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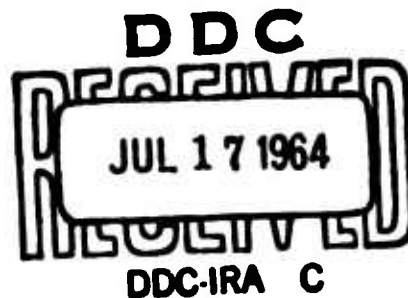
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Bayesian Single Sampling Attribute Plans
for Discrete Prior Distributions.

By

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Contents.

	Page
1. Introduction and summary.	1 - 2
2. The model.	2 - 10
3. The exact solution and the tables.	10 - 15
4. The asymptotic solution.	15 - 20
5. Comparison of exact and approximate solution.	20 - 28
6. Proportional change of (p_r, p_g, p_1, p_2) for fixed w_2 .	28 - 31
7. Change of p_g for fixed (p_r, p_1, p_2, w_2) .	32 - 33
8. Proportional change of (p_r, p_1, p_2) and change of w_2 .	33 - 34
9. Change of w_2 for fixed (p_r, p_g, p_1, p_2) .	35 - 39
10. Change of $p_r = p_g$ for fixed (p_1, p_2, w_2) .	39 - 41
11. Change of all parameters.	41 - 43
12. Efficiency.	43 - 49
13. An example.	49 - 51
14. General remarks.	51 - 54

References.

55

Appendix.

	1 - 33
Master tables for $p_r = 0.10$.	2 - 10
Master tables for $p_r = 0.01$.	11 - 26
Tables of conversion factors.	27 - 32
Summary of conversion formulas.	33

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1. Introduction and summary.

The main purpose of the present paper is to give a rather complete tabulation and discussion of properties of a system of single sampling attribute plans obtained by minimizing average costs under the assumptions that costs are linear in p , the fraction defective, and that the distribution of lot quality is a double binomial distribution.

Starting from a cost function containing 6 parameters and a mixed binomial prior distribution it is shown how the average costs may be written in a standard form containing only two parameters, p_r and p_s , besides the parameters defining the prior distribution. The one parameter, p_r , is the economic break-even quality and depends on the costs of acceptance and rejection only, whereas the second parameter, p_s , also depends on the costs of sampling inspection and the average quality. In a simple and practically important case p_r and p_s denote the costs of rejection and the costs of sampling inspection, respectively, divided by the costs of accepting a defective item.

Specializing the prior distribution to a double binomial distribution defined by the two quality levels (p_1, p_2) and the weights (w_1, w_2) , $w_1 + w_2 = 1$, it will be seen that the optimum sampling plan (n, c) depends on the 6 parameters $(N, p_r, p_s, p_1, p_2, w_2)$ where N denotes lot size. It may be shown, however, that the weights combine with the p 's in such a way that only 5 (independent) parameters are left.

A procedure to obtain the exact solution of the problem has been developed in a previous paper and this has been used for computing a set of master tables in which $p_r = p_s = 0.01$ and 0.10 , $w_2 = 0.05$, (p_1, p_2) take on suitably chosen values in relation to the value of p_r , and $1 \leq N \leq 200,000$.

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In the remaining part of the paper the properties of the optimum plans are studied with the purpose to derive simple conversion formulas which will make it possible to find the optimum plan for an arbitrary set of parameters from a plan in the master table with a "corresponding" set of parameters. The main tool for this investigation is the asymptotic expressions for the acceptance number and for the sample size, viz.

$$c = np_0 + a + o(1) \text{ and } n = \frac{1}{\varphi_0} (\ln N - \frac{1}{2} \ln \ln N + \ln \lambda + \frac{3}{2} \ln \varphi_0) + o(1),$$

where p_0 and φ_0 are functions of (p_1, p_2) only, whereas a and λ depend on the other parameters also. It is furthermore shown that the minimum value of the standardized costs per lot asymptotically equals the costs of sampling inspection plus a constant (depending on (p_1, p_2)) and that the producer's and the consumer's risks tend to zero inversely proportional to lot size. Numerical investigations show that the asymptotic expressions give good approximations to the optimum plan even for quite small values of N .

By means of the asymptotic formulas it is possible to find out how (n, c) vary with the individual parameters. One of the most important results is found by letting all the p 's tend to zero which leads to "the proportionality law": The optimum sampling plan corresponding to $(N, \lambda p_r, \lambda p_s, \lambda p_1, \lambda p_2, w_2)$ is approximately equal to $(n^*/\lambda, c^*)$ where (n^*, c^*) is the plan corresponding to $(N^*, p_r, p_s, p_1, p_2, w_2)$ with $N^* = N\lambda$.

This theorem combined with other similar results regarding the effect of varying the individual parameters lead to two general conversion formulas stated in sections 8 and 11. A summary of these formulas is given at the end of the paper in connection with the tables.

Efficiency of a sampling plan is defined as the ratio of the standardized costs (loss) of the optimum plan and the costs of the plan in question. Efficiency is discussed for various alternative systems and the efficiency of using optimum plans determined from wrong values of the parameters is studied.

Finally the present system is discussed in relation to other systems and it is pointed out that from an economic point of view it is not advisable to fix the consumer's or the producer's risk. If one wants a system with a fixed risk then the risk should be fixed to 50 per cent at a point between p_1 and p_2 . Two such IQL systems are then briefly discussed.

2. The model.

Several authors have studied economic models, mostly linear, for the determination of single sampling inspection plans by attributes, see for instance [1] and [2].

We shall here start from the formulation proposed by Guthrie and Johns [3] and show how the model may be reduced to a standard form as previously used by Hald [4].

Let N and n denote lot size and sample size and let X and x denote number of defectives in the lot and the sample, respectively. The acceptance number is denoted by c .

Let the costs be

$$nS_1 + xS_2 + (N-n)A_1 + (X-x)A_2 \quad \text{for } x \leq c \quad (1)$$

and

$$nS_1 + xS_2 + (N-n)R_1 + (X-x)R_2 \quad \text{for } x > c. \quad (2)$$

The interpretation of the six cost parameters depends on the kind of inspection envisaged, i.e. whether inspection is a consumers receiving inspection, a producers inspection of finished goods, or "internal inspection" by delivery of goods from one department to another within the same firm. The cost parameters may have quite different values when considered exclusively from a producer's or a consumer's point of view because certain costs are borne primarily by one of the parties involved. The values of the cost parameters also depend on whether the inspection is rectifying or non-rectifying, destructive or non-destructive. In the following the two cost expressions are discussed and a few examples of interpretation are given.

Costs associated with the sample, $nS_1 + xS_2$, for brevity called "costs of sampling inspection" consist of two parts: one part, nS_1 , proportional to the number of items in the sample so that S_1 includes sampling and testing costs per item, and another part, xS_2 , proportional to the number of defectives in the sample, i.e. S_2 denotes additional costs for an inspected defective item. If defective items found in the sample are repaired, say, then S_2 may include the repair costs per item.

"Costs of acceptance" are similarly composed of a part, $(N-n)A_1$, proportional to the number of items in the remainder of the lot, and another part, $(X-x)A_2$, proportional to the number of defective items accepted. Whereas A_1 usually will be zero or negligible, A_2 will often be considerable. If accepted items are used as parts in an assembly operation, say, A_2 may include the manufacturing costs (or the price) of an item, the costs of handling the defective item in assembling and disassembling, and the damage done to other parts used in the assembly. In case of inspection of finished goods A_2 may include costs of repair, service and guarantees plus loss of good-will.

"Costs of rejection" consist of a part, $(N-n)R_1$ proportional to the number of items in the remainder of the lot, and another part, $(X-x)R_2$, proportional to the number of defective items rejected. Rejection is here taken in a broad sense meaning only that

the lot cannot be accepted according to the sampling plan used. Rejection may therefore lead to sorting, price reduction, scrapping, or salvaging. If rejection means sorting, say, then R_1 includes sorting costs per item and R_2 denotes additional costs for defective items found, for example costs of repair or replacement.

It is obvious that from a practical point of view it will in general be easiest to obtain information on the values of the cost parameters in the case of "internal inspection".

Denoting the hypergeometric probability by

$$p(x|X) = \frac{\binom{n}{x} \binom{N-n}{X-x}}{\binom{N}{X}}$$

the average costs for lots of size N with X defectives become

$$K(N, n, c, X) = \sum_{x=0}^n (nS_1 + xS_2) p(x|X) + \sum_{x=0}^c ((N-n)A_1 + (X-x)A_2) p(x|X) + \sum_{x=c+1}^n ((N-n)R_1 + (X-x)R_2) p(x|X) \quad (3)$$

Let $f_N(X)$ denote the (prior) distribution of X , i.e. the distribution of lot quality. The average costs then become

$$K(N, n, c) = \sum_X K(N, n, c, X) f_N(X). \quad (4)$$

As shown in [4] this expression becomes linear in N for the important class of mixed binomial distributions, i.e. for

$$f_N(X) = \int_0^1 \binom{N}{X} p^X q^{N-X} dW(p) \quad (5)$$

where $W(p)$ denotes a cumulative distribution function (independent of N).

From (3) - (5) we find

$$K(N, n, c) = \int_0^1 K(N, n, c, p) dW(p) \quad (6)$$

where

$$K(N, n, c, p) = n(S_1 + S_2 p) + (N-n)((A_1 + A_2 p)P(p) + (R_1 + R_2 p)Q(p)), \quad (7)$$

$$P(p) = B(c, n, p) = \sum_{x=0}^c \binom{n}{x} p^x q^{n-x}, \quad (8)$$

and $Q(p) = 1 - P(p)$.

For convenience the frequency function corresponding to $W(p)$ will be called the distribution of the process average or the distribution of p as distinct from $f_N(X)$

where

which gives the distribution of X/N , i.e. the distribution of lot quality. (The following discussion will be in terms of p).

Limiting the prior distributions to mixed binomials (6) shows that the average costs may be considered as an average of the cost function (7), which is a function of p , with respect to the distribution of p . It should be noted that this result is valid for any (N, n) for a mixed binomial prior distribution and that a similar result holds for $N \rightarrow \infty$, $n \rightarrow \infty$, and $n/N \rightarrow 0$, for any prior distribution. The limit theorems derived in the following may therefore be applied in general.

The sampling plans discussed are obtained by minimizing $K(N, n, c)$ according to (6) with respect to (n, c) for given cost parameters and prior distribution and they will be called Bayesian single sampling plans or optimum plans.

Starting from (7) we introduce the three cost functions

$$k_s(p) = S_1 + S_2 p, \quad (9)$$

$$k_a(p) = \Lambda_1 + \Lambda_2 p, \quad (10)$$

and

$$k_r(p) = R_1 + R_2 p, \quad (11)$$

defined for $0 \leq p \leq 1$. We shall make the following assumptions regarding these functions:

1. All three functions are non-negative and none of them is identical zero.
2. $k_a(0) < k_r(0)$ and $k_a(1) > k_r(1)$, from which follows that the equation $k_a(p) = k_r(p)$ has the solution

$$p_r = (R_1 - \Lambda_1) / (\Lambda_2 - R_2), \quad 0 < p_r < 1, \quad (12)$$

p_r being called the (economic) break-even quality.

3. $k_s(p) \geq k_m(p)$ for $0 \leq p \leq 1$, where

$$k_m(p) = \begin{cases} k_a(p) & \text{for } p \leq p_r \\ k_r(p) & \text{for } p > p_r. \end{cases} \quad (13)$$

The function $k_m(p)$ gives the unavoidable (minimum) costs, i.e. the costs corresponding to the situation where perfect knowledge of quality exists without costs and all lots (processes) are classified correctly on that basis, viz. accepted for $p \leq p_r$ and rejected for $p > p_r$.

Averages over the prior distribution are denoted by k_s, k_a , etc., i.e.

$$k_a = \int_0^1 k_a(p) dW(p) = k_a(\bar{p}) = A_1 + A_2 \bar{p}, \quad (14)$$

and

$$k_m = \int_0^1 k_m(p) dW(p) = \int_0^{P_r} k_a(p) dW(p) + \int_{P_r}^1 k_r(p) dW(p). \quad (15)$$

Costs per item are denoted by k , costs per lot by the corresponding K , i.e. $K = Nk$.

The average costs for the three cases without sampling inspection, i.e. the cases where

- (a) all lots are classified correctly,
- (b) all lots are accepted, and
- (c) all lots are rejected,

then become k_m , k_a , and k_r , respectively. These cases are useful "reference cases" since sampling inspection is justified only if $k - k_m < \min \{k_a - k_m, k_r - k_m\}$, where $k = K(N, n, c)/N$.

Case (a) will usually be considered as the basic reference case and average costs for other cases will therefore be reduced by k_m , since k_m represents the average fixed costs per item which will be incurred irrespective of the decision made. The cost differences

$$k_a - k_m = \int_{P_r}^1 (k_a(p) - k_r(p)) dW(p)$$

and

$$k_r - k_m = \int_0^{P_r} (k_r(p) - k_a(p)) dW(p)$$

represent average decision losses in case (b) and (c) respectively, and $k_s - k_m$ represents the average "loss" by inspection.

From (6) and (15) we find

$$K = nk_s + (N-n) \int_0^1 (k_a(p)P(p) + k_r(p)Q(p)) dW(p) \quad (16)$$

and

$$K_m = nk_m + (N-n) \left\{ \int_0^{P_r} k_a(p) dW(p) + \int_{P_r}^1 k_r(p) dW(p) \right\}$$

leading to

$$\begin{aligned} K - K_m &= n(k_s - k_m) + (N-n) \left\{ \int_0^{P_r} (k_r(p) - k_a(p)) Q(p) dW(p) + \int_{P_r}^1 (k_a(p) - k_r(p)) P(p) dW(p) \right\} \\ &= n(k_s - k_m) + (N-n) (A_2 - R_2) \left\{ \int_0^{P_r} (P_r - p) Q(p) dW(p) + \int_{P_r}^1 (p - P_r) P(p) dW(p) \right\}, \end{aligned} \quad (17)$$

the two terms giving the average costs of sampling inspection and the average

Instead of minimizing K with respect to (n, c) we might just as well minimize $K - K_m$, $(K - K_m)/(A_2 - R_2)$, or $(K - K_m)/(k_s - k_m)$, since K_m , $A_2 - R_2$, and $k_s - k_m$ are independent of (n, c) . It will be seen from (17) that it is practical to use $A_2 - R_2$ or $k_s - k_m$ as "economic unit".

Defining

$$p_m = \int_0^{p_r} p dW(p) + \int_{p_r}^1 p_r dW(p) = p_r - \int_0^{p_r} (p_r - p) dW(p), \quad (18)$$

we find $0 \leq p_m \leq p_r$ and

$$p_r - p_m = (k_r - k_m)/(A_2 - R_2). \quad (19)$$

Defining p_s by means of

$$p_s - p_m = (k_s - k_m)/(A_2 - R_2) \quad (20)$$

we find

$$p_s - p_r = (k_s - k_r)/(A_2 - R_2) \quad (21)$$

and

$$p_s = \{(S_1 - A_1) + (S_2 - R_2)\bar{p}\}/(A_2 - R_2). \quad (22)$$

Introducing

$$R^*(N, n, c) = [K(N, n, c) - K_m]/(A_2 - R_2) \quad (23)$$

we find the standard form

$$R^* = n(p_s - p_m) + (N - n) \left\{ \int_0^{p_r} (p_r - p) Q(p) dW(p) + \int_{p_r}^1 (p - p_r) P(p) dW(p) \right\} \quad (24)$$

containing only two parameters p_r and $p_s - p_m$, instead of the six cost parameters in the original model, see [4]. It should be noted that $p_s - p_m$ depends on the prior distribution besides on the cost parameters.

Consider the special case given by $k_a(p) = A_2 p$, $k_r(p) = R_1$, and $k_s(p) = S_1$, which is a model commonly used in practice. It follows that $p_r = R_1/A_2$ and $p_s = S_1/A_2$, i.e. p_r and p_s are the costs of rejection and of sampling and testing, respectively, measured with the cost of accepting a defective item. This simple interpretation of p_r and p_s is one of the reasons for using them as parameters.

It is often useful to discuss the problem in terms of the simple cost functions $k_a(p) = p$, $k_r(p) = p_r$, and $k_s(p) = p_s$, which immediately lead to the form (24). The corresponding form of (7) becomes

$$K_0(p) = np_s + (N - n)(pP(p) + p_r Q(p))$$

from which the general form may be found as

$$K(p) = (A_2 - R_2)K_0(p) + (nS_2 + (N - n)R_2)p + (NA_1 - n(S_2 - R_2)\bar{p}).$$

A sketch of the cost functions for a typical case has been given in Fig. 1, which is based on the data in section 13.

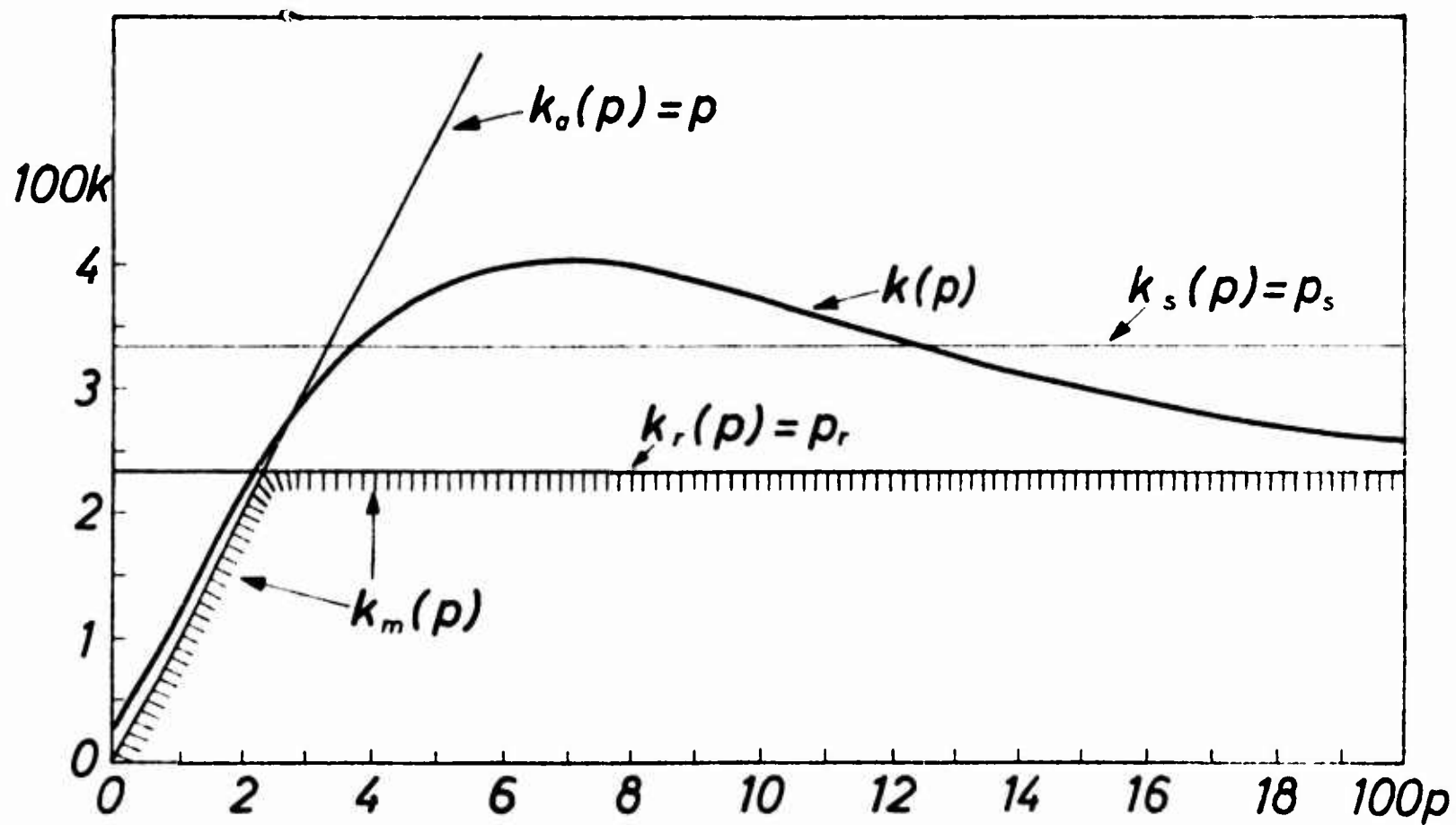
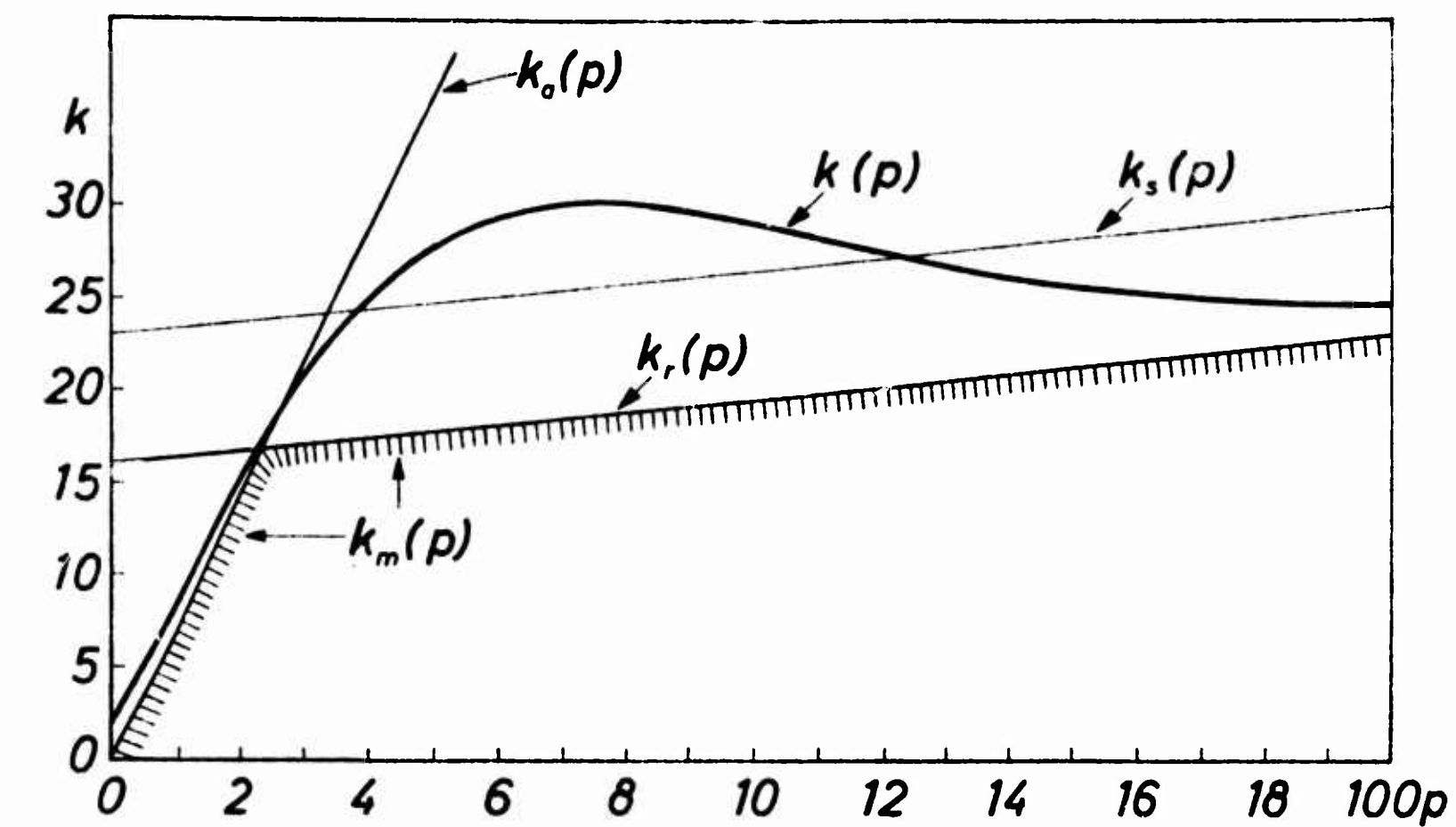


Fig. 1
Example of cost functions.

For some purposes it is useful to use $k_s - k_m$ as economic unit instead of $A_2 - R_2$.
Putting

$$R(N, n, c) = \{K(N, n, c) - K_m\} / (k_s - k_m),$$

i.e.

$$R = R^* / (p_s - p_m),$$

we find

$$R = n + \frac{N - n}{p_s - p_m} \left\{ \int_0^{p_r} (p_r - p) Q(p) dW(p) + \int_{p_r}^1 (p - p_r) P(p) dW(p) \right\}, \quad (25)$$

the two terms again giving the costs of sampling inspection and the average decision losses, respectively, but here using the average costs of sampling inspection (minus k_m) per item in the sample as economic unit.

In the next section we shall discuss the determination of (n, c) for a double binomial distribution as prior distribution. This means that p is a random variable taking on only two values, $p_1 < p_r < p_2$, with probabilities w_1 and $w_2 = 1 - w_1$, respectively. From (25) we then find

$$R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2)) \quad (26)$$

where

$$\gamma_i = |p_i - p_r| w_i / (p_s - p_m) = |k_a(p_i) - k_r(p_i)| w_i / (k_s - k_m), \quad i=1, 2, \quad (27)$$

$$p_m = p_1 w_1 + p_r w_2, \quad (28)$$

i.e. R depends on four parameters only, viz. $p_1, p_2, \gamma_1, \gamma_2$.

The correspondingly standardized costs for the cases of acceptance and rejection without inspection are

$$R_a = N(k_a - k_m) / (k_s - k_m) = N\gamma_2 \quad (29)$$

and

$$R_r = N(k_r - k_m) / (k_s - k_m) = N\gamma_1. \quad (30)$$

These results may also be obtained from (26) for $n = 0$ by setting $P(p) = 1$ and 0 , respectively.

If acceptance without inspection is cheaper than rejection without inspection, i.e. $k_a < k_r$ we find $\bar{p} < p_r$ and $\gamma_2 < \gamma_1$.

In the special case $k_s = k_r$ we have $p_s = p_r$ and $\gamma_1 = 1$ so that the model contains only three parameters.

It should be noted that

$$\gamma_2 = \frac{\bar{p} - p_m}{p_s - p_m} = 1 - \frac{p_s - \bar{p}}{p_s - p_m} \quad (31)$$

and

$$\gamma_1 = \frac{p_r - p_m}{p_s - p_m} = 1 - \frac{p_s - p_r}{p_s - p_m}. \quad (32)$$

3. The exact solution and the tables.

In a previous paper [4] we have proved the following theorem:

For a double binomial (prior) distribution of lot quality given by the parameters (p_1, p_2, w_2) and for linear cost functions (1) and (2) the Bayesian single sampling plan may be found by minimizing $R(N, n, c)$, see (26), with respect to (n, c) . The solution satisfies the two inequalities

$$\alpha + \beta c \leq n < \alpha + \beta(c + 1) \quad (33)$$

and

$$F(n - 1, c) \leq N < F(n, c) \quad (34)$$

where

$$\alpha = \log \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)} / \log \frac{q_1}{q_2} = \log \frac{\gamma_2}{\gamma_1} / \log \frac{q_1}{q_2}, \quad (35)$$

$$\beta = \log \frac{p_2 q_1}{q_2 p_1} / \log \frac{q_1}{q_2}, \quad (36)$$

and

$$F(n, c) = n + 1 + \frac{p_s - p_r + \sum_i w_i (p_r - p_i) B(c, n, p_i)}{\sum_i w_i (p_i - p_r) p_i b(c, n, p_i)}. \quad (37)$$

For two plans (n_1, c_1) and (n_2, c_2) , $c_1 < c_2$ say, satisfying (33) and having overlapping N-intervals according to (34) $R(N, n_1, c_1) \leq R(N, n_2, c_2)$ for $N \leq N_{12}$ where

$$N_{12} = \frac{(p_s - p_r)(n_2 - n_1) + n_2 \gamma(n_2, c_2) - n_1 \gamma(n_1, c_1)}{\gamma(n_2, c_2) - \gamma(n_1, c_1)} \quad (38)$$

and

$$\gamma(n, c) = \sum_i w_i (p_r - p_i) B(c, n, p_i). \quad (39)$$

In [4] the theorem was derived as a special case of a more general one. We shall here derive the theorem directly from (26) using the same method as in [4].

Values of (n, c) minimizing R must satisfy the two inequalities

$$\Delta_c R(N, c-1, n) \leq 0 < \Delta_c R(N, c, n), \quad 0 \leq c \leq n, \quad (40)$$

and

$$\Delta_n R(N, c, n-1) \leq 0 < \Delta_n R(N, c, n), \quad c \leq n \leq N, \quad (41)$$

Δ denoting the usual forward difference operator.

Noting that $\Delta_c B(c, n, p) = b(c+1, n, p)$ and $\Delta_n B(c, n, p) = -pb(c, n, p)$ we find from (26)

$$\Delta_c R(N, n, c) = (N-n) \{-\gamma_1 b(c+1, n, p_1) + \gamma_2 b(c+1, n, p_2)\} \quad (42)$$

and

$$\Delta_n R(N, n, c) = 1 - \{\gamma_1 Q(p_1) + \gamma_2 P(p_2)\} + (N-n-1) \{\gamma_1 p_1 b(c, n, p_1) - \gamma_2 p_2 b(c, n, p_2)\} \quad (43)$$

Inserting these expressions into (40) and (41) and solving for n and N , respectively, immediately leads to (33) and (34). From $R(N, n_1, c_1) = R(N, n_2, c_2)$ we next determine N_{12} by solving for N .

A sketch of R as function of n and c for fixed N has been given in Fig. 2 for a typical case.

The economic interpretation of (40) and (42) is the following: For given n the optimum value of c is determined such that a change of c , an increase by 1 say, will give nearly no change of the total decision loss, since the loss due to the increased consumers' risk is nearly balanced by the gain due to the smaller producers' risk.

Similarly the interpretation of (41) and (43) is that for given c the optimum value of n is determined such that a change of n , an increase by 1 say, will give nearly no change of total costs, since the increase of sampling inspection costs by 1 minus the average decision loss for one item is nearly balanced by the decrease in decision losses for the remainder of the lot.

Tabulation of optimum plans may be carried out by starting from the smallest value of c giving a positive n ($n \geq c$) according to (33), i.e. $c_m = \lceil -\alpha/(\beta-1) \rceil$, $\lceil \cdot \rceil$ denoting "the integer part of". For consecutive values of c , n - and N -intervals are computed from (33) and (34) and in case of overlapping N -intervals costs are compared by means of (38). A detailed example may be found in [4]. The tables have been computed by this method on an electronic computer.

The sampling plans have been tabulated for two "quality levels", viz. $p_r = p_s = 0.01$ and 0.10 , for one value of the weight function $w_2 = 0.05$, for 8 values of p_1/p_r , and for 10, respectively 5, values of p_2/p_r , giving a total of 120 tables. Each table gives (n, c) as function of N for $N \leq 200,000$.

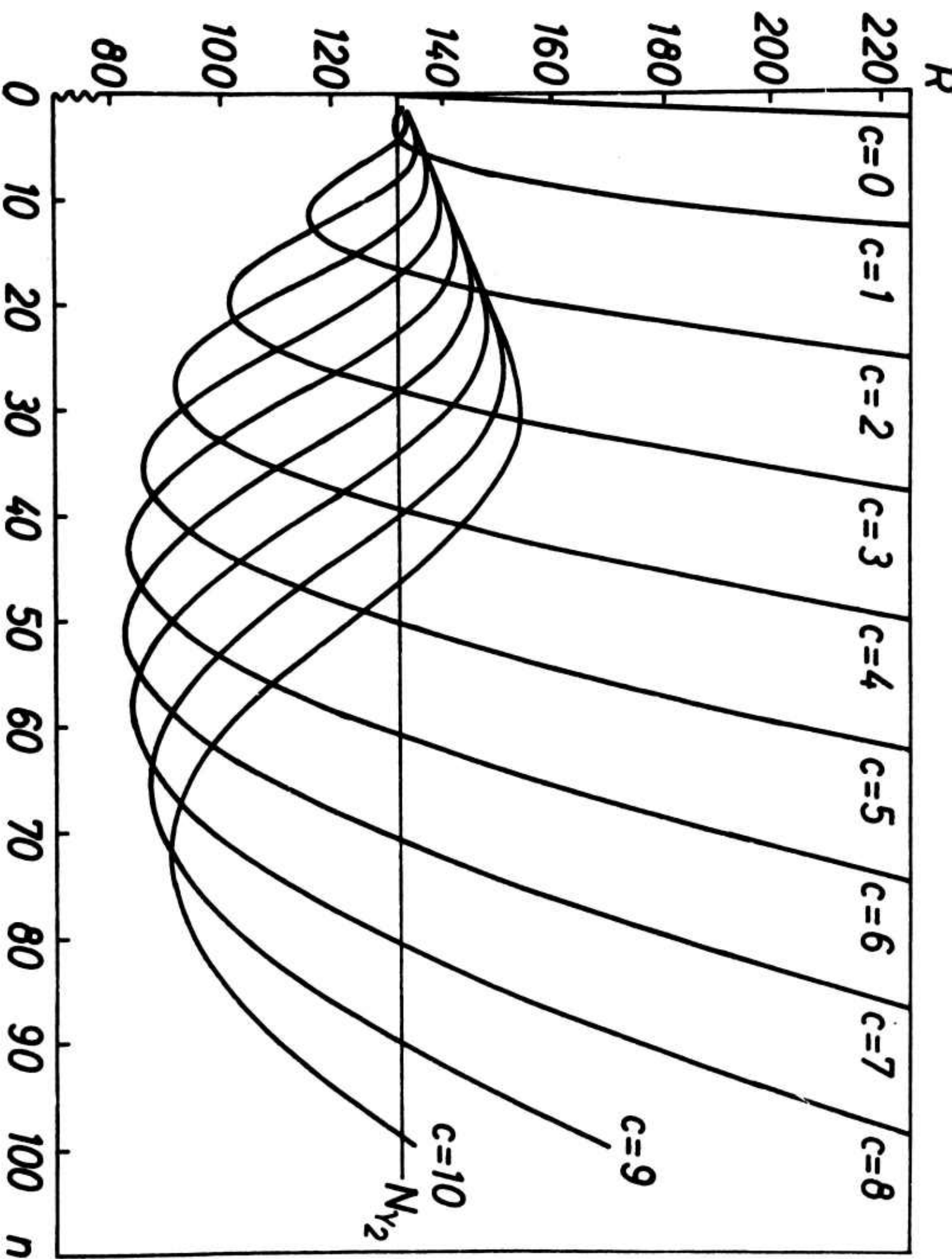


Fig. 2

$R(N, n, c)$ as function of n and c for $N = 1000$, $P_r = P_s = 0.10$, $P_1 = 0.06$, $P_2 = 0.20$, and $w_2 = 0.05$.

For $p_r = 0.01$ the search for optimum plans has been limited to values of n which are multiples of 5.

These tables will be referred to as "master tables" since optimum plans for other values of the parameters may easily be found from the tabulated ones by means of conversion formulas developed in the following sections.

The exact solution has been modified in one respect. For a given value of c the first and last N -interval may be rather short as compared to the other intervals. As an example consider the following section of the original table for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.020$, $w_2 = 0.05$:

N	n	c	ΔN
4010 - 4370	165	3	360
4370 - 4420	170	3	50
4420 - 4430	240	4	10
4430 - 4920	245	4	490
4920 - 5570	250	4	650
5570 - 5590	255	4	20
5590 - 5610	325	5	20
5610 - 6250	330	5	640

The example shown is an extreme one with small intervals occurring at the beginning as well as at the end of each section of the table. It is naturally without any interest to use the sampling plan (240,4) for $4420 < N < 4430$ and then change to (245,4) for $4430 < N < 4920$. To eliminate such small intervals from the final table it was decided to discard the first and the last sampling plan for a given c if the length of the corresponding N -interval was less than $1/5$ of the length of the neighbouring interval. In such cases the value of N according to (38) was computed for the new neighbouring plans, (165,3) and (245,4) say, to find the optimum N -intervals for the remaining plans. The result of this procedure is in most cases practically equal to incorporating the small N -intervals into the larger neighbouring intervals, for example using (245,4) for $4420 < N < 4920$.

To save space every second N -interval for a given value of c has been omitted because the corresponding sampling plans may be found by adding 1 ($p_r = 0.10$) and 5 ($p_r = 0.01$), respectively, to n for the preceding interval.

Values of N have been rounded to 3 significant figures and tabulation has been stopped at $N = 200,000$.

As mentioned above the tables were designed as master tables from which optimum plans may be derived for other values of the parameters and for this reason it was decided to tabulate the complete solution with respect to N to make interpolation superfluous.

The user of the tables in practice may easily derive a simplified set of tables from the given ones, either by using a set of fixed N-intervals, or a set of fixed N-arguments. An example has been given in the following table.

Single Sampling Tables for $100p_r = 100p_s = 1.0$, $100p_1 = 0.5$, and $w_2 = 0.05$.

$100p_2$	1.5		2.0		3.0		4.0		5.0		6.0		7.0	
N	n	c	n	c	n	c	n	c	n	c	n	c	n	c
20											Accept		5	0
30									Accept		5	0	5	0
50									5	0	10	0	10	0
70									5	0	10	0	15	0
100							Accept		10	0	15	0	15	0
200							10	0	20	0	20	0	25	0
300							15	0	50	1	50	1	50	1
500					Accept		55	1	60	1	60	1	55	1
700					45	1	65	1	65	1	65	1	60	1
1000					55	1	110	2	105	2	100	2	90	2
2000					125	2	170	3	155	3	140	3	100	2
3000			Accept		195	3	180	3	160	3	145	3	135	3
5000			185	3	265	4	235	4	210	4	185	4	140	3
7000			275	4	330	5	290	5	215	4	190	4	170	4
10000			450	6	400	6	295	5	260	5	195	4	175	4
20000	Accept		640	8	475	7	355	6	310	6	240	5	215	5
30000	755	9	735	9	545	8	410	7	315	6	280	6	220	5
50000	1080	12	920	11	620	9	470	8	365	7	285	6	255	6
70000	1300	14	1095	13	690	10	475	8	410	8	325	7	260	6
100000	1520	16	1190	14	760	11	530	9	415	8	330	7	290	7

The 'natural' parameters of the model are (p_1, p_2, w_2) , which characterize the prior distribution, and (p_r, p_s) , which depend on the costs. The tables and the properties of the solution will be discussed in terms of these parameters on basis of the results in the next section. However, one property may be stated immediately from the observation that the solution depends on four parameters only, viz. $(p_1, p_2, \gamma_1, \gamma_2)$. The three parameters (p_r, p_s, w_2) may therefore in respect to the solution be considered as functionally related, i.e. combinations of (p_r, p_s, w_2) giving the same (γ_1, γ_2) will lead to the same sampling plan.

From

$$\frac{\gamma_2}{\gamma_1} = \frac{(p_2 - p_r)w_2}{(p_r - p_1)w_1}$$

we find

$$p_r - p_1 = (p_2 - p_1) / (1 + \frac{\gamma_2 w_1}{\gamma_1 w_2}). \quad (44)$$

From

$$\gamma_1 (p_s - p_m) = (p_r - p_1)w_1$$

and

$$p_s - p_m = p_s - p_1 - w_2 (p_r - p_1)$$

we find

$$p_s - p_1 = (p_r - p_1) (\frac{w_1}{\gamma_1} + w_2). \quad (45)$$

2 4 6 8 10 12 14 R-11 36 38 40 42 44 46 48 50

These formulas show how p_r and p_s depend on w_2 for given $(p_1, p_2, \gamma_1, \gamma_2)$. To use them in connection with the master tables we put $p_r = p_s$ and $w_2 = 0.05\lambda$ which leads to

$$p_r(\lambda) = p_{10} + (p_{20} - p_{10}) / (1 - \gamma_{20} + \frac{20\gamma_{20}}{\lambda})$$

where

$$\gamma_{20} = \frac{p_{20} - p_{ro}}{19(p_{ro} - p_{10})} = \frac{\rho_2 - 1}{19(1 - \rho_1)},$$

the index o denoting an argument in the master table, $p_{ro} = 0.01$ or 0.10 , $\rho_i = p_{io}/p_{ro}$. Dividing by p_{ro} gives

$$p_r(\lambda)/p_{ro} = \rho_1 + (\rho_2 - \rho_1) / (1 - \gamma_{20} + \frac{20\gamma_{20}}{\lambda}) = f(w_2, \rho_1, \rho_2) \quad (46)$$

which has been tabulated in the appendix.

The field of application of the master tables may therefore be considerably enlarged by making use of the following rule:

The optimum sampling plan for $(N, p_{ro}, p_{10}, p_{20}, w_2 = 0.05)$, $p_{ro} = p_{so}$, is the same as the plan for $(N, p_{ro} f(w_2, \rho_1, \rho_2), p_{10}, p_{20}, w_2)$.

Consider for example the case with $p_{ro} = p_{so} = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, and $w_2 = 0.05$ for which the optimum plans have been given in the master table. The same plans are also optimum for $w_2 = 0.20$, say, and $p_r = p_s = 0.019$, $p_1 = 0.006$, and $p_2 = 0.040$ which may be seen by interpolation in the table of $f(w_2, \rho_1, \rho_2)$ for $\rho_1 = 0.6$ and $\rho_2 = 4.0$.

4. The asymptotic solution.

In this section we shall give a somewhat simpler and more direct proof of the asymptotic results found by Guthrie and Johns [3] and by Hald [4], and furthermore carry the asymptotic expansion so far that we get a useful approximation to the exact solution also for small values of c .

The proof is based on the following lemma which is a special case of a theorem proved by Blackwell and Hodges [5]:

For $c/n = h = p_o + \varepsilon$, p_o being a constant and $\varepsilon \rightarrow 0$ for $n \rightarrow \infty$, we have

$$P(p) \approx \frac{1}{\sqrt{2\pi p_o q_o}} \frac{q_o p}{p - p_o} e^{-n\varphi(h, p)} (1 + O(\sqrt{\varepsilon})) \text{ for } p_o < p, \quad (47)$$

where

$$\varphi(h, p) = h \ln \frac{h}{p} + (1 - h) \ln \frac{1-h}{1-p}. \quad (48)$$

For $p_0 > p$ the same expression is valid for $Q(p)$ if only $p - p_0$ is replaced by $p_0 - p$.

Writing

$$\varphi(h, p) = \varphi(p_0, p) + \varepsilon \varphi'(p_0, p) + O(\varepsilon^2) \quad (49)$$

where

$$\varphi'(p_0, p) = \ln \frac{p_0 q}{q_0 p} \quad (50)$$

we find from (26) and (47) the asymptotic expression

$$R = n + (N-n) \frac{q_0}{\sqrt{2\pi p_0 q_0}} \sum_{i=1}^2 \frac{\gamma_i p_i}{|p_0 - p_i|} e^{-n\varphi(p_0, p_i) - n\varepsilon \varphi'(p_0, p_i)} (1 + O(\sqrt{\varepsilon})) \quad (51)$$

on the assumption that $p_1 < p_0 < p_2$. (As will be shown later $\varepsilon = O(1/n)$, and we may therefore disregard $n\varepsilon^2$). We shall first determine the value of $h = p_0 + \varepsilon$ which minimize R for given n and next determine the value of n giving the absolute minimum by treating R as a differentiable function of h and n .

The essential feature of (51) is that the two binomial risks, $Q(p_1)$ and $P(p_2)$, have been expressed as functions tending exponentially to zero for $n \rightarrow \infty$.

As explained in [4] the optimum plan must have the property that $R/N \rightarrow 0$ for $N \rightarrow \infty$, $n \rightarrow \infty$, and $n/N \rightarrow 0$. It follows that p_0 must satisfy the inequality $p_1 < p_0 < p_2$ because otherwise R/N would not tend to zero but to γ_1 or γ_2 .

We shall state the theorem to be proved for the double binomial distribution only, but it is valid for a more general class of distributions, viz. for a distribution having probability density $w(p) = 0$ for $p_1 < p < p_2$, $w(p_1) = w_1 > 0$, $w(p_2) = w_2 > 0$, $w_1 + w_2 \leq 1$, and

$$\int_0^{p_1^*} dW(p) + \int_{p_2^*}^1 dW(p) = 1 - w_1 - w_2$$

for $0 \leq p_1^* < p_1$ and $p_2 < p_2^* \leq 1$, which means that the probability distribution may be arbitrary outside the interval $p_1^* < p < p_2^*$. The result of such a generalization will only be to add a term to (51) of form

$$\frac{N-n}{p_s - p_m} \frac{q_0}{\sqrt{2\pi p_0 q_0}} \int_I \frac{(p_r - p)p}{p_0 - p} e^{-n\varphi(h, p)} dW(p),$$

(I denoting the intervals $(0 \leq p \leq p_1^*)$ and $(p_2^* \leq p \leq 1)$) which obviously is $O(e^{-n})$

times the last term of (51) since $\varphi(h,p) > \varphi(h,p_1)$ for $p < p_1$ and $\varphi(h,p) > \varphi(h,p_2)$ for $p > p_2$.

Because of the factor $p_r - p$ in the cost function we might also have assumed that $w(p_r) > 0$ without altering the result.

It is reasonable to assume that the two exponential terms in (51) tend to zero with the same speed, i.e. that p_0 is determined from

$$\varphi(p_0, p_1) = \varphi(p_0, p_2)$$

which gives

$$p_0 = \ln \frac{q_1}{q_2} / \ln \frac{p_2 q_1}{q_2 p_1} = \frac{1}{\beta} \quad (52)$$

and

$$\varphi_0 = p_0 \ln \frac{p_0}{p_i} + q_0 \ln \frac{q_0}{q_i}, \quad i = 1 \text{ or } 2. \quad (53)$$

Under this assumption we shall determine ϵ by minimization of (51). The part of R depending on ϵ is

$$f(\epsilon) = \sum_i \frac{\gamma_i p_i}{|p_0 - p_i|} e^{-n\epsilon \varphi'(p_0, p_i)}.$$

From $f'(\epsilon) = 0$ we find

$$\sum_{i=1}^2 \frac{\gamma_i p_i \varphi'_i}{|p_0 - p_i|} e^{-n\epsilon \varphi'_i} = 0 \quad (54)$$

where - according to (50) -

$$\varphi'_i = \ln \frac{p_0 q_i}{q_0 p_i}. \quad (55)$$

Solving for $a = n\epsilon$ we find

$$a\delta'_0 = \ln \frac{\gamma_1 p_1 (p_2 - p_0) \varphi'_1}{\gamma_2 p_2 (p_0 - p_1) (-\varphi'_2)} \quad (56)$$

where

$$\delta'_0 = \varphi'_1 - \varphi'_2 = \ln \frac{p_2 q_1}{q_2 p_1}. \quad (57)$$

We thus have the result that $c = np_0 + a + o(1)$ in accordance with what could be expected from (33).

Inserting these results into (51) we find

$$R = n + (N-n) \frac{\lambda}{\sqrt{n}} e^{-n\varphi_0} \quad (58)$$

with

$$\lambda = \frac{q_0}{\sqrt{2\pi p_0 q_0}} \sum_{i=1}^2 \frac{\gamma_i p_i}{|p_0 - p_i|} e^{-a\varphi'_i}. \quad (59)$$

To prove (indirectly) that $h = p_0 + \varepsilon$ minimizes R let us assume that $h = p_0 + \varepsilon$, given by (52) and (56), does not minimize R but that $\min R$ is obtained for $h = h_0 + \varepsilon_0$, $h_0 \neq p_0$ and $\varepsilon_0 \rightarrow 0$. Denoting the part of R depending on h by $g(h)$ we find for sufficiently large n and for $h_0 < p_0$, say, that

$$g(h_0) = \lambda_1(h_0) e^{-n\varphi(h_0, p_1)} (1 + o(e^{-n}))$$

since $\varphi(h_0, p_2) > \varphi(h_0, p_1)$ for $h_0 < p_0$. However, $g(h_0)$ cannot be $\min g(h)$ since $\varphi(h_0, p_1) < \varphi(p_0, p_1)$, i.e. we have reached a contradiction by assuming $h_0 \neq p_0$.

From $dR/dn = 0$ we find

$$1 - (N-n) \frac{\lambda}{\sqrt{n}} e^{-n\varphi_0} (\varphi_0 + \frac{1}{2n}) - \frac{\lambda}{\sqrt{n}} e^{-n\varphi_0} = 0 \quad (60)$$

or

$$\ln(N-n) = \varphi_0 n + \frac{1}{2} \ln n - \ln(\lambda \varphi_0) + o(1). \quad (61)$$

From (58) and (60) we also have that

$$\min_{(n, c)} R = n + \frac{1}{\varphi_0} + o(1) \quad (62)$$

where n may be determined by inversion of (61), i.e.

$$n = \frac{1}{\varphi_0} (\ln N - \frac{1}{2} \ln \ln N + \ln \lambda + \frac{3}{2} \ln \varphi_0) + o(1).$$

We have thus found that asymptotically c is a linear function of n and n is proportional to $\ln N - \frac{1}{2} \ln \ln N$ plus a constant. Furthermore it follows from (62) that the average decision loss per lot tends to a constant $1/\varphi_0$ so that for large lots decision losses divided by sampling inspection costs tend to zero.

To investigate the two risks asymptotically we find from (54)

$$\frac{\gamma_1 p_1 \varphi'_1}{p_0 - p_1} e^{-a\varphi'_1} = \frac{\gamma_2 p_2 (-\varphi'_2)}{p_2 - p_0} e^{-a\varphi'_2}$$

so that (59) gives

$$\lambda = \frac{q_0}{\sqrt{2\pi p_0 q_0}} \frac{\gamma_1 p_1 \varphi'_0}{(p_0 - p_1)(-\varphi'_2)} e^{-a\varphi'_1}$$

which together with (60) may be used to reduce

$$Q(p_1) = \frac{1}{\sqrt{2\pi p_0 q_0}} \frac{q_0 p_1}{p_0 - p_1} e^{-a\varphi'_1} \frac{1}{\sqrt{n}} e^{-n\varphi_0}$$

$$Q(p_1) = \frac{-\varphi_2}{\varphi_0 \gamma_1 \delta'_0} \frac{1}{N-n} \quad (63)$$

Similarly we have

$$P(p_2) = \frac{\varphi'_1}{\varphi_0 \gamma_2 \delta'_0} \frac{1}{N-n} \quad (64)$$

so that

$$P(p_2)/Q(p_1) = \gamma_1 \varphi'_1 / \gamma_2 (-\varphi'_2). \quad (65)$$

We have thus proved the following theorem:

Asymptotically the optimum sampling plan is given by

$$c = np_0 + a + o(1) \quad (66)$$

and

$$n = \frac{1}{\varphi_0} (\ln N - \frac{1}{2} \ln \ln N + \ln \lambda + \frac{3}{2} \ln \varphi_0) + o(1) \quad (67)$$

which lead to

$$\min R = \frac{1}{\varphi_0} (\ln N - \frac{1}{2} \ln \ln N + \ln \lambda + \frac{3}{2} \ln \varphi_0 + 1) + o(1), \quad (68)$$

$$Q(p_1) = \frac{-\varphi'_2}{\varphi_0 \gamma_1 \delta'_0} \frac{1}{N-n} + o\left(\frac{1}{N}\right)$$

and

$$P(p_2) = \frac{\varphi'_1}{\varphi_0 \gamma_2 \delta'_0} \frac{1}{N-n} + o\left(\frac{1}{N}\right).$$

It will be noted that p_0 and φ_0 depend on (p_1, p_2) only, i.e. they are independent of the cost parameters and of w_2 .

The asymptotic solution supplements the exact one in several respects. Since the optimum plan is a function of 5 parameters $(N, p_1, p_2, \gamma_1, \gamma_2)$ a complete tabulation is rather hopeless even if a program has been worked out for an electronic computer. Furthermore the properties of the exact solution are not easily to be found from the procedure by which the solution is obtained. The advantages of the asymptotic solution are that

- (1) it clearly shows how the optimum plan and various derived quantities depend on the parameters,
- (2) it may be used as starting point for developing approximations which are valid also for small N ,
- (3) it may be used for developing interpolation and extrapolation formulas in connection with "master tables" of the exact solution, and
- (4) it shows the sensitivity of the solution with respect to changes of the parameters.

These aspects of the solution will be discussed in the following sections.

5. Comparison of exact and approximate solution.

Looking at the relation between n and c in the tables it will be seen that the optimum values of n for a given value of c tend to cluster around

$$n_c = \alpha + \beta \left(c + \frac{1}{2} \right) \quad (69)$$

as might be expected from (33). Comparing with the asymptotic result $c = np_0 + a$, $p_0 = 1/\beta$ and a being defined by (56), agreement between the two expressions would require that

$$\left(\ln \frac{p_2(p_0 - p_1)(-\phi_2')}{p_1(p_2 - p_0)\phi_1'} \right) / \left(\ln \frac{p_2 q_1}{q_2 p_1} \right) = \frac{1}{2}.$$

It can be proved that the ratio on the left hand side above is positive and less than 1. Numerical investigations show that in typical cases in practice the ratio does not deviate much from 1/2. As examples consider the following results:

$100p_1$	$100p_2$	p_2/p_1	Ratio
0.2	4.0	20	0.528
0.2	2.0	10	0.517
0.6	4.0	6.7	0.512
0.6	2.0	3.3	0.505

The ratio depends primarily on p_2/p_1 and practically the same results will be found for values of (p_1, p_2) which are 10 times as large or 1/10 of the values considered. We shall therefore in the following use the simpler expression (69) instead of $c = np_0 + a$ as the starting point for finding n from c or reversely.

The asymptotic formulas may be used in two ways

- (1) Starting from c we may determine the corresponding N -interval and within that the relation between n and N .
- (2) Starting from N we may determine the corresponding n and from n determine c .

The first method is useful for making a systematic tabulation of sampling plans whereas the second is suitable for computing "isolated" plans for a given N .

Starting from an integer value of c we first find n_c from (69) and the corresponding N_c from (61). Similarly we find $N_{c-0.5}$ and $N_{c+0.5}$, being the lower and upper limit for N having c as optimum acceptance number.

In the asymptotic solution we have disregarded the discreteness of c and n . We may, however, afterwards try to take the effect of the discreteness of c into account by investigating the relationship between n and N for given (integer) value of c . From $dR(N, n, c)/dn = 0$ it can be found that n is approximately a linear function of $\log N$ with slope $-1/\log q_2$ *). Within the interval $(N_{c-0.5}, N_{c+0.5})$ we may therefore determine n from the approximate formula

$$n = n_c - (\log N - \log N_c) / \log q_2, \quad N_{c-0.5} < N < N_{c+0.5}, \quad (70)$$

which for small p_2 and small intervals may be replaced by

$$n = n_c + (N - N_c) / N_c p_2, \quad N_{c-0.5} < N < N_{c+0.5}. \quad (71)$$

It follows that the values of n belong to the interval

$$n_c \pm \beta(\varphi_0 + \frac{1}{2n_c}) / 2p_2.$$

For applications in practice we give the formula corresponding to (61) with logarithms to base 10, i.e.

$$\log (N_c - n_c) = \varphi n_c + \frac{1}{2} \log n_c + \delta \quad (72)$$

where

$$\varphi = p_0 \log \frac{p_0}{p_i} + q_0 \log \frac{q_0}{q_i}, \quad i = 1 \text{ or } 2, \quad (73)$$

$$\delta = -\log (\lambda \varphi_0), \quad (74)$$

and

$$\lambda \varphi_0 = 10^{\varphi(\alpha + \frac{\beta}{2})} \frac{\varphi}{\log e} \sqrt{\frac{q_0}{2\pi p_0}} \sum_{i=1}^2 \frac{\gamma_i p_i}{|p_0 - p_i|} \left(\frac{q_i}{q_0}\right)^{\alpha + \frac{\beta}{2}} \quad (75)$$

-a having been replaced by $\frac{\alpha}{\beta} + \frac{1}{2}$ in $\lambda \varphi_0$.

In the following we shall make much use of (72) with $N_c - n_c$ replaced by N_c which only means that we disregard terms of order n_c/N_c and less.

The approximation obtained by using (69), (70), and (72) is usually very good even for quite small values of c . Normally the approximate value of c will deviate at most 1 from the correct value. The approximation depends essentially on p_2/p_1 , being good for large values of p_2/p_1 and poorer for small values. Two examples for $p_2/p_1 = 6.7$ and 3.3 , respectively, will show the results obtained for a typical good and poor case. Table 1 and Fig. 3 show that the approximate and the exact solution are practically identical in the first

*) This result is due to Mrs. K. West Andersen.

case whereas the approximate solution in the second case often will lead to a value of c being 1 too large and a corresponding value of n .

Table 1.

Comparisons of exact and approximate sampling plans computed from (69), (70), and (72).

$$p_r = p_s = 0.010, p_1 = 0.006, p_2 = 0.040, w_2 = 0.05.$$

$$\alpha = -26.7, \beta = 55.509, \varphi = 0.0034156, \delta = 1.5499, -1/\log q_2 = 56.405.$$

c	Approximation			Exact	
	n_c	n	$N_{c \pm 0.5}$	n	N
1	57	43-66	269-714	45-65	280-714
2	112	104-120	715-1400	105-120	715-1420
4	223	216-230	2490-4300	220-230	2550-4390
6	334	328-340	7190-12000	330-340	7390-12300
8	445	439-451	19700-32300	440-450	20200-33000
10	556	550-562	52400-85300	550-560	53600-87000
12	667	661-673	137000-200000	665-670	140000-200000

$$p_r = p_s = 0.010, p_1 = 0.006, p_2 = 0.020, w_2 = 0.05.$$

$$\alpha = -143.0, \beta = 85.879, \varphi = 0.0009088, \delta = 2.0785, -1/\log q_2 = 113.97.$$

c	Approximation			Exact	
	n_c	n	$N_{c \pm 0.5}$	n	N
2	72	44-83	715-1750	-	-
4	243	234-251	2790-3970	245-250	4420-5590
6	415	408-422	5410-7150	415-420	7100-8980
8	587	581-593	9270-11900	585-595	11300-14200
10	759	753-765	15100-19000	755-765	17700-22000
12	931	925-936	23800-29600	930-935	27300-33700
14	1102	1097-1107	36800-45700	1100-1105	41500-51100
16	1274	1269-1279	56500-69700	1270-1280	62800-77200
18	1446	1441-1451	86000-106000	1445-1450	94600-116000
20	1618	1613-1623	130000-159000	1615-1620	142000-173000

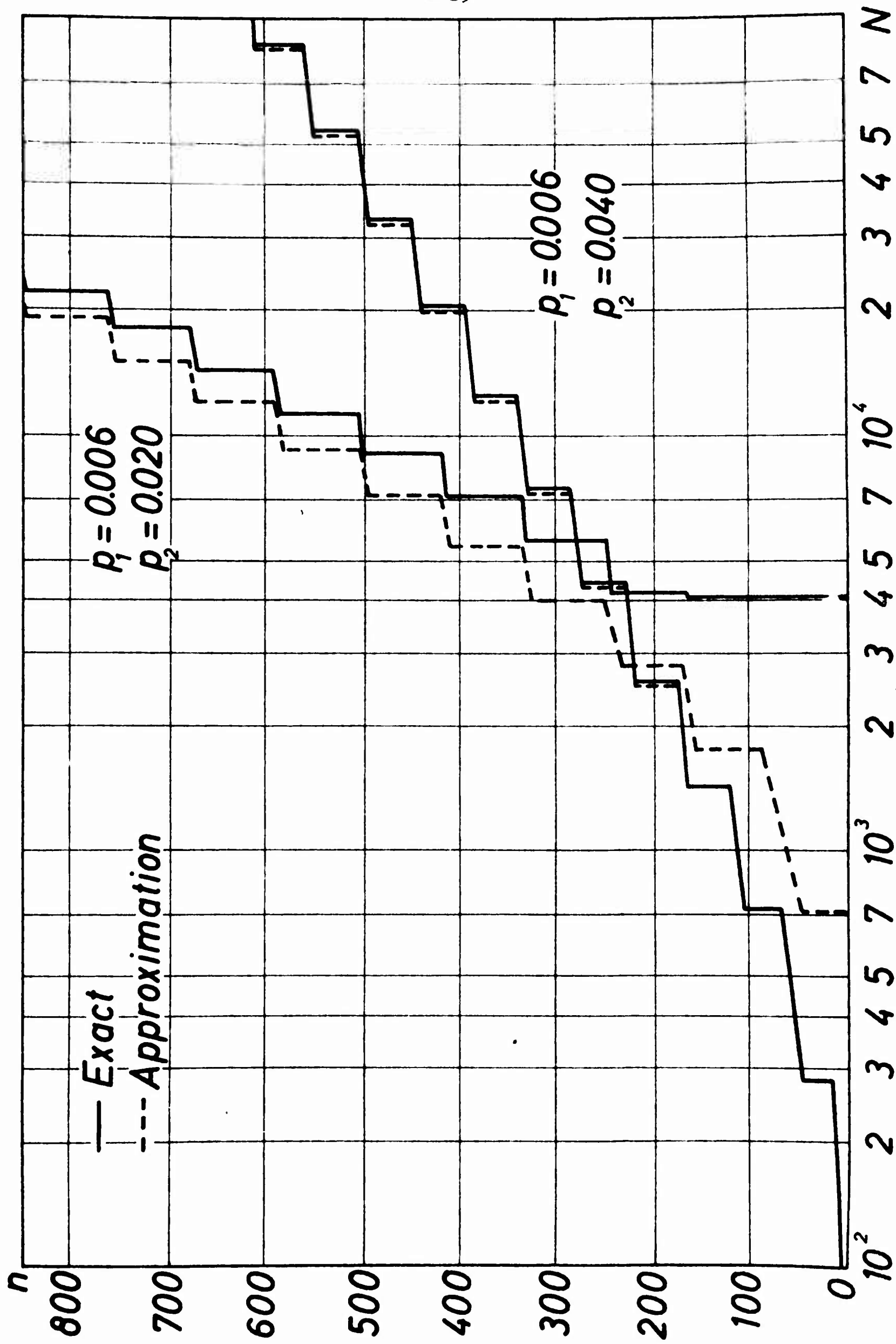


Fig. 3

Comparisons of exact and approximate sampling plans.

It is essential for the efficiency of the approximation to use the right relation between n and c , see the discussion in section 12, and it is therefore fortunate that this relation is a simple linear one.

The approximation formula breaks down for values of N for which the cheapest solution is acceptance without inspection (or rejection without inspection). As will be seen from Table 1 the approximation formula may in such cases lead to a sampling plan even if no optimum plan exists. The difference in costs by using such a plan instead of accepting without sampling inspection will, however, normally be small.

Turning to the inverse formula (67) numerical investigations show that the results are not as accurate as those found from (61). Taking one more term in the inversion of (61) and changing to logarithms with base 10 we find

$$n_N = \frac{1}{\varphi} \left\{ \log N - \left(\frac{1}{2} \log \log N + d \right) \left(1 - \frac{1}{3 \log N} \right) \right\} \quad (76)$$

where

$$d = - \log \lambda \varphi_0 - \frac{1}{2} \log \varphi = \delta - \frac{1}{2} \log \varphi. \quad (77)$$

The exact inversion leads to the correction term $(\log e)/2 \log N = 0.22/\log N$ which, however, on the basis of numerical investigations has been replaced by $1/3 \log N$. If (76) is to be used extensively it pays to tabulate

$$g(N) = \log N - \frac{1}{2} \left(1 - \frac{1}{3 \log N} \right) \log \log N \quad (78)$$

and use (76) in the form

$$n_N = \frac{1}{\varphi} \left(g(N) - d \left(1 - \frac{1}{3 \log N} \right) \right). \quad (79)$$

From n we may then find

$$c_N = p_0 (n_N - \alpha) - \frac{1}{2}$$

and round to the nearest integer. To obtain more accurate results n_c may be computed from the rounded value of c_N and n could then be found from (70) or (71).

Table 2 shows that (76) leads to good results for the two previously discussed typical examples.

As a general conclusion of the many numerical comparisons which have been carried out we may state that the asymptotic formulas give sufficiently good approximations to the optimum sampling plans for most practical purposes. If one wants to be sure to find the optimum plan one may start from the approximation and compare the costs of

this plan with the costs of suitably chosen neighbouring plans thus finding the optimum one by trial and error.

Table 2.

Comparisons of exact and approximate sampling plans computed from (76).

$$p_r = p_s = 0.010, \quad w_2 = 0.05.$$

N	$p_1 = 0.006, \quad p_2 = 0.040.$				$p_1 = 0.006, \quad p_2 = 0.020.$			
	Approx.		Exact		Approx.		Exact	
	n_N	c_N	n	c	n_N	c_N	n	c
300	Accept		50	1				
500	20	0	60	1				
700	55	1	65	1				
1000	90	2	115	2				
2000	165	3	170	3				
3000	210	4	220	4	Accept		Accept	
5000	265	5	275	5	180	3	250	4
7000	300	5	285	5	320	5	335	5
10000	345	6	335	6	465	7	505	7
20000	420	8	395	7	755	10	760	10
30000	470	8	450	8	930	12	930	12
50000	525	9	505	9	1145	14	1105	14
70000	565	10	560	10	1290	16	1275	16
100000	610	11	610	11	1445	18	1445	18
200000	690	12	670	12	1750	22	1705	21

The formulas (72) and (77) have, however, the serious drawback from the point of view of application that the constants δ and d are rather hard to compute. The asymptotic formulas have therefore in the following only been used to derive relationships between sampling plans under variation of the parameters. It is to be expected that these relationships will prove to be rather accurate in view of the good approximation demonstrated above.

According to (62) we have for the optimum plans that the average decision loss asymptotically is constant, i.e. $R - n \sim 1/\varphi_0$. For small N this gives an upper limit to the decision loss but the formula is not of much value as an approximation.

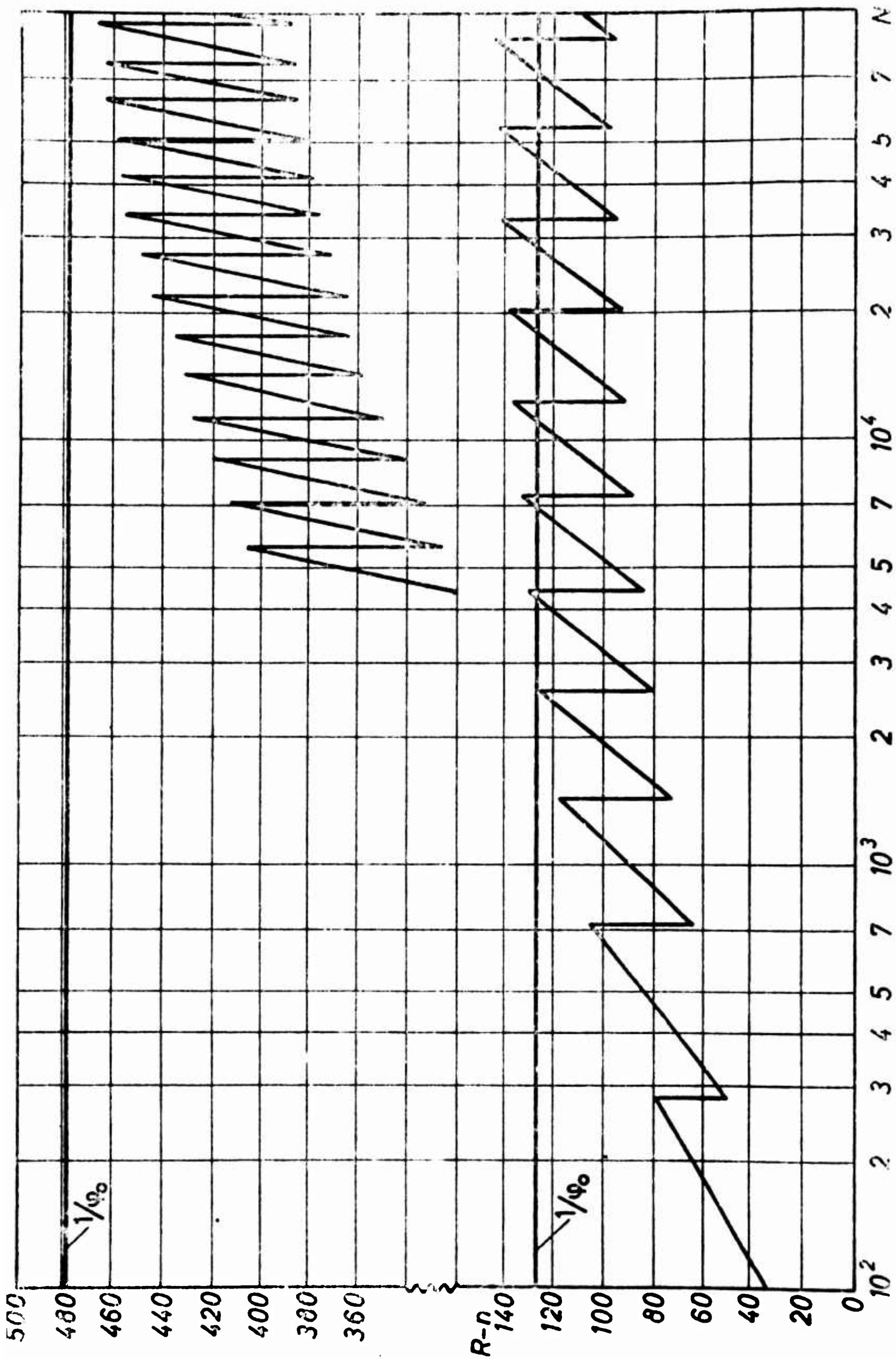


Fig. 4

Average decision loss as function of N and α .

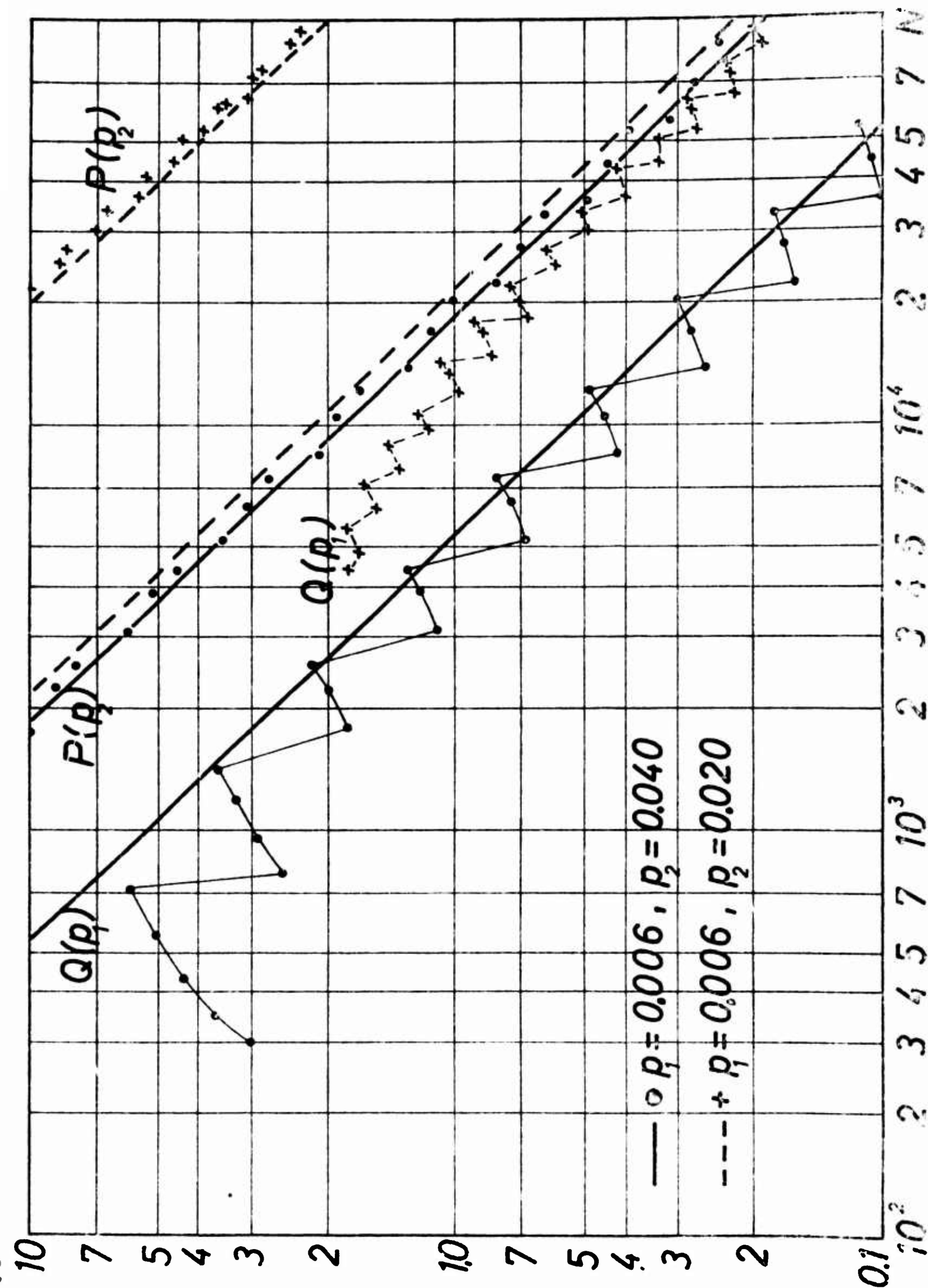


Fig. 5

Probabilities of wrong decisions as functions of lot size.

Fig. 4 sketches for the two previously considered examples $R - n$ as function of N . The discontinuities correspond to changes in c ; each time c is increased by 1 n increases approximately by β and $R - n$ decreases with the same quantity. The asymptotic result corresponds to the mid-points of the intervals. It will be seen that the asymptote is nearly being reached for $N = 100,000$ in the case with $p_2/p_1 = 6.7$ but not for $p_2/p_1 = 3.3$.

For small N a useful upper limit to the average decision loss may be obtained by noticing that $R < Ny_2$ if an optimum plan exists and the alternative is acceptance without inspection.

According to (63) and (64) the probabilities of wrong decisions, $Q(p_1)$ and $P(p_2)$, are asymptotically inversely proportional to N . Fig. 5 sketches for the two examples $Q(p_1)$ and $P(p_2)$ as functions of N . The asymptotic formula gives a reasonable approximation to $P(p_2)$ in both cases, whereas the approximation to $Q(p_1)$ is rather poor, particularly for the case $p_2/p_1 = 3.3$. The discontinuities resulting from changes of c are very pronounced for $Q(p_1)$.

6. Proportional change of (p_r, p_s, p_1, p_2) for fixed w_2 .

We shall first study the asymptotic formulas for all "quality levels" tending to zero with the same speed. Introducing the auxiliary quantities

$$\rho_s = \frac{p_s}{p_r}, \quad \rho_1 = \frac{p_1}{p_r}, \quad \rho_2 = \frac{p_2}{p_r}, \quad \rho_m = \frac{p_m}{p_r}, \quad \rho = \frac{p_2}{p_1}, \quad (80)$$

we find for $p_r \rightarrow 0$ and fixed $(\rho_s, \rho_1, \rho_2, w_2)$

$$\alpha p_r \rightarrow \left(\ln \frac{w_2(\rho_2 - 1)}{w_1(1 - \rho_1)} \right) / (\rho_2 - \rho_1) = \alpha_0,$$

$$\beta p_r \rightarrow \left(\ln \frac{\rho_2}{\rho_1} \right) / (\rho_2 - \rho_1) = \beta_0,$$

$$p_0/p_r \rightarrow 1/\beta_0 = \rho_0,$$

$$\varphi_0/p_r \rightarrow \rho_0 \ln \frac{\rho_0}{\rho_i} + (\rho_i - \rho_0) = \varphi^*, \quad i = 1 \text{ or } 2,$$

and

$$\lambda \varphi_0 / \sqrt{p_r} \rightarrow \exp\left\{\varphi^*\left(\alpha_0 + \frac{\beta_0}{2}\right)\right\} \frac{\varphi^*}{\sqrt{2\pi\rho_0}} \sum_{i=1}^2 \frac{w_i \rho_i (\rho_i - 1)}{(\rho_s - \rho_m)(\rho_i - \rho_0)} \exp\{(\rho_0 - \rho_i)\left(\alpha_0 + \frac{\beta_0}{2}\right)\}$$

$$= \exp\{-\delta_0\},$$

where in the last expression $-a$ has been replaced by $\frac{\alpha}{\beta} + \frac{1}{2}$ as in (75).

Inserting these results into (69) and (72) we find

$$n_c p_r \rightarrow \alpha_c + \beta_o (c + \frac{1}{2}) = n_o(c)$$

and

$$\ln(N_c p_r) \rightarrow \varphi * n_o(c) + \frac{1}{2} \ln n_o(c) + \delta_o = \ln N_o(c).$$

It follows that for small p_r we have approximately

$$n_c \sim n_o(c)/p_r$$

and

$$N_c \sim N_o(c)/p_r$$

where $n_o(c)$ and $N_o(c)$ are independent of p_r , i.e. n and N vary inversely proportional to p_r for given c .

Suppose that the optimum sampling plans have been tabulated for a small value of p_r , $p_r = 0.01$ say, and certain values of $(\rho_s, \rho_1, \rho_2, w_2)$. The above result may then be used to find the optimum plans for λp_r , say, from the plans in the given table. Denoting the quantities required by $n_c(\lambda p_r)$ and $N_c(\lambda p_r)$ we have for given c

$$n_c(\lambda p_r) \sim n_c(p_r)/\lambda \quad (81)$$

and

$$N_c(\lambda p_r) \sim N_c(p_r)/\lambda, \quad (82)$$

i.e. we have found the following important "proportionality law":

The optimum sampling plan corresponding to $(N, \lambda p_r, \lambda p_s, \lambda p_1, \lambda p_2, w_2)$ is approximately equal to $(n^*/\lambda, c^*)$ where (n^*, c^*) is the plan corresponding to $(N^*, p_r, p_s, p_1, p_2, w_2)$ with $N^* = N\lambda$.

The theorem has been illustrated in Fig. 6 which shows that the approximation holds good also for quite large values of p_r .

This theorem greatly enlarges the field of application of the two master tables. The table with $p_r = 0.01$ may be used for $\lambda < 5$ and the table with $p_r = 0.10$ for $0.5 < \lambda < 2$, in that way covering all cases with $p_r < 0.20$ which is the domain of practical interest.

A large number of numerical investigations has shown that the proportionality law gives rather accurate results. The value of c found will seldom deviate more than 1 from the correct value. For $\lambda > 1$ the formula will normally tend to give too large a value of c and for $\lambda < 1$ too small a value.

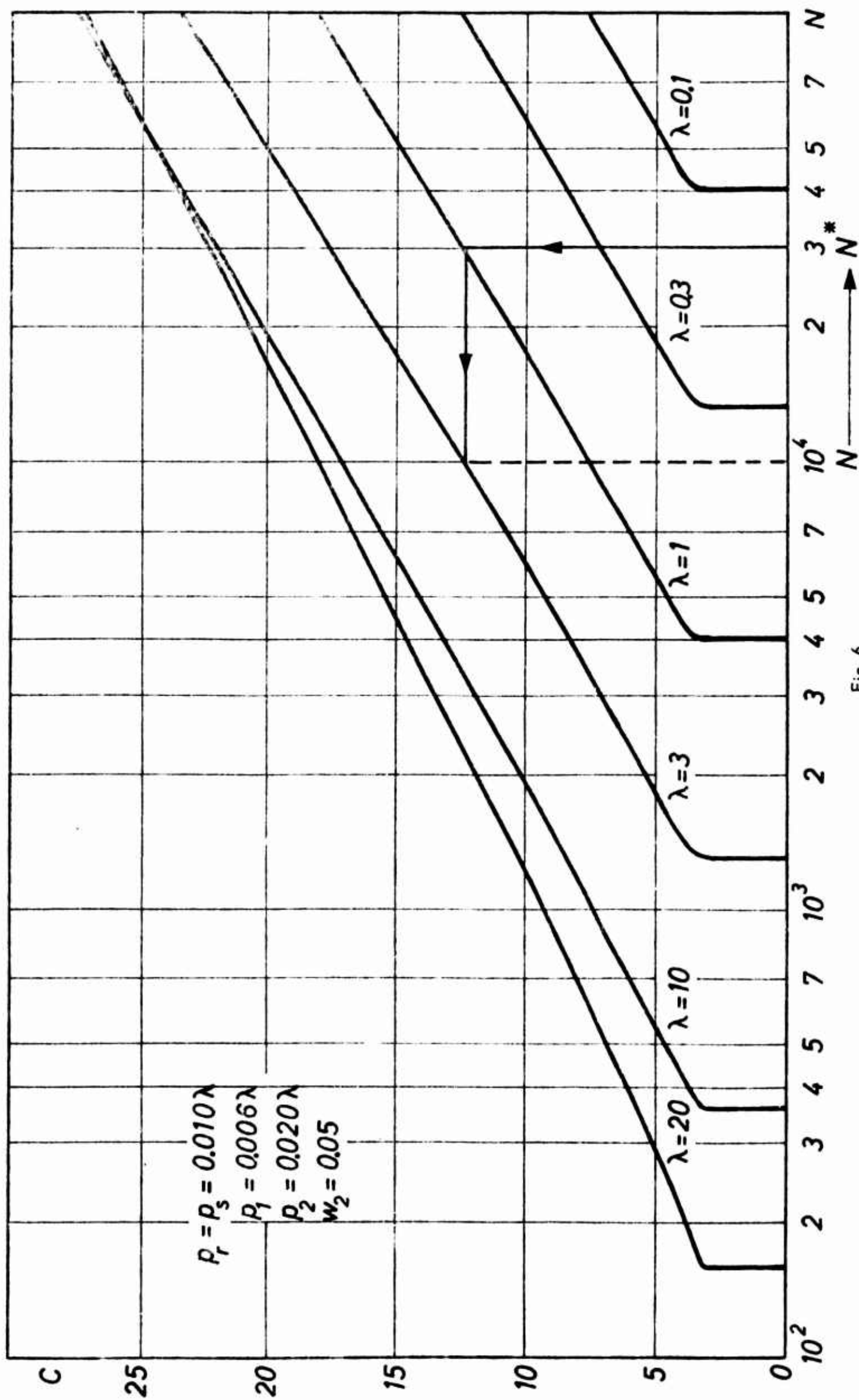


Fig. 6

Relation between lot size and acceptance number by proportional change of (p_r, p_s, p_1, p_2) for fixed w_2 .

Table 3.

Comparisons of exact sampling plans for $p_r = p_s = 0.030$, $p_1 = 0.018$, $p_2 = 0.060$, $w_2 = 0.05$, and approximate plans derived from the master tables by the proportionality law.

N	Exact		Derived from $p_r = 0.01 (\lambda = 3)$			Derived from $p_r = 0.10 (\lambda = 0.3)$		
	n	c	$N^* = 3N$	$n^*/3$	c^*	$N^* = 0.3N$	$n^*/0.3$	c^*
1000	Accept		3000	Accept		300	Accept	
2000	110	5	6000	110	5	600	115	5
3000	140	6	9000	165	7	900	145	6
5000	225	9	15000	225	9	1500	200	8
7000	255	10	21000	255	10	2100	255	10
10000	310	12	30000	310	12	3000	285	11
20000	395	15	60000	395	15	6000	365	14
30000	455	17	90000	455	17	9000	425	16
50000	510	19	150000	540	20	15000	480	18
70000	570	21	210000	570	21	21000	535	20
100000	625	23	-	-	-	30000	565	21
200000	710	26	-	-	-	60000	675	25

The example in Table 3 shows the derivation of sampling plans with a break-even quality of $p_r = 0.03$, partly from the first master table using $\lambda = 3$ and partly from the second using $\lambda = 0.3$. Both results are remarkably close to the exact solution, see also Fig. 6.

Consider now the inverse formula (79).

From

$$d + \log p_r \rightarrow d_0 \log e - \frac{1}{2} \log \varphi^* = d_0$$

we find

$$n_N(\lambda p_r) \sim \frac{n_N(p_r)}{\lambda} + (1 - \frac{1}{3 \log N}) \frac{\log \lambda}{\lambda \varphi(p_r)} \quad (83)$$

where $\varphi(p_r)$ denotes the value of φ for the given (basic) set of parameters. This formula shows how the sample size for a given lot size changes with the "quality level". This result is, however, not as accurate as the previous one for small N and it is neither as convenient for use in connection with the tables.

An example has been given in the following table for $N = 50,000$, $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, and $w_2 = 0.05$.

Comparisons of exact and approximate sampling plans derived from (83).

Exact			Approximation	
λ	n	c	n	c
0.1	1850	3	2330	4
0.3	1300	7	1210	7
1.0	505	9	-	-
3.0	205	11	210	11
10.0	63	12	78	15

7. Change of p_s for fixed (p_r, p_1, p_2, w_2) .

The master tables contain sampling plans for $p_s = p_r$ only, because a simple and rather accurate rule exists for deriving plans for $p_s \neq p_r$ from the tabulated ones.

From (69) and (72) it will be seen that p_s influences N_c only through δ . Writing

$$\begin{aligned} p_s - p_m &= p_s - p_r + w_1(p_r - p_1) \\ &= w_1(p_r - p_1) \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)} \right) \end{aligned}$$

it follows from (72) that

$$\log N_c(p_r, p_s) = \log N_c(p_r, p_r) + \log \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)} \right)$$

or

$$N_c(p_r, p_s) = N_c(p_r, p_r) / \lambda_s, \quad (84)$$

say, where

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)} \right)^{-1}. \quad (85)$$

We have thus proved the following theorem:

The optimum sampling plan corresponding to $(N, p_r, p_s, p_1, p_2, w_2)$ is approximately equal to the plan (n^*, c^*) corresponding to $(N^*, p_r, p_r, p_1, p_2, w_2)$ with $N^* = N\lambda_s$.

This theorem makes it possible to use the master tables also for $p_s \neq p_r$ if only N is replaced by N^* . The error in c by using this procedure will seldom be more than ± 1 . An example has been given in Table 4 with

$$\lambda_s = \left(1 + \frac{2 - 1}{0.95(1 - 0.6)} \right)^{-1} = 0.275.$$

Table 4.

Comparisons of exact sampling plans for $p_r = 0.010$, $p_s = 0.020$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$ with approximate plans derived from the master table.

N	Exact		Approximation		
	n	c	$N^* = 0.275N$	n^*	c^*
300	Accept		83	5	0
500	5	0	138	10	0
700	10	0	193	15	0
1000	15	0	275	15	0
2000	60	1	550	60	1
3000	110	2	825	110	2
5000	120	2	1380	120	2
7000	170	3	1930	170	3
10000	220	4	2750	220	4
20000	280	5	5500	280	5
30000	330	6	8250	330	6
50000	390	7	13800	390	7
70000	395	7	19300	395	7
100000	450	8	27500	445	8
200000	505	9	55000	550	10

The corresponding "inverse" formula becomes

$$n_N(p_r, p_s) = n_N(p_r, p_r) + \frac{1}{\varphi} \left(1 - \frac{1}{3 \log N} \right) \log \lambda_s. \quad (86)$$

Using this result for $N = 50000$ and the parameters given in Table 4 we find

$$n = 505 - 293 \times 0.9291 \times 0.5607 = 350$$

as compared to the exact solution 390.

In the following sections we shall limit ourselves to consider cases with $p_s = p_r$ since we may always begin the analysis by replacing N by N^* if $p_s \neq p_r$. The "conversion factor" λ_s depends on w_2 and the ratios (ρ_3, ρ_1) , i.e. λ_s is independent of p_2 and the general quality level.

8. Proportional change of (p_r, p_1, p_2) and change of w_2 .

Consider the problem of finding the optimum plans for an arbitrary set of parameter values (p_r, p_1, p_2, w_2) by combining the proportionality law with the relation between p_r and w_2 for given γ_2 and using the tabulated plans in the master table for parameter values $(p_{ro}, p_{lo}, p_{2o}, w_{2o})$, say.

The problem is to determine λ so that $(p_r, p_1, p_2) = (\lambda p_{ro}, \lambda p_{lo}, \lambda p_{2o})$ and $(p_{ro}^*, p_{lo}, p_{2o}, w_2)$ give the same value of γ_2 as $(p_{ro}, p_{lo}, p_{2o}, w_{2o})$. For this value of λ we may find the plans for (p_r, p_1, p_2, w_2) from the plans for $(p_{ro}^*, p_{lo}, p_{2o}, w_2)$ by means of the proportionality law, and the plans for $(p_{ro}^*, p_{lo}, p_{2o}, w_2)$ are identical to the plans for $(p_{ro}, p_{lo}, p_{2o}, w_{2o})$. (It will be noted that p_{ro}^*/p_{ro} is identical to the function defined by (46)).

Since the value of γ_2 is the same for (p_r, p_1, p_2, w_2) and $(p_{ro}^*, p_{lo}, p_{2o}, w_2)$ we have the equation $\gamma_{2o} = \gamma_2$ for the determination of λ , i.e.

$$\frac{w_{2o}(p_{2o} - p_{ro})}{w_{1o}(p_{ro} - p_{lo})} = \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)}.$$

Introducing $p_{2o} = p_2/\lambda$ and $p_{lo} = p_1/\lambda$ we find

$$\lambda p_{ro} = \left(p_2 + \frac{w_{1o}}{w_{2o}} \gamma_2 p_1 \right) / \left(1 + \frac{w_{1o}}{w_{2o}} \gamma_2 \right). \quad (87)$$

For the master table with $p_{ro} = 0.01$ and $w_{2o} = 0.05$ the result is

$$\lambda = 100(p_2 + 19\gamma_2 p_1) / (1 + 19\gamma_2). \quad (88)$$

For the other master table ($p_{r0} = 0.10$) the factor 100 should be replaced by 10.

The results of sections 6-8 may be combined to the following theorem:

The optimum sampling plan corresponding to $(N, p_r, p_s, p_1, p_2, w_2)$ is approximately equal to $(n^*/\lambda, c^*)$ where (n^*, c^*) may be found in the master table for $N^* = N\lambda_s\lambda$, $p_{10} = p_1/\lambda$, and $p_{20} = p_2/\lambda$, the conversion factors being equal to

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)} \right)^{-1}$$

and

$$\lambda = 100(p_2 + 19 \gamma_2 p_1) / (1 + 19 \gamma_2), \quad \gamma_2 = \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)},$$

for the 0.01-table, 100 being replaced by 10 for the 0.10-table.

By means of this theorem it is rather easy to find the optimum plan corresponding to an arbitrary set of parameter values if only p_1/λ and p_2/λ fall within the range of arguments in the master tables. If that is not the case the method given in the next section may be used.

Usually p_1/λ and p_2/λ will not be equal to the arguments used in the master tables. One might then interpolate but this is hardly worth while since the arguments in the table have been chosen in such a way that by rounding to the nearest argument the rounding error will ordinarily be less than 10 %.

If one wants to be sure to obtain a sufficiently large sample the value of p_1/λ should be rounded up and the value of p_2/λ rounded down.

As an example consider the problem of finding the sampling plans for $(p_r, p_1, p_2, w_2) = (0.03, 0.01, 0.07, 0.08)$ and $p_s = p_r$. Since $p_r < 0.05$ say, we choose to use the 0.01-table. From

$$\gamma_2 = \frac{8}{92} \frac{7-3}{3-1} = 0.174, \quad 19 \gamma_2 = 3.31,$$

we find $\lambda = (7 + 3.31) / (1 + 3.31) = 2.39$, $p_1/\lambda = 0.01/2.39 = 0.0042$, and $p_2/\lambda = 0.07/2.39 = 0.029$. The master table should thus be entered with $p_{10} = 0.004$ and $p_{20} = 0.030$. For $N = 2000$, say, we find $N^* = 4780$ and $(n^*, c^*) = (210, 3)$ leading to $(n, c) = (210/2.39, 3) = (90, 3)$ which is the correct solution.

If $w_2 = 0.02$ instead of 0.08 we find similarly $\lambda = 4.38$, $p_1/\lambda = 0.0023 \cong 0.0025$ and $p_2/\lambda = 0.0160 \cong 0.0150$. For $N = 2000$ we get $N^* = 8760$ leading to acceptance without inspection as the most economical decision.

9. Change of w_2 for fixed (p_r, p_s, p_1, p_2) .

In the following we shall develop a method for evaluating the effect of changing one of the five parameters only, and use it first for w_2 and then for p_r .

From (69) and (72) we find for given c

$$\frac{\partial n_c}{\partial \log w_2} = \frac{\partial \alpha}{\partial \log w_2} = 1/(w_1 \log \frac{q_1}{q_2}) \quad (89)$$

and

$$\frac{\partial \log N_c}{\partial \log w_2} = \varphi \frac{\partial n_c}{\partial \log w_2} + \frac{1}{2} \frac{\partial \log n_c}{\partial \log w_2} + \frac{\partial \delta}{\partial \log w_2}. \quad (90)$$

The last term on the right hand side is a rather complicated function of the parameters. Tabulation of δ and graphical analysis of δ as a function of $\log w_2$ has shown, however, that at least for $w_2 \leq 0.20$ and $p_r \leq 0.10$ (and corresponding values of p_s, p_1, p_2) δ is approximately a linear function of $\log w_2$ with a slope depending slightly on (ρ_s, ρ_1, ρ_2) and being practically independent of p_r .

Limiting ourselves to the case $p_s = p_r$ we thus have

$$\frac{\partial \delta}{\partial \log w_2} \approx -b_1(\rho_1, \rho_2),$$

say, where $b_1(\rho_1, \rho_2)$ has been tabulated in the appendix.

Writing $\delta = \delta(p_r, \rho_1, \rho_2, w_2)$ and putting $w_2 = 0.02$ and 0.20 respectively, so that so that $\Delta \log w_2 = \log 0.20 - \log 0.02 = 1$, an approximation to $\partial \delta / \partial \log w_2$ may be found as $\delta(p_r, \rho_1, \rho_2, 0.20) - \delta(p_r, \rho_1, \rho_2, 0.02)$. This approximation has been computed for both $p_r = 0.01$ and 0.10 , and finally the average of the two has been taken as $-b_1$.

For small p_r we also have

$$\varphi / \log \frac{q_1}{q_2} \approx \varphi^* / (\rho_2 - \rho_1) = b_2(\rho).$$

The values given for b_2 have been computed as averages of $\varphi / (\log \frac{q_1}{q_2})$ for $p_r = 0.01$ and $p_r = 0.10$.

For large n we have that $(\log e)/2n$ is small as compared to φ and we shall therefore disregard the second term on the right hand side of (90). We then have approximately

$$\frac{\partial \log N_c}{\partial \log w_2} = \frac{b_2(\rho)}{w_1} - b_1(\rho_1, \rho_2)$$

which gives

$$N_c(w_2) = Aw_2^{-b_1} \left(\frac{w_1}{w_2} \right)^{-b_2}$$

where A denotes a constant of integration. Changing from w_2 to λw_2 we get

$$N_c(\lambda w_2) = N_c(w_2) / f_1(\lambda) \quad (91)$$

where

$$f_1(\lambda) = \lambda^{b_1 - b_2} \left(1 - (\lambda - 1) \frac{w_2}{w_1} \right)^{b_2} \quad (92)$$

From (69) we further have

$$n_c(\lambda w_2) = n_c(w_2) + g_1(\lambda) \quad (93)$$

where

$$g_1(\lambda) = \left(\log \frac{\lambda w_1}{1 - \lambda w_2} \right) / \left(\log \frac{q_1}{q_2} \right) \cong \left(\ln \frac{\lambda w_1}{1 - \lambda w_2} \right) / (\rho_2 - \rho_1) p_r \quad (94)$$

For convenience f_1 and g_1 have been written as functions of λ only, even if they both depend also on other parameters. The function f_1 which will be called the conversion factor for N due to a change in w_2 has been tabulated in the appendix for $w_2 = 0.05$ as a function of $(\lambda, \rho_1, \rho_2)$. The function g_1 which gives the correction to n due to a change in w_2 has similarly been tabulated in the appendix as function of $(\lambda, \rho_1, \rho_2)$ for $w_2 = 0.05$ and $p_r = 0.01$. Values of this function for other values of p_r may be obtained as $g_1/100p_r$ where g_1 represents the tabulated values.

The above results may be formulated as the following theorem:

The optimum sampling plan corresponding to $(N, p_r, p_s, p_1, p_2, \lambda w_2)$, $p_r = p_s$, is approximately equal to $(n^* + g_1(\lambda), c^*)$ where (n^*, c^*) is the plan corresponding to $(N^*, p_r, p_s, p_1, p_2, w_2)$ with $N^* = N f_1(\lambda)$.

The theorem has been illustrated in Fig. 7.

This theorem enlarges the field of application of the two master tables with respect to values of w_2 in a similar manner as the law of proportionality does with respect to the other parameters. The results of using the approximation have been compared with the exact solutions in a large number of cases and the deviations found between the approximate and the correct value of c have never exceeded 1 for $\lambda < 4$. There is a tendency for the approximation to give too small a value of c for $\lambda > 1$ and too large a value for $\lambda < 1$, in particular for small N .

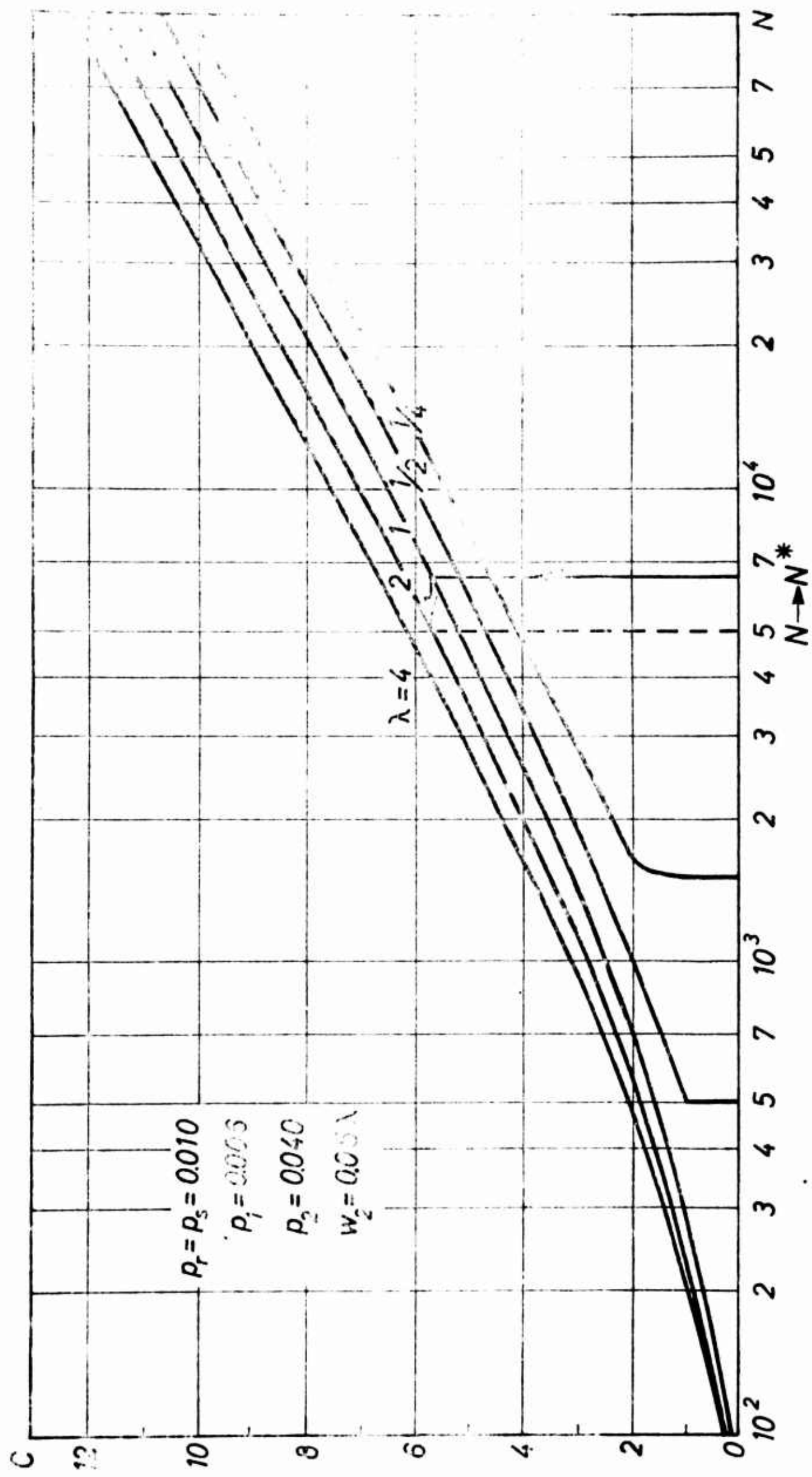


Fig. 7

Relation between lot size and acceptance number by change of w_2 for fixed (P_r, P_s, P_1, P_2)

It should be noted that the formula breaks down in some cases for small N . Let N_a denote the largest N for which acceptance without inspection is cheaper than sampling inspection for the master table used. If $\lambda > 1$ and $Nf_1(\lambda) = N^* < N_a$ then the formula does not lead to a sampling plan even if there may exist a plan which for λw_2 is cheaper than acceptance without inspection. Similarly, for $\lambda < 1$ and $Nf_1(\lambda) = N^* > N_a$ there may be some cases where the approximation formula leads to a sampling plan even if the cheapest solution is acceptance without inspection.

An example has been shown in Table 5. The approximation is remarkably good. Since $N_a = 74$ the approximation formula leads to acceptance without inspection for all $N \leq 57$. Sampling plans cheaper than acceptance without inspection do, however, exist for $12 \leq N \leq 57$.

Using the method of section 8 we find $\gamma_2 = 0.833$, $\lambda = 0.804$, $p_1/\lambda = 0.0075$, and $p_2/\lambda = 0.050$. Since p_1/λ falls outside the range of arguments in the master table the method does not apply. Using $p_1/\lambda = 0.007$ gives, however, a rather good approximation.

Table 5.

Comparisons of exact sampling plans for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.10$, and approximate plans derived from the master table. $f_1 = 1.29$, $g_1 = 20$.

N	Exact		Approximate		
	n	c	$N^* = 1.29N$	$n^* + 20$	c^*
50	15	0	65	Accept	
70	20	0	90	25	0
100	25	0	129	30	0
200	35	0	258	35	0
300	75	1	387	75	1
500	85	1	645	85	1
700	130	2	903	130	2
1000	140	2	1290	140	2
2000	240	4	2580	240	4
3000	250	4	3870	250	4
5000	305	5	6450	300	5
7000	355	6	9030	355	6
10000	405	7	12900	405	7
20000	465	8	25800	465	8
30000	520	9	38700	520	9
50000	575	10	64500	575	10
70000	630	11	90300	630	11
100000	635	11	129000	635	11
200000	740	13	-	-	-

From the inverse formula (79) we get

$$\frac{\partial n_N}{\partial \log w_2} = \frac{b_1(\rho_1, \rho_2)}{\varphi} \left(1 - \frac{1}{3 \log N}\right)$$

and consequently

$$n_N(w_2) = \Lambda + \frac{b_1}{\varphi} \left(1 - \frac{1}{3 \log N}\right) \log w_2$$

or

$$n_N(\lambda w_2) = n_N(w_2) + \frac{b_1}{\varphi} \left(1 - \frac{1}{3 \log N}\right) \log \lambda. \quad (95)$$

This shows that the difference between $n(\lambda w_2)$ and $n_N(w_2)$ for given N is proportional to $\log \lambda$. This formula is, however, not as accurate as (93) for small N .

An example has been given in the following table for $N = 20,000$, $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.020$, and $w_2 = 0.05$, which gives $b_1 = 0.59$ and $1/\varphi = 1100$.

Comparisons of exact and approximate sampling plans derived from (95).

100 w_2	λ	Exact		Approx.	
		n	c	n	c
2.5	0.5	540	8	580	9
5.0	1.0	760	10	-	-
10.0	2.0	980	12	940	11
20.0	4.0	1130	13	1120	13

10. Change of $p_r = p_s$ for fixed (p_1, p_2, w_2) .

From (69) and (72) we find for given c

$$\frac{\partial n_c}{\partial \log p_r} = \frac{\partial \alpha}{\partial \log p_r} = -p_r \left(\frac{1}{p_r - p_1} + \frac{1}{p_2 - p_r} \right) / \left(\log \frac{q_1}{q_2} \right) \quad (96)$$

and

$$\frac{\partial \log N_c}{\partial \log p_r} = \varphi \frac{\partial n_c}{\partial \log p_r} + \frac{1}{2} \frac{\partial \log n_c}{\partial \log p_r} + \frac{\partial \delta}{\partial \log p_r}. \quad (97)$$

Numerical investigations show that - for $w_2 < 0.20$, $p_r < 0.20$, and $p_r = p_s - \delta$ is approximately a linear function of $\log p_r$ with a slope depending on c and being practically independent of "the level of (p_1, p_2) " and of w_2 if only p_r does not come too close to p_1 or p_2 , i.e.

$$\frac{\partial \delta}{\partial \log p_r} \approx b_3(\rho) \quad \text{for} \quad p_1^{\rho^{1/5}} < p_r < p_2^{\rho^{-1/5}},$$

say, where $b_3(\rho)$ has been tabulated. (Another limitation of no practical importance is that p_2 must not be too close to 1). An approximation to $\partial\delta/\partial\log p_r$ may be computed as the corresponding difference - quotient setting $p_r = p_1\rho^{1/5}$ and $p_r = p_2\rho^{-1/5}$ respectively. This has been done for $w_2 = 0.05$ and for the "standard" values of p_1 and p_2 , partly at the 1 % and partly at the 10 % level. The value of b_3 given in the table is the average of the two values found.

Proceeding as in section 9 we have approximately

$$\frac{\partial \log N_c}{\partial \log p_r} = -b_2(\rho)p_r \left(\frac{1}{p_r - p_1} + \frac{1}{p_2 - p_r} \right) + b_3(\rho)$$

which on integration gives

$$N_c(p_r) = Ap_r^{b_3} \left(\frac{p_2 - p_r}{p_r - p_1} \right)^{b_2}$$

and

$$N_c(\lambda p_r) = N_c(p_r)/f_2(\lambda) \quad (98)$$

where

$$f_2(\lambda) = \lambda^{-b_3} \left(\frac{(\rho_2 - 1)(\lambda - \rho_1)}{(1 - \rho_1)(\rho_2 - \lambda)} \right)^{b_2} \quad (99)$$

From (69) we further have

$$n_c(\lambda p_r) = n_c(p_r) + g_2(\lambda) \quad (100)$$

where

$$g_2(\lambda) = \left(\ln \frac{(1 - \rho_1)(\rho_2 - \lambda)}{(\rho_2 - 1)(\lambda - \rho_1)} \right) / (\rho_2 - \rho_1)p_r \quad (101)$$

The conversion factor for N due to a change in p_r , $f_2(\lambda)$, has been tabulated in the appendix as function of $(\lambda, \rho_1, \rho_2)$, and the correction to n due to a change in p_r , $g_2(\lambda)$, has been tabulated as function of $(\lambda, \rho_1, \rho_2)$ for $p_r = 0.01$. Values of $g_2(\lambda)$ for other values of p_r may be found from the tabulated ones by dividing by $100p_r$.

The above results may be formulated as the following theorem:

The optimum sampling plan corresponding to $(N, \lambda p_r, \lambda p_s, p_1, p_2, w_2)$, $p_r = p_s$, is approximately equal to $(n^* + g_2(\lambda), c^*)$ where (n^*, c^*) is the plan corresponding to $(N^*, p_r, p_s, p_1, p_2, w_2)$ with $N^* = Nf_2(\lambda)$.

With the given set of tables this theorem is, however, not as important in practice as the previous ones, because the tables contain the optimum plans for so many combinations of (p_r, p_1, p_2) that an adjustment of the relative position of p_r within the interval (p_1, p_2) will seldom be felt necessary from a practical point of view.

In table 6 an example has been shown of the effect of changing $p_r = p_s$ from 0.010 to 0.020 within the interval $(p_1, p_2) = (0.006, 0.040)$.

Table 6.

Comparisons of exact sampling plans for $p_r = p_s = 0.020$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$ with approximate plans derived from the master table. $\lambda = 2$, $f_2 = 0.52$, $g_2 = -55$.

N	Exact		Approximate		c*
	n	c	$N^* = 0.52N$	$n^* - 55$	
2000	Accept		1040	60	2
3000	75	2	1560	110	3
5000	130	3	2600	165	4
7000	180	4	3640	170	4
10000	230	5	5200	225	5
20000	290	6	10400	280	6
30000	345	7	15600	335	7
50000	400	8	26000	390	8
70000	450	9	36400	445	9
100000	460	9	52000	450	9
200000	565	11	104000	555	11

From the inverse formula (79) we get

$$\frac{\partial n_N}{\partial \log p_r} = - \frac{b_3(\rho)}{\varphi} \left(1 - \frac{1}{3 \log N}\right)$$

which leads to

$$n_N(\lambda p_r) = n_N(p_r) - \frac{b_3}{\varphi} \left(1 - \frac{1}{3 \log N}\right) \log \lambda. \quad (102)$$

An example of the application of this formula has been given in the following table for $N = 50,000$, $p_r = p_s = 0.010$, $\lambda = 0.5$ and 2.5 , $p_1 = 0.002$, $p_2 = 0.040$, and $w_2 = 0.05$, which give $b_3 = 1.09$ and $1/\varphi = 177$.

Comparisons of exact and approximate sampling plans derived from (102).

p_r	λ	Exact		Approximate	
		n	c	n	c
0.005	0.5	350	4	370	4
0.010	1.0	315	4	-	-
0.025	2.5	250	4	245	4

11. Change of all parameters.

The results of the preceding sections may be combined into a "chain formula" of the type

$$N_c(\lambda p_r, \rho_s \lambda p_r, \lambda p_1, \lambda p_2, \lambda_1 w_2) = N_c(p_r, p_r, p_1, p_2, w_2) / \lambda_s f_1 \lambda \quad (103)$$

and

$$n_c(\lambda p_r, \rho_s \lambda p_r, \lambda p_1, \lambda p_2, \lambda_1 w_2) = (n_c(p_r, p_r, p_1, p_2, w_2) + g_1) / \lambda \quad (104)$$

where

$$\lambda_s = \left(1 + \frac{\rho_s - 1}{(1 - \lambda_1 w_2)(1 - \rho_1)} \right)^{-1},$$

$f_1(\lambda_1)$ and $g_1(\lambda_1)$ being defined by (92) and (94) for $\rho_1 = p_1/p_r$ and $\rho_2 = p_2/p_r$.

In the master tables $p_r = p_s = 0.01$ (or 0.10) and $w_2 = 0.05$ have been used as reference values. What has been denoted by λ and λ_1 in the above formulas become $100p_r$ (or $10p_r$) and $20w_2$ if p_r and w_2 denote the values for which the optimum plan is required.

We thus get the following rule for using the master table with $p_r = 0.01$:

The optimum plan for $(N, p_r, p_s, p_1, p_2, w_2)$ with $p_r < 0.05$ is approximately equal to $((n^* + g_1)/100p_r, c^*)$ where (n^*, c^*) may be found by entering the master table with

$$N^* = N(100p_r)f_1(20w_2, \rho_1, \rho_2) / \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)} \right), \quad (105)$$

$\rho_1 = p_1/p_r$, $\rho_2 = p_2/p_r$, and $g_1 = g_1(20w_2, \rho_1, \rho_2)$, the arguments for (p_1, p_2) in the master table being $(\rho_1/100, \rho_2/100)$.

For $0.05 < p_r < 0.20$ the master table with $p_r = 0.10$ should be used accordingly.

If $(\rho_1/100, \rho_2/100)$ are not to be found in the table then use the "nearest" argument or interpolate. One may also use the results in section 10 to change p_r in the master table so that the relations between (p_r, p_1, p_2) in the table become closer to the ones for which the sampling plan is required. From a practical point of view, however, the master tables combined with the rule above will normally suffice.

An example has been given in Table 7. The conversion factor for N is found as

$$3f_1(2, 0.6, .4.0) / \left(1 + \frac{30}{0.90 \times 12} \right) = 3 \times 1.29/3.78 = 1.02.$$

The agreement between the approximate and the exact solution is very good.

Using instead the method of section 8 we get $\lambda_s = 1/3.78 = 0.265$, $\lambda = 2.40$, $p_1/\lambda = 0.0075$, and $p_2/\lambda = 0.050$, i.e. $N^* = 0.636N$ and $n = n^*/2.40$. Since the master table does not contain the argument 0.0075 we may as an approximation use 0.0070 which, however, will tend to give too small values of c .

The corresponding inverse formula takes the form

$$n_N(p_r, p_s, p_1, p_2, w_2) = \frac{1}{100p_r} \left[n_0 + \frac{1}{\varphi} \left\{ \log(100p_r) + b_1 \log(20w_2) - \log \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)} \right) \right\} \left(1 - \frac{1}{3 \log N} \right) \right] \quad (106)$$

where n_0 denotes the sample size to be found in the master table for $p_r = 0.01$, $\rho_1 = p_1/p_r$, $\rho_2 = p_2/p_r$, corresponding to the given lot size N .

Table 7.
Comparisons of exact sampling plans for $p_r = 0.030$, $p_s = 0.060$,
 $p_1 = 0.018$, $p_2 = 0.120$, $w_2 = 0.10$ and approximate plans derived from
the master table for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$.

N	Exact		Approximate		
	n	c	$N^* = 1.02N$	$(n^* + 20)/3$	c^*
50	Accept		51	Accept	
70	5	0	71	Accept	
100	5	0	102	10	0
200	10	0	204	10	0
300	10	0	306	25	1
500	25	1	510	25	1
700	30	1	714	30	1
1000	45	2	1020	45	2
2000	65	3	2040	65	3
3000	80	4	3060	80	4
5000	100	5	5100	100	5
7000	100	5	7140	100	5
10000	120	6	10200	120	6
20000	135	7	20400	155	8
30000	155	8	30600	155	8
50000	175	9	51000	175	9
70000	190	10	71400	195	10
100000	210	11	102000	210	11
200000	225	12	204000	230	12

As an example consider the determination of n for $N = 50,000$ and the parameters given in Table 7. The value of $1/\phi$ is 293 for $p_1 = 0.006$ and $p_2 = 0.040$, and $(1 - \frac{1}{3 \log N}) = 0.929$, so that we find

$$n = \frac{1}{3}(505 + 293(\log 3 + 0.61 \log 2 - \log 3.78)0.929)$$

$$= \frac{1}{3}(505 + 23) = 176$$

in agreement with the (rounded) exact solution, $n = 175$, given in Table 7.

12. Efficiency.

In a previous paper [6] it has been proposed to define the efficiency of a sampling plan as

$$e(N, n, c) = R_0(N)/R(N, n, c) \quad (107)$$

where $R_0(N)$ denotes the costs of the optimum plan and $R(N, n, c)$ denotes the costs of the plan in question.

We shall first discuss the efficiency of a sampling plan on the assumption that the optimum relationship between n and c has been used so that the loss in efficiency is due to using a wrong relationship between N and n . Looking at Fig. 2 it will be seen that it does not matter much whether we use the value of c giving the absolute minimum of R or a neighbouring value of c provided n is chosen such that a (relative) minimum of R is obtained.

For a given set of parameters let (n_0, c_0) be optimum for N_0 and (n_1, c_1) be optimum for N_1 . From (26) it follows that

$$R(N, n, c) = n + (N - n)h(n, c)$$

where

$$h(n, c) = \gamma_1 Q(p_1) + \gamma_2 P(p_2).$$

Using the plan (n_1, c_1) for lot size N_0 (instead of N_1) we find

$$\begin{aligned} R(N_0, n_1, c_1) &= n_1 + (N_0 - n_1)h(n_1, c_1) \\ &= n_1 + (N_0 - n_1)(R_0(N_1) - n_1)/(N_1 - n_1). \end{aligned} \quad (108)$$

It is therefore rather simple by means of the function $R_0(N)$ to evaluate the efficiency of plans contained in the master table in case such plans are used for the wrong value of N . The resulting efficiency is

$$e(N_0, n_1, c_1) = \frac{R_0(N_0)}{n_1 + (N_0 - n_1)(R_0(N_1) - n_1)/(N_1 - n_1)} \quad (109)$$

Since $R_0(N) \sim n + 1/\varphi_0$ we have asymptotically

$$e(N_0, n_1, c_1) \sim \left(n_0 + \frac{1}{\varphi_0} \right) / \left(n_1 + \frac{N_0 - n_1}{N_1 - n_1} \frac{1}{\varphi_0} \right). \quad (110)$$

Introducing $n_0 = (\ln N_0)/\varphi_0 + o(\ln N_0)$ and considering n_1 as an arbitrary function of N_0 , $n_1 = g(N_0)/\varphi_0$ say, we find

$$e(N, n_1, c_1) \sim (\ln N) / (g(N) + N e^{-g(N)}), \quad (111)$$

for $n_1 = o(N)$, which is the result given without proof in [6].

For $g(N) = \lambda \ln N$ we get $e \rightarrow 1/\lambda$ for $\lambda \geq 1$ but $e \rightarrow 0$ for $0 < \lambda < 1$, i.e. if we use a semilogarithmic relationship between n and N differing from the correct one then it is important to use too large a sample.

For $g(N) = N^\lambda$, $\lambda > 0$, we get $e \rightarrow 0$. A more accurate expression than (111) may be found by using all three term of (61) which leads to

$$e(N_0, n_1, c_1) \sim (n_0 + \frac{1}{\varphi_0}) / \left(n_1 + \frac{1}{\varphi_0} (e^{\varphi_0(n_0 - n_1)} \sqrt{\frac{n_0}{n_1}} + \frac{n_0 - n_1}{N_1 - n_1}) \right).$$

This formula is, however, not of direct value because it contains N_1 which is unknown in practice. A simple and practically useful approximation is the following

$$e(N_0, n_1, c_1) \sim (n_0 + \frac{1}{\varphi_0}) / \left(n_1 + \frac{1}{\varphi_0} e^{\varphi_0(n_0 - n_1)} \right). \quad (112)$$

This formula will, however, give too large efficiencies for small values of n because the decision loss has been overestimated.

In connection with the various approximations developed in the preceding sections is has repeatedly been stated that the value of c found by using the approximations will normally not deviate more than 1 from the correct value (for $N < 200,000$). It is therefore of importance to know the efficiency of a plan for which $|c_1 - c_0| = 1$.

If $|c_1 - c_0| = a$ (constant) then $|n_1 - n_0| = a\beta$ and $e \rightarrow 1$ for $N_0 \rightarrow \infty$. Expanding the denominator of (112) we find for small values of $\varphi_0 a\beta$

$$e(N_0, n_1, c_1) \sim (n_0 + \frac{1}{\varphi_0}) / (n_0 + \frac{1}{\varphi_0} + \frac{1}{2} \varphi_0 a^2 \beta^2) \quad (113)$$

which converges rather fast to 1 for $n_0 \rightarrow \infty$ and $a = 1$. By means of the results of section 6 it will be seen that this asymptotic efficiency (as a function of c_0) is independent of the "quality level".

An example has been given in Table 8. The costs for each optimum plan have been compared with the costs of using a neighbouring plan, i.e. $c_1 = c_0 \pm 1$. The efficiency has been compared with the asymptotic efficiency found from (112). It will be seen that the efficiency is larger than 0.90 for $c \geq 2$ and that the asymptotic formula gives too high an efficiency for small N . (N has been chosen as the geometric mean of the smallest and largest N for each c). This conclusion is typical for the cases investigated.

Table 8.

Investigation of efficiency for sampling plans with an acceptance number deviating 1 from the optimum. (e^* = asymptotic efficiency).

$p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$.

N	n_o	c_o	R	n_1	c_1	100e	100e*
145	10	0	53	60	1	72	94
447	60	1	126	10	0	84	94
				115	2	86	95
1010	115	2	200	60	1	90	95
				170	3	92	96
1900	170	3	265	115	2	93	96
				225	4	95	97
3350	225	4	326	170	3	95	96
				280	5	96	97
5700	280	5	386	225	4	96	97
				335	6	97	98
9530	335	6	444	280	5	96	97
				390	7	97	98
15800	390	7	502	335	6	97	97
				445	8	98	98
25800	445	8	559	390	7	97	98
				500	9	98	98
42100	500	9	616	445	8	97	98
				555	10	98	98
68300	555	10	672	500	9	98	98
				615	11	98	98

The conversion formulas and tables show how sensitive the solution is to changes of the parameters. A change of w_2 from 0.05 to 0.10, say, means, that N has to be multiplied by a factor of about 1.3 and the corresponding n should be increased by about 30. (In most systems of sampling plans in practical use to-day the same plan is used for a rather large N-interval, the ratio between endpoints usually being 1.5 or larger). As an example consider the case with $p_r = p_s = 0.010$, $p_1 = 0.006$ and $p_2 = 0.040$ as shown in the following table.

Optimum sampling plans.

N	$w_2 = 0.05$		$w_2 = 0.10$	
	n	c	n	c
500	60	1	85	1
1000	115	2	140	2
5000	275	5	305	5
10000	335	6	405	7
50000	505	9	575	10
100000	610	11	635	11

For most lot sizes we find the same value of c and a difference in n of about 25, in other cases the difference in c is 1 and the difference in n correspondingly larger. It is immediately clear that using the plans corresponding to $w_2 = 0.05$ if the true value of w_2 is 0.10 does not lead to an essential loss in efficiency.

The conclusion is that even if the value of w_2 used deviates from the true value by a factor of 2 the method will nevertheless lead to a sampling plan of very high efficiency.

Similar conclusions may be drawn for the other parameters by studying the conversion formulas.

The main reason why changes of p_r and w_2 does not affect the optimum solution seriously is that p_o and φ_o are independent of p_r and w_2 .

Since the most important relation in the system is

$$c + \frac{1}{2} = p_o(n - \alpha)$$

it is of importance to know how p_o depends on p_1 and p_2 .

From

$$\frac{\partial \ln p_o}{\partial \ln p_1} = \frac{p_o - p_1}{q_1 \ln \frac{q_1}{q_2}} > 0 \text{ and } \frac{\partial \ln p_o}{\partial \ln p_2} = \frac{p_2 - p_1}{q_2 \ln \frac{q_1}{q_2}} > 0$$

it follows that p_o is an increasing function of as well p_1 as p_2 . Furthermore we have approximately

$$\frac{\partial \ln p_o}{\partial \ln p_1} + \frac{\partial \ln p_o}{\partial \ln p_2} \approx 1.$$

Within the domain of variation tabulated the first term is on the average 0.35 and the second 0.65.

The coefficient $p_o \alpha$ varies rather slowly with (p_1, p_2) .

It follows that p_0 is known with a relative error of about the same size as the relative errors of p_1 and p_2 .

If the choice of p_1 and p_2 is doubtful then p_1 should be chosen too large and p_2 too small (by about half of the percentage error in p_1) because the two errors will tend to counterbalance one another and thus give the correct p_0 . The reason for bringing the two parameters closer together in case of doubt lies also in the fact that φ_0 is a decreasing function of p_1 and an increasing function of p_2 . Since $n \sim (\log N)/\varphi_0$ the proposed rule will lead to a larger sample size than the optimum one which normally gives a better efficiency than too small a sample. However, with a wrong p_0 (and φ_0) the efficiency will tend to zero because the average decision loss becomes $O(N^\epsilon)$, $0 < \epsilon < 1$, instead of $O(1)$ as for the optimum plan.

Table 9 shows the efficiency of using a plan obtained by entering the master table by a wrong value of p_1, p_2 or both. It is assumed that the true values of (p_1, p_2) are $(0.006, 0.040)$ and optimum plans have been substituted by plans obtained by using neighbouring values of (p_1, p_2) in the tables, i.e. the relative error of p_1 is 17% and the relative error of p_2 is 12.5% downwards and 25% upwards. The table shows that the efficiency in all cases is larger than 90% for $N < 10,000$. For $N = 200,000$, however, the efficiency falls to 58% in the worst case, i.e. the case where p_2 is chosen 25% too large.

The results in the table support the statement above that in case of doubt it is important to use a large value of p_1 and a small value of p_2 .

13. An example.

Consider now an example starting from the original cost functions. To show the various aspects of the method the example will be worked out in more detail than is necessary for routine applications.

Let the three cost functions be $k_s(p) = 23 + 35p$, $k_r(p) = 16 + 35p$, $k_a(p) = 720p$, the coefficients denoting costs per item in cents, say, i.e. the costs of sampling and testing is 23 cents per item in the sample and the costs of accepting a defective item is 720 cents etc., see section 2.

Let us further assume that lots are generated with probability $w_1 = 0.93$ from a binomially controlled process with $p_1 = 0.009$ and with probability $w_2 = 0.07$ from a process with $p_2 = 0.080$.

The costs may then be described as in the following table:

w	p	$k_s(p)$	$k_r(p)$	$k_a(p)$	$k_m(p)$	$ k_r(p) - k_a(p) $
0.93	0.009	23.315	16.315	6.480	6.480	9.835
0.07	0.080	25.800	18.800	57.600	18.800	38.800
Average	0.014	23.489	16.489	10.058	7.342	11.863

From (12) we find

$$p_r = (16-0)/(720-35) = 0.0234,$$

from (28)

$$p_m = 0.93 \times 0.009 + 0.07 \times 0.0234 = 0.0100,$$

and from (22)

$$p_s = (23-0)/(720-35) = 0.0336.$$

To find the optimum plan for $N = 500$ from the master table with $p_r = p_s = 0.010$ we first have to find the conversion factor λ_s which corrects for the difference between p_s and p_r , i.e.

$$\lambda_s^{-1} = 1 + \frac{336-234}{0.93(234-90)} = 1.76.$$

To use the method of section 3 we find $\gamma_2 = 0.296$, $\lambda = 1.97$, $p_1/\lambda = 0.0046 \cong 0.005$, $p_2/\lambda = 0.041 \cong 0.040$, and $N^* = 1.97 N/1.76 = 1.12 N = 560$. From the master table we read $(n^*, c^*) = (60, 1)$ which gives $n = 60/1.97 = 30$ as the optimum sample size.

To illustrate the method of section 11 we have to find the conversion factor f_1 and the correction g_1 corresponding to the change from $w_2 = 0.05$ to 0.07 . Since $\rho_1 = 90/234 = 0.385 \cong 0.40$ and $\rho_2 = 800/234 = 3.42 \cong 3.50$ we have $f_1 = 1.13$ and $g_1 = 15$. We then enter the master table with

$$N^* = N \times 2.34 \times 1.13/1.76 = 1.50N = 750$$

and find $(n^*, c^*) = (60, 1)$ which finally gives $(n, c) = (30, 1)$ since $(60 + 15)/2.34 = 32$.

To find the corresponding value of R we first compute

$$\gamma_1 = 0.93(234-90)/(336-100) = 0.567$$

and

$$\gamma_2 = 0.07(800-234)/(336-100) = 0.168$$

which lead to

$$R = n + (N-n)(0.567Q(p_1) + 0.168P(p_2)).$$

From a table of the binomial distribution one finds for $(n, c) = (30, 1)$ that $Q(p_1) = 0.02982$ and $P(p_2) = 0.29579$ and consequently

$$R = 30 + 470 \times 0.0666 = 61.3.$$

The costs of sampling inspection and the average decision losses per lot are thus of nearly the same size.

Returning to the original monetary unit we find

$$k - k_m = R(k_s - k_m)/500 = 1.98$$

and finally

$$k = 7.34 + 1.98 = 9.32.$$

We thus have the following conclusion:

The quality of submitted lots is such that on the average costs per item will be 7.34 cents if all lots are classified correctly, i.e. all lots from process No. 1 are accepted and all lots from process No. 2 are rejected. To decide whether to accept or reject we inspect a sample of 30 items at the average costs of 0.97 cents per item of the lot. The decision losses will be 1.01 cents per item of the lot on the average. The first part of the costs, 7.34, depends on the prior distribution and can only be reduced by producing (or buying) lots of better quality. The second part, 1.98, depends on the sampling plan used. Since we have here used the optimum plan any change in sample size or acceptance number will result in increased costs. The average costs of accepting all lots without inspection is 10.06 cents per item.

The two functions $k_o(p) = K_o(p)/N$ and $k(p) = K(p)/N$ have been shown in Fig. 1 for the example above.

14. General remarks.

There exists already a great body of theories and tables for constructing single sampling attribute plans based on two specified quality levels (p_1, p_2) and some further requirements. To see how the present paper fits into this the most important systems have been listed below by stating the "further requirements" for each system:

(a). Specification of the producer's and the consumer's risks, see for instance Peach and Littauer [7] and Grubbs [8].

(b). Specification of the consumer's risk and minimization of the average amount of inspection for lots of process average quality (p_1) in the case of rectifying inspection, see Dodge and Romig [9].

(c). Specification of the consumer's risk and minimization of the average costs for lots of process average quality (p_1), i.e. a generalization of the Dodge-Romig LTPD system requiring specification of one cost parameter, see Hald [10].

(d). Specification of two cost parameters, p_r and p_s , and a weight, w_2 , and minimization of the average costs, as for instance in the present paper.

It follows from the results of the present paper that from an economic point of view it is not advisable to fix the consumer's or the producer's risk. On the contrary the producer's and the consumer's risks should both tend to zero with increasing lot size. This theorem is valid not only for the double binomial prior distribution but for any prior distribution, and it is valid not only for the Bayes solution but also for the minimax solution ($p_1 < p_r < p_2$), the only difference being the speed of the convergence. For a discrete prior distribution the risks tend to zero inversely proportional to N , see (63) and (64). These considerations lead to the result that if one wants a system with a fixed risk then the risk should be fixed to 50 per cent at a point between p_1 and p_2 . We may therefore increase the list of systems of sampling plans above by the following item:

(e). Minimization of average costs for lots of process average quality (p_1) under the restriction that $P(p_0) = 1/2$. Such a system, named the IQL system (Indifference Quality Level) has been discussed by Hald, see [6] and [10], and will be further discussed in a forthcoming paper. This system requires the specification of p_0 and a cost parameter. In view of the asymptotic relation (66) between c and n it is clear that p_0 should be determined from (52).

The simplest possible system based on the specification of two risks and having the same properties as the Bayes solution may be formulated as follows:

(f). Specification of the consumer's or the producer's risk as inversely proportional to lot size, and $P(p_0) = 1/2$.

This system requires only the specification of one parameter (besides the two quality levels) and it is extremely simple to handle both mathematically and numerically. This is due to the fact that the equation $P(p_0) = 1/2$ has the solution $c = np_0 + (p_0 - 2)/3$ (with sufficient accuracy for all practical applications, perhaps apart from the case $c = 0$ where the exact solution may be easily found) and that the other equation, $Q(p_1) = \alpha/N$ say, may be solved with respect to N for related values of (n, c) from the first equation. Setting $c = 0, 1, 2, \dots$ and solving the first equation for n , the second equation gives $N = \alpha / (1 - B(c, n, p_1))$ which may easily be found by means of a table of the binomial (or the Poisson) distribution. The only difficulty lies in the choice of α . If the problem is fully specified one may naturally choose α as the coefficient of $1/N$ in (63) and the system will then asymptotically give an approximation to the Bayes solution. The reason for using the simple system will,

however, usually be that some of the parameters in the problem are unknown and in that case the choice of α will to some extent be arbitrary, just as in the other cases the choice of the producer's or the consumer's risk is arbitrary. This system of sampling plans will be discussed in more detail in the forthcoming paper on the IQL system.

Turning to applications it is important to notice that a system of sampling plans in practice often is required to serve several purposes. In particular we shall here stress (a) that the system should protect the consumer against deterioration of the prior distribution, (b) that the system should work as an incentive for the producer to produce better quality or at least to keep to the quality agreed upon, see Hill [11], and (c) that (average) costs should be minimized. The first two requirements are concerned with consequences of changes of the prior distribution and the problem should therefore really be formulated as a dynamic one. However, since a dynamic model at present is lacking we shall try to indicate how the Bayesian solution may be modified to take requirements (a) and (b) into account.

One of the arguments advanced against the Bayesian method in general has been that a prior distribution does normally not exist. This may be true in many fields but certainly not for industrial mass production with its effective planning and control of operations. Admittedly the prior distribution may change, but changes are usually rather small and slow within a given production period in which the same machinery, techniques, and raw materials are being used. We are here not concerned about isolated very poor lots which may occasionally occur since any sampling plan will detect such lots.

Published data on prior distributions are scarce. Whether the double binomial distribution is a reasonable approximation to distributions occurring in practice is not known. According to the experience of the author mixed binomial distributions with beta-distributions as weight functions are rather common. (A paper analogous to the present one will present the corresponding theory and tables for the beta-distribution).

One of the drawbacks of the Bayesian solution from a practical point of view is that the solution may be acceptance (or rejection) without inspection. If one is not completely confident that the prior distribution used is the right one and is stable, then a sampling plan is required to guard against deterioration of the prior distribution. One possibility is to use the first or one of the first Bayesian sampling plans in the appropriate table. If that is not satisfactory one may in such cases use an IQL plan.

The same procedure may be used to satisfy requirement (a) above. It should first of all be noted that if a Bayesian sampling plan exists then some protection against deterioration of the prior distribution is automatically obtained and the protection may in the usual way be expressed by means of the OC curve. It is always easy when the plan has been found to compute the consumer's risk and then to decide whether the risk is sufficiently small. If the consumer's risk is too large one may again find a sufficiently large sample in the same table or turn to an IQL plan or a LTPD plan.

The price to be paid for obtaining the required protection is naturally that the plan used will not minimize costs if the prior distribution holds. If the change in the value of c is not large the increase in costs will, however, be small.

For large lots the consumer's risk for the Bayesian sampling plan will usually be much smaller than 10 per cent so that the problem does only exist for small lots.

The incentive for the producer to keep to the specified quality is usually obtained by alternating between normal and tightened inspection in a specific way such that the system reacts upon observed changes in the prior distribution. If it was possible to estimate in what way the distribution had changed the reaction could be made to depend on the change. In practice, however, one wants to install tightened inspection as soon as possible on the basis of some over-all criterion, for example when the number of lots rejected exceeds some critical limit. A thorough theory does not exist but some rules have been found to work satisfactory in practice. The Military Standard 105 D uses the same sample size for normal and tightened inspection and a reduced acceptance number, c_T , for tightened inspection. The difference between the two acceptance numbers, $c_N - c_T$, equals 1 for $2 \leq c_N \leq 4$, 2 for $5 \leq c_N \leq 20$, and 3 for $c_N \geq 21$. For $c_N = 0$ or 1, c_T is usually equal to c_N but the sample size is increased for tightened inspection. Similar rules may be used for the present tables although it has to be realized that the resulting plans will not be minimum-cost plans. The main point is, that under normal conditions the plans will minimize costs and that the plans may be adjusted to changes in the prior distribution so that costs are minimized under the new conditions. If, however, the incentive aspect of sampling inspection is more important for the user of the system than to minimize costs in case of change of the distribution then some form of tightened inspection may be introduced with the result that during periods of tightened inspection the plans will not minimize costs.

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Appendix.

Master Tables of Sampling Plans.

Tables of Conversion Factors.

Summary of Conversion Formulas.

All sampling plans in the master tables assume $p_r = p_s$ and $w_2 = 0.05$.

In the first set of tables $p_r = 10\%$ and (p_1, p_2) take on the values

$p_1 = 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, 6.0, 7.0\%$

$p_2 = 15.0, 17.5, 20.0, 25.0, 30.0\%$.

In the second set of tables $p_r = 1\%$ and (p_1, p_2) take on the values

$p_1 = 0.20, 0.25, 0.30, 0.35, 0.40, 0.50, 0.60, 0.70\%$

$p_2 = 1.50, 1.75, 2.00, 2.50, 3.00, 3.50, 4.00, 5.00, 6.00, 7.00\%$.

Single Sampling Tables for $p_1 = 2.0\%$

$p_2 = 15.0\%$			$p_2 = 17.5\%$			$p_2 = 20.0\%$			$p_2 = 25.0\%$			$p_2 = 30.0\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1	1460	Accept	1	665	Accept	1	334	Accept	1	140	Accept	1	45	Accept
1460	16	2	666	16	2	335	414	7	141	149	6	46	92	1
1600	29	3	829	18	2	479	586	17	191	259	8	93	103	6
2160	31	3	1170	29	3	727	946	19	358	396	16	139	199	8
2630	43	4	1510	31	3	1020	1240	29	509	682	18	316	346	15
3180	45	4	2270	43	4	1540	1980	31	945	1200	27	471	679	17
4560	58	5	3280	45	4	2200	2530	41	1590	2240	29	974	1400	25
5530	60	5	4410	56	5	3120	3990	43	2380	2720	37	2030	2800	27
7990	74	6	5760	58	5	4690	5040	53	3560	4880	39	2800	3970	34
11300	76	6	8530	70	6	6200	7850	55	5910	7790	48	5750	7850	36
14000	89	7	12200	72	6	9860	12100	66	10500	14200	50	7850	11000	43
19500	91	7	16400	83	7	15300	20400	68	14200	16800	58	15900	21600	45
24300	104	8	20900	85	7	20400	23600	78	22200	30800	60	21600	29800	52
33300	106	8	31300	97	8	29400	37800	80	34600	47100	69	43000	58900	54
42500	119	9	43400	99	8	42500	45600	90	63800	81900	71	58900	80000	61
56700	121	9	59700	111	9	56400	71700	92	81900	99600	79	115000	159000	63
73600	134	10	90200	113	9	87600	108000	103	132000	200000	81	159000	200000	70
96200	136	10	113000	124	10	136000	178000	105						
128000	148	11	151000	126	10	178000	200000	115						
163000	151	11												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 2.5\%$

$p_2 = 15.0\%$			$p_2 = 17.5\%$			$p_2 = 20.0\%$			$p_2 = 25.0\%$			$p_2 = 30.0\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1	1600	3	1	698	2	1	367	1	139	6	1	1	51	0
1600	1870	3	699	791	2	368	417	6	140	161	1	52	84	1
2050	2440	4	944	1040	3	418	485	15	221	291	8	85	105	6
2930	3120	4	1250	1610	3	605	777	17	292	347	15	150	246	8
3120	3230	5	1610	1960	4	778	876	26	459	673	17	247	276	14
3840	4640	5	2410	2780	4	1090	1410	28	674	867	25	382	570	16
4860	5150	6	2780	3050	5	1490	1890	38	1170	1520	27	670	856	23
6060	7330	6	3700	4790	5	2410	2810	40	1520	2070	35	1230	1690	25
7620	8070	7	4790	5670	6	2810	3220	49	2860	3290	37	1690	1840	31
9520	11500	7	6980	8250	6	4050	5260	51	3290	3660	44	2580	4100	33
12000	12600	8	8250	8690	7	5260	5750	61	4850	7000	46	4100	5330	40
14900	18100	8	10500	13200	7	8680	9750	63	7000	8350	54	7730	10200	42
18700	19700	9	14100	15900	8	9750	11200	72	11200	15000	56	10200	15400	49
23300	28200	9	19500	24000	8	14200	18000	74	15000	18900	64	23800	30900	57
29400	30600	10	24000	29100	9	18000	23200	84	25700	31700	66	45000	57800	59
36200	43900	10	36100	40800	9	29900	33000	86	31700	42500	74	57800	87700	66
46000	56200	11	40800	43700	10	33000	37900	95	58700	66100	76	133000	173000	74
58100	71800	11	53300	68800	10	48000	60400	97	66100	71800	83			
71800	87000	12	68800	79300	11	60400	77600	107	95300	137000	85			
105000	112000	12	97700	117000	11	100000	110000	109	137000	159000	93			
112000	134000	13	117000	144000	12	110000	126000	118						
163000	174000	13	179000	200000	12	159000	200000	120						
174000	200000	14												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 3.0\%$

$p_2 = 15.0\%$			$p_2 = 17.5\%$			$p_2 = 20.0\%$			$p_2 = 25.0\%$			$p_2 = 30.0\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1-1760	35	4	1-749	14	2	1-396	14	2	1-138	6	1	1-57	1	0
1760-2100	48	5	750-823	24	3	397-481	24	3	139-178	14	2	58-78	6	1
2420-2960	61	6	824-980	35	4	636-730	26	3	248-299	16	2	79-109	8	1
3450-4180	74	7	1250-1340	37	4	923-1110	35	4	404-523	23	3	167-209	14	2
4960-5910	87	8	1630-1990	47	5	1110-1340	46	5	524-602	25	3	210-298	16	2
7160-8350	100	9	1990-2200	49	5	1730-1920	56	6	807-1060	32	4	454-503	22	3
10300-11800	102	9	2710-3150	59	6	1920-2440	61	6	1060-1150	34	4	504-708	30	4
14200-15000	113	10	3150-3600	61	6	3250-3470	67	7	1530-2110	42	5	1130-1590	38	5
15000-16500	115	10	4460-4970	71	7	4370-5550	69	7	2110-2830	51	6	2520-3460	46	6
20000-21700	126	11	4970-5840	73	7	5550-6120	78	8	4080-5140	53	6	5500-7410	48	6
21700-23200	128	11	7300-7820	83	8	7760-9380	80	8	7120-7910	60	7	11100-12000	54	7
27900-31400	140	12	7820-9440	95	9	9380-10700	89	9	7910-1280	62	7	12000-15700	56	7
31400-39000	153	13	12200-15200	107	10	13700-15800	91	9	12700-15200	69	8	23500-25700	62	8
45200-54400	166	14	19100-24400	118	11	15800-18700	100	10	15200-16600	71	9	25700-33000	64	8
65000-75800	179	15	29700-31700	120	11	24200-26400	111	11	22400-29000	79	9	49100-54800	70	9
93300-106000	181	15	39100-46300	130	12	26400-32600	121	12	29000-39500	88	10	54800-68800	72	9
128000-134000	192	16	46300-50400	132	12	44000-56500	123	12	54600-69500	90	10	102000-116000	78	10
134000-147000	194	16	62500-71800	142	13	73100-77200	132	13	97000-103000	97	11	116000-143000		
178000-193000	205	17	71800-80000	144	13	97800-121000	134	13	103000-122000	99	11			
193000-200000			99600-111000	154	14	121000-133000	166	15	168000-194000	106	12			
			111000-127000	156	14	169000-200000			194000-200000					
			159000-172000											
			172000-200000											

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 3.5\%$

$p_2 = 15.0\%$		$p_2 = 17.5\%$		$p_2 = 20.0\%$		$p_2 = 25.0\%$		$p_2 = 30.0\%$	
N	n Accept	N	n Accept	N	n Accept	N	n Accept	N	n Accept
1-1850	32	1-787	22	1-399	13	1-138	6	1-62	1
1850-2030	43	788-906	33	400-470	22	139-215	13	63-71	5
2030-2150	45	1060-1290	44	549-591	24	216-252	15	72-80	7
2150-2290	56	1550-1840	55	746-886	33	342-426	22	115-175	13
2290-2430	68	2270-2610	66	887-1150	43	427-531	24	176-227	15
2430-2570	81	3340-3670	68	1420-1750	53	741-804	31	340-398	21
2570-2710	93	4580-4960	78	2250-2620	55	805-1070	40	399-569	28
2710-2850	105	4960-6360	89	3400-3570	63	1480-2110	48	833-940	30
2850-3000	107	7260-8830	100	3570-3890	65	2660-2990	50	1370-1710	36
3000-3150	118	10600-12200	111	4990-5660	74	4110-4760	57	1710-2140	38
3150-3300	130	15400-16900	113	5660-7300	84	4760-5680	59	3230-3450	44
3300-3450	143	21200-22600	123	8840-10700	94	7930-8430	66	3450-4850	52
3450-3600	155	22600-29000	134	13800-15500	96	8430-10700	75	6950-11000	59
3600-3750	168	32800-39700	145	20100-21600	105	14900-20200	84	13600-16600	61
3750-3900	180	47600-54400	156	21600-28900	115	26400-38000	92	24900-26600	67
3900-4050	193	68700-74600	158	33400-41600	125	45700-51700	94	26600-36500	75
4050-4200	205	93600-100000	168	51600-59900	135	71400-79600	101	52400-80700	82
4200-4350	218	100000-127000	179	79400-86400	137	79600-95600	110	101000-120000	84
4350-4500	230	144000-173000		112000-123000	146	138000-177000	110	180000-196000	90
				123000-159000	156			196000-200000	12
				190000-200000					

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 4.0\%$

$p_2 = 15.0\%$		$p_2 = 17.5\%$		$p_2 = 20.0\%$		$p_2 = 25.0\%$		$p_2 = 30.0\%$	
N	n Accept	N	n Accept	N	n Accept	N	n Accept	N	n Accept
1 - 1950	40	1 - 806	21	1 - 402	12	1 - 134	5	1 - 65	5
1950 - 2130	52	807 - 923	30	403 - 447	21	135 - 151	13	66 - 79	7
2260 - 2850	63	924 - 981	41	487 - 576	31	198 - 276	21	119 - 156	13
2850 - 3090	75	1260 - 1470	52	736 - 944	40	359 - 456	29	157 - 244	20
3670 - 4130	87	1770 - 2210	62	1110 - 1190	42	632 - 719	31	326 - 442	27
4730 - 5520	99	2470 - 2650	73	1530 - 1670	50	996 - 1100	38	640 - 760	29
6120 - 7390	111	3420 - 3930	84	1670 - 1880	60	1100 - 1490	46	1140 - 1240	35
7920 - 10200	122	4780 - 5830	95	2490 - 2950	70	1860 - 2220	55	1240 - 1860	42
10200 - 10800	134	6660 - 8660	105	3730 - 4610	80	3130 - 4530	63	2320 - 3010	49
13200 - 14400	146	9170 - 10100	116	5550 - 7190	90	5210 - 6550	71	4300 - 4870	51
17000 - 19100	158	12700 - 14900	127	8210 - 11200	99	8620 - 9530	73	7280 - 7980	57
22000 - 25500	170	17500 - 21900	137	12000 - 13200	109	13300 - 14200	80	7980 - 11400	64
28400 - 33900	182	24100 - 25500	148	17500 - 20300	119	14200 - 18900	88	14600 - 18000	72
36600 - 45100	194	33000 - 37200	159	25700 - 31100	129	23400 - 27100	97	26800 - 42500	79
47200 - 60400	205	45500 - 54300	170	37700 - 47600	139	38300 - 53700	105	48500 - 65700	86
60400 - 64700	217	62600 - 79400	180	55200 - 73000	148	62500 - 76100	114	87600 - 103000	94
77800 - 85600	229	85500 - 91700	191	79900 - 85600	158	103000 - 151000	122	158000 - 200000	13
100000 - 113000	241	117000 - 133000	202	115000 - 130000	168	165000 - 200000	15		
129000 - 150000	253	160000 - 200000	202	168000 - 200000	168				
165000 - 200000	253								

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 5.0\%$

$p_2 = 15.0\%$			$p_2 = 17.5\%$			$p_2 = 20.0\%$			$p_2 = 25.0\%$			$p_2 = 30.0\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1-2090	45	6	1-822	27	4	1-404	19	3	1-123	5	1	1-56	5	1
2090-2110	55	7	823-949	36	5	405-506	28	4	124-160	12	2	57-74	12	2
2110-2240	66	8	950-1000	46	6	553-716	37	5	161-230	19	3	121-176	19	3
2240-2750	77	9	1001-1340	56	7	766-998	46	6	268-304	27	4	236-359	26	4
2750-3400	88	10	1341-1790	66	8	1060-1370	55	7	432-526	35	5	424-713	32	5
3400-4210	99	11	1791-2000	76	9	1450-1860	64	8	685-888	43	6	714-825	39	6
4210-5050	109	12	2001-2390	86	10	1980-2510	73	9	1060-1480	51	7	1220-1470	46	7
5050-5230	120	13	2391-2570	96	11	2690-3360	82	10	1630-2420	58	8	2050-2600	53	8
5230-6060	131	14	2571-3280	105	12	3650-4480	91	11	2420-2770	66	9	3420-4570	60	9
6060-7270	142	15	3281-4210	115	13	4920-5950	100	12	3660-4450	74	10	5650-7990	67	10
7270-8730	153	16	4211-5350	125	14	6630-7880	109	13	5500-7130	82	11	9290-14000	74	11
8730-9850	164	17	5351-6820	135	15	8910-10400	118	14	8230-11400	90	12	15200-24300	80	12
9850-10500	175	18	6821-8680	145	16	11900-13700	127	15	12300-17900	97	13	24300-27300	87	13
10500-12600	186	19	8681-11000	155	17	16000-18000	136	16	17900-20300	105	14	39700-46700	94	14
12600-15000	196	20	11001-14000	165	18	21400-23700	145	17	26600-32000	113	15	64500-79700	101	15
15000-18100	207	21	14001-17700	174	19	28500-31100	154	18	39400-50500	121	16	105000-136000	108	16
18100-21600	218	22	17701-22400	184	20	38000-40700	164	19	53200-80000	129	17	169000-200000		
21600-22700	229	23	22401-28400	194	21	50900-67500	173	20	85700-124000	136	18			
22700-25900	240	24	28401-35900	204	22	67500-89600	182	21	124000-139000	144	19			
25900-29900	251	25	35901-45300	214	23	89600-119000	191	22	183000-200000					
29900-30900	262	26	45301-57200	224	24	119000-157000	200	23						
30900-37000	273	27	57201-72200	234	25	157000-200000								
37000-44200	283	28	72201-91000	244	26									
44200-52800	294	29	91001-115000	253	27									
52800-63200	305	30	115001-144000											
63200-75700	316	31	144001-181000											
75700-90100	327	32	181001-189000											
90100-94900														
108000-116000														
128000-143000														
153000-175000														
183000-200000														

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $P_1 = 6.0\%$

$P_2 = 15.0\%$			$P_2 = 17.5\%$			$P_2 = 20.0\%$			$P_2 = 25.0\%$			$P_2 = 30.0\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1-2110	59	8	1-784	33	5	1-361	18	3	1-103	5	1	1-39	1	0
2110-2160	69	9	785-872	42	6	362-435	26	4	104-130	11	2	40-46	5	1
2170-2390	79	10	917-1020	51	7	436-563	34	5	131-161	18	3	47-63	11	2
2450-2670	89	11	1110-1190	61	8	564-679	42	6	208-236	26	4	97-111	18	3
2770-2980	99	12	1350-1630	70	9	737-783	51	7	319-465	33	5	175-248	24	4
3160-3340	109	13	1630-1970	79	10	955-1220	59	8	466-608	40	6	294-341	31	5
3600-3730	120	14	1970-2380	88	11	1220-1350	68	9	677-787	48	7	475-693	37	6
4110-4680	130	15	2380-2760	97	12	1580-2000	76	10	975-1360	55	8	739-888	44	7
4680-5330	140	16	2870-3170	106	13	2000-2260	85	11	1360-1890	62	9	1160-1750	50	8
5330-6070	150	17	3470-3640	116	14	2550-3210	93	12	1890-2260	70	10	1750-2180	57	9
6070-6920	160	18	4190-5010	125	15	3210-3730	102	13	2690-3700	77	11	2710-4050	63	10
6920-7880	170	19	5010-6000	134	16	4090-5120	110	14	3700-5060	84	12	4050-5230	70	11
7880-8960	180	20	6000-7160	143	17	5120-6070	119	15	5060-6150	91	13	6220-9220	76	12
8960-10200	190	21	7160-7980	152	18	6480-8100	127	16	7050-7590	99	14	9220-12300	82	13
10200-11200	200	22	8630-9080	162	19	8100-10100	135	17	9720-13200	106	15	13800-15200	89	14
11600-12500	210	23	10300-12300	171	20	10100-10800	144	18	13200-16300	113	16	20700-28700	95	15
13300-13900	220	24	12300-14700	180	21	12700-15900	152	19	18200-19900	121	17	30800-35000	102	16
15100-15500	231	25	14700-17500	189	22	15900-17200	161	20	25100-33900	128	18	46200-67600	108	17
17200-19600	241	26	17500-19500	208	24	20000-24800	178	22	33900-42500	135	19	67600-80200	115	18
19600-22200	251	27	20900-22100	217	25	24800-27500	186	23	46500-51600	143	20	102000-149000	121	19
22200-25200	261	28	25000-29700	226	26	31100-38600	195	24	63800-86100	150	21	149000-183000		
25200-28600	271	29	29700-35300	235	27	38600-43600	203	25	86100-109000	157	22			
28600-32500	281	30	35300-41900	244	28	48400-60000	212	26	117000-132000	165	23			
32500-36800	291	31	41900-46700	254	29	60000-69200	220	27	161000-200000					
36800-40600	301	32	50100-52700	263	30	75000-92900	229	28						
41900-45000	311	33	59700-70800	272	31	92900-109000	237	29						
47600-49900	321	34	70800-83900	281	32	116000-144000	245	30						
54100-55400	332	35	83900-99300	290	33	144000-177000								
61500-69700	342	36	99300-110000	300	34	177000-187000								
69700-79000	352	37	118000-124000	309	35									
79000-89400	362	38	141000-167000											
89400-101000	372	39	167000-200000											
101000-115000														

$P_2 = 15.0\%$		
N	n	c
115000-130000	382	40
130000-142000	392	41
147000-157000	402	42
167000-174000	412	43
190000-193000	422	44

Continued

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 7.0\%$

$p_2 = 15.0\%$			$p_2 = 17.5\%$			$p_2 = 20.0\%$			$p_2 = 25.0\%$			$p_2 = 30.0\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1-1960	Accept		1-668	Accept		1-287	Accept		1-73	Accept		1-19	Accept	
1960-2030	73 10		669-706	31 5		288-335	17 3		74-101	5 1		20-36	1 0	
2030-2220	82 11		707-735	39 6		336-357	24 4		102-129	11 2		37-45	5 1	
2220-2260	91 12		823-958	48 7		424-462	32 5		161-211	18 3		76-88	11 2	
2440-2680	101 13		959-1120	57 8		532-591	40 6		236-331	25 4		135-213	18 3	
2680-2780	110 14		1120-1220	65 9		663-746	48 7		332-460	32 5		214-324	24 4	
2940-3240	120 15		1300-1520	74 10		818-934	56 8		461-627	39 6		325-478	30 5	
3240-3430	129 16		1520-1760	83 11		1000-1160	64 9		628-837	46 7		479-662	36 6	
3560-3920	139 17		1760-1970	91 12		1220-1430	72 10		838-892	52 8		701-912	42 7	
3920-4230	148 18		2040-2360	100 13		1490-1800	80 11		1110-1220	59 9		1010-1240	48 8	
4310-4750	158 19		2360-2450	108 14		1800-2170	88 12		1470-1660	66 10		1460-1690	54 9	
4750-5210	167 20		2720-3130	117 15		2170-2620	96 13		1930-2250	73 11		2080-2280	60 10	
5210-5740	177 21		3130-3610	126 16		2620-3150	104 14		2520-3040	80 12		2980-4180	67 11	
5740-6310	186 22		3610-3820	134 17		3150-3780	112 15		3290-4100	87 13		4180-5860	73 12	
6310-6920	196 23		4150-4760	143 18		3780-4540	120 16		4280-5550	94 14		5860-8180	79 13	
6920-7620	205 24		4760-5470	152 19		4540-5430	128 17		5550-7220	101 15		8180-11400	85 14	
7620-8350	215 25		5470-5880	160 20		5430-6490	136 18		7220-9390	108 16		11400-15100	91 15	
8350-9170	224 26		6260-7190	169 21		6490-7760	144 19		9390-12200	115 17		16000-20000	97 16	
9170-9420	233 27		7190-8220	178 22		7760-9260	152 20		12200-15800	122 18		22500-26400	103 17	
10000-11000	243 28		8220-8990	186 23		9260-11000	160 21		15800-20300	129 19		31500-34900	109 18	
11000-11500	252 29		9380-10900	195 24		11000-13100	168 22		20300-21500	135 20		44400-61500	116 19	
12100-13300	262 30		10800-12300	204 25		13100-15600	176 23		26100-28500	142 21		61500-85100	122 20	
13300-14000	271 31		12300-13700	212 26		15600-18600	184 24		33500-37800	149 22		85100-117000	128 21	
14500-15900	281 32		14000-16000	221 27		19600-22100	192 25		43100-50200	156 23		117000-162000	134 22	
15900-17100	290 33		16000-16400	229 28		22100-26300	200 26		55300-66700	153 24		162000-200000	140 23	
17400-19100	300 34		18200-20700	233 29		26300-31200	208 27		70900-90500	170 25				
19100-20900	309 35		20700-23600	247 30		31200-37000	216 28		90500-117000	177 26				
20900-22900	319 36		23600-24800	255 31		37000-43800	224 29		117000-150000	184 27				
22900-25100	328 37		27000-30800	264 32		43800-45200	231 30		150000-200000	191 28				
25100-27500	338 38		30800-35100	273 33		51900-54000	239 31							
27500-30100	347 39		35100-37200	281 34		61400-64600	247 32							
						72700-77100	255 33							
						86000-92000	263 34							
						Continued								

For H between two intervals adjacent in the table find (n, c) for the first of these intervals and use $(n+1, c)$ as optimum plan.

Single Sampling Tables for $p_1 = 7.0\%$

$p_2 = 15.0\%$			$p_2 = 17.5\%$			$p_2 = 20.0\%$			$p_2 = 25.0\%$			$p_2 = 30.0\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
00 - 32900	357	40	39900 - 45500	290	35	102000 - 110000	271	35						
00 - 36000	366	41	45500 - 51800	299	36	120000 - 131000	279	36						
00 - 39300	376	42	51800 - 55700	307	37	142000 - 156000	287	37						
00 - 43000	385	43	55700 - 67000	316	38	168000 - 186000	295	38						
00 - 44100	394	44	67000 - 76100	325	39									
00 - 51500	404	45	76100 - 83300	333	40									
00 - 53400	413	46	83300 - 98600	342	41									
00 - 61500	423	47	98600 - 112000	351	42									
00 - 64700	432	48	112000 - 126000	359	43									
00 - 73400	442	49	126000 - 144000	368	44									
00 - 78300	451	50	144000 - 148000	376	45									
00 - 87600	461	51	164000 - 185000	385	46									
00 - 95400	470	52	185000 - 200000	394	47									
00 - 104000	480	53												
00 - 114000	489	54												
00 - 125000	499	55												
00 - 136000	508	56												
00 - 148000	518	57												
00 - 162000	527	58												
00 - 177000	537	59												
00 - 193000	546	60												
00 - 197000	555	61												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.20 \%$

$p_2 = 1.50 \%$			$p_2 = 1.75 \%$			$p_2 = 2.00 \%$			$p_2 = 2.50 \%$			$p_2 = 3.00 \%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1	15700	Accept	1	7060	Accept	1	3630	Accept	1	1510	Accept	1	508	Accept
00	16100	155 2	7060	7200	155 2	3630	3770	60 1	1510	1640	60 1	509	728	5 0
00	16600	275 3	7710	8320	165 2	4210	4770	70 1	1810	2020	70 1	943	967	55 1
00	18700	285 3	9060	9950	175 2	4770	5140	160 2	2290	2640	80 1	1070	1200	65 1
00	21600	295 3	11100	11400	185 2	5550	6040	170 2	3100	3440	90 1	1360	1560	75 1
00	25400	305 3	11400	11900	285 3	6620	7320	180 2	3440	3710	160 2	1820	2170	85 1
00	26300	425 4	12800	13800	295 3	8190	9270	190 2	4090	4550	170 2	2640	2960	95 1
00	29800	435 4	15000	16300	305 3	9670	10500	285 3	5100	5780	180 2	2960	3250	155 2
00	34500	445 4	17900	19900	315 3	11400	12400	295 3	6630	7720	190 2	3670	4190	165 2
00	41000	455 4	21100	22700	425 4	13700	15100	305 3	8500	9260	270 3	4820	5620	175 2
00	44900	580 5	24500	26600	435 4	16900	19200	315 3	10300	11500	280 3	6670	8250	185 2
00	51500	590 5	29000	31800	445 4	19800	20700	410 4	13000	14800	290 3	8250	8710	250 3
00	60000	600 5	35100	39400	455 4	22600	24700	420 4	17100	20100	300 3	9860	11300	260 3
00	71800	610 5	39400	40000	560 5	27200	30100	430 4	20100	22200	380 4	13000	15100	270 3
00	77100	735 6	43100	46600	570 5	33600	37900	440 4	24300	27800	390 4	17900	21900	280 3
00	88800	745 6	50700	55400	580 5	40200	44000	540 5	31500	36000	400 4	21900	25200	350 4
00	104000	755 6	60300	67500	590 5	48100	52900	550 5	41600	46500	410 4	28800	33200	360 4
00	122000	765 6	73100	75000	700 6	58500	65100	560 5	46500	52000	490 5	38700	45700	370 4
00	124000	885 7	81000	87500	710 6	73200	80500	570 5	58100	65500	500 5	56200	62800	445 5
00	142000	895 7	95700	105000	720 6	80500	84500	665 6	74300	85200	510 5	71800	82600	455 5
00	165000	905 7	116000	128000	730 6	92400	101000	675 6	99000	106000	520 5	96200	113000	465 5
00	196000	915 7	135000	140000	840 7	112000	124000	685 6	106000	120000	600 6	136000	143000	475 5
			151000	164000	850 7	139000	160000	695 6	135000	152000	610 6	143000	154000	540 6
			179000	200000	860 7	160000	176000	795 7	173000	200000	620 6	176000	200000	550 6
						193000	200000	805 7						

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.20\%$

$p_2 = 3.50\%$		$p_2 = 4.00\%$		$p_2 = 5.00\%$		$p_2 = 6.00\%$		$p_2 = 7.00\%$	
N	n c Accept	N	n c Accept	N	n c Accept	N	n c Accept	N	n c Accept
- 252	5 0	157 - 218	5 0	80 - 118	5 0	49 - 78	5 0	33 - 58	5 0
- 348	15 0	309 - 447	15 0	174 - 256	15 0	123 - 188	15 0	97 - 156	15 0
- 740				393 - 533	25 0	299 - 468	25 0	259 - 428	25 0
- 840	60 1	640 - 730	60 1						
- 1110	70 1	857 - 1020	70 1	534 - 666	60 1	469 - 554	55 1	429 - 465	50 1
- 1560	80 1	1230 - 1510	80 1	825 - 1040	70 1	716 - 941	65 1	625 - 857	60 1
- 2380	90 1	1900 - 2580	90 1	1340 - 1780	80 1	1270 - 1800	75 1	1220 - 1820	70 1
- 3230	150 2	2580 - 2900	140 2	2470 - 2700	125 2	2450 - 2950	115 2	2470 - 3020	105 2
- 4400	160 2	3420 - 4100	150 2	3330 - 4180	135 2	3840 - 5100	125 2	4110 - 5780	115 2
- 6360	170 2	4980 - 6170	160 2	5340 - 7040	145 2	7010 - 10600	135 2	8470 - 11900	125 2
- 8400	180 2	7860 - 8710	170 2						
- 9460	235 3	8710 - 10100	220 3	9580 - 11900	195 3	10600 - 14100	175 3	11900 - 13200	155 3
- 12900	245 3	12000 - 14500	230 3	14900 - 19000	205 3	18600 - 25000	185 3	17900 - 24900	165 3
- 18600	255 3	17800 - 22300	240 3	24900 - 34800	215 3	35000 - 44000	195 3	36000 - 54600	175 3
- 24400	265 3								
- 26400	320 4	27600 - 33800	300 4	34800 - 40900	260 4	44000 - 45900	230 4	54600 - 74700	210 4
- 36100	330 4	40500 - 49200	310 4	51000 - 64700	270 4	65000 - 86400	240 4	103000 - 147000	220 4
- 51800	340 4	60800 - 76900	320 4	84000 - 113000	280 4	118000 - 175000	250 4		
- 69100	350 4	84500 - 92700	375 5	124000 - 137000	325 5	175000 - 200000	290 5		
- 72000	405 5	110000 - 133000	385 5	170000 - 200000	335 5				
- 98200	415 5	163000 - 200000	395 5						
- 140000	425 5								
- 192000	435 5								
- 200000	495 6								

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.25\%$

$p_2 = 1.50\%$		$p_2 = 1.75\%$		$p_2 = 2.00\%$		$p_2 = 2.50\%$		$p_2 = 3.00\%$	
N	n c Accept	N	n c Accept	N	n c Accept	N	n c Accept	N	n c Accept
-17100	250 3	7520 - 7670	140 2	4100 - 4290	55 1	1510 - 1610	55 1	1 - 586	5 0
-18600	375 4	6380 - 9380	150 2	4290 - 4480	140 2	1800 - 2040	65 1	587 - 863	50 1
-21400	385 4	9380 - 9550	250 3	4840 - 5280	150 2	2370 - 2830	75 1	864 - 884	60 1
-24700	395 4	10200 - 11100	260 3	5820 - 6500	160 2	2830 - 3040	145 2	983 - 1110	70 1
-29200	515 5	12000 - 13200	270 3	7490 - 7920	255 3	3370 - 3760	155 2	1260 - 1470	80 1
-31300	525 5	14600 - 15300	280 3	8620 - 9450	265 3	4250 - 4860	165 2	1740 - 2130	140 2
-36200	535 5	15300 - 16200	380 4	10400 - 11700	275 3	5660 - 6200	175 2	2350 - 2430	150 2
-42700	655 6	17500 - 19100	390 4	13200 - 13700	285 3	6200 - 6420	245 3	2730 - 3110	160 2
-46100	665 6	20900 - 23100	400 4	13700 - 15000	375 4	7130 - 7980	255 3	3590 - 4190	170 2
-53200	675 6	25500 - 27600	510 5	16500 - 18200	385 4	9010 - 10300	265 3	5000 - 5840	235 3
-62700	800 7	30000 - 32700	520 5	20200 - 22700	395 4	12000 - 13000	275 3	5840 - 6710	245 3
-72700	810 7	36000 - 39900	530 5	24700 - 25700	490 5	13000 - 14400	350 4	7680 - 8880	255 3
-84500	820 7	42400 - 43100	635 6	28100 - 30900	500 5	16100 - 18200	360 4	10400 - 12500	325 4
-100000	940 8	46700 - 50800	645 6	34300 - 38400	510 5	20700 - 24000	370 4	13700 - 15700	335 4
-107000	950 8	55700 - 61400	655 6	44200 - 47400	610 6	26700 - 28300	450 5	18000 - 20900	345 4
-124000	960 8	68300 - 70300	665 6	52000 - 57500	620 6	31600 - 35600	460 5	24500 - 29400	415 5
-146000	1085 9	70300 - 72400	765 7	64000 - 72000	630 6	40600 - 46800	470 5	31000 - 35600	425 5
-168000	1095 9	78600 - 85700	775 7	78800 - 86800	730 7	54100 - 61100	555 6	41000 - 47600	435 5
-196000		94000 - 104000	785 7	95600 - 106000	740 7	68800 - 78200	565 6	56200 - 69200	505 6
		116000 - 121000	895 8	119000 - 134000	750 7	89800 - 104000	575 6	69200 - 79800	515 6
		132000 - 144000	905 8	139000 - 144000	845 8	108000 - 117000	655 7	91900 - 107000	525 6
		158000 - 175000	915 8	158000 - 175000	855 8	132000 - 149000	665 7	126000 - 153000	595 7
		191000 - 200000	1025 9	194000 - 200000	865 8	170000 - 200000	675 7	153000 - 177000	

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.25\%$

$p_2 = 3.50\%$			$p_2 = 4.00\%$			$p_2 = 5.00\%$			$p_2 = 6.00\%$			$p_2 = 7.00\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1 - 259	5	0	1 - 154	5	0	1 - 75	5	0	1 - 45	5	0	1 - 30	5	0
60 - 370	15	0	155 - 220	15	0	76 - 114	15	0	46 - 74	15	0	31 - 55	15	0
54 - 657	55	1	320 - 486	55	1	171 - 258	55	1	119 - 185	25	0	93 - 151	25	0
58 - 736	65	1	556 - 618	65	1	414 - 453	65	1	304 - 404	55	1	260 - 365	55	1
46 - 984	75	1	728 - 866	75	1	454 - 533	75	1	405 - 542	65	1	366 - 451	65	1
70 - 1410	85	1	1050 - 1310	85	1	660 - 831	85	1	712 - 960	75	1	615 - 864	85	1
50 - 2120	135	2	1680 - 2000	135	2	1080 - 1440	135	2	1360 - 1830	110	2	1280 - 1820	110	2
120 - 2210	145	2	2000 - 2160	145	2	1870 - 2230	145	2	1830 - 2310	120	2	1820 - 2230	120	2
150 - 2980	155	2	2560 - 3070	155	2	2780 - 3550	155	2	3030 - 4100	130	2	3060 - 4360	130	2
130 - 4260	165	2	3750 - 4700	165	2	4660 - 6290	165	2	5800 - 6920	180	3	6570 - 7600	180	3
170 - 5790	205	3	5880 - 6570	205	3	6290 - 6640	205	3	6920 - 8730	165	3	7600 - 9960	165	3
190 - 6690	215	3	7830 - 9480	215	3	8230 - 10400	215	3	11500 - 15700	175	3	13800 - 19900	175	3
320 - 9270	225	3	11700 - 14800	225	3	13500 - 18100	225	3	22300 - 24600	185	3	18000 - 20000	185	3
200 - 13800	280	4	16300 - 19100	280	4	20000 - 23300	280	4	24600 - 31700	220	4	30000 - 42800	220	4
300 - 16600	290	4	22900 - 27900	290	4	29200 - 37400	290	4	42000 - 57300	230	4	60000 - 87800	230	4
500 - 23100	300	4	34800 - 43700	300	4	49400 - 62100	300	4	84900 - 112000	275	5	115000 - 131000	275	5
300 - 34300	350	5	43700 - 45500	350	5	62100 - 80400	350	5	149000 - 200000	285	5	180000 - 200000	285	5
700 - 40300	360	5	54200 - 65500	360	5	102000 - 132000	360	5						
200 - 55900	370	5	80400 - 101000	370	5	186000 - 200000	370	6						
400 - 82800	425	6	116000 - 127000	425	6									
700 - 96200	435	6	152000 - 184000	435	6									
200 - 133000	470	6												
200 - 200000	480	6												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.30\%$

$p_1 = 1.50\%$			$p_1 = 1.75\%$			$p_1 = 2.00\%$			$p_1 = 2.50\%$			$p_1 = 3.00\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
-18700	335	4	8220 - 8270	125	2	4270 - 4510	130	2	1520 - 1570	50	1	1 - 675	50	1
-19500	345	4	8360 - 8630	225	3	4940 - 5500	140	2	1760 - 2040	60	1	676 - 812	50	1
-22900	460	5	9310 - 10100	235	3	6260 - 6920	235	3	2450 - 2710	135	2	813 - 889	60	1
-24000	470	5	11100 - 12200	245	3	7590 - 8400	245	3	3020 - 3410	145	2	1000 - 1150	70	1
-27700	480	5	12200 - 13000	345	4	9420 - 10400	255	3	3900 - 4540	155	2	1350 - 1620	135	2
-32700	595	6	14100 - 15400	355	4	10400 - 11500	345	4	4880 - 5340	230	3	1970 - 2260	145	2
-34400	605	6	17000 - 18600	365	4	12600 - 14000	355	4	5970 - 6760	240	3	2580 - 2990	155	2
-40100	615	6	18600 - 19700	465	5	15700 - 17200	365	4	7750 - 9040	250	3	3530 - 4260	220	3
-45900	730	7	21500 - 23500	475	5	17200 - 18700	455	5	9360 - 10000	325	4	4450 - 5070	230	3
-49600	740	7	26000 - 28500	485	5	20600 - 22900	465	5	11300 - 12700	335	4	5810 - 6770	240	3
-58300	860	8	28500 - 29800	585	6	25700 - 28300	475	5	14600 - 17000	345	4	8010 - 9470	305	4
-66500	870	8	32400 - 35500	595	6	28300 - 30200	565	6	17500 - 18400	420	5	9470 - 10800	315	4
-77600	880	8	39200 - 43500	605	6	33300 - 36900	575	6	20600 - 23400	430	5	12400 - 14500	325	4
-91200	995	9	43500 - 44700	705	7	41400 - 46300	585	6	26800 - 31200	440	5	17200 - 19600	390	5
-95800	1005	9	48700 - 53300	715	7	46300 - 48200	675	7	32400 - 37200	520	6	19600 - 22300	400	5
-112000	1015	9	58800 - 66300	725	7	53100 - 59000	685	7	42100 - 48200	530	6	25700 - 30100	410	5
-129000	1130	10	66300 - 72700	830	8	66000 - 75300	695	7	56000 - 59200	540	6	35800 - 39700	495	6
-138000	1140	10	79500 - 87700	840	8	75300 - 84300	790	8	59200 - 66300	615	7	39700 - 45300	560	7
-163000	1265	11	97600 - 101000	850	8	93500 - 104000	800	8	75000 - 85800	625	7	52400 - 61400	570	7
-200000			101000 - 108000	950	9	118000 - 122000	810	8	99400 - 107000	635	7	73300 - 79000	580	7
			118000 - 130000	960	9	122000 - 133000	900	9	107000 - 117000	710	8	79800 - 91200	645	8
			145000 - 153000	970	9	147000 - 164000	910	9	133000 - 151000	720	8	106000 - 124000		
			153000 - 160000	1070	10	185000 - 196000	920	9	175000 - 193000	730	8	148000 - 159000		
			175000 - 193000	1080	10	196000 - 206000	1010	10	193000 - 200000	805	9	159000 - 182000		

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.30 \%$

$p_2 = 3.50 \%$			$p_2 = 4.00 \%$			$p_2 = 5.00 \%$			$p_2 = 6.00 \%$			$p_2 = 7.00 \%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1 - 262	50	1	1 - 149	50	1	1 - 70	50	1	1 - 41	50	1	1 - 27	50	1
33 - 386	60	1	150 - 217	15	0	71 - 109	15	0	42 - 70	15	0	28 - 51	15	0
30 - 638	70	1	326 - 492	50	1	166 - 256	60	1	113 - 179	50	1	88 - 145	60	1
33 - 853	80	1	493 - 519	70	1	398 - 423	70	1	303 - 348	50	1	256 - 321	50	1
10 - 1230	130	2	609 - 724	80	1	521 - 653	125	2	349 - 408	60	1	322 - 436	125	2
50 - 1750	140	2	881 - 1100	135	2	842 - 1120	145	2	530 - 707	135	2	605 - 874	145	2
50 - 1980	150	2	1420 - 1620	195	3	1500 - 1830	175	3	986 - 1440	175	3	1410 - 1640	175	3
10 - 2740	205	3	1620 - 1870	205	3	2290 - 2960	215	3	1440 - 1790	215	3	2240 - 3190	215	3
10 - 4120	215	3	2240 - 2720	265	4	3970 - 4550	265	4	2350 - 3210	265	4	4850 - 5320	265	4
20 - 4620	285	4	3400 - 4320	275	4	4550 - 5570	285	4	4820 - 5320	285	4	5320 - 7480	285	4
90 - 6380	295	4	4320 - 4970	335	5	7040 - 9150	335	5	6960 - 9380	335	5	10600 - 15700	335	5
90 - 9520	305	4	5960 - 7290	345	5	12900 - 16200	355	5	13200 - 15500	355	5	18600 - 24000	355	5
00 - 11900	360	5	9160 - 10800	405	6	20600 - 27000	405	6	15500 - 19800	405	6	33500 - 49200	405	6
00 - 16900	370	5	10800 - 12500	415	6	35700 - 46200	415	6	26400 - 36500	415	6	63400 - 75300	415	6
00 - 22500	380	5	15100 - 18500	475	7	59000 - 77700	475	7	47900 - 55000	475	7	104000 - 151000	475	7
00 - 25500	435	6	23400 - 26200	485	7	96700 - 103000	485	7	72600 - 98900	485	7			
00 - 36100	445	6	26200 - 30700			130000 - 166000			145000 - 200000					
00 - 49700	455	6	37200 - 45900											
00 - 54000	510	7	58100 - 62400											
00 - 75900	520	7	62400 - 74400											
00 - 108000	530	7	90200 - 112000											
00 - 113000			146000 - 178000											
00 - 158000														
00 - 200000														

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.35\%$

$p_1 = 1.50\%$	$p_2 = 1.75\%$	$p_2 = 2.00\%$	$p_2 = 2.50\%$	$p_2 = 3.00\%$
n Accept c	N 1 - 8500 n Accept c	N 1 - 4350 n Accept c	N 1 - 1520 n Accept c	N 1 - 761 n Accept c
- 20000	8500 - 8990	4350 - 4510	1520 - 1700	762 - 794
- 20400	9880 - 10500	5020 - 5480	1980 - 2180	893 - 1020
- 21900	10500 - 11000	5480 - 5900	2180 - 2370	1200 - 1460
- 26200	11900 - 13100	6480 - 7200	2650 - 3000	1700 - 1830
- 28300	14800 - 15900	8110 - 8430	3460 - 4020	2090 - 2410
- 33500	17400 - 19200	8430 - 9270	4020 - 4340	2850 - 3440
- 36900	21100 - 23200	10200 - 11400	4870 - 5530	3560 - 3730
- 43800	25400 - 28100	13000 - 14400	6390 - 7190	4270 - 4940
- 48300	30300 - 33600	16000 - 17900	7190 - 7590	5830 - 7050
- 57200	36900 - 41000	20000 - 22100	8520 - 9690	7050 - 8230
- 63200	43200 - 44300	24500 - 27500	11200 - 12600	9550 - 11300
- 74800	48500 - 53400	30600 - 33700	12600 - 12900	13500 - 15400
- 82600	59400 - 61600	37400 - 41900	14500 - 16500	17900 - 21100
- 97600	61600 - 63700	46600 - 50900	19000 - 21600	25400 - 28400
- 117000	69700 - 76900	56600 - 63500	21600 - 24200	32900 - 38800
- 141000	85700 - 87700	70500 - 76600	27500 - 31700	47100 - 51600
- 152000	87700 - 91200	85100 - 95600	36600 - 40100	59800 - 70400
- 182000	100000 - 110000	106000 - 115000	45500 - 52200	86600 - 93100
- 200000	124000 - 130000	128000 - 143000	61700 - 65800	108000 - 127000
	143000 - 158000	160000 - 171000	74500 - 85400	152000 - 159000
	176000 - 186000	190000 - 200000	99300 - 104000	159000 - 167000
			104000 - 107000	193000 - 200000
			121000 - 139000	
			161000 - 173000	
			173000 - 200000	

For N between two intervals adjacent in the table find (n, c) for the first of these intervals and use $(n+5, c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.35\%$

$p_2 = 3.50\%$			$p_2 = 4.00\%$			$p_2 = 5.00\%$			$p_2 = 6.00\%$			$p_2 = 7.00\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
- 259	50	1	143	50	1	66	50	1	38	50	1	25	50	1
- 394	60	1	324	60	1	158	60	1	106	60	1	82	60	1
- 624	70	1	447	70	1	357	70	1	308	70	1	248	70	1
- 861	125	2	596	125	2	506	125	2	516	125	2	421	125	2
- 1320	135	2	895	135	2	851	135	2	1010	135	2	884	135	2
- 1750	145	2	1360	145	2	1230	145	2	1190	145	2	1170	145	2
- 2470	195	3	1920	195	3	1850	195	3	1790	195	3	1620	195	3
- 3400	205	3	3010	205	3	3260	205	3	3620	205	3	2280	205	3
- 3630	215	3	3360	215	3	3450	215	3	5480	215	3	3950	215	3
- 5060	270	4	4420	270	4	4640	270	4	10500	270	4	7880	270	4
- 7300	280	4	6830	280	4	7980	280	4	16000	280	4	12500	280	4
- 8290	340	5	7770	340	5	9010	340	5	29800	340	5	18200	340	5
- 11800	350	5	11700	350	5	14100	350	5	45800	350	5	26200	350	5
- 15700	360	5	17300	360	5	22700	360	5	82500	360	5	38500	360	5
- 22100	415	6	24700	415	6	32700	415	6	129000	415	6	58700	415	6
- 30800	425	6	37600	425	6	56200	425	6	179000	425	6	118000	425	6
- 34300	490	7	51200	490	7	74800	490	7	130000	490	7	186000	490	7
- 49200	500	7	81200	500	7	130000	500	7	138000	500	7	200000	500	7
- 74500	560	8	105000	560	8	138000	560	8	169000	560	8			
- 109000	570	8	164000	570	8									
- 136000			175000											
- 200000														

For N between two intervals adjacent in the table find (n, c) for the first of these intervals and use $(n+5, c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.40\%$

1.50 %	$P_2 = 1.75 \%$			$P_2 = 2.00 \%$			$P_2 = 2.50 \%$			$P_2 = 3.00 \%$		
n Accept	N 1 - 8820	n Accept	c Accept	N 1 - 4450	n Accept	c Accept	N 1 - 1510	n Accept	c Accept	N 1 - 722	n Accept	c Accept
100	8820 - 9320	195	3	4450 - 4940	115	2	1510 - 1590	45	1	723 - 781	45	1
2500	9320 - 9710	290	4	4940 - 5360	200	3	1860 - 1980	55	1	892 - 1040	55	1
4200	10600 - 11700	300	4	5920 - 6630	210	3	1980 - 2040	115	2	1260 - 1490	65	1
7800	12400 - 13500	400	5	7130 - 7970	300	4	2270 - 2560	125	2	1490 - 1650	120	2
9500	14800 - 16700	410	5	8850 - 9970	310	4	2950 - 3420	135	2	1890 - 2210	130	2
1800	16700 - 17200	505	6	10400 - 10600	395	5	3420 - 3850	205	3	2650 - 2960	140	2
5800	18800 - 20800	515	6	11700 - 13100	405	5	4350 - 5010	215	3	2960 - 3060	195	3
2400	22700 - 23900	615	7	14800 - 15200	415	5	5780 - 6230	290	4	3510 - 4080	205	3
5500	26200 - 29100	625	7	15200 - 17100	500	6	7040 - 8070	300	4	4840 - 5510	215	3
4100	30800 - 33100	725	8	19100 - 22000	510	6	9520 - 9810	375	5	5510 - 6110	275	4
7900	36500 - 40600	735	8	22000 - 24700	600	7	11100 - 12600	385	5	7070 - 8350	285	4
9100	41800 - 45800	835	9	27600 - 31700	610	7	14700 - 15500	395	5	9910 - 10300	350	5
3600	50600 - 56500	845	9	31700 - 35400	700	8	15500 - 17100	465	6	11900 - 14000	360	5
9300	56500 - 63200	945	10	39600 - 45400	710	8	19400 - 22500	475	6	16700 - 17500	370	5
3700	70000 - 76300	955	10	45400 - 50500	800	9	24800 - 26100	550	7	17500 - 19700	430	6
13000	76300 - 9100	1050	11	56500 - 64900	810	9	29600 - 34000	560	7	23000 - 27300	440	6
19000	87000 - 96700	1060	11	64900 - 71700	900	10	39400 - 44700	640	8	30600 - 32200	505	7
44000	103000 - 108000	1160	12	80300 - 92500	910	10	51200 - 59500	650	8	37400 - 44100	515	7
51000	120000 - 133000	1170	12	92500 - 102000	1000	11	62200 - 67100	725	9	52800 - 60500	585	8
82000	138000 - 149000	1270	13	114000 - 131000	1010	11	76600 - 88500	735	9	70900 - 84700	595	8
92000	164000 - 186000	1280	13	131000 - 143000	1100	12	98000 - 114000	815	10	90900 - 97500	660	9
	186000 - 200000	1380	14	160000 - 181000	1110	12	131000 - 153000	825	10	114000 - 134000	670	9
				187000 - 200000	1200	13	153000 - 170000	900	11	156000 - 181000	740	10

For N between two intervals adjacent in the table find $\bar{n}(n,c)$ for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.40\%$

$P_2 = 3.50\%$		$P_2 = 4.00\%$		$P_2 = 5.00\%$		$P_2 = 6.00\%$		$P_2 = 7.00\%$	
N	n c Accept	N	n c Accept	N	n c Accept	N	n c Accept	N	n c Accept
1 - 249	5 0	1 - 133	5 0	1 - 59	5 0	1 - 33	5 0	1 - 21	5 0
250 - 389	45 1	134 - 199	15 0	60 - 94	15 0	34 - 59	15 0	22 - 43	15 0
497 - 526	55 1	407 - 482	50 1	147 - 235	50 1	98 - 159	50 1	76 - 128	45 1
608 - 714	65 1	579 - 710	60 1	324 - 387	60 1	281 - 373	60 1	249 - 293	55 1
864 - 1080	120 2	903 - 1170	70 1	488 - 630	70 1	497 - 691	100 2	402 - 575	65 1
1290 - 1520	130 2	1170 - 1350	115 2	854 - 1050	105 2	1000 - 1340	110 2	889 - 972	90 2
1800 - 2180	190 3	1620 - 2000	125 2	1050 - 1170	115 2	1800 - 2570	145 3	973 - 1150	100 2
2770 - 3270	200 3	2560 - 2720	135 2	1460 - 1890	125 2	2860 - 3190	155 3	1600 - 2350	135 3
3890 - 4730	260 4	2720 - 3200	180 3	2570 - 2730	160 3	4220 - 5830	195 4	3050 - 4030	145 3
5600 - 6610	270 4	3890 - 4860	190 3	2730 - 3020	170 3	7680 - 9530	205 4	5730 - 8830	175 4
7880 - 9640	330 5	5860 - 7200	245 4	3800 - 4950	215 4	12900 - 18200	245 5	8830 - 9820	185 4
10900 - 13000	340 5	8840 - 11200	255 4	6600 - 7400	225 4	20100 - 28000	255 5	13600 - 20000	220 5
15500 - 19100	400 6	12100 - 13000	305 5	9390 - 12300	270 5	38600 - 51000	290 6	25500 - 32200	230 5
20700 - 25000	410 6	15800 - 19600	315 5	15500 - 17700	280 5	51000 - 60900	300 6	45700 - 71500	265 6
30000 - 37000	470 7	24700 - 28000	370 6	22600 - 29800	325 6	81900 - 115000	340 7	71500 - 105000	275 6
39000 - 47600	480 7	34100 - 42700	380 6	35800 - 41700	335 6	130000 - 174000		153000 - 200000	
57400 - 72300	535 8	50000 - 59500	435 7	53500 - 71000	380 7				
72300 - 75900	545 8	73200 - 92500	445 7	81500 - 97500	390 7				
90000 - 109000	605 9	100000 - 126000	500 8	125000 - 167000	435 8				
134000 - 142000	615 9	156000 - 200000	510 8	184000 - 200000					
169000 - 200000									

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.50 \%$

$p_2 = 1.50 \%$				$p_2 = 1.75 \%$				$p_2 = 2.00 \%$				$p_2 = 2.50 \%$				$p_2 = 3.00 \%$			
N	n	c	Accept	N	n	c	Accept	N	n	c	Accept	N	n	c	Accept	N	n	c	Accept
1 - 22700	535	7	260	1 - 9030	260	4	4420	1 - 4420	180	3	1410	1 - 1410	40	1	626	1 - 625	45	1	
22700 - 23300	645	8	350	9030 - 9630	350	5	5500	4420 - 4950	265	4	1410 - 1460	1410 - 1460	105	2	851	626 - 725	55	1	
23300 - 24700	750	9	360	9630 - 10300	360	5	6490	5500 - 5830	275	4	1650 - 1720	1650 - 1720	115	2	1190	725 - 1030	110	2	
24700 - 27400	760	9	450	10300 - 11900	450	6	7370	6490 - 7370	355	5	1930 - 2210	1930 - 2210	185	3	1150	1190 - 1260	120	2	
27400 - 29700	860	10	460	11900 - 13000	460	6	8450	7370 - 7610	365	5	2610 - 2780	2610 - 2780	195	3	2060	1260 - 1700	130	2	
29700 - 32500	870	10	550	13000 - 14900	550	7	9900	8450 - 9530	450	6	3150 - 3640	3150 - 3640	265	4	2170	1700 - 2170	185	3	
32500 - 33700	970	11	560	14900 - 16400	560	7	12200	9900 - 10900	460	6	4060 - 4310	4060 - 4310	275	4	2940	2170 - 2510	195	3	
33700 - 35300	1080	12	650	16400 - 18200	650	8	13200	12200 - 13200	540	7	4900 - 5670	4900 - 5670	345	5	3690	2510 - 3540	255	4	
35300 - 39700	1190	13	750	18200 - 20700	750	9	15600	13200 - 14000	550	7	6150 - 6490	6150 - 6490	355	5	4710	3540 - 4080	265	4	
39700 - 42000	1300	12	850	20700 - 23500	850	10	17600	15600 - 17600	635	8	7390 - 8570	7390 - 8570	425	6	6060	4080 - 5690	325	5	
42000 - 46800	1410	15	950	23500 - 29500	950	11	22300	17600 - 19800	645	8	9140 - 9580	9140 - 9580	435	6	7450	5690 - 6400	335	5	
46800 - 50000	1520	16	1050	29500 - 36900	1050	12	23300	22300 - 23300	725	9	10900 - 12700	10900 - 12700	505	7	9750	6400 - 7450	400	6	
50000 - 55300	1630	17	1150	36900 - 51500	1150	13	25100	23300 - 25000	735	9	13400 - 14000	13400 - 14000	515	7	13500	7450 - 8860	470	7	
55300 - 60900	1740	18	1250	51500 - 57600	1250	14	30300	25100 - 30300	820	10	15900 - 18500	15900 - 18500	585	8	15400	8860 - 11400	480	7	
60900 - 69500	1845	19	1350	57600 - 71900	1350	15	40400	30300 - 35200	910	11	19500 - 20100	19500 - 20100	595	8	20400	11400 - 15400	540	8	
69500 - 77300	1855	19	1450	71900 - 89500	1450	16	49500	40400 - 43900	920	11	23000 - 26800	23000 - 26800	670	9	24100	15400 - 25900	550	8	
77300 - 89100	1955	20	1550	89500 - 111000	1550	17	53000	49500 - 53000	1000	12	28200 - 33000	28200 - 33000	680	9	30400	25900 - 37600	615	9	
89100 - 101000			1650	111000 - 138000	1650	18	61500	53000 - 54900	1010	12	35400 - 40500	35400 - 40500	750	10	37600	37600 - 45200	625	9	
101000 - 119000				138000 - 172000			69400	61500 - 69400	1095	13	40500 - 47200	40500 - 47200	760	10	54000	45200 - 58400	685	10	
119000 - 151000							89500	69400 - 76400	1105	13	54800 - 58000	54800 - 58000	830	11	58400	58400 - 66900	695	10	
151000 - 169000							172000	86400 - 90600	1185	14	58000 - 67000	58000 - 67000	840	11	79400	66900 - 90000	755	11	
169000 - 178000								90600 - 94200	1195	14	77900 - 82700	77900 - 82700	910	12	90000	90000 - 98800	765	11	
								107000 - 118000	1280	15	82700 - 95000	82700 - 95000	920	12	117000	98800 - 138000	825	12	
								118000 - 132000	1290	15	110000 - 118000	110000 - 118000	990	13	138000	138000 - 145000	835	12	
								149000 - 154000	1370	16	118000 - 134000	118000 - 134000	1000	13	171000	145000 - 200000			
								154000 - 163000	1380	16	156000 - 167000	156000 - 167000	1070	14					
								184000 - 200000			167000 - 189000	167000 - 189000							

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.50 \%$

$p_2 = 3.50 \%$			$p_2 = 4.00 \%$			$p_2 = 5.00 \%$			$p_2 = 6.00 \%$			$p_2 = 7.00 \%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1-207	Accept		1-106	Accept		1-46	Accept		1-25	Accept		1-16	Accept	
208-330	5	0	107-163	5	0	47-75	5	0	26-47	5	0	17-34	5	0
417-467	45	1	259-337	15	0	120-193	15	0	80-131	15	0	62-105	15	0
550-662	55	1	338-349	45	1	271-340	50	1	232-250	45	1	193-211	25	0
829-1000	65	1	429-523	55	1	436-578	60	1	329-445	55	1	212-255	45	1
1000-1080	110	2	662-899	65	1	798-874	100	2	643-755	65	1	356-523	55	1
1270-1540	120	2	900-1080	110	2	1100-1440	110	2	756-946	95	2	737-1060	90	2
1970-2120	175	3	1310-1560	120	2	1870-2390	155	3	1280-1900	105	2	1560-2010	100	2
2500-3040	195	3	1910-2280	170	3	3120-4030	165	3	1900-2360	140	3	2010-2820	130	3
3620-3870	240	4	2800-3680	180	3	4030-4900	205	4	3210-4510	150	3	4130-5180	140	3
4600-5610	250	4	3680-4530	230	4	6370-8370	215	4	4510-5610	185	4	5180-7150	170	4
6370-6850	305	5	620-6920	240	4	8370-9780	255	5	7670-10400	195	4	10500-12900	180	4
8100-10000	315	5	6920-8730	290	5	12700-17100	265	5	10400-13000	230	5	12900-17700	210	5
10900-11900	370	6	10900-12600	300	5	17100-19200	305	6	17900-23400	240	5	26000-31900	220	5
14300-17600	380	6	12600-13500	345	6	24800-34400	315	6	23400-29800	275	6	31900-43300	250	6
18500-20400	435	7	16600-20900	355	6	34400-37300	355	7	41100-52400	285	6	63600-77600	260	6
24600-31000	445	7	22700-25200	405	7	48000-64200	365	7	52400-67500	320	7	77600-105000	290	7
31000-34800	500	8	31100-40500	415	7	69700-92200	410	8	93500-116000	330	7	154000-187000	300	7
42000-51800	510	8	40500-46800	465	8	123000-138000	420	8	116000-152000	365	8	187000-200000	330	8
51800-58900	565	9	58000-72200	475	8	138000-176000	460	9						
71400-86200	575	9	72200-86300	525	9									
86200-99300	630	10	108000-128000	535	9									
121000-143000	640	10	128000-159000	585	10									
143000-167000	695	11												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.60 \%$

$P_2 = 1.50 \%$			$P_2 = 1.75 \%$			$P_2 = 2.00 \%$			$P_2 = 2.50 \%$			$P_2 = 3.00 \%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1-23100	670	9	1-8630	320	5	1-4010	165	3	1-1190	40	1	1-493	40	1
23100-23800	770	10	8630-9290	405	6	4010-4420	245	4	1190-1320	100	2	493-524	50	1
24500-25900	870	11	9290-9710	500	7	4420-4920	330	5	1360-1440	110	2	524-606	60	1
27400-28200	975	12	11000-12100	590	8	5590-6250	415	6	1630-1900	175	3	606-720	105	2
30700-33800	1075	13	13000-13700	685	9	7100-7860	500	7	2050-2230	185	3	720-895	115	2
34500-36900	1175	14	15500-17100	775	10	8980-9810	585	8	2550-3010	250	4	895-952	175	3
36800-40300	1280	15	17500-19200	870	11	11300-12100	670	9	3010-3300	260	4	952-987	185	3
43700-49200	1380	16	21000-22000	960	12	13700-14200	755	10	3790-4320	325	5	987-1140	240	4
49200-53000	1480	17	22000-23900	1055	13	14200-15000	845	11	4320-4720	335	5	1140-1350	250	4
55500-57900	1585	18	26100-26700	1130	14	16900-17700	930	12	5440-6040	400	6	1350-1650	310	5
62500-70400	1685	19	29700-30900	1270	15	17700-18300	1015	13	6040-6620	410	6	1650-1840	320	5
70400-76100	1785	20	30900-33100	1335	16	20600-22000	1100	14	7650-8340	475	7	1840-2160	375	6
79300-82900	1890	21	33100-36600	1425	17	22000-25200	1185	15	8340-9110	525	8	2160-2640	445	7
82900-101000	1995	22	36600-43200	1520	18	27300-30500	1270	16	10600-11400	560	8	2640-2760	510	8
101000-109000	2090	23	43200-51000	1615	19	33700-37000	1360	17	11400-12500	625	9	2760-3220	520	8
113000-118000	2195	24	51000-60400	1705	20	37000-41500	1445	18	14500-15400	635	9	3220-3860	575	9
128000-143000	2295	25	60400-70800	1800	21	41500-44700	1530	19	15400-16900	700	10	3860-4080	585	9
143000-155000	2395	26	70800-83500	1895	22	51100-53900	1615	20	20700-22800	710	10	4080-4650	615	10
161000-168000	2500	27	83500-115000	1995	23	61200-62800	1700	21	26600-27800	775	11	4650-5520	710	11
182000-200000			115000-135000	2075	24	62800-65000	1785	22	27900-30700	795	11	5520-6120	740	12
			135000-150000			73600-77200	1865	23	35800-37100	850	12	6120-6560	845	13
			150000-172000			77200-83300	1950	24	37100-41000	860	12	6560-7730	855	13
			172000-197000			83300-94600	2035	25	47900-49400	925	13	7730-9040	910	14
			197000-220000			94600-116000	2115	26	49400-54800	1000	14	9040-9400	920	14
			220000-260000			116000-142000	2195	27	54800-65500	1075	15	9400-11000	980	15
			260000-300000			142000-173000	2275	28	65500-72900	1150	16	11000-122000		
			300000-350000			173000-200000	2355	29	72900-86900	1225	17	122000-163000		
			350000-400000			200000-230000	2435	30	86900-96200			163000-193000		
			400000-450000			230000-260000	2515	31	96200-115000					
			450000-500000			260000-290000	2595	32	115000-128000					
			500000-550000			290000-320000	2675	33	128000-152000					
			550000-600000			320000-350000	2755	34	152000-170000					
			600000-650000			350000-380000	2835	35						
			650000-700000			380000-410000	2915	36						
			700000-750000			410000-440000	3000	37						
			750000-800000			440000-470000	3080	38						
			800000-850000			470000-500000	3160	39						
			850000-900000			500000-530000	3240	40						
			900000-950000			530000-560000	3320	41						
			950000-1000000			560000-590000	3400	42						
			1000000-1050000			590000-620000	3480	43						
			1050000-1100000			620000-650000	3560	44						
			1100000-1150000			650000-680000	3640	45						
			1150000-1200000			680000-710000	3720	46						
			1200000-1250000			710000-740000	3800	47						
			1250000-1300000			740000-770000	3880	48						
			1300000-1350000			770000-800000	3960	49						
			1350000-1400000			800000-830000	4040	50						
			1400000-1450000			830000-860000	4120	51						
			1450000-1500000			860000-890000	4200	52						
			1500000-1550000			890000-920000	4280	53						
			1550000-1600000			920000-950000	4360	54						
			1600000-1650000			950000-980000	4440	55						
			1650000-1700000			980000-1010000	4520	56						
			1700000-1750000			1010000-1040000	4600	57						
			1750000-1800000			1040000-1070000	4680	58						
			1800000-1850000			1070000-1100000	4760	59						
			1850000-1900000			1100000-1130000	4840	60						
			1900000-1950000			1130000-1160000	4920	61						
			1950000-2000000			1160000-1190000	5000	62						
			2000000-2050000			1190000-1220000	5080	63						
			2050000-2100000			1220000-1250000	5160	64						
			2100000-2150000			1250000-1280000	5240	65						
			2150000-2200000			1280000-1310000	5320	66						
			2200000-2250000			1310000-1340000	5400	67						
			2250000-2300000			1340000-1370000	5480	68						
			2300000-2350000			1370000-1400000	5560	69						
			2350000-2400000			1400000-1430000	5640	70						
			2400000-2450000			1430000-1460000	5720	71						
			2450000-2500000			1460000-1490000	5800	72						
			2500000-2550000			1490000-1520000	5880	73						
			2550000-2600000			1520000-1550000	5960	74						
			2600000-2650000			1550000-1580000	6040	75						
			2650000-2700000			1580000-1610000	6120	76						
			2700000-2750000			1610000-1640000	6200	77						
			2750000-2800000			1640000-1670000	6280	78						
			2800000-2850000			1670000-1700000	6360	79						
			2850000-2900000			1700000-1730000	6440	80						
			2900000-2950000			1730000-1760000	6520	81						
			2950000-3000000			1760000-1790000	6600	82						
			3000000-3050000			1790000-1820000	6680	83						
			3050000-3100000			1820000-1850000	6760	84						
			3100000-3150000			1850000-1880000	6840	85						
			3150000-3200000			1880000-1910000	6920	86						
			3200000-3250000			1910000-1940000	7000	87						
			3250000-3300000			1940000-1970000	7080	88						
			3300000-3350000			1970000-2000000	7160	89						
			3350000-3400000			2000000-2030000	7240	90						
			3400000-3450000			2030000-2060000	7320	91						
			3450000-3500000			2060000-2090000	7400	92						
			3500000-3550000			2090000-2120000	7480	93						
			3550000-3600000			2120000-2150000	7560	94						
			3600000-3650000			2150000-2180000	7640	95						
			3650000-3700000			2180000-2210000	7720	96						
			3700000-3750000			2210000-2240000	7800	97						
			3750000-3800000			2240000-2270000	7880	98						
			3800000-3850000			2270000-2300000	7960	99						
			3850000-3900000			2300000-2330000	8040	100						
			3900000-3950000			2330000-2360000	8120	101						
			3950000-4000000			2360000-2390000	8200	102						
			4000000-4050000			2390000-2420000	8280	103						
			4050000-4100000			2420000-2450000	8360	104						

Single Sampling Tables for $P_1 = 0.50\%$

$P_2 = 3.50\%$			$P_2 = 4.00\%$			$P_2 = 5.00\%$			$P_2 = 6.00\%$			$P_2 = 7.00\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1 - 144	Accept		1 - 74	Accept		1 - 31	Accept		1 - 16	Accept		1 - 9	Accept	
145 - 225	5 0		75 - 115	5 0		32 - 54	5 0		17 - 34	5 0		10 - 25	5 0	
340 - 382	45 1		181 - 279	15 0		88 - 139	15 0		60 - 97	15 0		47 - 80	15 0	
451 - 552	55 1								167 - 202	25 0		142 - 182	25 0	
702 - 800	65 1		280 - 292	45 1		227 - 280	50 1		203 - 271	50 1		183 - 211	45 1	
801 - 960	110 2		352 - 432	55 1		361 - 483	60 1		371 - 541	60 1		295 - 437	55 1	
1160 - 1480	120 2		553 - 714	65 1		633 - 759	100 2		597 - 825	95 2		569 - 662	85 2	
1480 - 1700	170 3		715 - 786	105 2		981 - 1360	110 2		1150 - 1370	105 2		935 - 1400	95 2	
2050 - 2540	180 3		957 - 1200	115 2		1360 - 1740	150 3		1370 - 1620	135 3		1400 - 1820	125 3	
2540 - 2810	230 4		1420 - 1780	165 3		2300 - 2690	160 3		2200 - 2960	145 3		2650 - 3320	135 3	
3380 - 4150	240 4		2230 - 2550	175 3		2690 - 2950	195 4		2960 - 4040	180 4		3320 - 4840	165 4	
4150 - 4470	290 5		2550 - 3090	220 4		3810 - 5100	205 4		5730 - 6120	190 4		7340 - 8830	200 5	
5370 - 6610	300 5		3370 - 4390	230 4		5100 - 6200	245 5		6120 - 7290	220 5		12600 - 16500	210 5	
6610 - 6910	350 6		4390 - 5170	275 5		8180 - 9620	255 5		10100 - 12700	230 5		15600 - 22600	240 6	
8350 - 10400	360 6		6470 - 7390	285 5		9620 - 12900	295 6		12700 - 17700	265 6		35700 - 40500	275 7	
10400 - 12600	415 7		7390 - 8490	330 6		17500 - 20400	340 7		25200 - 31000	305 7		58200 - 79100	285 7	
16100 - 19400	475 8		10600 - 12300	340 6		26900 - 32300	350 7		43200 - 51500	315 7		79100 - 102000	315 8	
24300 - 29300	535 9		12300 - 13800	385 7		32300 - 41800	390 8		51500 - 74000	350 8		151000 - 172000	325 8	
36000 - 38100	545 9		17200 - 20200	395 7		58000 - 65400	435 9		101000 - 128000	390 9		172000 - 180000	350 9	
38100 - 43900	595 10		20200 - 22100	440 8		85700 - 106000	445 9		179000 - 200000	400 9				
53900 - 58200	605 10		27600 - 33000	450 8		106000 - 132000	485 10							
58200 - 65600	655 11		33000 - 35400	495 9		177000 - 190000	495 10							
60400 - 88700	665 11		44000 - 53600	505 9		190000 - 200000	530 11							
88700 - 97700	715 12		53600 - 56300	550 10										
119000 - 135000	725 12		69800 - 87000	560 10										
			87000 - 110000	610 11										
			140000 - 174000	665 12										
135000 - 145000	775 13													
177000 - 200000	785 13													

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.70 \%$

$p_2 = 1.50 \%$			$p_2 = 1.75 \%$			$p_2 = 2.00 \%$			$p_2 = 2.50 \%$			$p_2 = 3.00 \%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1 - 21600	Accept		1 - 7440	Accept		1 - 3220	Accept		1 - 826	Accept		1 - 220	Accept	
21600 - 22400	800	11	7440 - 8330	380	6	3220 - 3300	155	3	827 - 902	40	1	221 - 365	5	0
22400 - 23500	890	12	8330 - 9140	465	7	3470 - 3630	230	4	1050 - 1120	100	2	366 - 425	45	1
23500 - 25500	985	13	9140 - 10100	550	8	4090 - 4250	240	4	1290 - 1520	110	2	503 - 612	55	1
25500 - 27700	1080	14	10100 - 12600	640	9	4250 - 4470	310	5	1580 - 1680	170	3	752 - 885	110	2
27700 - 30100	1175	15	12600 - 13600	725	10	5050 - 5220	320	5	1930 - 2260	180	3	1050 - 1270	120	2
30100 - 32800	1270	16	13600 - 15000	810	11	5220 - 5460	390	6	2260 - 2360	240	4	1270 - 1320	170	3
32800 - 35700	1365	17	15000 - 16300	900	12	6170 - 6380	400	6	2700 - 3130	250	4	1550 - 1860	180	3
35700 - 38800	1460	18	16300 - 18900	985	13	6380 - 6600	470	7	3130 - 3660	315	5	1950 - 2130	235	4
38800 - 42200	1555	19	18900 - 19900	1075	14	7450 - 7760	480	7	4200 - 4820	385	6	2520 - 2880	245	4
42200 - 45900	1650	20	19900 - 24600	1160	15	7760 - 8930	555	8	5540 - 6260	455	7	2880 - 3290	300	5
45900 - 49900	1745	21	24600 - 26400	1250	16	9360 - 10600	635	9	7230 - 8020	525	8	3950 - 4120	310	5
49900 - 54200	1840	22	26400 - 31800	1335	17	11200 - 12600	715	10	9340 - 10200	595	9	4120 - 5000	365	6
54200 - 58900	1935	23	31800 - 34700	1420	18	13400 - 14800	795	11	12000 - 12800	665	10	5750 - 6280	425	7
58900 - 63900	2030	24	34700 - 37500	1510	19	16000 - 17300	875	12	15300 - 16100	735	11	7500 - 7970	435	7
63900 - 69300	2125	25	37500 - 45400	1595	20	19000 - 20200	955	13	18800 - 19500	745	11	7970 - 9250	490	8
69300 - 75200	2220	26	45400 - 48900	1685	21	22400 - 23600	1035	14	19500 - 23400	810	12	10900 - 11100	550	9
75200 - 87800	2320	27	48900 - 59400	1770	22	26500 - 27400	1115	15	24700 - 29200	880	13	13600 - 14900	560	9
87800 - 95500	2415	28	59400 - 63600	1860	23	31300 - 36800	1200	16	31200 - 36200	950	14	14900 - 16600	615	10
95500 - 104000	2510	29	63600 - 75900	1945	24	36800 - 41900	1280	17	39400 - 44800	1020	15	20200 - 24000	680	11
104000 - 113000	2605	30	75900 - 82500	2030	25	43300 - 48400	1360	18	49500 - 55400	1090	16	27200 - 29100	740	12
113000 - 123000	2700	31	82500 - 88500	2120	26	50800 - 55800	1440	19	62200 - 68300	1160	17	34900 - 36700	750	12
123000 - 133000	2795	32	88500 - 107000	2205	27	59500 - 64400	1520	20	78000 - 84200	1230	18	36700 - 41800	805	13
133000 - 145000	2890	33	107000 - 114000	2295	28	69700 - 74200	1600	21	97700 - 104000	1300	19	43400 - 60300	870	14
145000 - 158000	2985	34	114000 - 138000	2380	29	81600 - 85400	1680	22	122000 - 127000	1370	20	66100 - 72200	930	15
158000 - 171000	3080	35	138000 - 147000	2470	30	95400 - 98200	1760	23	153000 - 183000	1445	21	88300 - 103000	995	16
171000 - 186000	3175	36	147000 - 156000	2555	31	112000 - 130000	1845	24	192000 - 200000	1515	22	118000 - 124000	1055	17
186000 - 200000	3270	37	156000 - 176000	2640	32	130000 - 148000	1925	25				149000 - 158000	1065	17
			176000 - 190000	2730	33	152000 - 169000	2005	26				158000 - 176000	1120	18
			190000 - 200000	2820	34	177000 - 194000	2085	27						

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.70 \%$

$p_2 = 3.50 \%$			$p_2 = 4.00 \%$			$p_2 = 5.00 \%$			$p_2 = 6.00 \%$			$p_2 = 7.00 \%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1-80	Accept		1-42	Accept		1-17	Accept		1-8	Accept		1-5	Reject	
81-122	5 0		43-68	5 0		18-33	5 0		9-21	5 0		6-16	5 0	
130-272	15 0		105-162	15 0		56-87	15 0		40-65	15 0		33-55	15 0	
273-324	50 1		233-256	50 1		140-194	25 0		107-172	25 0		95-160	25 0	
393-489	60 1		314-394	60 1		195-209	50 1		173-205	50 1		161-223	50 1	
634-746	110 2		520-572	70 1		267-351	60 1		277-394	60 1		326-455	60 1	
907-1140	120 2		573-613	105 2		505-595	100 2		480-649	95 2		456-520	85 2	
1140-1190	165 3		748-945	115 2		771-1020	110 2		914-1030	105 2		738-1050	95 2	
1420-1760	175 3		1080-1210	160 3		1020-1120	145 3		1030-1330	135 3		1050-1490	125 3	
1850-2090	225 4		1190-1830	170 3		1450-1900	155 3		1870-2040	145 3		2220-2890	160 4	
2540-2880	235 4		1830-2220	215 4		1900-2570	195 4		2040-2570	175 4		4560-5490	195 5	
2880-3550	285 5		2810-2960	225 4		3360-4420	240 5		3620-3910	185 4		8080-9600	205 5	
4310-4860	340 6		2960-3210	265 5		5820-7450	285 6		3910-4840	215 5		9600-15000	235 6	
5970-6400	350 6		3980-4700	275 5		9940-12400	330 7		6830-7390	225 5		19300-27600	270 7	
6400-7990	400 7		4700-5570	320 6		16800-20600	375 8		7390-9000	255 6		38400-50700	305 8	
9320-10600	455 8		7260-7750	370 7		23400-33900	420 9		12700-13800	265 6		76000-93000	340 9	
13500-14100	510 9		9670-11300	380 7		47500-55700	465 10		13800-15600	295 7		138000-152000	350 9	
17200-19500	520 9		11300-13200	425 8		74500-80000	475 10		23400-25600	305 7		152000-170000	375 10	
19500-22600	570 10		17200-22600	480 9		80000-91000	510 11		25600-30300	335 8				
27900-29700	625 11		26200-30500	530 10		121000-134000	520 11		42700-47100	345 8				
36400-40100	635 11		39700-51700	585 11		134000-148000	555 12		47100-55200	375 9				
40100-47300	685 12		59900-69200	635 12					77700-86500	385 9				
57100-61600	740 13		90200-117000	690 13					86100-100000	415 10				
75900-81400	750 13		136000-155000	740 14					140000-158000	425 10				
81400-97800	800 14								158000-181000	455 11				
115000-127000	855 15													
157000-164000	865 15													
164000-200000	915 16													

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Relation between p_r and w_2 for fixed $(p_{10}, p_{20}, \gamma_2)$.

Use the same sampling plan for $w_2 = 0.05$ and $p_{ro} = 0.01$ (0.10) as for w_2 and $p_r = 0.01f$ (0.10f) where f is given in the table.

w_2	$p_2 = p_{20}/p_{ro}$					ρ_1 = p_{10}/p_{ro}
	1.5	2.0	3.0	5.0	7.0	
1	.51	.44	.40	.38	.37	.2
	.78	.77	.76	.76	.76	.7
2	.70	.63	.58	.55	.53	.2
	.85	.84	.83	.82	.82	.7
3	.83	.78	.73	.70	.69	.2
	.91	.89	.89	.88	.88	.7
4	.93	.90	.87	.86	.85	.2
	.96	.95	.94	.94	.94	.7
5	1.00	1.00	1.00	1.00	1.00	.2
	1.00	1.00	1.00	1.00	1.00	.7
6	1.06	1.09	1.11	1.14	1.15	.2
	1.04	1.05	1.05	1.06	1.06	.7
7	1.10	1.16	1.22	1.27	1.29	.2
	1.07	1.09	1.11	1.12	1.12	.7
8	1.14	1.22	1.31	1.39	1.43	.2
	1.10	1.13	1.16	1.17	1.18	.7
9	1.18	1.28	1.40	1.51	1.56	.2
	1.12	1.17	1.21	1.23	1.24	.7
10	1.20	1.33	1.48	1.63	1.69	.2
	1.15	1.20	1.25	1.29	1.30	.7
12	1.25	1.41	1.63	1.84	1.95	.2
	1.19	1.27	1.34	1.40	1.42	.7
14	1.28	1.48	1.75	2.03	2.19	.2
	1.22	1.33	1.43	1.51	1.54	.7
16	1.31	1.54	1.86	2.22	2.41	.2
	1.25	1.38	1.51	1.62	1.67	.7
18	1.33	1.58	1.95	2.38	2.63	.2
	1.27	1.42	1.59	1.72	1.79	.7
20	1.35	1.63	2.03	2.54	2.84	.2
	1.29	1.46	1.66	1.83	1.91	.7

Table of b_1 , b_2 , and b_3 .

ρ_1 $= p_1/p_r$	$\rho_2 = p_2/p_r$					
	1.5	2.0	3.0	5.0	7.0	
0.2	0.63	0.64	0.65	0.66	0.67	b_1
	0.24	0.27	0.31	0.35	0.37	b_2
	1.42	1.29	1.15	1.03	0.96	b_3
0.3	0.61	0.62	0.64	0.65	0.65	b_1
	0.19	0.23	0.27	0.31	0.34	b_2
	1.69	1.48	1.28	1.11	1.03	b_3
0.4	0.60	0.61	0.63	0.64	0.64	b_1
	0.16	0.19	0.24	0.29	0.32	b_2
	1.99	1.69	1.41	1.19	1.08	b_3
0.5	0.59	0.60	0.62	0.63	0.63	b_1
	0.13	0.17	0.21	0.27	0.30	b_2
	2.33	1.91	1.54	1.27	1.14	b_3
0.6	0.58	0.59	0.61	0.62	0.62	b_1
	0.11	0.15	0.19	0.25	0.28	b_2
	2.74	2.15	1.68	1.34	1.19	b_3
0.7	0.58	0.59	0.60	0.62	0.62	b_1
	0.09	0.13	0.18	0.23	0.26	b_2
	3.24	2.42	1.82	1.42	1.25	b_3

Conversion factor f_2 for N due to a change in $p_r = p_s$ for fixed (p_1, p_2, w_2) .

Use $N^* = Nf_2$ as argument in the master table to find (n^*, c^*) .

$p_r = p_s = 0.01\lambda$ or 0.10λ , (p_1, p_2, w_2) are given in the master tables,

$\rho_1 = 100p_1$ or $10p_1$, $\rho_2 = 100p_2$ or $10p_2$.

ρ_2	ρ_1	$\lambda = 0.50$	0.60	0.70	0.80	0.90	1.00	1.25	1.50	1.75	2.00	3.00
1.5	0.2	1.80	1.52	1.33	1.18	1.08	1.00	-	-	-	-	-
	0.3	2.21	1.79	1.50	1.28	1.12	1.00	-	-	-	-	-
	0.4	-	2.11	1.69	1.38	1.16	1.00	-	-	-	-	-
	0.5	-	-	1.90	1.50	1.21	1.00	-	-	-	-	-
	0.6	-	-	-	1.64	1.27	1.00	-	-	-	-	-
	0.7	-	-	-	-	-	1.00	-	-	-	-	-
2.0	0.2	1.69	1.47	1.30	1.18	1.08	1.00	0.87	-	-	-	-
	0.3	1.92	1.63	1.41	1.24	1.10	1.00	0.82	-	-	-	-
	0.4	-	1.79	1.52	1.30	1.13	1.00	0.78	-	-	-	-
	0.5	-	-	1.62	1.36	1.16	1.00	0.73	0.58	-	-	-
	0.6	-	-	-	1.42	1.19	1.00	0.69	0.52	-	-	-
	0.7	-	-	-	-	1.21	1.00	0.65	0.47	-	-	-
3.0	0.2	1.53	1.38	1.25	1.15	1.07	1.00	0.88	0.80	-	-	-
	0.3	1.64	1.46	1.31	1.19	1.08	1.00	0.85	0.74	0.67	-	-
	0.4	-	1.52	1.36	1.22	1.10	1.00	0.82	0.70	0.62	0.56	-
	0.5	-	-	-	1.24	1.11	1.00	0.80	0.66	0.57	0.50	-
	0.6	-	-	-	-	1.12	1.00	0.78	0.63	0.53	0.46	-
	0.7	-	-	-	-	-	1.00	0.76	0.60	0.49	0.41	-
5.0	0.2	1.39	1.28	1.19	1.12	1.05	1.00	0.89	0.82	0.76	0.72	-
	0.3	-	1.31	1.22	1.13	1.06	1.00	0.88	0.79	0.72	0.67	-
	0.4	-	-	1.23	1.14	1.07	1.00	0.86	0.76	0.69	0.63	0.50
	0.5	-	-	-	1.14	1.07	1.00	0.85	0.74	0.66	0.60	0.46
	0.6	-	-	-	-	-	1.00	0.85	0.73	0.65	0.58	0.42
	0.7	-	-	-	-	-	1.00	0.85	0.73	0.63	0.56	0.39
7.0	0.2	1.31	1.23	1.16	1.10	1.05	1.00	0.91	0.84	0.79	0.74	0.65
	0.3	-	1.24	1.17	1.11	1.05	1.00	0.89	0.81	0.75	0.70	0.59
	0.4	-	-	-	1.11	1.05	1.00	0.89	0.80	0.74	0.68	0.55
	0.5	-	-	-	-	1.05	1.00	0.89	0.79	0.72	0.66	0.52
	0.6	-	-	-	-	-	1.00	0.89	0.79	0.72	0.65	0.50
	0.7	-	-	-	-	-	1.00	0.90	0.80	0.72	0.65	0.48

Correction g_2 to n^* due to a change in $p_r = p_s$.

Reference value $p_r = p_s = 0.010$, $n = n^* + g_2$.

ρ_2	ρ_1	$\lambda = 0.50$	0.60	0.70	0.80	0.90	1.00	1.25	1.50	1.75	2.00	3.00
1.5	0.2	130	100	70	50	25	0	-	-	-	-	-
	0.3	160	120	85	55	30	0	-	-	-	-	-
	0.4	-	150	105	65	35	0	-	-	-	-	-
	0.5	-	-	135	85	40	0	-	-	-	-	-
	0.6	-	-	-	115	50	0	-	-	-	-	-
	0.7	-	-	-	-	-	0	-	-	-	-	-
	0.7	-	-	-	-	-	0	-	-	-	-	-
2.0	0.2	75	55	40	25	15	0	-30	-	-	-	-
	0.3	95	70	50	30	15	0	-35	-	-	-	-
	0.4	-	90	60	35	15	0	-40	-	-	-	-
	0.5	-	-	80	45	20	0	-45	-90	-	-	-
	0.6	-	-	-	60	25	0	-55	-105	-	-	-
	0.7	-	-	-	-	40	0	-70	-125	-	-	-
	0.7	-	-	-	-	40	0	-70	-125	-	-	-
3.0	0.2	40	30	20	15	5	0	-15	-25	-	-	-
	0.3	55	40	25	15	5	0	-15	-30	-45	-	-
	0.4	-	50	30	20	10	0	-20	-35	-50	-65	-
	0.5	-	-	-	25	10	0	-20	-40	-55	-70	-
	0.6	-	-	-	-	15	0	-25	-45	-60	-80	-
	0.7	-	-	-	-	-	0	-30	-55	-75	-90	-
	0.7	-	-	-	-	-	0	-30	-55	-75	-90	-
5.0	0.2	20	15	10	5	5	0	- 5	-15	-20	-20	-
	0.3	-	20	15	10	5	0	-10	-15	-20	-25	-
	0.4	-	-	15	10	5	0	-10	-15	-20	-25	-45
	0.5	-	-	-	10	5	0	-10	-20	-25	-30	-50
	0.6	-	-	-	-	-	0	-10	-20	-30	-35	-55
	0.7	-	-	-	-	-	0	-15	-25	-35	-40	-60
	0.7	-	-	-	-	-	0	-15	-25	-35	-40	-60
7.0	0.2	15	10	5	5	0	0	- 5	-10	-10	-15	-25
	0.3	-	15	10	5	0	0	- 5	-10	-10	-15	-25
	0.4	-	-	-	5	5	0	- 5	-10	-15	-15	-25
	0.5	-	-	-	-	5	0	- 5	-10	-15	-20	-30
	0.6	-	-	-	-	-	0	-10	-15	-20	-20	-35
	0.7	-	-	-	-	-	0	-10	-15	-20	-25	-35
	0.7	-	-	-	-	-	0	-10	-15	-20	-25	-35

For $p_r = p_s = 0.10$ the correction is $g_2/10$ (rounded down).

Conversion factor f_1 for N due to a change in w_2 .

Reference value of $w_2 = 0.05$, $p_s = p_r$.

Use $N^* = Nf_1$ as argument in the master table to find (n^*, c^*) .

$100w_2$	p_2/p_r					p_1/p_r
	1.5	2.0	3.0	5.0	7.0	
1	0.54	0.56	0.58	0.61	0.63	0.2
	0.46	0.48	0.51	0.54	0.57	0.7
2	0.70	0.72	0.74	0.76	0.77	0.2
	0.65	0.66	0.68	0.71	0.73	0.7
3	0.82	0.83	0.84	0.86	0.87	0.2
	0.78	0.79	0.81	0.83	0.84	0.7
4	0.92	0.92	0.93	0.94	0.94	0.2
	0.90	0.90	0.91	0.92	0.93	0.7
5	1.00	1.00	1.00	1.00	1.00	0.2
	1.00	1.00	1.00	1.00	1.00	0.7
6	1.07	1.07	1.06	1.06	1.05	0.2
	1.09	1.09	1.08	1.07	1.06	0.7
7	1.14	1.13	1.12	1.10	1.09	0.2
	1.17	1.16	1.15	1.13	1.12	0.7
8	1.19	1.18	1.16	1.15	1.13	0.2
	1.25	1.24	1.21	1.19	1.17	0.7
9	1.25	1.23	1.21	1.18	1.17	0.2
	1.32	1.30	1.27	1.24	1.22	0.7
10	1.30	1.27	1.25	1.22	1.20	0.2
	1.39	1.37	1.33	1.29	1.26	0.7
12	1.38	1.36	1.32	1.28	1.25	0.2
	1.51	1.48	1.43	1.38	1.34	0.7
14	1.46	1.43	1.38	1.33	1.30	0.2
	1.63	1.58	1.52	1.45	1.40	0.7
16	1.53	1.49	1.44	1.38	1.34	0.2
	1.73	1.68	1.61	1.52	1.46	0.7
18	1.60	1.55	1.49	1.42	1.38	0.2
	1.83	1.77	1.68	1.58	1.52	0.7
20	1.65	1.60	1.53	1.46	1.41	0.2
	1.92	1.85	1.75	1.64	1.56	0.7

Correction g_1 to n^* due to a change in w_2 .

Reference value of $w_2 = 0.05$, $p_g = p_r = 0.01$, $n = n^* + g_1$.

$100w_2$	p_2/p_r					p_1/p_r
	1.5	2.0	3.0	5.0	7.0	
1	-125	-90	-60	-35	-25	0.2
	-205	-125	-70	-35	-25	0.7
2	-70	-50	-35	-20	-15	0.2
	-115	-70	-40	-20	-15	0.7
3	-40	-30	-20	-10	-10	0.2
	-65	-40	-25	-10	-10	0.7
4	-20	-15	-10	-5	-5	0.2
	-30	-20	-10	-5	-5	0.7
5	0	0	0	0	0	0.2
	0	0	0	0	0	0.7
6	15	10	5	5	5	0.2
	25	15	10	5	5	0.7
7	25	20	15	5	5	0.2
	45	25	15	10	5	0.7
8	40	30	20	10	5	0.2
	60	40	20	10	10	0.7
9	50	35	20	15	10	0.2
	80	50	25	15	10	0.7
10	55	40	25	15	10	0.2
	90	55	30	15	10	0.7
12	75	50	35	20	15	0.2
	120	70	40	20	15	0.7
14	85	60	40	25	15	0.2
	140	85	50	25	15	0.7
16	100	70	45	25	20	0.2
	160	100	55	30	20	0.7
18	110	80	50	30	20	0.2
	175	110	60	30	20	0.7
20	120	85	55	30	20	0.2
	195	120	65	35	25	0.7

For $p_g = p_r = 0.10$ the correction is $g_1/10$ (rounded down).

Summary of conversion formulas

to find

(n, c) corresponding to $(N, p_r, p_s, p_1, p_2, w_2)$

from

(n^*, c^*) in the master table for $(N^*, p_{ro}, p_{1o}, p_{2o})$.

For $\begin{cases} p_r \leq 0.05 \\ p_r > 0.05 \end{cases}$ use master table with $p_{ro} = \begin{cases} 0.01 \\ 0.10 \end{cases}$.

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)} \right)^{-1}.$$

Formula 1.

$$\gamma_2 = \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)} \quad \text{and} \quad \lambda p_{ro} = \frac{p_2 + 19\gamma_2 p_1}{1 + 19\gamma_2}.$$

Use

$$N^* = N\lambda_s \lambda, \quad p_{ro}, \quad p_{1o} = p_1/\lambda, \quad p_{2o} = p_2/\lambda$$

as arguments to find (n^*, c^*) in the master table.

$$(n, c) = (n^*/\lambda, c^*).$$

If (p_{1o}, p_{2o}) fall outside the tabulated range use formula 2.

Formula 2.

$$\lambda = p_r/p_{ro}, \quad \rho_1 = p_1/p_r, \quad \rho_2 = p_2/p_r.$$

Use

$$N^* = N\lambda_s \lambda f_1(w_2, \rho_1, \rho_2), \quad p_{ro}, \quad p_{1o} = \rho_1 p_{ro}, \quad p_{2o} = \rho_2 p_{ro}$$

as arguments to find (n^*, c^*) in the master table.

$$(n, c) = ((n^* + g_1(w_2, \rho_1, \rho_2))/\lambda, c^*).$$

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