u, \quad \left(\frac{y'''}{2\alpha^3}\right)_{0,1} = v$$

*$m$ now indicates the number of frames from any starting bay $i$.

**See Appendix A.

and neglecting the identity equation, Equation [21] reduces to

$$\alpha_1 u + \beta_1 v + \gamma_1 = 0 \qquad [22a]$$

$$\alpha_2 u + \beta_2 v + \gamma_2 = 0 \qquad [22b]$$

$$\alpha_3 u + \beta_3 v + \gamma_3 = 0 \qquad [22c]$$

$$\alpha_4 u + \beta_4 v + \gamma_4 = 0 \qquad [22d]$$

The method of least-squares shows that $u$ and $v$ are given by

$$u = \frac{\Sigma\alpha\beta\Sigma\beta\gamma - \Sigma\alpha\gamma\Sigma\beta^2}{\Sigma\alpha^2\Sigma\beta^2 - (\Sigma\alpha\beta)^2} \qquad [23]$$

$$v = \frac{\Sigma\alpha\beta\Sigma\alpha\gamma - \Sigma\beta\gamma\Sigma\alpha^2}{\Sigma\alpha^2\Sigma\beta^2 - (\Sigma\alpha\beta)^2} \qquad [24]$$

Combining $u$ and $v$ with the known values $y$, $y''/2\alpha^2$, and $y_u$ gives the complete column matrix $|B|_{0,1}$, and the column matrix $|B|_{0,i}$ at any other frame is then found by multiplication by the $|F, S|_{i,1}$ matrix. Equation [10] then completely determines the deformations, and hence the stresses, in any bay.

## OPTIMUM DESIGN FOR END BAY

The analysis of the previous section is applicable to an axially symmetric structure composed of circular cylindrical bays with a common axis and radius. The length of bay, thickness of shell, and Young's modulus may vary from bay to bay; the size of stiffeners may also change.

In actual practice, however, all these parameters are usually constant except for isolated interruptions such as bulkheads, heavy frames, and conical reducer sections. Analyses made on several cylinders in the range of current submarine design indicate that there is usually a bay near any bulkhead or other variation of geometry which has larger elastic circumferential membrane stresses than those bays farther from the bulkhead. The following design method has been developed to eliminate this weakness.

For this design procedure a long, ring-stiffened cylinder of uniform geometry composed of a ductile material and terminating at a relatively rigid bulkhead or other discontinuity is assumed. It is desired that the stress pattern caused by the bulkhead shall not cause any bay to be weaker than a bay more distant from the bulkhead. It is apparent that, if Equation [21] is exactly satisfied for some $m$, then all subsequent bays will have the Salerno and Pulos[1] stress distribution. To satisfy Equation [21], two additional parameters are required. If they are selected from the geometry of the first bay and first frame, and Equation [21] can be satisfied for $m = 1$, the stress pattern of the second and all subsequent bays will be identical. The length $l_1$ of the first bay and the area $A_{F_1}$ of the first frame are selected as the parameters to be determined. Since stresses are nonlinear functions of pressure, this determination should be made for $P = P_c$, the expected collapse pressure.

It is more convenient now to select the origin, $x = 0$, of the first bay at the first frame, while the origin for the second or typical bay will remain as it was. The nomenclature for this case is shown in Figure 3.

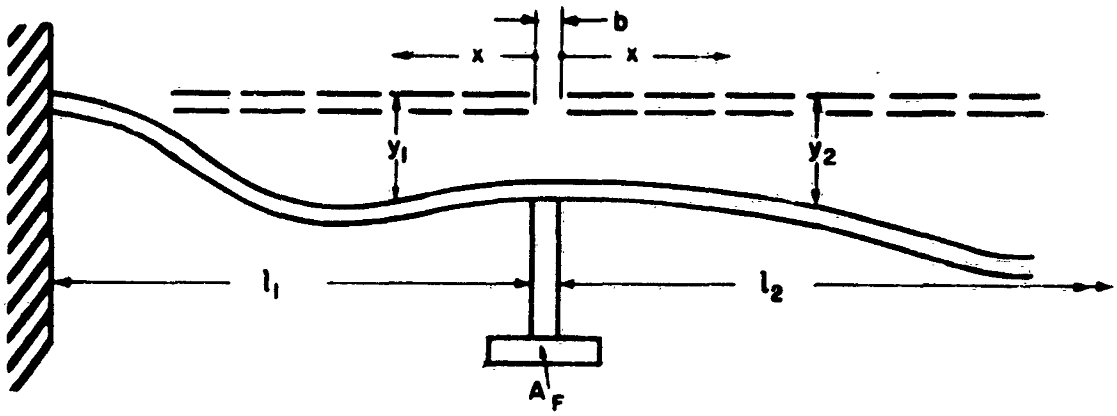


Figure 3 – Nomenclature for End Bay Design

At the right of the frame we set

$$|B|_{0,2} = |B|_{0,\,S\&P} \qquad [25]$$

From Equation [16]

$$|B|_{0,1} = |F^*|_{2,1} \times |B|_{0,2} \qquad [26]$$

where $|F^*|_{2,1}$ is identical to $|F|_{2,1}$ with the second and fourth columns multiplied by (–1).

Since $y'_{S\&P}(0) = 0$, substituting Equation [25] into Equation [26] and rearranging yields

$$y_1(0) - y_{S\&P}(0) = 0 \qquad [27a]$$

$$\frac{y_1'(0)}{\alpha} - \frac{y'_{S\&P}(0)}{\alpha} = 0 \qquad [27b]$$

$$\frac{y_1''(0)}{2\alpha^2} - \frac{y_{S\&P}''(0)}{2\alpha^2} = -(b\alpha)^2 \frac{A_F + bh}{bh} y_{S\&P}(0) - (b\alpha)\, y_{S\&P}'''(0) + (b\alpha)^2\, y_u \qquad [27c]$$

$$\frac{y_1'''(0)}{2\alpha^3} - \frac{y_{S\&P}'''(0)}{2\alpha^3} = -2\,(b\alpha) \frac{A_F + bh}{bh} y_{S\&P}(0) - 2y_{S\&P}'''(0) + 2\,(b\alpha) y_u \qquad [27d]$$

It is now convenient to define $\delta$ and $\Delta V$ as follows:

$$\delta = \frac{\Delta V}{4\alpha^3 D} = \frac{V_1(0) - V_{S\&P}(0)}{4\alpha^3 D} \qquad [28]$$

whence Equations [27c] and [27d] may be written as

$$\frac{y_1''(0)}{2\alpha^2} - \frac{y_{S\&P}''(0)}{2\alpha^2} = -(b\alpha)\,\delta \qquad [27c']$$

$$\frac{y_1'''(0)}{2\alpha^3} - \frac{y_{S\&P}'''(0)}{2\alpha^3} = -2\,\delta \qquad [27d']$$

The difference between Equation [10] for Bay 1 and Equation [10] for Bay 2 is

$$|B|_{x,1} - |B|_{x,2} = |S|_{x,i} \times \quad |B|_{0,1} - |B|_{0,2} \qquad [29]$$

since under the assumptions of this section $|S|_{x,i}$ is the same for all bays. From Equations [27]

$$|B|_{0,1} - |B|_{0,2} = \begin{vmatrix} 0 \\ 0 \\ -(b\alpha)\delta \\ -2\delta \\ 0 \end{vmatrix} \qquad [30]$$

and, for $x = l_1$, Equation [29] becomes

$$|B|_{l_1,1} - |B|_{l_1,2} = |S|_{l_1,i} \times \begin{vmatrix} 0 \\ 0 \\ -(b\alpha)\delta \\ -2\delta \\ 0 \end{vmatrix} \qquad [31]$$

Now, since $|S|_{l_1,i}$ and $|B|_{l_1,2}$ are completely known from the geometry and Equations [11] and [5], respectively, and since there must exist two boundary relations on $|B|_{l_1,1}$,* Equation [31] represents four equations in the two unknown boundary conditions and the two parameters $l_1$ and $\delta$. Equation [28] defines $\delta$ as a function of the difference between the shear on one side of the first frame and that on the other side. This indicates that the area of the first frame

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*This subject is discussed in Appendix A.

must be altered to maintain the assumed conditions. Since the S&P frame is in equilibrium except for the load $\Delta V$, a corresponding $\Delta A_{F_1}$ is computed from the equilibrium expression

$$\Delta A_{F_1} = \frac{\Delta V R^2}{E y_2(0)} \qquad [32]$$

and Equations [5a] and [28] as

$$\Delta A_{F_1} = \frac{\delta \lambda}{\alpha y_u \left[1 - \dfrac{A_F}{H} f_1(0)\right]} \qquad [33]^*$$

assuming a solution to Equation [31] exists then Equation [31] and the two known boundary relations will completely determine the geometry necessary for the condition that the second and all subsequent bays will have identical stress patterns at the collapse pressure. This is developed in detail in Appendix A. A sample solution is presented in Appendix B.

To determine the strength of the first bay, Equation [29] is written, after substitution of Equation [30], as follows:

$$y_1(x) - y_2(x) = -\delta g_1(x) \qquad [34a]$$

$$\frac{y_1'(x) - y_2'(x)}{\alpha} = -\delta g_2(x) \qquad [34b]$$

$$\frac{y_1''(x) - y_2''(x)}{2\alpha^2} = -\delta g_3(x) \qquad [34c]$$

$$-\frac{y_1'''(x) - y_2'''(x)}{2\alpha^3} = -\delta g_4(x) \qquad [34d]$$

where

$$g_1(x) = \frac{\cosh K_1 x \sin K_2 x}{\sqrt{1+\alpha^2\beta^2}} - \frac{\sinh K_1 x \cos K_2 x}{\sqrt{1-\alpha^2\beta^2}} + (b\alpha)\frac{\sinh K_1 x \sin K_2 x}{\sqrt{1-\alpha^4\beta^4}} \qquad [35a]$$

---

*The actual additional area required will, of course, be given by $\left(\Delta A_{F_1}\right)\left(\dfrac{R_{F_1}}{R}\right)$

$$g_2(x) = \frac{g_1'(x)}{\alpha} = \frac{2 \sinh K_1 x \sin K_2 x}{\sqrt{1-\alpha^4\beta^4}} + (b\alpha)\left(\frac{\cosh K_1 x \sin K_2 x}{\sqrt{1+\alpha^2\beta^2}} + \frac{\sinh K_1 x \cos K_2 x}{\sqrt{1-\alpha^2\beta^2}}\right) \quad [35b]$$

$$g_3(x) = \frac{g_1''(x)}{2\alpha^2} = \frac{\cosh K_1 x \sin K_2 x}{\sqrt{1+\alpha^2\beta^2}} + \frac{\sinh K_1 x \cos K_2 x}{\sqrt{1-\alpha^2\beta^2}} + b\alpha\left(\cosh K_1 x \cos K_2 x - \frac{\alpha^2\beta^2}{\sqrt{1-\alpha^4\beta^4}} \sinh K_1 x \sin K_2 x\right) \quad [35c]$$

$$g_4(x) = \frac{g_1'''(x)}{2\alpha^3} = 2 \cosh K_1 x \cos K_2 x - \frac{2\alpha^2\beta^2}{\sqrt{1-\alpha^4\beta^4}} \sinh K_1 x \sin K_2 x + b\alpha\left(\frac{1-2\alpha^2\beta^2}{\sqrt{1-\alpha^2\beta^2}} \sinh K_1 x \cos K_2 x - \frac{1+2\alpha^2\beta^2}{\sqrt{1+\alpha^2\beta^2}} \cosh K_1 x \sin K_2 x\right) \quad [35d]$$

For cylinders of interest here, it will be true that $y_1(l_1) < y_2(l_1)$, $y_1''(l_1) > y_2''(l_1)$, and $l_1 < \frac{3.5}{K_2}$. If this is so, then $g_1(x) > 0$ for $0 < x \leq l_1$ and $g_3(x) > 0$ near the middle of the bay. Thus the maximum longitudinal stress near the middle of the first bay will be somewhat higher than that of a typical bay. However, the average and also the maximum circumferential membrane stresses of the first bay are less than those of all subsequent bays. Hence the first bay should withstand a higher pressure than the typical bays.

## DISCUSSION

The analysis of this report is, of course, limited to small deflections in axially symmetric, thin-walled cylinders. Therefore, cylinders with the ends designed by the method outlined here which fail by an axisymmetric shell yield should have the maximum obtainable strength. It is logical, also, to expect such cylinders which fail by shell instability to have at least as high a collapse pressure as can be obtained with any other end design.

In applying the analysis of this report, there are a few points to be remembered. This analysis, which is analogous to the Salerno and Pulos solution for the infinite stiffened cylinders, gives results which are nonlinear functions of pressure. Familiarity with the Salerno and Pulos solution should not be allowed to lead one to believe that this nonlinearity is negligible, since the variation in deflection near a rigid or nearly rigid bulkhead may easily be five or six times that in a bay remote from the bulkhead, thus greatly magnifying the "beam-column" effect. In addition, at pressures near collapse there will be a change of boundary conditions due to higher local bending stresses near the bulkhead which produce a "plastic hinge" at a pressure well below that which causes collapse of the cylinder. Both of these nonlinear effects are even more important where an outward negative deflection is expected, such as at the juncture of a cylinder with the large end of a cone. For these reasons caution must also be exercised in interpretation of experimental strain data from a cylinder designed by the methods presented here, since the data taken at low pressures will always show greater strains near the middle of the second bay than in any other bay. It is only when the pressure nears the collapse pressure that the strains in the second bay will approach those of subsequent bays.

## ACKNOWLEDGMENT

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## APPENDIX A

## BOUNDARY CONDITIONS AND SPECIFIC PROBLEMS

The usefulness of this analysis in design is, of course, limited by the ability of the designer to estimate boundary conditions. Although it is not necessary that the boundary condition be known absolutely, an attempt should be made to limit the sum of the absolute values of the errors in each of the pair of $y_1(l_1)$; $\frac{y_1'(l_1)}{\alpha}$; $\frac{y_1''(l_1)}{2\alpha^2}$; $\frac{y_1'''(l_1)}{2\alpha^3}$ which are used to 5 percent of $y_u$. Although it will usually be necessary for the designer to exercise judgment in selecting proper boundary values, a few more common cases will be discussed here. In all the examples that follow it will be assumed that the material is ductile and of the ideally plastic type. In other words, it has a stress-strain curve which can be approximated by a linear elastic curve terminating in a well-defined plateau-type yield.* It will be assumed that, for these cases, only $l_1$ and $A_{F_1}$ can be varied to attain the maximum collapse pressure.

### SYMMETRICAL HEAVY FRAME OR BULKHEAD

By a bulkhead is meant a structure for which it may be assumed that all deflections are zero, i. e., $y_1(l_1) = 0$. It is, therefore, only a special case of a heavy frame located so that it may be assumed to be symmetrically loaded. For the heavy frame there are two possibilities to consider; (1) a "plastic hinge" is formed in the shell at the frame, or (2) partial or no yielding occurs near the frame prior to collapse. Since in case (1) only local yielding will occur, it is assumed for the first case that the tangent to a generator will rotate at the bulkhead or heavy frame i. e., $y_1'(l_1) \neq 0$. For the second case it is assumed that no rotation occurs and $y_1'(l_1) = 0$.

For the first case, then, two conditions at the frame are known. The first, which comes from equilibrium (see Equation [13c]) and symmetry (i.e., equal deflections and shear forces on either side of the frame), is

$$-\frac{y_1'''(l_1)}{2\alpha^3} + b_0\alpha \frac{(A_F + bh)_0}{b_0 h} y_1(l_1) - b_0\alpha y_u - 2\alpha^2\beta^2 \frac{y_1'(l_1)}{\alpha} = 0 \qquad [36]$$

Here the subscript 0 refers to the heavy frame. The second condition is based on the maximum strength of the material at the frame which occurs under the assumed equilibrium stress distribution of Figure 4. It is

$$\frac{y_1''}{2\alpha^2}(l_1) = h\,\frac{\alpha^2\beta^2}{2}\left(-\frac{2h\sigma_1}{P_c R} + \frac{P_c R}{2h\sigma_1}\right)\left[1 - \left(\frac{1 + \frac{P_c R}{2h\sigma_1}}{1 - \frac{P_c R}{2h\sigma_1}}\right)\left(\frac{\sigma_1 + \sigma_2}{\sigma_1 - \sigma_2}\right)\right] \qquad [37]$$

*Other types of material are beyond the scope of this report.

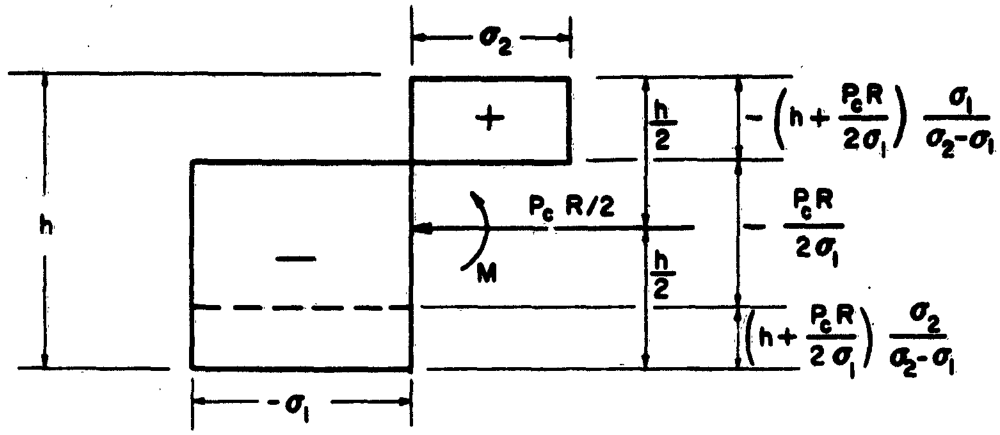


**Figure 4 – Assumed Stress Condition at Formation of Plastic Hinge**

If shear stresses are neglected, the Hencky-Von Mises criterion of yielding gives:

$$\sigma_1 = \frac{1}{2(1-\nu+\nu^2)}\left[-\frac{Ey_1(l_1)}{R}(1-2\nu) - \sqrt{4\sigma_y^2(1-\nu+\nu^2) - 3\left(\frac{Ey_1(l_1)}{R}\right)^2}\right] \quad [38a]$$

$$\sigma_2 = \frac{1}{2(1-\nu+\nu^2)}\left[-\frac{Ey_1(l_1)}{R}(1-2\nu) - \sqrt{4\sigma_y^2(1-\nu+\nu^2) - 3\left(\frac{Ey_1(l_1)}{R}\right)^2}\right] \quad [38b]$$

where $\sigma_y$ is the yield stress in a one-dimensional stress field (assumed to be the same in tension and compression). Since $\sigma_1$ and $\sigma_2$ are nonlinear functions of $y_1(l_1)$, it is easier to make a preliminary estimate of $y_1(l_1)$ and then to compute these stresses directly. Stresses so obtained will usually be sufficiently accurate.

Combining Equations [36], [34a], [34b], [34d], and [5] with $x = l_1$ gives:

$$y_u\left\{1-\frac{A_F}{H}f_1(l_1)+\frac{h}{\alpha(A_F+bh)_0}\left[\frac{A_F}{H}\left(2\alpha^2\beta^2 f_2(l_1)+f_4(l_1)\right)-b_0\alpha\right]\right\} =$$

$$\delta\left\{g_1(l_1)-\frac{h}{\alpha(A_F+bh)_0}\left[2\alpha^2\beta^2 g_2(l_1)+g_4(l_1)\right]\right\} \quad [39]$$

Combining Equation [37] with Equations [34c] and [5c] gives:

$$y_u\left\{\frac{\sqrt{3(1-\nu^2)}}{2(2-\nu)}\left(T-\frac{1}{T}\right)\left[1-\left(\frac{T-1}{T+1}\right)\left(\frac{\sigma_1+\sigma_2}{\sigma_1-\sigma_2}\right)\right]+\frac{A_F}{H}f_3(l_1)\right\} = -\delta g_3(l_1) \quad [40]$$

where $T = -2h\sigma_1/P_c R$. Elimination of $\delta$ from Equations [39] and [40] yields

$$\phi_1(l_1) = \frac{\sqrt{3(1-\nu^2)}}{2(2-\nu)}\left(T - \frac{1}{T}\right)\left[1 - \left(\frac{T-1}{T+1}\right)\left(\frac{\sigma_1+\sigma_2}{\sigma_1-\sigma_2}\right)\right]$$

$$+\left\{1 + \frac{h}{\alpha(A_F + bh)_0}\left[\frac{A_F}{H}\left(2\alpha^2\beta^2 f_2(l_1) + f_4(l_1)\right) - b_0\alpha\right] - \frac{A_F}{H} f_1(l_1)\right\}$$

$$\left[\frac{g_3(l_1)}{g_1(l_1) - \frac{h}{\alpha(A_F+bh)_0}\left(2\alpha^2\beta^2 g_2(l_1) + g_4(l_1)\right)}\right] + \frac{A_F}{H} f_3(l_1) = 0 \qquad [41]$$

The smallest value of $l_1$ which satisfies Equation [41] is the desired solution. Since for $l_1$ near $l$, $\phi_1$ is an approximately linearly decreasing function of $l_1$, Equation [41] can usually be solved most easily as follows:

Step 1: Compute $\phi_1(l)$.

Step 2: If $\phi_1(l) < 0$; compute $\phi_1(0.9\,l)$.
If $\phi_1(l) > 0$; compute $\phi_1(1.1\,l)$.

Step 3: Compute $l_1$ by a linear interpolation.*

If $\phi_1(l) < 0$, then

$$l_1 = \left[1 - \frac{0.1\,\phi_1(l)}{\phi_1(l) - \phi_1(0.9\,l)}\right] l < l \qquad [42a]$$

If $\phi_1(l) > 0$, then

$$l_1 = \left[1 + \frac{0.1\,\phi_1(l)}{\phi_1(l) - \phi_1(1.1\,l)}\right] l > l \qquad [42b]$$

This process may, of course, be repeated for more accuracy. For the case of partial or no yielding at the heavy frame, we assume $y_1'(l_1) = 0$. It follows from Equation [34b] that Equation [40] can be replaced by

$$y_u \frac{A_F}{H} f_2(l_1) = -\delta g_2(l_1) \qquad [43]$$

*More accuracy may be obtained by computing an intermediate value of $\phi_1$ and passing a circle through the three values graphically.

Then the expression analogous to Equation [41] is

$$\phi_2(l_1) = \left\{1 + \frac{h}{\alpha(A_F + bh)_0}\left[\frac{A_F}{H} f_4(l_1) - b_0\alpha\right] - \frac{A_F}{H} f_1(l_1)\right\}$$

$$\left[\frac{g_2(l_1)}{g_1(l_1) - \frac{h}{\alpha(A + bh)_0} g_4(l_1)}\right] + \frac{A_F}{H} f_2(l_1) = 0 \qquad [44]$$

The solution is by the same method as that used for Equation [41]. The correct value of $l_1$ is then, of course, the smaller of the two values satisfying Equations [41] and [44]. It is obvious that Equation [41] will usually apply to bulkheads and very heavy frames, while Equation [44] will apply to moderately heavy frames. The transition point will vary depending on the geometry of the cylinders and the lighter-frames. Having $l_1$, $\Delta A_{F_1}$ is obtained from Equations [33] and [39] as:

$$\Delta A_{F_1} = \frac{h}{\alpha}\left\{1 - \frac{A_F}{H} f_1(l_1) + \frac{h}{\alpha(A_F + bh)_0}\left[\frac{A_F}{H}\left(2\alpha^2\beta^2 f_2(l_1) + f_4(l_1)\right) - b_0\alpha\right]\right\}$$

$$\left\{\left[1 - \frac{A_F}{H} f_1(0)\right]\left[g_1(l_1) - \frac{h}{\alpha(A_F + bh)_0}\left(2\alpha^2\beta^2 g_2(l_1) + g_4(l_1)\right)\right]\right\}^{-1} \qquad [45]$$

For the rigid bulkhead $\frac{h}{\alpha(A_F + bh)_0}$ is allowed to go to zero in Equations [41] and [45]. Sometimes, where a nearly rigid bulkhead is present, the actual radial deflection of the bulkhead may be known from previous experimental data more accurately than an effective area can be computed. Then it is simply a matter of replacing

$$\frac{h}{\alpha(A_F + bh)_0}\left\{y_u\left[b_0\alpha - \frac{A_F}{h}\left(2\alpha^2\beta^2 f_2(l_1) + f_4(l_1)\right)\right] - \delta\left[2\alpha^2\beta^2 g_2(l_1) + g_4(l_1)\right]\right\}$$

by $y_1(l_1)$ in Equation [39]. Then an equation analogous to Equation [41] results:

$$\phi_3(l_1) = \frac{\sqrt{3(1-\nu^2)}}{2(2-\nu)}\left(T - \frac{1}{T}\right)\left[1 - \left(\frac{T-1}{T+1}\right)\left(\frac{\sigma_1 + \sigma_2}{\sigma_1 - \sigma_2}\right)\right] + \frac{g_3(l_1)}{g_1(l_1)}\left[1 - \frac{A_F}{H} f_1(l_1) - \frac{y_1(l_1)}{y_u}\right]$$

$$+ \frac{A_F}{H} f_3(l_1) = 0 \qquad [46]$$

and Equation [45] is replaced by

$$\Delta A_{F_1} = \frac{h}{\alpha}\left[1 - \frac{A_F}{H} f_1(l_1) - \frac{y_1(l_1)}{y_u}\right]\left\{\left[1 - \frac{A_F}{H} f_1(0)\right] g_1(l_1)\right\}^{-1} \qquad [47]$$

## INTERSECTIONS

At an axisymmetric intersection of a cylinder with another shell such as a transition cone or an ellipsoidal or hemispherical cap, the simplest method of design is to estimate the deflection at the intersection by some approximate analysis or to compare the proposed structure with available experimental results. Then Equations [46] and [47] can be used, or, for relatively large positive (inward) deflections, Equation [46] should be replaced by a modified Equation [44].

$$\phi_4 (l_1) - \frac{g_2 (l_1)}{g_1 (l_1)} \left[ 1 - \frac{A_F}{H} f_1 (l_1) - \frac{y_1 (l_1)}{y_u} \right] + \frac{A_F}{H} f_2 (l_1) = 0 \qquad [48]$$

Of course, if there is sufficient knowledge of the contiguous shell near the intersection at high pressure, a more detailed analysis can be made from conditions of continuity and equilibrium at the intersection. However, this will usually involve more work than is practical.

## APPENDIX B

## NUMERICAL EXAMPLE

A numerical example is provided to illustrate the procedure for designing an end bay and frame. The pertinent scantlings and material properties are

| | |
|---|---|
| $R = 8.4180$ in. | $R_F = 8.8464$ in. |
| $h = 0.0858$ in. | $\nu = 0.30$ |
| $l = 1.3217$ in. | $E = 3 \times 10^7$ psi |
| $b = 0.0443$ in. | $\sigma_y = 66{,}400$ psi |
| $A_F = 0.0516$ in. | |

The cylinder is closed by a rigid bulkhead $[y_1(l_1) = 0]$, and $P_c$ is estimated to be 1000 psi. The computation is facilitated by use of an instruction sheet. The first column of the instruction sheet contains the item number in parentheses. The second column indicates the operation, the result of which is to be recorded in the third column. Numbers in parentheses in the second column are item numbers and indicate that the operation is to be performed on the quantity recorded in that item, e.g., (2) (34) means multiply the quantity in Item 2 by the quantity in Item 34, whereas 2(34) means multiply the number 2 by the quantity in Item 34. The first quantity in Item 84 is always $\pm$ 0.1 depending upon the sign of $\phi_3(l)$, Item 83. The subsequent values are entered from the previous interpolation in Item 121. The last value in Item 86 is the correct value of $l_1$, and Item 126 contains the ratio $A_{F_1} / A_F$ . Thus $L_1$, the distance from the bulkhead to the center of the first frame, is obtained from $l_1$ by adding $b/2$, and $A_{F_1}$ is obtained by multiplying $A_F$ by Item 126. In the example which follows the interpolation was carried out three times in order to demonstrate that the first interpolation is sufficiently accurate for design purposes.

## End Bay Design Calculation Varying $L_1$ and $A_{F_1}$

| Item | Operation | Result | Item | Operation | Result |
|---|---|---|---|---|---|
| (1)* | $R$ | 8.4180 | (44) | (15)(43)/(7) | 0.265078 |
| (2)* | $h$ | 0.0858 | (45) | 1−(44) | 0.734922 |
| (3)* | $l$ | 1.3217 | (46) | 1+(44) | 1.265078 |
| (4)* | $b$ | 0.0443 | (47) | $\sqrt{(45)}$ | 0.857276 |
| (5)* | $A_F \frac{R}{R_F}$ | 0.0491 | (48) | $\sqrt{(46)}$ | 1.124757 |
| (6)* | $\nu$ | 0.3 | (49) | (47)(48) | 0.964227 |
| (7)* | $E$ | $3\times 10^7$ | (50) | (3)(42) | 1.99906 |
| (8)* | $\sigma_y$ | 66,400 | (51) | (47)(50) | 1.7137 |
| (9)* | $P_c$ | 1,000 | (52) | (48)(50) | 2.2485 |
| (10)* | $y_1(l_1)$ | 0 | (53) | cosh (51) | 2.86483 |
| (11) | $(6)^2$ | 0.09 | (54) | sinh (51) | 2.68463 |
| (12) | 1−(6)+(11) | 0.79 | (55) | cos (52) | −0.62701 |
| (13) | 1−2(6) | 0.4 | (56) | sin (52) | 0.77901 |
| (14) | 3−3(11) | 2.73 | (57) | (48)(54) | 3.01969 |
| (15) | $\sqrt{(14)}$ | 1.65227 | (58) | (47)(56) | 0.66785 |
| (16) | 2−(6) | 1.7 | (59) | (57)+(58) | 3.68754 |
| (17) | (7)(10)/(1) | 0 | (60) | (58)−(57) | −2.35184 |
| (18) | $(17)^2$ | 0 | (61) | (53)−(55) | 3.49191 |
| (19) | (13)(17)/(12) | 0 | (62) | (49)(61) | 3.36699 |
| (20) | $(8)^2$ | $44.0896\times 10^8$ | (63) | (2)(4) | 0.0038 |
| (21) | (12)(20) | $34.8308\times 10^8$ | (64) | (5)+(63) | 0.0529 |
| (22) | 4(21)−3(18) | $139.3232\times 10^8$ | (65) | 2(2)/(42) | 0.11346 |
| (23) | $\sqrt{(22)}$ | $11.8035\times 10^4$ | (66) | (59)(64)+(65)(62) | 0.57709 |
| (24) | (23)/(12) | $14.9411\times 10^4$ | (67) | (5)/(66) | 0.085082 |
| (25) | (24)+(19) | $14.9411\times 10^4$ | (68) | (4)(42) | 0.067003 |
| (26) | (1)/(2) | 98.112 | (69) | (53)(56)/(48) | 1.98434 |
| (27) | (9)(26) | $9.8112\times 10^4$ | (70) | (54)(55)/(47) | −1.96349 |
| (28) | (25)/(27) | 1.52286 | (71) | (53)(55) | −1.79623 |
| (29) | (28)−1 | 0.52286 | (72) | (54)(56) | 2.09153 |
| (30) | (28)+1 | 2.52286 | (73) | (68)(72)/(49) | 0.14534 |
| (31) | (29)(30)/(28) | 0.86620 | (74) | (44)(72)/(47) | 0.64672 |
| (32) | (19)/(24) | 0 | (75) | (71)−(74) | −2.44295 |
| (33) | (29)(32)/(30) | 0 | (76) | (68)(75) | −0.16368 |
| (34) | 1−(33) | 1 | (77) | (69)−(70)+(73) | 4.09317 |
| (35) | (31)(34) | 0.86620 | (78) | (69)+(70)+(76) | −0.14283 |
| (36) | (15)/(16) | 0.97192 | (79) | (59)(67) | 0.31374 |
| (37) | (35)(36)/2 | 0.42094 | (80) | (60)(67) | −0.20010 |
| (38) | (17)/(27) | 0 | (81) | 1−(39)−(79) | 0.68626 |
| (39) | 2(38)/(16) | 0 | (82) | (78)(81)/(77) | −0.02395 |
| (40) | (1)(2) | 0.722264 | (83) | (37)+(80)+(82) | 0.19689 |
| (41) | (15)/(40) | 2.28763 | | | |
| (42) | $\sqrt{(41)}$ | 1.51249 | | | |
| (43) | (26)(27)/2 | $4.81298\times 10^6$ | | | |

*Data

| Item | Operation | | Results | | |
|---|---|---|---|---|---|
| (84) | (121) | -0.1 | -0.06974 | -0.07035 | -0.07033 |
| (85) | 1-(84) | 1.1 | 1.06974 | 1.07035 | 1.07033 |
| (86) | (3) (85) | - | 1.4139 | 1.4147 | 1.4147 |
| (87) | (51) (85) | 1.8851 | 1.8332 | 1.8343 | 1.8342 |
| (88) | (51) (84) | -0.1714 | -0.1195 | -0.1206 | -0.1205 |
| (89) | (52) (85) | 2.4734 | 2.4053 | 2.4067 | 2.4066 |
| (90) | (52) (84) | -0.2249 | -0.1568 | -0.1582 | -0.1581 |
| (91) | cosh (87) | 3.3694 | 3.2069 | 3.2102 | 3.2099 |
| (92) | sinh (87) | 3.2176 | 3.0470 | 3.0505 | 3.0502 |
| (93) | cosh (88) | 1.01472 | 1.00715 | 1.00728 | 1.00727 |
| (94) | sinh (88) | -0.17224 | -0.11978 | -0.12089 | -0.12079 |
| (95) | cos (89) | -0.78494 | -0.74096 | -0.74190 | -0.74183 |
| (96) | sin (89) | 0.61957 | 0.67155 | 0.67051 | 0.67059 |
| (97) | cos (90) | 0.97482 | 0.98773 | 0.98751 | 0.98753 |
| (98) | sin (90) | -0.22301 | -0.15616 | -0.15754 | -0.15744 |
| (99) | (91) (98)+(93) (96) | -0.12272 | 0.17556 | 0.16966 | 0.17010 |
| (100) | (92) (97)+(94) (95) | 3.27178 | 3.09837 | 3.10209 | 3.10177 |
| (101) | (91) (96)/(48) | 1.85602 | 1.91471 | 1.91372 | 1.91377 |
| (102) | (92) (95)/(47) | -2.94610 | -2.63358 | -2.63995 | -2.63944 |
| (103) | (91) (95) | -2.64478 | -2.37618 | -2.38165 | -2.38120 |
| (104) | (92) (96) | 1.99353 | 2.04621 | 2.04539 | 2.04543 |
| (105) | (68) (104)/(49) | 0.13853 | 0.14219 | 0.14213 | 0.14213 |
| (106) | (44) (104)/(47) | 0.61642 | 0.63271 | 0.63245 | - |
| (107) | (103)-(106) | -3.26120 | -3.00889 | -3.01410 | - |
| (108) | (68) (107) | -0.21851 | -0.20160 | -0.20195 | - |
| (109) | (47) (99) | -0.10520 | 0.15050 | 0.14545 | 0.14582 |
| (110) | (48) (100) | 3.67996 | 3.48491 | 3.48910 | 3.48874 |
| (111) | (109)+(110) | 3.57476 | 3.63541 | 3.63455 | 3.63456 |
| (112) | (109)-(110) | -3.78516 | -3.33441 | -3.34365 | - |
| (113) | (101)-(102)+(105) | 4.94065 | 4.69048 | 4.69580 | 4.69534 |
| (114) | (101)+(102)+(108) | -1.30859 | -0.92047 | -0.92818 | - |
| (115) | (67) (111) | 0.30415 | 0.30931 | 0.30923 | 0.30924 |
| (116) | (67) (112) | -0.32205 | -0.28370 | -0.28446 | - |
| (117) | 1-(39)-(115) | 0.69585 | 0.69069 | 0.69077 | 0.69076 |
| (118) | (114) (117)/(113) | -0.18430 | -0.13554 | -0.13654 | - |
| (119) | (37)+(116)+(118) | -0.08541 | 0.00170 | -0.00006 | - |
| (120) | (83)-(119) | 0.28230 | 0.19519 | 0.19695 | - |
| (121) | (84) (83)/(120) | -0.06974 | -0.07035 | -0.07033 | - |
| (122) | 1-(79) | - | 0.68626 | 0.68626 | 0.68626 |
| (123) | (122) (113) | - | 3.21889 | 3.22254 | 3.22222 |
| (124) | (65)/2 | - | 0.05673 | 0.05673 | 0.05673 |
| (125) | (117) (124)/(123) | - | 0.01217 | 0.01216 | 0.01216 |
| (126) | 1+(125)/5 | - | 1.248 | 1.248 | 1.248 |

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