ANALYTICAL APPROXIMATIONS

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Analytical Approximations

Chi-Square Integral: The table below gives details of cross-section approximations holding over $m < \chi^2 < \infty$ and pertaining to

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-t^2/2} d(t^2)$$

$$\doteq 1 - \frac{A}{\left[1+a_1 t+a_2 t^2+a_3 t^3\right]^4};$$

$$t = \sqrt{\chi^2} - \sqrt{m}.$$

m	A	a_1	a_2	a_3	ϵ
2	.3679	.3622	.1384	.1169	.0006
3	.3916	.3497	.1414	.1214	.0007
4	.4060	.3424	.1421	.1251	.0007
5	.4159	.3374	.1426	.1275	.0007
6	.4232	.3335	.1432	.1294	.0008
7	.4289	.3307	.1427	.1316	.0008
8	.4335	.3286	.1416	.1340	.0008
9	.4373	.3265	.1416	.1353	.0008
10	.4405	.3250	.1410	.1367	.0008
30	.4657	.3123	.1367	.1493	.0009
∞	.5	.2952	.1235	.1749	.0012

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Analytical Approximation

Chi-Square Integral: To better than .0013 over $m \leq \chi^2 < \infty$ and $2 \leq m < \infty$, m being considered a continuous parameter,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-t^2/2} d(t^2)$$

$$\doteq 1 - \frac{A}{\left[1+a_1t+a_2t^2+a_3t^3\right]^4}.$$

$$t = \sqrt{\chi^2} - \sqrt{m}$$

$$A = .5 - .133\sqrt{\frac{2}{m}}$$

$$a_1 = .295 + .067\sqrt{\frac{2}{m}}$$

$$a_2 = .124 + .057\sqrt{\frac{2}{m}} - .043\left(\frac{2}{m}\right)$$

$$a_3 = .174 - .105\sqrt{\frac{2}{m}} + .049\left(\frac{2}{m}\right)$$

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