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ACOUSTICAL RAY TRACING

BY

ROBERT P. LEE

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WHITE SANDS MISSILE RANGE
NEW MEXICO

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ENVIRONMENTAL SCIENCES DIRECTORATE
U. S. ARMY ELECTRONICS RESEARCH AND DEVELOPMENT ACTIVITY
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ABSTRACT

A mathematical model is derived for acoustic ray tracing which takes into account the effects of temperature, winds, and earth curvature, and a calculation procedure is outlined.

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INTRODUCTION

The new air and space systems have placed a requirement for acoustic ray tracing to altitudes of 60 km. At the same time, a far greater amount of wind and temperature data are available up to 60 km than ever before, and large-scale computers with ever increasing speed and capacity are a standard fixture at all major installations. This report was undertaken to determine the feasibility of using more sophisticated mathematical models for acoustic ray tracing, utilizing advanced computer facilities and rocketsonde meteorological information.

THEORY

Figure 1 illustrates a vertical plane containing a ray path R. The earth is taken to be a sphere with no local anomalies to deflect the vertical axis, Z, i.e., geodetic, astronomic, and geocentric zenith points coincide. In terms of the moving Y-Z coordinate system shown in Figure 1, this acoustic ray trace model assumes that the zenith angle, θ, made by the normal to the wave front with the local vertical to the earth's surface, can be obtained at any point, r, along the ray path, R, by the following line integral:

$$\theta = \theta_0 + \int_{r_0}^{r} \left(\frac{G\sin\theta}{2T} + \frac{H\sin\theta}{C} - \frac{C\sin\theta + W}{C(N+z)} \right) dr \qquad (1)$$

where θ_0 = Initial zenith angle at r_0

G = Vertical temperature gradient (degrees Kelvin per meter)

T = Absolute temperature (degrees Kelvin)

C = Velocity of acoustic propagation (meters per second)

W = Component of the horizontal wind velocity in the direction of the Y-axis (meters per second)

H = Vertical rate of change of W (meters per second per meter)

N = Earth's radius (meters)

z = Elevation of a given point above sea-level (meters)

It is assumed that temperature is a function of height only, that there are no vertical wind components, that the only effect of a constant wind velocity is to translate the wave front with no change in the direction of its normal, and that the only wind gradient having any effect on θ is that defined as H above. It is well within the precision of the

VERTICAL PLANE CONTAINING A RAY PATH R.

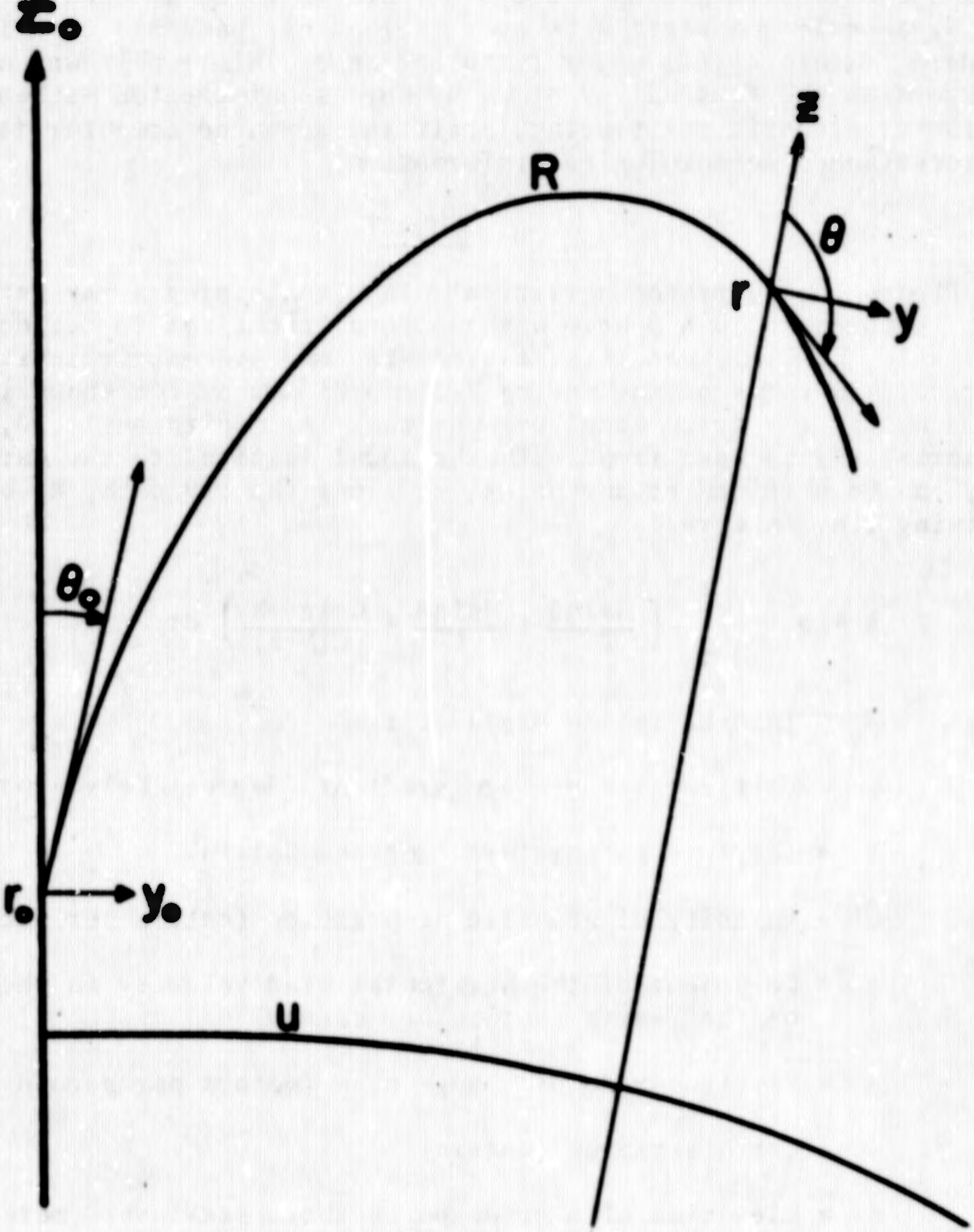

Figure I

meteorological data available to assume that the temperature and winds vary linearly from one reported layer to another, and therefore, the gradients are constant throughout each layer. It is necessary to use Snell's law in the form

$$\frac{\sin\theta}{C} = L, \tag{2}$$

and considerable simplification is obtained if the velocity of acoustic propagation is taken to be directly proportional to the square root of the absolute temperature

$$C = MT^{\frac{1}{2}} \tag{3}$$

where M is a constant.

In addition to the symbols used in the derivation of Equation 1, the following definitions are used:

Q = Time associated with each point on the ray path, R (seconds)

U = Sea-level projection of the ray path, R (meters)

X = Component of the horizontal wind velocity at right angles to the Y-Z plane (meters per second)

V = Sea-level projection of the displacement (translation) of the wave front at right angles to the Y-Z plane due to X above (meters).

An acoustic ray passing through a region in which there is a vertical temperature gradient, H, will refract, and the change in θ as previously defined can be determined by considering the properties of a plane curve (Figure 2) and applying Snell's law.

The curvature of a plane curve (the reciprocal of the radius of curvature) in the Y-Z plane is given by

$$K = \frac{\dfrac{d^2y}{dz^2}}{\left[1 + \left(\dfrac{dy}{dz}\right)^2\right]^{\frac{3}{2}}} \tag{4}$$

If the assumption is made (Snell's law) that for all the points on the curve R

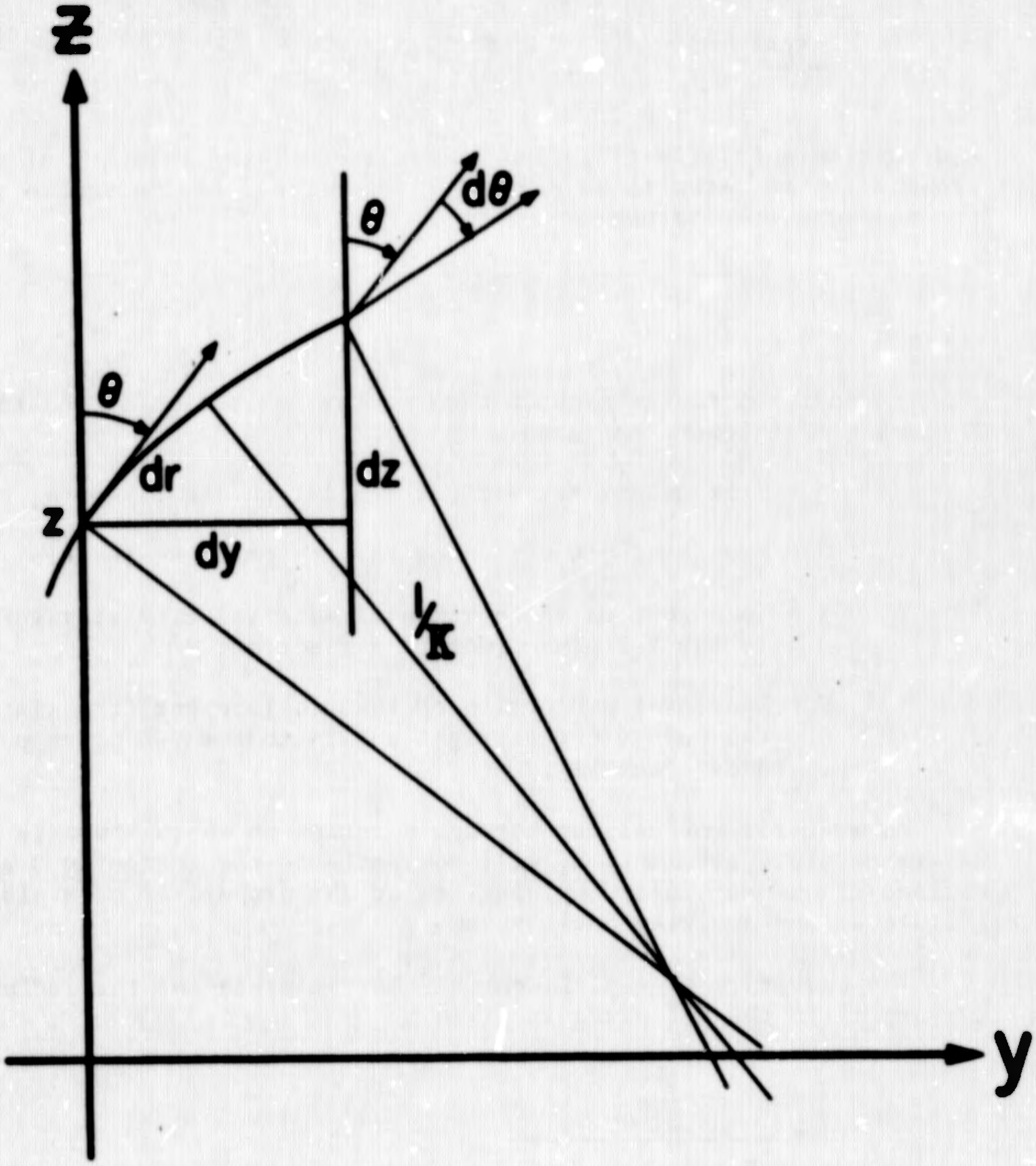

Figure 2

$$\frac{\text{Sin}\theta}{C} = L, \qquad (2)$$

where L is an unknown constant of proportionality, then

$$\text{Sin}\theta = CL \qquad (5)$$

and by differentiation

$$\text{Cos}\theta \frac{d\theta}{dz} = L \frac{dC}{dz} = \frac{\text{Sin}\theta}{C} \frac{dC}{dz} \qquad (6)$$

or

$$\frac{d\theta}{dz} = \frac{\text{Tan}\theta}{C} \frac{dC}{dz} \quad . \qquad (7)$$

Since acoustic propagation velocity is assumed to be related to temperature by

$$C = MT^{\frac{1}{2}}, \qquad (3)$$

it follows that

$$\frac{dC}{dz} = \frac{M}{2T^{\frac{1}{2}}}\left(\frac{dT}{dz}\right) \quad . \qquad (8)$$

After substituting Equations 3 and 8 into 7, there results

$$\frac{d\theta}{dz} = \frac{\text{Tan}\theta}{2T} \left(\frac{dT}{dz}\right) . \qquad (9)$$

Also by definition of derivative (Figure 2)

$$\frac{dy}{dz} = \text{Tan}\theta \qquad (10)$$

and therefore,

$$\frac{d^2y}{dz^2} = \text{Sec}^2\theta \frac{d\theta}{dz} \quad . \qquad (11)$$

Substituting (9), (10), and (11) into (4) and simplifying using trigonometric identities gives

$$K = \frac{\mathrm{Sin}\theta}{2T}\left(\frac{dT}{dz}\right). \tag{12}$$

By definition

$$d\theta = Kdr \tag{13}$$

and, therefore,

$$d\theta = \frac{G\sin\theta}{2T}\,dr. \tag{14}$$

This is the first fraction in Equation 1 and represents the change in zenith angle of the normal to the wave front as the acoustic wave is refracted by a vertical temperature gradient.

Assume (Figure 3) that a differential element of wave front, AB, lying in the Y-Z plane is displaced in distance $dy = OO'$ due to the mean horizontal wind component, but due to a gradient, H, in the horizontal wind component the element is twisted through a small angle, $d\theta$, and falls along A'B' instead A"B". Then

$$d\theta = \frac{BB' - AA'}{AB}. \tag{15}$$

Let dq = arc AB. By geometry

$$\frac{dW}{dq} = \frac{dW}{dz}\,\mathrm{Sin}\theta. \tag{16}$$

Also

$$dy = WdQ. \tag{17}$$

Then

$$AA' = \left(W - \frac{dW}{dq}\frac{dq}{2}\right) dQ \tag{18}$$

$$BB' = \left(W + \frac{dW}{dq}\frac{dq}{2}\right) dQ \tag{19}$$

$$d\theta = \frac{dW}{dq}\,dQ = \frac{dW}{dz}\,\mathrm{Sin}\theta dQ. \tag{20}$$

The time and velocity of propagation along the ray path will be related by the usual equation

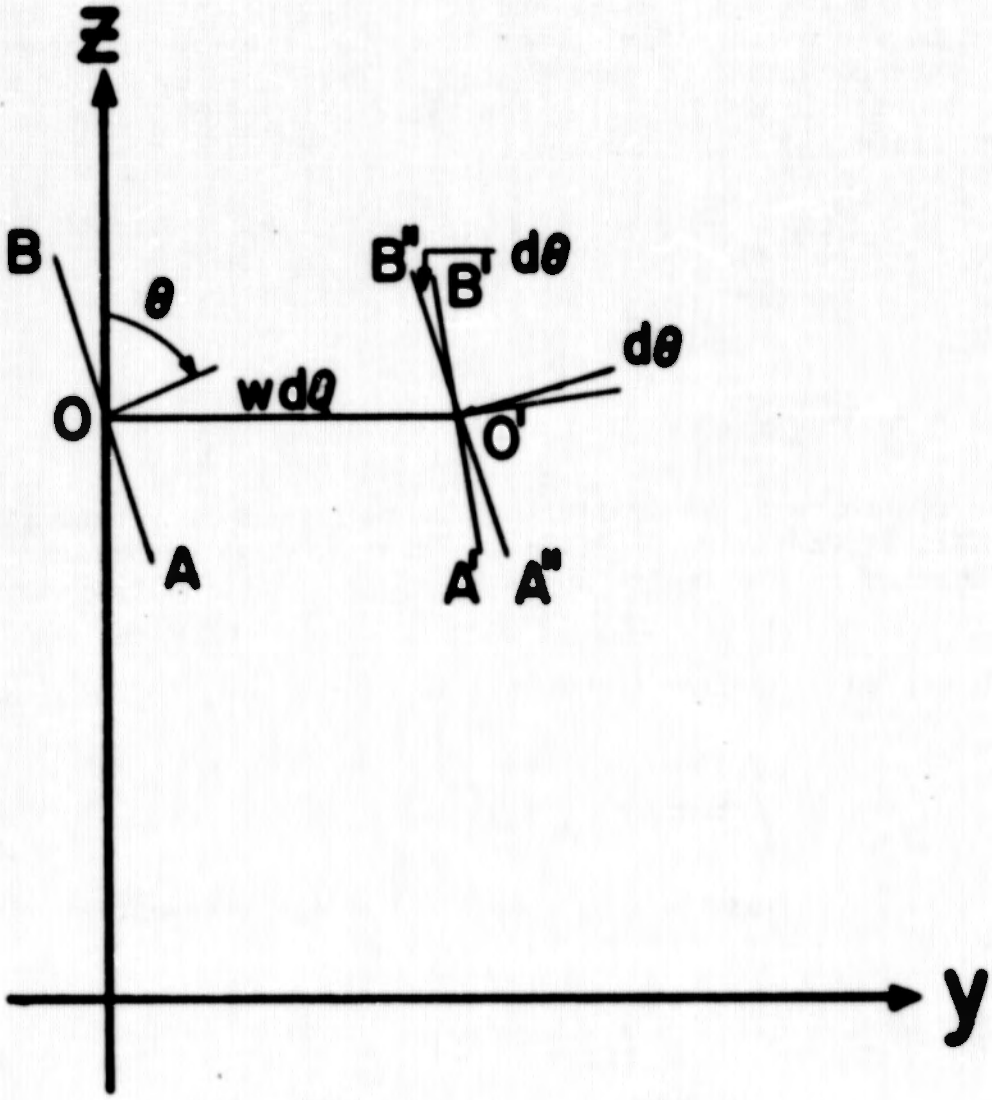

Figure 3

$$dQ = \frac{dr}{C} , \qquad (21)$$

so

$$d\theta = \frac{H\sin\theta}{C} dr \qquad (22)$$

This is the second fraction in Equation 1 and represents the deflection of the normal due to a vertical gradient in the down-range horizontal winds.

Since the zenith angle is defined with respect to a local vertical, an apparent deflection will take place as the wave front moves over the surface of a curved earth. During the time, dQ, the ray propagates a horizontal distance Sinθdr (Figure 2) and is displayed by W, (Figure 3) an additional distance WdQ. Taking the value of dQ from Equation 21 and considering Figure 4,

$$dy = \frac{C\sin\theta+W}{C} dr \qquad (23)$$

and

$$d\theta = \frac{C\sin\theta+W}{C(N+z)} dr. \qquad (24)$$

That this correction must be subtracted can be seen by an inspection of Figure 3. This is the third fraction in Equation 1 and represents the apparent reduction in the zenith angle due to rotation of the vertical axis.

It should also be apparent that the sea-level arc, dU, will be given by

$$dU = \left(\frac{N}{N+z}\right) \left(\frac{C\sin\theta+W}{C}\right) dr \qquad (25)$$

and that if X is the cross wind at a point, the displacement at right angles to dU will be, when reduced to sea level,

$$dV = \left(\frac{N}{N+z}\right) X\, dQ = \left(\frac{N}{N+z}\right) \left(\frac{X}{C}\right) dr \qquad (26)$$

thus allowing an evaluation of a translational effect due to winds not in the plane of propagation.

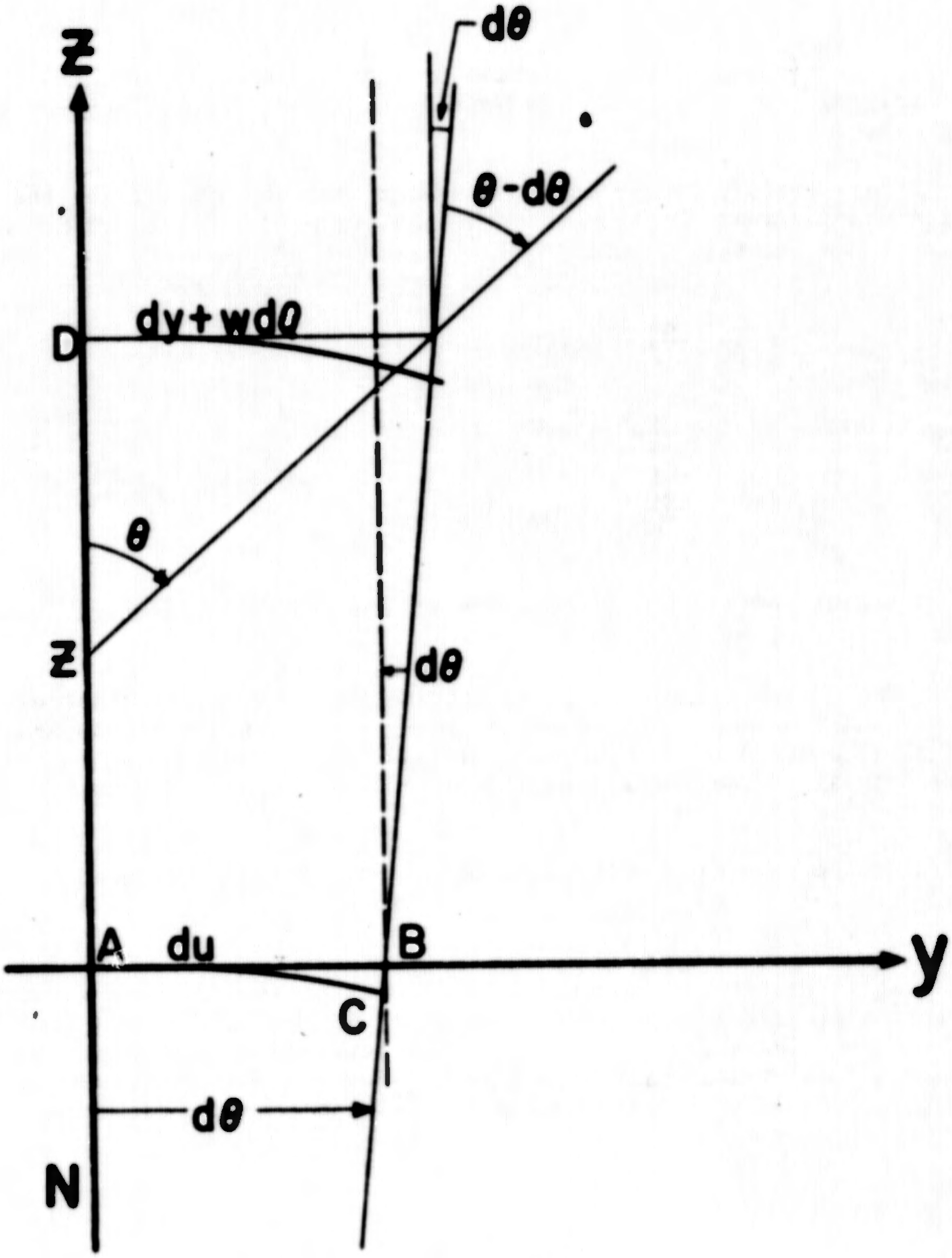

Figure 4

APPLICATION

For computational purposes the integrals in Equation 1 are replaced by sums giving

$$\theta = \theta_0 + \sum_R \left[\frac{G\sin\bar{\theta}}{2\bar{T}} + \frac{H\sin\bar{\theta}}{\bar{C}} - \frac{\bar{C}\sin\bar{\theta}+\bar{W}}{\bar{C}(N+\bar{z})} \right] \Delta r \qquad (27)$$

where the bar indicates a weighted or mean value over the interval, Δr. By varying Δr, Equation 27 can be made to approximate (1) by any desired accuracy.

Very satisfactory results have been obtained by imposing the condition that Δr shall be less than the smaller of $0.1/G$ and $0.2/H$. Substituting the first into Equation 14

$$d\theta < \frac{0.1}{G} \frac{G\sin\theta}{2T} \sim .0002 \text{ radians.} \qquad (28)$$

Substituting the second into Equation 22

$$d\theta < \frac{0.2}{H} \frac{H\sin\theta}{C} \sim .0002 \text{ radians.} \qquad (29)$$

The third fraction of Equation 27 always gives a $d\theta$ much less than the first two.

One possible approach is to iterate into a consistent set of values for $\bar{\theta}$ and $\bar{T}$ using the first of the three fractions since both are functions of r. The calculation of the second and third fractions will then follow directly since they will depend on quantities related directly to $\bar{\theta}$, $\bar{T}$, and r.

At the same time that summation of $d\theta$ along R is carried out the summation of dQ, dU, dV, and dz can be made to determine time, down-range distance, off-range drift, and elevation.

An inspection of Figure 4 will show that the effect of the third fraction (correction for earth curvature) is equivalent to a translation of the coordinate origin from A to B, a rotation through angle $d\theta$, about B and a final translation from B to C. For the small values of $d\theta$ given by Equations 28 and 29 the effect of these on computed elevations is completely negligible.

If V is the total cross-wind drift and U the total down-range distance, the azimuth angle through which the wave front has been translated will be given approximately by dA = arc tan (V/U). This means that if A is the azimuth from the sound source to the detector the ray leaving at an angle A = dA will be the one which actually crosses the microphone.

Finally, by substituting the terminal value of θ in Equation 27 and making Δr negative, it is possible to backtrack along the ray path, duplicating in reverse the calculations made in the forward direction, subject to the usual minor rounding and computational errors.

Ray trace curves comparing the flat earth and curved earth models for a typical atmosphere are shown in Figure 5. Each curve is labeled with the initial zenith angle (upper) and initial elevation angle (lower). An examination will show that the range of initial zenith angles for which the acoustic ray would be turned back began at 77 degrees on the flat earth model while on the curved earth model the range started at 79 degrees.

In general, the differences in computed values on the two models are quite minor except for those rays which return to earth on the flat earth model and do not return on the curved earth model.

Table I shows the results of a ray trace forward followed by feeding the final values in as starting values for a new trace, setting Δr negative and tracing backwards. This is, of course, a measure of the ability of the program to obtain substantially the same results starting at either end of an acoustic ray, rather than a measure of overall accuracy. Comparisons have been made with existing "layer type" acoustic ray trace programs and the results are in good agreement.

CONCLUSIONS

The equations presented here have been used with success for acoustic ray tracing. One ray trace generally required between 30 and 120 seconds on the IBM 7044 computer.

The forward and backward calculations agree to about one part in ten thousand in time, distances, and zenith angles. Thus, knowing the direction and zenith angle of the normal to an acoustic wave front entering a microphone array, it is possible to backtrack to determine the points from which it could have originated.

In general, the curved earth model gives results very similar to the flat earth. However, the zenith angle correction of 1 and 2 degrees due to earth curvature in traversing 50 to 100 nautical miles may be sufficient to prevent trapping where the flat earth model would indicate a return to earth.

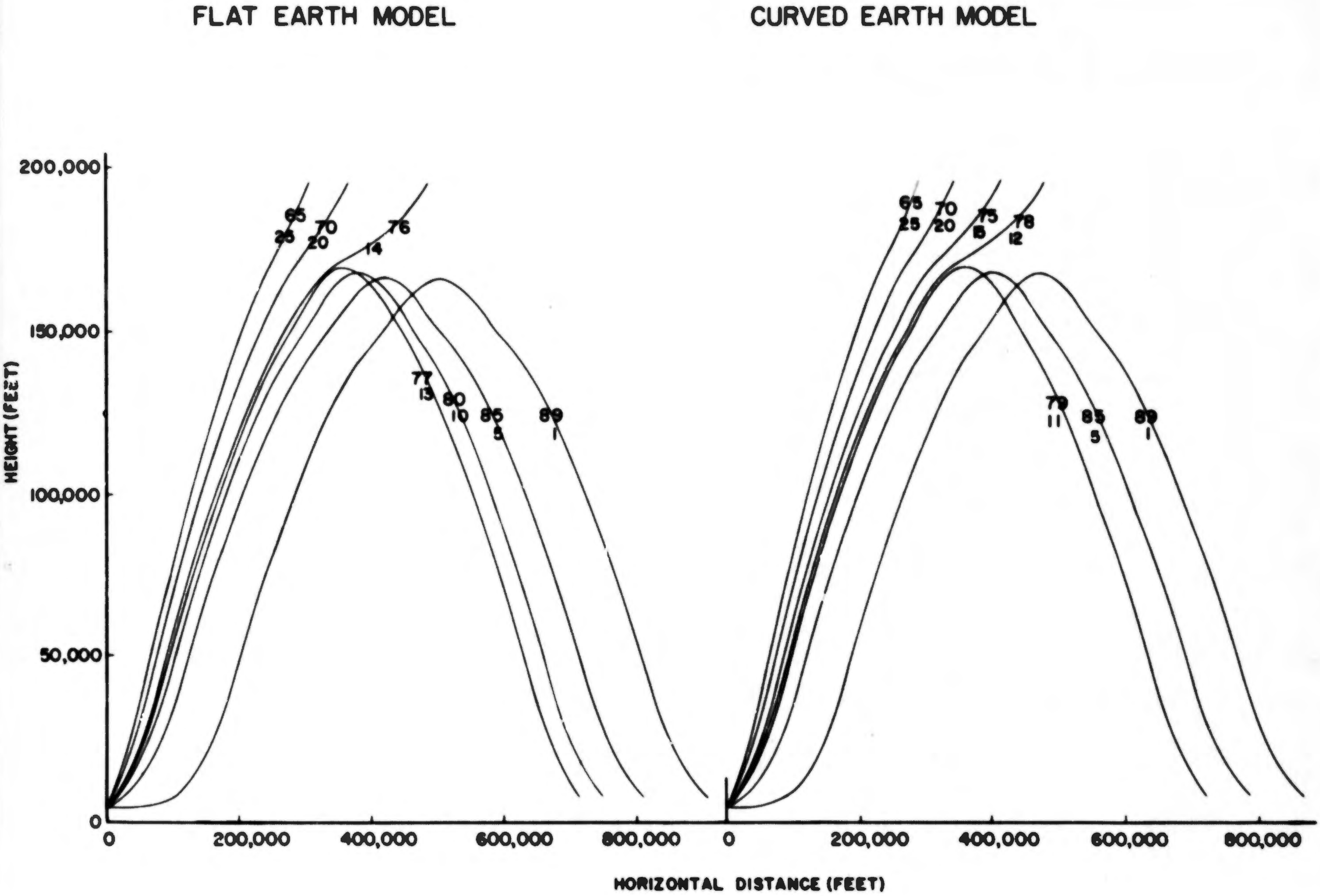
FLAT EARTH MODEL
CURVED EARTH MODEL
HEIGHT (FEET)
200,000
150,000
100,000
50,000
0
0
200,000
400,000
600,000
800,000
0
200,000
400,000
600,000
800,000
HORIZONTAL DISTANCE (FEET)
65
25
70
20
76
14
77
13
80
10
85
5
89
1
65
25
70
20
75
15
78
12
79
11
85
5
89
1

FIGURE 5

TABLE I

Z	U	V	ϕ	Q
5000	0	0	71.000	0
200000	372600	-23262	59.103	404.69
369573	615558	-23262	90.009	717.29
200000	858525	-23262	120.881	1029.89
8000	1222677	-46577	110.602	1426.64
8000	1222677	-46577	110.602	1426.64
200000	858638	-23272	120.899	1029.91
369576	615693	-23272	89.995	717.28
200000	372725	-23272	59.117	404.64
5000	109	-6	70.993	-.11

RAY TRACE RESULTS

Thus, for detailed acoustic propagation studies these procedures allow a more realistic evaluation of travel paths and time relationships and a method for estimating apparent azimuth shifts where applicable distances are involved.

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