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ANALYTICAL APPROXIMATIONS

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Analytical Approximation

Incomplete Elliptic Integral: To better than .0000,0003 over $45^\circ \leq \phi \leq 90^\circ$ and $45^\circ \leq \alpha \leq 90^\circ$,

$$F(\phi,\alpha) = \int_0^{\phi} \frac{d\phi}{\sqrt{1-\sin^2\alpha \sin^2\phi}}$$

$$\doteq A(\alpha) + B(\phi,\alpha)\sqrt{1-\sin^2\alpha\sin^2\phi} + C(\alpha)\ln\left[\frac{1}{\cos\phi\sin\alpha+\sqrt{1-\sin^2\alpha\sin^2\phi}}\right] .$$

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$A(\alpha) = 1.3862,9436$
$+.0965,7350(\frac{\pi}{2}-\alpha)^2$
$-.0013,0437(\frac{\pi}{2}-\alpha)^4$
$-.0013,7074(\frac{\pi}{2}-\alpha)^6$
$-.0002,7227(\frac{\pi}{2}-\alpha)^8$
$-.0000,9495(\frac{\pi}{2}-\alpha)^{10}$

$C(\alpha) = 1.0$
$+.2499,9980(\frac{\pi}{2}-\alpha)^2$
$+.0572,9764(\frac{\pi}{2}-\alpha)^4$
$+.0149,5866(\frac{\pi}{2}-\alpha)^6$
$+.0047,0849(\frac{\pi}{2}-\alpha)^8$
$+.0008,2724(\frac{\pi}{2}-\alpha)^{10}$
$+.0010,8108(\frac{\pi}{2}-\alpha)^{12}$

Analytical Approximation

$$B(\phi,\alpha) \approx f_1(\phi)g_1(\alpha) + f_2(\phi)g_2(\alpha) + f_3(\phi)g_3(\alpha)$$

$$f_1(\phi) = -.3277,5088(\tfrac{\pi}{4}-\phi) - .0638,1682(\tfrac{\pi}{4}-\phi)^3 - .0080,8686(\tfrac{\pi}{4}-\phi)^5 - .0008,5024(\tfrac{\pi}{4}-\phi)^7 - .0000,9551(\tfrac{\pi}{4}-\phi)^9$$

$$g_1(\alpha) = .7674,7222 + .3184,7443(\tfrac{\pi}{4}-\alpha)^2 + .0820,7991(\tfrac{\pi}{4}-\alpha)^4 + .0179,8074(\tfrac{\pi}{4}-\alpha)^6 + .0034,7561(\tfrac{\pi}{4}-\alpha)^8 + .0013,5976(\tfrac{\pi}{4}-\alpha)^{10}$$

$$f_2(\phi) = .0298,811(\tfrac{\pi}{4}-\phi) - .0606,009(\tfrac{\pi}{4}-\phi)^3 - .0206,017(\tfrac{\pi}{4}-\phi)^5 - .0040,129(\tfrac{\pi}{4}-\phi)^7 - .0007,920(\tfrac{\pi}{4}-\phi)^9$$

$$g_2(\alpha) = .0516,668 - .1689,896(\tfrac{\pi}{4}-\alpha)^2 - .0998,444(\tfrac{\pi}{4}-\alpha)^4 - .0344,697(\tfrac{\pi}{4}-\alpha)^6 - .0065,684(\tfrac{\pi}{4}-\alpha)^8 - .0043,974(\tfrac{\pi}{4}-\alpha)^{10}$$

$$f_3(\phi) = -.000727(\tfrac{\pi}{4}-\phi) + .004444(\tfrac{\pi}{4}-\phi)^3 - .003986(\tfrac{\pi}{4}-\phi)^5 - .002520(\tfrac{\pi}{4}-\phi)^7 - .000621(\tfrac{\pi}{4}-\phi)^9$$

$$g_3(\alpha) = .005752 - .075208(\tfrac{\pi}{4}-\alpha)^2 + .073873(\tfrac{\pi}{4}-\alpha)^4 + .088197(\tfrac{\pi}{4}-\alpha)^6 - .014601(\tfrac{\pi}{4}-\alpha)^8 + .044161(\tfrac{\pi}{4}-\alpha)^{10}$$

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