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THE STRESS-STRAIN RELATIONS OF SPIRALLOY WITH FILAMENTS PARALLEL OR NORMAL TO THE APPLIED FORCE
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THE STRESS-STRAIN RELATIONS OF SPIRALLOY WITH FILAMENTS PARALLEL OR NORMAL TO THE APPLIED FORCE

by

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FOREWORD

This report, written by Bernard W. Shaffer, Consultant to Hercules Powder Company, was prepared by the Publications Group of the Research and Development Department, Bacchus Plant Chemical Propulsion Division, Hercules Powder Company, in accordance with requirements established by Contract AF 04(647)-243, Exhibit B, paragraph J. This report presents research data on the stress-strain relations of Spiralloy with filaments parallel or normal to the applied force.

ABSTRACT

Moduli of elasticity are derived from stress-strain relations for Spiralloy with windings either parallel or normal to the applied force.

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INTRODUCTION

To evaluate its material properties, tensile tests have been conducted on thin, flat Spiralloy specimens such as those shown in figures 1a, 1b. The specimens are taken from large diameter Spiralloy tubes which are cut and flattened to form sheets having all filaments oriented in a single direction. To minimize the effect of the residual stresses created during the flattening process, the specimens are cut from the sheets in such a manner that the axis along which they will subsequently be loaded is originally parallel to the axis of the basic tube.

The tensile tests have been conducted to evaluate such important material properties as the moduli of elasticity and the ultimate stresses of Spiralloy with all filaments either parallel or normal to the direction of the applied load. Whereas, the tests provide data which describe the macroscopic behavior of Spiralloy, they do not clearly explain the influence of the filament and resin material properties on the characteristics of the composite material. The purpose of this report is to shed some light on the latter problem. At the same time, a method of analysis is developed which may prove useful in subsequent studies of multilayered Spiralloy having different winding angles.

For the present analysis it is assumed that the resin obeys the stress-strain relation shown in figure 2. The stress-strain relation is linear within the elastic range, where it obeys Hooke's Law; it may deform without requiring any significant additional force once its yield stress Y_r has been reached. The numerical value of Y_r includes size effect which reflects the very large ratio of resin surface area to volume present in Spiralloy because of its many internal filaments. Once its yield stress is reached, the true behavior of resin is open to speculation. It may yield, fracture, or display a combination of both characteristics. Yet, because we are concerned only with strains of the order of magnitude of elastic strains, it is reasonable to assume that some straining beyond the yield stress can occur without requiring any additional load.

The filaments are also elastic and obey Hooke's Law up to the yield stress Y_f. The numerical value of Y_f used in the analysis includes the influence of a very thin coating of resin, which seals the microscopic surface imperfections of the filaments and thereby inhibits the initiation of surface cracks. The filament behavior beyond its yield stress is not specified because our analysis terminates once Y_f is reached.

The analysis is divided into two parts. First, we discuss the behavior of Spiralloy with filaments parallel to the applied force; then we discuss the behavior of Spiralloy with filaments normal to the applied force.

FILAMENTS PARALLEL TO THE APPLIED FORCE

Let us evaluate the behavior of Spiralloy with filaments parallel to the applied force by studying a model which exhibits the basic characteristics of the actual material. Such a model is shown in figure 3. It consists of two parallel bars joined to rigid end plates. One bar, of cross-sectional area A_f, has a modulus of elasticity E_f and a yield stress Y_f equal to that of the filament. The other bar, of cross-sectional area A_r, has a modulus of elasticity E_r and a yield stress Y_r equal to that of the resin. The cross-sectional areas A_f and A_r are chosen so that the percentage of filament by volume in the model and in the specimen are equal.

The model is designed so that when a force P is applied to its end plates, the force separates between the filament and resin as it does in the actual material. Thus

$$P = P_f + P_r \tag{1}$$

where P_f is the force in the filament and P_r is the force in the resin. The corresponding axial strains are

$$\varepsilon_f = \frac{P_f}{A_f E_f} = \frac{\sigma_f}{E_f} \tag{2}$$

in the filament and

$$\varepsilon_r = \frac{P_r}{A_r E_r} = \frac{\sigma_r}{E_r} \tag{3}$$

in the resin, where σ_f and σ_r are their respective stresses. The model is also designed so that its filament and resin bars are strained equally when an external force is applied. Thus:

$$\frac{P_f}{A_f E_f} = \frac{P_r}{A_r E_r} \quad (4)$$

Equations (1) and (4) show that the forces in the resin and in the filament are

$$P_r = \frac{P}{1 + \frac{A_f E_f}{A_r E_r}} \quad (5a)$$

and

$$P_f = \frac{P}{1 + \frac{A_r E_r}{A_f E_f}} \quad (5b)$$

respectively. Equations (2) and (3) show that because of equal straining, the stresses in the filament and in the resin are proportional to their respective moduli of elasticity:

$$\sigma_f = \frac{E_f}{E_r} \sigma_r \quad (6)$$

The modulus of elasticity of the filament is greater than that of the resin. Therefore, according to equation (6), the stress in the filament is greater than that of the resin as long as both elements remain elastic. This conclusion remains valid as long as the yield stress of the resin is reached before that of the filament, because the resin stress then remains fixed at

Y_r while the filament stress may continue to increase. The yield stress requirement is contained in the inequality:

$$\frac{Y_f}{Y_r} > \frac{E_f}{E_r} \qquad (7)$$

Inspection of the material properties of filament and resin currently used in Spiralloy shows that the previous inequality is always satisfied.

As the external force on the model increases, the stress in each bar also increases. The stress in the resin reaches its yield stress first, for reasons explained in the previous paragraph. At the instant the yield stress of the resin is reached:

$$\sigma_r = \frac{P_r}{A_r} = Y_r \qquad (8)$$

The corresponding external force, to be designated as P_1, may be evaluated from equations (5a) and (8) as:

$$P_1 = Y_r A_r \left[1 + \frac{A_f E_f}{A_r E_r} \right] \qquad (9)$$

The corresponding nominal stress, equal to the total external force P_1 divided by the total cross-sectional area A, is:

$$\sigma_1 = \frac{P_1}{A} = Y_r \left[1 + \frac{A_f}{A} \left(\frac{E_f}{E_r} - 1 \right) \right] \qquad (10)$$

The associated nominal strain ε_1 is numerically equal to the strain of the resin, and is given by equations (3) and (8) as:

$$\varepsilon_1 = \frac{Y_r}{E_r} \tag{11}$$

The nominal stress and the nominal strain are of interest because they are the quantities measured during a tensile test.

The relation between nominal stress and nominal strain is shown in figure 4. The resin and filament are elastic; therefore, the stress-strain relation is linear as long as the nominal stress is less than that indicated by equation (10), marked point 1 on figure 4. The slope of the line from the origin to point 1 is the modulus of elasticity of Spiralloy. It is given by equations (10) and (11) as

$$E_s = \frac{\sigma_1}{\varepsilon_1} = E_r \left[1 + \frac{A_f}{A}\left(\frac{E_f}{E_r} - 1\right)\right] \tag{12}$$

and may be verified experimentally. The previous expression shows that the modulus of elasticity of Spiralloy is directly proportional to the modulus of elasticity of the resin, the percentage of filament in the Spiralloy and the moduli of elasticity ratio E_f/E_r. The difference between the modulus of elasticity of the filament and that of Spiralloy is:

$$E_f - E_s = \left(1 - \frac{A_f}{A}\right)(E_f - E_r) \tag{13}$$

Since both terms in parenthesis are positive, it is apparent that Spiralloy has a smaller modulus of elasticity than the filament it contains. The difference between the moduli of elasticity, $E_f - E_s$, is directly proportional to $(1 - A_f/A)$, the percentage of resin present in Spiralloy.

Once the yield stress of the resin is reached, its stress remains constant, at Y_r, as the external force P increases, and:

$$P = Y_r A_r + P_f \tag{14}$$

The stress in the filament

$$\sigma_f = \frac{P_f}{A_f} = \frac{P}{A_f} - Y_r \left(\frac{A_r}{A_f} \right) \tag{15}$$

may continue to increase until its yield stress Y_f is reached. The filament reaches its yield stress when the nominal stress

$$\sigma_2 = \frac{P_2}{A} = Y_r + \frac{A_f}{A} (Y_f - Y_r) \tag{16}$$

and the associated nominal strain, numerically equal to the strain of the filament, is given by the expression:

$$\varepsilon_2 = \frac{\sigma_f}{E_f} = \frac{Y_f}{E_f} \tag{17}$$

The stress and strain at the instant the filament yields is depicted in figure 4, by the coordinate of point 2. The nominal stress-strain relation is linear between points 1 and 2 with a slope given by equations (10), (11), (16) and (17), as:

$$C = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{A_f}{A} E_f \tag{18}$$

The previous expression shows that the slope C is equal to the product of the modulus of elasticity of the filament and the percentage of filament present in the Spiralloy. The difference between E_s and C is given by equations (12) and (18) as:

$$E_s - C = \left(1 - \frac{A_f}{A}\right) E_r \qquad (19)$$

It can be seen that E_s is larger than C because both terms to the right of the previous expression are positive quantities.

The relative position of point 1 to point 2 on the stress-strain curve of figure 4 may be of interest in evaluating the range over which the stresses are completely elastic. According to equations (10) and (16):

$$\frac{\sigma_1}{\sigma_2} = \frac{1 + \frac{A_f}{A}\left(\frac{E_f}{E_r} - 1\right)}{1 + \frac{A_f}{A}\left(\frac{Y_f}{Y_r} - 1\right)} \qquad (20)$$

As the ratios of yield stress and moduli of elasticity approach each other, point σ_1 approaches σ_2 and the section of the stress-strain curve from point 1 to point 2 decreases. It completely disappears when

$$\frac{Y_f}{Y_r} = \frac{E_F}{E_r} \qquad (21)$$

which, according to equation (7), is the limit of applicability of the present solution.

The relative difference between the slopes E_s and C may also be of interest. According to equations (12) and (19):

$$\frac{E_s - C}{E_s} = \frac{1 - \frac{A_f}{A}}{\left(1 - \frac{A_f}{A}\right) + \frac{A_f}{A}\frac{E_f}{E_r}} \qquad (22)$$

Obviously, then, the difference in slopes may be minimized, if desired, by maximizing the percentage of filament and the ratio of moduli of elasticity E_f/E_r used in a Spiralloy specimen.

It is also worthwhile noticing that according to equations (17) and (11), the difference in strain

$$\varepsilon_2 - \varepsilon_1 = \frac{Y_r}{E_r}\left(\frac{Y_f}{Y_r}\frac{E_r}{E_f} - 1\right) \tag{23}$$

is of the order of magnitude of elastic strains. Thus the horizontal section of the stress-strain relation of resin included in the present analysis is indeed small.

Some interest also exists in the secant modulus of elasticity E_s', evaluated at the instant the filament reaches its yield stress. The latter is equal to the slope of the line from the origin to point 2 on figure 4. According to equations (16) and (17):

$$E_s' = \frac{\sigma_2}{\varepsilon_2} = \frac{Y_r}{Y_f}\left[1 + \frac{A_f}{A}\left(\frac{Y_f}{Y_r} - 1\right)\right] E_f \tag{24}$$

It is interesting to observe that the secant modulus is a function of the yield stress of the filament and resin, whereas the modulus of elasticity E_s, defined by equation (12), is a function of their moduli of elasticity. Obvisously E_s' is less than E_s as long as the inequality of equation (7) holds.

FILAMENTS NORMAL TO THE APPLIED FORCE

A specimen of Spiralloy with filaments normal to the applied force is shown in figure 1b. Section A-A, taken through the specimen, is shown in figure 5. In the latter view, the circular cross-sections of the filaments are shown enlarged. The filaments are of diameter d, equally spaced and separated by the resin.

Evaluation of the exact stress distribution in the specimen of figure 5 would be extremely complicated, because from a mathematical point of view the resin is a multiply-connected domain with statically indeterminate boundary conditions, while the filaments are subjected to statically indeterminate boundary conditions. However, the situation is not hopeless, if initially one is content with a qualitative examination of the problem. Then, a number of assumptions can be made which lead to relatively simple but reasonable mathematical expressions. The assumptions may be extensive, in the initial analysis of the problem, as long as the consequences are properly recognized. Some attempt will be made to balance the effect of major assumptions, so that the derived results may also be used for reasonable first engineering approximations. With the aforementioned point of view in mind, let us replace the structure of figure 5 with a relatively simple model which has the basic characteristics of Spiralloy, but is more amenable to mathematical analysis.

The model should account for a difference in the volume occupied by the filament and by the resin. This difference may be expressed in terms of the area ratio of filament to resin. To find an expression for this ratio, let us examine the Spiralloy sheet of unit thickness shown in figure 5. The sheet contains a uniform distribution of filaments of diameter d, spaced a distance p apart. The triangular region marked abc is a typical region which is repeated throughout the specimen. Its total area is

$$A = \frac{\sqrt{3}}{4} p^2 \tag{25}$$

while the area occupied by the filament is only:

$$A_f = \frac{1}{2}\left(\frac{\pi d^2}{4}\right) \tag{26}$$

Thus, the filament area ratio present in the composite material is:

$$\frac{A_f}{A} = \frac{\pi}{2\sqrt{3}} \frac{d^2}{p^2} \tag{27}$$

If one were to draw lines nn and oo tangent to the filaments, in the direction of the applied force, it would be observed that the lines are separated a distance s, where:

$$s = \frac{\sqrt{3}}{2} p - d \tag{28}$$

The width of such a strip of continuous resin decreases as the percentage of filament increases. According to equations (27) and (28), s vanishes when A_f/A is equal to 68 percent. In the first analysis we will assume that s is always positive. Thus the analysis will be applicable to Spiralloy having less than 68 percent filament by volume. The analysis will then be modified to describe the behavior of Spiralloy having more than 68 percent filament by volume.

With the distance $s > 0$, one can imagine a stress flow in the direction of the external axial force which lies entirely within the resin strips, and is uninterrupted by the filaments. Most of the stress, however, does not flow without interruption from one external boundary to the other. It continually meets and alternately crosses regions of resin and filament.

To average the effect of the aforementioned stress flow, let us design our model with two bars, one of pure resin and the other consisting of filament and resin in series. Such a model is shown in figure 6. Each bar is of length p and of unit width perpendicular to the page. The pure resin bar is marked r and is of width s. The other bar, to be referred to as the equivalent bar, is marked e, and is of width d. The equivalent bar contains filament and resin in the same area ratio as its counterpart shown in figure 5, as the region between the lines mm and nn. Thus the length of filament in the model:

$$\frac{\ell_f}{p} = \left(\frac{\pi d^2}{4}\right)\left(\frac{1}{pd}\right)$$

or (29)

$$\ell_f = \frac{\pi d}{4}$$

The resin bar has a modulus of elasticity E_r and a yield stress Y_r. The modulus of elasticity and yield stress of the equivalent bar is yet to be evaluated.

To evaluate the modulus of elasticity of the equivalent bar E_e, let us study its stress-strain characteristics. When a force P is applied to the model, some of it, to be designated P_e, passes through the equivalent bar which then elongates by an amount:

$$e_e = \frac{P_e}{d}\left[\frac{\ell_f}{E_f} + \left(\frac{p - \ell_f}{E_r}\right)\right] \tag{30}$$

Its ratio of stress to strain is its modulus of elasticity. Thus

$$E_e = \frac{P_e/d}{e_e/p} = \frac{E_r}{1 - \frac{\ell_f}{p}\left(1 - \frac{E_r}{E_f}\right)} \tag{31}$$

or in view of equation (29):

$$E_e = \frac{E_r}{1 - \frac{\pi}{4}\frac{d}{p}\left(1 - \frac{E_r}{E_f}\right)} \tag{32}$$

The modulus of elasticity of the equivalent bar is larger than that of the resin because $E_f > E_r$.

The force applied to the model of figure 6 distributes itself between the resin and equivalent bar so that the sum of the forces in each bar is equal to the total applied force P:

$$P_r + P_e = P \tag{33}$$

Within the elastic range, the resin bar, elongates by an amount

$$e_r = \frac{P_r p}{sE_r} = \frac{\sigma_r p}{E_r} \tag{34}$$

while the equivalent bar elongates by an amount

$$e_e = \frac{P_e p}{E_e d} = \frac{\sigma_e p}{E_e} \tag{35}$$

where σ_r and σ_e are their respective stresses. Since the elongations e_r and e_e are equal:

$$\sigma_e = \frac{E_e}{E_r}\sigma_r \tag{36}$$

We see, therefore, that the stress in the equivalent bar is larger than that of the resin bar because $E_e > E_r$.

Equations (33), (34) and (35) also show that since $e_r = e_e$, the stress in the equivalent bar:

$$\sigma_e = \frac{P_e}{d} = \frac{P}{d\left(1 + \frac{s}{d}\frac{E_r}{E_e}\right)} \tag{37}$$

In view of equations (28) and (32), the previous expression may be rewritten to read:

$$\sigma_e = \frac{P}{d\left[\frac{\sqrt{3}}{2}\frac{p}{d} - \frac{\pi}{4}\left(1 - \frac{E_r}{E_f}\right)\left(\frac{\sqrt{3}}{2} - \frac{d}{p}\right)\right]} \tag{38}$$

The stress σ_e is equal to the stress in the resin and in the filament of the equivalent bar. It increases as the external force P increases. When the external force is large enough, the yield stress of the equivalent bar is reached. Yielding begins in the resin region of the equivalent bar, because the yield stress of the resin Y_r is less than that of the filament. According to equation (38) the yield stress of the resin Y_r is reached when the external force is P_1, where

$$P_1 = Y_r\, d\left[\frac{\sqrt{3}}{2}\frac{p}{d} - \frac{\pi}{4}\left(1 - \frac{E_r}{E_f}\right)\left(\frac{\sqrt{3}}{2} - \frac{d}{p}\right)\right] \tag{39}$$

and the force in the equivalent bar:

$$P_e = Y_r d \tag{40}$$

The nominal stress in the model at the initiation of yield is designated

σ_1 where in view of equations (28) and (39):

$$\sigma_1 = \frac{P_1}{d + s} = Y_r \left[1 - \frac{\pi}{4} \frac{d}{p} \left(1 - \frac{E_r}{E_f} \right) \left(1 - \frac{2}{\sqrt{3}} \frac{d}{p} \right) \right] \tag{41}$$

The nominal strain at the initiation of yield is equal to e_e/p, the strain of the equivalent bar. In view of equations (29), (30) and (40) the nominal strain at the instant of initial yield may be expressed as:

$$\varepsilon_1 = \frac{Y_r}{E_r} \left[1 - \frac{\pi}{4} \frac{d}{p} \left(1 - \frac{E_r}{E_f} \right) \right] \tag{42}$$

The nominal stress-strain relation of the model is linear up to the stress σ_1 of equation (41), as shown in figure 4. The slope of the linear region between the origin and point 1 is equal to the ratio of stress to strain at initial yield. It is the modulus of elasticity of Spiralloy and may be evaluated from equations (41) and (42), as:

$$E_s = \frac{\sigma_1}{\varepsilon_1} = E_r \left[\frac{1 - \frac{\pi}{4} \frac{d}{p} \left(1 - \frac{E_r}{E_f} \right) \left(1 - \frac{2}{\sqrt{3}} \frac{d}{p} \right)}{1 - \frac{\pi}{4} \frac{d}{p} \left(1 - \frac{E_r}{E_f} \right)} \right] \tag{43}$$

The previous expression may also be written in terms of the filament ratio by making use of equation (27), so that:

$$E_s = E_r \left[\frac{1 - \left(1 - \frac{E_r}{E_f} \right) \left(0.8247 \sqrt{\frac{A_f}{A}} - \frac{A_f}{A} \right)}{1 - 0.8247 \sqrt{\frac{A_f}{A}} \left(1 - \frac{E_r}{E_f} \right)} \right] \tag{44}$$

The equations for the modulus of elasticity of Spiralloy with a 90 degree winding angle show an increase as either the percentage of filament increases or the ratio E_r/E_f decreases. They also reveal a very simple expression for E_s when $2d/\sqrt{3}$ is equal to unity, corresponding to a filament ratio of 68 percent, namely:

$$E_s = \frac{E_r}{1 - \frac{\pi\sqrt{3}}{8}\left(1 - \frac{E_r}{E_f}\right)} \tag{45}$$

Once the external force applied to the model shown in figure 6 is of sufficient intensity to cause yielding of the resin in the equivalent bar, the force within the equivalent bar remains constant and the external force P separates in accordance with the expression:

$$P = P_r + Y_r\, a \tag{46}$$

The stress in the resin bar is then

$$\sigma_r = \frac{P}{s} - \frac{d}{s} Y_r \tag{47}$$

and the associated strain:

$$\varepsilon_r = \frac{1}{E_r}\left(\frac{P}{s} - \frac{d}{s} Y_r\right) \tag{48}$$

As the external force P is increased, so is the stress in the resin bar. The stress may increase until yielding starts in the resin bar when Y_r is reached. According to equations (28) and (47) the external force on the model is then

$$P_2 = \frac{\sqrt{3}}{2} Y_r\, p \tag{49}$$

and the nominal stress:

$$\sigma_2 = \frac{P_2}{s + d} = Y_r \tag{50}$$

The corresponding nominal strain is then numerically equal to the strain of the resin bar. According to equations (28), (48) and (49) the nominal strain when the resin bar starts to yield is given by the expression:

$$\varepsilon_2 = \frac{Y_r}{E_r} \tag{51}$$

The nominal stress-strain behavior of Spiralloy is also linear when forces are applied between P_1 and P_2. It is shown on figure 4 as the straight line from point 1 to point 2. The slope of this line is equal to the ratio $(\sigma_2 - \sigma_1)/(\varepsilon_2 - \varepsilon_1)$, which in view of equations (41), (42), (50) and (51) may be expressed as:

$$C = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \left(1 - \frac{2}{\sqrt{3}}\frac{d}{p}\right) E_r \tag{52}$$

The previous expression is applicable as long as there exists a positive distance s, as shown in figure 5. When s vanishes, the term in the parenthesis also vanishes, as one can see by checking with equation (28) and equation (52).

The force that can be carried beyond initial yield is given by equations (39) and (49) as:

$$P_2 - P_1 = \frac{\pi}{4}\left(1 - \frac{E_r}{E_f}\right)\left(\frac{\sqrt{3}}{2} - \frac{d}{p}\right) Y_r d \tag{53}$$

It is relatively small, but may be increased by decreasing the modulus ratio E_r/E_f and increasing the yield stress of the resin. It disappears entirely when the filament ratio is 68 percent, for then $(\sqrt{3}/2 - d/p)$ vanishes.

The secant modulus associated with point 2 on figure 4 is equal to the ratio of equation (50) to (51). Thus:

$$E_s = \frac{\sigma_2}{\epsilon_2} = E_r \tag{54}$$

The secant modulus corresponding to final yield is equal to the modulus of elasticity of the resin and is independent of all the other material properties.

The previous analysis applies when there exists less than 68 percent filament by volume. When more filament exists, the model of figure 6 should be modified by removing the bar of pure resin. The appropriate model is then only a single bar with filament and resin in series. Its modulus of elasticity was previously expressed by equation (31). When more then 68 percent filament exists the length ratio ℓ_f/p is equal to the area ratio A_f/A, and equation (31) should be rewritten to read:

$$E_e = \frac{E_r}{1 - \frac{A_f}{A}\left(1 - \frac{E_r}{E_f}\right)} \tag{55}$$

Under an externally applied force, the resin and filament are equally stressed. Again the resin is the first to yield. This time, however, there is no secondary member to restrain the elongation, so that the model can extend without any additional external force until fracture occurs. Thus, there is no discontinuity of slope in the stress-strain curve when Spiralloy contains more than 68 percent of filament by volume.

DISCUSSION OF RESULTS

An analysis has been presented which describes the stress-strain relations of Spiralloy having filaments which lie either parallel to or normal to the applied load. To simplify what would otherwise be an extremely complex analysis, the actual materials were replaced by equivalent models whose physical characteristics are similar to Spiralloy. Even though the models were carefully selected, some limitations exist that restrict the use of the results that were obtained. These limitations arise by virtue of the fact that substitution of the models in our analyses permitted us to find an average stress field instead of the actual complex situation. Nevertheless, the external results that were obtained, such as the shape of the stress-strain curves and the expressions for the moduli of elasticity, do represent a good engineering approximation of the actual situation. On the other hand, details of the internal results, such as a description of the stress field or the stresses at which the stress-strain curves change slope, are not nearly as accurate and should be used only from a qualitative viewpoint.

There have been some reports published in the open literature which point to the reasonableness of the results obtained. For example, Kitzmiller, DeHaven and Young (1)+ have presented experimental evidence which shows the stress-strain curve of Spiralloy to be of the general form shown in figure 4. Yet other investigators have expressed the belief, privately, that the stress-strain curve is linear up to the failure. The discrepancy may now be resolved by virtue of the fact that the present analysis shows that either possibility may occur, depending upon the material composition. Furthermore, since the present analysis shows the upper region of the curve to be relatively small for some Spiralloy specimens, it is suggested that additional data be taken to identify the shape of the curve in the yield

(+) Superscript numbers in squared brackets refer to the references listed in the bibliography.

region before a positive conclusion is reached.

It is felt that sufficient data has been published for us to confirm the reasonableness of the present theory with regard to its prediction of the moduli of elasticity of Spiralloy. For example, if we chose as representative values, E_r equal to $0.4x10^6$ and E_f equal to $10x10^6$, equation (12) shows that the resulting modulus of elasticity of Spiralloy with 68 percent filament by volume and with filaments which lie parallel to the external force should be $6.93x10^6$, while equation (45) shows that it should be only $1.15x10^6$, when the filaments lie normal to the external force. If, on the other hand, we assume that the modulus of elasticity of the resin is $0.35x10^6$, while the ratio between that of the filament and resin remains 25, the computed moduli of elasticity become $6.06x10^6$ and $1.01x10^6$ respectively. Such large differences in the moduli of elasticity have been observed, and the numerical values are consistent with experimental evidence.

Let us now study some of the relations between the moduli of elasticity of Spiralloy and its material properties, from a graphical representation of equations (12) and (44) and (55). The relations are shown in figure 7 where the filament ratio by volume is taken as the abscissa while the moduli of elasticity ratio of Spiralloy to resin is taken as the ordinate. It can be seen that for a fixed resin, the moduli of elasticity of Spiralloy increase with the percentage of filament by volume and with E_f/E_r, the moduli of elasticity ratio of filament to resin. The influence of E_f/E_r is significant when the filament windings are parallel to the applied load, but it has relatively little significance when the filament windings are normal to the applied load.

It may be interesting to compare the moduli of elasticity of identical Spiralloy obtained from tensile tests of specimens having filaments parallel and normal to the applied loads. This may be done by computing the ratios of equation (12) to equation (44) and equation (12) to equation (55). The results are also shown in figure 7. It is found that for Spiralloy with less than 68 percent filament by volume the ratios are almost linear between A_f/A equal to 0.40 and 0.68. To a very reasonable degree of accuracy, one can approximate the ratio

$$\frac{E_{parallel}}{E_{normal}} = 11.5 \frac{A_f}{A} + 0.58 \qquad (56a)$$

for $E_f/E_r = 20$ and

$$\frac{E_{parallel}}{E_{normal}} = 13.01 \frac{A_f}{A} + 0.73 \qquad (56b)$$

for $E_f/E_r = 25$.

As indicated earlier in this section, the description of the internal stress distribution should be viewed with caution. The description may serve as a first approximation when the Spiralloy has windings parallel to the applied load, but no such statement can be made for the other case. At most, the str ss analysis of Spiralloy with windings normal to the applied load may be used for qualitative discussions.

The stress analysis of the previous section shows two different situations that may occur, depending upon the filament ratios. When the filament ratio is relatively small, the model of figure 6 indicates yielding of the resin in the sections between filaments, before the clear strip of resin of width s reaches its yield stress. The stress-strain curve was found to consist of two linear sections which meet at a point which corresponds to the attainment of the resin yield stress in the equivalent bar. No such sharp deviation is expected in actuality. Instead, as individual particles of resin between filaments continuously reach their yield stress, the slope of the stress-strain curve changes continuously until all the resin eventually yields.

Throughout the present analysis it was tacitly assumed that the resin can be strained beyond the yield stress by an amount which is of the order of magnitude of elastic stresses. In the event that resins are used which do not obey the latter requirement, the analysis of events between points 0 and 1 of figure 4 will not change, but those between points 1 and 2 will be affected. The change will be small and will result in a decrease of the slope C. For Spiralloy with filaments which lie parallel to the applied force, the slope C will become equal to E_f, the modulus of elasticity of the filament; but the slope C will become equal to E_r for Spiralloy with filaments which lie normal to the applied force.

It was also tacitly assumed that the bond stress between the filament and resin is greater than the yield stress of the resin. No change in the analysis of Spiralloy with windings parallel to the external force would occur if this were not true. However, for Spiralloy with the other type of winding, the slope of the stress-strain curve would change from E_s to E_r as separation takes place between the resin and the filament.

The models of figure 3 and figure 6 were used to study the stress-strain relations of Spiralloy under a uniaxial tensile force. They may also be used to study its behavior under a uniaxial compressive force. The major difference between the tensile and the compressive characteristics of the model of figure 3 is associated with the buckling behavior of the filament bar. One's first impression is that the filament bar can offer virtually no resistance to a compressive force. However, this impression is modified when it is realized that the filaments are imbedded in a resin, thereby increasing its buckling load. Even though some compressive force can be carried, we suspect that it is less than the tensile yield stress of the material, although there is no supporting experimental evidence. If the latter is true, the modulus of elasticity of Spiralloy in compression is smaller than it is in tension, provided the resin characteristics are not affected by the direction of load. Naturally, any difference between the tensile and compressive material characteristics of the resin would also affect the final results.

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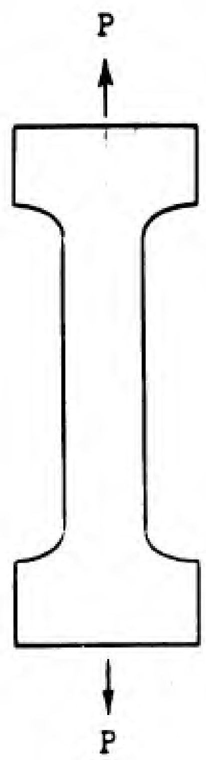

Fig. 1a A Spiralloy specimen with filaments parallel to the applied force

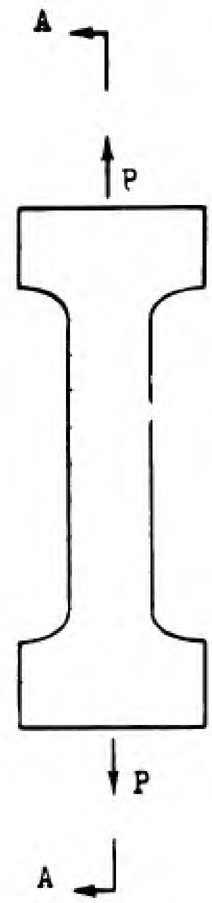

Fig. 1b A Spiralloy specimen with filaments normal to the applied force

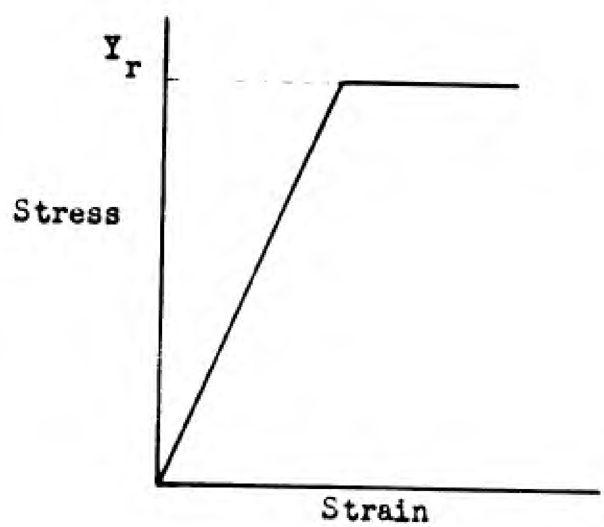

Fig.2 The stress-strain relation of resin

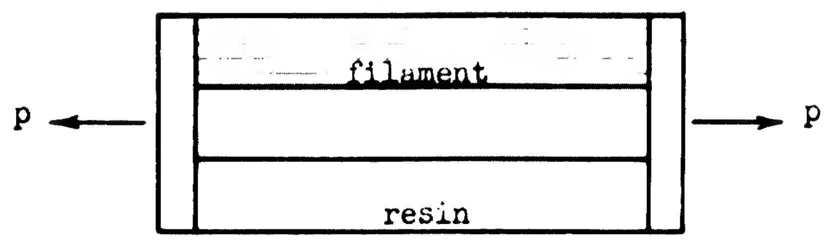

Fig. 3 A model used in the analysis of Spiralloy with filaments parallel to the applied force.

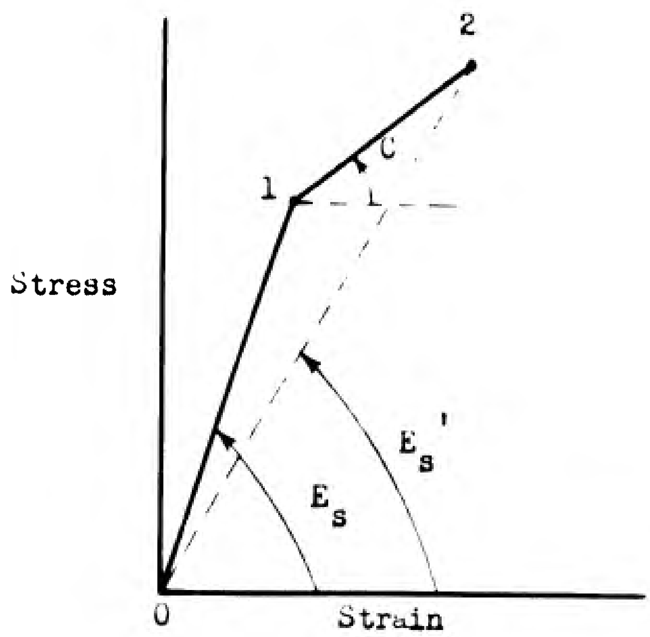

Fig. 4 The stress-strain relation of Spiralloy

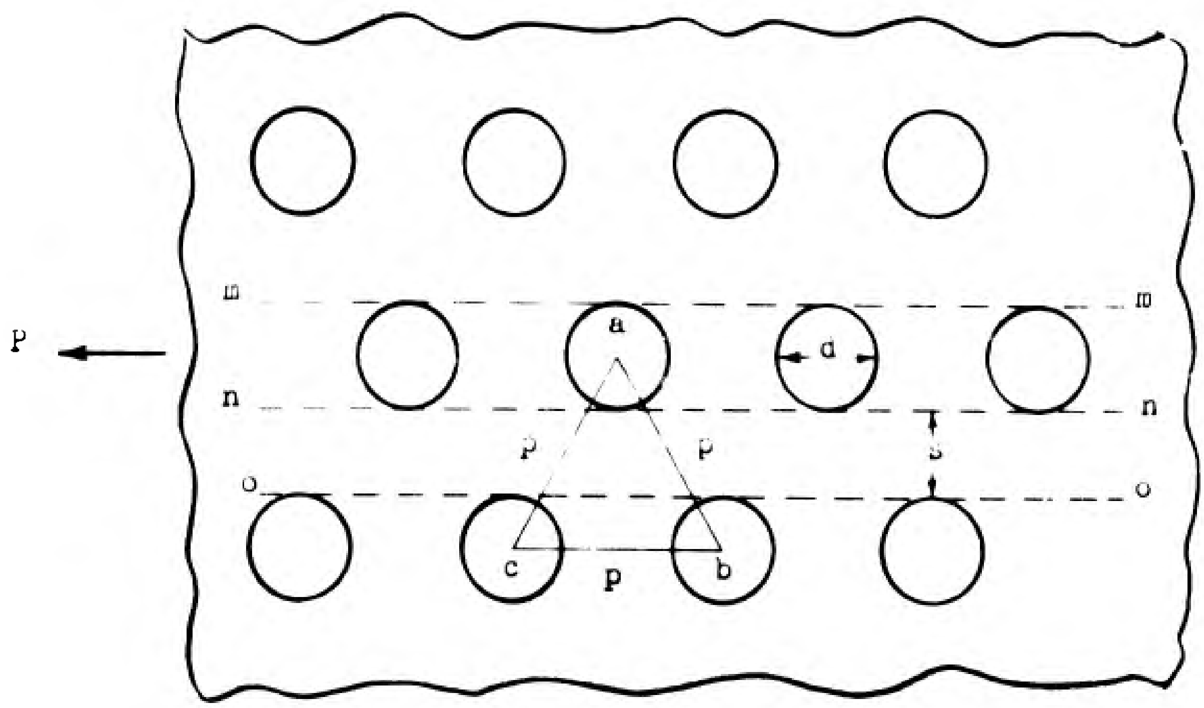

Fig. 5 Section A-A of the Spiralloy specimen shown in figure 1b

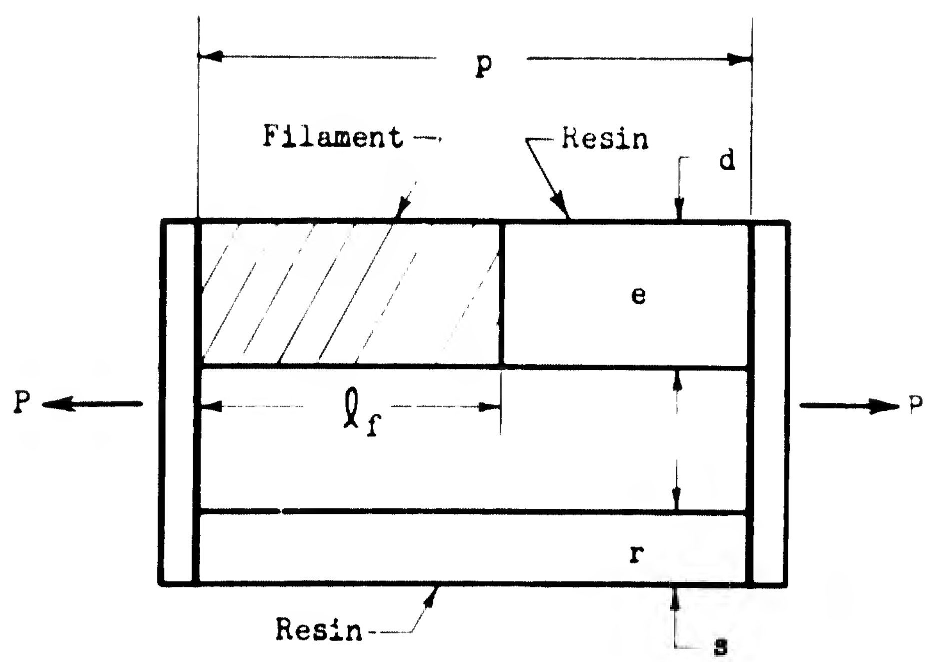

Fig. 6 A model used in the analysis of Spiralloy with filaments normal to the applied force

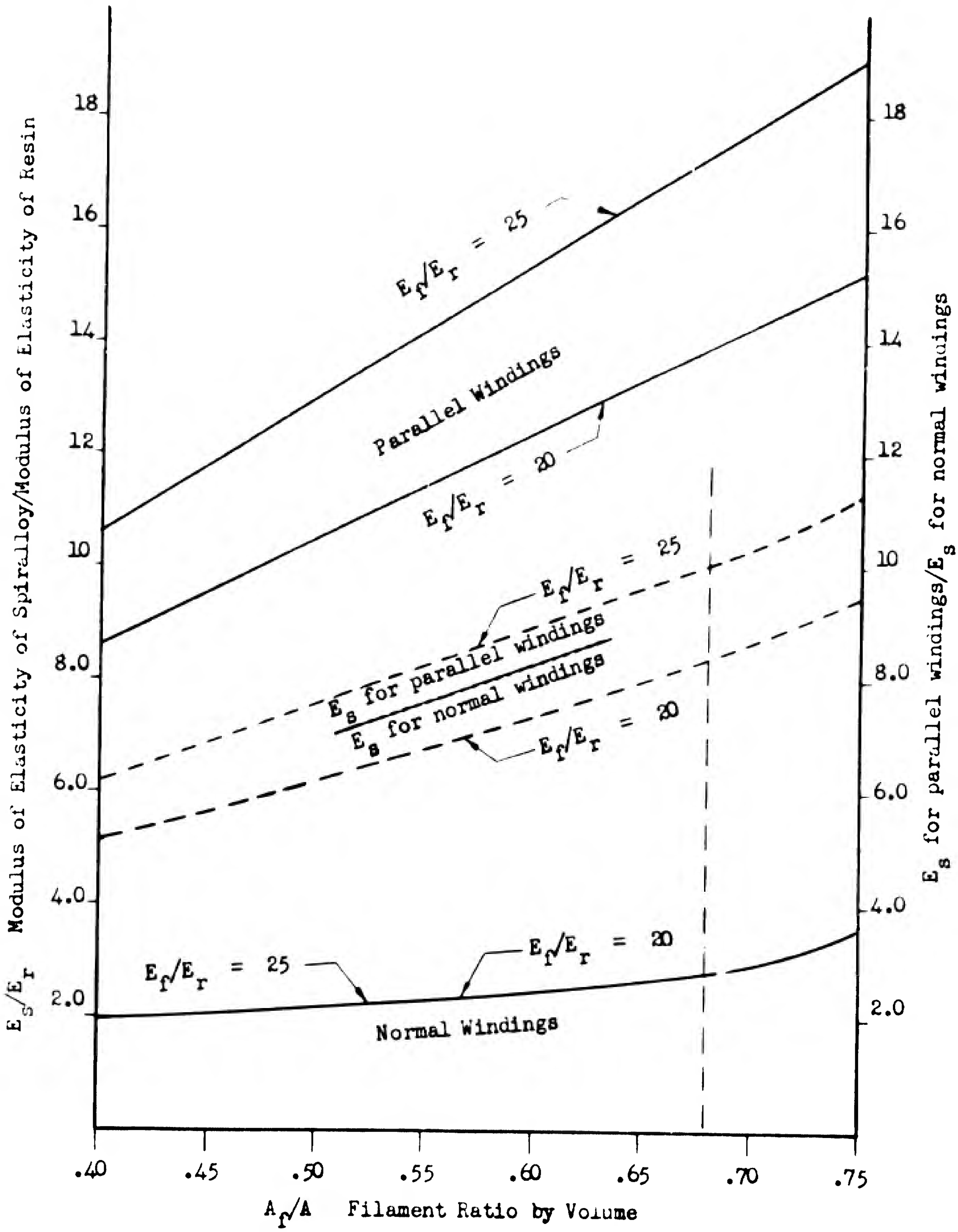

Fig. 7 Relation between the moduli of elasticity of Spiralloy and its filament ratio by volume

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