STABILITY OF A PRISMATIC BAR UNDER THE INFLUENCE OF A COMPRESSIONAL STRAIN WAVE

by

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In this paper a study is made of critical strain distributions introduced into a bar by dynamical loading in an axial direction. Transverse vibrations of the bar which may result from this type of loading are not investigated. It is considered that they are really involved in an extension of the problem.

The theory is developed in some detail for cases of bars of elastic material having Hookean or Non-Hookean stress-strain laws. Also, consideration is given to the case of the bar in the plastically deformed state.

One of the conclusions from the study is that the proposed method for determining critical states of strain suggests a means for experimentally investigating various plastic wave theories. Assuming tentatively a given theory of plastic strain propagation into the bar the critical strain distribution may be calculated. Indications of plastic buckling may then be investigated. The buckled shapes should also provide information on the theory of wave propagation.

INTRODUCTION

There is a growing interest in the problem of structures subjected to dynamic loads of the transient type. These loads are sometimes produced by collision or explosion. The column is an important structural element that may be subjected to this kind of force. The case of periodic axial loads of the steady-state vibration type has been considered by Stoker and Lubkin (1). The column compressed by the moving head of a testing machine has been treated by Hoff (2). In these cases the load is considered to vary with time but to be the same at all sections of the column at any one time. In other words, waves of compressional strain are not considered. In the present paper a study will be made of the compressional strain waves.

DIFFERENTIAL EQUATIONS OF INSTABILITY

If the stress is not proportional to strain we may define the one-dimensional strain in the manner of von Kármán (3):

$$\frac{\partial \sigma}{\partial \epsilon} \cdot \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots [1]$$

If stress is proportional to strain the one-dimensional wave equation may be written in the usual manner:

$$E \cdot \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots [2]$$

With given initial and boundary conditions for a bar these equations may be solved. At any time t a determinable stress distribution σ_x exists in the bar.

As can easily be shown the differential equation for the transverse vibration of a longitudinally loaded bar of variable stiffness per unit length may be written:

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 w}{\partial x^2}\right) + \frac{\partial}{\partial x}\left(P \frac{\partial w}{\partial x}\right) + \rho A \frac{\partial^2 w}{\partial t^2} = 0 \quad \ldots\ldots [3]$$

If we consider [1] or [2] and [3] simultaneously we define possible transverse displacements induced by the compressional strain wave. In order to obtain a first criterion of instability it is proposed to neglect the acceleration term in [3] and determine P which is a function of x and of t from [1] or [2]. The merit of these assumptions will be investigated in a series of experiments at a later date.

The D'Alembert principle can be used and the $\rho \frac{\partial^2 u}{\partial t^2}$ considered as reversed effective forces putting the bar in statical equilibrium along its length. This principle provides a means of determining the load distribution along the bar or column at any time. We may then consider a force at the wave front, a force at the externally loaded end, and a distribution of forces between these two points. The force corresponding to each differential length is $\rho A dx \frac{\partial^2 u}{\partial t^2}$. This theory will be illustrated by three cases.

CASE I

ELASTIC CONDITION, HOOKEAN STRESS-STRAIN LAW

Assume a straight bar initially unstressed and with certain given boundary conditions. The material of the bar is elastic with a linear stress-strain law. A compressive load as a known function of time is suddenly applied at one end. If numerical methods of solution are to be avoided, and they would be quite clumsy in a problem of this type, analytical solutions must be obtained. Many of these can be found in the literature. The simplest will now be considered (4).

In Fig. 1 a simply supported bar of length l is shown with a force P applied at one end and the front of the strain wave progressed to a distance x_0 from that end. Assume that the force P is caused by a large mass M moving with constant velocity v_0. Then the strain ϵ_x will be constant and of magnitude given by

$$\epsilon_x = \epsilon_0 = \frac{v_0}{c}$$

where c is the constant velocity with which the strain moves into the bar (5).

The principle of D'Alembert will now enable us to translate this problem into an equivalent static one for any given time t. The corresponding loading for this case is then shown in Fig. 2.

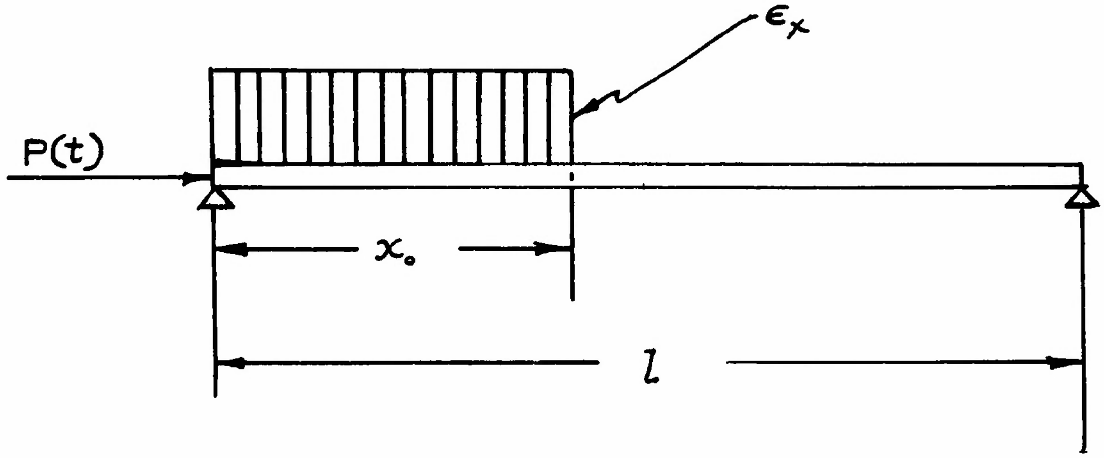

Fig. 1. Simple strain wave in bar

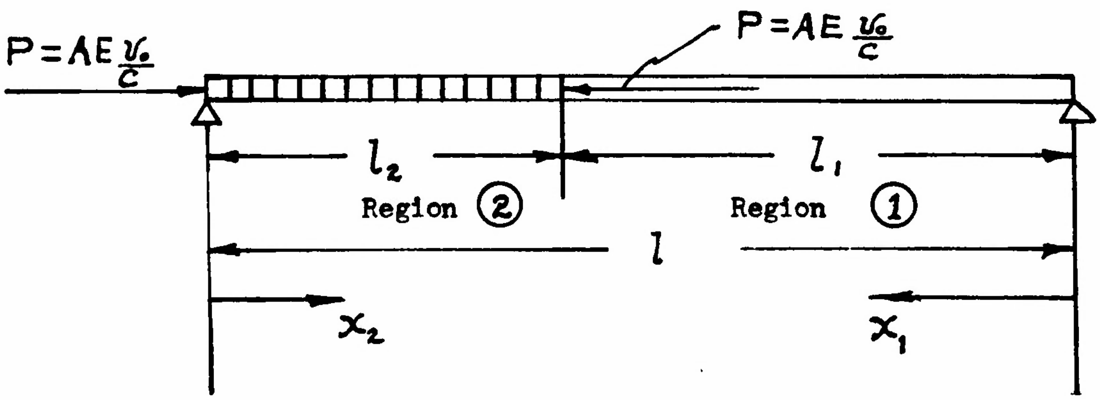

Fig. 2. Equivalent loading

In region ① there is no strain since the strain wave has not progressed into that portion of the bar as yet. In Region ②, there is constant strain of magnitude $\frac{v_o}{c}$ at each point and loads of magnitude $AE\frac{v_o}{c}$ at each of its extremities.

The two differential equations for bending of the bar are:

$$EI\frac{\partial^4 w}{\partial x^4} = 0 \qquad \text{Region ①} \ldots\ldots [4]$$

$$EI\frac{\partial^4 w}{\partial x^4} + P\frac{\partial^2 w}{\partial x^2} = 0 \qquad \text{Region ②} \ldots\ldots [5]$$

Solution of [4] is:

$$w_1 = A_1 x^3 + B_1 x^2 + C_1 x + D$$

and solution of [5] is

$$w_2 = A_2 + B_2 x + C_2 \sin\lambda x + D_2 \cos\lambda x$$

The Boundary and continuity conditions are:

$$\left.\begin{array}{llll} W = 0, & \frac{\partial^2 w}{\partial x^2} = 0 & \text{for} & x_1 = 0 \\ W = 0, & \frac{\partial^2 w}{\partial x^2} = 0 & \text{for} & x_2 = 0 \end{array}\right. \quad \text{-----} [6]$$

$$\left.\begin{array}{l} W_1 = W_2 \\ \frac{\partial w_1}{\partial x} = -\frac{\partial w_2}{\partial x} \\ \frac{\partial^2 w_1}{\partial x^2} = \frac{\partial^2 w_2}{\partial x^2} \\ \frac{\partial^3 w_1}{\partial x^3} = -\frac{\partial^3 w_2}{\partial x^3} \end{array}\right\} \text{for } \begin{array}{l} x_1 = \ell_1 \\ x_2 = \ell_2 \end{array} \quad \text{-----} [7]$$

The vanishing of the determinant of the system of the 8 linear equations in the 8 constants A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2 gives the instability equation:

$$\ell_1 \lambda \cos \lambda \ell_2 + \sin \lambda \ell_2 = 0 \qquad \ldots\ldots\ldots\ldots [8]$$

For convenience we may put

$$\ell_1 + \ell_2 = \ell, \quad \ell_2 = f\ell, \quad \text{and} \quad \lambda\ell = \beta$$

then equation [8] becomes:

$$(1-f)\beta \cos \beta f + \sin \beta f = 0 \qquad \ldots\ldots\ldots\ldots [9]$$

By means of equation [9] the following table of critical loads is determined.

TABLE I.

f	β	$\beta^2 = \frac{P_{cr}\ell^2}{EI}$
0.	∞	∞
.1	16.35	268.3
.2	8.55	73.0
.3	6.01	36.1
.4	4.76	22.7
.5	4.06	16.4
.6	3.62	13.1
.7	3.36	11.2
.8	3.21	10.3
.9	3.15	9.9
1.0	3.14	9.9

$$P_{cr} = \beta^2 \frac{EI}{\ell^2} \qquad \ldots\ldots\ldots\ldots [10]$$

It is readily seen that a relationship has been established between the fractional length factor f and the critical load P_{cr} given by [10]. If reflections of the strain wave occur before the critical stress is attained, the process must be repeated with appropriate strain distributions and another table of f values.

CASE II

ELASTIC CONDITION, NON-HOOKEAN STRESS-STRAIN LAW

In thecase of bars made of non-Hookean materials the stress-strain law is non-

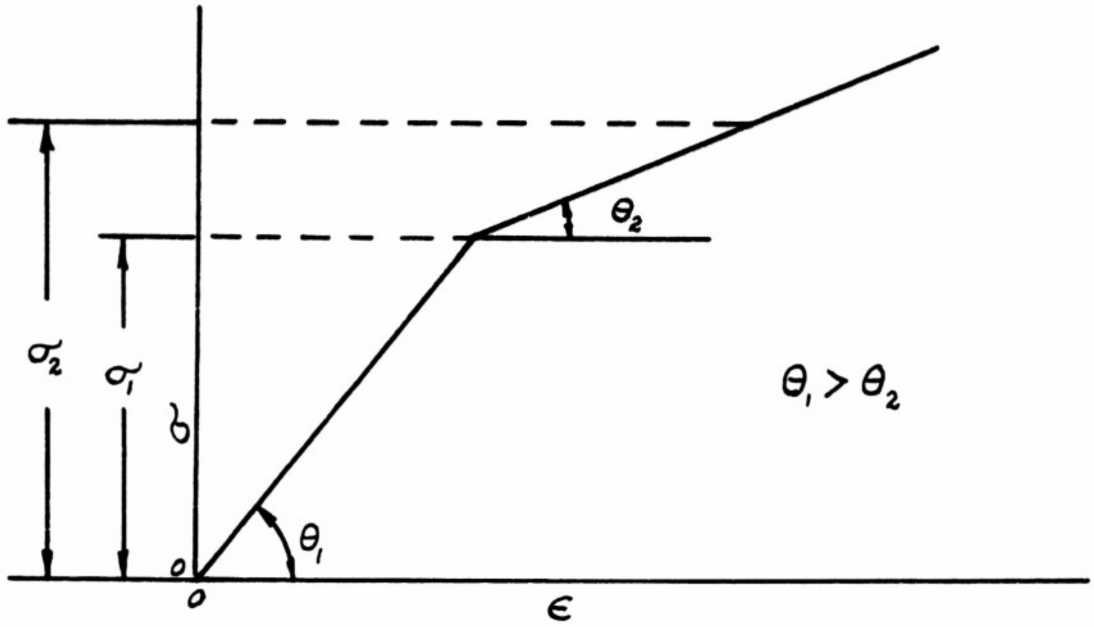

Fig. 3. Stress-strain law as two straight line segments

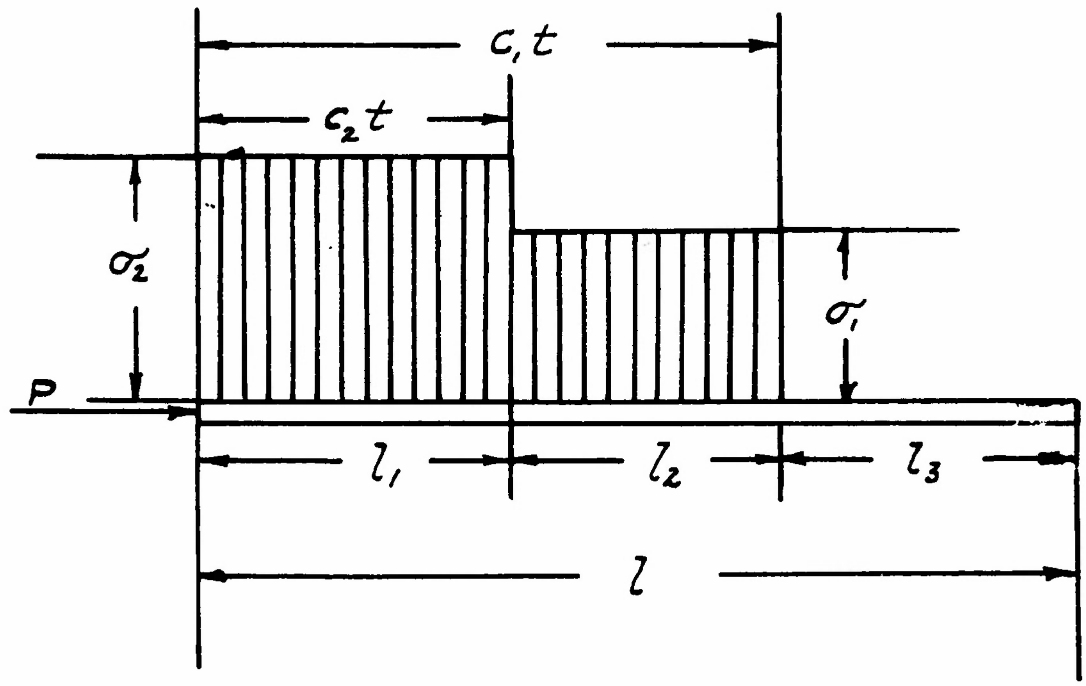

Fig. 4. Strain distribution in bar

linear, but the materials may still be elastic. A simple example of such a stress-strain relation is shown in Fig. 3.

As an example of this case suppose a force is applied at one end of a straight bar so that the load is constant in time and produces a stress, at the end of the bar, which lies between σ_1 and σ_2 in Fig. 3. In this case a wave of stress of magnitude σ_1 propagates at a known speed into the bar. This wave is followed by a slower one of magnitude σ_2 as shown in Fig. 4. In Fig. 5 the equivalent static loading at some time t is shown.

In region (1) the load is a force (P_1+P_2) in region (2) the load is P_1 and in region (3) there is no load as the wave front has not penetrated this region as yet. This strain propagation satisfies the differential equation [1] . The differential equations of lateral bending are:

$$\left.\begin{aligned} EI\frac{\partial^4 w}{\partial x^4} + (P_1+P_2)\frac{\partial^2 w}{\partial x^2} &= 0 && \text{Region (1)} \\ EI\frac{\partial^4 w}{\partial x^4} + P_1\frac{\partial^2 w}{\partial x^2} &= 0 && \text{Region (2)} \\ EI\frac{\partial^4 w}{\partial x^4} &= 0 && \text{Region (3)} \end{aligned}\right\} \quad \ldots [8]$$

The solutions are:

$$\left.\begin{aligned} W_1 &= A_1 + B_1 x + C_1 \sin\lambda_1 x + D_1 \cos\lambda_1 x \\ W_2 &= A_2 + B_2 x + C_2 \sin\lambda_2 x + D_2 \cos\lambda_2 x \\ W_3 &= A_3 x^3 + B_3 x^2 + C_3 x + D \end{aligned}\right\} \quad \cdots\cdots [9]$$

The boundary and continuity conditions are:

$$\left.\begin{aligned} W = 0, \quad \frac{\partial^2 W}{\partial x^2} = 0 \quad \text{for } x_1 = 0 \\ W = 0, \quad \frac{\partial^2 W}{\partial x^2} = 0 \quad \text{for } x_3 = 0 \end{aligned}\right\} \quad \cdots [10]$$

and

$$\left.\begin{aligned} W_1\Big]_{x=\ell_1} &= W_2\Big]_{x=\ell_2} \\ \frac{\partial W_1}{\partial x}\Big]_{x=\ell_1} &= -\frac{\partial W_2}{\partial x}\Big]_{x=\ell_2} \\ E_2 \frac{\partial^2 W_1}{\partial x^2}\Big]_{x=\ell_1} &= E_1 \frac{\partial^2 W_2}{\partial x^2}\Big]_{x=\ell_2} \\ E_2 \frac{\partial^3 W_1}{\partial x^3}\Big]_{x=\ell_1} &= -E_1 \frac{\partial^3 W_2}{\partial x^3}\Big]_{x=\ell_2} \\ W_3\Big]_{x_3=\ell_3} &= W_2\Big]_{x_2=0} \\ \frac{\partial W_3}{\partial x}\Big]_{x_3=\ell_3} &= \frac{\partial W_2}{\partial x^2}\Big]_{x_2=0} \\ \frac{\partial^2 W_3}{\partial x^2}\Big]_{x_3=\ell_3} &= \frac{\partial^2 W_2}{\partial x^2}\Big]_{x_2=0} \\ \frac{\partial^3 W_3}{\partial x^3}\Big]_{x_3=\ell_3} &= \frac{\partial^3 W_2}{\partial x^3}\Big]_{x_2=0} \end{aligned}\right\} \quad \cdots\cdots [11]$$

The venishing of the determinant of the system of 12 lineer homogeneous equations in the 12 constents gives the instability equation:

$$\lambda_2^2 \left(\lambda_1^2 \lambda_2^3 \sin \lambda_1 \ell_1 \cos \lambda_2 \ell_2 + \lambda_1^3 \lambda_2^2 \sin \lambda_2 \ell_2 \cos \lambda_1 \ell_1 \right)$$

$$+ \ell_3 \lambda_2^3 \left(- \lambda_1^2 \lambda_2^3 \sin \lambda_1 \ell_1 \sin \lambda_2 \ell_2 + \lambda_1^3 \lambda_2^2 \cos \lambda_1 \ell_1 \cos \lambda_2 \ell_2 \right)$$

$$= 0 \quad \text{. . . . .} \quad [12]$$

We may divide equation [12] by $\lambda_1^2 \lambda_2^2$ and determine the roots for various ratios σ_2 / σ_1 and E_2 / E_1. As an example suppose:

$$\sigma_2 / \sigma_1 = 2 \quad \text{and} \quad E_2 / E_1 = 1/2$$

Then we may write as follows:

$$c_1 t = \ell_1 + \ell_2 , \quad c_2 t = \ell_1$$

$$c_2 = \sqrt{\frac{E_2}{\rho}} , \quad c_1 = \sqrt{\frac{E_1}{\rho}}$$

$$c_1 / c_2 = \frac{\ell_1 + \ell_2}{\ell_1} = \sqrt{\frac{E_1}{E_2}} = 1 + \frac{\ell_2}{\ell_1}$$

so $$\frac{l_2}{l_1} = \sqrt{\frac{E_1}{E_2}} - 1$$

$$l_1 + l_2 + l_3 = l \quad \text{and so} \quad l_3 = l - l_1 - l_2$$

Then $$l_2 = \left[\sqrt{\frac{E_1}{E_2}} - 1\right] l_1$$

$$l_3 = l - l_1 - \left[\sqrt{\frac{E_1}{E_2}} - 1\right] l_1$$

Then let $l_1 = f l$ where f is a fractional parameter.

so $$l_2 = \left[\sqrt{\frac{E_1}{E_2}} - 1\right] f l$$

$$l_3 = \left\{1 - f - \left(\sqrt{\frac{E_1}{E_2}} - 1\right) f\right\} l$$

For the assumed ratios E_1/E_2 and σ_1/σ_2 we have

$$P_1 + P_2 = 2P_1$$

$$\lambda_1^2 = 4\frac{P_1}{E_1}$$

$$\lambda_2^2 = \frac{P_1}{E_1}$$

so

$$\lambda_1 = 2\lambda_2$$

and

$$l_1 = f l, \quad l_2 = [\sqrt{2} - 1] f l,$$

$$l_3 = \{1 - \sqrt{2}\, f\} l$$

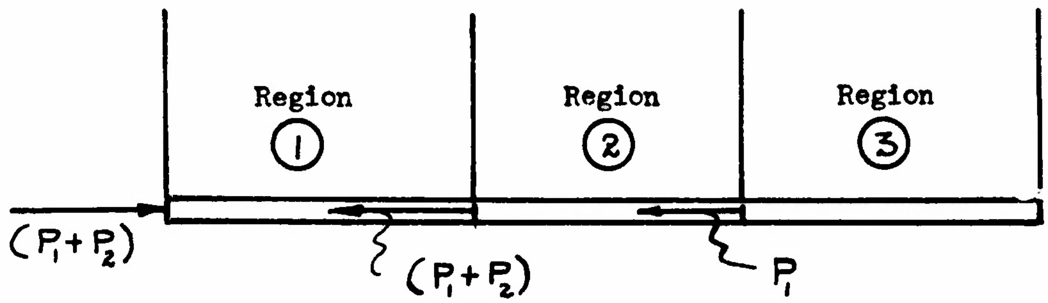

Fig. 5. Equivalent static loading

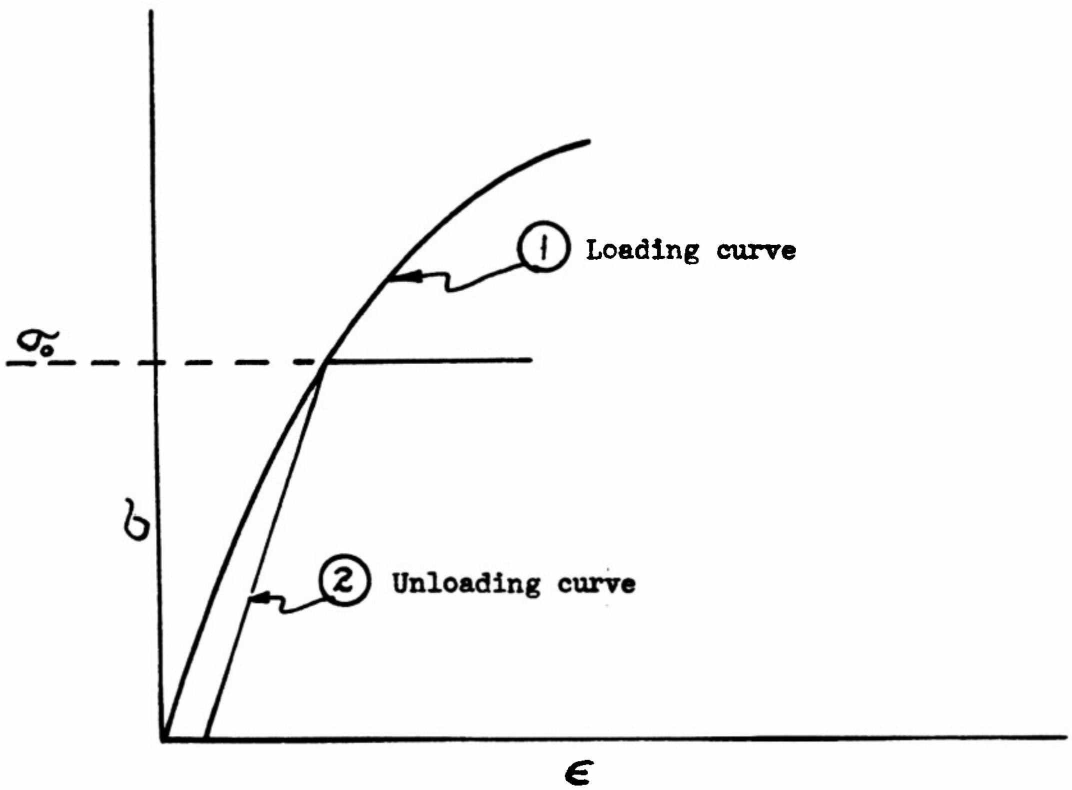

Fig. 6. Non-linear stress-strain curve of strain hardenable material

Obviously the maximum allowable f in this formulation is given by

$$1 - \sqrt{2}\, f = 0$$

Then we may construct a table of values relating critical loads to lengths.

TABLE II

f	$\lambda_2 \ell$	$(\lambda_2 \ell)^2 = \frac{P_1 \ell^2}{E_1 I}$	$\lambda_1 \ell = 2\lambda_2 \ell$	$(\lambda_1 \ell)^2 = \frac{(P_1+P_2)\ell^2}{E_2 I}$
.1	7.48	55.90	14.96	223.6
.2	3.94	15.40	7.88	61.6
.3	2.78	7.70	5.56	30.8
.4	2.24	5.00	4.48	20.0
.5	1.94	3.75	3.88	15.0
.6	1.76	3.09	3.52	12.4
.7	1.65	2.71	3.30	10.8
.7071	1.64	2.68	3.28	10.7

Obviously for any given force P, the critical fractional length f is determined. The shape of the bent bar is given by equations [9] . Again in this case if reflections of the strain wave occur before the critical stress is attained the process must be repeated with appropriate strain distributions and another table of f values.

CASE III

PLASTIC CONDITION

The differential equation of the strain in bar in the plastic region exhibiting unloading phenomenon may be written as noted by T. von Kármán

$$\frac{\partial \sigma}{\partial \epsilon} \cdot \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

where $\frac{\partial \sigma}{\partial \epsilon}$ is the slope of the stress strain curve and ρ is the density.

It is claimed that the velocity of such a plastic wave is

$$c = \sqrt{\frac{1}{\rho} \frac{\partial \sigma}{\partial \epsilon}}$$

and the static engineering stress-strain curve for the material may be used to determine $\frac{\partial \sigma}{\partial \epsilon}$ for any given ϵ.

As an example suppose that a bar shown in Fig. 1 is subjected to a constant stress σ_o at one end. Then it is asserted that a plastic wave of intensity σ_o progresses into the bar at velocity

$$c = \sqrt{\frac{1}{\rho} \frac{\partial \sigma}{\partial \epsilon}}$$

In order to determine instability in this case for a material exhibiting strain-hardening, we may use the method of Engesser (6) to take account of unloading.

Recalling the method of Engesser an effective modulus is determined for the bar section as in Fig. 7.

$$\frac{E_2 h_2}{\rho^*} \cdot \frac{h_2}{2} = \frac{E_1 h_1}{\rho^*} \cdot \frac{h_1}{2}$$

so $$E_2 h_2^2 = E_1 h_1^2$$ which is the force equation

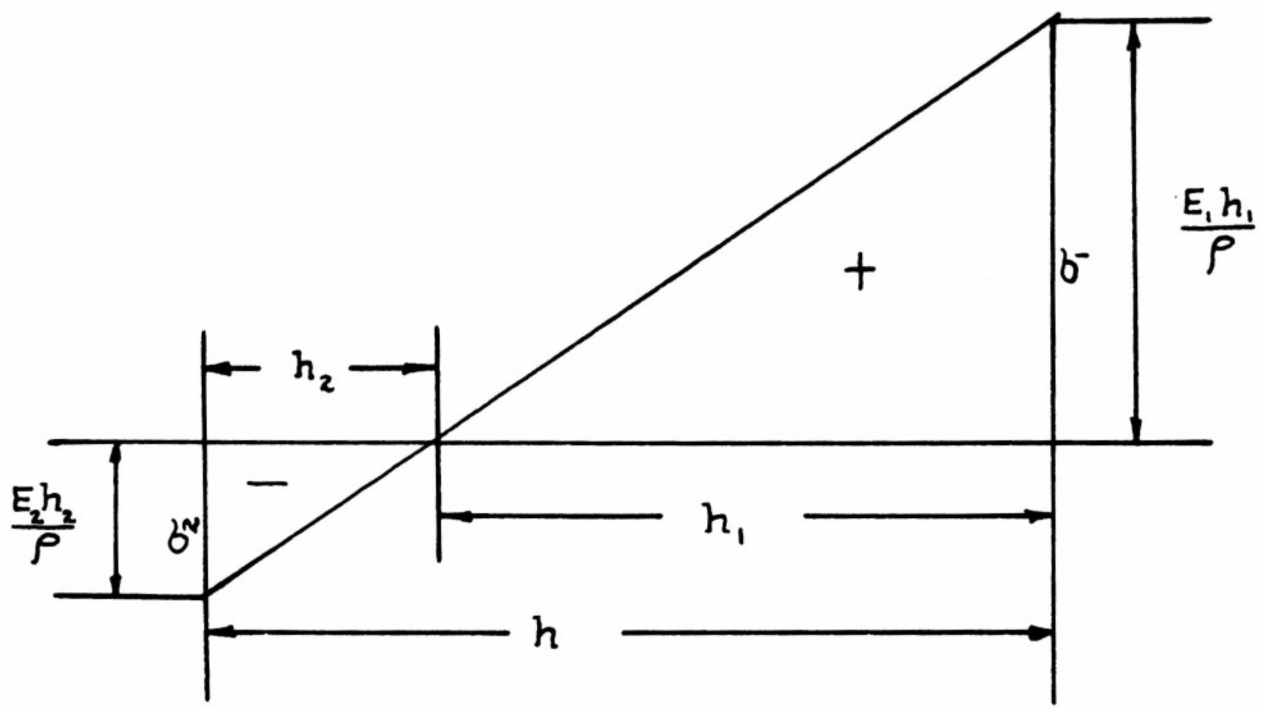

Fig. 7. Distribution of stress in cross-section of bar

$$h_1 + h_2 = h$$

$$\frac{1}{2} h_1 \cdot \frac{E_1 h_1}{\rho^*} \cdot \frac{2h_1}{3} + \frac{1}{2} h_2 \cdot \frac{E_2 h_2}{\rho^*} \cdot \frac{2h_2}{3} = M$$

M is the moment at section.

If b is the width of the section

$$M = \frac{bh^3}{12\rho} \cdot \frac{4 E_1 \cdot E_2}{\left[\sqrt{E_1} + \sqrt{E_2}\right]^2}$$

putting $$E_r = \frac{4 E_1 \cdot E_2}{\left[\sqrt{E_1} + \sqrt{E_2}\right]^2}$$

and $$M = \frac{E_r \cdot I}{\rho^*}$$

If we now replace E by E_r we may revert to Case I for determining the length of loading section for any given applied end stress σ_o which is critical.

DISCUSSION

The theory presented herein is proposed as a first criterion of instability of a prismatic bar with a transient force applied at its end. Although a complete and accurate equation describing strain wave propagation in a bar must include lateral contraction as well as other effects in the bar it is considered that the simplest one-dimensional wave theory serves the purpose of the present exposition. For a similar reason, the transverse inertia forces related to bending are neglected in order to get an approximate criterion of instability. It is considered that

The merit of these assumptions should be experimentally investigated, because very few data on the compression impact loading of bars occur in the literature. Some experiments by D. S. Clark (7) are quite suggestive of what may be expected in the way of sidewise plastic deformation of relatively long bars of lead subjected to dynamic loads on one end. A series of experiments will be conducted now by the author with the specific purpose of investigating sidewise deformation of bars subjected to impact loading on one end.

ACKNOWLEDGMENT

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