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Resource Management of Flexible Sensors

Part 1 - Phased Array Radar Simulator
[Unclassified Title]

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ABSTRACT

A preliminary simulation model of the search mode of a phased array radar has been developed. The basic components are a search strategy for the radar, an evasion policy for the target, and a simple radar environment. At present there is no attempt at adaptive modification of either component. A particular search strategy has been chosen. There is provision in the simulator to modify the procedure, however. The evasion policy of the target is one of changing course at random times with random deviations.

It is planned to develop the tracking package next. The approach will be through a Kalman-Bucy filter.

PROBLEM STATUS

This is an interim report; work on this problem is continuing.

AUTHORIZATION

NRL Problem No. B01-07
NAVSHIPSYSCOM Project SF 001-02-05-6284

RESOURCE MANAGEMENT OF FLEXIBLE SENSORS

I. PHASED ARRAY RADAR SIMULATOR

INTRODUCTION

A computer program has been developed for the CDC 3800 computer to simulate a radar detection system and target. The radar detection system is located, by assumption, on a ship at a specified position and orientation on the earth and will be referred to as the observer. This system scans particular ranges, elevations, and azimuths according to a predetermined strategy. The pulse rate and power transmitted by the system at any given time are also specified by the strategy. The target is assumed airborne and flying at constant altitude and speed, heading by way of an evasion course towards the observer. The observer and target are considered adversaries, and the present goal is for the observer to detect the target before the target is close enough to destroy the observer.

DESCRIPTION OF SIMULATOR

Figure 1 is a general block diagram of the simulator. It will be described briefly in the remainder of this section. The portions of the block diagram indicated by asterisks are each detailed procedures and will be described in subsequent sections.

For simplicity, the initial model of the radar beam is a conical surface with apex at the observer, bounded above and below by an upper and lower elevation (δ_u and δ_ℓ), and bounded to the sides by an upper and lower azimuth (ν_u and ν_ℓ), where $\delta_u - \delta_\ell = \nu_u - \nu_\ell = 1.7^\circ$.

This model will be increased in complexity after other more primary goals are attained, in particular the incorporation of target tracking into the simulator.

The target is considered a point in space, with range, elevation, and azimuth coordinates at time t expressed as $R_T(t)$, $\delta_T(t)$, and $\nu_T(t)$ respectively. The sequence of operations is roughly as follows. The main program calls on a strategy subroutine, and the latter yields the output parameters of time t, beam position ($\delta_\ell, \delta_u, \nu_\ell$, and ν_u) at time t, and the relative power radiated (mode of operation). The main program then summons the target simulator subroutine to obtain the target coordinates $R_T(t)$, $\delta_T(t)$, $\nu_T(t)$ at the time t specified by the strategy. A test is then performed to see if $\delta_T > 65^\circ$. If this is the case, then the target is considered within destruct distance of the observer and, accordingly, the observer is assumed destroyed (it should be remarked that $\delta_T > 65^\circ$ is an arbitrary and perhaps somewhat inaccurate criterion for destruction, but it is being used temporarily for expediency). This terminates the program. If, on the other hand, $\delta_T \leq 65^\circ$, the program continues and next determines whether the target is inside or outside of the radar beam. It is inside if both inequalities $\delta_\ell \leq \delta_T \leq \delta_u$ and $\nu_\ell \leq \nu_T \leq \nu_u$ are satisfied. If this is the case, then a probability of detection is computed based on the range of the target and its position in the beam, and on the mode of the beam. A random number between 0 and 1 is chosen and compared with the probability of detection: if the former is no greater in value the target is assumed discovered, and the program terminates; otherwise, control cycles back to the calling of the strategy subroutine to obtain the next set of parameters. Control also cycles back to this point if the target is outside the beam.

The following sections will describe in more detail the strategy and target simulator subroutines, and the computation of the probability of detection. The results of a series of computer tests of the general simulator can be seen in the Appendix.

TARGET SIMULATOR

The method of operation of the target simulator subroutine will now be described. This involves a set of assumptions about the earth and target, a method of choosing the type of motion of the target, including evasive action, and finally the derivation of the equations of target position at some particular time t.

Assumptions about Target and Environment

The earth is assumed to be a sphere of radius $R_E = 3963.34$ statute miles (the term "mile(s)" will subsequently mean "statute mile(s)" unless otherwise specified), and the observer is postulated to be at the approximate location of Washington, D. C.: colatitude 52°, east longitude 283°.

Because the target remains at a constant height h above the earth's surface, its motion is restricted to the surface of a sphere having radius $R_E + h$ and concentric with the earth. If we radially project the observer's coordinates onto this sphere, hereafter referred to as the target sphere, we can analyze the relation between the target and projected observer using spherical trigonometry. It is stipulated that the target first appears on the north horizon of the observer; it thus has a colatitude of less than 52° and an east longitude of 283°. The evasion course of the target consists of a series of connected great-circle paths or segments; two consecutive segments will be said to intersect at a pivot point. While the direction of the target changes abruptly at these pivot points its speed is constant. At any particular pivot point the angular deviation of the next great-circle path from a direct great-circle path to the projected observer is chosen at random according to a Gaussian distribution. The time that the target adheres to any particular great-circle path (i.e., the time between pivot points) is also chosen at random according to the waiting time distribution of a Poisson process.

Equations of Target Position

The following symbols are used in developing the equations of position of the target at some time t:

$t_o, t_1, t_2, \ldots, t_i, \ldots$ are the times, in sequence, at which the target arrives at each pivot point. At time t_o the target is at the north horizon of the observer; t_1 is the time of arrival at the first pivot point thereafter; t_2 the time of arrival at the next pivot point, and so forth. Clearly $t_o < t_1 < t_2 < t_3 < \ldots < t_i < \ldots$.

The point (θ_o^i,ϕ_o^i) gives the coordinates of the target at the i^{th} pivot point (occurring at time t_i), where θ_o^i and ϕ_o^i are the target colatitude and east longitude, respectively. Thus the target path is a series of great-circle arcs joining the points $(\theta_o^{i-1},\phi_o^{i-1})$ and (θ_o^i,ϕ_o^i), $i = 1, 2, \dots$.

α_i is the deviation the target path makes at the point (θ_o^i,ϕ_o^i) from a direct great-circle path joining (θ_o^i,ϕ_o^i) and the projected observer. If the deviation is counterclockwise (as viewed from outside the target sphere) $\alpha_i \geq 0$, if clockwise then $\alpha_i < 0$.

The point (θ_B,ϕ_B) gives the coordinates, colatitude and east longitude respectively, of the observer (and thus also the projected observer). Hence $\theta_B = 52^o$ and $\phi_B = 283^o$.

h is the height of the target in miles.

V_T is the speed of the target in mph.

ω is the angular velocity of the target in rad/sec. We have that $\omega = \frac{V_T}{R_E+h} \left(\frac{1}{3600} \right)$.

The point $(\theta,\phi) = (\theta(t),\phi(t))$ gives the colatitude and east longitude of the target at time t.

The range, elevation, and azimuth of the target at time t relative to the observer are denoted respectively by $R_T(t)$, $\delta_T(t)$, and $\nu_T(t)$. The argument "(t)" will sometimes be omitted for convenience.

Suppose at time t the target has travelled a slight distance beyond its most recent pivot point (θ_o^i,ϕ_o^i), which it reached at time t_i. Thus $t_i < t \le t_{i+1}$, and the great-circle path the target is now traversing makes an angle α_i with the great-circle arc joining (θ_o^i, ϕ_o^i) and (θ_B, ϕ_B). Figure 2 is the representation of this situation on the surface of the target sphere. We can calculate the target coordinates $\theta(t)$, $\phi(t)$ using the known values θ_B, ϕ_B, θ_o^i, ϕ_o^i, ω, t, t_i, and α_i. The following procedure accomplishes this:

Step 1: Calculate μ from the relation

$$\mu = \cos^{-1}[\cos\theta_B \cos\theta_o^i + \sin\theta_B \sin\theta_o^i \cos(\phi_o^i - \phi_B)].$$

Step 2a: Calculate $\cos\beta$ from the relation

$$\cos\beta = \frac{\cos\theta_B - \cos\mu \cos\theta_o^i}{\sin\mu \sin\theta_o^i} .$$

If $|\cos\beta| \le 0.999$, β is computed as the arccosine of $\cos\beta$, noting that $0 \le \beta \le 180^\circ$, and we proceed to step 3; otherwise we perform

Step 2b: If $|\cos\beta| > 0.999$, calculate β from the relation

$$(1) \qquad \beta = 90^\circ + [\mathrm{sgn}(\cos\beta)] \left[\sin^{-1}\left(\frac{\sin\theta_B |\sin(\phi_o^i - \phi_B)|}{\sin\mu}\right) - 90^\circ\right] ,$$

in order to reduce numerical error. The principal arcsine is used, and the "sgn" function is defined as follows:

$$\mathrm{sgn}\,\lambda \equiv +1 \text{ if } \lambda \ge 0,$$
$$\equiv -1 \text{ if } \lambda < 0,$$

where λ is any real number. If relation (1) is used, β satisfies the inequality $0 \le \beta \le 180^\circ$.

Step 3: Compute γ from the formula

$$\gamma = \beta + \alpha_i \cdot \{\mathrm{sgn}[\sin(\phi_o^i - \phi_B)]\}, \quad 0 \le \gamma < 360^\circ.$$

Step 4: Compute $\theta(t)$ from the formula

$$\theta(t) = \cos^{-1}[\cos \theta_o^i \cos \omega(t-t_i) + \sin \theta_o^i \sin \omega(t-t_i) \cos \gamma]$$

Step 5a: Compute $\cos \psi$ from the relation

$$\cos \psi = \frac{\cos \omega(t-t_i) - \cos \theta(t) \cos \theta_o^i}{\sin \theta(t) \sin \theta_o^i}$$

If $|\cos \psi| \leq 0.999$, ψ is computed as the arccosine of $\cos \psi$, noting that $0 \leq \psi \leq 180^o$, and we proceed to step 6; otherwise we perform

Step 5b: If $|\cos \psi| > 0.999$, calculate ψ from the relation

$$\psi = 90^o + [\mathrm{sgn}(\cos \psi)] \left[\sin^{-1}\left(\frac{\sin \omega(t-t_i)|\sin \gamma|}{\sin \theta(t)}\right) - 90^o\right],$$

where, as in eq. (1), the arcsine is taken in the principal region. Again we have $0 \leq \psi \leq 180^o$. We assume also that the pivot points occur sufficiently often to insure that $\omega(t-t_i) < \pi$ radians.

Step 6: Compute $\phi(t)$ from the relation

$$\phi(t) = \phi_o^i - \psi \cdot [\mathrm{sgn}(\sin \gamma)] \cdot \{\mathrm{sgn}[\sin(\phi_o^i - \phi_B)]\},$$

$\phi(t)$ should satisfy $0 \leq \phi(t) < 360^o$.

We note that if $t = t_{i+1}$ we have computed $\theta_o^{i+1} = \theta(t_{i+1})$ and $\phi_o^{i+1} = \phi(t_{i+1})$, the coordinates of the (i+1)st pivot point; these coordinates are stored for possible use in later calcualtions.

We digress at this point to discuss how the times between pivot points, $\Delta t_i = t_{i+1} - t_i$ $(i=0,1,2,\ldots)$, and the deviations α_i $(i=0,1,2,\ldots)$, are determined. To choose a value of Δt_i for any i, we must first have access to a preselected value of the mean free time of traverse between pivot points, denoted by t_f, which remains constant for the entire simulation. A random number r satisfying $0 < r < 1$ is then chosen from a uniform distribution. The value of Δt_i is then computed as $\Delta t_i = t_f \ell n \left(\frac{1}{1-r} \right)$. The value of t_f currently being used is 20 sec., which guarantees with 95% probability that $\Delta t_i \leq 60$ sec. The deviations α_i are chosen from a Gaussian distribution having a preselected standard deviation σ_α, which remains constant for the entire simulation. To date we have performed simulations using $\sigma_\alpha = 15^\circ$ and $\sigma_\alpha = 30^\circ$, using the CDC CØØP subroutine G5 WISC RANSS to generate the α_i.

We now calculate the values $R_T(t)$, $\delta_T(t)$, and $\nu_T(t)$ at time t using the parameters we have previously specified or calculated. Figure 3 shows a meridian plane of the earth, the particular plane being the one containing the target. The following procedure then follows immediately:

Step 1: Calculate ξ from the relation (cf. Fig. 2)

$$\xi = \cos^{-1} [\cos \theta_B \cos \theta(t) + \sin \theta_B \sin \theta(t) \cos (\phi(t) - \phi_B)].$$

Step 2: Calculate R_T from the relation

$$R_T = \sqrt{h^2 + 2R_E(R_E+h)(1 - \cos \xi)}.$$

Step 3: Observing $\delta_T = \eta - 90^\circ$, we obtain an equation for computing δ_T,

$$\delta_T = \sin^{-1} \left[\frac{h(2R_E+h) - R_T^2}{2R_E R_T} \right], \quad -90^\circ \leq \delta_T \leq 90^\circ,$$

although cases where $\delta_T < 0^o$ mean that the target is beneath the horizon.

Step 4: Referring to Fig. 2, we obtain

$$\cos \nu_T = \frac{\cos \theta(t) - \cos \theta_B \cos \xi}{\sin \theta_B \sin \xi} .$$

If $|\cos \nu_T| \leq 0.999$, we define ν' to be the principal arccosine of $\cos \nu_T$, so that

$$\nu' = \cos^{-1} (\cos \nu_T), \quad 0 \leq \nu' \leq 180^o.$$

We then compute ν_T from the relation

$$\nu_T = 180^o + \{\text{sgn}[\sin(\phi(t) - \phi_B)]\} \cdot (\nu' - 180^o).$$

If $|\cos \nu_T| > 0.999$, then ν_T is computed using the formula

$$\nu_T = 90^o + [\text{sgn}(\cos \nu_T)] \left[\sin^{-1}\left(\frac{\sin \theta(t) \sin (\phi(t) - \phi_B)}{\sin \xi}\right) - 90^o\right],$$

where the arcsine is taken to be in the principal region $[-90^o, 90^o]$. ν_T is normalized, if necessary, so that $0 \leq \nu_T < 360^o$.

STRATEGY

We subsequently describe the present strategy, keeping in mind that all aspects of it are flexible and can be completely modified for future experimentation. The present strategy was based on conventional procedure and was employed to hasten the initial testing of the simulator.

Antenna Structure and Search Modes

The present model of the observer is a ship with four antennas, each with a 90° span in azimuth, mounted in two units with an antenna pair in each unit in such a way that the units span non-overlapping 180° sectors in azimuth. Figure 4 indicates the positions of these antennas on the ship. The antenna pair shown on the right-hand part of the diagram spans an azimuth range from 0° to 180°. This pair shall be subsequently considered as one antenna for simplicity; in fact, it will be denoted as antenna #1. Similarly, the remaining pair spans an azimuth range from 180° to 360° and will be denoted as antenna #2.

Three search modes are used in the strategy, and these shall be designated as modes I, II, and III. The region of space that is searched by each mode will also be designated by the Roman numeral of the mode. Thus we can refer to regions and modes interchangeably. We stipulate that the power transmitted for searching in modes I and III is the same, and equal to one-fourth that transmitted in mode II. This condition, of course, will penalize the radar's performance. It may arise, however, under certain types of energy management, e.g., purposely transmitting several main antenna lobes simultaneously. Future versions of the simulator will provide for varying degrees of such management.

We define R_T^{I}, R_T^{II}, and R_T^{III}, corresponding to the respective modes, as those ranges at which the target, when in the center of the beam, can be detected with a 50% (0.5) probability. We specify that $R_T^{II} = 150$ nautical miles (n.m.). The above conditions determine R_T^I and R_T^{III} uniquely. R_T^I is calculated by use of the radar equation, which in its fundamental form is

$$P_r = \frac{P_t G_t A_r \sigma}{(4\pi R_T^2)^2} ,$$

where P_r = echo power received at the radar,

P_t = power transmitted by the radar,

G_t = transmitting gain of the antenna,

A_r = effective capture area of the receiving antenna,

σ = radar cross section of the target,

and R_T = range of the target.

Let superscripts I, II, and III refer to quantities relating to modes (or regions) I, II, and III, respectively. The variables A_r and σ are independent of the mode number, and the antenna gain G_t is kept independent by considering the target to be in the center of the beam in all cases. Thus $P_r^j = K \dfrac{P_t^j}{(R_T^j)^4}$, where j = I, II, or III, and K is a lumped constant independent of j. Since equal detection probabilities mean that equal echo power is received at the radar, we have $P_r^I = P_r^{II} = P_r^{III}$. We then get for the radar equation

$$\frac{P_t^I}{(R_T^I)^4} = \frac{P_t^{II}}{(R_T^{II})^4} , \quad \text{or} \quad R_T^I = R_T^{II} \left(\frac{P_t^I}{P_t^{II}} \right)^{1/4} . \qquad (2)$$

Since $P_t^{II} = 4P_t^I$ and $R_T^{II} = 150$ n.m., it follows from (2) that $R_T^I = 106.1$ n.m. $= R_T^{III}$, the last equality occurring because $P_t^I = P_t^{III}$.

Figure 5 depicts these regions in polar coordinates, with range as the modulus and elevation the argument. The minimum and maximum elevations searched in each region are indicated, as are the ranges $R_T^I (= R_T^{III})$ and R_T^{II}. The diagram also shows the frequency with which each region is searched. For any particular elevation within the bounds searched in any of the modes, all azimuths from 0° to 360° are scanned. Region I is completely searched every second, regions II and III every 12 and 6 seconds respectively. Elevations from 0° to 1.7° (one beamwidth) are scanned in region I, 0° to 4.5° in region II, and 1.7° to 65° in region III.

The pulse rate used in each region is a function of the maximum range to be searched in that region. This dependency is brought about by the conflicting goals of having as high a pulse rate as possible to minimize search time, yet slow enough so that the return pulse will be received before the next transmitted pulse occurs and thus not create ambiguity as to the source of the return pulse and the value of target range. The maximum unambiguous pulse rate f_R is related to the maximum range R_{MAX} by the formula

$$f_R = \frac{C}{2R_{MAX}},$$

where C is the velocity of light. We use as the value of R_{MAX} the ranges R_T^I, R_T^{II}, and R_T^{III}, at which the probability of detection in each respective mode is 0.5. It follows that $f_R^I = f_R^{III} = 761$ pulses/second, and $f_R^{II} = 538$ pulses/second. Accordingly a pulse rate of 537/sec. was used for region II and, through an oversight, a pulse rate of 1253/sec. was used for regions I and III (this is not expected to influence significantly the results of our simulator tests, because we are not determining target range at this stage of the project; however, this error will be corrected prior to subsequent tests). These rates suggest we create a new unit of time, denoted as a <u>snarf</u>, such that 1 second = 3,759 snarfs. Then for region II, with a maximum pulse rate of 537/sec., we must have at least 7 snarfs between pulses. For regions I and III, with a maximum pulse rate of 1253/sec., there must be at least 3 snarfs between pulses. Our strategy operates at these maximum pulse rates.

Representation of Beam Positions

The beam positions used in the present strategy are considered discrete; that is, the beam does not move continuously through a region, but instead scans a region by incremental steps in position, either in elevation, azimuth, or both. Thus at most a finite number of possible beam positions can be assumed by the radar. We mention, however, that the particular choice of discrete beam postions used here is temporary and probably will be modified in the future to provide a more tightly packed scan of each region.

The azimuth incremental positions used are independent of the region. For any particular azimuth scanned by antenna #1, an azimuth 180 degrees greater is being scanned by antenna #2. Thus if at any instant of time the beam center of antenna #1 is pointing at an azimuth ν, the beam center of antenna #2 is pointing at azimuth $\nu + 180^\circ$. Each azimuth position can be represented by the format $[\nu_\ell, \nu_u]$, where ν_ℓ and ν_u are expressed in degrees. We use this in spite of the redundancy arising from the relation $\nu_u = \nu_\ell + 1.7^\circ$. There are 106 positions in azimuth that each antenna can assume under the present strategy, covering, in slightly overlapping segments, the span of 0° to 180°. These are the positions

$$[1.6981(n-1),\ 0.0019 + 1.6981n], \qquad n = 1, \ldots, 106.$$

This is the minimum number of positions necessary for an antenna with 1.7° beamwidth to span 180° in azimuth. Corresponding to the positions of antenna #1, antenna #2 assumes the positions

$$[180 + 1.6981(n-1),\ 180.0019 + 1.6981n], \quad n = 1, \ldots, 106.$$

Thus an azimuth position is described by the antenna number and the integer n.

The elevation incremental positions used are dependent on the region. Similar to the azimuth format, we represent the elevation by $[\delta_\ell, \delta_u]$, where δ_ℓ and δ_u are expressed in degrees and $\delta_u = \delta_\ell + 1.7^\circ$. In region I there is only one possible elevation position, and that is [0,1.7]. In region II there are three possible elevation positions:

$$[1.4(m-1),\ 0.3 + 1.4m], \qquad \text{for } m = 1, 2, 3.$$

In region III there are 38 possible elevation positions, and they are

$$[1.6658(m-1),\ 0.0342 + 1.6658m], \quad \text{for } m = 2, 3, \ldots, 39.$$

In each of regions II and III the positions are slightly overlapping and scan the total interval using the minimum number of positions necessary for a beamwidth of 1.7^o. Antennas #1 and #2, although 180^o apart in azimuth, are always searching at the same elevation.

The preceding discussion indicates that the beam position and power transmitted can be completely specified by an ordered quadruple of integers: (1) the region no., (2) the antenna no., (3) m, and (4) n (for region I, m is set to 1).

Sequence of Beam positions and Modes for Current Strategy

The search sequence used in the present strategy has a period of 12 seconds. At the beginning of the 0^{th} second, the scan of region I begins at pulse rate f_R^I and continues to completion. 7 snarfs later, the scan of region II begins at pulse rate f_R^{II} and continues until one-twelth of this region has been searched. 3 snarfs later, the scan of region III begins at pulse rate $f_R^{III} = f_R^I$ and continues until one-sixth of this region has been searched. At this point about 0.7 sec. have elapsed, and no further searching is done until the beginning of the 1st second. Then the scan of region I begins again and is carried to completion as before. 7 snarfs later the scan of region II resumes at the point at which it was interrupted during the 0^{th} second, and another twelth of this region is searched. 3 snarfs thereafter, the scan of region III resumes at the point of interruption during the 0^{th} second, and another sixth of this region is searched. Then searching halts until the beginning of the 2nd second, when the process continues. The scan of region III is completed during the 5th second and starts again in a new cycle during the 6th second. The same occurs for region II (and region III again) during the 11th and 12th seconds, respectively. This search cycle is shown diagrammatically in Fig. 6. The order of search within a fixed mode is to increment the azimuth steps of the beam position (alternately switching between antennas #1 and #2) while keeping the elevation fixed. At the end of the azimuth scan, the elevation is increased by one step and the azimuth reset.

COMPUTATION OF PROBABILITY OF DETECTION

When the target satisfies the criteria for being inside the beam, i.e., $\delta_\ell \le \delta_T \le \delta_u$ and $\nu_\ell \le \nu_T \le \nu_u$, a probability of detection P_{det} is computed based on the mode of the radar, the target range, and the position of the target relative to the center of the beam. Preparatory to this computation a value for the probability of a false alarm, P_{fa}, must be selected. This is the probability that the radar system will indicate the presence of a target at a particular location when, in fact, no target is there. For this model, P_{fa} has been set equal to 10^{-6}, although it will probably be lowered to 10^{-9} for subsequent tests. We next define x to be the solution of the equation

$$P_{fa} = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt.$$

If $P_{fa} = 10^{-6}$ then $x = 4.7615$ is the solution. We also define $R_o^j \equiv R_T^j \sqrt{x}$, where j is the mode number (j = I, II, or III).

We now suppose the target is inside the radar beam and has coordinates R_T, δ_T, and ν_T, and the radar is searching in mode j. We compute the deviation ϵ_T of the target from the center of the beam by the equation

$$\epsilon_T = \cos^{-1}[\sin \delta_T \sin \delta_c + \cos \delta_T \cos \delta_c \cos(\nu_T - \nu_c)],$$

where $\delta_c \equiv \frac{1}{2}(\delta_\ell + \delta_u)$ and $\nu_c \equiv \frac{1}{2}(\nu_\ell + \nu_u)$ are the elevation and azimuth, respectively, of the beam center. The effects of the target range and deviation from beam center appear in the value of the parameter d, which is calculated as

$$d = \left(\frac{R_o^j}{R_T}\right)^2 \cos(52.941 \epsilon_T).$$

The factor 52.941 insures that the cosine term equals $\frac{1}{\sqrt{2}}$ when $\epsilon_T = 0.85^\circ$. The probability of detection is evaluated as

$$P_{det} = \frac{1}{\sqrt{2\pi}} \int_{x-d}^{\infty} e^{-t^2/2}\, dt.$$

The value of P_{det} is computed by using an approximation for the error function Φ defined by

$$\Phi(y) \equiv \frac{2}{\sqrt{\pi}} \int_0^y e^{-u^2}\, du, \qquad y \geq 0.$$

Once Φ is computed, P_{det} can be found from the relations

$$P_{det} = \frac{1}{2}\left[1 - \Phi\left(\frac{x-d}{\sqrt{2}}\right)\right] \qquad \text{if } x \geq d,$$

$$= \frac{1}{2}\left[1 + \Phi\left(\frac{d-x}{\sqrt{2}}\right)\right] \qquad \text{if } x < d.$$

The approximation we used for Φ is found in [2, p. 167] and is the following:

$$\Phi(y) \approx 1 - (a_1\eta + a_2\eta^2 + a_3\eta^3)\frac{2}{\sqrt{\pi}}\, e^{-y^2}, \qquad y \geq 0,$$

where $\eta = \dfrac{1}{1+py}$,

and $p = 0.47047$, $a_1 = 0.3084284$, $a_2 = -0.0849713$,

$a_3 = 0.6627698.$

APPENDIX. COMPUTER TESTS OF SIMULATOR

Table 1 indicates the results obtained from 20 computer runs of the general simulator, consisting of four sets with five trial runs in each set. A set corresponds to one of four possible combinations of target height h and standard deviation σ_α (see definition p. 8). These latter parameters assume the values h = 1.26 and 5.05 miles (target crosses horizon at ranges of 100 and 200 miles, respectively), and $\sigma_\alpha = 0^\circ$ (target heads directly for projected observer) and 30° (target heads for projected observer via evasion course). For all 20 runs, mean free time t_f (see definition p. 7) is 20 seconds and target speed is 2000 mph.

The parameters tabulated for each run are (1) the time that elapses between the horizon crossing of the target and its detection, (2) the target range at detection, and (3) the search mode in which detection occurred. The numerical results of each set of 5 runs are averaged.

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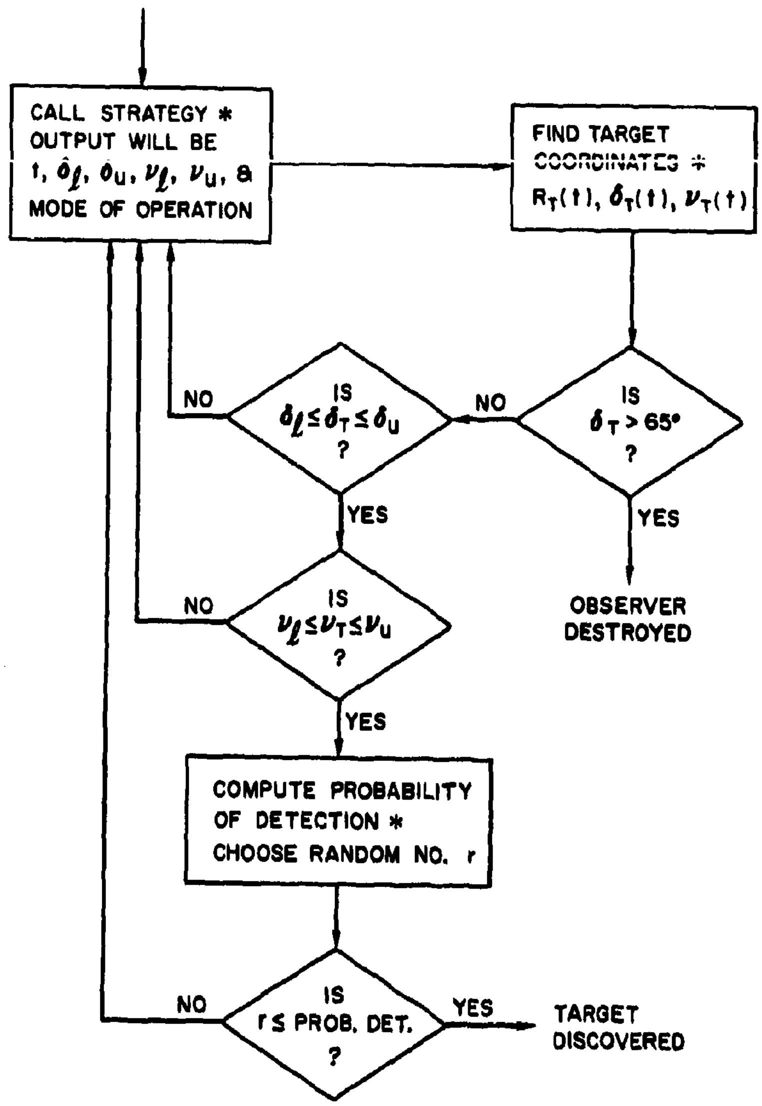
2. Hastings, Cecil. Jr., Approximations for Digital Computers, Princeton University Press, Princeton, New Jersey, 1955.

Standard Deviation	TARGET HEIGHT = 1.26 STATUTE MILES: Time from Horizon Crossing to Detection	Range at Detection	Region of Detection	TARGET HEIGHT = 5.05 STATUTE MILES: Time from Horizon Crossing to Detection	Range at Detection	Region of Detection	
$\sigma_\alpha = 0^o$	2.59 sec	98.5 st.mi.	II	57.99 sec	168.0 st.mi.	II	
	2.00	98.8	II	55.80	169.2	II	
	0.89	99.4	I	79.78	155.9	II	
	8.48	95.2	II	43.47	176.0	II	
	3.47	98.0	II	91.56	149.3	II	
	3.49 sec(avg)	98.0 (avg)		65.72 sec(avg)	163.7 (avg)		←averages
$\sigma_\alpha = 30^o$	1.70 sec	99.7 st.mi.	I	55.96 sec	171.5 st.mi.	II	
	1.06	99.4	II	80.95	159.2	I	
	1.66	99.4	I	87.74	165.2	II	
	3.89	97.8	I	39.98	180.6	II	
	0.04	99.9	II	[illegible]2.69	168.7	II	
	1.67 sec(avg)	99.2 (avg)		65.46 sec(avg)	169.0 (avg)		←averages

Table 1. Results of general simulator tests performed for different values of target height h and standard deviation σ_α for the angular great-circle path deviations. Averages are taken based on five independent computer runs for each combination of h and σ_α. For all runs, target speed = 2000 mph, and mean free time between pivot points = 20 sec.



RADAR BEAM

δ_ℓ – LOWER ELEVATION
δ_u – UPPER ELEVATION
ν_ℓ – LOWER AZIMUTH
ν_u – UPPER AZIMUTH

TARGET

$R_T(t)$ – RANGE AT TIME t
$\delta_T(t)$ – ELEVATION AT TIME t
$\nu_T(t)$ – AZIMUTH AT TIME t

Fig. 1 - Block diagram of target and target-detection simulator

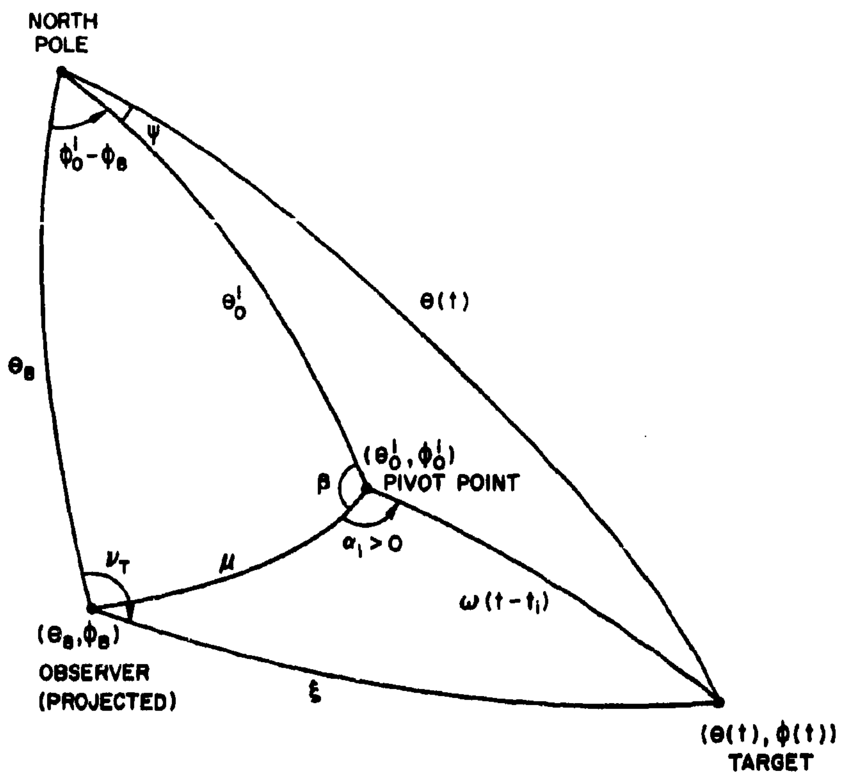


Fig. 2 - Representation of target-simulator parameters on surface of target sphere

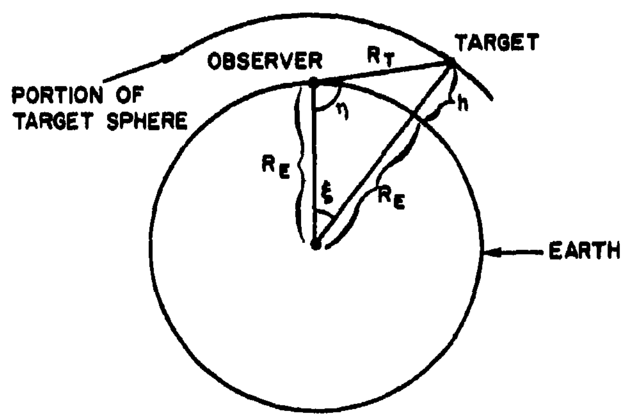


Fig. 3 - Meridian cross-section of earth containing target

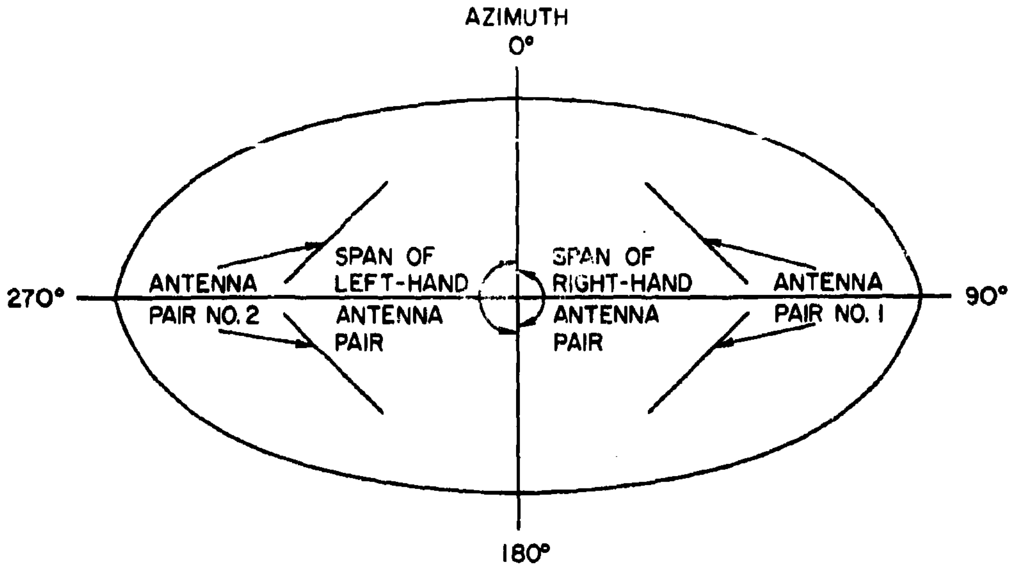


Fig. 4 - Configuration of antennas aboard ship

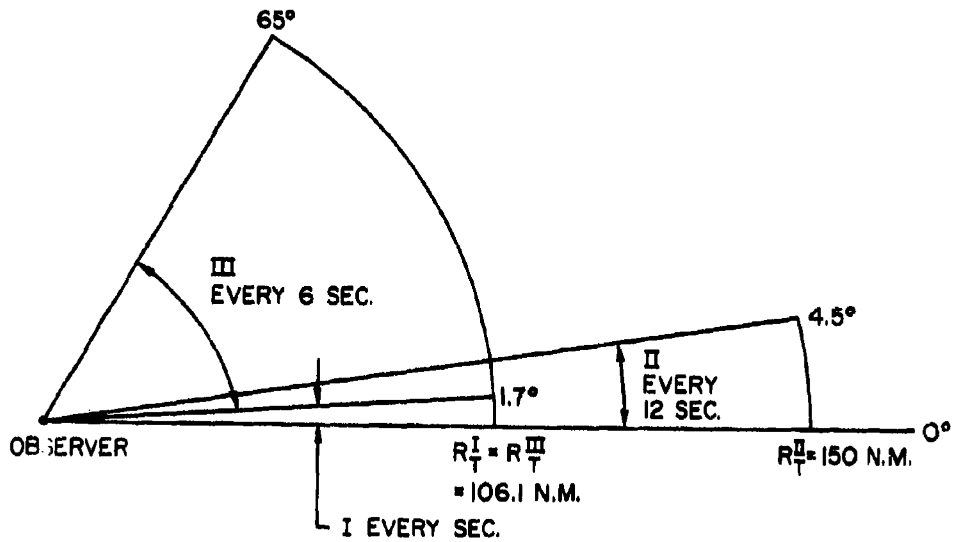


Fig. 5 - Regions searched under present strategy

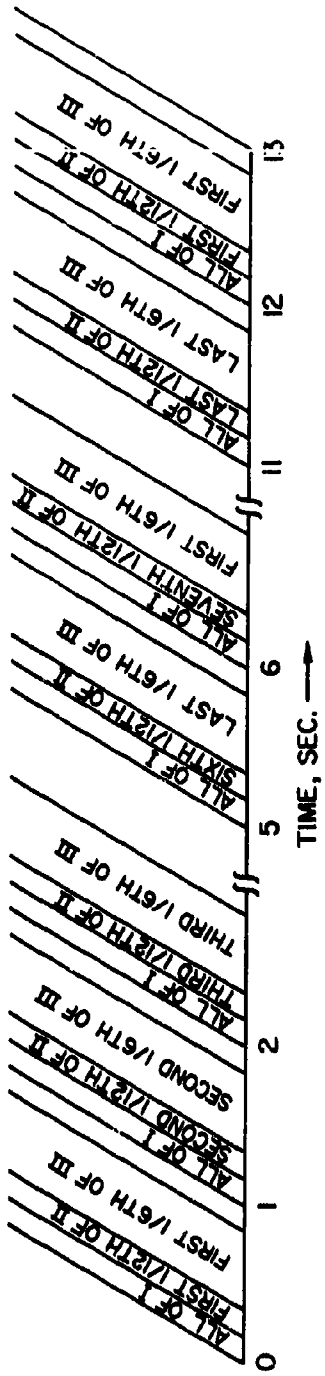


Fig. 6 - Time plot of current search strategy

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13. ABSTRACT

A preliminary simulation model of the search mode of a phased array radar has been developed. The basic components are a search strategy for the radar, an evasion policy for the target, and a simple radar environment. At present there is no attempt at adaptive modification of either component. A particular search strategy has been chosen. There is provision in the simulator to modify the procedure, however. The evasion policy of the target is one of changing course at random times with random deviations.

It is planned to develop the tracking package next. The approach will be through a Kalman-Bucy filter.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Computer program						
Phased array radar						
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Target simulator						
Search strategy						
Target evasion policy						
Simulator computer tests						

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